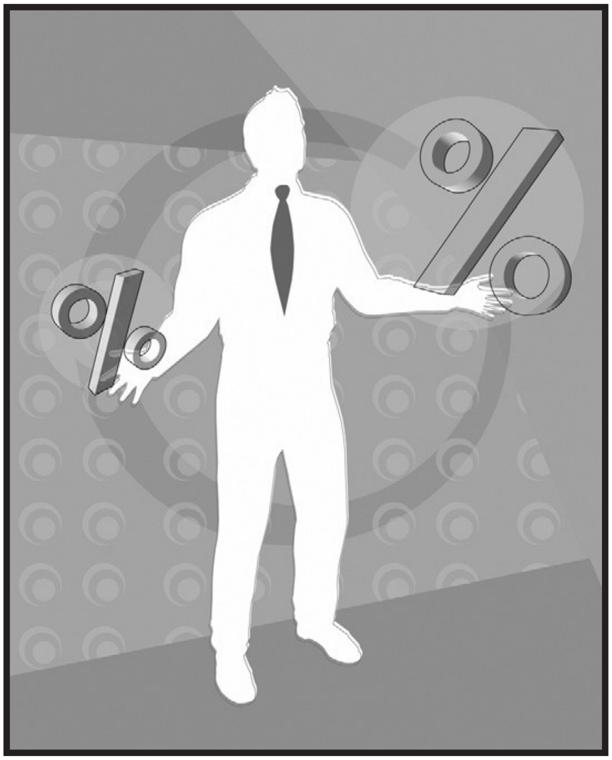
Chapter 6

Rate of Return Analysis



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Although the most commonly quoted measure of economic worth for a project or alternative is the rate of return (ROR), its meaning is easily misinterpreted, and the methods to determine ROR are often applied incorrectly. This chapter presents the methods by which one, two, or more alternatives are evaluated using the ROR procedure. Though the computations vary slightly, ROR is known by several other names: internal rate of return (IRR), return on investment (ROI), and profitability index (PI), to name three.

In some cases, more than one ROR value may satisfy the rate of return equation. This chapter describes how to recognize this possibility and presents an approach to find the multiple values. Alternatively, one unique ROR value can be obtained by using information determined independently of the project's cash flows.

Purpose: Understand the meaning of rate of return (ROR) and perform ROR calculations when considering one or more alternatives.

LEARNING OUTCOMES

1. State the meaning of rate of return.	**Definition of ROR**
2. Calculate the rate of return using a present worth, annual worth, or future worth equation.	**ROR using PW, AW, or FW**
3. Understand the difficulties of using the ROR method, relative to PW, AW, and FW methods.	**Cautions about ROR**
4. Tabulate incremental cash flows and interpret ROR on the incremental investment.	**Incremental analysis**
5. Select the best of multiple alternatives using an incremental ROR analysis.	**Alternative selection**
6. Determine the maximum number of possible ROR values and their values for a cash flow series.	**Multiple RORs**
7. Calculate the external rate of return using reinvestment and/or borrowing rates.	**External ROR**
8. Use a spreadsheet to perform ROR analysis of one or more alternatives.	**Spreadsheets**

6.1 INTERPRETATION OF ROR VALUES

As stated in Chapter 1, interest rate and rate of return refer to the same thing. We commonly use the term *interest rate* when discussing borrowed money and *rate of return* when dealing with investments.

From the perspective of someone who has *borrowed money,* the interest rate is applied to the *unpaid balance* so that the total loan amount and interest due are paid in full exactly with the last loan payment. From the perspective of the *investor* (or lender) there is an *unrecovered balance* at each time period. The interest rate is the return on this unrecovered balance so that the total amount lent and interest earned are recovered exactly with the last receipt. Calculation of the *rate of return* describes both of these perspectives.

Rate of return (ROR) is the rate paid on the unpaid balance of borrowed money, or the rate earned on the unrecovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with interest considered.

The rate of return is expressed as a percent per period, for example, $i = 10\%$ per year. It is stated as a positive percentage; the fact that interest paid on a loan is actually a negative rate of return from the borrower's perspective is not considered. The numerical value of i can range from -100% to infinity, that is, $-100\% < i < \infty$. In terms of an investment, a return of $i = -100\%$ means the entire amount is lost.

The definition above does *not* state that the rate of return is on the initial amount of the investment; rather it is on the *unrecovered balance,* which changes each time period. Example 6.1 illustrates this difference.

EXAMPLE 6.1

Jefferson Bank lent a newly graduated engineer \$1000 at $i = 10\%$ per year for 4 years to buy home office equipment. From the bank's perspective (the lender), the investment in this young engineer (the borrower) is expected to produce an equivalent net cash flow of \$315.47 for each of 4 years.

$$A = \$1000(A/P,10\%,4) = \$315.47$$

This represents a 10% per year rate of return on the bank's unrecovered balance. Compute the amount of the unrecovered investment for each of the 4 years using (*a*) the rate of return on the unrecovered balance (the correct basis) and (*b*) the return on the initial \$1000 investment (the incorrect basis).

Solution

a. Table 6.1 shows the unrecovered balance at the end of each year in column 6 using the 10% rate on the *unrecovered balance at the beginning of the year.* After 4 years, the total \$1000 is recovered, and the balance in column 6 is exactly zero.

TABLE 6.1 Unrecovered Balances Using a Rate of Return of 10% on the Unrecovered Balance

(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) − (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Unrecovered Balance	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	$−1,000.00	—	—
1	$−1,000.00	$100.00	+315.47	$215.47	$−784.53
2	−784.53	78.45	+315.47	237.02	−547.51
3	−547.51	54.75	+315.47	260.72	−286.79
4	−286.79	28.68	+315.47	286.79	0
		$261.88		$1,000.00	

TABLE 6.2 Unrecovered Balances Using a 10% Return on the Initial Amount

(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) − (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Initial Amount	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	$−1,000.00	—	—
1	$−1,000.00	$100	+315.47	$215.47	$−784.53
2	−784.53	100	+315.47	215.47	−569.06
3	−569.06	100	+315.47	215.47	−353.59
4	−353.59	100	+315.47	215.47	−138.12
		$400		$861.88	

b. Table 6.2 shows the unrecovered balance if the 10% return is always figured on the *initial $1000.* Column 6 in year 4 shows a remaining unrecovered amount of $138.12, because only $861.88 is recovered in the 4 years (column 5).

Because rate of return is the interest rate on the unrecovered balance, the computations in *Table 6.1 present a correct interpretation of a 10% rate of return.* An interest rate applied to the original principal represents a higher rate than stated. From the standpoint of the borrower, it is better that interest is charged on the unpaid balance than on the initial amount borrowed.

6.2 ROR CALCULATION

The basis for calculating an unknown rate of return is an *equivalence* relation in PW, AW, or FW terms. The objective is to *find the interest rate,* represented as i^*, at which the cash flows are equivalent. The calculations are the reverse of those made in previous chapters, where the interest rate was known. For example, if you invest \$1000 now and are promised payments of \$500 three years from now and \$1500 five years from now, the rate of return relation using PW factors is

$$1000 = 500(P/F,i^*,3) + 1500(P/F,i^*,5) \qquad \textbf{[6.1]}$$

The value of i^* is sought (see Figure 6.1). Move the \$1000 to the right side in Equation [6.1].

$$0 = -1000 + 500(P/F,i^*,3) + 1500(P/F,i^*,5)$$

The equation is solved to obtain $i^* = 16.9\%$ per year. The rate of return will always be greater than zero if the total amount of receipts is greater than the total amount of disbursements.

It should be evident that rate of return relations are merely a rearrangement of a present worth equation. That is, if the above interest rate is known to be 16.9%, and it is used to find the present worth of \$500 three years from now and \$1500 five years from now, the PW relation is

$$\text{PW} = 500(P/F,16.9\%,3) + 1500(P/F,16.9\%,5) = \$1000$$

This illustrates that rate of return and present worth equations are set up in exactly the same fashion. The only differences are what is given and what is sought. The PW-based ROR equation can be generalized as

$$\mathbf{0 = -PW_D + PW_R} \qquad \textbf{[6.2]}$$

where PW_D = present worth of disbursements or cash outflows
PW_R = present worth of receipts or cash inflows

Annual worth or future worth values can also be used in Equation [6.2].

There are different ways to determine i^* once the PW relation is established: solution via trial and error with tabulated factors and solution by calculator or spreadsheet function. The first helps in understanding how ROR computations work; the other two are faster.

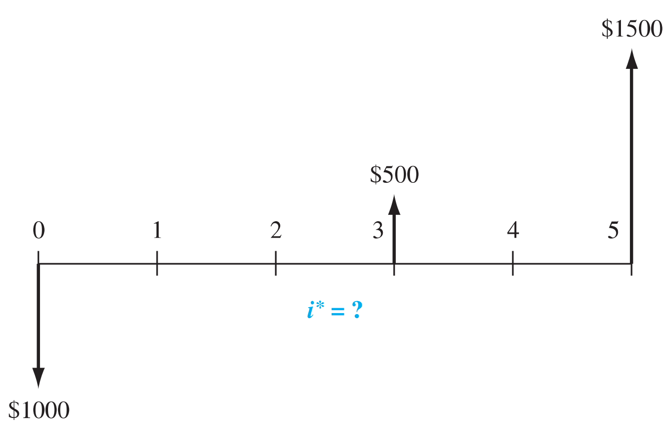


FIGURE 6.1
Cash flow for which a value of i^* is to be determined.

*i** Using Tabulated Factors

The general procedure for using a PW-based equation is

1. Draw a cash flow diagram.
2. Set up the rate of return equation in the form of Equation [6.2].
3. Select values of i by trial and error until the equation is balanced.

The next two examples illustrate PW and AW equivalence relations to find i^*.

*i** by Calculator or Spreadsheet

The fastest way to determine an i^* value, when there is a series of equal cash flows (A series), is to apply the i function on a calculator or the RATE function on a spreadsheet. These are powerful functions, where it is acceptable to have a separate P value in year 0 and an F value in year n that is separate from the series A amount. The spreadsheet format is = RATE(n, A, P, F) and the calculator format is $i(n,A,P,F)$. If cash flows vary over the years, the IRR function on a spreadsheet is used to determine i^*; there is no similar calculator function. These functions are illustrated in the last section of this chapter.

EXAMPLE 6.2

The HVAC engineer for a company that constructed one of the world's tallest buildings (Burj Khalifa in the United Arab Emirates) requested that \$500,000 be spent on software and hardware to improve the efficiency of the environmental control systems. This is expected to save \$10,000 per year for 10 years in energy costs and \$700,000 at the end of 10 years in equipment refurbishment costs. Find the rate of return.

Solution

For trial-and-error use the procedure based on a PW equation.

1. Figure 6.2 shows the cash flow diagram.
2. Use Equation [6.2] format for the ROR equation.

$$0 = -500{,}000 + 10{,}000(P/A,i^*,10) + 700{,}000(P/F,i^*,10)$$

3. Try $i = 5\%$.

$$0 = -500{,}000 + 10{,}000(P/A,5\%,10) + 700{,}000(P/F,5\%,10)$$
$$0 < \$6946$$

The result is positive, indicating that the return is more than 5%. Try $i = 6\%$.

$$0 = -500{,}000 + 10{,}000(P/A,6\%,10) + 700{,}000(P/F,6\%,10)$$
$$0 > \$-35{,}519$$

Since 6% is too high, linearly interpolate between 5% and 6%.

$$i^* = 5.00 + \frac{6946 - 0}{6946 - (-35{,}519)}(1.0)$$
$$= 5.00 + 0.16 = 5.16\%$$

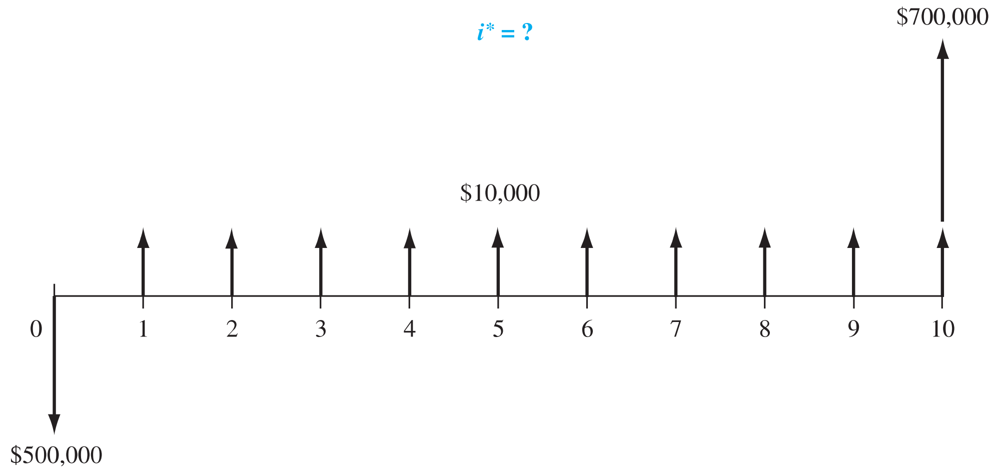


FIGURE 6.2 Cash flow diagram, Example 6.2.

