8

Magnetic Circuits

Introduction

We have seen that magnetic lines of force form closed loops around and through the magnetic material. The closed path followed by magnetic flux is called a magnetic circuit just as the closed path followed by current is called an electric circuit. Many electrical devices (*e.g.* generator, motor, transformer etc.) depend upon magnetism for their operation. Therefore, such devices have magnetic circuits *i.e.* closed flux paths. In order that these devices function efficiently, their magnetic circuits must be properly designed to obtain the required magnetic conditions. In this chapter, we shall focus our attention on the basic principles of magnetic circuits and methods to obtain their solution.

8.1. Magnetic Circuit

The closed path followed by magnetic flux is called a **magnetic circuit.**

In a magnetic circuit, the magnetic flux leaves the *N*-pole, passes through the entire circuit, and returns to the starting point. A magnetic circuit usually consists of materials having high permeability *e.g.* iron, soft steel *etc*. It is because these materials offer very small opposition to the 'flow' of magnetic flux. The most usual way of producing magnetic flux is by passing electric current through a wire of number of turns wound over a magnetic material. This helps in exercising excellent control over the magnitude, density and direction of magnetic flux.

Consider a coil of N turns wound on an iron core as shown in Fig. 8.1. When current I is passed through the coil, magnetic flux ϕ is set up in the core. The flux follows the closed path *ABCDA* and hence *ABCDA* is the magnetic circuit. The following points may be noted carefully :

(*i*) The amount of magnetic flux set up in the core depends upon current (I) and number of turns (N). If we increase the current or number of turns, the amount of magnetic flux also increases and *vice-versa*. The product *$N\,I$ is called the **magnetomotive force (m.m.f.)** and determines the amount of flux set up in the magnetic circuit.

$$\text{m.m.f.} = N\,I \text{ ampere-turns}$$

It can just be compared to electromotive force (e.m.f.) which sends current in an electric circuit.

(*ii*) The opposition that the magnetic circuit offers to the magnetic flux is called **reluctance.** It depends upon length of magnetic circuit (*i.e.* length *ABCDA* in this case), area of *X*-section of the circuit and the nature of material that makes up the magnetic circuit.

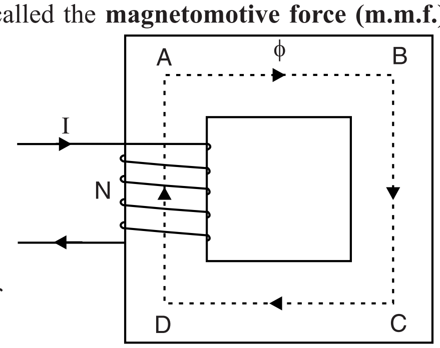

Fig. 8.1

8.2. Analysis of Magnetic Circuit

Consider the magnetic circuit shown in Fig. 8.1. Suppose the mean length of the magnetic circuit (*i.e.* length *ABCDA*) is l metres, cross-sectional area of the **core is 'a' m^2 and relative

* Coiling a conductor into two or more turns has the effect of using the same current for more than once. For example, 5-turn coil carrying a current of 10A produces the same magnetic flux in a given magnetic circuit as a 1-turn coil carrying a current of 50A. Hence m.m.f. is equal to the product of N and I.

** The arrangement of magnetic materials to form a magnetic circuit is generally called a *core*.

permeability of core material is μ_r. When current I is passed through the coil, it will set up flux ϕ in the material.

Flux density in the material, $B = \dfrac{\phi}{a}\,\text{Wb/m}^2$

Magnetising force in the material, $H = \dfrac{B}{\mu_0\mu_r} = \dfrac{\phi}{a\mu_0\mu_r}\,\text{AT/m}$

According to work law, the work done in moving a unit magnetic pole once around the magnetic circuit (*i.e.* path *ABCDA* in this case) is equal to the ampere-turns enclosed by the magnetic circuit.

$$\therefore \qquad *H \times l = N I$$

$$\text{or} \qquad \frac{\phi}{a\mu_0\mu_r} \times l = N I$$

$$\text{or} \qquad \phi = \frac{NI}{(l/a\mu_0\mu_r)}$$

The quantity *NI* which produces the magnetic flux is called the magnetomotive force (m.m.f.) and is measured in ampere-turns. The quantity $l/a\ \mu_0\ \mu_r$ is called the reluctance of the magnetic circuit. Reluctance is the opposition that the magnetic circuit offers to magnetic flux.

$$\therefore \qquad \text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} \qquad \ldots(i)$$

Note that the relationship expressed in eq. (*i*) has a strong resemblance to Ohm's law of electric circuit ($I = E/R$). The m.m.f. is analogous to e.m.f. in the electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, eq. (*i*) is sometimes referred to as **Ohm's law of magnetic circuit.**

8.3. Important Terms

In the study of magnetic circuits, we generally come across the following terms :

(*i*) Magnetomotive force (m.m.f.). It is a magnetic pressure which sets up or tends to set up flux in a magnetic circuit and may be defined as under :

The work done in moving a unit magnetic pole once around the magnetic circuit is called the magnetomotive force (m.m.f.). It is equal to the product of current and number of turns of the coil *i.e.*

$$\text{m.m.f.} = N\,I \text{ ampere-turns (or AT)}$$

Magnetomotive force in a magnetic circuit corresponds to e.m.f. in an electric circuit. The only change in the definition is the substitution of unit magnetic pole in place of unit charge.

(*ii*) Reluctance. *The opposition that the magnetic circuit offers to magnetic flux is called reluctance.* The reluctance of a magnetic circuit depends upon its length, area of *X*-section and permeability of the material that makes up the magnetic circuit. Its unit is †AT/Wb.

$$\text{Reluctance, } S = \frac{l}{a\,\mu_0\,\mu_r}$$

Reluctance in a magnetic circuit corresponds to resistance ($R = \rho\, l/a$) in an electric circuit. Both of them vary as length ÷ area and are dependent upon the nature of material of the circuit. Magnetic materials (*e.g.* iron, steel *etc.*) have a low reluctance because the value of μ_r is very large in their case. On the other hand, non-magnetic materials (*e.g.* air, wood, copper, brass *etc.*) have a high reluctance because they possess least value of μ_r ; being 1 in case of all non-magnetic materials.

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* You may recall that *H* means force acting on a unit magnetic pole. If the unit pole is moved once around the magnetic circuit (*i.e.* distance covered is *l*), then work done = $H \times l$.

† Reluctance $= \dfrac{\text{m.m.f.}}{\text{flux}} = \dfrac{\text{AT}}{\text{Wb}} = \text{AT/Wb}$

The reciprocal of permeability $\mu (= \mu_0 \mu_r)$ corresponds to resistivity ρ of the electrical circuit and is called *reluctivity. It may be noted that magnetic permeability (μ) is the analog of electrical conductivity.*

(iii) **Permeance.** *It is the reciprocal of reluctance* and is a measure of the ease with which flux can pass through the material. Its unit is Wb/AT.

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{a\mu_0\mu_r}{l}$$

Permeance of a magnetic circuit corresponds to conductance (reciprocal of resistance) in an electric circuit.

8.4. Comparison Between Magnetic and Electric Circuits

There are many points of similarity between magnetic and electric circuits. However, the two circuits are not anologous in all respects. A comparison of the two circuits is given below in the tabular form.

Magnetic Circuit

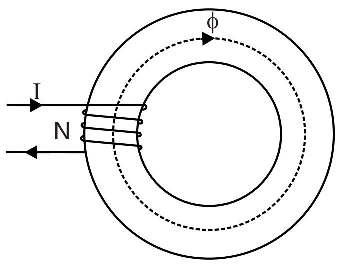


Fig. 8.2

Electric Circuit

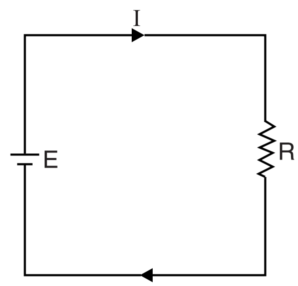


Fig. 8.3

Similarities

1. The closed path for magnetic flux is called a magnetic circuit.	**1.** The closed path for electric current is called an electric circuit.
2. Flux, $\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$	**2.** Current, $I = \dfrac{\text{e.m.f.}}{\text{resistance}}$
3. m.m.f. (ampere-turns)	**3.** e.m.f. (volts)
4. Reluctance, $S = \dfrac{l}{a\mu_0\mu_r}$	**4.** Resistance, $R = \rho \dfrac{l}{a}$
5. Flux density, $B = \dfrac{\phi}{a} \text{Wb/m}^2$	**5.** Current density, $J = \dfrac{I}{a} \text{A/m}^2$
6. m.m.f. drop = $\phi\, S$	**6.** Voltage drop = $I\, R$
7. Magnetic intensity, $H = N\, I/l$	**7.** Electric intensity, $E = V/d$
8. Permeance	**8.** Conductance.
9. Permeability	**9.** Conductivity

Dissimilarities

1. Truly speaking, magnetic flux does not flow.	**1.** The electric current acutally flows in an electric circuit.
2. There is no magnetic insulator. For example, flux can be set up even in air (the best known magnetic insulator) with reasonable m.m.f.	**2.** There are a number of electric insulators. For instance, air is a very good insulator and current cannot pass through it.

3. The value of μ_r is not constant for a given magnetic material. It varies considerably with flux density (B) in the material. This implies that reluctance of a magnetic circuit is not constant rather it depends upon B.	**3.** The value of resistivity (ρ) varies very slightly with temperature. Therefore, the resistance of an electric circuit is practically constant. This salient feature calls for different approach to the solution of magnetic and electric circuits.
4. No energy is expended in a magnetic circuit. In other words, energy is required in creating the flux, and not in maintaining it.	**4.** When current flows through an electric circuit, energy is expended so long as the current flows. The expended energy is dissipated in the form of heat.

8.5. Calculation of Ampere-Turns

In any magnetic circuit, flux produced is given by ;

$$\text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{AT}{(l/a\mu_0\mu_r)}$$

$$\therefore \quad AT \text{ required} = \phi \times \frac{l}{a\mu_0\mu_r} = \frac{\phi}{a} \times \frac{l}{\mu_0\mu_r}$$

$$= \frac{B}{\mu_0\mu_r} \times l \qquad \left(\because \; B = \frac{\phi}{a}\right)$$

$$= H \times l \qquad (\because \; H = B/\mu_0\,\mu_r)$$

i.e. **AT required for any part = Field strength H in that part × length of that part of magnetic circuit**

8.6. Series Magnetic Circuits

In a series magnetic circuit, the same flux ϕ flows through each part of the circuit. It can just be compared to a series electric circuit which carries the same current throughout.

Consider a ***composite series magnetic circuit** consisting of three different magnetic materials of different relative permeabilities along with an air gap as shown in Fig. 8.4. Each part of this series magnetic circuit will offer reluctance to the magnetic flux ϕ. The reluctance offered by each part will depend upon dimensions and μ_r of that part. Since the different parts of the circuit are in series, the total reluctance is equal to the sum of reluctances of individual parts, *i.e.*

$$\text{Total reluctance} = \frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}^{**}$$

$$\text{Total m.m.f.} = \text{Flux} \times \text{Total reluctance}$$

$$= \phi\left[\frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}\right]$$

$$= \frac{\phi}{a_1\,\mu_0\,\mu_{r1}} \times l_1 + \frac{\phi}{a_2\,\mu_0\,\mu_{r2}} \times l_2 + \frac{\phi}{a_3\,\mu_0\,\mu_{r3}} \times l_3 + \frac{\phi}{a_g\,\mu_0} \times l_g$$

$$= \frac{B_1}{\mu_0\,\mu_{r1}} \times l_1 + \frac{B_2}{\mu_0\,\mu_{r2}} \times l_2 + \frac{B_3}{\mu_0\,\mu_{r3}} \times l_3 + \frac{B_g}{\mu_0} \times l_g$$

$$= H_1\,l_1 + H_2\,l_2 + H_3\,l_3 + H_g\,l_g \qquad (\because \; H = B/\mu_0\,\mu_r)$$

* A series magnetic circuit that has parts of different dimensions and materials is called a composite series circuit.

** For air, $\mu_r = 1$.

Hence the total ampere-turns required for a series magnetic circuit can be found as under :

(*i*) Find H for each part of the series magnetic circuit. For air, $H = B/\mu_0$ whereas for magnetic material, $H = B/\mu_0\mu_r$.

(*ii*) Find the mean length (l) of magnetic path for each part of the circuit.

(*iii*) Find AT required for each part of the magnetic circuit using the relation, $AT = H \times l$.

(*iv*) The total AT required for the entire series circuit is equal to the sum of AT for various parts.

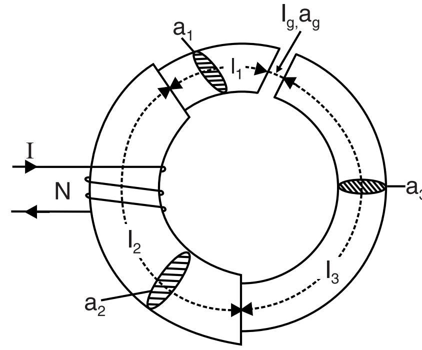


Fig. 8.4

8.7. Air Gaps in Magnetic Circuits

In many practical magnetic circuits, air gap is indispensable. For example, in electromechanical conversion devices like electric motors and generators, the magnetic flux must pass through stator as well as rotor. This necessitates to have a small air gap between the stator and rotor to permit mechanical clearance.

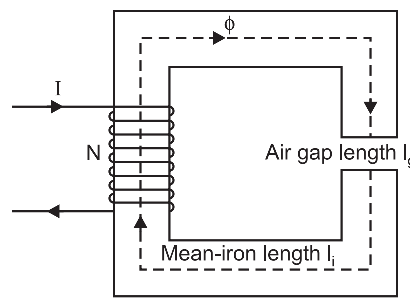


Fig. 8.5

The magnitude of AT required for air gap is much greater than that required for iron part of the magnetic circuit. It is because reluctance of air is very large compared to that offered by iron. Consider a magnetic circuit of uniform cross-sectional area a with an air gap as shown in Fig. 8.5. The length of the air gap is l_g and the mean length of iron part is l_i. The flux density $B(= \phi/a)$ is constant in the magnetic circuit.

$$\therefore \qquad \text{Reluctance of air gap} = \frac{l_g}{a\mu_0}$$

$$\text{Reluctance of iron part} = \frac{l_i}{a\mu_0\mu_r}$$

Now relative permeability μ_r of iron is very high (> 6000) so that reluctance of iron part is very small as compared to that of air gap inspite of the fact that $l_i > l_g$. In fact, most of ampere-turns (AT) are required in a magnetic circuit to force the flux through the air gap than through the iron part. In some magnetic circuits, we neglect reluctance of iron part compared to the air gap/gaps. This assumption leads to reasonable accuracy.

8.8. Parallel Magnetic Circuits

A magnetic circuit which has more than one path for flux is called a parallel magnetic circuit. It can just be compared to a parallel electric circuit which has more than one path for electric current.

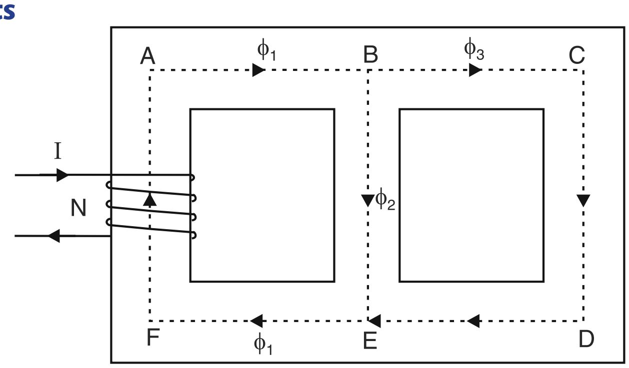


Fig. 8.6

The concept of parallel magnetic circuit is illustrated in Fig. 8.6. Here a coil of N turns wounded on limb AF carries a current of I amperes. The flux ϕ_1 set up by the coil divides at B into two paths, namely ;

(i) flux ϕ_2 passes along the path *BE*

(ii) flux ϕ_3 follows the path *BCDE*

$$\text{Clearly, } \phi_1 = \phi_2 + \phi_3$$

The magnetic paths *BE* and *BCDE* are in parallel and form a parallel magnetic circuit. The *AT* required for this parallel circuit is equal to *AT* required for any *one of the paths.

Let S_1 = reluctance of path *EFAB*

S_2 = reluctance of path *BE*

S_3 = reluctance of path *BCDE*

$\therefore$ Total m.m.f. required = m.m.f. for path *EFAB* + m.m.f. for path *BE* or path *BCDE*

or $NI = \phi_1 S_1 + \phi_2 S_2$

$= \phi_1 S_1 + \phi_3 S_3$

The reluctances S_1, S_2 and S_3 must be determined from a calculation of $l/a\mu_0\mu_r$ for those paths of the magnetic circuit in which ϕ_1, ϕ_2 and ϕ_3 exist respectively.

8.9. Magnetic Leakage and Fringing

The flux that does not follow the desired path in a magnetic circuit is called a **leakage flux.**

In most of practical magnetic circuits, a large part of flux path is through a magnetic material and the remainder part of flux path is through air. The flux in the air gap is known as *useful flux* because it can be utilised for various useful purposes. Fig. 8.7 shows an iron ring wound with a coil and having a narrow air gap. The total flux produced by the coil does not pass through the air gap as some of it **leaks through the air (path at '*a*') surrounding the iron. These flux lines as at '*a*' are called leakage flux.

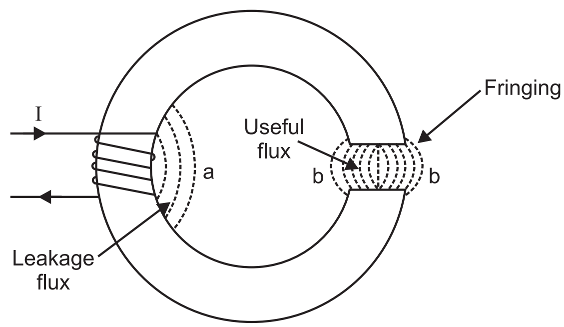


Fig. 8.7

Let ϕ_i = total flux produced *i.e.,* flux in the ***iron ring

ϕ_g = useful flux across the air gap

$\therefore$ Leakage flux, $\phi_{leak} = \phi_i - \phi_g$

$$\text{Leakage coefficient, } \lambda = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_i}{\phi_g}$$

The value of leakage coefficients for electrical machines is usually about 1.15 to 1.25.

Magnetic leakage is undesirable in electrical machines because it increases the weight as well as cost of the machine. Magnetic leakage can be greatly reduced by placing source of m.m.f. close to the air gap.

Fringing. When crossing an air gap, magnetic lines of force tend to bulge out such as lines of force at *bb* in Fig. 8.7. It is because lines of force repel each other when passing through non-

* This means that we may consider either path, say path *BE*, and calculate *AT* required for it. The same *AT* will also send the flux (ϕ_3 in this case) through the other parallel path *BCDE*. The situation is similar to that of two resistors R_1 and R_2 in parallel in an electric circuit. The voltage V required to send currents (say I_1 and I_2) in the resistors is equal to that appearing across either resistor *i.e.* $V = I_1 R_1 = I_2 R_2$.

** Air is not a good magnetic insulator. Therefore, leakage of flux from iron to air takes place easily.

*** The flux ϕ_i is not constant all around the ring. However, for reasonable accuracy, it is assumed that the iron carries the whole of the flux produced by the coil.

magnetic material such as air. This effect is known as *fringing*. The result of bulging or fringing is to increase the effective area of air gap and thus decrease the flux density in the gap. The longer the air gap, the greater is the fringing and *vice-versa*.

Note. In a short air gap with large cross-sectional area, the fringing may be insignificant. In other situations, 10% is added to the air gap's cross-sectional area to allow for fringing.

