5
Electrostatics

Introduction

So far we have discussed that if two oppositely charged bodies are connected through a conductor, electrons will flow from the negative charge (excess of electrons) to the positive charge (deficiency of electrons). This directed flow of electrons is called electric current. The electric current will continue to flow so long as the 'excess' and 'deficiency' of electrons exist in the bodies. In other words, electric current will continue to flow so long as we maintain the potential difference between the bodies. The branch of engineering which deals with the flow of electrons (*i.e.* electric current) is called **current electricity** and is important in many ways. For example, it is the electric current by means of which electrical energy can be transferred from one point to another for utilisation.

There can be another situation where charges (*i.e.* electrons) do not move but remain static or stationary on the bodies. Such a situation will arise when the charged bodies are separated by some insulating medium, disallowing the movement of electrons. This is called **static electricity** and the branch of engineering which deals with static electricity is called **electrostatics**. Although current electricity is of greater practical use, yet the importance of static electricity cannot be ignored. Many of the advancements made in the field of electricity owe their developments to the knowledge scientists obtained from electrostatics. *The most useful outcomes of static electricity are the development of* **lightning rod** *and the* **capacitor.** In this chapter, we shall confine our attention to the behaviour and applications of static electricity.

5.1. Electrostatics

The branch of engineering which deals with charges at rest is called **electrostatics**.

When a glass rod is rubbed with silk and then separated, the former becomes positively charged and the latter attains equal negative charge. It is because during rubbing, some electrons are transferred from glass to silk. Since glass rod and silk are separated by an insulating medium (*i.e.*, air), they retain the charges. In other words, the charges on them are static or stationary. Note that the word 'electrostatic' means electricity at rest.

5.2. Importance of Electrostatics

During the past century, there was considerable increase in the practical importance of electrostatics. A few important applications of electrostatics are given below :

(*i*) Electrostatic generators can produce voltages as high as 10^6 volts. Such high voltages are required for X-ray work and nuclear bombardment.

(*ii*) We use principles of electrostatics for spray of paints, powder, etc.

(*iii*) The principles of electrostatics are used to prevent pollution.

(*iv*) The problems of preventing sparks and breakdown of insulators in high voltage engineering are essentially electrostatic.

(*v*) *The development of lightning rod and capacitor are the outcomes of electrostatics.*

5.3. Methods of Charging a Conductor

An uncharged conductor can be charged by the following two methods :

(*i*) By conduction (*ii*) By induction

(*i*) **By conduction.** *In this method, a charged body is brought in contact with the uncharged conductor.* Fig. 5.1 (*i*) shows the uncharged conductor *B* kept on an insulating stand. When the positively charged conductor *A* provided with insulating handle is touched with uncharged conductor *B* [See Fig. 5.1 (*ii*)], free electrons from conductor *B* move to conductor *A*. As a result, there occurs a deficit of electrons in conductor *B* and it becomes positively charged. Similarly, if the conductor *A* is negatively charged, the conductor *B* will also get negatively charged.

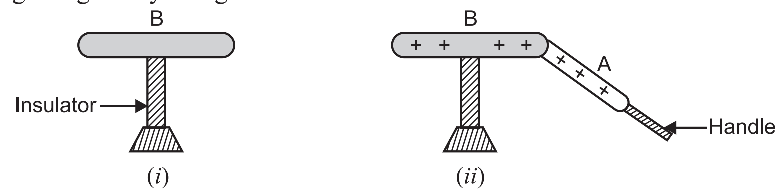

Fig. 5.1

It may be noted that conductor *A* is provided with an insulting handle so that its charge does not escape to the ground through our body. For the same reason, the conductor *B* is kept on the insulating stand.