Comment: For a spreadsheet solution, a single-cell entry of the function = RATE(n,A,P,F), which is = RATE(10,10000, −500000,700000) in this case, displays $i^* = 5.16\%$. The i function on a calculator has the same contents.

EXAMPLE 6.3

Allied Materials needs \$8 million in new capital for expanded composites manufacturing. It is offering small-denomination corporate bonds at a deep discount price of \$800 for a 4% \$1000 face value bond that matures in 20 years and pays the dividend semiannually. Find the nominal and effective annual rates, compounded semiannually, that Allied is paying an investor.

Solution

By Equation [4.1], the semiannual income from the bond dividend is $I = 1000(0.04)/2 = \$20$. This will be received by the investor for a total of 40 6-month periods. The AW-based relation to calculate the effective semiannual rate is

$$0 = -800(A/P,i^*,40) + 20 + 1000(A/F,i^*,40)$$

By trial-and-error and linear interpolation, $i^* = 2.87\%$ semiannually. The nominal annual rate is i^* times 2.

$$\text{Nominal } i = 2.87(2) = 5.74\% \text{ per year compounded semiannually}$$

Using Equation [3.2], the effective annual rate is

$$\text{Effective } i = (1.0287)^2 - 1 = 0.0582 \qquad (5.82\% \text{ per year})$$

Comment: The calculator function i(40,20,−800,1000) results in $i^* = 2.84\%$ semiannually. This can be compared with the trial-and-error result of 2.87%.

6.3 CAUTIONS WHEN USING THE ROR METHOD

The rate of return method is commonly used in engineering and business settings to evaluate one project, as discussed above, and to select one alternative from two or more, as explained later. When applied correctly, the ROR technique will always result in a good decision, indeed, the same one as with a PW, AW, or FW analysis. However, there are some assumptions and difficulties with ROR analysis that must be considered when calculating i^* and in interpreting its real-world meaning for a particular project. The summary that follows applies for all solution methods.

- *Computational difficulty versus understanding.* Especially in obtaining a trial-and-error solution, the computations rapidly become very involved. Spreadsheet solution is easier; however, there are no spreadsheet functions that offer the same level of understanding to the learner as that provided by hand (or calculator) solution of PW, AW, and FW relations.
- *Special procedure for multiple alternatives.* To correctly use the ROR method to choose from two or more mutually exclusive alternatives requires an analysis procedure significantly different from that used in other methods. Section 6.5 explains this procedure.
- *Multiple i^* values.* Depending upon the sequence of cash flow disbursements and receipts, there may be more than one real-number root to the ROR equation, resulting in more than one i^* value. There are procedures to use the ROR method and obtain one unique i^* value. Sections 6.6 and 6.7 cover these aspects of ROR analysis.
- *Reinvestment at i^*.* The PW, AW, and FW methods assume that any net positive investment (i.e., net positive cash flows once the time value of money is considered) are *reinvested at the MARR*. But the ROR method assumes reinvestment at the *i^* rate*. When i^* is not close to the MARR (e.g., if i^* is substantially larger than MARR), or if multiple i^* values exist, this is an unrealistic assumption. In such cases, the i^* value is not a good basis for decision making.

In general, it is good practice to use the MARR to determine PW, AW, or FW. If the ROR value is needed, find i^* while taking these cautions into consideration. As an illustration, if a project is evaluated at MARR $= 15\%$ and has PW < 0, there is no need to calculate i^*, because $i^* < 15\%$. However, if PW > 0, then calculate the exact i^* and report it along with the conclusion that the project is financially justified.

6.4 UNDERSTANDING INCREMENTAL ROR ANALYSIS

From previous chapters, we know that the PW (or AW or FW) value calculated at the MARR identifies the one mutually exclusive alternative that is best from the economic viewpoint. The best alternative is simply the one that has the numerically largest PW value. (This represents the equivalently lowest net cost or highest net revenue cash flow.) In this section, we learn that the ROR can also be used to identify the best alternative; however, it is *not* always as simple as selecting the highest rate of return alternative.

Let's assume that a company uses a MARR of 16% per year, that the company has \$90,000 available for investment, and that two alternatives (A and B) are being evaluated. Alternative A requires an investment of \$50,000 and has an internal rate of return i_A^* of 35% per year. Alternative B requires \$85,000 and has an i_B^* of 29% per year. Intuitively we may conclude that the better alternative is the one that has the larger ROR value, A in this case. However, this is not necessarily so. While A has the higher projected ROR, it requires an initial investment that is much less than the total money available (\$90,000). What happens to the investment capital that is left over? It is generally assumed that excess funds will be invested at the company's MARR, as we learned earlier. Using this assumption, it is possible to determine the consequences of the alternative investments. If alternative A is selected, \$50,000 will return 35% per year. The \$40,000 left over will be invested at the MARR of 16% per year. The rate of return on the total capital available, then, will be the weighted average. Thus, if alternative A is selected,

$$\text{Overall ROR}_A = \frac{50{,}000(0.35) + 40{,}000(0.16)}{90{,}000} = 26.6\%$$

If alternative B is selected, \$85,000 will be invested at 29% per year, and the remaining \$5000 will earn 16% per year. Now the weighted average is

$$\text{Overall ROR}_B = \frac{85{,}000(0.29) + 5000(0.16)}{90{,}000} = 28.3\%$$

These calculations show that even though the i^* for alternative A is higher, alternative B presents the better overall ROR for the \$90,000. If either a PW, AW, or FW comparison is conducted using the MARR of 16% per year as i, alternative B will be chosen.

This example illustrates a major dilemma of the rate of return method when comparing alternatives: Under some circumstances, alternative ROR (i^*) values do not provide the same ranking of alternatives as do the PW, AW, and FW analyses. To resolve the dilemma, conduct an *incremental analysis* between two alternatives at a time and base the alternative selection on the *ROR of the incremental cash flow series.*

A standardized format (Table 6.3) simplifies the incremental analysis. If the alternatives have *equal lives,* the year column will go from 0 to n. If the alternatives have *unequal lives,* the year column will go from 0 to the LCM (least common multiple) of the two lives. The use of the LCM is necessary because *incremental ROR analysis requires equal-service comparison* between alternatives. Therefore, all the assumptions and requirements developed earlier apply for any incremental ROR evaluation. When the LCM of lives is used, the salvage value and reinvestment in each alternative are shown at their respective times. If a study period is defined, the cash flow tabulation is for the specified period.

For the purpose of simplification, use the convention that between two alternatives, the one with the *larger initial investment* will be regarded as *alternative B.* Then, for each year in Table 6.3,

$$\textbf{Incremental cash flow} = \textbf{cash flow}_B - \textbf{cash flow}_A \qquad \textbf{[6.3]}$$

TABLE 6.3 Format for Incremental Cash Flow Tabulation

Year	Cash Flow: Alternative A (1)	Cash Flow: Alternative B (2)	Incremental Cash Flow (3) = (2) − (1)
0			
1			
.			
.			
.			

As discussed in Chapter 4, an alternative's cash flow series is one of two types:

Revenue alternative, where there are both negative and positive cash flows.

Cost alternative, where all cash flow estimates are negative.

In either case, Equation [6.3] is used to determine the incremental cash flow series with the sign of each cash flow carefully determined. The next two examples illustrate incremental cash flow tabulation of cost alternatives of equal and different lives. A later example treats revenue alternatives.

EXAMPLE 6.4

A tool and die company in Sydney is considering the purchase of a drill press with fuzzy-logic software to improve accuracy and reduce tool wear. The company has the opportunity to buy a slightly used machine for \$15,000 or a new one for \$21,000. Because the new machine is a more sophisticated model, its operating cost is expected to be \$7000 per year, while the used machine is expected to require \$8200 per year. Each machine is expected to have a 25-year life with a 5% salvage value. Tabulate the incremental cash flow.

Solution

Incremental cash flow is tabulated in Table 6.4 using Equation [6.3]. The subtraction performed is (new − used) since the new machine has a larger initial cost. The salvage values are separated from the year 25 cash flow for clarity.

TABLE 6.4 Cash Flow Tabulation for Example 6.4

Year	Cash Flow: Used Press	Cash Flow: New Press	Incremental Cash Flow (New − Used)
0	\$ −15,000	\$ −21,000	\$ −6,000
1–25	−8,200	−7,000	+1,200
25	+750	+1,050	+300
Total	\$−219,250	\$−194,950	\$+24,300

Comment: When the cash flow columns are subtracted, the difference between the totals of the two cash flow series should equal the total of the incremental cash flow column. This merely provides a check of the addition and subtraction in preparing the tabulation.

EXAMPLE 6.5

Sandersen Meat Processors has asked its lead process engineer to evaluate two different types of conveyors for the beef cutting line. Type A has an initial cost of \$70,000 and a life of 3 years. Type B has an initial cost of \$95,000 and a life expectancy of 6 years. The annual operating cost (AOC) for type A is expected to be \$9000, while the AOC for type B is expected to be \$7000. If the salvage values are \$5000 and \$10,000 for type A and type B, respectively, tabulate the incremental cash flow using their LCM.

Solution

The LCM of 3 and 6 is 6 years. In the incremental cash flow tabulation for 6 years (Table 6.5), note that the reinvestment and salvage value of A is shown in year 3.

TABLE 6.5 Incremental Cash Flow Tabulation, Example 6.5

	Cash Flow		Incremental Cash Flow
Year	Type A	Type B	(B − A)
0	\$ −70,000	\$ −95,000	\$−25,000
1	−9,000	−7,000	+2,000
2	−9,000	−7,000	+2,000
3	−70,000 −9,000 +5,000	−7,000	+67,000
4	−9,000	−7,000	+2,000
5	−9,000	−7,000	+2,000
6	−9,000 +5,000	−7,000 +10,000	+7,000
	\$−184,000	\$−127,000	\$+57,000

Once the incremental cash flows are tabulated, determine the incremental rate of return on the extra amount required by the larger investment alternative. This rate, termed Δi^*, represents the return over n years expected on the optional extra investment in year 0. The general selection guideline is to make the extra investment if the incremental rate of return meets or exceeds the MARR. Briefly stated,

If $\Delta i^* \geq$ MARR, select the larger investment alternative (labeled B). Otherwise, select the lower investment alternative (labeled A).

Use of this guideline is demonstrated in Section 6.5. The best rationale for understanding incremental ROR analysis is to think of only *one alternative* under consideration, that alternative being represented by the incremental cash flow series. Only if the return on the extra investment, which is the Δi^* value, meets or exceeds the MARR is it financially justified, in which case the larger investment alternative should be selected.

As a matter of efficiency, if the analysis is between *multiple revenue alternatives,* an acceptable procedure is to initially determine each alternative's i^* and remove those alternatives with $i^* <$ MARR, since their return is too low. Then complete the incremental analysis for the remaining alternatives. If no alternative i^* meets or exceeds the MARR, the do-nothing alternative is economically the best. This initial "weeding out" can't be done for cost alternatives since they have no positive cash flows.

When *independent projects* are compared, no incremental analysis is necessary. All projects with $i^* \geq$ MARR are acceptable. Limitations on the initial investment amount are considered separately, as discussed in Section 4.5.

6.5 ROR EVALUATION OF TWO OR MORE MUTUALLY EXCLUSIVE ALTERNATIVES

When selecting from two or more mutually exclusive alternatives on the basis of ROR, equal-service comparison is required, and an incremental ROR analysis must be used. The incremental ROR value between two alternatives (B and A) is correctly identified as $\Delta i^*_{\mathrm{B-A}}$, but it is usually shortened to Δi^*. The selection guideline, as introduced in Section 6.4, is:

Select the alternative that:

1. **requires the largest initial investment, and**
2. **has a $\Delta i^* \geq$ MARR, indicating that the extra initial investment is economically justified.**

If the higher initial investment is not justified, it should not be made as the extra funds could be invested elsewhere.

