7

Magnetism and Electromagnetism

Introduction

In the ancient times people believed that the invisible force of magnetism was purely a magical quality and hence they showed little practical interest. However, with steadily increasing scientific knowledge over the passing centuries, magnetism assumed a larger and larger role. Today magnetism has attained a place of pride in electrical engineering. Without the aid of magnetism, it is impossible to operate such devices as electric generators, electric motors, transformers, electrical instruments etc. Without the use of magnetism, we should be deprived of such valuable assets as the radio, television, telephone, telegraph and the ignition systems of our cars, airplanes, trucks etc. In fact, electrical engineering is so much dependent on magnetism that without it a very few of our modern devices would be possible. The purpose of this chapter is to present the salient features of magnetism.

7.1. Poles of a Magnet

If we take a bar magnet and dip it into iron filings, it will be observed that the iron filings cluster about the ends of the bar magnet. The ends of the bar magnet are apparently points of maximum magnetic effect and for convenience we call them the **poles* of the magnet. A magnet has two poles *viz* north pole and south pole. In order to determine the polarity of a magnet, suspend or pivot it at the centre. The magnet will then come to rest in north-south direction. The end of the magnet pointing north is called *north pole* of the magnet while the end pointing south is called the *south pole.* The following points may be noted about the poles of a magnet :

(*i*) The poles of a magnet cannot be separated. If a bar magnet is broken into two parts, each part will be complete magnet with poles at its ends. No matter how many times a magnet is broken, each piece will contain *N*-pole at one end and *S*-pole at the other.

(*ii*) The two poles of a magnet are of equal strength. The pole strength is represented by *m*.

(*iii*) Like poles repel each other and unlike poles attract each other.

7.2. Laws of Magnetic Force

Charles Coulomb, a French scientist observed that when two **isolated poles are placed near each other, they experience a force. He performed a number of experiments to study the nature and magnitude of force between the magnetic poles. He summed up his conclusions into two laws, known as Coulomb's laws of magnetic force. These laws give us the magnitude and nature of magnetic force between two magnetic poles.

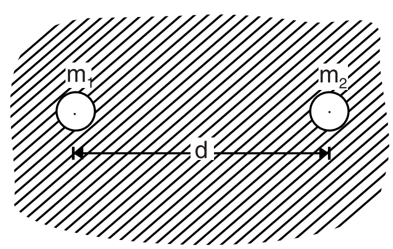

Fig. 7.1

* Magnetic poles have no physical reality, but the concept enables us to appreciate magnetic effects more easily.

** It is not possible to get an isolated pole because magnetic poles exist in pairs. However, if we take thin and long steel rods (about 50 cm long) with a small steel ball on either end and then magnetise them, *N* and *S* poles become concentrated in the steel balls. Such poles may be assumed point poles for all practical purposes.

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(*i*) *Like poles repel each other while unlike poles attract each other.*

(*ii*) *The force between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of distance between their centres.*

Consider two poles of magnetic strength m_1 and m_2 placed at a distance d apart in a medium as shown in Fig. 7.1. According to Coulomb's laws, the force between the two poles is given by ;

$$F \propto \frac{m_1 m_2}{d^2}$$

$$= K\frac{m_1 m_2}{d^2}$$

where K is a constant whose value depends upon the surrounding medium and the system of units employed. In SI units, force is measured in newtons, pole strength in *weber, distance in metres and the value of K is given by ;

$$K = \frac{1}{4\pi \mu_0 \mu_r}$$

where μ_0 = Absolute permeability of vacuum or air

μ_r = Relative permeability of the surrounding medium. For vacuum or air, its value is 1.

The value of $\mu_0 = 4\pi \times 10^{-7}$ H/m and the value of μ_r is different for different media.

$$\therefore \quad F = \frac{m_1 m_2}{4\pi \mu_0 \mu_r d^2} \text{ newtons} \qquad \textit{...in a medium}$$

$$= \frac{m_1 m_2}{4\pi \mu_0 d^2} \text{ newtons} \qquad \textit{...in air}$$

Unit of pole strength. By unit pole strength we mean 1 weber. It can be defined from Coulomb's laws of magnetic force. Suppose two equal point poles placed 1 m apart in *air* exert a force of 62800 newtons *i.e.*

$$m_1 = m_2 = m \; ; \quad d = 1 \text{ m} \; ; F = 62800 \text{ N}$$

$$\therefore \quad F = \frac{m_1 m_2}{4\pi\mu_0 d^2} \qquad (\because \text{ For air, } \mu_r = 1)$$

or
$$62800 = \frac{m^2}{4\pi \times 4\pi \times 10^{-7} \times (1)^2}$$

or
$$m^2 = (62800) \times (4\pi \times 4\pi \times 10^{-7} \times 1) = 1$$

$$\therefore \quad m = \pm 1 \text{ Wb}$$

Hence a **pole of unit strength** *(i.e. 1 Wb) is that pole which when placed in air 1 m from an identical pole, repels it with a force of 62800 newtons.*

In vector form :
$$\vec{F} = \frac{m_1 m_2}{4\pi\mu_0 \mu_r d^2}\hat{d}$$

where $\hat{d}$ is a unit vector to indicate the direction of d.

Example 7.1. *Two magnetic S poles are located 5 cm apart in air. If each pole has a strength of 5 mWb, find the force of repulsion between them.*

Solution.
$$F = \frac{m_1 m_2}{4\pi\mu_0 d^2} \qquad (\because \text{ For air, } \mu_r = 1)$$

* The unit of magnetic flux is named after Wilhelm Weber (1804–1890), the founder of electrical system of measurements.

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Here $m_1 = m_2 = 5$ mWb $= 5 \times 10^{-3}$ Wb ; $d = 5$ cm $= 0{\cdot}05$ m

$$\therefore \qquad F = \frac{(5\times10^{-3})\times(5\times10^{-3})}{4\pi\times4\pi\times10^{-7}\times(0.05)^2} = \mathbf{633\ N}$$

7.3. Magnetic Field

Just as electric field exists near a charged object, similarly magnetic field exists around a magnet. If an isolated magnetic pole is brought near a magnet, it experiences a force according to Coulomb's laws. The region near the magnet where forces act on magnetic poles is called a magnetic field. The magnetic field is strongest near the pole and goes on decreasing in strength as we move away from the magnet.

The space (or field) in which a magnetic pole experiences a force is called a **magnetic field.**

The magnetic field around a magnet is represented by imaginary lines called *magnetic lines of force.* By convention, the direction of these lines of force at any point is the direction along which an *isolated unit *N*-pole (*i.e.* *N*-pole of 1 Wb) placed at that point would move or tends to move. Following this convention, it is clear that magnetic lines of force would emerge from *N*-pole of the magnet, pass through the surrounding medium and re-enter the *S*-pole. Inside the magnet, each line of force passes from *S*-pole to *N*-pole (See Fig. 7.2), thus forming a closed loop or magnetic circuit. Although magnetic lines of force have no real existence and are purely imaginary, yet they are a useful concept to describe the various magnetic effects.

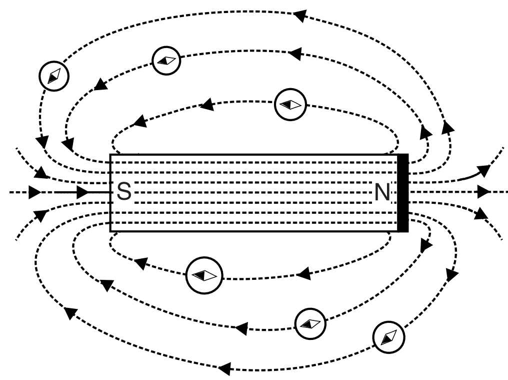

Fig. 7.2

Properties of magnetic lines of force. The important properties of magnetic lines of force are :

(*i*) *Each magnetic line of force forms a closed loop i.e.* outside the magnet, the direction of a magnetic line of force is from north pole to south pole and it continues through the body of the magnet to form a closed loop (See Fig. 7.2).

(*ii*) *No two magnetic lines of force intersect each other*. If two magnetic lines of force intersect, there would be two directions of magnetic field at that point which is not possible.

(*iii*) *Where the magnetic lines of force are close together, the magnetic field is strong and where they are well spaced out, the field is weak.*

(*iv*) *Magnetic lines of force contract longitudinally and widen laterally.*

(*v*) *Magnetic lines of force are always ready to pass through magnetic materials like iron in preference to pass through non-magnetic materials like air.*

It may be noted that in practice, magnetic fields are produced by (*a*) current carrying conductor or coil or (*b*) a permanent magnet. Both these means of producing magnetic fields are widely used in electrical engineering.

7.4. Magnetic Flux

The number of magnetic lines of force in a magnetic field determines the value of magnetic flux. The more the magnetic lines of force, the greater the magnetic flux and the stronger the magnetic field.

* Theoretically, it is not possible to get an isolated *N*-pole. However, a small compass needle well approximates to an isolated *N*-pole. The marked end (*N*-pole) of the compass needle indicates the direction of magnetic lines of force as shown in Fig. 7.2.

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The total number of magnetic lines of force produced by a magnetic source is called **magnetic flux**. It is denoted by Greek letter ϕ (phi).

A unit N-pole is supposed to radiate out a flux of one weber. Therefore, the magnetic flux coming out of N-pole of m weber is

$$\phi = m \text{ Wb}$$

Now $$1\text{Wb} = 10^8 \text{ lines of force}$$

Sometimes we have to use smaller unit of magnetic flux *viz* microweber (μWb).

$$1\ \mu\text{Wb} = 10^{-6}\text{ Wb} = 10^{-6} \times 10^8 \text{ lines} = 100 \text{ lines}$$

7.5. Magnetic Flux Density

The **magnetic flux density** *is defined as the magnetic flux passing normally per unit area i.e.*

$$\text{Magnetic flux density, } B = \frac{\phi}{A} \text{ Wb/m}^2$$

where ϕ = flux in Wb

A = area in m^2 normal to flux

The SI unit of magnetic flux density is Wb/m^2 or *tesla. Flux density is a measure of field concentration *i.e.* amount of flux in each square metre of the field. In practice, it is much more important than the total amount of flux. Magnetic flux density is a *vector quantity*.

(*i*) When the plane of the coil is perpendicular to the flux direction [See Fig. 7.3], maximum flux will pass through the coil *i.e.*

$$\text{Maximum flux, } \phi_m = B\,A \text{ Wb}$$

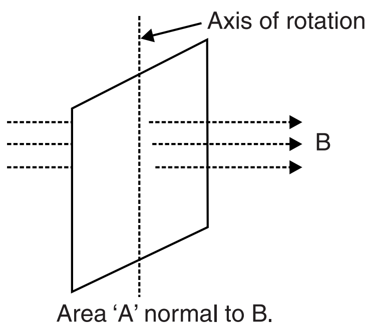

Fig. 7.3

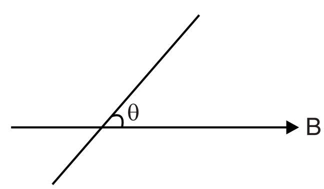

Fig. 7.4

(*ii*) When the plane of the coil is inclined at an angle θ to the flux direction [See Fig. 7.4], then flux ϕ through the coil is

$$\phi = B\,A \sin\theta \text{ Wb}$$

(*iii*) When the plane of the coil is parallel to the flux direction, $\theta = 0°$ so that no flux will pass through the coil ($\phi = BA \sin 0° = 0$).

Example 7.2. *A circular coil of 100 turns and diameter 3·18 cm is mounted on an axle through a diameter and placed in a uniform magnetic field, where the flux density is 0·01 Wb/m^2, in such a manner that axle is normal to the field direction. Calculate :*

(*i*) *the maximum flux through the coil and the coil position at which it occurs.*

(*ii*) *the minimum flux and the coil position at which it occurs.*

(*iii*) *the flux through the coil when its plane is inclined at 60° to the flux direction.*

* Named in honour of Nikola Tesla (1857–1943), an American electrician and inventor.

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Solution. Fig. 7.5 shows the conditions of the problem.

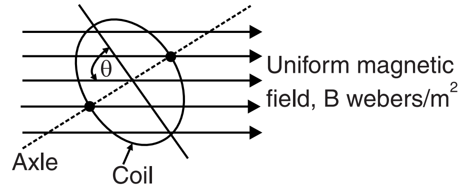

Fig. 7.5

(*i*) The maximum flux will pass through the coil when the plane of the coil is perpendicular to the flux direction.

$\therefore$ Maximum flux, $\phi_m = B \times \text{Total coil area}$

$$= (0{\cdot}01) \times \pi r^2$$

$$= 0{\cdot}01 \times \pi \times \left(\frac{3{\cdot}18}{2}\right)^2 \times 10^{-4} = \mathbf{0{\cdot}795 \times 10^{-5}\ Wb}$$

(*ii*) When the plane of the coil is parallel to the flux direction, no flux will pass through the coil. This is the minimum flux coil position and the minimum flux is **zero.**

(*iii*) When the plane of the coil is inclined at an angle θ to the flux direction, the flux ϕ through the coil is

$$\phi = B\,A \sin\theta = (B\,A)\sin\theta = (0{\cdot}795 \times 10^{-5}) \times \sin 60^\circ$$

$$= \mathbf{0{\cdot}69 \times 10^{-5}\ Wb}$$

Example 7.3. *The total flux emitted from the pole of a bar magnet is 2×10^{-4} Wb (See Fig. 7.6).*

(i) If the magnet has a cross-sectional area of 1 cm^2, determine the flux density within the magnet.

(ii) If the flux spreads out so that a certain distance from the pole, it is distributed over an area of 2 cm by 2 cm, find the flux density at that point.

