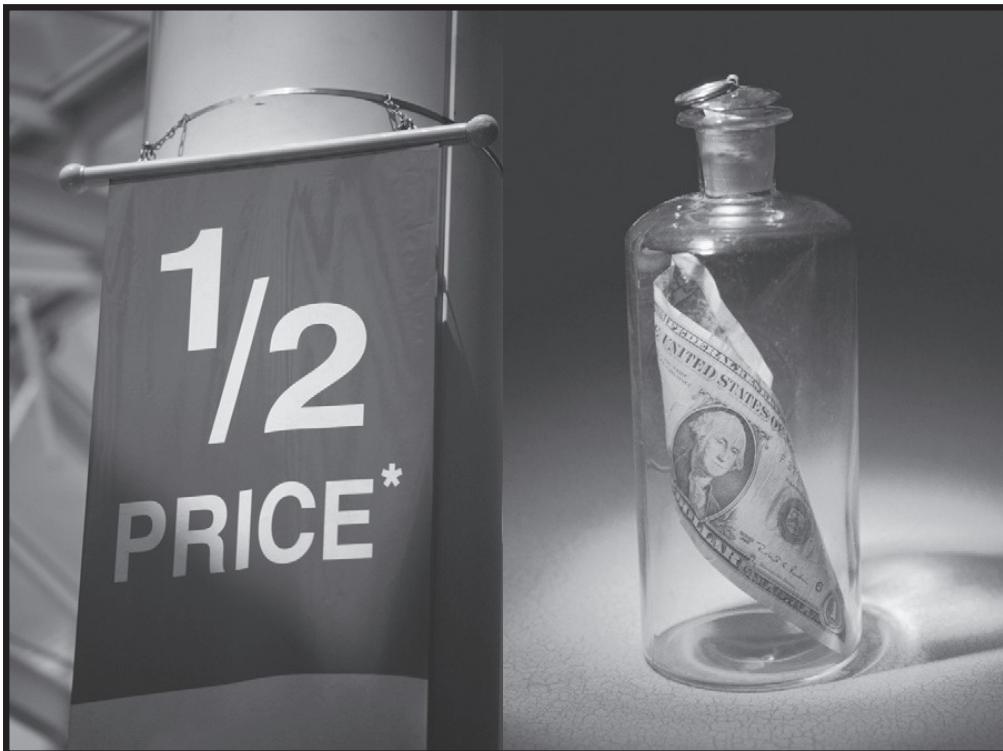


Annual Worth Analysis



Imagestate Media (John Foxx); Ingram Publishing

An AW analysis is commonly preferred over a PW analysis because the AW value is easy to calculate; the measure of worth—AW in monetary units (dollars) per year—is understood by most individuals; and its assumptions are essentially the same as those of the PW method.

Annual worth is known by other titles. Some are equivalent annual worth (EAW), equivalent annual cost (EAC), annual equivalent (AE), and EUAC (equivalent uniform annual cost). The alternative selected by the AW method will always be the same as that selected by the PW method, and all other alternative evaluation methods, provided they are performed correctly.

Purpose: Compare alternatives using the annual worth method.

LEARNING OUTCOMES

1. Calculate capital recovery and AW over one life cycle.

AW calculation

2. Select the best alternative on the basis of an AW analysis.

Alternative selection by AW

3. Select the best long-life (infinite-life) investment alternative using AW values.

Long-life investment AW

4. Use a spreadsheet to perform an AW evaluation.

Spreadsheets

5.1 AW VALUE CALCULATIONS

The annual worth (AW) method is commonly used for comparing alternatives. All cash flows are converted to an equivalent uniform annual amount over one life cycle of the alternative. The AW value is easily understood by all since it is stated in terms of dollars per year. The major advantage over all other methods is that the equal service requirement is met without using the least common multiple (LCM) of alternative lives. The AW value is calculated over one life cycle and is assumed to be exactly the same for any succeeding cycles, provided all cash flows change with the rate of inflation or deflation. If this cannot be reasonably assumed, a study period and specific cash flow estimates are needed for the analysis. The repeatability of the AW value over multiple cycles is demonstrated in Example 5.1.



New digital scanning graphics equipment is expected to cost \$20,000, to be used for 3 years, and to have an annual operating cost (AOC) of \$8000. Determine the AW values for one and two life cycles at $i = 22\%$ per year.

EXAMPLE 5.1

Solution

First use the cash flows for one life cycle (Figure 5.1) to determine AW.

$$AW = -20,000(A/P, 22\%, 3) - 8000 = \$-17,793$$

For two life cycles, calculate AW over 6 years. Note that the purchase for the second cycle occurs at the end of year 3, which is year zero for the second life cycle (Figure 5.1).

$$\begin{aligned} AW &= -20,000(A/P, 22\%, 6) - 20,000(P/F, 22\%, 3)(A/P, 22\%, 6) - 8000 \\ &= \$-17,793 \end{aligned}$$

The same AW value can be obtained for any number of life cycles, thus demonstrating that the AW value for one cycle represents the equivalent annual worth of the alternative for every cycle.

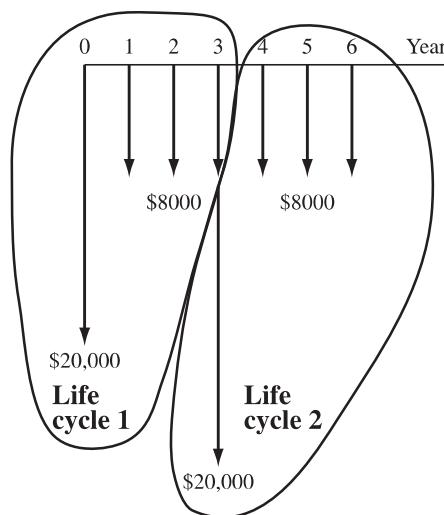


FIGURE 5.1
Cash flows over
two life cycles of
an alternative.