8.10. Solenoid

A long coil of wire consisting of closely packed loops is called a **solenoid.**

The word solenoid comes from Greek word meaning 'tube-like'. By a long solenoid we mean that length of the solenoid is very large as compared to its diameter. When current is passed through the coil of air-cored solenoid, magnetic field is set up as shown in Fig. 8.8. The path of the magnetic flux is made up of two components :

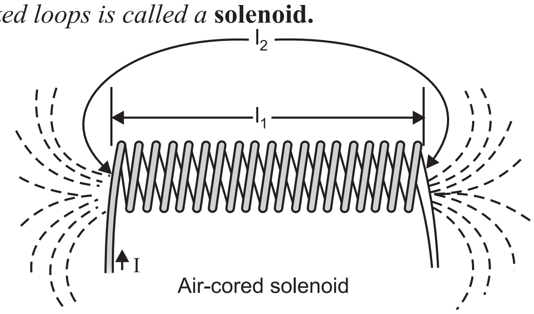


Fig. 8.8

(*i*) length l_1 of the path within the coil

(*ii*) length l_2 of the path outside the coil.

The total m.m.f. required for the solenoid is the sum of m.m.f.s required for these two paths *i.e.*

Total m.m.f. = m.m.f. for path l_1 + m.m.f. for path l_2

But m.m.f. for path l_1 *>> m.m.f. for path l_2

$\therefore$ Total m.m.f. = m.m.f. for path l_1

Hence, for a solenoid (air-cored or iron-cored), the length of the magnetic circuit is the coil length l_1. We can use right-hand rule to determine the direction of magnetic field in the core of the solenoid.

Example 8.1. *A cast steel electromagnet has an air gap length of 3 mm and an iron path of length 40 cm. Find the number of ampere-turns necessary to produce a flux density of 0.7 Wb/m² in the gap. Neglect leakage and fringing. Assume ampere-turns required for air gap to be 70% of the total ampere-turns.*

Solution. Air-gap length, $l_g = 3 \text{ mm} = 3 \times 10^{-3}$ m

Flux density in air gap, $B_g = 0.7$ Wb/m²

$$\therefore \quad \text{Magnetising force, } H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{0.7}{4\pi \times 10^{-7} \times 1} = 5.57 \times 10^5 \text{ AT/m}$$

$$AT \text{ required for air gap, } AT_g = H_g \times l_g = 5.57 \times 10^5 \times 3 \times 10^{-3} = 1671 \text{ AT}$$

It is given that : AT_g = 70% of total AT

$$\therefore \quad \text{Total } AT = \frac{AT_g}{0.7} = \frac{1671}{0.7} = \mathbf{2387\ AT}$$

Example 8.2. *An iron ring has a cross-sectional area of 400 mm² and a mean diameter of 25 cm. It is wound with 500 turns. If the value of relative permeability is 250, find the total flux set up in the ring. The coil resistance is 474 Ω and the supply voltage is 240 V.*

* The lengths l_2 and l_1 do not differ very much. However, the cross-sectional area of path l_2 is very large as compared to that of path l_1. Therefore, reluctance of path l_2 is very small as compared to that of path l_1.

Now, m.m.f. = flux × reluctance

Since reluctance of path l_2 is very small, the m.m.f. required for this path is negligible compared to that for path l_1.

Solution. The conditions of the problem are represented in Fig. 8.9.

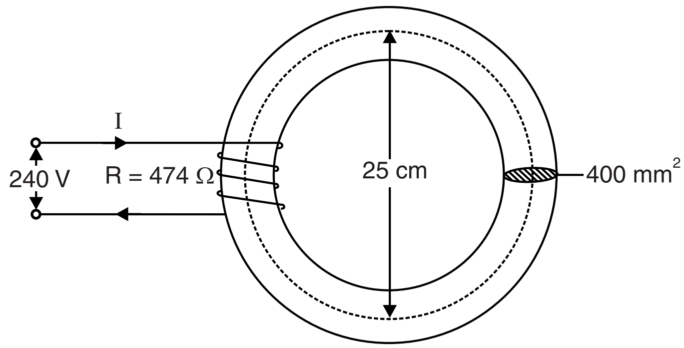


Fig. 8.9

Current through the coil, $I = V/R = 240/474 = 0{\cdot}506$ A

Mean length of magnetic circuit is given by ;

$$l = \pi \times (25 \times 10^{-2}) = 0{\cdot}7854 \text{ m}$$

$$\text{Magnetising force, } H = \frac{Nl}{l} = \frac{500 \times 0.506}{0.7854} = 322.13 \text{ AT/m}$$

$$\text{Flux density, } B = \mu_0 \mu_r H = (4\pi \times 10^{-7}) \times 250 \times 322{\cdot}13 = 0{\cdot}1012 \text{ Wb/m}^2$$

$\therefore$ Flux in the ring, $\phi = B \times a = 0{\cdot}1012 \times (400 \times 10^{-6}) =$ **$40{\cdot}48 \times 10^{-6}$ Wb**

Example 8.3. *An iron ring of cross-sectional area 6 cm² is wound with a wire of 100 turns and has a saw cut of 2 mm. Calculate the magnetising current required to produce a flux of 0·1 mWb if mean length of magnetic path is 30 cm and relative permeability of iron is 470.*

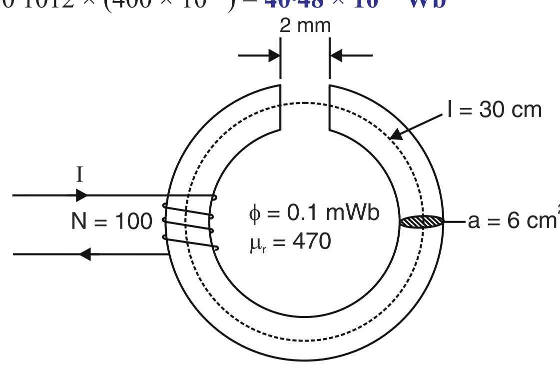


Fig. 8.10

Solution. The conditions of the problem are represented in Fig. 8.10. It will be assumed that flux density in the air gap is equal to the flux density in the core *i.e.* fringing is neglected. This assumption is quite reasonable in this case.

$$\text{Flux density, } B = \frac{\phi}{a} = \frac{0.1 \times 10^{-3}}{6 \times 10^{-4}} = 0.167 \text{ Wb/m}^2$$

Ampere-turns required for iron will be :

$$AT_i = H_i \times l_i$$

$$= \frac{B}{\mu_0 \mu_r} \times l_i = \frac{0.167}{4\pi \times 10^{-7} \times 470} \times 0.3 = 84.83 \text{ AT}$$

Ampere-turns required for air will be :

$$AT_g = \frac{B}{\mu_0} \times l_g = \frac{0{\cdot}167}{4\pi \times 10^{-7}} \times (2 \times 10^{-3}) = 265{\cdot}8 \text{ AT}$$

$\therefore$ Total $AT = 265{\cdot}8 + 84{\cdot}83 = 350{\cdot}63$ AT

$\therefore$ Magnetising current, $I = 350{\cdot}63/N = 350{\cdot}63/100 =$ **3·51 A**

It may be seen that many more ampere-turns are required to produce the magnetic flux through 2 mm of air gap than through the iron part. This is expected because reluctance of air is much more than that of iron.

Example 8.4. *A circular iron ring has a mean circumference of 1·5 m and a cross-sectional area of 0·01 m². A saw-cut of 4 mm wide is made in the ring. Calculate the magnetising current required to produce a flux of 0·8 mWb in the air gap if the ring is wound with a coil of 175 turns. Assume relative permeability of iron as 400 and leakage factor 1·25.*

Solution. $\phi_g = 0{\cdot}8 \times 10^{-3}$ Wb ; $a = 0{\cdot}01$ m² ; $l_i = 1{\cdot}5$m ; $l_g = 4 \times 10^{-3}$ m

AT for air gap

$$B_g = \frac{\phi_g}{a} = \frac{0.8\times10^{-3}}{0.01} = 0.08 \text{ Wb/m}^2$$

$$H_g = \frac{B_g}{\mu_0} = \frac{0\cdot08}{4\pi\times10^{-7}} = 63662 \text{ AT/m}$$

$$\therefore \quad AT_g = H_g \times l_g = 63662 \times (4 \times 10^{-3}) = 254{\cdot}6 \text{ AT}$$

AT for iron path

$$\phi_i = \phi_g \times \lambda = 0{\cdot}8 \times 10^{-3} \times 1{\cdot}25 = 10^{-3} \text{ Wb}$$

$$B_i = \phi_i/a = 10^{-3}/0{\cdot}01 = 0{\cdot}1 \text{ Wb/m}^2$$

$$H_i = \frac{B_i}{\mu_0\mu_r} = \frac{0\cdot1}{4\pi\times10^{-7}\times400} = 199 \text{ AT/m}$$

$$\therefore \quad AT_i = H_i \times l_i = 199 \times 1{\cdot}5 = 298{\cdot}5 \text{ AT}$$

$$\therefore \quad \text{Total } AT = 254{\cdot}6 + 298{\cdot}5 = 553{\cdot}1 \text{ AT}$$

$$\therefore \quad \text{Magnetising current, } I = 553{\cdot}1/N = 553{\cdot}1/175 = \mathbf{3{\cdot}16\ A}$$

Example 8.5. *A shunt field coil is required to develop 1500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be 2 × 10⁻⁶ Ω-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm².*

Solution. Suppose the number of turns of coil is N.

Then the total length of the coil, $l = 50 \times N$ cm

$$\text{Current in coil, } I = V/R = 60/R$$

$$\text{Resistance of coil, } R = \rho\frac{l}{A} = 2\times10^{-6}\times\frac{50\times N}{A} = \frac{N\times10^{-4}}{A} \quad ...(i)$$

Also $NI = 1500$ or $N \times (60/R) = 1500$ $\therefore$ $R = N/25$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\dfrac{N}{25} = \dfrac{N\times10^{-4}}{A}$ or $A = 25 \times 10^{-4}$ cm² $= 0{\cdot}25$ mm²

If D is the diameter of the wire, then,

$$\frac{\pi}{4}D^2 = 0{\cdot}25 \quad \text{or} \quad D = \mathbf{0{\cdot}568\ mm}$$

In order to operate the coil at a current density of 3 A/mm², the current in the coil is

$$I' = A \times \text{current density} = 0{\cdot}25 \times 3 = 0{\cdot}75 \text{ A}$$

$$\therefore \quad N'I' = 1500 \quad \text{or} \quad N' = 1500/I' = 1500/0{\cdot}75 = \mathbf{2000}$$

Example 8.6. *An iron ring has a mean diameter of 15 cm, a cross-section of 20 cm² and a radial gap of 0·5 mm cut in it. It is uniformly wound with 1500 turns of insulated wire and a magnetising current of 1 A produces a flux of 1 mWb. Neglecting the effect of magnetic leakage and fringing, calculate (i) reluctance of the magnetic circuit, (ii) relative permeability of iron and (iii) inductance of the winding.*

Solution. (*ii*) $a = 20 \times 10^{-4}$ m² ; $l_i = \pi \times 0{\cdot}15 = 0{\cdot}471$ m ; $l_g = 0{\cdot}5 \times 10^{-3}$ m

$$\text{Flux density in air gap, } B = \frac{\phi}{a} = \frac{1\times10^{-3}}{20\times10^{-4}} = 0{\cdot}5 \text{ Wb/m}^2$$

Magnetising force in air gap, $H_g = B/\mu_0 = 0{\cdot}5/4\pi \times 10^{-7} = 398 \times 10^3$ AT/m

Ampere-turns for air gap, $AT_g = H_g \times l_g = (398 \times 10^3) \times 0{\cdot}5 \times 10^{-3} = 199$ AT

Total AT provided $= NI = 1500 \times 1 = 1500$ AT

$\therefore$ AT available for iron part, $AT_i = 1500 - 199 = 1301$ AT

Magnetising force in iron, $H_i = \dfrac{AT_i}{l_i} = \dfrac{1301}{0.471} = 2762$ AT/m

Now, $$B = \mu_0\mu_r H_i$$

$\therefore$ $$\mu_r = \frac{B}{\mu_0 H_i} = \frac{0.5}{4\pi\times10^{-7}\times2762} = \mathbf{144}$$

(*i*) Reluctance of air gap $$= \frac{l_g}{a\mu_0} = \frac{0\cdot5\times10^{-3}}{(20\times10^{-4})\times4\pi\times10^{-7}} = 1{\cdot}99 \times 10^5 \text{ AT/Wb}$$

Reluctance of iron part $$= \frac{l_i}{a\mu_0\mu_r} = \frac{0.471}{(20\times10^{-4})\times4\pi\times10^{-7}\times144} = 13{\cdot}01 \times 10^5 \text{ AT/Wb}$$

$\therefore$ Total circuit reluctance $= 10^5\,(1.99 + 13.01) = \mathbf{15 \times 10^5}$ **AT/Wb**

(*iii*) Inductance of winding $$= \frac{N\phi}{I} = \frac{(1500)\times(1\times10^{-3})}{1} = \mathbf{1.5\ H}$$

Example 8.7. *A magnetic circuit is constructed as shown in Fig. 8.11. Both sections A and B are of 20 mm by 20 mm square cross-section and the mean dimensions are 100 mm by 80 mm. The relative permeability of section A is 250 and of section B is 500. Find the reluctance of each section and the total circuit reluctance.*

If the joints between sections A and B have an air gap of 0·5 mm at each joint, find the total reluctance of the circuit.

Solution. The conditions of the problem are represented in Fig. 8.11. The area of X-section of the core, $a = 20 \times 20 = 400\ \text{mm}^2 = 4 \times 10^{-4}\ \text{m}^2$.

Section A

Length of magnetic path, $l_A = 80 + 10 + 10 = 100$ mm $= 0{\cdot}1$ m

$\therefore$ Reluctance of section A $$= \frac{l_A}{a\mu_0\mu_r} = \frac{0\cdot1}{(4\times10^{-4})\times4\pi\times10^{-7}\times250} = \mathbf{0{\cdot}796 \times 10^6\ AT/Wb}$$

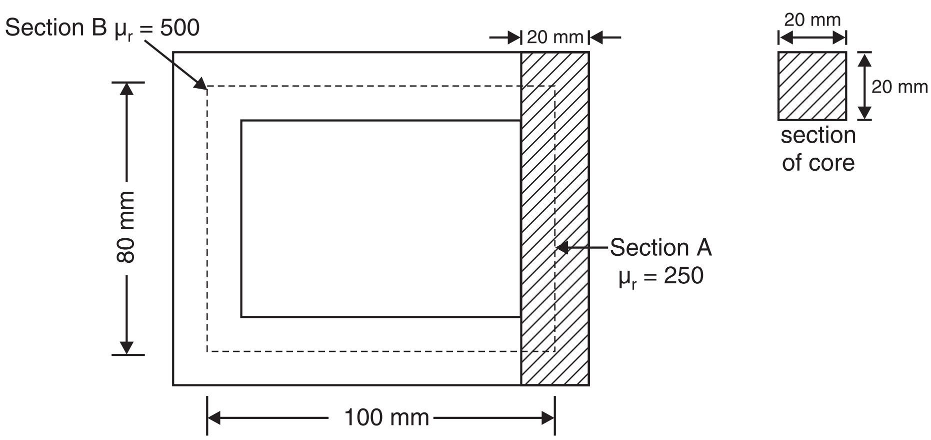


Fig. 8.11

Section B

Length of magnetic path, $l_B = 80 + 90 + 90 = 260$ mm $= 0{\cdot}26$ m

$\therefore$ Reluctance of section $B = \dfrac{l_B}{a\mu_0\mu_r} = \dfrac{0{\cdot}26}{(4\times10^{-4})\times4\pi\times10^{-7}\times500}$ $=$ **$1{\cdot}035 \times 10^6$ AT/Wb**

$\therefore$ Total circuit reluctance $= 10^6\,(0{\cdot}796 + 1{\cdot}035) =$ **$1{\cdot}831 \times 10^6$ AT/Wb**

Regarding the second part of the problem, the total length of air gaps is $l_g = 2 \times 0{\cdot}5 = 1$ mm $= 0{\cdot}001$ m.

$\therefore$ Reluctance of air gaps $= \dfrac{l_g}{a\mu_0} = \dfrac{0{\cdot}1}{(4\times10^{-4})\times4\pi\times10^{-7}}$ $= 1{\cdot}99 \times 10^6$ AT/Wb

$\therefore$ Total circuit reluctance $= 10^6\,(1{\cdot}831 + 1{\cdot}99) =$ **$3{\cdot}821 \times 10^6$ AT/Wb**

The reader may note that the reluctance of even small air gaps is very large. It is very important, therefore, that the joints of magnetic circuits — for example, the core of a transformer — should be tightly bolted together.

Note. The air gap is very small. Therefore, the magnetic length of iron part is the same in the two cases.

Example 8.8. *A rectangular iron core is shown in Fig. 8.12. It has a mean length of magnetic path of 100 cm, cross-section of 2 cm × 2 cm, relative permeability of 1400 and an air gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$ and the respective currents are 1·6 A, 4 A and 3 A. The directions of the currents are as shown in Fig. 8.12. Find the flux in the air gap.*

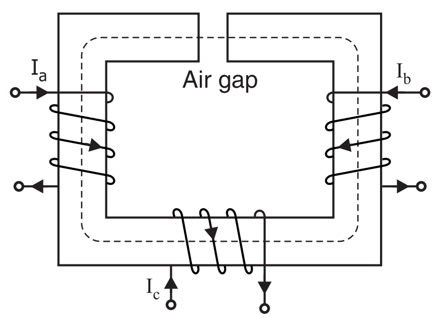


Fig. 8.12

Solution. By applying right-hand rule for the coil, it is easy to see that fluxes produced by currents I_a and I_b are in the clockwise direction through the iron core while the flux produced by current I_c is in the anticlockwise direction through the core.

$\therefore$ Net m.m.f. $= N_aI_a + N_bI_b - N_cI_c = 335 \times 1{\cdot}6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of air gap $= \dfrac{l_g}{\mu_0 a} = \dfrac{5\times10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}}$ $= 9{\cdot}946 \times 10^6$ AT/Wb

Reluctance of iron path $= \dfrac{l_i}{\mu_0\mu_r a} = \dfrac{(100-0{\cdot}5)\times10^{-2}}{4\pi\times10^{-7}\times1400\times4\times10^{-4}}$ $= 1{\cdot}414 \times 10^6$ AT/Wb

$\therefore$ Total reluctance $= (9{\cdot}946 + 1{\cdot}414) \times 10^6 = 11{\cdot}36 \times 10^6$ AT/Wb

The statement of the example suggests that there is no leakage flux. Therefore, flux in the air gap is the same as in the iron core.

$\therefore$ Flux in air gap $= \dfrac{\text{Net m.m.f.}}{\text{Total reluctance}} = \dfrac{1136}{11{\cdot}36\times10^6}$ $= 100 \times 10^{-6}$ Wb $=$ **100 μWb**

Example 8.9. *An angular ring of wood has a cross-sectional area of 4 cm^2 and a mean diameter of 30 cm. It is uniformly wound with 1200 turns of wire having a resistance of 6 Ω. The core of the second ring, with same dimensions and similarly wound, is made of a magnetic material of relative permeability 50. When the two windings are connected in parallel to a battery, the sum of the two fluxes in the two cores is 0·2 mWb [See Fig. 8.13]. Find the terminal voltage of the battery.*

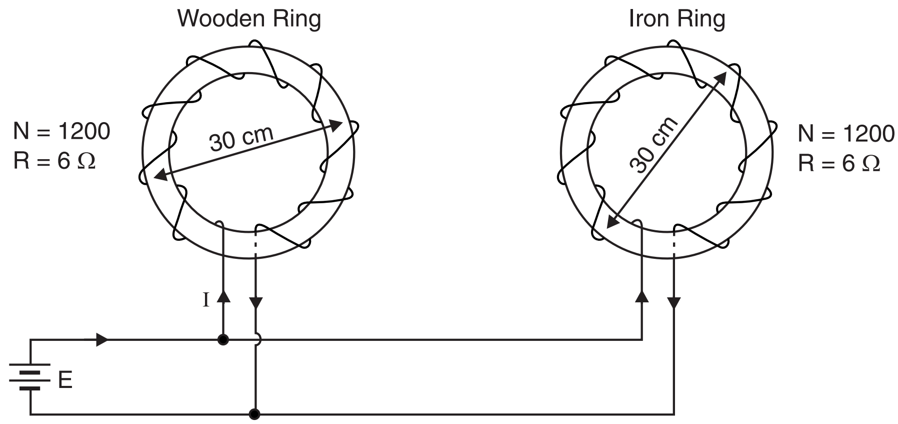


Fig. 8.13

Solution. The windings will carry the same current I as their resistances are equal. Moreover, each ring has the same mean magnetic length $l = \pi \times 0{\cdot}3 = 0{\cdot}942$ m.

