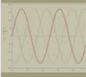
Revised Edition

Basic Electrical Engineering

(For B.E./B.Tech. and other Engineering Examinations)

V.K. MEHTA
ROHIT MEHTA



S. CHAND

BASIC ELECTRICAL ENGINEERING

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Preface to Sixth Edition

The general response to the Fifth Edition of the book was very encouraging. Authors feel that their work has been amply rewarded and wish to express their deep sense of gratitude to the large number of readers who have used it and in particular to those of them who have sent helpful suggestions from time to time for the improvement of the book.

The popularity of the book is judged from the fact that authors frequently receive feedback from many quarters including teachers, students and serving engineers. This feedback helps the authors to make the book up-to-date. In the present edition, many new topics/numericals/illustrations have been added to make the book more useful.

Authors lay no claim to the original research in preparing the book. Liberal use of materials available in the works of eminent authors has been made. What they may claim, in all modesty, is that they have tried to fashion the vast amount of material available from primary and secondary sources into coherent body of description and analysis.

The authors wish to thank their colleagues and friends who have contributed many valuable suggestions regarding the scope and content sequence of the book. Authors are also indebted to S. Chand & Company Ltd., New Delhi for bringing out this revised edition in a short time and pricing the book moderately inspite of heavy cost of paper and printing.

Errors might have crept in despite utmost care to avoid them. Authors shall be grateful if these are pointed out along with other suggestions for the improvement of the book.

V.K. MEHTA

ROHIT MEHTA

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Basic Concepts

Introduction

Everybody is familiar with the functions that electricity can perform. It can be used for lighting, heating, traction and countless other purposes. The question always arises, "What is electricity" ? Several theories about electricity were developed through experiments and by observation of its behaviour. The only theory that has survived over the years to explain the nature of electricity is the *Modern Electron theory of matter*. This theory has been the result of research work conducted by scientists like Sir William Crooks, J.J. Thomson, Robert A. Millikan, Sir Earnest Rutherford and Neils Bohr. In this chapter, we shall deal with some basic concepts concerning electricity.

1.1. Nature of Electricity

We know that matter is electrical in nature *i.e.* it contains particles of electricity *viz*. protons and electrons. The positive charge on a proton is equal to the negative charge on an electron. Whether a given body exhibits electricity (*i.e.* charge) or not depends upon the relative number of these particles of electricity.

(*i*) If the number of protons is equal to the number of electrons in a body, the resultant charge is zero and the body will be electrically neutral. Thus, the paper of this book is electrically neutral (*i.e.* paper exhibits no charge) because it has the same number of protons and electrons.

(*ii*) If from a neutral body, some *electrons are removed, there occurs a deficit of electrons in the body. Consequently, the body attains a *positive charge*.

(*iii*) If a neutral body is supplied with electrons, there occurs an excess of electrons. Consequently, the body attains a *negative charge*.

1.2. Unit of Charge

The charge on an electron is so small that it is not convenient to select it as the unit of charge. In practice, *coulomb* is used as the unit of charge *i.e.* SI unit of charge is coulomb abbreviated as C. *One coulomb of charge is equal to the charge on 625 × 10^{16} electrons, i.e.*

$$1 \text{ coulomb} = \text{Charge on } 625 \times 10^{16} \text{ electrons}$$

Thus when we say that a body has a positive charge of one coulomb (*i.e.* +1 C), it means that the body has a deficit of 625 × 10^{16} electrons from normal due share. The charge on one electron is given by ;

$$\text{Charge on electron} = -\frac{1}{625\times10^{16}} = -1.6 \times 10^{-19} \text{ C}$$

1.3. The Electron

Since electrical engineering generally deals with tiny particles called electrons, these small particles require detailed study. We know that an electron is a negatively charged particle having negligible mass. Some of the important properties of an electron are :

(*i*) Charge on an electron, $e = 1.602 \times 10^{-19}$ coulomb

(*ii*) Mass of an electron, $m = 9.0 \times 10^{-31}$ kg

(*iii*) Radius of an electron, $r = 1.9 \times 10^{-15}$ metre

* Electrons have very small mass and, therefore, are much more mobile than protons. On the other hand, protons are powerfully held in the nucleus and cannot be removed or detached.

The ratio *e*/*m* of an electron is 1.77×10^{11} coulombs/kg. This means that mass of an electron is very small as compared to its charge. It is due to this property of an electron that it is very mobile and is greatly influenced by electric or magnetic fields.

1.4. Energy of an Electron

An electron moving around the nucleus possesses two types of energies *viz*. kinetic energy due to its motion and potential energy due to the charge on the nucleus. The total energy of the electron is the sum of these two energies. The energy of an electron increases as its distance from the nucleus increases. Thus, an electron in the second orbit possesses more energy than the electron in the first orbit ; electron in the third orbit has higher energy than in the second orbit. It is clear that electrons in the last orbit possess very high energy as compared to the electrons in the inner orbits. These last orbit electrons play an important role in determining the physical, chemical and electrical properties of a material.

1.5. Valence Electrons

The electrons in the outermost orbit of an atom are known as **valence electrons**.

The outermost orbit can have a maximum of 8 electrons *i.e.* the maximum number of valence electrons can be 8. The valence electrons determine the physical and chemical properties of a material. These electrons determine whether or not the material is chemically active; metal or non-metal or, a gas or solid. These electrons also determine the electrical properties of a material.

On the basis of electrical conductivity, materials are generally classified into *conductors, insulators* and *semi-conductors*. As a rough rule, one can determine the electrical behaviour of a material from the number of valence electrons as under :

(*i*) *When the number of valence electrons of an atom is less than 4* (*i.e.* half of the maximum eight electrons), the material is usually *a metal and a conductor*. Examples are sodium, magnesium and aluminium which have 1, 2 and 3 valence electrons respectively.

(*ii*) *When the number of valence electrons of an atom is more than 4*, the material is usually *a non-metal and an insulator*. Examples are nitrogen, sulphur and neon which have 5, 6 and 8 valence electrons respectively.

(*iii*) *When the number of valence electrons of an atom is 4* (*i.e.* exactly one-half of the maximum 8 electrons), the material has both metal and non-metal properties and is usually a *semi-conductor*. Examples are carbon, silicon and germanium.

1.6. Free Electrons

We know that electrons move around the nucleus of an atom in different orbits. The electrons in the inner orbits (*i.e.*, orbits close to the nucleus) are tightly bound to the nucleus. As we move away from the nucleus, this binding goes on decreasing so that electrons in the last orbit (called valence electrons) are quite loosely bound to the nucleus. In certain substances, especially metals (*e.g.* copper, aluminium etc.), the valence electrons are so weakly attached to their nuclei that they can be easily removed or detached. Such electrons are called free electrons.

Those valence electrons which are very loosely attached to the nucleus of an atom are called **free electrons.**

The free electrons move at random from one atom to another in the material. Infact, they are so loosely attached that they do not know the atom to which they belong. It may be noted here that all valence electrons in a metal are not free electrons. It has been found that one atom of a metal can provide at the most one free electron. Since a small piece of metal has billions of atoms, one can expect a very large number of free electrons in metals. For instance, one cubic centimetre of copper has about 8.5×10^{22} free electrons at room temperature.

(i) A substance which has a large number of free electrons at room temperature is called a **conductor** of electricity *e.g.* all metals. If a voltage source (*e.g.* a cell) is applied across the wire of a conductor material, free electrons readily flow through the wire, thus constituting electric current. The best conductors are silver, copper and gold in that order. Since copper is the least expensive out of these materials, it is widely used in electrical and electronic industries.

(ii) A substance which has very few free electrons is called an **insulator** of electricity. If a voltage source is applied across the wire of insulator material, practically no current flows through the wire. Most substances including plastics, ceramics, rubber, paper and most liquids and gases fall in this category. Of course, there are many practical uses for insulators in the electrical and electronic industries including wire coatings, safety enclosures and power-line insulators.

(iii) There is a third class of substances, called **semi-conductors**. As their name implies, they are neither conductors nor insulators. These substances have crystalline structure and contain very few free electrons at room temperature. Therefore, at room temperature, a semiconductor practically behaves as an insulator. However, if suitable controlled impurity is imparted to a semi-conductor, it is possible to provide controlled conductivity. Most common semi-conductors are silicon, germanium, carbon etc. However, *silicon* is the principal material and is widely used in the manufacture of electronic devices (*e.g.* crystal diodes, transistors etc.) and integrated circuits.

1.7. Electric Current

The directed flow of free electrons (*or charge*) *is called* **electric current**. The flow of electric current can be beautifully explained by referring to Fig. 1.1. The copper strip has a large number of free electrons. When electric pressure or voltage is applied, then free electrons, being negatively charged, will start moving towards the positive terminal around the circuit as shown in Fig. 1.1. This directed flow of electrons is called electric current.

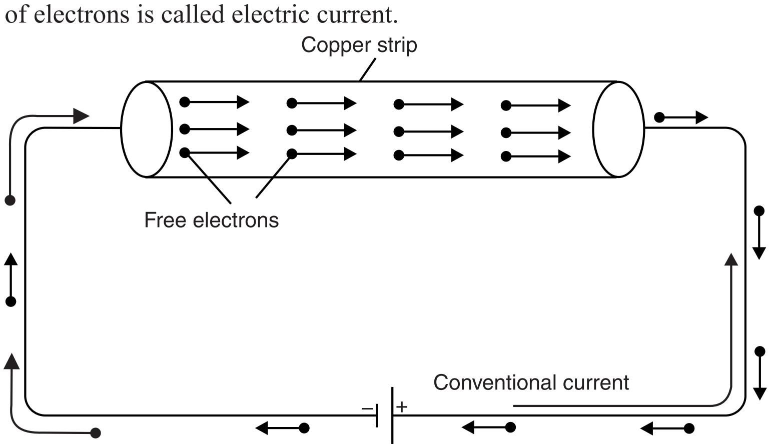

Fig. 1.1

The reader may note the following points :

(i) Current is flow of electrons and electrons are the constituents of matter. Therefore, electric current is matter (*i.e.* free electrons) in motion.

(ii) The actual direction of current (*i.e.* flow of electrons) is from negative terminal to the positive terminal through that part of the circuit external to the cell. However, prior to Electron theory, it was assumed that current flowed from positive terminal to the negative terminal of the cell *via* the circuit. This convention is so firmly established that it is still in use. This assumed direction of current is now called *conventional current*.

Unit of Current. The strength of electric current I is the rate of flow of electrons *i.e.* charge flowing per second.

$$\therefore \qquad \text{Current, } I = \frac{Q}{t}$$

The charge Q is measured in coulombs and time t in seconds. Therefore, the unit of electric current will be *coulombs/sec or ampere*. If $Q = 1$ coulomb, $t = 1$ sec, then $I = 1/1 = 1$ ampere.

One ampere *of current is said to flow through a wire if at any cross-section one coulomb of charge flows in one second.*

Thus, if 5 amperes current is flowing through a wire, it means that 5 coulombs per second flow past any cross-section of the wire.

Note. 1 C = charge on 625×10^{16} electrons. Thus when we say that current through a wire is 1 A, it means that 625×10^{16} electrons per second flow past any cross-section of the wire.

$$\therefore \qquad I = \frac{Q}{t} = \frac{ne}{t} \qquad \text{where } e = -1.6 \times 10^{-19}\text{ C ; } n = \text{number of electrons}$$

1.8. Electric Current is a Scalar Quantity

(*i*) Electric current, $I = \dfrac{Q}{t}$

As both charge and time are scalars, electric current is a scalar quantity.

(*ii*) We show electric current in a wire by an arrow to indicate the direction of flow of positive charge. But such arrows are not vectors because they do not obey the laws of vector algebra. This point can be explained by referring to Fig. 1.2. The wires *OA* and *OB* carry currents of 3 A and 4 A respectively. The total current in the wire *CO* is 3 + 4 = 7 A irrespective of the angle between the wires *OA* and *OB*. This is not surprising because the charge is conserved so that the magnitudes of currents in wires *OA* and *OB* must add to give the magnitude of current in the wire *CO*.

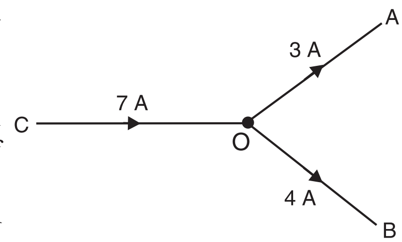

Fig. 1.2

1.9. Types of Electric Current

The electric current may be classified into three main classes: (*i*) steady current (*ii*) varying current and (*iii*) alternating current.

(*i*) Steady current. *When the magnitude of current does not change with time, it is called a steady current.* Fig. 1.3 (*i*) shows the graph between steady current and time. Note that value of current remains the same as the time changes. The current provided by a battery is almost a steady current (*d.c.*).

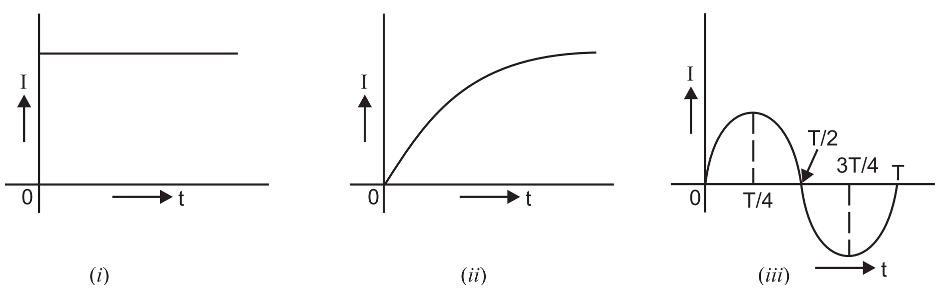

Fig. 1.3

(*ii*) Varying current. *When the magnitude of current changes with time, it is called a varying current*. Fig. 1.3 (*ii*) shows the graph between varying current and time. Note that value of current varies with time.

(*iii*) Alternating current. *An alternating current is one whose magnitude changes continuously with time and direction changes periodically.* Due to technical and economical reasons, we produce alternating currents that have sine waveform (or cosine waveform) as shown in Fig. 1.3 (*iii*). It is called *alternating current* because current flows in alternate directions in the circuit, *i.e.,* from 0 to $T/2$ second (T is the time period of the wave) in one direction and from $T/2$ to T second in the opposite direction. The current provided by an a.c. generator is alternating current that has sine (or cosine) waveform.