It is always possible to determine the AW, PW, and FW values from each other using the following relation.

$$AW = PW(A/P,i,n) = FW(A/F,i,n) \quad [5.1]$$

The equal-service requirement necessary for a PW comparison means that the n value in this equation is the LCM of lives.

The AW value of an alternative is the addition of two distinct components: *capital recovery (CR)* of the initial investment and the *equivalent A value* of the annual operating costs (AOC).

$$AW = CR + A \text{ of AOC} \quad [5.2]$$

The recovery of an amount of capital P committed to an asset, plus the time value of the capital at a particular interest rate, is a fundamental principle of economic analysis. *Capital recovery is the equivalent annual cost of owning the asset plus the return on the initial investment.* The A/P factor is used to convert P to an equivalent annual cost. If there is some anticipated positive salvage value S at the end of the asset's useful life, its equivalent annual value is removed using the A/F factor. This action reduces the equivalent annual cost of owning the asset. Accordingly, CR is

$$CR = -P(A/P,i,n) + S(A/F,i,n) \quad [5.3]$$

The annual amount (A of AOC) is determined from uniform recurring costs (and possibly receipts) and nonrecurring amounts. The P/A and P/F factors may be necessary to first obtain a present worth amount, then the A/P factor converts this amount to the A value in Equation [5.2].

EXAMPLE 5.2

Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of \$13 million. Annual operating costs for the system are expected to start the first year and continue at \$0.9 million per year. The useful life of the tracker is 8 years with a salvage value of \$0.5 million. Calculate the AW value for the system if the corporate MARR is currently 12% per year.

Solution

The cash flows (Figure 5.2a) for the tracker system must be converted to an equivalent AW cash flow sequence over 8 years (Figure 5.2b). (All amounts are expressed in \$1 million units.) The AOC is $A = -0.9$ per year, and the capital recovery is calculated by using Equation [5.3].

$$\begin{aligned} CR &= -13(A/P,12\%,8) + 0.5(A/F,12\%,8) \\ &= -13(0.2013) + 0.5(0.0813) \\ &= \$-2.576 \end{aligned}$$

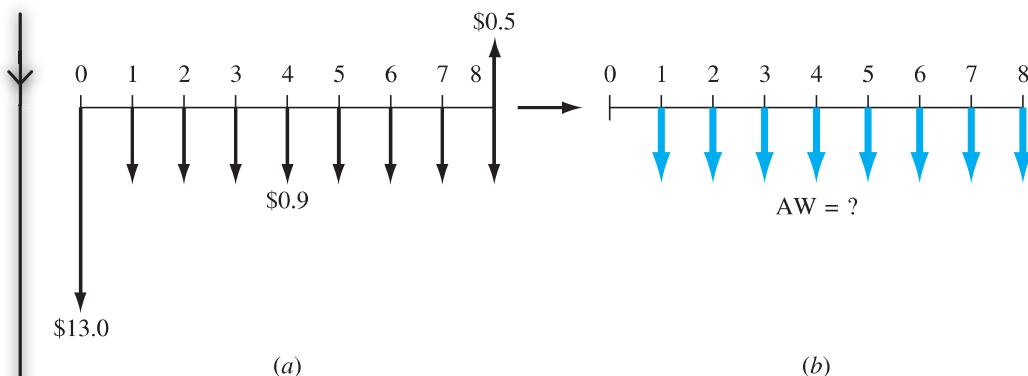


FIGURE 5.2 (a) Cash flow diagram for satellite tracker costs, and (b) conversion to an equivalent AW (in \$1 million), Example 5.2.

The correct interpretation of this result is very important to Lockheed Martin. It means that each and every year for 8 years, the equivalent total revenue from the tracker must be at least \$2,576,000 *just to recover the initial present worth investment plus the required return of 12% per year*. This does not include the AOC of \$0.9 million each year. Total AW is found by Equation [5.2].

$$AW = -2.576 - 0.9 = \$-3.476 \text{ million per year}$$

This is the AW for all future life cycles of 8 years, provided the costs rise at the same rate as inflation, and the same costs and services apply for each succeeding life cycle.

For solution by computer, use the PMT function to determine CR only in a single spreadsheet cell. The format is $= \text{PMT}(i\%, n, P, -S)$. As an illustration, the CR in Example 5.2 is displayed when $= \text{PMT}(12\%, 8, 13, -0.5)$ is entered.

The annual worth method is applicable in any situation where PW, FW, or Benefit/Cost analysis can be utilized. The AW method is especially useful in certain types of studies: asset replacement and retention studies to minimize overall annual costs, breakeven studies and make-or-buy decisions (all covered in later chapters), and all studies dealing with production or manufacturing where cost/unit is the focus.

5.2 EVALUATING ALTERNATIVES BASED ON ANNUAL WORTH

The annual worth method is typically the easiest of the evaluation techniques to perform, when the MARR is specified. The alternative selected has the lowest equivalent annual cost (cost alternatives), or highest equivalent income (revenue alternatives). The selection guidelines for the AW method are the same as for the PW method.



One alternative: $AW \geq 0$, the alternative is financially viable.

Two or more alternatives: Choose the numerically largest AW value (lowest cost or highest income).

If a study period is used to compare two or more alternatives, the AW values are calculated using cash flow estimates over only the study period. For a study period shorter than the alternative's expected life, use an estimated market value for the salvage value.

