# 3

# D.C. Network Theorems

## Introduction

Any arrangement of electrical energy sources, resistances and other circuit elements is called an electrical network. The terms *circuit* and *network* are used synonymously in electrical literature. In the text so far, we employed two network laws *viz* Ohm's law and Kirchhoff's laws to solve network problems. Occasions arise when these laws applied to certain networks do not yield quick and easy solution. To overcome this difficulty, some network theorems have been developed which are very useful in analysing both simple and complex electrical circuits. Through the use of these theorems, it is possible either to simplify the network itself or render the analytical solution easy. In this chapter, we shall focus our attention on important d.c. network theorems and techniques with special reference to their utility in solving network problems.

## 3.1. Network Terminology

While discussing network theorems and techniques, one often comes across the following terms:

**(*i*) Linear circuit.** A linear circuit is one whose parameters (*e.g.* resistances) are constant *i.e.* they do not change with current or voltage.

**(*ii*) Non-linear circuit.** A non-linear circuit is one whose parameters (*e.g.* resistances) change with voltage or current.

**(*iii*) Bilateral circuit.** A bilateral circuit is one whose properties are the same in either direction. For example, transmission line is a bilateral circuit because it can be made to perform its function equally well in either direction.

**(*iv*) Active element.** An active element is one which supplies electrical energy to the circuit. Thus in Fig. 3.1, $E_1$ and $E_2$ are the active elements because they supply energy to the circuit.

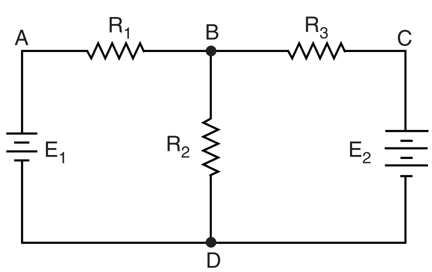



**Fig. 3.1**

**(*v*) Passive element.** A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). In Fig. 3.1, there are three passive elements, namely $R_1$, $R_2$ and $R_3$. These passive elements (*i.e.* resistances in this case) receive energy from the active elements (*i.e.* $E_1$ and $E_2$) and convert it into heat.

**(*vi*) Node.** A node of a network is an equipotential surface at which *two or more* circuit elements are joined. Thus in Fig. 3.1, circuit elements $R_1$ and $E_1$ are joined at $A$ and hence $A$ is the node. Similarly, $B$, $C$ and $D$ are nodes.

**(*vii*) Junction.** A junction is that point in a network where *three or more* circuit elements are joined. In Fig. 3.1, there are only two junction points *viz.* $B$ and $D$. That $B$ is a junction is clear from the fact that three circuit elements $R_1$, $R_2$ and $R_3$ are joined at it. Similarly, point $D$ is a junction because it joins three circuit elements $R_2$, $E_1$ and $E_2$.

**(*viii*) Branch.** A branch is that part of a network which lies between two junction points. Thus referring to Fig. 3.1, there are a total of three branches *viz.* $BAD$, $BCD$ and $BD$. The branch

*BAD* consists of $R_1$ and $E_1$ ; the branch *BCD* consists of $R_3$ and $E_2$ and branch *BD* merely consists of $R_2$.

**(ix) Loop.** A loop is any closed path of a network. Thus in Fig. 3.1, *ABDA*, *BCDB* and *ABCDA* are the loops.

**(x) Mesh.** A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 3.1, both loops *ABDA* and *BCDB* qualify as meshes because they cannot be further divided into other loops. However, the loop *ABCDA* cannot be called a mesh because it encloses two loops *ABDA* and *BCDB*.

**(xi) Network and circuit.** Strictly speaking, the term network is used for a circuit containing passive elements only while the term circuit implies the presence of both active and passive elements. However, there is no hard and fast rule for making these distinctions and the **terms "network" and "circuit" are often used interchangeably**.

**(xii) Parameters.** The various elements of an electric circuit like resistance (*R*), inductance (*L*) and capacitance (*C*) are called parameters of the circuit. These parameters may be lumped or distributed.

**(xiii) Unilateral circuit.** A unilateral circuit is one whose properties change with the direction of its operation. For example, a diode rectifier circuit is a unilateral circuit. It is because a diode rectifier cannot perform rectification in both directions.

**(xiv) Active and passive networks.** An active network is that which contains active elements as well as passive elements. On the other hand, a passive network is that which contains passive elements only.

## 3.2. Network Theorems and Techniques

Having acquainted himself with network terminology, the reader is set to study the various network theorems and techniques. In this chapter, we shall discuss the following network theorems and techniques :

(*i*) Maxwell's mesh current method
(*ii*) Nodal analysis
(*iii*) Superposition theorem
(*iv*) Thevenin's theorem
(*v*) Norton's theorem
(*vi*) Maximum power transfer theorem
(*vii*) Reciprocity theorem
(*viii*) Millman's theorem
(*ix*) Compensation theorem
(*x*) Delta/star or star/delta transformation
(*xi*) Tellegen's theorem

## 3.3. Important Points About Network Analysis

While analysing network problems by using network theorems and techniques, the following points may be noted :

(*i*) There are two general approaches to network analysis *viz.* (*a*) **direct method** (*b*) **network reduction method**. In direct method, the network is left in its original form and different voltages and currents in the circuit are determined. This method is used for simple circuits. Examples of direct method are Kirchhoff's laws, Mesh current method, nodal analysis, superposition theorem etc. In network reduction method, the original network is reduced to a simpler equivalent circuit. This method is used for complex circuits and gives a better insight into the performance of the circuit. Examples of network reduction method are Thevenin's theorem, Norton's theorem, star/delta or delta/star transformation etc.

(*ii*) *The above theorems and techniques are applicable only to networks that have linear, bilateral circuit elements.*

(*iii*) The network theorem or technique to be used will depend upon the network arrangement. The general rule is this. *Use that theorem or technique which requires a smaller number of independent equations to obtain the solution or which can yield easy solution.*

(*iv*) *Analysis of a circuit usually means to determine all the currents and voltages in the circuit.*

## 3.4. Maxwell's Mesh Current Method

*In this method, Kirchhoff's voltage law is applied to a network to write mesh equations in terms of* **mesh currents** *instead of branch currents*. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow *clockwise* around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is then applied to write equations in terms of unknown mesh currents. The branch currents are then found by taking the algebraic sum of the mesh currents which are common to that branch.

**Explanation.** Maxwell's mesh current method consists of following steps :

(*i*) Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in *clockwise direction. For example, in Fig. 3.2, meshes *ABDA* and *BCDB* have been assigned mesh currents $I_1$ and $I_2$ respectively. The mesh currents take on the appearance of a mesh fence and hence the name mesh currents.

(*ii*) If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of the two. Thus in Fig. 3.2, there are two mesh currents $I_1$ and $I_2$ flowing in $R_2$. If we go from *B* to *D*, current is $I_1 - I_2$ and if we go in the other direction (*i.e.* from *D* to *B*), current is $I_2 - I_1$.

(*iii*) **Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember, while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.

(*iv*) If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise *i.e.* opposite to the assumed clockwise direction.

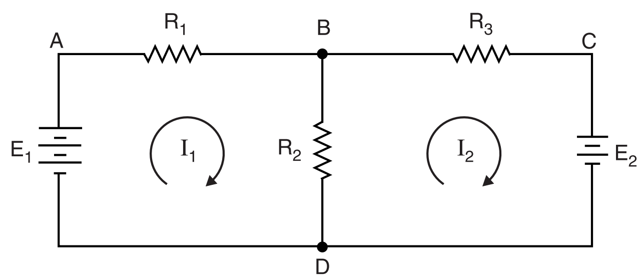



**Fig. 3.2**

Applying Kirchhoff's voltage law to Fig. 3.2, we have,

**Mesh ABDA.**

$$-I_1R_1 - (I_1 - I_2)\,R_2 + E_1 = 0$$

or $$I_1\,(R_1 + R_2) - I_2R_2 = E_1 \qquad ...(i)$$

* It is convenient to consider all mesh currents in one direction (clockwise or anticlockwise). The same result will be obtained if mesh currents are given arbitrary directions.

** Since the circuit unknowns are currents, the describing equations are obtained by applying *KVL* to the meshes.

**Mesh BCDB.**

$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

or`
$$-I_1R_2 + (R_2 + R_3)I_2 = -E_2 \quad ...(ii)$$

Solving eq. (*i*) and eq. (*ii*) simultaneously, mesh currents $I_1$ and $I_2$ can be found out. Once the mesh currents are known, the branch currents can be readily obtained. *The advantage of this method is that it usually reduces the number of equations to solve a network problem*.

**Note.** Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured except in those instances where they happen to be identical with branch currents. Thus in branch *DAB*, branch current is the same as mesh current and both can be measured. But in branch *BD*, mesh currents ($I_1$ and $I_2$) cannot be measured. Hence mesh current is a concept rather than a reality. However, it is a useful concept to solve network problems as it leads to the reduction of number of mesh equations.

## 3.5. Shortcut Procedure for Network Analysis by Mesh Currents

We have seen above that Maxwell mesh current method involves lengthy mesh equations. Here is a shortcut method to write mesh equations simply by inspection of the circuit. Consider the circuit shown in Fig. 3.3. The circuit contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be $I_1$, $I_2$ and $I_3$ flowing in the clockwise direction.

**Loop 1.** Applying *KVL* to this loop, we have,

$$100 - 20 = I_1(60 + 30 + 50) - I_2 \times 50 - I_3 \times 30$$

or
$$80 = 140I_1 - 50I_2 - 30I_3 \quad ...(i)$$

We can write eq. (*i*) in a shortcut form as :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

Here $E_1$ = Algebraic sum of e.m.f.s in Loop (1) in the direction of $I_1$
= 100 – 20 = 80 V

$R_{11}$ = Sum of resistances in Loop (1)
= Self*-resistance of Loop (1)
= 60 + 30 + 50 = 140 Ω

$R_{12}$ = Total resistance common to Loops (1) and (2)
= Common resistance between Loops (1) and (2) = 50 Ω

$R_{13}$ = Total resistance common to Loops (1) and (3) = 30 Ω

It may be seen that the sign of the term involving self-resistances is positive while the sign of common resistances is negative. It is because the positive directions for mesh currents were all chosen clockwise. Although mesh currents are abstract currents, yet mesh current analysis offers the advantage that resistor polarities do not have to be considered when writing mesh equations.

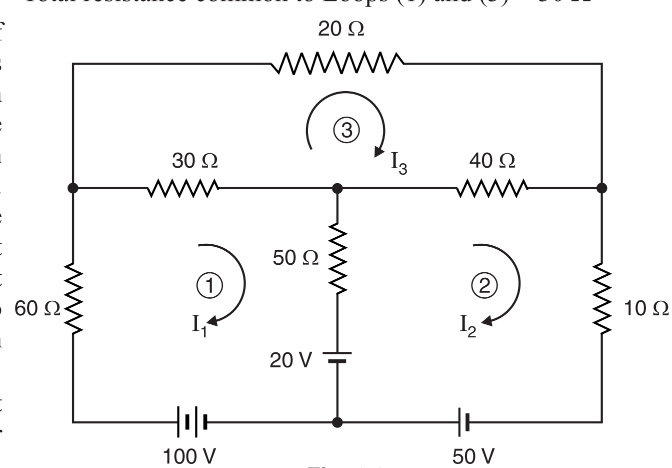



Fig. 3.3

**Loop 2.** We can use shortcut method to find the mesh equation for Loop (2) as under :

* The sum of all resistances in a loop is called self-resistance of that loop. Thus in Fig. 3.3, self-resistance of Loop (1) = 60 + 30 + 50 = 140 Ω.

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

or $$50 + 20 = -50I_1 + 100I_2 - 40I_3 \quad ...(ii)$$

Here, $E_2$ = Algebraic sum of e.m.f.s in Loop (2) in the direction of $I_2$

$= 50 + 20 = 70$ V

$R_{21}$ = Total resistance common to Loops (2) and (1) = 50 Ω

$R_{22}$ = Sum of resistances in Loop (2) = 50 + 40 + 10 = 100 Ω

$R_{23}$ = Total resistance common to Loops (2) and (3) = 40 Ω

Again the sign of self-resistance of Loop (2) ($R_{22}$) is positive while the sign of the terms of common resistances ($R_{21}$, $R_{23}$) is negative.

**Loop 3.** We can again use shortcut method to find the mesh equation for Loop (3) as under :

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

or $$0 = -30I_1 - 40I_2 + 90I_3 \quad ...(iii)$$

Again the sign of self-resistance of Loop (3) ($R_{33}$) is positive while the sign of the terms of common resistances ($R_{31}$, $R_{32}$) is negative.

**Mesh analysis using matrix form.** The three mesh equations are rewritten below :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

The matrix equivalent of above given equations is :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It is reminded again that (*i*) all self-resistances are positive (*ii*) all common resistances are negative and (*iii*) by their definition, $R_{12} = R_{21}$ ; $R_{23} = R_{32}$ and $R_{13} = R_{31}$.

**Example 3.1.** *In the network shown in Fig. 3.4 (i), find the magnitude and direction of each branch current by mesh current method.*

**Solution.** Assign mesh currents $I_1$ and $I_2$ to meshes *ABDA* and *BCDB* respectively as shown in Fig. 3.4 (*i*).

**Mesh ABDA.** Applying *KVL*, we have,

$$-40I_1 - 20(I_1 - I_2) + 120 = 0$$

or $$60I_1 - 20I_2 = 120 \quad ...(i)$$

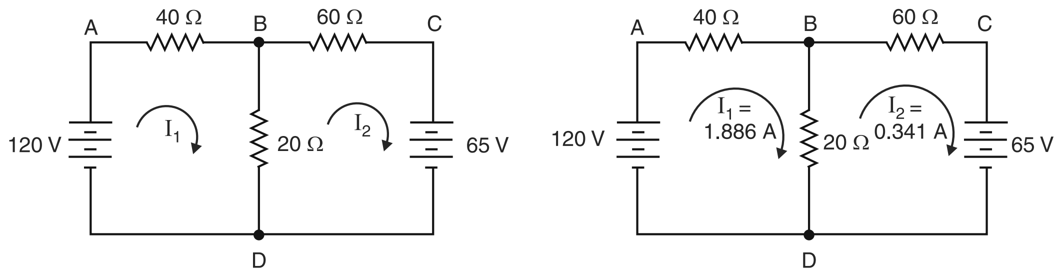
**Mesh BCDB.** Applying *KVL*, we have,

$$-60I_2 - 65 - 20(I_2 - I_1) = 0$$

or $$-20I_1 + 80I_2 = -65 \quad ...(ii)$$

Multiplying eq. (*ii*) by 3 and adding it to eq. (*i*), we get,

$$220I_2 = -75 \qquad \therefore \quad I_2 = -75/220 = -0{\cdot}341 \text{ A}$$




(*i*) (*ii*)

**Fig. 3.4**

The minus sign shows that true direction of $I_2$ is anticlockwise. Substituting $I_2 = -\,0{\cdot}341$A in eq. (*i*), we get, $I_1 = 1{\cdot}886$ A. The actual direction of flow of currents is shown in Fig. 3.4 (*ii*).

**By determinant method**

$$60I_1 - 20I_2 = 120$$

$$-20I_1 + 80I_2 = -65$$

$$\therefore \qquad I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = \frac{(120\times 80)-(-65\times -20)}{(60\times 80)-(-20\times -20)} = \frac{8300}{4400} = 1{\cdot}886 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\text{Denominator}} = \frac{(60\times -65)-(-20\times 120)}{4400} = \frac{-1500}{4400} = -0{\cdot}341 \text{ A}$$

Referring to Fig. 3.4 (*ii*), we have,

Current in branch $DAB = I_1 =$ **1·886 A** ; Current in branch $DCB = I_2 =$ **0·341 A**

Current in branch $BD = I_1 + I_2 = 1{\cdot}886 + 0{\cdot}341 =$ **2·227 A**

**Example 3.2.** *Calculate the current in each branch of the circuit shown in Fig. 3.5.*

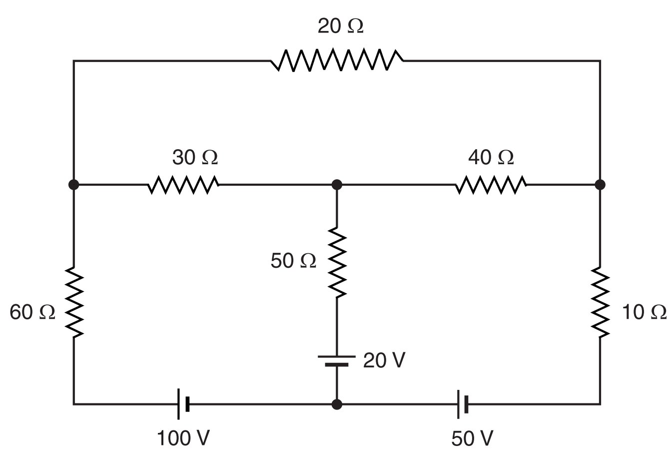



**Fig. 3.5**

**Solution.** Assign mesh currents $I_1$, $I_2$ and $I_3$ to meshes *ABHGA*, *HEFGH* and *BCDEHB* respectively as shown in Fig. 3.6.

**Mesh ABHGA.** Applying *KVL*, we have,

$$-60I_1 - 30(I_1 - I_3) - 50(I_1 - I_2) - 20 + 100 = 0$$

or $$140I_1 - 50I_2 - 30I_3 = 80$$

or $$14I_1 - 5I_2 - 3I_3 = 8 \quad ...(i)$$

**Mesh GHEFG.** Applying *KVL*, we have,

$$20 - 50(I_2 - I_1) - 40(I_2 - I_3) - 10I_2 + 50 = 0$$

or $$-50I_1 + 100I_2 - 40I_3 = 70$$

or $$-5I_1 + 10I_2 - 4I_3 = 7 \quad ...(ii)$$

**Mesh BCDEHB.** Applying *KVL*, we have,

$$-20I_3 - 40(I_3 - I_2) - 30(I_3 - I_1) = 0$$

or $$30I_1 + 40I_2 - 90I_3 = 0$$

or $$3I_1 + 4I_2 - 9I_3 = 0 \quad ...(iii)$$

Solving for equations (*i*), (*ii*) and (*iii*), we get, $I_1 = 1{\cdot}65$ A ; $I_2 = 2{\cdot}12$ A ; $I_3 = 1{\cdot}5$ A

**By determinant method**

$$14I_1 - 5I_2 - 3I_3 = 8$$
$$-5I_1 + 10I_2 - 4I_3 = 7$$
$$3I_1 + 4I_2 - 9I_3 = 0$$

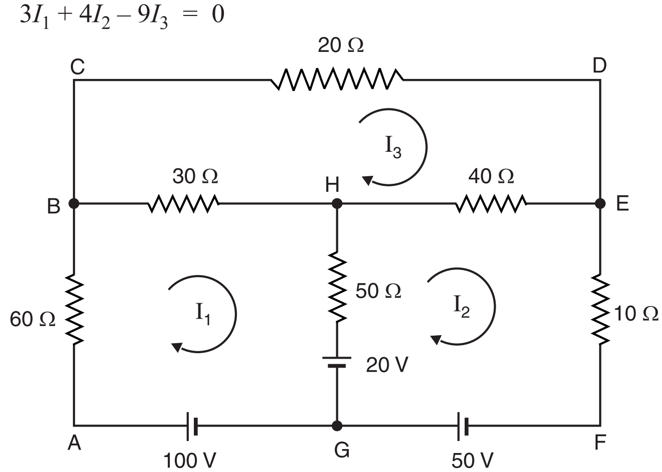



**Fig. 3.6**

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 8 & -5 & -3 \\ 7 & 10 & -4 \\ 0 & 4 & -9 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{vmatrix}} = \frac{8\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} 7 & -4 \\ 0 & -9 \end{vmatrix} - 3\begin{vmatrix} 7 & 10 \\ 0 & 4 \end{vmatrix}}{14\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} -5 & -4 \\ 3 & -9 \end{vmatrix} - 3\begin{vmatrix} -5 & 10 \\ 3 & 4 \end{vmatrix}}$$

$$= \frac{8[(10\times -9) - (4\times -4)] + 5[(7\times -9) - (0\times -4)] - 3[(7\times 4) - (0\times 10)]}{14[(10\times -9) - (4\times -4)] + 5[(-5\times -9) - (3\times -4)] - 3[(-5\times 4) - (3\times 10)]}$$

$$= \frac{-592 - 315 - 84}{-1036 + 285 + 150} = \frac{-991}{-601} = 1{\cdot}65 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 14 & 8 & -3 \\ -5 & 7 & -4 \\ 3 & 0 & -9 \end{vmatrix}}{\text{Denominator}} = \frac{14[(-63)-(0)]-8[(45)-(-12)]-3[(0)-(21)]}{-601}$$

$$= \frac{-882-456+63}{-601} = \frac{-1275}{-601} = 2{\cdot}12 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 14 & -5 & 8 \\ -5 & 10 & 7 \\ 3 & 4 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{14[(0)-(28)]+5[(0)-(21)]+8[(-20)-(30)]}{-601}$$

$$= \frac{-392-105-400}{-601} = \frac{-897}{-601} = 1{\cdot}5 \text{ A}$$

$\therefore$ Current in 60 Ω $= I_1 =$ **1·65 A** from $A$ to $B$

Current in 30 Ω $= I_1 - I_3 = 1{\cdot}65 - 1{\cdot}5 =$ **0·15 A** from $B$ to $H$

Current in 50 Ω $= I_2 - I_1 = 2{\cdot}12 - 1{\cdot}65 =$ **0·47 A** from $G$ to $H$

Current in 40 Ω $= I_2 - I_3 = 2{\cdot}12 - 1{\cdot}5 =$ **0·62 A** from $H$ to $E$

Current in 10 Ω $= I_2 =$ **2·12 A** from $E$ to $F$

Current in 20 Ω $= I_3 =$ **1·5 A** from $C$ to $D$

**Example 3.3.** *By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown in Fig. 3.7.*

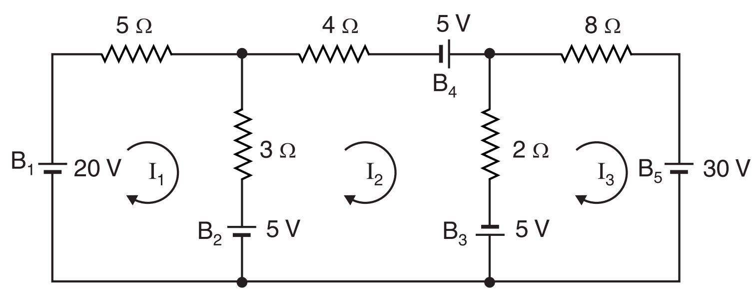



**Fig. 3.7**

**Solution.** Since there are three meshes, let the three mesh currents be $I_1$, $I_2$ and $I_3$, all assumed to be flowing in the clockwise direction. The different quantities of the mesh-resistance matrix are :

$$R_{11} = 5+3 = 8\ \Omega \quad ; \quad R_{22} = 4+2+3 = 9\ \Omega \quad ; \quad R_{33} = 8+2 = 10\ \Omega$$

$$R_{12} = R_{21} = -3\ \Omega \quad ; \quad R_{13} = R_{31} = 0 \quad ; \quad R_{23} = R_{32} = -2\ \Omega$$

$$E_1 = 20-5 = 15 \text{ V} \quad ; \quad E_2 = 5+5+5 = 15 \text{ V} \quad ; \quad E_3 = -30-5 = -35 \text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or
$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}} = \frac{1530}{598} = 2{\cdot}56 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix}}{\text{Denominator}} = \frac{1090}{598} = 1{\cdot}82 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix}}{\text{Denominator}} = \frac{-1875}{598} = -3{\cdot}13 \text{ A}$$

The negative sign with $I_3$ indicates that actual direction of $I_3$ is opposite to that assumed in Fig. 3.7. Note that batteries $B_1$, $B_3$, $B_4$ and $B_5$ are discharging while battery $B_2$ is charging.

$\therefore$ Current supplied by battery $B_1 = I_1 =$ **2·56 A**

Current supplied to battery $B_2 = I_1 - I_2 = 2{\cdot}56 - 1{\cdot}82 =$ **0·74 A**

Current supplied by battery $B_3 = I_2 + I_3 = 1{\cdot}82 + 3{\cdot}13 =$ **4·95 A**

Current supplied by battery $B_4 = I_2 =$ **1·82 A**

Current supplied by battery $B_5 = I_3 =$ **3·13 A**

**Example 3.4.** *By using mesh resistance matrix, calculate the current in each branch of the circuit shown in Fig. 3.8.*

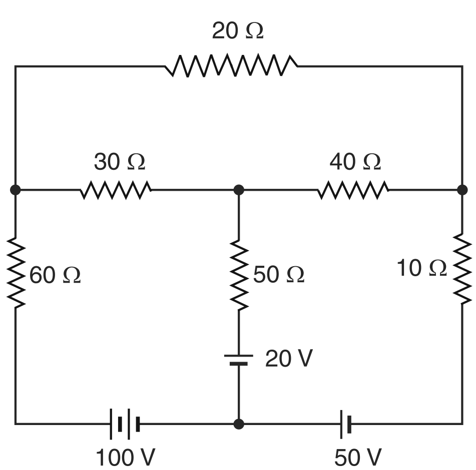



Fig. 3.8

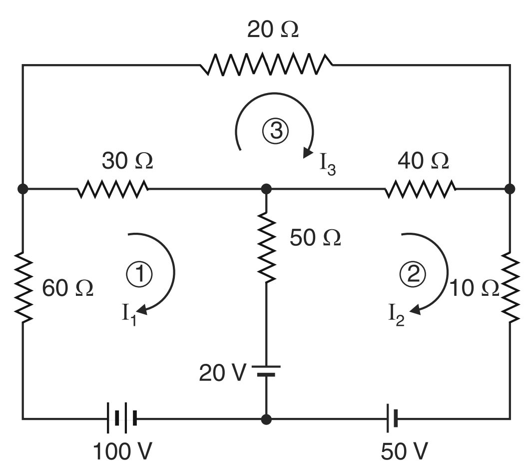



Fig. 3.9

**Solution.** Since there are three meshes, let the three mesh currents be $I_1$, $I_2$ and $I_3$, all assumed to be flowing in the clockwise direction as shown in Fig. 3.9. The different quantities of the mesh resistance-matrix are :

$$R_{11} = 60 + 30 + 50 = 140\ \Omega\ ;\ R_{22} = 50 + 40 + 10 = 100\ \Omega\ ;\ R_{33} = 30 + 20 + 40 = 90\ \Omega$$

$$R_{12} = R_{21} = -50\ \Omega\quad ;\quad R_{13} = R_{31} = -30\ \Omega\quad ;\quad R_{23} = R_{32} = -40\ \Omega$$

$$E_1 = 100 - 20 = 80\ \text{V}\quad ;\quad E_2 = 50 + 20 = 70\ \text{V}\quad ;\quad E_3 = 0\ \text{V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or
$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix}} = \frac{991000}{601000} = 1{\cdot}65\ \text{A}$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ -30 & 0 & 90 \end{vmatrix}}{\text{Denominator}} = \frac{1275000}{601000} = 2{\cdot}12\ \text{A}$$

$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ -30 & -40 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{897000}{601000} = 1{\cdot}5\ \text{A}$$

∴ Current in 60 Ω = $I_1$ = **1·65 A in the direction of $I_1$**

Current in 30 Ω = $I_1 - I_3$ = **0·15 A in the direction of $I_1$**

Current in 50 Ω = $I_2 - I_1$ = **0·47 A in the direction of $I_2$**

Current in 40 Ω = $I_2 - I_3$ = **0·62 A in the direction of $I_2$**

Current in 10 Ω = $I_2$ = **2·12 A in the direction of $I_2$**

Current in 20 Ω = $I_3$ = **1·5 A in the direction of $I_3$**

**Example 3.5.** *Find mesh currents $i_1$ and $i_2$ in the electric circuit shown in Fig. 3.10.*

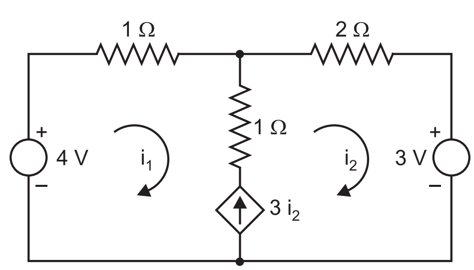



Fig. 3.10

**Solution.** We shall use mesh current method for the solution. Mesh analysis requires that all the sources in a circuit be voltage sources. If a circuit contains any current source, convert it into equivalent voltage source.

**Outer mesh.** Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - 2i_2 - 3 + 4 = 0 \quad \text{or} \quad i_1 + 2i_2 = 1 \qquad ...(i)$$

**First mesh.** Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - (i_1 - i_2) \times 1 - 3i_2 + 4 = 0 \quad \text{or} \quad i_1 + i_2 = 2 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we have $i_1 = \mathbf{3A}$ ; $i_2 = \mathbf{-1\ A}$

**Example 3.6.** *Using mesh current method, determine current $I_x$ in the circuit shown in Fig. 3.11.*

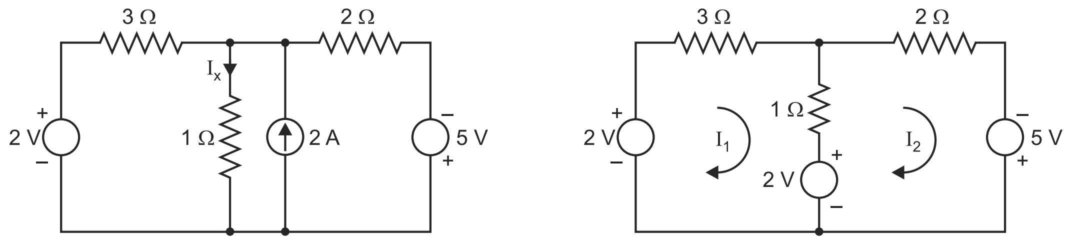



**Fig. 3.11** **Fig. 3.12**

**Solution.** First convert 2A current source in parallel with 1Ω resistance into equivalent voltage source of voltage 2A × 1Ω = 2V in series with 1Ω resistance. The circuit then reduces to that shown in Fig. 3.12. Assign mesh currents $I_1$ and $I_2$ to meshes 1 and 2 in Fig. 3.12.

**Mesh 1.** Applying *KVL* to this mesh, we have,

$$-3I_1 - 1 \times (I_1 - I_2) - 2 + 2 = 0 \quad \text{or} \quad I_2 = 4I_1$$

**Mesh 2.** Applying *KVL* to this mesh, we have,

$$-2I_2 + 5 + 2 - (I_2 - I_1) \times 1 = 0$$

or $$-2(4I_1) + 7 - (4I_1 - I_1) = 0 \quad (\because I_2 = 4I_1)$$

$$\therefore \quad I_1 = \frac{7}{11}\text{A and } I_2 = 4I_1 = 4 \times \frac{7}{11} = \frac{28}{11}\text{A}$$

$$\therefore \quad \text{Current in } 3\Omega \text{ resistance, } I_1 = \frac{7}{11}\text{A} \ ; \ \text{Current in } 2\Omega \text{ resistance, } I_2 = \frac{28}{11}\text{A}$$

Referring to the original Fig. 3.11, we have,

$$I_x = I_1 + (2 - I_2) = \frac{7}{11} + \left(2 - \frac{28}{11}\right) = \mathbf{\frac{1}{11}\ A}$$

**Example 3.7.** *Using mesh current method, find the currents in resistances $R_3$, $R_4$, $R_5$ and $R_6$ of the circuit shown in Fig. 3.13 (i).*

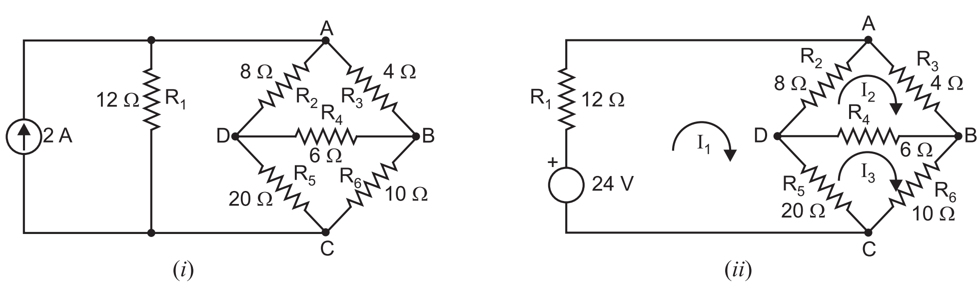



**Fig. 3.13**

**Solution.** First convert 2 A current source in parallel with 12Ω resistance into equivalent voltage source of voltage = 2A × 12Ω = 24V in series with 12Ω resistance. The circuit then reduces to the one shown in Fig. 3.13 (*ii*). Assign the mesh currents $I_1$, $I_2$ and $I_3$ to three meshes 1, 2 and 3 shown in Fig. 3.13 (*ii*).

**Mesh 1.** Applying *KVL* to this mesh, we have,

$$-12I_1 - 8 \times (I_1 - I_2) - 20 \times (I_1 - I_3) + 24 = 0$$

or $$10I_1 - 2I_2 - 5I_3 = 6 \qquad ...(i)$$

**Mesh 2.** Applying *KVL* to this mesh, we have,

$$-4I_2 - 6 \times (I_2 - I_3) - 8(I_2 - I_1) = 0$$

or $$-4I_1 + 9I_2 - 3I_3 = 0 \qquad ...(ii)$$

**Mesh 3.** Applying *KVL* to this mesh, we have,

$$-10I_3 - 20 \times (I_3 - I_1) - 6 \times (I_3 - I_2) = 0$$

or $$-10I_1 - 3I_2 + 18I_3 = 0 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $I_1 = 1.125$ A ; $I_2 = 0.75$ A ; $I_3 = 0.75$ A

∴ Current in $R_3$ (= 4Ω) = $I_2$ = **0.75 A *from* A to B**

Current in $R_4$ (= 6Ω) = $I_2 - I_3 = 0.75 - 0.75$ = **0A**

Current in $R_5$ (= 20Ω) = $I_1 - I_3 = 1.125 - 0.75$ = **0.375A *from* D to C**

Current in $R_6$ (= 10Ω) = $I_3$ = **0.75A *from* B to C**

**Example 3.8.** *Use mesh current method to determine currents through each of the components in the circuit shown in Fig. 3.14 (i).*

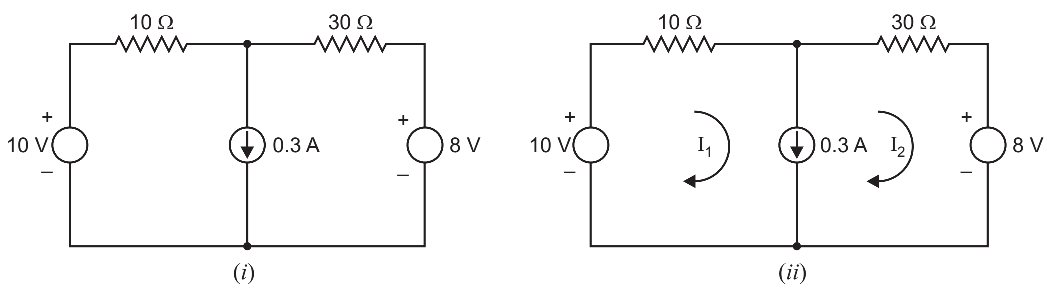



**Fig. 3.14**

**Solution.** Suppose voltage across current source is $v$. Assign mesh currents $I_1$ and $I_2$ in the meshes 1 and 2 respectively as shown in Fig. 3.14 (*ii*).

**Mesh 1.** Applying *KVL* to this mesh, we have,

$$10 - 10I_1 + v = 0 \qquad ...(i)$$

**Mesh 2.** Applying *KVL* to this mesh, we have,

$$-30I_2 - 8 - v = 0 \qquad ...(ii)$$

Adding eqs. (*i*) and (*ii*), $2 - 10I_1 - 30I_2 = 0$ ...(*iii*)

Also current in the branch containing current source is

$$I_1 - I_2 = 0.3 \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_1 = 0.275$ A ; $I_2 = -0.025$A

∴ Current in 10Ω = $I_1$ = **0.275A**

Current in 30Ω = $I_2$ = **– 0.025 A**

Current in current source = $I_1 - I_2 = 0.275 - (-0.025)$ = **0.3A**

Note that negative sign means current is in the opposite direction to that assumed in the circuit.

## Tutorial Problems

1. Use mesh analysis to find the current in each resistor in Fig. 3.15.

**[in 100 Ω = 0·1 A from *L* to *R* ; in 20 Ω = 0·4 A from *R* to *L* ; in 10 Ω = 0·5 A downward]**

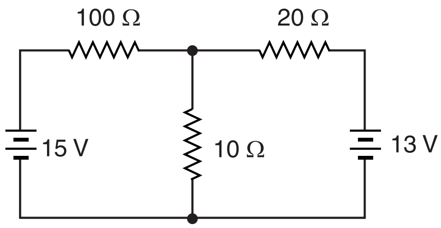



**Fig. 3.15**

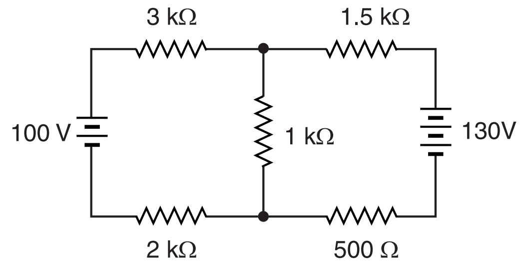



**Fig. 3.16**

2. Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. 3.16. **[50 V]**
3. Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. 3.17. **[*I*(50 Ω) = 0·171 A → ; *I*(250 Ω) = 0·237 A ← ; *I*(100 Ω) = 0·408 A ↓]**

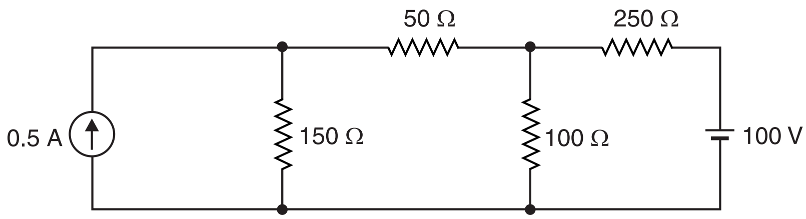



**Fig. 3.17**

4. For the network shown in Fig. 3.18, find the mesh currents $I_1$, $I_2$ and $I_3$. **[5 A, 1 A, 0·5 A]**

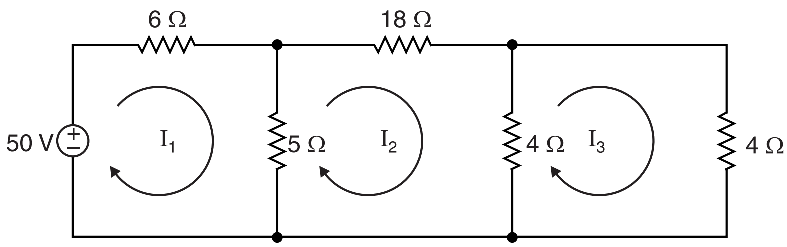



**Fig. 3.18**

5. In the network shown in Fig. 3.19, find the magnitude and direction of current in the various branches by mesh current method. **[*FAB* = 4 A ; *BF* = 3 A ; *BC* = 1 A ; *EC* = 2 A ; *CDE* = 3 A]**

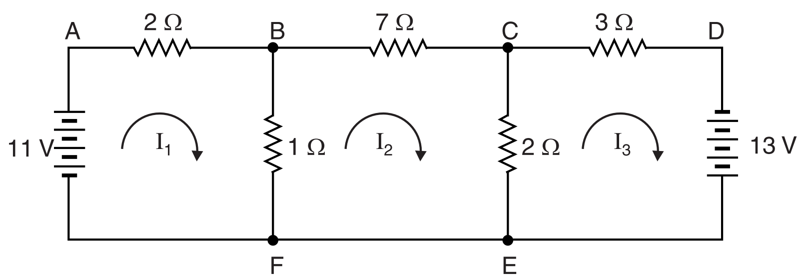



**Fig. 3.19**

## 3.6. Nodal Analysis

Consider the circuit shown in Fig. 3.20. The branch currents in the circuit can be found by Kirchhoff's laws or Maxwell's mesh current method. There is another method, called *nodal analysis* for determining branch currents in a circuit. In this method, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the *reference node*. The potentials of all the points in the circuit are measured w.r.t. this reference node. In Fig. 3.20, *A*, *B*, *C* and *D* are four nodes and the node *D* has been taken as the *reference node. The fixed-voltage nodes are called *dependent nodes*. Thus in Fig. 3.20, *A* and *C* are fixed nodes because $V_A = E_1 = 120$ V and $V_C = 65$ V. The voltage from *D* to *B* is $V_B$ and its magnitude depends upon the parameters of circuit elements and the currents through these elements. Therefore, node *B* is called *independent node*. Once we calculate the potential at the independent node (or nodes), each branch current can be determined because the voltage across each resistor will then be known.

*Hence* **nodal analysis** *essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node*. For a circuit containing *N* nodes, there will be *N*–1 node voltages, some of which may be known if voltage sources are present.

**Circuit analysis.** The circuit shown in Fig. 3.20 has only one independent node *B*. Therefore, if we find the voltage $V_B$ at the independent node *B*, we can determine all branch currents in the circuit. We can express each current in terms of e.m.f.s, resistances (or conductances) and the voltage $V_B$ at node *B*. Note that we have taken point *D* as the reference node.

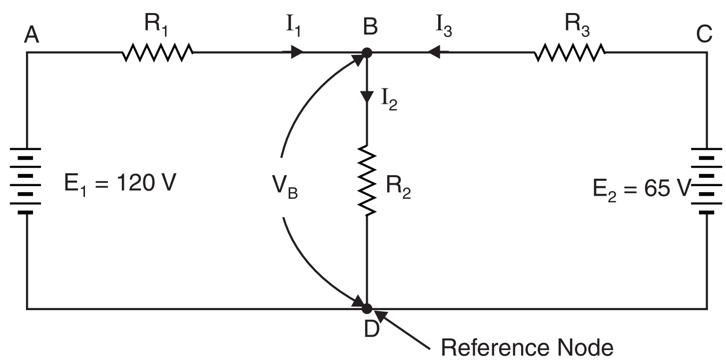



**Fig. 3.20**

The voltage $V_B$ can be found by applying **Kirchhoff's current law at node *B*.

$$I_1 + I_3 = I_2 \quad ...(i)$$

In mesh *ABDA*, the voltage drop across $R_1$ is $E_1 - V_B$.

$$\therefore \quad I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh *CBDC*, the voltage drop across $R_3$ is $E_2 - V_B$.

$$\therefore \quad I_3 = \frac{E_2 - V_B}{R_3}$$

Also $$I_2 = \frac{V_B}{R_2}$$

Putting the values of $I_1$, $I_2$ and $I_3$ in eq. (*i*), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \quad ...(ii)$$

All quantities except $V_B$ are known. Hence $V_B$ can be found out. Once $V_B$ is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation [eq. (*ii*)] for determining the branch currents in this circuit. However, Kirchhoff's or Maxwell's solution would have needed two equations.

* An obvious choice would be ground or common, if such a point exists.

** Since the circuit unknowns are voltages, the describing equations are obtained by applying *KCL* at the nodes.

**Notes.**

*(i)* We can mark the directions of currents at will. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed.

*(ii)* We can also express the currents in terms of conductances.

$$I_1 = \frac{E_1 - V_B}{R_1} = (E_1 - V_B)G_1 \ ; \ I_2 = \frac{V_B}{R_2} = V_B G_2 \ ; \ I_3 = \frac{E_2 - V_B}{R_3} = (E_2 - V_B)G_3$$

## 3.7. Nodal Analysis with Two Independent Nodes

Fig. 3.21 shows a network with two independent nodes $B$ and $C$. We take node $D$ (or $E$) as the reference node. We shall use Kirchhoff's current law for nodes $B$ and $C$ to find $V_B$ and $V_C$. Once the values of $V_B$ and $V_C$ are known, we can find all the branch currents in the network.

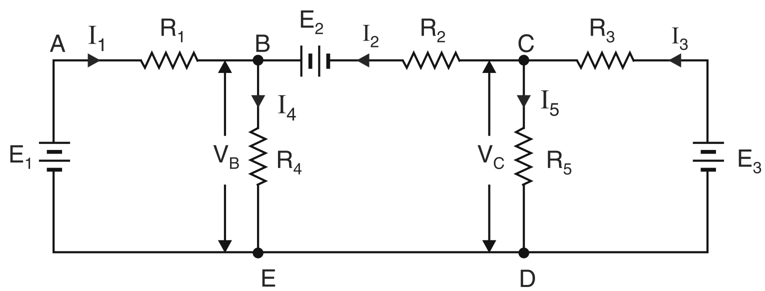



**Fig. 3.21**

Each current can be expressed in terms of e.m.f.s, resistances (or conductances), $V_B$ and $V_C$.

$$E_1 = V_B + I_1R_1 \quad \therefore I_1 = \frac{E_1 - V_B}{R_1}$$

$$E_3 = V_C + I_3R_3 \quad \therefore I_3 = \frac{E_3 - V_C}{R_3}$$

$$E_2{}^* = V_B - V_C + I_2R_2 \quad \therefore I_2 = \frac{E_2 - V_B + V_C}{R_2}$$

Similarly, $$I_4 = \frac{V_B}{R_4} \ ; I_5 = \frac{V_C}{R_5}$$

**At node B.** $$I_1 + I_2 = I_4$$

or $$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B + V_C}{R_2} = \frac{V_B}{R_4} \qquad ...(i)$$

**At node C.** $$I_2 + I_5 = I_3$$

or $$\frac{E_2 - V_B + V_C}{R_2} + \frac{V_C}{R_5} = \frac{E_3 - V_C}{R_3} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we can find $V_B$ and $V_C$ since all other quantities are known. Once we know the values of $V_B$ and $V_C$, we can find all the branch currents in the network.

**Note.** We can also express currents in terms of conductances as under :

$I_1 = (E_1 - V_B)\,G_1 \ ; \ I_2 = (E_2 - V_B + V_C)\,G_2$

$I_3 = (E_3 - V_C)\,G_3 \ ; \ I_4 = V_B\,G_4 \ ; \ I_5 = V_C\,G_5$

* As we go from $C$ to $B$, we have,

$$V_C - I_2R_2 + E_2 = V_B$$

$$\therefore \quad E_2 = V_B - V_C + I_2R_2$$

**Example 3.9.** *Find the currents in the various branches of the circuit shown in Fig. 3.22 by nodal analysis.*

**Solution.** Mark the currents in the various branches as shown in Fig. 3.22. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point $E$ (or $F$) as the reference node. We shall find the voltages at nodes $B$ and $C$.

**At node B.** $$I_2 + I_3 = I_1$$

or $$\frac{V_B}{10} + \frac{{}^*V_B - V_C}{15} = \frac{100 - V_B}{20}$$

or $$13V_B - 4V_C = 300 \qquad ...(i)$$

**At node C.** $$I_4 + I_5 = I_3$$

or $$\frac{V_C}{10} + \frac{V_C + 80}{10} = \frac{V_B - V_C}{15}$$

or $$V_B - 4V_C = 120 \qquad ...(ii)$$

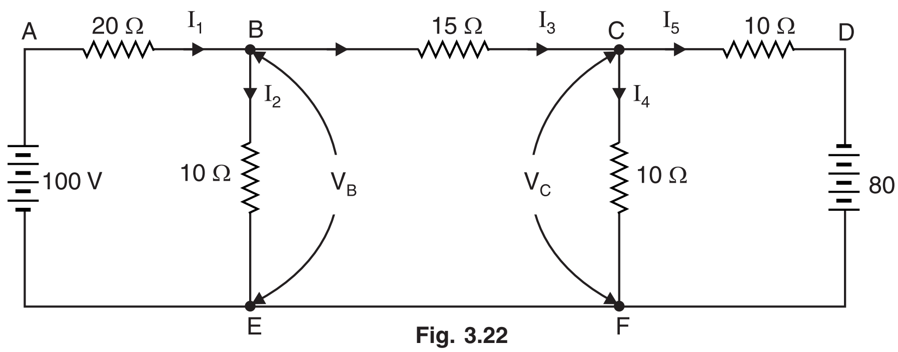



**Fig. 3.22**

Subtracting eq. ($ii$) from eq. ($i$), we get, $12V_B = 180$ $\quad\therefore\quad V_B = 180/12 = 15$ V

Putting $V_B = 15$ volts in eq. ($i$), we get, $V_C = -26.25$ volts.

**By determinant method**

$$13V_B - 4V_C = 300$$

$$V_B - 4V_C = 120$$

$$\therefore \qquad V_B = \frac{\begin{vmatrix} 300 & -4 \\ 120 & -4 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ 1 & -4 \end{vmatrix}} = \frac{(300 \times -4) - (120 \times -4)}{(13 \times -4) - (1 \times -4)} = \frac{-720}{-48} = 15 \text{ V}$$

$$\text{and} \qquad V_C = \frac{\begin{vmatrix} 13 & 300 \\ 1 & 120 \end{vmatrix}}{\text{Denominator}} = \frac{(13 \times 120) - (1 \times 300)}{-48} = \frac{1260}{-48} = -26{\cdot}25 \text{ V}$$

$$\therefore \qquad \text{Current } I_1 = \frac{100 - V_B}{20} = \frac{100 - 15}{20} = \mathbf{4{\cdot}25\ A}$$

$$\text{Current } I_2 = V_B/10 = 15/10 = \mathbf{1{\cdot}5\ A}$$

$$\text{Current } I_3 = \frac{V_B - V_C}{15} = \frac{15 - (-26.25)}{15} = \mathbf{2{\cdot}75\ A}$$

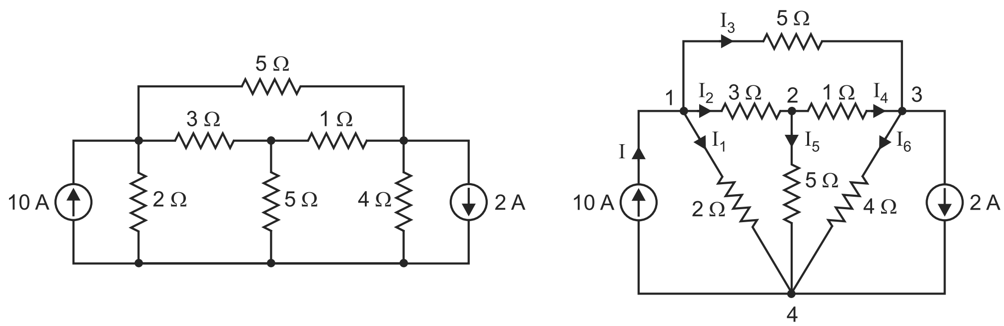
* Note that the current $I_3$ is assumed to flow from $B$ to $C$. Therefore, with this assumption, $V_B > V_C$.

$$\text{Current } I_4 = V_C/10 = -26{\cdot}25/10 = \mathbf{-2{\cdot}625\ A}$$

$$\text{Current } I_5 = \frac{V_C + 80}{10} = \frac{-26.25 + 80}{10} = \mathbf{5{\cdot}375\ A}$$

The negative sign for $I_4$ shows that actual current flow is opposite to that of assumed.

**Example 3.10.** *Use nodal analysis to find the currents in various resistors of the circuit shown in Fig. 3.23 (i).*




(*i*) (*ii*)

**Fig. 3.23**

**Solution.** The given circuit is redrawn in Fig. 3.23 (*ii*) with nodes marked 1, 2, 3 and 4. Let us take node 4 as the reference node. We shall apply *KCL* at nodes 1, 2 and 3 to obtain the solution.

**At node 1.** Applying *KCL*, we have,

$$I_1 + I_2 + I_3 = I$$

or $$\frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 10$$

or $$31V_1 - 10V_2 - 6V_3 = 300 \qquad ...(i)$$

**At node 2.** Applying *KCL*, we have,

$$I_2 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{3} = \frac{V_2 - V_3}{1} + \frac{V_2}{5}$$

or $$5V_1 - 23V_2 + 15V_3 = 0 \qquad ...(ii)$$

**At node 3.** Applying *KCL*, we have,

$$I_3 + I_4 = I_6 + 2$$

or $$\frac{V_1 - V_3}{5} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} + 2$$

or $$4V_1 + 20V_2 - 29V_3 = 40 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $V_1 = \dfrac{6572}{545}\text{V}\ ;\ V_2 = \dfrac{556}{109}\text{V}\ ;\ V_3 = \dfrac{2072}{545}\text{V}$

$\therefore$ $$\text{Current } I_1 = \frac{V_1}{2} = \frac{6572}{545} \times \frac{1}{2} = \mathbf{6.03\ A}$$

$$\text{Current } I_2 = \frac{V_1 - V_2}{3} = \frac{1}{3}\left[\frac{6572}{545} - \frac{556}{109}\right] = \mathbf{2.32A}$$

$$\text{Current } I_3 = \frac{V_1 - V_3}{5} = \frac{1}{5}\left[\frac{6572}{545} - \frac{2072}{545}\right] = \mathbf{1.65\ A}$$

$$\text{Current } I_4 = \frac{V_2 - V_3}{1} = \frac{556}{109} - \frac{2072}{545} = \mathbf{1.3A}$$

$$\text{Current } I_5 = \frac{V_2}{5} = \frac{556}{109} \times \frac{1}{5} = \mathbf{1.02A}$$

$$\text{Current } I_6 = \frac{V_3}{4} = \frac{2072}{545} \times \frac{1}{4} = \mathbf{0.95A}$$

**Example 3.11.** *Find the total power consumed in the circuit shown in Fig. 3.24.*

**Solution.** Mark the direction of currents in the various branches as shown in Fig. 3.24. Take $D$ as the reference node. If voltages $V_B$ and $V_C$ at nodes $B$ and $C$ respectively are known, then all the currents can be calculated.

**At node B.** $I_1 + I_3 = I_2$

or $$\frac{15 - V_B}{1} + \frac{V_C - V_B}{0.5} = \frac{V_B}{1}$$

or $$15 - V_B + 2(V_C - V_B) - V_B = 0$$

or $$4V_B - 2V_C = 15 \qquad \text{...}(i)$$

**At node C.** $I_3 + I_4 = I_5$

or $$\frac{V_C - V_B}{0.5} + \frac{V_C}{2} = \frac{20 - V_C}{1}$$

or $$2(V_C - V_B) + 0{\cdot}5V_C - (20 - V_C) = 0$$

or $$3{\cdot}5V_C - 2V_B = 20$$

or $$4V_B - 7V_C = -40 \qquad \text{...}(ii)$$

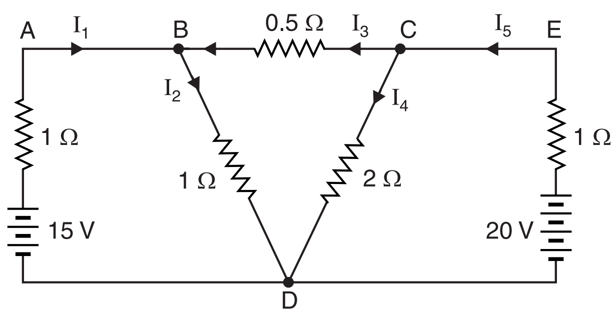



**Fig. 3.24**

Subtracting eq. (*ii*) from eq. (*i*), we get, $5V_C = 55$

$\therefore$ $V_C = 55/5 = 11$ volts

Putting $V_C = 11$ V in eq. (*i*), we get, $V_B = 9{\cdot}25$ V

$$\therefore \quad \text{Current } I_1 = \frac{15 - V_B}{1} = \frac{15 - 9.25}{1} = 5{\cdot}75 \text{ A}$$

$$\text{Current } I_2 = V_B/1 = 9{\cdot}25/1 = 9{\cdot}25 \text{ A}$$

$$\text{Current } I_3 = \frac{V_C - V_B}{0.5} = \frac{11 - 9.25}{0.5} = 3{\cdot}5 \text{ A}$$

$$\text{Current } I_4 = V_C/2 = 11/2 = 5{\cdot}5 \text{ A}$$

$$\text{Current } I_5 = \frac{20 - V_C}{1} = \frac{20 - 11}{1} = 9 \text{ A}$$

$\therefore$ Power loss in the circuit $= I_1^2 \times 1 + I_2^2 \times 1 + I^2{}_3 \times 0{\cdot}5 + I_4{}^2 \times 2 + I_5^2 \times 1$

$= (5{\cdot}75)^2 \times 1 + (9{\cdot}25)^2 \times 1 + (3{\cdot}5)^2 \times 0{\cdot}5 + (5{\cdot}5)^2 \times 2 + (9)^2 \times 1$

$=$ **266·25 W**

**Example 3.12.** *Using nodal analysis, find node-pair voltages $V_B$ and $V_C$ and branch currents in the circuit shown in Fig. 3.25. Use conductance method.*

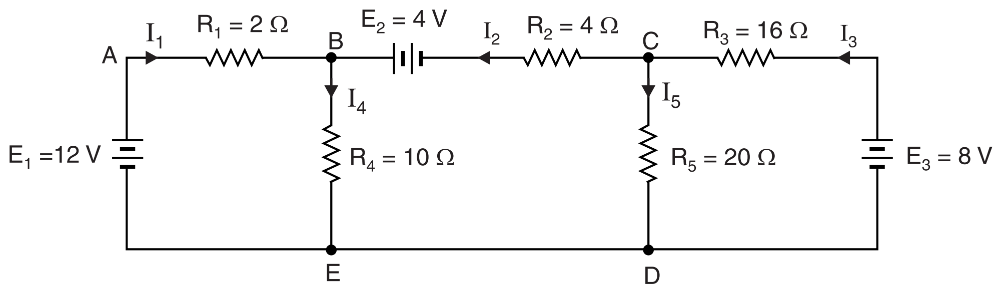



**Fig. 3.25**

**Solution.** Mark the currents in the various branches as shown in Fig. 3.25. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point $D$ (or $E$) as the reference node. We shall find the voltages at nodes $B$ and $C$ and hence the branch currents.

$$G_1 = \frac{1}{R_1} = \frac{1}{2} = 0{\cdot}5 \text{ S} \; ; \; G_2 = \frac{1}{R_2} = \frac{1}{4} = 0{\cdot}25 \text{ S} \; ; \; G_3 = \frac{1}{R_3} = \frac{1}{16} = 0{\cdot}0625 \text{ S} \; ;$$

$$G_4 = \frac{1}{R_4} = \frac{1}{10} = 0.1 \text{ S} \; ; \quad G_5 = \frac{1}{R_5} = \frac{1}{20} = 0{\cdot}05 \text{ S}$$

**At node *B*.** $I_1 + I_2 = I_4$

or $(E_1 - V_B)G_1 + (E_2 - V_B + V_C)G_2 = V_B G_4$

or $E_1G_1 + E_2G_2 = V_B(G_1 + G_2 + G_4) - V_C G_2$

or $(12 \times 0.5) + (4 \times 0.25) = V_B(0{\cdot}5 + 0{\cdot}25 + 0{\cdot}1) - V_C \times 0{\cdot}25$

or $7 = 0{\cdot}85\, V_B - 0{\cdot}25\, V_C$ ...(*i*)

**At node *C*.** $I_3 = I_2 + I_5$

or $(E_3 - V_C)G_3 = (E_2 - V_B + V_C)G_2 + V_C \times G_5$

or $E_3G_3 - E_2G_2 = -V_BG_2 + V_C(G_2 + G_3 + G_5)$

or $(8 \times 0.0625) - (4 \times 0.25) = -V_B(0{\cdot}25) + V_C(0{\cdot}25 + 0{\cdot}0625 + 0{\cdot}05)$

or $-0{\cdot}5 = -0{\cdot}25\, V_B + 0{\cdot}362\, V_C$ ...(*ii*)

From equations (*i*) and (*ii*), we get, $V_B =$ **9.82 V** ; $V_C =$ **5·4V**

$\therefore$ $I_1 = (E_1 - V_B)G_1 = (12 - 9{\cdot}82) \times 0{\cdot}5 =$ **1·09 A**

$I_2 = (E_2 - V_B + V_C)G_2 = (4 - 9{\cdot}82 + 5{\cdot}4) \times 0{\cdot}25 =$ **–0·105A**

$I_3 = (E_3 - V_C)G_3 = (8 - 5{\cdot}4) \times 0{\cdot}0625 =$ **0·162A**

$I_4 = V_BG_4 = 9{\cdot}82 \times 0{\cdot}1 =$ **0·982A**

$I_5 = V_CG_5 = 5{\cdot}4 \times 0.05 =$ **0·27A**

The negative sign for $I_2$ means that the actual direction of this current is opposite to that shown in Fig. 3.25.

**Example 3.13.** *Using nodal analysis, find the different branch currents in the circuit shown in Fig. 3.26 (i).*

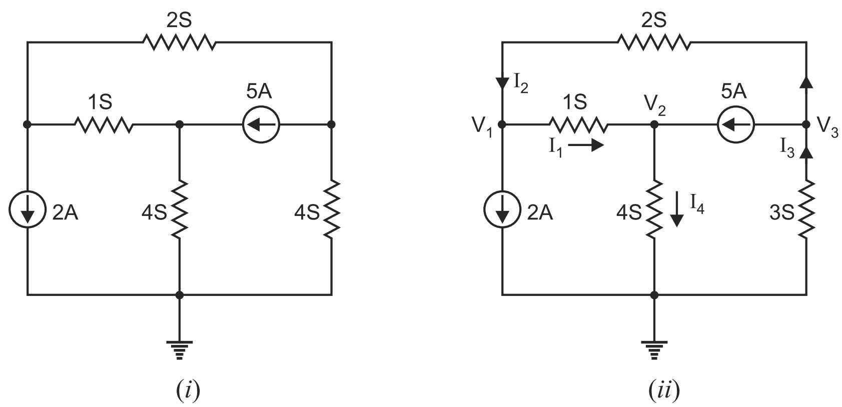



**Fig. 3.26**

**Solution.** Mark the currents in the various branches as shown in Fig. 3.26 (*ii*). Take ground as the reference node. We shall find the voltages at the other three nodes.