Wooden ring. Reluctance $= \dfrac{l}{a\mu_0\mu_r} = \dfrac{0{\cdot}942}{(4\times10^{-4})\times4\pi\times10^{-7}\times1} = 18{\cdot}74 \times 10^8$ AT/Wb

Now, m.m.f. = flux × reluctance

∴ Flux in wooden ring, $\phi_1 = \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1200I}{18{\cdot}74\times10^8} = 6{\cdot}4 \times 10^{-7}\, I$ Wb

Iron ring. Reluctance $= \dfrac{l}{a\mu_0\mu_r} = \dfrac{0{\cdot}942}{(4\times10^{-4})\times4\pi\times10^{-7}\times50} = 0{\cdot}375 \times 10^8$ AT/Wb

∴ Flux in the iron ring, $\phi_2 = \dfrac{1200I}{0{\cdot}375\times10^8} = 320 \times 10^{-7}\, I$ Wb

∴ Total flux in the two rings $= (6{\cdot}4 + 320)\, 10^{-7}\, I = 326{\cdot}4 \times 10^{-7}\, I$ Wb

But the sum of two fluxes in the rings is given to be $0{\cdot}2 \times 10^{-3}$ Wb.

∴ $326{\cdot}4 \times 10^{-7}\, I = 0{\cdot}2 \times 10^{-3}$ or $I = \dfrac{0{\cdot}2\times10^{-3}}{326{\cdot}4\times10^{-7}} = 6{\cdot}13$ A

∴ Battery terminal voltage $= IR = 6{\cdot}13 \times 6 =$ **36·78 V**

Example 8.10. *In the magnetic circuit shown in Fig. 8.14, find (i) the total reluctance of the magnetic circuit and (ii) value of flux linking the coil. Assume that the relative permeability of the magnetic material is 800. The exciting coil has 1000 turns and carries a current of 1·25 A.*

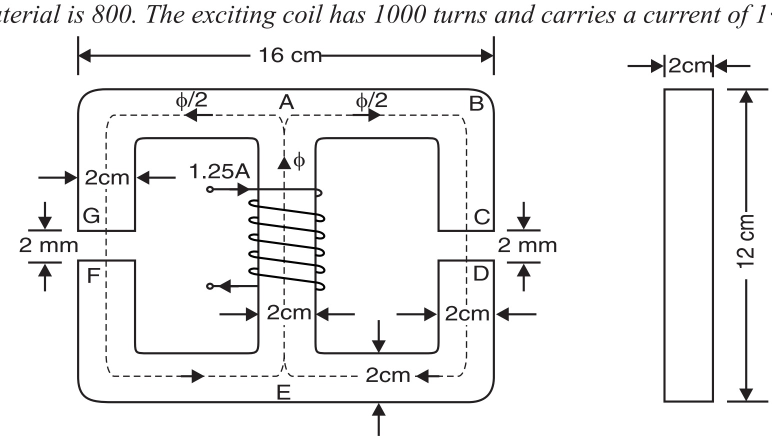


Fig. 8.14

Solution. The total flux ϕ set up by the exciting coil is divided into two parallel paths *viz*. path *AGFE* and path *ABCDE*. Since the two parallel paths are identical, each path will carry a flux = $\phi/2$ and that each parallel path has the same reluctance.

$l_{AE} = 10$ cm ; $l_{AG} = l_{FE} = 12$ cm ; $l_{GF} = 2$ mm ; $a = 2 \times 2 = 4$ cm^2

(*i*) Reluctance of magnetic path *AGFE*

$$= 2^* \text{ (Reluct. of path } AG) + \text{Reluct. of air gap } GF$$

$$= 2\left(\frac{l_{AG}}{a\mu_0\mu_r}\right) + \frac{l_{GF}}{a\mu_0}$$

$$= 2\left[\frac{0 \cdot 12}{(4\times10^{-4})\times4\pi\times10^{-7}\times800}\right] + \frac{2\times10^{-3}}{4\times10^{-4}\times4\pi\times10^{-7}}$$

$$= 5{\cdot}968 \times 10^5 + 39{\cdot}788 \times 10^5 = 45{\cdot}756 \times 10^5 \text{ AT/Wb}$$

Reluctance of magnetic path *AE* will be

$$= \frac{l_{AE}}{a\mu_0\mu_r} = \frac{0 \cdot 1}{(4\times10^{-4})\times4\pi\times10^{-7}\times800} = 2{\cdot}486 \times 10^5 \text{ AT/Wb}$$

Total reluctance of magnetic circuit will be

$$= 45{\cdot}75 \times 10^5 + 2{\cdot}486 \times 10^5 = \mathbf{48{\cdot}242 \times 10^5 \text{ AT/Wb}}$$

(*ii*) m.m.f. = flux × reluctance

or $1000 \times 1{\cdot}25 = \phi \times (48.242 \times 10^5)$

$$\therefore \quad \phi = \frac{1000\times1\cdot25}{48\cdot242\times10^5} = \mathbf{25{\cdot}9 \times 10^{-5} \text{ Wb}}$$

Example 8.11. *A magnetic circuit consists of three parts in series, each of uniform cross-sectional area. They are :*

(*a*) *a length of 80 mm and cross-sectional area 50 mm^2*

(*b*) *a length of 60 mm and cross-sectional area 90 mm^2*

(*c*) *an air gap of length 0·5 mm and cross-sectional area 150 mm^2.*

A coil of 4000 turns is wound on part (b) and the flux density in the air gap is 0·3 Wb/m^2. Assuming that all the flux passes through the given circuit, and that relative permeability μ_r is 1300, estimate the coil current to produce such a flux density.

Solution. Flux in the circuit, $\phi = B_g \times a_g = 0{\cdot}3 \times 1{\cdot}5 \times 10^{-4} = 0{\cdot}45 \times 10^{-4}$ Wb/m^2

$$\text{m.m.f. required for part } (a) = \phi\, S_a = \phi\times\frac{l_a}{\mu_0\mu_r a_a}$$

$$= 0\cdot45\times10^{-4}\times\frac{80\times10^{-3}}{4\pi\times10^{-7}\times1300\times50\times10^{-6}} = 44{\cdot}07 \text{ AT}$$

$$\text{m.m.f. required for part } (b) = \phi\, S_b = \phi\times\frac{l_b}{\mu_0\mu_r a_b}$$

$$= 0\cdot45\times10^{-4}\times\frac{60\times10^{-3}}{4\pi\times10^{-7}\times1300\times90\times10^{-6}} = 18{\cdot}4 \text{ AT}$$

$$\text{m.m.f. required for part } (c) = \phi\, S_c = \phi\times\frac{l_c}{\mu_0 a_c}$$

$$= 0\cdot45\times10^{-4}\times\frac{0\cdot5\times10^{-3}}{4\pi\times10^{-7}\times150\times10^{-6}} = 119{\cdot}3 \text{ AT}$$

$$\text{Total m.m.f. required} = 44{\cdot}07 + 18{\cdot}4 + 119{\cdot}3 = 181{\cdot}77 \text{ AT}$$

* Reluctance of path *AG* = Reluctance of path *FE*

$\therefore \quad NI = 181 \cdot 77 \quad$ or $\quad I = 181 \cdot 77/N = 181 \cdot 77/4000 = 45 \cdot 4 \times 10^{-3}$ A = **45·4 mA**

Since the absolute permeability of air (μ_0) is much smaller than that of a ferromagnetic material, the value of reluctance of air gap ($= l_g/a_g\mu_0$) is much greater than the reluctance of adjacent magnetic material ($= l_i/a_i\mu_0\mu_r$). Therefore, the m.m.f. required to force flux through the air gap can be quite large.

Example 8.12. *A laminated soft-iron ring has a mean circumference of 600 mm, cross-sectional area 500 mm² and has a radial air gap of 1 mm cut through it. It is wound with a coil of 1000 turns. Estimate the current in the coil to produce a flux of 0·5 mWb in the air gap assuming :*

(*i*) *the relative permeability of the soft iron is 1000,* (*ii*) *the leakage factor is 1·2,* (*iii*) *fringing is negligible,* (*iv*) *the space factor is 0·9.*

Solution. AT for air-gap

$\phi_g = 0 \cdot 5 \text{ mWb} = 5 \times 10^{-4} \text{ Wb} \quad ; \quad l_g = 1 \times 10^{-3} \text{ m} \quad ; \quad a_g = 500 \times 10^{-6} \text{ m}^2$

$$\text{m.m.f. for air gap} = \phi_g S_g = \phi_g \times \frac{l_g}{\mu_0 a_g}$$

$$= 5 \times 10^{-4} \times \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 500 \times 10^{-6}} = 795 \cdot 7 \text{ AT}$$

AT for iron part

$\phi_i = \phi_g \times 1 \cdot 2^* = 5 \times 10^{-4} \times 1 \cdot 2 \text{ Wb} \; ; \; l_i = 600 \times 10^{-3} \text{ m} \; ; \; a_i = 500 \times 10^{-6} \times 0 \cdot 9^{**} \text{m}^2$

$$\therefore \quad \text{m.m.f. for iron part} = \phi_i S_i = \phi_i \times \frac{l_i}{\mu_0 \mu_r a_i}$$

$$= 5 \times 10^{-4} \times 1 \cdot 2 \times \frac{600 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000 \times 500 \times 10^{-6} \times 0 \cdot 9}$$

$$= 636 \cdot 6 \text{ AT}$$

$$\therefore \quad \text{Total m.m.f. required} = 795 \cdot 7 + 636 \cdot 6 = 1432 \cdot 3 \text{ AT}$$

Now $\quad NI = 1432 \cdot 3 \quad \therefore \quad I = 1432 \cdot 3/N = 1432.3/1000 =$ **1·432 A**

Note that *AT* for air-gap are comparable to that for iron part. It is because length of air gap is very small.

Example 8.13. *The ring-shaped core shown in Fig. 8.15 is made of material having relative permeability 1000. The flux density in the thicker section is 1·5 T. If the current through the coil is not to exceed 0·5 A, find the number of turns of the coil.*

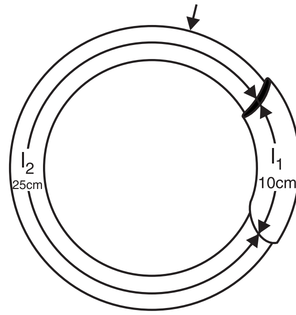


Fig. 8.15

Solution. The statement of the problem suggests that flux in the thicker as well as in thin section is the same *i.e.* it is a series magnetic circuit.

Flux in the magnetic circuit is

$$\phi = 1 \cdot 5 \times 6 \times 10^{-4}$$
$$= 9 \times 10^{-4} \text{ Wb}$$

AT for thick section

$$H_1 = \frac{B_1}{\mu_0 \mu_r} = \frac{1 \cdot 5}{4\pi \times 10^{-7} \times 1000} = 1194 \text{ AT/m}$$

m.m.f. for thick section $= H_1 l_1 = (1194) \times (10 \times 10^{-2})$

* The leakage factor refers to the flux leakage in the iron part of the magnetic circuit.

$$\text{Leakage factor} = \frac{\text{Total flux}}{\text{Useful flux}}$$

** $$\text{Space factor} = \frac{\text{Useful area}}{\text{Total area}}$$

$= 119{\cdot}4$ AT

AT for thin section $B_2 = \frac{\phi}{a} = \frac{9 \times 10^{-4}}{4 \times 10^{-4}} = 2{\cdot}25$ T

$$H_2 = \frac{B_2}{\mu_0 \mu_r} = \frac{2 \cdot 25}{4\pi \times 10^{-7} \times 1000} = 1790 \text{ AT/m}$$

m.m.f. for thin section $= H_2 l_2 = (1790) \times (25 \times 10^{-2}) = 448$ AT

$\therefore$ Total m.m.f. required $= 119{\cdot}4 + 448 = 567{\cdot}4$ AT

Now $NI = 567{\cdot}4$ or $N = 567{\cdot}4/I = 567{\cdot}4/0{\cdot}5 =$ **1135**

Example 8.14. *A steel ring 30 cm mean diameter and of circular section 2 cm in diameter has an air gap 1 mm long. It is wound uniformly with 600 turns of wire carrying current of 2·5 A. Find (i) total m.m.f., (ii) total reluctance and (iii) flux. Neglect magnetic leakage. The iron path takes 40% of the total m.m.f.*

Solution. (*i*) Total m.m.f. $= NI = 600 \times 2{\cdot}5 =$ **1500 AT**

(*ii*) Let M_1 and M_2 be the m.m.f. for iron part and air gap respectively and S_1 and S_2 their corresponding reluctances.

$M_1 = 40\%$ of $1500 = (40/100) \times 1500 = 600$ AT

$M_2 = 1500 - 600 = 900$ AT

Now, $M_1 = \phi S_1$ and $M_2 = \phi S_2$

$$\therefore \quad \frac{S_1}{S_2} = \frac{M_1}{M_2} = \frac{600}{900} = 0{\cdot}67$$

$$S_2 = \frac{l_g}{a\mu_0} = \frac{1 \times 10^{-3}}{\pi(1 \times 10^{-2})^2 \times 4\pi \times 10^{-7}} = 2{\cdot}5 \times 10^6 \text{ AT/Wb}$$

$\therefore$ $S_1 = 0{\cdot}67 S_2 = 0{\cdot}67 \times (2{\cdot}5 \times 10^6) = 1{\cdot}675 \times 10^6$ AT/Wb

Total reluctance $= S_1 + S_2 = (1{\cdot}675 + 2{\cdot}5) \times 10^6 =$ **$4{\cdot}175 \times 10^6$ AT/Wb**

(*iii*)

$$\text{Flux} = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{1500}{4 \cdot 175 \times 10^6}$$

$= 0{\cdot}36 \times 10^{-3}$ Wb $=$ **0·36 mWb**

Example 8.15. *A cast steel magnetic structure made of a bar of section 2 cm × 2 cm is shown in Fig. 8.16. Determine the current that the 500 turn magnetising coil on the left limb should carry so that a flux of 2mWb is produced in the right limb. Take μ_r = 600 and neglect leakage.*

Solution. The magnetising coil on the left limb produces flux ϕ which divides into two parallel paths; ϕ_1 in path B and ϕ_2 in path C. Since paths B and C are in parallel, AT required for path B ($= \phi_1 S_B$) are equal to that required for path C ($= \phi_2 S_c$) *i.e.*

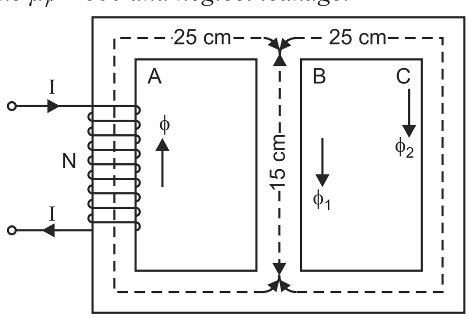


Fig. 8.16

$$\phi_1 S_B = \phi_2 S_c$$

or

$$\phi_1 \times \frac{l_B}{\mu_0 \mu_r a} = \phi_2 \times \frac{l_C}{\mu_0 \mu_r a}$$

$$\therefore \quad \phi_1 = \phi_2 \times \frac{l_C}{l_B} = 2 \times \frac{25}{15} = \frac{10}{3} \text{mWb} \qquad (\because \ \phi_2 = 2\text{mWb})$$

Total flux in path A, $\phi = \phi_1 + \phi_2 = \frac{10}{3} + 2 = \frac{16}{3}$ mWb

Total *AT* required for the whole magnetic circuit are equal to the sum of (*i*) *AT* required for path *A* and (*ii*) *AT* required for one of the parallel paths *B* or *C*.

$$\text{Flux density in path } A,\ B_A = \frac{\phi}{a} = \frac{(16/3)\times 10^{-3}}{4\times 10^{-4}} = \frac{40}{3}\,\text{Wb/m}^2$$

$$AT \text{ required for path } A = \frac{B_A}{\mu_0\mu_r}\times l_A \times \frac{(40/3)}{4\pi\times 10^{-7}\times 600}\times 0.25 = 4420\text{ AT}$$

$$\text{Flux density in path } B,\ B_B = \frac{\phi_1}{a} = \frac{(10/3)\times 10^{-3}}{4\times 10^{-4}} = 8.33\text{ Wb/m}^2$$

$$AT \text{ required for path } B = \frac{B_B}{\mu_0\mu_r}\times l_B = \frac{8.33}{4\pi\times 10^{-7}\times 600}\times 0.15 = 1658\text{ AT}$$

$$\therefore \quad \text{Total } AT \text{ required} = 4420 + 1658 = 6078\text{ AT}$$

$$\text{Now,} \quad NI = 6078 \quad \therefore I = \frac{6078}{N} = \frac{6078}{500} = \mathbf{12.16\ A}$$

Example 8.16. *A magnetic core made of annealed sheet steel has the dimensions as shown in Fig. 8.17. The X-section is 25 cm² everywhere. The flux in branches A and B is 3500 μWb but that in the branch C is zero. Find the required ampere-turns for coil A and for coil C. Relative permeability of sheet steel is 1000.*

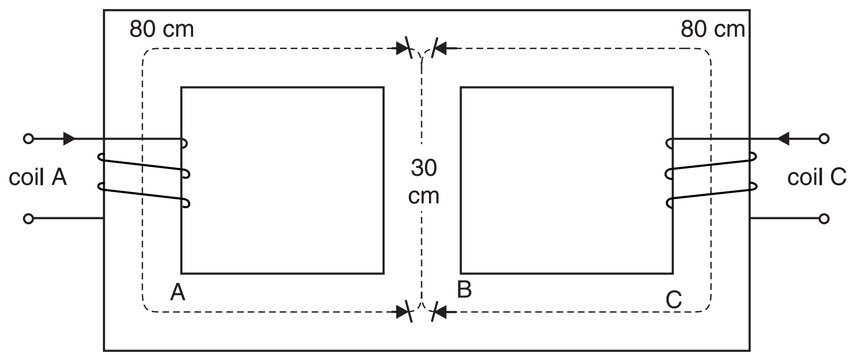


Fig. 8.17

Solution. AT for coil *A*. Flux paths *B* and *C* are in parallel. Therefore, AT required for coil *A* is equal to AT for path *A* plus AT for path *B* or path *C*.

$$\text{AT for path } A = \text{flux} \times \text{reluctance} = (3500\times 10^{-6})\times \frac{0\cdot 8}{(25\times 10^{-4})\times 4\pi\times 10^{-7}\times 1000} = 891\cdot 3\text{ AT}$$

$$\text{AT for path B} = \text{flux} \times \text{reluctance} = (3500\times 10^{-6})\times \frac{0\cdot 3}{(25\times 10^{-4})\times 4\pi\times 10^{-7}\times 1000} = 334\cdot 2\text{ AT}$$

Total AT for coil $A = 891\cdot 3 + 334\cdot 2 =$ **1225·5 AT**

AT for coil *C*. The coil *C* produces flux ϕ_C μWb in the opposite direction to that produced by coil *A*.

$$\text{m.m.f. of path } B = \text{m.m.f. of path } C$$

$$\phi_B S_B = \phi_C S_C$$

$$\text{or} \quad (3500\times 10^{-6})\times \frac{l_B}{a\mu_0\mu_r} = \phi_C \times \frac{l_C}{a\mu_0\mu_r}$$

$$\therefore \quad \phi_C = (3500 \times 10^{-6}) \times l_B/l_C = (3500 \times 10^{-6}) \times 0\cdot 3/0\cdot 8 = 1312\cdot 5\ \mu\text{Wb}$$

$$\text{Total AT for coil } C = \phi_C \times \text{reluctance}$$

$$= (1312\cdot 5\times 10^{-6})\times \frac{0\cdot 8}{(25\times 10^{-4})\times 4\pi\times 10^{-7}\times 1000} = \mathbf{334.22\ AT}$$