EXAMPLE 5.3

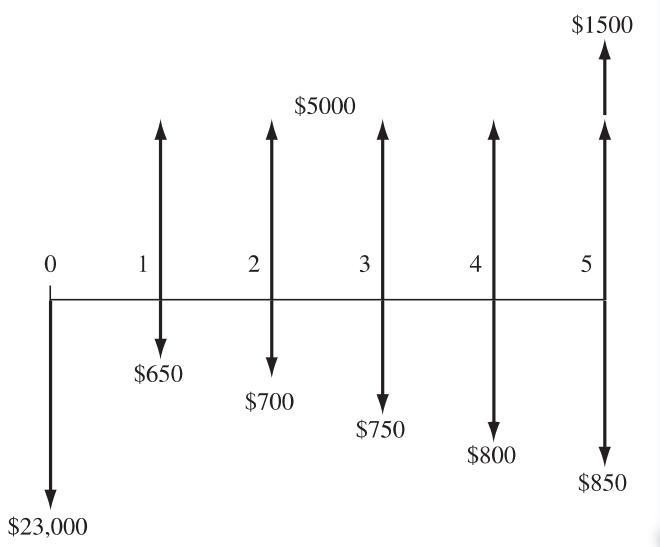
PizzaRush, which is located in the general Los Angeles area, fares very well with its competition in offering fast delivery. Many students at the area universities and community colleges work part-time delivering orders made via the web at PizzaRush.com. The owner, a software engineering graduate of USC, plans to purchase and install five portable, in-car systems to increase delivery speed and accuracy. The systems provide a link between the web order-placement software and the in-car GPS system for satellite-generated directions to any address in the Los Angeles area. The expected result is faster, friendlier service to customers, and more income for PizzaRush.

Each system costs \$4600, has a 5-year useful life, and may be salvaged for an estimated \$300. Total operating cost for all systems is \$650 for the first year, increasing by \$50 per year thereafter. The MARR is 10% per year. Perform an annual worth evaluation that answers the following questions:

- How much new annual revenue is necessary to recover only the initial investment at an MARR of 10% per year?
- The owner conservatively estimates increased income of \$5000 per year for all five systems. Is this project financially viable at the MARR? See cash flow diagram in Figure 5.3.
- Based on the answer in part (b), determine how much new income PizzaRush must have to economically justify the project. Operating costs remain as estimated.

FIGURE 5.3

Cash flow diagram used to compute AW, Example 5.3.



Solution

- a. The CR value will answer this question. Use Equation [5.3] at 10%.

$$\begin{aligned} \text{CR} &= -5(4600)(A/P, 10\%, 5) + 5(300)(A/F, 10\%, 5) \\ &= \$-5822 \end{aligned}$$

- b. The financial viability could be determined now without calculating the AW value, because the \$5000 in new income is lower than the CR of \$5822, which does not yet include the annual costs. So, the project is not economically justified. However, to complete the analysis, determine the total AW. The annual operating costs and incomes form an arithmetic gradient series with a base of \$4350 in year 1, decreasing by \$50 per year for 5 years. The AW relation is

$$\begin{aligned} \text{AW} &= \text{capital recovery} + A \text{ of net income} \\ &= -5822 + 4350 - 50(A/G, 10\%, 5) \\ &= \$-1562 \end{aligned} \tag{5.4}$$

This shows conclusively that the alternative is not financially viable at MARR = 10%.

- c. An equivalent of the projected \$5000 plus the AW amount are necessary to make the project economically justified at a 10% return. This is $5000 + 1562 = \$6562$ per year in new revenue. At this point AW will equal zero based on Equation [5.4].

A quarry outside of Austin, Texas, wishes to evaluate two similar pieces of equipment by which the company can meet new state environmental requirements for dust emissions. The MARR is 12% per year. Determine which alternative is economically better using (a) the AW method, and (b) AW method with a 3-year study period.

EXAMPLE 5.4

Equipment	X	Y
First cost, \$	-40,000	-75,000
AOC, \$ per year	-25,000	-15,000
Life, years	4	6
Salvage value, \$	10,000	7,000
Estimated value after 3 years, \$	14,000	20,000

Solution

- a. Calculating AW values over the respective lives indicates that Y is the better alternative.

$$\begin{aligned} AW_X &= -40,000(A/P, 12\%, 4) - 25,000 + 10,000(A/F, 12\%, 4) \\ &= \$-36,077 \end{aligned}$$

$$\begin{aligned} AW_Y &= -75,000(A/P, 12\%, 6) - 15,000 + 7,000(A/F, 12\%, 6) \\ &= \$-32,380 \end{aligned}$$

- b. All n values are 3 years and the “salvage values” become the estimated market values after 3 years. Now X is economically better.

$$\begin{aligned} AW_X &= -40,000(A/P, 12\%, 3) - 25,000 + 14,000(A/F, 12\%, 3) \\ &= \$-37,505 \end{aligned}$$

$$\begin{aligned} AW_Y &= -75,000(A/P, 12\%, 3) - 15,000 + 20,000(A/F, 12\%, 3) \\ &= \$-40,299 \end{aligned}$$

5.3 AW OF A LONG-LIFE OR INFINITE-LIFE INVESTMENT

The annual worth equivalent of a very long-lived project is the AW value of its capitalized cost (CC), discussed in Section 4.4. The AW value of the first cost, P , or present worth, PW, of the alternative uses the same relation as Equation [4.2].

$$AW = CC(i) = PW(i) \quad [5.5]$$

Cash flows that occur at regular intervals are converted to AW values over one life cycle of their occurrence. All other nonregular cash flows are first converted to a P value and then multiplied by i to obtain the AW value over infinity.