(*ii*) **By Induction.** *In this method, a charged body is brought close to the uncharged conductor but does not touch it.* Fig. 5.2 (*i*) shows a negatively charged plastic rod (provided with insulating handle) kept near an uncharged metal sphere. The free electrons of the sphere near the rod are repelled to the farther end. As a result, the region of the sphere near the rod becomes positively charged and the farthest end of sphere becomes equally negatively charged. If now the sphere is connected to the ground through a wire as shown in Fig. 5.2 (*ii*), its free electrons at the farther end flow to the ground. On removing the wire to the ground [See Fig. 5.2 (*iii*)], the positive charge at the near end of sphere remains held there due to the attractive force of external negative charge. Finally, when the plastic rod is removed [See Fig. 5.2 (*iv*)], the positive charge spreads uniformly on the sphere. Thus, the sphere is positively charged by induction. Note that in the process, the negatively charged plastic rod loses none of its negative charge. Similarly, the metal sphere can be negatively charged by bringing a positively charged rod near it.

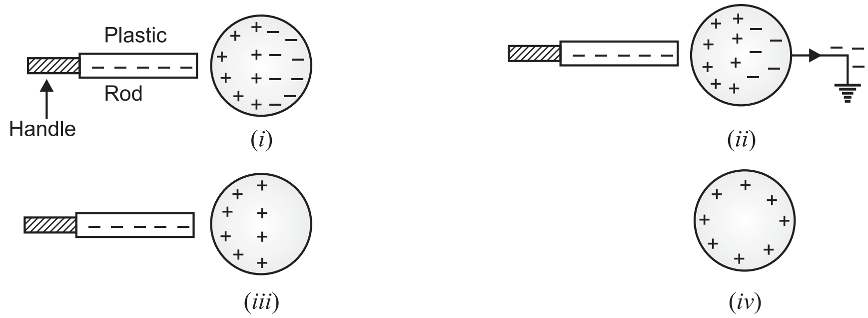

Fig. 5.2

Note that charging a body by induction requires no contact with the body inducing the charge. This is in contrast to charging a body by conduction which does require contact between the two bodies.

5.4. Coulomb's Laws of Electrostatics

Charles Coulomb, a French scientist, observed that when two charges are placed near each other, they experience a force. He performed a number of experiments to study the nature and magnitude of the force between the charged bodies. He summed up his conclusions into two laws, known as Coulomb's laws of electrostatics.

First law. This law relates to the nature of force between two charged bodies and may be stated as under :

Like charges repel each other while unlike charges attract each other.

In other words, if two charges are of the same nature (*i.e.* both positive or both negative), the force between them is repulsion. On the other hand, if one charge is positive and the other negative, the force between them is an attraction.

Second law. This law tells about the magnitude of force between two charged bodies and may be stated as under :

*The force between two *point charges is directly proportional to the product of their magnitudes and inversely proportional to the square of distance between their centres.*

Mathematically, $$F \propto \frac{Q_1 Q_2}{d^2}$$

or $$F = k\frac{Q_1 Q_2}{d^2} \qquad \text{...}(i)$$

where k is a constant whose value depends upon the medium in which the charges are placed and the system of units employed. In SI units, force is measured in newtons, charge in coulombs, distance in metres and the value of k is given by ;

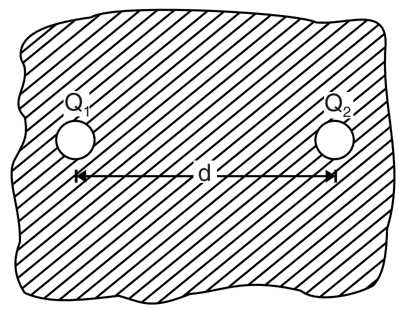

Fig. 5.3

$$k = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$$

where ε_0 = Absolute permittivity of vacuum or air.

ε_r = Relative permittivity of the medium in which the charges are placed. For vacuum or air, its value is 1.

