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CHAPTER 7

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ELECTRO-MAGNETIC INDUCTION

The above figure shows the picture of a hydro-electric generator. Electric generators, motors, transformers, etc., work based on the principle of electromagnetic induction

7.1. Relation Between Magnetism and Electricity

It is well known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. It can be said that when electrons are in motion, they produce a magnetic field. The converse of this is also true *i.e.* when a magnetic field embracing a conductor moves *relative* to the conductor, it produces a flow of electrons in the conductor. This phenomenon whereby an e.m.f. and hence current (*i.e.* flow of electrons) is induced in any conductor which is cut across or is cut by a magnetic flux is known as ***electromagnetic induction***. The historical background of this phenomenon is this :

After the discovery (by Oersted) that electric current produces a magnetic field, scientists began to search for the converse phenomenon from about 1821 onwards. The problem they put to themselves was how to 'convert' magnetism into electricity. It is recorded that Michael Faraday* was in the habit of walking about with magnets in his pockets so as to constantly remind him of the problem. After nine years of continuous research and experimentation, he succeeded in producing electricity by 'converting magnetism'. In 1831, he formulated basic laws underlying the phenomenon of electromagnetic induction (known after his name), upon which is based the operation of most of the commercial apparatus like motors, generators and transformers etc.

7.2. Production of Induced E.M.F. and Current

In Fig. 7.1 is shown an insulated coil whose terminals are connected to a sensitive galvanometer *G*. It is placed close to a stationary bar magnet initially at position *AB* (shown dotted). As seen, some flux from the *N*-pole of the magnet is linked with or threads through the coil but, as yet, there is no deflection of the galvanometer. Now, suppose that the magnet is ***suddenly*** brought closer to the coil in position *CD* (see figure). Then, it is found that there is a jerk or a sudden but a momentary deflection

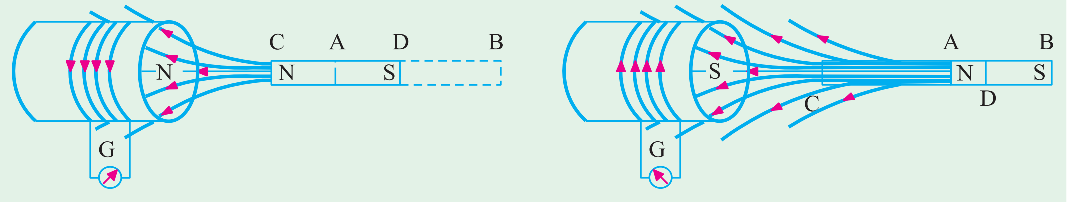


Fig. 7.1. **Fig. 7.2.**

in the galvanometer and that this ***lasts so long as the magnet is in motion relative to the coil, not otherwise***. The deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*. It should be noted that due to the approach of the magnet, flux linked with the coil is increased.

Next, the magnet is ***suddenly*** withdrawn away from the coil as in Fig. 7.2. It is found that again there is a ***momentary*** deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary. It is important to note that this deflection is in a direction opposite to that of Fig. 7.1. Obviously, due to the withdrawal of the magnet, flux linked with the coil is decreased.

The deflection of the galvanometer indicates the production of e.m.f. in the coil. The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of flux linking with the coil. This e.m.f. exists so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f. in a stationary conductor. In fact, the same results can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

* Michael Faraday (1791-1867), an English physicist and chemist.

The direction of this electromagnetically-induced e.m.f. is as shown in the two figures given on back page.

The production of this electromagnetically-induced e.m.f. is further illustrated by considering a conductor *AB* lying within a magnetic field and connected to a galvanometer as shown in Fig. 7.3. It is found that whenever this conductor is moved up or down, *a* ***momentary*** deflection is produced in the galvanometer. It means that some transient e.m.f. is induced in *AB*. The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) ***depends on the quickness of the movement of AB.***

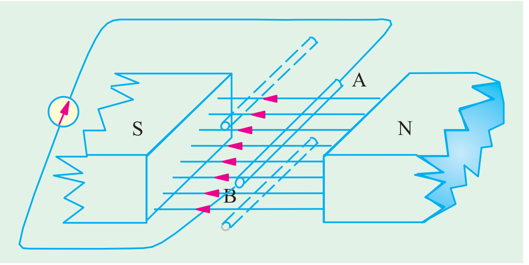


Fig. 7.3

From this experiment we conclude that whenever a conductor cuts or *shears* the magnetic flux, an e.m.f. is always induced in it.

It is also found that if the conductor is moved parallel to the direction of the flux so that it does not cut it, then no e.m.f. is induced in it.

7.3. Faraday's Laws of Electromagnetic Induction

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction.

First Law. It states :

Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

or

Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

Second Law. It states :

The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds. Then, remembering that by flux-linkages mean the product of number of turns and the flux linked with the coil, we have

Initial flux linkages = $N\Phi_1$, add Final flux linkages = $N\Phi_2$

$\therefore$ induced *e.m.f.* $e = \dfrac{N\Phi_2 - N\Phi_1}{t}$ Wb/s or volt or $e = N\dfrac{\Phi_2 - \Phi_1}{t}$ volt

Putting the above expression in its differential form, we get

$$e = \frac{d}{dt}(N\,\Phi) = N\,\frac{d}{dt}\text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it (Art. 7.5).

$$e = -N\,\frac{d\,\Phi}{dt}\text{ volt}$$

Example 7.1. *The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and residual magnetism is 0.002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.*

Solution. Total number of turns, $N = 6 \times 500 = 3000$

Total initial flux $= 6 \times 0.02 = 0.12$ Wb

Total residual flux $= 6 \times 0.002 = 0.012$ Wb

Change in flux, $d\Phi = 0.12 - 0.012 = 0.108$ Wb

Time of opening the circuit, $dt = 0.02$ second

$\therefore$ Induced e.m.f. $= N\dfrac{d\,\Phi}{dt}$ volt $= 3000 \times \dfrac{0.108}{0.02}$ = **16,200 V**

The direction of this induced e.m.f. is the same as the initial direction of the exciting current.

Example 7.2. *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.*

Solution. Induced e.m.f. $= N\,.\dfrac{d\,\Phi}{dt}$ volt

Here $d\,\Phi = 1 - 0.2 = 0.8$ mWb $= 0.8 \times 10^{-3}$ Wb

$dt = 1/10 = 0.1$ second ; $N = 100$

$e = 100 \times 0.8 \times 10^{-3}/0.1$ = **0.8 V**

Total circuit resistance $= 100 + 400 = 500\ \Omega$

$\therefore$ Current induced $= 0.8/500 = 1.6 \times 10^{-3}$ A = **1.6 mA**

Example 7.3. *The time variation of the flux linked with a coil of 500 turns during a complete cycle is as follows :*

$$\Phi = 0.04\,(1 - 4\,t/T)\ \text{Weber} \qquad 0 < t < T/2$$
$$\Phi = 0.04\,(4t/T - 3)\ \text{Weber} \qquad T/2 < t < T$$

where T represents time period and equals 0.04 second. Sketch the waveforms of the flux and induced e.m.f. and also determine the maximum value of the induced e.m.f..

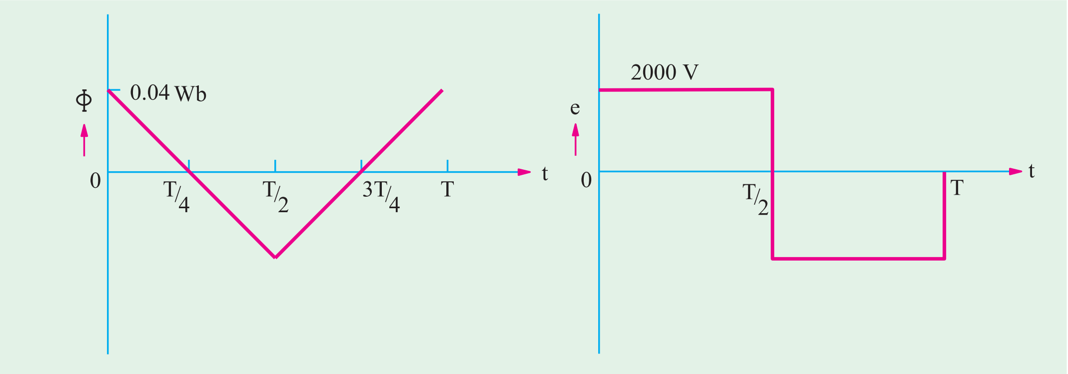


Fig. 7.4.