**At first node.** Applying *KCL* to the first node from left,

$$I_2 = I_1 + 2$$

or $$(V_3 - V_1)2 = (V_1 - V_2)1 + 2$$

or $$3V_1 - V_2 - 2V_3 = -2 \qquad ...(i)$$

**At second node.** Applying *KCL* to the second node from left,

$$I_1 + 5 = I_4$$

or $$(V_1 - V_2)1 + 5 = V_2 \times 4$$

or $$V_1 - 5V_2 = -5 \qquad ...(ii)$$

**At third node.** Applying *KCL* to the third node from left,

$$I_3 = 5 + I_2$$

or $$-V_3 \times 3 = 5 + (V_3 - V_1)2$$

or $$2V_1 - 5V_3 = 5 \qquad ...(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we have, $V_1 = -\frac{3}{2}\text{V}$ ; $V_2 = \frac{7}{10}\text{V}$ and $V_3 = \frac{-8}{5}\text{V}$

$$\therefore \quad I_1 = (V_1 - V_2)1 = \left(-\frac{3}{2} - \frac{7}{10}\right)1 = \mathbf{-2.2A}$$

$$I_2 = (V_3 - V_1)2 = \left(-\frac{8}{5} + \frac{3}{2}\right)2 = \mathbf{-0.2A}$$

$$I_3 = -V_3 \times 3 = \frac{8}{5} \times 3 = \mathbf{4.8\ A}$$

$$I_4 = V_2 \times 4 = \frac{7}{10} \times 4 = \mathbf{2.8A}$$

The negative value of any current means that actual direction of current is opposite to that originally assumed.

**Example 3.14.** *Find the current I in Fig. 3.27 (i) by changing the two voltage sources into their equivalent current sources and then using nodal method. All resistances are in ohms.*

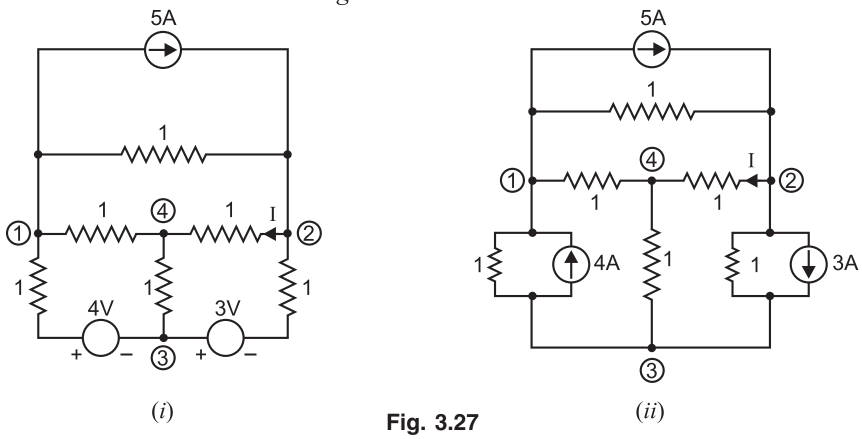



**Fig. 3.27**

**Solution.** Since we are to find $I$, it would be convenient to take node 4 as the reference node. The two voltage sources are converted into their equivalent current sources as shown in Fig. 3.27. (*ii*). We shall apply *KCL* at nodes 1, 2 and 3 in Fig. 3.27 (*ii*) to obtain the required solution.

**At node 1.** Applying *KCL*, we have,

$$\frac{V_3 - V_1}{1} + 4 = \frac{V_1}{1} + \frac{V_1 - V_2}{1} + 5$$

or $\quad 3V_1 - V_2 - V_3 = -1 \quad$ ...(*i*)

**At node 2.** Applying *KCL*, we have,

$$5 + \frac{V_1 - V_2}{1} = \frac{V_2}{1} + \frac{V_2 - V_3}{1} + 3$$

or $\quad V_1 - 3V_2 + V_3 = -2 \quad$ ...(*ii*)

**At node 3.** Applying *KCL*, we have,

$$\frac{V_2 - V_3}{1} + 3 - \frac{V_3}{1} = \frac{V_3 - V_1}{1} + 4$$

or $\quad V_1 + V_2 - 3V_3 = 1 \quad$ ...(*iii*)

From eqs. (*i*), (*ii*) and (*iii*), we get, $V_2 = 0.5$ V.

$$\therefore \quad \text{Current } I = \frac{V_2 - 0}{1} = \frac{0.5 - 0}{1} = \mathbf{0.5A}$$

**Example 3.15.** *Use nodal analysis to find the voltage across and current through 4 Ω resistor in Fig. 3.28 (i).*

**Solution.** We must first convert the 2V voltage source to an equivalent current source. The value of the equivalent current source is $I = 2\text{V}/2\Omega = 1$ A. The circuit then becomes as shown in Fig. 3.28 (*ii*).

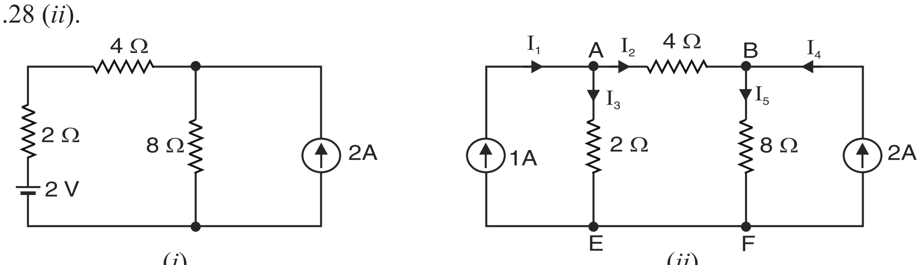



**Fig. 3.28**

Mark the currents in the various branches as shown in Fig. 3.28 (*ii*). Take point $E$ (or $F$) as the reference node. We shall calculate the voltages at nodes $A$ and $B$.

**At node A.** $I_1 = I_2 + I_3$

or $$1 = \frac{{}^*V_A - V_B}{4} + \frac{V_A}{2}$$

or $$3V_A - V_B = 4 \quad ...(i)$$

**At node B.** $I_2 + I_4 = I_5$

or $$\frac{V_A - V_B}{4} + 2 = \frac{V_B}{8}$$

or $$2V_A - 3V_B = -16 \quad ...(ii)$$

Solving equations (*i*) and (*ii*), we find $V_A = 4$V and $V_B = 8$V. Note that $V_B > V_A$, contrary to our initial assumption. Therefore, actual direction of current is from node $B$ to node $A$.

**By determinant method**

$$3V_A - V_B = 4$$
$$2V_A - 3V_B = -16$$

$$\therefore \quad V_A = \frac{\begin{vmatrix} 4 & -1 \\ -16 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}} = \frac{(-12)-(16)}{(-9)-(-2)} = \frac{-28}{-7} = 4\text{V}$$

$$V_B = \frac{\begin{vmatrix} 3 & 4 \\ 2 & -16 \end{vmatrix}}{\text{Denominator}} = \frac{(-48)-(8)}{-7} = \frac{-56}{-7} = 8\text{V}$$

Voltage across 4Ω resistor = $V_B - V_A = 8 - 4 =$ **4V**

Current through 4Ω resistor $= \frac{4\text{V}}{4\Omega} =$ **1A**

We can also find the currents in other resistors.

$$I_3 = \frac{V_A}{2} = \frac{4}{2} = 2\text{A}$$

$$I_5 = \frac{V_B}{8} = \frac{8}{8} = 1\text{A}$$

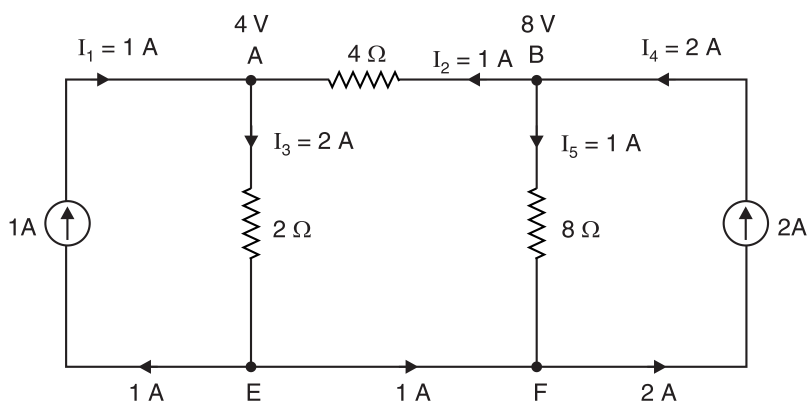



**Fig. 3.29**

* We assume that $V_A > V_B$. On solving the circuit, we shall see whether this assumption is correct or not.

Fig. 3.29 shows the various currents in the circuit. You can verify Kirchhoff's current law at each node.

**Example 3.16.** *Use nodal analysis to find current in the 4 kΩ resistor shown in Fig. 3.30.*

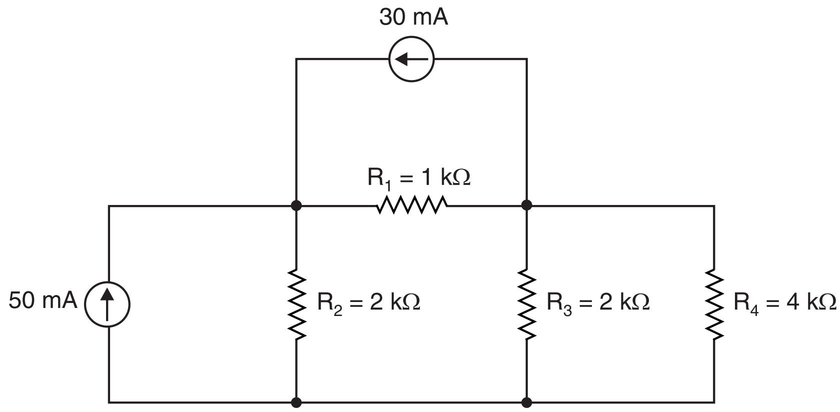



**Fig. 3.30**

**Solution.** We shall solve this example by expressing node currents in terms of conductance than expressing them in terms of resistance. The conductance of each resistor is

$$G_1 = \frac{1}{R_1} = \frac{1}{1 \times 10^3} = 10^{-3}\text{ S} \;;\; G_2 = \frac{1}{R_2} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3}\text{ S}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3}\text{ S} \;;\; G_4 = \frac{1}{R_4} = \frac{1}{4 \times 10^3} = 0.25 \times 10^{-3}\text{ S}$$

Mark the currents in the various branches as shown in Fig. 3.31. Take point *E* (or *F*) as the reference node. We shall find voltages at nodes *A* and *B*.

**At node A.** $I_5 + I_6 = I_1 + I_2$

or $50 \times 10^{-3} + 30 \times 10^{-3} = G_1(V_A - V_B) + G_2 V_A$

or $80 \times 10^{-3} = 10^{-3}(V_A - V_B) + 0.5 \times 10^{-3} V_A$

or $1.5V_A - V_B = 80$ ...(*i*)

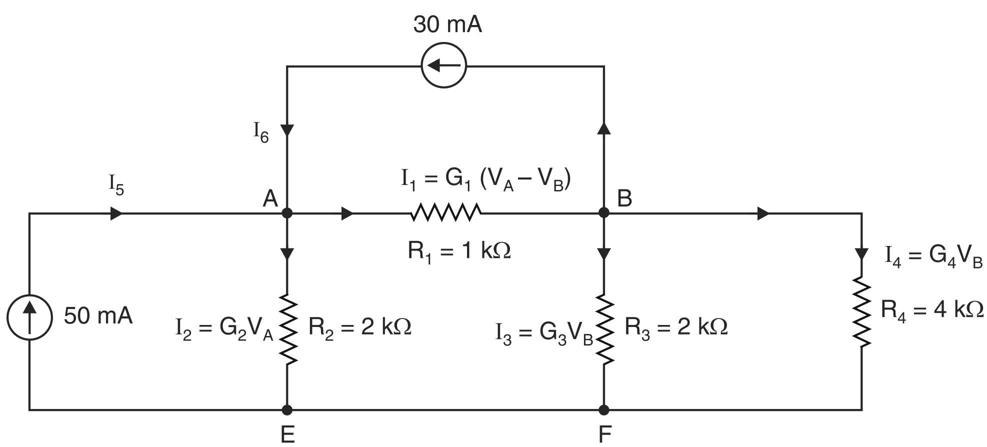



**Fig. 3.31**

**At node B.** $I_1 = I_6 + I_3 + I_4$

or $G_1(V_A - V_B) = 30 \times 10^{-3} + G_3 V_B + G_4 V_B$

or $$10^{-3}(V_A - V_B) = 30 \times 10^{-3} + 0.5 \times 10^{-3} V_B + 0{\cdot}25 \times 10^{-3} V_B$$

or $$V_A - 1{\cdot}75\, V_B = 30 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get, $V_B = 21{\cdot}54$ V.

**By determinant method**

$$1{\cdot}5\, V_A - V_B = 80$$

$$V_A - 1{\cdot}75\, V_B = 30$$

$$\therefore \quad V_B = \frac{\begin{vmatrix} 1.5 & 80 \\ 1 & 30 \end{vmatrix}}{\begin{vmatrix} 1.5 & -1 \\ 1 & -1.75 \end{vmatrix}} = \frac{(45)-(80)}{(-2.625)-(-1)} = \frac{-35}{-1.625} = 21{\cdot}54 \text{ V}$$

$\therefore$ Current in 4 kΩ resistor, $I_4 = G_4 V_B = 0{\cdot}25 \times 10^{-3} \times 21{\cdot}54 = 5{\cdot}39 \times 10^{-3}$ A = **5·39 mA**

**Example 3.17.** *For the circuit shown in Fig. 3.32 (i), find (i) voltage v and (ii) current through 2Ω resistor using nodal method.*

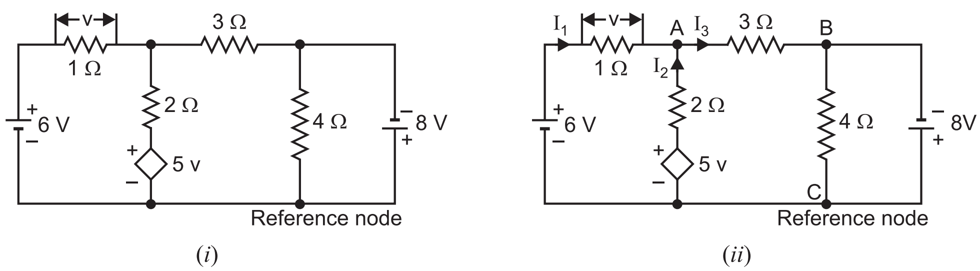



**Fig. 3.32**

**Solution.** Mark the direction of currents in the various branches as shown in Fig. 3.32 (*ii*). Let us take node *C* as the reference node. It is clear from Fig. 3.32 (*ii*) that $V_B = -8$V ($\because V_C = 0$V). Also, $v = 6 - V_A$.

Applying *KCL* to node A, we have,

$$I_1 + I_2 = I_3$$

or $$\frac{6 - V_A}{1} + \frac{5v - V_A}{2} = \frac{V_A - V_B}{3}$$

or $$\frac{6 - V_A}{1} + \frac{5(6 - V_A) - V_A}{2} = \frac{V_A - (-8)}{3}$$

On solving, we get, $V_A = \dfrac{55}{13}$ V

(*i*) Voltage $v = 6 - V_A = 6 - \dfrac{55}{13} = \mathbf{\dfrac{23}{13}}$ **V**

(*ii*) Current through 2Ω, $I_2 = \dfrac{5v - V_A}{2} = \dfrac{5(23/13) - (55/13)}{2} = \mathbf{\dfrac{30}{13}}$ **A**

## 3.8. Shortcut Method for Nodal Analysis

There is a shortcut method for writing node equations similar to the form for mesh equations. Consider the circuit with three independent nodes *A*, *B* and *C* as shown in Fig. 3.33.

The node equations in shortcut form for nodes *A*, *B* and *C* can be written as under :

$$V_A G_{AA} + V_B G_{AB} + V_C G_{AC} = I_A$$
$$V_A G_{BA} + V_B G_{BB} + V_C G_{BC} = I_B$$
$$V_A G_{CA} + V_B G_{CB} + V_C G_{CC} = I_C$$

Let us discuss the various terms in these equations.

$$G_{AA} = \text{Sum of all conductances connected to node } A$$
$$= G_1 + G_2 \text{ in Fig. 3.33.}$$

The term $G_{AA}$ is called *self-conductance* at node $A$. Similarly, $G_{BB}$ and $G_{CC}$ are self-conductances at nodes $B$ and $C$ respectively. *Note that product of node voltage at a node and self-conductance at that node is always a* **positive** *quantity.* Thus $V_A G_{AA}$, $V_B G_{BB}$ and $V_C G_{CC}$ are all positive.

$$G_{AB} = \text{Sum of all conductances } \textit{directly} \text{ connected between nodes } A \text{ and } B$$
$$= G_2 \text{ in Fig. 3.33}$$

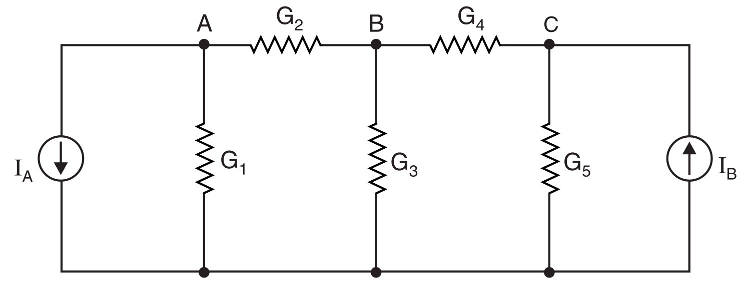



**Fig. 3.33**

The term $G_{AB}$ is called *common conductance* between nodes $A$ and $B$. Similarly, the term $G_{BC}$ is common conductance between nodes $B$ and $C$ and $G_{CA}$ is common conductance between nodes $C$ and $A$. *The product of connecting node voltage with common conductance is always a* **negative** *quantity.* Thus $V_B G_{AB}$ is a negative quantity. Here connecting node voltage is $V_B$ and common conductance is $G_{AB}$. Note that $G_{AB} = G_{BA}$, $G_{AC} = G_{CA}$ and so on.

Note the direction of current provided by current source connected to the node. *A current leaving the node is shown as negative and a current entering a node is positive. If a node has no current source connected to it, set the term equal to zero.*

**Node A.** Refer to Fig. 3.33. At node $A$, $G_{AA} = G_1 + G_2$ and is a positive quantity. The product $V_B G_{AB}$ is a negative quantity. The current $I_A$ is leaving the node $A$ and will be assigned a negative sign. Therefore, node equation at node $A$ is

$$V_A G_{AA} - V_B G_{AB} = -I_A$$

or
$$V_A(G_1 + G_2) - V_B(G_2) = -I_A$$

Similarly, for **nodes B and C,** the node equations are :

$$V_B(G_2 + G_3 + G_4) - V_A(G_2) - V_C(G_4) = 0$$
$$V_C(G_4 + G_5) - V_B(G_4) = I_B$$

**Example 3.18.** *Solve the circuit shown in Fig. 3.34 using nodal analysis.*

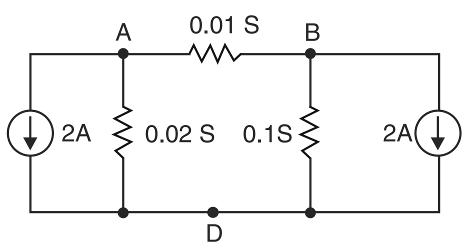



**Fig. 3.34**

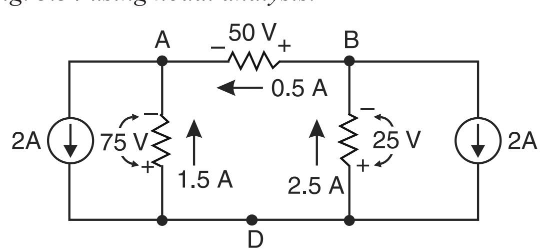



**Fig. 3.35**

**Solution.** Here point $D$ is chosen as the reference node and $A$ and $B$ are the independent nodes.

**Node A.** $V_A(0.02 + 0.01) - V_B(0.01) = -2$

or $\quad 0.03\ V_A - 0.01\ V_B = -2 \quad$ ...(*i*)

**Node B.** $V_B(0.01 + 0.1) - V_A\ (0.01) = -2$

or $\quad -0.01\ V_A + 0.11\ V_B = -2 \quad$ ...(*ii*)

From equations (*i*) and (*ii*), we have, $V_A = -75\text{V}$ and $V_B = -25\text{V}$

Fig. 3.35 shows the circuit redrawn with solved voltages.

$$\text{Current in } 0.02\ \text{S} = VG = 75 \times 0{\cdot}02 = \mathbf{1{\cdot}5A}$$
$$\text{Current in } 0.1\ \text{S} = VG = 25 \times 0{\cdot}1 = \mathbf{2{\cdot}5A}$$
$$\text{Current in } 0.01\ \text{S} = VG = 50 \times 0{\cdot}01 = \mathbf{0{\cdot}5\ A}$$

The directions of currents will be as shown in Fig. 3.35.

**Example 3.19.** *Solve the circuit shown in Fig. 3.36 using nodal analysis.*

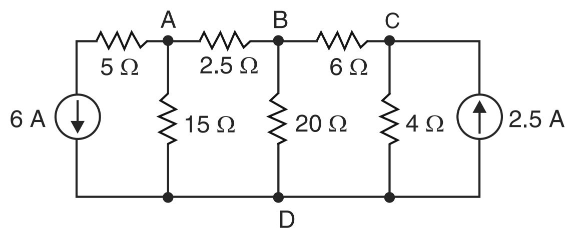



**Fig. 3.36**

**Solution.** Here $A$, $B$ and $C$ are the independent nodes and $D$ is the reference node.

**Node A.** $$V_A^{*}\left(\frac{1}{15}+\frac{1}{2.5}\right)-V_B\left(\frac{1}{2.5}\right) = -6$$

or $\quad 0{\cdot}467\ V_A - 0{\cdot}4\ V_B = -6 \quad$ ...(*i*)

**Node B.** $$V_B\left(\frac{1}{2.5}+\frac{1}{20}+\frac{1}{6}\right)-V_A\left(\frac{1}{2.5}\right)-V_C\left(\frac{1}{6}\right) = 0$$

or $\quad -0{\cdot}4\ V_A + 0{\cdot}617\ V_B - 0{\cdot}167\ V_C = 0 \quad$ ...(*ii*)

**Node C.** $$V_C\left(\frac{1}{6}+\frac{1}{4}\right)-V_B\left(\frac{1}{6}\right) = 2{\cdot}5$$

or $\quad -0{\cdot}167\ V_B + 0{\cdot}417\ V_C = 2{\cdot}5 \quad$ ...(*iii*)

From equations (*i*), (*ii*) and (*iii*), $V_A = -30\ \text{V}$ ; $V_B = -20\ \text{V}$ ; $V_C = -2\ \text{V}$

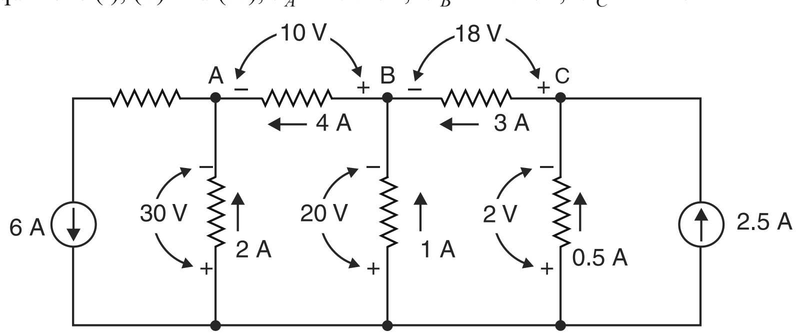



**Fig. 3.37**

* Note that 5Ω is omitted from the equation for node $A$ because it is in series with the current source.

Fig. 3.37 shows the circuit redrawn with solved voltages.

$$\text{Current in } 15\ \Omega = 30/15 = \mathbf{2\ A}$$
$$\text{Current in } 20\ \Omega = 20/20 = \mathbf{1\ A}$$
$$\text{Current in } 4\ \Omega = 2/4 = \mathbf{0.5\ A}$$
$$\text{Current in } 6\ \Omega = 18/6 = \mathbf{3\ A}$$
$$\text{Current in } 2.5\ \Omega = 10/2.5 = \mathbf{4\ A}$$
$$\text{Current in } 5\ \Omega = 4 + 2 = \mathbf{6\ A}$$

The directions of currents will be as shown in Fig. 3.37.

**Example 3.20.** *Find the value of $I_x$ in the circuit shown in Fig. 3.38 using nodal analysis. The various values are :*

*$G_u = 10\ S$ ; $G_v = 1S$ ; $G_w = 2S$ ;*

*$G_x = 1S$ ; $G_y = 1S$ ; $G_z = 1S$ and $I = 100\ A$.*

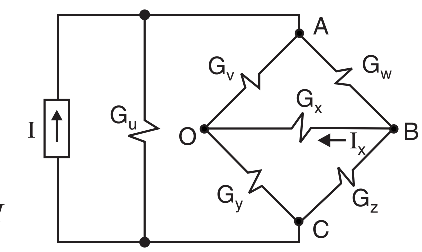



Fig. 3.38

**Solution.**

**Node A.** $(G_u + G_v + G_w)V_A - G_wV_B - G_uV_C = I$

**Node B.** $-G_wV_A + (G_w + G_x + G_z)V_B - G_zV_C = 0$

**Node C.** $-G_uV_A - G_zV_B + (G_u + G_y + G_z)V_C = -I$

Putting the various values in these equations, we have,

$$13\ V_A - 2\ V_B - 10\ V_C = I$$
$$-2\ V_A + 4\ V_B - V_C = 0$$
$$-10\ V_A - V_B + 12\ V_C = -I$$

Now $V_B$ can be calculated as the ratio of two determinants $N_B/D$ where

$$D = \begin{vmatrix} 13 & -2 & -10 \\ -2 & 4 & -1 \\ -10 & -1 & 12 \end{vmatrix} = 624 - 20 - 20 - (400 + 48 + 13) = 123$$

and

$$N_B = \begin{vmatrix} 13 & I & -10 \\ -2 & 0 & -1 \\ -10 & -I & 12 \end{vmatrix} = 10I - 20I - (13I - 24I) = I$$

$$\therefore \qquad V_B = \frac{N_B}{D} = \frac{I}{123}$$

$$\text{Current } I_x = G_xV_B = 1 \times \frac{I}{123} = 1 \times \frac{100}{123} = \mathbf{0.813A}$$

## Tutorial Problems

1. Using nodal analysis, find the voltages at nodes $A$, $B$ and $C$ w.r.t. the reference node shown by the ground symbol in Fig. 3.39.
**[$V_A$ = –30V ; $V_B$ = –20V ; $V_C$ = –2V]**

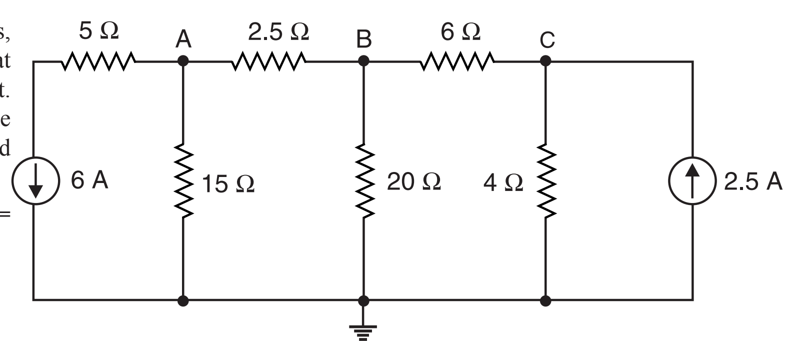



Fig. 3.39

2. Using nodal analysis, find the current through 0.05 S conductance in Fig. 3.40. **[0.264 A]**

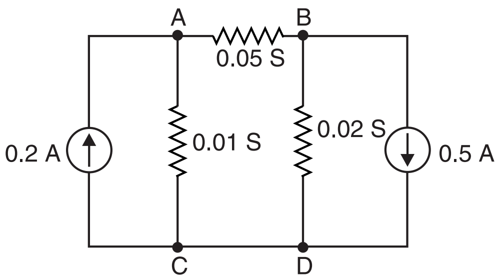



**Fig. 3.40**

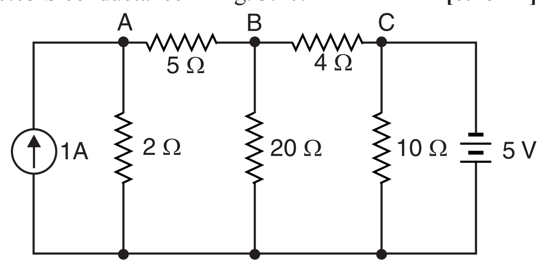



**Fig. 3.41**

3. Using nodal analysis, find the current flowing in the battery in Fig. 3.41. **[1.21 A]**

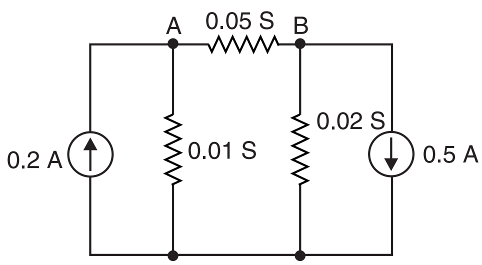



**Fig. 3.42**

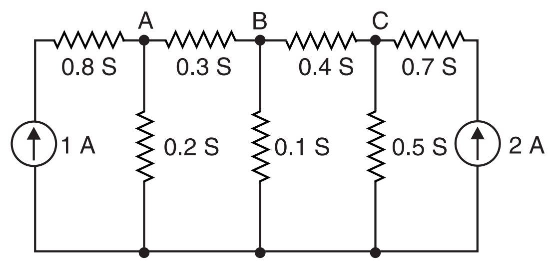



**Fig. 3.43**

4. In Fig. 3.42, find the node voltages. **[$V_A$ = –6.47 V; $V_B$ = – 11.8V]**
5. In Fig. 3.42, find current through 0.05 S conductance. Use nodal analysis. **[264 mA]**
6. In Fig. 3.43, find the node voltages. **[$V_A$ = 4.02 V; $V_B$ = 3.37 V ; $V_C$ = 3.72 V]**
7. By using nodal analysis, find current in 0.3 S in Fig. 3.43. **[196 mA]**
8. Using nodal analysis, find current in 0.4 S conductance in Fig. 3.43. **[141 mA]**

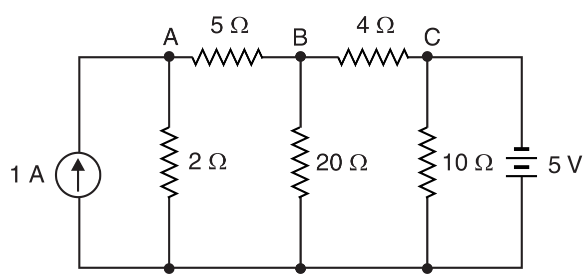



**Fig. 3.44**

9. Find node voltages in Fig. 3.44. **[$V_A$ = 0.806 V; $V_B$ = –2.18 V; $V_C$ = –5 V]**
10. Using nodal analysis, find current through the battery in Fig. 3.44. **[1. 21A]**

## 3.9. Superposition Theorem

*Superposition* is a general principle that allows us to determine the effect of several energy sources (voltage and current sources) acting simultaneously in a circuit by considering the effect of each source acting alone, and then combining (superposing) these effects. This theorem as applied to d.c. circuits may be stated as under :

*In a linear, bilateral d.c. network containing more than one energy source, the resultant potential difference across or current through any element is equal to the algebraic sum of potential differences or currents for that element produced by each source acting alone with all other independent ideal voltage sources replaced by short circuits and all other independent ideal current sources replaced by open circuits (non-ideal sources are replaced by their internal resistances).*

**Procedure.** The procedure for using this theorem to solve d.c. networks is as under :

(*i*) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

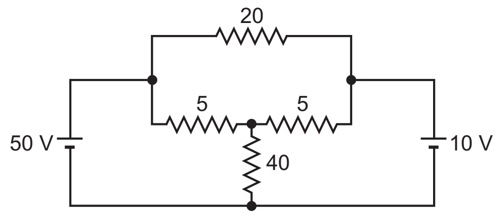
(*ii*) Determine the voltage across or current through the desired element/branch due to single source selected in step (*i*).

(*iii*) Repeat the above two steps for each of the remaining sources.

(*iv*) *Algebraically* add all the voltages across or currents through the element/branch under consideration. The sum is the actual voltage across or current through that element/branch when all the sources are acting simultaneously.

**Note.** This theorem is called *superposition* because we superpose or algebraically add the components (currents or voltages) due to each independent source acting alone to obtain the total current in or voltage across a circuit element.

**Example 3.21.** *Using superposition theorem, find the current through the 40 Ω resistor in the circuit shown in Fig. 3.45 (i). All resistances are in ohms.*

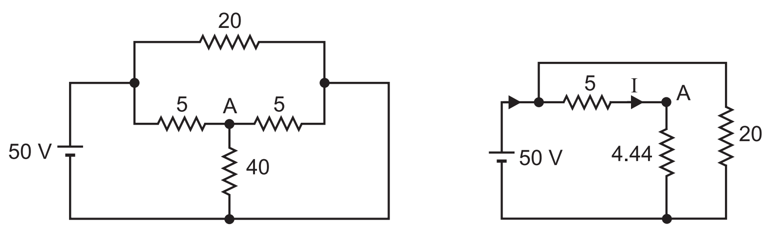



(*i*)

**Fig. 3.45**

**Solution.** In Fig. 3.45 (*ii*), 10V battery is replaced by a short so that 50V battery is acting alone. It can be seen that right-hand 5 Ω resistance is in parallel with 40 Ω resistance and their combined resistance = 5 Ω || 40 Ω = 4.44 Ω as shown in Fig. 3.45 (*iii*). The 4.44 Ω resistance is in series with left-hand 5 Ω resistance giving total resistance of (5 + 4.44) = 9.44 Ω to this path. As can be seen from Fig. 3.45 (*iii*), there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 50 V battery. Therefore, current through 9.44 Ω branch is $I = 50/9.44 = 5.296$ A. Thus in Fig. 3.45 (*ii*), the current $I$ (= 5.296 A) at point A divides between 5 Ω resistance and 40 Ω resistance. By current-divider rule, current $I_1$ in 40 Ω resistance is

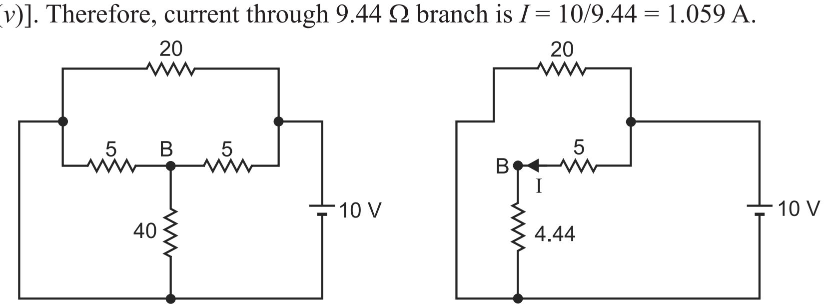
$$I_1 = I \times \frac{5}{5+40} = 5.296 \times \frac{5}{45} = 0.589 \text{ A downward}$$




(*ii*) (*iii*)

**Fig. 3.45**

In Fig. 3.45 (*iv*), the 50 V battery is replaced by a short so that 10 V battery is acting alone. Again, there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 10V battery [See Fig. 3.45 (*v*)]. Therefore, current through 9.44 Ω branch is $I = 10/9.44 = 1.059$ A.




(*iv*) (*v*)

**Fig. 3.45**

Thus in Fig. 3.45 (*iv*), the current $I$ (= 1.059 A) at point $B$ divides between 5 Ω resistance and 40 Ω resistance. By current-divider rule, current in 40 Ω resistance is

$$I_2 = 1.059 \times \frac{5}{5+40} = 0.118 \text{ A downward}$$

∴ By superposition theorem, the total current in 40 Ω

$$= I_1 + I_2 = 0.589 + 0.118 = \mathbf{0.707\ A\ \ downward}$$

**Example 3.22.** *In the circuit shown in Fig. 3.46 (i), the internal resistances of the batteries are 0·12 Ω and 0·08 Ω. Calculate (i) current in load (ii) current supplied by each battery.*

**Solution.** In Fig. 3.46 (*ii*), the right-hand 12 V source is replaced by its internal resistance so that left-hand battery of 12 V is acting alone. The various branch currents due to left-hand battery of 12 V alone [See Fig. 3.46 (*ii*)] are :

$$\text{Total circuit resistance} = 0.12 + \frac{0.08 \times 0.5}{0.08 + 0.5} = 0{\cdot}189\ \Omega$$

$$\text{Total circuit current, } I_1' = 12/0{\cdot}189 = 63{\cdot}5 \text{ A}$$

$$\text{Current in } 0{\cdot}08\ \Omega,\ I_2' = 63.5 \times \frac{0.5}{0.08+0.5} = 54{\cdot}74 \text{ A}$$

$$\text{Current in } 0{\cdot}5\ \Omega,\ I_3' = 63.5 \times \frac{0.08}{0.08+0.5} = 8{\cdot}76 \text{ A}$$

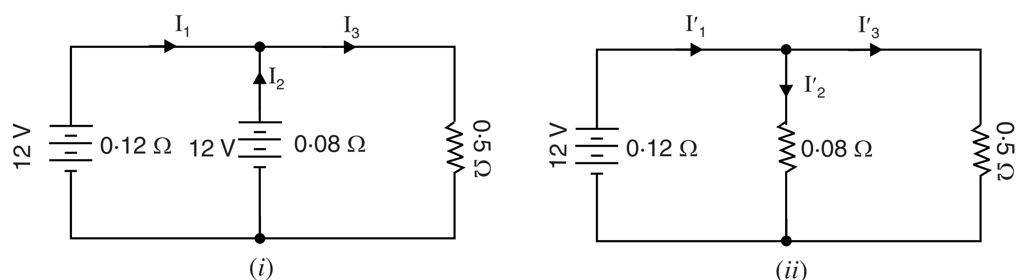



**Fig. 3.46**

In Fig. 3.46 (*iii*), left-hand 12 V source is replaced by its internal resistance so that now right-hand 12 V source is acting alone.

$$\text{Total circuit resistance} = 0.08 + \frac{0.12 \times 0.5}{0.12+0.5}$$

$$= 0{\cdot}177\ \Omega$$

$$\text{Total circuit current, } I_2'' = 12/0{\cdot}177 = 67{\cdot}8 \text{ A}$$

$$\text{Current in } 0{\cdot}12\ \ \Omega,\ I_1'' = 67.8 \times \frac{0.5}{0.12+0.5}$$

$$= 54{\cdot}6 \text{ A}$$

$$\text{Current in } 0{\cdot}5\ \ \Omega,\ I_3'' = 67.8 \times \frac{0.12}{0.12+0.5} = 13{\cdot}12 \text{ A}$$

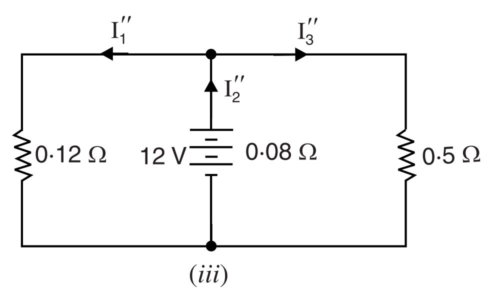



**Fig. 3.46**

The actual current values of $I_1$ (current in first battery), $I_2$ (current in second battery) and $I_3$ (load current) can be found by algebraically adding the component values.

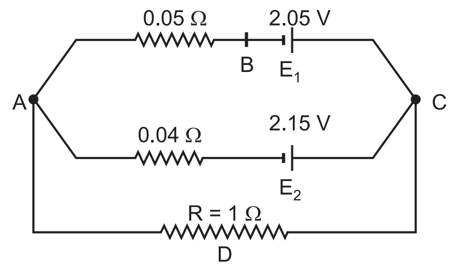
$$I_1 = I_1' - I_1'' = 63{\cdot}5 - 54{\cdot}6 = \mathbf{8{\cdot}9\ A}$$

$$I_2 = I_2'' - I_2' = 67{\cdot}8 - 54{\cdot}74 = \mathbf{13{\cdot}06\ A}$$

$$I_3 = I_3' + I_3'' = 8{\cdot}76 + 13{\cdot}12 = \mathbf{21{\cdot}88\ A}$$

**Example 3.23.** *By superposition theorem, find the current in resistance R in Fig. 3.47 (i).*

**Solution.** In Fig. 3.47 (*ii*), battery $E_2$ is replaced by a short so that battery $E_1$ is acting alone. It is clear that resistances of 1Ω (= $R$) and 0.04Ω are in parallel across points $A$ and $C$.




(*i*)

**Fig. 3.47**

$$\therefore \quad R_{AC} = 1\Omega \,||\, 0.04\Omega = \frac{1 \times 0.04}{1 + 0.04} = 0.038\ \Omega$$

This resistance (*i.e.,* $R_{AC}$) is in series with 0.05 Ω.

Total resistance to battery $E_1 = 0.038 + 0.05 = 0.088\Omega$

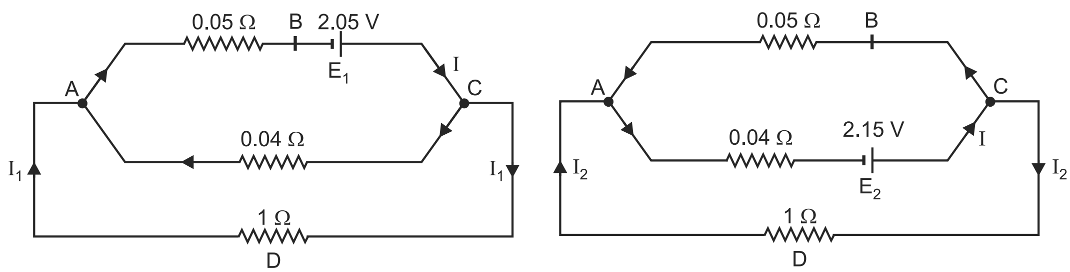
$\therefore$ Current supplied by battery $E_1$ is

$$I = \frac{E_1}{0.088} = \frac{2.05}{0.088} = 23.2\text{A}$$

The current $I$(= 23.2A) is divided between the parallel resistances of 1Ω (= $R$) and 0.04Ω.

$\therefore$ Current in 1Ω (= $R$) resistance is

$$I_1 = 23.2 \times \frac{0.04}{1 + 0.04} = 0.892\text{ A from } C \text{ to } A$$




(*ii*) (*iii*)

**Fig. 3.47**

In Fig. 3.47 (*iii*), battery $E_1$ is replaced by a short so that battery $E_2$ is acting alone.

Total resistance offered to battery $E_2$

$$= (1\Omega \,||\, 0.05\Omega) + 0.04\Omega$$

$$= \frac{1 \times 0.05}{1 + 0.05} + 0.04 = 0.088\Omega$$

$\therefore$ Current supplied by battery $E_2$ is

$$I = \frac{2.15}{0.088} = 24.4\text{A}$$

The current $I$(= 24.4A) is divided between two parallel resistances of 1Ω (= $R$) and 0.05Ω.

$\therefore$ Current in 1Ω (= $R$) resistance is

$$I_2 = 24.4 \times \frac{0.05}{1 + 0.05} = 1.16\text{A from } C \text{ to } A$$

$\therefore$ Current through 1Ω resistance when both batteries are present

$$= I_1 + I_2 = 0.892 + 1.16 = \mathbf{2.052A}$$

**Example 3.24.** *Using the superposition principle, find the voltage across 1k$\Omega$ resistor in Fig. 3·48. Assume the sources to be ideal.*

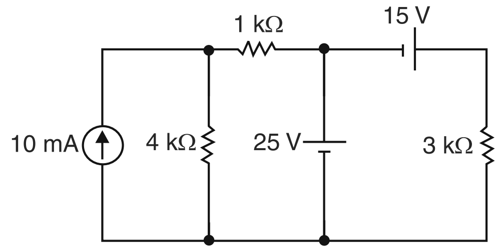



**Fig. 3.48**

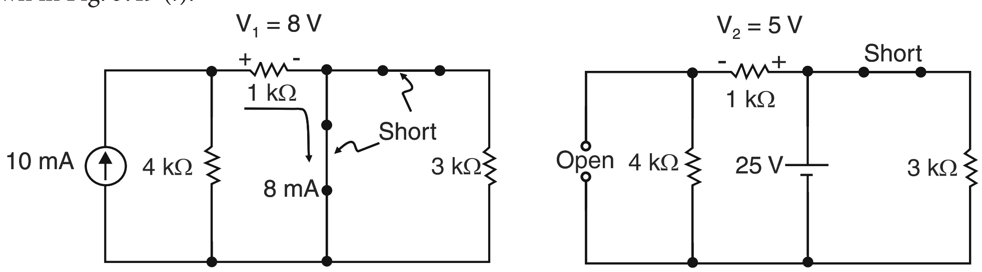
**Solution. (*i*)** The voltage across 1k$\Omega$ resistor due to **current source acting alone** is found by replacing 25-V and 15-V sources by short circuit as shown in Fig. 3.49 (*i*). Since 3 k$\Omega$ resistor is shorted out, the current in 1 k$\Omega$ resistor is, by current divider rule,

$$I_{1\,k\,\Omega} = \left(\frac{4}{1+4}\right)10 = 8 \text{ mA}$$

$\therefore$ Voltage $V_1$ across 1 k$\Omega$ resistor is

$$V_1 = (8 \text{ mA})\,(1 \text{ k}\Omega) = {}^{+}8\text{V}^{-}$$

The + and – symbols indicate the polarity of the voltage due to current source acting alone as shown in Fig. 3.49 (*i*).




Voltage across the 1-k Ω resistor due to the 10-mA current source acting alone.

(*i*)

Voltage across the 1-k Ω resistor due to the 25-V voltage source acting alone.

(*ii*)

**Fig. 3.49**

**(*ii*)** The voltage across the 1 k $\Omega$ resistor due to **25 V source acting alone** is found by replacing the 10 mA current source by an open circuit and 15 V source by a short circuit as shown in Fig. 3.49 (*ii*). Since the 25 V source is across the series combination of the 1 k$\Omega$ and 4 k $\Omega$ resistors, the voltage $V_2$ across 1 k$\Omega$ resistor can be found by the voltage divider rule.

$$\therefore \qquad V_2 = \left(\frac{1}{4+1}\right)25 = {}^{-}5\text{V}^{+}$$

Note that 3 k$\Omega$ resistor has no effect on this computation.

**(*iii*)** The voltage $V_3$ across 1 k$\Omega$ resistor due to **15 V source acting alone** is found by replacing the 25 V source by a short circuit and the 10 mA current source by an open circuit as shown in Fig. 3.49 (*iii*). The short circuit prevents any current from flowing in the 1 k $\Omega$ resistor.

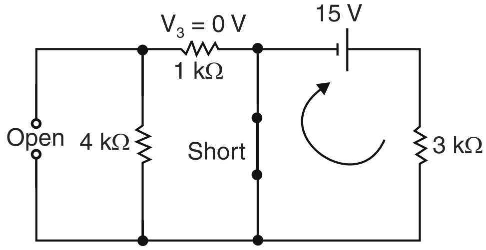
$$\therefore \qquad V_3 = 0$$

**(*iv*)** Applying superposition principle, the voltage across the 1k$\Omega$ resistor due to all the three sources acting simultaneously [See Fig. 3.49 (*iv*)] is

$$\begin{aligned} V_{1\,k\,\Omega} &= V_1 + V_2 + V_3 \\ &= {}^{+}8\text{ V}^{-} + {}^{-}5\text{ V}^{+} + 0\text{ V} \\ &= {}^{+}\mathbf{3\ V}^{-} \end{aligned}$$

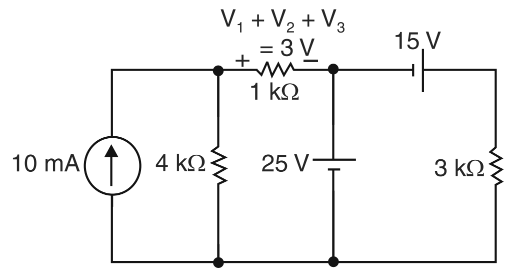
Note that $V_1$ and $V_2$ have opposite polarities so that the sum (net) voltage is actually

$$= 8 - 5 = 3 \text{ V}$$




Voltage across the 1-k Ω resistor due to the 15-V voltage source acting alone.

(*iii*)




Voltage across the 1-k Ω resistor due to all sources acting simultaneously.

(*iv*)

**Fig. 3.49**

**Example 3.25.** *To what voltage should adjustable source E be set in order to produce a current of 0.3 A in the 400 Ω resistor shown in Fig. 3.50?*

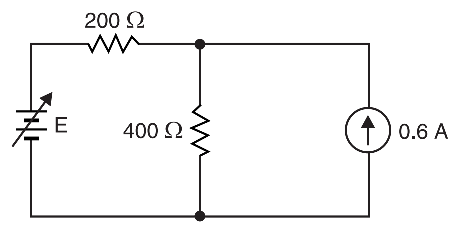
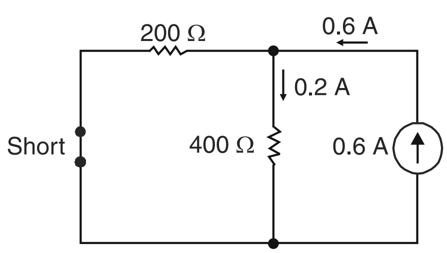


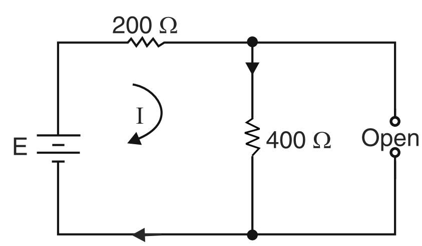

**Fig. 3.50**

**Solution.** We first find the current $I_1$ in 400 Ω resistor due to the 0.6 A current source alone. This current can be found by replacing $E$ by a short circuit as shown in Fig. 3.51 (*i*). Applying current divider rule to Fig. 3.51 (*i*),

$$I_1 = \left(\frac{200}{200 + 400}\right) 0.6 = 0.2 \text{ A}$$




(*i*)




(*ii*)

**Fig. 3.51**

In order that current in the 400 Ω resistor is equal to 0.3 A, the current produced in the resistor by the voltage source acting alone must be = 0.3 − 0.2 = 0.1 A. The current in the 400 Ω resistor due to voltage source alone can be calculated by open-circuiting the current source as shown in Fig 3.51 (*ii*). Referring to Fig. 3.51 (*ii*) and applying Ohm's law, we have,

$$I = \frac{E}{200 + 400} = \frac{E}{600}$$

or

$$0.1 = \frac{E}{600} \qquad \therefore E = 600 \times 0.1 = \mathbf{60 \text{ V}}$$

**Example 3.26.** *Use superposition theorem to find current I in the circuit shown in Fig. 3.52 (i). All resistances are in ohms.*

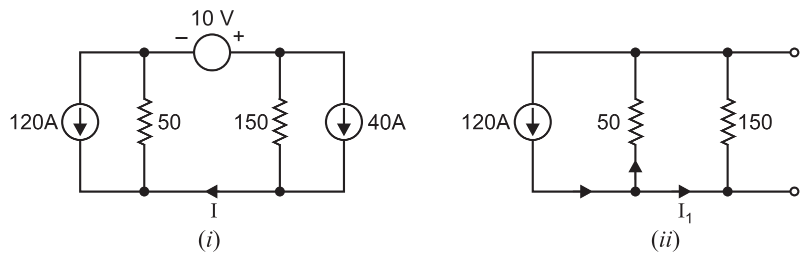



**Fig. 3.52**

**Solution.** In Fig. 3.52 (*ii*), the 10V voltage source has been replaced by a short and the 40A current source by an open so that now only 120A current source is acting alone. By current-divider rule, $I_1$ is given by ;

$$I_1 = 120 \times \frac{50}{50+150} = 30 \text{ A}$$

In Fig. 3.52 (*iii*), 40A current source is acting alone; 10 V voltage source being replaced by a short and 120A current source by an open. By current-divider rule, $I_2$ is given by ;

$$I_2 = 40 \times \frac{150}{50+150} = 30\text{A}$$

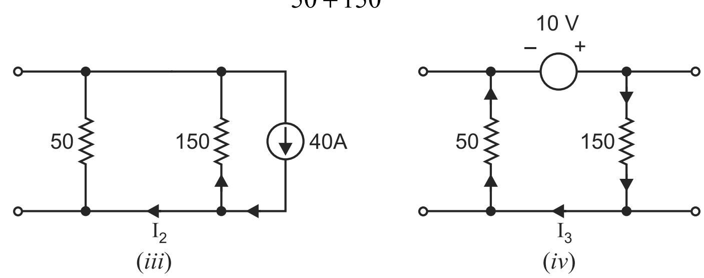



**Fig. 3.52**

In Fig. 3.52 (*iv*), 10V voltage source is acting alone. By Ohm's law, $I_3$ is given by ;

$$I_3 = \frac{10}{50+150} = 0.05\text{A}$$

Currents $I_1$ and $I_2$, being equal and opposite, cancel out so that :

$$I = I_3 = \mathbf{0.05\ A}$$

**Example 3.27.** *Using superposition theorem, find the current in the branch AC of the network ABCD shown in Fig. 3.53 (i).*

**Solution.** Let the current in section *AC* be *I* as shown in Fig. 3.53 (*i*). We shall determine the value of this current by superposition theorem.

**First consider 20A load acting alone**

Let $I_1$ and $I_2$ be the currents through *AB* and *AC* respectively as shown in Fig. 3.53 (*ii*). Then the current distribution will be as shown. We shall apply Kirchhoff's voltage law to loops *ADCA* and *ABCA*.

**Loop *ADCA*.** Applying *KVL*, we have,

$$-(20 - I_1 - I_2) \times 0.15 + 0.1\, I_2 = 0$$

or $$0.15\, I_1 + 0.25\, I_2 = 3 \quad ...(i)$$

**Loop *ABCA*.** Applying *KVL*, we have,

$$-0.1\, I_1 + (20 - I_1) \times 0.05 + 0.1\, I_2 = 0$$

or $$0.15\, I_1 - 0.1\, I_2 = 1 \quad ...(ii)$$

From equations (*i*) and (*ii*), we get, $I_2 = 40/7$A.

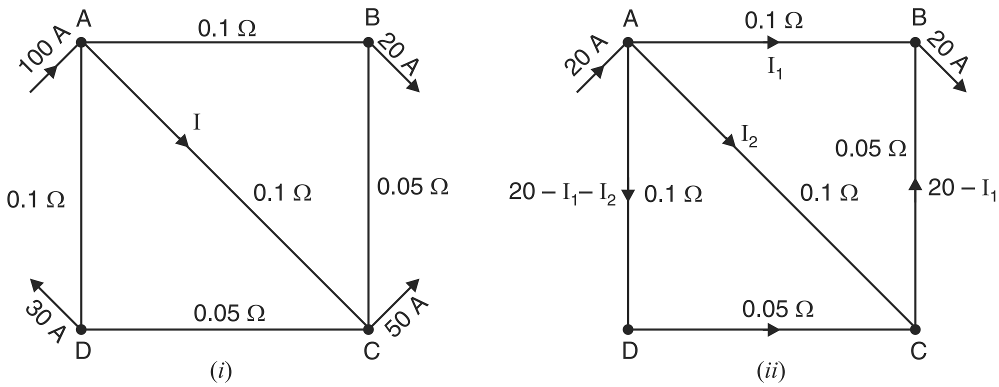



**Fig. 3.53**

### Consider now 50 A load acting alone

Let $I_1'$ and $I_2'$ be the currents through *AB* and *AC* respectively. Then the current distribution will be as shown in Fig. 3.54 (*i*).

**Loop *ABCA*.** Applying *KVL*, we have,

$$-0.15\, I_1' + 0.1\, I_2' = 0$$

or $$0.15\, I_1' - 0.1\, I_2' = 0 \quad ...(iii)$$

**Loop *ADCA*.** Applying *KVL*, we have,

$$-(50 - I_1' - I_2') \times 0.15 + 0.1\, I_2' = 0$$

or $$0.15\, I_1' + 0.25\, I_2' = 7.5 \quad ...(iv)$$

From equations (*iii*) and (*iv*), we get, $I_2' = 150/7$A.

### Consider now 30A load acting alone

Let the currents circulate as shown in Fig. 3.54 (*ii*). It is required to find $I_2''$.

**Loop *ABCA*.** Applying *KVL*, we have,

$$-0.15\, I_1'' + 0.1\, I_2'' = 0$$

or $$0.15\, I_1'' - 0.1\, I_2'' = 0 \quad ...(v)$$

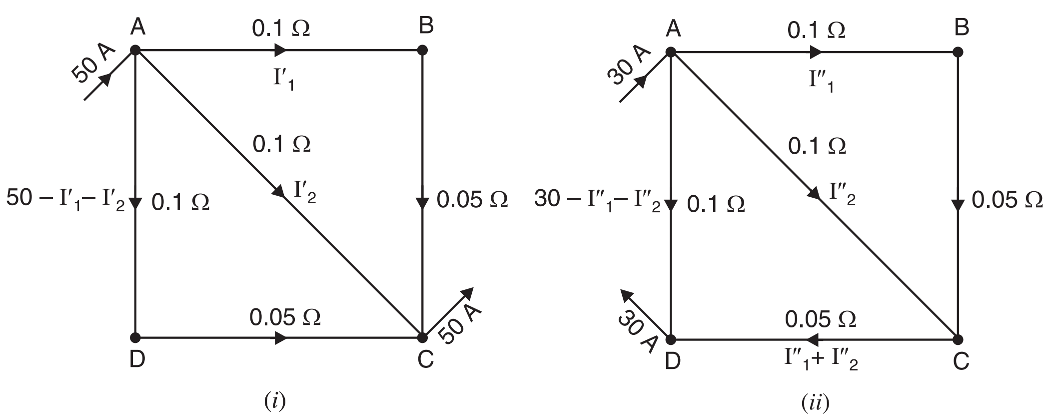



**Fig. 3.54**

**Loop *ADCA*.** Applying *KVL*, we have,

$$-(30 - I_1'' - I_2'') \times 0.1 + 0.05\,(I_1'' + I_2'') + 0.1 I_2'' = 0$$

or $$0.15\, I_1'' + 0.25\, I_2'' = 3 \qquad ...(vi)$$

From equations ($v$) and ($vi$), we get, $I_2'' = 60/7$A.

According to superposition theorem, the total current in $AC$ is equal to the algebraic sum of the component values.

$$I = I_2 + I_2' + I_2''$$
$$= 40/7 + 150/7 + 60/7$$
$$= 250/7 = \mathbf{35.7A}$$

**Example 3.28.** *Using superposition theorem, find the current in the each branch of the network shown in Fig. 3.55 (i).*

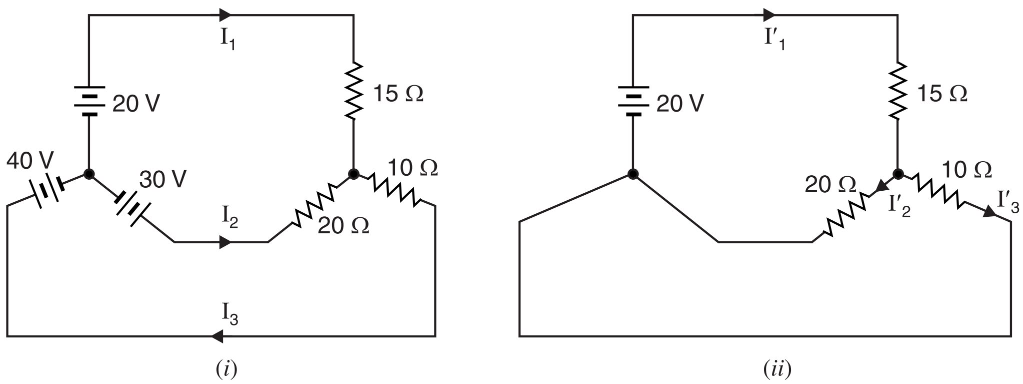



**Fig. 3.55**

**Solution.** Since there are three sources of e.m.f., three circuits [Fig. 3.55 ($ii$), Fig. 3.56 ($i$) and ($ii$)] are required for analysis by superposition theorem.