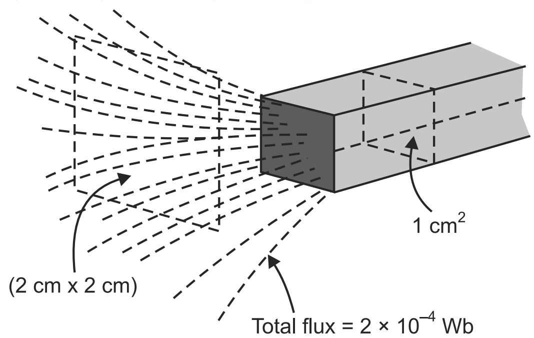

Fig. 7.6

Solution. (*i*) *Flux density within magnet.* $\phi = 2 \times 10^{-4}$ Wb ; $A = 1\ \text{cm}^2 = 1 \times 10^{-4}\ \text{m}^2$

$$\therefore \quad \text{Flux density, } B = \frac{\phi}{A} = \frac{2\times10^{-4}}{1\times10^{-4}} = \mathbf{2\ Wb/m^2}$$

(*ii*) *Flux density away from the pole.*

$$\phi = 2 \times 10^{-4}\ \text{Wb} \ ; \ A = 2 \times 2 = 4\ \text{cm}^2 = 4 \times 10^{-4}\ \text{m}^2$$

$$\therefore \quad \text{Flux density, } B = \frac{\phi}{A} = \frac{2\times10^{-4}}{4\times10^{-4}} = \mathbf{0.5\ Wb/m^2}$$

Example 7.4. *Flux density in the air gap between N and S poles is 2.5 Wb/m^2. The poles are circular with a diameter of 5.6 cm. Calculate the total flux crossing the air gap.*

Solution. $B = 2.5\ \text{Wb/m}^2$; Area of each pole, $A = \pi r^2 = \pi \times (5.6/2)^2 = 24.63\ \text{cm}^2 = 24.63 \times 10^{-4}\ \text{m}^2$

$\therefore$ Flux crossing the air gap is given by ;

$$\phi = B \times A = 2.5 \times 24.63 \times 10^{-4} = 6.16 \times 10^{-3}\ \text{Wb} = \mathbf{6.16\ mWb}$$

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7.6. Magnetic Intensity or Magnetising Force (H)

Magnetic intensity (or field strength) at a point in a magnetic field is the force acting on a unit *N*-pole (*i.e.*, *N*-pole of 1 Wb) placed at that point. Clearly, the unit of *H* will be N/Wb.

Suppose it is desired to find the magnetic intensity at a point *P* situated at a distance *d* metres from a pole of strength *m* webers (See Fig. 7.7). Imagine a unit north pole (*i.e.* *N*-pole of 1 Wb) is placed at *P*. Then, by definition, magnetic intensity at *P* is the force acting on the unit *N*-pole placed at *P i.e.*

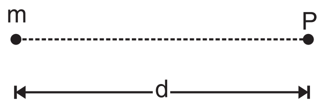

Fig. 7.7

Magnetic intensity at *P*, *H* = Force on unit *N*-pole placed at *P*

or $$H = \frac{m \times 1}{4\pi\mu_0 d^2} \text{ N/Wb} \qquad [\because \mu_r = 1 \text{ for air}]$$

or $$H = \frac{m}{4\pi\mu_0 d^2} \text{ N/Wb}$$

The reader may note the following points carefully :

(*i*) Magnetic intensity is a vector quantity, possessing both magnitude and direction. In vector form, it is given by ;

$$\vec{H} = \frac{m}{4\pi\mu_0 d^2}\hat{d}$$

(*ii*) If a pole of *m* Wb is placed in a uniform magnetic field of strength *H* newtons/Wb, then force acting on the pole, *F* = *m H* newtons.

7.7. Magnetic Potential

The **magnetic potential** *at any point in the magnetic field is measured by the work done in moving a unit N-pole (i.e. 1 Wb strength) from infinity to that point against the magnetic force.*

Consider a magnetic pole of strength *m* webers placed in a medium of relative permeability μ_r. At a point at a distance *x* metres from it, the force on unit *N*-pole is

$$F = \frac{m}{4\pi\mu_0\mu_r x^2}$$

If the unit *N*–pole is moved towards *m* through a small distance *dx*, then work done is

$$dW = \frac{m}{4\pi\mu_0\mu_r x^2} \times (-dx)$$

The negative sign is taken because *dx* is considered in the negative direction of *x*.

Therefore, the total work done (*W*) in bringing a unit *N*-pole from infinity to any point which is *d* metres from *m* is

$$W = \int_{x=\infty}^{x=d} -\frac{m}{4\pi\mu_0\mu_r x^2}\,dx = \frac{m}{4\pi\mu_0\mu_r d} \text{ J/Wb}$$

By definition, *W* = Magnetic potential *V* at that point.

$\therefore$ Magnetic potential, $V = \dfrac{m}{4\pi\mu_0\mu_r d}$ J/Wb

Note that magnetic potential is a scalar quantity.

7.8. Absolute and Relative Permeability

Permeability of a material means its conductivity for magnetic flux. The greater the permeability of a material, the greater is its conductivity for magnetic flux and *vice-versa.* Air or vacuum is the poorest conductor of magnetic flux. The absolute (or actual) permeability *μ_0 (Greek letter "*mu*")

* The absolute (or actual) permeability of all non-magnetic materials is also $4\pi \times 10^{-7}$ H/m.

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of air or vacuum is $4\pi \times 10^{-7}$ H/m. The absolute (or actual) permeability μ of magnetic materials is much greater than μ_0. The ratio μ/μ_0 is called the relative permeability of the material and is denoted by μ_r *i.e.*

$$\mu_r = \frac{\mu}{\mu_0}$$

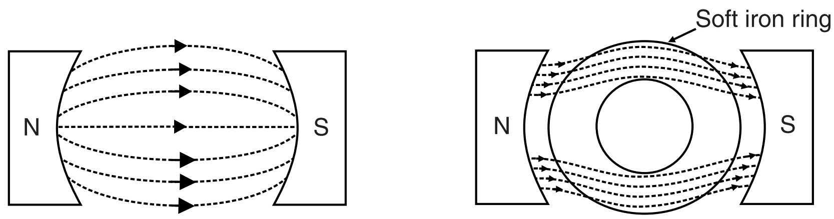
where μ = absolute (or actual) permeability of the material

μ_0 = absolute permeability of air or vacuum

μ_r = relative permeability of the material

Obviously, the relative permeability for air or vacuum would be $\mu_0/\mu_0 = 1$. The value of μ_r for all non-magnetic materials is also 1. However, relative permeability of magnetic materials is very high. For example, soft iron (*i.e.* pure iron) has a relative permeability of 8,000 whereas its value for permalloy (an alloy containing 22% iron and 78% nickel) is as high as 50,000.

Concept of relative permeability. The relative permeability of a material is a measure of the relative ease with which that material conducts magnetic flux compared with the conduction of flux in air. Fig. 7.8 illustrates the concept of relative permeability. In Fig. 7.8 (*i*), the magnetic flux passes between the poles of a magnet in air. Consider a soft iron ring (μ_r = 8,000) placed between the same poles as shown in Fig. 7.8 (*ii*). Since soft iron is a very good conductor of magnetic flux, the flux follows a path entirely within the soft iron itself. The flux density in the soft iron is much greater than it is in air. In fact, flux density in soft iron will be 8,000 times (*i.e.* μ_r times) the flux density in air.



(*i*) (*ii*)

Fig. 7.8

Due to high relative permeability of magnetic materials (*e.g.* iron, steel and other magnetic alloys), they are widely used for the cores of all electromagnetic equipment.

7.9. Relation Between B and H

The flux density B produced in a material is directly proportional to the applied magnetising force H. In other words, the greater the magnetising force, the greater is the flux density and *vice-versa i.e.*

$$B \propto H$$

or $$\frac{B}{H} = \text{Constant} = \mu$$

The ratio B/H in a material is always constant and is equal to the absolute permeability μ ($= \mu_0\,\mu_r$) of the material. This relation gives yet another definition of absolute permeability of a material.

Obviously, $B = \mu_0\,\mu_r H$ ...*in a medium*

$= \mu_0 H$ ...*in air*

Suppose a magnetising force H produces a flux density B_0 in air. Clearly, $B_0 = \mu_0 H$. If air is replaced by some other material (relative permeability μ_r) and the same magnetising force H is applied, then flux density in the material will be $B_{mat} = \mu_0\,\mu_r H$.

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$$\therefore \qquad \frac{B_{mat}}{B_0} = \frac{\mu_0 \mu_r H}{\mu_0 H} = \mu_r$$

Hence **relative permeability** *of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetising force.*

Thus when we say that μ_r of soft iron is 8000, it means that for the same magnetising force, flux density in soft iron will be 8000 times its value in air. In other words, for the same cross-sectional area and H, the magnetic lines of force will be 8000 times greater in soft iron than in air.

7.10. Important Terms

(*i*) Intensity of magnetisation (I). When a magnetic material is subjected to a magnetising force, the material is magnetised. Intensity of magnetisation is a measure of the extent to which the material is magnetised and depends upon the nature of the material. It is defined as under :

The intensity of magnetisation of a magnetic material is defined as the magnetic moment developed per unit volume of the material.

$$\therefore \quad \text{Intensity of magnetisation, } I = \frac{M}{V}$$

where M = magnetic moment developed in the material

V = volume of the material

If m is the pole strength developed, a is the area of X-section of the material and $2l$ is the magnetic length, then,

$$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a}$$

Hence intensity of magnetisation of a material may be defined as the pole strength developed per unit area of cross-section of the material.

$$I = \frac{\text{magnetic moment}}{\text{volume}} = \frac{\text{Amp. (metre)}^2}{\text{(metre)}^3} = \text{A m}^{-1}$$

$\therefore$ SI units of I are A m^{-1}.

(*ii*) Magnetic susceptibility (χ_m). The magnetic susceptibility of a material indicates how easily the material can be magnetised. It is defined as under :

The magnetic susceptibility of a material is defined as the ratio of intensity of magnetisation (I) developed in the material to the applied magnetising force (H). It is represented by χ_m (Greek alphabet Chi).

$$\therefore \quad \text{Magnetic susceptibility, } \chi_m = \frac{I}{H}$$

The unit of I is the same as that of H so that χ_m is a number. Since I is magnetic moment per unit volume, χ_m is also called *volume susceptibility* of the material.

7.11. Relation Between μ_r and χ_m

Consider a current carrying toroid having core material of relative permeability μ_r. The total magnetic flux density in the material is given by ;

$$B = B_0 + B_M$$

where B_0 = magnetic flux density due to current in the coils.

B_M = magnetic flux density due to the magnetisation of the material.

Now $B_0 = \mu_0 H$ and $B_M = \mu_0 I$*

* We can imagine that B_M is produced by a fictitious current I_M in the coils.

$$\therefore \quad B_M = \mu_0\, n\, I_M = \mu_0 \frac{N}{l} I_M = \mu_0 \frac{N I_M A}{Al} = \mu_0 I$$

where $N I_M A$ = magnetic dipole moment developed and $A\,l$ is the volume of the specimen.

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$\therefore \quad B = \mu_0 H + \mu_0 I = \mu_0 (H + I)$

or $\quad B = \mu_0 (H + I)$

Now $\quad \chi_m = \dfrac{I}{H}$ so that $I = \chi_m H$

$\therefore \quad B = \mu_0 (H + \chi_m H) = \mu_0 H (1 + \chi_m)$

But $\quad B = \mu H = \mu_0 \mu_r H$

$\therefore \quad \mu_0 \mu_r H = \mu_0 H (1 + \chi_m)$

or $\quad \mu_r = 1 + \chi_m$

Example 7.5. *The magnetic moment of a magnet (10 cm × 2 cm × 1 cm) is 1 Am^2. What is the intensity of magnetisation ?*

Solution. Volume of the magnet, $V = 10 \times 2 \times 1 = 20 \text{ cm}^3 = 20 \times 10^{-6} \text{ m}^3$

Magnetic moment of magnet, $M = 1 \text{ Am}^2$

$\therefore$ Intensity of magnetisation, $I = \dfrac{M}{V} = \dfrac{1}{20 \times 10^{-6}} = \mathbf{5 \times 10^4 \text{ A/m}}$

Example 7.6. *A specimen of iron is uniformly magnetised by a magnetising field of 500 A/m. If the magnetic induction in the specimen is 0·2 Wb/m^2, find the relative permeability and susceptibility.*

Solution. $\quad B = \mu H = \mu_0 \mu_r H$

$\therefore$ Relative permeability of the specimen is

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0 \cdot 2}{4\pi \times 10^{-7} \times 500} = \mathbf{318{\cdot}5}$$

Now $\quad \mu_r = 1 + \chi_m$

$\therefore$ Susceptibility, $\chi_m = \mu_r - 1 = 318{\cdot}5 - 1 = \mathbf{317{\cdot}5}$

7.12. Refraction of Magnetic Flux

When magnetic flux passes from one medium to another of different permeabilities, the magnetic flux gets refracted at the boundary of the two media [See Fig. 7.9]. Under this condition, the following two conditions exist at the boundary (called **boundary conditions**) :

(*i*) The normal components of magnetic flux density are equal *i.e.*

$$B_{1n} = B_{2n}$$

(*ii*) The tangential components of magnetic field intensities are equal *i.e.*

$$H_{1t} = H_{2t}$$

As proved in Art. 5.25, in a similar way, it can be proved that :

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This relation is called law of magnetic flux refraction.

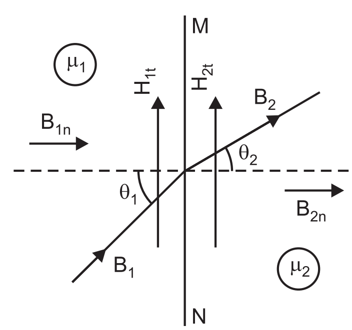

Fig. 7.9

7.13. Molecular Theory of Magnetism

The molecular theory of magnetism was proposed by Weber in 1852 and modified by Ewing in 1890. *According to this theory, every molecule of a magnetic substance (whether magnetised or not) is a complete magnet in itself having a north pole and a south pole of equal strength.*

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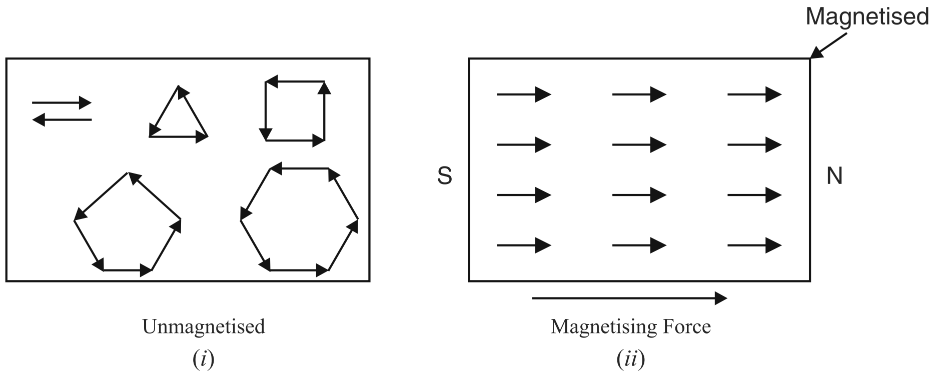

Fig. 7.10

(i) In an unmagnetised substance, the molecular magnets are randomly oriented and form closed chains as shown in Fig. 7.10 (*i*). The north pole of one molecular magnet cancels the effect of the south pole of the other so that the substance does not show any net magnetism.