1.10. Mechanism of Current Conduction in Metals

Every metal has a large number of free electrons which wander randomly within the body of the conductor somewhat like the molecules in a gas. The average speed of free electrons is sufficiently high ($\simeq 10^5$ ms^{-1}) at room temperature. During random motion, the free electrons collide with positive ions (positive atoms of metal) again and again and after each collision, their direction of motion changes. When we consider all the free electrons, their random motions average to zero. In other words, there is no net flow of charge (electrons) in any particular direction. Consequently, no current is established in the conductor.

When potential difference is applied across the ends of a conductor (say copper wire) as shown in Fig. 1.4, electric field is applied at every point of the copper wire. The electric field exerts force on the free electrons which start accelerating towards the positive terminal (*i.e.,* opposite to the direction of the field). As the free electrons move, they *collide again and again with positive ions of the metal. Each collision destroys the extra velocity gained by the free electrons.

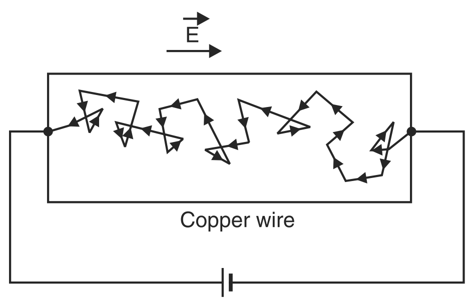

Fig. 1.4

The average time that an electron spends between two collisions is called the **relaxation time** (τ). Its value is of the order of 10^{-14} second.

Although the free electrons are continuously accelerated by the electric field, collisions prevent their velocity from becoming large. The result is that electric field provides a small constant velocity towards positive terminal which is superimposed on the random motion of the electrons. This constant velocity is called the drift velocity.

The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called **drift velocity** $(\vec{v}_d)$. The drift velocity of free electrons is of the order of 10^{-5} ms^{-1}.

Thus when a metallic conductor is subjected to electric field (or potential difference), free electrons move towards the positive terminal of the source with drift velocity $\vec{v}_d$. Small though it is, the drift velocity is entirely responsible for electric current in the metal.

Note. The reader may wonder that if electrons drift so slowly, how room light turns on quickly when switch is closed ? The answer is that propagation of electric field takes place with the speed of light. When we apply electric field (*i.e.,* potential difference) to a wire, the free electrons everywhere in the wire begin drifting almost at once.

* What happens to an electron after collision with an ion ? It moves off in some new and quite random direction. However, it still experiences the applied electric field, so it continues to accelerate again, gaining a velocity in the direction of the positive terminal. It again encounters an ion and loses its directed motion. This situation is repeated again and again for every free electron in a metal.

1.11. Relation Between Current and Drift Velocity

Consider a portion of a copper wire through which current I is flowing as shown in Fig. 1.5. Clearly, copper wire is under the influence of electric field.

Let A = area of X-section of the wire

n = electron density, *i.e.,* number of free electrons per unit volume

e = charge on each electron

v_d = drift velocity of free electrons

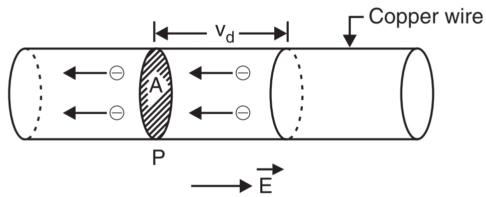

Fig. 1.5

In one second, all those free electrons within a distance v_d to the right of cross-section at P (*i.e.,* in a volume Av_d) will flow through the cross-section at P as shown in Fig. 1.5. This volume contains $n\,Av_d$ electrons and, hence, a charge $(nAv_d)e$. Therefore, a charge of $neAv_d$ per second passes the cross-section at P.

$$\therefore \qquad I = n\,e\,A\,v_d$$

Since A, n and e are constant, $I \propto v_d$.

Hence, current flowing through a conductor is directly proportional to the drift velocity of free electrons.

(i) The drift velocity of free electrons is very small. Since the number of free electrons in a metallic conductor is very large, even small drift velocity of free electrons gives rise to sufficient current.

(ii) The current density J is defined as current per unit area and is given by ;

Current density, $J = \dfrac{I}{A} = \dfrac{n\,e\,Av_d}{A} = n\,e\,v_d$

The SI unit of current density is amperes/m^2.

Note. Current density is a vector quantity and is denoted by the symbol $\vec{J}$. Therefore, in vector notation, the relation between I and $\vec{J}$ is $I = \vec{J}.\vec{A}$

where $\vec{A}$ = Area vector

Example 1.1. *A 60 W light bulb has a current of 0.5 A flowing through it. Calculate (i) the number of electrons passing through a cross-section of the filament (ii) the number of electrons that pass the cross-section in one hour.*

Solution. **(i)**
$$I = \frac{Q}{t} = \frac{n\,e}{t}$$

$$\therefore \qquad n = \frac{I\,t}{e} = \frac{0.5\times 1}{1.6\times 10^{-19}} = \mathbf{3.1 \times 10^{18}\ electrons/s}$$

(ii) Charge passing the cross-section in one hour is

$$Q = I\,t = (0.5) \times (60 \times 60) = 1800 \text{ C}$$

Now, $\qquad Q = n\,e$

$$\therefore \qquad n = \frac{Q}{e} = \frac{1800}{1.6\times 10^{-19}} = \mathbf{1.1 \times 10^{22}\ electrons/hour}$$

Example 1.2. *A copper wire of area of X-section 4 mm^2 is 4 m long and carries a current of 10 A. The number density of free electrons is 8×10^{28} m^{-3}. How much time is required by an electron to travel the length of wire ?*

Solution. $\qquad I = n\,A\,e\,v_d$

Here $I = 10\text{ A} ; A = 4\text{ mm}^2 = 4 \times 10^{-6}\text{ m}^2 ; e = 1.6 \times 10^{-19}\text{ C} ; n = 8 \times 10^{28}\text{ m}^{-3}$

$\therefore$ Drift velocity, $$v_d = \frac{I}{n\,A\,e} = \frac{10}{8\times10^{28}\times(4\times10^{-6})\times1.6\times10^{-19}} = 1.95\times10^{-4}\text{ ms}^{-1}$$

$\therefore$ Time taken by the electron to travel the length of the wire is

$$t = \frac{l}{v_d} = \frac{4}{1.95\times10^{-4}} = 2.05\times10^4\text{s} = \mathbf{5.7\ hours}$$

Example 1.3. *The area of X-section of copper wire is 3×10^{-6} m^2. It carries a current of 4.2 A. Calculate (i) current density in the wire and (ii) the drift velocity of electrons. The number density of conduction electrons is 8.4×10^{28} m^{-3}.*

Solution. (*i*) Current density, $$J = \frac{I}{A} = \frac{4.2}{3\times10^{-6}} = \mathbf{1.4 \times 10^6\ A/m^2}$$

(*ii*) $$I = n\,e\,A\,v_d$$

$\therefore$ Drift velocity, $$v_d = \frac{I}{n\,A\,e} = \frac{4.2}{(8.4\times10^{28})\times(1.6\times10^{-19})\times3\times10^{-6}} = \mathbf{1.04 \times 10^{-4}\ ms^{-1}}$$

Tutorial Problems

1. How much current is flowing in a circuit where 1.27×10^{15} electrons move past a given point in 100 ms ? **[2.03 A]**
2. The current in a certain conductor is 40 mA.
   (*i*) Find the total charge in coulombs that passes through the conductor in 1.5 s.
   (*ii*) Find the total number of electrons that pass through the conductor in that time.
   [(*i*) 60 mC (*ii*) 3.745×10^{17} electrons]
3. The density of conduction electrons in a wire is 10^{22} m^{-3}. If the radius of the wire is 0.6 mm and it is carrying a current of 2 A, what will be the average drift velocity ? **[1.1×10^{-3} ms^{-1}]**
4. Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section 1 cm^2 and length 10 km. Free electron density of copper = 8.5×10^{28} per m^3. How long will it take the electric charge to travel from one end of the conductor to the other ? **[0.735 μm/s; 431 years]**

1.12. Electric Potential

When a body is charged, work is done in charging it. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges either by attraction or repulsion. The ability of the charged body to do work is called electric potential.

The capacity of a charged body to do work is called its **electric potential.**

The greater the capacity of a charged body to do work, the greater is its electric potential. Obviously, the work done to charge a body to 1 coulomb will be a measure of its electric potential *i.e.*

Electric potential, $$V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

The work done is measured in joules and charge in coulombs. Therefore, the unit of electric potential will be *joules/coulomb* or *volt*. If $W = 1$ joule, $Q = 1$ coulomb, then $V = 1/1 = 1$ volt.

Hence a body is said to have an electric potential of **1 volt** *if 1 joule of work is done to give it a charge of 1 coulomb.*

Thus, when we say that a body has an electric potential of 5 volts, it means that 5 joules of work has been done to charge the body to 1 coulomb. In other words, every coulomb of charge possesses an energy of 5 joules. The greater the joules/coulomb on a charged body, the greater is its electric potential.

1.13. Potential Difference

The difference in the potentials of two charged bodies is called **potential difference.**

If two bodies have different electric potentials, a potential difference exists between the bodies. Consider two bodies A and B having potentials of 5 volts and 3 volts respectively as shown in Fig. 1.6 (*i*). Each coulomb of charge on body A has an energy of 5 joules while each coulomb of charge on body B has an energy of 3 joules. Clearly, body A is at higher potential than the body B.

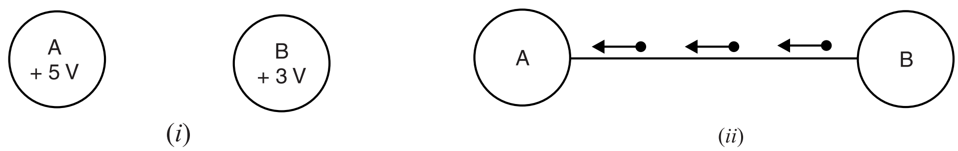

Fig. 1.6

If the two bodies are joined through a conductor [See Fig. 1.6 (*ii*)], then electrons will *flow from body B to body A. When the two bodies attain the same potential, the flow of current stops. Therefore, we arrive at a very important conclusion that current will flow in a circuit if potential difference exists. No potential difference, no current flow. It may be noted that potential difference is sometimes called voltage.

Unit. Since the unit of electric potential is volt, one can expect that unit of potential difference will also be *volt*. It is defined as under :

The potential difference between two points is **1 volt** *if one joule of work is **done or released in transferring 1 coulomb of charge from one point to the other.*

1.14. Maintaining Potential Difference

A device that maintains potential difference between two points is said to develop electromotive force (e.m.f.). A simple example is that of a cell. Fig. 1.7 shows the familiar voltaic cell. It consists of a copper plate (called anode) and a zinc rod (called cathode) immersed in dilute H_2SO_4.

The chemical action taking place in the cell removes electrons from copper plate and transfers them to the zinc rod. This transference of electrons takes place through the agency of dil. H_2SO_4 (called electrolyte). Consequently, the copper plate attains a positive charge of $+Q$ coulombs and zinc rod a charge of $-Q$ coulombs. The chemical action of the cell has done a certain amount of work (say W joules) to do so. Clearly, the potential difference between the two plates will be W/Q volts. If the two plates are joined through a wire, some electrons from zinc rod will be attracted through the wire to copper plate. The chemical action of the cell now transfers an equal amount of electrons from copper plate to zinc rod internally through the cell to maintain original potential difference (*i.e.* W/Q). This process continues so long as the

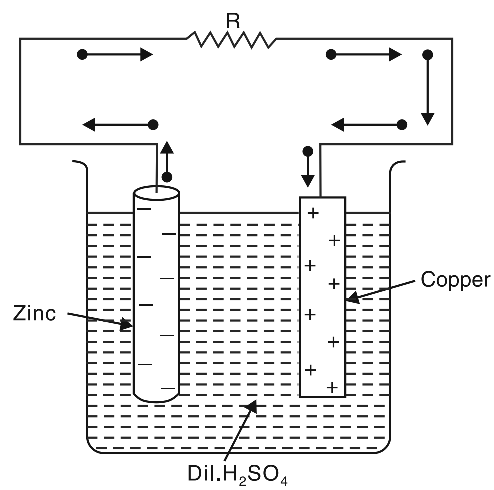

Fig. 1.7

* The conventional current flow will be in the opposite direction *i.e.* from body A to body B.

** 1 joule of work will be done in the case if 1 coulomb is transferred from point of lower potential to that of higher potential. However, 1 joule of work will be released (as heat) if 1 coulomb of charge moves from a point of higher potential to a point of lower potential.

circuit is complete or so long as there is chemical energy. The flow of electrons through the external wire from zinc rod to copper plate is the electric current.

Thus potential difference causes current to flow while an *e.m.f.* maintains the potential difference. Although both *e.m.f.* and *p.d.* are measured in volts, they do not mean exactly the same thing.

1.15. Concept of E.M.F. and Potential Difference

There is a distinct difference between *e.m.f.* and potential difference. The *e.m.f.* of a device, say a battery, is a measure of the energy the battery gives to each coulomb of charge. Thus if a battery supplies 4 joules of energy per coulomb, we say that it has an *e.m.f.* of 4 volts. The energy given to each coulomb in a battery is due to the chemical action.

The potential difference between two points, say *A* and *B*, is a measure of the energy used by one coulomb in moving from *A* to *B*. Thus if potential difference between points *A* and *B* is 2 volts, it means that each coulomb will give up an energy of 2 joules in moving from *A* to *B*.

Illustration. The difference between e.m.f. and p.d. can be made more illustrative by referring to Fig. 1.8. Here battery has an e.m.f. of 4 volts. It means battery supplies 4 joules of energy to each coulomb continuously. As each coulomb travels from the positive terminal of the battery, it gives up its most of energy to resistances (2 Ω and 2 Ω in this case) and remaining to connecting wires. When it returns to the negative terminal, it has lost all its energy originally supplied by the battery. The battery now supplies fresh energy to each coulomb (4 joules in the present case) to start the journey once again.

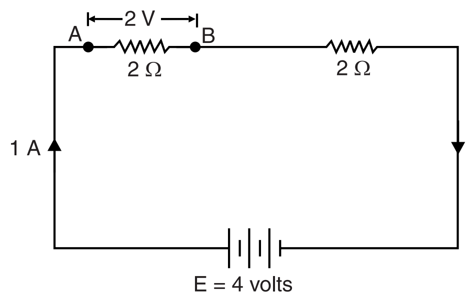

Fig. 1.8

The *p.d.* between any two points in the circuit is the energy used by one coulomb in moving from one point to another. Thus in Fig. 1.8, p.d. between *A* and *B* is 2 volts. It means that 1 coulomb will give up an energy of 2 joules in moving from *A* to *B*. This energy will be released as heat from the part *AB* of the circuit.