EXAMPLE 5.5

If you receive an inheritance of \$10,000 today, how long do you have to invest it at 8% per year to be able to withdraw \$2000 every year forever? Assume the 8% per year is a return that you can depend on forever.

Solution

Cash flow is detailed in Figure 5.4. Solving Equation [5.5] for PW indicates that it is necessary to have \$25,000 accumulated at the time that the \$2000 annual withdrawals start.

$$PW = 2000/0.08 = \$25,000$$

Find $n = 11.91$ years using the relation $\$25,000 = 10,000(F/P, 8\%, n)$.

Comment: It is easy to use a spreadsheet to solve this problem. In any cell write the function =NPER(8%, -10000, 25000) to display the answer of 11.91 years. The financial calculator function $n(8,0,-10000,25000)$ displays the same n value.

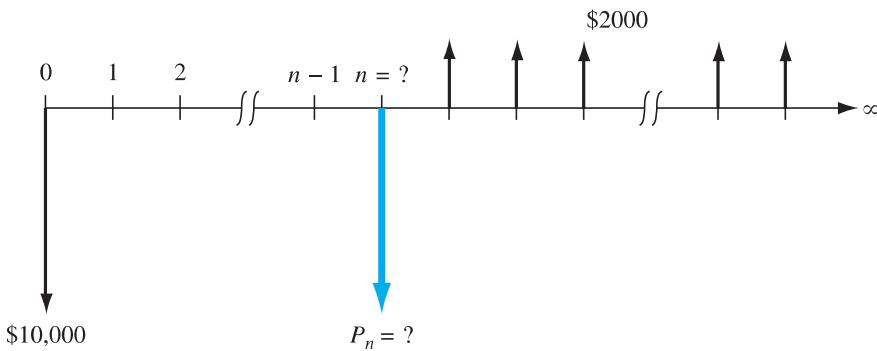


FIGURE 5.4 Diagram to determine n for a perpetual withdrawal, Example 5.5.

The state is considering three proposals for increasing the capacity of the main drainage canal in an agricultural region. Proposal A requires dredging the canal. The state is planning to purchase the dredging equipment and accessories for \$650,000. The equipment is expected to have a 10-year life with a \$17,000 salvage value. The annual operating costs are estimated to total \$50,000. To control weeds in the canal itself and along the banks, environmentally safe herbicides will be sprayed during the irrigation season. The yearly cost of the weed control program is expected to be \$120,000.

Proposal B is to line the canal walls with concrete at an initial cost of \$4 million. The lining is assumed to be permanent, but minor maintenance will be required every year at a cost of \$5000. In addition, lining repairs will have to be made every 5 years at a cost of \$30,000.

Proposal C is to construct a new pipeline along a different route. Estimates are: an initial cost of \$6 million, annual maintenance of \$3000 for right-of-way, and a life of 50 years.

Compare the alternatives on the basis of annual worth, using an interest rate of 5% per year.

EXAMPLE 5.6

Solution

Since this is an investment for a long-life project, compute the AW for one cycle of all recurring costs. For proposals A and C, the CR values are found using Equation [5.3], with $n_A = 10$ and $n_C = 50$, respectively. For proposal B, the CR is simply $P(i)$.

Proposal A

CR of dredging equipment:	
$-650,000(A/P,5\%,10) + 17,000(A/F,5\%,10)$	\$ -82,824
Annual cost of dredging	-50,000
Annual cost of weed control	-120,000
	<hr/>
	\$-252,824

Proposal B

CR of initial investment: $-4,000,000(0.05)$	\$-200,000
Annual maintenance cost	-5,000
Lining repair cost: $-30,000(A/F,5\%,5)$	-5,429
	<hr/>
	\$-210,429

Proposal C

CR of pipeline: $-6,000,000(A/P,5\%,50)$	\$-328,680
Annual maintenance cost	-3,000
	<hr/>
	\$-331,680

Proposal B is selected.

Comment: The A/F factor is used instead of A/P in B because the lining repair cost begins in year 5, not year 0, and continues indefinitely at 5-year intervals.

If the 50-year life of proposal C is considered infinite, $CR = P(i) = \$-300,000$, instead of $\$-328,680$ for $n = 50$. This is a small economic difference. How expected lives of 40 or more years are treated economically is a matter of local practice.

5.4 USING SPREADSHEETS FOR AW ANALYSIS

Annual worth evaluation of equal or unequal life, mutually exclusive alternatives is simplified using the PMT function. The general format for determining an alternative's AW is

$$= \text{PMT}(i\%, n, P, F) - A$$

PMT determines the required capital recovery (CR). Usually F is the estimated salvage value entered as $-S$, and $-A$ is the annual operating cost (AOC). Since AW analysis does not require evaluation over the least common multiple of lives, the n values can be different for each alternative.

As with the PV function, the + and - signs on PMT values must be correctly entered to ensure the appropriate sense on the result. To obtain a negative AW value for a cost alternative, enter $+P$, $-S$, and $-A$. The next two examples illustrate

spreadsheet-based AW evaluations for unequal life alternatives, including a long-life investment discussed in Section 5.3.

Herme, the quarry supervisor for Espinosa Stone, wants to select the more economical of two alternatives for quarry dust control. Help him perform a spreadsheet-based AW evaluation for the estimates with MARR = 12% per year.

EXAMPLE 5.7

Equipment	X	Y
First cost, \$	-40,000	-75,000
AOC, \$ per year	-25,000	-15,000
Life, years	4	6
Salvage value, \$	10,000	7,000

Solution

Because these are the same estimates as Example 5.4a, you can see what is required to hand-calculate AW values by referring back. Spreadsheet evaluation is much faster and easier; it is possible to enter the two single-cell PMT functions for X and Y using the general format presented earlier. Figure 5.5 includes details. The decision is to select Y with the lower-cost AW value.