Before conducting the incremental evaluation, classify the alternatives as *cost* or *revenue* alternatives. The incremental comparison will differ slightly for each type.

Cost: Evaluate alternatives only against each other.

Revenue: First evaluate against do-nothing (DN), then against each other.

The following procedure for comparing multiple, mutually exclusive alternatives, using a PW-based equivalence relation, can now be applied.

1. Order the alternatives by increasing initial investment. *For revenue alternatives* add DN as the first alternative.
2. Determine the incremental cash flow between the first two ordered alternatives (B – A) over their least common multiple of lives. (For revenue alternatives, the first ordered alternative is DN.)

3. Set up a PW-based relation of this incremental cash flow series and determine Δi^*, the incremental rate of return.
4. If $\Delta i^* \geq$ MARR, eliminate A; B is the survivor. Otherwise, A is the survivor.
5. Compare the survivor to the next alternative. Continue to compare alternatives using steps (2) through (4) until only one alternative remains as the survivor.

The next two examples illustrate this procedure for cost and revenue alternatives, respectively, as well as for equal and different-life alternatives.

For completeness's sake, it is important to understand the procedural difference for comparing *independent projects.* If the projects are independent rather than mutually exclusive, the preceding procedure does not apply. As mentioned in Section 6.4, no incremental evaluation is necessary; all projects with $i^* \geq$ MARR are selected, thus comparing each project's i^* against the MARR, not each other.

EXAMPLE 6.6

As the film of an oil spill from an at-sea tanker moves ashore, great losses occur for aquatic life as well as shoreline feeders and dwellers, such as birds. Environmental engineers and lawyers from several international petroleum corporations and transport companies—Exxon-Mobil, BP, Shell, and some transporters for OPEC producers—have developed a plan to strategically locate throughout the world newly developed equipment that is substantially more effective than manual procedures in cleaning crude oil residue from bird feathers. The Sierra Club, Greenpeace, and other international environmental interest groups are in favor of the initiative. Alternative machines from manufacturers in Asia, America, Europe, and Africa are available with the cost estimates in Table 6.6. Annual cost estimates are expected to be high to ensure readiness at any time. The company representatives have agreed to use the average of the corporate MARR values, which results in MARR = 13.5%. Use incremental ROR analysis to determine which manufacturer offers the best economic choice.

Solution

Follow the procedure for incremental ROR analysis.

1. These are *cost alternatives* and are arranged by increasing first cost.
2. The lives are all the same at $n = 8$ years. The B − A incremental cash flows are indicated in Table 6.7. The estimated salvage values are shown separately in year 8.

TABLE 6.6 Costs for Four Alternative Machines, Example 6.6

	Machine A	Machine B	Machine C	Machine D
First cost, $	−5,000	−6,500	−10,000	−15,000
AOC, $/year	−3,500	−3,200	−3,000	−1,400
Salvage value, $	+500	+900	+700	+1,000
Life, years	8	8	8	8

TABLE 6.7 Incremental Cash Flow for Comparison of Machine B-to-A

Year	Cash Flow Machine A	Cash Flow Machine B	Incremental Cash Flow for (B − A)
0	\$−5000	\$−6500	\$−1500
1–8	−3500	−3200	+300
8	+500	+900	+400

3. The following PW relation for (B − A) results in $\Delta i^* = 14.57\%$.

$$0 = -1500 + 300(P/A,\Delta i^*,8) + 400(P/F,\Delta i^*,8)$$

4. Since this return exceeds MARR = 13.5%, A is eliminated and B is the survivor.
5. The comparison of C-to-B results in the elimination of C based on $\Delta i^* = -18.77\%$ from the incremental relation

$$0 = -3500 + 200(P/A,\Delta i^*,8) - 200(P/F,\Delta i^*,8)$$

The D-to-B incremental cash flow PW relation for the final evaluation is

$$0 = -8500 + 1800(P/A,\Delta i^*,8) + 100(P/F,\Delta i^*,8)$$

With $\Delta i^* = 13.60\%$, machine D is the overall, though marginal, survivor of the evaluation; it should be purchased and located in the event of oil spill accidents.

EXAMPLE 6.7

Harold owns a construction company that subcontracts to international power equipment corporations such as GE, ABB, Siemens, and LG. For the last 4 years he has leased crane and lifting equipment for \$32,000 annually. He now wishes to purchase similar equipment. Use an MARR of 12% per year to determine if any of the options detailed in Table 6.8 are financially justified.

Solution

Apply the incremental ROR procedure with MARR = 12% per year.

1. Because these are revenue alternatives, add the do-nothing option as the first alternative and order the remaining ones. The comparison order is DN, 4, 2, 1, 3.

TABLE 6.8 Estimates for Alternative Equipment, Example 6.7

Alternative	1	2	3	4
First cost, \$	−80,000	−50,000	−145,000	−20,000
Annual cost, \$/year	−28,000	−26,000	−16,000	−21,000
Annual revenue, \$/year	61,000	43,000	51,000	29,000
Life, years	4	4	8	4

2. Each annual cash flow for the DN alternative is \$0. Therefore, the incremental cash flows for comparing 4-to-DN are the same as those for alternative 4.
3. The Δi^* for the comparison 4-to-DN is actually the project ROR. Since n = 4 years, the PW relation and return are

$$0 = -20{,}000 + (29{,}000 - 21{,}000)(P/A,\Delta i^*,4)$$
$$(P/A,\Delta i^*,4) = 2.5$$
$$\Delta i^* = 21.9\%$$

4. Since 21.9% > 12%, eliminate DN and proceed with the 2-to-4 comparison.
5. Both alternatives 2 and 4 have $n = 4$. The incremental cash flows are \$−30,000 in year 0 and (43,000−26,000) − (29,000−21,000) = \$+9000 in years 1 to 4. Incremental analysis results in $\Delta i^* = 7.7\%$ from the PW relation

$$0 = -30{,}000 + 9000(P/A,\Delta i^*,4)$$
$$\Delta i^* = 7.7\%$$

Alternative 4 is, again, the survivor. Continue with the comparison 1-to-4 to obtain

$$0 = -60{,}000 + 25{,}000(P/A,\Delta i^*,4)$$
$$\Delta i^* = 24.1\%$$

Now alternative 1 is the survivor. The final comparison of 3 to 1 must be conducted over the LCM of 8 years for equal service. Table 6.9 details the incremental cash flows, including the alternative 1 repurchase in year 4. The PW relation is

$$0 = -65{,}000 + 2000(P/A,\Delta i^*,8) + 80{,}000(P/F,\Delta i^*,4)$$
$$\Delta i^* = 10.1\%$$

Since 10.1% < 12%, eliminate 3; declare alternative 1 the survivor and select it as the one that is economically justified.

TABLE 6.9 Incremental Cash Flows for the Comparison of Alternatives 3-to-1, Example 6.7

	Alternative Cash Flows		
Year	Alternative 1	Alternative 3	Incremental Cash Flow for (3–1)
0	\$−80,000	\$−145,000	\$−65,000
1	+33,000	+35,000	+2,000
2	+33,000	+35,000	+2,000
3	+33,000	+35,000	+2,000
4	−47,000	+35,000	+82,000
5	+33,000	+35,000	+2,000
6	+33,000	+35,000	+2,000
7	+33,000	+35,000	+2,000
8	+33,000	+35,000	+2,000

The previous incremental analyses were performed using the PW relations. It is equally correct to apply an AW-based or FW-based analysis; however, the LCM of lives must be used since ROR analysis requires an equal-service comparison. Consequently, there is usually no advantage to developing AW relations to find Δi^* for different-life alternatives.

The use of the IRR spreadsheet function can greatly speed up incremental ROR comparison of multiple alternatives, especially for those with unequal lives. This is fully illustrated in the last section of this chapter.

6.6 MULTIPLE ROR VALUES

For some cash flow series (net for one project or incremental for two alternatives) it is possible that more than one unique rate of return i^* exists. This is referred to as *multiple i^* values.* It is difficult to complete the economic evaluation when multiple i^* values are present, since none of the values may be the *correct* rate of return. The discussion that follows explains how to predict the number of i^* values in the range -100% to infinity, how to determine their values, and how to resolve the difficulty of knowing the "true" ROR value (if this is important). If using ROR evaluation is not absolutely necessary, a simple way to avoid this dilemma is to use the PW, AW, or FW evaluation method at the MARR.

In actuality, finding the rate of return is solving for the root(s) of an *n*th order polynomial. *Conventional* or *simple* cash flows have only one sign change over the entire series, as shown in Table 6.10. Commonly, this is negative in year 0 to positive at some time during the series. There is a unique, real number i^* value for a conventional series. A *nonconventional* series (Table 6.10) has more than one sign change and multiple roots may exist. The *cash flow rule of signs* (based upon Descartes' rule) states:

The maximum number of i^* values is equal to the number of sign changes in the cash flow series.

When applying this rule, zero cash flow values are disregarded.

TABLE 6.10 Examples of Conventional and Nonconventional Net or Incremental Cash Flows for a 6-year Period

	Sign on Cash Flow							
Type of Series	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Number of Sign Changes**
Conventional	−	+	+	+	+	+	+	1
Conventional	−	−	−	+	+	+	+	1
Conventional	+	+	+	+	+	−	−	1
Nonconventional	−	+	+	+	−	−	−	2
Nonconventional	+	+	−	−	−	+	+	2
Nonconventional	−	+	−	−	+	+	+	3

Prior to determining the multiple i^* values, a second rule can be applied to indicate that a unique nonnegative i^* value exists for a nonconventional series. It is the *cumulative cash flow test* (also called Norstom's criterion). It states:

There is one real-number, positive i^* value if the cumulative cash flow series $S_0, S_1, \ldots, S_n$ changes sign only once and $S_0 < 0$.

To perform the test, observe the sign on S_0 and count the number of sign changes in the S_t series, where

$$S_t = \text{cumulative cash flows through period } t$$

More than one sign change provides no information, and the rule of signs is applied to indicate the possible number of i^* values.

Now the unique or multiple i^* value(s) can be determined using trial-and-error with tabulated factors, by graphical interpolation using a PW-based relation, or using the IRR spreadsheet function that incorporates the "guess" option to search for the multiple i^* values. A calculator function is equally useful if the cash flow series is not too complex. The next example illustrates the two rules on sign changes and solution using PW-based relations. The use of spreadsheets is shown in the last section of the chapter.

EXAMPLE 6.8

The engineering design and testing group for Honda Motor Corp. does contract-based work for automobile manufacturers throughout the world. During the last 3 years, the net cash flows for contract payments have varied widely, as shown below, primarily due to a large manufacturer's inability to pay its contract fee.

Year	0	1	2	3
Cash Flow ($1000)	+2000	−500	−8100	+6800

a. Determine the maximum number of i^* values that may satisfy the ROR relation.

b. Write the PW-based ROR relation and approximate the i^* value(s) by plotting PW versus i.

Solution

a. Table 6.11 shows the annual cash flows and cumulative cash flows. Since there are two sign changes in the cash flow sequence, the rule of signs indicates a maximum of two i^* values. The *cumulative* cash flow sequence has two sign changes and $S_0 > 0$, indicating there is not just one nonnegative root. The conclusion is that as many as two i^* values can be found.

b. The PW relation is

$$PW = 2000 - 500(P/F,i,1) - 8100(P/F,i,2) + 6800(P/F,i,3)$$

Select values of i to find the two i^* values, and plot PW versus i. The PW values are shown on the next page and plotted in Figure 6.3 (using a smooth

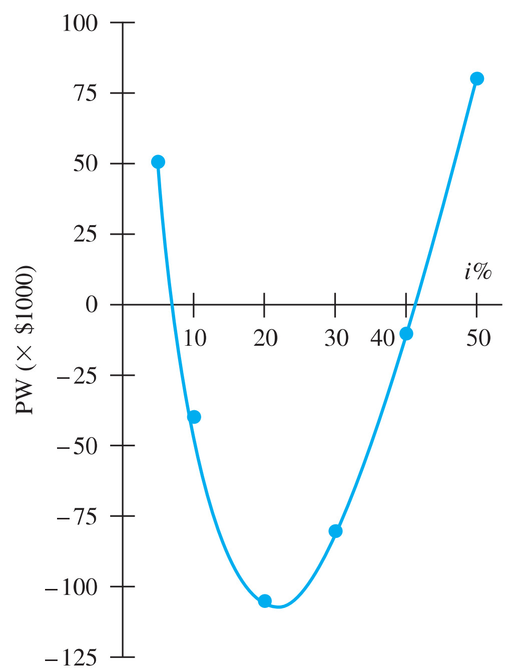
TABLE 6.11 Cash Flow and Cumulative Cash Flow Sequences, Example 6.8

Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	−500	S_1	+1500
2	−8100	S_2	−6600
3	+6800	S_3	+200

approximation) for i values of 0, 5, 10, 20, 30, 40, and 50%. The characteristic parabolic shape for a second-degree polynomial is obtained, with PW crossing the i axis at approximately $i_1^* = 8$ and $i_2^* = 41\%$.

i%	0	5	10	20	30	40	50
PW ($1000)	+200	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85

FIGURE 6.3 Present worth of cash flows at several interest rates, Example 6.8.