In Fig. 3.55 ($ii$), it is shown that only 20 V source is acting.

$$\text{Total resistance across source} = 15 + \frac{20 \times 10}{20 + 10} = 21.67\Omega$$

$\therefore$ $$\text{Total circuit current, } I'_1 = 20/21.67 = 0.923 \text{ A}$$
$$\text{Current in 20 } \Omega,\ I'_2 = 0.923 \times 10/30 = 0.307\text{A}$$
$$\text{Current in 10 } \Omega,\ I'_3 = 0.923 \times 20/30 = 0.616 \text{ A}$$

In Fig. 3.56 ($i$), only 40V source is acting in the circuit.

$$\text{Total resistance across source} = 10 + \frac{20 \times 15}{20 + 15} = 18.57\Omega$$

$$\text{Total circuit current, } I_3'' = 40/18.57 = 2.15\text{A}$$
$$\text{Current in 20 } \Omega,\ I_2'' = 2.15 \times 15/35 = 0.92 \text{ A}$$
$$\text{Current in 15 } \Omega,\ I_1'' = 2.15 \times 20/35 = 1.23 \text{ A}$$

In Fig. 3.56 ($ii$), only 30 $V$ source is acting in the circuit.

$$\text{Total resistance across source} = 20 + 10 \times 15/(10 + 15) = 26\ \Omega$$
$$\text{Total circuit current, } I_2''' = 30/26 = 1.153 \text{ A}$$
$$\text{Current in 15 } \Omega,\ I_1''' = 1.153 \times 10/25 = 0.461 \text{ A}$$
$$\text{Current in 10 } \Omega,\ I_3''' = 1.153 \times 15/25 = 0.692 \text{ A}$$

The actual values of currents $I_1$, $I_2$ and $I_3$ shown in Fig. 3.55 (*i*) can be found by algebraically adding the component values.

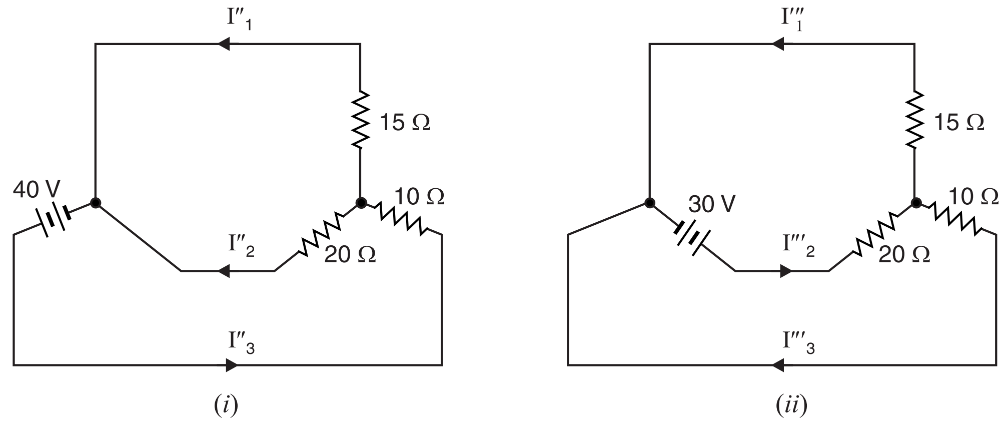



**Fig. 3.56**

$$I_1 = I_1' - I_1'' - I_1''' = 0.923 - 1.23 - 0.461 = \mathbf{-0.768\ A}$$

$$I_2 = -I_2' - I_2'' + I_2''' = -0.307 - 0.92 + 1.153 = \mathbf{-0.074\ A}$$

$$I_3 = I_3' - I_3'' + I_3''' = 0.616 - 2.15 + 0.692 = \mathbf{-0.842\ A}$$

The negative signs with $I_1$, $I_2$ and $I_3$ show that their actual directions are opposite to that assumed in Fig. 3.55 (*i*).

**Example 3.29.** *Use superposition theorem to find the voltage V in Fig. 3.57 (i).*

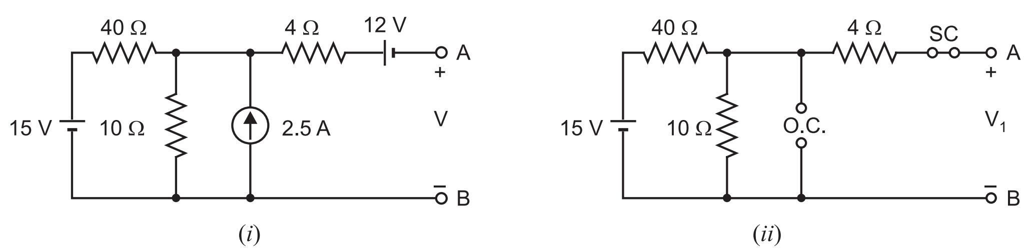



**Fig. 3.57**

**Solution.** In Fig. 3.57 (*ii*), 12 V battery is replaced by a short and 2.5A current source by an open so that 15V battery is acting alone. Therefore, voltage $V_1$ across open terminals *A* and *B* is

$$V_1 = \text{Voltage across } 10\Omega \text{ resistor}$$

By voltage-divider rule, $V_1$ is given by ;

$$V_1 = 15 \times \frac{10}{40+10} = 3\text{V}$$

In Fig. 3.57 (*iii*), 15 V and 12 V batteries are replaced by shorts so that 2.5A current source is acting alone. Therefore, voltage $V_2$ across open terminals *A* and *B* is

$$V_2 = \text{Voltage across } 10\ \Omega \text{ resistor}$$

By current-divider rule, current in 10 Ω = $2.5 \times \frac{40}{50} = 2\text{A}$

$$\therefore \quad V_2 = 2 \times 10 = 20\text{V}$$

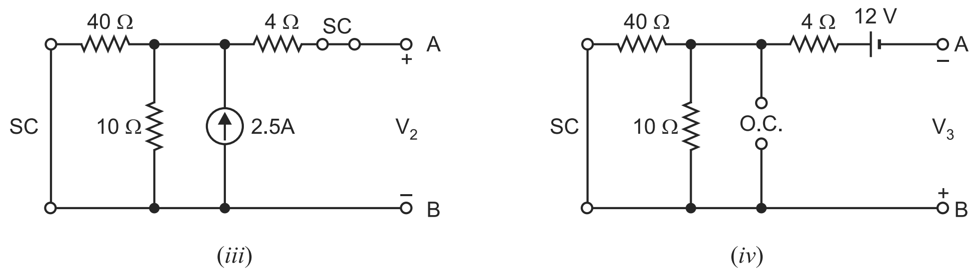



**Fig. 3.57**

In Fig. 3.57 (*iv*), 15 V battery is replaced by a short and 2.5 A current source by an open so that 12V battery is acting alone. Therefore, voltage $V_3$ across open terminals $A$ and $B$ is

$$V_3 = -^{*}12\text{V}$$

The minus sign is given because the negative terminal of the battery is connected to point $A$ and positive terminal to point $B$.

∴ Voltage across open terminals $AB$ when all sources are present is

$$V = V_1 + V_2 + (-V_3) = 3 + 20 - 12 = \mathbf{11V}$$

**Example 3.30.** *Using superposition theorem, find the current in 23 Ω resistor in the circuit shown in Fig. 3.58.*

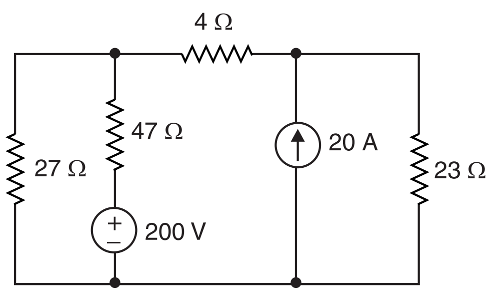



**Fig. 3.58**

**Solution.**

**200 V source acting alone.** We first consider the case when 200 V voltage source is acting alone as shown in Fig. 3.59. Note that current source is replaced by an open. The total resistance $R_T$ presented to the voltage source is 47 Ω in series with the parallel combination of 27 Ω and (23 + 4) Ω. Therefore, the value of $R_T$ is given by ;

$$R_T = 47 + [27 \parallel (23 + 4)] = 47 + \frac{27 \times 27}{27 + 27} = 47 + 13.5 = 60.5\ \Omega$$

∴ Current supplied by 200 V source is given by ;

$$I_T = \frac{V}{R_T} = \frac{200}{60.5} = 3.31\text{ A}$$

At the node $A$, $I_T$ (= 3.31 A) divides between the parallel resistors of 27 Ω and (23 + 4) Ω.

∴ Current through 23 Ω, $I_1 = 3.31 \times \frac{27}{27 + 27} = 1.65$ A downward

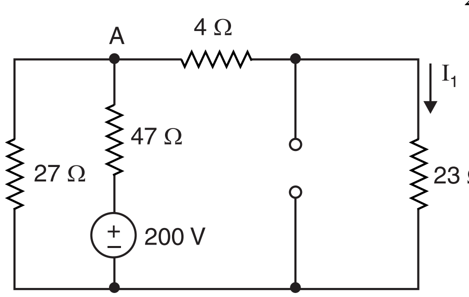



**Fig. 3.59**

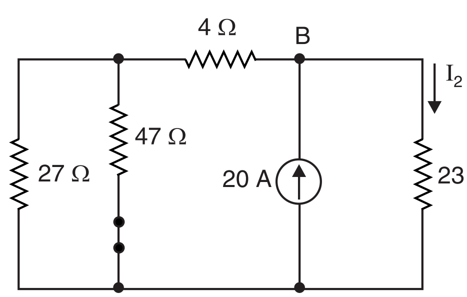



**Fig. 3.60**

* The total circuit resistance at terminals $AB = 4 + (40\|10) = 12\Omega$. The circuit behaves as a 12V battery having internal resistance of 12Ω with terminals $A$ and $B$ open.

**20 A current source acting alone.** We now consider the case when the current source is acting alone as shown in Fig. 3.60. Note that voltage source is replaced by a short because its internal resistance is assumed zero. The equivalent resistance $R_{eq}$ to the left of the current source is

$$R_{eq} = 4 + (27 \parallel 47) = 4 + \frac{27 \times 47}{27 + 47} = 4 + 17.15 = 21.15\ \Omega$$

At node $B$, 20 A divides between two parallel resistors $R_{eq}$ and 23 Ω. By current divider rule,

$$\text{Current in } 23\Omega \text{ resistor, } I_2 = 20 \times \frac{R_{eq}}{R_{eq} + 23} = 20 \times \frac{21.15}{21.15 + 23} = 9.58 \text{ A}$$

Note that $I_2$ in 23Ω resistor is downward.

∴ Total current in 23 Ω = $I_1 + I_2$ = 1.65 + 9.58 = **11.23 A**

**Example 3.31.** *Fig. 3.61 shows the circuit with two independent sources and one dependent source. Find the power delivered to the 3 Ω resistor.*

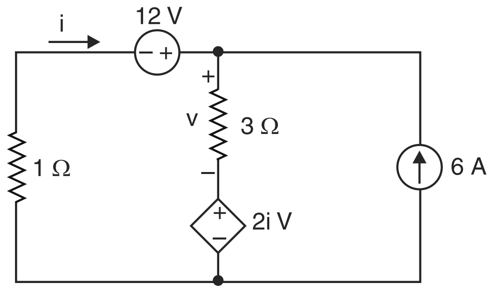



Fig. 3.61

**Solution.** While applying superposition theorem, two points must be noted carefully. First, we *cannot* find the power due to each independent source acting alone and add the results to obtain total power. It is because the relation for power is non-linear ($P = I^2R$ or $V^2/R$). Secondly, when the circuit also has dependent source, only independent sources act one at a time while dependent sources remain unchanged. Let us come back to the problem. Suppose $v_1$ is the voltage across 3 Ω resistor when 12 V source is acting alone and $v_2$ is the voltage across 3 Ω resistor when 6 A source is acting alone. Therefore, $v = v_1 + v_2$.

**When 12 V source is acting alone.** When 12 V source is acting alone, the circuit becomes as shown in Fig. 3.62. Note that 6A source is replaced by an open. Applying $KVL$ to the loop $ABCDA$ in Fig. 3.62, we have,

$$12 - v_1 - 2i_1 - i_1 \times 1 = 0$$

or $$12 - 3i_1 - 2i_1 - i_1 = 0 \quad \therefore \quad i_1 = 12/6 = 2 \text{ A}$$

∴ $$v_1 = 3i_1 = 3 \times 2 = 6\text{V}$$

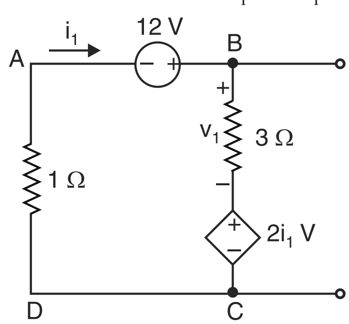



Fig. 3.62

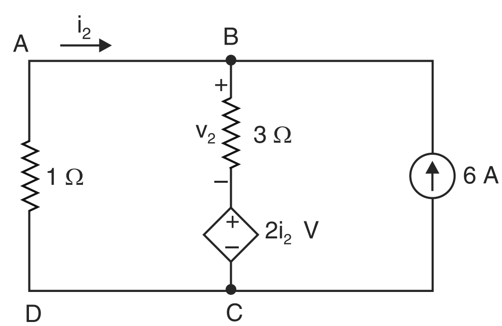



Fig. 3.63

**When 6A source is acting alone.** When 6A source is acting alone, the circuit becomes as shown in Fig. 3.63. Note that 12V source is replaced by a short because internal resistance of the source is assumed zero. Applying $KVL$ to the loop $ABCDA$ in Fig. 3.63, we have,

$$-3(i_2 + 6) - 2i_2 - i_2 \times 1 = 0$$

or $$-3i_2 - 18 - 2i_2 - i_2 = 0 \quad \therefore\ i_2 = -18/6 = -3 \text{ A}$$

∴ $$v_2 = 3(i_2 + 6) = 3 \times 3 = 9 \text{ V}$$

$\therefore \quad v = v_1 + v_2 = 6 + 9 = 15 \text{ V}$

$\therefore$ Power delivered to 3Ω, $P = \dfrac{v^2}{3} = \dfrac{(15)^2}{3} =$ **75 W**

**Example 3.32.** *Using superposition principle, find the current through $G_C$ conductance in the circuit shown in Fig. 3.64. Given that $G_A = 0.3$ S ; $G_B = 0.4$ S and $G_C = 0.1$ S.*

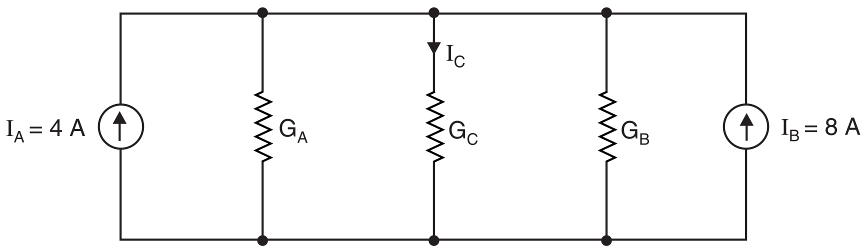



**Fig. 3.64**

**Solution.**

**Current source $I_A$ acting alone.** We first consider the case when current source $I_A$ is acting alone as shown in Fig. 3.65. Note that current source $I_B$ is replaced by an open.

Total conductance, $G_T = G_A + G_C + G_B = 0.3 + 0.1 + 0.4 = 0.8\text{S}$

Voltage across $G_C$, $V' = \dfrac{I_A}{G_T} = \dfrac{4}{0.8} = 5 \text{ V}$

$\therefore$ Current through $G_C$, $I'_C = V'G_C = 5 \times 0.1 = 0.5 \text{ A}$

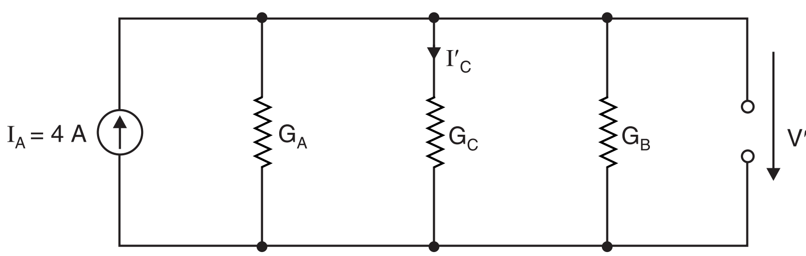



**Fig. 3.65**

**Current source $I_B$ acting alone.** We now consider the case when current source $I_B$ acts alone as shown in Fig. 3.66. Note that current source $I_A$ is replaced by an open.

Voltage across $G_C$, $V'' = \dfrac{I_B}{G_T} = \dfrac{8}{0.8} = 10 \text{ V}$

Current through $G_C$, $I''_C = V''G_C = 10 \times 0.1 = 1 \text{ A}$

$\therefore$ Total current through $G_C$, $I_C = I'_C + I''_C = 0.5 + 1 =$ **1·5 A**

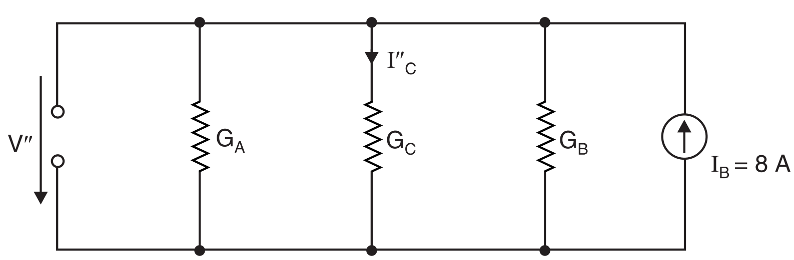



**Fig. 3.66**

**Note.** It is important to note that superposition theorem applies to currents and voltages; it does not mean that powers from two sources can be superimposed. It is because power varies as the *square* of the voltage or the current and this relationship is nonlinear.

**Example 3.33.** *Using superposition theorem, find the value of output voltage $V_0$ in the circuit shown in Fig. 3.67.*

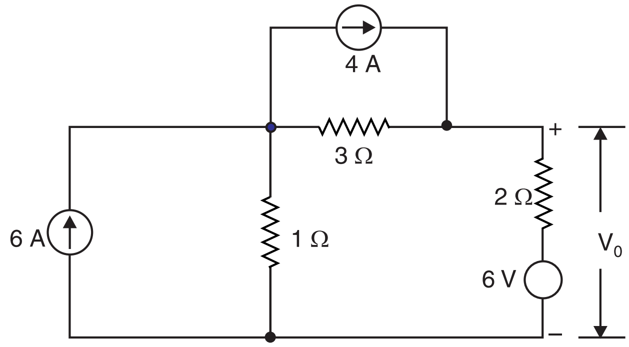



**Fig. 3.67**

**Solution.** The problem will be divided into three parts using one source at a time.

**6A source acting alone.** We first consider the case when 6 A source is acting alone as shown in Fig. 3.68. Note that voltage source is replaced by a short and the current source of 4 A is replaced by an open. According to current-divider rule, current $i_1$ through 2 Ω resistor is

$$i_1 = 6 \times \frac{1}{1+2+3} = 1\text{A} \quad \therefore \quad V_{01} = 1 \times 2 = 2\text{V}$$

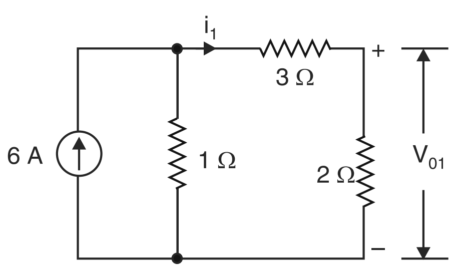



**Fig. 3.68**

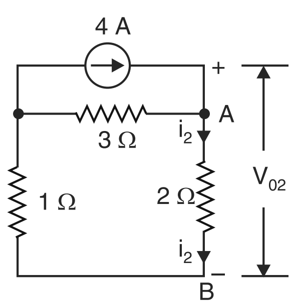



**Fig. 3.69**

**4A source acting alone.** We now consider the case when 4A source is acting alone as shown in Fig. 3.69. Note that voltage source is replaced by a short and current source of 6A is replaced by an open. At point *A*, the current 4A finds two parallel paths; one of resistance 3 Ω and the other of resistance = 2 + 1 = 3 Ω. Therefore, current $i_2$ through 2 Ω resistor is

$$i_2 = 4/2 = 2\text{A} \quad \therefore \quad V_{02} = 2 \times 2 = 4\text{V}$$

**6 V source acting alone.** Finally, we consider the case when 6 V source is acting alone as shown in Fig. 3.70. Note that each current source is replaced by an open. The circuit current is 1A and voltage drop across 2 Ω resistor = 2 × 1 = 2V.

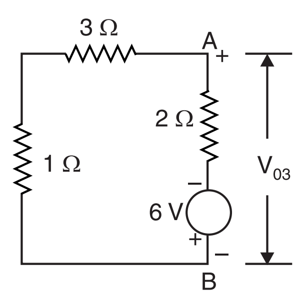



**Fig. 3.70**

It is clear from Fig. 3.70 that :

$$V_A - 2\text{V} + 6\text{V} = V_B \quad \therefore \quad V_A - V_B = V_{03} = -4\text{V}$$

According to superposition theorem, we have,

$$V_0 = V_{01} + V_{02} + V_{03} = 2 + 4 - 4 = \mathbf{2V}$$

**Example 3.34.** *Using superposition theorem, find voltage across 4Ω resistance in Fig. 3.71 (i).*

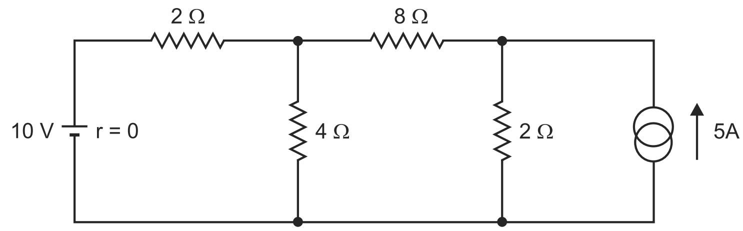



**Fig. 3.71** (*i*)

**Solution.** In Fig. 3.71 (*ii*), the 5A current source is replaced by an open so that 10V source is acting alone. Referring to Fig. 3.71 (*ii*), the total circuit resistance $R_T$ offered to 10V source is

$$R_T = 2\Omega + [4\Omega \parallel (2+8)\Omega] = 2 + \frac{4 \times 10}{4+10} = 4.857\Omega$$

∴ Current $I$ supplied by 10 V source is given by ;

$$I = \frac{10\text{V}}{R_T} = \frac{10\text{V}}{4.857\Omega} = 2.059 \text{ A}$$

At point $A$ in Fig. 3.71 (*ii*), the current 2.059 A divides into two parallel paths consisting of 4Ω resistance and (8 + 2) = 10Ω resistance.

∴ By current-divider rule, current $I_1$ in 4Ω due to 10 V alone is

$$I_1 = 2.059 \times \frac{10}{4+10} = 1.471 \text{ A in downward direction}$$

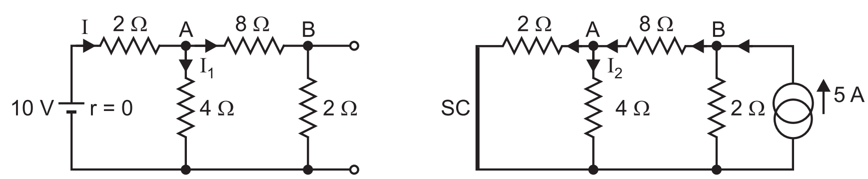



**Fig. 3.71**

In Fig. 3.71 (*iii*), the 10V battery is replaced by a short so that 5A current source is acting alone. At point $B$ in Fig. 3.71 (*iii*), current 5A divides into two parallel paths consisting of 2Ω resistance and 8Ω + (2Ω||4Ω) = 8 + (2 × 4)/(2 + 4) = 9.333Ω.

∴ By current-divider rule, current in 8Ω resistance is

$$I_{8\Omega} = 5 \times \frac{2}{2+9.333} = 0.8824 \text{ A}$$

At point A in Fig. 3.71 (*iii*), current 0.8824A divides into two parallel paths consisting of 2Ω resistance and 4Ω resistance.

∴ By current-divider rule, current $I_2$ in 4Ω due to 5A alone is

$$I_2 = 0.8824 \times \frac{2}{2+4} = 0.294 \text{ A in downward direction}$$

By superposition theorem, total current in 4 Ω

$$= I_1 + I_2 = 1.471 + 0.294 = 1.765\text{A in downward direction}$$

$$\therefore \quad \text{Voltage across } 4\Omega = 1.765 \times 4 = \mathbf{7.06V}$$

**Note.** We can also find $I_2$ in another way. Current in left-hand side $2\Omega$ resistance will be $2I_2$ because $2\Omega \parallel 4\Omega$. By *KCL*, current in $8\Omega$ resistance is

$$I_{8\Omega} = I_2 + 2I_2 = 3I_2$$

$$\text{Resistance to } I_{8\Omega} \text{ flow} = 8\Omega + (4\Omega \parallel 2\Omega) = 8 + \frac{2 \times 4}{2 + 4} = 9.333\ \Omega$$

Now 5A divides between two parallel paths of resistances 9.333 Ω and 2 Ω.

$$\therefore \quad I_{8\Omega} = 5 \times \frac{2}{2 + 9.333} = 0.8824\text{ A}$$

$$\text{or} \quad 3I_2 = 0.8824 \quad \therefore I_2 = \frac{0.8824}{3} = 0.294\text{ A}$$

## Tutorial Problems

1. Use the superposition theorem to find the current in $R_1$ (= 60 Ω) in the circuit shown in Fig. 3.72. **[0.125 A from left to right]**
2. Use the superposition theorem to find the current through $R_1$ (= 1k Ω) in the circuit shown in Fig 3.73. **[2 mA from right to left]**

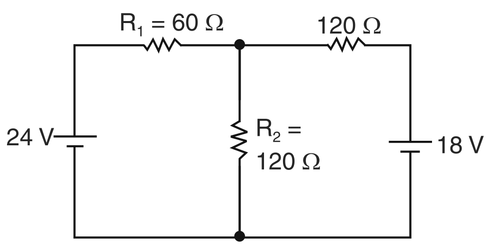


**Fig. 3.72**

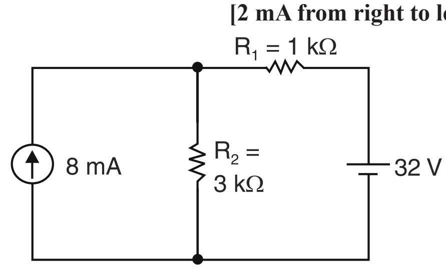


**Fig. 3.73**

3. Use the superposition theorem to find the current through $R_1$ (= 10 Ω) in the circuit shown in Fig. 3.74. **[4.6 A from left to right]**

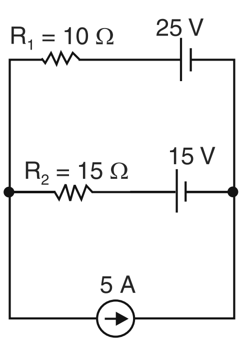


**Fig. 3.74**

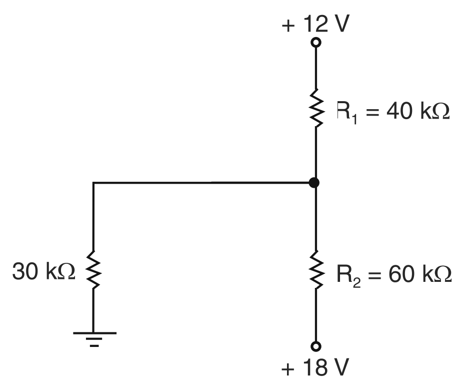


**Fig. 3.75**

4. Use superposition principle to find the current through resistance $R_1$ (= 40 kΩ) in the circuit shown in Fig. 3.75. **[1 mA downward]**
5. Use superposition principle to find the voltage across $R_1$ (= 1 k Ω) in the circuit shown in Fig. 3.76. Be sure to indicate the polarity of the voltage. **[− (11 V) + ]**

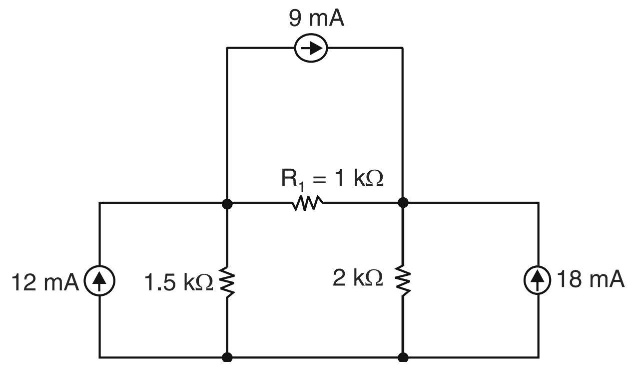



**Fig. 3.76**

**6.** Using superposition principle, find the current through 10 Ω resistor in Fig. 3.77. **[ 0.5 A↓ ]**

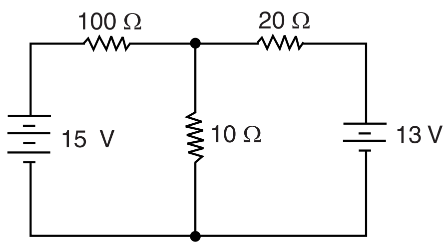



**Fig. 3.77**

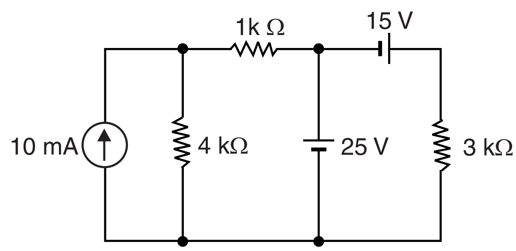



**Fig. 3.78**

**7.** Using superposition principle, find the voltage across 4 kΩ resistor in Fig. 3.78. **[28 $V^{+}_{-}$ ]**

**8.** Referring to Fig. 3.79, the internal resistance $R_S$ of the current source is 100 Ω. The internal resistance $R_S$ of the voltage source is 10 Ω. Use superposition principle to find the power dissipated in 50 Ω resistor. **[8.26 W]**

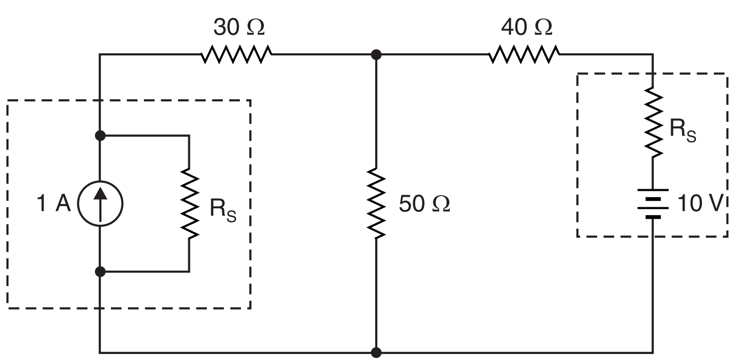



**Fig. 3.79**

**9.** Find $v$ using superposition principle if $R = 2\Omega$ in Fig. 3.80. **[8 V]**

**10.** State whether true or false.

(*i*) Superposition theorem is applicable to multiple source circuits.

(*ii*) Superposition theorem is restricted to linear circuits. [(*i*) **True** (*ii*) **True**]

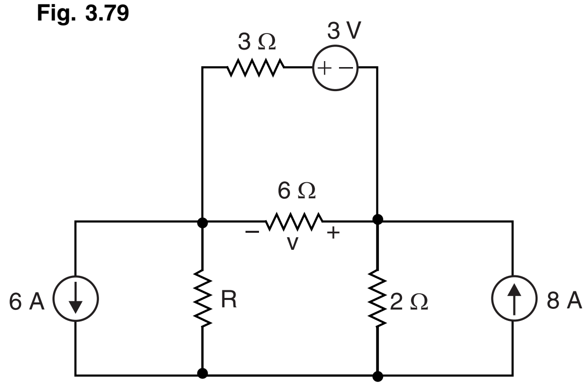



**Fig. 3.80**

**11.** Find $i$ using superposition theorem in Fig. 3.81. **[ –6 A]**

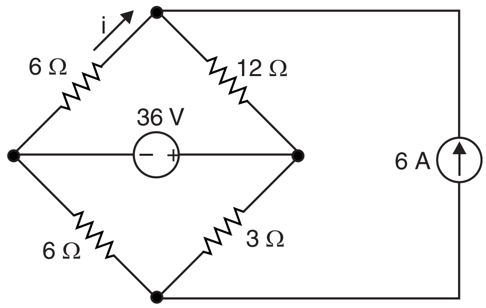
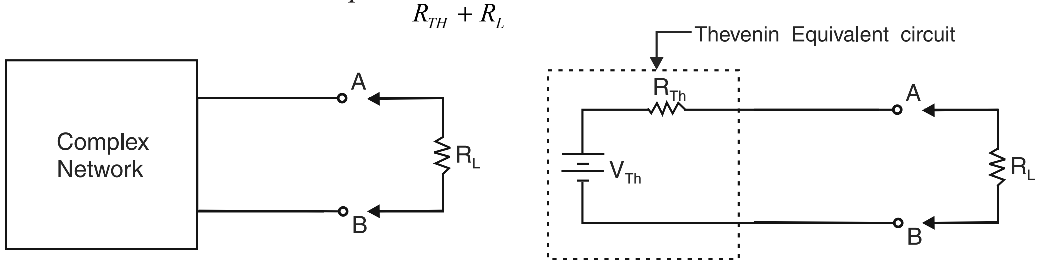



**Fig. 3.81**

## 3.10. Thevenin's Theorem

Fig. 3.82 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may consist of any number of resistors and e.m.f. sources connected in any manner. But according to Thevenin, the entire circuit behind terminals *A* and *B* can be replaced by a single source of e.m.f. $V_{Th}$ (called Thevenin voltage) in series with a single resistance $R_{Th}$ (called Thevenin resistance) as shown in Fig. 3.82 (*ii*). The values of $V_{Th}$ and $R_{Th}$ are determined as mentioned in Thevenin's theorem. Once *Thevenin's equivalent circuit* is obtained [See Fig. 3.82 (*ii*)], then current *I* through any load resistance $R_L$ connected across *AB* is given by ;

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$




(*i*) (*ii*)

**Fig. 3.82**

Thevenin's theorem as applied to d.c. circuits is stated below :

*Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. $V_{Th}$ in series with a single resistance $R_{Th}$.*

(*i*) *The e.m.f. $V_{Th}$ is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.*

(*ii*) *The resistance $R_{Th}$ is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Note how truly remarkable the implications of this theorem are. No matter how complex the circuit and no matter how many voltage and / or current sources it contains, it is equivalent to a single voltage source in series with a single resistance (*i.e.* **equivalent to a single real voltage source).** *Although Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of output voltage and current.*

**Explanation**. Consider the circuit shown in Fig. 3.83 (*i*). As far as the circuit behind terminals *AB* is concerned, it can be replaced by a single source of e.m.f. $V_{Th}$ in series with a single resistance $R_{Th}$ as shown in Fig. 3.84 (*ii*).

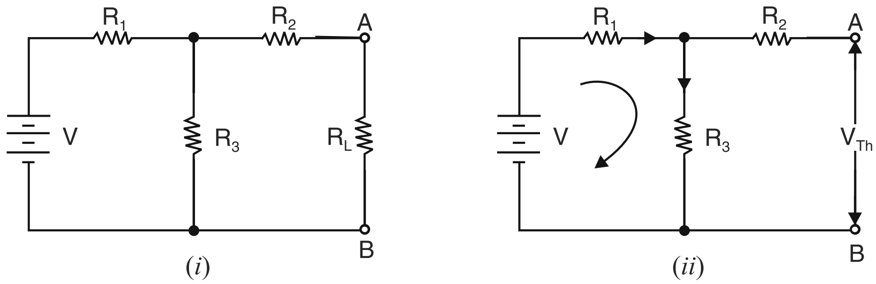



**Fig. 3.83**

***(i)* Finding $V_{Th}$.** The e.m.f. $V_{Th}$ is the voltage across terminals *AB* with load (*i.e.* $R_L$) removed as shown in Fig. 3.83 (*ii*). With $R_L$ disconnected, there is no current in $R_2$ and $V_{Th}$ is the voltage appearing across $R_3$.

$$\therefore \quad V_{Th} = \text{Voltage across } R_3 = \frac{V}{R_1 + R_3} \times R_3$$

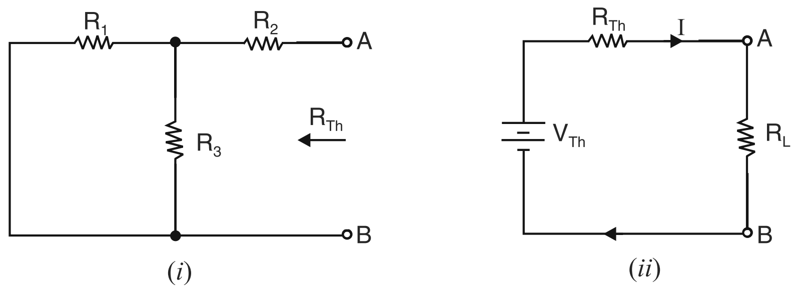



**Fig. 3.84**

***(ii)* Finding $R_{Th}$.** To find $R_{Th}$, remove the load $R_L$ and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance between terminals *A* and *B* is equal to $R_{Th}$ as shown in Fig. 3.84 (*i*). Obviously, at the terminals *AB* in Fig. 3.84 (*i*), $R_1$ and $R_3$ are in parallel and this parallel combination is in series with $R_2$.

$$\therefore \quad R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

When load $R_L$ is connected between terminals *A* and *B* [See Fig. 3.84 (*ii*)], then current in $R_L$ is given by ;

$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

## 3.11. Procedure for Finding Thevenin Equivalent Circuit

**(*i*)** Open the two terminals (*i.e.*, remove any load) between which you want to find Thevenin equivalent circuit.

**(*ii*)** Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage $V_{Th}$.

**(*iii*)** Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Thevenin resistance $R_{Th}$.

**(*iv*)** Connect $V_{Th}$ and $R_{Th}$ in series to produce Thevenin equivalent circuit between the two terminals under consideration.

**(*v*)** Place the load resistor removed in step (*i*) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.

**Note.** Thevenin's theorem is sometimes called *Helmholtz's theorem*.

**Example 3.35.** *Using Thevenin's theorem, find the current in 6 Ω resistor in Fig. 3·85 (i).*

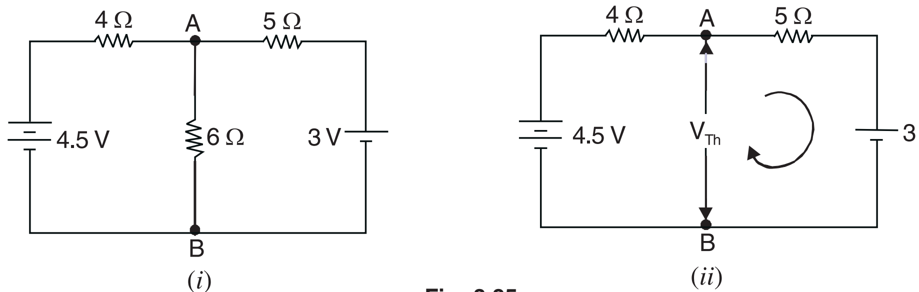



**Fig. 3.85**

**Solution.** Since internal resistances of batteries are not given, it will be assumed that they are zero. We shall find Thevenin's equivalent circuit at terminals *AB* in Fig. 3.85 (*i*).

$V_{Th}$ = Voltage across terminals *AB* with load (*i.e.* 6 Ω resistor) removed as shown in Fig. 3·85 (*ii*).

$$= {}^*4{\cdot}5 - 0.167 \times 4 = 3{\cdot}83 \text{ V}$$

$R_{Th}$ = Resistance at terminals *AB* with load (*i.e.* 6 Ω resistor) removed and battery replaced by a short as shown in Fig. 3·86 (*i*).

$$= \frac{4 \times 5}{4+5} = 2.22\ \Omega$$

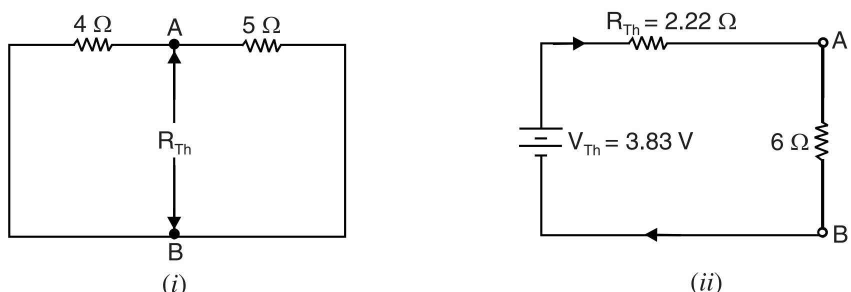



**Fig. 3.86**

Thevenin's equivalent circuit at terminals *AB* is $V_{Th}$ (= 3·83 *V*) in series with $R_{Th}$ (= 2·22 Ω). When load (*i.e.* 6 Ω resistor) is connected between terminals *A* and *B*, the circuit becomes as shown in Fig. 3·86 (*ii*).

$$\therefore \quad \text{Current in 6 Ω resistor} = \frac{V_{Th}}{R_{Th}+6} = \frac{3.83}{2.22+6} = \mathbf{0.466A}$$

**Example 3.36.** *Using Thevenin's theorem, find p.d. across terminals AB in Fig. 3·87 (i).*

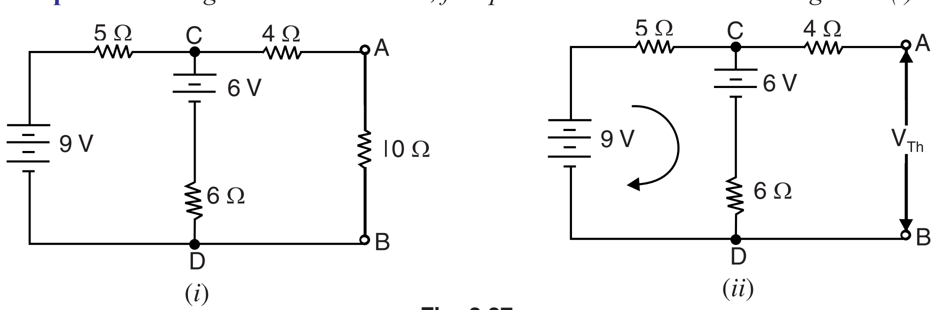



**Fig. 3.87**

* Net e.m.f. in the circuit shown in Fig. 3·85 (*ii*) is 4·5 − 3 = 1·5 V and total circuit resistance is 9 Ω.

∴ Circuit current = 1·5/9 = 0·167 A

The voltage across *AB* is equal to 4·5 V *less* drop in 4 Ω resistor.

$$\therefore \quad V_{Th} = 4{\cdot}5 - 0{\cdot}167 \times 4 = 3{\cdot}83 \text{ V}$$

**Solution.** We shall find Thevenin's equivalent circuit at terminals *AB* in Fig. 3.87 (*i*).

$V_{Th}$ = Voltage across terminals *AB* with load (*i.e.* 10 Ω resistor) removed as shown in Fig. 3.87 (*ii*).

= Voltage across terminals *CD*

= 9 – drop in 5 Ω resistor

= 9* – 5 × 0·27 = 7·65 V

$R_{Th}$ = Resistance at terminals *AB* with load (*i.e.* 10 Ω resistor) removed and batteries replaced by a short as shown in Fig. 3.88 (*i*).

$$= 4 + \frac{5 \times 6}{5 + 6} = 6.72 \Omega$$

Thevenin's equivalent circuit to the left of terminals *AB* is $V_{Th}$ (= 7·65 *V*) in series with $R_{Th}$ (= 6·72 Ω). When load (*i.e.* 10 Ω resistor) is connected between terminals *A* and *B*, the circuit becomes as shown in Fig. 3.88 (*ii*).

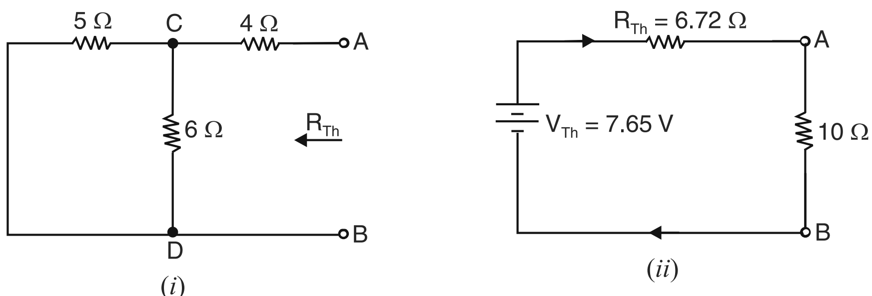



**Fig. 3.88**

$$\therefore \quad \text{Current in } 10\ \Omega \text{ resistor} = \frac{V_{Th}}{R_{Th} + 10} = \frac{7.65}{6.72 + 10} = 0.457 \text{ A}$$

P.D. across 10 Ω resistor = 0·457 × 10 = **4·57 V**

**Example 3.37.** *Using Thevenin's theorem, find the current through resistance R connected between points a and b in Fig. 3.89* (*i*).

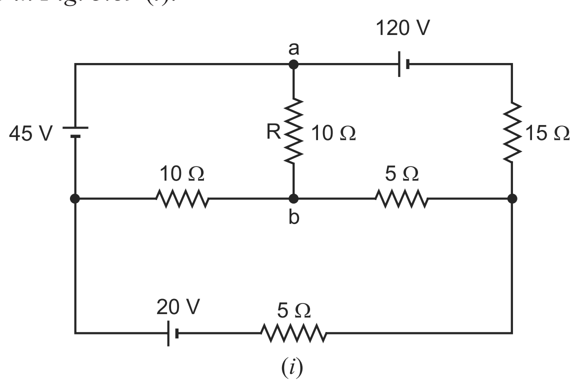


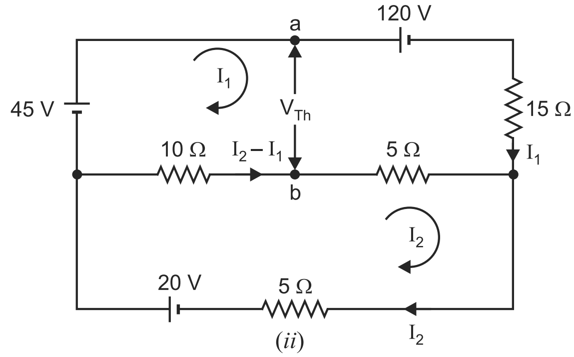

**Fig. 3.89**

**Solution.** **(*i*) Finding $V_{Th}$.** Thevenin voltage $V_{Th}$ is the voltage across terminals *ab* with

* The net e.m.f. in the loop of circuit shown in Fig. 3·87 (*ii*) is 9 – 6 = 3V and total resistance is 5 + 6 = 11 Ω.

∴ Circuit current = 3/11 = 0·27 A

resistance $R$ (= 10Ω) removed as shown in Fig. 3.89 (*ii*). It can be found by Maxwell's mesh current method.




(*ii*)

**Fig. 3.89**

**Mesh 1.** $45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$

or $\qquad 30I_1 - 15I_2 = -75 \qquad$ ...(*i*)

**Mesh 2.** $-10(I_2 - I_1) - 5(I_2 - I_1) - 5I_2 + 20 = 0$

or $\qquad -15I_1 + 20I_2 = 20 \qquad$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\quad I_1 = -3.2\text{A} \;;\; I_2 = -1.4$ A

Now, $\quad V_a - 45 - 10(I_2 - I_1) = V_b$

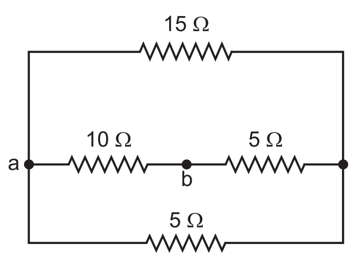
or $\quad V_a - V_b = 45 + 10\,(I_2 - I_1) = 45 + 10\,[-1.4 - (-3.2)] = 63\text{V}$

$\therefore \quad V_{Th} = V_{ab} = V_a - V_b = 63\text{V}$

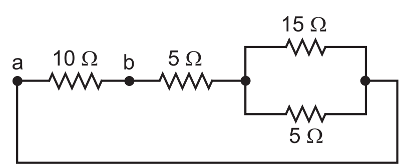
***(ii)* Finding $R_{Th}$.** Thevenin resistance $R_{Th}$ is the resistance at terminals *ab* with resistance $R$ (= 10Ω) removed and batteries replaced by a short as shown in Fig. 3.89 (*iii*). Using laws of series and parallel resistances, the circuit is reduced to the one shown in Fig. 3.89 (*iv*).

$\therefore \quad R_{Th}$ = Resistance at terminals *ab* in Fig 3.89 (*iv*).

$$= 10\Omega \parallel [5\Omega + (15\Omega \parallel 5\Omega)] = 10\Omega \parallel (5\Omega + 3.75\Omega) = \frac{14}{3}\Omega$$




(*iii*)




(*iv*)

**Fig. 3.89**

$$\therefore \text{ Current in } R\,(= 10\Omega) = \frac{V_{Th}}{R_{Th} + R} = \frac{63}{(14/3) + 10} = \mathbf{4.295A}$$

**Example 3.38.** *A Wheatstone bridge ABCD has the following details : AB = 10 Ω, BC = 30 Ω, CD = 15 Ω and DA = 20 Ω. A battery of e.m.f. 2 V and negligible resistance is connected between A and C with A positive. A galvanometer of 40 Ω resistance is connected between B and D. Using Thevenin's theorem, determine the magnitude and direction of current in the galvanometer.*

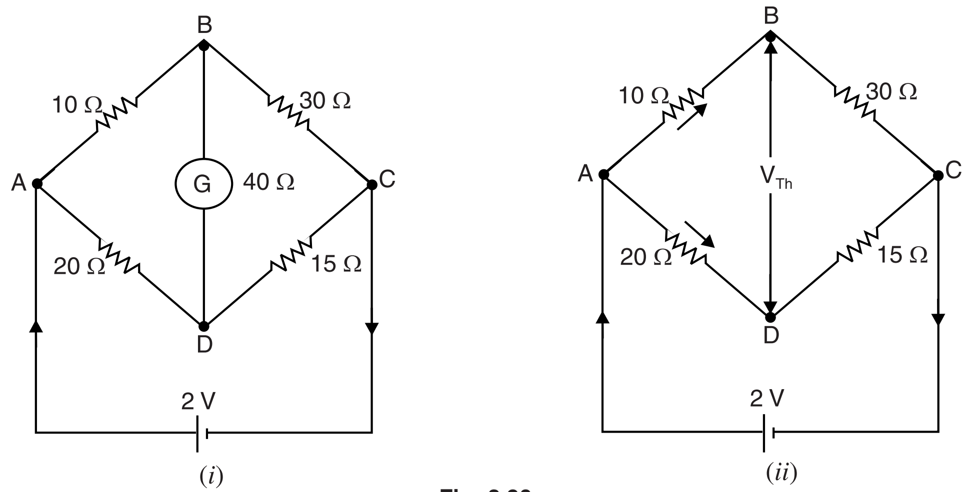



**Fig. 3.90**

**Solution.** We shall find Thevenin's equivalent circuit at terminals *BD* in Fig. 3.90 (*i*).

***(i)* Finding $V_{Th}$.** To find $V_{Th}$ at terminals *BD*, remove the load (*i.e.* 40 Ω galvanometer) as shown in Fig. 3.90 (*ii*). The voltage between terminals *B* and *D* is equal to $V_{Th}$.

$$\text{Current in branch } ABC = \frac{2}{10+30} = 0.05 \text{ A}$$

$$\text{P.D. between } A \text{ and } B,\ V_{AB} = 10 \times 0.05 = 0.5 \text{ V}$$

$$\text{Current in branch } ADC = \frac{2}{20+15} = 0.0571\text{A}$$

$$\text{P.D. between } A \text{ and } D,\ V_{AD} = 0.0571 \times 20 = 1.142 \text{ V}$$

$$\therefore \quad \text{P.D. between } B \text{ and } D,\ V_{Th} = V_{AD} - V_{AB} = 1.142 - 0.5 = 0.642 \text{ V}$$

Obviously, point *B** is positive w.r.t. point *D i.e.* current in the galvanometer, when connected between *B* and *D*, will flow from *B* to *D*.

***(ii)* Finding $R_{Th}$.** In order to find $R_{Th}$, remove the load (*i.e.* 40 Ω galvanometer) and replace the battery by a short (as its internal resistance is assumed zero) as shown in Fig. 3.91 (*i*). Then resistance measured between terminals *B* and *D* is equal to $R_{Th}$.

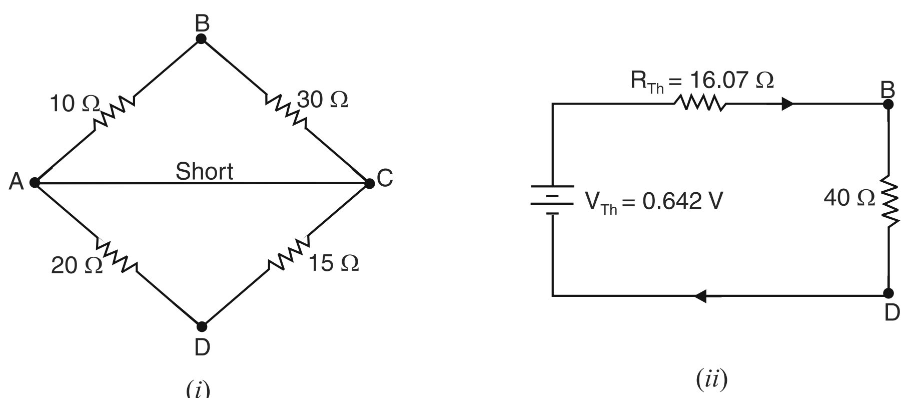



**Fig. 3.91**

* The potential at point *D* is 1·142 V lower than at *A*. Also potential of point *B* is 0·5 V lower than *A*. Hence point *B* is at higher potential than point *D*.

$$R_{Th} = \text{Resistance at terminals } BD \text{ in Fig. 3.91 } (i).$$

$$= \frac{10 \times 30}{10+30} + \frac{20 \times 15}{20+15} = 7.5 + 8.57 = 16.07\Omega$$

Thevenin's equivalent circuit at terminals *BD* is $V_{Th}$ (= 0.642 V) in series with $R_{Th}$ (= 16·07 Ω). When galvanometer is connected between *B* and *D*, the circuit becomes as shown in Fig. 3.91 (*ii*).

$$\therefore \quad \text{Galvanometer current} = \frac{V_{Th}}{R_{Th}+40} = \frac{0.642}{16.07+40}$$

$$= 11.5 \times 10^{-3} \text{ A} = \mathbf{11.5\ mA} \text{ from } B \text{ to } D$$

**Example 3·39.** *Find the Thevenin equivalent circuit lying to the right of terminals x − y in Fig. 3.92.*

**Solution.** In this example, there is no external circuitry connected to $x - y$ terminals.

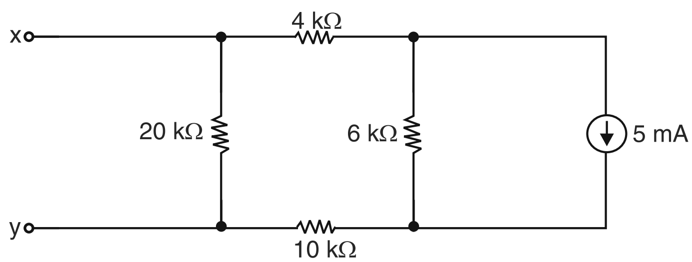



**Fig. 3.92**

(*i*) **Finding $R_{Th}$.** To find Thevenin equivalent resistance $R_{Th}$, we open-circuit the current source as shown in Fig. 3.93 (*i*). Note that 4 kΩ, 6 kΩ and 10 kΩ resistors are then in series and have a total resistance of 20 kΩ. Thus $R_{Th}$ is the parallel combination of that 20 kΩ resistance and the other 20 kΩ resistor as shown in Fig. 3·93 (*ii*).

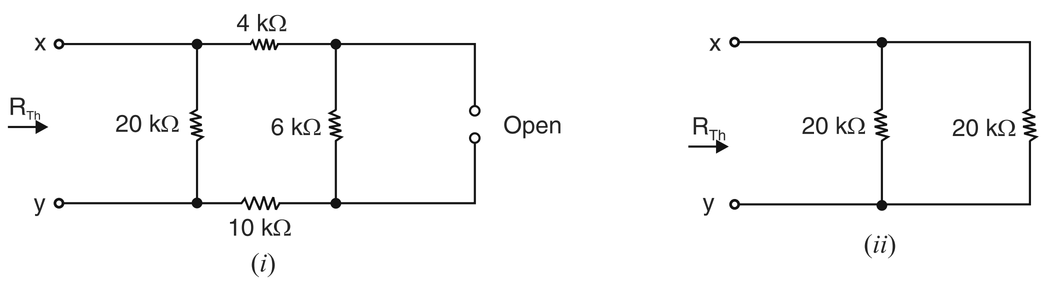
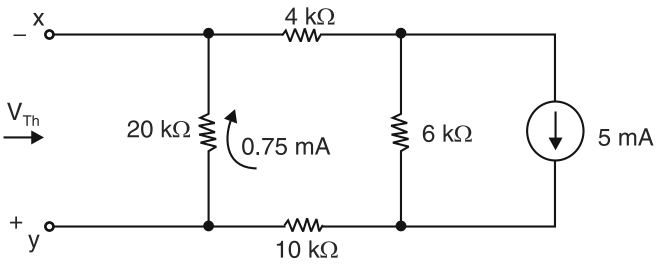



**Fig. 3.93**

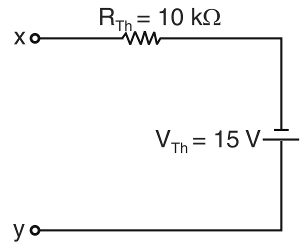
$$\therefore \quad R_{Th} = 20 \text{ k}\Omega \parallel 20 \text{ k}\Omega = \frac{20 \times 20}{20+20} = 10 \text{ k}\Omega$$

(*ii*) **Finding $V_{Th}$.** Fig. 3.94 (*i*) shows the computation of Thevenin equivalent voltage $V_{Th}$. Note that $V_{Th}$ is the voltage drop across the 20 kΩ resistor. The current from the 5 mA source divides between 6 kΩ resistor and the series string of 10 kΩ + 20 kΩ + 4 kΩ = 34 kΩ. Thus, by the current divider-rule, the current in 20 kΩ resistor is

$$I_{20}\text{ k}\Omega = \left(\frac{6}{34+6}\right) \times 5 = 0.75 \text{ mA}$$




(*i*)




(*ii*)

**Fig. 3.94**

Voltage across 20 kΩ resistor is given by ;

$$V_{Th} = (0.75 \text{ mA})(20 \text{ k}\Omega) = 15 \text{ V}$$

Notice that terminal *y* is positive with respect to terminal *x*. **Fig. 3.94 (*ii*) shows the Thevenin equivalent circuit.** The polarity of $V_{Th}$ is such that terminal *y* is positive with respect to terminal *x*, as required.

**Example 3.40.** *Calculate the power which would be dissipated in a 50 Ω resistor connected across xy in the network shown in Fig. 3.95.*

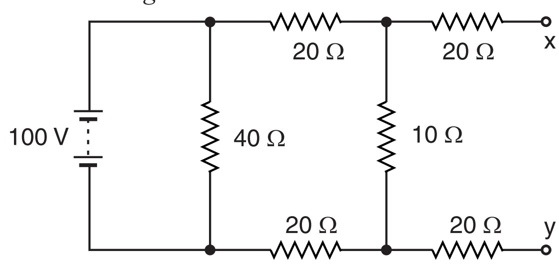



**Fig. 3.95**

**Solution.** We shall find Thevenin equivalent circuit to the left of terminals *xy*. With *xy* terminals open, the current in 10 Ω resistor is given by ;

$$^*I = \frac{100}{20+10+20} = 2\text{A}$$

∴ Open circuit voltage across *xy* is given by ;

$$V_{Th} = I \times 10 = 2 \times 10 = 20\text{V}$$

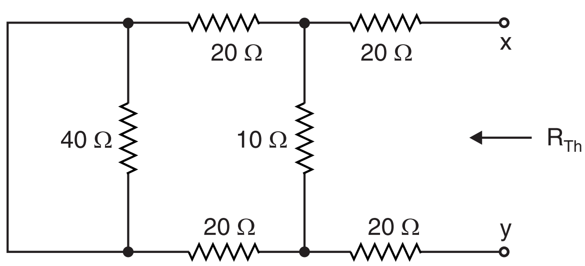



**Fig. 3.96**

In order to find $R_{Th}$ replace the battery by a short since its internal resistance is assumed to be zero [See Fig. 3.96].

$$R_{Th} = \text{Resistance looking into the terminals } xy \text{ in Fig. 3.96.}$$
$$= 20 + [(20 + 20) \parallel 10] + 20$$

* It is clear that (20 + 10 + 20) Ω is in parallel with 40 Ω resistor across 100 V source.

$$= \ 20 + \frac{*40 \times 10}{40 + 10} + 20 \ = 20 + 8 + 20 = 48\ \Omega$$

Therefore, Thevenin's equivalent circuit behind terminals *xy* is $V_{Th}$ ( = 20V) in series with $R_{Th}$ (= 48 Ω). When load $R_L$ (= 50 Ω) is connected across *xy*, the circuit becomes as shown in Fig. 3.97.