A	B	C
1	Equipment X	Equipment Y
2		
3 AW value	-\$36,077	-\$32,379
4		
5 Function	= - PMT(12%,4,-40000,10000)-25000	= - PMT(12%,6,-75000,7000)-15000
6		

FIGURE 5.5 Evaluation by AW method using PMT functions, Example 5.7.

Perform a spreadsheet-based AW evaluation of the three proposals in Example 5.6. Note that the lives vary from 10 years for A, to 50 for C, to “infinite” for B.

EXAMPLE 5.8

Solution

Figure 5.6 summarizes the estimates on the left. Obtaining AW values is easy using PMT and a few computations shown on the right. The PMT function

determines capital recovery of first cost at 5% per year for A and C. For B, capital recovery is simply P times i , according to Equation [5.5], because proposal B (line walls with cement) is considered permanent. (Remember to enter $-4,000,000$ for P to ensure a minus sign on the CR amount.)

All annual costs are uniform series except proposal B's periodic lining repair cost of \$30,000 every 5 years, which is annualized (in cell G7) using a PMT function. Adding all costs (row 8) indicates that proposal B has the lowest AW of costs.

	A	B	C	D	E	F	G	H	I	J	K
1	Recap of Estimates			Calculation of AW values							
2	Proposal	A	B	C	Proposal	A	B	C			
3	First cost, \$	-650,000	-4,000,000	-6,000,000	Capital recovery, CR	-82,826	-200,000	-328,660	= -PMT(5%,50,-6000000)		
4	Life, years	10	Infinite	50					= -4000000*(0.05)		
5	Annual costs, \$	-170,000	-5000	-3000					= -PMT(5%,10,-650000,17000)		
6	Periodic cost, \$	None	-30,000 every 5 years	None	Annual costs, A	-170,000	-5,000	-3,000			
7	Salvage, \$	17,000	0	0					= -PMT(5%,5,-30000)		
8					AW value, CR + A	-252,826	-210,429	-331,660			
9											

FIGURE 5.6 Use of PMT functions to perform an AW evaluation of three proposals with different lives, Example 5.8.

When estimated annual costs are not a uniform series A , the cash flows must be entered individually (using the negative sign for cash outflows) and the use of single-cell functions is somewhat limited. In these cases, first use the NPV function to find the PW value over one life cycle, followed by the PMT function to determine the AW value. Alternatively, though it is more involved, embed the NPV function directly into PMT using the general format

$$= \text{PMT}(i\%, n, P + \text{NPV}(i\%, \text{year_1_cell:year_n_cell}))$$

SUMMARY

The annual worth method of comparing alternatives is preferred to the present worth method, because the AW comparison is performed for only one life cycle. This is a distinct advantage when comparing different-life alternatives. When a study period is specified, the AW calculation is determined for that time period only, and

the estimated value remaining in the alternative at the end of the study period becomes the salvage value.

For infinite-life alternatives, the initial cost is annualized by multiplying P by i . For finite-life alternatives, the AW through one life cycle is equal to the perpetual equivalent annual worth.

PROBLEMS

Capital Recovery and AW

- 5.1** Heyden Motion Solutions ordered \$7 million worth of seamless tubes for manufacturing their high performance and precision linear motion products. If their annual operating costs are \$860,000 per year, how much annual revenue is required over a 3-year planning period to recover the initial investment and operating costs at the company's MARR of 15% per year?
- 5.2** NRG Energy plans to construct a giant solar plant in Santa Teresa, NM to supply electricity to 30,000 southern NM and western TX homes. The plant will have 390,000 heliostats to concentrate sunlight onto 32 water towers to generate steam. NRG will spend \$560 million in constructing the plant and \$430,000 per year in operating it. If a salvage value of 20% of the initial cost is assumed, how much will the company have to make each year for 15 years in order to recover its investment at a MARR of 18% per year?
- 5.3** Environmental recovery company RexChem Partners plans to finance a site reclamation project that will require a 4-year cleanup period. The company will borrow \$3.8 million now to finance the project. How much will the company have to receive in annual payments for 4 years, provided it will also receive a final lump sum payment after 4 years in the amount of \$500,000? The MARR is 20% per year on this investment.
- 5.4** U.S. Steel is planning a plant expansion to produce austenitic, precipitation-hardened, duplex and martensitic stainless steel round bar (as well as various nickels) that is expected to cost \$13 million now and another \$10 million 1 year from now. If total operating costs will be \$1.2 million per year starting 1 year from now, how much must the company realize in annual revenue in years 1 through 10 to recover its investment plus 15% per year?
- 5.5** A small metal plating company wants to become involved in electronic commerce. A modest e-commerce package is available for \$20,000. Semiannual updates and site maintenance will cost \$300. The salvage value of the package is estimated to be \$1500 after 3 years. If the company wants to recover its total cost in 3 years, what is the equivalent semiannual amount of new income that must be realized at an interest rate of 5% per 6-month period?
- 5.6** Toro Company is expanding its U.S.-based plastic molding plant as it continues to transfer work from Juarez, Mexico contractors. The plant bought a \$1.1 million precision injection molding machine to make plastic parts for Toro lawn mowers, trimmers, and snow blowers. The plant also spent \$275,000 for three smaller plastic injection molding machines to make plastic parts for a new line of sprinkler systems. The plant expects to hire 13 people, including some engineers for the expansion. If the average loaded cost (i.e., including benefits) of each employee is \$100,000 per year, determine the annual worth of the new systems over a five-year planning period at an interest rate of 10% per year. Assume a 25% salvage value for the new equipment.
- 5.7** In an effort to retain troops who are proficient with weapons and who can speak the languages of Middle Eastern countries, the Pentagon offered bonuses of \$150,000 to specialized personnel who were near or already eligible for retirement. If 400 enlisted personnel accepted the bonus in year one, 300 in year two, and 600 in year three, what was the equivalent annual cost of the program over the 3-year period at an interest rate of 6% per year?
- 5.8** A company that manufactures magnetic flow meters expects to undertake a project that will have the cash flows below. At an interest rate of 10% per year, what is the equivalent annual cost of the project? Find the AW value using (a) tabulated factors, (b) calculator functions, and (c) a spreadsheet. Which method did you find the easiest to use?
- | | |
|---|----------|
| First cost, \$ | -800,000 |
| Equipment replacement cost
in year 2, \$ | -300,000 |
| Annual operating cost, \$/year | -950,000 |
| Salvage value, \$ | 250,000 |
| Life, years | 4 |
- 5.9** A small commercial building contractor purchased a used crane 2 years ago for \$60,000. Its operating cost was \$2500 in month one, \$2550 in month two, and amounts increasing by \$50 per