EXAMPLE 6.9 An American-Australian joint venture has been contracted to provide the train cars for a 25-mile subway system using new tunnel-boring and track-design technologies. Austin, Texas was selected as the proof-of-concept site based on its variety in landscape features (hilly terrain, lake and green space areas, and relatively low precipitation) and its public environmental mindedness. Selection of either a Swiss

or Japanese contractor to provide the gears and power components for the electric transfer motor assemblies resulted in two cost alternatives. Table 6.12 gives the incremental cash flow estimates (in $1000) over the expected 10-year life of the motors. Determine the number of i^* values and estimate them graphically.

TABLE 6.12 Incremental and Cumulative Cash Flow Series, Example 6.9

	Cash Flow, $1000			Cash Flow, $1000	
Year	**Incremental**	**Cumulative**	**Year**	**Incremental**	**Cumulative**
0	−500	−500	6	+800	+200
1	−2000	−2500	7	+400	+600
2	−2000	−4500	8	+300	+900
3	+2500	−2000	9	+200	+1100
4	+1500	−500	10	+100	+1200
5	−100	−600			

Solution

The incremental cash flows form a nonconventional series with three sign changes in years 3, 5, and 6. The cumulative series starts with $S_0 < 0$ and has one sign change in year 6. This test indicates a single nonnegative root. The incremental ROR is determined from a PW relation (in $1000).

$$0 = -500 - 2000(P/F,\Delta i^*,1) - 2000(P/F,\Delta i^*,2) \cdots + 100(P/F,\Delta i^*,10)$$

Calculation of PW at various i values is plotted (Figure 6.4) to estimate the unique Δi^* of 8% per year.

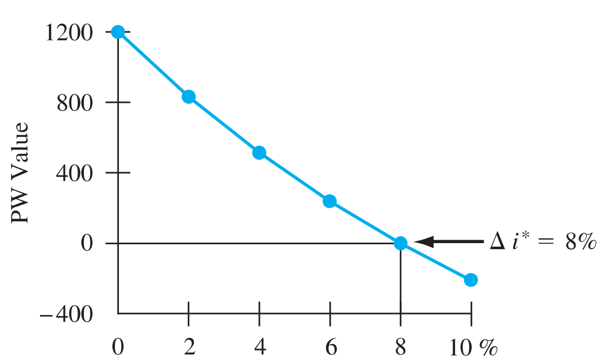


FIGURE 6.4 Graphical estimation of Δi^* using PW values, Example 6.9.

Commonly when multiple i^* or Δi^* values are indicated, there is only one realistic root. Others may be large negative or positive numbers that make no real-world sense and can be neglected. (One clear advantage of using a spreadsheet or calculator for ROR evaluation, as described in Section 6.8, is that realistic i^* values are commonly determined first by the functions.) Here are some useful guidelines for retaining and discarding multiple rates. Assume there are two i^* values for a cash flow series.

If i^* values are	**Do this**
Both $i^* < 0$	Discard both values
Both $i^* > 0$	Discard both values
One $i^* > 0$; one $i^* < 0$	Use $i^* > 0$ as ROR

As mentioned previously, the use of a PW, AW, or FW analysis eliminates the multiple ROR dilemma since the MARR is used in all equivalence relations, and excess funds are assumed to earn the MARR. (See Section 6.4 for a quick review.) It is because of the complexities of the ROR method, namely, incremental analysis, the use of LCM for equal service, reinvestment rate assumption, and possible multiple i^* values, that other methods are preferred to ROR. Yet, the ROR result is important in that some people wish to know the estimated return on proposed project(s).

6.7 TECHNIQUES TO REMOVE MULTIPLE ROR VALUES

All ROR values calculated thus far can be termed *internal rates of return (IRR).* As discussed in Section 6.1, the IRR guarantees that the last receipt or payment brings the balance to exactly zero with interest considered. No excess funds are generated in any year, so all funds are kept internal to the project. However, a project can generate excess funds prior to the end of the project's life when the net cash flow in any year is positive ($NCF_t > 0$). This can result in a nonconventional series, as we learned in Section 6.6. The ROR method assumes these excess funds can earn at any one of the multiple i^* values. This generates ambiguity when the ROR method is used to evaluate alternatives. For example, assume an alternative's nonconventional cash flows have two ROR roots at -2% and 40%. Further, assume that neither is a realistic reinvestment assumption. Instead, excess funds will likely earn at the MARR of 15%. The question "Is the project economically justified?" is not answered by the ROR analysis that resulted in the multiple IRR values. Finding the external rate of return provides a more definitive answer to the question.

The *external rate of return* (EROR), different from the IRR, is highly influenced by parameters *outside* the project's cash flows. Two of these parameters are the cost to borrow money and the earnings rate on invested funds. Determination of the EROR is the correct way to obtain a useful and unique rate of return when multiple i^* values are indicated by a project's net cash flow (NCF) series. Each year a project will generate excess funds (positive NCF) that can be reinvested, or negative NCF, which indicates that funds must be borrowed from a source external to the project. The value and accuracy of the EROR is a function of: (1) the reinvestment rate earned by excess funds; (2) the interest rate paid on borrowed funds; and (3) the reliability of these estimates. Two different methods that determine an EROR are discussed below. The resulting EROR is different for each method, and the EROR is *not* equal to any of the multiple i^* values determined from the cash flow series, since i^* is an *internal* ROR. The benefit is that the EROR determined by either method can be used to make a sound decision about the economic viability of a project. First, let's define the external rates necessary to apply one or both methods.

> **Reinvestment rate i_r –** The rate at which extra funds are invested in some source external to the project. Also called the *investment rate*, this rate is applied to all positive NCF. It is common that i_r is set equal to the MARR.
>
> **Borrowing rate i_b –** The interest rate at which funds are borrowed from an external source to provide capital to the project. This applies to all negative annual NCF. The cost of capital (CoC was introduced in Section 1.3) or weighted average cost of capital (WACC is discussed in Chapter 13) can be used for this rate.

Though the two rates can be set equal to each other, it is not a good idea. Setting $i_r = i_b$ implies that the company is willing to borrow funds and reinvest funds at the same rate; this means no profit margin over time. The company can't survive for long. Commonly MARR > CoC, which means that $i_r > i_b$.

6.7.1 MIRR—Modified ROR Method

This is the easier approach to apply and it has a spreadsheet function that can display the EROR value, which is identified by the symbol i'. The reinvestment and borrowing rates must be reliably estimated, since the resulting i' using the MIRR method may be quite sensitive to them. Figure 6.5 is a reference diagram. The net cash flows change sign several times; multiple i^* values are likely. The modified ROR method uses the following procedure to determine the unique external rate of return i'.

1. *For all negative NCF:* Determine the PW in year 0 at the *borrowing rate* i_b. (Gray shaded area and resulting PW_0 value in Figure 6.5.)
2. *For all positive NCF:* Determine the FW value in year n at the *reinvestment rate* i_r. (Green shaded area and resulting FW_n value.)
3. Determine the external rate of return i' at which PW_0 *and* FW_n *are equivalent* using the relation

$$\mathbf{FW}_n = \mathbf{PW}_0(F/P,i',n) \qquad [6.4]$$

4. The selection guideline is the same as applied previously:

If $i' \geq$ MARR, the project is economically justified
If $i' <$ MARR, the project is not economically justified

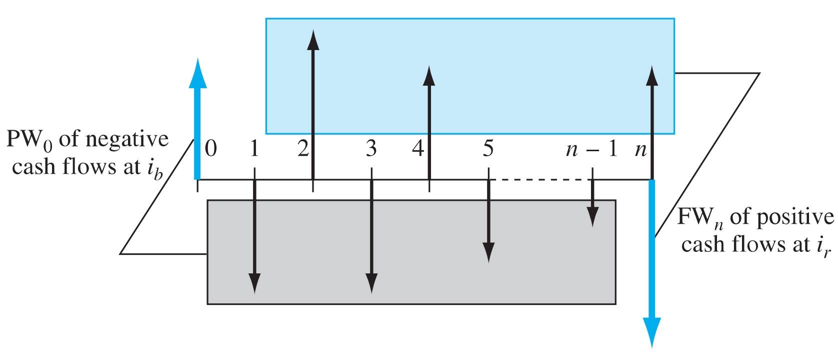


FIGURE 6.5 Sample cash flow series used to determine the external ROR by the modified ROR (MIRR) method.

The MIRR spreadsheet function displays i' directly once the NCF values are entered into adjacent cells. The borrowing rate i_b is called the finance rate in the MIRR function. The format is:

$$= \text{MIRR}(\text{first\_cell:last\_cell}, i_b, i_r)$$

6.7.2 ROIC—Return on Invested Capital Method

From a project perspective, the return on invested capital is a measure of how effectively the project utilizes funds invested in it. From the viewpoint of an entire corporation, ROIC is a measure of how effectively all of its resources (facilities, equipment, people, systems, process, and all other assets) are utilized in the conduct of business.

The symbol for the ROIC rate is i''. The ROIC method utilizes the *reinvestment rate* i_r for excess funds generated by a project in any year. Often, i_r is set equal to the MARR. It is not necessary to estimate a borrowing rate for the ROIC method. The EROR is determined using a technique called the *net-investment procedure,* which involves developing a series of future worth relations (FW values) moving forward 1 year at a time from time $t = 0$ to $t = n$ with time value of money considered. In those years that the net balance of the project cash flows is positive (extra funds generated by the project), the funds are reinvested at the rate i_r. When the net balance is negative, the ROIC rate i'' accounts for the time value of money. The procedure follows.

1. Develop a series of future worth relations for each year t ($t = 1, 2, \ldots, n$).

$$\text{FW}_t = \text{FW}_{t-1}(1 + k) + \text{NCF}_t \qquad [6.5]$$

where FW_t = future worth in year t based on previous year and time value of money

NCF_t = net cash flow in year t

$$k = \begin{cases} i_r & \text{if FW}_{t-1} > 0 \quad \text{(extra funds available)} \\ i'' & \text{if FW}_{t-1} < 0 \quad \text{(project uses all available funds)} \end{cases}$$

2. Set the future worth relation for the last year n equal to 0, that is, $\text{FW}_n = 0$, and solve for i''. The i'' value is the ROIC for the specified reinvestment rate i_r.
3. The selection guideline is the same as above.

If $i'' \geq$ MARR, the project is economically justified
If $i'' <$ MARR, the project is not economically justified

The FW_t series and solution for i'' can be become involved mathematically when hand solution is performed. Fortunately, the GOAL SEEK spreadsheet tool, coupled with logical IF statements determine i'' rapidly, because this is the only unknown in the relation and the target value is $\text{FW}_n = 0$.

The next example illustrates how multiple i^* values can be removed using either hand or spreadsheet solution utilizing the MIRR and ROIC methods. However, prior to presenting an example, a reminder is in order. The EROR values determined by these two methods are highly dependent upon the reinvestment rate and/or borrowing rate estimates. Additionally, remember that the EROR values determined are not the same as any multiple i^* rate, but each EROR value is unique for the estimated i_r and i_b rates, as required by the method.

Remember: The MIRR method and ROIC method are used when multiple i^* values are indicated. Multiple i^* values are present when a nonconventional cash flow series does not have a single, unique root. Finally, it is important to remember that these procedures are unnecessary if the PW, AW or FW method is used to perform the economic evaluation at the MARR.