Example 8.17. *A magnetic circuit is shown in Fig. 8.18. It is made of cast steel 0.05 m thick. The length of air gap is 0.003 m. Find the m.m.f. to establish a flux of 5 × 10^{-4} Wb in the air gap. The relative permeability for the material is 800.*

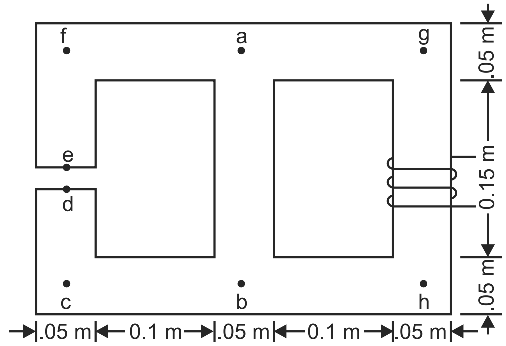


Fig. 8.18

Solution. The flux ϕ set up by the current-carrying coil in the path *bhga* divides into two parallel paths *viz* path *ab* and path *aedb*. Therefore, total m.m.f. required is equal to *AT* required for path *bhga plus AT* required for one of the parallel paths (*i.e.* path *aedb* or path *ab*) *i.e.*

Total m.m.f = *AT* for path *aedb* + *AT* for path *bhga*

1. AT for path aedb. The m.m.f. required for this path is equal to *AT* required for air gap *ed plus AT* required for steel path (*ae* + *db*)

(*i*) *AT for air gap.* $\phi_g = 5 \times 10^{-4}$ Wb ; $a_g = 0.05 \times 0.05 = 0.0025$ m^2 ; $l_g = 0.003$ m

$$\therefore \quad B_g = \frac{\phi_g}{a_g} = \frac{5\times10^{-4}}{0.0025} = 0.2 \text{ Wb/m}^2$$

$$\text{Now,} \quad H_g = \frac{B_g}{\mu_0} = \frac{0.2}{4\pi\times10^{-7}} = 15.92 \times 10^4 \text{ AT/m}$$

$$\therefore \quad AT_g = H_g \times l_g = 15.92 \times 10^4 \times 0.003 = 477.6 \text{ AT}$$

(*ii*) *AT for steel path (ae + db)*. The flux density in this path is also 0.2 Wb/m^2.

$$l_{ae} + l_{bd} = 0.5 - 0.003 = 0.497 \text{ m}$$

$$\text{Magnetising force, } H_s = \frac{0.2}{\mu_0\mu_r} = \frac{0.2}{4\pi\times10^{-7}\times800} = 198.94 \text{ AT/m}$$

$$\therefore \quad AT_s = 198.94 \times 0.497 = 98.87 \text{ AT}$$

$$\therefore \quad AT \text{ required for path } aedb = 477.6 + 98.87 = 576.47 \text{ AT} = AT_{ab}$$

2. AT for path *bhga*. We first find flux ϕ in this path. Now, $l_{ab} = 0.2$ m.

$$\text{Also,} \quad AT_{ab} = 576.47 \text{ AT ... Calculated above}$$

$$\text{Flux density, } B_{ab} = \frac{AT_{ab}\times\mu_0\mu_r}{l_{ab}} = \frac{576.47\times4\pi\times10^{-7}\times800}{0.2} = 2.898 \text{ Wb/m}^2$$

$$\text{Flux, } \phi_{ab} = B_{ab} \times a = 2.898 \times 0.0025 = 0.007245 \text{ Wb}$$

$$\therefore \quad \phi = \phi_g + \phi_{ab} = 5 \times 10^{-4} + 0.007245 = 0.007745 \text{ Wb}$$

$$\text{Flux density in path } bhga = \frac{\phi}{a} = \frac{0.007745}{0.0025} = 3.098 \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{3.098}{\mu_0\mu_r} = \frac{3.098}{4\pi\times10^{-7}\times800} = 3081.63 \text{ AT/m}$$

$$\text{Length of path } bhga, \; l = 0.5 \text{ m}$$

AT for path *bhga* = $H \times l$ = 3081 .63 × 0.5 = 1540.815 AT

∴ Total m.m.f. required = 576.47 + 1540.815 = **2117.285 AT**

Example 8.18. *The magnetic core shown in Fig. 8.19 has the following dimensions :*

l_1 = 10 cm ; l_2 = l_3 = 18 cm ; cross-sectional area of l_1 path = 6.25 × 10^{-4} m^2 ; cross-sectional areas of l_2 and l_3 paths = 3 × 10^{-4} m^2 ; length of air gap, l_4 = 2mm.

Determine the current that must be passed through the 600-turn coil to produce a total flux of 100 μWb in the air gap. Assume that the metal has relative permeability of 800.

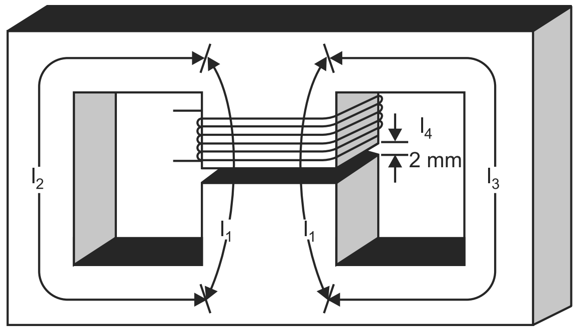


Fig. 8.19

Solution. $\phi_g = 100\ \mu\text{Wb} = 100 \times 10^{-6}$ Wb ; $a_g = 6.25 \times 10^{-4}$ m^2

AT for air gap. $B_g = \dfrac{\phi_g}{a_g} = \dfrac{100\times10^{-6}}{6.25\times10^{-4}} = 0.16$ Wb/m^2

Now, $H_g = \dfrac{B_g}{\mu_0} = \dfrac{0.16}{4\pi\times10^{-7}} = 1.27 \times 10^5$ AT/m

$\therefore$ $AT_g = H_g \times l_g = 1.27 \times 10^5 \times 2 \times 10^{-3} = 254$ AT

AT for path l_1. $B_1 = 0.16$ Wb/m^2 ; $l_1 = 10 \times 10^{-2}$ m

Now, $H_1 = \dfrac{B_1}{\mu_0\mu_r} = \dfrac{0.16}{4\pi\times10^{-7}\times800} = 159$ AT/m

$\therefore$ $AT_1 = H_1 \times l_1 = 159 \times 10 \times 10^{-2} = 15.9$ AT

Here, we neglect l_g, being very small, compared to iron path. Paths l_2 and l_3 are similar so that total flux (= 100×10^{-6} Wb) divides equally between these two paths. Since paths l_2 and l_3 are in parallel, it is necessary to consider m.m.f. for only one of them. Let us find AT for path l_2.

AT for path l_2. $\phi_2 = 50 \times 10^{-6}$ Wb ; $\mu_r = 800$; $l_2 = 18 \times 10^{-2}$ m

$\therefore$ $B_2 = \dfrac{\phi_2}{a} = \dfrac{50\times10^{-6}}{3\times10^{-4}} = 0.167$ Wb/m^2

Now, $H_2 = \dfrac{B_2}{\mu_0\mu_r} = \dfrac{0.167}{4\pi\times10^{-7}\times800} = 166$ AT/m

$\therefore$ $AT_2 = H_2 \times l_2 = 166 \times 18 \times 10^{-2} = 29.9$ AT

$\therefore$ Total $AT = 254 + 15.9 + 29.9 = 300$ AT

Now, $NI = 300$ or $I = \dfrac{300}{N} = \dfrac{300}{600} = 0.5$ A = **500 mA**

Tutorial Problems

1. It is required to produce a flux density of 0·6 Wb/m^2 in an air gap having a length of 8 mm. Calculate the m.m.f. required. **[480 × 10^3 AT/m]**
2. A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 60 cm and a uniform cross-sectional area of 5 cm^2. If the current through the coil is 4A, calculate (*i*) the magnetising force (*ii*) the flux density and (*iii*) the total flux. **[(*i*) 1333 AT/m (*ii*) 1675 μWb/m^2 (*iii*) 0·8375 μWb]**
3. A core forms a closed magnetic loop of path length 32 cm. Half of this path has a cross-sectional area of 2 cm^2 and relative permeability 800. The other half has a cross-sectional area of 4 cm^2 and relative

permeability 400. Find the current needed to produce a flux of 0·4 Wb in the core if it is wound with 1000 turns of insulated wire. Ignore leakage and fringing effects. **[636·8 A]**

4. An iron ring has a cross-sectional area of 400 mm^2 and a mean diameter of 250 mm. An air gap of 1 mm has been made by a saw-cut across the section of the ring. If a magnetic flux of 0·3 mWb is required in the air gap, find the current necessary to produce this flux when a coil of 400 turns is wound on the ring. The iron has a relative permeability of 500. **[3·84 A]**

5. An iron ring has a mean circumferential length of 60 cm and a uniform winding of 300 turns. An air gap has been made by a saw-cut across the section of the ring. When a current of 1 A flows through the coil, the flux density in the air gap is found to be 0·126 Wb/m^2. How long is the air gap ? Assume iron has a relative permeability of 300. **[1 mm]**

6. An iron magnetic circuit has a uniform cross-sectional area of 5 cm^2 and a length of 25 cm. A coil of 120 turns is wound uniformly over the magnetic circuit. When the current in the coil is 1·5 A, the total flux is 0·3 Wb. Find the relative permeability of iron. **[663]**

7. The uneven ring-shaped core shown in Fig. 8.20 has μ_r = 1000 and the flux density in the thicker section is to be 0.75 T. If the current through a coil wound on the core is to be 500 mA, determine number of coil turns required. **[567]**

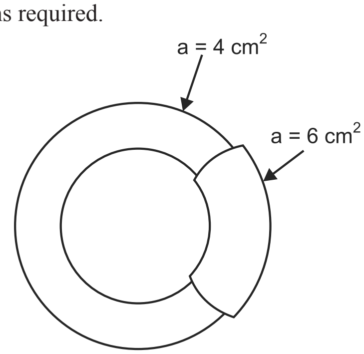


Fig. 8.20

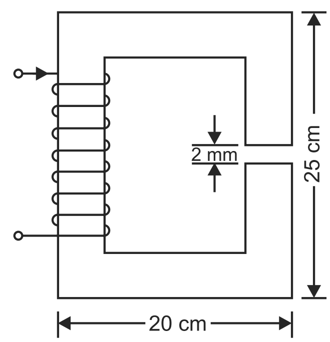


Fig. 8.21

8. A rectangular magnetic core shown in Fig. 8.21 has square cross section of area 16 cm^2. An air gap of 2 mm is cut across one of its limbs. Find the exciting current needed in the coil having 1,000 turns wound on the core to create an air-gap flux of 4 mWb. The relative permeability of the core is 2000. **[4.713 A]**

9. The magnetic circuit of Fig. 8.22 is energised by a current of 3A. If the coil has 1500 turns, find the flux produced in the air gap. The relative permeability of the core material is 3000. **[65.25 × 10^{-4} Wb]**

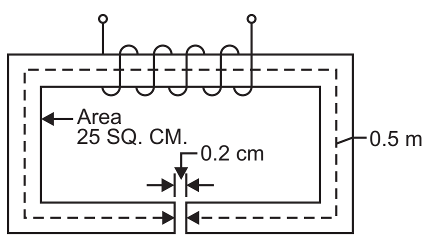


Fig. 8.22

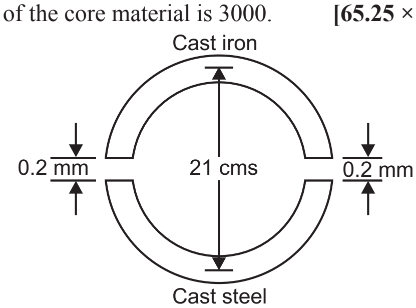


Fig. 8.23

10. A ring [See Fig. 8.23] has a diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the ampere turns required to produce a flux of 8 × 10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect leakage and fringing effects. **[1783 AT]**

8.11. B-H Curve

The *B-H* curve (or magnetisation curve) indicates the manner in which the flux density (*B*) varies with the magnetising force (*H*).

***(i)* For non-magnetic materials.** For non-magnetic materials (*e.g.* air, copper, rubber, wood *etc.*), the relation between B and H is given by ;

$$B = \mu_0 H$$

Since μ_0 (= $4\pi \times 10^{-7}$H/m) is constant,

$$\therefore \qquad B \propto H$$

Hence, the *B-H* curve of a non-magnetic material is a straight line passing through the origin as shown in Fig. 8.24. Two things are worth noting. First, the curve never saturates no matter how great the flux density may be. Secondly, a large m.m.f. is required to produce a given flux in the non-magnetic material *e.g.* air.

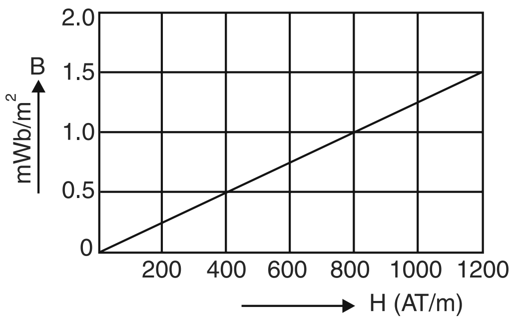
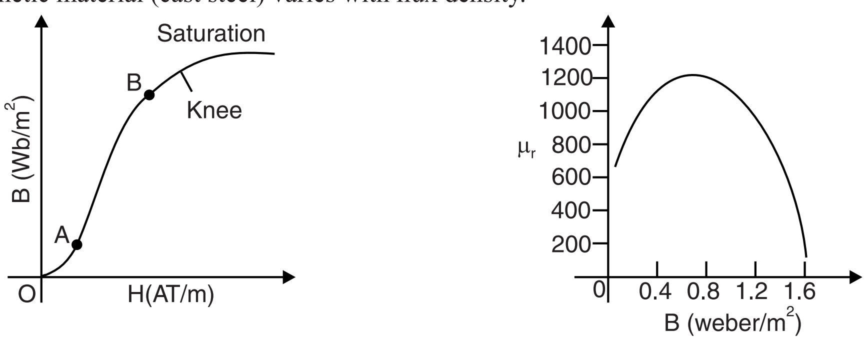


Fig. 8.24

***(ii)* For magnetic materials.** For magnetic materials (*e.g.* iron, steel *etc.*), the relation between B and H is given by ;

$$B = \mu_0 \mu_r H$$

Unfortunately, μ_r is not constant but varies with the flux density. Consequently, the *B-H* curve of a magnetic material is not linear. Fig. 8.25 (*i*) shows the general *shape of *B-H* curve of a magnetic material. The non-linearity of the curve indicates that relative permeability μ_r (= $B/\mu_0 H$) of a material is not constant but depends upon the flux density. Fig. 8.25 (*ii*) shows how relative permeability μ_r of a magnetic material (cast steel) varies with flux density.



(*i*) (*ii*)

Fig. 8.25

While carrying out magnetic calculations, it should be ensured that the values of μ_r and H are taken at the working flux density. For this purpose, the *B-H* curve of the material in question may be very helpful. In fact, the use of *B-H* curves permits the calculations of magnetic circuits with a fair degree of ease.

8.12. Magnetic Calculations From B-H Curves

The solution of magnetic circuits can be easily obtained by the use of *B-H* curves. The procedure is as under :

(*i*) Corresponding to the flux density B in the material, find the magnetising force H from the *B-H* curve of the material.

* Note the shape of the curve. It is slightly concave up for 'low' flux densities (portion *OA*) and exhibits a straight line character (portion *AB*) for 'medium' flux densities. In the portion *AB* of the curve, the μ_r of the material is almost constant. For higher flux 'densities', the curve concaves down (called the *knee* of the curve). After knee of the curve, any further increase in H does not increase B. From now onwards, the curve is almost flat and the material is said to be *saturated.* In terms of molecular theory, saturation can be explained as the point at which all the molecular magnets are oriented in the direction of applied H.

(*ii*) Compute the magnetic length *l*.

(*iii*) m.m.f. required = $H \times l$

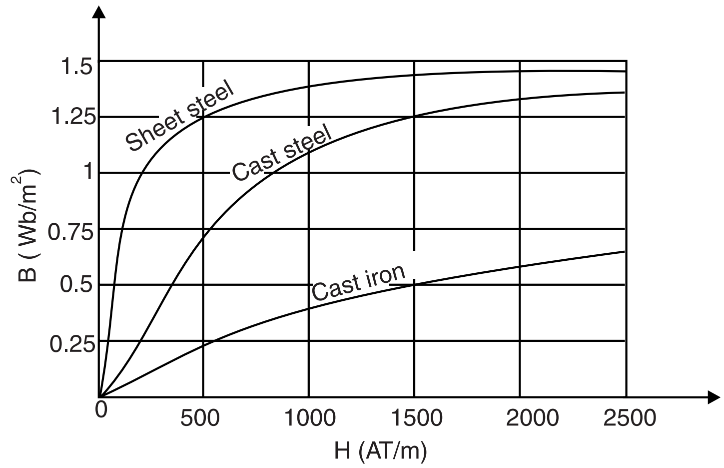


Fig. 8.26

The reader may note that the use of *B-H* curves for magnetic calculations saves a lot of time. Fig. 8.26 shows the *B-H* curves for sheet steel, cast steel and cast iron.

Note. We do not use *B-H* curve to find m.m.f. for air gap. We can find H_g directly from B_g/μ_0 and hence the m.m.f. = $H_g \times l_g$. However, in a magnetic material, *$H_i = B_i/\mu_0\,\mu_r$. Since the value of μ_r depends upon the working flux density, this realtion will not yield correct result. Instead, we find H_i corresponding to B_i in the material from the *B-H* curve. Then m.m.f. required for iron path = $H_i \times l_i$.

Example 8.19. *A cast steel ring of mean diameter 30 cm having a circular cross-section of 5 cm² is uniformly wound with 500 turns. Determine the magnetising current required to establish a flux of 5 × 10⁻⁴ Wb (i) with no air gap (ii) with a radial air gap of 1 mm.*

The magnetisation curve for cast steel is given by the following :

B(*Wb*/*m*²)	0·2	0·4	0·6	0·8	1	1·2
H(*AT*/*m*)	175	300	400	600	850	1250

Solution. Plot the *B-H* curve from the given data as shown in Fig. 8.27.

(*i*) With no air gap

$$B_i = \frac{\phi}{a} = \frac{5\times10^{-4}}{5\times10^{-4}} = 1\ \text{Wb/m}^2$$

From the *B-H* curve, we find that for a flux density of 1 Wb/m², the value of

$$H_i = 850\ \text{AT/m}$$

Now, $l_i = \pi D = \pi \times 30 \times 10^{-2}$

$= 0{\cdot}942$ m

∴ Total *AT* required $= H_i \times l_i$

$= 850 \times 0{\cdot}942 = 800{\cdot}7$ AT

∴ Magnetising current, $I = 800{\cdot}7/500 =$ **1·6 A**

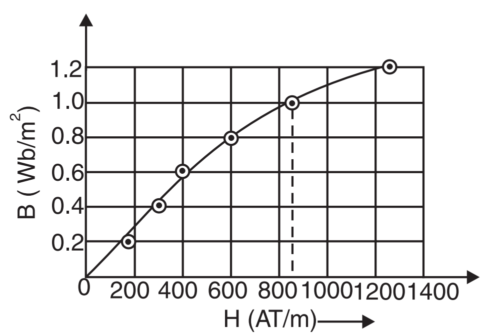


Fig. 8.27

(*ii*) With air gap of 1 mm

Flux density in air gap, $B_g = 1$ Wb/m² (same as in steel)

* The suffix *i* denotes iron part while suffix *g* denotes air gap.

$$\text{Magnetising force required, }{}^*H_g = \frac{B}{\mu_0} = \frac{1}{4\pi\times10^{-7}} = 7{\cdot}96 \times 10^5 \text{ AT/m}$$

$$AT \text{ required for air gap} = H_g \times l_g = (7{\cdot}96 \times 10^5) \times (1 \times 10^{-3}) = 796 \text{ AT}$$

$$\text{Total } AT \text{ required} = 800{\cdot}7 + 796 = 1596{\cdot}7 \text{ AT}$$

$\therefore$ Magnetising current, I = 1596·7/500 = **3·19 A**

Example 8.20. *A magnetic circuit made of wrought iron is arranged as shown in Fig. 8.28. The central limb has a cross-sectional area of 8 cm² and each of the side limbs has a cross-sectional area of 5 cm². Calculate the ampere-turns required to produce a flux of 1 mWb in the central limb, assuming the magnetic leakage is negligible. Given that for wrought iron (from B-H curve), H = 500 AT/m at B = 1·25 Wb/m² and H = 200 AT/m at B = 1 Wb/m².*

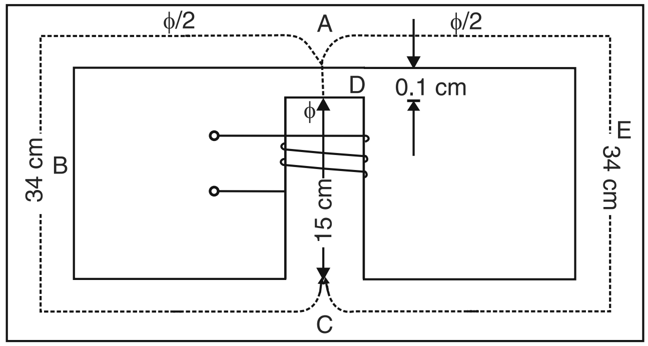


Fig. 8.28

Solution. The flux ϕ set up in the central limb divides equally into two identical parallel paths *viz.* path *ABC* and path *AEC*. The toal m.m.f. required for the entire circuit is the sum of the following three m.m.fs' :

(*i*) that required for path *CD*

(*ii*) that required for air gap *DA*

(*iii*) that required for either of parallel paths (*i.e.* path *ABC* or path *AEC*).