The value of $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ F/m and the value of ε_r is different for different media.

$$\therefore \quad F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2} \qquad \text{...}(ii)$$

Now $$\frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9$$

$$\therefore \quad F = 9\times10^9 \frac{Q_1 Q_2}{\varepsilon_r d^2} \quad \text{...in a medium}$$

$$= 9\times10^9 \frac{Q_1 Q_2}{d^2} \quad \text{...in air}$$

Unit of charge. The unit of charge (*i.e.* 1 coulomb) can also be defined from Coulomb's second law of electrostatics. Suppose two equal charges placed 1 m apart in *air* exert a force of 9×10^9 newtons *i.e.*

$$Q_1 = Q_2 = Q \ ; \ d = 1\text{m} \ ; \ F = 9 \times 10^9 \text{ N}$$

$$\therefore \quad F = 9\times10^9 \frac{Q_1 Q_2}{d^2}$$

or $$9 \times 10^9 = 9\times10^9 \frac{Q^2}{(1)^2}$$

* Charged bodies approximate to point charges if they are small compared to the distance between them.

or $\qquad Q^2 = 1$

or $\qquad Q = \pm 1 = 1\text{coulomb}$

Hence **one coulomb** *is that charge which when placed in air at a distance of one metre from an equal and similar charge repels it with a force of 9×10^9 N.*

Note that coulomb is very large unit of charge in the study of electrostatics. In practice, charges produced experimentally range between pico-coulomb (pC) and micro-coulomb (μC).

$$1\text{pC} = 10^{-12}\text{C} \quad ; \qquad 1\,\mu\text{C} = 10^{-6}\text{C}$$

Note. One disadvantage of SI units is that coulomb is an inconveniently large unit. This is clear from the fact that the force exerted by a charge of 1C on another equal charge at a distance of 1m is 9×10^9N. Could you hold two one-coulomb charges a metre apart ?

5.5. Absolute and Relative Permittivity

Permittivity is the property of a medium and affects the magnitude of force between two point charges. The greater the permittivity of a medium, the lesser the force between the charged bodies placed in it and *vice-versa*. Air or vacuum has a minimum value of permittivity. The absolute (or actual) permittivity ε_0 (Greek letter 'epsilon') of air or vacuum is $8{\cdot}854 \times 10^{-12}$ F/m. The absolute (or actual) permittivity ε of all other insulating materials is greater than ε_0. The ratio $\varepsilon/\varepsilon_0$ is called the **relative permittivity* of the material and is denoted by ε_r *i.e.*

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where $\qquad \varepsilon$ = absolute (or actual) permittivity of the material

ε_0 = absolute (actual) permittivity of air or vacuum ($8{\cdot}854 \times 10^{-12}$ F/m)

ε_r = relative permittivity of the material.

Obviously, ε_r for air would be $\varepsilon_0/\varepsilon_0 = 1$.

Permittivity of a medium plays an important role in electrostatics. For instance, the relative permittivity of insulating oil is 3. It means that for the same charges (Q_1 and Q_2) and distance (d), the force between the two charges in insulating oil will be one-third of that in air [See eq. (*ii*) in Art.5·4].

5.6. Coulomb's Law in Vector Form

Consider two like point charges Q_1 and Q_2 separated by distance d in vacuum. Clearly, charges will repel each other [See Fig. 5.4].

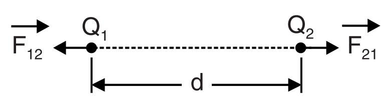

Fig. 5.4

Let $\qquad \vec{F}_{21}$ = force on Q_2 due to Q_1

$\vec{F}_{12}$ = force on Q_1 due to Q_2

$\hat{d}_{12}$ = unit vector pointing from Q_1 to Q_2

$\hat{d}_{21}$ = unit vector pointing from Q_2 to Q_1

According to Coulomb's law,

$$\vec{F}_{21} = k\frac{Q_1Q_2}{d^2}\hat{d}_{12}$$

or
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{12} \qquad ...(i)$$

Similarly,
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{21} \qquad ...(ii)$$

Eqs. (*i*) and (*ii*) express Coulomb's law in vector form.

