HM321 Engineering Economics
Fall 2024 – Lecture 7

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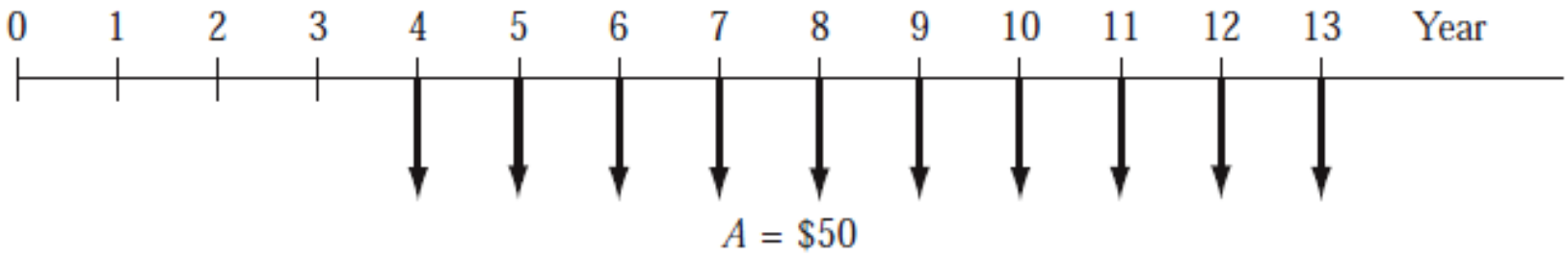
Bring Calculator Always

- Always bring your calculator with you in lectures
- Without practice you will not be able to do the calculations in your exams

CALCULATIONS FOR CASH FLOWS THAT ARE SHIFTED

Shifted Uniform Series

- The first payment of a uniform series occurs at a time other than 1
- In the following cash flow diagram the first payment of a uniform series occurs at the end of year 4



Present Worth of Shifted Uniform Series

- There are several ways in which present worth at year 0 can be calculated for a shifted uniform series:
  - By discounting each payment in the series individually using P/F factor to time 0 and then adding the discounted amounts
  - By compounding each payment in the series individually to the time of last payment and adding them. Present worth at time 0 is then calculated using P/F factor

Present Worth of Shifted Uniform Series - 2

- By calculating future worth of the series using F/A factor and then discounting it to time 0 using P/F factor
- By calculating "present worth" at one interest period before the first payment in the series using P/A factor and then discounting this amount by P/F factor (note quote marks)
  - The present worth within quotes does not occur at time 0

- Typically the last method is used most often

Remember

- When P/A factor is used, the present worth of a uniform series occurs at one period before the first payment in the series
- When F/A factor is used, the future worth of a uniform series occurs at the time of last payment in the series

Mixed Uniform Series and Randomly Placed Single Amounts

- For uniform series the present worth at time 0 is calculated by using one of the methods described in the previous slides
- For single amounts the present worth at time 0 is calculated by using P/F factor for each single amount
- Total present worth at time 0 is sum of all the present worth values calculated

Shifted Arithmetic Gradient Series

- When arithmetic gradient series starts at time other than end of year 1 then the arithmetic gradient series is shifted
- Present worth at time 0 of a shifted arithmetic gradient series can be calculated using similar approach as in case of shifted uniform series

Shifted Geometric Gradient Series

- This is also handled in a similar manner as in case of shifted uniform series

Annual Equivalent of Mixed Shifted Cash Flows

- The given cash flows may be a combination of single amounts, shifted uniform series, shifted arithmetic and/or geometric gradient series
- To calculate annual equivalent of such cash flows first calculate present worth at time 0 and then annualize using A/P factor

Reference

- Basics of Engineering Economy by Leland Blank and Anthony Tarquin, 2nd edition, McGraw-Hill