(ii) When a magnetising force is applied to the substance (*e.g.* by rubbing a magnet or by passing electric current through a wire wound over it), the molecular magnets are turned and tend to align in the same direction with *N*-pole of one molecular magnet facing the *S*-pole of other as shown in Fig. 7.10 (*ii*). The result is that magnetic fields of the molecular magnets aid each other and two definite *N* and *S* poles are developed near the ends of the specimen ; the strength of the two poles being equal. Hence the substance gets magnetised.

(iii) The extent of magnetisation of the substance depends upon the extent of alignment of molecular magnets. When all the molecular magnets are fully aligned, the substance is said to be *saturated* with magnetism.

(iv) When a magnetised substance (or a magnet) is heated, the molecular magnets acquire kinetic energy and some of them go back to the closed chain arrangement. For this reason, a magnet loses some magnetism on heating.

Curie temperature. The magnetisation of a magnetised substance decreases with the increase in temperature. It is because when a magnetised substance is heated, random thermal motions tend to destroy the alignment of molecular magnets. As a results, the magnetisation of the substance decreases. At sufficiently high temperature, the magnetic property of the substance suddenly disappears and the substance loses magnetism.

The temperature at which a magnetised substance loses its magnetism is called **Curie temperature** *or* **Curie point** *of the substance*.

For example, the curie temperature of iron is 770°C. Therefore, if the temperature of the magnetised iron piece becomes 770°C, it will loose its magnetism. Similarly, the curie temperatures of nickel and cobalt are 358°C and 1121°C respectively.

7.14. Modern View about Magnetism

According to modern view, the magnetic properties of a substance are attributed to the motions of electrons (orbital and spin) in the atoms. We know that an atom consists of central nucleus with electrons revolving around the nucleus in different orbits. This motion of electrons is called *orbital motion* [See Fig. 7.11 (*i*)]. The electrons also rotate around their own axis. This motion of electrons is called *spin motion* [See Fig. 7.11 (*ii*)]. Due to these two motions, each atom is equivalent to a current loop *i.e.* each atom behaves as a magnetic dipole.

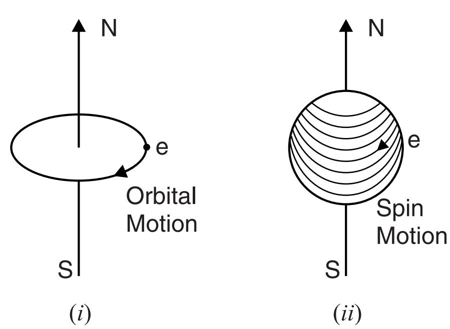

Fig. 7.11

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(*i*) In the unmagnetised substances, the magnetic dipoles are randomly oriented so that magnetic fields mutually cancel. When the substance is magnetised, the magnetic dipoles are aligned in the same direction. Hence the substance shows net magnetism.

(*ii*) Since the revolving and spinning electrons in each atom cause magnetism, no substance is non-magnetic.

(*iii*) It is important to note that spinning motion of electrons in particular is responsible for magnetism of a substance.

7.15. Magnetic Materials

We can classify materials into three categories *viz*. **diamagnetic**, **paramagnetic** and **ferromagnetic**. The behaviour of these three classes of substances is different in an external magnetic field.

(*i*) When a diamagnetic substance (*e.g*. copper, zinc, bismuth etc.) is placed in a magnetic field, the substance if *feebly* magnetised in a direction opposite to that of the applied field. Therefore, a diamagnetic substance is feebly repelled by a strong magnet.

(*ii*) When a paramagnetic substance (*e.g*. aluminium, antimony etc.) is placed in a magnetic field, the substance is *feebly* magnetised in the direction of the applied field. Therefore, a paramagnetic substance is feebly attracted by a strong magnet.

(*iii*) When a ferromagnetic substance (*e.g*. iron, nickel, cobalt etc.) is placed in a magnetic field, the substance is *strongly* magnetised in the direction of the applied field. Therefore, a ferromagnetic substance is strongly attracted by a magnet.

Note that diamagnetism and paramagnetism are weak forms of magnetism. However, ferromagnetic substances exhibit very strong magnetic effects.

7.16. Electromagnetism

The first discovery of any connection between electricity and magnetism was made by Hans Christian Oersted, a Danish physicist in 1819. On one occasion at the end of his lecture, he inadvertently placed a wire carrying current parallel to a compass needle. To his surprise, needle was deflected. Upon reversing the current in the wire, the needle deflected in the opposite direction.

Oersted found that the compass deflection was due to a magnetic field established around the current carrying conductor. This accidental discovery was the first evidence of a long suspected link between electricity and magnetism. The production of magnetism from electricity (which we call electromagnetism) has opened a new era. The operation of all electrical machinery is due to the applications of magnetic effects of electric current in one form or the other.

7.17. Magnetic Effect of Electric Current

When an electric current flows through a conductor, magnetic field is set up all along the length of the conductor. Fig. 7.12 shows the magnetic field produced by the current flowing in a straight wire. The magnetic lines of force are in the form of *concentric circles around the conductor.

The direction of lines of force depends upon the direction of current and may be determined by **right-hand rule.** *Hold the conductor in the right-hand with the thumb pointing in the direction of current (See Fig. 7.12). Then the fingers will point in the direction of magnetic field around the*

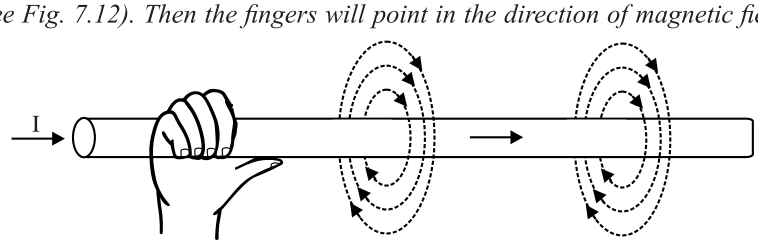

Fig. 7.12

* This can be readily established with a compass needle. If a compass needle is placed near the conductor and it is progressively moved in the direction of its north pole, it will be seen that the paths of magnetic lines of force are concentric circles.

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conductor. Applying this rule to Fig. 7.12, it is clear that when viewed from left-hand side, the direction of magnetic lines of force will be clockwise.

The following points may be noted about the magnetic effect of electric current :

(*i*) The greater the current through the conductor, the stronger the magnetic field and *vice-versa.*

(*ii*) The magnetic field neart the conductor is stronger and becomes weaker and weaker as we move away from the conductor.

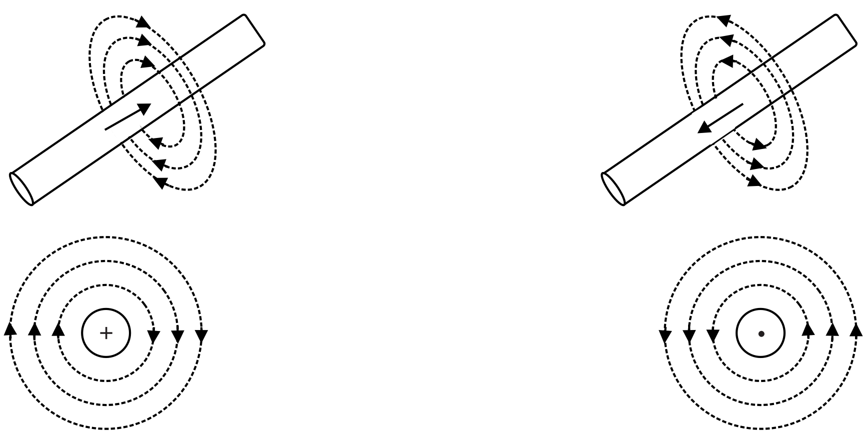
(*iii*) The magnetic lines of force around the conductor will be either clockwise or anticlockwise, depending upon the direction of current. One may use *right-hand rule* to determine the direction of magnetic field around the conductor.

(*iv*) The shape of the magnetic field depends upon the shape of the conductor.

7.18. Typical Electromagnetic Fields

The current carrying conductor may be in the form of a straight wire, a loop of one turn, a coil of several turns. The shape of the magnetic field would eventually depend upon the shape of conductor. By way of illustration, we shall discuss magnetic fields produced by some current carrying conductor arrangements.

(*i*) Long straight conductor. If a straight long conductor is carrying current, the magnetic lines of force will be concentric circles around the conductor as shown in Fig. 7.13. In Fig. 7.13 (*i*), the conductor is carrying current into the plane of paper (usually represented by a cross inside the *X*-section of the conductor). Applying right-hand rule, it is clear that direction of magnetic lines of force will be clockwise. In Fig. 7.13 (*ii*), the conductor is carrying current out of the plane of paper (usually represented by a dot inside the *X*-section of the conductor). Clearly, the direction of magnetic lines of force will be anticlockwise.



(*i*) (*ii*)

Fig. 7.13

(*ii*) Parallel conductors. Consider two parallel conductors *A* and *B* placed close together and carrying current into the plane of the paper as shown in Fig. 7.14 (*i*). The magnetic lines of force will be clockwise around each conductor. In the space between *A* and *B*, the lines of force due to the conductors are in the opposite direction and hence they cancel out each other. This results in a field that entirely surrounds the conductors as shown in Fig. 7.14 (*ii*).

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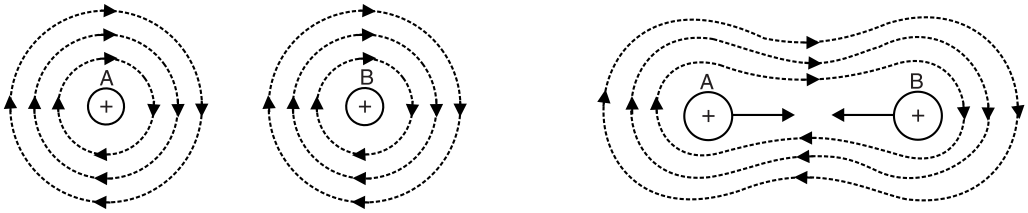


(*i*) (*ii*)

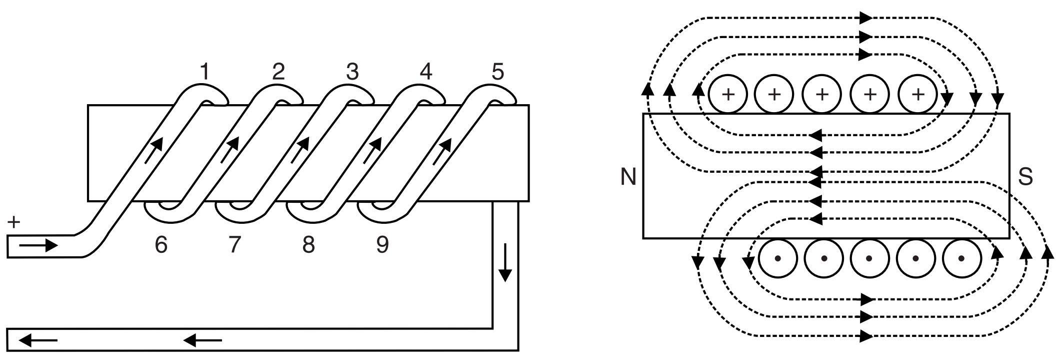
Fig. 7.14

If there are several parallel conductors placed close together and carrying current into the plane of the paper as shown in Fig. 7.15 (*i*), the magnetic field envelops the conductors. If the direction of current is reversed, the direction of field is also reversed as shown in Fig. 7.15 (*ii*).

(*i*) (*ii*)

Fig. 7.15

(*iii*) Coil of several turns. Consider a coil of several truns wound on a hollow tube or iron bar as shown in Fig. 7.16 (*i*). Such an arrangement is called a *solenoid. Suppose current flows through the coil in the direction shown. In the upper part of each turn (at points 1, 2, 3, 4 and 5), the current is flowing into the plane of the paper and in the lower part of each turn (at points 6, 7, 8 and 9), current is flowing out of the plane of paper. This is shown in the cross-sectional view of the coil in Fig. 7.16 (*ii*). It is clear that a clockwise field entirely surrounds the conductors 1, 2, 3, 4 and 5 while anticlockwise field completely envelops the conductors 6, 7, 8 and 9. As a result, the field becomes similar to that of a bar magnet with flux emerging from one end of the coil and entering the other.



(*i*) (*ii*)

Fig. 7.16

It is clear that left-hand face of the coil [See Fig. 7.16 (*ii*)] becomes a *N*-pole and right-hand face *S*-pole. The magnetic polarity of the coil can also be determined by the **right-hand rule for coil.** *Grasp the whole coil with right-hand so that the fingers are curled in the direction of current. Then thumb stretched parallel to the axis of the coil will point towards the N-pole end of the coil (See*

* Solenoid is Greek word meaning "tube-like."

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Fig. 7.17). It may be noted that both right-hand rules (for a conductor and for a coil) discussed so far can be applied in reverse. If we know the direction of magnetic field encircling a conductor or the magnetic polarity of a coil, we can determine the directiron of current by applying appropriate right-hand rule.

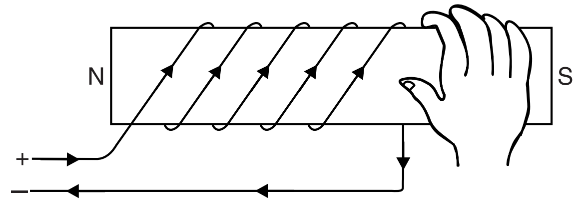

Fig. 7.17

7.19. Magnetising Force (H) Produced by Electric Current

The magnetic flux (ϕ) can be produced by (*i*) current-carrying conductor or coil or (*ii*) a permanent magnet. We generally use current-carrying conductor or coil to produce magnetic flux. Experiments show that magnetic flux (ϕ) produced by a current-carrying coil is directly proportional to the product of number of turns (N) of the coil and electric current (I) which the coil carries. The quantity NI is called *magnetomotive force (m.m.f)* and is measured in *ampere-turns* (AT) or **amperes* (A)

$$\therefore \qquad \text{m.m.f.} = NI \text{ Ampere-turns (AT)}$$

Just as e.m.f. (electromotive force) is required to produce electric current in an electric circuit, similarly, m.m.f. is required to produce magnetic flux in a **magnetic circuit. The greater the m.m.f., the greater is the magnetic flux produced in the magnetic circuit and *vice-versa*.