The following points may be noted carefully :

(i) The name e.m.f. at first sight implies that it is a force that causes current to flow. This is not correct because it is not a force but energy supplied to charge by some active device such as a battery.

(ii) *Electromotive force (e.m.f.) maintains potential difference while p.d. causes current to flow.*

1.16. Potential Rise and Potential Drop

Fig. 1.9 shows a circuit with a cell and a resistor. The cell provides a potential difference of 1.5 V. Since it is an energy source, there is a *rise* in potential associated with a cell. The cell's potential difference represents an e.m.f. so that symbol *E* could be used. The resistor is also associated with a potential difference. Since it is a consumer (converter) of energy, there is a *drop* in potential across the resistor. We can combine the idea of potential rise or drop with the popular term "voltage". It is customary to refer to the potential difference across the cell as a *voltage rise* and to the potential difference across the resistor as a *voltage drop*.

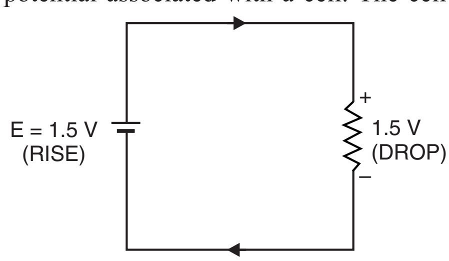

Fig. 1.9

Note. The term voltage refers to a potential difference across two points. There is no such thing as a voltage at one point. In cases where a single point is specified, some reference must be used as the other point. Unless stated otherwise, the ground or common point in any circuit is the reference when specifying a voltage at some other point.

Example 1.4. *A charge of 4 coulombs is flowing between points A and B of a circuit. If the potential difference between A and B is 2 volts, how many joules will be released by part AB of the circuit ?*

Solution. The p.d. of 2 volts between points A and B means that each coulomb of charge will give up an energy of 2 joules in moving from A to B. As the charge flowing is 4 coulombs, therefore, total energy released by part AB of the circuit is $= 4 \times 2 =$ **8 joules.**

Example 1.5. *How much work will be done by an electric energy source with a potential difference of 3 kV that delivers a current of 1 A for 1 minute ?*

Solution. We know that 1 A of current represents a charge transfer rate of 1 C/s. Therefore, the total charge for a period of 1 minute is $Q = It = 1 \times 60 = 60$ C.

$$\text{Total work done, } W = Q \times V = 60 \times (3 \times 10^3) = 180 \times 10^3 \text{ J} = \mathbf{180\ kJ}$$

Tutorial Problems

1. Calculate the potential difference of an energy source that provides 6.8 J for every milli-coulomb of charge that it delivers. **[6.8 kV]**
2. The potential difference across a battery is 9 V. How much charge must it deliver to do 50 J of work ? **[5.56 C]**
3. A 300 V energy source delivers 500 mA for 1 hour. How much energy does this represent ? **[540 kJ]**

1.17. Resistance

The opposition offered by a substance to the flow of electric current is called its **resistance.**

Since current is the flow of free electrons, resistance is the opposition offered by the substance to the flow of free electrons. This opposition occurs because atoms and molecules of the substance obstruct the flow of these electrons. Certain substances (*e.g.* metals such as silver, copper, aluminium etc.) offer very little opposition to the flow of electric current and are called conductors. On the other hand, those substances which offer high opposition to the flow of electric current (*i.e.* flow of free electrons) are called insulators *e.g.* glass, rubber, mica, dry wood etc.

It may be noted here that resistance is the electric friction offered by the substance and causes production of heat with the flow of electric current. The moving electrons collide with atoms or molecules of the substance ; each collision resulting in the liberation of minute quantity of heat.

Unit of resistance. The practical unit of resistance is ohm and is represented by the symbol Ω. It is defined as under :

A wire is said to have a resistance of **1 ohm** *if a p.d. of 1 volt across its ends causes 1 ampere to flow through it* (See Fig. 1.10).

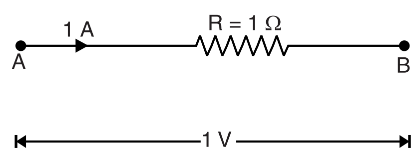

Fig. 1.10

There is another way of defining ohm.

A wire is said to have a resistance of **1 ohm** *if it releases 1 joule (or develops 0.24 calorie of heat) when a current of 1 A flows through it for 1 second.*

A little reflection shows that second definition leads to the first definition. Thus 1 A current flowing for 1 second means that total charge flowing is $Q = I \times t = 1 \times 1 = 1$ coulomb. Now the charge flowing between A and B (See Fig. 1.10) is 1 coulomb and energy released is 1 joule (or 0.24 calorie). Obviously, by definition, p.d. between A and B should be 1 volt.

1.18. Factors Upon Which Resistance Depends

The resistance R of a conductor

(*i*) is directly proportional to its length *i.e.*

$$R \propto l$$

(*ii*) is inversely proportional to its area of X-section *i.e.*

$$R \propto \frac{1}{a}$$

(*iii*) depends upon the nature of material.

(*iv*) depends upon temperature.

From the first three points (leaving temperature for the time being), we have,

$$R \propto \frac{l}{a} \quad \text{or} \quad R = \rho\frac{l}{a}$$

where ρ (Greek letter 'Rho') is a constant and is known as *resistivity* or *specific resistance* of the material. Its value depends upon the nature of the material.

1.19. Specific Resistance or Resistivity

We have seen above that $R = \rho\frac{l}{a}$

$$\text{If} \quad l = 1 \text{ m}, \; a = 1 \text{ m}^2, \text{ then}, \quad R = \rho$$

Hence **specific resistance** *of a material is the resistance offered by 1 m length of wire of material having an area of cross-section of 1 m*^{2} [See Fig. 1.11 (*i*)].

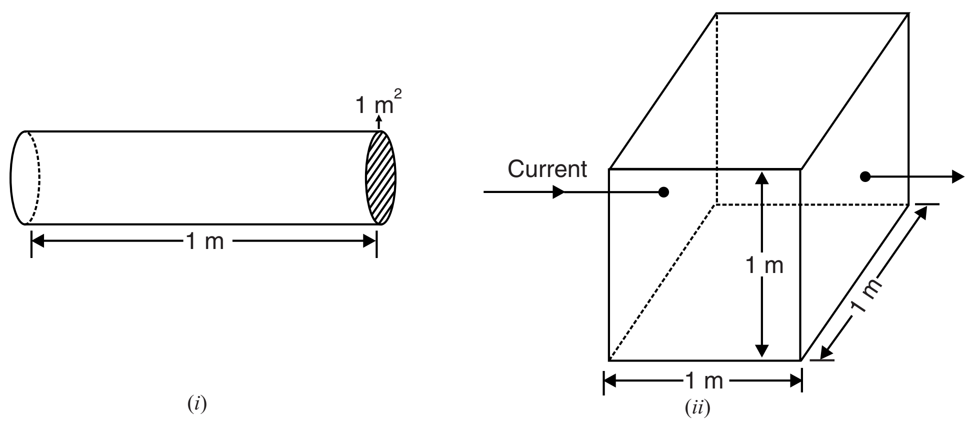

Fig. 1.11

Specific resistance can also be defined in another way. Take a cube of the material having each side 1 m. Considering any two opposite faces, the area of cross-section is 1 m^2 and length is 1 m [See Fig. 1.11 (*ii*)] *i.e.* $l = 1$ m, $a = 1$ m^2.

Hence **specific resistance** *of a material may be defined as the resistance between the opposite faces of a metre cube of the material.*

Unit of resistivity. We know $R = \frac{\rho l}{a}$ or $\rho = \frac{Ra}{l}$

Hence the unit of resistivity will depend upon the units of area of cross-section (a) and length (l).

(*i*) If the length is measured in metres and area of cross-section in m^2, then unit of resistivity will be ohm-metre (Ω m).

$$\rho = \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm-m}$$

(ii) If length is measured in cm and area of cross-section in cm^2, then unit of resistivity will be ohm-cm (Ω cm).

$$\rho = \frac{\text{ohm} \times \text{cm}^2}{\text{cm}} = \text{ohm-cm}$$

The resistivity of substances varies over a wide range. To give an idea to the reader, the following table may be referred :

S.No.	Material	Nature	Resistivity (Ω-m) at room temperature
1	Copper	metal	1.7×10^{-8}
2	Iron	metal	9.68×10^{-8}
3	Manganin	alloy	48×10^{-8}
4	Nichrome	alloy	100×10^{-8}
5	Pure silicon	semiconductor	2.5×10^{3}
6	Pure germanium	semiconductor	0.6
7	Glass	insulator	10^{10} to 10^{14}
8	Mica	insulator	10^{11} to 10^{15}

The reader may note that resistivity of metals and alloys is very small. Therefore, these materials are good conductors of electric current. On the other hand, resistivity of insulators is extremely large. As a result, these materials hardly conduct any current. There is also an intermediate class of semiconductors. The resistivity of these substances lies between conductors and insulators.

1.20. Conductance

The reciprocal of resistance of a conductor is called its **conductance** *(G).* If a conductor has resistance R, then its conductance G is given by ;

$$G = 1/R$$

Whereas resistance of a conductor is the opposition to current flow, the conductance of a conductor is the inducement to current flow.

The SI unit of conductance is mho (*i.e.*, ohm spelt backward). These days, it is a usual practice to use **siemen** as the unit of conductance. It is denoted by the symbol S.

Conductivity. *The reciprocal of resistivity of a conductor is called its* **conductivity.** It is denoted by the symbol σ. If a conductor has resistivity ρ, then its conductivity is given by ;

$$\text{Conductivity,} \quad \sigma = \frac{1}{\rho}$$

We know that $G = \frac{1}{R} = \frac{a}{\rho l} = \sigma \frac{a}{l}\cdot$ Clearly, the SI unit of conductivity is *Siemen metre*$^{-1}$ ($S\ m^{-1}$).

Example 1.6. *A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm*^{2}*. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.*

Solution. Length of coil, $l = 0.8 \times 2000 = 1600$ m; cross-sectional area of coil, $a = 0.8\ mm^2 = 0.8 \times 10^{-6} m^2$; Resistivity of copper, $\rho = 0.02 \times 10^{-6}\ \Omega m$

$$\therefore \quad \text{Resistance of coil,} \quad R = \rho \frac{l}{a} = 0.02 \times 10^{-6} \frac{1600}{0.8 \times 10^{-6}} = \mathbf{40\ \Omega}$$

$$\text{Power absorbed,} \quad P = \frac{V^2}{R} = \frac{(110)^2}{40} = \mathbf{302.5\ W}$$

Example 1.7. *Find the resistance of 1000 metres of a copper wire 25 sq. mm in cross-section. The resistance of copper is 1/58 ohm per metre length and 1 sq. mm cross-section. What will be the resistance of another wire of the same material, three times as long and one-half area of cross-section ?*

Solution. For the first case, $R_1 =$? ; $a_1 = 25$ mm^2 ; $l_1 = 1000$ m

For the second case, $R_2 = 1/58\ \Omega$; $a_2 = 1$ mm^2 ; $l_2 = 1$ m

$$R_1 = \rho\,(l_1/a_1)\ ;\quad R_2 = \rho\,(l_2/a_2)$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1}{l_2}\times\frac{a_2}{a_1} = \left(\frac{1000}{1}\right)\times\left(\frac{1}{25}\right) = 40$$

$$\text{or} \quad R_1 = 40\,R_2 = 40 \times \frac{1}{58} = \mathbf{\frac{20}{29}\,\Omega}$$

For the third case, $R_3 =$? ; $a_3 = a_1/2$; $l_3 = 3l_1$

$$\therefore \quad \frac{R_3}{R_1} = \left(\frac{l_3}{l_1}\right)\times\left(\frac{a_1}{a_3}\right) = (3)\times(2) = 6$$

$$\text{or} \quad R_3 = 6R_1 = 6 \times \frac{20}{29} = \mathbf{\frac{120}{29}\,\Omega}$$

Example 1.8. *A copper wire of diameter 1 cm had a resistance of 0.15 Ω. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire ?*

Solution. Area of wire before drawing, $a_1 = \dfrac{\pi}{4}\,(1)^2 = 0.785$ cm^2

Area of wire after drawing, $a_2 = \dfrac{\pi}{4}\,(0.5)^2 = 0.196$ cm^2

As the volume of wire remains the same before and after drawing,

$$\therefore \quad a_1 l_1 = a_2 l_2$$

$$\text{or} \quad l_2/l_1 = a_1/a_2 = 0.785/0.196 = 4$$

For the first case, $R_1 = 0.15\ \Omega$; $a_1 = 0.785$ cm^2 ; $l_1 = l$

For the second case, $R_2 =$? ; $a_2 = 0.196$ cm^2 ; $l_2 = 4l$

$$\text{Now} \quad R_1 = \rho\frac{l_1}{a_1};\quad R_2 = \rho\frac{l_2}{a_2}$$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)\times\left(\frac{a_1}{a_2}\right) = (4)\times(4) = 16$$

$$\text{or} \quad R_2 = 16R_1 = 16 \times 0.15 = \mathbf{2.4\ \Omega}$$

Example 1.9. *Two wires of aluminium and copper have the same resistance and same length. Which of the two is lighter? Density of copper is 8.9×10^3 kg/m^3 and that of aluminium is 2.7×10^3 kg/m^3. The resistivity of copper is 1.72×10^{-8} Ω m and that of aluminium is 2.6×10^{-8} Ω m.*

Solution. That wire will be lighter which has less mass. Let suffix 1 represent aluminium and suffix 2 represent copper.

$$R_1 = R_2 \quad \text{or} \quad \rho_1\frac{l_1}{A_1} = \rho_2\frac{l_2}{A_2}$$

$$\text{or} \quad \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2} \qquad (\because\ l_1 = l_2)$$

$$\text{or} \quad \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2} = \frac{2.6\times10^{-8}}{1.72\times10^{-8}} = 1.5$$

Now $$\frac{m_1}{m_2} = \frac{(A_1 l_1)d_1}{(A_2 l_2)d_2} = \frac{A_1 d_1}{A_2 d_2} \qquad (\because l_1 = l_2)$$

or $$\frac{m_1}{m_2} = \left(\frac{A_1}{A_2}\right) \times \left(\frac{d_1}{d_2}\right) = 1.5 \times \frac{2.7\times10^3}{8.9\times10^3} = 0.46$$

or $$m_1/m_2 = 0.46$$

It is clear that for the same length and same resistance, **aluminium wire is lighter than copper wire**. For this reason, aluminium wires are used for overhead power transmission lines.