EXAMPLE 6.10

Large oil exploration corporations are using better machinery and technology to cap offshore oil spills before they become major disasters. Marine Wells, a company experienced in providing containment response equipment, has estimated the net annual savings shown below (in \$ million of cash flow) over the current and next 3 years if their equipment is contracted for by international offshore exploration corporations such as BP, Exxon-Mobil, Chevron, and Total SA. The negative amount in year 1 assumes no oil spill is experienced; the cost is that of the annual contract. Find a unique rate of return using (a) the MIRR method, and (b) the ROIC method, if the following external rates are estimated.

MARR = 12% per year

Borrowing rate for extra funds = 10% per year

Reinvestment rate for excess funds = 15% per year

Year	Cash Flow, \$ million
0	50
1	−200
2	50
3	100

Solution

This is a nonconventional cash flow series and does have multiple i^* values as indicated by the cash flow sign test (2 changes) and cumulative cash flow test (inconclusive and $S_0 > 0$). Mathematically, two positive i^* values can be determined: 0% and 256%, neither of which is useful for economic decision making.

a. For the MIRR method, reinvestment is at $i_r = 15\%$ for any excess fund years, and the borrowing rate is $i_b = 10\%$. For a hand solution of the external rate of return i', use the MIRR procedure and utilize Figure 6.5 as a general reference.

1. Negative NCF in year 1 at borrowing rate:

$$PW_0 = -200(P/F,10\%,1) = \$-181.82$$

2. Positive NCF in years 0, 2 and 3 at reinvestment rate:

$$FW_3 = 50[(F/P,15\%,3) + (F/P,15\%,1)] + 100 = \$233.54$$

3. Per Equation [6.4], set $FW_3 = PW_0$ considering the time value of money and solve for i'.

$$233.54 = 181.82(F/P,i',3)$$
$$181.82(1 + i')^3 = 233.54$$
$$(1 + i')^3 = 1.2845$$
$$i' = 0.0870 \qquad (8.70\%)$$

4. The external rate of return of $i' = 8.70\%$ is less than MARR $= 12\%$. The project is not economically viable.

If a spreadsheet has the cash flows entered into cells B2 through B5, the function = MIRR(B2:B5,10%,15%) will display $i' = 8.70\%$.

b. The ROIC method uses the reinvestment rate of $i_r = 15\%$ to determine a significantly lower EROR value of $i'' = 3.13\%$. The hand and spreadsheet procedures are summarized here.

Hand solution: Apply the 3-step procedure to develop the FW series and find i''. Figure 6.6*a* shows the original cash flows, and the remaining diagrams track the development year-by-year.

1. Develop the FW relations for years 0 through 3 using $i_r = 15\%$ only when $FW_{t-1} > 0$.

Year 0:	$FW_0 = \$50$	(Reinvest at 15%)
Year 1:	$FW_1 = 50(1.15) - 200$ $= \$-142.50$	(Figure 6.6*b*; use i'' for year 2)
Year 2:	$FW_2 = -142.50(1 + i'') + 50$	(Figure 6.6*c*; use i'' for year 3)
Year 3:	$FW_3 = [-142.50(1 + i'') + 50]$ $\times (1 + i'') + 100$	(Figure 6.6*d*)

2. Set $FW_3 = 0$ and solve for i'' using the quadratic equation.

$$-142.50(1 + i'')^2 + 50(1 + i'') + 100 = 0$$

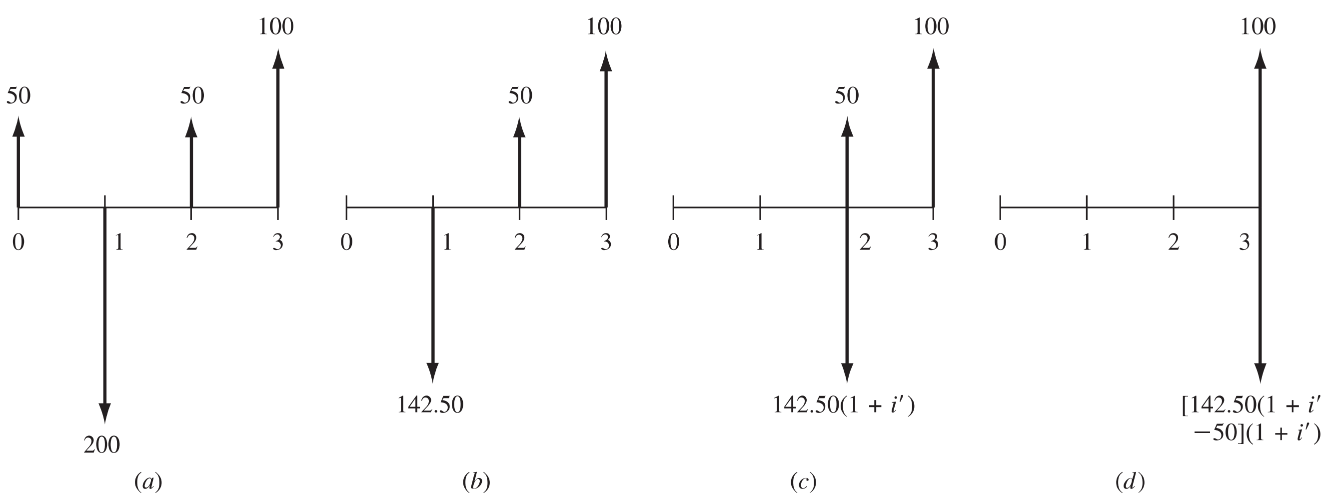


FIGURE 6.6 Cash flow series for which the external rate of return i'' is computed using the ROIC method: (*a*) original form; equivalent form in (*b*) year 1, (*c*) year 2, and (*d*) year 3.

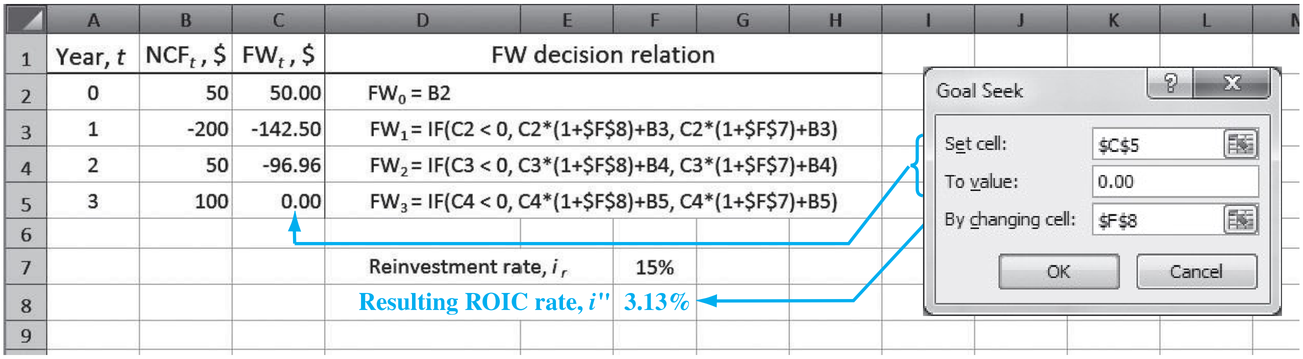


FIGURE 6.7 Application of ROIC method using logical IF statements and the GOAL SEEK tool, Example 6.10.

The two roots for $1 + i''$ are -0.68 and 1.0313. This translates into the rates of -168% and 3.13%. Discard the negative 168%, since it is below the -100% lower limit for a rate of return. We conclude that the EROR is $i'' = 3.13\%$.

3. Since 3.13% is much less than the MARR of 12%, again the project is not economically viable.

Spreadsheet solution: Figure 6.7 details the logical IF statements and resulting FW values (column C) for each year. As shown in the IF statement of column D, when $FW_{t-1} < 0$ the logic statement is "true" and the ROIC rate in cell F8 is applied to the next FW. Alternatively, when $FW_{t-1} > 0$, the reinvestment rate of 15% (cell F7) accounts for the time value of money. The GOAL SEEK tool is used to force FW_3 to equal 0 by changing the trial ROIC value, thus resulting in $i'' = 3.13\%$.

Comment

Note that the two EROR rates—8.70% by the MIRR method and 3.13% by the ROIC method—are different; plus they are both different from the multiple i^* rates (0% and 256%). This illustrates how dependent the different methods are upon the additional information provided by i_b and i_r.

The two methods discussed above remove multiple i^* values. They are very helpful when the multiple i^* values are unrealistic and the reinvestment assumption that excess cash flows are reinvested at these rates makes no sense. Here are some interesting relations between the multiple i^* values for a nonconventional cash flow series, the external reinvestment rate i_r, the borrowing rate i_b, and the resulting EROR rates i' and i''.

MIRR method – When both i_b and i_r are exactly equal to any one of the multiple i^* values, the MIRR-method rate i' equals that i^* value. In this case, all four parameters have the same value; if $i^* = i_b = i_r$, then $i' = i^*$.

ROIC method – Similarly, if i_r equals a multiple i^* value, the ROIC-method rate is $i'' = i^*$.

6.8 USING SPREADSHEETS AND CALCULATORS TO DETERMINE ROR VALUES

Spreadsheets greatly reduce the time needed to perform a rate of return analysis through the use of the RATE or IRR functions. Coupled with the NPV function to develop a spreadsheet plot of PW versus i, the IRR function can perform virtually any analysis for one project, perform incremental analysis of multiple alternatives, and find multiple i^* values for nonconventional cash flows and incremental cash flows between two alternatives.

If the annual cash flows are all equal with separate P and/or F values, find i^* using the spreadsheet function

$$= \text{RATE}(n,A,P,F)$$

For series that are not too complex, the financial calculator function $i(n,A,P,F)$ is a very rapid way to find the i^* value. It is the same as the RATE function on a spreadsheet.

If cash flows vary throughout the n years, a spreadsheet must be used to find one or multiple i^* values using

$$= \text{IRR}(\text{first\_cell:last\_cell, guess})$$

For IRR, each cash flow must be entered in succession by spreadsheet row or column. A "zero" cash flow year must be entered as "0" so the year is accounted for. "Guess," an optional entry that starts the ROR analysis, is used most commonly to find multiple i^* values for nonconventional cash flows, or if the #NUM error is displayed when IRR is initiated without a guess entry. The next two examples illustrate RATE, IRR, and NPV use as follows:

One project—RATE, IRR, and NPV for single and multiple i^* values (Example 6.11)

Multiple alternatives—IRR for incremental evaluation (Example 6.12)

EXAMPLE 6.11 Two brothers, Gerald and Henry, own the Edwards Service Company in St. Johns, Newfoundland. It provides onshore services for spent lubricants from North Atlantic offshore platforms. The company needs immediate cash flow. Because the Edwards have an excellent reputation among major oil producers, they have been offered an 8-year contract that pays $200,000 total with 50% upfront and 50% at the end of the suggested 8-year contract. The estimated annual cost for Edwards to provide the services is $30,000. Assume you are the financial person for Edwards. Is the project justified if the brothers want to make at least 8% per year? Use calculator and spreadsheet functions and charts to perform a thorough ROR analysis.

Solution

The analysis can be accomplished in several ways, some more thorough than others. The approaches illustrated here are of increasing thoroughness.

1. The $i(n,A,P,F)$ calculator function is fast and easy to use. The function $i(8,-30000,100000,100000)$ will display one of several responses for i^*, depending upon the method used by the calculator to solve for i in Equation [2.3]. The normal display is $i^* = 15.91\%$. However, some calculators return the statement "Divided by 0" and provide no numerical answer, and some display a negative rate of return of $i^* = -23.98\%$. In this case, it is better to rely on a spreadsheet, as illustrated below.
2. Refer to Figure 6.8 (left). The easiest and quickest spreadsheet approach is to develop the RATE function for $n = 8$, $A = \$-30{,}000$, $P = \$100{,}000$, and $F = \$100{,}000$ in a single cell. The value $i^* = 15.91\%$ is displayed. The project is definitely justified since MARR = 8%.
3. Refer to Figure 6.8 (right). Alternatively, to obtain the same answer, enter the years and associated net cash flows in adjacent cells. Develop the IRR function for the 9 entries to display $i^* = 15.91\%$; the project is justified.
4. Refer to Figure 6.9 (top). The cash flow (upper left of the figure) is a nonconventional series with two sign changes. The cumulative series has one sign change, but $S_0 > 0$. However, there is one real-number, positive i^*; it is 15.91%, displayed when the function = IRR(B3:B11) is input. However, as predicted by the rule of signs, there is a second, though negative, i^* value at -23.98% found by using the guess option. A variety of guess percentages can be used; Figure 6.9 shows only four, but the results will always be $i_1^* = -23.98\%$ and $i_2^* = 15.91\%$.