(*i*) AT for path CD

$$B = \frac{\phi}{a} = \frac{1\times10^{-3}}{8\times10^{-4}} = 1.25 \text{ Wb/m}^2$$

Now H at 1·25 Wb/m² = 500 AT/m (*given*)

$\therefore$ AT required for path CD = 500 × 0·15 = 75 AT

(*ii*) AT for air gap DA

$$H \text{ in air gap} = \frac{B}{\mu_0} = \frac{1\cdot25}{4\pi\times10^{-7}} = 994{\cdot}7 \times 10^3 \text{ AT/m}$$

$$\therefore \quad AT \text{ required for air gap} = (994{\cdot}7 \times 10^3) \times (0{\cdot}1 \times 10^{-2}) = 994{\cdot}7 \text{ AT}$$

(*iii*) AT for path ABC

$$\text{Flux in path } ABC = \phi/2 = 1/2 = 0{\cdot}5 \text{ mWb}$$

$$\text{Flux density in path } ABC = \frac{0.5\times10^{-3}}{5\times10^{-4}} = 1 \text{ Wb/m}^2$$

Now H at 1 Wb/m² = 200 AT/m (*given*)

$\therefore$ AT required for path ABC = 200 × 0·34 = 68 AT

$\therefore$ Total AT required = 75 + 994·7 + 68 = **1137·7 AT**

The reader may note that air gap "grabs" 87 per cent of the applied ampere-turns.

* We do not use *B-H* curve to find *AT* for air gap. It is because μ_r for air (in fact for all non-magnetic materials) is constant, being equal to 1, and *AT* can be calculated directly.

Example 8.21. *A series magnetic circuit comprises three sections (i) length of 80 mm with cross-sectional area 60 mm², (ii) length of 70 mm with cross-sectional area 80 mm² and (iii) air gap of length 0.5 mm with cross-sectional area 60 mm². Sections (i) and (ii) are of a material having magnetic characteristics given by the following table.*

H(AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B(Tesla)	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 T in the air gap. Neglect magnetic leakage.

Solution. Air-gap flux density, $B_g = 0.7$ T ; Air-gap area, $a_g = 60 \times 10^{-6}$ m²

Air-gap, flux, $\phi_g = B_g \times a_g = 0.7 \times 60 \times 10^{-6} = 42 \times 10^{-6}$ Wb

Since it is a series magnetic circuit, the flux in each of the three sections will be the same $(=\phi_g = 42 \times 10^{-6}$ Wb) but flux density will depend on the area of *X*-section of the section.

AT for section (*i*). $B = 0.7$ T because it has the same cross-sectional area as the air gap. If we plot the $B - H$ curve, it will be found that corresponding to $B = 0.7$ T, $H = 415$ AT/m.

$\therefore$ *AT* required for section (*i*) $= H \times l = 415 \times 80 \times 10^{-3} = 33.2$ AT

AT for section (*ii*). $B = \dfrac{\phi_g}{a} = \dfrac{42 \times 10^{-6}}{80 \times 10^{-6}} = 0.525$ T

From B–H curve, corresponding to $B = 0.525$ T, $H = 285$ AT/m.

$\therefore$ *AT* required for section (*ii*) $= H \times l = 285 \times 70 \times 10^{-3} = 19.95$ AT

AT for section (*iii*). This section is air gap.

$$B_g = 0.7 \text{ T and } H_g = \frac{B_g}{\mu_0} = \frac{0.7}{4\pi \times 10^{-7}} = 0.557 \times 10^6 \text{ AT/m}$$

$\therefore$ *AT* required for air gap $= H_g \times l_g = 0.557 \times 10^6 \times 0.5 \times 10^{-3} = 278.5$ AT

Total *AT* required $= 33.2 + 19.95 + 278.5 = 331.6$ AT

Now, $NI = 331.6$ or $I = \dfrac{331.6}{N} = \dfrac{331.6}{4000} =$ **0.083 A**

Example 8.22. *A magnetic circuit is made of mild steel arranged as shown in Fig. 8.29. The central limb is wound with 500 turns and has a cross-sectional area of 8cm². Each of the outer limbs has a cross-sectional area of 5 cm². The air gap has a length of 1 mm. Calculate the current required to set up a flux of 1·3 mWb in the central limb, assuming no magnetic leakage and fringing. The mean lengths of the various magnetic paths are shown in the diagram. Given that for mild steel (from B-H curve) H = 3800 AT/m at B = 1·625 T and H = 850 AT/m at B = 1·3 T.*

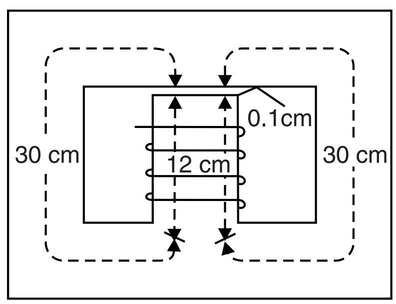


Fig. 8.29

Solution. Flux density in the central limb

$$= \frac{\text{Flux}}{\text{cross-sectional area}} = \frac{1.3 \times 10^{-3}}{8 \times 10^{-4}} = 1.625 \text{ T}$$

Given that $H = 3800$ AT/m at $B = 1{\cdot}625$ T

$\therefore$ m.m.f. for central limb $= H_1 l_1 = 3800 \times 0{\cdot}12 = 4{\cdot}56$ AT

Since half the flux returns through one outer limb and half through the other, the two outer limbs are magnetically equivalent to a single limb having a cross-sectional area of 10 cm² and a length of 30 cm.

$\therefore$ Flux density in outer limbs $= \dfrac{1.3 \times 10^{-3}}{10 \times 10^{-4}} = 1.3$ T

Given that $H = 850$ AT/m at $B = 1{\cdot}3$ T

$\therefore$ m.m.f. for outer limbs $= H_2 l_2 = 850 \times 0{\cdot}3 = 255$ AT

Flux density in airgap, B=1·625 T

Magnetising force for air gap is given by ;

$$H_3 = \frac{B}{\mu_0} = \frac{1{\cdot}625}{4\pi \times 10^{-7}} = 1{\cdot}294 \times 10^6 \text{ AT/m}$$

m.m.f. for air gap $= H_3 l_3 = (1{\cdot}294 \times 10^6) \times (1 \times 10^{-3}) = 1294$ AT

Total m.m.f. $= 456 + 255 + 1294 = 2005$ AT

$$\therefore \quad \text{Magnetising current, } I = \frac{\text{Total m.m.f.}}{\text{Turns}} = \frac{2005}{500} \simeq \mathbf{4A}$$

Example 8.23. *Fig. 8.30 shows the cross-section of a simple relay. Calculate the ampere-turns required on the coil for a flux density of 0·1 Wb/m² in the air gaps from the following data :*

Cross-sectional area of yoke	=	*2 cm²*
Magnetic length of yoke	=	*25 cm*
Cross-sectional area of armature	=	*3 cm²*
Magnetic length of armature	=	*12 cm*
Air gap area	=	*6 cm²*
Each air gap length	=	*5 mm*
Leakage coefficient	=	*1·33*

The yoke and armature material have the following magnetic characteristics :

H (AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B (Wb/m²)	*0·2*	*0·4*	*0·6*	*0·8*	*1·0*	*1·2*

Solution. Plot the *B-H* curve from the given data as shown in Fig. 8.31.

Flux in air gap, $\phi_g = 6 \times 10^{-4} \times 0{\cdot}1 = 6 \times 10^{-5}$ Wb = Flux in armature

Flux in yoke, $\phi_y = \lambda\, \phi_g = 1{\cdot}33 \times 6 \times 10^{-6} = 7{\cdot}98 \times 10^{-5}$ Wb

AT for armature

$$\text{Flux density in armature} = \frac{6 \times 10^{-5}}{3 \times 10^{-4}} = 0{\cdot}2 \text{ Wb/m}^2$$

Corresponding to $B = 0{\cdot}2$ Wb/m² (See *B-H* curve), $H = 100$ AT/m.

$\therefore$ AT required for armature $= 100 \times 0{\cdot}12 = 12$ AT

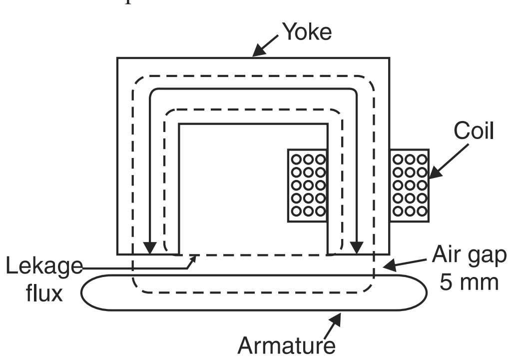


Fig. 8.30

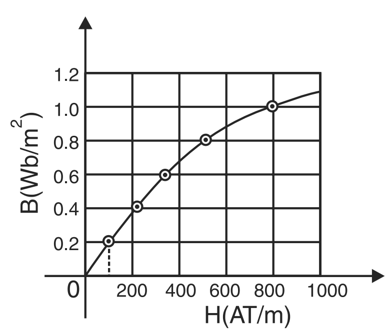


Fig. 8.31

AT for yoke

$$\text{Flux density in the yoke} = \frac{7\cdot98\times10^{-5}}{2\times10^{-4}} = 0\cdot4 \text{ Wb/m}^2$$

$$\text{Corresponding to } B = 0\cdot4 \text{ Wb/m}^2, \quad H = 210 \text{ AT/m.}$$

$$\therefore \quad \text{AT required for yoke} = 210 \times 0\cdot25 = 52\cdot5 \text{ AT}$$

AT for air gaps

$$\text{Magnetising force in air gaps} = \frac{0\cdot1}{\mu_0} = \frac{0\cdot1}{4\pi\times10^{-7}} = 7\cdot96 \times 10^4 \text{ AT/m}$$

$$\text{AT for two air gaps} = (7\cdot96 \times 10^4) \times (10 \times 10^{-3}) = 796 \text{ AT}$$

$$\text{Total AT required} = 12 + 52\cdot5 + 796 = \mathbf{860\cdot5\ AT}$$

Example 8.24. *An iron ring of mean diameter 19.1 cm and having cross-sectional area of 4 cm^2 is required to produce a flux of 0.44 mWb. Find the coil m.m.f. required. If a saw-cut 1 mm wide is made in the ring, how many extra ampere-turns are required to maintain the same flux ? $B - \mu_r$ curve is as follows :*

B(Wb/m^2)	*0.8*	*1.0*	*1.2*	*1.4*
μ_r	*2300*	*2000*	*1600*	*1100*

Solution. $D_m = 0.191$ m ; $a = 4 \times 10^{-4}$ m^2; $\phi = 0.44 \times 10^{-3}$ Wb

Length of mean path, $l_m = \pi D_m = \pi \times 0.191 = 0.6$ m

$$\text{Flux density in ring, } B_i = \frac{\phi}{a} = \frac{0.44\times10^{-3}}{4\times10^{-4}} = 1.1 \text{ Wb/m}^2$$

By *interpolation, for flux density of 1.1 Wb/m^2, $\mu_r = 1800$.

$$\therefore \quad \text{Magnetising force, } H_i = \frac{B_i}{\mu_0\mu_r} = \frac{1.1}{4\pi\times10^{-7}\times1800} = 486.5 \text{ AT/m}$$

$$\therefore \quad \text{m.m.f. required} = H_i \times l_m = 486.5 \times 0.6 = \mathbf{292\ AT}$$

If a saw-cut of 1 mm wide is made in the ring, we require extra *AT* to maintain the same flux (= 0.44×10^{-3}Wb).

$$\text{Now } H_g = \frac{B_g}{\mu_0} = \frac{1.1}{4\pi\times10^{-7}} = 875352 \text{ AT/m} ;\ l_g = 1 \times 10^{-3} \text{ m}$$

$$\therefore \quad \text{Extra m.m.f. required} = H_g \times l_g = 875352 \times 1 \times 10^{-3} = \mathbf{875\ AT}$$

Example 8.25. *A transformer core made of annealed steel sheet has the form and dimensions shown in Fig. 8.32. A coil of N turns is wound on the central limb. The average length of magnetic circuit (i.e. path ABCDA or path EFGHE) is 30 cm. Determine the ampere-turns of the coil required to produce a flux density of 1 Wb/m^2 in the central leg. What will be the total amount of flux in the central leg and in each outside leg ? Given that for annealed sheet steel (from B-H curve), H = 200 AT/m at 1 Wb/m^2.*

---

* For $B = 1.0$ Wb/m^2, $\mu_r = 2000$ and for $B = 1.2$ Wb/m^2, $\mu_r = 1600$. By interpolation, we are to find μ_r for $B = 1.1$Wb/m^2.

If increase in B is 0.2 Wb/m^2 (= 1.2 – 1.0 = 0.2), then decrease in μ_r is 400 (2000 – 1600 = 400). If increase in B is 0.1/Wb/m^2 (1.1 – 1.0 = 0.1), then decrease in μ_r

$$= \frac{400}{0.2} \times 0.1 = 200$$

$\therefore$ μ_r at 1.1 Wb/m^2 = 2000 – 200 = 1800

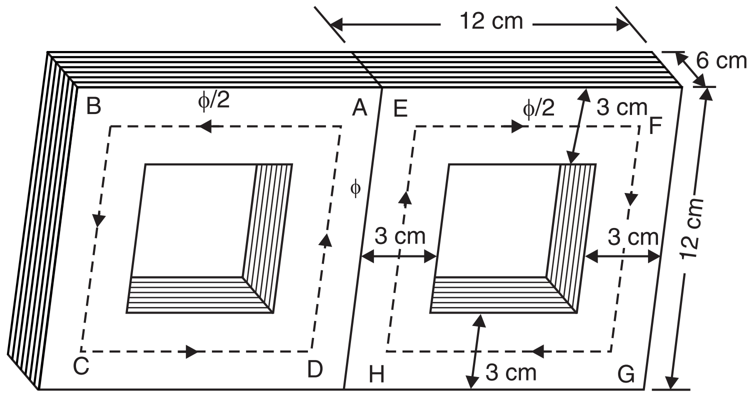


Fig. 8.32

Solution. It is a case of parallel magnetic circuit. It is clear from Fig. 8.32 that central leg has twice the area of an outside leg. The flux ϕ set up in the central limb divides equally into two parallel identical paths *viz*. path *ABCD* and path *EFGH*. It may be noted very carefully that flux density is the *same in the central leg, each outside leg and other parts.

Mean length of magnetic path (*i.e.* path *ABCDA* and *EFGHE*)

$$= 30 \text{ cm} = 0{\cdot}3 \text{ m}$$

$$\therefore \quad \text{AT required} = 200 \times 0{\cdot}3 = \mathbf{60\ AT}$$

$$\text{Area of central leg} = 0{\cdot}06 \times 0{\cdot}06 = 0{\cdot}0036 \text{ m}^2$$

$$\text{Flux in central leg} = \text{Flux density} \times \text{Area} = 1 \times 0{\cdot}0036 = \mathbf{0{\cdot}0036\ Wb}$$

$$\text{Area of each outside leg} = 0{\cdot}03 \times 0{\cdot}06 = 0{\cdot}0018 \text{ m}^2$$

$$\text{Flux in each outside leg} = 1 \times 0{\cdot}0018 = \mathbf{0{\cdot}0018\ Wb}$$

Alternatively, flux in each outside leg will be half that in the central leg *i.e.* $0{\cdot}0036/2 =$ **0·0018 Wb.**

Example 8.26. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and cross the ring, an ordinary steel bar 18 cm × 3 cm × 0·4 cm is fitted with negligible gap. Calculate the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1·0 weber per metre^2 in the other half. Neglect leakage. The B-H characteristics are as below :*

	For Cast Steel			*For Ordinary Plate*			
B in Wb/m^2	*1·0*	*1·1*	*1·2*	*B in Wb/m^2*	*1·2*	*1·4*	*1·45*
Amp-turn/m	*900*	*1020*	*1220*	*Amp-turn/m*	*590*	*1200*	*1650*

Solution. The conditions of the problem lead to the magnetic circuit shown in Fig. 8.33. The equivalent electrical circuit is shown in Fig. 8.34. Note that m.m.f. is shown as a battery and reluctances as resistances. Referring to Fig. 8.33, the flux paths *D* and *C* are in parallel. Therefore, total AT required is equal to AT for path *A* plus AT for path *C* or path *D*.

* The area of central leg is '*a*' and flux is ϕ so that $B = \phi/a$. The area of each outside and other part of flux path is $a/2$ and flux is $\phi/2$ so that B is again $= \phi/a$.