* Thus when we say that relative permittivity of a material is 10, it means that its absolute or acutal permittivity $\varepsilon = \varepsilon_0\,\varepsilon_r = 8.854 \times 10^{-12} \times 10 = 8.854 \times 10^{-11}$ F/m.

Importance of vector form. The reader may wonder about the utility of Coulomb's law in vector form over the scalar form. The answer will be readily available from the following discussion :

(*i*) The vector form shows at a glance that forces $\vec{F}_{21}$ and $\vec{F}_{12}$ are equal and opposite.

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{21}$$

As $\hat{d}_{12} = -\hat{d}_{21}$

$\therefore$ $\vec{F}_{21} = -\vec{F}_{12}$

That is $\vec{F}_{21}$ is equal in magnitude to $\vec{F}_{12}$ but opposite in direction. The scalar form does not show this fact. This is a distinct advantage over the scalar form.

(*ii*) $\vec{F}_{21} = -\vec{F}_{12}$

This means that $\vec{F}_{21}$ and $\vec{F}_{12}$ act along the same line *i.e.* along the line joining charges Q_1 and Q_2. In other words, the electrostatic force between two charges is a central force *i.e.* it acts along the line joining the centres of the two charges. However, scalar form does not show such a nature of electrostatic force between two charges.

5.7. The Superposition Principle

If we are given two charges, the electrostatic force between them can be found by using Coulomb's laws. However, if a number of charges are present, the force on any charge due to the other charges can be found by superposition principle stated below :

When a number of charges are present, the total force on a given charge is equal to the vector sum of the forces due to the remaining other charges on the given charge.

This simply means that we first find the force on the given charge (by Coulomb's laws) due to each of the other charges in turn. We then determine the total or net force on the given charge by finding the vector sum of all the forces.

Notes. (*i*) Consider two charges Q_1 and Q_2 located in air. If a third charge Q_3 is brought nearby, it has been found experimentally that presence of the third charge (Q_3) has no effect on the force between Q_1 and Q_2. This fact permits us to use superposition principle for electric forces.

(*ii*) The superposition principle holds good for electric forces and electric fields. This fact has made the mathematical description of electrostatic phenomena simpler than it otherwise would be.

(*iii*) We can use superposition principle to find (*a*) net force (*b*) net field (*c*) net flux (*d*) net potential and (*e*) net potential energy due to a number of charges.

Example 5.1. *A small sphere is given a charge of + 20μC and a second sphere of equal diameter is given a charge of −5 μC. The two spheres are allowed to touch each other and are then spaced 10 cm apart. What force exists between them ? Assume air as the medium.*

Solution. When the two spheres touch each other, the resultant charge = (20) + (−5) = 15 μC. When the spheres are separated, charge on each sphere, $Q_1 = Q_2 = 15/2 = 7{\cdot}5$ μC.

$$\therefore \quad \text{Force, } F = 9\times10^9\times\frac{Q_1Q_2}{d^2}$$

$$= 9\times10^9\times\frac{(7.5\times10^{-6})\,(7.5\times10^{-6})}{(0.1)^2} = \textbf{50.62 N } \textbf{\textit{repulsive}}$$

Example 5.2. *A charge q is divided into two parts in such a way that they repel each other with a maximum force when held at a certain distance apart. Find the distribution of the charge.*

Solution. Let the two parts be q' and $(q - q')$. Therefore, force F between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q'(q-q')}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{qq'-q'^2}{d^2}$$

For maximum value of F, $\frac{dF}{dq'} = 0 \quad \therefore \quad \frac{dF}{dq'} = \frac{1}{4\pi\varepsilon_0 d^2}(q-2q') = 0$

or $\qquad q - 2q' = 0 \quad \therefore \quad q' = \mathbf{\frac{q}{2}}$

Hence in order to have maximum force, q should be divided into two equal parts.