month through the end of year two (now). If the crane was sold for \$48,000 now, what was its equivalent monthly cost at an interest rate of 1% per month?

- 5.10** A 600-ton press used to produce composite-material fuel cell components for automobiles using proton exchange membrane (PEM) technology can reduce the weight of enclosure parts up to 75%. At MARR = 12% per year, calculate (a) capital recovery and (b) annual revenue required. (c) Solve using a spreadsheet.

Installed cost = \$−3.8 million $n = 12$ years

Salvage value = \$250,000

Annual operating costs = \$−350,000 in year 1, increasing by \$25,000 per year

Evaluating Alternatives Using AW

- 5.11** Two machines with the following cost estimates are under consideration for a dishwasher assembly process. Using an interest rate of 10% per year, determine which alternative should be selected on the basis of an annual worth analysis.

	Machine X	Machine Y
First cost, \$	−300,000	−430,000
Annual operating cost, \$/year	−60,000	−40,000
Salvage value, \$	70,000	95,000
Life, years	4	6

- 5.12** A public water utility is replacing its service trucks with more fuel-efficient vehicles. Two types of trucks are under consideration: Type 1 uses “start-stop” technology that turns the engine off when the vehicle comes to a halt in traffic or at a stop light. These trucks will get 22 miles per gallon of gasoline and will cost \$27,000 to buy. At the end of the truck’s 5-year service life, it is expected to sell for \$10,000. Type 2 trucks use a system called Variable Cylinder Management wherein the engine operates on fewer cylinders when the vehicle is cruising under light load conditions. These trucks will cost \$29,500 and will get 25 miles per gallon. Their salvage value is expected to be \$12,000 after 5 years. The trucks are driven an average of 21,000 miles per year and the wholesale

price of gasoline is \$4.00 per gallon. Which type of truck should the utility buy on the basis of an annual worth comparison at an interest rate of 6% per year?

- 5.13** An engineer is considering two different liners for an evaporation pond that will receive salty concentrate from a brackish water desalting plant. A plastic liner will cost \$0.90 per square foot and will have to be replaced in 20 years when precipitated solids have to be removed from the pond using heavy equipment. A rubberized elastomeric liner is tougher and, therefore, is expected to last 30 years, but it will cost \$2.20 per square foot. The pond covers 110 acres (1 acre = 43,560 square feet). Which liner is more cost effective on the basis of an annual worth analysis at an interest rate of 8% per year? Solve using (a) tabulated factors, and (b) a calculator.

- 5.14** One of two methods will produce solar panels for electric power generation. Method 1 will have an initial cost of \$550,000, an annual operating cost of \$160,000 per year, and a \$125,000 salvage value after its three-year life. Method 2 will cost \$830,000 with an annual operating cost of \$120,000, and a \$240,000 salvage value after its five-year life. The company has asked you to determine which method is economically better, but it wants the analysis done over a three-year planning period. The salvage value of Method 2 will be 35% higher after 3 years than it is after 5 years. If the company’s MARR is 10% per year, which method should the company select?

- 5.15** An environmental engineer is considering three methods for disposing of a non-hazardous chemical sludge: land application, fluidized-bed incineration, and private disposal contract. The estimates for each method are below. (a) Determine which has the least cost on the basis of an annual worth comparison at 10% per year. (b) Determine the equivalent present worth value of each alternative using its AW value.

	Land Application	Incineration	Contract
First cost, \$	−150,000	−900,000	0
Annual cost, \$/year	−95,000	−60,000	−170,000
Salvage value, \$	25,000	300,000	0
Life, years	4	6	2

- 5.16** BP Oil is in the process of replacing sections of its Prudhoe Bay, Alaska oil transit pipeline. This will reduce corrosion problems, while allowing higher line pressures and flow rates to downstream processing facilities. The installed cost is expected to be about \$170 million. Alaska imposes a 22.5% tax on annual profits (net revenue over costs), which are estimated to average \$85 million per year for a 20-year period. Use tabulated factors and a spreadsheet to answer the following: (a) At a corporate MARR of 10% per year, does the project AW indicate it will make at least the MARR? (b) Recalculate the AW at MARR values increasing by 10% per year, that is, 20%, 30%, etc. At what required return does the project become financially unacceptable?