Example 1.10. *A rectangular metal strip has the dimensions x = 10 cm, y = 0.5 cm and z = 0.2 cm. Determine the ratio of the resistances R_x, R_y and R_z between the respective pairs of opposite faces.*

Solution. $$R_x : R_y : R_z = \frac{\rho x}{yz} : \frac{\rho y}{zx} : \frac{\rho z}{xy} = \frac{10}{0.5\times0.2} : \frac{0.5}{0.2\times10} : \frac{0.2}{10\times0.5}$$

$$= \frac{10}{0.1} : \frac{1}{4} : 0.04 = \mathbf{2500 : 6.25 : 1}$$

Example 1.11. *Calculate the resistance of a copper tube 0.5 cm thick and 2 m long. The external diameter is 10 cm. Given that resistance of copper wire 1 m long and 1 mm^2 in cross-section is 1/58 Ω.*

Solution. External diameter, $D = 10$ cm

Internal diameter, $d = 10 - 2 \times 0.5 = 9$ cm

Area of cross-section, $$a = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[(10)^2 - (9)^2\right] \text{cm}^2$$

$$= \frac{\pi}{4}\left[(10)^2 - (9)^2\right] \times 100 \text{ mm}^2$$

∴ Resistance of copper tube $$= \frac{\rho l}{a} = \frac{1}{58} \times \frac{\text{length in metres}}{\text{area of X-section in mm}^2}$$

$$= \frac{1}{58} \times \frac{2}{\frac{\pi}{4}\left[(10)^2 - (9)^2\right] \times 100} = 23.14 \times 10^{-6}\ \Omega = \mathbf{23.14\ \mu\Omega}$$

Example 1.12. *A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance ?*

Solution. $$R = \rho\frac{l}{a};\ R' = \rho\frac{l'}{a'}$$

Now $$l' = l + \frac{0.1}{100} \times l = 1.001\, l$$

As the volume remains the same, $al = a'l'$.

∴ $$a' = a\frac{l}{l'} = \frac{a}{1.001}$$

∴ $$\frac{R'}{R} = \left(\frac{l'}{l}\right) \times \left(\frac{a}{a'}\right) = (1.001) \times (1.001) = 1.002$$

or $$\frac{R' - R}{R} = 0.002$$

∴ Percentage increase $$= \frac{R' - R}{R} \times 100 = 0.002 \times 100 = \mathbf{0.2\%}$$

Example 1.13. *A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80% more current than the latter and the latter 47% longer than the former. Determine the ratio of their cross-sectional areas.*

Solution. Let us represent lead and iron by suffixes 1 and 2 respectively. Then as per the conditions of the problem, we have,

$$\frac{\rho_1}{\rho_2} = \frac{49}{24} \;;\; I_1 = 1.8\, I_2 \;;\; l_2 = 1.47\, l_1$$

Now
$$R_1 = \rho_1 \frac{l_1}{a_1} \;;\; R_2 = \rho_2 \frac{l_2}{a_2}$$

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

$$\therefore \quad \frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\rho_1 l_1}{a_1} \times \frac{a_2}{\rho_2 l_2} = \left(\frac{\rho_1}{\rho_2}\right) \times \left(\frac{l_1}{l_2}\right) \times \left(\frac{a_2}{a_1}\right)$$

or
$$\frac{1}{1.8} = \frac{49}{24} \times \frac{1}{1.47} \times \frac{a_2}{a_1}$$

$$\therefore \quad \frac{a_2}{a_1} = \frac{1}{1.8} \times \frac{24}{49} \times 1.47 = \mathbf{0.4}$$

Example 1.14. *An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is 0.017 µΩm ; that of the aluminium is 0.028 µΩ m.*

Solution. Let us assign subscripts *a* and *c* to aluminium and copper respectively.

Current through *Al* wire, $I_a = 3$ A

$\therefore$ Current through Cu wire, $I_c = 5 - 3 = 2$ A

Since R_a and R_c are in parallel, the voltage across them is the same [See Fig. 1.12] *i.e.*

$$I_a R_a = I_c R_c \quad \text{or} \quad \frac{R_a}{R_c} = \frac{I_c}{I_a} = \frac{2}{3}$$

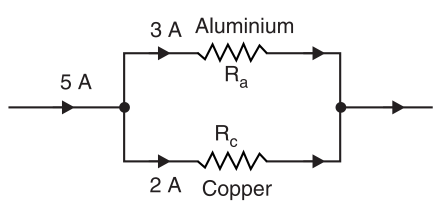

Fig. 1.12

Now
$$R_a = \frac{\rho_a l_a}{a_a} \;;\; R_c = \frac{\rho_c l_c}{a_c}$$

$$\therefore \quad \frac{R_c}{R_a} = \frac{\rho_c}{\rho_a} \times \frac{l_c}{l_a} \times \frac{a_a}{a_c}$$

Here
$$\frac{R_c}{R_a} = \frac{3}{2} \;;\; \frac{\rho_c}{\rho_a} = \frac{0.017}{0.028} \;;\; \frac{l_c}{l_a} = \frac{6}{7.5} \;;$$

$$a_a = \frac{\pi}{4} d^2 = \frac{\pi \times (1)^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

$$\therefore \quad \frac{3}{2} = \frac{0.017}{0.028} \times \frac{6}{7.5} \times \frac{\pi/4}{a_c}$$

or
$$a_c = \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} \times \frac{\pi}{4} = 0.2544 \text{ mm}^2$$

or
$$\frac{\pi}{4} d_c^2 = 0.2544 \qquad \therefore \quad d_c = \mathbf{0.569 \text{ mm}}$$

Example 1.15. *A transmission line cable consists of 19 strands of identical copper conductors, each 1.5 mm in diameter. The length of the cable is 2 km but because of the twist of the strands, the actual length of each conductor is increased by 5 percent. What is resistance of the cable ? Take the resistivity of the copper to be 1.78 × 10^{-8} Ω m.*

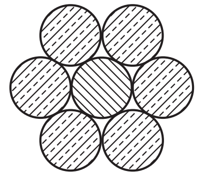

Fig. 1.13

Solution. Fig. 1.13 shows the general shape of a stranded conductor. Allowing for twist, the length of the strands is

$$l = 2000 \text{ m} + 5\% \text{ of } 2000 \text{ m} = 2100 \text{ m}$$

Area of X-section of 19 strands, $a = (19)\left(\dfrac{\pi}{4}\right) \times (1.5 \times 10^{-3})^2 = 33.576 \times 10^{-6} \text{ m}^2$

∴ Resistance of line, $R = \rho \dfrac{l}{a} = 1.72 \times 10^{-8} \times \dfrac{2100}{33.576 \times 10^{-6}}$ = **1.076 Ω**

Example 1.16. *The resistance of the wire used for telephone is 35 Ω per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is 1.95 × 10^{-8} Ω m, what is the cross-sectional area of the wire ? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used?*

Solution. For the first case, $R = 35\ \Omega$; $l = 1000$ m ; $\rho = 1.95 \times 10^{-8}\ \Omega$ m

Now $R = \rho \dfrac{l}{a}$ ∴ $a = \dfrac{\rho l}{R} = \dfrac{1.95 \times 10^{-8} \times 1000}{35}$ = **55.7 × 10^{-8} m²**

Since weight of conductor is directly proportional to the area of cross-section, for the second case, we have,

$$a = \frac{20}{5} \times 55.7 \times 10^{-8} = 222.8 \times 10^{-8} \text{ m}^2 \;;\; l = 2 \times 8 = 16 \text{ km} = 16000 \text{ m}$$

∴ $R = \rho \dfrac{l}{a} = 1.95 \times 10^{-8} \times \dfrac{16000}{222.8 \times 10^{-8}}$ = **140.1 Ω**

Example 1.17. *Find the resistance of a cubic centimetre of copper (i) when it is drawn into a wire of diameter 0.32 mm and (ii) when it is hammered into a flat sheet of 1.2 mm thickness, the current flowing through the sheet from one face to another, specific resistance of copper is 1.6 × 10^{-8} Ω-m.*

Solution. Volume of copper wire, $v = 1 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$

(*i*) **Resistance when drawn into wire.**

Area of X-section, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (0.32 \times 10^{-3})^2 = 0.804 \times 10^{-7} \text{ m}^2$

Length of wire, $l = \dfrac{v}{a} = \dfrac{1 \times 10^{-6}}{0.804 \times 10^{-7}} = 12.43 \text{ m}$

∴ Resistance of wire, $R = \rho \dfrac{l}{a} = 1.6 \times 10^{-8} \dfrac{12.43}{0.804 \times 10^{-7}}$ = **2.473 Ω**

(*ii*) **Resistance when hammered into flat sheet.**

Length of flat sheet, $l = 1.2 \times 10^{-3}$ m ; Area of cross-section of flat sheet is

$$a = \frac{v}{l} = \frac{1 \times 10^{-6}}{1.2 \times 10^{-3}} = \frac{10^{-3}}{1.2} \text{ m}^2$$

∴ Resistance of copper flat sheet is $R = \rho \dfrac{l}{a} = 1.6 \times 10^{-8} \dfrac{1.2 \times 10^{-3}}{10^{-3}/1.2}$ = **2.3 × 10^{-8} Ω**

Tutorial Problems

1. Calculate the resistance of 915 metres length of a wire having a uniform cross-sectional area of 0.77 cm^2 if the wire is made of copper having a resistivity of 1.7×10^{-6} Ω cm. **[0.08 Ω]**
2. A wire of length 1 m has a resistance of 2 ohms. What is the resistance of second wire, whose specific resistance is double the first, if the length of wire is 3 metres and the diameter is double of the first? **[3 Ω]**
3. A rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (*i*) opposite ends (*ii*) opposite sides. The resistivity of copper is 1.7×10^{-6} Ω cm. **[(*i*) 0.85 × 10^{-4} Ω (*ii*) 0.212 × 10^{-6} Ω]**
4. A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same material has a length of 9 cm and a uniform cross-sectional area 1 cm^2, what will be the resistance of this length ? **[0.009 Ω]**
5. An aluminium wire 10 metres long and 2 mm in diameter is connected in parallel with a copper wire 6 metres long. A total current of 2 A is passed through the combination and it is found that current through the aluminium wire is 1.25 A. Calculate the diameter of copper wire. Specific resistance of copper is 1.6×10^{-6} Ω cm and that of aluminium is 2.6×10^{-6} Ω cm. **[0.94 mm]**
6. A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance ? **[0.2%]**

1.21. Types of Resistors

A component whose function in a circuit is to provide a specified value of resistance is called a **resistor**. The principal applications of resistors are to limit current, divide voltage and in certain cases, generate heat. Although there are a variety of different types of resistors, the following are the commonly used resistors in electrical and electronic circuits :

(*i*) Carbon composition types (*ii*) Film resistors
(*iii*) Wire-wound resistors (*iv*) Cermet resistors

(*i*) Carbon composition type. This type of resistor is made with a mixture of finely ground carbon, insulating filler and a resin binder. The ratio of carbon and insulating filler decides the resistance value [See Fig. 1.14]. The mixture is formed into a rod and lead connections are made. The entire resistor is then enclosed in a plastic case to prevent the entry of moisture and other harmful elements from outside.

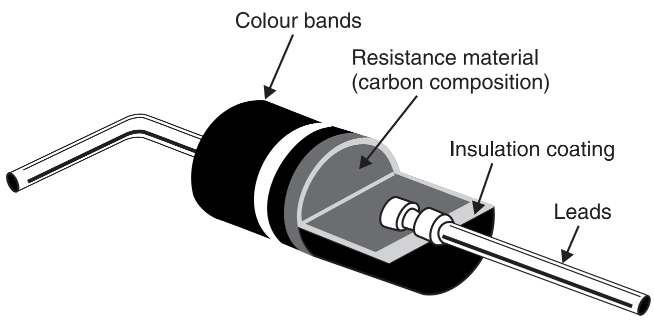

Fig. 1.14

Carbon resistors are relatively inexpensive to build. However, they are highly sensitive to temperature variations. The carbon resistors are available in power ratings ranging from 1/8 to 2 W.

(*ii*) Film resistors. In a film resistor, a resistive material is deposited uniformly onto a high-grade ceramic rod. The resistive film may be carbon (carbon film resistor) or nickel-chromium (metal film resistor). In these types of resistors, the desired resistance value is obtained by removing a part of the resistive material in a helical pattern along the rod as shown in Fig. 1.15.

Metal film resistors have better characteristics as compared to carbon film resistors.

***(iii)* Wire-wound resistors.** A wire-wound resistor is constructed by winding a resistive wire of some alloy around an insulating rod. It is then enclosed in an insulating cover. Generally, nickle-chromium alloy is used because of its very small temperature coefficient of resistance. Wire-wound resistors can safely operate at higher temperatures than carbon types. These resistors have high power ratings ranging from 12 to 225 W.

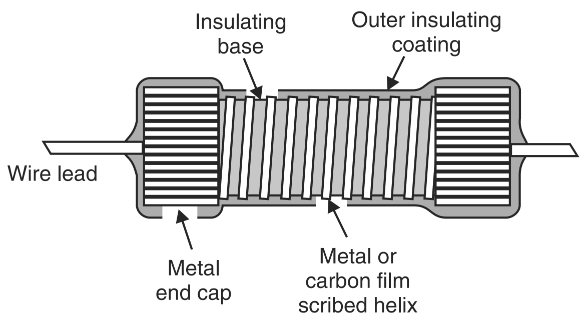

Fig. 1.15

***(iv)* Cermet resistors.** A cermet resistor is made by depositing a thin film of metal such as nichrome or chromium cobalt on a ceramic substrate. They are cermet which is a contraction for ceramic and metal. These resistors have very accurate values.