Example 5.3. *Three point charges of + 5μC, + 5μC and + 5μC are placed at the vertices of an equilateral triangle which has sides 10 cm long. Find the force on each charge.*

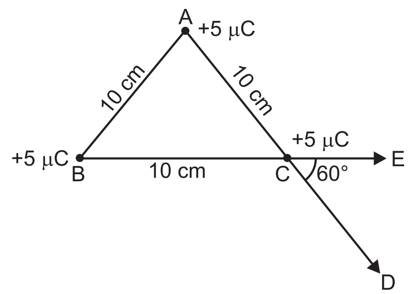

Fig. 5.5

Solution. The conditions of the problem are represented in Fig. 5.5. Consider + 5μC placed at the corner C. It is being repelled by the charges at A and B along ACD and BCE respectively. These two forces are equal, each being given by ;

$$F = 9\times10^9\frac{(5\times10^{-6})\,(5\times10^{-6})}{(0.1)^2} = 22.5 \text{ N}$$

Resultant force at $C = 2F \cos 30° = 2 \times 22.5 \times \frac{\sqrt{3}}{2} = \mathbf{38.97\ N}$

The forces acting on the charges placed at A and B will also be the same (*i.e.,* 38.97 N)

Example 5.4. *Two small spheres, each having a mass of 0·1g are suspended from a point by threads 20 cm long. They are equally charged and they repel each other to a distance of 24cm. What is the charge on each sphere ?*

Solution. Fig. 5.6 shows the conditions of the problem. Let B and C be the spheres, each carrying a charge q. The force of repulsion between the spheres is given by ;

$$F = 9\times10^9\frac{q^2}{(0.24)^2}$$
$$= 156.25 \times 10^9\, q^2$$

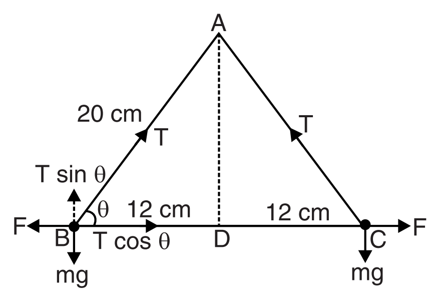

Fig. 5.6

Each sphere is under the action of three forces :

(*i*) weight $m\,g$ acting vertically downward, **(*ii*)** tension T, and **(*iii*)** electrostatic force F. Considering the sphere B and resolving T into rectangular components, we have,

$$m\,g = T\sin\theta \ ; \ F = T\cos\theta$$

$\therefore \qquad \tan\theta = mg/F$

Now, $\qquad AD = \sqrt{AB^2 - BD^2} = \sqrt{(20)^2 - (12)^2} = 16 \text{ cm}$

$\therefore \qquad \tan\theta = \frac{AD}{BD} = \frac{16}{12} \quad \therefore \quad \frac{16}{12} = \frac{mg}{F}$

or $$F = \frac{12}{16}mg = 0.75\ mg = 0.75 \times 10^{-4} \times 9.8 = 7.4 \times 10^{-4}\ \text{N}$$

But $$F = 156{\cdot}25 \times 10^{9}\ q^2$$

$\therefore$ $$156{\cdot}25 \times 10^{9}\ q^2 = 7{\cdot}4 \times 10^{-4} \quad \text{or} \quad q^2 = \frac{7.4\times10^{-4}}{156.25\times10^{9}} = 4.8 \times 10^{-15}$$

$\therefore$ $$q = \mathbf{6{\cdot}9 \times 10^{-8}\ C}$$

Example 5.5. *Two point charges +Q and +4Q are placed at a distance 'a' apart on a horizontal plane. Where should the third charge be placed for it to be in equilibrium ?*

Solution. Let the point charge $+q$ be placed at a distance x from the charge $+4Q$ [See Fig. 5.7].