Solution. The variation of flux is linear as seen from the following table.

t (second) :	0	$T/4$	$T/2$	$3T/4$	T
F (Weber) :	0.04	0	−0.04	0	0.04

The induced e.m.f. is given by $e = -Nd\,\Phi/dt$

From $t = 0$ to $t = T/2$, $d\Phi/dt = -0.04 \times 4/T = -4$ Wb/s $\therefore e = -500\,(-4) = 2000$ V

From $t = T/2$ to $t = T$, $d\Phi/dt = 0.04 \times 4/T = 4$ Wb/s $\therefore e = -500 \times 4 = -2000$ V.

The waveforms are selected in Fig. 7.4.

7.4. Direction of induced e.m.f. and currents

There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or Flat-hand rule or Lenz's Law. Fleming's rule

(Fig. 7.5) is used where induced e.m.f. is due to flux-cutting (*i.e.*, dynamically induced e.m.f.) and Lenz's when it is used to change by flux-linkages (*i.e.*, statically induced e.m.f.).

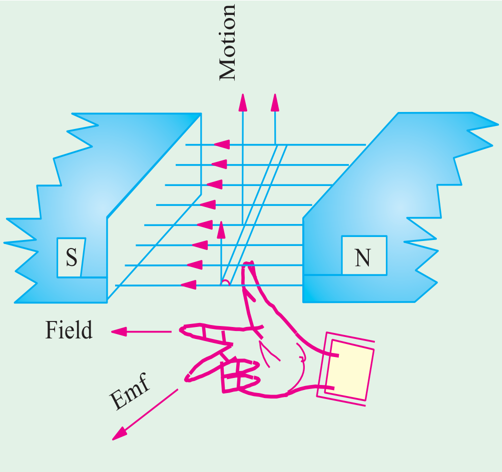


Fig. 7.5.

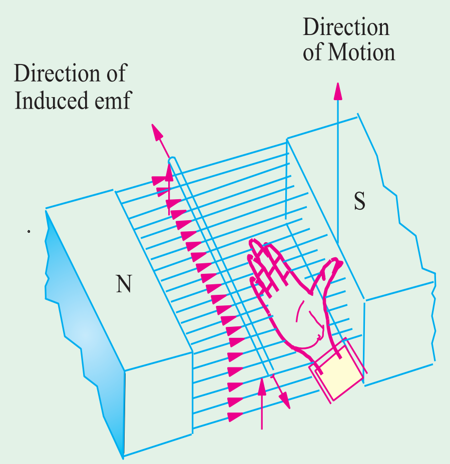


Fig. 7.6.

Fig. 7.6 shows another way of finding the direction of the induced e.m.f. It is known as Right Flat-hand rule. Here, the front side of the hand is held perpendicular to the incident flux with the thumb pointing in the direction of the motion of the conductor. The direction of the fingers give the direction of the induced e.m.f. and current.

7.5. Lenz's Law

The direction of the induced current may also be found by this law which was formulated by Lenz* in 1835. This law states, in effect, that electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

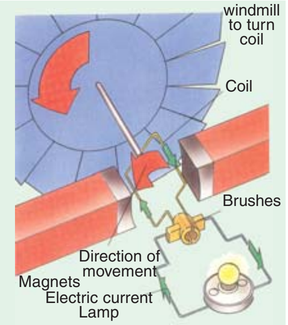
This statement will be clarified with reference to Fig. 7.1 and 7.2. It is found that when N-pole of the bar magnet approaches the coil, the induced current set up by induced e.m.f. flows in the ***anti-clockwise*** direction in the coil as seen from the magnet side. The result is that face of the coil becomes a N-pole and so tends to oppose the onward approach of the *N*-Pole of the magnet (like poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

When the magnet is withdrawn as in Fig. 7.2, the induced current flows in the clockwise direction thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to withdrawn against this attractive force of the *S*-pole of coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy. Imagine for a moment that when *N*-pole of the magnet (Fig. 7.1) approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then, due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause which produces it. In the present case, it is relative motion of the magnet with magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direction to oppose this relative motion *i.e.*, the approach or withdrawal of the magnet.

* After the Russian born geologist and physicist Heinrich Friedrich Emil Lenz (1808 - 1865).

7.6. Induced e.m.f.



The principle of electric generation

Induced e.m.f. can be either **(*i*) dynamically induced** or **(*ii*) statically induced.** In the first case, usually the field is stationary and conductors cut across it (as in d.c. generators). But in the second case, usually the conductors or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).

7.7. Dynamically induced e.m.f.

In Fig. 7.7. a conductor A is shown in cross-section, lying m^2 within a uniform magnetic field of flux density B Wb/m^2. The arrow attached to A shows its direction of motion. Consider the conditions shown in Fig. 7.7 (*a*) when A cuts across at right angles to the flux. Suppose 'l' is its length lying within the field and let it move a distance dx in time dt. Then area swept by it is = ldx. Hence, flux cut = $l.dx \times B$ webers.

$$\text{Change in flux} = Bldx \text{ weber}$$
$$\text{Time taken} = dt \text{ second}$$

Hence, according to Faraday's Laws (Art. 7.3.) the e.m.f. induced in it (known as dynamically induced e.m.f.) is

rate of change of flux linkages $= \dfrac{Bldx}{dt} = Bl\dfrac{dx}{dt} = Blv$ volt where $\dfrac{dx}{dt}$ = velocity

If the conductor A moves at an angle θ with the direction of flux [Fig. 7.7 (*b*)] then the induced e.m.f. is $e = Bl\upsilon \sin\theta$ volts = $l\upsilon \times B$ (*i.e.* as cross product vector υ and B).

The direction of the induced e.m.f. is given by Fleming's Right-hand rule (Art. 7.5) or Flat-hand rule and most easily by vector cross product given above.

It should be noted that generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

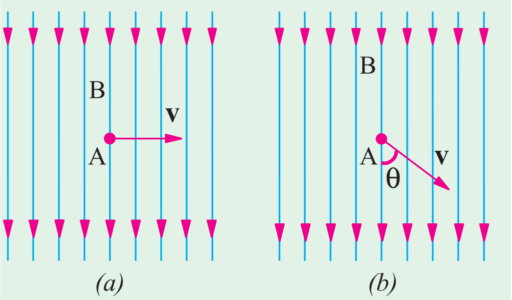


Fig. 7.7

Example 7.4. *A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density 1.5 Wb/m^2 with a velocity of 50 metre/second. Calculate the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of 30º to the direction of the field.*

Solution. Here $B = 1.5$ Wb/m^2 $l = 1$ m $\upsilon = 50$ m/s ; $e = ?$

Now $e = Bl\upsilon = 1.5 \times 1 \times 50 =$ **75 V.**

In the second case $\theta = 30º$ $\therefore \sin 30º = 0.5$ $\therefore e = 75 \times 0.5 =$ **37.5 V**

Example 7.5. *A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 500 rpm about an axis at right angle to a uniform field of 0.5 Wb/m^2. Calculate the instantaneous value of induced e.m.f. when the plane of the coil is* ***(i)*** *at right angle to the plane of the field.* ***(ii)*** *in the plane of the field.* ***(iii)*** *at 45º with the field direction.* **(Elect. Engg. A.M.Ae. S.I. Dec. 1991)**

Solution. As seen from Art. 12.2, e.m.f. induced in the coil would be zero when its plane is at right angles to the plane of the field, even though it will have maximum flux linked with it. However, the coil will have maximum e.m.f. induced in it when its plane lies parallel to the plane of the field even though it will have minimum flux linked with it. In general, the value of the induced e.m.f. is given by $e = \omega N\Phi_m \sin\theta = E_m \sin\theta$ where θ is the angle between the axis of zero *e.m.f.* and the plane of the coil.

Here, $f = 500/60 = 25/3$ r.p.s ; $N = 100$; $B = 0.5$ Wb/ m^2 ; $A = (10 \times 10) \times 10^{-4} = 10^{-2}$ m^2.