1.22. Effect of Temperature on Resistance

In general, the resistance of a material changes with the change in temperature. The effect of temperature upon resistance varies according to the type of material as discussed below :

(i) The resistance of pure metals (*e.g.* copper, aluminium) increases with the increase of temperature. The change in resistance is fairly regular for normal range of temperatures so that temperature/resistance graph is a straight line as shown in Fig. 1.16 (for copper). Since the resistance of metals increases with the rise in temperature, they have *positive temperature co-efficient of resistance.*

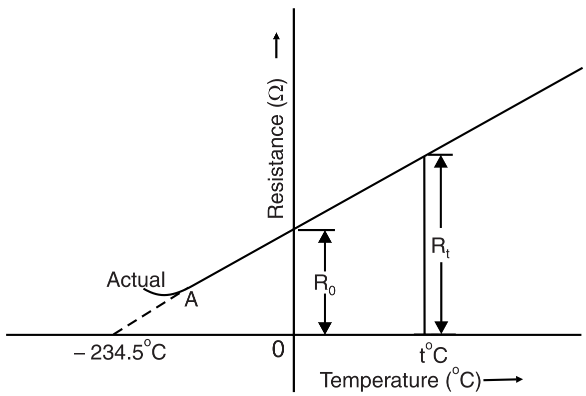

Fig. 1.16

(ii) The resistance of electrolytes, insulators (*e.g.* glass, mica, rubber etc.) and semiconductors (*e.g.* germanium, silicon etc.) decreases with the increase in temperature. Hence these materials have *negative temperature co-efficient of resistance*.

(iii) The resistance of alloys increases with the rise in temperature but this increase is very small and irregular. For some high resistance alloys (*e.g.* Eureka, manganin, constantan etc.), the change in resistance is practically negligible over a wide range of temperatures.

Fig. 1.16 shows temperature/resistance graph for copper which is a straight line. If this line is extended backward, it would cut the temperature axis at −234.5°C. It means that theoretically, the resistance of copper wire is zero at −234.5°C. However, in actual practice, the curve departs (point *A*) from the straight line path at very low temperatures.

1.23. Temperature Co-efficient of Resistance

Consider a conductor having resistance R_0 at 0°C and R_t at t °C. It has been found that in the normal range of temperatures, the increase in resistance (*i.e.* $R_t - R_0$)

(i) is directly proportional to the initial resistance *i.e.*

$$R_t - R_0 \propto R_0$$

(*ii*) is directly proportional to the rise in temperature *i.e.*

$$R_t - R_0 \propto t$$

(*iii*) depends upon the nature of material.

Combining the first two, we get,

$$R_t - R_0 \propto R_0\, t$$

or
$$R_t - R_0 = {}^*\alpha_0\, R_0\, t \quad ...(i)$$

where α_0 is a constant and is called temperature co-efficient of resistance at 0°C. Its value depends upon the nature of material and temperature.

Rearranging eq. (*i*), we get,

$$R_t = R_0\,(1 + \alpha_0\, t) \quad ...(ii)$$

Definition of α_0. From eq. (*i*), we get,

$$\alpha_0 = \frac{R_t - R_0}{R_0 \times t}$$

= Increase in resistance/ohm original resistance/°C rise in temperature

Hence **temperature co-efficient of resistance** *of a conductor is the increase in resistance per ohm original resistance per °C rise in temperature.*

A little reflection shows that unit of α will be ohm/ohm°C *i.e.*/°C. Thus, copper has a temperature co-efficient of resistance of 0.00426/°C. It means that if a copper wire has a resistance of 1 Ω at 0°C, then it will increase by 0.00426 Ω for 1°C rise in temperature *i.e.* it will become 1.00426 Ω at 1°C. Similarly, if temperature is raised to 10°C, then resistance will become 1 + 10 × 0.00426 = 1.0426 ohms.

The following points may be noted carefully :

(*i*) Those substances (*e.g.* pure metals) whose resistance increases with rise in temperature are said to have *positive* temperature co-efficient of resistance. On the other hand, those substances whose resistance decreases with increase in temperature are said to have *negative* temperature co-efficient of resistance.

(*ii*) If a conductor has a resistance R_0, R_1 and R_2 at 0°C, t_1°C and t_2°C respectively, then,

$$R_1 = R_0\,(1 + \alpha_0\, t_1)$$

$$R_2 = R_0\,(1 + \alpha_0\, t_2)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1 + \alpha_0\, t_2}{1 + \alpha_0 t_1} \quad ...(iii)$$

This relation is often utilised in determining the rise of temperature of the winding of an electrical machine. The resistance of the winding is measured both before and after the test run. Let R_1 and t_1 be the resistance and temperature before the commencement of the test. After the operation of the machine for a given period, let these values be R_2 and t_2. Since R_1 and R_2 can be measured and t_1 (ambient temperature) and α_0 are known, the value of t_2 can be calculated from eq. (*iii*). The average rise in temperature of the winding will be $(t_2 - t_1)$°C.

Note. The life expectancy of electrical apparatus is limited by the temperature of its insulation; the higher the temperature, the shorter the life. The useful life of electrical apparatus reduces approximately by half every time the temperature increases by 10°C. This means that if a motor has a normal life expectancy of eight years

* It will be shown in Art. 1.25 that value of α depends upon temperature. Therefore, it is referred to the original temperature *i.e.* 0°C in this case. Hence the symbol α_0.

at a temperature of 100°C, it will have a life expectancy of only four years at a temperature of 110°C, of two years at a temperature of 120°C and of only one year at 130°C.

1.24. Graphical Determination of α

The value of temperature co-efficient of resistance can also be determined graphically from temperature/resistance graph of the material. Fig. 1.17 shows the temperature/resistance graph for a conductor. The graph is a straight line *AX* as is the case with all conductors. The resistance of the conductor is R_0 (represented by *OA*) at 0°C and it becomes R_t at t°C. By definition,

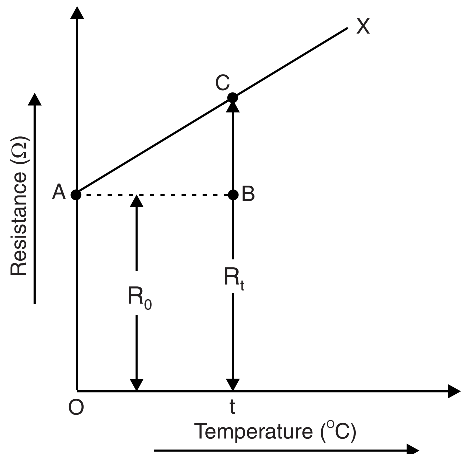

Fig. 1.17

$$\alpha_0 = \frac{R_t - R_0}{R_0 \times t}$$

But $R_t - R_0 = BC$

and t = rise in temperature = AB

$$\therefore \quad \alpha_0 = \frac{BC}{R_0 \times AB}$$

But *BC*/*AB* is the slope of temperature/resistance graph.

$$\therefore \quad \alpha_0 = \frac{\text{Slope of temp./resistance graph}}{\text{Original resistance}} \quad ...(i)$$

Hence, **temperature co-efficient** *of resistance of a conductor at 0°C is the slope of temp./resistance graph divided by resistance at 0°C (i.e.* R_0*).*

The following points may be particularly noted :

(i) The value of α depends upon temperature. At any temperature, α can be calculated by using eq. (*i*).

Thus, $$\alpha_0 = \frac{\text{Slope* of temperature/resistance graph}}{R_0}$$

and $$\alpha_t = \frac{\text{Slope of temperature/resistance graph}}{R_t}$$

(ii) The value of α_0 is maximum and it decreases as the temperature is increased. This is clear from the fact that the slope of temperature/resistance graph is constant and R_0 has the minimum value.

1.25. Temperature Co-efficient at Various Temperatures

Consider a conductor having resistances R_0 and R_1 at temperatures 0°C and t_1°C respectively. Let α_0 and α_1 be the temperature co-efficients of resistance of the conductor at 0°C and t_1°C respectively. It is desired to establish the relationship between α_1 and α_0. Fig. 1.18 shows the temperature/resistance graph of the conductor. As proved in Art. 1.24,

$$\alpha_0 = \frac{\text{Slope of graph}}{R_0}$$

$$\therefore \quad \text{Slope of graph} = \alpha_0 R_0$$

* The slope of temp./resistance graph of a conductor is always constant (being a straight line).

Similarly, $\alpha_1 = \dfrac{\text{Slope of graph}}{R_1}$

or Slope of graph $= \alpha_1 R_1$

Since the slope of temperature/resistance graph is constant,

$\therefore \quad \alpha_0 R_0 = \alpha_1 R_1$

or $\alpha_1 = \dfrac{\alpha_0 R_0}{R_1} = \dfrac{\alpha_0 R_0}{R_0(1+\alpha_0 t_1)}$

$[\because R_1 = R_0 (1 + \alpha_0 t_1)]$

$$\therefore \quad \alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \qquad ...(i)$$

Similarly,* $$\alpha_2 = \frac{\alpha_0}{1+\alpha_0 t_2} \qquad ...(ii)$$

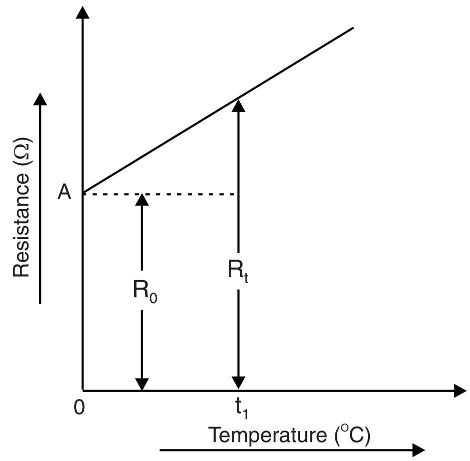

Fig. 1.18

Subtracting the reciprocal of eq. (*i*) from the reciprocal of eq. (*ii*),

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_2}{\alpha_0} - \frac{1+\alpha_0 t_1}{\alpha_0} = t_2 - t_1$$

$$\therefore \quad \alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)} \qquad ...(iii)$$

Eq. (*i*) gives the relation between α_1 and α_0 while Eq. (*iii*) gives the relation between α_2 and α_1.

1.26. Summary of Temperature Co-efficient Relations

(*i*) If R_0 and α_0 are the resistance and temperature co-efficient of resistance of a conductor at 0°C, then its resistance R_t at t°C is given by ;

$$R_t = R_0(1 + \alpha_0 t)$$

(*ii*) If α_0, α_1 and α_2 are the temperature co-efficients of resistance at 0°C, t_1°C and t_2°C respectively, then,

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \; ; \quad \alpha_2 = \frac{\alpha_0}{1+\alpha_0 t_2} \; ; \quad \alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)}$$

(*iii*) Suppose R_1 and R_2 are the resistances of a conductor at t_1°C and t_2°C respectively. If α_1 is the temperature co-efficient of resistance at t_1°C, then,

$$R_2^{**} = R_1[1 + \alpha_1(t_2 - t_1)]$$

1.27. Variation of Resistivity With Temperature

Not only resistance but resistivity or specific resistance of a material also changes with temperature. The change in resistivity per °C change in temperature is called *temperature*

* $\alpha_0 R_0 = \alpha_2 R_2$ where R_2 is the resistance at t_2°C

or $\alpha_2 = \dfrac{\alpha_0 R_0}{R_2} = \dfrac{\alpha_0 R_0}{R_0(1+\alpha_0 t_2)} = \dfrac{\alpha_0}{1+\alpha_0 t_2}$

** Slope of graph, $\tan\theta = R_0 \alpha_0 = R_1 \alpha_1 = R_2 \alpha_2$

Increase in resistance as temperature is raised from t_1°C to t_2°C

$= \tan\theta(t_2 - t_1) = R_1\alpha_1(t_2 - t_1)$

$\therefore$ Resistance at t_2°C, $R_2 = R_1 + R_1\alpha_1(t_2 - t_1) = R_1[1 + \alpha_1(t_2 - t_1)]$

coefficient of resistivity. In case of metals, the resistivity increases with increase in temperature and vice-versa. It is found that resistivity of a metallic conductor increases linearly over a wide range of temperatures and is given by ;

$$\rho_t = \rho_0(1 + \alpha_0 t)$$

where ρ_0 = resistivity of metallic conductor at 0°C

ρ_t = resistivity of metallic conductor at temperature t°C

Note that temperature coefficient of resistivity is equal to temperature coefficient of resistance α_0.

Example 1.18. *A coil has a resistance of 18 Ω when its mean temperature is 20°C and of 20 Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21Ω and the surrounding temperature is 15°C.*

Solution. Let R_0 be the resistance of the coil at 0°C and α_0 be its temperature coefficient of resistance at 0°C. Then,

$$18 = R_0(1 + \alpha_0 \times 20) \quad \text{and} \quad 20 = R_0(1 + \alpha_0 \times 50)$$

$$\therefore \quad \frac{20}{18} = \frac{1+50\alpha_0}{1+20\alpha_0} \quad \text{or} \quad \alpha_0 = \frac{1}{250} = 0.004/°C$$

If t°C is the temperature of the coil when its resistance is 21Ω, then,

$$21 = R_0(1 + 0.004\,t)$$

$$\therefore \quad \frac{21}{18} = \frac{R_0(1+0.004\,t)}{R_0(1+0.004\times 20)} \quad \text{or} \quad t = 65°\text{C}$$

$\therefore$ Temperature rise = $t - 15 = 65° - 15° =$ **50°C**

Example 1.19. *The resistance of the field coils of a dynamo is 173 Ω at 16°C. After working for 6 hours on full-load, the resistance of the coils increases to 212 Ω. Calculate (i) the temperature of the coils (ii) mean rise of temperature of the coils. Assume temperature co-efficient of resistance of copper is 0.00426/°C at 0°C.*

Solution. ***(i)*** Let t°C be the final temperature.

$$\frac{R_{16}}{R_t} = \frac{R_0(1+\alpha_0\times 16)}{R_0(1+\alpha_0\times t)}$$

or
$$\frac{173}{212} = \frac{1+0.00426\times 16}{1+0.00426\times t}$$

or
$$0.816 = \frac{1.068}{1+0.00426\,t} \quad \therefore \quad t = \textbf{72.5°C}$$

(ii) Rise in temperature = $t - 16 = 72.5 - 16 =$ **56.5°C**

Example 1.20. *The resistance of a transformer winding is 460 Ω at room temperature of 25°C. When the transformer is running and the final temperature is reached, the resistance of the winding increases to 520 Ω. Find the average temperature rise of winding, assuming that α_{20} = 1/250 per °C.*

Solution.
$$\alpha_{25} = \frac{1}{1/\alpha_{20} + (25-20)} = \frac{1}{250+5} = \frac{1}{255}/°C$$

Let t°C be the final temperature of the winding. Then, the rise in temperature is $t - 25$.