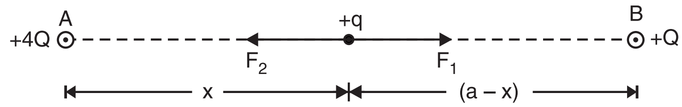

Fig. 5.7

Force on charge $+q$ due to charge $+4Q$ is

$$F_1 = \frac{q(4Q)}{4\pi\varepsilon_0 x^2} \quad \text{from } A \text{ to } B$$

Force on charge $+q$ due to charge $+Q$ is

$$F_2 = \frac{q(Q)}{4\pi\varepsilon_0 (a-x)^2} \quad \text{from } B \text{ to } A$$

In order that charge $+q$ is in equilibrium, $F_1 = F_2$.

$\therefore$ $$\frac{q(4Q)}{4\pi\varepsilon_0 x^2} = \frac{q(Q)}{4\pi\varepsilon_0 (a-x)^2} \quad \text{or} \quad x = \mathbf{2a/3}$$

Example 5.6. *Two point charges of +16 μC and −9 μC are 8 cm apart in air. Where can a third charge be located so that no net electrostatic force acts on it ?*

Solution. Let the third charge $+Q$ be located at P at a distance x from the charge −9μC as shown in Fig. 5.8.

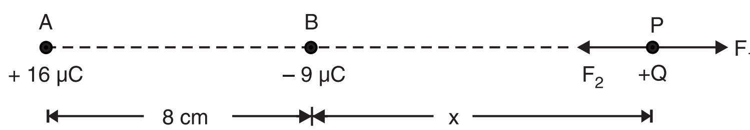

Fig. 5.8

Force at P due to charge + 16 μC at A is

$$F_1 = k\frac{16\times10^{-6}\times Q}{(x+0.08)^2} \quad \text{along } AP$$

Force at P due to charge −9 μC at B is

$$F_2 = k\frac{9\times10^{-6}\times Q}{x^2} \quad \text{along } PB$$

For zero electrostatic force at P, $F_1 = F_2$.

$\therefore$ $$k\frac{16\times10^{-6}\times Q}{(x+0.08)^2} = k\frac{9\times10^{-6}\times Q}{x^2}$$

or $$\frac{16}{(x+0.08)^2} = \frac{9}{x^2} \quad \text{or} \quad \frac{4}{x+0.08} = \frac{3}{x}$$

$\therefore$ $$x = 0.24\ \text{m} = \mathbf{24\ cm}$$

Example 5.7. *Two small balls are having equal charge Q (coulomb). The balls are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity.*

(*i*) *What is the angle between the two strings ?*

(*ii*) *What is the tension in each string ?*

Solution. **(*i*)** In the absence of gravity, the tension in the strings is only due to Coulomb's repulsive force. Therefore, the strings become horizontal due to the electric force between the charges. Consequently, the angle between the strings is **180°**.

(*ii*) $$F = 9 \times 10^9 \times \frac{Q_1 Q_2}{d^2}$$

Here $$Q_1 = Q_2 = Q \;;\quad d = 2L$$

$$\therefore \quad F = \mathbf{9 \times 10^9 \frac{Q^2}{4L^2}}$$

Example 5.8. *Two identical charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 800 kg m^{-3}, the angle remains the same. What is the dielectric constant of the liquid ? The density of the material of the sphere is 1600 kg m^{-3}.*

Solution. Fig. 5.9 shows the conditions of the problem. Suppose the mass of each sphere is m kg, the charge on each q coulomb and in equilibrium, the distance between them is r. Each sphere is in equilibrium under the action of three forces as shown. Considering the sphere A,

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}$$

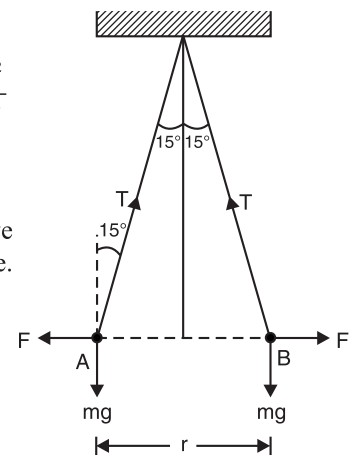

Fig. 5.9

Now $$T\cos 15° = mg \;;\; T\sin 15° = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\therefore \quad \tan 15° = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{mg\,r^2} \qquad \text{...}(i)$$