The **magnetising force** (*H*) *produced by an electric current is defined as the m.m.f. set up per unit length of the magnetic circuit i.e.*

$$\text{Magnetising force, } H = \frac{NI}{l} \text{ AT/m}$$

where NI = m.m.f. (AT)

l = length of magnetic circuit in m

Different current-carrying conductor arrangements produce different magnetising force. Magnetising force (H) is known by different names such as *magnetic field strength, magnetic intensity* and *magnetic potential gradient*.

Example 7.7. *A toroidal coil has a magnetic path length of 33 cm and a magnetic field strength of 650 A/m. The coil current is 250 mA. Determine the number of coil turns.*

Solution. $$H = \frac{NI}{l}$$

Here, $H = 650$ A/m ; $I = 250$ mA $= 0.25$A ; $l = 33$ cm $= 0.33$ m

$$\therefore \qquad 650 = \frac{N \times 0.25}{0.33} \quad \text{or} \quad N = \frac{650 \times 0.33}{0.25} = \textbf{858 turns}$$

Example 7.8. *Determine the m.m.f. required to generate a total flux of 100µWb in an air gap 0.2 cm long. The cross-sectional area of the air gap is 25 cm^2.*

Solution. $\phi = 100\ \mu\text{Wb} = 100 \times 10^{-6}$ Wb ; $l = 0.2 \times 10^{-2}$ m ; $A = 25 \times 10^{-4}$ m^2

$$\text{Flux density, } B = \frac{\phi}{A} = \frac{100 \times 10^{-6}}{25 \times 10^{-4}} = 4 \times 10^{-2} \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{B}{\mu_0} = \frac{4 \times 10^{-2}}{4\pi \times 10^{-7}} = 3.18 \times 10^4 \text{ AT/m}$$

* Since number of turns is dimensionless, ampere turns and amperes are the same as for as dimensions are concerned.

** The closed path followed by magnetic flux is called a magnetic circuit; just as the closed path followed by electric current is called an electric circuit.

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Now, $$H = \frac{\text{m.m.f.}}{l}$$

$\therefore$ $$\text{m.m.f.} = H \times l = 3.18 \times 10^4 \times 0.2 \times 10^{-2} = \mathbf{63.7\ AT}$$

An air gap is a necessity in a rotating machine such as a motor or a generator. It provides mechanical clearance between the fixed and moving parts. Air gaps are also used to prevent saturation in some magnetic devices.

7.20. Force on Current-carrying Conductor Placed in a Magnetic Field

When a current-carrying conductor is placed at right angles to a magnetic field, it is found that the conductor experiences a force which acts in a direction perpendicular to the direction of both the field and the current. Consider a straight current-carrying conductor placed in a uniform magnetic field as shown in Fig. 7.18.

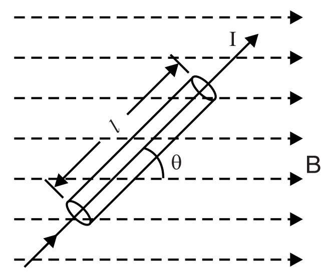

Fig. 7.18

Let B = magnetic flux density in Wb/m^2

I = current through the conductor in amperes

l = effective length of the conductor in metres *i.e.* the length of the conductor lying in the magnetic field

θ = angle which the conductor makes with the direction of the magnetic field

It has been found experimentally that the magnitude of force (F) acting on the conductor is directly proportional to the magnitudes of flux density (B), current (I), length (l) and sin θ *i.e.*

$$F \propto BIl \sin\theta \text{ newtons}$$

or $$F = k\,BIl \sin\theta$$

where k is a constant of proportionality. Now SI unit of B is so defined that value of k becomes unity.

$\therefore$ $$F = BIl \sin\theta$$

By experiment, it is found that the direction of the force is always perpendicular to the plane containing the conductor and the magnetic field.

Both magnitude and direction of the force will be given by the following vector equation :

$$\vec{F} = I\left(\vec{l} \times \vec{B}\right)$$

The direction of this force is perpendicular to the plane containing $\vec{l}$ and $\vec{B}$. It can be found by using right-hand rule for cross product.

Special Cases. $F = BIl \sin\theta$

(i) When θ = 0° or 180° ; sin θ = 0

$\therefore$ $$F = BIl \times 0 = 0$$

Therefore, if a current-carrying conductor is placed parallel to the direction of magnetic field, the conductor will experience no force.

(ii) When θ = 90° ; sin θ = 1

$\therefore$ $$F = BIl \quad \text{...maximum value}$$

Therefore, a current-carrying conductor will experience a maximum force when it is placed at right angles to the direction of the magnetic field.

Direction of force. The direction of force $\vec{F}$ is always perpendicular to the plane containing $\vec{l}$ and $\vec{B}$ and can be determined by *right-hand rule for cross product* stated below :

Orient your right hand so that your outstretched fingers point along the direction of the conventional current; the orientation should be such that when you bend your fingers, they must

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point along the direction of the magnetic field $(\vec{B})$. *Then your extended thumb will point in the direction of the force on the conductor.*

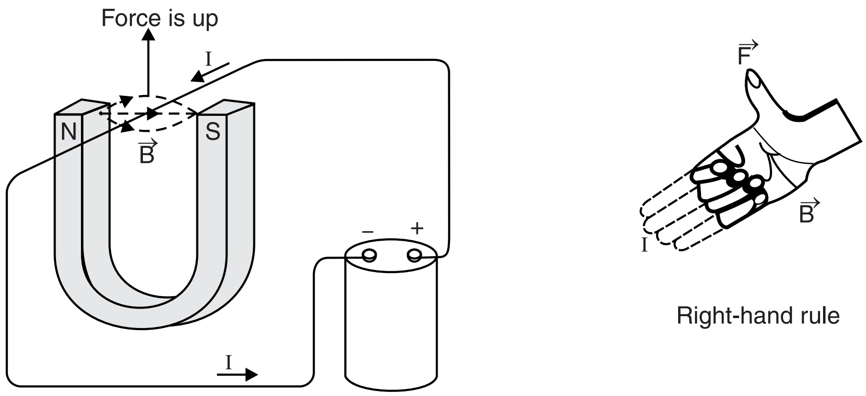

Fig. 7.19

Thus applying right-hand rule for cross product to Fig. 7.19, it is clear that magnetic force on the conductor is vertically upward.

Note. If the current-carrying conductor is at right angles to the magnetic field, the direction of force can also be found by Fleming's Left-hand rule stated below :

Fleming's Left-hand Rule. Stretch out the First finger, seCond finger and thuMb of your left hand so that they are at right angles to one another. If the first finger points in the direction of magnetic field (North to South) and second finger (*i.e.* middle finger) points towards the direction of current, then the thumb will point in the direction of motion of the conductor.

Example 7.9. *A conductor of length 100 cm and carrying 100 A is situated in and at right angles to a uniform magnetic field produced by the pole core of an electrical machine. If the pole core has a circular cross-section of 120 mm diameter and the total flux in the core is 16 mWb, find (i) the mechanical force on the conductor and (ii) power required to move the conductor at a speed of 10 m/s in a plane at right angles to the magnetic field.*

Solution. In this case, mechanical force acts on the conductor.

X-sectional area of pole core $= (\pi/4) \times (0{\cdot}12)^2 = 0{\cdot}0113 \text{ m}^2$

$$\text{Flux density of field, } B = \frac{\text{Flux}}{\text{Pole core area}} = \frac{16\times10^{-3}}{0.0113} = 1{\cdot}416 \text{ Wb/m}^2$$

(*i*) Force on the conductor is given by ;

$$F = B\,I\,l = 1{\cdot}416 \times 100 \times 1 = \mathbf{141{\cdot}6\ N}$$

(*ii*) Power required $=$ Force $\times$ distance/second

$$= 141{\cdot}6 \times 10 = \mathbf{1416\ watts}$$

Example 7.10. *The plane of a rectangular coil makes an angle of 60° with the direction of a uniform magnetic field of flux density* 4×10^{-2} *Wb/m*^{2}. *The coil is of 20 turns, measuring 20 cm by 10 cm, and carries a current of 0·5 A. Calculate the torque acting on the coil.*

Solution. Consider a rectangular coil, measuring b by l, of N turns carrying a current of I amperes and placed in a uniform magnetic field of B Wb/m^2. The coil is pivoted about the mid points of the sides b and is free to rotate about an axis in its own plane ; this axis being at right angles to the field density B [See Fig. 7.20 (*i*)]. When current is passed through the coil, forces acting on the coil sides are :

(*i*) The forces developed on each half of coil sides b are equal and produce torques of opposing sense. They, therefore, cancel each other.

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(*ii*) The coil sides l always remain at right angles to the field as the coil rotates. The force F acting on each of the coil sides l gives rise to a torque as shown in Fig. 7.20 (*ii*).

Force on each coil side l, $F = B\,I\,l\,N$ newtons

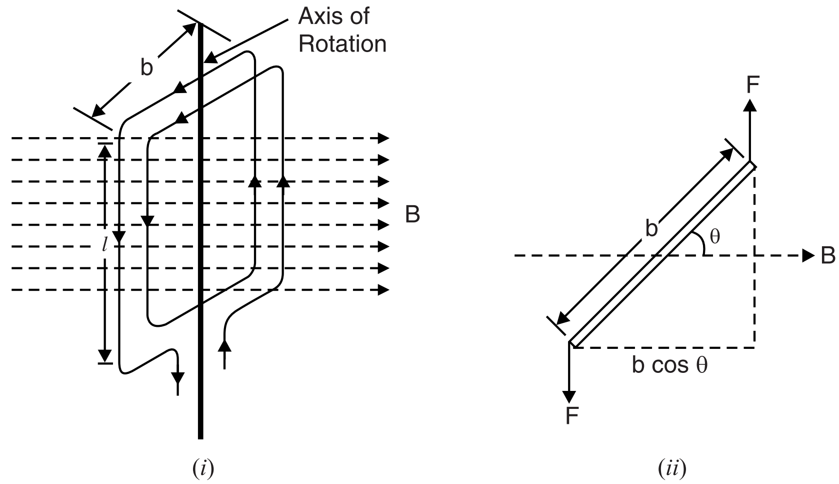

Fig. 7.20

The perpendicular distance between the lines of action of the two forces is $b \cos \theta$.

$\therefore$ Torque, $T = F \times b \cos \theta = (B\,I\,l\,N)\, b \cos \theta$

or $T = BINA \cos \theta$ newton-metre

where $A\ (= l \times b)$ is the area of the coil. By an extension of this reasoning, the expression may be proved quite generally for a coil of area A and of any shape.

In the given problem, the data is

$$B = 4 \times 10^{-2}\ \text{Wb/m}^2\ ;\ A = 20 \times 10 = 200\ \text{cm}^2 = 2 \times 10^{-2}\ \text{m}^2\ ;\ I = 0{\cdot}5\ \text{A}\ ;\ \theta = 60°\ ;\ N = 20$$

$\therefore$ Torque, $T = (4 \times 10^{-2}) \times (0{\cdot}5) \times (20) \times (2 \times 10^{-2}) \times \cos 60° =$ **4×10^{-3} Nm**

Tutorial Problems

1. A straight conductor 0·4m long carries a current of 12 A and lies at right angles to a uniform field of 2·5 Wb/m^2. Find the mechanical force on the conductor when (*i*) it lies in the given position (*ii*) it lies in a position such that it is inclined at an angle of 30º to the direction of field. **[(*i*) 12 N (*ii*) 6 N]**
2. A conductor of length 100 cm and carrying 100 A is situated in and at right angles to a uniform magnetic field of strength 1 Wb/m^2. Calculate the force and power required to move the conductor at a speed of 100 m/s in a plane at right angles to the magnetic field. **[100 N ; 1000 watts]**
3. A d.c. motor consists of an armature winding of 400 turns (equivalent to 800 conductors). The effective lengths of conductor in the field is 160 mm and the conductors are situated at a radius of 100 mm from the centre of the motor shaft. The magnetic flux density is 0·6 Wb/m^2 and a current of 25 A flows through the winding. Calculate the torque available at the motor shaft. **[192 Nm]**
4. A d.c. motor is to provide a torque of 540 Nm. The armature winding consists of 600 turns (equivalent to 1200 conductors). The effective length of a conductor in the field is 250 mm and the conductors are situated at a radius of 150 mm from the centre of the motor shaft. Each conductor carries a current of 10 A. Calculate the flux density which must be provided by the radial field in which the conductors lie. **[1·2 Wb/m^2]**

7.21. Ampere's Work Law or Ampere's Circuital Law

The magnetising force (H) at any point in an electromagnetic field is the force experienced by a unit N-pole placed at that point. If the unit N-pole is made to move in a complete path around N current-carrying conductors, then work is done provided the unit N-pole is moved in opposition to

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the lines of force. Conversely, if the unit N-pole moves in the direction of magnetic field, then work will be done by the magnetic force on whatever force is restraining the movement of the pole. In either case, unit N-pole makes one complete loop around the N conductors. The work done is given by Ampere's work law stated below :

The work done on or by a unit N-pole in moving once around any complete path is equal to the product of current and number of turns enclosed by that path i.e.

$$^{*}\oint \vec{H_r} \cdot \vec{dr} = N\,I$$

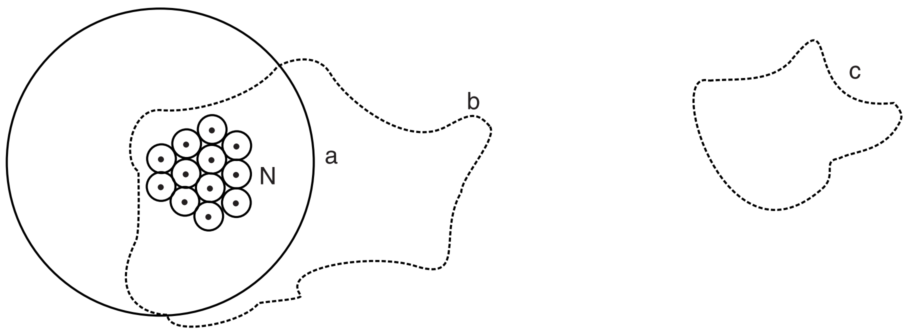

Fig. 7.21

where $\vec{H_r}$ is the magnetising force at a distance r. The circle around the integral sign indicates that the integral is around a complete path.