Now, $R_{25} = 460\ \Omega$; $R_t = 520\ \Omega$

$$R_t = R_{25}[1 + \alpha_{25}(t - 25)]$$

or
$$t - 25 = \frac{1}{\alpha_{25}}\left(\frac{R_t}{R_{25}} - 1\right) = 255(520/460 - 1) = 33.26°\text{C}$$

$\therefore$ Temperature rise = $t - 25$ = **32.26°C**

Example 1.21. *The filament of a 60 watt, 230 V lamp has a normal working temperature of 2000°C. Find the current flowing in the filament at the instant of switching, when the lamp is cold. Assume the temperature of cold lamp to be 15°C and* $\alpha_{15} = 0.005/°C$.

Solution. Resistance of lamp at 2000°C is

$$R_{2000} = V^2/P = (230)^2/60 = 881.67\ \Omega$$

$$R_{2000} = R_{15}[1 + \alpha_{15}(2000 - 15)]$$

$$\therefore \quad R_{15} = \frac{R_{2000}}{1+0.005(1985)} = \frac{881.67}{10.925} = 80.7\ \Omega$$

$\therefore$ Current taken by cold lamp (*i.e.* at the time of switching) is

$$I = V/R_{15} = 230/80.7 = \mathbf{2.85\ A}$$

Example 1.22. *Two coils connected in series have resistances of 600 Ω and 300 Ω and temperature coefficients of 0.1% and 0.4% per °C at 20°C respectively. Find the resistance of combination at a temperature of 50°C. What is the effective temperature coefficient of the combination at 50°C ?*

Solution. Resistance of 600 Ω coil at 50°C

$$= 600\ [1 + 0.001(50 - 20)] = 618\ \Omega$$

Resistance of 300 Ω coil at 50°C

$$= 300\ [1 + 0.004\ (50 - 20)] = 336\ \Omega$$

Resistance of series combination at 50°C is

$$R_{50} = 618 + 336 = \mathbf{954}\ \Omega$$

Resistance of series combination at 20°C is

$$R_{20} = 600 + 300 = 900\ \Omega$$

Now $$R_{50} = R_{20}\ [1 + \alpha_{20}\ (t_2 - t_1)]$$

$$\therefore \quad \alpha_{20} = \frac{\dfrac{R_{50}}{R_{20}} - 1}{t_2 - t_1} = \frac{\dfrac{954}{900} - 1}{50 - 20} = 0.002$$

Now $$\alpha_{50} = \frac{1}{1/\alpha_{20} + (t_2 - t_1)} = \frac{1}{1/0.002 + (50 - 20)} = \mathbf{\frac{1}{530}\ /°C}$$

Example 1.23. *The coil of a relay takes a current of 0.12 A when it is at the room temperature of 15°C and connected across a 60 V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 0°C.*

Solution. Resistance of relay coil at 15°C, $R_{15} = 60/0.12 = 500\ \Omega$

If the temperature increases, the resistance of relay coil increases and current in relay coil decreases. Let t°C be the temperature at which the current in relay coil becomes 0.1 A (= the minimum relay coil current for its operation). Clearly, $R_t = 60/0.1 = 600\ \Omega$.

Now, $$R_{15} = R_0\ (1 + 15\ \alpha_0) = R_0\ (1 + 15 \times 0.0043)$$

$$R_t = R_0\ (1 + \alpha_0 t) = R_0\ (1 + 0.0043\ t)$$

$$\therefore \quad \frac{R_t}{R_{15}} = \frac{1 + 0.0043\,t}{1.0645}$$

or $$\frac{600}{500} = \frac{1 + 0.0043\,t}{1.0645}$$

On solving, $t = \mathbf{64.5°C}$

If the temperature of relay coil increases above 64.5°C, the resistance of relay coil will increase and the relay coil current will be less than 0.1 A. As a result, the relay will fail to operate.

Example 1.24. *Two materials, A and B, have resistance temperature coefficients of 0.004 and 0.0004 respectively at a given temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001 ?*

Solution. Let the resistance of A be 1 Ω and that of B be x Ω *i.e.* $R_A = 1\ \Omega$ and $R_B = x\ \Omega$.

Resistance of series combination $= R_A + R_B = (1 + x)\ \Omega$

Suppose the temperature rises by t°C.

Resistance of series combination at the raised temperature $= (1 + x)(1 + 0.001\,t)$ ...(*i*)

Resistance of A at the raised temperature $= 1\,(1 + 0.004\,t)$ ...(*ii*)

Resistance of B at the raised temperature $= x\,(1 + 0.0004\,t)$ ...(*iii*)

As per the conditions of the problem, we have, (*ii*) + (*iii*) = (*i*)

or $1\,(1 + 0.004\,t) + x\,(1 + 0.0004\,t) = (1 + x)(1 + 0.001\,t)$

or $0.004\,t + 0.0004\,t\,x = (1 + x) \times 0.001\,t$

Dividing by t and multiplying throughout by 10^4, we have,

$$40 + 4x = 10\,(1 + x) \quad \therefore\ x = 5$$

$\therefore$ $R_A : R_B =$ **1 : 5** *i.e.* R_B should be 5 times R_A.

Example 1.25. *A resistor of 80 Ω resistance, having a temperature coefficient of 0.0021/°C is to be constructed. Wires of two materials of suitable cross-sectional areas are available. For material A, the resistance is 80 Ω per 100 m and the temperature coefficient is 0.003/°C. For material B, the corresponding figures are 60 Ω per 100 m and 0.0015/°C. Calculate suitable lengths of wires of materials A and B to be connected in series to construct the required resistor. All data are referred to the same temperature.*

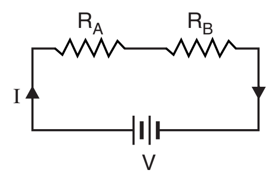

Fig. 1.19

Solution. Let R_A and R_B be the required resistances of materials A and B respectively which when joined in series have a combined temperature coefficient of 0.0021 [See Fig. 1.19].

Resistance of series combination $= R_A + R_B$

Resistance of series combination at raised temperature $= (R_A + R_B)(1 + 0.0021\,t)$ ...(*i*)

Resistance of A at raised temperature $= R_A\,(1 + 0.003\,t)$ ...(*ii*)

Resistance of B at raised temperature $= R_B\,(1 + 0.0015\,t)$ ...(*iii*)

As per conditions of the problem, (*ii*) + (*iii*) = (*i*).

$\therefore$ $R_A\,(1 + 0.003\,t) + R_B\,(1 + 0.0015\,t) = (R_A + R_B)(1 + 0.0021\,t)$

On solving, $$\frac{R_B}{R_A} = \frac{3}{2} \quad \text{...}(iv)$$

Now, $R_A + R_B = 80$ ...(*v*)

From eqs. (*iv*) and (*v*), $R_A = 32\ \Omega$ and $R_B = 48\ \Omega$

$\therefore$ Length of wire A, $L_A = (100/80) \times 32 =$ **40 m**

Length of wire B, $L_B = (100/60) \times 48 =$ **80 m**

Example 1.26. *Two wires A and B are connected in series at 0°C and resistance of B is 3.5 times that of A. The resistance temperature coefficient of A is 0.4% and that of combination is 0.1%. Find the resistance temperature coefficient of B.*

Solution. Let the temperature coefficient of resistance of wire B be α_B. If R is the resistance of wire A, then,

$$R_A = R \;;\; R_B = 3.5\,R$$

Total resistance of two wires at 0°C = $R_A + R_B = R + 3.5\,R = 4.5\,R$

Increase in resistance of wire A per °C rise = $\alpha_A\,R = 0.004\,R$

Increase in resistance of wire B per °C rise = $\alpha_B \times 3.5\,R = 3.5\,R\,\alpha_B$

Total increase in the resistance of combination per °C rise = 0.004 R + 3.5 $R\,\alpha_B$ ... (*i*)

Also, total increase in the resistance of combination per °C rise = α_C × Total resistance of combination = 0.001 × 4.5 R = 0.0045 R ... (*ii*)

From eqs. (*i*) and (*ii*), $0.004\,R + 3.5\,R\alpha_B = 0.0045\,R$

$$\therefore \qquad \alpha_B = \frac{0.0045\,R - 0.004\,R}{3.5\,R} = \mathbf{0.000143/°C} \text{ or } \mathbf{0.0143\%}$$

Example 1.27. *Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at 25°C. What proportion of current will pass through each if the temperature is raised to 100°C? The temperature co-efficients of resistance at 0°C are 0.0043/°C and 0.0063/°C for copper and iron respectively.*

Solution. Since copper and iron conductors carry equal currents at 25°C, their resistances are the same at this temperature. Let their resistance be R ohms at 25°C. If R_1 and R_2 are the resistances of copper and iron conductors respectively at 100°C, then,

$$R_1 = R\,[1 + 0.0043\,(100 - 25)] = 1.3225\,R$$
$$R_2 = R\,[1 + 0.0063\,(100 - 25)] = 1.4725\,R$$

If I is the total current at 100°C, then,

$$\text{Current in copper conductor} = I \times \frac{R_2}{R_1 + R_2} = I \times \frac{1.4725\,R}{1.3225\,R + 1.4725\,R} = 0.5268\,I$$

$$\text{Current in iron conductor} = I \times \frac{R_1}{R_1 + R_2} = I \times \frac{1.3225\,R}{1.3225R + 1.4725\,R} = 0.4732\,I$$

Therefore, at 100°C, the copper conductor will carry **52.68%** of total current and the remaining **47.32%** will be carried by iron conductor.

Example 1.28. *A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of copper at 20°C = 1.724 × 10^{-6} Ω-cm and resistance temperature coefficient of copper at 0°C = 0.0043/°C.*

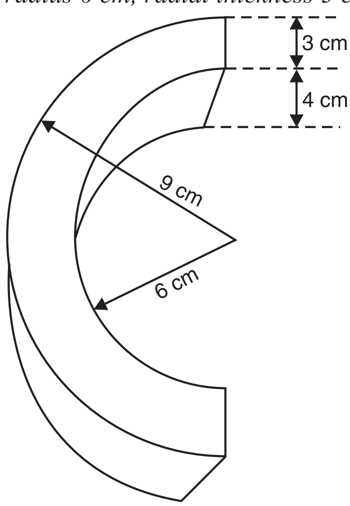

Fig. 1.20

Solution. Fig. 1.20 shows the semi-circular ring.

Mean radius of the ring, $r_m = (6 + 9)/2 = 7.5$ cm

Mean length between end faces is

$$l_m = \pi r_m = \pi \times 7.5 = 23.56 \text{ cm}$$

Cross-sectional area of the ring is

$$a = 3 \times 4 = 12 \text{ cm}^2$$

Now
$$\alpha_{20} = \frac{\alpha_0}{1 + \alpha_0\,t} = \frac{0.0043}{1 + 0.0043 \times 20}$$
$$= 0.00396/°C$$

Also $\rho_{50} = \rho_{20}[1 + \alpha_{20}(t - 20)]$

$$= 1.724 \times 10^{-6}[1 + 0.00396 \times (50 - 20)]$$
$$= 1.93 \times 10^{-6}\ \Omega\ \text{cm}$$

$$\therefore \quad R_{50} = \frac{\rho_{50}\, l_m}{a} = \frac{1.93 \times 10^{-6} \times 23.56}{12} = \mathbf{3.79 \times 10^{-6}\ \Omega}$$

This example shows that resistivity of a conductor increases with the increase in temperature and vice-versa.

Example 1.29. *A copper conductor has its specific resistance of 1.6 ×10^{-6} Ω cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find (i) specific resistance and (ii) the resistance temperature coefficient at 60°C.*

Solution. $\alpha_{20} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20}$ or $\dfrac{1}{254.5} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20}$ $\therefore\ \alpha_0 = \dfrac{1}{234.5}$ /°C

(*i*) $\rho_{60} = \rho_0(1 + \alpha_0 \times 60) = 1.6 \times 10^{-6}(1 + 60/234.5) = \mathbf{2.01 \times 10^{-6}\ \Omega\ cm}$

(*ii*) $$\alpha_{60} = \frac{1}{\dfrac{1}{\alpha_{20}} + (t_2 - t_1)} = \frac{1}{254.5 + (60 - 20)} = \mathbf{\frac{1}{294.5}}\ \mathbf{/°C}$$

Example 1.30. *The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20° C of 4.3 μΩ cm. If α_{20} = 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 W at a filament temperature of 2420°C ?*

Solution. Power to be dissipated by the lamp at 2420°C is

$$\frac{V^2}{R_{2420}} = 60 \qquad \therefore \quad R_{2420} = \frac{V^2}{60} = \frac{(240)^2}{60} = 960\,\Omega$$

Now $R_{2420} = R_{20}[1 + \alpha_{20}(2420 - 20)]$

or $960 = R_{20}[1 + 0.005(2420 - 20)]$

$\therefore$ $R_{20} = 960/13\ \Omega$

Now $\rho_{20} = 4.3 \times 10^{-6}\ \Omega\ \text{cm}$; $a = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.02 \times 10^{-1})^2\ \text{cm}^2$

$$\therefore \quad \text{Length of filament is } l = \frac{a \times R_{20}}{\rho_{20}} = \frac{\pi}{4} \times \frac{(0.02 \times 10^{-1})^2 \times 960}{4.3 \times 10^{-6} \times 13} = \mathbf{54\ cm}$$

Tutorial Problems

1. The shunt winding of a motor has a resistance of 35.1 Ω at 20°C. Find its resistance at 32.6°C. The temperature co-efficient of copper is 0.00427/°C at 0°C. **[39.6 Ω]**
2. The resistance of a coil of wire increases from 40 Ω at 10°C to 48.25 Ω at 60°C. Find the temperature coefficient at 0°C of the conductor material. **[0.0043/°C]**
3. The coil of an electromagnet, made of copper wire, has resistance of 4 Ω at a temperature of 22°C. After operating for 2 days, the coil current is 42 A at a terminal voltage of 210 V. Calculate the average temperature of the coil at that time. **[86.1°C]**
4. The filament of a 60 watt incandescent lamp possesses a cold resistance of 17.6 Ω at 20°C. The lamp draws a current of 0.25 A when connected to a 240 V source. Calculate the temperature of hot filament. Take temperature co-efficient at 0°C as 0.0055/°C. **[2571°C]**

5. A nichrome heater is operated at 1500°C. What is the percentage increase in its resistance over that at room temperature (20°C) ? Temperature co-efficient of nichrome is 0.00016/°C at 0°C. **[23.6%]**
6. Two wires A and B are connected in series at 0°C and resistance of B is 3.5 times that of A. The resistance temperature coefficient of A is 0.4% and that of the combination is 0.1%. Find the resistance temperature coefficient of B. **[0.0143%]**
7. A d.c. shunt motor after running for several hours on constant voltage mains of 400 V takes a field current of 1.6 A. If the temperature rise is known to be 40°C, what value of extra circuit resistance is required to adjust the field current to 1.6 A when starting from cold at 20°C ? Temperature coefficient = 0.0043/°C at 20°C. **[36.69 Ω]**
8. A potential difference of 250 V is applied to a copper field coil at a temperature of 15°C and the current is 5 A. What will be the mean temperature of the coil when the current has fallen to 3.91 A, the applied voltage being the same as before ? **[85°C]**
9. An insulating material has an insulation resistance of 100% at 0°C. For each rise in temperature of 5°C its resistance is reduced by 10%. At what temperature is the insulation resistance halved ? **[33°C]**
10. A carbon electrode has a resistance of 0.125 Ω at 20°C. The temperature coefficient of carbon is –0.0005 at 20°C. What will the resistance of the electrode be at 85°C ? **[0.121 Ω]**

1.28. Ohm's Law

The relationship between voltage (V), the current (I) and resistance (R) in a d.c. circuit was first discovered by German scientist George Simon *Ohm. This relationship is called Ohm's law and may be stated as under :

The ratio of potential difference (V) between the ends of a conductor to the current (I) flowing between them is constant, provided the physical conditions (e.g. temperature etc.) do not change i.e.

$$\frac{V}{I} = \text{Constant} = R$$

where R is the resistance of the conductor between the two points considered.