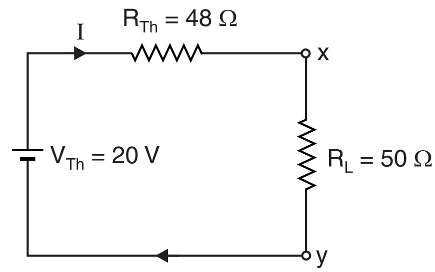


**Fig. 3.97**

∴ Current $I$ in 50 Ω resistor is

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{20}{48 + 50} = \frac{20}{98}\text{A}$$

∴ Power dissipated in 50 Ω resistor is

$$P = I^2 R_L = \left(\frac{20}{98}\right)^2 \times 50 = \mathbf{2{\cdot}08\ W}$$

**Example 3.41.** *Calculate the current in the 50 Ω resistor in the network shown in Fig. 3.98.*

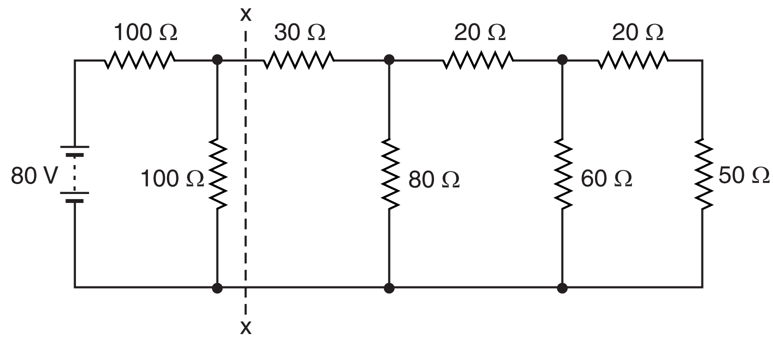
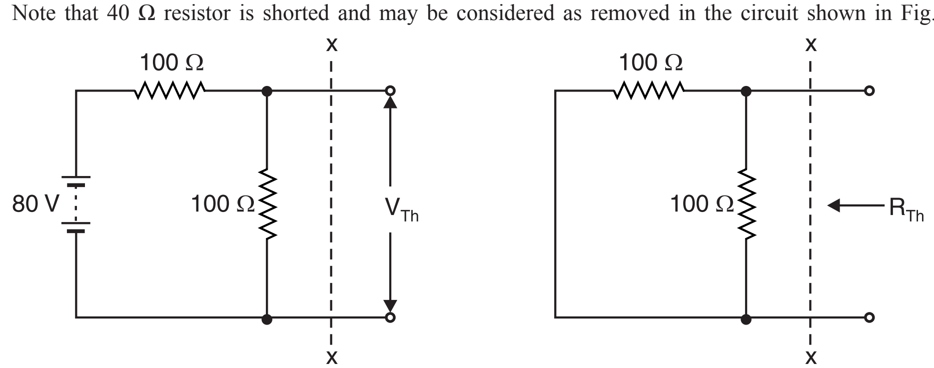


**Fig. 3.98**

**Solution.** We shall simplify the circuit shown in Fig. 3.98 by the repeated use of Thevenin's theorem. We first find Thevenin's equivalent circuit to the left of ***xx*.

* Note that 40 Ω resistor is shorted and may be considered as removed in the circuit shown in Fig. 3.96.

**



(*a*) (*b*)

$$V_{Th} = \text{Current in } 100\ \Omega \times 100\ \Omega = \frac{80}{100 + 100} \times 100 = 40\text{V}$$

$R_{Th}$ = Resistance looking into the open terminals in Fig. (*b*)

$$= 100 \parallel 100 = \frac{100 \times 100}{100 + 100} = 50\Omega$$

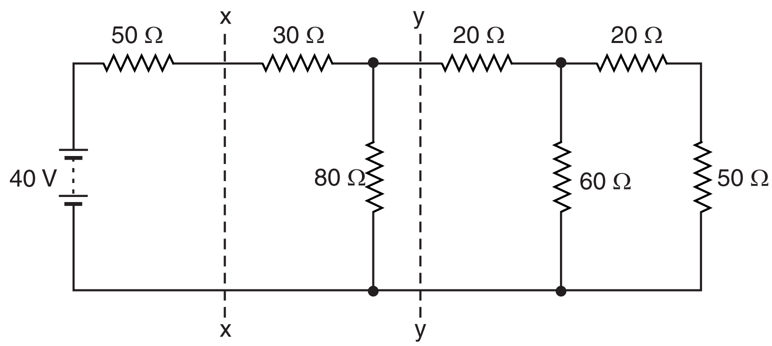



**Fig. 3.99**

$$V_{Th} = \frac{80}{100+100} \times 100 = 40\text{V}$$

$$R_{Th} = 100 \parallel 100 = \frac{100 \times 100}{100+100} = 50\Omega$$

Therefore, we can replace the circuit to the left of *xx* in Fig. 3.98 by its Thevenin's equivalent circuit *viz*. $V_{Th}$ (= 40V) in series with $R_{Th}$ (= 50 Ω). The original circuit of Fig. 3.98 then reduces to the one shown in Fig. 3.99.

We shall now find Thevenin's equivalent circuit to the left of *yy* in Fig. 3.99.

$$V'_{Th} = \frac{40}{50+30+80} \times 80 = 20 \text{ V}$$

$$R'_{Th} = (50+30) \parallel 80 = \frac{80 \times 80}{80+80} = 40 \ \Omega$$

We can again replace the circuit to the left of *yy* in Fig. 3.99 by its Thevenin's equivalent circuit. Therefore, the original circuit reduces to that shown in Fig. 3.100.

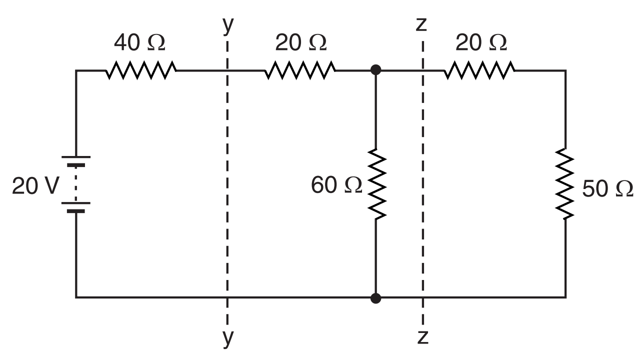



**Fig. 3.100**

Using the same procedure to the left of *zz*, we have,

$$V''_{Th} = \frac{20}{40+20+60} \times 60 = 10 \text{ V}$$

$$R''_{Th} = (40+20) \parallel 60 = \frac{60 \times 60}{60+60} = 30\Omega$$

The original circuit then reduces to that shown in Fig. 3.101.

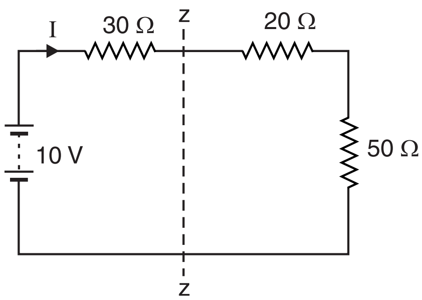



**Fig. 3.101**

By Ohm's law, current $I$ in 50 Ω resistor is

$$I = \frac{10}{30+20+50} = \mathbf{0.1\ A}$$

**Example 3.42.** *Calculate the current in the 10 Ω resistor in the network shown in Fig. 3·102.*

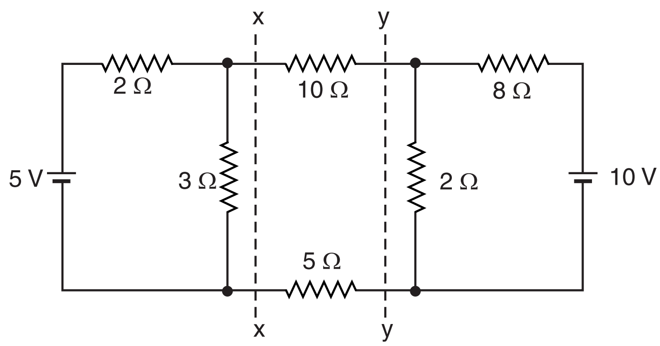



**Fig. 3.102**

**Solution**. We can replace circuits to the left of *xx* and right of *yy* by the Thevenin's equivalent circuits. It is easy to see that to the left of *xx*, the Thevenin's equivalent circuit is a voltage source of 3V (= $V_{Th}$) in series with a resistor of *1·2 Ω (= $R_{Th}$). Similarly, to the right of *yy*, the Thevenin's equivalent circuit is a voltage source of 2V ( = $V_{Th}$) in series with a resistor of **1·6 Ω (= $R_{Th}$). The original circuit then reduces to that shown in Fig. 3.103.

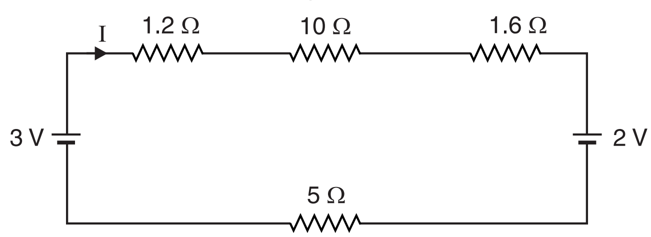



**Fig. 3.103**

∴ Current through 10 Ω resistor is given by ;

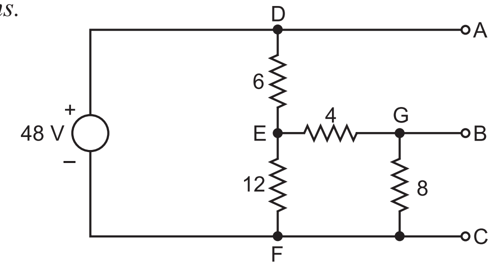
$$I = \frac{\text{Net voltage}}{\text{Total resistance}} = \frac{3-2}{1.2+10+1.6+5} = 56{\cdot}2 \times 10^{-3}\ \text{A} = \mathbf{56{\cdot}2\ mA}$$

---

* $R_{Th} = 2 \parallel 3 = \dfrac{2\times3}{2+3} = 1.2\Omega$

** $R_{Th} = 2 \parallel 8 = \dfrac{2\times8}{2+8} = 1.6\Omega$

**Example 3.43.** *Calculate the values of $V_{Th}$ and $R_{Th}$ between terminals A and B in Fig. 3.104 (i). All resistances are in ohms.*




(*i*)

**Fig. 3.104**

**Solution. (*i*) Finding $V_{Th}$.** Between points *E* and *F* [See Fig. 3.104 (*i*)], 12Ω || (4 + 8)Ω.

$$\therefore \quad R_{EF} = 12\Omega \parallel (4+8)\Omega = 12\Omega \parallel 12\Omega = 6\Omega$$

By voltage-divider rule, we have,

$$V_{DE} = 48 \times \frac{6}{6+6} = 24\text{V} \;;\; V_{EF} = 48 \times \frac{6}{6+6} = 24\text{V}$$

Now $V_{EF}$ (= 24V) is divided between 4Ω and 8Ω resistances in series.

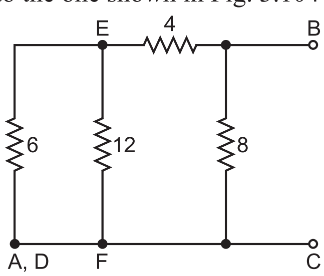
$$\therefore \quad V_{EG} = 24 \times \frac{4}{4+8} = 8\text{V}$$

In going from *A* to *B* via *D, E* and *G,* there is fall in potential from *D* to *E*, fall in potential from *E* to *G* and rise in potential from *B* to *A*. Therefore, by *KVL*,

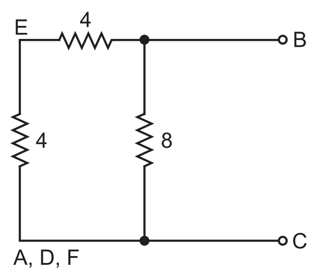
$$V_{BA} - V_{DE} - V_{EG} = 0 \quad \text{or} \quad V_{BA} = V_{DE} + V_{EG} = 24 + 8 = 32\text{V}$$

$$\therefore \quad V_{Th} = V_{BA} = \mathbf{32V} \;;\; A \text{ positive } w.r.t\ B.$$

**(*ii*) Finding $R_{Th}$.** $R_{Th}$ is the resistance between open terminals *AB* with voltage source replaced by a short as shown in Fig. 3.104 (*ii*). Shorting voltage source brings points *A*, *D* and *F* together. Now combined resistance of parallel combination of 6Ω and 12Ω = 6Ω || 12Ω = 4Ω and the circuit reduces to the one shown in Fig. 3.104 (*iii*).




(*ii*)




(*iii*)

**Fig. 3.104**

$$\therefore \quad R_{Th} = R_{AB} \text{ in Fig. 3.104 } (iii) = 8\Omega \parallel (4+4)\Omega = \mathbf{4\Omega}$$

**Example 3.44.** *The circuit shown in Fig. 3.105 consists of a current source I = 10 A paralleled by G = 0·1S and a voltage source E = 200 V with a 10 Ω series resistance. Find Thevenin equivalent circuit to the left of terminals AB.*

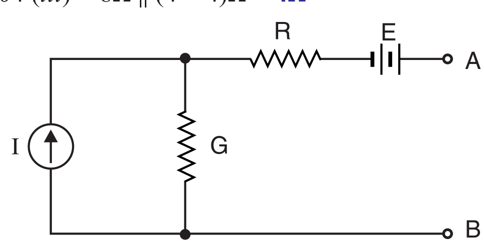



**Fig. 3.105**

**Solution.** With terminals *A* and *B* open-circuited, the current source will send a current through conductance *G* as shown in Fig. 3.106.

$\therefore$ Voltage across $G$, $V_G = \dfrac{I}{G} = \dfrac{10}{0.1} = 100$ V

Thevenin voltage, $V_{Th}$ = Open-circuited voltage at terminals $AB$ in Fig. 3.106.

$= E + V_G = 200 + 100 = 300$ V

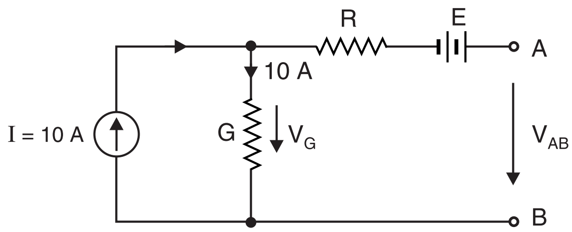



**Fig. 3.106**

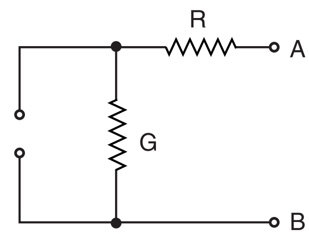



**Fig. 3.107**

In order to find Thevenin resistance $R_{Th}$, replace the voltage source by a short and current source by an open. The circuit then becomes as shown in Fig. 3.107.

$R_{Th}$ = Resistance looking into terminals $AB$ in Fig. 3.107.

$$= R + \frac{1}{G} = 10 + \frac{1}{0.1} = 10 + 10 = 20\Omega$$

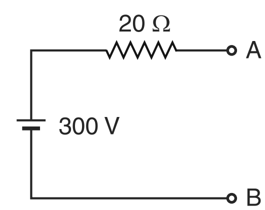



**Fig. 3.108**

Therefore, Thevenin equivalent circuit consists of **300V voltage source in series with a resistance of 20 Ω** as shown in Fig. 3.108.

**Example 3.45.** *Using Thevenin's theorem, find the voltage across 3Ω resistor in Fig. 3.109 (i).*

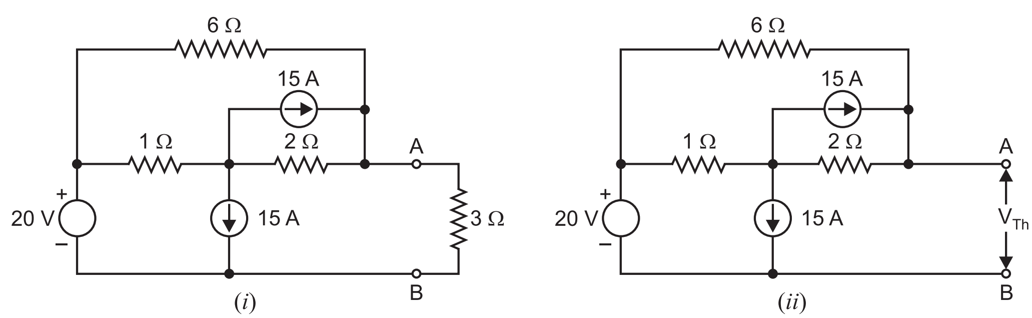


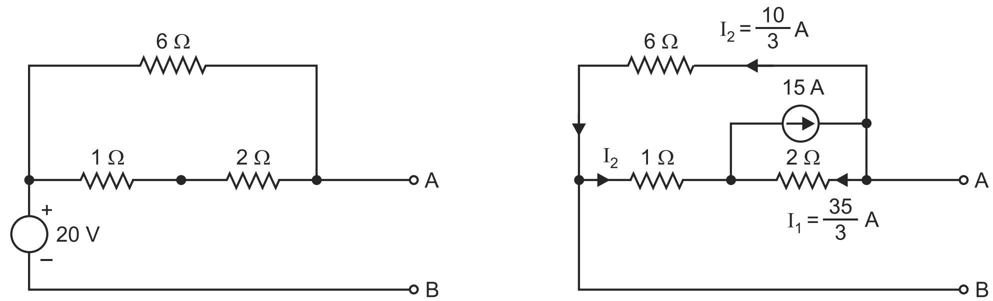

**Fig. 3.109**

**Solution. (*i*) Finding $V_{Th}$.** Thevenin voltage $V_{Th}$ is the voltage at the open-circuited load terminals $AB$ (*i.e.,* when 3Ω is removed) as shown in Fig. 3.109 (*ii*). It can be found by superposition theorem. First, open circuit both 15A current sources so that 20V voltage source is acting alone as shown in Fig. 3.109 (*iii*). It is clear that :

$$V_{AB1} = {}^{*}20\text{V}$$

Next, open one 15A current source and replace 20V source by a short so that the second 15A source is acting alone as shown in Fig. 3.109 (*iv*). By current-divider rule, the currents in the various branches will be as shown in Fig. 3.109 (*iv*).

* The circuit behaves as a 20V source having internal resistance of (1 + 2)Ω || 6Ω with terminals $AB$ open.




(*iii*) (*iv*)

**Fig. 3.109**

Referring to Fig. 3.109 (*iv*), we have,

$$V_A - I_1 \times 2 + I_2 \times 1 = V_B$$

$$\therefore \quad V_A - V_B = I_1 \times 2 - I_2 \times 1 = \frac{35}{3}\times 2 - \frac{10}{3}\times 1 = 20 \text{ V}$$

$$\therefore \quad V_{AB2} = V_A - V_B = 20\text{V}$$

Finally, open the second 15A source and replace the 20V source by a short as shown in Fig. 3.109 (*v*). By current-divider rule, the currents in the various branches will be as shown in Fig. 3.109 (*v*).

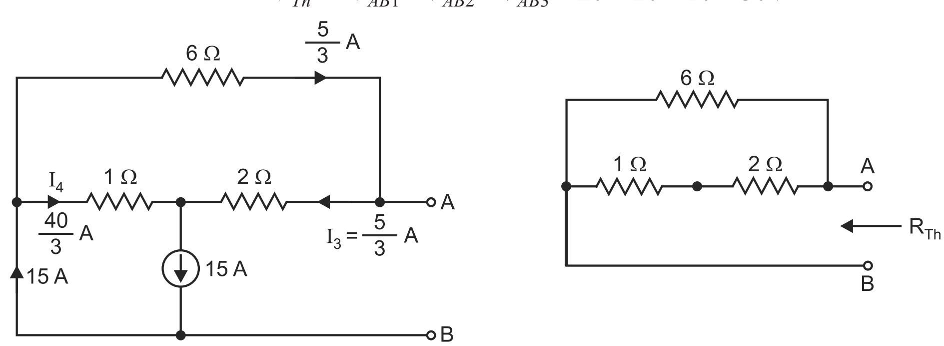
Now, $$V_A - I_3 \times 2 + I_4 \times 1 = V_B$$

$$\therefore \quad V_A - V_B = I_3 \times 2 - I_4 \times 1 = \frac{5}{3}\times 2 - \frac{40}{3}\times 1 = -10\text{V}$$

$$\therefore \quad V_{AB3} = V_A - V_B = -10\text{V}$$

By superposition theorem, the open-circuited voltage at terminals *AB* (*i.e.*, $V_{Th}$) with all sources present is

$$V_{Th} = V_{AB1} + V_{AB2} + V_{AB3} = 20 + 20 - 10 = 30\text{V}$$




(*v*) (*vi*)

**Fig. 3.109**

***(ii)* Finding $R_{Th}$.** Thevenin resistance $R_{TH}$ is the resistance at terminals *AB* when 3Ω is removed and current sources replaced by open and voltage source replaced by short as shown in Fig. 3.109 (*vi*).

$$\therefore \quad R_{Th} = (1\Omega + 2\Omega) \parallel 6\Omega = 2\Omega$$

$$\therefore \quad \text{Current in } 3\Omega,\ I = \frac{V_{Th}}{R_{Th}+3} = \frac{30}{2+3} = 6\text{A}$$

$$\therefore \quad \text{Voltage across } 3\Omega = I \times 3 = 6 \times 3 = \mathbf{18V}$$

**Example 3.46.** *Using Thevenin's theorem, determine the current in 1 Ω resistor across AB of the network shown in Fig. 3.110 (i). All resistances are in ohms.*

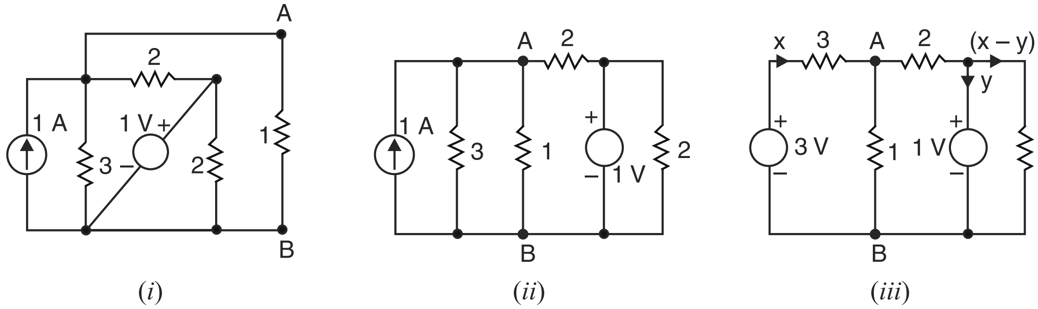



**Fig. 3.110**

**Solution.** The circuit shown in Fig. 3.110 (*i*) can be redrawn as shown in Fig. 3.110 (*ii*). If we convert the current source into equivalent voltage source, the circuit becomes as shown in Fig. 3.110 (*iii*). In order to find $V_{Th}$, remove 1 Ω resistor from the terminals *AB*. Then voltage at terminals *AB* is equal to $V_{Th}$ (See Fig. 3.111 (*i*)). Applying *KVL* to the first loop in Fig. 3.111 (*i*), we have,

$$3 - (3 + 2)\,x - 1 = 0 \qquad \therefore \qquad x = 0{\cdot}4 \text{ A}$$

$$\therefore \qquad V_{Th} = V_{AB} = 3 - 3x = 3 - 3 \times 0{\cdot}4 = 1{\cdot}8 \text{ V}$$

In order to find $R_{Th}$, replace the voltage sources by short circuits and current sources by open circuits in Fig. 3.110 (*ii*). The circuit then becomes as shown in Fig. 3.111 (*ii*). Then resistance at terminals *AB* is equal to $R_{Th}$.

Clearly, $$R_{Th} = 2 \,||\, 3 = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

Thevenin's equivalent circuit is 1·8 V voltage source in series with 1·2 Ω resistor. When 1 Ω resistor is connected across the terminals *AB* of the Thevenin's equivalent circuit, the circuit becomes as shown in Fig. 3.111 (*iii*).

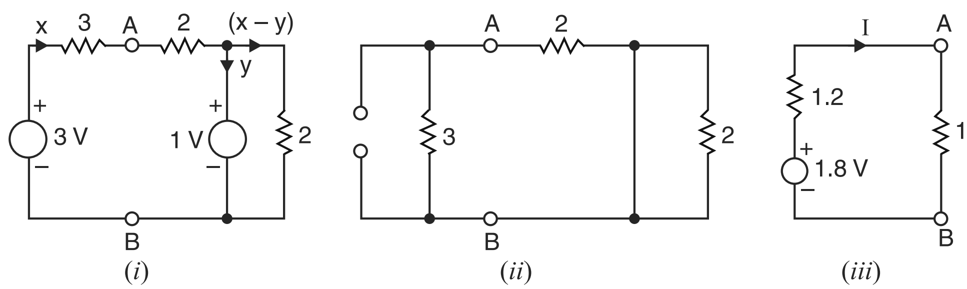



**Fig. 3.111**

$$\therefore \qquad \text{Current in } 1\ \Omega = \frac{V_{Th}}{R_{Th} + 1} = \frac{1.8}{1.2 + 1} = \mathbf{0{\cdot}82\ A}$$

**Example 3.47.** *At no-load, the terminal voltage of a d.c. generator is 120 V. When delivering its rated current of 40 A, its terminal voltage drops to 112 V. Represent the generator by its Thevenin equivalent.*

**Solution.** If *R* is the internal resistance of the generator, then,

$$E = V + IR \qquad \text{or} \qquad R = \frac{E - V}{I} = \frac{120 - 112}{40} = 0.2\Omega$$

Therefore, $V_{Th}$ = No-load voltage = 120 V and $R_{Th} = R = 0{\cdot}2\ \Omega$.

Hence Thevenin equivalent circuit of the generator is **120 V source in series with 0·2 Ω resistor.**

**Example 3.48.** *Calculate $V_{Th}$ and $R_{Th}$ between the open terminals A and B of the circuit shown in Fig. 3.112 (i). All resistance values are in ohms.*

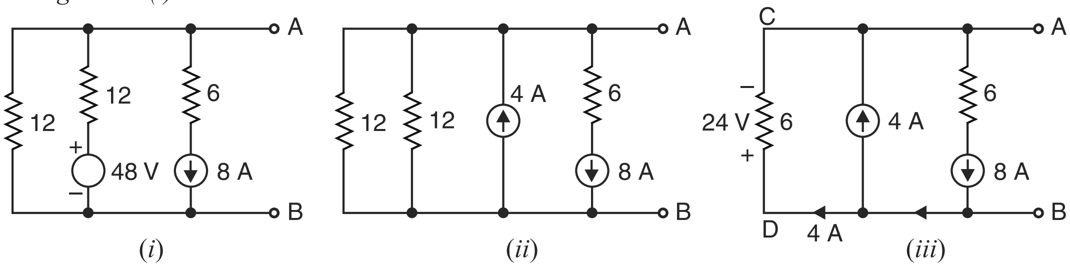



**Fig. 3.112**

**Solution.** If we replace the 48 V voltage source into equivalent current source, the circuit becomes as shown in Fig. 3.112 (*ii*). The two 12 Ω resistors are in parallel and can be replaced by 6 Ω resistor. The circuit then reduces to the one shown in Fig. 3.112 (*iii*). It is clear that 4 A current flows through 6 Ω resistor.

∴ $V_{Th}$ = Voltage across terminals *AB* in Fig. 3.112 (*iii*)

= Voltage across 6 Ω resistor = 4 × 6 = 24 V

Note that terminal *A* is negative w.r.t. *B*. Therefore, $V_{Th}$ = **– 24 V.**

$R_{Th}$ = Resistance between terminals *AB* in Fig. 3.112 (*i*) with 48V source replaced by a short and 8 A source replaced by an open

= 12 || 12 = **6 Ω**

**Example 3.49.** *Find the voltage across $R_L$ in Fig. 3.113 when (i) $R_L$ = 1 kΩ (ii) $R_L$ = 2 kΩ (iii) $R_L$ = 9 kΩ. Use Thevenin's theorem to solve the problem.*

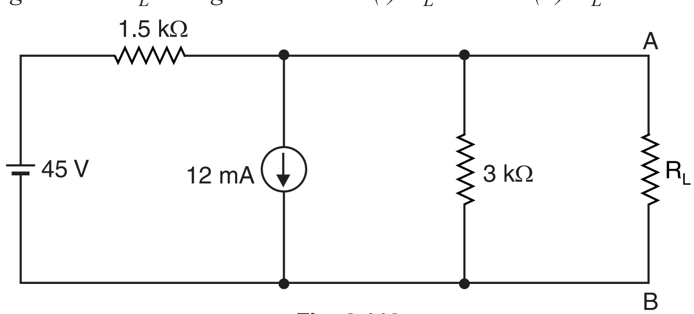



**Fig. 3.113**

**Solution.** It is required to find the voltage across $R_L$ when $R_L$ has three different values. We shall find Thevenin's equivalent circuit to the left of the terminals *AB*. The solution involves two steps.

The first step is to find the open-circuited voltage $V_{Th}$ at terminals *AB*. For this purpose, we shall use the superposition principle. With the current source removed (opened), we find voltage $V_1$ due to the 45 V source acting alone as shown in Fig. 3.114 (*i*). Since $V_1$ is the voltage across the 3 kΩ resistor, we have by voltage-divider rule :

$$V_1 = 45 \times \frac{3\,\text{k}\Omega}{1.5\,\text{k}\Omega + 3\,\text{k}\Omega} = 30\text{V}^{+}_{-}$$

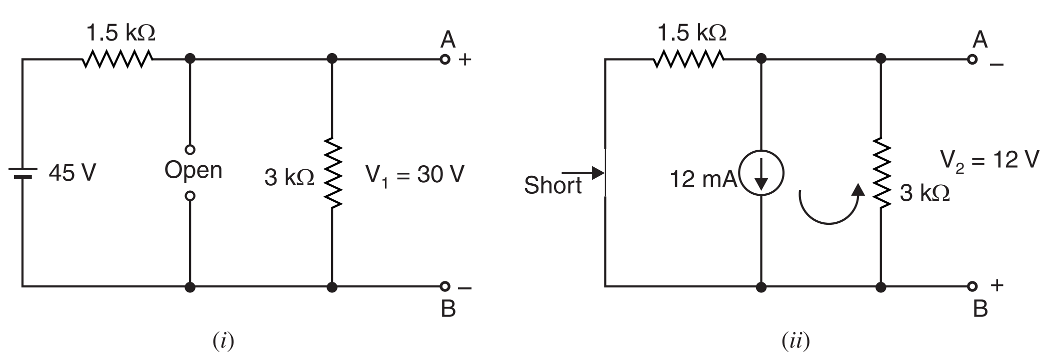



**Fig. 3.114**

The voltage $V_2$ due to the current source acting alone is found by shorting 45 V voltage source as shown in Fig. 3.114 (*ii*). By current-divider rule,

$$\text{Current in 3 k}\Omega\text{ resistor} = 12 \times \frac{1.5\text{ k}\Omega}{1.5\text{ k}\Omega + 3\text{ k}\Omega} = 4\text{ mA}$$

$$\therefore \quad V_2 = 4\text{ mA} \times 3\text{ k}\Omega = 12\text{ V}_{+}^{-}$$

Note that $V_1$ and $V_2$ have opposite polarities.

$$\therefore \quad \text{Thevenin's voltage, } V_{Th} = V_1 - V_2 = 30 - 12 = 18\text{ V}_{-}^{+}$$

The second step is to find Thevenin's resistance $R_{Th}$. For this purpose, we replace the 45 V voltage source by a short circuit and the 12 mA current source by an open circuit as shown in Fig. 3.115. As can be seen in the figure, $R_{Th}$ is equal to parallel equivalent resistance of 1·5 kΩ and 3 kΩ resistors.

$$\therefore \quad R_{Th} = 1{\cdot}5\text{ k}\Omega \parallel 3\text{ k}\Omega = 1\text{ k}\Omega$$

Fig. 3.116 shows Thevenin's equivalent circuit.

$$\text{Voltage across } R_L, \quad V_L = 18 \times \frac{R_L}{1\text{ k}\Omega + R_L}$$

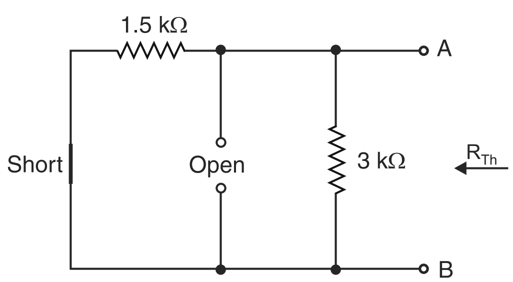



**Fig. 3.115**

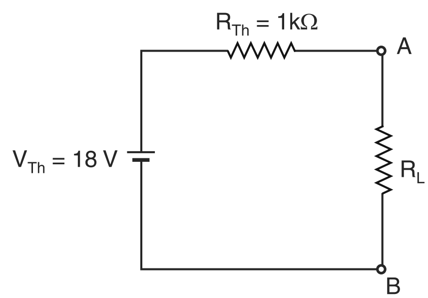



**Fig. 3.116**

(*i*) When $R_L = 1$ kΩ ; $V_L = 18 \times \frac{1\text{k}\Omega}{1\text{k}\Omega + 1\text{k}\Omega} = \mathbf{9\ V}$

(*ii*) When $R_L = 2$ kΩ ; $V_L = 18 \times \frac{2\text{k}\Omega}{1\text{k}\Omega + 2\text{k}\Omega} = \mathbf{12\ V}$

(*iii*) When $R_L = 9$ kΩ ; $V_L = 18 \times \frac{9\text{k}\Omega}{1\text{k}\Omega + 9\text{k}\Omega} = \mathbf{16{\cdot}2\ V}$

**Example 3.50.** *Find Thevenin's equivalent circuit to the left of terminals AB in Fig. 3.117.*

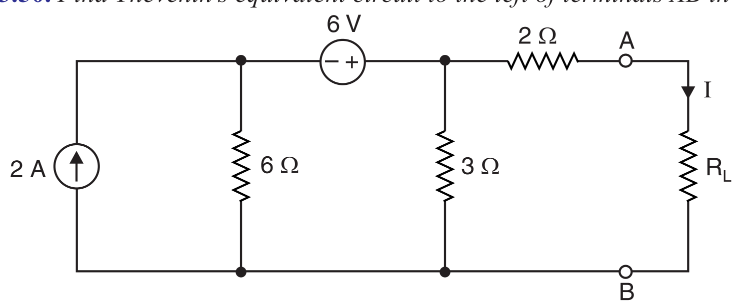



**Fig. 3.117**

**Solution.** To find $V_{Th}$, remove $R_L$ from terminals $AB$. The circuit then becomes as shown in Fig. 3.118 (*i*).

$\therefore$ $V_{Th}$ = Voltage across terminals $AB$ in Fig. 3.118 (*i*)

= Voltage across 3 Ω resistor in Fig. 3.118 (*i*)

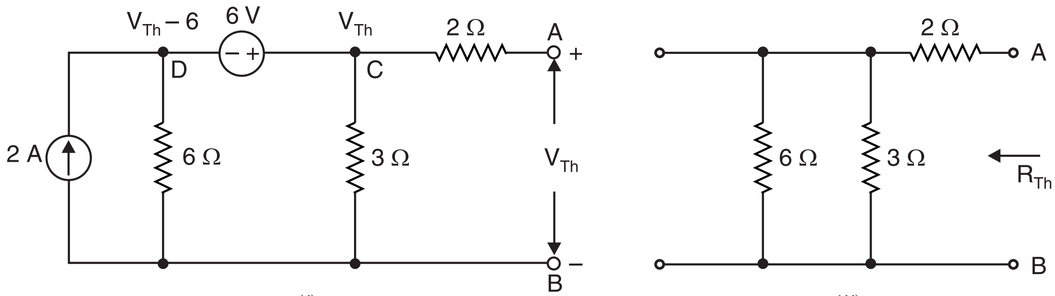



**Fig. 3.118**

Note that voltage at point $C$ is $V_{Th}$ and voltage at point $D$ is $V_{Th} - 6$. Therefore, nodal equation becomes :

$$\frac{V_{Th} - 6}{6} + \frac{V_{Th}}{3} = 2 \qquad \text{or} \qquad V_{Th} = 6 \text{ V}$$

In order to find $R_{Th}$, remove $R_L$ and replace voltage source by a short and current source by an open in Fig. 3.117. The circuit then becomes as shown in Fig. 3.118 (*ii*).

$\therefore$ $R_{Th}$ = Resistance looking into terminals $AB$ in Fig. 3.118 (*ii*).

$$= 2 + (3 \,||\, 6) = 2 + \frac{3 \times 6}{3 + 6} = 4\,\Omega$$

Therefore, Thevenin equivalent circuit to the left of terminals $AB$ is a **voltage source of 6 V (= $V_{Th}$) in series with a resistor of 4 Ω (= $R_{Th}$).** When load $R_L$ is connected across the output terminals of Thevenin equivalent circuit, the circuit becomes as shown in Fig. 3.119. We can use Ohm's law to find current in the load $R_L$.

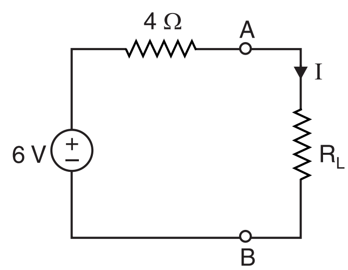



**Fig. 3.119**

$$\therefore \qquad \text{Current in } R_L,\ I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{6}{4 + R_L}$$

**Example 3.51.** *Find Thevenin's equivalent circuit in Fig. 3.120 when we view from (i) between points A and C (ii) between points B and C.*

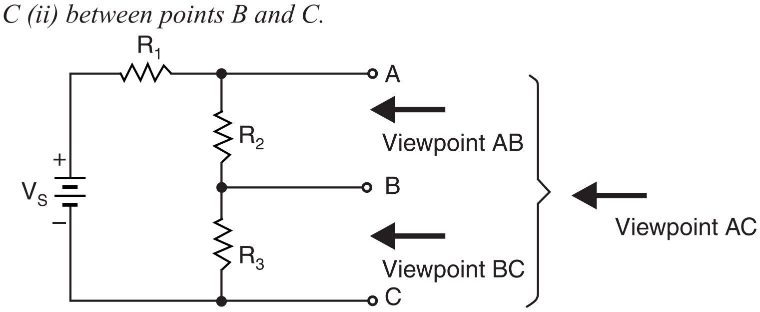



**Fig. 3.120**

**Solution.** The Thevenin equivalent for any circuit depends on the location of the two points from between which circuit is "viewed". Any given circuit can have more than one Thevenin equivalent, depending on how the viewpoints are designated. For example, if we view the circuit in Fig. 3.120

from between points $A$ and $C$, we obtain a completely different result than if we view it from between points $A$ and $B$ or from between points $B$ and $C$.

**(*i*) Viewpoint AC.** When the circuit is viewed from between points $A$ and $C$,

$V_{Th}$ = Voltage between open-circuited points $A$ and $C$ in Fig. 3.121 (*i*).

= Voltage across $(R_2 + R_3)$ in Fig. 3.121 (*i*)

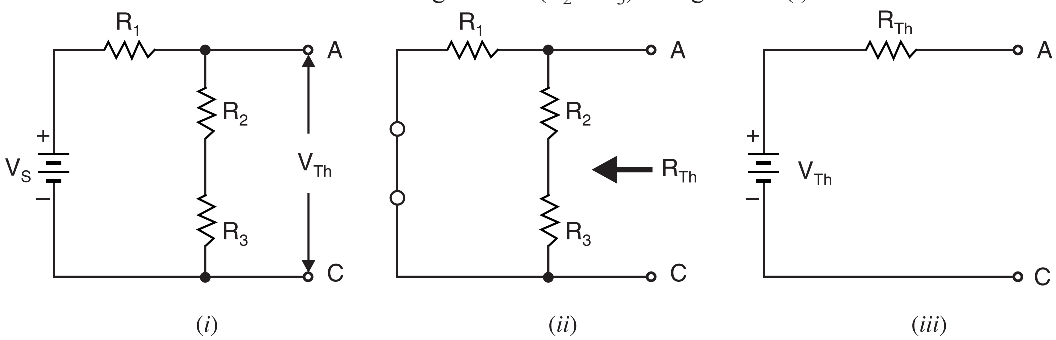



**Fig. 3.121**

$$= \frac{V_S}{R_1 + R_2 + R_3} \times (R_2 + R_3) = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right) V_S$$

In order to find $R_{Th}$, replace the voltage source by a short. Then resistance looking into the open-circuited terminals $A$ and $C$ [See Fig. 3.121 (*ii*)] is equal to $R_{Th}$.

$$\therefore \qquad R_{Th} = R_1 \parallel (R_2 + R_3) = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}$$

The resulting Thevenin equivalent circuit is shown in Fig. 3.121 (*iii*).

**(*ii*) Viewpoint BC.** When the circuit is viewed from between points $B$ and $C$,

$V_{Th}$ = Voltage between open-circuited points $B$ and $C$ in Fig.3.122 (*i*).

= Voltage across $R_3$

$$= \frac{V_S}{R_1 + R_2 + R_3} \times R_3 = \left(\frac{R_3}{R_1 + R_2 + R_3}\right) V_S$$

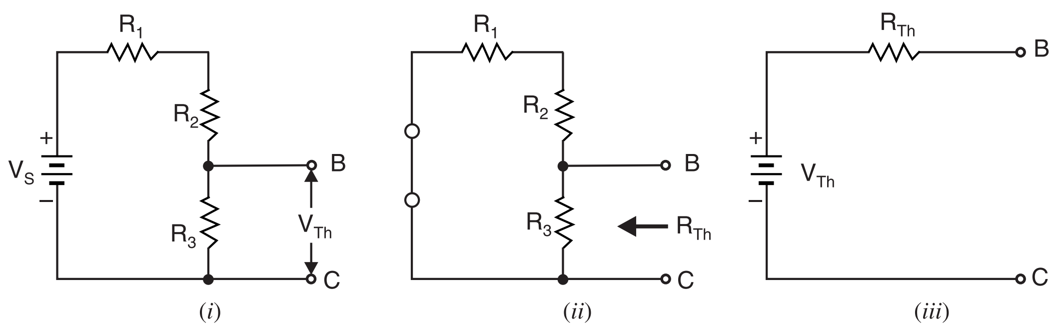



**Fig. 3.122**

In order to find $R_{Th}$, replace the voltage source by a short. Then resistance looking into the open-circuited terminals $B$ and $C$ [See Fig. 3.122 (*ii*)] is equal to $R_{Th}$.

$$\therefore \qquad R_{Th} = (R_1 + R_2) \parallel R_3 = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$$

The resulting Thevenin equivalent circuit is shown in Fig. 3.122 (*iii*).

**Example 3.52.** *Calculate (i) $V_{Th}$ and (ii) $R_{Th}$ between the open terminals A and B in the circuit shown in Fig. 3.123 (i). All resistance values are in ohms.*

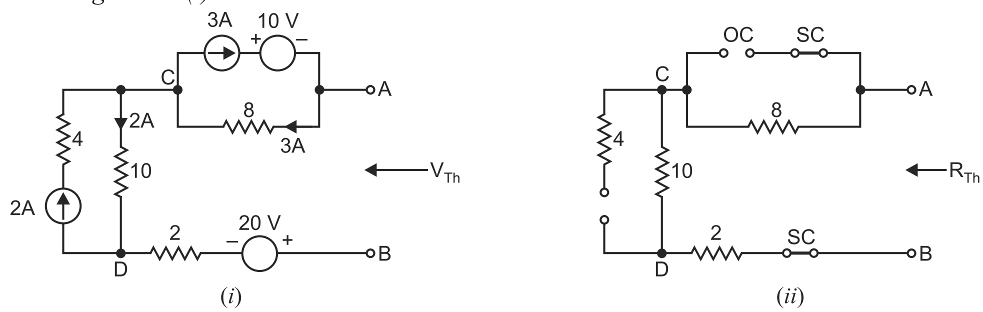



**Fig. 3.123**

**Solution.** Since terminals *A* and *B* are open, it is clear from the circuit that 10V and 20V voltage sources are ineffective in producing current in the circuit. However, current sources will circulate currents in their respective loops. Therefore, 2A current circulating in its loop will produce a voltage drop across 10 Ω resistance = 2A × 10 Ω = 20 V. Similarly, 3A current will produce a voltage drop across 8 Ω resistance = 3A × 8Ω = 24V. Tracing the circuit from *A* to *B via* points *C* and *D* [See Fig. 3.123 (*i*)], we have,

$$V_A - 24 - 20 + 20 = V_B$$

or $$V_A - V_B = 24 + 20 - 20 = 24\text{V}$$

∴ $$V_{Th} = V_{AB} = V_A - V_B = \mathbf{24V}$$

In order to find $R_{Th}$, open circuit the current sources and replace the voltage sources by a short as shown in Fig. 3.123 (*ii*). The resistance at the open-circuited terminals *AB* is $R_{Th}$.

∴ $$R_{Th} = \text{Resistance at terminals } AB \text{ in Fig. 3.123 } (ii)$$

$$= 8\Omega + 10\Omega + 2\Omega = \mathbf{20\Omega}$$

**Example 3.53.** *Find the current in the 25 Ω resistor in Fig. 3.124 (i) when E = 3 V.*

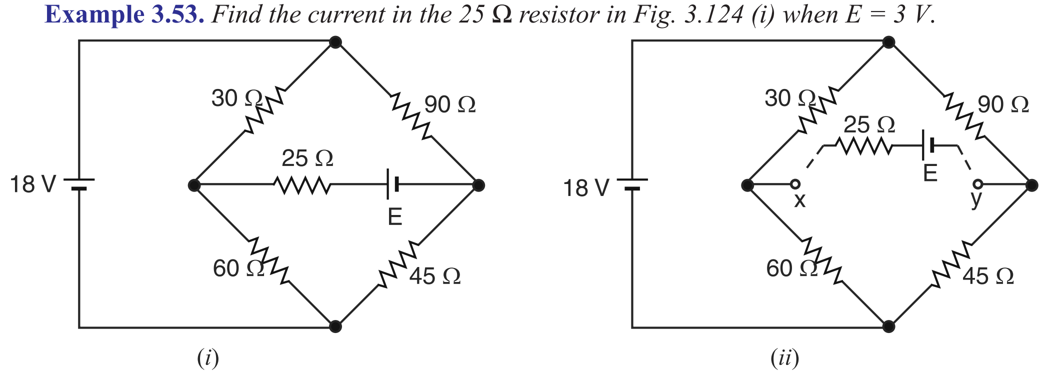



**Fig. 3.124**

**Solution. Finding $V_{Th}$.** Remove the voltage source *E* and the 25 Ω resistor, leaving the terminals *x – y* open-circuited as shown in Fig. 3.124 (*ii*). The circuit shown in Fig. 3.124 (*ii*) can be redrawn as shown in Fig. 3.125. The voltage between terminals *xy* in Fig. 3.125 is equal to $V_{Th}$. We can use voltage-divider rule to find voltage drops across 60 Ω and 45 Ω resistors.

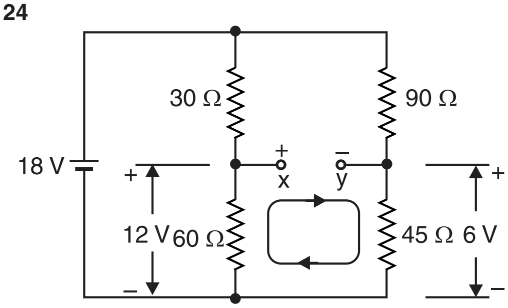



**Fig. 3.125**

$$\text{Voltage across } 60\ \Omega = 18 \times \frac{60}{60+30} = 12\text{V}$$

$$\text{Voltage across } 45\ \Omega = 18 \times \frac{45}{90+45} = 6\text{V}$$

Applying *KVL* around the loop shown in Fig. 3.125, we have,

$$12 - V_{xy} - 6 = 0 \qquad \therefore \qquad V_{xy} = 6\text{ V}$$

$$\text{But} \quad V_{xy} = V_{Th}. \text{ Therefore, } \quad V_{Th} = 6\text{ V}.$$

**Finding $R_{Th}$.** In order to find $R_{Th}$, replace the voltage source by a short. Then resistance at open-circuited terminals *xy* (See Fig. 3.126) is equal to $R_{Th}$. Note that in Fig. 3.126, 30 Ω and 60 Ω resistors are in parallel and so are 90 Ω and 45 Ω resistors.

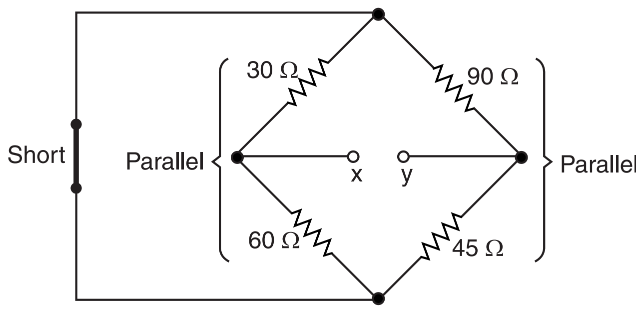



**Fig. 3.126**

The circuit shown in Fig. 3.126 can be redrawn as shown in Fig. 3.127 (*i*). This further reduces to the circuit shown in Fig. 3.127 (*ii*).

$$\therefore \qquad R_{Th} = 20 + 30 = 50\ \Omega$$

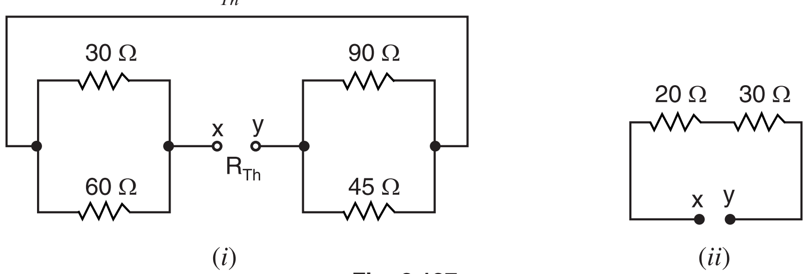



**Fig. 3.127**

Therefore, the Thevenin equivalent circuit is a voltage source of 6 V in series with 50 Ω resistor. When we reconnect *E* and 25 Ω resistor, the circuit becomes as shown in Fig. 3.128. Note that $V_{Th}$ and *E* are in series opposition.

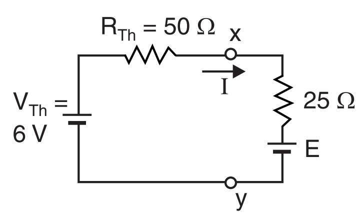



**Fig. 3.128**

$$\therefore \qquad \text{Current in } 25\ \Omega, I = \frac{V_{Th} - E}{R_{Th} + 25} = \frac{6-3}{50+25}$$

$$= 40 \times 10^{-3}\text{ A} = \mathbf{40\ mA}$$

**Example 3.54.** *Find the current in the feeder AC of the distribution circuit shown in Fig. 3.129 (i) by using Thevenin's theorem. Also determine the currents in other branches.*

**Solution.** To determine current in the feeder *AC*, we shall find Thevenin voltage $V_{Th}$ and Thevenin resistance $R_{Th}$ at terminals *AC*.

(*i*) With *AC* removed, the voltage between *A* and *C* will be equal to $V_{Th}$ as shown in Fig. 3.129 (*ii*). Assuming that current *I* flows in *AB*, then current distribution in the network will be as shown in Fig. 3.129 (*ii*).

$$\text{Voltage drop along } ADC = \text{Voltage drop across } ABC$$

or $$0{\cdot}05\,(100 - I) + 0{\cdot}05\,(80 - I) = 0.1\,I + 0{\cdot}1\,(I - 30)$$

or $$0{\cdot}3\,I = 12 \quad \therefore \quad I = 12/0{\cdot}3 = 40 \text{ A}$$

$\therefore$ $$\text{P.D. between } A \text{ and } C,\ V_{Th} = \text{Voltage drop from } A \text{ to } C$$
$$= 0{\cdot}05\,(100 - 40) + 0{\cdot}05\,(80 - 40) = 5 \text{ V}$$

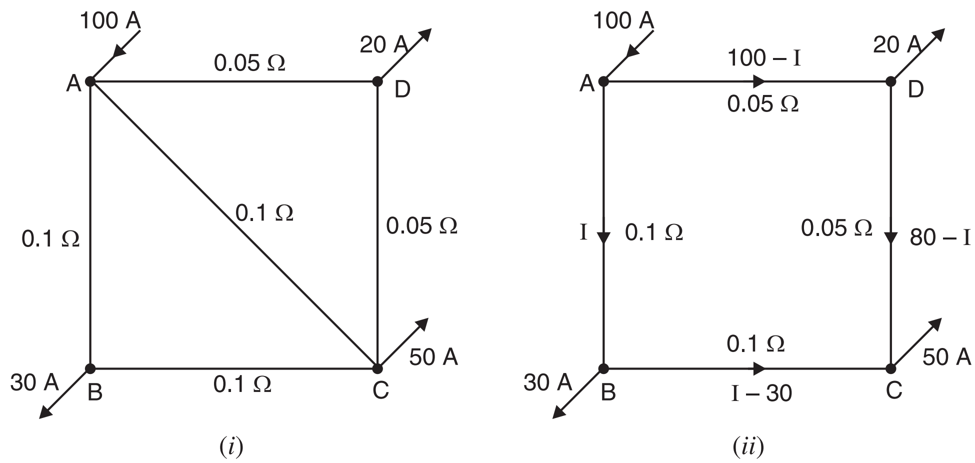



**Fig. 3.129**

**(*ii*)** With *AC* removed, the resistance between terminals *A* and *C* is equal to $R_{Th}$. Referring to Fig. 3.129 (*ii*), there are two parallel paths *viz ADC* (= 0·05 + 0·05 = 0·1 Ω) and *ABC* (= 0·1 + 0·1 = 0·2 Ω) between terminals *A* and *C*.

$\therefore$ $$R_{Th} = \frac{0.2 \times 0.1}{0.2 + 0.1} = 0{\cdot}067\ \Omega$$

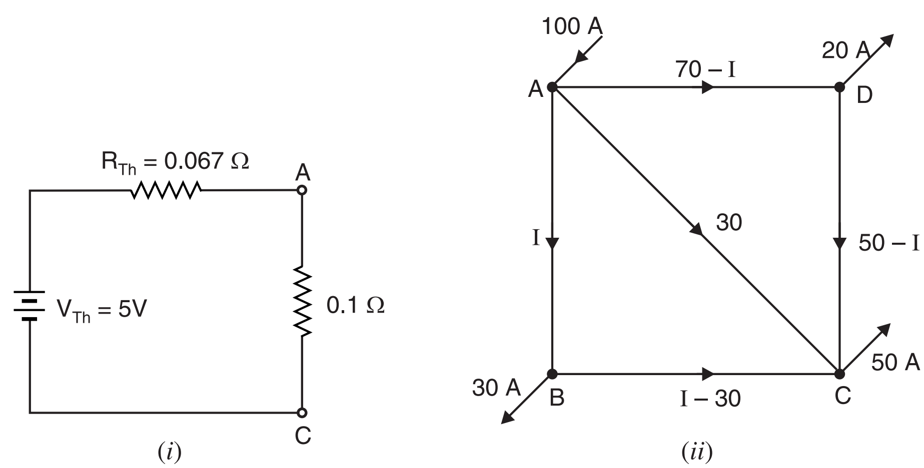



**Fig. 3.130**

The Thevenin equivalent circuit at terminals *AC* will be $V_{Th}$ (= 5 V) in series with $R_{Th}$ (= 0·067 Ω). When feeder *AC* (= 0·1 Ω) is connected between *A* and *C*, the circuit becomes as shown in Fig. 3.130 (*i*).

$\therefore$ $$\text{Current in } AC = \frac{V_{Th}}{R_{Th} + 0.1} = \frac{5}{0.067 + 0.1} = \mathbf{30\ A}$$

To find currents in other branches, refer to Fig. 3.130 (*ii*). With current in *AC* calculated (*i.e.* 30A) and current in *AB* assumed to be *I*, the current distribution will be as shown in Fig. 3.130 (*ii*). It is clear that voltage drop along the path *ADC* is equal to the voltage drop along the path *ABC i.e.*

$$0{\cdot}05\,(70 - I) + 0{\cdot}05\,(50 - I) = 0{\cdot}1\, I + 0{\cdot}1\,(I - 30)$$

or $\quad 0{\cdot}3\, I = 9$

$\therefore \quad I = 9/0{\cdot}3 =$ **30 A**

The current distribution in the various branches will be as shown in Fig. 3.131. Note that branch *BC* of the circuit carries no current.

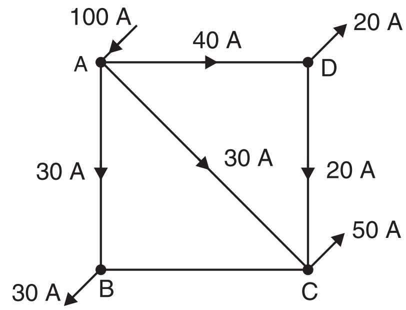



**Fig. 3.131**

**Example 3.55.** *Using Thevenin's theorem, calculate current in 1000Ω resistor connected between terminals A and B in Fig. 3.132 (i).*

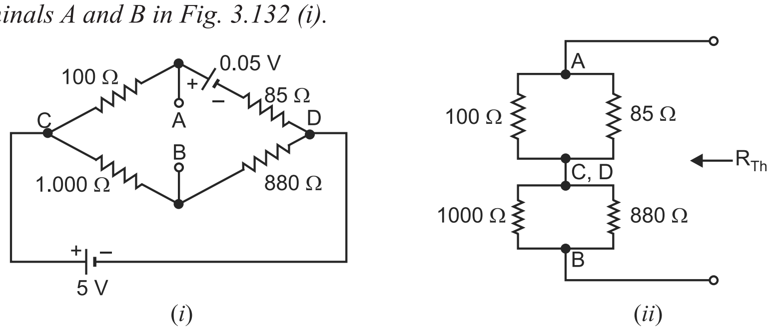



**Fig. 3.132**

**Solution. (*i*) Finding $V_{Th}$.** Thevenin voltage $V_{Th}$ is the voltage across open circuited terminals *AB* in Fig. 3.132 (*i*). Refer to Fig. 3.132 (*i*).

By voltage-divider rule, we have,

$$V_{BD} = 5 \times \frac{880}{1000 + 880} = 2.340426\text{V}$$

Current in branch *CAD* is $I = \dfrac{5 - 0.05}{100 + 85} = 0.026757\text{A}$

Now, $V_A - 0.05 - 0.026757 \times 85 = V_D$

$$\therefore \quad V_{AD} = V_A - V_D = 0.05 + 0.026757 \times 85 = 2.324324 \text{ V}$$

Clearly, point *B* is at higher potential than point *A*.

$$\therefore \quad V_{Th} = V_{BA} = 2.340426 - 2.324324 = 0.0161\text{V}$$

**(*ii*) Finding $R_{Th}$.** Thevenin resistance $R_{Th}$ is the resistance at open circuited terminals *AB* with 5V battery replaced by a short as shown in Fig. 3.132 (*ii*).

$$\therefore \quad R_{Th} = (100\Omega \,||\, 85\Omega) + (1000\Omega \,||\, 880\Omega)$$

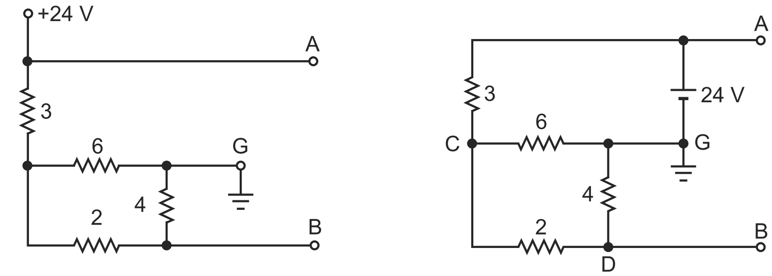
$$= \frac{100 \times 85}{100 + 85} + \frac{1000 \times 880}{1000 + 880} = 514\Omega$$

$\therefore$ Current in 1000Ω connected between terminals *A* and *B*

$$= \frac{V_{Th}}{R_{Th} + 1000} = \frac{0.0161}{514 + 1000} = 10.634 \times 10^{-6} \text{ A}$$

$$= \mathbf{10.634\ \mu A} \textbf{\textit{ from B to A}}$$

**Example 3.56.** *Calculate the values of $V_{Th}$ and $R_{Th}$ between the open terminals A and B of the circuit shown in Fig. 3.133 (i). All resistance values are in ohms.*




(*i*) (*ii*)

**Fig. 3.133**

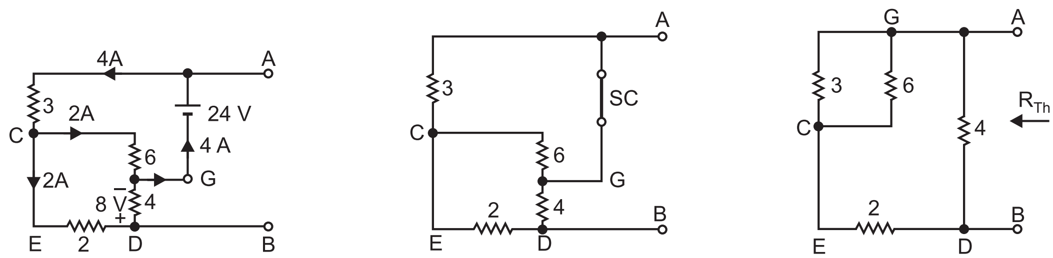
**Solution.** If we eliminate the ground symbols in the circuit shown in Fig. 3.133 (*i*), we get the circuit shown in Fig. 3.133 (*ii*). Referring to Fig. 3.133 (*ii*),

Total resistance offered to 24V battery

$$= 3\Omega + (6\Omega \parallel 6\Omega) = 3\Omega + 3\Omega = 6\Omega$$

Current delivered by 24V battery = 24/6 = 4A

The distribution of currents in the various branches of the circuit is shown in Fig. 3.133 (*iii*).