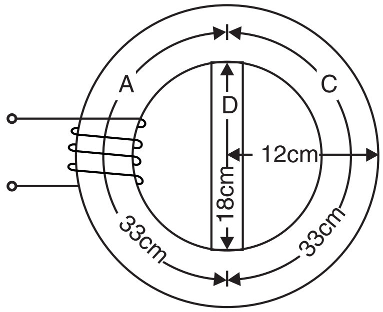


Fig. 8.33

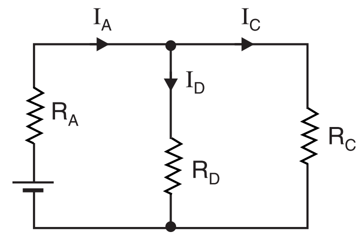


Fig. 8.34

$$\text{Mean diameter of ring} = \frac{24+18}{2} = 21 \text{ cm}$$

$$\text{Mean circumference} = \pi \times 21 = 66 \text{ cm}$$

$$\text{Length of path } A \text{ or } C = 66/2 = 33 \text{ cm} = 0{\cdot}33 \text{ m}$$

AT for path *C*. We shall first determine AT required for path C because flux density in this path is known (1 Wb/m^2). From the B-H characteristic, H corresponding to 1 Wb/m^2 is 900 AT/m.

$$\therefore \quad \text{AT required for path } C = H \times \text{Length of path } C$$

$$= 900 \times 0{\cdot}33 = 297 \text{ AT}$$

AT for path *D*. Since paths C and D are in parallel, AT required for path D = 297 AT and H = 297/0·18 = 1650 AT/m. From the B-H characteristic, B corresponding to 1650 AT/m is 1·45 Wb/m^2.

$$\text{Flux through } C, \phi_C = B \times A = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4} \text{ Wb}$$

$$\text{Flux through } D, \phi_D = (1{\cdot}45) \times (3 \times 0{\cdot}4 \times 10^{-4}) = 1{\cdot}74 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad \text{Flux through } A, \phi_A = \phi_C + \phi_D = (9 + 1{\cdot}74) \times 10^{-4} = 10{\cdot}74 \times 10^{-4} \text{ Wb}$$

$$\text{Flux density in } A = \frac{10{\cdot}74 \times 10^{-4}}{9 \times 10^{-4}} = 1{\cdot}193 \text{ Wb/m}^2$$

From the B-H characteristics, H corresponding to 1·193 Wb/m^2 is 1200 AT/m (approx.).

$$\therefore \quad \text{AT for path } A = 1200 \times 0{\cdot}33 = 396 \text{ AT}$$

$$\therefore \quad \text{Total AT required} = \text{AT for path } C + \text{AT for path } A$$

$$= 297 + 396 = \mathbf{693 \text{ AT}}$$

Tutorial Problems

1. A cast iron-cored toroidal coil has 3000 turns and carries a current of 0.1A . The length of the magnetic circuit is 15 cm and cross-sectional area of the coil is 4 cm^2. Find H, B and total flux. Use the following B–H curve for cast iron :

H(AT/m) :	200	400	1000	2000	3000
B(T) :	0.1	0.19	0.375	0.57	0.625

[2000 AT/m ; 0.57 T; 2.28 × 10^{-4} Wb]

2. A series magnetic circuit has an iron path of length 50 cm and an air gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :

B(Wb/m^2) :	1.2	1.35	1.45	1.55
H(AT/m) :	500	1000	2000	4500

[6.35 A]

3. A cast-steel ring of mean circumference 50 cm has a cross-section of 0.52 cm^2. It has a saw-cut of 1 mm at one place. Given the following data :

B(Wb/m^2) :	1.0	1.25	1.46	1.60
μ_r :	714	520	360	247

Calculate how many ampere-turns are required to produce a flux of 0.052 mWb if leakage factor is 1.2. **[1647 AT]**

4. A magnetic circuit with a uniform cross-sectional area of 6 cm^2 consists of a cast steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetisation curve of cast steel are :

B(Wb/m^2) :	0.12	0.14	0.16	0.18	0.20
H(AT/m) :	200	230	260	290	320

[0.1128 mWb]

8.13. Determination of B/H or Magnetisation Curve

The variation of permeability μ (= $\mu_0\mu_r$) with flux density creates a design problem. Permeability must be known in order to find the flux density ($B = \mu H$) but permeability changes with flux density. This necessitates a graphical approach to magnetic circuit design. We plot *B-H* curves or magnetisation curves for various magnetic materials. The value of permeability is determined from the *B-H* curve of the material. The *B-H* curve can be determined by the following two methods provided the material is in the form of a ring : (*i*) By means of ballistic galvanometer, (*ii*) By means of fluxmeter.

8.14. B-H Curve by Ballistic Galvanometer

A ballistic galvanometer is similar in principle to the permanent moving coil instrument. It has a moving coil suspended between the poles of a permanent magnet. The coil is wound on a *non-metallic* former so that there is very little damping. The first deflection or 'throw' is proportional to the charge passed through the galvanometer if the duration of the charge passed is short compared with the time of one oscillation.

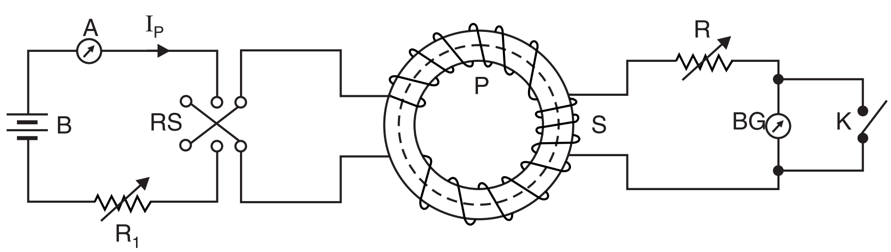


Fig. 8.35

Fig. 8.35 shows the circuit arrangement for the determination of *B-H* curve of a magnetic material by ballistic galvanometer. The specimen ring of uniform cross-section is wound uniformly with a coil *P*, thereby eliminating magnetic leakage. The primary coil *P* is connected to a battery through a reversing switch *RS*, an ammeter *A* and a variable resistor R_1. Another secondary coil *S* (called *search coil*) is wound over a small portion of the ring and is connected through a resistance *R* to the ballistic galvanometer *BG*.

Theory. We shall use subscript *P* for primary and subscript *S* for secondary.

Let θ = first deflection or 'throw' of the galvanometer when primary current I_P is reversed

k = ballistic constant of the galvanometer *i.e.* charge per unit deflection

$\therefore$ Charge passing through $BG = k\,\theta$ coulombs ...(*i*)

If ϕ is the flux produced in the ring by I_P amperes through primary *P* and *t* the time in seconds of *reversal of flux, then,

* The flux changes from ϕ to $-\phi$ by changing reversing switch *RS*. Therefore, change in flux is 2ϕ Wb.

$$\text{Rate of change of flux} = \frac{2\phi}{t} \text{ Wb/s}$$

If N_S is the number of turns in the secondary or search coil, then,

$$\text{Average e.m.f. induced in } S = N_S \times \frac{2\phi}{t} \text{ volts}$$

If R_S is the total resistance in the secondary circuit, then,

$$\text{Current through secondary or } BG,\ I_S = \frac{2N_S\,\phi}{R_S t} \text{ amperes}$$

$$\therefore \quad \text{Charge through } BG = I_S \times t = \frac{2N_S\,\phi}{R_S\,t} \times t = \frac{2N_S\,\phi}{R_S} \text{ coulombs} \quad ...(ii)$$

From eqs. (*i*) and (*ii*), we get, $k\theta = \dfrac{2N_S\,\phi}{R_S} \quad \therefore \quad \phi = \dfrac{k\,\theta\,R_S}{2N_S}$ Wb

If A is the area of cross-section of the ring in m^2, then,

$$\text{Flux density in the ring, } B = \frac{\phi}{A} = \frac{k\theta\,R_S}{2N_S\,A} \text{ Wb/m}^2$$

If N_P is the number of turns on coil P, l the mean circumference of the ring and I_P is the current through coil P, then,

$$\text{Magnetising force, } H = \frac{N_P I_P}{l}$$

The above experiment is repeated with different values of primary current and from the data obtained, the *B-H* curve can be plotted.

8.15. B-H Curve by Fluxmeter

In this method, the *BG* is replaced by the fluxmeter which is a special type of ballistic galvanometer. Its operation is based on the change in flux linkages.

Theory. Let θ = fluxmeter deflection when current through P is reversed

c = fluxmeter constant *i.e.* Wb-turns per unit deflection

$\therefore$ Change of flux linkages with coil $S = c\,\theta$ ...(*i*)

If the flux in the ring changes from ϕ to $-\phi$ when the current through the coil P is reversed and N_S is the number of turns on coil S, then,

Change of flux linkages with coil $S = 2\phi\,N_S$ ...(*ii*)

From eqs. (*i*) and (*ii*), we get, $2\phi\,N_S = c\,\theta \quad \therefore \quad \phi = \dfrac{c\,\theta}{2N_S}$ Wb

If A is the cross-sectional area of the ring in m^2, then,

$$\text{Flux density, } B = \frac{\phi}{A} = \frac{c\,\theta}{2N_S A} \text{ Wb/m}^2$$

Also, $$H = \frac{N_P I_P}{l}$$

where l = mean circumference of the ring in metres

Thus we can plot the *B-H* curve.

Example 8.27. *A fluxmeter is connected to a search coil having 600 turns and mean area of 4 cm^2. The search coil is placed at the centre of an air-cored solenoid 1 m long and wound with 1000 turns. When a current of 4A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.*

Solution. Here, $N_P = 1000$ turns ; $I_P = 4$A ; $l = 1$m ; $N_S = 600$ turns ; $A = 4 \times 10^{-4}$ m^2.

Since the length of the solenoid is large compared to its diameter, the magnetising force inside the solenoid is uniform. Therefore, magnetising force H at the centre of the solenoid is

$$H = \frac{N_P I_P}{l} = \frac{1000 \times 4}{1} = 4000 \text{ AT/m}$$

$\therefore$ Flux density, $B = \mu_0 H = 4\pi \times 10^{-7} \times 4000 = 16\pi \times 10^{-4}$ Wb/m^2

Flux linked with search coil, $\phi = BA = 16\pi \times 10^{-4} \times 4 \times 10^{-4} = 64\pi \times 10^{-8}$ Wb

When current in the solenoid is reversed, the change in flux linkages with search coil

$$= 2N_S\phi = 2 \times 600 \times 64\pi \times 10^{-8} = 7.68\pi \times 10^{-4} \text{ Wb-turns}$$

It c is the fluxmeter constant, then, value of c is given by ;

$$c = \frac{\text{Change in flux linkages}}{\text{Deflection produced}}$$

$$= \frac{7.68\pi \times 10^{-4}}{20} = \mathbf{1.206 \times 10^{-4} \text{ Wb-turns/division}}$$

Example 8.28. *A solenoid 1·2 m long is uniformly wound with a coil of 800 turns. A short coil of 50 turns, having a mean diameter of 30 mm, is placed at the centre of the solenoid and is connected to a ballistic galvanometer. The total resistance of the galvanometer circuit is 2000 Ω. When a current of 5 A through the solenoid primary winding is reversed, the initial deflection of the ballistic galvanometer is 85 divisions. Determine the ballistic constant.*

Solution. Within the solenoid, we have,

$$H = \frac{N_P I_P}{l}; \quad B = \mu_0 H = \frac{\mu_0 N_P I_P}{l}$$

$\therefore$ Flux passing through the secondary or search coil of area A is

$$\phi = B \times A = \frac{\mu_0 N_P I_P A}{l}$$

Here, $N_P = 800$; $I_P = 5$ A ; $A = \pi \times (15)^2 \times 10^{-6}$ m^2 ; $l = 1{\cdot}2$ m

$\therefore$
$$\phi = \frac{4\pi \times 10^{-7} \times 800 \times 5 \times (\pi \times 15^2 \times 10^{-6})}{1 \cdot 2} = 2{\cdot}96 \times 10^{-6} \text{ Wb}$$

$$\text{Ballistic constant, } k = \frac{2N_S\phi}{R_S\,\theta} = \frac{2 \times 50 \times 2 \cdot 96 \times 10^{-6}}{2000 \times 85}$$

$$= 1{\cdot}74 \times 10^{-9} \text{ C/div} = \mathbf{1740 \text{ pC/div.}}$$

Example 8.29. *A steel ring, 400 mm^2 cross-sectional area with a mean length 800 mm, is wound with a magnetising winding of 1000 turns. A secondary coil with 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 μC/div. The total resistance of the secondary circuit is 2 kΩ. On reversing a current of 1 A in the magnetising coil, the galvanometer gives a throw of 100 scale divisions. Calculate :*

(*i*) *The flux density in the specimen.*

(*ii*) *The relative permeability at this flux density.*

Solution.

(*i*) As proved in Art. 8·14, the flux density B within the ring is given by ;

$$B = \frac{k\,\theta\,R_S}{2N_S\,A}$$

Here, $k = 1$ μC/div $= 1 \times 10^{-6}$ C/div ; $\theta = 100$ divisions ;

$R_S = 2$ kΩ $= 2000$ Ω ; $N_S = 200$; $A = 400$ mm^2 $= 400 \times 10^{-6}$ m^2

$\therefore$
$$B = \frac{(1 \times 10^{-6}) \times (100) \times (2000)}{2 \times 200 \times 400 \times 10^{-6}} = \mathbf{1{\cdot}25 \text{ T}}$$

(*ii*) $$H = \frac{N_P I_P}{l} = \frac{1000 \times 1}{800 \times 10^{-3}} = 1{\cdot}25 \times 10^3 \text{ AT/m}$$

Now $$B = \mu_0 \mu_r H$$

$\therefore$ Relative permeability, $$\mu_r = \frac{B}{\mu_0 H} = \frac{1.25}{4\pi \times 10^{-7} \times 1.25 \times 10^3} = \mathbf{796}$$

Example 8.30. *An iron ring has a mean diameter of 0·1 m and a cross-section of 33·5 × 10^{-6} m^2. It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert magnetic standard with 10 turns and a flux of 2·5 × 10^{-4} Wb gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen.*

Solution. Within the iron ring, we have,

Length of magnetic path, $l = \pi D = 0{\cdot}1\pi$ m

$$H = \frac{N_P I_P}{l} = \frac{320 \times 10}{0.1\,\pi} = 10186 \text{ AT/m}$$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times \mu_r \times 10186 = 0.0128\, \mu_r \qquad ...(i)$$

From Hilbert's magnetic standard, we have,

$$2.5 \times 10^{-4} \times 10 = k \times 102 \quad \therefore \quad k = 2.45 \times 10^{-5} \text{ Wb-turn/div.}$$

On reversing a current of 10 A in the primary coil, change in terms of Wb-turn is

$$2\phi N_S = k\theta \quad \text{or} \quad 2 \times \phi \times 220 = 2.45 \times 10^{-5} \times 272$$

$\therefore$ $$\phi = \frac{2.45 \times 10^{-5} \times 272}{2 \times 220} = 1.51 \times 10^{-5} \text{ Wb}$$

$$B = \frac{\phi}{A} = \frac{1.51 \times 10^{-5}}{33.5 \times 10^{-6}} = 0.45 \text{ Wb/m}^2$$

But $B = 0{\cdot}0128\, \mu_r$ as is evident from eq. (*i*).

$\therefore$ $$0.45 = 0.0128\, \mu_r \quad \text{or} \quad \mu_r = 0.45/0.0128 = \mathbf{35.1}$$

Example 8.31. *A coil of 120 turns is wound uniformly over a steel ring having a mean circumference of 1 m and a cross-sectional area of 500 mm^2. A search coil of 15 turns, wound on the ring, is connected to a fluxmeter having a constant of 300 μWbt/div. When a current of 6 A through the 120-turn coil is reversed, the fluxmeter deflection is 64 divisions. Calculate :*

(*i*) *The flux density in the ring.*

(*ii*) *The corresponding value of relative permeability.*

Solution. (*i*) Fluxmeter constant, $c = \dfrac{2N_S\,\phi}{\theta}$

Here $c = 300 \times 10^{-6}$ Wbt/div. ; $N_S = 15$; $\theta = 64$ div.

$\therefore$ $$\phi = \frac{c\theta}{2N_S} = \frac{300 \times 10^{-6} \times 64}{2 \times 15} = 0{\cdot}64 \times 10^{-3} \text{ Wb}$$

Note that ϕ is the flux passing through the search coil.

$\therefore$ Flux density, $$B = \frac{\phi}{A} = \frac{0.64 \times 10^{-3}}{500 \times 10^{-6}} = \mathbf{1{\cdot}28\ Wb/m^2}$$

(*ii*) Within the ring, $$H = \frac{N_P I_P}{l} = \frac{120 \times 6}{1} = 720 \text{ AT/m}$$

Now, $$B = \mu_0 \mu_r H$$

$$\therefore \qquad \mu_r = \frac{B}{\mu_0 H} = \frac{1 \cdot 28}{4\pi \times 10^{-7} \times 720} = \mathbf{1400}$$

Tutorial Problems

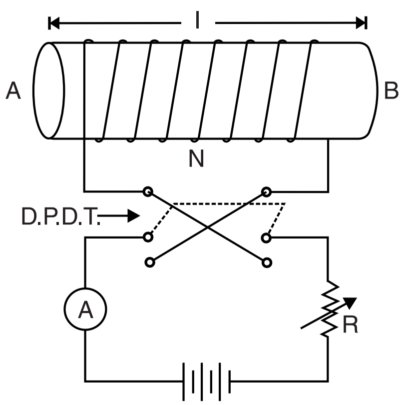
1. A moving coil ballistic galvanometer of 150Ω resistance gives a throw of 75 divisions when the flux through a search coil to which it is connected is reversed. Find the flux density given that the galvanometer constant is 110 μC per scale division and the search coil has 1400 turns, a mean area of 50 cm^2 and a resistance of 20Ω. **[0.1T]**
2. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the fluxmeter constant. **[10^{-4} Wb-turn/division]**
3. A ballistic galvanometer connected to a search coil for measuring flux density in a core gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180Ω. The galvanometer constant is 100μC per scale division. The search coil has an area of 50 cm^2 wound with 1000 turns having a resistance of 20Ω . Calculate the flux density in the core. **[0.2 T]**

8.16. Magnetic Hysteresis

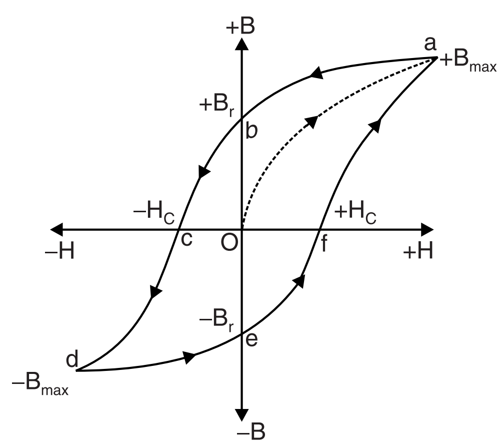
When a magnetic material is subjected to a cycle of magnetisation (*i.e.* it is magnetised first in one direction and then in the other), it is found that flux density B in the material lags behind the applied magnetising force H. This phenomenon is known as hysteresis.

The phenomenon of lagging of flux density (B) behind the magnetising force (H) in a magnetic material subjected to cycles of magnetisation is known as **magnetic hysteresis.**

The term 'hysteresis' is derived from the Greek word *hysterein* meaning to lag behind. If a piece of magnetic material is subjected to *one cycle of magnetisation, the resultant *B-H* curve is a closed loop *abcdefa* called *hysteresis loop* [See Fig. 8.36 (*ii*)]. Note that B always lags behind H. Thus at point '*b*', H is zero but flux density B has a positive finite value *ob*. Similarly at point '*e*', H is zero, but flux density has a finite negative value *oe*. This tendency of flux density B to lag behind magnetising force H is known as magnetic hysteresis.



(*i*)



(*ii*)

Fig. 8.36

* If we start with unmagnetised iron piece, then magnetise it in one direction and then in the other direction and finally demagnetise it (*i.e.* obtain the original condition we started with), the piece is said to go through one cycle of magnetisation. Compare it with one cycle of alternating current or voltage.

Hysteresis Loop. Consider an unmagnetised iron bar *AB* wound with *N* turns as shown in Fig. 8.36 (*i*). The magnetising force H (= NI/l) produced by this solenoid can be changed by varying the current through the coil. The double-pole, double-throw switch (DPDT) is used to reverse the direction of current through the coil. We shall see that *when the iron piece is subjected to a cycle of magnetisation, the resultant B-H curve traces a loop abcdefa called hysteresis loop.*

(*i*) We start with unmagnetised solenoid *AB*. When the current in the solenoid is zero, $H = 0$ and hence B in the iron piece is 0. As H is increased (by increasing solenoid current), the flux density (+ B) also increases until the point of maximum flux density (+ B_{max}) is reached. The material is saturated and beyond this point, the flux density will not increase regardless of any increase in current or magnetising force. Note that *B-H* curve of the iron follows the path *oa*.

(*ii*) If now H is gradually reduced (by reducing solenoid current), it is found that the flux density B does not decrease along the same line by which it had increased but follows the path *ab*. At point *b*, the magnetising force H is zero but flux density in the material has a finite value + B_r (= *ob*) called **residual flux density.** It means that after the removal of H, the iron piece still retains some magnetism (*i.e.* + B_r). In other words, B lags behind H. The greater the lag, the greater is the residual magnetism (*i.e.* ordinate *ob*) retained by the iron piece. The power of retaining residual magnetism is called **retentivity** of the material.

The hysteresis effect (*i.e.* lagging of B behind H) in a magnetic material is due to the opposition offered by the magnetic domains (or molecular magnets) to the turning effect of magnetising force. Once arranged in an orderly position by the magnetising force, the magnetic domains do not return exactly to the original positions. In other words, the material retains some magnetism even after the removal of magnetising force. This results in the lagging of B behind H.

(*iii*) To demagnetise the iron piece (*i.e.* to remove the residual magnetism *ob*), the magnetising force H is reversed by reversing the current through the coil. When H is graudally increased in the reverse direction, the *B-H* curve follows the path *bc* so that when H = *oc*, the residual magnetism is zero. The value of H (= *oc*) required to wipe out residual magnetism is known as **coercive force** (H_c).