$\therefore$ $E_m = 2\pi f NBA = 2\pi (25/3) \times 100 \times 0.5 \times 10^{-2} = 26.2$ V (*i*) since $\theta = 0$; $\sin\theta = 0$; therefore, $e = 0$. (*ii*) Here, $\theta = 90°$; $e = E_m \sin 90º = 26.2 \times 1 = 26.2$ V (*iii*) $\sin 45º = 1/\sqrt{2}$; $e = 26.2 \times 1/\sqrt{2}$ = **18.5 V**

Example 7.6. *A conducting rod AB (Fig. 7.8) makes contact with metal rails AD and BC which are 50 cm apart in a uniform magnetic field of B = 1.0 Wb/m^2 perpendicular to the plane ABCD. The total resistance (assumed constant) of the circuit ABCD is 0.4 Ω*

(a) *What is the direction and magnitude of the e.m.f. induced in the rod when it is moved to the left with a velocity of 8 m/s ?*

(b) *What force is required to keep the rod in motion ?*

(c) *Compare the rate at which mechanical work is done by the force F with the rate of development of electric power in the circuit.*

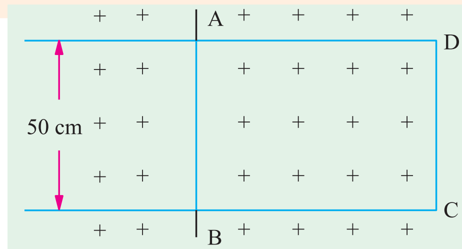


Fig. 7.8

Solution. **(*a*)** Since *AB* moves to the left, direction of the induced current, as found by applying Fleming's Right-hand rule is from *A* to *B*. Magnitude of the induced e.m.f. is given by

$$e = \beta l \upsilon \text{ volt} = 1 \times 0.5 \times 8 = \mathbf{4 \text{ volt}}$$

(*b*) Current through *AB* = 4/0.4 = 10 A

Force on *AB i.e.* $F = BIl = 1 \times 10 \times 0.5 =$ **5 N**

The direction of this force, as found by applying Fleming's left-hand rule, is to the right.

(*c*) Rate of doing mechanical work $= F \times \upsilon = 5 \times 8 =$ **40 J/s or W**

Electric power produced $= e\, i = 4 \times 10 = 40$ W

From the above, it is obvious that the mechanical work done in moving the conductor against force *F* is converted into electric energy.

Example 7.7 *In a 4-pole dynamo, the flux/pole is 15 mWb. Calculate the average e.m.f. induced in one of the armature conductors, if armature is driven at 600 r.p.m.*

Solution. It should be noted that each time the conductor passes under a pole (whether *N* or *S*) it cuts a flux of 15 mWb. Hence, the flux cut in one revolution is 15 × 4 = 60 mWb. Since conductor is rotating at 600/60 = 10 r.p.s. time taken for one revolution is 1/10 = 0.1 second.

$$\therefore \quad \text{average e.m.f. generated} = N\frac{d\Phi}{dt} \text{ volt}$$

$$N = 1; \qquad d\Phi = 60 \text{ mWb} = 6 \times 10^{-2} \text{ Wb} ;\ dt = 0.1 \text{ second}$$

$$\therefore \qquad e = 1 \times 6 \times 10^{-2}/0.1 = \mathbf{0.6\ V}$$

Tutorial Problems No. 7.1

1. A conductor of active length 30 cm carries a current of 100 A and lies at right angles to a magnetic field of strength 0.4 Wb/m^2. Calculate the force in newtons exerted on it. If the force causes the conductor to move at a velocity of 10 m/s, calculate (*a*) the e.m.f. induced in it and (*b*) the power in watts developed by it. **[12 N; 1.2 V, 120 W]**
2. A straight horizontal wire carries a steady current of 150 A and is situated in a uniform magnetic field of 0.6 Wb/m^2 acting vertically downwards. Determine the magnitude of the force in kg/metre length of conductor and the direction in which it works. **[9.175 kg/m horizontally]**
3. A conductor, 10 cm in length, moves with a uniform velocity of 2 m/s at right angles to itself and to a uniform magnetic field having a flux density of 1 Wb/m^2. Calculate the induced e.m.f. between the ends of the conductor. **[0.2 V]**

7.8. Statically Induced E.M.F.

It can be further sub-divided into **(*a*)** ***mutually induced e.m.f. and (b) self-induced e.m.f.***

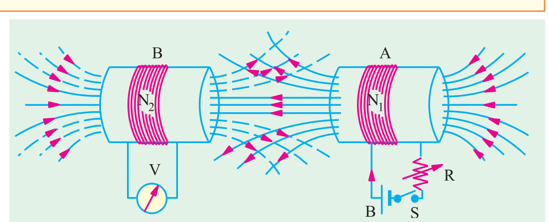


Fig. 7.9

(*a*) Mutually-induced e.m.f. Consider two coils *A* and *B* lying close to each other (Fig. 7.9).

Coil *A* is joined to a battery, a switch and a variable resistance *R* whereas coil *B* is connected to a sensitive voltmeter V. When current through A is established by closing the switch, its magnetic field is set up which partly links with or threads through the coil B. As current through A is changed, the flux linked with B is also changed. Hence, mutually induced e.m.f. is produced in B whose magnitude is given by Faraday's Laws (Art. 7.3) and direction by Lenz's Law (Art. 7.5).

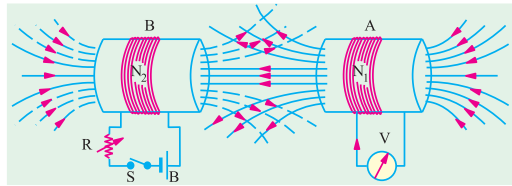


Fig. 7.10

If, now, battery is connected to B and the voltmeter across A (Fig. 7.10), then the situation is reversed and now a change of current in B will produce mutually-induced e.m.f. in A.

It is obvious that in the examples considered above, there is no movement of any conductor, the flux variations being brought about by variations in current strength only. Such an **e.m.f.** induced in one coil by the influence of the other coil is called (statically but) mutually induced e.m.f.

(*b*) Self-induced e.m.f. This is the e.m.f. induced in a coil due to ***the change of its own flux linked with it.*** If current through the coil (Fig. 7.11) is changed, then the flux linked with its own turns will also change, which will produce in it what is called *self-induced* e.m.f. The direction of this induced e.m.f. (as given by Lenz's law) would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter e.m.f. of self-induction.

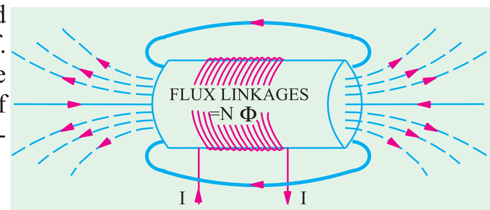


Fig. 7.11

7.9. Self-inductance

Imagine a coil of wire similar to the one shown in Fig. 7.11 connected to a battery through a rheostat. It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery. As will be fully explained later on, this energy is stored in the additional flux produced.

If, now an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction. This property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as ***self-inductance***. It is quantitatively measured in terms of coefficient of self induction L. This property is analogous to inertia in a material body. We know by experience that initially it is difficult to set a heavy body into motion, but once in motion, it is equally difficult to stop it. Similarly, in a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it. Hence, self-induction is sometimes analogously called ***electrical inertia or electromagnetic inertia.***

7.10. Coefficient of Self-induction (L)

It may be defined in any one of the three ways given below :

(*i*) First Method for L

The coefficient of self-induction of a coil is defined as

the weber-turns per ampere in the coil

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil.

Consider a solenoid having N turns and carrying a current of I amperes. If the flux produced is Φ webers, the weber-turns are $N\Phi$. Hence, weber-turns per ampere are $N\,\Phi/I$.

By definition, $L = \dfrac{N\,\Phi}{L}$. The unit of self-induction is henry*.

* After the American scientist Joseph Henry (1797 - 1878), a company of Faraday.

If in the above relation, $N\Phi = 1$ Wb-turn, $I = 1$ ampere, then $L = 1$ henry (H)

Hence a coil is said to have a self-inductance of one henry if a current of 1 ampere when flowing through it produced flux-linkages of 1 Wb-turn in it.

Therefore, the above relation becomes $L = \dfrac{N\,\Phi}{I}$ henry

Example 7.8. *The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.*

Solution. Formula used : $L = \dfrac{N\,\Phi}{L}H$

Current through coil = 230/50 = 4.6 A $\quad \Phi = 0.005$ Wb ; $N = 960$

$L = \dfrac{960 \times 0.005}{4.6}$ = **1.0435 H**. Energy stored = $\dfrac{1}{2}L\,I^2 = 1\dfrac{1}{2} \times 1.0435 \times 4.6^2$ = **11.04 J**

Second Method for L

We have seen in Art. 6.20 that flux produced in a solenoid is

$$\Phi = \frac{NI}{l/\mu_0\mu_r A} \quad \therefore \frac{\Phi}{I} = \frac{N}{I/\mu_0\mu_r A} \quad \text{Now } L = N\frac{\Phi}{I} = N \cdot \frac{N}{I/\mu_0\mu_r A}H$$

$$\therefore \quad L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S}\text{H} \quad \text{or} \quad L = \frac{\mu_0\mu_r AN^2}{l}\text{H}$$

It gives the value of self-induction in terms of the dimensions of the solenoid*.