(*iii*) (*iv*) (*v*)

**Fig. 3.133**

Referring to Fig. 3.133 (*iii*) and tracing the circuit from point *A* to point *B* *via* points *C* and *D*, we have,

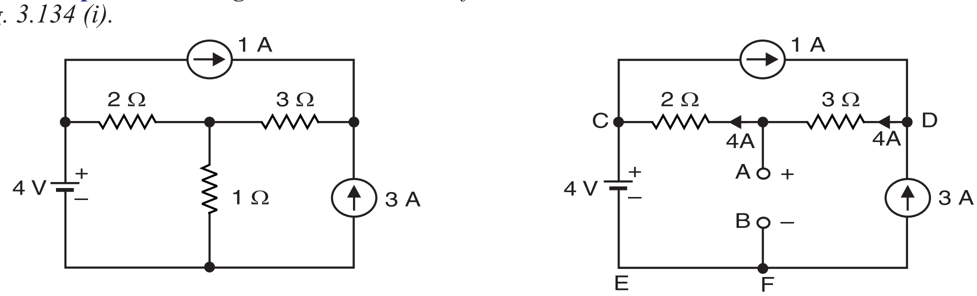
$$V_A - 3 \times 4 - 2 \times 6 + 4 \times 2 = V_B \quad \therefore \quad V_A - V_B = 3 \times 4 + 2 \times 6 - 4 \times 2 = 16\text{V}$$

$$\therefore \qquad V_{Th} = V_{AB} = V_A - V_B = \mathbf{16V}$$

In order to find $R_{Th}$, we replace the 24V source by a short and the circuit becomes as shown in Fig. 3.133 (*iv*). This circuit further reduces to the one shown in Fig. 3.133 (*v*).

$$\therefore \qquad R_{Th} = R_{AB} = [(3\Omega \parallel 6\Omega) + 2\Omega] \parallel 4\Omega = [2\Omega + 2\Omega] \parallel 4\Omega = \mathbf{2\Omega}$$

**Example 3.57.** *Using Thevenin theorem, find current in 1 Ω resistor in the circuit shown in Fig. 3.134 (i).*



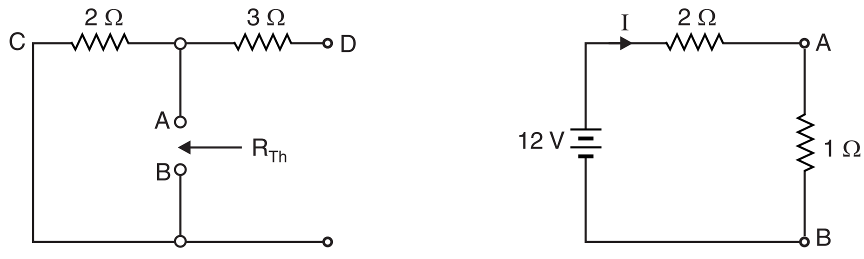

(*i*) (*ii*)

**Fig. 3.134**

**Solution.** In order to find $V_{Th}$, remove the load as shown in Fig. 3.134 (*ii*). Then voltage between the open-circuited terminals *A* and *B* is equal to $V_{Th}$. It is clear from Fig. 3.134 (*ii*) that 4 A (= 3 + 1) flows from *D* to *C*. Applying *KVL* to the loop *ECABFE*, we have,

$$4 + 2 \times 4 - V_{AB} = 0 \qquad \therefore \qquad V_{AB} = V_{Th} = 12 \text{ V}$$

$$R_{Th} = \text{Resistance looking into terminals } AB \text{ in Fig. 3.134 } (iii) = 2\ \Omega$$




(*iii*) (*iv*)

**Fig. 3.134**

When load (*i.e.* 1 Ω resistor) is reconnected, circuit becomes as shown in Fig. 3.134 (*iv*).

$$\therefore \qquad \text{Current in } 1\ \Omega = \frac{12}{2+1} = \mathbf{4\ A}$$

## 3.12. Thevenin Equivalent Circuit

**(Circuits containing both independent and dependent sources)**

Sometimes we come across circuits which contain both independent and dependent sources. One such example is shown in Fig. 3.135. The procedure for finding Thevenin equivalent circuit (*i.e.* finding $v_{Th}$ and $R_{Th}$) in such cases is as under :

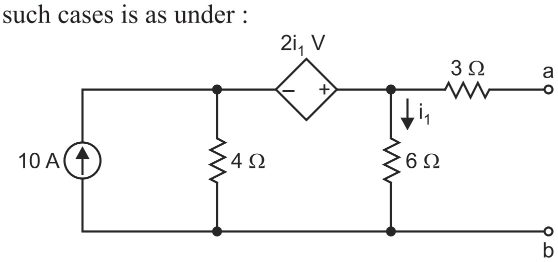



**Fig. 3.135**

(*i*) The open-circuit voltage $v_{oc}$ (= $v_{Th}$) at terminals *ab* is determined as usual with sources present.

(*ii*) We cannot find $R_{Th}$ at terminals *ab* simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals *ab* and find the value of short-circuit current $i_{sc}$ at terminals *ab*.

(*iii*) Therefore, Thevenin resistance *$R_{Th} = v_{oc}/i_{sc}(= v_{Th}/i_{sc})$. It is the same procedure as adopted for Norton's theorem.

**Note.** In case the circuit contains dependent sources *only*, the procedure of finding $v_{oc}(= v_{Th})$ and $R_{Th}$ is as under :

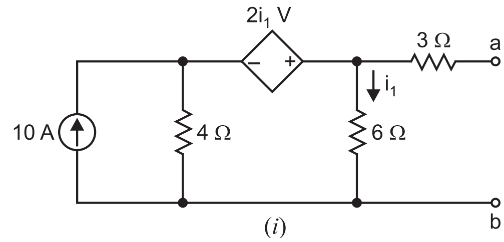
(*a*) In this case, $v_{oc} = 0$ and $i_{sc} = 0$ because no independent source is present.

(*b*) We cannot use the relation $R_{Th} = v_{oc}/i_{sc}$ as we do in case the circuit contains both independent and dependent sources.

* Alternatively, we can find $R_{Th}$ in another way. We excite the circuit at terminals *ab* from external 1A current source and measure $v_{ab}$. Then $R_{Th} = v_{ab}/1\Omega$.

(*c*) In order to find $R_{Th}$, we excite the circuit at terminals *ab* by connecting 1A source to the terminals *a* and *b* and calculate the value $v_{ab}$. Then $R_{Th} = v_{ab}/1\Omega$.

**Example 3.58.** *Find the values of $v_{Th}$ and $R_{Th}$ at terminals ab for the circuit shown in Fig. 3.136 (i).*



(*i*)

**Fig. 3.136**

**Solution.** We first put a short circuit across terminals *a* and *b* and find short-circuit current $i_{sc}$ at terminals *ab* as shown in Fig. 3.136 (*ii*). Applying *KCL* at node *C*,

$$10 = i_1 + i_2 + i_{sc}$$

or $$i_2 = 10 - i_1 - i_{sc}$$

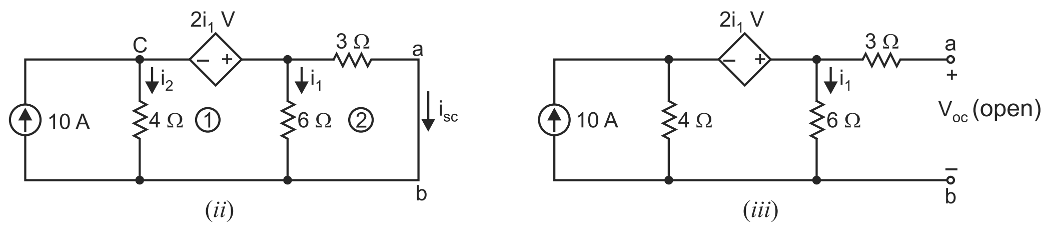
Applying *KVL* to loops 1 and 2, we have,

$$-4i_2 + 6i_1 - 2i_1 = 0 \qquad \text{... Loop 1}$$

or $$-4(10 - i_1 - i_{sc}) + 4i_1 = 0 \qquad ...(i)$$

Also $$-6i_1 + 3i_{sc} = 0 \qquad ...(ii) \text{ ... Loop 2}$$

From eqs. (*i*) and (*ii*), $i_{sc} = 5\text{A}$.



(*ii*) (*iii*)

**Fig. 3.136**

In order to find $v_{oc}$ (= $v_{Th}$), we refer to Fig. 3.136 (*iii*) where we have,

$$v_{oc} = 6i_1 \qquad ...(iii)$$

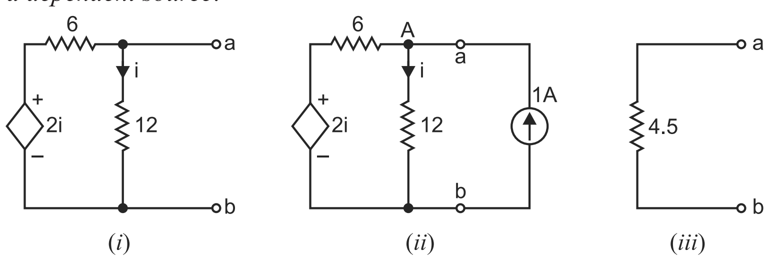
Applying *KVL* to the central loop in Fig. 3.136 (*iii*),

$$-4(10 - i_1) + 6i_1 - 2i_1 = 0 \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), we have, $v_{oc} = v_{Th} = \mathbf{30V}$

Also $$R_{Th} = \frac{v_{oc}}{i_{sc}} = \frac{30}{5} = \mathbf{6\Omega}$$

**Example 3.59.** *Find Thevenin equivalent circuit for the network shown in Fig. 3.137 (i) which contains only a dependent source.*



(*i*) (*ii*) (*iii*)

**Fig. 3.137**

**Solution.** In order to find $R_{Th}$, we connect 1A current source to terminals *a* and *b* as shown in Fig. 3.137 (*ii*). Then by finding the value of $v_{ab}$, we can determine the value of $R_{Th} = v_{ab}/1\Omega$. It may be seen that potential at point *A* is the same as that at *a*.

$\therefore$ $$v_{ab} = \text{Voltage across } 12\Omega \text{ resistor}$$

Applying *KCL* to point *A*, we have,

$$\frac{2i - v_{ab}}{6} + 1 = \frac{v_{ab}}{12}$$

or $$4i - 3v_{ab} = -12 \quad \text{or} \quad 4\left(\frac{v_{ab}}{12}\right) - 3v_{ab} = -2 \quad \therefore\ v_{ab} = 4.5\text{V}$$

$\therefore$ $$R_{Th} = 4.5/1 = \mathbf{4.5\Omega}$$

Fig. 3.137 (*iii*) shows the Thevenin equivalent circuit.

**Example 3.60.** *Find Thevenin equivalent circuit at terminals ab for the circuit shown in Fig. 3.138.*

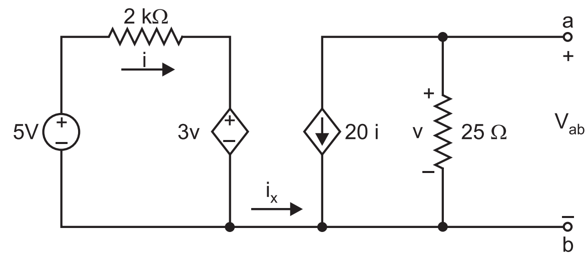



**Fig. 3.138**

**Solution.** The current $i_x$ is zero because there is no return path for $i_x$. The Thevenin voltage $v_{Th}$ will be the voltage across 25Ω resistor.

With $i_x = 0$, $v_{Th} = v = v_{ab} = (-20i)(25) = -500i$

The current $i$ is, $i = \dfrac{5 - 3v}{2 \times 1000} = \dfrac{5 - 3v_{Th}}{2000}$ $\qquad (\because v = v_{Th})$

$\therefore$ $$v_{Th} = -500\left(\frac{5 - 3v_{Th}}{2000}\right) \quad \text{or} \quad v_{Th} = -5\text{V}$$

In order to find Thevenin resistance $R_{Th}$, we find the short-circuit current $i_{sc}$ at terminals *ab*. Then,

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$

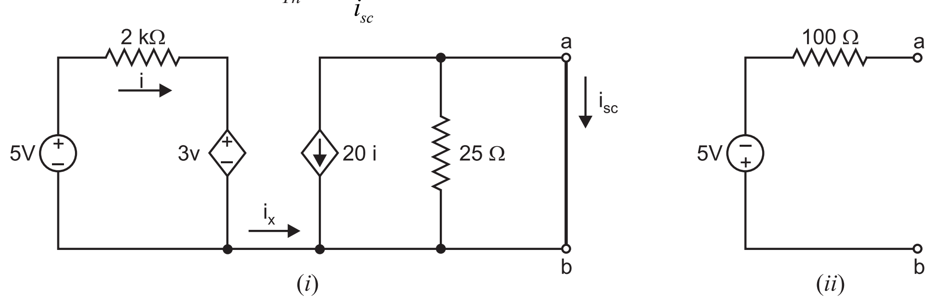



**Fig. 3.139**

To find $i_{sc}$, we short circuit the terminals *ab* as shown in Fig. 3.139 (*i*). It is clear that all the current from the dependent current source will pass through the short circuit ($\because$ 25Ω resistor is shunted by the short circuit).

$\therefore$ $$i_{sc} = -20i$$

Now, $i = \dfrac{5}{2000} = 2.5$ mA so that $i_{sc} = -20 \times 2.5 = -50$ mA

$\therefore$ $$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{-5}{-50 \times 10^{-3}} = 100\ \Omega$$

Fig. 3.139 (*ii*) shows the **Thevenin equivalent circuit at terminals *ab*.**

## Tutorial Problems

1. Using Thevenin's theorem, find the current in 10 Ω resistor in the circuit shown in Fig. 3.140. **[0.481 A]**

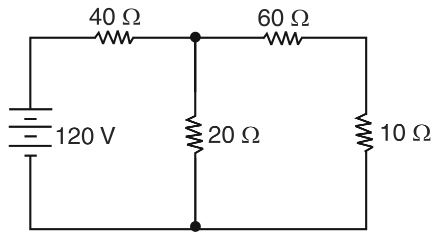



Fig. 3.140

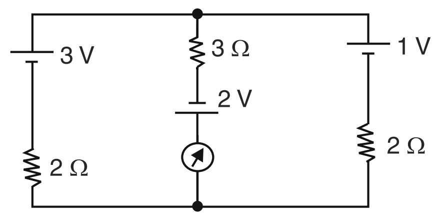



Fig. 3.141

2. Using Thevenin's theorem, find current in the ammeter shown in Fig. 3.141. **[1 A]**
3. Using Thevenin's theorem, find p.d. across branch *AB* of the network shown in Fig. 3.142. **[4.16 V]**

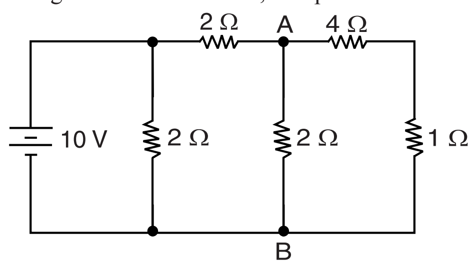



Fig. 3.142

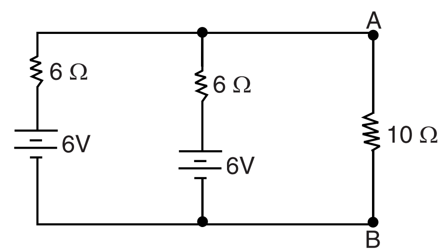



Fig. 3.143

4. Determine Thevenin's equivalent circuit to the left of *AB* in Fig. 3.143.

**[A 6 V source in series with 3 Ω]**

5. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 100 Ω, *BC* = 99 Ω, *CD* = 1000 Ω and *DA* = 1000 Ω. A battery of e.m.f. 10 V and negligible resistance is connected between *A* and *C* with *A* positive. A galvanometer of resistance 100 Ω is connected between *B* and *D*. Using Thevenin's theorem, determine the galvanometer current. **[38.6 μA]**
6. Find the Thevenin equivalent circuit of the circuitry, excluding $R_1$, connected to the terminals $x - y$ in Fig. 3.144. **[10 V in series with 9Ω ; *x* positive w.r.t. *y*]**

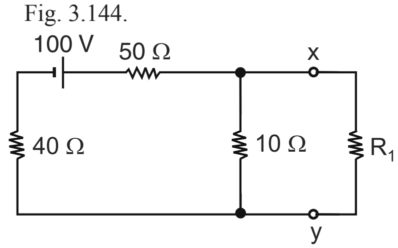



Fig. 3.144

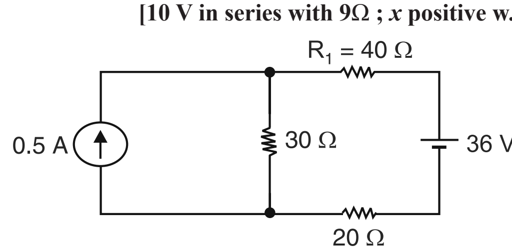



Fig. 3.145

7. Find the voltage across $R_1$ in Fig. 3.145 by constructing Thevenin equivalent circuit at the $R_1$ terminals. Be sure to indicate the polarity of the voltage. **[– (9.33V) + ]**
8. By using Thevenin Theorem, find current *I* in the circuit shown in Fig. 3.146. **[2·5 A]**

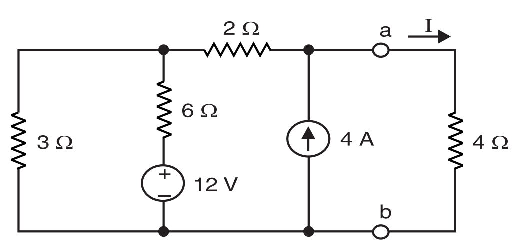



Fig. 3.146

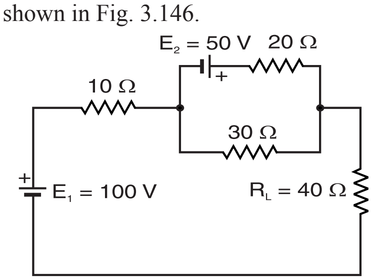



Fig. 3.147

**9.** Find Thevenin equivalent circuit in Fig. 3.147. **[$V_{Th}$ = 130 V; $R_{Th}$ = 22 Ω]**

**10.** Find the Thevenin equivalent circuit of the circuitry, excluding $R_1$, connected to terminals $x - y$ in Fig. 3.148. **[$V_{Th}$ = 23·1 V; $R_{Th}$ = 69 kΩ]**

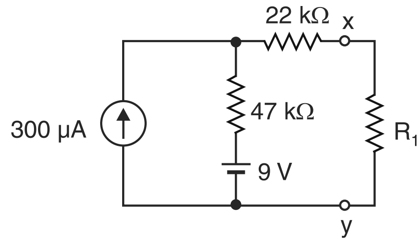



**Fig. 3.148**

**11.** Using Thevenin's theorem, find the magnitude and direction of current in 2Ω resistor in the circuit shown in Fig. 3.149. **[0.25A from *D* to *B*]**

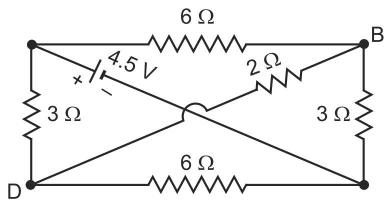



**Fig. 3.149**

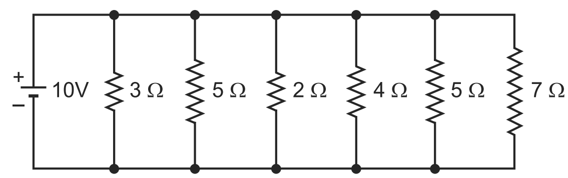



**Fig. 3.150**

**12.** Using Thevenin's theorem, find the current flowing and power dissipated in the 7Ω resistance branch in the circuit shown in Fig. 3.150. **[1.43A; 14.3W]**

**13.** Find Thevenin's equivalent circuit at terminals *BC* of Fig. 3.151. Hence determine current through the resistor $R$ = 1Ω. **[$V_{Th}$ = 76/7 V; $R_{Th}$ = 32/7Ω ; 76/39A]**

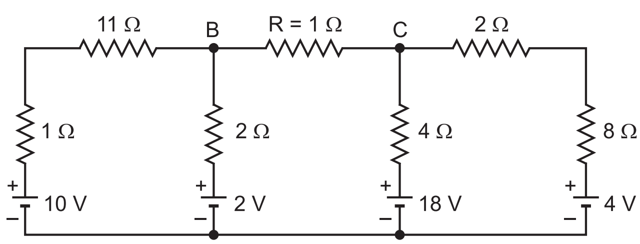
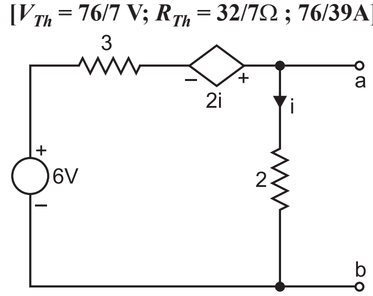



**Fig. 3.151**




**3.152**

**14.** Find the Thevenin equivalent circuit of the network shown in Fig. 3.152. All resistances are in ohms. **[$v_{Th}$ = 4V; $R_{Th}$ = 8Ω]**

**15.** Replace the circuit (See Fig. 3.153) to the left of terminals $a - b$ by its Thevenin equivalent and use the result to find $v$. **[$v_{Th}$ = 12V ; $R_{Th}$ = 8Ω ; $v$ = 4V]**

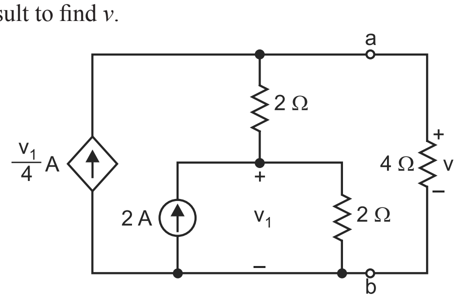



**Fig. 3.153**

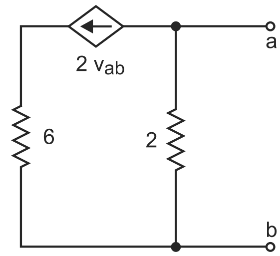



**Fig. 3.154**

**16.** Find the Thevenin equivalent circuit for the network shown in Fig. 3.154. All resistances are in ohms. **[$v_{Th}$ = 0V; $R_{Th}$ = 2/5Ω]**

## 3.13. Advantages of Thevenin's Theorem

The Thevenin equivalent circuit is *always* an equivalent voltage source ($V_{Th}$) in series with an equivalent resistance ($R_{Th}$) regardless of the original circuit that it replaces. Although the Thevenin equivalent is not the same as its original circuit, it acts the same in terms of output voltage and current. It is worthwhile to give the advantages of Thevenin's theorem.

(*i*) It reduces a complex circuit to a simple circuit *viz.* a single source of e.m.f. $V_{Th}$ in series with a single resistance $R_{Th}$.

(*ii*) It greatly simplifies the portion of the circuit of lesser interest and enables us to view the action of the output part directly.

(*iii*) This theorem is particularly useful to find current in a particular branch of a network as the resistance of that branch is varied while all other resistances and sources remain constant.

(*iv*) Thevenin's theorem can be applied in successive steps. Any two points in a circuit can be chosen and all the components to one side of these points can be reduced to Thevenin's equivalent circuit.

## 3.14. Norton's Theorem

Fig. 3.155 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind *AB* can be replaced by a current source $I_N$ in parallel with a resistance $R_N$ as shown in Fig. 3.155 (*ii*). The resistance $R_N$ is the same as Thevenin resistance $R_{Th}$. The value of $I_N$ is determined as mentioned in Norton's theorem. Once *Norton's equivalent circuit* is determined [See Fig. 3.155 (*ii*)], then current in any load $R_L$ connected across *AB* can be readily obtained.

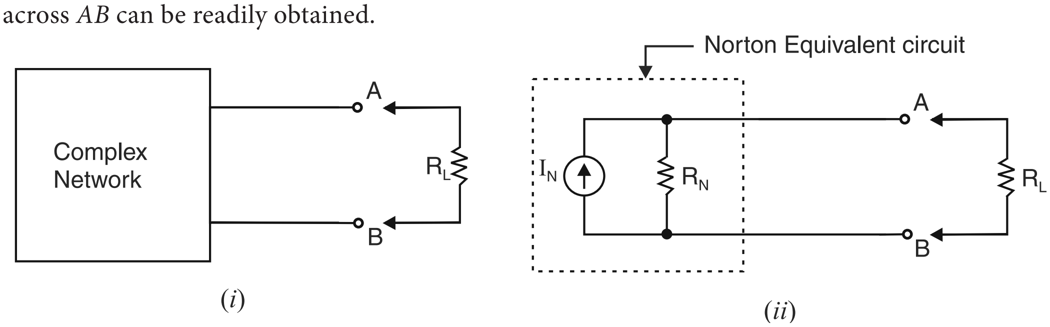



**Fig. 3.155**

Hence Norton's theorem as applied to d.c. circuits may be stated as under :

*Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output $I_N$ in parallel with a resistance $R_N$.*

(*i*) *The output $I_N$ of the current source is equal to the current that would flow through AB when A and B are short-circuited.*

(*ii*) *The resistance $R_N$ is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Norton's Theorem is *converse* of Thevenin's theorem in that Norton equivalent circuit uses a current generator instead of voltage generator and the resistance $R_N$ (which is the same as $R_{Th}$) in parallel with the generator instead of being in series with it. *Thus the use of either of these theorems enables us to replace the entire circuit seen at a pair of terminals by an equivalent circuit made up of a single source and a single resistor.*

**Illustration.** Fig. 3.156 illustrates the application of Norton's theorem. As far as the circuit behind terminals *AB* is concerned [See Fig. 3.156 (*i*)], it can be replaced by a current source $I_N$ in parallel with a resistance $R_N$ as shown in Fig. 3.156 (*iv*). The output $I_N$ of the current generator is equal to the current that would flow through *AB* when terminals *A* and *B* are short-circuited as shown in Fig. 3.156 (*ii*). The load on the source when terminals *AB* are short-circuited is given by ;

$$R' = R_1 + \frac{R_2R_3}{R_2 + R_3} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2 + R_3}$$

$$\text{Source current, } I' = \frac{V}{R'} = \frac{V(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$\text{Short-circuit current, } I_N = \text{Current in } R_2 \text{ in Fig. 3.156 } (ii)$$

$$= I' \times \frac{R_3}{R_2 + R_3} = \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

To find $R_N$, remove the load $R_L$ and replace battery by a short because its internal resistance is assumed zero [See Fig. 3.156 (*iii*)].

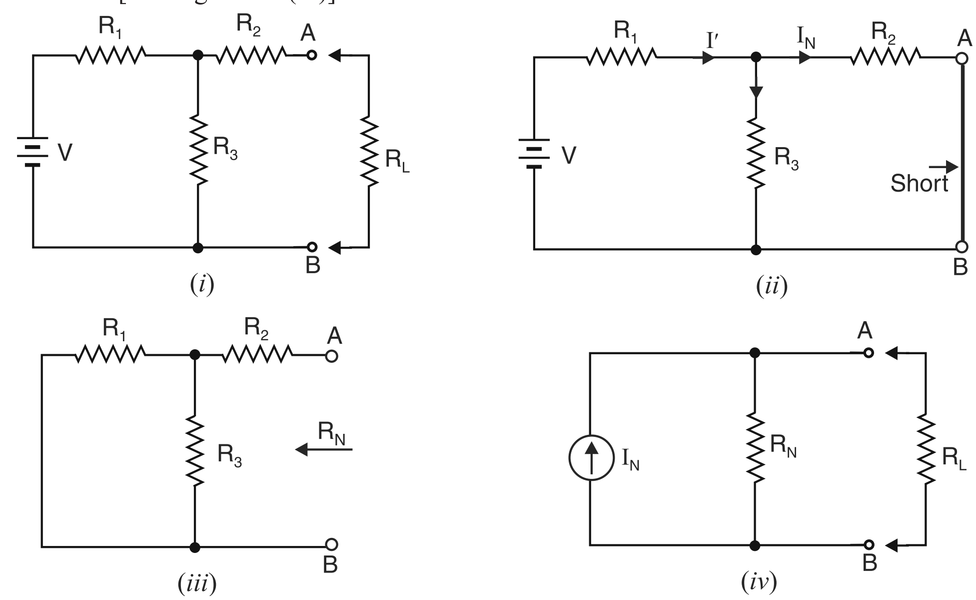



**Fig. 3.156**

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.156 } (iii).$$

$$= R_2 + \frac{R_1R_3}{R_1 + R_3}$$

Thus the values of $I_N$ and $R_N$ are known. The Norton equivalent circuit will be as shown in Fig. 3.156 (*iv*).

## 3.15. Procedure for Finding Norton Equivalent Circuit

(*i*) Open the two terminals (*i.e.* remove any load) between which we want to find Norton equivalent circuit.

(*ii*) Put a short-circuit across the terminals under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current $I_N$.

**(iii)** Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Norton's resistance $R_N$. It is easy to see that $R_N = R_{Th}$.

**(iv)** Connect $I_N$ and $R_N$ in parallel to produce Norton equivalent circuit between the two terminals under consideration.

**(v)** Place the load resistor removed in step (*i*) across the terminals of the Norton equivalent circuit. The load current can now be calculated by using current-divider rule. This load current will be the same as the load current in the original circuit.

**Example 3.61.** *Show that when Thevenin's equivalent circuit of a network is converted into Norton's equivalent circuit, $I_N = V_{Th}/R_{Th}$ and $R_N = R_{Th}$. Here $V_{Th}$ and $R_{Th}$ are Thevenin voltage and Thevenin resistance respectively.*

**Solution.** Fig. 3.157 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. Thevenin's equivalent circuit of this network will be as shown in Fig. 3.157 (*ii*). To find Norton's equivalent circuit, we are to find $I_N$ and $R_N$. Referring to Fig. 3.157 (*ii*),

$I_N$ = Current flowing through short-circuited *AB* in Fig. 3.157 (*ii*)

$= V_{Th}/R_{Th}$

$R_N$ = Resistance at terminals *AB* in Fig. 3.157 (*ii*)

$= R_{Th}$

Fig. 3.157 (*iii*) shows Norton's equivalent circuit. Hence we arrive at the following two important conclusions :

**(i)** To convert Thevenin's equivalent circuit into Norton's equivalent circuit,

$$I_N = V_{Th}/R_{Th} \ ; \ R_N = R_{Th}$$

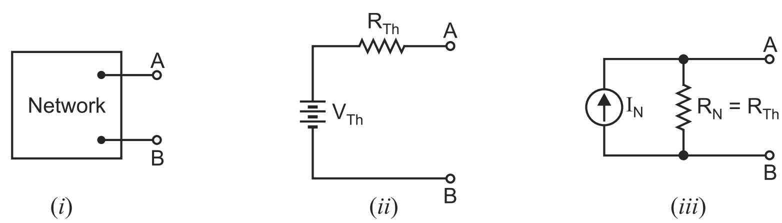



**Fig. 3.157**

**(ii)** To convert Norton's equivalent circuit into Thevenin's equivalent circuit,

$$V_{Th} = I_N R_N \ ; \ R_{Th} = R_N$$

**Example 3.62.** *Find the Norton equivalent circuit at terminals x − y in Fig. 3.158.*

**Solution.** We shall first find the Thevenin equivalent circuit and then convert it to an equivalent current source. This will then be Norton equivalent circuit.

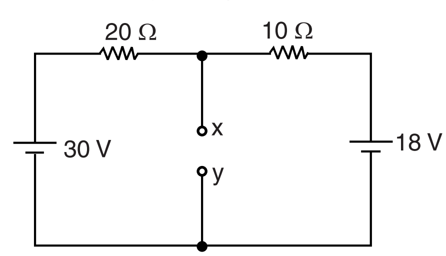



**Fig. 3.158**

**Finding Thevenin equivalent circuit.** To find $V_{Th}$, refer to Fig. 3.159 (*i*). Since 30 V and 18 V sources are in opposition, the circuit current *I* is given by ;

$$I = \frac{30-18}{20+10} = \frac{12}{30} = 0.4 \text{ A}$$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$$30 - 20 \times 0.4 - V_{Th} = 0 \quad \therefore \quad V_{Th} = 30 - 8 = 22 \text{ V}$$

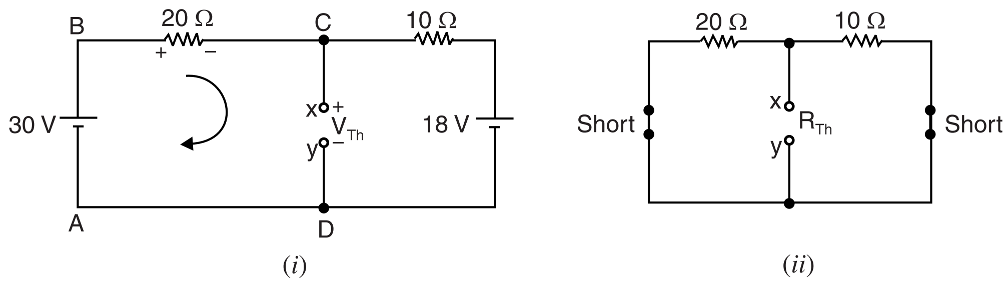



**Fig. 3.159**

To find $R_{Th}$, we short both voltage sources as shown in Fig. 3.159 (*ii*). Notice that 10 Ω and 20 Ω resistors are then in parallel.

$$\therefore \qquad R_{Th} = 10\ \Omega \parallel 20\ \Omega = \frac{10 \times 20}{10 + 20} = 6.67\ \Omega$$

Therefore, Thevenin equivalent circuit will be as shown in Fig. 3.160 (*i*). Now it is quite easy to convert it into equivalent current source.

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{22}{6.67} = 3.3\text{A} \qquad \text{[See Fig. 3.160 (}ii\text{)]}$$

$$R_N = R_{Th} = 6.67\ \Omega$$

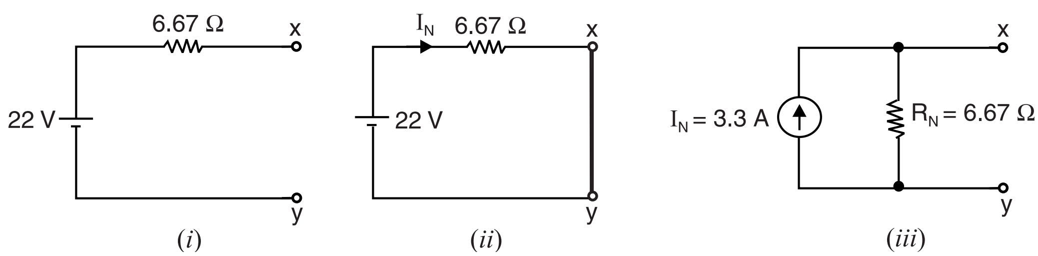



**Fig. 3.160**

Fig. 3.160 (*iii*) shows **Norton equivalent circuit.** Observe that the Norton equivalent resistance has the same value as the Thevenin equivalent resistance. Therefore, $R_N$ is found exactly the same way as $R_{Th}$.

**Example 3.63.** *Using Norton's theorem, calculate the current in the 5 Ω resistor in the circuit shown in Fig. 3.161.*

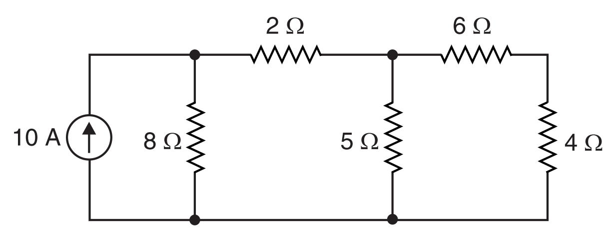



**Fig. 3.161**

**Solution.** Short the branch that contains 5 Ω resistor in Fig. 3.161. The circuit then becomes as shown in Fig. 3.162 (*i*). Referring to Fig. 3.162 (*i*), the 6 Ω and 4 Ω resistors are in series and this series combination is in parallel with the short. Therefore, these resistors have no effect on Norton current and may be considered as removed from the circuit. As a result, 10 A divides between parallel resistors of 8 Ω and 2 Ω.

$$\therefore \qquad \text{Norton current, } I_N = \text{Current in 2 Ω resistor}$$

$$= 10 \times \frac{8}{8+2} = 8\text{ A} \qquad \text{... Current-divider rule}$$

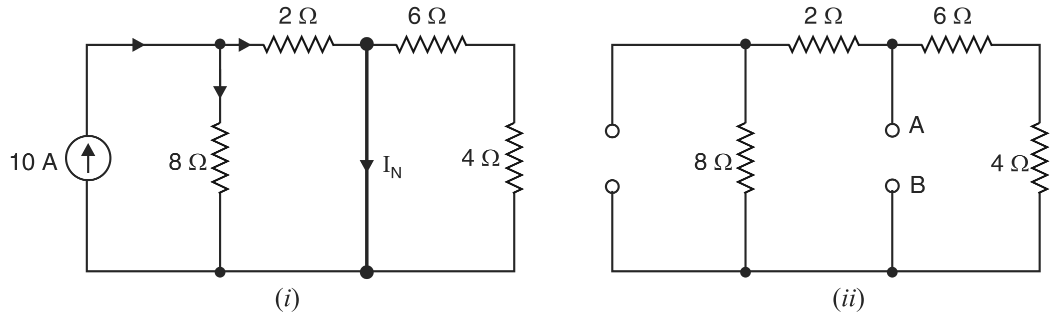



**Fig. 3.162**

In order to find Norton resistance $R_N (= R_{Th})$, open circuit the branch containing the 5 Ω resistor and replace the current source by an open in Fig. 3.161. The circuit then becomes as shown in Fig. 3.162 (*ii*).

Norton resistance, $R_N$ = Resistance at terminals *AB* in Fig. 3.162 (*ii*).

$$= (2 + 8) \parallel (4 + 6) = 10 \parallel 10 = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 8 A (= $I_N$) in parallel with a resistance of 5 Ω (= $R_N$) as shown in Fig. 3.163 (*i*). When the branch containing 5 Ω resistor is connected across the output terminals of Norton's equivalent circuit, the circuit becomes as shown in Fig. 3.163 (*ii*).

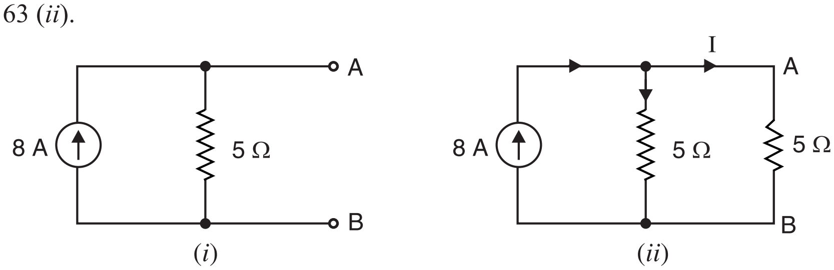



**Fig. 3.163**

By current-divider rule, the current *I* in 5 Ω resistor is

$$I = 8 \times \frac{5}{5 + 5} = \mathbf{4\ A}$$

**Example 3.64.** *Find Norton equivalent circuit for Fig. 3.164 (i). Also solve for load current and load voltage.*

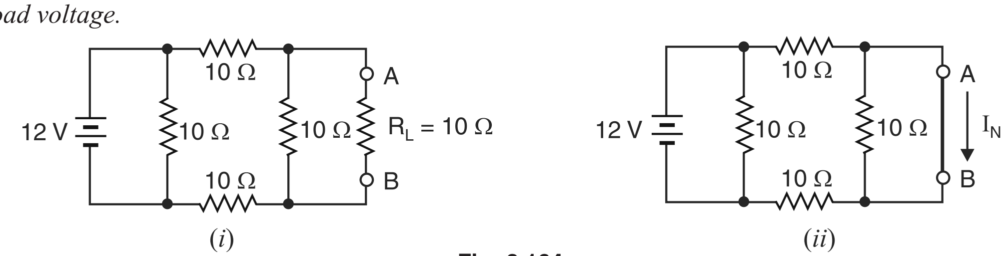



**Fig. 3.164**

**Solution.** Short the branch that contains $R_L$ (= 10 Ω) in Fig. 3.164 (*i*). The circuit then becomes as shown in Fig. 3.164 (*ii*). The resistor that is in parallel with the battery has no effect on the Norton current ($I_N$). The resistor in parallel with the short also has no effect. Therefore, these resistors may be considered as removed from the circuit shown in Fig. 3.164 (*ii*). The circuit then contains two 10 Ω resistors in series.

$\therefore$ Norton current, $I_N = \dfrac{12}{10+10} = 0{\cdot}6$ A

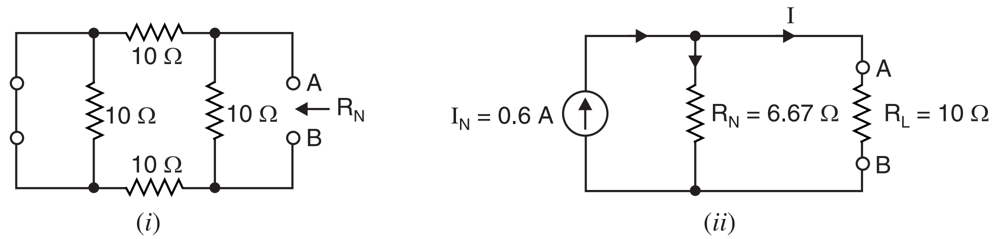



**Fig. 3.165**

In order to find Norton resistance $R_N$ (= $R_{Th}$), open circuit the branch containing $R_L$ and replace the voltage source by a short ($\because$ internal resistance of the voltage source is assumed zero) in Fig. 3.164 (*i*). The circuit then becomes as shown in Fig. 3.165 (*i*).

Norton resistance, $*R_N$ = Resistance at terminals *AB* in Fig. 3.165 (*i*)

$$= (10 + 10) \parallel 10 = \frac{20 \times 10}{20+10} = 6{\cdot}67\ \Omega$$

Therefore, Norton equivalent circuit consists of a **current source of 0·6 A (= $I_N$) in parallel with a resistance of 6·67 Ω (=$R_N$).** When the branch containing $R_L$ (= 10 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. 3.165 (*ii*).

By current-divider rule, the current $I$ in $R_L$ is

$$I = 0.6 \times \frac{6.67}{6.67+10} = \mathbf{0{\cdot}24\ A}$$

Voltage across $R_L$ = $I R_L = 0{\cdot}24 \times 10 = \mathbf{2{\cdot}4\ V}$

**Example 3.65.** *Find the Norton current for the unbalanced Wheatstone bridge shown in Fig. 3.166.*

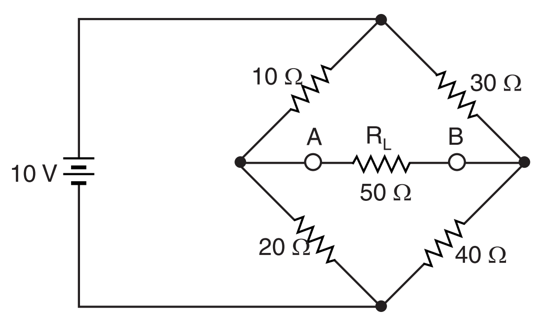



**Fig. 3.166**

**Solution.** The Norton current is found by shorting the load terminals as shown in Fig. 3.167 (*i*). This situation is more complicated than finding the Thevenin voltage. Here is an easy way to find $I_N$ in the circuit of Fig. 3.167 (*i*). First determine the total current and then use Ohm's law to find current in the four resistors. Once the currents in the four resistors are known, Kirchhoff's current law can be used to determine Norton current $I_N$.

* The resistor 10 Ω that is in parallel with short is ineffective and may be considered as removed from the circuit of Fig. 3.165 (*i*). Therefore, two 10 Ω resistors are in series and this series combination is in parallel with 10 Ω resistor.

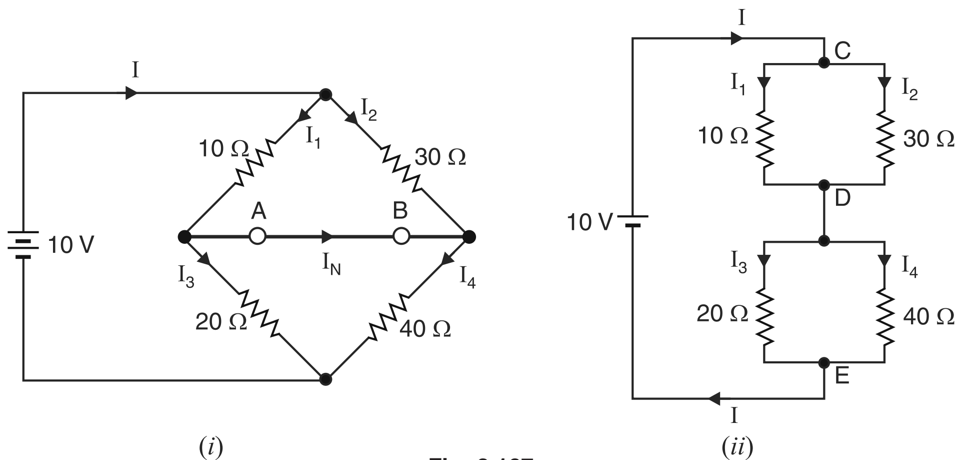



**Fig. 3.167**

Fig. 3.167 (*ii*) shows the equivalent circuit of Fig. 3.167 (*i*). The total circuit resistance $R_T$ to 10 V source is

$$R_T = \frac{10 \times 30}{10+30} + \frac{20 \times 40}{20+40} = 7{\cdot}5 + 13{\cdot}33 = 20{\cdot}83\ \Omega$$

$$\text{Total circuit current, } I = \frac{10}{20.83} = 0{\cdot}48 \text{ A}$$

Referring to Fig. 3.167 (*ii*), we have,

$$V_{CD} = I \times R_{CD} = 0{\cdot}48 \times 7{\cdot}5 = 3{\cdot}6 \text{ V}$$

$$V_{DE} = I \times R_{DE} = 0{\cdot}48 \times 13{\cdot}33 = 6{\cdot}4 \text{ V}$$

$$\therefore \quad I_1 = \frac{V_{CD}}{10} = \frac{3.6}{10} = 0{\cdot}36 \text{ A} \ ; \quad I_2 = \frac{V_{CD}}{30} = \frac{3.6}{30} = 0{\cdot}12 \text{ A}$$

$$I_3 = \frac{V_{DE}}{20} = \frac{6.4}{20} = 0{\cdot}32 \text{ A} \ ; \quad I_4 = \frac{V_{DE}}{40} = \frac{6.4}{40} = 0{\cdot}16 \text{ A}$$

Referring to Fig. 3.167 (*i*), it is now clear that $I_1 (= 0{\cdot}36 \text{ A})$ is greater than $I_3 (= 0{\cdot}32 \text{ A})$. Therefore, current $I_N$ will flow from $A$ to $B$ and its value is

$$I_N = I_1 - I_3 = 0{\cdot}36 - 0{\cdot}32 = \mathbf{0{\cdot}04 \text{ A}}$$

**Example 3.66.** *Two batteries, each of e.m.f. 12 V, are connected in parallel to supply a resistive load of 0·5 Ω. The internal resistances of the batteries are 0·12 Ω and 0·08 Ω. Calculate the current in the load and the current supplied by each battery.*

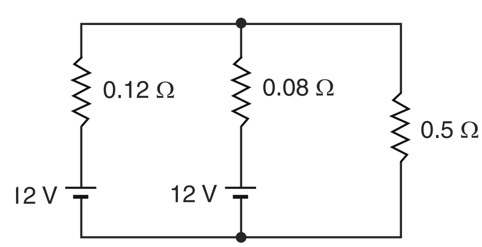



**Fig. 3.168**

**Solution.** Fig. 3.168 shows the conditions of the problem. If a short circuit is placed across the load, the circuit becomes as shown in Fig. 3.169 (*i*). The total short circuit current is given by ;

$$I_N = \frac{12}{0.12} + \frac{12}{0.08}$$

$$= 100 + 150 = 250 \text{ A}$$

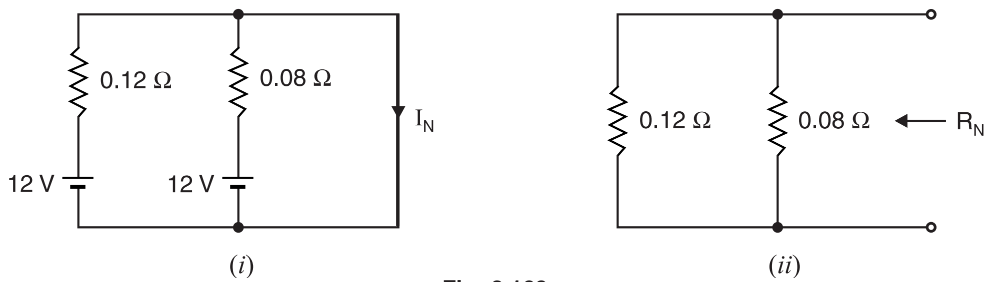



**Fig. 3.169**

In order to find Norton resistance $R_N (= R_{Th})$, open circuit the load and replace the batteries by their internal resistances. The circuit then becomes as shown in Fig. 3.169 (*ii*). Then resistance looking into the open-circuited terminals is the Norton resistance.

∴ Norton resistance, $R_N$ = Resistance looking into the open-circuited load terminals in Fig. 3.169 (*ii*)

$$= 0{\cdot}12 \parallel 0{\cdot}08 = \frac{0.12 \times 0.08}{0.12 + 0.08} = 0{\cdot}048\ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 250 A (= $I_N$) in parallel with a resistance of 0·048 Ω (= $R_N$). When load (= 0·5 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. 3.170. By current-divider rule, the current $I$ in load (= 0·5 Ω) is given by ;

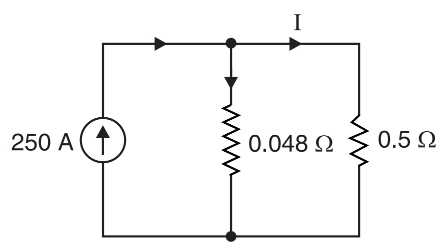



**Fig. 3.170**

$$I = 250 \times \frac{0.048}{0.048 + 0.5} = \mathbf{21{\cdot}9\ A}$$

$$\text{Battery terminal voltage} = I R_L = 21{\cdot}9 \times 0{\cdot}5$$

$$= 10{\cdot}95\ \text{V}$$

$$\text{Current in first battery} = \frac{12 - 10.95}{0.12} = \mathbf{8{\cdot}8\ A}$$

$$\text{Current in second battery} = \frac{12 - 10.95}{0.08} = \mathbf{13{\cdot}1\ A}$$

**Example 3.67.** *Represent the network shown in Fig. 3.171 between the terminals A and B by one source of current $I_N$ and internal resistance $R_N$. Hence calculate the current that would flow in a 6 Ω resistor connected across AB.*

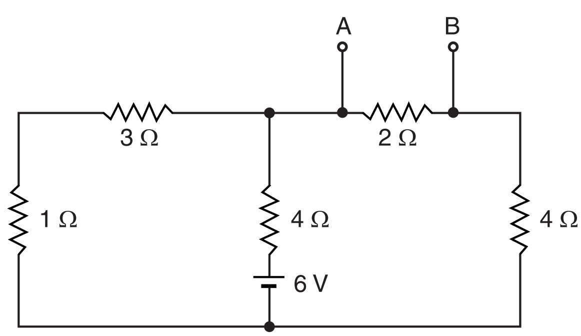



**Fig. 3.171**

**Solution.** Place short circuit across *AB* in Fig. 3.171. Then the circuit becomes as shown in Fig. 3.172 (*i*). Note that 2 Ω resistor is shorted and may be considered as removed from the circuit. The total resistance $R_T$ presented to the 6 V source is a parallel combination of (3 + 1) Ω and 4 Ω in series with 4 Ω. Therefore, the value of $R_T$ is given by ;

$$R_T = [(3 + 1) \parallel 4] + 4 = \frac{4 \times 4}{4 + 4} + 4 = 2 + 4 = 6\ \Omega$$

$\therefore$ Current supplied by 6 V source, $I = 6/6 = 1$ A

At node $D$, 1 A current divides between two parallel resistors of (3 + 1) Ω and 4 Ω.

$$\therefore \quad \text{Norton current, } I_N = 1 \times \frac{4}{4+4} = 0{\cdot}5 \text{ A}$$

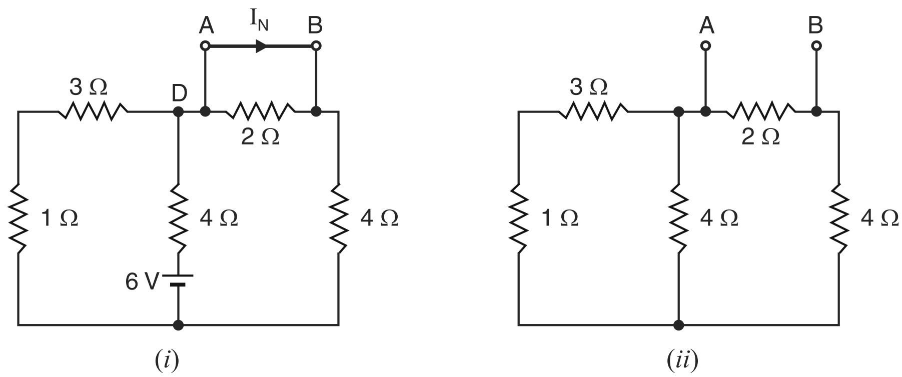


Fig. 3.172

Now Norton resistance $R_N (= R_{Th})$ is the resistance between open-circuited terminals $AB$ with voltage source replaced by a short as shown in Fig. 3.172 (*ii*). Referring to Fig. 3.172 (*ii*), (3 + 1) Ω resistance is in parallel with 4 Ω, giving equivalent resistance of 2 Ω. Now (2 + 4) Ω resistance is in parallel with 2 Ω.

$$\therefore \quad R_N = (2+4) \parallel 2 = 6 \parallel 2$$

$$= \frac{6 \times 2}{6+2} = \frac{12}{8} = 1{\cdot}5\ \Omega$$

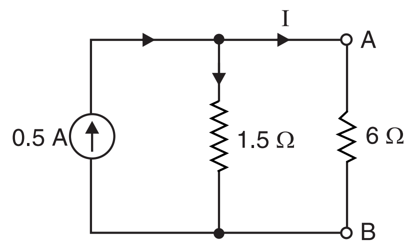


Fig. 3.173

Therefore, Norton equivalent circuit is a **current source of 0·5 A in parallel with resistance of 1·5 Ω.** When a 6 Ω resistor is connected across $AB$, the circuit becomes as shown in Fig. 3.173. By current-divider rule, current in 6 Ω,

$$I = 0.5 \times \frac{1.5}{1.5+6} = 0.5 \times \frac{1.5}{7.5} = \mathbf{0{\cdot}1\ A}$$

**Example 3.68.** *For the circuit shown in Fig. 3.174, calculate the potential difference between the points O and N and what current would flow in a 50 Ω resistor connected between these points?*

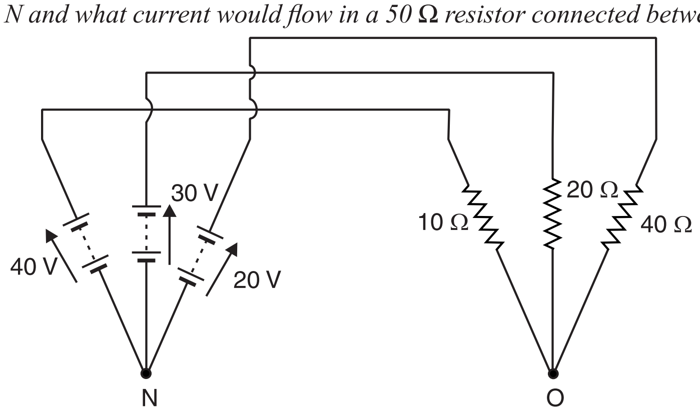


Fig. 3.174

**Solution.** Place a short circuit across $ON$ in Fig. 3.174. Then total short circuit current in $ON$ is

$$I_N = \frac{40}{10}+\frac{30}{20}+\frac{20}{40} = 4 + 1{\cdot}5 + 0{\cdot}5 = 6 \text{ A}$$

In order to find $R_N (= R_{Th})$, replace the voltage sources by short. Then $R_N$ is equal to the resistance looking into open circuited terminals $ON$. It is easy to see that the resistors 10 Ω, 20 Ω and 40 Ω are in parallel across $ON$.

$$\therefore \qquad \frac{1}{R_N} = \frac{1}{10}+\frac{1}{20}+\frac{1}{40}=\frac{7}{40}$$

or $$R_N = \frac{40}{7} = 5{\cdot}71\ \Omega$$

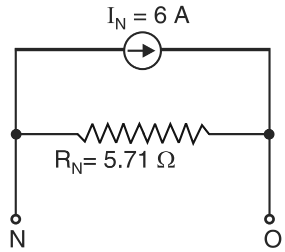



**Fig. 3.175**

Therefore, the original circuit reduces to that shown in Fig. 3.175.

$\therefore$ Open-circuited voltage across $ON = I_N R_N = 6 \times 5{\cdot}71 =$ **34·26 V**

When 50 Ω resistor is connected between points $O$ and $N$,

Current in 50 Ω connected between $ON = 6\times\dfrac{5.71}{5.71+50}$ = **0·62 A**

**Example 3.69.** *Find Norton equivalent circuit to the left of terminals AB in the circuit shown in Fig. 3.176. The current sources are $I_1$ = 10 A and $I_2$ = 15 A. The conductances are $G_1 = 0{\cdot}2$ S, $G_2 = 0{\cdot}3$ S and $G_3$ is variable.*

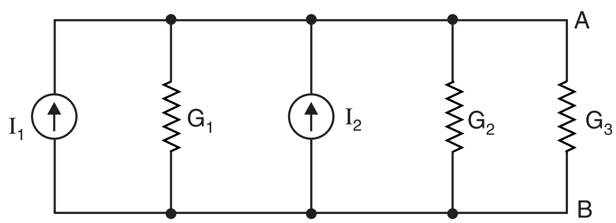



**Fig. 3.176**

**Solution.** First, disconnect branch $G_3$ and short circuit the terminals $AB$ as shown in Fig. 3.177 (*i*). Since the short circuit has infinite conductance, the total current of 25 A $(= I_1 + I_2)$ supplied by the two sources would pass through the short-circuited terminals *i.e.*

Norton current, $I_N = I_1 + I_2 = 10 + 15 = 25$ A

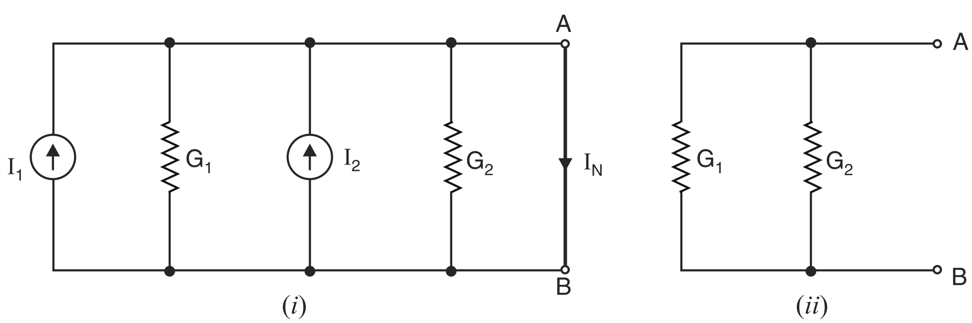



**Fig. 3.177**

Next, remove the short-circuit and replace the current sources by open. The circuit then becomes as shown in Fig. 3.177 (*ii*).

Norton conductance, $G_N$ = Conductance at terminals $AB$ in Fig. 3.177 (*ii*).

$$= G_1 + G_2 = 0{\cdot}2 + 0{\cdot}3 = 0{\cdot}5 \text{ S}$$

Therefore, Norton equivalent circuit consists of a **25 A current source in parallel with a conductance of 0·5 S.** When conductance $G_3$ is connected across terminals $AB$, the circuit becomes as shown in Fig. 3·178. Although Norton equivalent circuit is not the same as its original circuit, it acts the same in terms of output voltage and current.

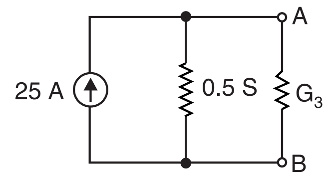



**Fig. 3.178**

**Example 3.70.** *The circuit shown in Fig. 3.179 consists of a current source I = 10 A paralleled by G = 0·1 S and a voltage source E = 200 V with 10 Ω series resistance. Find Norton equivalent circuit to the left of terminals AB.*

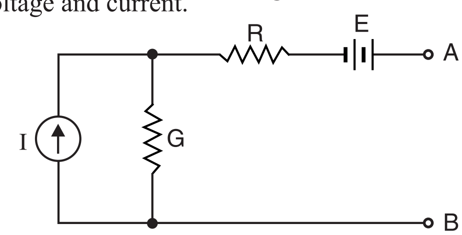



**Fig. 3.179**

**Solution.** We are to find Norton current and Norton resistance. In order to find Norton current $I_N$, short-circuit the terminals $AB$ as shown in Fig. 3.180 (*i*). Then current that flows in $AB$ is $I_N$. It is easy to see that current which flows in conductance $G$ is *$I_G$ = 5 A (upward).