   Is the project justified, knowing there are two roots? If reinvestment can be assumed to be at 15.91% rather than the MARR of 8%, the project is justified. If released funds are actually expected to make 8%, the project's

	A	B	C	D	E	F	G	H	I	J
1										
2	(2) *i** using RATE is		**15.91%**			Year	Cash flow, $	(3) *i** using IRR is		**15.91%**
3						0	100,000			
4						1	-30,000			
5		= RATE(8, −30000, 100000, 100000)				2	-30,000			
6						3	-30,000			
7						4	-30,000		= IRR(G3:G11)	
8						5	-30,000			
9						6	-30,000			
10						7	-30,000			
11						8	70,000			
12										
13										

FIGURE 6.8 ROR analysis of a project using the RATE and IRR functions, Example 6.11.

Year	Cash flow, $	Cumulative, $
0	100,000	100,000
1	-30,000	70,000
2	-30,000	40,000
3	-30,000	10,000
4	-30,000	-20,000
5	-30,000	-50,000
6	-30,000	-80,000
7	-30,000	-110,000
8	70,000	-40,000

(4) Multiple *i** using 'guess' option

Guess	-25%	-15%	0%	10%
*i** found	-23.98%	-23.98%	15.91%	15.91%

*i** result using guess of −25% = IRR($B$3:$B$11,−25%)

*i** result using guess of 10% = IRR($B$3:$B$11,10%)

(5) Multiple *i** using PW vs *i* graph

i value, %	PW at *i*, $
-30%	200,000
-25%	20,226
-20%	-48,023
-10%	-64,611
0%	-40,000
5%	-26,212
10%	-13,397
15%	-1,929
20%	8,142
25%	16,910
30%	24,518

Sample NPV function for $i = 30\%$ = NPV(B26,$B$4:$B$11) + $B$3

*i** values are approximately −25% and +15%

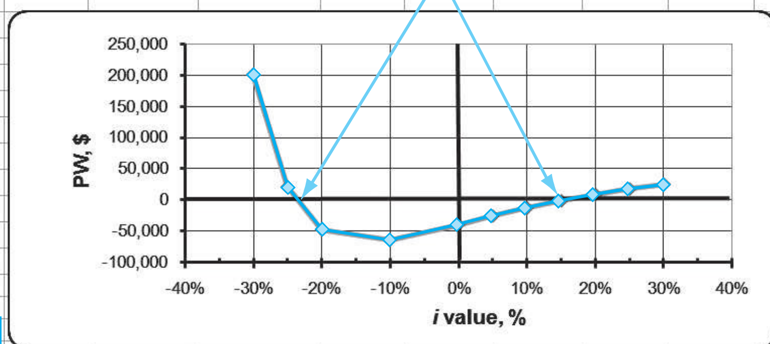


FIGURE 6.9 ROR analysis of a project with multiple *i** values using the IRR function with "guess" option and PW versus *i* graphical analysis, Example 6.11.

return is between 8 and 15.91%, and it is still justified. However, if funds are actually expected to make less than the MARR, it is not justified because the true rate is less than MARR, in fact between the MARR and −23.98%. *The conclusion is that the ROR analysis does not provide a conclusive answer.* If reinvestment is not assumed at either *i** value, one of the procedures of Section 6.7 should be performed.

5. Refer to Figure 6.9 (bottom). An excellent graphical way to approximate *i** values is to generate an *x*-*y* scatter chart of PW versus *i*. Use the NPV function with different *i* values to determine where the PW curve crosses the PW = 0 line. Here *i* values from −30% to +30% were chosen. For details, refer to the sample NPV function cell tag using the cash flows at the top of the figure and different *i* values. The approximate *i** values are −25% and +15%. Basically, this graphical analysis provides the same information as that of (4).

EXAMPLE 6.12

Perform a spreadsheet analysis of the four alternatives for Harold's construction company in Example 6.7.

Solution

First, review the situation in Example 6.7 for the four revenue alternatives. Figure 6.10 performs the complete analysis in one spreadsheet starting with the addition of the do-nothing alternative and the ordered alternatives DN, 4, 2, 1, and 3.

The top portion provides the estimates, including net annual cash flows in row 6. The middle portion calculates the incremental investment and cash flows for each comparison of two alternatives. Equal lives of 4 years are present for the first three comparisons. Details for the final comparison of 3-to-1 over the LCM of 8 years are included in the cell tags. (Multiple i^* values are not indicated for any incremental series.)

The bottom portion shows conclusions after using the IRR function to find the incremental ROR, comparing it to MARR $= 12\%$, and identifying the surviving alternative. The entire logic of the analysis is identical to that of the hand solution. Alternative 1 is the best economic choice.

	A	B	C	D	E	F	G	H	I
1									
2			DN	4	2	1	3		
3	Investment, $		0	-20,000	-50,000	-80,000	-145,000		
4	Revenue, $/year		0	29,000	43,000	61,000	51,000		
5	Cost, $/year		0	-21,000	-26,000	-28,000	-16,000		
6	Net cash flow, $/year		0	8000	17,000	33,000	35,000		
7	Life, n			4	4	4	8		
8									
9	Incremental evaluation	Year		4-to-DN	2-to-4	1-to-4	3-to-1		
10	Δ investment	0		-20,000	-30,000	-60,000	-65,000		
11	Δ cash flow	1		8000	9000	25,000	2000		
12		2		8000	9000	25,000	2000		
13		3		8000	9000	25,000	2000		
14		4		8000	9000	25,000	82,000		
15		5					2000		
16		6					2000		
17		7					2000		
18		8					2000		
19									
20	Conclusions								
21	Δ*i**			21.9%	7.7%	24.1%	10.1%		
22	Increment justified?			Yes	No	Yes	No		MARR = 12%
23	Alternative retained			4	4	1	1		
24									
25									

Revenue alternatives ordered after DN is added

Net cash flow: = G4+G5

Evaluation over LCM of 8 years

Δ investment: = G3-F3

Δ cash flow: = G6-F6

Repurchase of 1: = G6-F6 + 80000

Sample incremental computations for comparison of 3-to-1

IRR functions use LCM of compared alternative lives

The economically best alternative is 1

FIGURE 6.10 Incremental ROR analysis of four revenue alternatives, Example 6.12.

SUMMARY

Just as present worth, annual worth, and future worth methods find the best alternative from several, the ROR calculations serve the same purpose. With ROR analysis, however, the *incremental* cash flow series between two alternatives is evaluated. Alternatives are ordered by increasing initial investments, and the pairwise ROR evaluation proceeds from smallest to largest investment. The alternative with the larger incremental i^* value is selected as each comparison is conducted. Once eliminated, an alternative cannot be reconsidered.

Rate-of-return calculations performed by hand typically require trial-and-error solutions using tabulated factors. Spreadsheets and calculators greatly speed up this process. The analysis may result in more than one ROR value depending upon the number of sign changes present in the cash flow series. The cash flow rule of signs and cumulative cash flow test assist in determining if a unique ROR value does exist. The dilemma of multiple rates can be effectively dealt with by calculating the external rate of return (EROR) using the MIRR or ROIC method with either hand or spreadsheet procedures. In the end, if multiple rates are present, it is strongly recommended that the PW, AW, or FW value is determined at the MARR.

PROBLEMS

Understanding ROR

6.1 In percent, what is (*a*) the highest, and (*b*) the lowest rate of return that is possible?

6.2 When interest is charged on the unrecovered balance, if you borrow \$10,000 at 10% per year interest and repay the loan in equal payments over a 5-year period, the payment amount is \$2638 per year. How much will the annual payment be if the interest rate is charged on the *initial loan amount* instead of the unrecovered balance?

6.3 Spectra Scientific of Santa Clara, CA, manufactures Q-switched solid state industrial lasers for LED substrate scribing and silicon wafer dicing. The company got a \$60 million loan, amortized over a 5-year period at 8% per year interest. What is the amount of the unrecovered balance (*a*) immediately *before* the payment is made at the end of year 1, and (*b*) immediately *after* the first payment?

6.4 The production of polyamide from raw materials of plant origin, such as castor oil, requires 20% less fossil fuel than conventional production methods. Darvon Chemicals borrowed \$6 million to implement the process. If the interest rate on the loan is 10% per year for 10 years, what is the amount of interest for year 2?

6.5 General Dynamics obtained a 0.5%-per-month \$100 million loan to be repaid over a 5-year period. (*a*) What is the difference in the amount of interest in the second month's payment if interest is charged on the original principal of the loan rather than on the unrecovered balance? (*b*) As months pass, for which basis—principal only or unrecovered balance—does the monthly interest decrease in amount?

ROR Calculation

6.6 Use tabulated factors and a spreadsheet to determine the interest rate per period for the following rate of return equation: $0 = -40{,}000 + 8000(P/A,i^*,5) + 8000(P/F,i^*,8)$.

6.7 Determine the rate of return per year for the cash flows shown below. Use (*a*) tabulated factors, and (*b*) a spreadsheet.

Year	1	2	3	4
Cash Flow, \$	−80,000	9000	70,000	30,000

6.8 A company that manufactures brushless blowers invested \$650,000 in an automated quality control system for blower housings. The resultant savings was \$160,000 per year for 5 years. If the equipment had a salvage value of \$50,000, what rate of return per year did the company make?

6.9 A University of Massachusetts study found that married women who work outside the home do about one hour less of housework per week for every \$7500 they earn outside the home. Assume that they hire a housekeeper one time per week for \$120; that the \$7500 is received in uniform amounts of \$625 per month; and, that the housekeeper is paid weekly with these payments made before the \$625 is received. What rate of return are they making *per week* on their "investment" in the housekeeper?

6.10 A 473-foot, 7000-ton World War II troop carrier (once commissioned as the SS Excambion) was sunk in the Gulf of Mexico to serve as an underwater habitat and diving destination. The project took 10 years of planning and cost \$4 million, which was spent equally at \$400,000 in years 1 through 10. Fishing and recreation activities, estimated at \$270,000 per year, will begin in year 11 and are expected to continue in perpetuity. Determine the rate of return on the venture using (*a*) tabulated factors, and (*b*) the GOAL SEEK tool.

6.11 The Closing the Gaps initiative by the Texas Higher Education Coordinating Board established the goal of increasing the number of students in higher education in Texas from 1,064,247 in 2000 to 1,694,247 in 2015. If the increase occurs uniformly and is compounded annually, what rate of increase is required each year to meet the goal?

6.12 When Hurricane Katrina struck New Orleans, there was a significant loss of aquarium fish at the Audubon Aquarium of the Americas. FEMA originally stated that the aquarium needed to buy the fish from commercial vendors, a method the agency said would cost \$616,849 but would comply with disaster aid laws. FEMA later reversed their decision and allowed the aquarium staff to catch the fish themselves at a total cost of \$99,766. If it is assumed that the aquarium staff spent the \$99,766 equally over a 12-month period of time, what rate of return per month did their effort represent? Assume FEMA would have given the aquarium the \$616,849 at the end of month 12.

6.13 Texas Governor Rick Perry promised to put hundreds of cameras on the Texas-Mexico border and broadcast the video over the Web so that anyone, anywhere could become a border patroller, helping root out border crime and illegal crossings. As part of that project, Texas secured a federal grant for \$3 million that paid for 200 mobile cameras in strategic high-traffic areas. If the 200 cameras are considered to be equivalent to 20 border patrol agents, each with an annual salary of \$75,000, what is the rate of return over a 3-year project period?

6.14 The University of California at San Diego is considering a plan to build a 8-megawatt cogeneration plant to provide for part of its power needs. The cost of the plant is expected to be \$41 million. The university consumes 55,000 megawatt-hours per year at a cost of \$120 per megawatt-hour. (*a*) If the university will be able to produce power at half the cost that it now pays, what rate of return will it make on its investment for an expected power plant life of 30 years? (*b*) If, in addition, the university can sell an average of 12,000 megawatt-hours per year back to the utility at \$90 per megawatt-hour, what rate of return will it make?