(*iv*) If H is further increased in the reverse direction, the flux density increases in the reverse direction (− B). This process continues (curve *cd*) till the material is saturated in the reverse direction ($-B_{max}$ point) and can hold no more flux.

(*v*) If H is now gradually decreased to zero, the flux density also decreases and the curve follows the path *de*. At point *e*, the magnetising force is zero but flux density has a finite value $-B_r$ (= *oe*) — the residual magnetism.

(*vi*) In order to neutralise the residual magnetism *oe*, magnetising force is applied in the positive direction (*i.e.* original direction) so that when H = *of* (coercive force H_c), the flux density in the iron piece is zero. Note that the curve follows the path *ef*. If H is further increased in the positive direction, the curve follows the path *fa* to complete the loop *abcdefa.*

Thus when a magnetic material is subjected to one cycle of magnetisation, B always lags behind H so that the resultant B-H curve forms a closed loop, called **hysteresis loop.**

For the second cycle of magnetisation, a *similar loop *abcdefa* is formed. If a magnetic material is located within a coil through which alternating current (50 Hz frequency) flows, 50 loops will be formed every second. This hysteresis effect is present in all those electrical machines where the iron parts are subjected to cycles of magnetisation *e.g.* armature of a d.c. machine rotating in a stationary magnetic field, transformer core subjected to alternating flux etc.

* Owing to the nature of magnetic material, a second or even third cycle of H would not exactly lie on the tops of the first one. After a relatively few cycles, the successive loops would follow a fixed path.

8.17. Hysteresis Loss

When a magnetic material is subjected to a cycle of magnetisation (*i.e.* it is magnetised first in one direction and then in the other), an energy loss takes place due to the *molecular friction in the material. That is, the domains (or molecular magnets) of the material resist being turned first in one direction and then in the other. Energy is thus expended in the material in overcoming this opposition. This loss is in the form of heat and is called *hysteresis loss*. Hysteresis loss is present in all those electrical machines whose iron parts are subjected to cycles of magnetisation. The obvious effect of hysteresis loss is the rise of temperature of the machine.

(*i*) Transformers and most electric motors operate on alternating current. In such devices, the flux in the iron changes continuously, both in value and direction. Hence hysteresis loss occurs in such machines.

(*ii*) Hysteresis loss also occurs when an iron part rotates in a constant magnetic field *e.g.* d.c. machines.

8.18. Calculation of Hysteresis Loss

We will now show that area of hysteresis loop represents the †energy loss/m^3/cycle.

Let l = length of the iron bar

A = area of X-section of bar

N = No. of turns of wire of solenoid

Suppose at any instant the current in the solenoid is i. Then,

$$H = \frac{Ni}{l} \quad \text{or} \quad i = \frac{Hl}{N}$$

Suppose the current increases by di in a small time dt. This will cause the flux density to increase by dB [See Fig. 8.37] and hence an increase in flux $d\phi$ (= AdB). This causes an e.m.f. e to be induced in the solenoid.

$$\therefore \quad e = N\frac{d\phi}{dt} = NA\frac{dB}{dt}$$

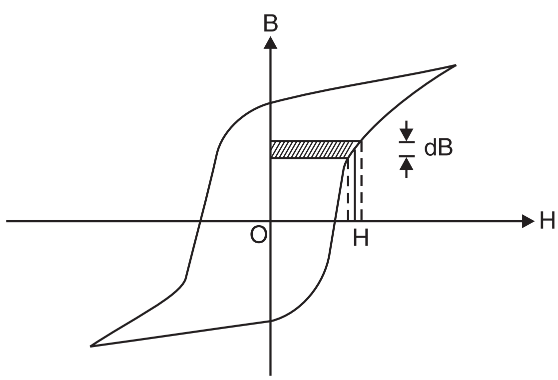


Fig. 8.37

By Lenz's law, this e.m.f. opposes the current i so that energy dW is spent in overcoming this opposing e.m.f.

* The opposition offered by the magnetic domains (or molecular magnets) to the turning effect of magnetising force is sometimes referred to as the molecular friction.

† In order to set up magnetic field, certain amount of energy has to be supplied which is stored in the field. If the field is in free space, the stored energy is returned to the circuit when the field collapses. If the field is in a magnetic material, not all the energy supplied can be returned ; part of it having been converted into heat due to hysteresis effect.

$$\therefore \qquad dW = ei\,dt \text{ joules}$$
$$= \left(NA\frac{dB}{dt}\right)\times\left(\frac{Hl}{N}\right)\times dt \text{ joules}$$
$$= Al \times H \times dB \text{ joules}$$
$$= V \times (H \times dB) \text{ joules}$$
$$\text{where} \quad Al = V = \text{volume of iron bar}$$

Now $H \times dB$ is the area of the shaded strip (See Fig. 8.37). For one cycle of magnetisation, the area $H \times dB$ will be equal to the area of hysteresis loop.

$\therefore$ Hysteresis energy loss/cycle, $W_h = V \times$ (area of loop) joules

If f is the frequency of reversal of magnetisation, then,

Hysteresis power loss, $P_h = W_h \times f = V \times$ (area of loop) $\times f$

Note. While calculating the area of hysteresis loop, proper scale factors of B and H must be considered.

For example, if the scales are : 1 cm $= x$ AT/m ...for H

1 cm $= y$ Wb/m^2 ...for B

Then, $W_h = xy \times$ (area of loop in cm^2) $\times V$ joules

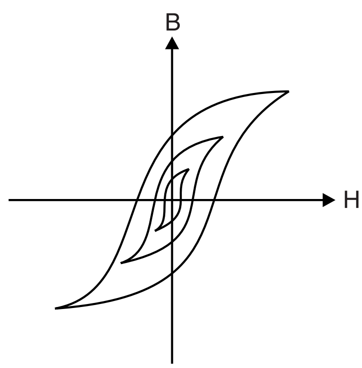
where x and y are the scale factors.

8.19. Factors Affecting the Shape and Size of Hysteresis Loop

There are three factors that affect the shape and size of hysteresis loop.

(*i*) **The material.** The shape and size of the hysteresis loop largely depends upon the nature of the material. If the material is easily magnetised, the loop will be narrow. On the other hand, if the material does not get magnetised easily, the loop will be wide. Further, different materials will saturate at different values of magnetic flux density thus affecting the height of the loop.

(*ii*) **The maximum flux density.** The loop area also depends upon the maximum flux density that is established in the material. This is illustrated in Fig. 8.38. It is clear that the loop area increases as the alternating magnetic field has progressively greater peak values.



Variation of peak flux density

Fig. 8.38

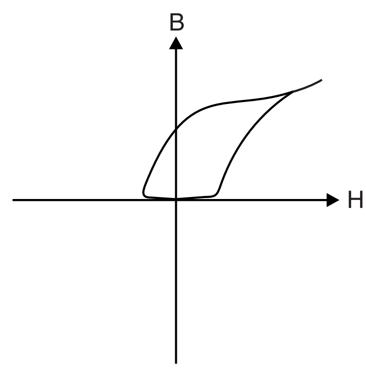


Fig. 8.39

(*iii*) **The initial state of the specimen.** The shape and size of the hysteresis loop also depends upon the initial state of the specimen. To illustrate this point, refer to Fig. 8.39. It is clear that the specimen is already saturated to start with. The magnetic flux density is then reduced to zero and finally the specimen is returned to the saturated condition.

8.20. Importance of Hysteresis Loop

The shape and size of the hysteresis loop *largely depends upon the nature of the material. The choice of a magnetic material for a particular application often depends upon the shape and size of the hysteresis loop. A few cases are discussed below by way of illustration.

(*i*) *The smaller the hysteresis loop area of a magnetic material, the less is the hysteresis loss*. The hysteresis loop for silicon steel has a very small area [See Fig. 8.40 (*i*)]. For this reason, silicon steel is widely used for making transformer cores and rotating machines which are subjected to rapid reversals of magnetisation.

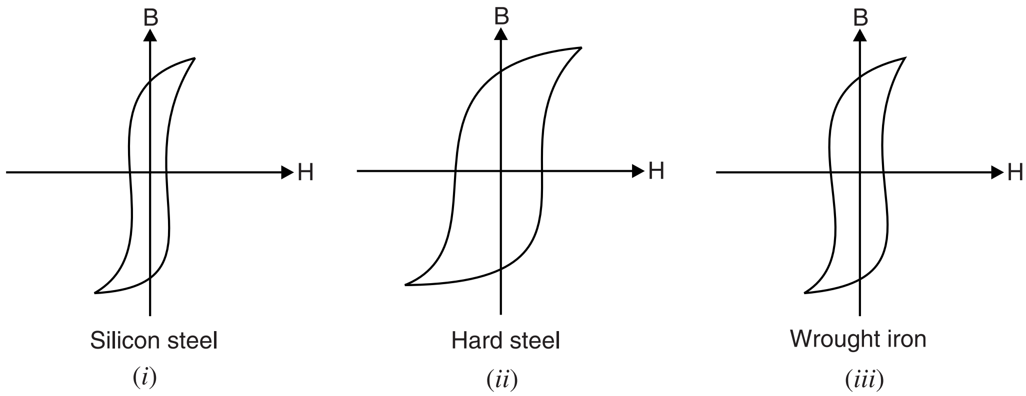


Fig. 8.40

(*ii*) The hysteresis loop for hard steel [See Fig. 8.40 (*ii*)] indicates that this material has high retentivity and coercivity. Therefore, hard steel is quite suitable for making permanent magnets. But due to the large area of the loop, there is greater hysteresis loss. For this reason, hard steel is not suitable for the construction of electrical machines.

(*iii*) The hysteresis loop for wrought iron [See Fig. 8.40 (*iii*)] shows that this material has fairly good residual magnetism and coercivity. Hence, it is suitable for making cores of electromagnets.

8.21. Applications of Ferromagnetic Materials

Ferromagnetic materials (*e.g.* iron, steel, nickel, cobalt etc.) are widely used in a number of applications. The choice of a ferromagnetic material for a particular application depends upon its magnetic properties such as retentivity, coercivity and area of the hysteresis loop. Ferromagnetic materials are classified as being either **soft** (soft iron) and **hard** (steel). Fig. 8.41 shows the hysteresis loop for soft and hard ferromagnetic materials. The table below gives the magnetic properties of hard and soft ferromagnetic materials.

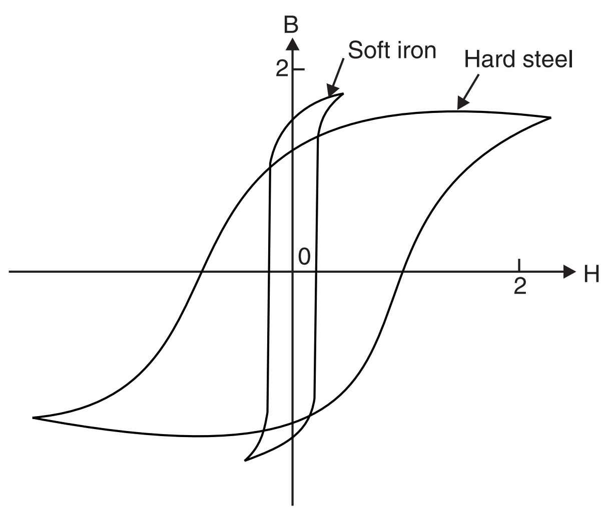


Fig. 8.41

* It also depends upon (*i*) the maximum value of flux density established and (*ii*) the initial magnetic state of the material.

Magnetic property	Soft Iron	Hard Steel
Hysteresis loop	narrow	large area
Retentivity	high	high
Coercivity	low	high
Saturation flux density	high	good

(*i*) The **permanent magnets** are made from hard ferromagnetic materials (steel, cobalt steel, carbon steel etc). Since these materials have high retentivity, the magnet is quite strong. Due to their high coercivity, they are unlikely to be demagnetised by stray magnetic fields.

(*ii*) The **electromagnets** or **temporary magnets** are made from soft ferromagnetic materials (*e.g.* soft iron). Since these materials have low coercivity, they can be easily demagnetised. Due to high saturation flux density, they make strong magnets.

(*iii*) The **transformer cores** are made from soft ferromagnetic materials. When a transformer is in use, its core is taken through many cycles of magnetisation. Energy is dissipated in the core in the form of heat during each cycle. The energy dissipated is known as *hysteresis loss* and is proportional to the area of hysteresis loop. Since the soft ferromagnetic materials have narrow hysteresis loop (*i.e.* smaller hysteresis loop area), they are used for making transformer cores.

Example 8.32. *A magnetic circuit is made of silicon steel and has a volume of 2×10^{-3} m^3. The area of hysteresis loop of silicon steel is found to be 7·25 cm^2 ; the scales being 1 cm = 10 AT/m and 1 cm = 4 Wb/m^2. Calculate the hysteresis power loss when the flux is alternating at 50 Hz.*

Solution. 1 cm = 10 AT/m on x-axis and 1 cm = 4 Wb/m^2 on y-axis.

$$\text{Area of hysteresis loop in J/m}^3\text{/cycle} = (\text{Area in cm}^2) \times (\text{Scale factors}) = (7{\cdot}25) \times (xy)$$

$$= (7{\cdot}25) \times (10 \times 4) = 290 \text{ J/m}^3\text{/cycle}$$

$$\therefore \quad \text{Hysteresis power loss, } P_h = \text{Volume} \times \text{area of loop} \times \text{frequency}$$

$$= (2 \times 10^{-3}) \times (290) \times (50) \text{ W} = \mathbf{29\ W}$$

Example 8.33. *The area of hysteresis loop obtained with a certain magnetic material was 9·3 cm^2. The co-ordinates were such that 1 cm = 1000 AT/m and 1 cm = 0·2 Wb/m^2. If the density of the given material is 7·8 g/cm^3, calculate the hysteresis loss in watts/kg at 50 Hz.*

Solution. 1 cm = 1000 AT/m on x-axis and 1 cm = 0.2 Wb/m^2 on y-axis.

$$\text{Volume of 1 kg of material, } V = \frac{10^3}{7 \cdot 8} 10^{-6} = 1{\cdot}282 \times 10^{-4} \text{ m}^3$$

$$\text{Area of hysteresis loop in J/m}^3\text{/cycle} = \text{Area in cm}^2 \times \text{scales factors}$$

$$= (9{\cdot}3) \times (1000 \times 0{\cdot}2) = 1860 \text{ J/m}^3\text{/cycle}$$

$$\text{Hysteresis energy loss, } W_h = V \times (\text{area of loop in J/m}^3\text{/cycle})$$

$$= (1{\cdot}282 \times 10^{-4}) \times 1860 = 0{\cdot}238 \text{ J/cycle}$$

$$\text{Hysteresis power loss, } P_h = W_h \times f = 0{\cdot}238 \times 50 = 11{\cdot}9 \text{ W}$$

Since we have considered 1 kg of material, $\therefore$ Hysteresis power loss, P_h = **11·9 W/kg**

Example 8.34. *Calculate the loss of energy caused by hysteresis in 1 hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of hysteresis loop is equivalent in area to 240 J/m^3/cycle and the density of iron is 7·8 g/cm^3.*

Solution. Hysteresis energy loss = 240 J/m^3/cycle

$$\text{Volume of iron} = \frac{\text{mass}}{\text{density}} = \frac{50 \times 10^3}{7 \cdot 8} 10^{-6} = 6{\cdot}41 \times 10^{-3} \text{ m}^3$$

$$\text{No. of cycles/hour} = 25 \times 60 \times 60 = 9 \times 10^4$$

$$\therefore \quad \text{Energy loss/hour} = \text{volume} \times (\text{area of loop in J/m}^3\text{/cycle}) \times \text{cycles/hour}$$

$$= (6{\cdot}41 \times 10^{-3}) \times (240) \times (9 \times 10^4) = \mathbf{138456\ J}$$

Example 8.35. *The armature of a 4-pole d.c. generator has a volume of 12×10^{-3} m^3. During rotation, the armature is taken through a hysteresis loop whose area is 20 cm^2 when plotted to a scale of 1 cm = 100 AT/m, 1 cm = 0·1 Wb/m^2. Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m.*

Solution. 1 cm = 100 AT/m on x–axis and 1 cm = 0.1 Wb/m^2 on y-axis. Since it is a 4-pole machine, two hysteresis loops will be formed in one revolution of the armature.

$$\therefore \quad \text{No. of loops generated/second}, f = 2 \times 900/60 = 30$$

$$\text{Hysteresis energy loss/cycle} = \text{Area of loop in cm}^2 \times \text{scale factors}$$

$$= 20 \times (100 \times 0{\cdot}1) = 200 \text{ J/m}^3\text{/cycle}$$

$$\text{Total hysteresis energy loss/second} = \text{volume} \times (\text{area of loop in J/m}^3\text{/cycle}) \times f$$

$$= (12 \times 10^{-3}) \times 200 \times 30 = 72 \text{ W}$$

i.e. Hysteresis power loss = **72 W**

Example 8.36. *A magnetic circuit core is made of silicon steel and has a volume of 1000000 mm^3. Using the hysteresis loop shown in Fig. 8.42, calculate the hysteresis power loss when the flux is alternating at 50 Hz.*

Solution. Hysteresis power loss, $P_h = V \times f \times$ (area of loop in J/m^3/cycle)

$$\text{Volume of material}, V = 1000000 \text{ mm}^3 = 1000000 \times 10^{-9} \text{ m}^3$$

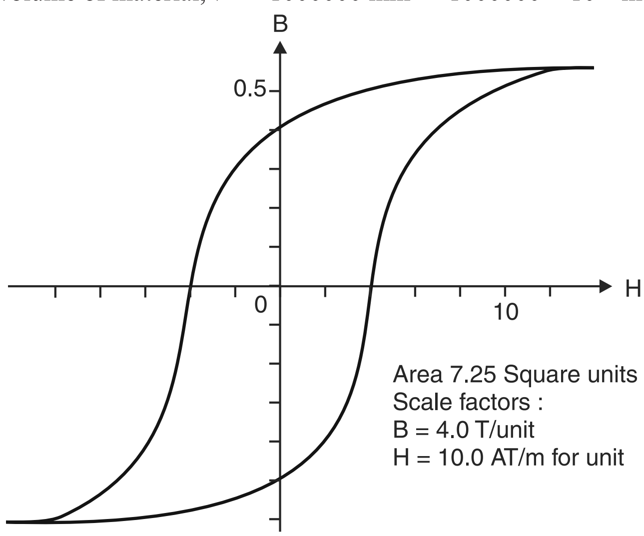


Fig. 8.42

$$\text{Area of loop in J/m}^3\text{/cycle} = \text{Area in square units} \times \text{scale factors}$$

$$= 7{\cdot}25 \times 4 \times 10 = 290 \text{ J/m}^3\text{/cycle}$$

$$\therefore \quad P_h = (1000000 \times 10^{-9}) \times 50 \times 290 = \mathbf{14{\cdot}5\ W}$$

Example 8.37. *A hysteresis loop is plotted with horizontal axis scale of 1 cm = 1000 AT/m and vertical axis scale of 5 cm = 1T. The area of the loop is 9 cm^2 and overall height is 14 cm. Find (i) hysteresis loss in J/m^3/cycle (ii) B_m and (iii) hysteresis loss in W/kg if density is 7800 kg/m^3. The frequency is 50 Hz.*

Solution. ***(i)*** 1 cm = 1000 AT/m on x-axis and 1 cm = 0.2T on y-axis.