∴ Norton current, $I_N = I + I_G = 10 + 5 = 15$ A

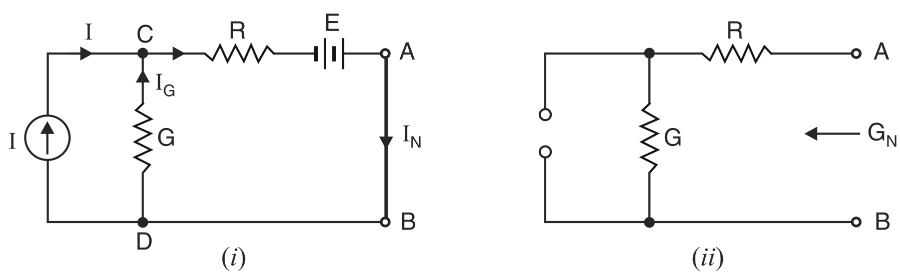



**Fig. 3.180**

In order to find Norton resistance, remove the short circuit and replace the voltage source by a short and current source by an open. The circuit then becomes as shown in Fig. 3.180 (*ii*).

$R_N$ = Resistance looking into terminals $AB$ in Fig. 3.180 (*ii*).

$$= R + \frac{1}{G} = 10 + \frac{1}{0.1} = 10 + 10 = 20\ \Omega$$

∴ Norton conductance, $G_N = \dfrac{1}{R_N} = \dfrac{1}{20} = 0{\cdot}05$ S

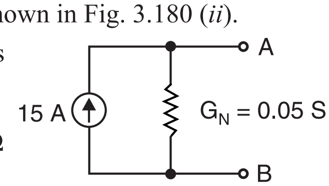



**Fig. 3.181**

Therefore, Norton equivalent circuit consists of a **15 A current source paralleled with 0·05 S conductance $G_N$** as shown in Fig. 3·181.

* Applying *KVL* to loop *CABDC*, we have,

$$-(I + I_G)\,R + 200 - \frac{I_G}{G} = 0$$

or $$-(10 + I_G)\,10 + 200 - \frac{I_G}{0.1} = 0$$

or $$-100 - 10I_G + 200 - 10\,I_G = 0$$

∴ $$I_G = 100/20 = 5 \text{ A}$$

**Example 3.71.** *Draw Norton's equivalent circuit at terminals AB and determine the current flowing through 12Ω resistor for the network shown in Fig. 3.182 (i).*

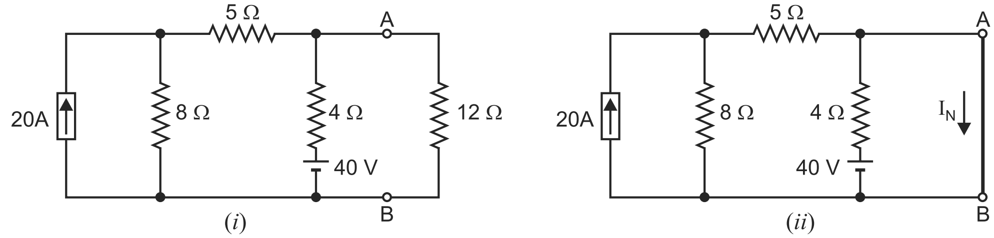



**Fig. 3.182**

**Solution.** In order to find Norton current $I_N$, short circuit terminals $A$ and $B$ after removing the load (= 12Ω). The circuit then becomes as shown in Fig. 3.182 (*ii*). The current flowing in the short circuit is the Norton current $I_N$. It can be found by using superposition theorem.

(*i*) **When current source is acting alone.** In this case, we short circuit the voltage source so that only current source acts in the circuit. The circuit then becomes as shown in Fig. 3.183 (*i*). It is clear that :

$$\text{Norton current, } I_{N1} = \text{*Current in } 5\Omega \text{ resistor}$$

$$= 20 \times \frac{8}{8+5} = \frac{160}{13}\text{A}$$

(*ii*) **When voltage source is acting alone.** In this case, we open circuit the current source so that only voltage source acts in the circuit. The circuit then becomes as shown in Fig. 3.183 (*ii*). It is clear that :

$$\text{Norton current, } I_{N2} = \frac{40}{4} = 10\text{A}$$

Therefore, when both voltage and current sources are present in the circuit, we have,

$$\text{Norton current, } I_N = I_{N1} + I_{N2} = \frac{160}{13} + 10 = \frac{290}{13}\text{A}$$

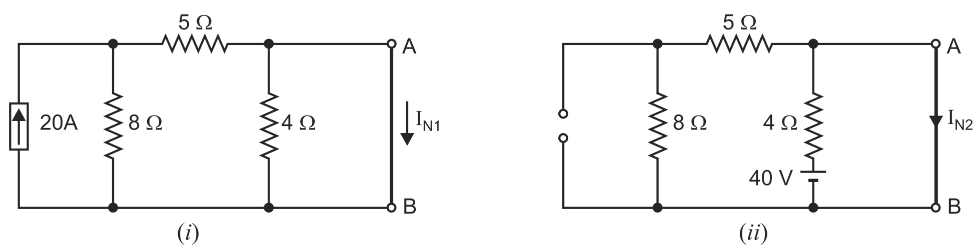



**Fig. 3.183**

In order to find $R_N$, open circuit 12Ω resistor and replace current source by open circuit and voltage source by short circuit. Then circuit becomes as shown in Fig. 3.184 (*i*).

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.184 } (i)$$

$$= 4 \,||\, (5+8) = 4 \,||\, 13 = \frac{4 \times 13}{4+13} = \frac{52}{17}\Omega$$

* No current flows in 4Ω resistor because it is short circuited at terminals $A$ and $B$. Therefore, 20A divides between 8Ω and 5Ω connected in parallel.

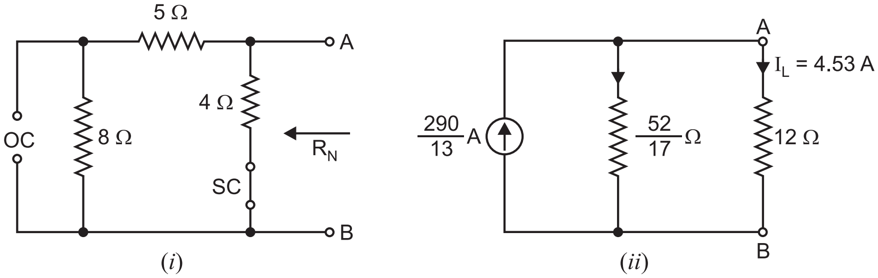



**Fig. 3.184**

Thus Norton equivalent circuit at terminals $AB$ is a current source of current 290/13 A in parallel with 52/17Ω resistance. When load resistor of 12Ω is connected across Norton's equivalent circuit, the circuit becomes as shown in Fig. 3.184 (*ii*).

$$\therefore \quad \text{Load current, } I_L = I_N \times \frac{R_N}{R_N + R_L} = \frac{290}{13} \times \frac{52/17}{52/17 + 12} = \mathbf{4.53\ A}$$

**Example 3.72.** *Determine the values of I and R in the circuit shown in Fig. 3.185.*

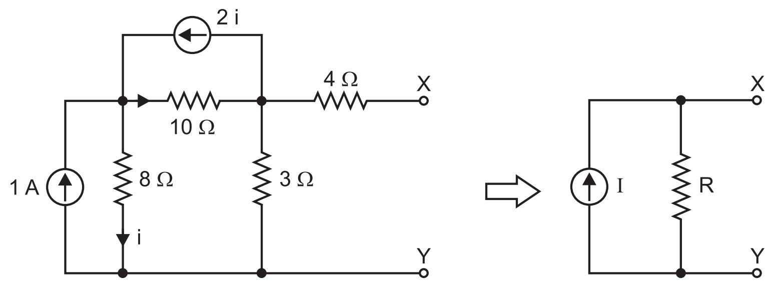



**Fig. 3.185**

**Solution.** Short the terminals $XY$ in Fig. 3.185 and we get the circuit shown in Fig. 3.186 (*i*). The currents in the various branches will be as shown. In order to find the short-circuit current $I_{sc}$ $(= I = I_N)$, we apply $KVL$ to loops 1 and 2 in Fig. 3.186 (*i*).

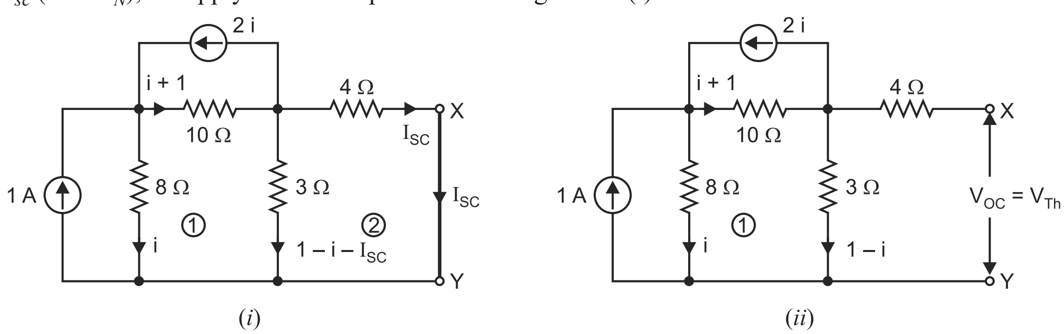



**Fig. 3.186**

**Loop 1.** $-10(i + 1) - 3(1 - i - I_{sc}) + 8i = 0$

or $\qquad i + 3I_{sc} = 13 \qquad$ ...(*i*)

**Loop 2.** $-4I_{sc} + 3(1 - i - I_{sc}) = 0$

or $\qquad 3i + 7I_{sc} = 3 \qquad$ ...(*ii*)

From eqs. (*i*) and (*ii*), we have, $I_{sc}$ = 18A.

In order to find the open-circuited voltage $V_{oc}$ (= $V_{Th}$) at terminals $X$ and $Y$, refer to Fig 3.186 (*ii*). The various branch currents are shown. Applying *KVL* to loop 1 in Fig. 3.186 (*ii*), we have,

$$-10\,(i+1) - 3(1-i) + 8i = 0 \quad \text{or} \quad i = 13\text{A}$$

$$\therefore \quad V_{oc} = \text{Voltage across } 3\Omega \text{ resistor}$$

$$= 3(1-i) = 3(1-13) = -36 \text{ V}$$

$$\text{Thevenin resistance, } R(=R_N) = \frac{V_{oc}}{I_{sc}} = \frac{36}{18} = \mathbf{2\Omega}$$

$$\text{Current } I = I_N = \mathbf{-18A}$$

Note the polarity of current source $I$ (= $I_N$).

**Example 3.73.** *With the help of Norton's theorem, find $V_o$ in the circuit shown in Fig. 3.187 (i). All resistances are in ohms.*

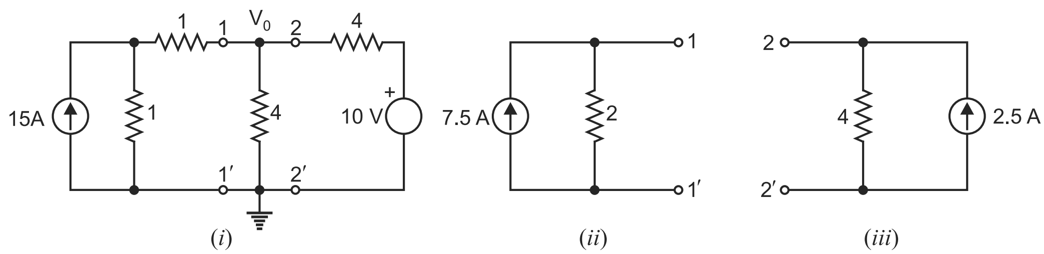



**Fig. 3.187**

**Solution.** In order to find $V_o$, it is profitable to find Norton equivalent circuit to the left of terminals 1 – 1′ and to the right of terminals 2 – 2′ in Fig. 3.187 (*i*). To the left of terminals 1 – 1′, $V_{oc} = 15 \times 1 = 15$ V and $R_N = 1 + 1 = 2\Omega$ so that $I_N = 15/2 = 7.5$A as shown in Fig. 3.187 (*ii*). To the right of terminals 2 – 2′, $V_{oc} = 10$ V and $R_{Th} = R_N = 4\Omega$ so that $I_N = 10/4 = 2.5$A as shown in Fig. 3.187 (*iii*). The two Norton equivalent circuits are put back at terminals 1 – 1′, and 2 – 2′ as shown in Fig. 3.187 (*iv*).

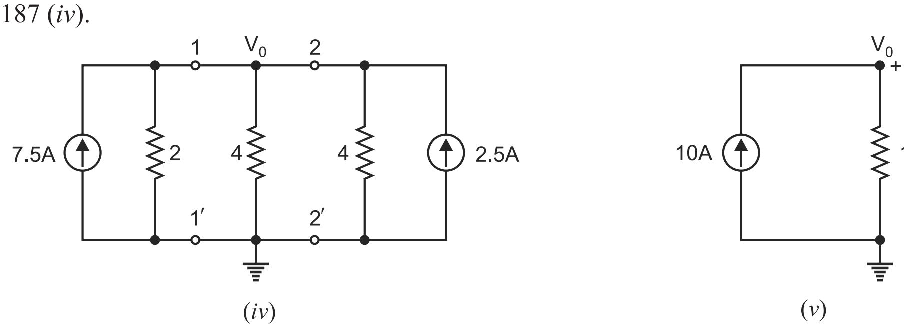



**Fig. 3.187**

In Fig. 3.187 (*iv*), the two current sources, being parallel and carrying currents in the same direction, can be combined into a single current source of 7.5 + 2.5 = 10A. The three resistances are in parallel and can be combined to give a single resistance = 2Ω || 4Ω || 4Ω = 1Ω. Therefore, the circuit of Fig. 3.187 (*iv*) reduces to the circuit shown in Fig. 3.187 (*v*).

$$\therefore \quad V_o = 10\text{A} \times 1\Omega = \mathbf{10V}$$

**Example 3.74.** *Find current in the 4 ohm resistor by any three methods for the circuit shown in Fig. 3.188(i).*

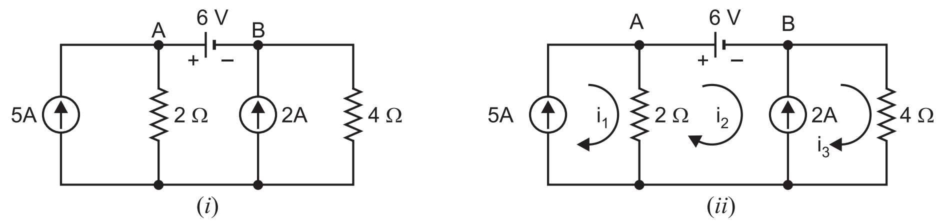



**Fig. 3.188**

**Solution. Method 1.** We shall find current in 4Ω resistor by **mesh current method.** Mark three mesh currents $i_1$, $i_2$ and $i_3$ in the three loops as shown in Fig. 3.188 (*ii*). The describing circuit equations are :

$$i_1 = 5\text{A due to the current source of 5A}$$

$$V_A - V_B = 6\text{V due to voltage source of 6V}$$

$$i_3 - i_2 = 2\text{A due to current source of 2A}$$

$$V_A = (i_1 - i_2)2 \;;\; V_B = i_3 \times 4$$

Now, $\quad -6 - 4i_3 - 2(i_2 - i_1) = 0 \qquad$ ... Applying *KVL*

or $\quad -6 - 4(2 + i_2) - 2(i_2 - 5) = 0$

or $\quad -6i_2 = 4$

$\therefore \quad i_2 = -\frac{4}{6} = -\frac{2}{3}\text{A}$ and $i_3 = i_2 + 2 = -\frac{2}{3} + 2 = \frac{4}{3}\text{A}$

$\therefore \quad$ Current in 4Ω resistor $= i_3 = \mathbf{\frac{4}{3}\,A}$

**Method 2.** We now find current in 4Ω resistor by **Thevenin's theorem.** Remove 4Ω resistor (*i.e.* load) and the circuit becomes as shown in Fig. 3.188 (*iii*).

Current in 2Ω resistor = 5 + 2 = 7A

It is because 6V source is ineffective in producing any current.

In going from point *X* to point *Y via B* and *A*, we have,

$$V_X + 6 - 7 \times 2 = V_Y$$

or $\quad V_X - V_Y = 7 \times 2 - 6 = 8\text{V}$

$\therefore \quad V_{Th} = V_{XY} = V_X - V_Y = 8\text{V}$

In order to find $R_{Th}$, short circuit the voltage source and open-circuit the current sources in Fig. 3.188 (*iii*). Then circuit becomes as shown in Fig. 3.188 (*iv*). The resistance at the open-circuited terminals *XY* in Fig. 3.188 (*iv*) is $R_{Th}$.

$\therefore \quad R_{Th} = 2\Omega$

$\therefore \quad$ Current in 4Ω resistor $= \frac{V_{Th}}{R_{Th} + 4} = \frac{8}{2 + 4} = \mathbf{\frac{4}{3}\,A}$

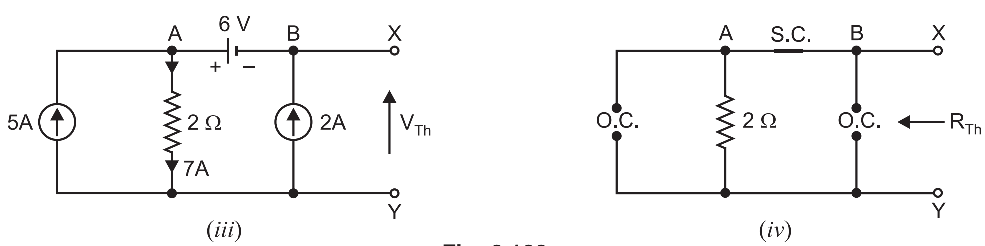



**Fig. 3.188**

**Method 3.** Finally, we find current in 4Ω resistor by **Norton's theorem.** To find $I_N$, short-circuit 4Ω resistor in Fig. 3.188 (*i*). The circuit then becomes as shown in Fig. 3.188 (*v*). The current distribution in the various branches will be as shown.

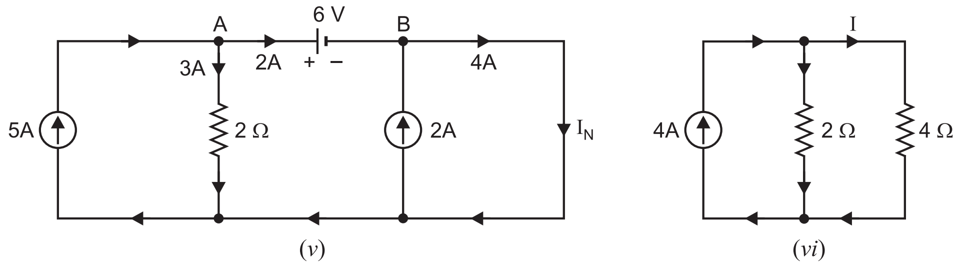



**Fig. 3.188**

It is clear from Fig. 3.188 (*v*) that :

$$I_N = 2 + 2 = 4\text{A}$$

$$R_N = R_{Th} = 2\Omega \qquad \text{...as calculated above}$$

When 4Ω resistor is connected to Norton equivalent circuit, it becomes as shown in Fig. 3.188 (*vi*).

∴ Current in 4Ω resistor is given by (current-divider rule) ;

$$I = 4 \times \frac{2}{2+4} = \frac{8}{6} = \mathbf{\frac{4}{3}\,A}$$

**Example 3.75.** *Using Norton's theorem, find current through 1Ω resistor in Fig. 3.189 (i). All resistances are in ohms,*

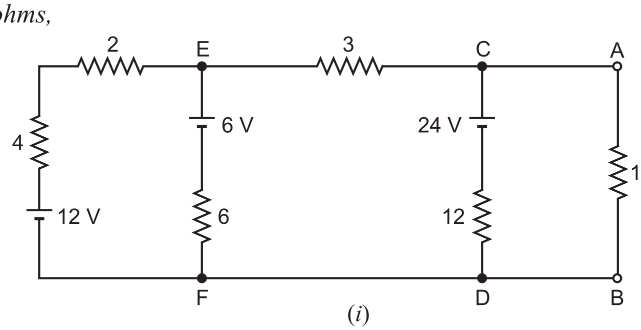



**Fig. 3.189**

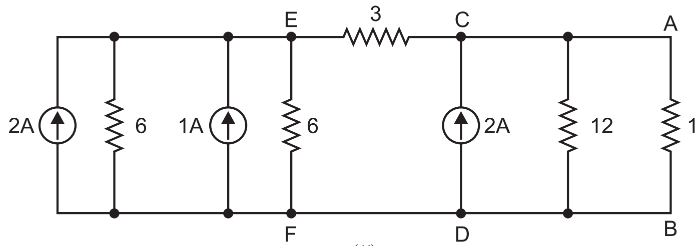
**Solution.** To find the answers, we convert the three voltage sources into their equivalent current sources.

(***a***) 12 V source in series with (4 + 2) = 6Ω resistance is converted into equivalent current source of 12V/6Ω = 2A in parallel with 6Ω resistance.

(***b***) 6V source in series with 6Ω resistance is converted into equivalent current source of 6V/6Ω = 1A in parallel with 6Ω resistance.

(***c***) 24V source in series with 12Ω resistance is converted into equivalent current source of 24V/12Ω = 2A in parallel with 12Ω resistance.

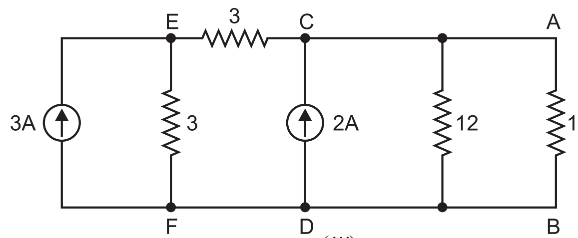
After the above source conversions, the circuit of Fig. 3.189 (*i*) becomes the circuit shown in Fig. 3.189 (*ii*).




(*ii*)

**Fig. 3.189**

Referring to Fig. 3.189 (*ii*), we can combine the two current sources to the left of *EF* but cannot combine 2A source across *CD* with them because 3Ω resistance is between *E* and *C*. Therefore, combining the two current sources to the left of *EF*, we have a single current source of 2 + 1 = 3A and a single resistance of 6Ω || 6Ω = 3Ω in parallel with it. As a result, Fig. 3.189 (*ii*) reduces to the circuit shown in Fig. 3.189 (*iii*).



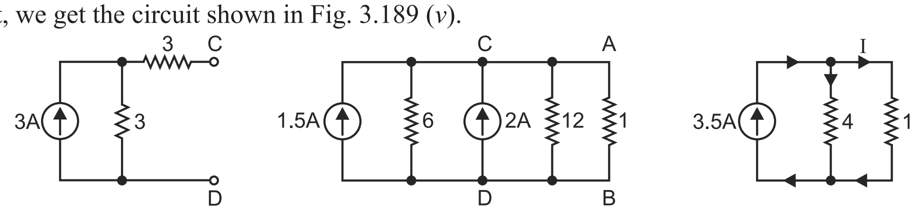

(*iii*)

**Fig. 3.189**

We now convert the circuit to the left of *CD* in Fig. 3.189 (*iii*) into Norton equivalent circuit. Fig. 3.189 (*iv*) shows this circuit to the left of *CD*. Its Norton equivalent circuit values are :

$$I_N = 3 \times \frac{3}{3+3} = 1.5\text{A} \ ; \ R_N = 3\Omega + 3\Omega = 6\Omega$$

Therefore, replacing the circuit to the left of *CD* in Fig. 3.189 (*iii*) by its Norton equivalent circuit, we get the circuit shown in Fig. 3.189 (*v*).




(*iv*) (*v*) (*vi*)

**Fig. 3.189**

Referring to Fig. 3.189 (*v*), we can combine the two current sources into a single current source of 1.5 + 2 = 3.5 A and a single resistance of 6Ω || 12Ω = 4Ω in parallel with it. The circuit then reduces to the one shown in Fig. 3.189 (*vi*). By current-divider rule [See Fig. 3.189 (*vi*)],

$$\text{Current in } 1\Omega \text{ resistor, } I = 3.5 \times \frac{4}{4+1} = \mathbf{2.8\ A}$$

## 3.16. Norton Equivalent Circuit

**(Circuits containing both independent and dependent sources)**

Sometimes we come across circuits which contain both independent and dependent sources.

One such example is shown in Fig. 3.190. The procedure for finding Norton equivalent circuit (*i.e.* finding $i_N$ and $R_N$) in such cases is as under :

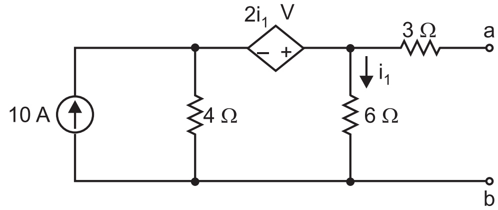



**Fig. 3.190**

(*i*) The open-circuited voltage $v_{oc}(= v_{Th})$ at terminals *ab* is determined as usual with sources present.

(*ii*) We cannot find $R_N$ $(= R_{Th})$ at terminals *ab* simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals *ab* and find the value of short-circuit current $i_{sc}$ $(= i_N)$ at terminals *ab*.

(*iii*) Norton resistance, $R_N = v_{oc}/i_{sc}$ $(= v_{Th}/i_{sc})$.

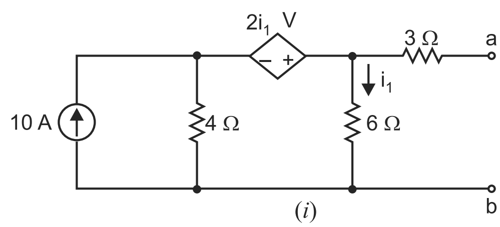
**Note.** In case the circuit contains dependent sources *only*, the procedure for finding $v_{oc}$ $(= v_{Th})$ and $R_N(= R_{Th})$ is as under :

(*a*) In this case, $v_{oc} = 0$ and $i_{sc} = 0$ because no independent source is present.

(*b*) We cannot use the relation $R_N = v_{oc}/i_{sc}$ as we do in case the circuit contains both independent and dependent sources.

(*c*) In order to find $R_N$, we excite the circuit at terminals *ab* by connecting 1A source to the terminals *a* and *b* and calculate the value of $v_{ab}$. Then $R_N$ $(= R_{Th}) = v_{ab}/1\Omega$.

**Example 3.76.** *Find the values of* $i_N$ *and* $R_N$ *at terminals ab for the circuit shown in Fig. 3.191 (i).*




(*i*)

**Fig. 3.191**

**Solution.** We first put a short circuit across terminals *a* and *b* to find short-circuit current $i_{sc}$ $(= i_N)$ at terminals *ab* as shown in Fig. 3.191 (*ii*). Applying *KCL* at node *c*, we have,

$$10 = i_1 + i_2 + i_{sc}$$

or $$i_2 = 10 - i_1 - i_{sc}$$

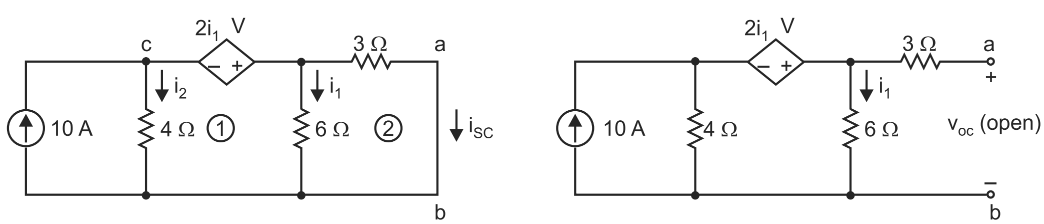
Applying *KVL* to loops 1 and 2, we have,

$$-4i_2 + 6i_1 - 2i_1 = 0 \quad \text{... Loop 1}$$

or $$-4(10 - i_1 - i_{sc}) + 4i_1 = 0 \quad ...(i)$$

Also $$-6i_1 + 3i_{sc} = 0 \quad ...(ii) \text{ ... Loop 2}$$

From eqs. (*i*) and (*ii*), $i_{sc} = i_N =$ **5A.**




(*ii*) (*iii*)

**Fig. 3.191**

In order to find $v_{oc}$ $(= v_{Th})$, we refer to Fig. 3.191 (*iii*) where we have,

$$v_{oc} = 6i_1 \quad ...(iii)$$

Applying $KVL$ to the central loop in Fig. 3.191 ($iii$),

$$-4(10-i_1)+6i_1-2i_1 = 0 \quad ...(iv)$$

From eqs. ($iii$) and ($iv$), we have, $v_{oc} = v_{Th} = 30V$.

Also $$R_N (= R_{Th}) = \frac{v_{oc}}{i_{sc}} = \frac{30}{5} = 6\Omega$$

## Tutorial Problems

**1.** Using Norton's theorem, find the current in 8 Ω resistor of the network shown in Fig. 3.192. **[1.55 A]**

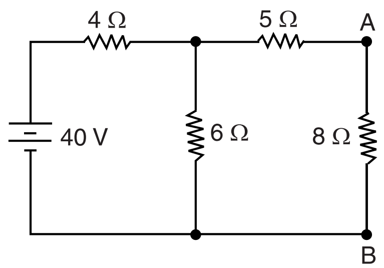



**Fig. 3.192**

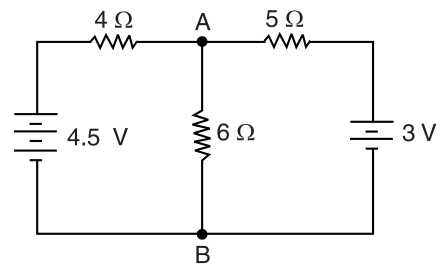



**Fig. 3.193**

**2.** Using Norton's theorem, find the current in the branch $AB$ containing 6 Ω resistor of the network shown in Fig. 3.193. **[0.466 A]**

**3.** Show that when Thevenin's equivalent circuit of a network is converted into Norton equivalent circuit, $I_N = V_{Th}/R_{Th}$ and $R_N = R_{Th}$.

**4.** Find the voltage between points $A$ and $B$ in the network shown in Fig. 3.194 using Norton's theorem. **[2·56 V]**

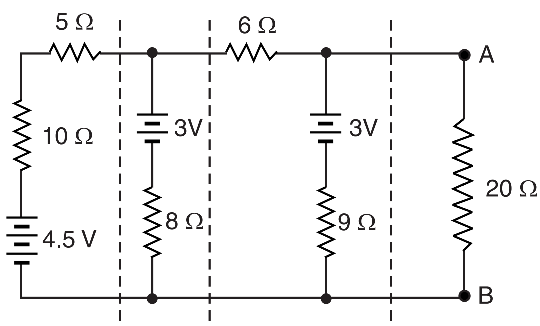



**Fig. 3.194**

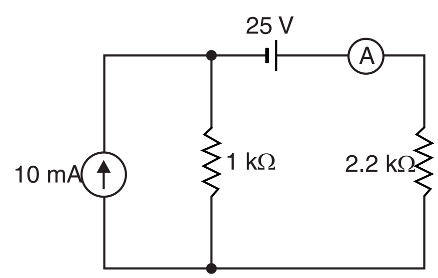



**Fig. 3.195**

**5.** The ammeter labelled A in Fig. 3.195 reads 35 mA. Is the 2·2 kΩ resistor shorted? Assume that ammeter has zero resistance. **[Shorted]**

**6.** Find Norton equivalent circuit to the left of terminals $a - b$ in Fig. 3.196. **[$I_N$ = 1·5 A; $R_N$ = 4 Ω]**

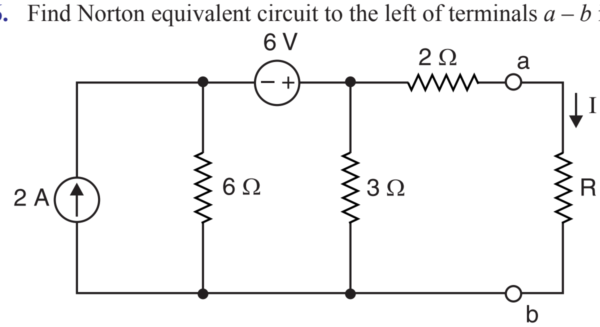



**Fig. 3.196**

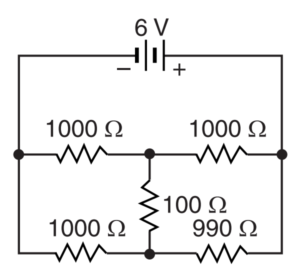



**Fig. 3.197**

7. What is the current in the 100 Ω resistor in Fig. 3.197 if the 990 Ω resistor is changed to 1010 Ω ? Use Norton theorem to obtain the result. **[13·45 μA]**

8. Determine the Norton equivalent circuit and the load current in $R_L$ in Fig. 3.198. The various circuit values are :

$$E' = 64\text{ V} \;;\quad R_1 = 230\ \Omega \;;\quad R_2 = 450\ \Omega\;;$$
$$R_3 = 260\ \Omega \;;\quad R_4 = 550\ \Omega \;;\quad R_5 = 440\ \Omega \;;\quad R_L = 360\ \Omega$$

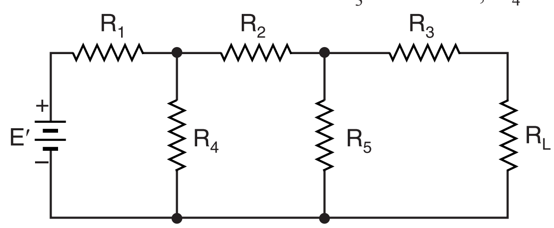



**Fig. 3.198**

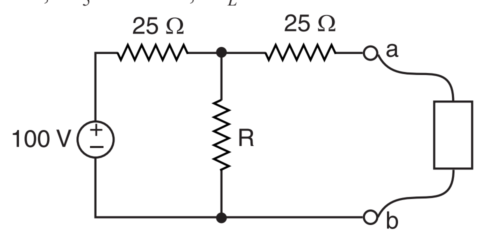



**Fig. 3.199**

9. In Fig. 3.199, replace the network to the left of terminals *ab* with its Norton equivalent.

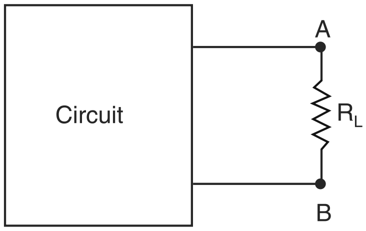
$$\left[I_N = \frac{2R}{R+12.5}\text{ A} \;;\; R_N = \frac{50\,R + 625}{R+25}\,\Omega\right]$$

10. When any source (voltage or current) is delivering maximum power to a load, prove that overall circuit efficiency is 50%.

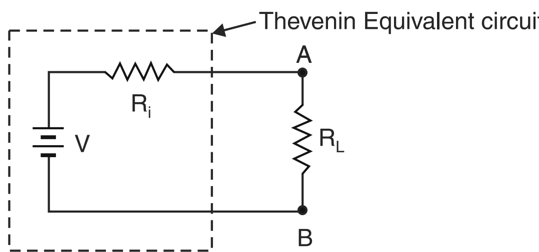
## 3.17. Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as under :

*In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.*




(*i*)




(*ii*)

**Fig. 3.200**

Fig. 3.200 (*i*) shows a circuit supplying power to a load $R_L$. The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin voltage $V = V_{Th}$ in series with Thevenin resistance $R_i (= R_{Th})$ as shown in Fig. 3.200 (*ii*). Clearly, resistance $R_i$ is the resistance measured between terminals *AB* with $R_L$ removed and e.m.f. sources replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when $R_L$ is made equal to $R_i$, the Thevenin resistance at terminals *AB*.

## 3.18. Proof of Maximum Power Transfer Theorem

Consider a voltage source $V$ of internal resistance $R_i$ delivering power to a load $R_L$. We shall prove that when $R_L = R_i$, the power delivered to $R_L$ is maximum. Referring to Fig. 3.201 (*i*), we have,

$$\text{Circuit current, } I = \frac{V}{R_L + R_i}$$

$$\text{Power delivered to load, } P = I^2 R_L$$

$$= \left(\frac{V}{R_L + R_i}\right)^2 R_L \qquad ...(i)$$

For a given source, generated voltage $V$ and internal resistance $R_i$ are constant. Therefore, power delivered to the load depends upon $R_L$. In order to find the value of $R_L$ for which the value of $P$ is maximum, differentiate eq. (*i*) w.r.t. $R_L$ and set the result equal to zero.

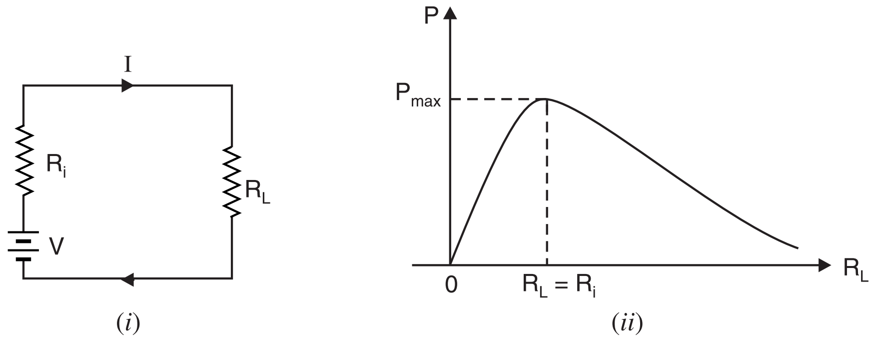



**Fig. 3.201**

Thus,
$$\frac{dP}{dR_L} = V^2\left[\frac{(R_L + R_i)^2 - 2R_L(R_L + R_i)}{(R_L + R_i)^4}\right] = 0$$

or
$$(R_L + R_i)^2 - 2\,R_L\,(R_L + R_i) = 0$$

or
$$(R_L + R_i)\,(R_L + R_i - 2R_L) = 0$$

or
$$(R_L + R_i)\,(R_i - R_L) = 0$$

Since $R_L + R_i$ cannot be zero,

$\therefore$
$$R_i - R_L = 0$$

or
$$R_L = R_i$$

or **Load resistance = Internal resistance of the source**

Thus, for maximum power transfer, load resistance $R_L$ must be equal to the internal resistance $R_i$ of the source. Fig. 3.201 (*ii*) shows the graph between power delivered ($P$) and $R_L$. We may extend the maximum power transfer theorem to a linear circuit rather than a single source by means of Thevenin's theorem as under :

*The maximum power is obtained from a linear circuit at a given pair of terminals when terminals are loaded by Thevenin's resistance ($R_{Th}$) of the circuit.*

The above statement is obviously true because by Thevenin's theorem, the circuit is equivalent to a voltage source in series with internal resistance ($R_{Th}$) of the circuit.

**Important Points.** The following points are worth noting about maximum power transfer theorem :

**(*i*)** *The circuit efficiency at maximum power transfer is only 50%* as one-half of the total power generated is dissipated in the internal resistance $R_i$ of the source.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{I^2 R_L}{I^2 (R_L + R_i)}$$

$$= \frac{R_L}{2R_L} = \frac{1}{2} = 50\% \qquad (\because\ R_L = R_i)$$

**(*ii*)** *Under the conditions of maximum power transfer, the load voltage is one-half of the open-circuited voltage at the load terminals.*

$$\text{Load voltage} = I\,R_L = \left(\frac{V}{R_L + R_i}\right) R_L = \frac{V\,R_L}{2\,R_L} = \frac{V}{2}$$

(*iii*) $$\text{Max. power transferred} = \left(\frac{V}{R_L + R_i}\right)^2 R_L = \left(\frac{V}{2\,R_L}\right)^2 R_L = \frac{V^2}{4\,R_L}$$

**Note.** In case of a practical current source, the maximum power delivered is given by ;

$$P_{max} = \frac{I_N^2 R_N}{4}$$

where $I_N$ = Norton current

$R_N$ = Norton resistance (= $R_{Th} = R_i$)

## 3.19. Applications of Maximum Power Transfer Theorem

This theorem is very useful in situations where transfer of maximum power is desirable. Two important applications are listed below :

(*i*) In communication circuits, maximum power transfer is usually desirable. For instance, in a public address system, the circuit is adjusted for maximum power transfer by making load (*i.e.* speaker) resistance equal to source (*i.e.* amplifier) resistance. When source and load have the same resistance, they are said to be *matched*.

In most practical situations, the internal resistance of the source is fixed. Also, the device that acts as a load has fixed resistance. In order to make $R_L = R_i$, we use a transformer. We can use the reflected-resistance characteristic of the transformer to make the load resistance appear to have the same value as the source resistance, thereby "fooling" the source into "thinking" that there is a match (*i.e.* $R_L = R_i$). This technique is called **impedance matching.**

(*ii*) Another example of maximum power transfer is found in starting of a car engine. The power delivered to the starter motor of the car will depend upon the effective resistance of the motor and internal resistance of the battery. If the two resistances are equal (as is the case when battery is fully charged), maximum power will be transferred to the motor to turn on the engine. This is particularly desirable in winter when every watt that can be extracted from the battery is needed by the starter motor to turn on the cold engine. If the battery is weak, its internal resistance is high and the car does not start.

**Note.** Electric power systems are never operated for maximum power transfer because the efficiency under this condition is only 50%. This means that 50% of the generated power will be lost in the power lines. This situation cannot be tolerated because power lines must operate at much higher than 50% efficiency.

**Example 3.77.** *Two identical cells connected in series deliver a maximum power of 1W to a resistance of 4 Ω. What is the internal resistance and e.m.f. of each cell ?*

**Solution.** Let $E$ and $r$ be the e.m.f. and internal resistance of each cell. The total internal resistance of the battery is $2r$. For maximum power transfer,

$$2\,r = R_L = 4 \quad \therefore \quad r = R_L/2 = 4/2 = \mathbf{2\ \Omega}$$

$$\text{Maximum power} = \frac{{}^*(2E)^2}{4\,R_L}$$

or $$1 = \frac{4\,E^2}{4\,R_L} \quad \therefore \quad E = \sqrt{R_L} = \sqrt{4} = \mathbf{2\ V}$$

* Here total voltage = $2E$.

**Example 3.78.** *Find the value of resistance R to have maximum power transfer in the circuit shown in Fig. 3.202 (i). Also obtain the amount of maximum power. All resistances are in ohms.*

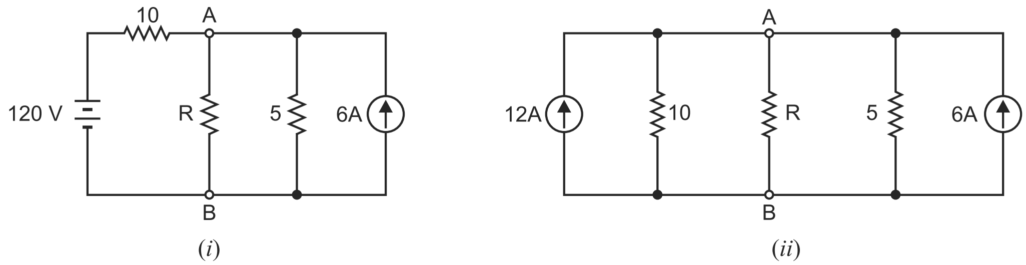



**Fig. 3.202**

**Solution.** To find the desired answers, we should find $V_{Th}$ and $R_{Th}$ at the load (*i.e. R*) terminals. For this purpose, we first convert 120V voltage source in series with 10Ω resistance into equivalent current source of 120/10 = 12A in parallel with 10Ω resistance. The circuit then becomes as shown in Fig. 3.202. (*ii*).

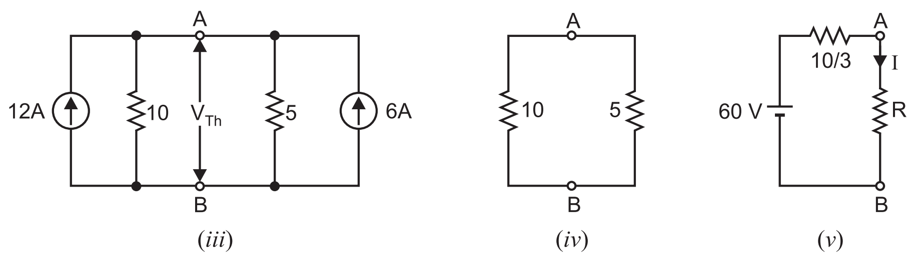



**Fig. 3.202**

To find $V_{Th}$, remove $R$ (*i.e.* load) from the circuit in Fig. 3.202 (*ii*), and the circuit becomes as shown in Fig. 3.202 (*iii*). Then voltage across the open-circuited terminals *AB* is $V_{Th}$. Referring to Fig. 3.202 (*iii*) and applying *KCL*, we have,

$$\frac{V_{Th}}{10} + \frac{V_{Th}}{5} = 12 + 6 \quad \text{or} \quad V_{Th} = 60\text{V}$$

In order to find $R_{Th}$, remove $R$ and replace the current sources by open in Fig. 3.202 (*ii*). Then circuit becomes as shown in Fig. 3.202 (*iv*). Then resistance at the open-circuited terminals *AB* is $R_{Th}$.

$$\therefore \quad R_{Th} = 10\Omega \parallel 5\Omega = \frac{10 \times 5}{10 + 5} = \frac{10}{3}\Omega$$

When $R$ is connected to the terminals of Thevenin equivalent circuit, the circuit becomes as shown in Fig. 3.202 (*v*).

For maximum power transfer, the condition is

$$R = R_{Th} = \mathbf{\frac{10}{3}\Omega}$$

Max. power transferred, $P_{max} = \dfrac{V_{Th}^2}{4R_L} = \dfrac{V_{Th}^2}{4R} = \dfrac{(60)^2}{4 \times (10/3)} =$ **270 W**

**Example 3.79.** *Calculate the value of R which will absorb maximum power from the circuit of Fig. 3.203 (i). Also find the value of maximum power.*

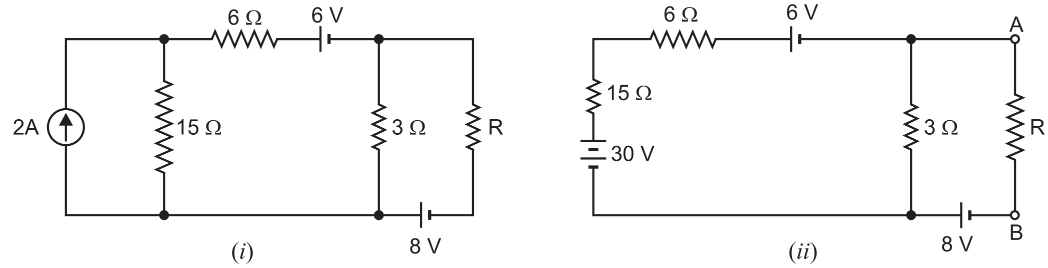



**Fig. 3.203**

**Solution.** To find the desired answers, we should find $V_{Th}$ and $R_{Th}$ at the load (*i.e. R*) terminals. For this purpose, we first convert 2A current source in parallel with 15Ω resistance into equivalent voltage source of 2A × 15Ω = 30 V in series with 15Ω resistance. The circuit then becomes as shown in Fig. 3.203 (*ii*).

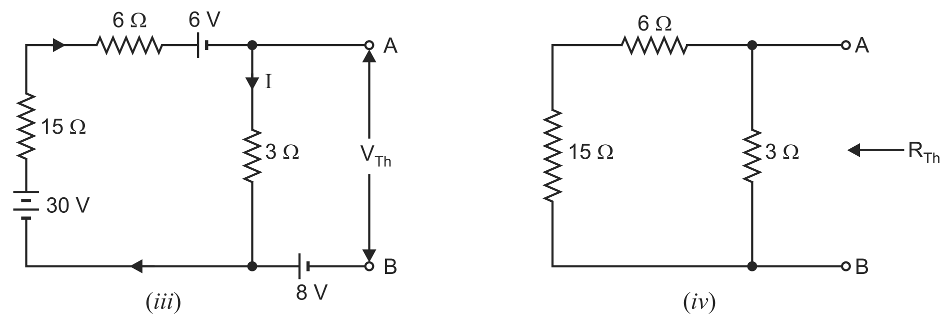



**Fig. 3.203**

To find $V_{Th}$, remove *R* (*i.e.* load) from the circuit in Fig. 3.203 (*ii*) and the circuit becomes as shown in Fig. 3.203 (*iii*). Then voltage across the open-circuited terminals *AB* is $V_{Th}$. Referring to Fig. 3.203 (*iii*),

Current in 3Ω resistor, $I = \dfrac{30-6}{15+6+3} = 1\text{A}$

In Fig. 3.203 (*iii*), as we go from point *A* to point *B via* 3Ω resistor, we have,

$$V_A - I \times 3 - 8 = V_B$$

or $$V_A - V_B = I \times 3 + 8 = 1 \times 3 + 8 = 11\text{V}$$

$\therefore$ $$V_{Th} = V_{AB} = V_A - V_B = 11\text{V}$$

In order to find $R_{Th}$, remove *R* and replace the voltage sources by short in Fig. 3.203 (*ii*). Then circuit becomes as shown in Fig. 3.203 (*iv*). Then resistance at open-circuited terminals *AB* is $R_{Th}$.

$\therefore$ $$R_{Th} = (15+6)\Omega \parallel 3\Omega = \frac{21\times 3}{21+3} = \frac{21}{8}\Omega$$

For maximum power transfer, the condition is

$$R = R_{Th} = \mathbf{\frac{21}{8}\Omega}$$

Max. power transferred, $P_{max} = \dfrac{V_{Th}^2}{4R_L} = \dfrac{V_{Th}^2}{4R} = \dfrac{(11)^2}{4\times(21/8)} = \mathbf{11.524\ W}$

**Example 3.80.** *Determine the value of $R_L$ in Fig. 3.204 (i) for maximum power transfer and evaluate this power.*

**Solution.** The three current sources in Fig. 3.204 (*i*) are in parallel and supply current in the same direction. Therefore, they can be replaced by a single current source supplying 0·8 + 1 + 0·9 = 2·7 A as shown in Fig. 3.204 (*ii*). The circuit to the left of $R_L$ in Fig. 3.204 (*ii*) can be replaced by Thevenin's equivalent circuit as under :

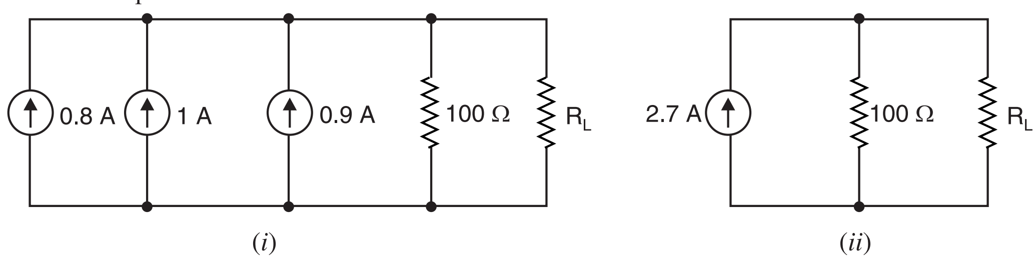



**Fig. 3.204**

$$V_{Th} = I_N R_N = 2{\cdot}7 \times 100 = 270 \text{ V}$$

$$R_i = R_N = 100\ \Omega$$

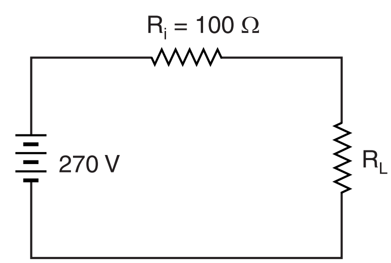



**Fig. 3.205**

The Thevenin's equivalent circuit to the left of $R_L$ is $V_{Th}$(= 270 V) in series with $R_i$ (= 100 Ω). When load $R_L$ is connected, the circuit becomes as shown in Fig. 3.205. It is clear that maximum power will be transferred when

$$R_L = R_i = \mathbf{100\ \Omega}$$

$$\text{Max. power} = \frac{V_{Th}^2}{4R_L} = \frac{(270)^2}{4 \times 100}$$

$$= \mathbf{182{\cdot}25 \text{ watts}}$$

**Example 3.81.** *Determine the maximum power that can be delivered by the circuit shown in Fig. 3.206 (i).*

**Solution.** Fig. 3.206 (*ii*) shows the Norton's equivalent circuit. Maximum power transfer occurs when $R_L = R_N = 300\ \Omega$.

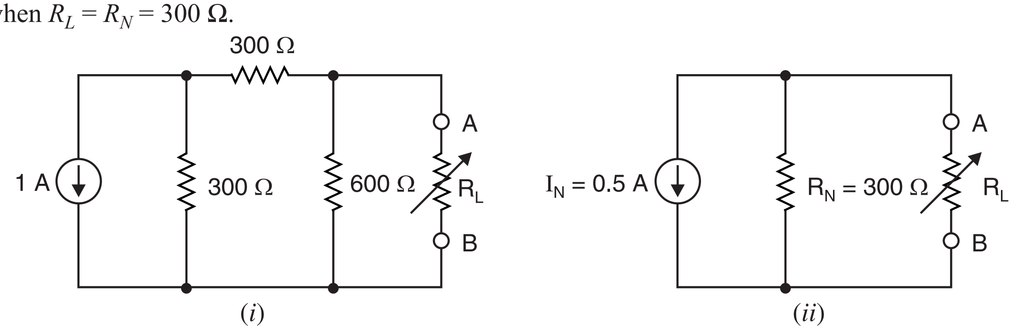



**Fig. 3.206**

Referring to Fig. 3.206 (*ii*), current in $R_L$ (= 300 Ω) = $I_N/2 = 0{\cdot}5/2 = 0{\cdot}25$ A

∴ Max. power transferred = $(0{\cdot}25)^2 \times R_L = (0{\cdot}25)^2 \times 300 =$ **18·8 W**

**Example 3.82.** *What percentage of maximum possible power is delivered to $R_L$ in Fig. 3.207 (i) when $R_L = 2\,R_{Th}$ ?*

**Solution.** Fig. 3.207 (*ii*) shows the circuit when $R_L = 2\,R_{Th}$.

$$\text{Circuit current} = \frac{V_{Th}}{R_{Th} + 2R_{Th}} = \frac{V_{Th}}{3R_{Th}}$$

$$\text{Voltage across load, } V_L = \frac{V_{Th}}{3R_{Th}} \times 2R_{Th} = \frac{2}{3}V_{Th}$$

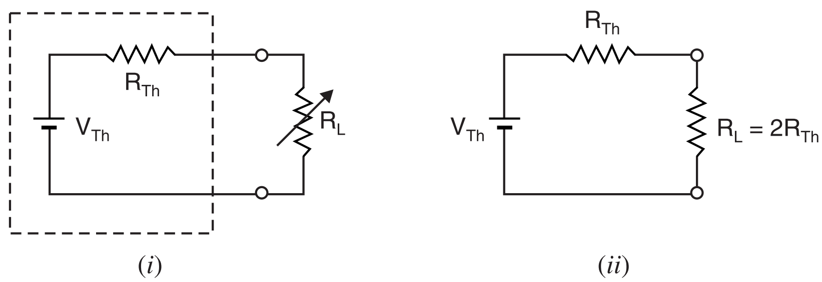



Fig. 3.207

$$\text{Power delivered to load, } P_L = \frac{V_L^2}{R_L} = \frac{\left(\frac{2}{3}V_{Th}\right)^2}{2R_{Th}} = \frac{4V_{Th}^2}{18R_{Th}}$$

Since $P_{max} = V_{Th}^2/4\,R_{Th}$, the ratio of $P_L/P_{max}$ is

$$\frac{P_L}{P_{max}} = \frac{\dfrac{4V_{Th}^2}{18R_{Th}}}{\dfrac{V_{Th}^2}{4R_{Th}}} = \frac{16}{18}$$

$$\therefore \quad P_L = \frac{16}{18}P_{max} \times 100 = \textbf{88·89\% of } \boldsymbol{P_{max}}$$

**Example 3.83.** *Find the maximum power in $R_L$ which is variable in the circuit shown in Fig. 3.208 (i).*

**Solution.** We shall use Thevenin theorem to obtain the result. For this purpose, remove the load $R_L$ as shown in Fig. 3.208 (*ii*). The open-circuited voltage at terminals *AB* in Fig. 3.208 (*ii*) is equal to $V_{Th}$. It is clear from Fig. 3.208 (*ii*) that current in the branch containing 40 Ω and 60 Ω resistors is 1 A. Similarly, current in the branch containing two 50 Ω resistors is 1 A. It is clear that point *A* is at higher potential than point *B*. Applying *KVL* to the loop *EABCDE*, we have,

$$-40 \times 1 - V_{AB} - 2 + 50 \times 1 = 0 \qquad \therefore \qquad V_{AB} = 8\text{ V}$$

Now $V_{AB}$ in Fig. 3.208 (*ii*) is equal to $V_{Th}$. Therefore, $V_{Th} = 8$ V.

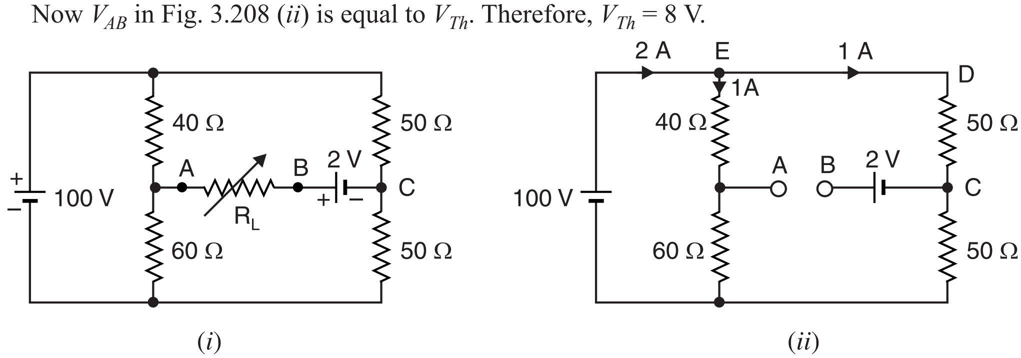



Fig. 3.208

In order to find Thevenin's resistance $R_{Th}$, replace 100V and 2V batteries by a short in Fig. 3.208 (*ii*). Then resistance at terminals *AB* is the $R_{Th}$. It is clear that 40 Ω and 60 Ω resistors are in parallel and so the two 50 Ω resistors.

$$\therefore \quad R_{Th} = (40 \,||\, 60) + (50 \,||\, 50) = \frac{40 \times 60}{40 + 60} + \frac{50 \times 50}{50 + 50} = 24 + 25 = 49\ \Omega$$

Therefore, for maximum power, $R_L$ should be 49 Ω. The Thevenin equivalent circuit is a voltage source of 8 V in series with a resistance of 49 Ω. When load $R_L$ is connected across the terminals of Thevenin equivalent circuit, the total circuit resistance = 49 + 49 = 98 Ω.

$$\therefore \quad \text{Circuit current, } I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{49 + 49} = \frac{8}{98} = 0{\cdot}08163 \text{ A}$$

$$\therefore \quad P_{max} = I^2 R_L = (0{\cdot}08163)^2 \times 49 = \mathbf{0{\cdot}3265\ W}$$

**Example 3.84.** *For the circuit shown in Fig. 3.209 (i), find the value of R that will receieve maximum power. Determine this power.*

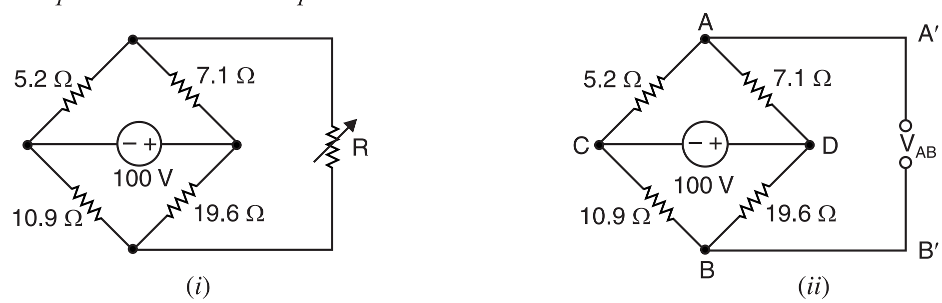



**Fig. 3.209**

**Solution.** We will use Thevenin's theorem to obtain the results. In order to find $V_{Th}$, remove the variable load $R$ as shown in Fig. 3.209 (*ii*). Then open-circuited voltage across terminals $AB$ is equal to $V_{Th}$.

$$\text{Current in branch } DAC = \frac{100}{7.1 + 5.2} = 8{\cdot}13 \text{ A}$$

$$\text{Current in branch } DBC = \frac{100}{19.6 + 10.9} = 3{\cdot}28 \text{ A}$$

It is clear from Fig. 3.209 (*ii*) that point $A$ is at higher* potential than point $B$. Applying $KVL$ to the loop $A'ACBB'A'$, we have,

$$-5{\cdot}2 \times 8{\cdot}13 + 10{\cdot}9 \times 3{\cdot}28 + V_{AB} = 0$$

$$\therefore \quad V_{AB} = 6{\cdot}52 \text{ V}$$

Now $V_{AB}$ in Fig. 3.209 (*ii*) is equal to $V_{Th}$ so that $V_{Th} = 6{\cdot}52$ V.