The work law is applicable regardless of the shape of complete path. Thus in Fig. 7.21, paths 'a' and 'b' completely enclose N conductors. If a unit N-pole is moved once around any of these complete paths, the work done in each case will be equal to $N\,I$. Although path 'c' is a complete path, it fails to enclose any current carrying conductor. Hence, no work is done in moving a unit N-pole around such a path.

Note. The work law is applicable for all magnetic fields, irrespective of the shape of the field or of the materials which may be present.

7.22. Applications of Ampere's Work Law

Ampere's work law can be used to find magnetising force (H) in simple conductor arrangements. We shall discuss two cases by way of illustration.

1. Magnetising force around a long straight conductor. Consider the case of a long straight conductor carrying a current of I amperes as shown in Fig. 7.22. The conductor will set up magnetic lines of force which encircle it. Consider a circular path of radius r metres. By symmetry, the field intensity H on all the points of this circular path will be the same. If a unit N-pole is moved once around this circular path, then work done is = $2\pi rH$. By work law, this must be equal to the product of current and number of turns enclosed by this circular path.

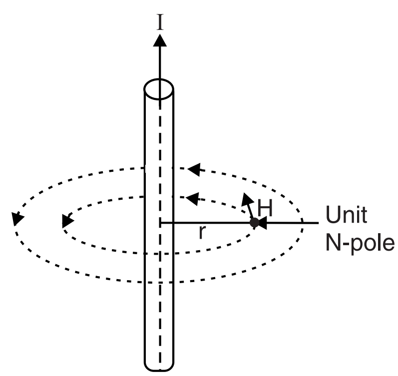

Fig. 7.22

$$\therefore \qquad 2\pi\, r\, H = I \qquad (\because N = 1)$$

$$\text{or} \qquad H = \frac{I}{2\pi r}$$

* This law can also be stated as *the closed line integral of magnetic field intensity (H) is equal to the encloed ampere-turns that produce the magnetic field.*

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Note that magnetic lines of force encircle the conductor like concentric circles. The direction of magnetic lines of force can be determined by right-hand rule.

If there had been N turns enclosed by the path, then,

$$H = \frac{NI}{2\pi r}$$

$$\text{Flux density, } B = \mu_0 H = \frac{\mu_0 NI}{2\pi r} \quad \text{... in air}$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \quad \text{... in a medium}$$

The following points may be noted carefully :

(*i*) If we choose a complete path for which r is smaller, H on that circle will be large. However, $2\pi\, r\, H$ will be still equal to NI.

(*ii*) Inspection of above expression reveals that H can also be expressed in ampere-turns per metre (AT/m).

(*iii*) It is reminded that the quantity NI (*i.e.* product of the number of turns in a winding and the current flowing through it) is called **magnetomotive force** (m.m.f.).

$$\text{m.m.f.} = NI \text{ ampere-turns}$$

2. Magnetising force due to long solenoid. Consider a long solenoid of length l and wound uniformly with N turns (See Fig. 7.23). The length of the solenoid is much greater than the breadth, say 10 times greater. The following assumptions are permissible :

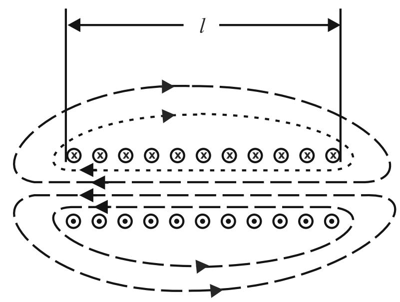

Fig. 7.23

(*i*) The field strength external to the solenoid is effectively zero.

(*ii*) The field strength inside the solenoid is uniform.

Suppose the current I flowing through the solenoid produces uniform magnetic field strength H within the solenoid. Applying work law to any closed path say dotted one shown in Fig. 7.23,

Total work done around closed path = Ampere turns linked

Since there is negligible field strength (H) outside the solenoid, the only work done will be in travelling length l within the solenoid.

$$\therefore \quad H \times l = NI$$

$$\text{or} \quad H = \frac{NI}{l} \text{ AT/m or A/m}$$

$$\text{Incidentally, } B = \mu_0 H = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \quad \text{...in air}$$

$$= \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \quad \text{...in a medium}$$

It is reminded that the magnetic field strength (H) is a vector quantity since it has magnitude and direction.

Example 7.11. *An air-cored toroidal coil shown in Fig. 7.24 has 3000 turns and carries a current of 0.1A. The cross-sectional area of the coil is 4 cm² and the length of the magnetic circuit is 15 cm. Determine the magnetic field strength, the flux density and the total flux within the coil.*

Solution. $N = 3000$ turns ; $I = 0.1$ A ; $A = 4 \times 10^{-4}$ m² ; $l = 15 \times 10^{-2}$ m

$$\text{Magnetic field strength, } H = \frac{NI}{l} = \frac{3000 \times 0.1}{15 \times 10^{-2}}$$

$$= \mathbf{2000\ AT/m}$$

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$$\text{Flux density, } B = \mu_0 H = 4\pi \times 10^{-7} \times 2000$$
$$= \mathbf{2.5 \times 10^{-3}\ Wb/m^2}$$
$$\text{Total flux, } \phi = B \times A = 2.5 \times 10^{-3} \times 4 \times 10^{-4}$$
$$= 1 \times 10^{-6}\ \text{Wb} = \mathbf{1\ \mu Wb}$$

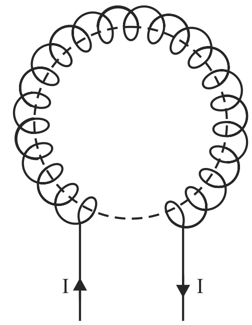

Fig. 7.24

Example 7.12. *An air-cored solenoid has length of 15 cm and inside diameter of 1.5 cm. If the coil has 900 turns, determine the total flux within the solenoid when the coil current is 100 mA.*

Solution. For a solenoid, the length of the magnetic circuit, l = coil length = 15×10^{-2} m.

$$D = 1.5 \times 10^{-2}\ \text{m} \quad ; \quad N = 900 \text{ turns} \ ; \ I = 100 \times 10^{-3}\ \text{A}$$
$$\therefore \quad \text{m.m.f.} = NI = 900 \times 100 \times 10^{-3} = 90\ \text{AT}$$
$$\text{Magnetising force, } H = \frac{\text{m.m.f.}}{l} = \frac{90}{15 \times 10^{-2}} = 600\ \text{AT/m}$$
$$\text{Magnetic flux density, } B = \mu_0 H = 4\pi \times 10^{-7} \times 600 = 24\pi \times 10^{-5}\ \text{Wb/m}^2$$
$$\therefore \quad \text{Total flux, } \phi = BA = 24\pi \times 10^{-5} \times \pi\frac{D^2}{4}$$
$$= 24\pi \times 10^{-5} \times \pi \times \frac{(1.5 \times 10^{-2})^2}{4} = \mathbf{1.33 \times 10^{-7}\ Wb}$$

If the solenoid were iron-cored, the magnitude of the magnetic flux within the solenoid would have been much greater than the calculated value because of very high relative permeability of iron.

7.23. Biot-Savart Law

A conductor carrying current I produces a magnetic field around it. We can consider the current carrying conductor to be consisting of infinitesimally small **current elements* $I\,\vec{dl}$; each current element contributing to magnetic field. *Biot-Savart law gives us expression for the magnetic field at a point due to a current element*.

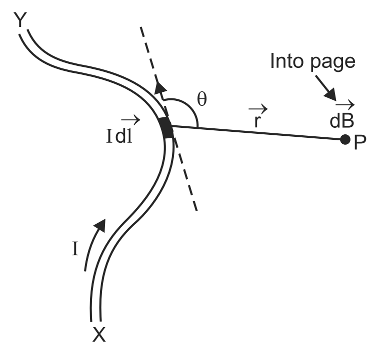

Fig. 7.25

Consider a current element $I\,\vec{dl}$ of a conductor XY carrying current I [See Fig. 7.25]. Let P be the point where the magnetic field $\vec{dB}$ due to the current element is to be found. Suppose $\vec{r}$ is the position vector of point P from the current element $I\,\vec{dl}$ and θ is the angle between $\vec{dl}$ and $\vec{r}$.

According to Biot-Savart law, the magnitude dB of magnetic field at point P due to the current element depends upon the following factors :

(*i*) $dB \propto I$ (*ii*) $dB \propto dl$ (*iii*) $dB \propto 1/r^2$ (*iv*) $dB \propto \sin\theta$

Combining all these four factors, we get,

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or
$$dB = K\frac{I\,dl \sin\theta}{r^2}$$

* The current element $I\,\vec{dl}$ is a vector. Its direction is tangent to the element and acts in the direction of flow of current in the conductor.

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where K is a constant of proportionality. Its value depends on the medium in which the conductor is situated and the system of units adopted.

$$\text{For free space and SI units, } K = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{Tm A}^{-1}$$

$$\text{where } \mu_0 = \text{Absolute permeability of free space} = 4\pi \times 10^{-7}\ \text{Tm A}^{-1}$$

$$\therefore \qquad dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2} \qquad \text{...}(i)$$

Eq. (i) is known as *Biot-Savart law* and gives the magnitude of the magnetic field at a point due to small current element $I\vec{dl}$. **Note that Biot-Savart law holds strictly for steady currents.**

In vector form.

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3} \qquad \text{...}(ii)$$

The Biot-Savart law is analogous to Coulomb's law. Just as the charge q is the source of electrostatic field, similarly, the source of magnetic field is the current element $I\vec{dl}$.

Direction of $\vec{B}$.

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$. By right-hand rule for the cross product, the field is directed *inward*.

Magnetic field due to whole conductor. Eq. (ii) gives the magnetic field at point P due to a small current element $I\vec{dl}$. The total magnetic field at point P is found by summing (integrating) over all current elements.

$$\vec{B} = \int \vec{dB} = \int \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

where the integration is taken over the entire conductor in which current I flows.

Special cases.

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin\theta}{r^2}$$

(i) When $\theta = 0°$ *i.e.,* point P lies on the axis of the conductor, then,

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin 0°}{r^2} = 0$$

Hence, there is no magnetic field at any point on the thin linear current carrying conductor.

(ii) When $\theta = 90°$ *i.e.,* point P lies at a perpendicular position *w.r.t.* current element, then,

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin 90°}{r^2} = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl}{r^2} \qquad \text{... Maximum value}$$

Hence magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

(iii) When $\theta = 0°$ or $180°$, $dB = 0$ *...Minimum value*

Important points about Biot-Savart law. This law has the following salient features :

(i) Biot-Savart law is valid for symmetrical current distributions.

(ii) Biot-Savart law cannot be proved experimentally because it is not possible to have a current carrying conductor of length dl.

(iii) Like Coulomb's law in electrostatics, Biot-Savart law also obeys inverse square law.

(iv) The direction of $\vec{dB}$ is perpendicular to the plane containing $I\vec{dl}$ and $\vec{r}$.

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7.24. Applications of Biot-Savart Law

Biot-Savart law is very useful in determining magnetic flux density B and hence magnetising force H (= B/μ_0) due to current-carrying conductor arrangements. We shall discuss the following cases by way of illustration.

(*i*) Magnetic flux density at the centre of current-carrying circular coil.

(*ii*) Magnetic flux density due to straight conductor carrying current.

(*iii*) Magnetic flux density on the axis of circular coil carrying current.

7.25. Magnetic Field at the Centre of Current-Carrying Circular Coil

This is a practical case because the operation of many devices depends upon the magnetic field produced by the current-carrying circular coil. Consider a circular coil of radius r and carrying current I in the direction shown in Fig. 7.26. Suppose the loop lies in the plane of paper. It is desired to find the magnetic field at the centre O of the coil. Suppose the entire circular coil is divided into a large number of current elements, each of length dl. According to Biot-Savart law, the magnetic field $\vec{dB}$ at the centre O of the coil due to current element $I\,\vec{dl}$ is given by ;

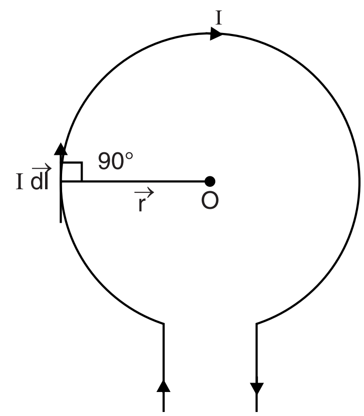

Fig. 7.26

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,(\vec{dl}\times\vec{r})}{r^3}$$

where $\vec{r}$ is the position vector of point O from the current element.

The magnitude of $\vec{dB}$ at the centre O is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \qquad \ldots(i)$$

The direction of $\vec{dB}$ is perpendicular to the plane of the coil and is directed inwards. Since each current element contributes to the magnetic field in the same direction, the total magnetic field B at the centre O can be found by integrating eq. (*i*) around the loop *i.e.*

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

For each current element, angle between $\vec{dl}$ and $\vec{r}$ is 90°. Also distance of each current element from the centre O is r.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I\sin 90^\circ}{r^2}\int dl$$

Now, $$\int dl = \text{Total length of the coil} = 2\pi r$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}(2\pi r)$$

or $$B = \frac{\mu_0 I}{2r}$$

Also, $$H = \frac{B}{\mu_0} = \frac{1}{\mu_0}\times\frac{\mu_0 I}{2r} = \frac{I}{2r}$$

If the coil has N turns, each carrying current in the same direction, then contributions of all the turns are added up. Therefore, the magnetic field at the centre of the coil is greatly increased and is given by ;

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$$B = \frac{\mu_0 NI}{2r}$$

Also, $$H = \frac{B}{\mu_0} = \frac{NI}{2r}$$

Direction of $\vec{B}$. The direction of magnetic field $\vec{B}$ is perpendicular to the plane of the coil and for Fig. 7.27, magnetic field inside the coil is directed inwards. The magnetic lines of force are circular near the wire but practically straight near the centre of the coil. In the middle *M* of the coil, the magnetic field is uniform for a short distance on either side. The direction of magnetic field at the centre of a current-carrying circular coil can be determined by *right-hand palm rule*.