For example, if in Fig. 1.21 (*i*), the voltage between points A and B is V volts and current flowing is I amperes, then V/I will be constant and equal to R, the resistance between points A and B. If the voltage is doubled up, the current will also be doubled up so that the ratio V/I remains constant. If we draw a graph between V and I, it will be a straight line passing through the origin as shown in Fig. 1.21 (*ii*). The resistance R between points A and B is given by slope of the graph *i.e.*

$$R = \tan\theta = V/I = \text{Constant}$$

Ohm's law can be expressed in three forms *viz*.

$$I = V/R\ ;\ V = IR\ ;\ R = V/I$$

These formulae can be applied to any part of a d.c. circuit or to a complete circuit. It may be noted that if voltage is measured in volts and current in amperes, then resistance will be in ohms.

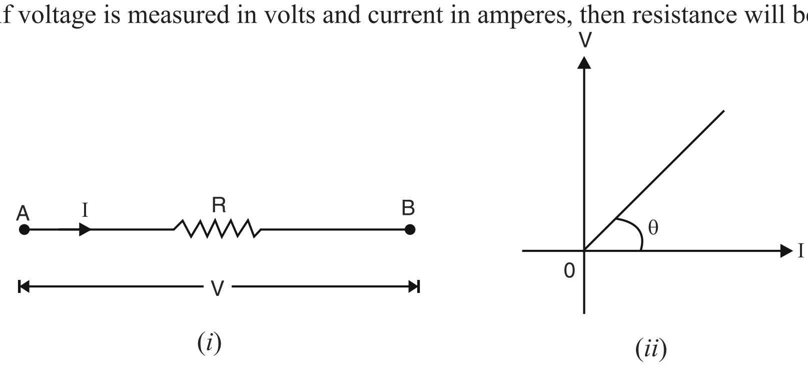

Fig. 1.21

* The unit of resistance (*i.e.* ohm) was named in his honour.

1.29. Non-ohmic Conductors

Those conductors which do not obey Ohm's law ($I \propto V$) are called non-ohmic conductors e.g., vacuum tubes, transistors, electrolytes, etc. A non-ohmic conductor may have one or more of the following properties :

(*i*) The *V-I* graph is non-linear *i.e. V/I* is variable.

(*ii*) The *V-I* graph may not pass through the origin as in case of an ohmic conductor.

(*iii*) A non-ohmic conductor may conduct poorly or not at all when the p.d. is reversed.

The non-linear circuit problems are generally solved by graphical methods.

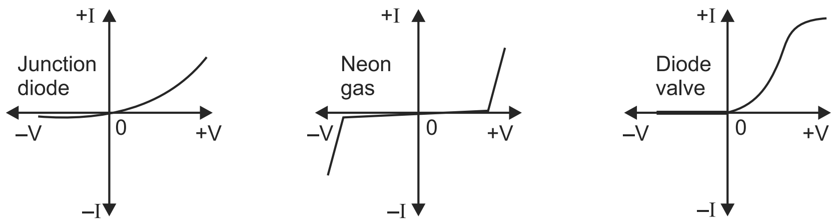

Fig. 1.22

Fig. 1.22 illustrates the graphs of non-ohmic conductors. Note that *V-I* graphs for these non-ohmic conductors are not a straight line.

Example 1.31. *What is the value of the unknown resistor R in Fig. 1.23 (i) if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.*

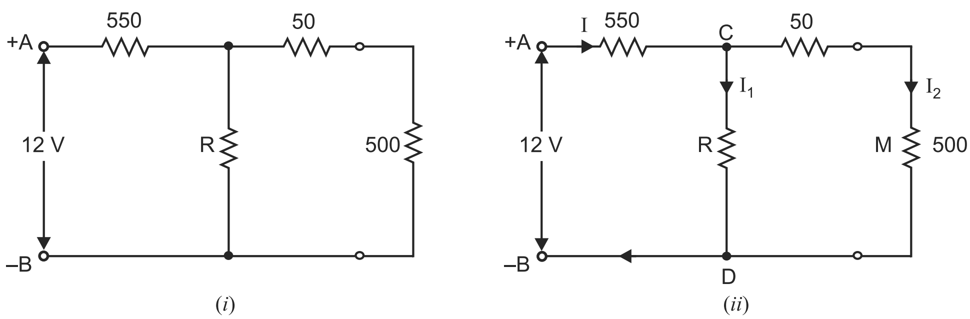

Fig. 1.23

Solution. Fig. 1.23 (*ii*) shows the various currents in the circuit.

$$I_2 = \frac{\text{Voltage drop across } 500\ \Omega}{500\ \Omega} = \frac{2.5}{500} = 0.005 \text{ A}$$

Voltage across *CMD* or *CD* is given by ;

$$V_{CMD} = V_{CD} = I_2\,(50 + 500) = 0.005 \times 550 = 2.75 \text{ V}$$

Now
$$I = \frac{12 - V_{CD}}{550} = \frac{12 - 2.75}{550} = 0.0168 \text{ A}$$

∴
$$I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

Now
$$V_{CD} = I_1 R \quad \therefore \quad R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = \mathbf{233\ \Omega}$$

Example 1.32. *A metal filament lamp takes 0.3 A at 230 V. If the voltage is reduced to 115 V, will the current be halved ? Explain your answer.*

Solution. No. It is because Ohm's law is applicable only if the resistance of the circuit does not change. In the present case, when voltage is reduced from 230 V to 115 V, the temperature of the lamp will decrease too much, resulting in an enormous decrease of lamp resistance. Consequently, Ohm's law ($I = V/R$) cannot be applied. To give an idea to the reader, the hot resistance (*i.e.* at normal operating temperature) of an incandescent lamp is more than 10 times its cold resistance.

Example 1.33. *A coil of copper wire has resistance of 90 Ω at 20°C and is connected to a 230 V supply. By how much must the voltage be increased in order to maintain the current constant if the temperature of the coil rises to 60°C? Take α_0 for copper = 0.00428/°C.*

Solution. $R_{20} = R_0(1 + \alpha_0 \times 20)$; $R_{60} = R_0(1 + \alpha_0 \times 60)$

$$\therefore \quad \frac{R_{60}}{R_{20}} = \frac{1+0.00428\times 60}{1+0.00428\times 20} = \frac{1.2568}{1.0856}$$

or
$$R_{60} = R_{20} \times \frac{1.2568}{1.0856} = 90\times\frac{1.2568}{1.0856} = 104.2\,\Omega$$

$$\text{Now, current at } 20°\text{C} = \frac{230}{90} = \frac{23}{9}\text{ A}$$

The wire resistance has become 104.2 Ω at 60°C. Therefore, in order to keep the current constant at the previous value, the new voltage required = (23/9) × 104.2 = 266.3 V.

∴ Required voltage increase = 266.3 – 230 = **36.3 V**

Tutorial Problems

1. A battery has an e.m.f. of 12.8 V and supplies a current of 3.2 A. What is the resistance of the circuit ? How many coulombs leave the battery in 5 minutes ? **[4 Ω ; 960 C]**
2. In a discharge tube, the number of hydrogen ions (*i.e.* protons) drifting across a cross-section per second is 1.2×10^{18} while the number of electrons drifting in the opposite direction is 2.8×10^{18} per second. If the supply voltage is 220 V, what is the effective resistance of the tube ? **[344 Ω]**
3. An electromagnet of resistance 12.4 Ω requires a current of 1.5 A to operate it. Find the required voltage. **[18.6 V]**
4. The cold resistance of a certain gas-filled tungsten lamp is 18.2 Ω and its hot resistance at the operating voltage of 220 V is 202 Ω. Find the current (*i*) at the instant of switching (*ii*) under normal operating conditions. **[(*i*) 12.08 A (*ii*) 1.09 A]**

1.30. Electric Power

The rate at which work is done in an electric circuit is called its **electric power** *i.e.*

$$\text{Electric power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current (*i.e.* electrons) to flow through it. Clearly, work is being done in moving the electrons in the circuit. This work done in moving the electrons in a unit time is called the electric power. Thus referring to the part *AB* of the circuit (See Fig. 1.24),

V = P.D. across *AB* in volts
I = Current in amperes
R = Resistance of *AB* in Ω
t = Time in sec. for which current flows

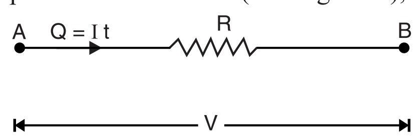

Fig. 1.24

The total charge that flows in t seconds is $Q = I \times t$ coulombs and by definition (See Art. 1.12),

$$V = \frac{\text{Work}}{Q}$$

or Work $= VQ = VIt$ $(\because Q = It)$

$$\therefore \quad \text{Electric power, } P = \frac{\text{Work}}{t} = \frac{VIt}{t} = VI \text{ joules/sec or watts}$$

$$\therefore \quad P = VI = I^2R = \frac{V^2}{R} \qquad [\because V = IR \text{ and } I = V/R]$$

The above three formulae are equally valid for calculation of electric power in a d.c. circuit. Which one is to be used depends simply on which quantities are known or most easily determined.

Unit of electric power. The basic unit of electric power is *joules/sec* or *watt*. The power consumed in a circuit is 1 watt if a p.d. of 1 V causes 1 A current to flow through the circuit.

$$\text{Power in watts} = \text{Voltage in volts} \times \text{Current in amperes}$$

The bigger units of electric power are kilowatts (kW) and megawatts (MW).

$$1 \text{ kW} = 1000 \text{ watts} \quad ; \quad 1 \text{ MW} = 10^6 \text{ watts or } 10^3 \text{ kW}$$

1.31. Electrical Energy

The total work done in an electric circuit is called **electrical energy** *i.e.*

$$\text{Electrical energy} = \text{Electrical power} \times \text{Time}$$
$$= V\,I\,t = I^2R\,t = \frac{V^2}{R}t$$

The reader may note that formulae for electrical energy can be readily derived by multiplying the electric power by 't', the time for which the current flows. The unit of electrical energy will depend upon the units of electric power and time.

(*i*) If power is taken in watts and time in seconds, then the unit of electrical energy will be *watt-sec. i.e.*

$$\text{Energy in watt-sec.} = \text{Power in watts} \times \text{Time in sec.}$$

(*ii*) If power is expressed in watts and time in hours, then unit of electrical energy will be *watt-hour i.e.*

$$\text{Energy in watt-hours} = \text{Power in watts} \times \text{Time in hours}$$

(*iii*) If power is expressed in kilowatts and time in hours, then unit of electrical energy will be *kilowatt-hour* (kWh) *i.e.*

$$\text{Energy in kWh} = \text{Power in kW} \times \text{Time in hours}$$

It may be pointed out here that in practice, electrical energy is measured in kilowatt-hours (kWh). Therefore, it is profitable to define it.

One kilowatt-hour (kWh) *of electrical energy is expended in a circuit if 1 kW (1000 watts) of power is supplied for 1 hour.*

The electricity bills are made on the basis of total electrical energy consumed by the consumer. The unit for charge of electricity is 1 kWh. One kWh is also called Board of Trade (B.O.T.) unit or simply unit. Thus when we say that a consumer has consumed 100 units of electricity, it means that electrical energy consumption is 100 kWh.

1.32. Use of Power and Energy Formulas

It has already been discussed that electric power as well as electrical energy consumed can be expressed by three formulas. While using these formulas, the following points may be kept in mind:

(*i*) $$\text{Electric power, } P = I^2 R = \frac{V^2}{R} \text{ watts}$$

$$\text{Electrical energy consumed, } W = I^2 Rt = \frac{V^2}{R}t \text{ joules}$$

The above formulas apply *only* to resistors and to devices (*e.g.* electric bulb, heater, electric kettle etc) where all electrical energy consumed is converted into heat.

(*ii*) Electric power, $P = VI$ watts

Electrical energy consumed, $W = VIt$ joules

These formulas apply to any type of load including the one mentioned in point (*i*).

Example 1.34. *A 100 V lamp has a hot resistance of 250 Ω. Find the current taken by the lamp and its power rating in watts. Calculate also the energy it will consume in 24 hours.*

Solution. Current taken by lamp, $I = V/R = 100/250 =$ **0.4 A**

Power rating of lamp, $P = VI = 100 \times 0.4 =$ **40 W**

Energy consumption in 24 hrs. = Power × time = 40 × 24 = **960 watt-hours**

Example 1.35. *A heating element supplies 300 kilojoules in 50 minutes. Find the p.d. across the element when current is 2 amperes.*

Solution. Total charge, $Q = I \times t = 2 \times 50 \times 60 = 6000$ C

$$\text{P.D., } V = \frac{\text{Work}}{\text{Charge}} = \frac{300 \times 10^3}{6000} = \mathbf{50\ V}$$

Example 1.36. *A 10 watt resistor has a value of 120 Ω. What is the rated current through the resistor ?*

Solution. Rated power, $P = I^2R$

$$\therefore \quad \text{Rated current, } I = \sqrt{\frac{P}{R}} = \sqrt{\frac{10}{120}} = \mathbf{0.2887\ A}$$

If current through the resistor exceeds this value, the resistor will be burnt due to excessive heat.

Note. Every electrical equipment has power and current ratings marked on its body. While the equipment is in operation, care should be taken that neither of these limits is exceeded, otherwise the equipment may be damaged/burnt due to excessive heat.