In order to find $R_{Th}$, replace the 100 V source in Fig. 3.209 (*ii*) by a short. The circuit becomes as shown in Fig. 3.209 (*iii*). The resistance across terminals $AB$ is the Thevenin resistance. Referring to Fig. 3.209 (*iii*),

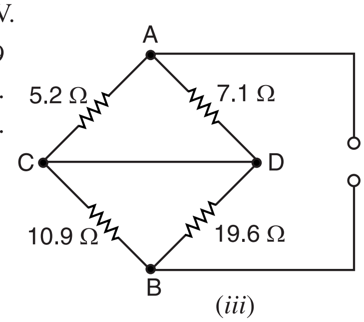



**Fig. 3.209**

$$R_{AB} = R_{Th} = (5{\cdot}2 \,||\, 7{\cdot}1) + (10{\cdot}9 \,||\, 19{\cdot}6)$$
$$= 3 + 7 = 10\ \Omega$$

Therefore, for maximum power transfer, $R = R_{Th} = \mathbf{10\ \Omega}$.

$$P_{max} = \frac{(V_{Th})^2}{4R} = \frac{(6.52)^2}{4 \times 10} = \mathbf{1{\cdot}06\ W}$$

* The fall in potential along $DA$ is less than the fall in potential along $DB$. Since point $D$ is common, point $A$ will be at higher potential than point $B$.

**Example 3.85.** *For the circuit shown in Fig. 3.210 (i), what will be the value of $R_L$ to get maximum power? Also find this power.*

**Solution.** We shall use Thevenin's theorem to obtain the results. In order to find $V_{Th}$, remove the load $R_L$ as shown in Fig. 3.210 (*ii*). Then voltage at the open-circuited terminals *AB* is equal to $V_{Th}$ *i.e.* $V_{AB} = V_{Th}$. The total load on 10 V source is

$$R_T = (90 \,\|\, 60 \,\|\, 180) + 20 = 30 + 20 = 50\ \Omega$$

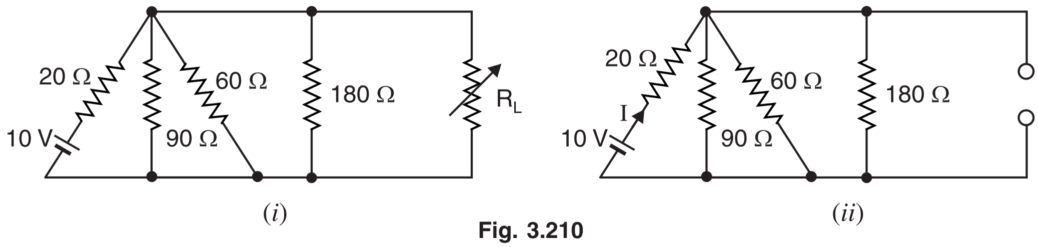



**Fig. 3.210**

Current supplied by source, $I = 10/50 = 0{\cdot}2$ A

$\therefore \quad V_{AB} = V_{Th} = 10 - 20 \times 0{\cdot}2 = 6$V

In order to find $R_{Th}$, replace the 10 V source by a short in Fig. 3.210 (*ii*). Then,

$$R_{Th} = 20 \,\|\, 90 \,\|\, 60 \,\|\, 180 = 12\ \Omega$$

Therefore, the variable load $R_L$ will receive maximum power when $R_L = R_{Th} =$ **12 Ω**.

$$\therefore \quad P_{max} = \frac{(V_{Th})^2}{4R_L} = \frac{(6)^2}{4 \times 12} = \mathbf{0{\cdot}75\ W}$$

## Tutorial Problems

1. Find the value of $R_L$ in Fig. 3.211 necessary to obtain maximum power in $R_L$. Also find the maximum power in $R_L$. **[150Ω ; 1.042 W]**

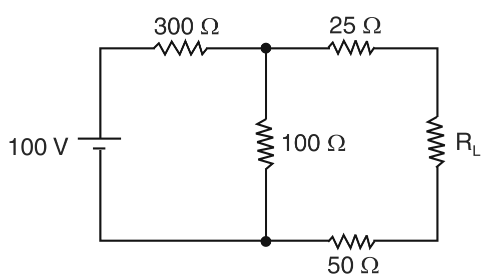



**Fig. 3.211**

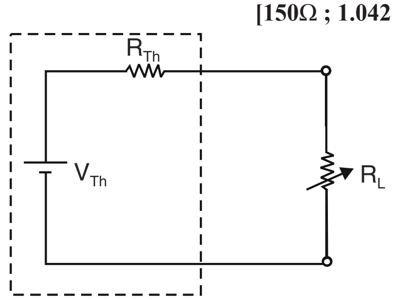



**Fig. 3.212**

2. If $R_L$ in Fig. 3.211 is fixed at 100 Ω, what alternation (s) can be made in the rest of the circuit to obtain maximum power in $R_L$? **[Short out 50 Ω resistor]**
3. What percentage of the maximum possible power is delivered to $R_L$ in Fig. 3.212, when $R_L = R_{Th}/2$ ? **[88.9%]**
4. Determine the value of $R_L$ for maximum power transfer in Fig. 3.213 and evaluate this power. **[100 Ω; 182·25 W]**

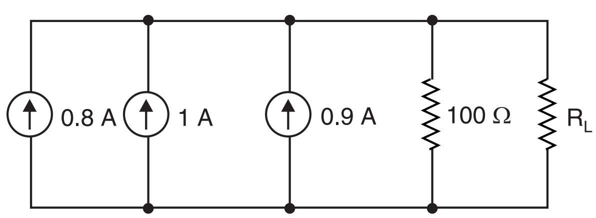



**Fig. 3.213**

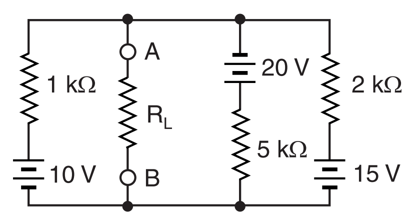



**Fig. 3.214**

5. What value should $R_L$ be in Fig. 3.214 to achieve maximum power transfer to the load? **[588 Ω]**
6. For the circuit shown in Fig. 3.215, find the value of $R_L$ for which power transferred is maximum. Also calculate this power. **[50 Ω; 0·72 W]**

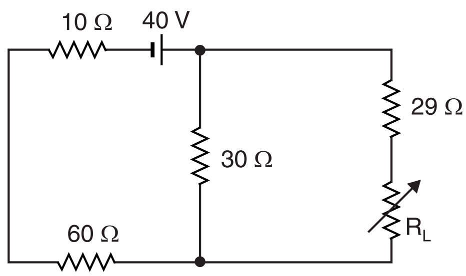



**Fig. 3.215**

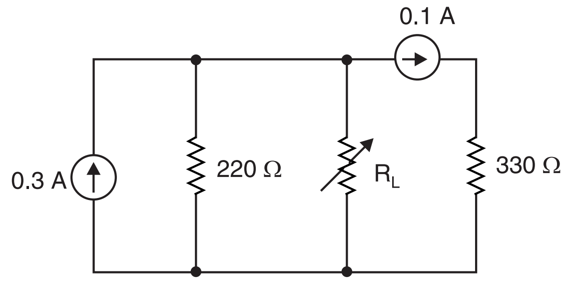



**Fig. 3.216**

7. Calculate the value of $R_L$ for transference of maximum power in Fig. 3.216. Evaluate this power. **[220 Ω; 2·2 W]**

## 3.20. Reciprocity Theorem

This theorem permits us to transfer source from one position in the circuit to another and may be stated as under :

*In any linear, bilateral network, if an e.m.f. E acting in a branch X causes a current I in branch Y, then the same e.m.f. E located in branch Y will cause a current I in branch X. However, currents in other parts of the network will not remain the same.*

**Explanation.** Consider the circuit shown in Fig. 3.217 (*i*). The e.m.f. $E$ (=100 V) acting in the branch *FAC* produces a current $I$ amperes in branch *CDF* and is indicated by the ammeter. According to reciprocity theorem, if the e.m.f. $E$ and ammeter are interchanged* as shown in Fig. 3.217 (*ii*), then the ammeter reading does not change *i.e.* the ammeter now connected in branch *FAC* will read $I$ amperes. In fact, the essence of this theorem is that $E$ and $I$ are interchangeable. The ratio $E/I$ is constant and is called *transfer resistance* (or impedance in case of a.c. system).

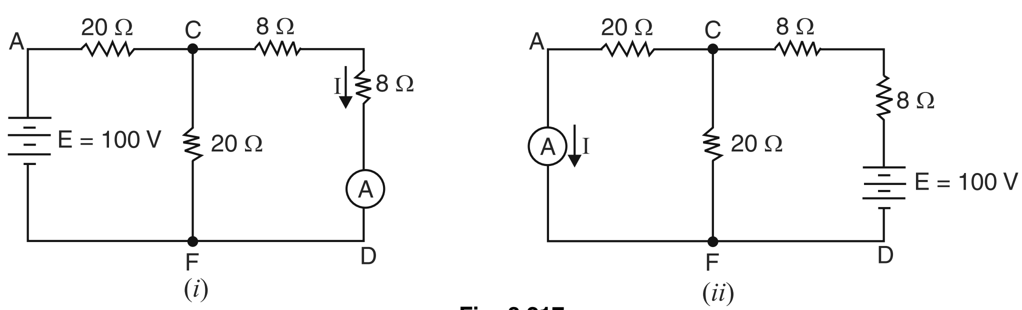



**Fig. 3.217**

**Note.** Suppose an ideal current source is connected across points *ab* of a network and this causes a voltage $v$ to appear across points *cd* of the network. The reciprocity theorem states that if the current source is now connected across *cd*, the same amount of voltage $v$ will appear across *ab*. This is sometimes stated as follows: An ideal current source and an ideal voltmeter can be interchanged without changing the reading of the voltmeter. However, voltages in other parts of the network will not remain the same.

**Example 3.86.** *Verify the reciprocity theorem for the network shown in Fig. 3.217 (i). Also find the transfer resistance.*

**Solution.** In Fig. 3.217 (*i*), e.m.f. $E$ ( = 100V) is in branch *FAC* and ammeter is in branch *CDF*. Referring to Fig. 3.217 (*i*),

* If the source of e.m.f in the original circuit has an internal resistance, this resistance must remain in the original branch and cannot be transferred to the new location of the e.m.f.

Resistance between $C$ and $F$ = 20 Ω || (8 + 8) Ω = 20 × 16/36 = 8·89 Ω

Total circuit resistance = 20 + 8·89 = 28·89 Ω

∴ Current supplied by battery = 100/28·89 = 3·46 A

The battery current is divided into two parallel paths *viz*. path *CF* of 20 Ω and path *CDF* of 8 + 8 = 16Ω.

Current in branch *CDF*, $I$ = 3·46 × 20/36 = 1·923 A

Now in Fig. 3.217 (*ii*), $E$ and ammeter are interchanged.

Referring to Fig. 3.217 (*ii*),

Resistance between $C$ and $F$ = 20 × 20/40 = 10 Ω

Total circuit resistance = 10 + 8 + 8 = 26 Ω

Current supplied by battery = 100/26 = 3·846 A

The battery current is divided into two parallel paths of 20 Ω each.

∴ Current in branch *CAF* = 3·846/2 = 1·923A

Hence, ammeter reading in both cases is the same. This verifies the reciprocity theorem.

Transfer resistance = $E/I$ = 100/1·923 = **52 Ω**

**Example 3.87.** *Find the currents in the various branches of the circuit shown in Fig. 3.218 (i). If a battery of 9V is added in branch BCD, find current in 4 Ω resistor using reciprocity theorem and superposition theorem.*

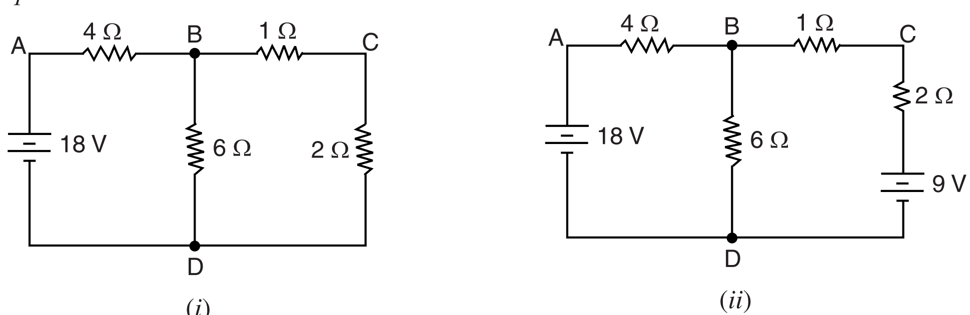



**Fig. 3.218**

**Solution.** Referring to Fig. 3.218 (*i*), we have,

Total resistance to source = 4 Ω + [6 Ω || (1 + 2) Ω] = 4 + 6 × 3/9 = 6 Ω

Current supplied by source (*i.e.* current in 4 Ω resistor or branch *DAB*)

= 18/6 = **3 A**

Current in branch *BD* = 3 × 3/9 = **1 A**

Current in branch *BCD* = 3 × 6/9 = **2 A**

In Fig. 3.218 (*i*), the current in branch *BCD* due to 18 V source acting alone is 2 A. If the 18V source is placed in branch *BCD*, then according to reciprocity theorem, the current in 4 Ω will be 2 A flowing from *B* to *A*. If a battery of 9 V is placed in branch *BCD*, then current in 4 Ω resistor due to it alone would be 2 × 9/18 = 1 A (By proportion).

Now referring to Fig. 3.218 (*ii*), the current in 4 Ω due to 18 V battery alone is 3 A flowing from *A* to *B*. The current in 4 Ω resistor due to 9 V acting alone in branch *BCD* is 1 A flowing from *B* to *A*. By superposition theorem, the current in 4 Ω is the algebraic sum of the two currents *i.e.*

Current in 4 Ω = 3 − 1 = **2 A** ***from A to B***

**Example 3.88.** *Prove the reciprocity theorem.*

**Solution.** We now prove the reciprocity theorem for the circuit shown in Fig. 3.219. In Fig. 3.219 (*i*), the e.m.f. $E$ is acting in the branch $FAC$ and the current in the branch $CDF$ is $I_2$. If the same e.m.f. $E$ now acts in branch $CDF$ [See Fig. 3.219 (*ii*)], then current $I_b$ in the branch $FAC$ will be equal to $I_2$. We now show that $I_b = I_2$. Referring to Fig. 3.219 (*i*), we have,

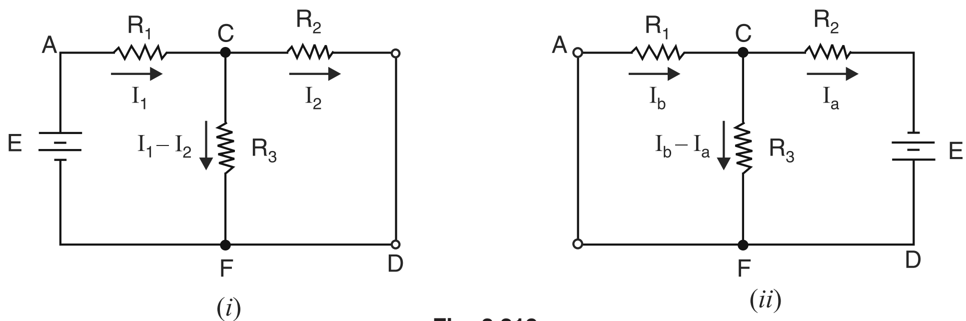



**Fig. 3.219**

$$E = I_1 R_T$$

where $$R_T = R_1 + (R_2 \parallel R_3) = \left[R_1 + \frac{R_2 R_3}{R_2 + R_3}\right]$$

$\therefore$ $$E = I_1\left[R_1 + \frac{R_2 R_3}{R_2 + R_3}\right]$$

$$= I_1\left[\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2 + R_3}\right] \quad ...(i)$$

Also in Fig. 3.219 (*i*), $$0 = -(I_1 - I_2) R_3 + I_2 R_2$$

or $$I_2 = I_1\left[\frac{R_3}{R_2 + R_3}\right] \quad ...(ii)$$

Dividing eq. (*i*) by eq. (*ii*), we have,

$$\frac{E}{I_2} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad ...(iii)$$

Similarly, it can be shown that in Fig. 3.219 (*ii*), we have,

$$\frac{E}{I_b} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_b = I_2$

Therefore, reciprocity theorem stands proved.

## 3.21. Millman's Theorem

Millman's theorem is a combination of Thevenin's and Norton's theorems. It is used to reduce any number of parallel voltage/current sources to an equivalent circuit containing only one source. It has the advantage of being easier to apply to some networks than mesh analysis, nodal analysis or superposition. This theorem can be stated in terms of voltage sources or current sources or both.

**1. Parallel voltage sources.** Millman's theorem provides a method of calculating the common voltage across different parallel-connected voltage sources and may be stated as under :

*The voltage sources that are directly connected in parallel can be replaced by a single equivalent voltage source.*

Obviously, the above statement is true by virtue of Thevenin's theorem. Fig 3.220 (*i*) shows three parallel-connected voltage sources $E_1$, $E_2$ and $E_3$. Then common terminal voltage $V_{AB}$ of these parallel voltage sources is given by ;

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G} \quad ...(i)$$

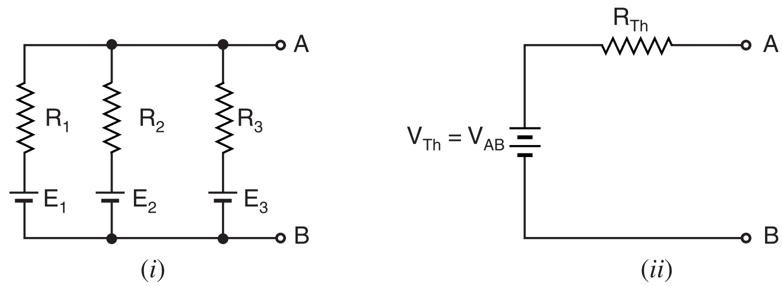



**Fig. 3.220**

This voltage represents the Thevenin's voltage $V_{Th}$. The denominator represents Thevenin's resistance $R_{Th}$ *i.e.*

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

Therefore, parallel-connected voltage sources in Fig 3.220 (*i*) can be replaced by a single voltage source as shown in Fig 3.220 (*ii*). If load $R_L$ is connected across terminals *AB*, then load current $I_L$ is given by ;

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

**Note.** If a branch does not contain any voltage source, the same procedure is used except that current in that branch will be zero. This is illustrated in example 3.89.

**2. Parallel current sources.** The Millman's theorem states as under :

*The current sources that are directly connected in parallel can be replaced by a single equivalent current source.* The current of this single current source is the *algebraic* sum of the individual source currents. The internal resistance of the single current source is equal to the combined resistance of the parallel combination of the source resistances.

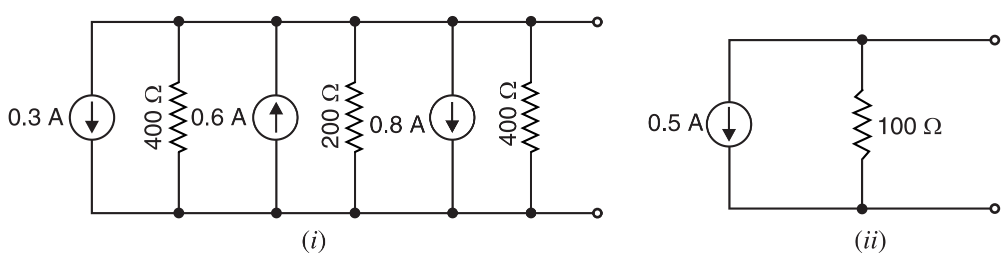



**Fig. 3.221**

Fig. 3.221 (*i*) shows three parallel connected current sources. The resultant current of the three sources is

$$0{\cdot}3 \downarrow + 0{\cdot}6 \uparrow + 0{\cdot}8 \downarrow = 0{\cdot}5 \text{ A} \downarrow$$

The internal resistance of the single current source is equal to the equivalent resistance of three parallel resistors.

$$400 \,||\, 200 \,||\, 400 = 100\ \Omega$$

Thus the single equivalent current source has value 0·5 A and internal resistance 100 Ω in parallel as shown in Fig. 3.221 (*ii*).

**3. Voltage sources and current sources in parallel.** The Millman's theorem is also applicable if the circuit has a mixture of parallel voltage and current sources. Each parallel-connected voltage source is converted to an equivalent current source. The result is a set of parallel-connected current sources and we can replace them by a single equivalent current source. Alternatively, each parallel-connected current source can be converted to an equivalent voltage source and the set of parallel-connected voltage sources can be replaced by an equivalent voltage source.

**Example 3.89.** *Using Millman's theorem, determine the common voltage $V_{xy}$ and the load current in the circuit shown in Fig. 3.222 (i).*

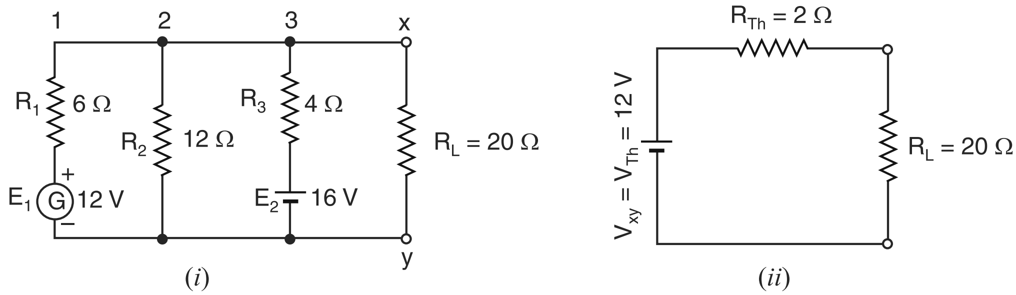



**Fig. 3.222**

**Solution.**
$$V_{xy} = V_{Th} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{12/6 + 0/2 + 16/4}{1/6 + 1/12 + 1/4} = \frac{2 + 0 + 4}{0.167 + 0.083 + 0.25} = \frac{6}{0.5} = \mathbf{12V}$$

$$R_{Th} = \frac{1}{1/6 + 1/12 + 1/4} = 2\Omega$$

Therefore, the circuit shown in Fig. 3.222 (*i*) can be replaced by the one shown in Fig. 3.222 (*ii*).

$$\text{Load current} = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{2 + 20} = \mathbf{0.545\ A}$$

**Example 3.90.** *Find the current in the 1 k Ω resistor in Fig. 3.223 by finding Millman equivalent voltage source with respect to terminals x – y.*

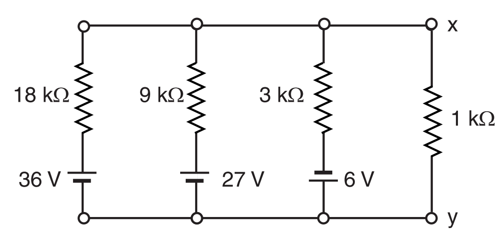



**Fig. 3.223**

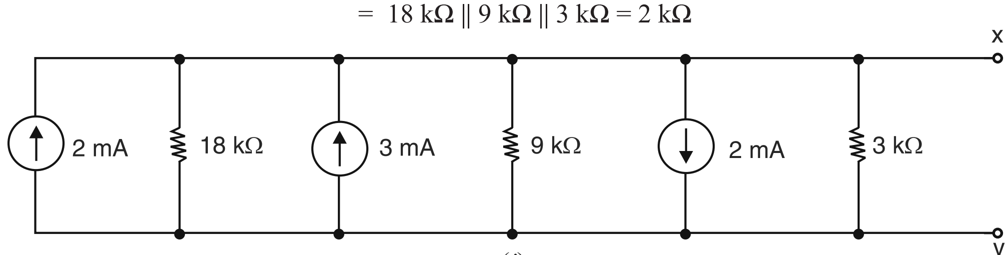
**Solution.** As shown Fig. 3.224 (*i*), each of the three voltage sources is converted to an equivalent current source. For example, the 36 V source in series with 18 kΩ resistor becomes a 36 V/18 kΩ = 2 mA current source in parallel with 18 kΩ. Note that the polarity of each current source is such that it produces current in the same direction as the voltage source it replaces.

The resultant current of the three current sources

$$= 2\text{ mA}\uparrow + 3\text{ mA}\uparrow + 2\text{ mA}\downarrow = 3\text{ mA}\uparrow$$

The parallel equivalent resistance of three resistors

$$= 18 \text{ k}\Omega \parallel 9 \text{ k}\Omega \parallel 3 \text{ k}\Omega = 2 \text{ k}\Omega$$

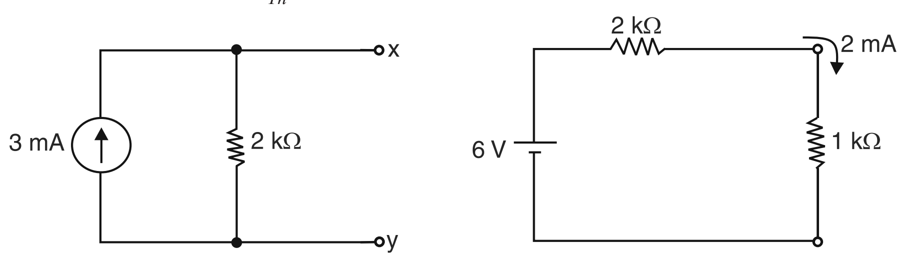



(*i*)

**Fig. 3.224**

Fig. 3.224 (*ii*) shows the single equivalent current source. Fig. 3.224 (*iii*) shows the voltage source that is equivalent to current source in Fig. 3.224 (*ii*).

$$V_{Th} = 3 \text{ mA} \times 2 \text{ k}\Omega = 6 \text{ V}$$



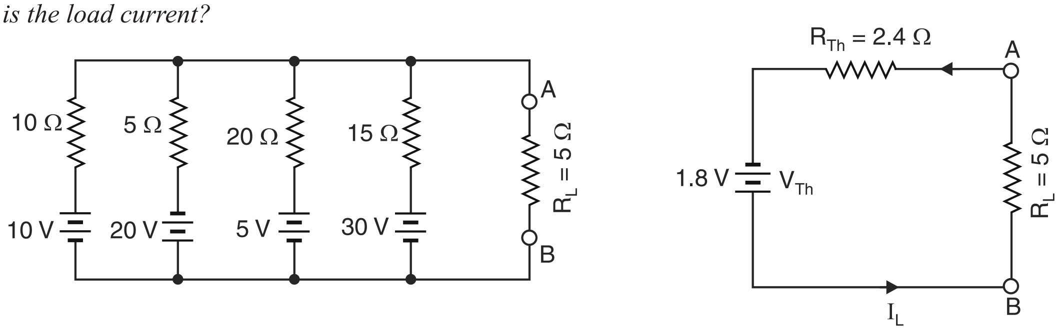

(*ii*) (*iii*)

**Fig. 3.224**

When the 1 kΩ resistor is connected across the $x - y$ terminals, the current is

$$I = \frac{6\text{V}}{3\text{k}\Omega} = \mathbf{2 \text{ mA}}$$

**Example 3.91.** *Find an equivalent voltage source for the circuit shown in Fig. 3.225 (i). What is the load current?*




(*i*) (*ii*)

**Fig. 3.225**

**Solution.**

$$V_{AB} = V_{Th} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3 + E_4/R_4}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4}$$

$$= \frac{10/10 - {}^*20/5 + 5/20 + 30/15}{1/10 + 1/5 + 1/20 + 1/15} = \frac{-0.75}{0.417} = -1{\cdot}8 \text{ V}$$

Negative sign shows that terminal *A* is negative *w.r.t.* terminal *B*.

* Note that polarity is opposite as compared to other sources.

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4}$$

$$= \frac{1}{1/10 + 1/5 + 1/20 + 1/15} = 2.4\Omega$$

Therefore, equivalent voltage source consists of **1·8 V source in series with 2·4 Ω resistor** as shown in Fig. 3.225 (*ii*).

$\therefore$ Load current, $I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1.8}{2.4 + 5} = \mathbf{0.24A}$

**Example 3.92.** *For the circuit shown in Fig. 3.225 (i) above, find the equivalent current source. Also find load current.*

**Solution.** Convert the voltage sources to current sources as shown in Fig. 3.226 (*i*). The arrow for each current source corresponds to the polarity of each voltage source in the original circuit.

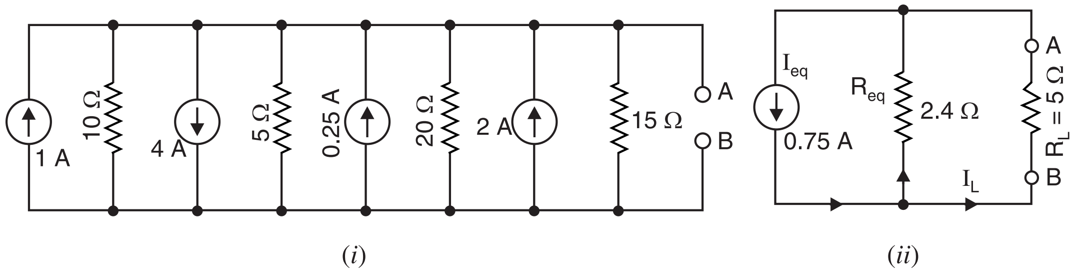



**Fig. 3.226**

The equivalent current source is found by algebraically adding the currents of individual sources.

$$I_{eq} = 1\text{ A}\uparrow + 4\text{ A}\downarrow + 0{\cdot}25\text{ A}\uparrow + 2\text{ A}\uparrow = 0{\cdot}75\text{ A}\downarrow$$

The downward arrow for $I_{eq}$ shows that terminal $A$ is negative *w.r.t.* terminal $B$.

$$R_{eq} = 10\ \Omega \parallel 5\ \Omega \parallel 20\ \Omega \parallel 15\ \Omega = 2{\cdot}4\ \Omega$$

Therefore, the equivalent current source consists of **0·75 A current source in parallel with 2·4 Ω resistor** as shown in Fig. 3.226 (*ii*). By current-divider rule, the load current $I_L$ is

$$I_L = 0.75 \times \frac{2.4}{2.4 + 5} = \mathbf{0.243A}$$

**Example 3.93.** *Find the load current for Fig. 3.227 (i) using the dual of Millman's theorem.*

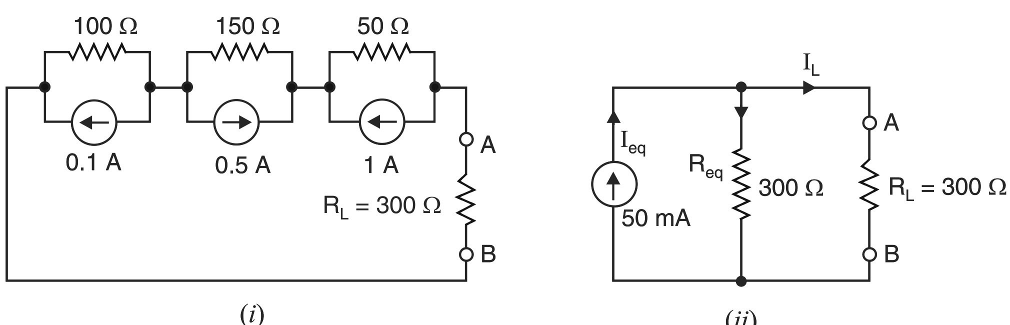



**Fig. 3.227**

**Solution.** There is a dual for Millman's theorem and it is useful for solving circuits with series current sources [See Fig. 3.227 (*i*)]. In such a case, the following equations are used to find the current and resistance of the equivalent circuit.

$$I_{eq} = \frac{I_1R_1 + I_2R_2 + I_3R_3}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

Thus referring to Fig. 3.227 (*i*), we have,

$$I_{eq} = \frac{-0.1\times100 + 0.5\times150 - 1\times50}{100+150+50} = \frac{15}{300}\text{A} = 50 \text{ mA}$$

$$R_{eq} = 100 + 150 + 50 = 300\ \Omega$$

The equivalent circuit is shown in Fig. 3.227 (*ii*). By current-divider rule, the load current $I_L$ is

$$I_L = 50\times\frac{300}{300+300} = \mathbf{25\ mA}$$

**Example 3.94.** *By constructing a Millman equivalent voltage source with respect to terminals x – y, find the voltage across 40 Ω resistor in Fig. 3.228 (i).*

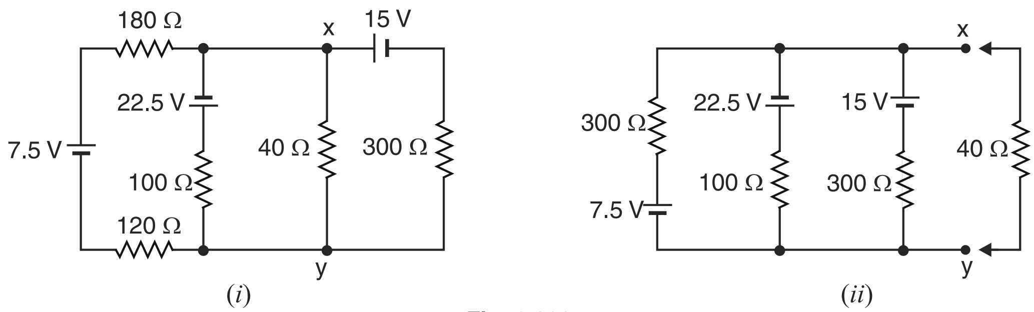



**Fig. 3.228**

**Solution.** Note that 120 Ω and 180 Ω resistors are in a series path and can therefore be combined into an equivalent resistance of 300 Ω. The circuit is *redrawn as shown in Fig. 3.228 (*ii*). It is clear that redrawn circuit has three parallel-connected voltage sources. Referring to Fig. 3.228 (*ii*), we have,

$$V_{xy} = V_{Th} = \frac{E_1/R_1 - E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{7.5/300 - 22.5/100 + 15/300}{1/300 + 1/100 + 1/300} = \frac{-0.15}{0.0167} = -9\text{V}$$

Negative sign shows that terminal $x$ is negative w.r.t. terminal $y$.

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3} = \frac{1}{1/300 + 1/100 + 1/300} = 60\ \Omega$$

Therefore, the equivalent voltage source consists of 9 V in series with 60 Ω resistor. When load is connected across the terminals of the equivalent voltage source, the circuit becomes as shown in Fig. 3.229.

$$\text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{9}{60+40} = 0.09 \text{ A}$$

$$\text{Voltage across } 40\ \Omega = I_L\, R_L = 0{\cdot}09 \times 40 = \mathbf{3{\cdot}6\ V}$$

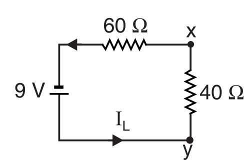


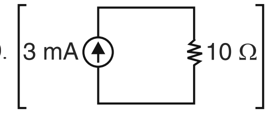

**Fig. 3.229**

Note that Millman's theorem is a powerful tool in the hands of engineers to solve many problems which cannot be solved easily by the usual methods of circuit analysis.

* It makes no difference on which side of each voltage source its series resistance is drawn.

## Tutorial Problems

1. Find the single equivalent current source for the circuit shown in Fig. 3.230.



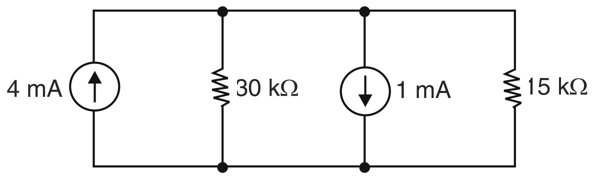




**Fig. 3.230**

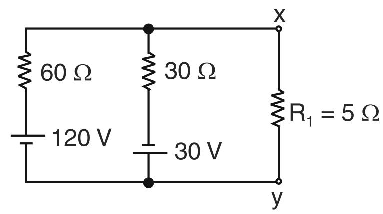
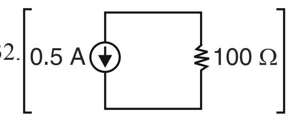



**Fig. 3.231**

2. By constructing a Millman equivalent voltage source at terminals $x - y$, find the voltage across $R_1$ (= 5 Ω) in the circuit shown in Fig. 3.231. **[4 V ±]**

3. Find the single equivalent current source for the circuit shown in Fig. 3.232.



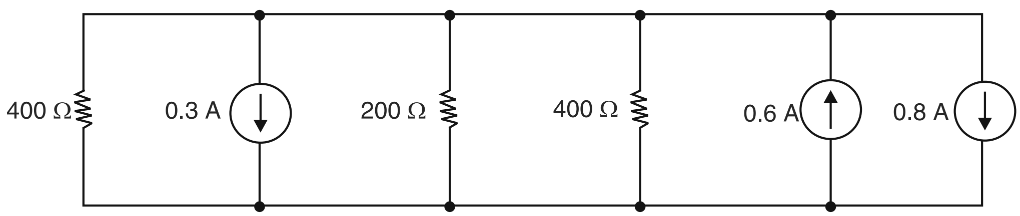




**Fig. 3.232**

4. What is the current flowing in the load resistor in Fig. 3.233 ? **[2·25 mA]**

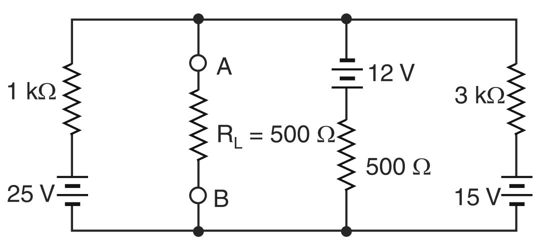



**Fig. 3.233**

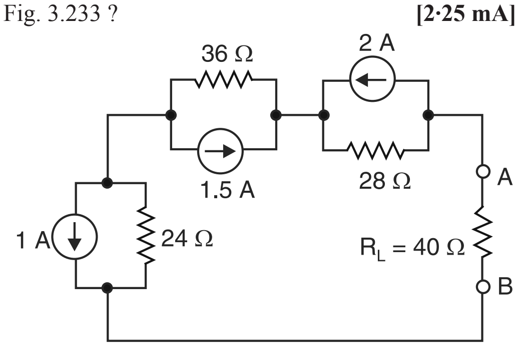



**Fig. 3.234**

5. What is the drop and polarity of the load in Fig. 3.234 ? [ **8·13V and terminal *A* is negative**]

## 3.22. Compensation Theorem

It is sometimes necessary to know, when making a change in one branch of a network, what effect this change will have on the various currents and voltages throughout the network. The compensation theorem deals with this situation and may be stated for d.c. circuits as under :

*The compensation theorem states that any resistance R in a branch of a network in which current I is flowing can be replaced, for the purpose of calculations, by a voltage equal to – IR*. It follows from Kirchhoff's voltage law that the current $I$ is unaltered if an e.m.f. $- IR$ is substituted for the voltage drop $IR$.

**Or**

*If the resistance of any branch of a network is changed from R to (R + ΔR) where the current was originally I, then the change of current at any point in the network may be calculated by assuming than an e.m.f. – IΔR has been introduced into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.*

**Illustration.** Let us illustrate the compensation theorem with a numerical example. Consider the circuit shown in Fig. 3.235 (*i*). The various branch currents in this circuit are :

$$I_1 = \frac{50}{20+5} = 2\text{ A} \quad ; \quad I_2 = I_3 = 1\text{ A}$$

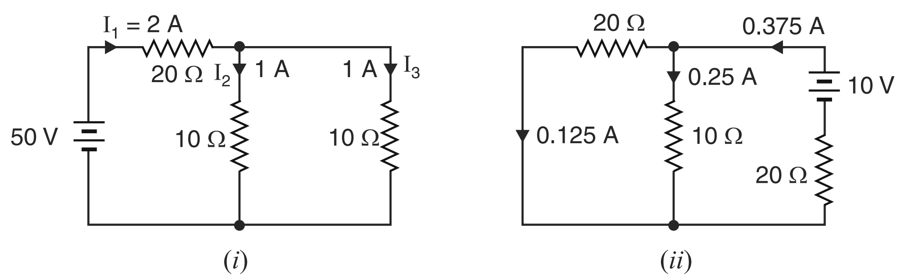



**Fig. 3.235**

Now suppose that the resistance of the right branch is increased to 20 Ω *i.e.* $\Delta R = 20 - 10 = 10\ \Omega$ and a voltage $V = -I_3\,\Delta R = -1 \times 10 = -10$ V is introduced in this branch and voltage source replaced by a short (∵ internal resistance is assumed zero). The circuit becomes as shown in Fig. 3.235 (*ii*). The compensating currents produced by this voltage are also indicated. When these compensating currents are algebraically added to the original currents in their respective branches, the new branch currents will be as shown in Fig. 3.236. The compensation theorem is useful in bridge and potentiometer circuits, where a slight change in one resistance results in a shift from a null condition.

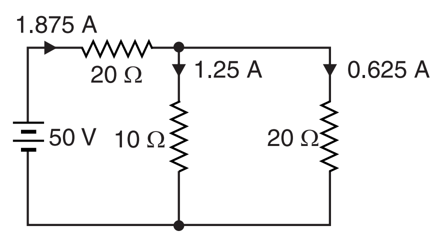



**Fig. 3.236**

## 3.23. Delta/Star and Star/Delta Transformation

There are some networks in which the resistances are neither in series nor in parallel. A familiar case is a three terminal network *e.g.* delta network or star network. In such situations, it is not possible to simplify the network by series and parallel circuit rules. However, converting delta network into star and *vice-versa* often simplifies the network and makes it possible to apply series-parallel circuit techniques.

## 3.24. Delta/Star Transformation

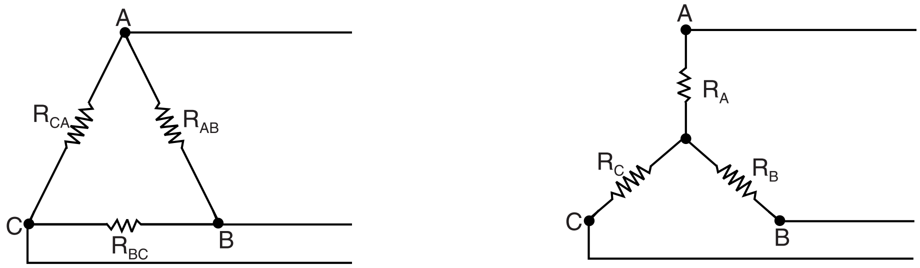
Consider three resistors $R_{AB}$, $R_{BC}$ and $R_{CA}$ connected in delta to three terminals *A, B* and *C* as shown in Fig. 3.237 (*i*). Let the equivalent star-connected network have resistances $R_A$, $R_B$ and $R_C$. Since the two arrangements are electrically equivalent, the resistance between any two terminals of one network is equal to the resistance between the corresponding terminals of the other network.

Let us consider the terminals *A* and *B* of the two networks.

Resistance between *A* and *B* for star = Resistance between *A* and *B* for delta

or $$R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$$

or $$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \quad \text{...(}i\text{)}$$




(*i*) (*ii*)

**Fig. 3.237**

Similarly, $$R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(ii)$$

and $$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(iii)$$

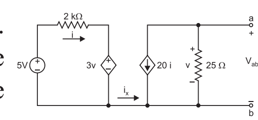
Subtracting eq. (*ii*) from eq. (*i*) and adding the result to eq. (*iii*), we have,

$$R_A = \frac{R_{AB}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(iv)$$

Similarly, $$R_B = \frac{R_{BC}\,R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(v)$$

and $$R_C = \frac{R_{CA}\,R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(vi)$$

**How to remember ?** There is an easy way to remember these relations. Referring to Fig. 3.238, star-connected resistances $R_A$, $R_B$ and $R_C$ are electrically equivalent to delta-connected resistances $R_{AB}$, $R_{BC}$ and $R_{CA}$. We have seen above that :




$$R_A = \frac{R_{AB}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

*i.e.* Any arm of star-connection $= \dfrac{\text{Product of two adjacent arms of } \Delta}{\text{Sum of arms of } \Delta}$

**Fig. 3.238**

Thus to find the star resistance that connects to terminal *A*, *divide the product of the two delta resistors connected to A by the sum of the delta resistors*. Same is true for terminals *B* and *C*.

## 3.25. Star/Delta Transformation

Now let us consider how to replace the star-connected network of Fig. 3.237 (*ii*) by the equivalent delta-connected network of Fig. 3.237 (*i*).

Dividing eq. (*iv*) by (*v*), we have,

$$R_A/R_B = R_{CA}/R_{BC}$$

$$\therefore \qquad R_{CA} = \frac{R_A\,R_{BC}}{R_B}$$

Dividing eq. (*iv*) by (*vi*), we have,

$$R_A / R_C = R_{AB} / R_{BC}$$

$\therefore \quad R_{AB} = \dfrac{R_A R_{BC}}{R_C}$

Substituting the values of $R_{CA}$ and $R_{AB}$ in eq. (*iv*), we have,

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

Similarly, $$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

and $$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

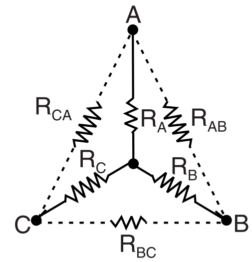



**Fig. 3.239**

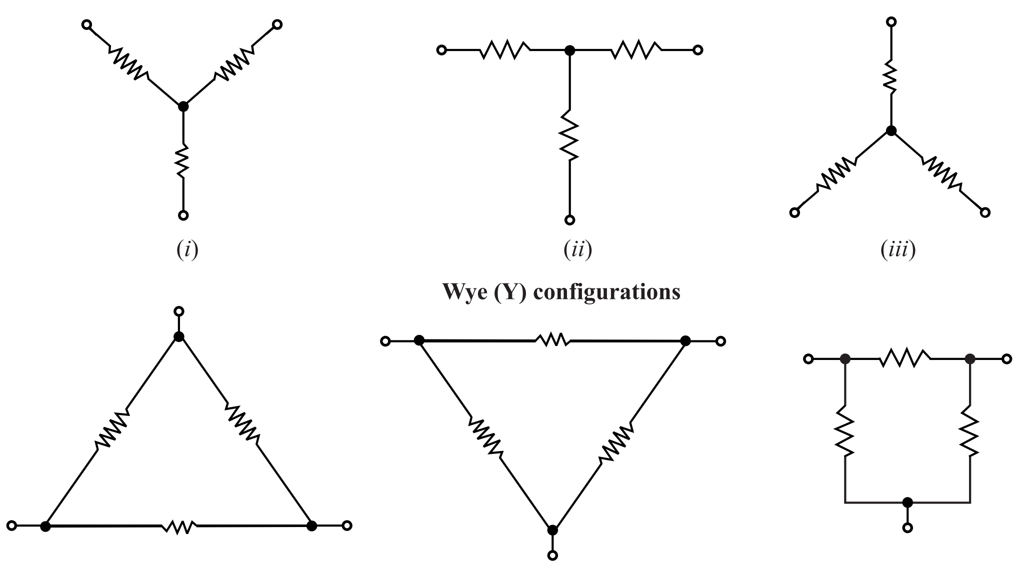
**How to remember ?** There is an easy way to remember these relations.

Referring to Fig. 3.239, star-connected resistances $R_A$, $R_B$ and $R_C$ are electrically equivalent to delta-connected resistances $R_{AB}$, $R_{BC}$ and $R_{CA}$. We have seen above that :

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

*i.e.* Resistance between two terminals of delta = Sum of star resistances connected to those terminals *plus* product of same two resistances divided by the third star resistance

**Note.** Figs. 3.240 (*i*) to (*iii*) show three ways that a wye (Y) arrangement might appear in a circuit. Because the wye-connected components may appear in the equivalent form shown in Fig. 3.240 (*ii*), the arrangement is also called a *tee* (*T*) arrangement. Figs. 3.240 (*iv*) to (*vi*) show equivalent delta forms. Because the delta (Δ) arrangement may appear in the equivalent form shown in Fig. 3.240 (*vi*), it is also called a *pi* (π) arrangement. The figures show only a few of the ways the wye (*Y*) and delta (Δ) networks might be drawn in a schematic diagram. Many equivalent forms can be drawn by rotating these basic arrangements through various angles. Note that each network has three terminals.



(*i*) (*ii*) (*iii*)

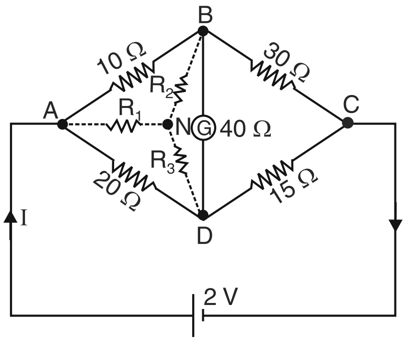
**Wye (Y) configurations**

(*iv*) (*v*) (*vi*)

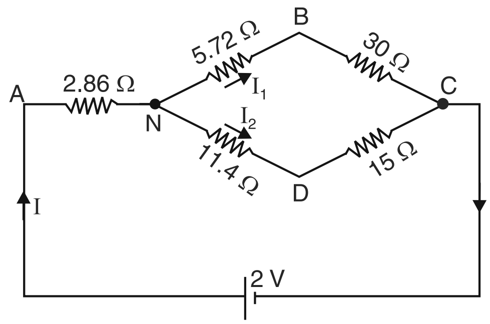
**Delta (Δ) configurations**

**Fig. 3.240**

**Example 3.95.** *Using delta/star transformation, find the galvanometer current in the Wheatstone bridge shown in Fig. 3.241 (i).*




(*i*)




(*ii*)

**Fig. 3.241**

**Solution.** The network *ABDA* in Fig. 3.241 (*i*) forms a delta. These delta-connected resistances can be replaced by equivalent star-connected resistances $R_1$, $R_2$ and $R_3$ as shown in Fig. 3.241 (*i*).

$$R_1 = \frac{R_{AB}\,R_{DA}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 20}{10+40+20} = 2.86\ \Omega$$

$$R_2 = \frac{R_{AB}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 40}{10+40+20} = 5.72\ \Omega$$

$$R_3 = \frac{R_{DA}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{20\times 40}{10+40+20} = 11.4\ \Omega$$

Thus the network shown in Fig. 3.241 (*i*) reduces to the network shown in Fig. 3.241 (*ii*).

$$R_{AC} = 2.86 + \frac{(30+5.72)\,(15+11.4)}{(30+5.72)+(15+11.4)} = 18.04\ \Omega$$

Battery current, $I = 2/18{\cdot}04 = 0{\cdot}11$ A

The battery current divides at $N$ into two parallel paths.

$\therefore$ Current in branch $NBC$, $I_1 = 0.11 \times \dfrac{26.4}{26.4+35.72} = 0{\cdot}047$ A

Current in branch $NDC$, $I_2 = 0.11 \times \dfrac{35.72}{26.4+35.72} = 0{\cdot}063$ A

Potential of $B$ *w.r.t.* $C$ = $30 \times 0{\cdot}047 = 1{\cdot}41$ V

Potential of $D$ *w.r.t.* $C$ = $15 \times 0{\cdot}063 = 0{\cdot}945$ V

Clearly, point $B$ is at higher potential than point $D$ by

$1{\cdot}41 - 0{\cdot}945 = 0{\cdot}465$ V

$\therefore$ Galvanometer current $= \dfrac{\text{P.D. between } B \text{ and } D}{\text{Galvanometer resistance}}$

$= 0{\cdot}465 / 40 = 11{\cdot}6 \times 10^{-3}$ A = **11·6 mA** *from B to D*

**Example 3.96.** *With the help of star/delta transformation, obtain the value of current supplied by the battery in the circuit shown in Fig. 3.242 (i).*

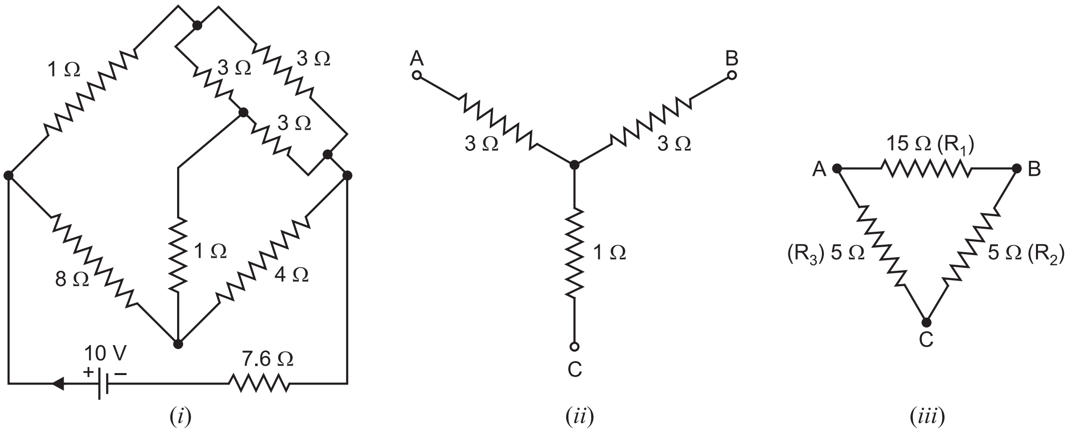



**Fig. 3.242**

**Solution.** The star-connected resistances 3 Ω, 3 Ω and 1 Ω in Fig. 3.242 (*i*), are shown separately in Fig. 3.242 (*ii*). These star-connected resistances are converted into equivalent delta-connected resistances $R_1$, $R_2$ and $R_3$ as shown in Fig. 3.242 (*iii*).

$$R_1 = 3+3+\frac{3\times 3}{1} = 15\ \Omega$$

$$R_2 = 3+1+\frac{3\times 1}{3} = 5\ \Omega$$

$$R_3 = 1+3+\frac{1\times 3}{3} = 5\ \Omega$$

After above star-delta conversion, the circuit reduces to the one shown in Fig. 3.242 (*iv*). This circuit can be further simplified by combining parallel resistances and the circuit becomes as shown in Fig. 3.242 (*v*).

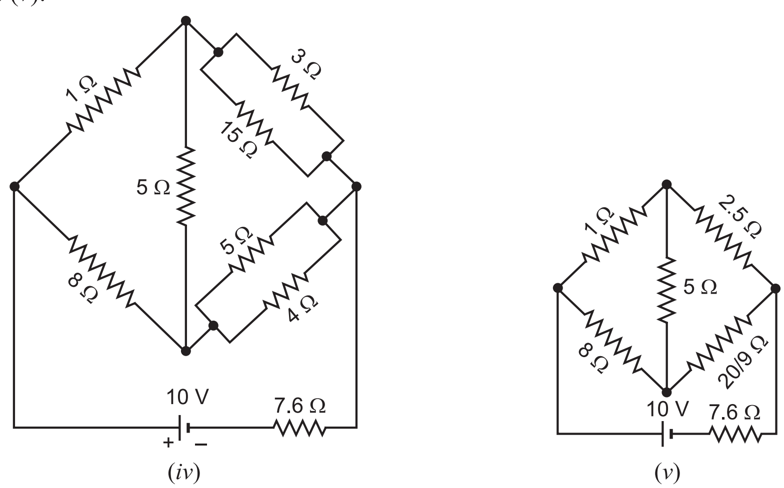



**Fig. 3.242**

The three delta-connected resistances 1 Ω, 5 Ω and 8 Ω in Fig. 3.242 (*v*) are shown separately in Fig. 3.242 (*vi*). These delta-connected resistances can be converted into equivalent star-connected resistances $R'_1$, $R'_2$ and $R'_3$ as shown in Fig. 3.242 (*vii*).

$$R'_1 = \frac{1\times 8}{1+5+8} = \frac{4}{7}\,\Omega$$

$$R'_2 = \frac{5\times 1}{1+5+8} = \frac{5}{14}\,\Omega$$

$$R'_3 = \frac{8\times 5}{1+5+8} = \frac{20}{7}\,\Omega$$

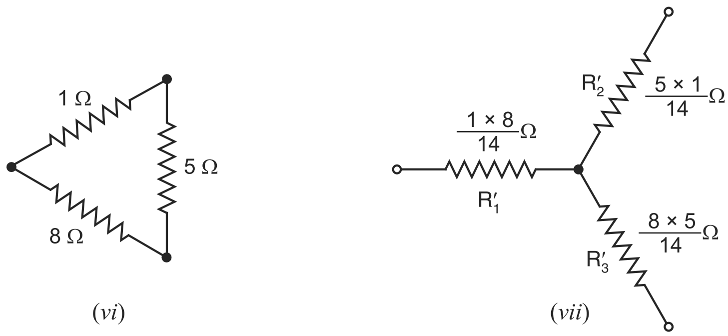



**Fig. 3.242**

After above delta-star conversion, the circuit reduces to the one shown in Fig. 3.242 (*viii*).

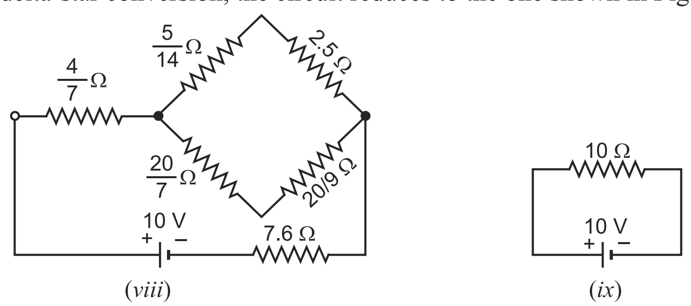



**Fig. 3.242**

Total resistance offered by the circuit to the battery is

$$R_T = \frac{4}{7} + \left[\left(\frac{5}{14} + 2.5\right) \| \left(\frac{20}{7} + \frac{20}{9}\right)\right] + 7.6$$

$$= \frac{4}{7} + \left(\frac{20}{7} \| \frac{320}{63}\right) + 7.6 = 10\,\Omega$$

$\therefore$ Current supplied by the battery [See Fig. 3.242 (*ix*)] is

$$I = \frac{V}{R_T} = \frac{10}{10} = \mathbf{1\ A}$$

**Example 3.97.** *A network of resistors is shown in Fig. 3.243 (i). Find the resistance (i) between terminals A and B (ii) B and C and (iii) C and A.*

**Solution.** The star-connected resistances 6 Ω, 3 Ω and 4 Ω in Fig. 3.243 (*i*) are shown separately in Fig. 3.243 (*ii*). These star-connected resistances can be converted into equivalent delta-connected resistances $R_1$, $R_2$ and $R_3$ as shown in Fig. 3.243 (*ii*).

$$R_1 = 4 + 6 + (4 \times 6/3) = 18\,\Omega$$

$$R_2 = 6 + 3 + (6 \times 3/4) = 13{\cdot}5\,\Omega$$

$$R_3 = 4 + 3 + (4 \times 3/6) = 9\,\Omega$$

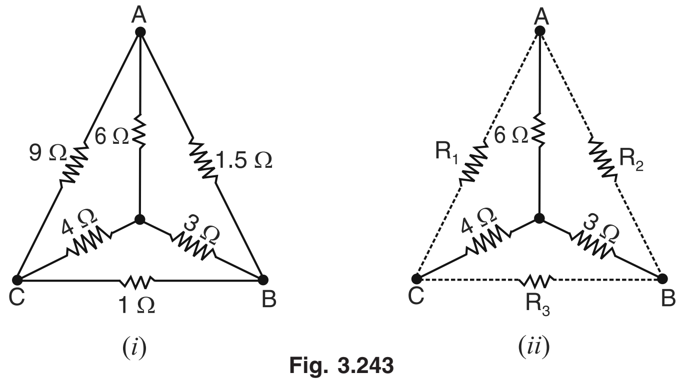



**Fig. 3.243**

These delta-connected resistances $R_1, R_2$ and $R_3$ come in parallel with the original delta-connected resistances. The circuit shown in Fig. 3.243 (*i*) reduces to the circuit shown in Fig. 3.244(*i*).