Area of hysteresis loop in J/m^3/cycle = (Area of loop in cm^2) × scale factors

$$= (9) \times (1000 \times 0.2) = 1800 \text{ J/m}^3\text{/cycle}$$

i.e. Hysteresis energy loss $=$ **1800 J/m³/cycle**

(ii) In a hysteresis loop, flux density varies from $+B_m$ to $-B_m$. The scale for B is 5 cm = 1T and the overall height of the loop is 14 cm.

$$\therefore \quad 2B_m = \frac{14}{5} = 2.8 \text{ T or } B_m = \frac{2.8}{2} = \mathbf{1.4\ T}$$

(iii) Volume of 1 kg of material, $V = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1}{7800} \text{m}^3$

$$\therefore \quad \text{Hysteresis power loss, } P_h = \text{Energy loss/m}^3\text{/cycle} \times V \times f$$

$$= 1800 \times \frac{1}{7800} \times 50 = 11.538 \text{ W}$$

Since we have considered 1 kg of material, $\therefore P_h =$ **11.538 W/kg**

Tutorial Problems

1. The hysteresis loop for a specimen of mass 12 kg is equivalent to 30 W/mm³. Find the loss of energy in kWh in one hour at 50 Hz. The density of the specimen is 7·8 g/cm³. **[0·024 kWh]**
2. A transformer is made of 200 kg of steel plate with a specific gravity of 7·5. It may be assumed that the maximum operating flux density is 1·1 Wb/m² for all parts of the steel. When a specimen of the steel was tested, it was found to have a hysteresis loop of area 100 cm² for a maximum flux density of 1·1 Wb/m². If the scales of the hysteresis loop graph were 1 cm = 50 AT/m and 1 cm = 0·1 Wb/m², calculate the hysteresis power loss when the transformer is operated on 50 Hz mains. **[667 W]**
3. A magnetic core is made from sheet steel, the hysteresis loop of which has an area of 2·1 cm²; the scales being 1 cm = 400 AT/m and 1 cm = 0·4 Wb/m². The core measures 100 cm long and has an average cross-sectional area of 10 cm². The hysteresis loss is 16·8 W. Calculate the frequency of alternating flux. **[50 Hz]**

8.22. Steinmetz Hysteresis Law

To eliminate the need of finding the area of hysteresis loop for computing the hysteresis loss, Steinmetz devised an empirical law for finding the hysteresis loss. He found that the area of hysteresis loop of a magnetic material is directly proportional to 1·6 the power of the maximum flux density established *i.e.*

$$\text{Area of hysteresis loop} \propto {}^*B_{max}^{1\cdot6}$$

$$\text{or} \quad \text{Hysteresis energy loss} \propto B_{max}^{1\cdot6} \text{ joules/m}^3\text{/cycle}$$

$$\text{or} \quad \text{Hysteresis energy loss} = \eta\, B_{max}^{1\cdot6} \text{ joules/m}^3\text{/cycle}$$

where η is a constant called **hysteresis coefficient**. Its value depends upon the nature of material. The smaller the value of η of a magnetic material, the lesser is the hysteresis loss. The armatures of electrical machines and transformer cores are made of magnetic materials having low hysteresis coefficient in order to reduce the hysteresis loss. The best transformer steels have η values around 130, for cast steel they are around 2500 and for cast iron about 3750.

If V is the volume of the material in m³ and f is the frequency of reversal of magnetisation, then,

$$\text{Hysteresis power loss, } P_h = \eta f B_{max}^{1\cdot6} V \text{ J/s or watts}$$

Example 8.38. *The volume of a transformer core built up of sheet steel laminations is 5000 cm³ and the gross cross-sectional area is 240 cm². Because of the insulation between the plates, the net cross-sectional area is 90% of the gross. The maximum value of flux is 22 mWb and the frequency is 50 Hz. Find (i) the hysteresis loss/m³/cycle and (ii) power loss in watts. Take hysteresis coefficient as 250.*

* The index 1·6 is called **Steinmetz index**. In fact, the value of this index depends upon the nature of material and may vary from 1·6 to 2·5. However, reasonable accuracy is obtained if it is taken as 1·6.

Solution. $a = 0{\cdot}9 \times 240 = 216 \text{ cm}^2$; $B_{max} = \dfrac{22 \times 10^{-3}}{216 \times 10^{-4}} = 1{\cdot}019 \text{ Wb/m}^2$

(*i*) Hysteresis energy loss $= \eta\, B^{1\cdot6}{}_{max} = 250 \times (1{\cdot}019)^{1\cdot6} =$ **257·6 J/m³/cycle**

(*ii*) Hysteresis power loss, $P_h = \eta\, f B^{1\cdot6}{}_{max} \times V = (257{\cdot}6) \times (50) \times (5000 \times 10^{-6}) =$ **64·4 W**

Example 8.39. *The area of hysteresis loop obtained with a certain specimen of iron was 9·3 cm². The co-ordinates were such that 1 cm = 1000 AT/m and 1 cm = 0·2 Wb/m². Calculate (i) the hysteresis loss in J/m³/cycle (ii) hysteresis loss in W/m³ at a frequency of 50 Hz. (iii) If the maximum flux density was 1·5 Wb/m², calculate the hysteresis loss/m³ for a maximum flux density of 1·2 Wb/m², and a frequency of 30 Hz, assuming the loss to be proportional to* $B^{1\cdot8}{}_{max}$.

Solution. 1 cm = 1000 AT/m on x-axis and 1 cm = 0.2 Wb/m² on y-axis.

(*i*) Hysteresis energy loss $= (xy) \times$ (area of loop) J/m³/cycle

$= (1000 \times 0{\cdot}2) \times 9{\cdot}3 =$ **1860 J/m³/cycle**

(*ii*) Hysteresis power loss $= 1860 \times 50 =$ **93,000 W/m³**

(*iii*) Hysteresis power loss/m³ $= \eta\, f\, (B_{max})^{1\cdot8}$

or $93000 = \eta \times 50 \times (1{\cdot}5)^{1\cdot8}$

$$\therefore \quad \eta = \frac{93000}{50 \times (1\cdot5)^{1\cdot8}} = 896{\cdot}5$$

For $B_{max} = 1{\cdot}2$ Wb/m² and $f = 30$ Hz,

Hysteresis loss/m³ $= \eta\, f\, (B_{max})^{1\cdot8}$ W $= 896{\cdot}5 \times 30 \times (1{\cdot}2)^{1\cdot8} =$ **37342 W**

Example 8.40. *A cylinder of iron of volume 8 × 10⁻³ m³ revolves for 20 min at a speed of 3000 r.p.m. in a two-pole field of flux density 0.8 Wb/m². If the hysteresis coefficient of iron is 753.6 J/m³, specific heat of iron is 0.11, the loss due to eddy current is equal to that due to hysteresis and 25% of heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as 7.8 × 10³ kg/m³.*

Solution. When an armature revolves in a multipolar field, one magnetic reversal occurs after it passes a pair of poles. If P is the number of poles, the number of magnetic reversals in one revolution is $P/2$. If the speed of the armature is N r.p.m., then number of revolutions/second = $N/60$.

∴ No. of magnetic reversals/second = Reversal in one sec. × No. of revolutions/sec.

or Frequency of magnetic reversals $= \dfrac{P}{2} \times \dfrac{N}{60} = \dfrac{2}{2} \times \dfrac{3000}{60} = 50$ cycles/sec

According to Steinmetz hysteresis law,

Hysteresis power loss, $P_h = \eta f B^{1.6}_{max}\, V$ joules/sec.

$= 753.6 \times 50 \times (0.8)^{1.6} \times 8 \times 10^{-3} = 211$ J/s

∴ Energy loss in 20 min. $= 211 \times (20 \times 60) = 253.2 \times 10^3$ J

Eddy current loss $= 253.2 \times 10^3$ J ... *given*

∴ Total energy loss $= 2 \times 253.2 \times 10^3 = 506.4 \times 10^3$ J

$$\text{Heat produced} = \frac{506.4 \times 10^3}{J} = \frac{506.4 \times 10^3}{4200} = 120.57 \text{ kcal}$$

It is given that 25% of heat produced is lost due to radiation.

∴ Heat used to heat iron cylinder = 0.75 × 120.57 = 90.43 kcal

Now, mass of iron cylinder, m = volume × density = $8 \times 10^{-3} \times 7.8 \times 10^3 = 62.4$ kg; specific heat, $S = 0.11$.

If θ°C is the rise of temperature of iron cylinder, then,

$$mS\theta = 90.43 \quad \text{or} \quad \theta = \frac{90.43}{62.4 \times 0.11} = \mathbf{13.17°C}$$

Example 8.41. *In a certain transformer, the hysteresis loss was found to be 160 watts when the maximum flux density was 1·1 Wb/m² and the frequency 60 Hz. What will be the loss when the maximum flux density is reduced to 0·9 Wb/m² and frequency to 50 Hz ?*

Solution. According to Steinmetz hysteresis law,

$$\text{Hysteresis loss, } P_h \propto f(B_{max})^{1\cdot6}$$

$$\text{For the first case, } P_1 \propto 60 \times (1\cdot1)^{1\cdot6}$$

$$\text{For the second case, } P_2 \propto 50 \times (0\cdot9)^{1\cdot6}$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{50 \times (0\cdot9)^{1\cdot6}}{60 \times (1\cdot1)^{1\cdot6}} = 0\cdot604$$

$$\therefore \quad P_2 = 0\cdot604\, P_1 = 0\cdot604 \times 160 = \mathbf{96\cdot64\ W}$$

Example 8.42. *Calculate the loss of energy caused by hysteresis in one hour in 11·25 kg of iron if maximum flux density reached is 1·3 Wb/m² and frequency is 50 Hz. Assume Steinmetz coefficient as 500 J/m³/cycle and density of iron as 7·5 g/cm³.*

What will be the area of B/H curve (i.e. hysteresis loop) of this specimen if 1 cm = 50 AT/m and 1 cm = 0·1 Wb/m² ?

Solution. Volume of iron, $V = \dfrac{11\cdot25}{7\cdot5 \times 10^3} = 1\cdot5 \times 10^{-3}\ \text{m}^3$

$$\text{Hysteresis power loss, } P_h = \eta f (B_{max})^{1\cdot6}\, V \text{ watts}$$

$$= 500 \times 50 \times (1\cdot3)^{1\cdot6} \times (1\cdot5 \times 10^{-3}) = 57\cdot06 \text{ W}$$

$\therefore$ Hysteresis energy loss in 1 hour

$$= 57\cdot06 \times 3600 = \mathbf{205416\ J}$$

According to Steinmetz hysteresis law,

$$\text{Hysteresis energy loss} = \eta\,(B_{max})^{1\cdot6} \text{ J/m}^3\text{/cycle}$$

1 cm = 50 AT/m on x-axis and 1 cm = 0.1 Wb/m² on y-axis.

$$\text{Hysteresis energy loss} = xy \times (\text{area of loop}) \text{ J/m}^3\text{/cycle}$$

Equating the two, we get,

$$500 \times (1\cdot3)^{1\cdot6} = (50 \times 0\cdot1) \times \text{Area of loop}$$

$$\therefore \quad \text{Area of loop} = \frac{500 \times (1\cdot3)^{1\cdot6}}{50 \times 0\cdot1} = \mathbf{152\cdot16\ cm^2}$$

Tutorial Problems

1. The hysteresis loss in an iron specimen is given by the expression; Hysteresis loss is J/m³/cycle = $\eta B_{max}^{1.7}$ where B_{max} is the maximum flux density. If loss is 5.215 W/kg at a frequency of 50 Hz and a maximum flux density is 1.1 Wb/m², find the constant η if density of iron is 7600 kg/m³. Also find the hysteresis loss at 60 Hz if B_{max} = 1.7 Wb/m². **[674.11; 13.117 W/kg]**
2. A sample of silicon steel has a hysteresis coefficient of 100 and a corresponding Steinmetz index of 1.6. Calculate the hysteresis power loss in 10^6 mm³ when the flux is alternating at 50 Hz, such that the maximum flux density is 2T. **[15.2 W]**
3. The hysteresis loss in an iron specimen is proportional to $(B_{max})^{1.7}$. At B_{max} = 1.1T, the hyteresis loss is 320W at 50 Hz. Find hysteresis loss at 60 Hz if B_{max} = 1.6 T. **[726.05 W]**

8.23. Comparison of Electrostatics and Electromagnetic Terms

It may be worthwhile to compare the terms and symbols used in electrostatics with the corresponding terms and symbols used in electromagnetism. (See table on page 427).

Electrostatics		Electromagnetism	
Term	**Symbol**	**Term**	**Symbol**
Electric flux	ψ	Magnetic flux	ϕ
Electric flux density	D	Magnetic flux density	B
Electric field strength	E	Magnetic field strength	H
Electromotive force	E	Magnetomotive force	—
Electric potential difference	V	Magnetic potential difference	—
Permittivity of free space	ε_0	Permeability of free space	μ_0
Relative permittivity	ε_r	Relative permeability	μ_r
Absolute permittivity $= \dfrac{\text{electric flux density}}{\text{electric field strength}}$		Absolute permeability $= \dfrac{\text{magnetic flux density}}{\text{magnetic field strength}}$	
i.e. $\varepsilon_0\varepsilon_r = \varepsilon = D/E$		*i.e.* $\mu_0\mu_r = \mu = B/H$	

Objective Questions

1. In Fig. 8.43, the magnetic circuit is the path

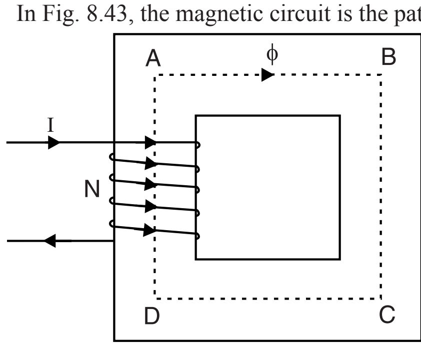


Fig. 8.43

(*i*) *DAB* (*ii*) *ABCDA*
(*iii*) *ABC* (*iv*) *ABCD*

2. If l is the magnetic path in Fig. 8.43, then magnetising force is

(*i*) NI (*ii*) $NI \times l$
(*iii*) l/NI (*iv*) NI/l

3. The reluctance of the magnetic circuit shown in Fig. 8.43 is

(*i*) NI/l (*ii*) ϕ/NI
(*iii*) NI/ϕ (*iv*) ϕ/l

4. The SI unit of reluctance is

(*i*) AT/Wb (*ii*) AT/m
(*iii*) AT (*iv*) N/Wb

5. A magnetic circuit has m.m.f. of 400 AT and reluctance of 2×10^5 AT/Wb. The magnetic flux in the magnetic circuit is

(*i*) 3×10^{-5} Wb (*ii*) 2×10^{-3} Wb
(*iii*) $1{\cdot}5 \times 10^{-2}$ Wb (*iv*) $2{\cdot}5 \times 10^{-4}$ Wb

6. A 2 cm long coil has 10 turns and carries a current of 750 mA. The magnetising force of the coil is

(*i*) 225 AT/m (*ii*) 675 AT/m
(*iii*) 450 AT/m (*iv*) 375 AT/m

7. A magnetic device has a core with cross-section of 1 inch^2. If the flux in the core is 1 mWb, then flux density (1 inch = 2.54 cm) is

(*i*) 2.5 T (*ii*) 1.3 T
(*iii*) 1.55 T (*iv*) 0.25 T

8. The reluctance of a magnetic circuit varies as ....

(*i*) length × area (*ii*) length ÷ area
(*iii*) area ÷ length (*iv*) (length)^2 + area

9. The reluctance of a magnetic circuit is ..... relative permeability of the material comprising the circuit.

(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of
(*iv*) none of the above

10. M.M.F. in a magnetic circuit corresponds to ...... in an electric circuit.

(*i*) voltage drop (*ii*) potential difference
(*iii*) electric intensity (*iv*) e.m.f.

11. Permeance of a magnetic circuit is .... area of *x*-section of the circuit.

(*i*) inversely proportional to
(*ii*) directly proportional to

(*iii*) independent of
(*iv*) none of the above.

12. The magnitude of AT required for air gap is much greater than that required for iron part of a magnetic circuit because ....
(*i*) air is a gas
(*ii*) air has the lowest relative permeability
(*iii*) air is a conductor of magnetic flux
(*iv*) none of the above

13. In electro-mechanical conversion devices (*e.g.* motors and generators), a small air gap is left between the rotor and stator in order to .....
(*i*) complete the magnetic path
(*ii*) decrease the reluctance of magnetic path
(*iii*) permit mechanical clearance
(*iv*) increase flux density in air gap

14. A magnetic circuit carries a flux ϕ_i in the iron part and a flux ϕ_g in the air gap. Then leakage coefficient is ......
(*i*) ϕ_i/ϕ_g (*ii*) ϕ_g/ϕ_i
(*iii*) $\phi_g \times \phi_i$ (*iv*) none of the above

15. The value of leakage coefficient for electrical machines is usually about.......
(*i*) 0·5 to 1 (*ii*) 4 to 10
(*iii*) above 10 (*iv*) 1·15 to 1·25

16. The reluctance of a magnetic circuit depends upon ....
(*i*) current in the coil
(*ii*) no. of turns of coil
(*iii*) flux density in the circuit
(*iv*) none of the above

17. The *B-H* curve for .... will be a straight line passing through the origin.
(*i*) air (*ii*) soft iron
(*iii*) hardened steel (*iv*) silicon steel

18. Whatever may be the flux density in ...., the material will never saturate.
(*i*) soft iron (*ii*) cobalt steel
(*iii*) air (*iv*) silicon steel

19. The *B-H* curve of .... will not be a straight line.
(*i*) air (*ii*) copper
(*iii*) wood (*iv*) soft iron

20. The *B-H* curve is used to find the m.m.f. of .... in a magnetic circuit.
(*i*) air gap (*ii*) iron part
(*iii*) both air gap and iron part
(*iv*) none of the above

21. A magnetising force of 800 AT/m will produce a flux density of .... in air.
(*i*) 1 mWb/m^2 (*ii*) 1 Wb/m^2
(*iii*) 10 mWb/m^2 (*iv*) 0·5 Wb/m^2

22. The saturation flux density for most magnetic materials is about ....
(*i*) 0·5 Wb/m^2 (*ii*) 10 Wb/m^2
(*iii*) 2 Wb/m^2 (*iv*) 1 Wb/m^2

23. Hysteresis is the phenomenon of .... in a magnetic circuit.
(*i*) lagging of *B* behind *H*
(*ii*) lagging of *H* behind *B*
(*iii*) setting up constant flux
(*iv*) none of the above

24. In Fig. 8.44, the point .... represents the saturation condition.
(*i*) *b* (*ii*) *c*
(*iii*) *a* (*iv*) *e*

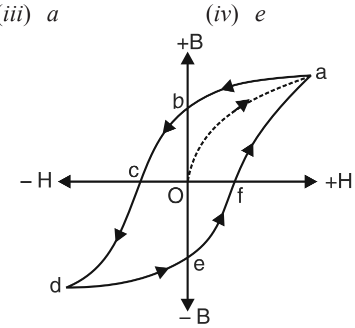


Fig. 8.44

25. In Fig. 8.44, ... represents the residual magnetism.
(*i*) of (*ii*) oc
(*iii*) ob (*iv*) none of the above

26. In Fig. 8.44, oc represents the ....
(*i*) residual magnetism
(*ii*) coercive force
(*iii*) retentivity (*iv*) none of the above

27. If a magnetic material is located within a coil through which alternating current (50 Hz frequency) flows, then .... hysteresis loops will be formed every second.
(*i*) 50 (*ii*) 25
(*iii*) 100 (*iv*) 150

28. Out of the following materials, the area of hysteresis loop will be least for ....
(*i*) wrought iron (*ii*) hard steel
(*iii*) silicon steel (*iv*) soft iron

29. The materials used for the core of a good relay should have .... hysteresis loop.

(*i*) large (*ii*) very large
(*iii*) narrow (*iv*) none of the above

30. The magnetic material used for .... should have a large hysteresis loop.

(*i*) transformers (*ii*) d.c. generators
(*iii*) a.c. motors (*iv*) permanent magnets

Answers

1. (*ii*)	**2.** (*iv*)	**3.** (*iii*)	**4.** (*i*)	**5.** (*ii*)
6. (*iv*)	**7.** (*iii*)	**8.** (*ii*)	**9.** (*ii*)	**10.** (*iv*)
11. (*ii*)	**12.** (*ii*)	**13.** (*iii*)	**14.** (*i*)	**15.** (*iv*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*iv*)	**20.** (*ii*)
21. (*i*)	**22.** (*iii*)	**23.** (*i*)	**24.** (*iii*)	**25.** (*iii*)
26. (*ii*)	**27.** (*i*)	**28.** (*iii*)	**29.** (*iii*)	**30.** (*iv*)