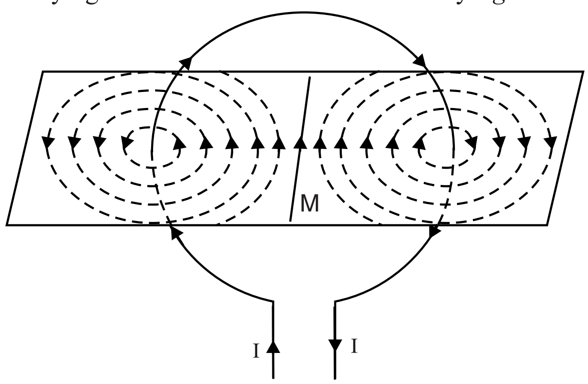

Fig. 7.27

Right-hand palm rule. *Orient the thumb of your right hand perpendicular to the grip of the fingers such that curvature of the fingers points in the direction of current in the circular coil. Then thumb will point in the direction of the magnetic field near the centre of the circular coil.*

7.26. Magnetic Field Due to Straight Conductor Carrying Current

Consider a straight conductor *XY* carrying current *I* in the direction shown in Fig. 7.28. It is desired to find the magnetic field at point *P* located at a perpendicular distance *a* from the conductor (*i.e.* $PQ = a$). Consider a small current element of length *dl*. Let $\vec{r}$ be the position vector of point *P* from the current element and θ be the angle between $\vec{dl}$ and $\vec{r}$ (*i.e.*, $\angle POQ = \theta$). Let us further assume that $QO = l$.

According to Biot-Savart law, the magnitude of magnetic field $\vec{dB}$ at point *P* due to the considered current element is given by ;

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin\theta}{r^2} \quad ...(i)$$

To get the total magnetic field *B*, we must integrate eq. (*i*) over the whole conductor. As we move along the conductor, the quantities *dl*, θ and *r* change. The integration becomes much easier if we express everything in terms of angle ϕ shown in Fig. 7.28.

In the right angled triangle *PQO*, $\theta = 90° - \phi$.

$\therefore \quad \sin\theta = \sin(90° - \phi) = \cos\phi$

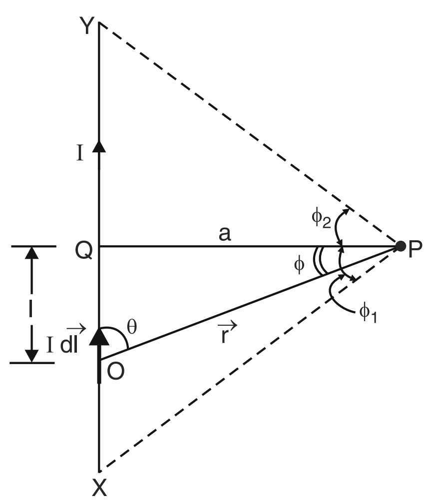

Fig. 7.28

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Also, $\cos\phi = \dfrac{a}{r}$ or $r = \dfrac{a}{\cos\phi}$

Further, $\tan\phi = \dfrac{l}{a}$ or $l = a\tan\phi$

or $dl = a\sec^2\phi\, d\phi$

Putting the values of sin θ, dl and r in eq. (i), we have,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,(a\sec^2\phi\, d\phi)\cos\phi}{(a/\cos\phi)^2}$$

or $$dB = \frac{\mu_0}{4\pi}\frac{I\cos\phi\, d\phi}{a} \quad ...(ii)$$

The direction of $\vec{dB}$ is perpendicular to the plane of the conductor and is directed inwards (Right-hand grip rule, See section 7.17). Since each current element contributes to the magnetic field in the same direction, the total magnetic field B at point P can be found by integrating eq. (ii) over the length XY *i.e.*

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0}{4\pi}\frac{I}{a}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi]_{-\phi_1}^{\phi_2} = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$$

∴ $$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1) \quad ...(iii)$$

Also, $$H = \frac{B}{\mu_0} = \frac{I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$$

Eq. (iii) gives the value of B at point P due to a conductor of *finite length*.

Special cases. We shall discuss a few important cases.

(*i*) *When the conductor XY is of infinite length and point P lies at the centre of the conductor.*

In this case, $\phi_1 = \phi_2 = 90° = \pi/2$.

∴ $$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\pi/2 + \sin\pi/2)$$

or $$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

Also, $$H = \frac{B}{\mu_0} = \frac{1}{4\pi}\cdot\frac{2I}{a} = \frac{I}{2\pi a}$$

(*ii*) *When conductor XY is of infinite length but point P lies near one end Y* (*or X*). In this case, $\phi_1 = 90°$ and $\phi_2 = 0°$.

∴ $$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 90° + \sin 0°)$$

or $$B = \frac{\mu_0 I}{4\pi a}$$

Note that it is half of that for case (i).

Also, $$H = \frac{B}{\mu_0} = \frac{I}{4\pi a}$$

(*iii*) *If the length of the conductor is finite (say l) and point P lies on the right bisector of the conductor*. In this case, $\phi_1 = \phi_2 = \phi$.

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Now, $$\sin\phi = \frac{l/2}{\sqrt{a^2 + (l/2)^2}} = \frac{l}{\sqrt{4a^2 + l^2}}$$

$\therefore$ $$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi + \sin\phi) = \frac{\mu_0}{4\pi}\frac{2I}{a}\sin\phi$$

or $$B = \frac{\mu_0}{4\pi}\frac{2I}{a}\frac{l}{\sqrt{4a^2 + l^2}}$$

Also $$H = \frac{B}{\mu_0} = \frac{1}{4\pi}\cdot\frac{2I}{a}\frac{l}{\sqrt{4a^2 + l^2}}$$

Direction of $\vec{B}$. For a long straight conductor carrying current, the magnetic lines of force are concentric circles with conductor as the centre ; the direction of magnetic lines of force can be found by *right-hand grip rule*. The direction of $\vec{B}$ at any point is along the tangent to field line at that point as shown in Fig. 7.29.

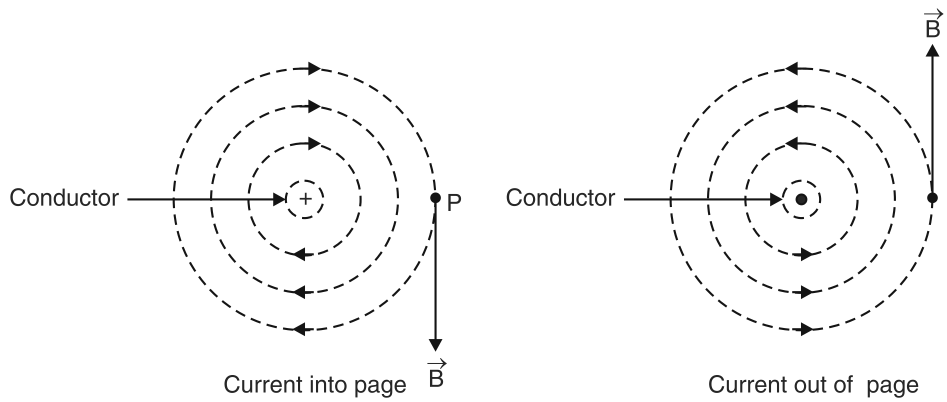

Fig. 7.29

Note. For a given current, $B \propto 1/a$ so that graph between B and a is a hyperbola.

7.27. Magnetic Field on the Axis of Circular Coil Carrying Current

Consider a circular coil of radius a, centre O and carrying a current I in the direction shown in Fig. 7.30. Let the plane of the coil be perpendicular to the plane of the paper. It is desired to find the magnetic field at a point P on the axis of the coil such that $OP = x$.

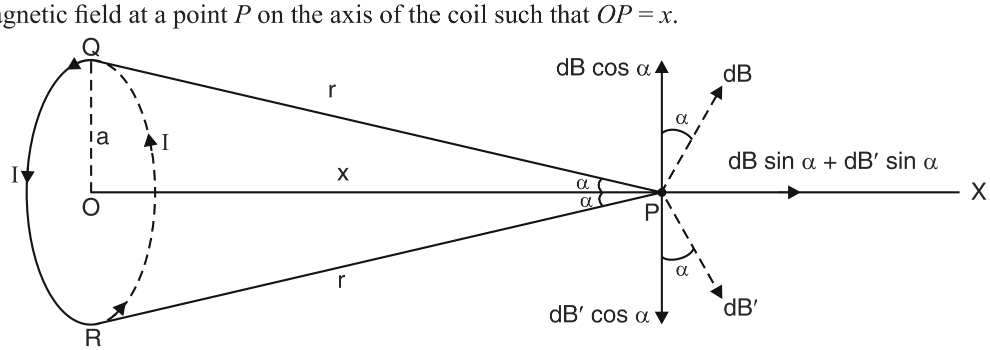

Fig. 7.30

Consider two small current elements, each of length dl, located diametrically opposite to each other at Q and R. Suppose the distance of Q or R from P is r *i.e.* $PQ = PR = r$.

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$\therefore \qquad r = \sqrt{a^2 + x^2}$

According to Biot-Savart law, the magnitude of magnetic field at P due to current element at Q is given by ;

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin 90°}{r^2} \qquad (\because \theta^* = 90°)$$

or
$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(i)$$

The magnetic field at P due to current element at Q is in the plane of paper and at right angles to $\vec{r}$ and in the direction shown.

Similarly, magnitude of magnetic field at point P due to current element at R is given by ;

$$dB' = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(ii)$$

It also acts in the plane of paper and at right angles to $\vec{r}$ but in opposite direction to dB.

From eqs. (i) and (ii), $\qquad dB = dB'$.

It is clear that vertical components ($dB \cos \alpha$ and $dB' \cos \alpha$) will be equal and opposite and thus cancel each other. However, components along the axis of the coil ($dB \sin \alpha$ and $dB' \sin \alpha$) are added and act in the direction PX. This is true for all the diametrically opposite elements of the circular coil. Therefore, when we sum up the contributions of all the current elements of the coil, the perpendicular components will cancel. Hence the resultant magnetic field at point P is the vector sum of all the components $dB \sin \alpha$ over the entire coil.

$$\therefore \qquad B = \int dB \sin\alpha = \int \frac{\mu_0\, I\, dl \sin\alpha}{4\pi\,(a^2 + x^2)} = \frac{\mu_0\, I \sin\alpha}{4\pi\,(a^2 + x^2)}\int dl$$

Now
$$\sin\alpha = \frac{a}{\sqrt{a^2 + x^2}} \quad \text{and} \quad \int dl = 2\pi\, a$$

$$\therefore \qquad B = \frac{\mu_0\, I\, a^2}{2(a^2 + x^2)^{3/2}} \quad \text{along } PX \qquad ...(iii)$$

Also,
$$H = \frac{B}{\mu_0} = \frac{Ia^2}{2(a^2 + x^2)^{3/2}}$$

If the circular coil has N turns, then,

$$B = \frac{\mu_0\, NI\, a^2}{2(a^2 + x^2)^{3/2}} \quad \text{along } PX \qquad ...(iv)$$

Also,
$$H = \frac{B}{\mu_0} = \frac{NIa^2}{2(a^2 + x^2)^{3/2}}$$

Different Cases. Let us discuss some special cases.

(i) *When point P is at the centre of the coil*. In this case, $x = 0$ and eq. (iv) becomes :

$$B = \frac{\mu_0 NIa^2}{2a^3} = \frac{\mu_0 NI}{2a}$$

This is the expression for the magnetic field at the centre of a current-carrying circular coil already derived in section 7.25. Note that the value of magnetic field is maximum at the centre of the coil.

* The radius vector QP of each current element is perpendicular to it so that $\theta = 90°$ in each case.

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Also, $$H = \frac{B}{\mu_0} = \frac{NI}{2a}$$

(*ii*) *When point P is far away from the centre of coil*. In this case, $x >> a$ so that $a^2 + x^2 \simeq x^2$.

$\therefore$ $$B = \frac{\mu_0 \, NI \, a^2}{2x^3}$$

Also, $$H = \frac{B}{\mu_0} = \frac{NIa^2}{2x^3}$$

The magnetic field is directed along the axis of the coil and falls off as the cube of the distance from the coil.

Direction of $\vec{B}$. The magnetic field at the centre of a coil carrying current is along the axis of the coil as shown in Fig. 7.31. The direction of magnetic field can be determined by using **right-hand fist rule.** *Hold the axis of the coil in the right-hand fist in such a way that fingers point in the direction of current in the coil. Then outstretched thumb gives the direction of the magnetic field.* Applying this rule to Fig. 7.31, it is clear that direction of magnetic lines of force is along the axis of the coil as shown.

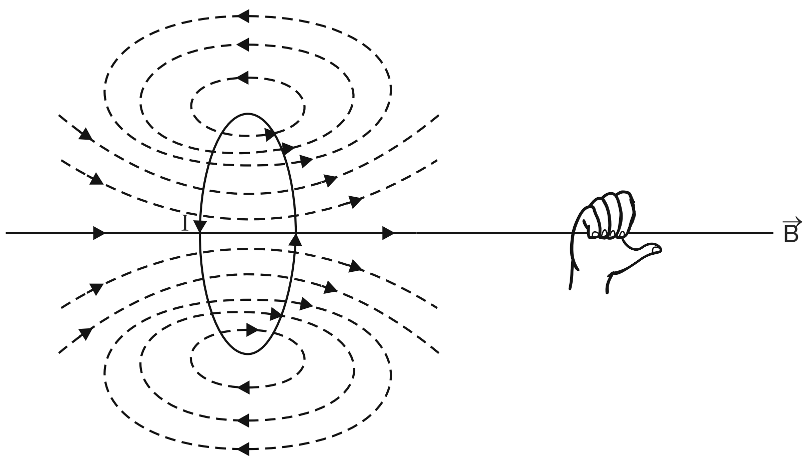

Fig. 7.31

Example 7.13. *How far from a compass should a wire carrying 1 A current be located if its magnetic field at the compass is not to exceed 1 percent of the *earth's magnetic field (3×10^{-5} Wb/m^2) ?*

Solution. Let r metre be the desired distance.

Required flux density at the compass is

$$B = 1\% \text{ of Earth's flux density}$$
$$= 0{\cdot}01 \times 3 \times 10^{-5} = 3 \times 10^{-7} \text{ Wb/m}^2$$

Required magnetising force at the compass is

$$H = \frac{B}{\mu_0} = \frac{3 \times 10^{-7}}{4\pi \times 10^{-7}} = 0{\cdot}239 \text{ AT/m}$$

Now, $$H = \frac{I}{2\pi r} \qquad \therefore \quad r = \frac{I}{2\pi H} = \frac{1}{2\pi \times 0 \cdot 239} = \mathbf{0{\cdot}67 \ m}$$

Example 7.14. *A horizontal overhead power line carries a current of 50 A in west to east direction. What is the magnitude and direction of the magnetic field 1·5 m below the line ?*

* **Earth's magnetic field.** The earth itself has a weak magnetic field. This is believed to be caused by electric currents circulating within its core. The currents are probably generated by convection in the liquid core maintained by radioactive heating of the earth's interior.