- 5.17** Equipment needed at a Valero Corporation refinery for the conversion of corn stock to ethanol, a cleaner burning gasoline additive, will cost \$175,000 and have net cash flows of \$35,000 the first year, increasing by \$10,000 per year over the life of 5 years. (a) Use a spreadsheet (and tabulated factors, if instructed to do so) to calculate the AW amounts at different MARR values to determine when the project switches from financially justified to unjustified. (b) Develop a spreadsheet chart that plots AW versus interest rate.
- 5.18** The TT Racing and Performance Motor Corporation wishes to evaluate two alternative machines for NASCAR motor tune-ups. (a) Use the AW method at 9% per year to select the better alternative. (b) Use spreadsheet single-cell functions to find the better alternative.

	Machine R	Machine S
First cost, \$	−250,000	−370,500
Annual operating cost, \$ per year	−40,000	−50,000
Life, years	3	5
Salvage value, \$	20,000	20,000

- 5.19** Estimates have been presented to Holly Farms, which is considering two environmental chambers for a project that will detail laboratory confirmations of on-line bacteria tests in chicken meat for the presence of *E. coli* 0157:H7 and *Listeriamonocytogenes*. (a) If the project will last for 6 years and $i = 10\%$ per year, perform an AW evaluation

to determine which chamber is more economical. (b) Chamber D103 can be purchased with different options and, therefore, at different installed costs. They range from \$300,000 to \$500,000. Will the selection change if one of these other models is installed? (c) Use single-cell spreadsheet functions to solve part (b).

	Chamber D103	Chamber 490G
Installed cost, \$	−400,000	−250,000
Annual operating cost, \$ per year	−4000	−3000
Salvage value at 10% of P , \$	40,000	25,000
Life, years	3	2

- 5.20** Blue Whale Moving and Storage recently purchased a warehouse building in Santiago. The manager has two good options for moving pallets of stored goods in and around the facility. Alternative 1 includes a 4000-pound capacity, electric forklift ($P = \$-30,000$; $n = 12$ years; AOC = \$−1000 per year; $S = \$8000$) and 500 new pallets at \$10 each. The forklift operator's annual salary and indirect benefits are estimated at \$32,000.

Alternative 2 uses two electric pallet movers ("walkies") each with a 3000-pound capacity (for each mover, $P = \$-2000$; $n = 4$ years; AOC = \$−150 per year; no salvage) and 800 pallets at \$10 each. The two operators' salaries and benefits will total \$55,000 per year. For both options, new pallets are purchased now and every two years that the equipment is in use. (a) If the MARR is 8% per year, use tabulated factors to determine which alternative is better. (b) Rework using a spreadsheet solution.

Evaluating Long-Life Alternatives

- 5.21** Calculate the equivalent annual cost for years 1 through infinity of \$1,000,000 now and \$1,000,000 three years from now at an interest rate of 10% per year.
- 5.22** Calculate the infinite-life equivalent annual cost of \$5,000,000 in year 0, \$2,000,000 in year 10, and \$100,000 in years 11 through infinity. The interest rate is 10% per year.

- 5.23** Compare the alternatives below using the annual worth method at an interest rate of 10% per year. Use (a) tabulated factors, and (b) calculator functions.

	A	B
First cost, \$	-60,000	-380,000
Annual cost, \$/year	-30,000	-5000
Salvage value, \$	10,000	25,000
Life, years	3	∞

- 5.24** For the cash flows below, use an annual worth comparison to determine which alternative is best at an interest rate of 1% per month.

	X	Y	Z
First cost, \$	-90,000	-400,000	-900,000
M&O costs, \$/month	-30,000	-20,000	-13,000
Overhaul every 10 years, \$	—	—	-80,000
Salvage value, \$	7000	25,000	200,000
Life, years	3	10	∞

- 5.25** Cheryl and Gunther wish to place into a retirement fund an equal amount each year for 20 consecutive years to accumulate just enough to withdraw \$24,000 per year starting exactly one year after the last deposit is made. The fund has a reliable return of 8% per year. Determine the annual deposit for two withdrawal plans: (a) forever (years 21 to infinity); (b) 30 years (years 21 through 50). (c) How much less per year is needed when the withdrawal horizon decreases from infinity to 30 years?

- 5.26** Baker|Trimline owned a specialized tools company for a total of 12 years when it was sold for \$38 million cash. During the ownership, annual net cash flow varied significantly as follows:

Year	1	2	3	4	5	6	7	8	9	10	11	12
Net Cash Flow, \$ million per year	4	0	-1	-3	-3	1	4	6	8	10	12	12

The company made 12% per year on its positive cash flows and paid 10% per year on short-term loans to cover the lean years. The president wants to use the cash accumulated after 12 years to improve capital investments starting in year 13 and forward. If an 8% per year return is expected after

the sale, what annual amount can Baker|Trimline invest forever?

- 5.27** A major repair on the suspension system of Jane's 3-year old car cost her \$2000 because the warranty expired after 2 years of ownership. Based on this experience, she will plan on additional \$2000 expenses every 3 years henceforth. Also, she spends \$800 every 2 years for maintenance now that the warranty is over. This is for years 2, 4, 6, 8, and 10, when she plans to donate the car to charity. Use these costs to determine Jane's equivalent annual cost for years 1 through infinity at $i = 5\%$ per year, if cars she owns in the future have the same cost pattern. Solve using tabulated factors and a spreadsheet, as requested by your instructor.

- 5.28** A West Virginia coal mining operation has installed an in-shaft monitoring system for oxygen tank and gear readiness for emergencies. Based on maintenance patterns for previous systems, costs are minimal for the first few years, increase for a time period, and then level off. Maintenance costs are expected to be \$150,000 in year 3, \$175,000 in year 4, and amounts increasing by \$25,000 per year through year 6 and remain constant thereafter for the expected 10-year life of this system. If similar systems will replace the current one, determine the perpetual equivalent annual maintenance cost at $i = 10\%$ per year. Solve using tabulated factors and a spreadsheet, as requested by your instructor.