6.15 The Camino Real Landfill was required to install a plastic liner to prevent leachate from migrating into the groundwater. The fill area was 50,000 m^2 and the installed liner cost was \$8 per square meter. In order to recover the investment, the owner charged \$10 for pick-up loads, \$25 for dump truck loads, and \$70 for compactor-truck loads. The annual distribution is 2400 pick-up loads, 600 dump truck loads, and 1200 compactor-truck loads. What rate of return will the landfill owner make on the investment if the fill area is adequate for 4 years?

6.16 U.S. Census Bureau statistics show that the annual earnings for a person with a high-school diploma are \$35,220 versus \$57,925 for someone with a bachelor's degree. If the cost of attending college is assumed to be \$30,000 per year for four years and the foregone earnings during those years is assumed to be \$35,220 per year, what rate of return does earning a bachelor's degree represent? Assume a 35-year study period.

6.17 Rubber sidewalks made from ground-up tires are said to be environmentally friendly and easier on peoples' knees. Rubbersidewalks, Inc. of Gardena, CA, manufactures the small rubberized squares that are installed where tree roots, freezing weather, and snow removal require sidewalk replacement or major repairs every three years. The District of Columbia spent \$60,000 for a rubber sidewalk to replace broken concrete in a residential neighborhood lined with towering willow oaks. If a concrete sidewalk costs \$28,000 and lasts only 3 years versus a 9-year life for the rubber sidewalks, what rate of return does this represent?

6.18 Steel cable barriers in highway medians are a low cost way to improve traffic safety without busting state department of transportation budgets. Cable barriers cost $44,000 per mile, compared with $72,000 per mile for guardrail and $419,000 per mile for concrete barriers. Furthermore, cable barriers tend to snag tractor-trailer rigs, keeping them from ricocheting back into same-direction traffic. The state of Ohio spent $4.97 million installing 113 miles of cable barriers. (*a*) If the cables prevent accidents totaling $1.3 million per year, determine the rate of return that this represents over a 10-year study period. Use all three methods—tabulated factors, a calculator, and a spreadsheet. (*b*) Now, determine the rate of return for 113 miles of guardrail if accident prevention is $1.1 million per year over a 10-year study period. To do so, first write the ROR relation and then find i^* using a single-cell spreadsheet function.

6.19 A broadband service company borrowed $2 million for new equipment and repaid the loan in amounts of $200,000 in years 1 and 2 plus a lump sum amount of $2.2 million at the end of year 3. What was the interest rate on the loan?

6.20 A new permanent endowment at the University of Alabama will award scholarships to engineering students twice per year (end of June and end of December). The first awards are to be made beginning 5-½ years after the $20 million lump sum donation is made. If the interest from the endowment is intended to fund 100 students each semester in the amount of $5000 twice per year, what semiannual rate of return must the endowment fund earn?

6.21 An Indium-Gallium-Arsenide-Nitrogen alloy developed at Sandia National Laboratory is said to have potential uses in electricity-generating solar cells. The new material is expected to have a longer life, and it is believed to have a 40% efficiency rate, which is nearly twice that of standard silicon solar cells. The useful life of a telecommunications satellite could be extended from 10 to 15 years by using the new solar cells. What rate of return could be realized if an extra investment now of $950,000 would result in extra revenues of $450,000 in year 11, $500,000 in year 12, and amounts increasing by $50,000 per year through year 15?

6.22 Barron Chemical used a thermoplastic polymer to enhance the appearance of certain RV panels. The initial cost of one process was $130,000 with annual costs of $49,000. Revenues were $78,000 in year 1, increasing by $1000 per year. A salvage value of $23,000 was realized when the process was discontinued after 8 years. What rate of return did the company make on the process?

Incremental Analysis

6.23 Why is an incremental analysis necessary when conducting a rate of return evaluation of cost alternatives?

6.24 What is the overall rate of return on a $100,000 investment that returns 20% on the first $30,000 and 14% on the remaining $70,000?

6.25 Alternatives X and Y have rates of return of 10% and 18%, respectively. What is known about the rate of return on the increment between X and Y if the investment required in Y is (*a*) larger than that required for X, and (*b*) smaller than that required for X? (*c*) Develop two spreadsheet examples that illustrate your responses to parts (*a*) and (*b*).

6.26 A company that manufactures rigid shaft couplings has $600,000 to invest. The company is considering three different projects that will yield the following rates of return:

Project X $i_X = 24\%$
Project Y $i_Y = 18\%$
Project Z $i_Z = 30\%$

The initial investment required for each project is $100,000, $300,000, and $200,000, respectively. If the company's MARR is 15% per year and the company invests in all three projects, what overall rate of return will the company make?

6.27 For each of the following scenarios, state whether an incremental investment analysis is required to select an alternative and state why or why not. Assume that alternative Y requires a larger initial investment than alternative X and that the MARR is 20% per year.

a. X has $i^* = 22\%$ per year, and Y has $i^* = 20\%$ per year.
b. X has $i^* = 19\%$ per year, and Y has $i^* = 21\%$ per year.
c. X has $i^* = 16\%$ per year, and Y has $i^* = 19\%$ per year.
d. X has $i^* = 25\%$ per year, and Y has $i^* = 23\%$ per year.
e. X has $i^* = 20\%$ per year, and Y has $i^* = 22\%$ per year.

6.28 For the cash flows shown and in preparation for a PW-based rate of return analysis, determine the incremental cash flow between machines B and A for (*a*) year 0, (*b*) year 3, and (*c*) year 6.

	Machine A	Machine B
First cost, $	−15,000	−25,000
Annual operating cost, $ per year	−1,600	−400
Salvage value, $	3,000	6,000
Life, years	3	6

6.29 Determine the sum of the cash flows in the incremental difference column (i.e., Y-X) for systems X and Y.

	System X	System Y
First cost, $	−45,000	−65,000
Annual operating cost, $	−21,800	−14,000
Salvage value, $	3,000	6,000
Life, years	5	5

6.30 For the alternatives shown, determine the sum of the cash flows in the Z-X difference column.

	System X	System Z
First cost, $	−40,000	−95,000
Annual operating cost, $/year	−12,000	−5,000
Salvage value, $	6,000	14,000
Life, years	3	6

ROR Evaluation of Two or More Alternatives

6.31 The incremental cash flows for alternatives P and Q are shown. Determine which should be selected using a FW-based rate of return analysis. The MARR is 15% per year and alternative Q requires the larger initial investment.

Year	Incremental Cash Flow (Q-P)
0	$−250,000
1–8	+50,000
8	+30,000

6.32 The Chem-Tex Chemical company is considering two additives for improving the dry-weather stability of its low-cost acrylic paint. Additive A will have a first cost of $110,000 and an annual operating cost of $60,000. Additive B will have a first cost of $175,000 and an annual operating cost of $35,000. If the company uses a three-year recovery period for paint products and a MARR of 20% per year, which process is favored on the basis of an incremental rate of return analysis?

6.33 Liquid Sleeve, Inc. is a company that makes a sealing solution for machine shaft surfaces that have been compromised by abrasion, high pressures, or inadequate lubrication. The manager is considering adding a metal-based nanoparticle (Type Al or Fe) to its solution to increase the product's performance at high temperatures. The costs associated with each type are estimated. If the company's MARR is 20% per year, which nanoparticle type should the company select? Utilize a rate of return analysis.

	Type Fe	Type Al
First cost, $	−150,000	−280,000
Annual operating cost, $/year	−92,000	−74,000
Salvage value, $	30,000	70,000
Life, years	2	4

6.34 A mechanical engineer is considering two robots for improving materials handling in the production of rigid shaft couplings that mate dissimilar drive components. Robot X has a first cost of $84,000, an annual maintenance and operation (M&O) cost of $31,000, a $40,000 salvage value, and will improve net revenues by $96,000 per year. Robot Y has a first cost of $146,000, an annual M&O cost of $28,000, a $47,000 salvage value, and will increase net revenues by $119,000 per year. Which one should be selected on the basis of a rate of return analysis if the company's MARR is 15% per year? Use a three-year study period.

6.35 Old Southwest Canning Co. has determined that any one of four machines can be used in a certain phase of its chili-canning operation. The first costs and annual operating costs (AOC) are estimated below, and all machines have a 5-year life. The MARR is 25% per year. (*a*) Determine which machine should be selected on the basis of a rate of return analysis. (*b*) Use a spreadsheet to perform PW analysis of each machine. Compare the machine selections with that of the ROR analysis.

Machine	First Cost, $	AOC, $
1	−28,000	−20,000
2	−51,000	−12,000
3	−32,000	−19,000
4	−33,000	−18,000

6.36 The four alternatives described below are being evaluated.

a. If the proposals are independent, which one(s) should be selected at a MARR of 17% per year?

b. If the proposals are mutually exclusive, which one should be selected at a MARR of 14.5% per year?

c. If the proposals are mutually exclusive, which one should be selected at a MARR of 10.0% per year?

Alternative	Initial Investment, $	Overall Rate of Return, %	Incremental Rate of Return, %, When Compared with Alternative		
			A	B	C
A	−60,000	11.7			
B	−90,000	22.2	43.3		
C	−140,000	17.9	22.5	10.0	
D	−190,000	15.8	17.8	10.0	10.0

6.37 A small manufacturing company expects to expand its operation by adding new product lines. Any or all of four new lines can be added. If the company uses a MARR of 15% per year and a 5-year project period, which products, if any, should the company manufacture? Monetary terms are in $1000.

	Product			
	1	2	3	4
Initial cost, $	−340	−500	−570	−620
Annual cost, $/year	−70	−64	−48	−40
Annual savings, $/year	180	190	220	205

6.38 A WiMAX wireless network integrated with a satellite network can provide connectivity to any location within 10 km of the base station. The number of sectors per base station can be varied to increase the bandwidth. An independent cable operator is considering the three bandwidth alternatives shown below (monetary values in $1000 units). Assume a life of 20 years and a MARR of 25% per year to determine which alternative is best using an incremental ROR analysis.

Bandwidth, Mbps	First Cost, $	Operating Cost, $ per year	Annual Income, $ per year
44	−40,000	−2000	+4000
55	−46,000	−1000	+5000
88	−61,000	−500	+8000

6.39 Ashley Foods, Inc. has determined that only one of five machines can be used in one phase of its dairy products operation. The first and annual costs are estimated; all machines are expected to have a 4-year useful life. If the MARR is 20% per year, determine which machine should be selected on the basis of rate of return.

Machine	First Cost, $	AOC, $ per year
1	−31,000	−16,000
2	−29,000	−19,300
3	−34,500	−17,000
4	−49,000	−12,200
5	−41,000	−15,500

6.40 Five *revenue* projects are under consideration by General Dynamics for improving material flow through an assembly line. The initial cost (in $1000 units) and life of each project are estimated. Income estimates are not known at this point.

	Project				
	A	B	C	D	E
Initial cost, $	−700	−900	−2300	−300	−1600
Life, years	5	5	5	5	5

An engineer determined the incremental ROR (Δi^*) values. From these results, determine which project, if any, should be undertaken, provided the company's MARR is (*a*) 13.5% per year, and (*b*) 16% per year. If other calculations must be made in order to make a decision, state which ones are necessary.

Comparison	Δi^*, %	Comparison	Δi^*, %
C to DN	24	B to A	53
C to B	25	B to DN	23
D to DN	15	A to DN	13
E to C	59	E to B	−16
E to D	4	B to D	26
E to A	0	D to C	25
E to DN	6	A to D	12

6.41 Four different machines are under consideration for improving material flow in a production process. An engineer performed an economic analysis to select the best machine, but some of his calculations were deleted from the report by a disgruntled employee. All machines are assumed to have a 10-year life. (*a*) Fill in the missing numbers in the report. (*b*) Which machine should the company select if its MARR is 18% per year and one of the machines must be selected?

	Machine			
	1	2	3	4
Initial cost, $	?	−60,000	−72,000	−98,000
Annual cost, $ per year	−70,000	−64,000	−61,000	−58,000
Annual savings, $ per year	+80,000	+80,000	+80,000	+82,000
Overall ROR, %	18.6%	?	23.1%	20.8%
Machines compared		2 to 1	3 to 2	4 to 3
Incremental investment, $		−16,000	?	−26,000
Incremental cash flow, $ per year		+6,000	+3,000	?
ROR on increment, %		35.7%	?	?