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Solution. Figure 7.32 shows the conditions of the problem. The magnitude of magnetic field at point P, 1·5 m below the wire is given by ;

$$B = \frac{\mu_0}{2\pi}\frac{I}{a}$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $I = 50$ A ; $a = 1{\cdot}5$ m

$$\therefore \quad B = \frac{4\pi\times10^{-7}}{2\pi}\times\frac{50}{1{\cdot}5} = \mathbf{6.7 \times 10^{-6}\ T}$$

According to right-hand grip rule, the direction of magnetic field below the wire is from south to north.

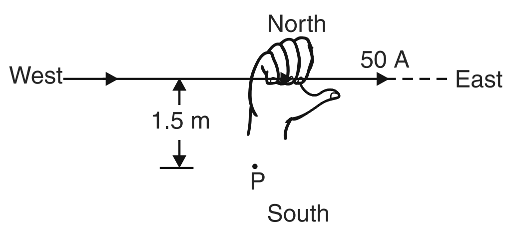

Fig. 7.32

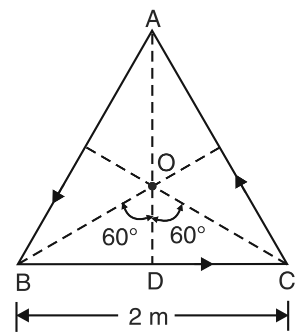

Fig. 7.33

Example 7.15. *A current of 1 A is flowing in the sides of an equilateral triangle of side 2 m. Find the magnetic field at the centroid of the triangle.*

Solution. It is clear that all the three sides of the triangle will produce magnetic field at the centroid O in the same direction. Therefore, total magnetic field at O is = 3 × magnetic field due to one side.

Magnetic field at O due to side BC [See Fig. 7.33] is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2)$$

Here, $I = 1$ A ; $\phi_1 = \phi_2 = 60°$; $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$a = OD = \frac{BD}{\tan 60°} = \frac{BC/2}{\tan 60°} = \frac{2/2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad B_1 = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{1}{1/\sqrt{3}}(\sin 60° + \sin 60°)$$

$$= 10^{-7}\times\sqrt{3}\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = 3 \times 10^{-7}\ \text{T}$$

∴ Magnetic field at O due to the whole triangle is

$$B = 3B_1 = 3(3\times10^{-7}) = \mathbf{9 \times 10^{-7}\ T}$$

Example 7.16. *A square loop of wire of side 2l carries a current I. What is the magnetic field at the centre of the square ? If the square wire is reshaped into a circle, would the magnetic field increase or decrease at the centre ?*

Solution. Square loop. Figure 7.34 (i) shows the conditions of the problem. It is clear that each side of the square produces magnetic field at the centre O of the square in the same direction. Therefore, total magnetic field at O = 4 × magnetic field due to one side.

Magnetic field at O due to side AB is given by ;

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$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2)$$

Here $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $\phi_1 = \phi_2 = 45^\circ$; $a = OM = AB/2 = l$

$$\therefore \quad B_1 = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{I}{l}\ (\sin 45^\circ + \sin 45^\circ)$$

$$= 10^{-7}\times\frac{I}{l}\left(\frac{2}{\sqrt{2}}\right) = \sqrt{2}\frac{I}{l}\times10^{-7}\,\text{T}$$

Magnetic field at O due to the whole square is

$$B = 4B_1 = \mathbf{4\sqrt{2}\frac{\textit{I}}{\textit{l}}\times10^{-7}\,\textit{T}} \qquad ...(i)$$

Circular loop. The total length of the square loop $= 4 \times 2l = 8l$. When this square loop is shaped into a circular loop of radius r, then [See Fig. 7.34 (*ii*)],

$$2\pi r = 8l \qquad \text{or} \qquad r = \frac{8l}{2\pi} = \frac{4l}{\pi}$$

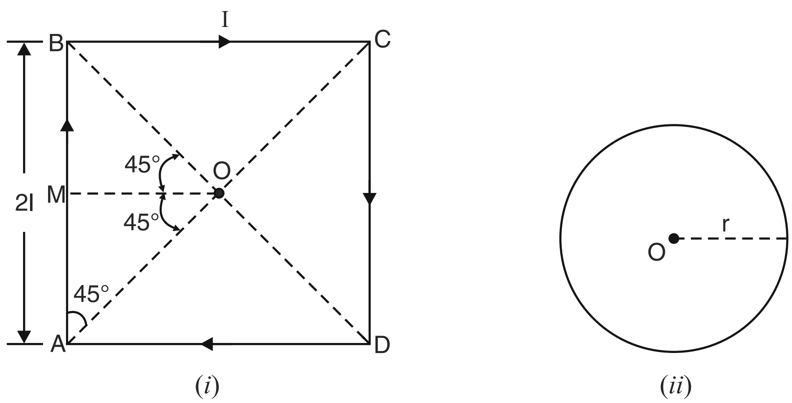

Fig. 7.34

Magnetic field at the centre of the circular loop is

$$B = \frac{\mu_0 I}{2r} = \frac{4\pi\times10^{-7}\times I}{2(4l/\pi)} = \frac{\pi^2}{2}\times\frac{I}{l}\times10^{-7}$$

$$\therefore \quad B = \mathbf{4.93\times\frac{\textit{I}}{\textit{l}}\times10^{-7}\,\textit{T}} \qquad ...(ii)$$

Comments. Inspection of eqs. (*i*) and (*ii*) reveals that magnetic field in case of square loop will be more.

Example 7.17. *A current of 15A is passing along a straight wire. Calculate the force on a unit N-pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit N-pole when carrying a current of 15A.*

Solution. By definition, the force on the unit N-pole is the magnetising force H. Therefore, force on a unit N-pole placed at a point 0.15 m (*i.e.* $a =$ 0.15m) from a long straight wire carrying current I(= 15A) is given by ;

$$H = \frac{I}{2\pi a} = \frac{15}{2\pi\times0.15} = \mathbf{\frac{50}{\pi}\ \text{AT/m or N/Wb}}$$

Force on a unit N-pole placed at the centre of a loop of radius r when the loop carries a current I(= 15 A) is

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$$H' = \frac{I}{2r} = \frac{15}{2r} \text{ AT/m}$$

As per the statement of the problem, $H' = H$.

$$\therefore \quad \frac{15}{2r} = \frac{50}{\pi} \quad \text{or} \quad r = \frac{15\pi}{2 \times 50} = 0.4713 \text{ m}$$

$\therefore$ Diameter of loop, $D = 2r = 2 \times 0.4713 = 0.9426$ m = **94.26 cm**

Tutorial Problems

1. A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of magnetic field due to the current 1·5 m below the wire ? **[1·2 × 10^{-5} T *towards south*]**

2. A long straight wire is turned into a loop of radius 10 cm as shown in Fig. 7.35. If a current of 8 A is passed, then find the value of magnetic field at the centre O of the loop.
[3·4 × 10^{-5} T *perpendicular to plane of paper pointing upward*]
[**Hint :** The magnetic field at O due to straight wire is perpendicular to the plane of paper and is directed downward. However, field due to circular loop is directed in opposite direction.]

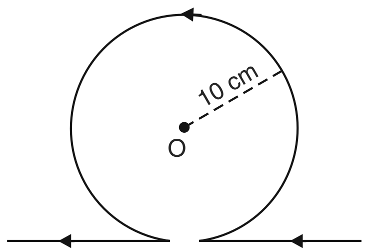

Fig. 7.35

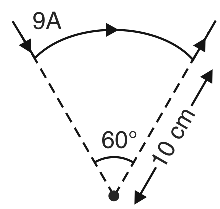

Fig. 7.36

3. A circular segment of radius 10 cm subtends an angle of 60° at its centre. A current of 9 A is flowing through it. Find the magnitude and direction of magnetic field produced at the centre [See Fig. 7.36].
[9·42 × 10^{-6} T *perpendicular to the plane of paper pointing downward*]
[**Hint :** The magnetic field at the centre of a single turn circular coil is

$$B = \frac{\mu_0 I}{2a} \qquad \text{... } a \text{ is the radius of coil.}$$

For the given arc, $$B = \frac{60°}{360°}\left(\frac{\mu_0 I}{2a}\right)$$

4. A long wire having a semicircular loop of radius a carries a current I amperes as shown in Fig. 7.37. Find the magnetic field at the centre of the semicircular arc. $\left[\boldsymbol{\frac{\mu_0 I}{4a}}\right]$
[**Hint :** The straight portions AB and DE do not contribute to any magnetic field at O. Therefore, magnetic field at O is only due to semicircular loop.]

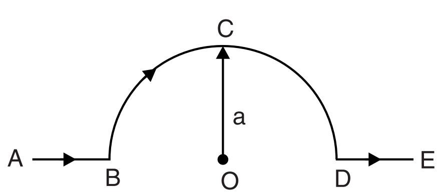

Fig. 7.37

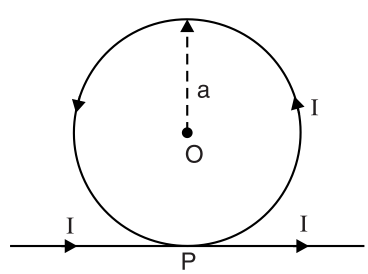

Fig. 7.38

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5. The wire shown in Fig. 7.38 carries a current I. What will be the magnitude and direction of magnetic field at the centre O ? Assume that various portions of wire do not touch each other at P.

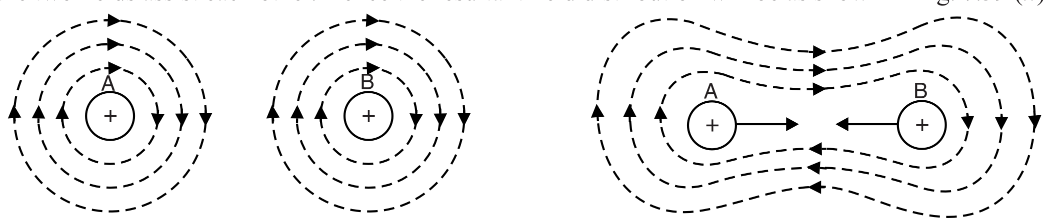
$$\left[\frac{\mu_0 I}{2a}\left(1+\frac{1}{\pi}\right) \textit{perpendicular to the plane of paper directed upward}\right]$$

[**Hint :** The magnetic field due to straight conductor and that due to circular part aid each other at O.]

7.28. Force Between Current-Carrying Parallel Conductors

When two current-carrying conductors are parallel to each other, a mechanical force acts on each of the conductors. This force is the result of each conductor being acted upon by the magnetic field produced by the other. *If the currents are in the same direction, the forces are attractive ; if currents are in opposite direction, the forces are repulsive.* This can be beautifully illustrated by drawing the magnetic field produced by each conductor.

***(i)* Currents in the same direction.** Consider two parallel conductors A and B carrying currents in the same direction (*i.e.* into the plane of paper) as shown in Fig. 7.39 (*i*). Each conductor will set up its own magnetic field as shown. It is clear that in the space between A and B, the two fields are in opposition and hence they tend to cancel each other. However, in the space outside A and B, the two fields assist each other. Hence the resultant field distribution will be as shown in Fig. 7.39 (*ii*).



(*i*) (*ii*)

Fig. 7.39

Since magnetic lines of force behave as stretched elastic cords, the two conductors are attracted towards each other. Alternatively, the conductors can be viewed as moving away from the relatively strong field (in the space outside A and B) into the weaker field between the conductors.

***(ii)* Currents in opposite direction.** Consider two parallel conductors A and B carrying currents in the opposite direction as shown in Fig. 7.40. Each conductor will set up its own field as shown. It is clear that in the space outside A and B, the two fields are in opposition and hence they tend to cancel each other. However, in the space between A and B, the two fields assist each other. The lateral pressure between lines of force exerts a force on the conductors tending to push them apart. In other words, the conductors experience a repulsive force.

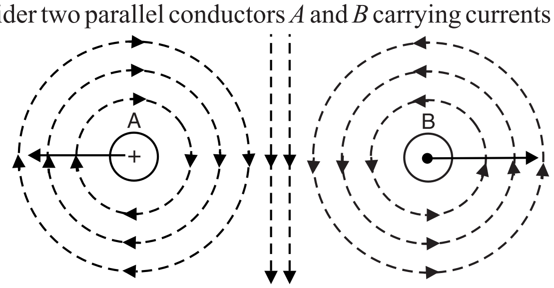


Fig. 7.40

If currents are in the same directions, the conductors attract each other ; if currents are in opposite directions, the conductors repel each other.

7.29. Magnitude of Mutual Force

Fig. 7.41 (*i*) shows two parallel conductors placed in air and carrying currents in the same direction. Here I_1 and I_2 are the currents in conductors 1 and 2 respectively, l is the length of each conductor in metres and d is the distance between conductors in metres. It is clear that each of the two parallel conductors lies in the magnetic field of the other conductor.

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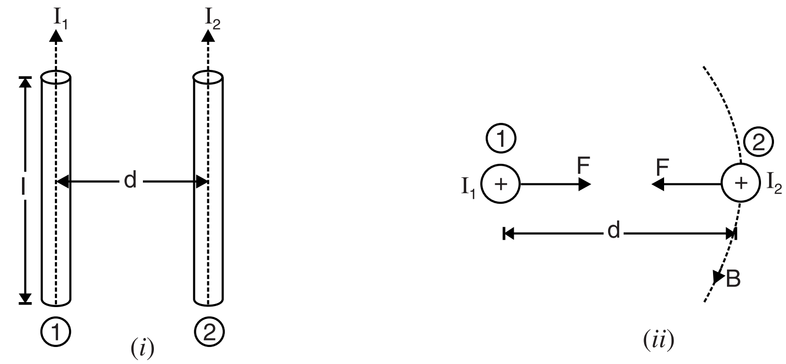

Fig. 7.41

In order to determine the magnitude of force, we can consider conductor 2 placed in the magnetic field produced by conductor 1 as shown in Fig. 7.41 (*ii*). Now field intensity H due to current I_1 in conductor 1 at the centre of conductor 2 is given by ;

$$H = \frac{I_1}{2\pi d}$$

$$\text{But} \quad B = \mu_0 \mu_r H = \mu_0 H = \frac{\mu_0 I_1}{2\pi d} \quad [\text{For air, } \mu_r = 1]$$

Force acting on conductor 2 is given by ;

$$F = B\, I_2\, l = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 l$$

$$= \frac{4\pi \times 10^{-7}\, I_1\, I_2\, l}{2\pi\, d} = \frac{2\, I_1\, I_2\, l}{d} \times 10^{-7} \text{ newtons}$$

$$\therefore \quad F = \frac{2\, I_1\, I_2\, l}{d} \times 10^{-7} \text{ N}$$

It can be easily shown that conductor 1 will experience an equal force in the opposite direction [See Fig. 7.41 (*ii*)].

