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# Question

Test for whether the 3rd, 8th, 13th, and so on, numbers in the sequence at the beginning of this section are autocorrelated using . Here, (beginning with the third number), (every five numbers), (30 numbers in the sequence), and (largest integer such that . Then,

# Solution:

Open Excel file,

Thus, the autocorrelation between the following numbers would be of interest: . The value M is the largest integer such that , where is the total number of values in the sequence. (Thus, a subsequence of length is being tested.)

A nonzero autocorrelation implies a lack of independence, so the following two-tailed test is appropriate:

For large values of M, the distribution of the estimator of , denoted  , is approximately normal if the values are uncorrelated. Then the test statistic can be formed as follows:

Which is distributed normally with a mean of zero and a variance of 1, under the assumption of independence, for large M.

After computing , do not reject the null hypothesis of independence if .

Where is the level of significance and is obtained from Table A.3.

**If** , the subsequence is said to exhibit positive autocorrelation. In this case, successive values at lag have a higher probability than expected of being close in value (i.e., high random numbers in the subsequence followed by high, and low followed by low).

**If** the subsequence is exhibiting negative autocorrelation, which means that low random numbers tend to be followed by high ones, and vice versa. The desired property, independence (which implies zero autocorrelation), means that there is no discernible relationship of the nature discussed here between successive random numbers at lag .

# For this Question:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Rith term |
| 1 | 0.12 | 0.01 | 0.23 | 0.28 | 0.89 | 0.31 | 0.64 | 0.28 | 0.83 | 0.93 |  |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | Rith term |
| 2 | 0.99 | 0.15 | 0.33 | 0.35 | 0.91 | 0.41 | 0.6 | 0.27 | 0.75 | 0.88 |  |
|  | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | Rith term |
| 3 | 0.68 | 0.49 | 0.05 | 0.43 | 0.95 | 0.58 | 0.19 | 0.36 | 0.69 | 0.87 |  |

The test statistic assumes the value

Critical value from Table A.3 is

Since the two – tailed test divides this into two equal parts, one for each tail:

Finding the Z-Value that corresponds to a cumulative probability of .

For :

The cumulative probability is .

Looking up to table A.3 in the Z-table,

1.9+0.06=1.96

In sense of probability:

Hence:

These values define the critical region for a two-tailed test at a 0.05 significance level:

1. Do not reject if .

Comments:

We will not reject this; it is within the range.

Problem Chapter 07:

Generate Random Number and Perform Null Hypothesis with the help of Table A.8. N=10, critical value

Question 15 (a):

After calculation, see Excel file Attach, P8-Q15-a.

|  |  |  |  |
| --- | --- | --- | --- |
| **Sr** |  | **Mod Value** | **Random Number** |
| **1** | **77** | **13** | **0.813** |
| **2** | **143** | **15** | **0.938** |
| **3** | **165** | **5** | **0.313** |
| **4** | **55** | **7** | **0.438** |
| **5** | **77** | **13** | **0.813** |
| **6** | **143** | **15** | **0.938** |
| **7** | **165** | **5** | **0.313** |
| **8** | **55** | **7** | **0.438** |
| **9** | **77** | **13** | **0.813** |
| **10** | **143** | **15** | **0.938** |

In order to perform frequency test, to check the uniformity, we will do .

Step 01:

Rank the data from smallest to largest.

|  |  |
| --- | --- |
| **R(i)** | **Random Number (Reorder)** |
| **1** | **0.313** |
| **2** | **0.313** |
| **3** | **0.438** |
| **4** | **0.438** |
| **5** | **0.813** |
| **6** | **0.813** |
| **7** | **0.813** |
| **8** | **0.938** |
| **9** | **0.938** |
| **10** | **0.938** |

From table A.8: Kolmogorov – Smirnov Critical Values

For N=10, N is degrees of freedom

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | 0.3125 | 0.315 | 0.4375 | 0.4375 | 0.8125 | 0.8125 | 0.8125 | 0.9375 | 0.9375 | 0.9375 |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
|  | - | - | - | - | - | - | - | - | - | - |
|  | 0.3125 | 0.2125 | 0.2375 | 0.1375 | 0.4125 | 0.3125 | 0.2125 | 0.2375 | 0.1375 | 0.0375 |

, , So, if the sample statistic *D* is greater than the critical value , the null hypothesis that the data are a sample from a uniform distribution is rejected. ***In this case, it is rejected.*** Otherwise, if , conclude that no difference has been detected between the true distribution of and the uniform distribution.

# Question no 15 (c):

Given

Generate random number:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Sr** | **Xn** | **Mod Value** | **Random Number** | **Convert to smallest to largest** | **R(i)** | **Random Number (Reorder)** |
| **1** | **49** | **1** | **0.063** | **1** | **0.063** |
| **2** | **7** | **7** | **0.438** | **2** | **0.063** |
| **3** | **49** | **1** | **0.063** | **3** | **0.063** |
| **4** | **7** | **7** | **0.438** | **4** | **0.063** |
| **5** | **49** | **1** | **0.063** | **5** | **0.063** |
| **6** | **7** | **7** | **0.438** | **6** | **0.438** |
| **7** | **49** | **1** | **0.063** | **7** | **0.438** |
| **8** | **7** | **7** | **0.438** | **8** | **0.438** |
| **9** | **49** | **1** | **0.063** | **9** | **0.438** |
| **10** | **7** | **7** | **0.438** | **10** | **0.438** |

From table A.8: Kolmogorov – Smirnov Critical Values

For N=10, N is degrees of freedom

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | 0.0625 | 0.0625 | 0.0625 | 0.0625 | 0.0625 | 0.4375 | 0.4375 | 0.4375 | 0.4375 | 0.4375 |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
|  | 0.0375 | 0.1375 | 0.2375 | 0.3375 | 0.4375 | 0.1625 | 0.2625 | 0.3625 | 0.4625 | 0.5625 |
|  | 0.0625 | - | - | - | - | - | - | - | - | - |

, , So, if the sample statistic *D* is greater than the critical value , the null hypothesis that the data are a sample from a uniform distribution is rejected. ***In this case, it is rejected.*** Otherwise, if , conclude that no difference has been detected between the true distribution of and the uniform distribution.