Example 1.37. *The following are the details of load on a circuit connected through a supply metre :*

(*i*) *Six lamps of 40 watts each working for 4 hours per day*

(*ii*) *Two flourescent tubes 125 watts each working for 2 hours per day*

(*iii*) *One 1000 watt heater working for 3 hours per day*

If each unit of energy costs 70 P, what will be the electricity bill for the month of June ?

Solution. Total wattage of lamps = 40 × 6 = 240 watts

Total wattage of tubes = 125 × 2 = 250 watts

Wattage of heater = 1000 watts

Energy consumed by the appliances per day

= (240 × 4) + (250 × 2) + (1000 × 3)

= 4460 watt-hours = 4.46 kWh

Total energy consumed in the month of June (*i.e.* in 30 days)

= 4.46 × 30 = 133.8 kWh

Bill for the month of June = Rs. 0.7 × 133.8 = **Rs. 93.66**

Tutorial Problems

1. A resistor of 50 Ω has a p.d. of 100 volts d.c. across it for 1 hour. Calculate (*i*) power and (*ii*) energy. **[(*i*) 200 watts (*ii*) 7.2 × 10^5 J]**
2. A current of 10 A flows through a resistor for 10 minutes and the power dissipated by the resistor is 100 watts. Find the p.d. across the resistor and the energy supplied to the circuit. **[10 V ; 6 × 10^4 J]**

3. A factory is supplied with power at 210 volts through a pair of feeders of total resistance 0.0225 Ω. The load consists of 354, 250 V, 60 watt lamps and 4 motors each taking 40 amperes. Find :
   (*i*) total current required
   (*ii*) voltage at the station end of feeders
   (*iii*) power wasted in feeders. **[(*i*) 231.4 A (*ii*) 215.78 V (*iii*) 1.4 kW]**
4. How many kilowatts will be required to light a factory in which 250 lamps each taking 1.3 A at 230 V are used ? **[74.75 kW]**

1.33. Power Rating of a Resistor

The ability of a resistor to dissipate power as heat without destructive temperature build-up is called **power rating** *of the resistor*.

Power rating of resistor = I^2R or V^2/R [See Fig. 1.25]

Suppose the power rating of a resistor is 2 W. It means that I^2R or V^2/R should not exceed 2 W. Suppose the quantity I^2R (or V^2/R) for this resistor becomes 4 W. The resistor is able to dissipate 2 W as heat and the remaining 2 W will start building up the temperature. In a matter of seconds, the resistor will burn out.

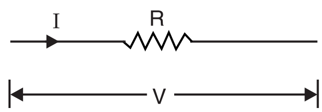

Fig. 1.25

The physical size of a resistor is not necessarily related to its resistance value but rather to its *power rating*. A large resistor is able to dissipate (throw off) more heat because of its large physical size. In general, the greater the physical size of a resistor, the greater is its power rating and vice-versa.

Example 1.38. *A 0.1 Ω resistor has a power rating of 5 W. Is this resistor safe when conducting a current of 10 A ?*

Solution. Power developed in the resistor is

$$P = I^2R = (10)^2 \times 0.1 = 10 \text{ W}$$

The resistor is **not safe** since the power developed in the resistor exceeds its dissipation rating.

Example 1.39. *What is the maximum safe current flow in a 47 Ω, 2 W resistor ?*

Solution. Power rating $= I^2R$

or $2 = I^2 \times 47$ $\therefore$ Maximum safe current, $I = \sqrt{\dfrac{2}{47}} =$ **0.21 A**

Example 1.40. *What is the maximum voltage that can be applied across a 100 Ω, 10 W resistor in order to keep within the resistor's power rating ?*

Solution. Power rating $= V^2/R$

or $10 = V^2/100$ $\therefore$ Max. safe voltage, $V = \sqrt{10 \times 100} =$ **31.6 volts**

Tutorial Problems

1. A 200 Ω resistor has a 2 W power rating. What is the maximum current that can flow in the resistor without exceeding the power rating ? **[100 mA]**
2. A 6.8 kΩ, 0.25 W resistor shows a potential difference of 40 V. Is the resistor safe ? **[Yes]**
3. A 1.5 kΩ resistor has 1 W power rating. What maximum voltage can be applied across the resistor without exceeding the power rating ? **[38.73 V]**

1.34. Nonlinear Resistors

A device or circuit element whose V/I characteristic is not a straight line is said to exhibit **nonlinear resistance.**

The examples of nonlinear resistors are thermistors, varistors, diodes, filaments of incandescent lamps etc.

1. Thermistors. *A* **thermistor** *is a heat sensitive device usually made of a semiconductor material whose resistance changes very rapidly with change of temperature.* A thermistor has the following important properties :

(*i*) The resistance of a thermistor changes very rapidly with change of temperature.

(*ii*) The temperature coefficient of a thermistor is very high.

(*iii*) The temperature co-efficient of a thermistor can be both positive and negative.

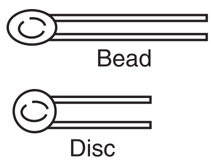

Fig. 1.26

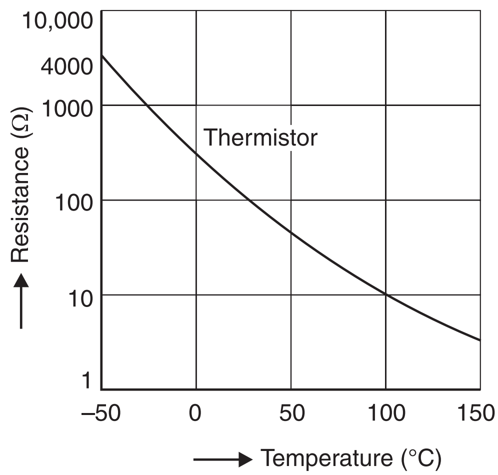

Fig. 1.27

Construction. Thermistors are made from semiconductor oxides of iron, nickel and cobalt. They are generally in the form of beads, discs or rods (See Fig. 1.26). A pair of platinum leads are attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed.

Fig. 1.27 shows the resistance/temperature characteristic of a typical thermistor with negative temperature coefficient. The resistance decreases progressively from 4000 Ω to 3 Ω as its temperature varies from – 50°C to +150°C.

Applications

(*a*) A thermistor with negative temperature coefficient of resistance may be used to safeguard against current surges in a circuit where this could be harmful *e.g.* in a circuit where the heaters of the radio valves are in series (See Fig. 1.28).

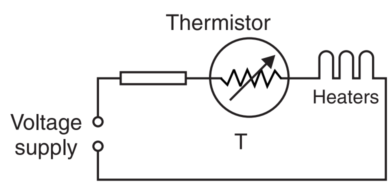

Fig. 1.28

A thermistor *T* is included in the circuit. When the supply voltage is switched on, the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warms up, the thermistor resistance drops appreciably and an increased current then flows through the heaters.

(*b*) A thermistor with a negative temperature coefficient can be used to issue an alarm for excessive temperature of winding of motors, transformers and generators [See Fig. 1.29].

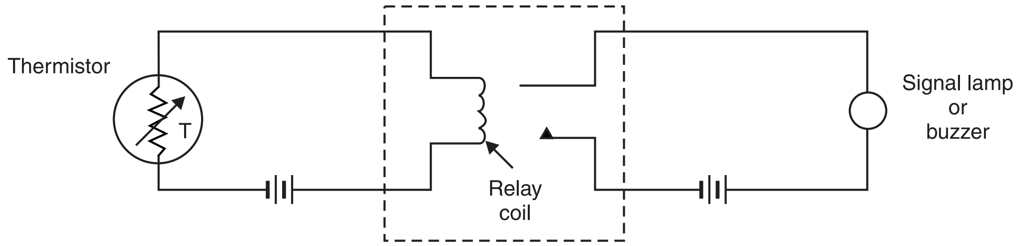

Fig. 1.29

When the temperature of windings is low, the thermistor is cool and its resistance is high. Therefore, only a small current flows through the thermistor and the relay coil. When the temperature of the windings is high, the thermistor is hot and its resistance is low. Therefore, a large current flows in the relay coil to close the contacts. This completes the circuit for the signal lamp or buzzer.

2. Varistor (Thyrite). A varistor is a nonlinear resistor whose resistance decreases as the voltage increases. Therefore, a varistor is a voltage-dependent resistor. It is made of silicon-carbide powder and is built in the shape of a disc. The *V-I* characteristic of a typical varistor is shown in Fig. 1.30. The curve shows that the current increases dramatically with increasing voltage. Thus when the voltage increases from 1.5 kV to 10 kV, the current rises from 1 mA to 100 A. Varistors are placed in parallel with critical components which might be damaged by high transient voltages. Under normal conditions, the varistor remains in high-resistance state and draws very little current. On the application of surge, the varistor is driven to its low-resistance state. The varistor then conducts a relatively large amount of current and dissipates much of the surge as heat. Thus the component is saved from damage.

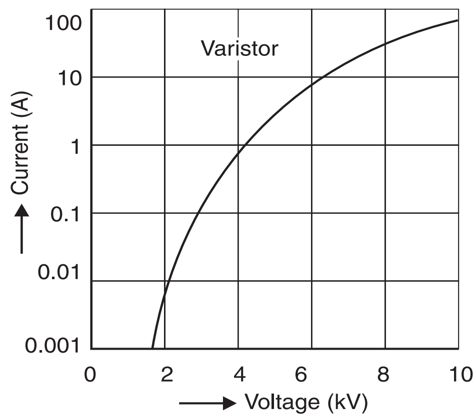

Fig. 1.30

OBJECTIVE QUESTIONS

1. The resistance of a wire is R ohms. It is stretched to double its length. The new resistance of the wire in ohms is
   (*i*) $R/2$ (*ii*) $2R$
   (*iii*) $4R$ (*iv*) $R/4$
2. The example of non-ohmic resistance is
   (*i*) copper wire
   (*ii*) carbon resistance
   (*iii*) tungsten wire
   (*iv*) diode
3. In which of the following substances, the resistance decreases with the increase of temperature
   (*i*) carbon (*ii*) constantan
   (*iii*) copper (*iv*) silver
4. The resistance of a wire of uniform diameter d and length l is R. The resistance of another wire of the same material but diameter $2d$ and length $4l$ will be
   (*i*) $2R$ (*ii*) R
   (*iii*) $R/2$ (*iv*) $R/4$
5. The temperature coefficient of resistance of a wire is 0.00125 $°C^{-1}$. At 300 K, its resistance is one ohms. The resistance of the wire will be 2 ohms at
   (*i*) 1154 K (*ii*) 1100 K
   (*iii*) 1400 K (*iv*) 1127 K
6. The resistance of 20 cm long wire is 5 ohms. The wire is stretched to a uniform wire of 40 cm length. The resistance now will be (in ohms)
   (*i*) 5 (*ii*) 10
   (*iii*) 20 (*iv*) 200

7. A current of 4.8 A is flowing in a conductor. The number of electrons flowing per second through the X-section of conductor will be

(*i*) 3×10^{19} electrons
(*ii*) 76.8×10^{20} electrons
(*iii*) 7.68×10^{20} electrons
(*iv*) 3×10^{20} electrons

8. A carbon resistor has coloured strips as brown, green, orange and silver respectively. The resistance is

(*i*) $15\ k\,\Omega \pm 10\%$ (*ii*) $10\ k\,\Omega \pm 10\%$
(*iii*) $15\ k\,\Omega \pm 5\%$ (*iv*) $10\ k\,\Omega \pm 5\%$

9. A wire has a resistance of 10 Ω. It is stretched by one-tenth of its original length. Then its resistance will be

(*i*) 10 Ω (*ii*) 12.1 Ω
(*iii*) 9 Ω (*iv*) 11 Ω

10. A 10 m long wire of resistance 20 Ω is connected in series with a battery of e.m.f. 3 V (negligible internal resistance) and a resistance of 10 Ω. The potential gradient along the wire in volt per metre is

(*i*) 0.02 (*ii*) 0.1
(*iii*) 0.2 (*iv*) 1.2

11. The diameter of an atom is about

(*i*) 10^{-10} m (*ii*) 10^{-8} m
(*iii*) 10^{-2} m (*iv*) 10^{-15} m

12. 1 cm^3 of copper at room temperature has about

(*i*) 200 free electrons
(*ii*) 20×10^{10} free electrons
(*iii*) 8.5×10^{22} free electrons
(*iv*) 3×10^{5} free electrons

13. The electric current is due to the flow of

(*i*) positive charges only
(*ii*) negative charges only
(*iii*) both positive and negative charges
(*iv*) neutral particles only

14. The quantity of charge that will be transferred by a current flow of 10 A over 1 hour period is

(*i*) 10 C (*ii*) 3.6×10^{4} C
(*iii*) 2.4×10^{3} C (*iv*) 1.6×10^{2} C

15. The drift velocity of electrons is of the order of

(*i*) 1 ms^{-1} (*ii*) 10^{-3} ms^{-1}
(*iii*) 10^{6} ms^{-1} (*iv*) 3×10^{8} ms^{-1}

16. Insulators have .... temperature co-efficient of resistance.

(*i*) zero (*ii*) positive
(*iii*) negative (*iv*) none of the above

17. Eureka has .... temperature co-efficient of resistance.

(*i*) almost zero (*ii*) negative
(*iii*) positive (*iv*) none of the above

18. Constantan wire is used for making standard resistances because it has ....

(*i*) low specific resistance
(*ii*) high specific resistance
(*iii*) negligibly small temperature co-efficient of resistance
(*iv*) high melting point

19. Two resistors *A* and *B* have resistances R_A and R_B respectively with $R_A < R_B$. The resistivities of their materials are ρ_A and ρ_B. Then,

(*i*) $\rho_A > \rho_B$ (*ii*) $\rho_A = \rho_B$
(*iii*) $\rho_A < \rho_B$
(*iv*) Information insufficient

20. In case of liquids, Ohm's law is ....

(*i*) fully obeyed
(*ii*) partially obeyed
(*iii*) there is no relation between current and p.d.
(*iv*) none of the above.

ANSWERS

1. (*iii*)	**2.** (*ii*) and (*iii*)	**3.** (*i*)	**4.** (*ii*)	**5.** (*ii*)
6. (*iii*)	**7.** (*i*)	**8.** (*i*)	**9.** (*ii*)	**10.** (*iii*)
11. (*i*)	**12.** (*iii*)	**13.** (*iii*)	**14.** (*ii*)	**15.** (*ii*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*iv*)	**20.** (*i*)