- 5.29** Harmony Auto Group sells and services imported and domestic cars. The owner is considering the outsourcing of all its new car warranty service work to Winslow, Inc., a private repair service that works on any make and year car. Both a 5-year contract basis or 10-year license agreement are available from Winslow. Revenue from the manufacturer will be shared with no added cost incurred by the car/warranty owner. Alternatively, Harmony can continue to do warranty work in-house. Use the estimates made by the Harmony owner to perform an annual worth evaluation at 10% per year to select the best option. All dollar values are in millions.

	Contract	License	In-house
First cost, \$	0	-2	-20
Annual cost, \$ per year	-1	-0.2	-4
Annual income, \$ per year	2.5	1.3	8
Life, years	5	10	∞

- 5.30** ABC Drinks purchases its 355 ml cans in large bulk from Wald-China Can Corporation. The finish on the anodized aluminum surface is produced by mechanical finishing technology called brushing or bead blasting. Engineers at Wald are switching to more efficient, faster, and cheaper machines to supply ABC. Use the estimates and MARR = 8% per year to select between two alternatives.

Brush alternative: $P = \$-400,000$;
 $n = 10$ years; $S = \$50,000$; nonlabor
 $AOC = \$-60,000$ in year 1, decreasing
by \$5000 annually starting in year 2.

Bead blasting alternative: $P = \$-400,000$;
 n is large, assume permanent; no salvage;
nonlabor $AOC = \$-70,000$ per year.

- 5.31** You are an engineer with Yorkshire Shipping in Singapore. Your boss, Zul, asks you to recommend one of two methods to reduce or eliminate rodent damage to silo-stored grain as it awaits shipment. Perform an AW analysis at 10% per year compounded quarterly. Dollar values are in millions.

	Alternative A Major Reduction	Alternative B Almost Eliminate
First cost, \$	-10	-35
Annual operating cost, \$ per year	-1.8	-0.6
Salvage value, \$	0.7	0.2
Life, years	5	Almost permanent

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 5.32** In comparing alternatives that have different lives by the annual worth method,

- a. the annual worth value of both alternatives must be calculated over a time period equal to the life of the shorter-lived one.
- b. the annual worth value of both alternatives must be calculated over a time period equal to the life of the longer-lived asset.
- c. the annual worth values must be calculated over a time period equal to the least common multiple of the lives.
- d. the annual worth values can be compared over one life cycle of each alternative.

- 5.33** If you have the present worth of an alternative with a 5-year life, you can obtain its annual worth by:

- a. multiplying the PW by i .
- b. multiplying the PW by $(A/F,i,5)$.
- c. multiplying the PW by $(P/A,i,5)$.
- d. multiplying the PW by $(A/P,i,5)$.

- 5.34** An automation asset with a high first cost of \$10 million has a capital recovery (CR) of \$1,985,000 per year. The correct interpretation of this CR value is that:

- a. the owner must pay an additional \$1,985,000 each year to retain the asset.
- b. each year of its expected life, a net revenue of \$1,985,000 must be realized to recover the

\$10 million first cost and the required rate of return on this investment.

- c. each year of its expected life, a net revenue of \$1,985,000 must be realized to recover the \$10 million first cost.
- d. the services provided by the asset will stop if less than \$1,985,000 in net revenue is reported in any year.

- 5.35** The AWs of three cost alternatives are \$-23,000 for Alternative A, \$-21,600 for B, and \$-27,300 for C. On the basis of AW values, the best economic choice is:

- a. select alternative A.
- b. select alternative B.
- c. select alternative C.
- d. select the do nothing alternative.

- 5.36** The initial cost of a packed-bed degassing reactor for removing trihalomethanes from potable water is \$84,000. The annual operating cost for power, site maintenance, etc. is \$13,000. If the salvage value of the pumps, blowers, and control systems is expected to be \$9000 at the end of 10 years, the AW of the packed-bed reactor at an interest rate of 8% per year is closest to:

- a. \$-26,140
- b. \$-25,520
- c. \$-24,900
- d. \$-13,140

- 5.37** The AW values of three revenue alternatives are \$−23,000 for A, \$−21,600 for B, and \$−27,300 for C. On the basis of these AW values, the correct decision is to:
- select alternative A.
 - select alternative B.
 - select alternative C.
 - select the do nothing alternative.

Problems 5.38 through 5.40 are based on the following estimates.

Use an interest rate of 10% per year.

Alternative	A	B
First cost, \$	−50,000	−80,000
Annual cost, \$/year	−20,000	−10,000
Salvage value, \$	10,000	25,000
Life, years	3	6

- 5.38** The equivalent annual worth of alternative A is closest to:
- \$−25,130
 - \$−37,100
 - \$−41,500
 - \$−42,900

- 5.39** The equivalent annual worth of alternative B is closest to:

- \$−25,130
- \$−28,190
- \$−37,080
- \$−39,100

- 5.40** The equivalent annual worth of alternative A over an infinite time period is closest to:

- \$−25,000
- \$−27,200
- \$−31,600
- \$−37,100

- 5.41** If you have the capitalized cost of an alternative that has an infinite life, you can get its annual cost over a very long number of years by:

- multiplying the capitalized cost by i .
- multiplying the capitalized cost by $(A/F,i,n)$.
- dividing the capitalized cost by $(P/A,i,n)$.
- dividing the capitalized cost by i .

- 5.42** If you have the annual worth of an alternative that has a 5-year life, you can obtain its perpetual annual worth by:

- doing no calculations, since perpetual annual worth equals the annual worth.
- multiplying the annual worth by $(A/P,i,5)$.
- dividing the annual worth by i .
- multiplying the annual worth by i .