Example 7.9. *An iron ring 30 cm mean diameter is made of square of iron of 2 cm × 2 cm cross-section and is uniformly wound with 400 turns of wire of 2 mm² cross-section. Calculate the value of the self-inductance of the coil. Assume μ_r = 800.* **(Elect. Technology. I, Gwalior Univ.)**

Solution. $L = \mu_0\,\mu_r\,AN^2/l$. Here $N = 400$; $A = 2 \times 2 = 4$ cm$^2 = 4 \times 10^{-4}$ m^2 ; $l = 0.3\,\pi$ m ; $\mu_r = 800$

$\therefore \quad L = 4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4}\,(400)^2/_{0.3\,\pi}$ = **68.3 mH**

Note. The cross-section of the wire is not relevant to the given question.

Third Method for L

It will be seen from Art. 7.10 (*i*) above that $L = \dfrac{N\,\Phi}{I} \quad \therefore\; N\,\Phi = LI$ or $-N\Phi = -L\,I$

Differentiating both sides, we get $-\dfrac{d}{dt}(N\Phi) = -L \cdot \dfrac{dI}{dt}$ (assuming L to be constant) ;

$$-N \cdot \frac{d\Phi}{dt} = -L \cdot \frac{dI}{dt}$$

As seen from Art. 7.3, $-N \cdot \dfrac{d\Phi}{dt}$ = self-induced e.m.f. $\qquad \therefore e_L = -L\dfrac{dI}{dt}$

If $\quad \dfrac{dI}{dt}$ = 1 ampere/second and $e_L = 1$ volt, then $L = 1\,H$

Hence, a coil has a self-inductance of one henry if one volt is induced in it when current through it changes at the rate of one ampere/second.

Example 7.10. *If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying current of 10 A, calculate the inductance of the coil. If this current is uniformly reversed in 0.01 second, calculating the induced electromotive force.*

Solution. $\quad L = N\Phi/I = 150 \times 0.01/10$ = **0.15 H**

Now, $\quad e_L = L\,dI/dt$; $dI = -10 -(-10) = 20$ A

$\therefore \quad e_L = 0.15 \times 20/0.01$ = **300 V**

* In practice, the inductance of a short solenoid is given by $L = K\mu_0\,\mu_r AN^2/l$ where K is Nagaoka's constant.

Example 7.11. *An iron rod, 2 cm in diameter and 20 cm long is bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0.5 A is passed through this coil, the flux density in the coil is 0.5 Wb/m^2. Assuming that all the flux is linked with every turn of the coil, what is* **(a)** *the B/H ratio for the iron* **(b)** *the inductance of the coil ? What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10 % of its former value in 0.001 second ?* **(Principle of Elect. Engg. Jadavpur Univ.)**

Solution. $H = NI/l = 3000 \times 0.2 = 7500$ AT/m $B = 0.5$ Wb/m^2

(a) Now, $\dfrac{B}{H} = \dfrac{0.5}{7500}$ = **6.67×10^{-5} H/m.** Also $\mu_r = B/\mu_a H = 6.67 \times 10^{-5}/4\pi \times 10^{-7} = 53$

(b) $L = \dfrac{N\Phi}{I} = \dfrac{300 \times \pi \times (0.02)^2 \times 0.5}{4 \times 0.5}$ = **0.94 H**

$e_L = N\dfrac{N\Phi}{dt}$ volt ; $d\Phi$ = 90 % of original flux = $\dfrac{0.9 \times \pi \times (0.02)^2 \times 0.5}{4} = 0.45\,\pi \times 10^{-4}$ Wb

$dt = 0.001$ second $\therefore\ e_L = 3000 \times 0.45\pi \times 10^{-4}/0.001$ = **424 V**

Example 7.12. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm^2 in section. With 4 A, the flux density is 1.0 Wb/m^2 and with 9A, it is 1.4 Wbm^2. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 seconds.* **(Elect. Engineering-1, Delhi Univ.)**

Solution. $L = N\dfrac{d\Phi}{dI} = N\dfrac{d}{dI}(BA) = NA\dfrac{dB}{dI}$ henry $= 1000 \times 20 \times 10^{-4}\,(1.4 - 1)/(9 - 4)$ = **0.16 H**

Now, $e_L = L.dI/dt$; $dI = (9 - 4) = 5$ A, $dt = 0.05$ s $\therefore\ e_L = 0.16 \times 5/0.05$ = **16 V**

Example 7.13. *A direct current of one ampere is passed through a coil of 5000 turns and produces a flux of 0.1mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil ? What would be the voltage developed across the coil if the current were interrupted in 10^{-3} second ? What would be the maximum voltage developed across the coil if a capacitor of 10μ F were connected across the switch breaking the d.c. supply ?*

Solution. $L = N\Phi/I = 5000 \times 10^{-4}$ = **0.5 H** ; Induced e.m.f. $= L\,.\,\dfrac{dI}{dt} = \dfrac{0.5 \times 1}{10^{-3}}$ = **500 V**

The energy stored in the coil is $= \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 0.5 \times I^2 = 0.25$ J

When the capacitor is connected, then the voltage developed would be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of the voltage,

then $\dfrac{1}{2}CV^2 \quad \dfrac{1}{2} \quad LI^2 ; \dfrac{1}{2} \quad 10 \quad 10^{\ 6}\,V^2 = 0.25$ or V = **224 volt**

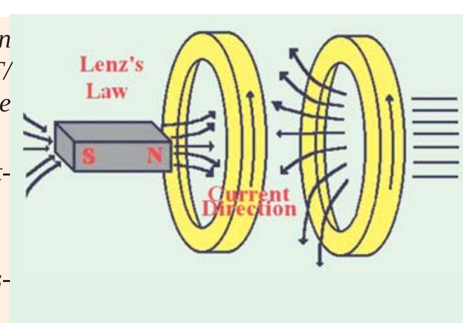
Example 7.14. (a) *A coil of 1000 turns is wound on a torroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5 A and is increasing at the rate of 200 A/s, determine.*

(i) *energy stored in the magnetic circuit* **(ii)** *voltage applied across the coil*

Assume coil resistance as zero.

(b) *How are your answers affected if the coil resistance is 2 Ω*

(Elect. Technology, Hyderabad Univ. 1991)



Solution. (a) $L = N^2/S = 1000^2/10^6 = 1$ H (*i*) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\times 1\times 5^2 =$ **12.5 J**

(*ii*) Voltage applied across coil= self-induced e.m.f. in the coil = $L.dI/dt = 1 \times 200 =$ **200 V**

(*b*) Though there would be additional energy loss of $5^2 \times 2 = 50$ W over the coil resistance, energy stored in the coil would remain the same. However, voltage across the coil would increase by an amount = 5 × 2 = 10 V *i.e.*, now its value would be **210 V**.

7.11. Mutual Inductance

In Art. 7.8 (Fig. 7.9) we have that any change of current in coil *A* is always accompanied by the production of mutually-induced e.m.f. in coil *B*. Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the coefficient of mutual induction *M*.

Example 7.15. *A single element has the current and voltage functions graphed in figure 7.12. (a) and (b). Determine the element.* **[Bombay University 2001]**

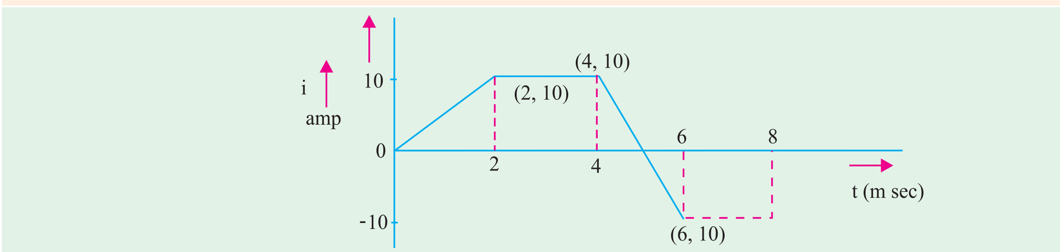


Fig. 7.12 (*a*)

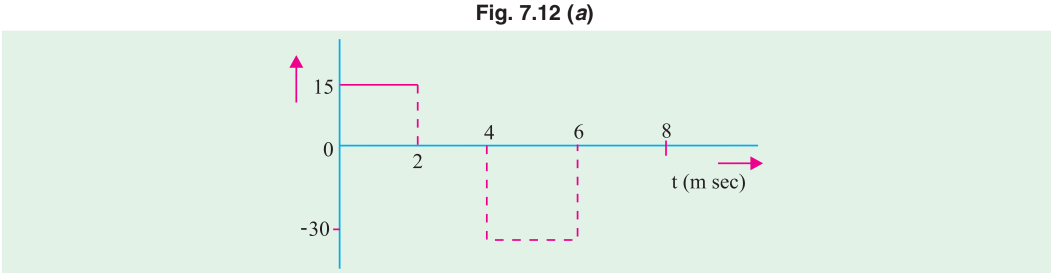


Fig. 7.12 (*b*)

Solution. Observations from the graph are tabulated below.