Force per metre run of the conductor is given by ;

$$F' = \frac{2 I_1 I_2}{d} \times 10^{-7} \text{ N/m}$$

According to Fleming's left-hand rule, the two conductors will attract each other.

7.30. Definition of Ampere

The force acting between two parallel conductors has led to the modern definition of an ampere. We have seen above that force between two parallel current-carrying conductors is

$$F = \frac{2\, I_1\, I_2\, l}{d} \times 10^{-7} \text{ newtons}$$

If $I_1 = I_2 = 1\text{A}$; $l = 1$ m ; $d = 1$ m, then,

$$F = \frac{2 \times 1 \times 1 \times 1}{1} \times 10^{-7} = 2 \times 10^{-7} \text{ N}$$

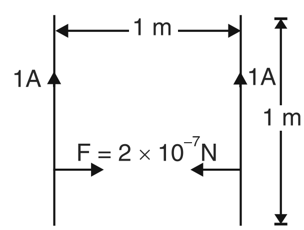

Fig. 7.42

Hence **one ampere** *is that current which, if maintained in two long parallel conductors, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2 × 10^{-7} newton per metre of length (See Fig. 7.42).*

Historically, the ampere was fixed originally in a very different way. The constant 2 × 10^{-7} that appears in the modern definition was chosen so as to keep the magnitude of ampere the same as formerly.

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Example 7.18. *Two long horizontal wires are kept parallel at a distance of 0·2 cm apart in a vertical plane. Both the wires have equal currents in the same direction. The lower wire has a mass of 0·05 kg/m. If the lower wire appears weightless, what is the current in each wire ?*

Solution. Let I amperes be the current in each wire. The lower wire is acted upon by two forces *viz* (*i*) upward magnetic force and (*ii*) downward force due to weight of the wire. Since the lower wire appears weightless, the two forces are equal over 1m length of the wire.

$$\text{Upward force/m length} = \frac{2I_1I_2}{d}\times 10^{-7} = \frac{2\times I\times I\times 10^{-7}}{0.2\times 10^{-2}} = 10^{-4}\,I^2\text{ N}$$

$$\text{Downward force/m length} = mg = 0{\cdot}05\times 9{\cdot}8 = 0{\cdot}49\text{ N}$$

$$\therefore\quad 10^{-4}\,I^2 = 0{\cdot}49 \quad\text{or}\quad I = \sqrt{0.49\times 10^4} = \mathbf{70\ A}$$

Example 7.19. *A rectangular loop ABCD carrying a current of 16A in clockwise direction is placed with its longer side parallel to a straight conductor 4 cm apart and carrying a current of 20A as shown in Fig. 7.43. The sides of the loop are 15 cm and 6 cm. What is the net force on the loop ? What will be the difference in force if the direction of current in the loop is reversed ?*

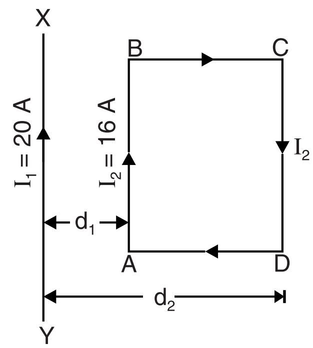

Fig. 7.43

Solution. Fig. 7.43 shows the arrangement. The long straight conductor XY will exert an attractive force on arm AB of the loop while arm CD will experience a repulsive force. The forces on the arms BC and AD will be equal and opposite and hence cancel out. Referring to Fig. 7.43,

$d_1 = 4$ cm $= 0{\cdot}04$ m ; $d_2 = 4 + 6 = 10$ cm $= 0{\cdot}1$ m

$$\text{Force on arm } AB,\ F_1 = \frac{2\,I_1\,I_2}{d_1}\times 10^{-7}\times \text{Length } AB \qquad \text{... towards } XY$$

$$= \frac{2\times 20\times 16}{0\cdot 04}\times 10^{-7}\times 0\cdot 15 = 2\cdot 4\times 10^{-4}\text{ N}$$

$$\text{Force on arm } CD,\ F_2 = \frac{2\,I_1\,I_2}{d_2}\times 10^{-7}\times \text{Length } CD \qquad \text{... away from } XY$$

$$= \frac{2\times 20\times 16}{0\cdot 1}\times 10^{-7}\times 0\cdot 15 = 0\cdot 96\times 10^{-4}\text{ N}$$

Net force on the loop is $F = F_1 - F_2 = 10^{-4}\,(2{\cdot}4 - 0{\cdot}96) = \mathbf{1{\cdot}44\times 10^{-4}\ N}$

Therefore, the net force on the loop is directed *towards* the current-carrying straight conductor XY. If the direction of current in the loop is reversed, the magnitude of net force on the loop remains the same (*i.e.* $F = 1{\cdot}44\times 10^{-4}$ N) but its direction will be away from the current-carrying straight conductor XY.

Example 7.20. *Two long straight parallel wires, standing in air 2m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is 2.4 × 10^{-4} N, evaluate I_1 and I_2.*

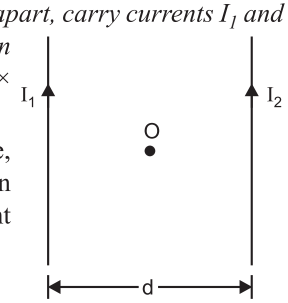

Fig. 7.44

Solution. Fig. 7.44 shows the conditions of the problem. Here, separation between the wires is $d = 2$ m and O is the point midway between the two wires. As proved in Art. 7.26, the magnetic intensity H at a point distant a from a long straight current-carrying wire is

$$H = \frac{I}{2\pi a}$$

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Since the two wires are carrying currents in the same direction, the net magnetic intensity H at O is the difference of the magnetic intensities at O due to two currents *i.e.*

$$H = H_1 - H_2$$

or $$7.95 = \frac{I_1}{2\pi\times 1} - \frac{I_2}{2\pi\times 1} \quad (\because \text{ point } O \text{ is 1m from each wire})$$

$\therefore$ $$I_1 - I_2 = 50 \quad ...(i)$$

As proved in Art. 7.29, force per unit length of the conductors is

$$F = \frac{2I_1I_2}{d}\times 10^{-7}$$

or $$2.4 \times 10^{-4} = \frac{2I_1I_2}{2}\times 10^{-7}$$

$\therefore$ $$I_1I_2 = 2400$$

Now, $$(I_1 + I_2)^2 = (I_1 - I_2)^2 + 4I_1I_2 = (50)^2 + 4 \times 2400 = 12100$$

$\therefore$ $$I_1 + I_2 = 110 \quad ...(ii)$$

From eqs. (*i*) and (*ii*), I_1 = **80A** ; I_2 = **30A**

Example 7.21. *A horizontal straight wire 5 cm long of mass 1·2 g/m is placed perpendicular to a uniform magnetic field of 0·6 T. If resistance of the wire is 3·8 Ω m*$^{-1}$*, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting.*

Solution. The current (I) in the wire is to be in such a direction that magnetic force acts on it vertically upward. To make the wire self-supporting, its weight should be equal to the upward magnetic force *i.e.*

$$B\,I\,l = m\,g \quad (\because \theta = 90°)$$

or $$I = \frac{mg}{Bl}$$

Here, $m = 1{\cdot}2 \times 10^{-3}\, l$; $B = 0{\cdot}6$ T ; $g = 9{\cdot}8$ ms^{-2}

$\therefore$ $$I = \frac{(1\cdot 2\times 10^{-3} l)\times 9\cdot 8}{0\cdot 6\times l} = 19{\cdot}6 \times 10^{-3}\text{ A}$$

Resistance of the wire, $R = 0{\cdot}05 \times 3{\cdot}8 = 0{\cdot}19\ \Omega$

$\therefore$ Required P.D., $V = I\,R = (19{\cdot}6 \times 10^{-3})\, 0{\cdot}19 =$ **$3{\cdot}7 \times 10^{-3}$ V**

Tutorial Problems

1. A pair of rising mains has a spacing of 200 mm between centres. If each conductor carries 500 A, determine the force between the conductors for each 10m length of run. **[2·5 N *repulsive*]**
2. Two busbars, each 20 m long, feed a circuit and are spaced at a distance of 80 mm inbetween centres. If a short-circuit current of 20,000 A flows through the conductors, calculate the force per metre between the bars. **[1000 N]**
3. Two long straight parallel conductors carry the same current I in the same direction. The conductors are placed 20 cm apart in air. The magnetic flux density between the conductors 5 cm from one of them is $1{\cdot}33 \times 10^{-5}$ Wb/m^2. If the force on each conductor per metre length is 25×10^{-6} N, find the current in each conductor. **[5 A]**
4. The wires that supply current to a 120 V, 2kW electric heater are 2 mm apart. What is the force per metre between the wires ? **[0·028 N/m]**
5. The busbars 10 cm apart are supported by insulators every metre along their length. The busbars each carry a current of 15 kA. What is the force acting on each insulator ? **[450 N]**

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Objective Questions

1. When a magnet is heated,
   (*i*) it gains magnetism
   (*ii*) it loses magnetism
   (*iii*) it neither loses nor gains magnetism
   (*iv*) none of the above
2. The magnetic material used in permanent magnets is
   (*i*) iron (*ii*) soft steel
   (*iii*) nickel (*iv*) hardened steel
3. The magnetic material used in temporary magnets is
   (*i*) hardened steel (*ii*) cobalt steel
   (*iii*) soft iron (*iv*) tungsten steel
4. Magnetic flux density is a
   (*i*) vector quantity (*ii*) scalar quantity
   (*iii*) phasor (*iv*) none of the above
5. The relative permeability of a ferromagnetic material is 1000. Its absolute permeability will be
   (*i*) 10^6 H/m (*ii*) $4\pi \times 10^{-3}$ H/m
   (*iii*) $4\pi \times 10^{-11}$ H/m (*iv*) none of the above
6. The main advantage of temporary magnets is that we can
   (*i*) change the magnetic flux
   (*ii*) use any magnetic material
   (*iii*) decrease the hysteresis loss
   (*iv*) none of the above
7. One weber is equal to
   (*i*) 10^6 lines (*ii*) $4\pi \times 10^{-7}$ lines
   (*iii*) 10^{12} lines (*iv*) 10^8 lines
8. Magnetic field intensity is a
   (*i*) scalar quantity (*ii*) vector quantity
   (*iii*) phasor (*iv*) none of the above
9. The absolute permeability of a material having a flux density of 1 Wb/m^2 is 10^{-3} H/m. The value of magnetising force is
   (*i*) 10^{-3} AT/m (*ii*) $4\pi \times 10^{-3}$ AT/m
   (*iii*) 1000 AT/m (*iv*) $4\pi \times 10^3$ AT/m
10. When the relative permeability of a material is slightly less than 1, it is called a
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
11. The greater percentage of substances are
    (*i*) diamagnetic (*ii*) paramagnetic
    (*iii*) ferromagnetic (*iv*) none of the above
12. When the relative permeability of material is much greater than 1, it is called
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
13. The magnetic flux density in an air-cored coil is 10^{-2} Wb/m^2. With a cast iron core of relative permeability 100 inserted, the flux density will become
    (*i*) 10^{-4} Wb/m^2 (*ii*) 10^4 Wb/m^2
    (*iii*) 10^{-2} Wb/m^2 (*iv*) 1 Wb/m^2
14. Which of the following is more suitable for the core of an electromagnet ?
    (*i*) soft iron (*ii*) air
    (*iii*) steel (*iv*) tungsten steel
15. The source of a magnetic field is
    (*i*) an isolated magnetic pole
    (*ii*) static electric charge
    (*iii*) magnetic substances
    (*iv*) current loop
16. A magnetic needle is kept in a uniform magnetic field. It experiences
    (*i*) a force and a torque
    (*ii*) a force but not a torque
    (*iii*) a torque but not a force
    (*iv*) neither a torque nor a force
17. The unit of pole strength is
    (*i*) A/m^2 (*ii*) Am
    (*iii*) Am^2 (*iv*) Wb/m^2
18. When the relative permeability of a material is slightly more than 1, it is called a
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
19. AT/m is the unit of
    (*i*) m.m.f.
    (*ii*) reluctance
    (*iii*) magnetising force
    (*iv*) magnetic flux density
20. A magnetic needle is kept in a non-uniform magnetic field. It experiences

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(i) a force and a torque
(ii) a force but not a torque
(iii) a torque but not a force
(iv) neither a force nor a torque

21. Magnetic flux passes more readily through
(i) air (ii) wood
(iii) vacuum (iv) iron

22. Iron is ferromagnetic
(i) above 770°C
(ii) below 770°C
(iii) at all temperatures
(iv) none of the above

23. The relative permeability of a material is 0·9998. It is
(i) diamagnetic (ii) paramagnetic
(iii) ferromagnetic (iv) none of the above

24. Magnetic lines of force
(i) intersect at infinity
(ii) intersect within the magnet
(iii) cannot intersect at all
(iv) none of the above

25. Demagnetising of magnets can be done by
(i) rough handling (ii) heating
(iii) magnetising in opposite direction
(iv) all of the above

Answers

1. (ii)	**2.** (iv)	**3.** (iii)	**4.** (i)	**5.** (ii)
6. (i)	**7.** (iv)	**8.** (ii)	**9.** (iii)	**10.** (i)
11. (ii)	**12.** (iii)	**13.** (iv)	**14.** (i)	**15.** (iv)
16. (iii)	**17.** (ii)	**18.** (ii)	**19.** (iii)	**20.** (i)
21. (iv)	**22.** (ii)	**23.** (i)	**24.** (iii)	**25.** (iv)

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