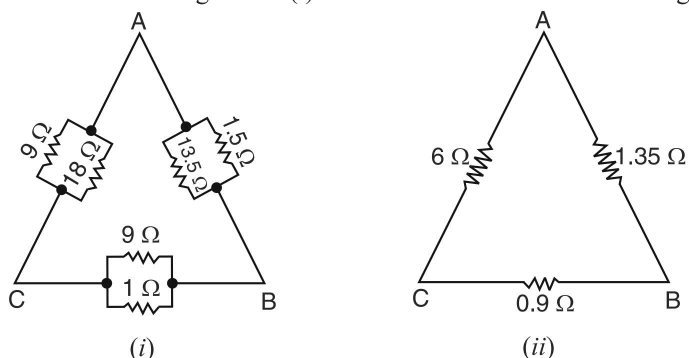



**Fig. 3.244**

The parallel resistances in each leg of delta in Fig. 3.244 (*i*) can be replaced by a single resistor as shown in Fig. 3.244 (*ii*) where

$$R_{AC} = 9 \times 18/27 = 6\ \Omega$$
$$R_{BC} = 9 \times 1/10 = 0{\cdot}9\ \Omega$$
$$R_{AB} = 1{\cdot}5 \times 13{\cdot}5/15 = 1{\cdot}35\ \Omega$$

**(*i*)** Resistance between $A$ and $B$ = $1{\cdot}35\ \Omega \parallel (6 + 0{\cdot}9)\ \Omega = 1{\cdot}35 \times 6{\cdot}9/8{\cdot}25 =$ **$1{\cdot}13\ \Omega$**

**(*ii*)** Resistance between $B$ and $C$ = $0{\cdot}9\ \Omega \parallel (6 + 1{\cdot}35)\ \Omega = 0{\cdot}9 \times 7{\cdot}35/8{\cdot}25 =$ **$0{\cdot}8\ \Omega$**

**(*iii*)** Resistance between $A$ and $C$ = $6\ \Omega \parallel (1{\cdot}35 + 0{\cdot}9)\ \Omega = 6 \times 2{\cdot}25/8{\cdot}25 =$ **$1{\cdot}636\ \Omega$**

**Example 3.98.** *Determine the load current in branch EF in the circuit shown in Fig. 3.245 (i).*

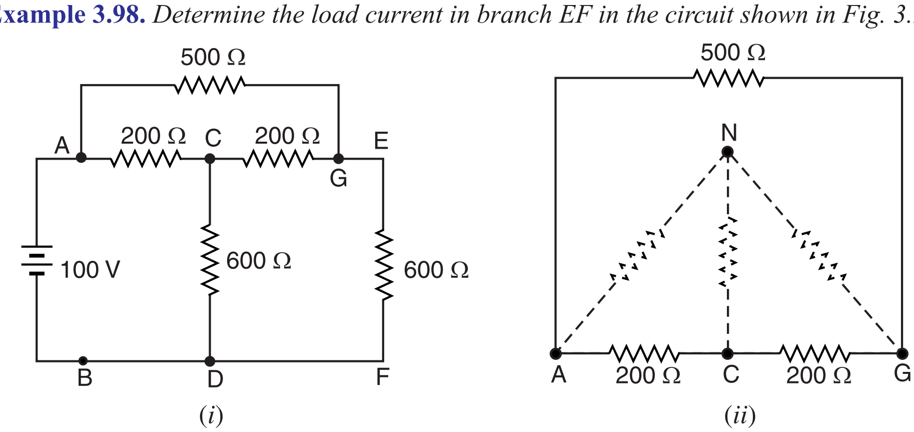



**Fig. 3.245**

**Solution.** The circuit *ACGA* forms delta and is shown separately in Fig. 3.245 (*ii*) for clarity. Changing this delta connection into equivalent star connection [See Fig. 3.245 (*ii*)], we have,

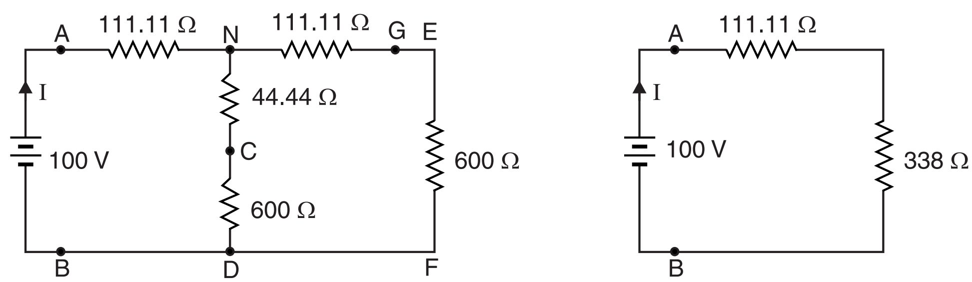
$$R_{AN} = \frac{500 \times 200}{500 + 200 + 200} = 111.11\ \Omega \quad ; \quad R_{CN} = \frac{200 \times 200}{500 + 200 + 200} = 44.44\ \Omega \ ;$$

$$R_{GN} = \frac{500 \times 200}{500 + 200 + 200} = 111.11\ \Omega$$

Thus the circuit shown in Fig. 3.245 (*i*) reduces to the circuit shown in Fig. 3.246 (*i*). The branch *NEF* ( = 111·11 + 600 = 711·11 **Ω**) is in parallel with branch *NCD* ( = 44·44 + 600 = 644·44 **Ω**) and the equivalent resistance of this parallel combination is

$$= \frac{711.11 \times 644.44}{711.11 + 644.44} = 338\ \Omega$$

The circuit shown in Fig. 3.246 (*i*) reduces to the circuit shown in Fig. 3.246 (*ii*).




(*i*) (*ii*)

**Fig. 3.246**

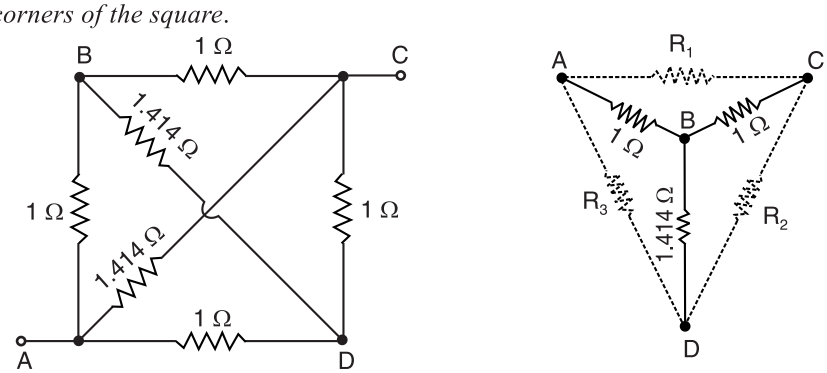
$$\therefore \qquad \text{Battery current, } I = \frac{100}{338 + 111.11} = 0.222 \text{ A}$$

This battery current divides into two parallel paths [See Fig. 3.246 (*i*)] *viz*. branch *NEF* and branch *NCD*.

∴ Current in branch *NEF i.e.* in branch *EF*

$$= 0.222 \times \frac{644.44}{711.11 + 644.44} = \mathbf{0.1055\ A}$$

**Example 3.99.** *A square and its diagonals are made of a uniform covered wire. The resistance of each side is 1* **Ω** *and that of each diagonal is 1·414* **Ω**. *Determine the resistance between two opposite corners of the square.*




(*i*) (*ii*)

**Fig. 3.247**

**Solution.** Fig. 3.247 (*i*) shows the given square. It is desired to find the resistance between terminals *A* and *C*. The star-connected resistances 1 Ω, 1 Ω and 1·414 Ω (with star point at *B*) are shown separately in Fig. 3.247 (*ii*). These star-connected resistances can be converted into equivalent delta connected resistances $R_1$, $R_2$ and $R_3$ as shown in Fig. 3.247 (*ii*) where

$$R_1 = R_{AB} + R_{BC} + \frac{R_{AB} \cdot R_{BC}}{R_{BD}}$$

$$= 1 + 1 + \frac{1 \times 1}{1.414} = 2.7\ \Omega$$

$$R_2 = 1 + 1.414 + \frac{1 \times 1.414}{1} = 3{\cdot}83\ \Omega$$

$$R_3 = 1 + 1.414 + \frac{1 \times 1.414}{1} = 3{\cdot}83\ \Omega$$

The circuit shown in Fig. 3.247 (*i*) then reduces to the circuit shown in Fig. 3.248 (*i*). Note that $R_1$ comes in parallel with 1·414 Ω connected between *A* and *C*; $R_2$ comes in parallel with 1 Ω connected between *C* and *D* and $R_3$ comes in parallel with 1 Ω connected between *A* and *D*.

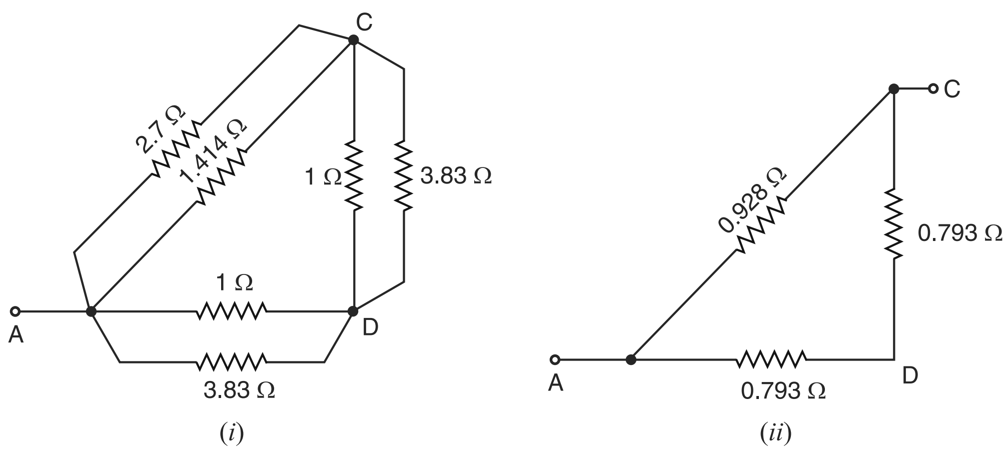



**Fig. 3.248**

In Fig. 3.248 (*i*), branch *AD* has 1 Ω and 3·83 Ω resistances in parallel.

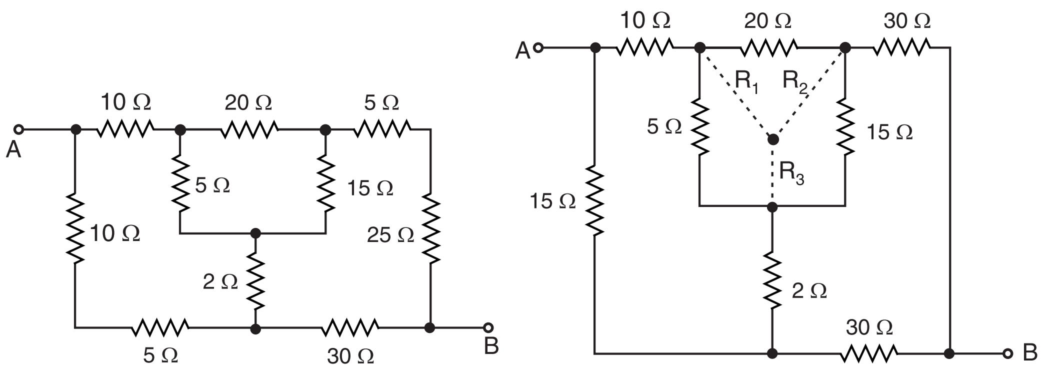
$$\therefore \quad R_{AD} = \frac{1 \times 3.83}{1 + 3.83} = 0.793\ \Omega\ ;\ R_{CD} = \frac{1 \times 3.83}{1 + 3.83} = 0.793\ \Omega\ ;$$

$$R_{AC} = \frac{2.7 \times 1.414}{2.7 + 1.414} = 0.928\ \Omega$$

∴ Resistance between terminals *A* and *C* [See Fig. 3.248 (*ii*)]

$$= 0{\cdot}928 \parallel (0{\cdot}793 + 0{\cdot}793) = 0{\cdot}928 \times 1{\cdot}586/2{\cdot}514 = \mathbf{0{\cdot}585\ \Omega}$$

**Example 3.100.** *Determine the resistance between the terminals A and B of the network shown in Fig. 3.249 (i).*




(*i*) (*ii*)

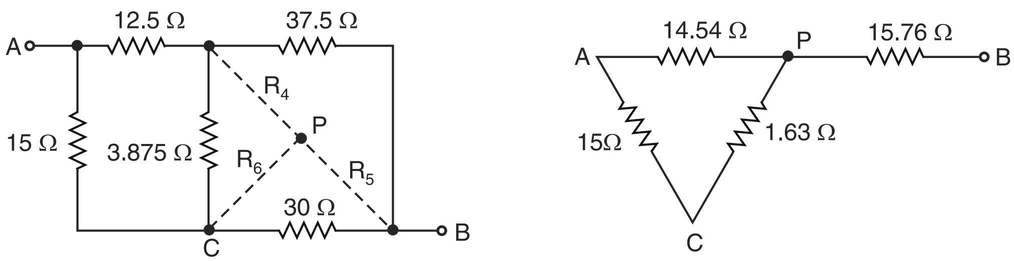
**Fig. 3.249**

**Solution.** We can combine series resistances on the right and left of Fig. 3.249 (*i*). The circuit then reduces to the one shown in Fig. 3.249 (*ii*). The resistances 5 Ω, 20 Ω and 15 Ω form a delta circuit and can be replaced by a star network where

$$R_1 = \frac{\text{Product of two adjacent arms of delta}}{\text{Sum of arms of delta}} = \frac{20 \times 5}{5 + 20 + 15} = \frac{100}{40} = 2.5\ \Omega\ ;$$

$$R_2 = \frac{20 \times 15}{40} = 7.5\ \Omega \quad ; \qquad R_3 = \frac{5 \times 15}{40} = 1.875\ \Omega$$

Referring to Fig. 3.249 (*ii*), $R_1$ is in series with 10 Ω resistor and their total resistance is $10 + R_1 = 10 + 2{\cdot}5 = 12{\cdot}5\ \Omega$. Similarly, we have $30 + R_2 = 30 + 7{\cdot}5 = 37{\cdot}5\ \Omega$ and $2 + R_3 = 2 + 1{\cdot}875 = 3{\cdot}875\ \Omega$. The circuit then reduces to the one shown in Fig. 3.249 (*iii*).




(*iii*) (*iv*)

**Fig. 3.249**

Referring to Fig. 3.249 (*iii*), 3·875 Ω, 37·5 Ω and 30 Ω form a delta network and can be reduced to star network where

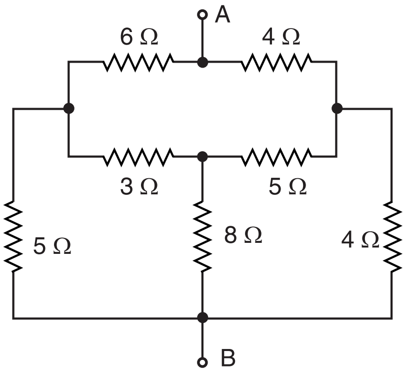
$$R_4 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = \frac{3.875 \times 37.5}{71.375} = 2.04\ \Omega\ ;$$

$$R_5 = \frac{37.5 \times 30}{71.375} = 15.76\ \Omega\ ;\ R_6 = \frac{3.875 \times 30}{71.375} = 1.63\ \Omega$$

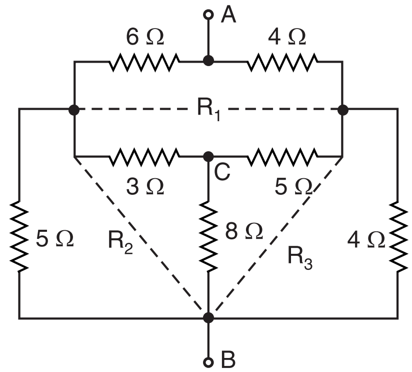
Referring to Fig. 3.249 (*iii*), $R_4$ is in series with 12·5 Ω resistor and their combined resistance = $R_4 + 12{\cdot}5 = 2{\cdot}04 + 12{\cdot}5 = 14{\cdot}54\ \Omega$. The circuit then reduces to the one shown in Fig. 3.249 (*iv*). The resistance between terminals *A* and *B* is given by ;

$$R_{AB} = 15.76 + [14.54 \,||\, (15 + 1.63)] = 15.76 + \frac{14.54 \times 16.63}{31.17} = \mathbf{23{\cdot}5\ \Omega}$$

**Example 3.101.** *Determine the resistance between points A and B in the network shown in Fig. 3.250 (i).*




(*i*)




(*ii*)

**Fig. 3.250**

**Solution.** The 3 Ω, 5 Ω and 8 Ω form star network and can be replaced by delta network where

Resistance between two terminals of delta = Sum of star resistances connected to those terminals *plus* product of same two resistances divided by the third star resistance.

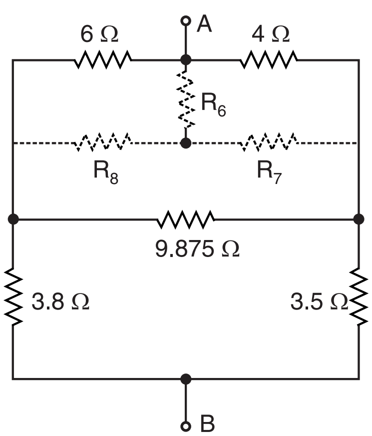
$$\therefore \quad R_1 = 3+5+\frac{3\times 5}{8} = 9\cdot875\ \Omega$$

$$R_2 = 3+8+\frac{3\times 8}{5} = 15\cdot8\ \Omega$$

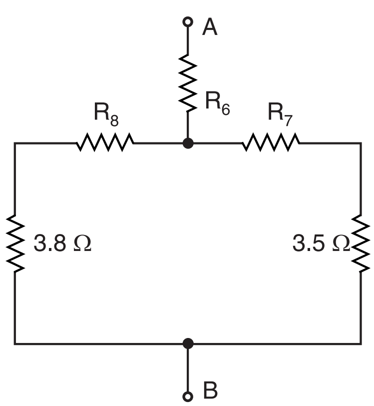
$$R_3 = 5+8+\frac{5\times 8}{3} = 26\cdot3\ \Omega$$

Referring to Fig. 3.250 (*ii*), 5 Ω resistor is in parallel with $R_2$ ( = 15·8 Ω) and their combined resistance is 3·8 Ω. Similarly, 4 Ω resistor is in parallel with $R_3$ (= 26·3 Ω) and their combined resistance is 3·5 Ω. The circuit then reduces to the one shown in Fig. 3.250 (*iii*). Referring to Fig. 3.250 (*iii*), 6 Ω, 4 Ω and 9·875 Ω form a delta network and can be replaced by star network where

$$R_6 = \frac{6\times 4}{6+4+9.875} = \frac{24}{19.875} = 1.2\ \Omega \ ; \ R_7 = \frac{9.875\times 4}{19.875} = 1.99\ \Omega \ ; \ R_8 = \frac{9.875\times 6}{19.875} = 2.98\ \Omega$$




(*iii*)




(*iv*)

**Fig. 3.250**

Therefore, the circuit shown in Fig. 3.250 (*iii*) reduces to the one shown in Fig. 3.250 (*iv*). It is clear that :

$$R_{AB} = (3{\cdot}8 + R_8) \parallel (R_7 + 3{\cdot}5) + R_6 = (3{\cdot}8 + 2{\cdot}98) \parallel (1{\cdot}99 + 3{\cdot}5) + 1{\cdot}2$$
$$= (6{\cdot}78 \parallel 5{\cdot}49) + 1{\cdot}2 = \mathbf{4{\cdot}23\ \Omega}$$

**Example 3.102.** *A π network is to be constructed as shown in Fig. 3.251 (i) so that the resistance $R_{XZ}$ looking into the X – Z terminals (with Y – Z open) equals the resistance $R_{YZ}$ looking into the Y – Z terminals (with X – Z open). If that resistance must equal 1 kΩ, find the value of $R_\Delta$ that should be used in the π network.*

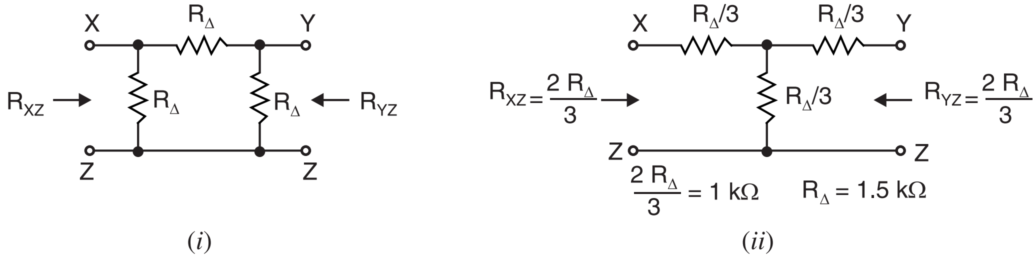



**Fig. 3.251**

**Solution.** The delta network shown in Fig. 3.251 (*i*) can be converted into star network as shown in Fig. 3.251 (*ii*). Note that the star network has equal-valued resistors $R_\Delta/3$. It is clear from this figure that :

$$R_{XZ} = R_{YZ} = \frac{R_\Delta}{3} + \frac{R_\Delta}{3} = \frac{2R_\Delta}{3}$$

or $$1\ \text{k}\Omega = \frac{2R_\Delta}{3} \quad \text{or} \quad R_\Delta = \mathbf{1.5\ k\Omega}$$

Therefore, the π network must have three 1.5 kΩ resistors as shown in Fig 3.251 (*iii*).

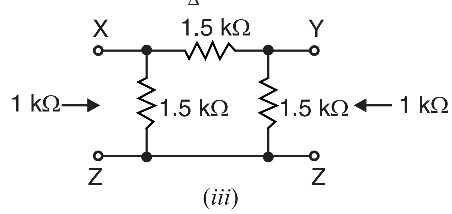



**Fig. 3.251**

**Example 3.103.** *Find the current distribution in the network shown in Fig. 3.252 (i).*

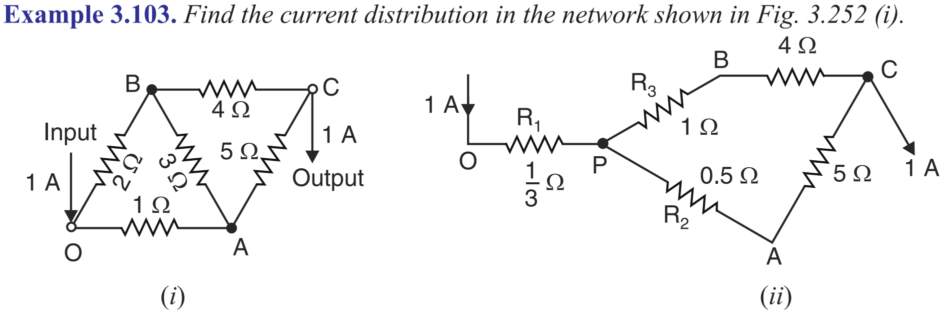



**Fig. 3.252**

**Solution.** The network *OAB* forms a delta and can be replaced by star where :

$$R_1 = \frac{1 \times 2}{6} = \frac{1}{3}\Omega \quad ; \quad R_2 = \frac{1 \times 3}{6} = 0.5\ \Omega \quad ; \quad R_3 = \frac{2 \times 3}{6} = 1\ \Omega$$

The network then reduces to the one shown in Fig. 3.252 (*ii*). The current through *OP* is 1 A and divides between two parallel paths at point *P*. By current-divider rule :

$$\text{Current in } PA = \text{Current in } AC = 1 \times \frac{5}{1 + 4 + 0.5 + 5} = 1 \times \frac{5}{10.5} = \mathbf{0.477\ A}$$

$$\text{Current in } PB = \text{Current in } BC = 1 - 0.477 = \mathbf{0.523A}$$

$$\text{Voltage drop in } PB = 1 \times 0{\cdot}523 = 0.523 \text{ V}$$

$$\text{Voltage drop in } PA = 0{\cdot}5 \times 0{\cdot}477 = 0.238 \text{ V}$$

$$\therefore \quad V_{AB} = 0{\cdot}523 - 0{\cdot}238 = 0.285 \text{ V}$$

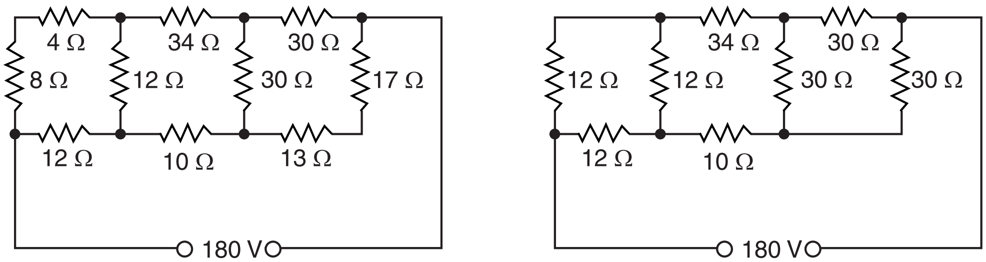
$$\therefore \quad I_{AB} = 0{\cdot}285/3 = \mathbf{0.095 \text{ A}}$$

$$\text{Current in } OB = \text{Current in } BC - \text{Current in } AB$$

$$= 0.523 - 0.095 = \mathbf{0.428 \text{ A}}$$

$$\text{Current in } OA = 1 - 0.428 = \mathbf{0.572 \text{ A}}$$

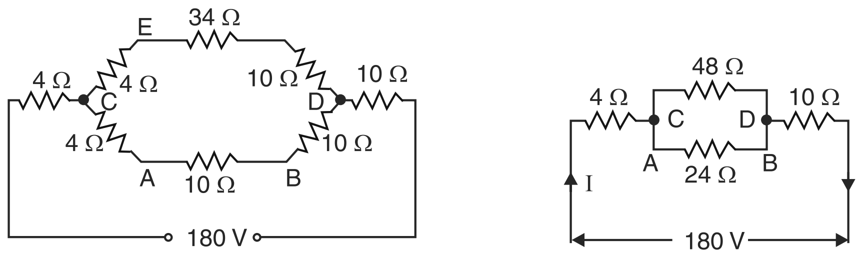
**Example 3.104.** *Find the current in 10 Ω resistor in the network shown in Fig. 3.253 (i) by star-delta transformation.*




(*i*) (*ii*)

**Fig. 3.253**

**Solution.** In Fig. 3.253 (*i*), the 4 Ω and 8 Ω resistors are in series and their total resistance is 8 + 4 = 12 Ω. Similarly, at the right end of figure, 17 Ω and 13 Ω are in series so that their total resistance becomes 17 + 13 = 30 Ω. The circuit then reduces to the one shown in Fig. 3.253 (*ii*). Replacing the two deltas at the left end and right end in Fig. 3.253 (*ii*) by their equivalent star, we get the circuit shown in Fig. 3.253 (*iii*).




(*iii*) (*iv*)

**Fig. 3.253**

Referring to Fig. 3.253 (*iii*), the path *CED* has resistance = 4 + 34 + 10 = 48 Ω and path *CABD* has resistance = 4 + 10 + 10 = 24 Ω. The circuit then reduces to the one shown in Fig. 3.253 (*iv*). The total resistance $R_T$ presented to 180V source is

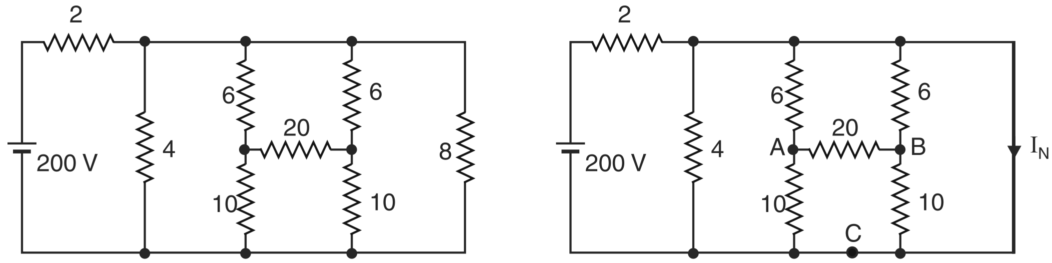
$$R_T = 4 + (48 \,||\, 24) + 10 = 30 \; \Omega$$

$$\therefore \quad \text{Circuit current, } I = 180/30 = 6 \text{ A}$$

$\therefore$ Voltage across parallel combination = $I \times (48 \,||\, 24) = 6 \times 16 = 96$ V

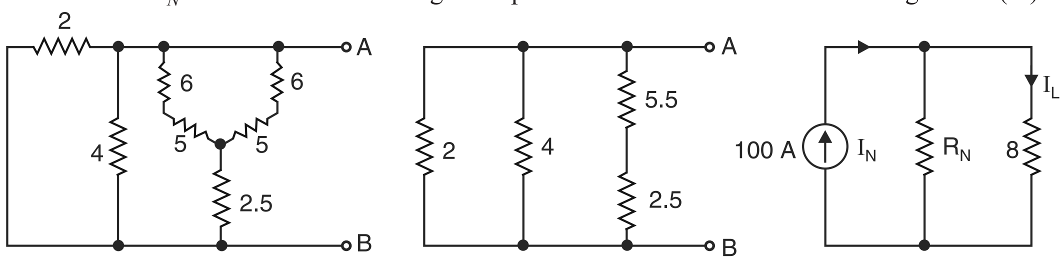
$\therefore$ Current in 10 Ω resistor [part of 24 Ω in Fig. 3.253 (*iv*)] = 96/24 = **4 A**

**Example 3.105.** *Using Norton's theorem, find the current through the 8 Ω resistor shown in Fig 3.254 (i). All resistance values are in ohms.*




(*i*) **Fig. 3.254** (*ii*)

**Solution.** In order to find Norton current $I_N$, place short circuit across the load of 8 Ω resistor. The circuit then becomes as shown in Fig. 3.254 (*ii*). The short circuit bypasses all the resistors except 2 Ω resistor. Therefore, $I_{SC} = I_N = 200/2 = 100$ A. In order to find $R_N$, replace 200 V source by a short. Then $R_N$ is the resistance looking into open-circuited terminals *A* and *B* in Fig. 3.254 (*iii*).




(*iii*) (*iv*) (*v*)

**Fig. 3.254**

In Fig. 3.254 (*ii*), *ABC* network forms a delta and can be replaced by equivalent star network as shown in Fig. 3.254 (*iii*). This circuit reduces to the one shown in Fig. 3.254 (*iv*).

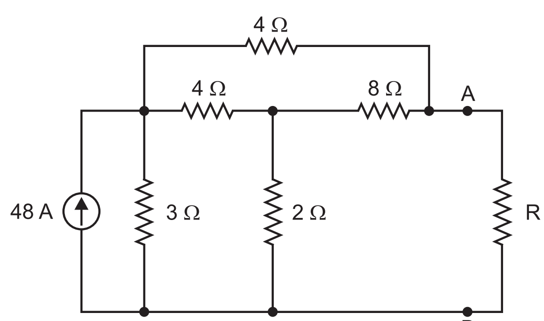
Norton's resistance, $R_N$ = Resistance at the open-circuited terminals in Fig. 3.254 (*iv*)

$$= 2 \parallel 4 \parallel (5{\cdot}5 + 2{\cdot}5) = 8/7\ \Omega$$

Therefore, Norton equivalent circuit consists of 100A current source in parallel with a resistance of 8/7 Ω. When load $R_L$ ( = 8 Ω) is connected at the output terminals of Norton's equivalent circuit, the circuit becomes as shown in Fig 3.254 (*v*). By current-divider rule, the load current $I_L$ through $R_L$ ( = 8 Ω) is given by ;

$$I_L = 100 \times \frac{8/7}{8 + (8/7)} = \mathbf{12{\cdot}5\ A}$$

**Example 3.106.** *In the network shown in Fig. 3.255 (i), find (i) Norton equivalent circuit at terminals AB (ii) the maximum power that can be provided to a resistor R connected between terminals A and B.*




(*i*)

**Fig. 3.255**

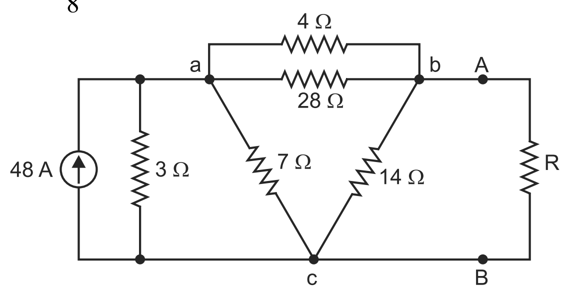
**Solution.** *(i)* The star- connected resistances 4 Ω, 8 Ω and 2 Ω in Fig. 3.255 (*i*) can be converted into equivalent delta-connected resistances $R_{ab}$, $R_{bc}$ and $R_{ca}$ as shown in Fig. 3.255 (*ii*).

$$R_{ab} = 4+8+\frac{4\times 8}{2} = 28\ \Omega$$

$$R_{bc} = 8+2+\frac{8\times 2}{4} = 14\ \Omega$$

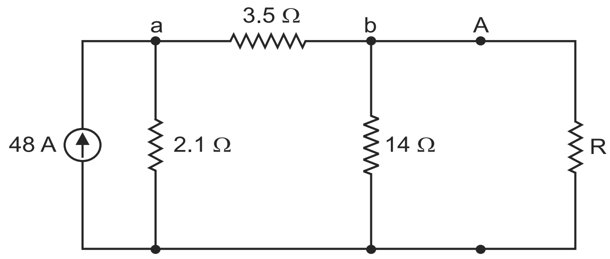
$$R_{ca} = 2+4+\frac{2\times 4}{8} = 7\ \Omega$$

After above star-delta conversion, the circuit reduces to the one shown in Fig. 3.255 (*ii*). We can further simplify the circuit in Fig. 3.255 (*ii*) by combining the parallel resistances (4 Ω || 28 Ω = 3.5 Ω and 3 Ω || 7 Ω = 2.1 Ω). The circuit then becomes as shown in Fig. 3.255 (*iii*). We now convert 48A current source in parallel with 2.1Ω resistance in Fig. 3.255 (*iii*) into equivalent voltage source of 48 A × 2.1 Ω = 100.8 V in series with 2.1Ω resistance. The circuit then becomes as shown in Fig. 3.255 (*iv*). In order to find Norton current $I_N$, we short circuit terminals *A* and *B* in Fig. 3.255 (*iv*) and get the circuit of Fig. 3.255 (*v*). Then current in the short-circuit is $I_N$. Referring to Fig. 3.255 (*v*) and applying Ohm's law, the value of $I_N$ is given by ;




(*ii*)

**Fig. 3.255**

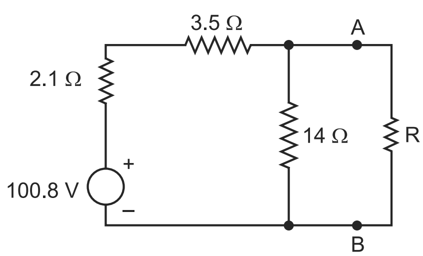



(*iii*)

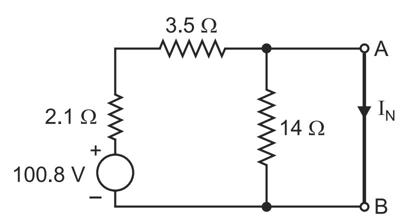
**Fig. 3.255**

$$I_N = \frac{100.8}{2.1+3.5} = 18\text{A}$$

Note that no current will pass through 14 Ω resistor in Fig. 3.255 (*v*). It is because there is a short across this resistor and the entire current ( = $I_N$) will pass through the short.




(*iv*)



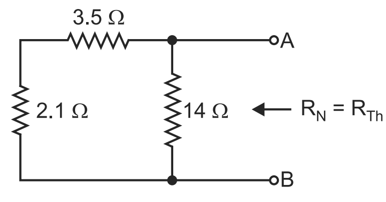

(*v*)

**Fig. 3.255**

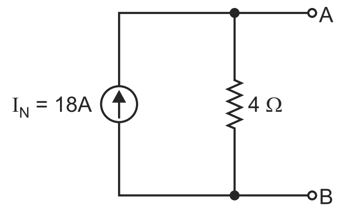
In order to find Norton resistance $R_N(= R_{Th})$, we open circuit the terminals *AB* and replace the voltage source by a short in Fig. 3.255 (*iv*). The circuit then becomes as shown in Fig. 3.255 (*vi*).

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.255 } (vi)$$

$$= (3.5 + 2.1)\ \Omega \parallel 14\ \Omega = 5.6\ \Omega \parallel 14\ \Omega = 4\ \Omega$$




(*vi*)




(*vii*)

**Fig. 3.255**

The Norton equivalent circuit at terminals *AB* is shown in Fig. 3.255 (*vii*).

**(*ii*)** Maximum power will be provided to resistance *R* connected between terminals *A* and *B* when resistance *R* is equal to Norton resistance $R_N$ *i.e.*

$$R = R_N = 4\ \Omega$$

When $R(= 4\ \Omega)$ is connected across terminals *A* and *B* in Fig. 3.255 (*vii*), then by current-divider rule,

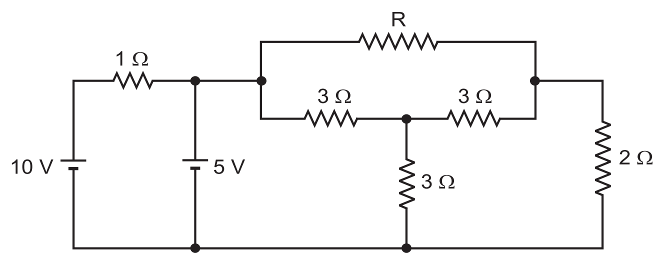
$$\text{Current in } R\ (= 4\ \Omega),\quad I = 18 \times \frac{4}{4+4} = 9\text{A}$$

∴ Maximum power ($P_{max}$) provided to *R* is

$$P_{max} = I^2R = (9)^2 \times 4 = \mathbf{324\ W}$$

Remember that under the condition of maximum power transfer, the circuit efficiency is *only* 50% and the remaining 50% is dissipated in the circuit.

**Example 3.107.** *Determine a non-negative value of R such that the power consumed by the 2 Ω resistor in Fig. 3.256 (i) is maximum.*



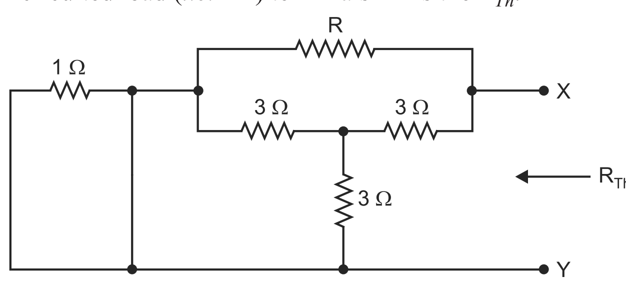

(*i*)

**Fig. 3.256**

**Solution.** In order to find maximum power consumed in 2 Ω resistor (*i.e.* load), we should find Thevenin resistance $R_{Th}$ at 2 Ω terminals. For this purpose, we open circuit the load terminals (*i.e.* remove 2 Ω resistor) and short circuit the voltage sources as shown in Fig. 3.256 (*ii*). The resistance at the open-circuited load (*i.e.* 2Ω) terminals *XY* is the $R_{Th}$.

$R_{Th}$ = Resistance at terminals *XY* in Fig. 3.256 (*ii*).

In order to facilitate the determination of $R_{Th}$, we convert delta-connected resistances *R* Ω, 3 Ω and 3 Ω in Fig. 3.256 (*ii*) into equivalent star-connected resistances $R_1$, $R_2$ and $R_3$ as shown in Fig. 3.256 (*iii*). The values of $R_1$, $R_2$ and $R_3$ are given by ;



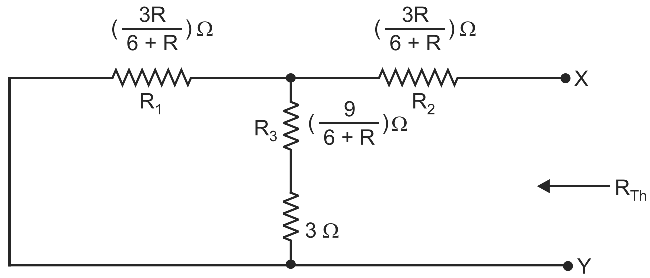

(*ii*)

**Fig. 3.256**

$$R_1 = \frac{3 \times R}{3+3+R} = \frac{3R}{6+R}$$

$$R_2 = \frac{3 \times R}{3+3+R} = \frac{3R}{6+R}$$

$$R_3 = \frac{3 \times 3}{3+3+R} = \frac{9}{6+R}$$




(*iii*)

**Fig. 3.256**

After above delta-star conversion, the circuit becomes as shown in Fig. 3.256 (*iii*). Then resistance at open-circuited terminals *XY* is $R_{Th}$.

Referring to Fig. 3.256 (*iii*),

$$R_{Th} = \left[\left(\frac{3R}{6+R}\right) \| \left(\frac{9}{6+R}+3\right)\right] + \frac{3R}{6+R}$$

$$= \left[\frac{3R}{6+R} \| \frac{27+3R}{6+R}\right] + \frac{3R}{6+R}$$

$$= \frac{3R \times (27+3R)}{(6+R)\,(27+3R+3R)} + \frac{3R}{6+R}$$

For maximum power in 2 Ω, the value of $R_{Th}$ should be equal to 2 Ω.

$$\therefore \quad \frac{3R \times (27+3R)}{(6+R)(27+3R+3R)} + \frac{3R}{6+R} = 2$$

$$\text{or} \quad \frac{3R \times (27+3R)}{27+6R} + 3R = 2(6+R)$$

$$\text{or} \quad 5R^2 + 12R - 108 = 0 \qquad \text{...after simplification}$$

$$\therefore \quad R = +3.6\,\Omega \quad \text{or} \quad -6\,\Omega$$

Accepting the positive value, $R$ = **3.6 Ω**.

## Tutorial Problems

1. Find the total current drawn from the voltage source and the current through $R_1$ ( = 1 Ω) in the circuit shown in Fig. 3.257. **[4 A ; 2 A]**
2. Convert the delta network shown in Fig. 3.258 into equivalent wye network.

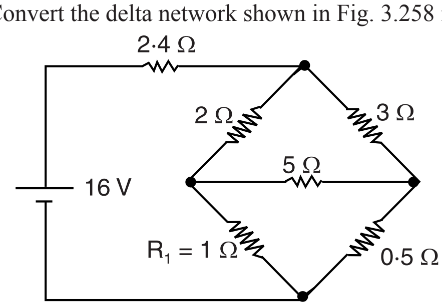



**Fig. 3.257**

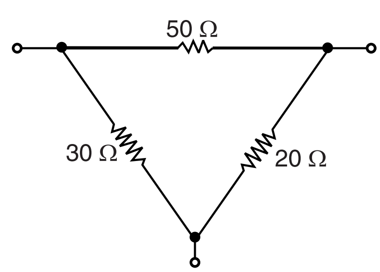



**Fig. 3.258**

**3.** Convert the wye network shown in Fig. 3.259 into equivalent delta network.

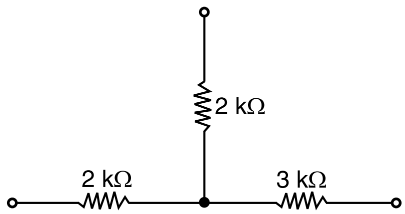



**Fig. 3.259**

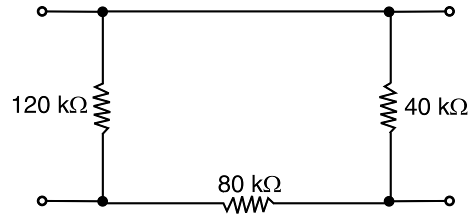



**Fig. 3.260**

**4.** Convert the delta network shown in Fig. 3.260 into the equivalent wye network.

**5.** In the network shown in Fig. 3.261, find the resistance between terminals *B* and *C* using star/delta transformation. **[17/12 Ω]**

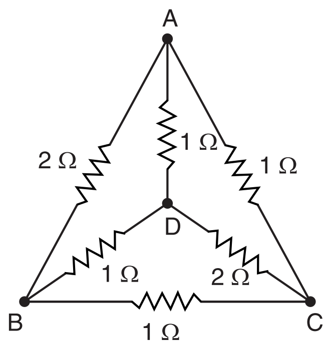



**Fig. 3.261**

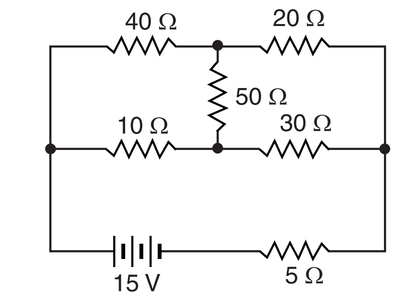



**Fig. 3.262**

**6.** In the network shown in Fig. 3.262, find the current supplied by the battery using star/delta transformation. **[0·452 A]**

**7.** What is the resistance between terminals *A* and *B* of the network shown in Fig. 3.263 ? **[274·2 Ω]**

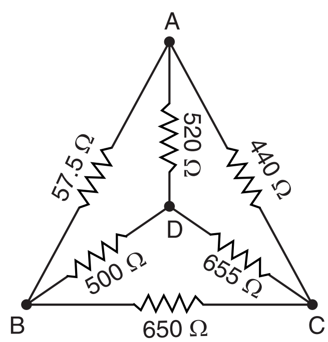



**Fig. 3.263**

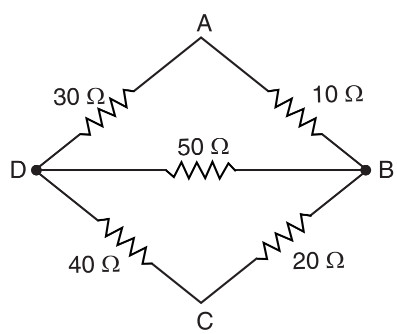



**Fig. 3.264**

**8.** Using delta/star transformation, find the resistance between terminals *A* and *C* of the network shown in Fig. 3.264.

**9.** Using star/delta transformation, determine the value of *R* for the network shown in Fig. 3.265 such that 4Ω resistor consumes the maximum power. **[*R* = 36Ω]**

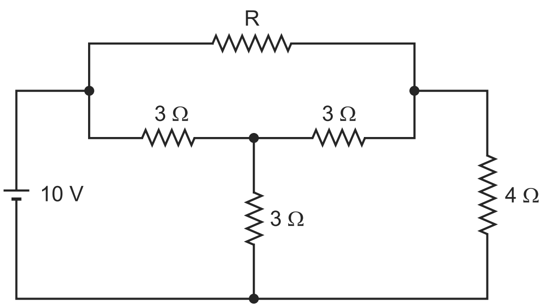



**Fig. 3.265**

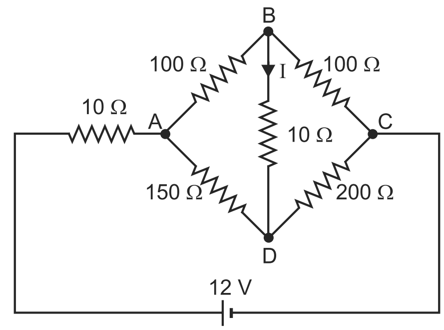
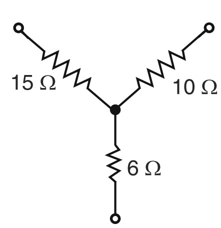


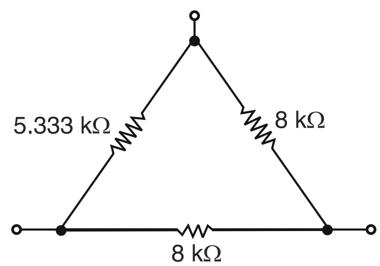

**Fig. 3.266**

**10.** Calculate the current $I$ flowing through the 10 Ω resistor in the circuit shown in Fig. 3.266. Apply Thevenin's theorem and star/delta transformation. **[5.45 mA from *D* to *B*]**

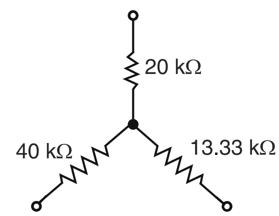
**ANSWERS TO PROBLEMS 2 TO 4**




**Prob. 2**




**Prob. 3**




**Prob. 4**

## 3.26. Tellegen's Theorem

This theorem has wide applications in electric networks and may be stated as under :

*For a network consisting of n elements if $i_1$, $i_2$, $i_3$ .... $i_n$ are the instantaneous currents flowing through the elements satisfying KCL and $v_1$, $v_2$, $v_3$ ... $v_n$ are the instantaneous voltages across these elements satisfying KVL, then,*

$$v_1 i_1 + v_2 i_2 + v_3 i_3 + \ldots.. + v_n i_n = 0$$

or

$$\sum_{n=1}^{n} v_n i_n = 0$$

Now $vi$ is the instantaneous power. Therefore, Tellegen's theorem can also be stated as under :

*The sum of instantaneous powers for n branches in a network is always * zero.*

This theorem is valid for any lumped network that contains elements linear or non-linear, passive or active, time variant or time invariant.

**Explanation.** Let us explain Tellegen's theorem with a simple circuit shown in Fig. 3.267. The total resistance offered to the battery = 8 Ω + (4 Ω || 4 Ω) = 10 Ω. Therefore, current supplied by battery is $I$ = 100/10 = 10A. This current divides equally at point $A$.

Voltage drop across 8 Ω = – (10 × 8) = – 80 V

Voltage drop across 4 Ω = – (5 × 4) = – 20 V

Voltage drop across 1 Ω = – (5 × 1) = – 5V

Voltage drop across 3 Ω = – (5 × 3) = – 15 V

* This is in accordance with the law of conservation of energy because power delivered by the battery is consumed in the circuit elements.

According to Tellegen's theorem,

Sum of instantaneous powers = 0

or $v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 = 0$

or $(100 \times 10) + (-80 \times 10) + (-20 \times 5) + (-5 \times 5) + (-15 \times 5) = 0$

or $1000 - 800 - 100 - 25 - 75 = 0$

or $0 = 0$ which is true

Thus Tellegen's theorem stands proved.

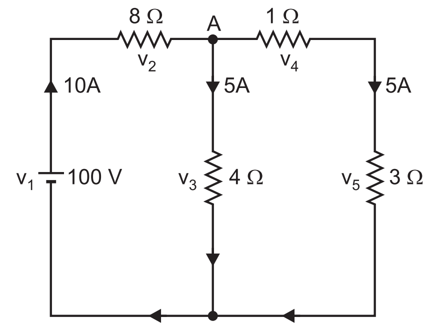



Fig. 3.267

## Objective Questions

**1.** An active element in a circuit is one which ..........

(*i*) receives energy

(*ii*) supplies energy

(*iii*) both receives and supplies energy

(*iv*) none of the above

**2.** A passive element in a circuit is one which ..........

(*i*) supplies energy

(*ii*) receives energy

(*iii*) both supplies and receives energy

(*iv*) none of the above

**3.** An electric circuit contains ..........

(*i*) active elements only

(*ii*) passive elements only

(*iii*) both active and passive elements

(*iv*) none of the above

**4.** A linear circuit is one whose parameters (*e.g.* resistances etc.) ..........

(*i*) change with change in current

(*ii*) change with change in voltage

(*iii*) do not change with voltage and current

(*iv*) none of the above

**5.** In the circuit shown in Fig. 3.268, the number of nodes is ..........

(*i*) one (*ii*) two

(*iii*) three (*iv*) four

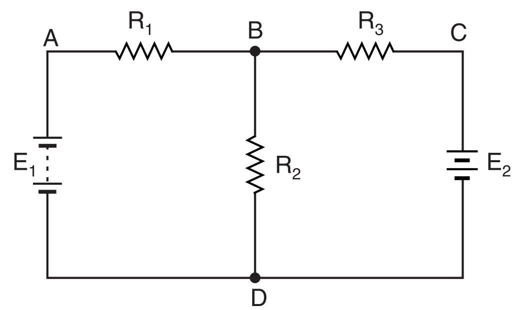



Fig. 3.268

**6.** In the circuit shown in Fig. 3.268, there are .......... junctions.

(*i*) three (*ii*) four

(*iii*) two (*iv*) none of the above

**7.** The circuit shown in Fig. 3.268 has .......... branches.

(*i*) two (*ii*) four

(*iii*) three (*iv*) none of these

**8.** The circuit shown in Fig. 3.268 has .......... loops.

(*i*) two (*ii*) four

(*iii*) three (*iv*) none of the above

**9.** In the circuit shown in Fig. 3.268, there are .......... meshes.

(*i*) two (*ii*) three

(*iii*) four (*iv*) five

**10.** To solve the circuit shown in Fig. 3.268 by Kirchhoff's laws, we require ..........

(*i*) one equation (*ii*) two equations

(*iii*) three equations (*iv*) none of the above

**11.** To solve the circuit shown in Fig. 3.268 by nodal analysis, we require ..........

(*i*) one equation (*ii*) two equations

(*iii*) three equations (*iv*) none of the above

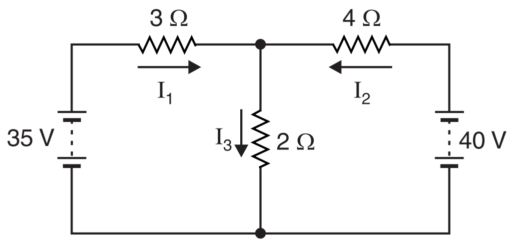



**Fig. 3.269**

**12.** To solve the circuit shown in Fig. 3.269 by superposition theorem, we require ..........

(*i*) one circuit (*ii*) two circuits
(*iii*) three circuits (*iv*) none of the above

**13.** To solve the circuit shown in Fig. 3.269 by Maxwell's mesh current method, we require ..........

(*i*) one equation (*ii*) three equations
(*iii*) two equations (*iv*) none of the above

**14.** In the circuit shown in Fig. 3.270, the voltage at node *B* *w.r.t.* *D* is calculated to be 15V. The current in 3 Ω resistor will be ..........

(*i*) 2 A (*ii*) 5 A
(*iii*) 2·5 A (*iv*) none of the above

**15.** The current in 2 Ω horizontal resistor in Fig. 3.270 is ..........

(*i*) 10 A (*ii*) 5 A
(*iii*) 2 A (*iv*) 2·5 A

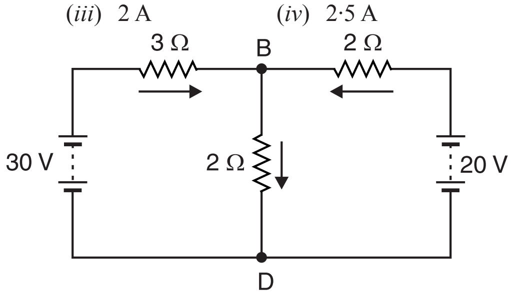



**Fig. 3.270**

**16.** In order to solve the circuit shown in Fig. 3.270 by nodal analysis, we require ..........

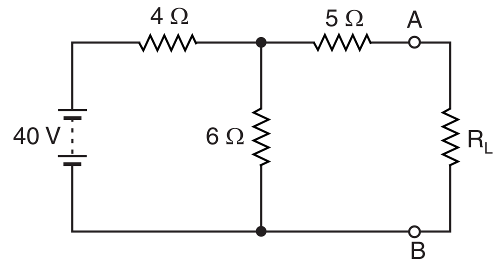
(*i*) one equation (*ii*) two equations
(*iii*) three equations (*iv*) none of the above

**17.** The superposition theorem is used when the circuit contains ..........

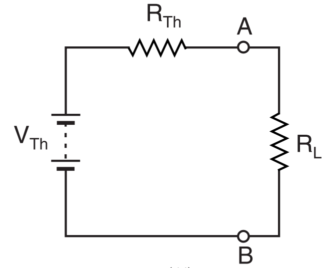
(*i*) a single voltage source
(*ii*) a number of voltage sources
(*iii*) passive elements only
(*iv*) none of the above

**18.** Fig. 3.271 (*ii*) shows Thevenin's equivalent circuit of Fig. 3.271 (*i*). The value of Thevenin's voltage $V_{Th}$ is ..........

(*i*) 20 V (*ii*) 24 V
(*iii*) 12 V (*iv*) 36 V




(*i*)




(*ii*)

**Fig. 3.271**

**19.** The value of $R_{Th}$ in Fig. 3.271 (*ii*) is ..........

(*i*) 15 Ω (*ii*) 3·5 Ω
(*iii*) 6·4 Ω (*iv*) 7·4 Ω

**20.** The open-circuited voltage at terminals *AB* in Fig. 3.271 (*i*) is ..........

(*i*) 12 V (*ii*) 20 V
(*iii*) 24 V (*iv*) 40 V

**21.** Find the value of $R_L$ in Fig. 3.272 to obtain maximum power in $R_L$.

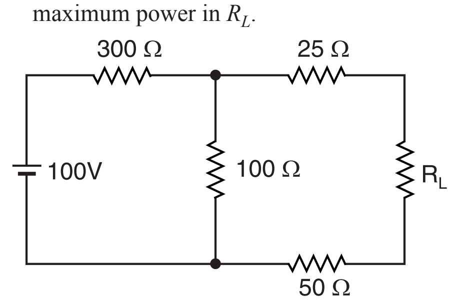



**Fig. 3.272**

(*i*) 100 Ω (*ii*) 75 Ω
(*iii*) 250 Ω (*iv*) 150 Ω

**22.** In Fig. 3.272, find the maximum power in $R_L$.

(*i*) 2 W (*ii*) 1·042 W
(*iii*) 2·34 W (*iv*) 4·52 W

**23.** What percent of the maximum power is delivered to $R_L$ in Fig. 3.273 when $R_L = 2R_{Th}$?

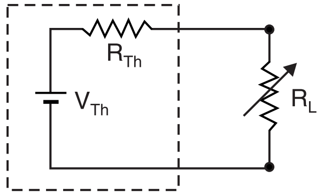



**Fig. 3.273**

(*i*) 79 % of $P_L$ (max)
(*ii*) 65 % of $P_L$ (max)
(*iii*) 88·89 % of $P_L$ (max)
(*iv*) none of above

**24.** What percent of the maximum power is delivered to $R_L$ in Fig. 3.273 when $R_L = R_{Th}/2$ ?

(*i*) 65 % (*ii*) 70 %
(*iii*) 88·89 % (*iv*) none of above

**25.** Find Millman's equivalent circuit *w.r.t.* terminals $x - y$ in Fig. 3.274.

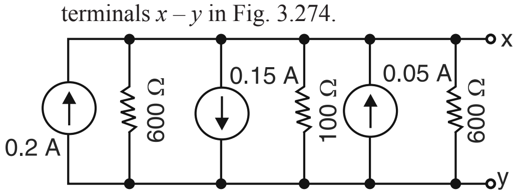



**Fig. 3.274**

(*i*) Single current source of 0·1A and resistance 75 Ω
(*ii*) Single current source of 2 A and resistance 50 Ω
(*iii*) Single current source of 1 A and resistance 25 Ω
(*iv*) none of above

**26.** Use superposition principle to find current through $R_1$ in Fig. 3.275.

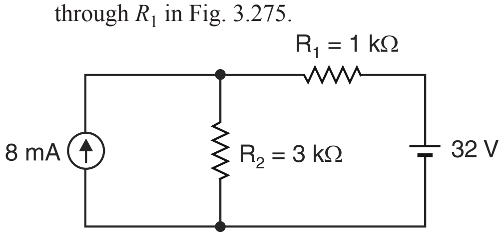



**Fig. 3.275**

(*i*) 1 mA ← (*ii*) 2 mA ←
(*iii*) 1·5 mA → (*iv*) 2·5 A ←

**27.** Use superposition principle to find current through $R_1$ in the circuit shown in Fig. 3.276.

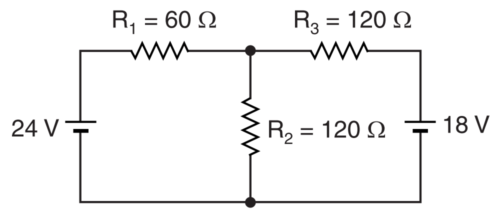



**Fig. 3.276**

(*i*) 0·2 A ← (*ii*) 0·25 A →
(*iii*) 0·125 A → (*iv*) 0·5 A →

**28.** Find Thevenin equivalent circuit to the left of terminals $x - y$ in Fig. 3.277.

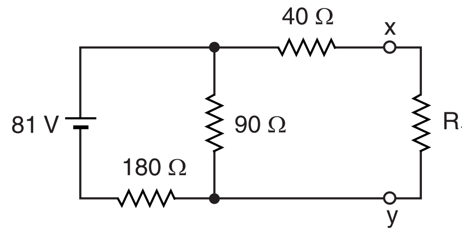



**Fig. 3.277**

(*i*) $V_{Th} = 5$ V ; $R_{Th} = 4{\cdot}5\ \Omega$
(*ii*) $V_{Th} = 6$ V ; $R_{Th} = 5\ \Omega$
(*iii*) $V_{Th} = 4{\cdot}5$ V ; $R_{Th} = 10\ \Omega$
(*iv*) $V_{Th} = 10$ V ; $R_{Th} = 9\ \Omega$

**29.** Convert delta network shown in Fig. 3.278 to equivalent Wye network.

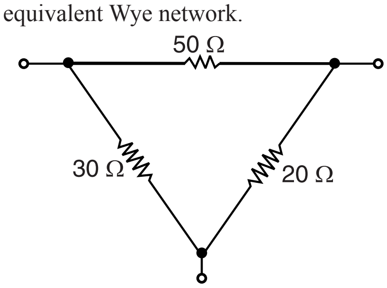
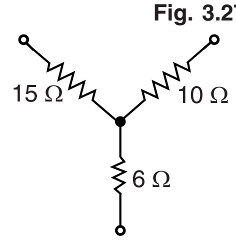
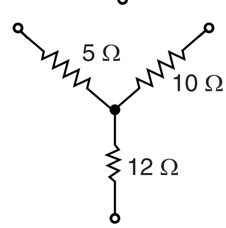



**Fig. 3.278**

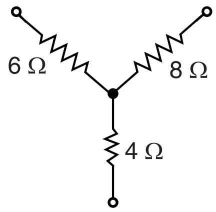
(*i*)



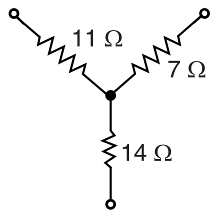

(*ii*)



(*iii*)




(*iv*)




**30.** What percentage of the maximum power is delivered to a load if load resistance is 10 times greater than the Thevenin resistance of the source to which it is connected ?

(*i*) 25 % (*ii*) 40 %
(*iii*) 35 % (*iv*) 33·06 %

## Answers

| | | | | |
|---|---|---|---|---|
| **1.** *(ii)* | **2.** *(ii)* | **3.** *(iii)* | **4.** *(iii)* | **5.** *(iv)* |
| **6.** *(iii)* | **7.** *(iii)* | **8.** *(iii)* | **9.** *(i)* | **10.** *(ii)* |
| **11.** *(i)* | **12.** *(ii)* | **13.** *(iii)* | **14.** *(ii)* | **15.** *(iv)* |
| **16.** *(i)* | **17.** *(ii)* | **18.** *(ii)* | **19.** *(iv)* | **20.** *(iii)* |
| **21.** *(iv)* | **22.** *(ii)* | **23.** *(iii)* | **24.** *(iii)* | **25.** *(i)* |
| **26.** *(ii)* | **27.** *(iii)* | **28.** *(iv)* | **29.** *(i)* | **30.** *(iv)* |