Sr. No.	Between time	*di/dt* amp/sec	*V*	*L*	
1	0 - 2 m Sec	5000	15	15/5000	= 3mH
2	2 - 4 m Sec	0	0	–	
3	4 - 6 m Sec	– 10,000	– 30	– 30 / (– 10,000)	= 3 mH
4	6 - 8 m Sec	0	0	–	

The element is a 3-mH inductor.

7.12. Coefficient of Mutual Inductance (M)

It can also be defined in three ways as given below :

(*i*) First Method for M

Let there be two magnetically-coupled coils having N_1 and N_2 turns respectively (Fig. 7.9). Coefficient of mutual inductance between the two coils is defined as

the weber-turns in one coil due to one ampere current in the other.

Let a current I_1 ampere when flowing in the first coil produce a flux Φ_1 webers in it. ***It is supposed that whole of this flux links with the turns of the second coil****. Then, flux-linkages *i.e.*, webers-turns in the *second* coil for unit current in the *first* coil are $N_2\,\Phi_1/I_1$. Hence, by definition

$$M = \frac{N_2\,\Phi_1}{I_1}$$

If weber-turns in *second* coil due to one ampere current in the *first* coil *i.e.* $N_2\,\Phi_1/I_1 = 1$ then, as seen from above, $M = 1$H.

Hence, two coils are said to have a mutual inductance of 1 henry is one ampere current when flowing in one coil produces flux-linkages of one Wb-turn in the other.

Example 7.16. *Two identical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in X produces a flux of 0.5 mWb in it, find the mutual inductance between X and Y.* **(Elect. Engg. A,M.Ae.S.I.)**

Solution. Formula used $M = \dfrac{N_2\Phi_1}{I_1}$ H ; Flux produced in $X = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb

Flux linked with $Y = 0.5 \times 10^{-3} \times 0.8 = 0.4 \times 10^{-3}$ Wb ; $M = \dfrac{1000 \times 0.4 \times 10^{-3}}{5}$ = **0.08 H**

Example 7.17. *A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 AT/metre. There is a small concentrated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils in each case if the concentrated coil has 120 turns on an effective diameter of* **(a)** *8 cm and* **(b)** *12 cm.*

(Elect. Science - II Allahabad Univ. 1992)

Solution. The two cases **(*a*)** and **(*b*)** are shown in Fig. 7.13 (*a*) and (*b*) respectively.

(*a*) Let I_1 be the current flowing through the solenoid. Then

$$B = \mu_0 H \times \mu_0 N I_1/l = 2500\,\mu_0 I_1 \text{ Wb/m}^2 \qquad \ldots l = 1 \text{ m}$$

Area of search coil $\quad A_1 = \dfrac{\pi}{4} \times 8^2 \times 10^{-4} = 16\pi \times 10^{-4}$ m^2

Flux linked with search coil is

$$\Phi = BA_1 = 2500\,\mu_0 I_1 \times 16\pi \times 10^{-4} = 15.79\,I_1 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad M = \frac{N_2\Phi_1}{I_1} = \frac{120 \times 15.39 I_1 \times 10^{6}}{I_1} = \mathbf{1.895 \times 10^{-3}\ H}$$

(*b*) Since the field strength outside the solenoid is negligible, the effective area of the search coil, in this case, equals the area of the long solenoid.

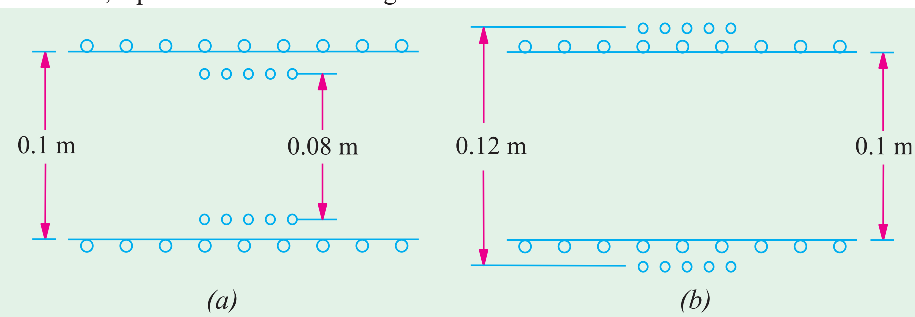


Fig. 7.13

$$A_2 = \frac{\pi}{4} \times 10^2 \times 10^{-4} = \frac{\pi}{4}\,10^{-2}\text{m}^2\ ;$$

* If whole of this flux does not link with turns of the second coil, then only that part of the flux which is actually linked is taken instead. (Ex. 7.13 and 7.17). In general, $M = N_2\Phi_2/I_1$.

$$\Phi = BA_2 = 2500\,\mu_0 I_1 \times \frac{\pi}{4} \times 10^{-2} = 24.68\,I_1 \times 10^{-6}\text{ Wb}$$

$$M = \frac{120 \times 24.68\,I_1 \times 10^{-6}}{I_1} = \mathbf{2.962 \times 10^{-3}\ H}$$

Example 7.18. *A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate* ***(i)*** *the inductance of the coil* ***(ii)*** *the e.m.f. induced in the coil when a current of 5 A is switched off, assuming the current to fall to zero in 1 milli second and* ***(iii)*** *the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.* **(F. E. Pune Univ.)**

Solution. **(*i*)** Inductance of the first coil $= \frac{N\Phi}{I} = \frac{900 \times 0.5 \times 10^{-3}}{3} = \mathbf{0.15\ H}$

(*ii*) e.m.f. induced $e_1 = L\frac{di}{dt} = 0.15 \times \frac{(5-0)}{1 \times 10^{-3}} = \mathbf{750\ V}$

(*iii*) $M \frac{N_2\Phi_1}{I_1} = \frac{600 \times 0.5 \times 10^{-3}}{3} = \mathbf{0.1\ H}$

(*ii*) Second Method for M

We will now deduce an expression for coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil $\Phi_1 = \frac{N_2 I_1}{l/\mu_0\mu_r A}$ Wb ; Flux/ampere $= \frac{\Phi_1}{I_1} = \frac{N_1}{l/\mu_0\mu_r A}$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having N_2 turns, the weber-turns in it due to the flux/ampere in the first coil is

$$M = \frac{N_2\Phi_1}{I_1} = \frac{N_2 N_1}{l/\mu_0\mu_r A} \quad \therefore\ M = \frac{\mu_0\mu_r A\, N_1 N_2}{l}\text{ H}$$

Also $M = \frac{N_1 N_2}{l/\mu_0\mu_r A} = \frac{N_1 N_2}{\text{reluctance}} = \frac{N_1 N_2}{S}$ H

Example 7.19. *If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A ; calculate the inductance of the coil. If this current is uniformly reversed in 0.1 second, calculate the induced e.m.f. If a second coil of 100 turns is uniformly wound over the first coil, find the mutual inductance between the coils.* **(F. E. Pune Univ.)**

Solution.

$$L_1 = N_1\Phi_1/I_1 = 150 \times 0.01/10 = \mathbf{0.15\ H}$$

$$e = L \times di/dt = 0.15 \times [10 - (-10)]/0.1 = 1 = \mathbf{30\ V}$$

$$M = N_2\Phi/I_1 = 100 \times 0.01/10 = \mathbf{0.1\ H}$$

(*iii*) Third Method for M

As seen from Art. 7.12 (*i*) $M = \frac{N_2\Phi_1}{I_1} \quad \therefore N_2\Phi_1 = MI_1$ or $-N_2\Phi_1 = -MI_1$

Differentiating both sides, we get : $-\frac{d}{dt}(N_2\Phi_1) = -M\frac{dI_1}{dt}$ (assuming M to be constant)

Now, $-\frac{d}{dt}(N_2\Phi_1)$ = mutually-induced e.m.f. in the second coil $= e_M \quad \therefore e_M = -M\frac{dI_1}{dt}$

If $dI_1/dt = 1$ A/s ; $e_M = 1$ volt, then $M =$ **1 H**.

Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other.