6.42 Allstate Insurance Company is considering adopting one of five fraud detection systems, all of which can be considered to last indefinitely. If the company's MARR is 14% per year, determine which one should be selected on the basis of a rate of return analysis.

	A	B	C	D	E
First cost, $	−10,000	−25,000	−15,000	−70,000	−50,000
Annual net income, $/year	2,000	4,000	2,900	10,000	6,000
Overall ROR, %	20	20	19.3	14.3	12

6.43 The four revenue proposals described below are being evaluated.

a. If the proposals are independent, which one(s) should be selected with MARR = 15.5% per year?
b. If the proposals are mutually exclusive, which one should be selected with MARR = 10% per year?
c. If the proposals are mutually exclusive, which one should be selected with MARR = 14% per year?

			Δi^* when Compared with Proposal, %		
Proposal	Initial Investment, $	Proposal i^*, %	A	B	C
A	−40,000	29			
B	−75,000	15	1		
C	−100,000	16	7	20	
D	−200,000	14	10	13	12

6.44 An engineer initiated a rate of return analysis for the infinite-life revenue proposals detailed below, but was unable to complete the evaluation.

a. Fill in the eleven blanks in the table.

b. What proposal(s) should be selected if they are independent and the MARR is 21% per year?

c. What proposal should be selected if they are mutually exclusive and the MARR is 13% per year.

			Δ*i** on Incremental Cash Flow when Compared with Proposal, %			
Proposal	**Investment, $**	**Proposal *i**, %**	**X1**	**X2**	**X3**	**X4**
X1	−20,000	?	—	?	?	?
X2	−30,000	13.33	2	—	?	?
X3	−50,000	?	14	20	—	?
X4	−75,000	12	?	?	?	—

Multiple ROR Values

6.45 For the following incremental cash flow series, what is the maximum number of *i** values according to the cash flow rule of signs?

Year	0	1	2	3	4	5	6	7	8
Cash Flow, $	−100	40	35	−15	−11	60	42	12	−10

6.46 Jenco Electric manufactures washdown adjustable speed drives in open loop, encoderless, and closed-loop servo configurations. The net cash flow associated with one phase of the production operation is shown below.

a. How many possible rate of return values are there according to the cash flow rule of signs?

b. How many changes of sign occur in the cumulative cash flow series? What does this mean?

Year	Net Cash Flow, $
0	−40,000
1	32,000
2	18,000
3	−2000
4	−1000

6.47 For the following incremental cash flow series, use a spreadsheet to find all rate of return values between 0% and 100%.

Year	Incremental Cash Flow, $
0	−50,000
1	+22,000
2	+38,000
3	−2000
4	−1000
5	+5000

6.48 According to Descartes' rule of signs, how many possible *i** values are there for net cash flows that have the following signs?: (*a*) − − − + + + − − + (*b*) − − − − − + + + + + (*c*) + + + + − − − − − − + − + − − −

6.49 The cash flow (in $1000 units) associated with a new method of manufacturing box cutters is shown below for a 2-year period. (*a*) Use Descartes' rule to determine the maximum number of possible rate of return values. (*b*) Use Norstrom's criterion to determine if there is only one positive rate of return value.

Quarter	Expenses, $1000	Revenue, $1000
0	−20	0
1	−20	5
2	−10	10
3	−10	25
4	−10	26
5	−10	20
6	−15	17
7	−12	15
8	−15	2

6.50 ARCI Instruments manufactures a ventilation controller designed for monitoring and controlling carbon monoxide in parking garages, boiler rooms, tunnels, etc. The net cash flow associated with one phase of the operation is shown on the next page. (*a*) How many possible rate of return values are there for this cash flow series? (*b*) Find all the rate of return values between 0 and 100% using tabulated factors and a spreadsheet.

Year	Net Cash Flow, $
0	−30,000
1	20,000
2	15,000
3	−2000

6.51 The cash flows associated with sales of handheld refractometers (instruments that measure the concentration of an aqueous solution by determining its refractive index) are shown. Determine the cumulative cash flow through year 4 and estimate the expected number of positive, real-number i^* values.

Year	Revenue, $	Costs, $
1	25,000	30,000
2	15,000	7,000
3	4,000	6,000
4	18,000	12,000

6.52 Boron nitride spray II (BNS II) from GE's Advanced Material Ceramics Division is a release agent and lubricant that prevents materials such as molten metal, rubber, plastics, and ceramic materials from sticking to or reacting with dies, molds, or other surfaces. A European distributor of BNS II and other GE products had the net cash flows shown. (*a*) Determine the number of possible rate of return values. (*b*) Find all rate of return values between −30% and 130%.

Year	Net Cash Flow, $
0	−17,000
1	−20,000
2	4,000
3	−11,000
4	32,000
5	47,000

6.53 Faro laser trackers are portable contact measurement systems that use laser technology to measure large parts and machinery to accuracies of 0.0002 inches across a wide range of industrial applications. A customer that manufactures and installs cell phone relay dishes and satellite receiving stations reported the cash flows (in $1000 units) for one of its product lines. (*a*) Determine the number of possible rate of return values. (*b*) Find all rate of return values between 0 and 150%.

Year	Expense, $	Receipts, $
0	−3000	0
1	−1500	2900
2	−4000	5700
3	−2000	5500
4	−1300	1100

Removing Multiple ROR Values

6.54 For the cash flow series shown, find the external rate of return using a reinvestment rate of 15% per year, using (*a*) the manual ROIC method, and (*b*) a spreadsheet to verify the answer.

Year	Incremental Cash Flow, $
0	+48,000
1	+20,000
2	−90,000
3	+50,000
4	−10,000

6.55 Carl, an engineer working for GE, invested his bonus money each year in company stock. His bonus at the end of each year 1 through 6 has been $5000. At the end of year 7, Carl received no bonus and he sold $9000 worth of stock to remodel his kitchen. In years 8 through 10, he again received a bonus and invested the $5000. Carl sold all the remaining stock for $50,000 immediately after the last investment at the end of year 10.

a. Determine the expected number of positive rate of return values.

b. Find the internal rate(s) of return.

c. Use hand solution and the MIRR spreadsheet function to determine the external rate of return using the modified rate of return approach with a borrowing rate of 8% and a reinvestment rate of 20% per year

d. Determine the external rate of return using the ROIC approach with a reinvestment rate of 20% per year. Apply both the net-investment procedure and spreadsheet functions to obtain the EROR.

6.56 A company that makes clutch disks for race cars had the cash flows shown for one department.

a. Calculate the internal rate of return.

b. Calculate the external rate of return using the ROIC method with a reinvestment rate of 15% per year.

c. Calculate the external rate of return using the MIRR method with a reinvestment rate of 15% per year and a borrowing rate of 8% per year.
d. Rework parts (*b*) and (*c*) using a spreadsheet.

Year	Cash Flow, $1000
0	−65
1	30
2	84
3	−10
4	−12

6.57 Gemini Products makes vitamin-enriched cereal products for large supermarket chains. They have forecasted the cash flows (in $1000 units) for this and the next 4 years. Develop hand and spreadsheet solutions that determine the EROR for (*a*) the ROIC method with $i_r = 14\%$ per year, and (*b*) the MIRR method with $i_r = 14\%$ and $i_b = 8\%$ per year. (*c*) Comment on these two EROR values compared to the IRR value(s) obtained using the IRR-function "guess" option to discover multiple roots.

Year	Cash Flow, $
0	3000
1	−2000
2	1000
3	−6000
4	3800

6.58 A public-private initiative in Florida will significantly expand the wind-generated energy throughout the state. The cash flow for one phase of the project involving Central Point Energy, a transmission utility company, is shown. Calculate the external rate of return (*a*) using the ROIC method and a reinvestment rate of 14% per year, and (*b*) using the modified ROR approach with a reinvestment rate of 14% and a borrowing rate of 7% per year. Solve by hand or spreadsheet, as requested by your instructor.

Year	Cash Flow, $1,000
0	5000
1	−2000
2	−1500
3	−7000
4	4000

6.59 A new advertising campaign by a company that manufactures products that rely on biometrics, surveillance, and satellite technologies resulted in the cash flows shown (in $1000 units). Develop one spreadsheet that displays the following: external rate of return using both the ROIC method with $i_r = 30\%$ per year, and the modified ROR approach with $i_r = 30\%$ and $i_b = 10\%$ per year; and the unique or multiple internal rate of return value(s) indicated by the two multiple-root sign tests.

Year	Cash Flow, $1,000
0	2000
1	1200
2	−4000
3	−3000
4	2000

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

6.60 The lowest rate of return possible is:
a. 0%
b. $-\infty$
c. −100%
d. the company's MARR

6.61 When calculating an i^* value, all net positive cash flows are assumed to be reinvested at:
a. the current market interest rate.
b. the i^* rate.
c. the company's MARR.
d. the company's cost of capital.

6.62 Alternative A has a rate of return of 14% and alternative B has a rate of return of 17%. If the investment required in B is larger than that required for A, the rate of return on the increment of investment between A and B is:
a. larger than 14%
b. larger than 17%
c. between 14% and 17%
d. smaller than 14%

6.63 A small manufacturing company borrowed $1 million and repaid the loan through monthly

payments of $20,000 for 2 years plus a single lump-sum payment of $1 million at the end of 2 years. The interest rate on the loan was closest to:
a. 0.5% per month
b. 2% per month
c. 2% per year
d. 8% per year

6.64 An investment of $60,000 resulted in uniform income of $10,000 per year for 10 years. The rate of return on the investment was closest to:
a. 10.6% per year
b. 14.2% per year
c. 16.4% per year
d. 18.6% per year

6.65 Assume you are told that by investing $100,000 now, you will receive $10,000 per year *starting in year 5* and continuing forever. If you accept the offer, the rate of return on the investment is:
a. 4% per year
b. between 6% and 7% per year
c. between 7% and 10% per year
d. over 12% per year

6.66 A chemical engineer working for a large chemical products company was asked to make a recommendation about which of three mutually exclusive revenue alternatives should be selected for improving the marketability of personal care products used for conditioning hair, cleansing skin, removing wrinkles, etc. The alternatives (X, Y, and Z) were ranked in order of increasing initial investment and then compared by incremental rate of return analysis. The rate of return on each increment of investment was less than the company's MARR of 17% per year. The alternative to select is:
a. DN
b. alternative X
c. alternative Y
d. alternative Z

Problems 6.67 and 6.68 are based on the following data.

The five alternatives are being evaluated by the rate of return method.

Proposal	Initial Investment, $	Overall i^*, %	Δi^*, %, when Compared with Proposal A	B	C	D	E
A	−25,000	9.6	—	27.3	19.4	35.3	25.0
B	−35,000	15.1		—	0	38.5	24.4
C	−40,000	13.4			—	46.5	27.3
D	−60,000	25.4				—	26.8
E	−75,000	20.2					—

6.67 If the alternatives are independent and the MARR is 15% per year, the one(s) that should be selected is (are):
a. only D
b. only D and E
c. only B, D, and E
d. only E

6.68 If the alternatives are mutually exclusive and the MARR is 15% per year, the alternative to select is:
a. either B, C, D or E
b. only B
c. only D
d. only E

6.69 Jewel-Osco evaluated three different pay-by-touch systems that identify customers by a finger scan and then deduct the amount of the bill directly from their checking accounts. The alternatives were ranked according to increasing initial investment and identified as alternatives A, B, and C.

Comparison	Δi^*, %
DN to A	23.4
DN to B	8.1
DN to C	16.6
B to A	−5.1
C to A	12.0
C to B	83.9

Based on the incremental rates of return and the company's MARR of 16% per year, the alternative that should be selected is:

a. alternative A
b. alternative B
c. alternative C
d. alternative DN

6.70 For the cash flows shown, the correct equation for FW_2 using the ROIC method at the reinvestment rate of 20% per year is:

a. $[10{,}000(1+ i'') + 6000](1.20) - 8000$
b. $[10{,}000(1.20) + 6000(1+i'')](1.20) - 8000$
c. $[10{,}000(1.20) + 6000](1.20) - 8000$
d. $[10{,}000(1.20) + 6000](1+ i'') - 8000$

Year	Cash Flow, $
0	10,000
1	6,000
2	−8,000
3	−19,000