Example 7.20. *Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of area of cross-section 100 sq.cm. and mean length 200 cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil.* **(Elect. Engg. I, JNT Univ., Warangal)**

Solution. Formula used : $M = \dfrac{N_1N_2}{l/\mu_0\mu_rA}$ **H**, $N_1 = 30$; $N_2 = 600$; $A = 100 \times 10^{-4} = 10^{-2}\text{m}^2$, $l = 2\text{m}$

$$\therefore \quad M = \mu_0\mu_rA\,N_1N_2/l = 4\pi \times 10^{-7} \times 2000 \times 10^{-2} \times 30 \times 600/2 = \mathbf{0.226\ H}$$

$$dI_1 = 20 - 0 = 20\text{ A} ; dt = 0.02\text{ s} ; e_M = MdI_1/dt = 0.226 \times 20/0.2 = \mathbf{226\ V}$$

Example 7.21. *Two coils A and B each having 1200 turns are placed near each other. When coil B is open-circuited and coil A carries a current of 5 A, the flux produced by coil A is 0.2 Wb and 30% of this flux links with all the turns of coil B. Determine the voltage induced in coil B on open-circuit when the current in the coil A is changing at the rate of 2 A/s.*

Solution. Coefficient of mutual induction between the two coils is $M = N_2\Phi_2/I_1$

Flux linked with coil B is 30 per cent of 0.2 Wb *i.e.* 0.06 Wb

$$\therefore \quad M = 1200 \times 0.06/5 = 14.4\text{ H}$$

Mutually-induced e.m.f. in coil B is $e_M = MdI_1/dt = 14.4 \times 2 =$ **28.8 V**

Example 7.22. *Two coils are wound side by side on a paper-tube former. An e.m.f. of 0.25 V is induced in coil A when the flux linking it changes at the rate of 10^3 Wb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance between the coils ?* **(Elect. Engg-I, Bombay Univ.)**

Solution. Induced e.m.f. in coil A is $e = N_1\dfrac{d\Phi}{dt}$ where N_1 is the number of turns of coil A.

$$\therefore \quad 0.25 = N_1 \times 10^{-3} \quad \therefore\ N_1 = 250$$

Now, flux linkages in coil A due to 2 A current in coil $B = 250 \times 10^{-5}$

$$\therefore \quad M = \frac{\text{flux linkages in coil } A}{\text{current in coil } B} = 250 \times 10^{-5}/2 = \mathbf{1.25\ mH}$$

7.13. Coefficient of Coupling

Consider two magnetically-coupled coils A and B having N_1 and N_2 turns respectively. Their individual coefficients of self-induction are,

$$L_1 = \frac{N_1^2}{l/\mu_0\mu_rA} \quad \text{and} \quad L_2 = \frac{N_2^2}{l/\mu_0\mu_rA}$$

The flux Φ_1 produced in A due to a current I_1 ampere is $\Phi_1 = \dfrac{N_1I_1}{l/\mu_0\mu_rA}$

Suppose a fraction k_1 of this flux *i.e.* $k_1\Phi_1$ is linked with coil B.

Then $$M = \frac{k_1\Phi_1 \times N_2}{I_1} \quad \text{where } k_1 \leqslant 1.$$

Substituting the value of Φ_1, we have, $M = k_1 \times \dfrac{N_1N_2}{l/\mu_0\mu_rA}$ ...*(i)*

Similarly, the flux Φ_2 produced in B due to I_2 ampere in it is $\Phi_2 = \dfrac{N_2I_2}{l/\mu_0\mu_rA}$

Suppose a fraction k_2 of this flux *i.e.* $k_2\Phi_2$ is linked with A.

Then $$M = \frac{k_2\Phi_2 \times N_1}{I_2} = k_2\frac{N_1N_2}{l/\mu_0\mu_rA} \quad \text{...(ii)}$$

Multiplying Eq. (*i*) and (*ii*), we get

$$M^2 = k_1 k_2 \frac{N_1{}^2}{l/\mu_0\mu_r A} \times \frac{N_2{}^2}{l/\mu_0\mu_r A} \quad \text{or} \quad M^2 = k_1 k_2 L_1 L_2$$

Putting $\sqrt{k_1 k_2} = k$, we have $M = k\sqrt{L_1 L_2}$ or $k = \dfrac{M}{\sqrt{L_1 L_2}}$

The constant k is called the ***coefficient of coupling*** and may be defined as the ratio of ***mutual inductance actually present between the two coils to the maximum possible value.*** If the flux due to one coil completely links with the other, then value of k is unity. If the flux of one coil does not at all link with the other, then $k = 0$. In the first case, when $k = 1$, coils are said to be tightly coupled and when $k = 0$, the coils are magnetically isolated from each other.

Example 7.23. *Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.*

Solution. Now, $e_M = M dI_1/dt$ ...Art. 7.12

$$M = \frac{e_M}{dI_1/dt} = \frac{11.25}{1500} = 7.5 \times 10^{-3} \text{ H} = \mathbf{7.5\ mH}$$

Now, $L_1 = \dfrac{N_1\Phi_1}{I_1}$ $\therefore$ $\dfrac{\Phi_1}{I_1} = \dfrac{L_1}{N_1} = \dfrac{15 \times 10^{-3}}{750} = \mathbf{2 \times 10^{-5}\ Wb/A}$ ...Art. 7.10

Now, $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{7.5 \quad 10^{\ 3}}{\sqrt{L^2}} \quad \dfrac{7.5 \quad 10^{\ 3}}{15 \quad 10^{\ 3}} = 0.5 = \mathbf{50\%}$ $(\because L_1 = L_2 = L)$ ...Art. 7.13

Example 7.24. *Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60 % of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0.6 mWb while the same current in B produces 0.8 mWb. Determine* **(i)** *mutual inductance and* **(ii)** *coupling coefficient.*

Solution. **(*i*)** Flux/ampere in A = $0.6/5 = 0.12$ mWb

Flux linked with B = $0.12 \times 0.6 = 0.072$ mWb

$\therefore$ $M = 0.072 \times 10^{-3} \times 16{,}000 = \mathbf{1.15\ H}$

Now, $L_1 = \dfrac{12{,}500 \times 0.6}{5} = 150 \times 10^{-3}$ H ; $L_2 = \dfrac{16{,}000 \times 0.8}{5} = 256 \times 10^{-3}$ H

(*ii*) $k = M/\sqrt{L_1 L_2} = 1.15/\sqrt{1.5 \times 2.56} = \mathbf{0.586}$

Note. We could find k in another way also. Value of $k_1 = 0.6$, that of k_2 could also be found, then $k = \sqrt{k_1 k_2}$.

Example 7.25. *Two magnetically-coupled coils have a mutual inductance of 32 mH. What is the average e.m.f. induced in one, if the current through the other changes from 3 to 15 mA in 0.004 second ? Given that one coil has twice the number of turns in the other, calculate the inductance of each coil. Neglect leakage.*

Solution. $M = 32 \times 10^{-3}$ H ; $dI_1 = 15 - 3 = 12$ mA $= 12 \times 10^{-3}$ A ; $dt = 0.004$ second

$$\text{Average e.m.f. induced} = M\frac{dI_1}{dt} = \frac{32 \times 10^{-3} \times 12 \times 10^{-3}}{0.004} = \mathbf{96 \times 10^{-3}\ V}$$

Now $L_1 = \mu_0 N^2 A/l = k N^2$ where $k = \mu_0 A/l$ (taking $\mu_r = 1$)

$$L_2 = \frac{(2N)^2\mu_0 A}{2\,l} = 2kN^2 \; ; \; \frac{L_2}{L_1} = \frac{2kN^2}{kN^2} = 2 \quad \therefore \; L_2 = 2L_1$$

Now $M = \sqrt{L_1 L_2} = \sqrt{2L_1 \times L_1} = 32$, $L_1 = 32/\sqrt{2} = \mathbf{16/\sqrt{2}\,mH}$; $L_2 = 2 \times 16/\sqrt{2} = \mathbf{32\sqrt{2}\ mH}$

Example 7.26. *Two coils, A and B, have self inductances of 120 μH and 300 μH respectively. A current of 1 A through coil A produces flux linkages of 100 μWb turns in coil B. Calculate **(i)** the mutual inductance between the coils **(ii)** the coupling coefficient and **(iii)** the average e.m.f. induced in coil B if a current of 1 A in coil A is reversed at a uniform rate in 0.1 sec.*

(F. E. Pune Univ.)

Solution. (*i*) $M = \dfrac{\text{flux-linkages of coil } B}{\text{current in coil } A} = \dfrac{100 \times 10^{-6}}{1} = \mathbf{100\ \mu H}$

(*ii*) $M = k\sqrt{L_1 L_2} \quad \therefore k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{100 \times 10^{-6}}{\sqrt{120 \times 10^{-6} \times 300 \times 10^{-6}}} = \mathbf{0.527}$

(*iii*) $e_2 = M \times di/dt = (100 \times 10^{-6}) \times 2/0.1 = 0.002 \text{ V}$ or **2 mV.**

7.14. Inductances in Series

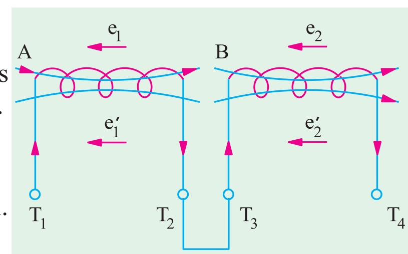


Fig. 7.14

(*i*) Let the two coils be so joined in series that their fluxes (or m.m.fs) are additive *i.e.*, in the same direction (Fig. 7.14).

Let M = coefficient of mutual inductance

L_1 = coefficient of self-inductance of 1st coil

L_2 = coefficient of self-inductance of 2nd coil.

Then, self induced e.m.f. in A is = $e_1 = -L_1 . \dfrac{di}{dt}$

Mutually-induced e.m.f. in A due to change of current in B is $e' = -M . \dfrac{di}{dt}$

Self-induced e.m.f. in B is = $e_2 = -L_2 . \dfrac{di}{dt}$

Mutually-induced e.m.f. in B due to change of current in A is = $e_2' = -M . \dfrac{di}{dt}$

(All have –ve sign, because both self and mutally induced e.m.fs. are in opposition to the applied e.m.f.). Total induced e.m.f. in the combination = $-\dfrac{di}{dt}(L_1 + L_2 + 2M)$ ...**(*i*)**

If L is the equivalent inductance then total induced e.m.f. in that single coil would have been

$$= -L\frac{di}{dt} \qquad \text{...(ii)}$$

Equating **(*i*)** and **(*ii*)** above, we have $L = L_1 + L_2 + 2M$

(*ii*) When the coils are so joined that their fluxes are in opposite directions (Fig. 7.15).

As before $e_1 = -L_1 \dfrac{di}{dt}$

$e_1' = +M . \dfrac{di}{dt}$ (mark this direction)

$e_2 = -L_2 \dfrac{di}{dt}$ and $e_2' = +M . \dfrac{di}{dt}$

Total induced e.m.f. $= -\dfrac{di}{dt}(L_1 + L_2 - 2M)$

∴ Equivalent inductance $L = L_1 + L_2 - 2M$

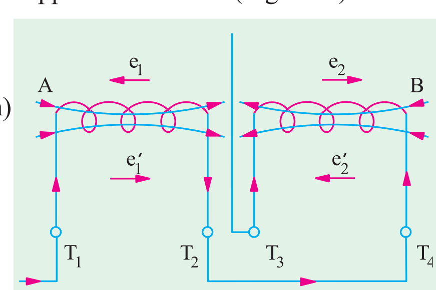


Fig. 7.15

In general, we have : $L = L_1 + L_2 + 2M$ ... if m.m.fs are additive

and $L = L_1 + L_2 - 2M$ ... if m.m.fs. are subtractive

Example 7.27. *Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.*

Solution. **(a)** $L = L_1 + L_2 + 2M$ or $1.9 = L_1 + L_2 + 2M$ ...(*i*)

(b) Here $L = L_1 + L_2 - 2M$ or $0.7 = L_1 + L_2 - 2M$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$1.2 = 4M \qquad \therefore \quad M = 0.3 \text{ H}$$

Putting this value in (*i*) above, we get $L_1 + L_2 = 1.3$ H ...(*iii*)

We know that, in general, $M = k\sqrt{L_1 L_2}$

$$\therefore \quad \sqrt{L_1 L_2} = \frac{M}{k} = \frac{0.3}{0.5} = 0.6 \qquad \therefore \; L_1 L_2 = \mathbf{0.36}$$

From (*iii*), we get $(L_1 + L_2)^2 - 4L_1L_2 = (L_1 - L_2)^2$

$$\therefore \quad (L_1 - L_2)^2 = 0.25 \quad \text{or} \quad L_1 - L_2 = 0.5 \qquad \text{...}(iv)$$

From (*iii*) and (*iv*), we get $L_1 = \mathbf{0.9\ H}$ and $L_2 = \mathbf{0.4\ H}$

Example 7.28. *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate (a) mutual inductance and (b) coupling coefficient.*

(Elect. Technology, Univ. of Indore)

Solution. **(*i*)** $L = L_1 + L_2 + 2M$ or $0.6 = L_1 + L_2 + 2M$ ...(*i*)

and $0.1 = L_1 + L_2 - 2M$ ...(*ii*)

(a) From (*i*) and (*ii*) we get, $M = \mathbf{0.125\ H}$

Let $L_1 = 0.2$ H, then substituting this value in (*i*) above, we get $L_2 = 0.15$ H

(b) Coupling coefficient $k = M\sqrt{L_1 L_2} = 0.125/\sqrt{0.2 \times 0.15} = \mathbf{0.72}$

Example 7.29. *Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80 mH. Calculate the self inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.*

(F. E. Pune Univ.)

Solution. If each coil has an inductance of L henry, then $L_1 = L_2 = L$; $M = k\sqrt{L_1 L_2} = k\sqrt{L \times L} = kL$

When connected in series comulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L\,(1 + 0.25) = 2.5L$$

$$\therefore \quad 2.5\,L = 80 \quad \text{or} \quad L = \mathbf{32\ mH}$$

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L\,(1 - k) = 2L\,(1 - 0.25)$$

$$\therefore \quad 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48\ mH}.$$

Example 7.30. *Two coils with terminals T_1, T_2 and T_3, T_4 respectively are placed side by side. When measured separately, the inductance of the first coil is 1200 mH and that of the second is 800 mH. With T_2 joined to T_3, the inductance between T_1 and T_4 is 2500 mH. What is the mutual inductance between the two coils ? Also, determine the inductance between T_1 and T_3 when T_2 is joined to T_4.*

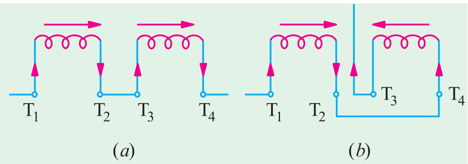


Fig. 7.16

(Electrical Circuit, Nagpur Univ. 1991)

Solution. $L_1 = 1200$ mH, $L_2 = 800$ mH

Fig. 7.16 (*a*) shows additive series.

$$\therefore \quad L = L_1 + L_2 + 2M$$

$$\text{or} \quad 2500 = 1200 + 800 + 2M \;;\; M = \mathbf{250\ mH}$$

Fig. 7.16 (*b*) shows the case of subtractive or opposing series.

Here, $L = L_1 + L_2 - 2M = 1200 + 800 - 2 \times 250 =$ **1500 mH**

Example 7.31. *The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2 H, depending on the relative directions of the current in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate*

(*a*) *the mutual inductance between the two coils*

(*b*) *the self-inductance of coil B*

(*c*) *the coupling factor between the coils.*

(*d*) *the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A per second in the series circuit.* **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. **(*a*)** Combined inductance is given by $L = L_1 + L_2 \pm 2M$

$\therefore \quad 0.5 = L_1 + L_2 + 2M \quad ...(i), \qquad 0.2 = L_1 + L_2 - 2M \quad ...(ii)$

Subtracting **(*ii*)** from **(*i*)**, we have $4M = 0.3$ or $M =$ **0.075 H**

(*b*) Adding **(*i*)** and **(*ii*)** we have $0.7 = 2 \times 0.2 + 2L_2 =$ **0.15 H**

(*c*) Coupling factor or coefficient is $k = M/\sqrt{L_1 L_2} \quad 0.075/\sqrt{0.2 \quad 0.15} = 0.433$ or **43.4%**

(*d*) $$e_1 = L_1 \frac{di}{dt} \pm M \frac{di}{dt}$$

$\therefore \quad e_1 = (0.2 + 0.075) \times 1000 =$ **275 V** ...'cumulative connection'

$= (0.2 - 0.075) \times 1000 =$ **125 V** ...'differential connection'

Example 7.32. *Find the equivalent inductance L_{AB} in Fig. 7.17* **(Bombay University, 2001)**

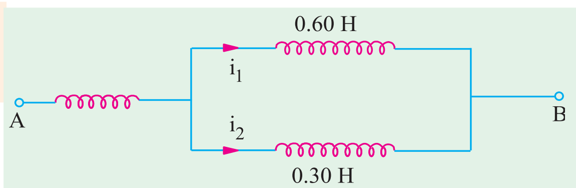


Fig. 7.17

Solution. Series Parallel combination of Inductors has to be dealt with. Note that there is no mutual coupling between coils.

$L_{AB} = 0.5 + [0.6 \times 0.3/(0.3 + 0.3)] = 0.7$ H

7.15. Inductance in Parallel

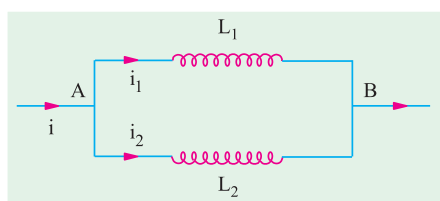


Fig. 7.18

In Fig. 7.18, two inductances of values L_1 and L_2 henry are connected in parallel. Let the coefficient of mutual inductance between the two be M. Let i be the main supply current and i_1 and i_2 be the branch currents

Obviously, $i = i_1 + i_2$

$$\therefore \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad ...(i)$$

In each coil, both self and mutually induced e.m.fs. are produced. Since the coils are in parallel, these e.m.fs. are equal. For a case when self-induced e.m.f.,

we get

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \therefore \quad L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\text{or} \quad \frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M) \quad \therefore \quad \frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \quad ...(ii)$$

$$\text{Hence, } (i) \text{ above becomes } \frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt} \quad ...(iii)$$

If L is the equivalent inductance, then $e = L.\dfrac{di}{dt}$ = induced e.m.f. in the parallel combination

$= \text{induced e.m.f. in any one coil} = L_1 . \dfrac{di_1}{dt} \quad M \dfrac{di_2}{dt}$

$$\therefore \qquad \frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right) \qquad ...(iv)$$

Substituting the value of di_1/dt from (*ii*) in (*iv*), we get $\dfrac{di}{dt} = \dfrac{1}{L}\left[L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M\right]\dfrac{di_2}{dt}$ ...(*v*)

Hence, equating (*iii*) to (*iv*), we have $\dfrac{L_2 - M}{L_1 - M} + 1 = \dfrac{1}{L}\left[L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M\right]$

or $$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L}\left(\frac{L_1L_2 - M^2}{L_1 - M}\right)$$

$\therefore \qquad L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$ when mutual field assists the separate fields.

Similarly, $\qquad L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$ when the two fields oppose each other.

Example 7.33. *Two coils of inductances 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if (i) mutual inductance assists the self-inductance (ii) mutual inductance opposes the self-inductance.*

Solution. (*i*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4\times 6 - 3^2}{4 + 6 - 2\times 3} = \frac{15}{4} = \textbf{3.75 H}$$

(*ii*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{24 - 9}{16} = \frac{15}{16} = \textbf{0.94 H (approx.)}$$

Tutorial Problems No. 7.2

1. Two coils are wound close together on the same paxolin tube. Current is passed through the first coil and is varied at a uniform rate of 500 mA per second, inducing an e.m.f. of 0.1 V in the second coil. The second coil has 100 turns. Calculate the number of turns in the first coil if its inductance is 0.4 H. **[200 turns]**
2. Two coils have 50 and 500 turns respectively are wound side by side on a closed iron circuit of section 50 cm^2 and mean length 120 cm. Estimate the mutual inductance between the coils if the permeability of iron is 1000. Also, find the self-inductance of each coil. If the current in one coil grows steadily from zero to 5A in 0.01 second, find the e.m.f. induced in the other coil. **[M = 0.131 H, L_1 = 0.0131 H, L_2 = 1.21 H, E = 65.4 V]**
3. An iron-cored choke is designed to have an inductance of 20 H when operating at a flux density of 1 Wb/m^2, the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding, given that the magnetic flux path has a mean length of 22 cm in the iron core and of 1 mm in air-gap that its cross-section is 10 cm^2. Neglect leakage and fringing. **[4100]**
4. A non-magnetic ring having a mean diameter of 30 cm and a cross-sectional area of 4 cm^2 is uniformly wound with two coils *A* and *B*, one over the other. *A* has 90 turns and *B* has 240 turns. Calculate from first principles the mutual inductance between the coils.
   Also, calculate the e.m.f. induced in *B* when a current of 6 A in *A* is reversed in 0.02 second. **[11.52 μH, 6.912 mV]**
5. Two coils *A* and *B*, of 600 and 100 turns respectively are wound uniformly around a wooden ring having a mean circumference of 30 cm. The cross-sectional area of the ring is 4 cm^2. Calculate (*a*) the mutual inductance of the coils and (*b*) the e.m.f. induced in coil *B* when a current of 2 A in coil *A* is reversed in 0.01 second. **[(*a*) 100.5 μH (*b*) 40.2 mV]**
6. A coil consists of 1,000 turns of wire uniformly wound on a non-magnetic ring of mean diameter 40 cm and cross-sectional area 20 cm^2.
   Calculate (*a*) the inductance of the coil (*b*) the energy stored in the magnetic field when the coil is carrying a current of 15 A (*c*) the e.m.f. induced in the coil if this current is completely interrupted in 0.01 second. **[(*a*) 2mH (*b*) 0.225 joule (*c*) 3V]**
7. A coil of 50 turns having a mean diameter of 3 cm is placed co-axially at the centre of a solenoid 60 cm long, wound with 2,500 turns and carrying a current of 2 A. Determine mutual inductance of the arrangement. **[0.185 mH]**
8. A coil having a resistance of 2 Ω and an inductance of 0.5 H has a current passed through it which

varies in the following manner ; (*a*) a uniform change from zero to 50 A in 1 second (*b*) constant at 50 A for 1 second (*c*) a uniform change from 50 A to zero in 2 seconds. Plot the current graph to a time base. Tabulate the p.d. applied to the coil during each of the above periods and plot the graph of p.d. to a time base. **[(*a*) 25 to 125 V (*b*) 100 V (*c*) 87.5 V to –12.5 V]**

9. A primary coil having an inductance of 100 μH is connected in series with a secondary coil of 240 μH and the total inductance of the combination is measured as 146 μH. Determine the coefficient of coupling.

10. Find the total inductance measured from A-B terminals, in Fig. 7.19. **[62.6%]** ***(Circuit Theory, Jadavpur Univ.)***

[**Hint :** $L = 100 + 50 - (2 \times 60) = 30$ μH, due to opposite senses of currents with respect to dot-markings.]

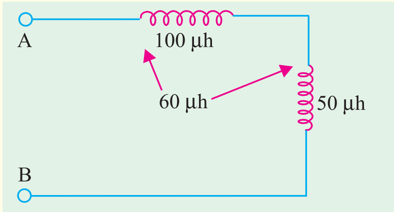


Fig. 7.19

11. Given that relative permeablility of cast iron as 200, that of cast steel is 1200 and for Copper $\mu_0 = 1$. ***(Nagpur University, Summer 2003)***

12. State Faraday's laws of electromagnetic induction. Distinguish between statically induced emf and dynamically induced emf with examples. ***(V.T.U., Belgaum Karnataka University, February 2002)***

13. State : (*i*) Flemming's right hand rule, and (*ii*) Fleming's left hand rule. Mention their applications. ***(V.T.U., Belgaum Karnataka University, Winter 2003)***

14. Define : (*i*) Self inductance, and (*ii*) Mutual inductance. Mention their units and formula to calculate each of them. Derive an expression for the energy stored in an inductor of self inductance 'L' henry carrying the current of 'I' amperes. ***(V.T.U., Belgaum Karnataka University, Winter 2003)***

15. State and explain Faraday's laws of electro magnetic induction, Lenz's Law. Fleming's right hand rule and Fleming's left hand rule. ***(V.T.U., Belgaum Karnataka University, Summer 2003)***

16. A coil of 300 turns wound on a core of non magnetic material has an inductance of 10mH. Calculate (*i*) the flux produced by a current of 5A (*ii*) the average value of the emf induced when a current of 5Amps is reversed in 8 mills seconds. ***(V.T.U., Belgaum Karnataka University, Summer 2003)***

OBJECTIVE TESTS – 7

1. According to Faraday's Laws of Electromagnetic Induction, an e.m.f. is induced in a conductor whenever it
(*a*) lies in a magnetic field
(*b*) cuts magnetic flux
(*c*) moves parallel to the direction of the magnetic field
(*d*) lies perpendicular to the magnetic flux.

2. A pole of driving point admittance function implies
(*a*) zero current for a finite value of driving voltage
(*b*) zero voltage for a finite value of driving current
(*c*) an open circuit condition
(*d*) None of (*a*), (*b*) and (*c*) mentioned in the question ***(ESE 2001)***

3. The inductance of a long solenoid of length 1000 mm wound uniformly with 3000 turns on a cylindrical paper tube of 60 mm diameter is
(a) 3.2 μH (b) 3.2 mH
(c) 32.0 mH (d) 3.2 H
(GATE 2004)

4. A moving iron ammeter produced a full scale torque of 240 μNm with a deflection of 120^0 at a current of 10 A. The rate of change of self inductance (μH/radian) of the instrument at full scale is
(a) 2.0 μH/radian
(b) 4.8 μH/radian
(c) 12.0 μH/radian
(d) 114.6 μH/radian
(GATE 2004)

5. The self-inductance of a long cylindrical conductor due to its internal flux linkages is k H/m. If the diameter of the conductor is doubled, then the selfinductance of the conductor due its internal flux linkages would be
(*a*) 0.5 K H/m (*b*) K H/m
(*c*) 1.414 K H/m (*d*) 4 K H/m
(GATE)

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