When the spheres are immersed in the liquid, the effective weight of each sphere and the force of repulsion both decrease. Consequently, tension also decreases.

$$\text{Weight of sphere in liquid} = mg\,{}^*\left(1 - \frac{800}{1600}\right) = \frac{mg}{2}$$

$$\text{Electric force in liquid,} \quad F' = \frac{1}{4\pi\varepsilon_0 K} \times \frac{q^2}{r^2}$$

Here K is the dielectric constant of the liquid. If the reduced tension is T', then for the equilibrium of sphere A, we have,

$$T'\cos 15° = \frac{mg}{2} \quad \text{and} \quad T'\sin 15° = \frac{1}{4\pi\varepsilon_0 K} \times \frac{q^2}{r^2}$$

$$\therefore \quad \tan 15° = \frac{1}{4\pi\varepsilon_0 K}\frac{2q^2}{mg\,r^2} \qquad \text{...}(ii)$$

From eqs. (*i*) and (*ii*), we have,

* Weight of sphere in liquid, W' = Weight in air – Weight of liquid displaced.

Now, Weight in air = mg

$$\text{Also, weight of liquid displaced} = m\left(\frac{\sigma}{\rho}\right)g = mg\left(\frac{\sigma}{\rho}\right) = mg\left(\frac{800}{1600}\right)$$

$$\therefore \quad W' = mg - mg\left(\frac{800}{1600}\right) = mg\left(1 - \frac{800}{1600}\right) = \frac{mg}{2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mg\,r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{2q^2}{mg\,r^2} \qquad \therefore K = \mathbf{2}$$

Tutorial Problems

1. Two copper spheres A and B have their centres separated by 50 cm. If charge on each sphere is $6{\cdot}5 \times 10^{-7}$C, what is the mutual force of repulsion between them ? The radii of the spheres are negligible compared to the distance of separation. What will be the magnitude of force if the two spheres are placed in water ? (Dielectric constant of water = 80). **[$1{\cdot}52 \times 10^{-2}$ N; $1{\cdot}9 \times 10^{-4}$ N]**
2. Charges q_1 and q_2 lie on the x-axis at points $x = -4$ cm and $x = +4$ cm respectively. How must q_1 and q_2 be related so that net electrostatic force on a charge placed at $x = +2$ cm is zero ? **[$q_1 = 9q_2$]**
3. Two small spheres of equal size are 10 cm apart in air and carry charges +1 μC and − 3 μC. Where should a third charge be located so that no net electrostatic force acts on it ? **[24 cm from −3 μC]**
4. Two identical spheres, having unequal and opposite charges are placed at a distance of 90 cm apart. After touching them mutually, they are again separated by same distance. Now they repel each other with a force of 0·025N. Find the final charge on each of them. **[1·5 μC on each]**
5. Two small spheres, each of mass 0·05 g are suspended by silk threads from the same point. When given equal charges, they separate the threads making an angle of 10° with each other. What is the force of repulsion acting on each sphere ? **[$4{\cdot}3 \times 10^{-5}$ N]**
6. Point charges of 2×10^{-9}C lie at each of the three corners of a square of side 20cm. Find the magnitude of force on a charge of -1×10^{-9}C placed at the centre of square. **[9×10^{7} N]**
7. The electrostatic force of repulsion between two positively charged ions carrying equal charge is $3{\cdot}7 \times 10^{-9}$ N. If their separation is 5 Å, how many electrons are missing from each ion ? **[2]**

5.8. Electric Field

The region surrounding a charged body is always under stress and strain because of the electrostatic charge. If a small charge is placed in this region, it will experience a force according to Coulomb's laws. This stressed region around a charged body is called electric field. Theoretically, electric field due to a charge extends upto infinity but its effect practically dies away very quickly as the distance from the charge increases.

The space (or field) in which a charge experiences a force is called an **electric field** or **electrostatic field.**

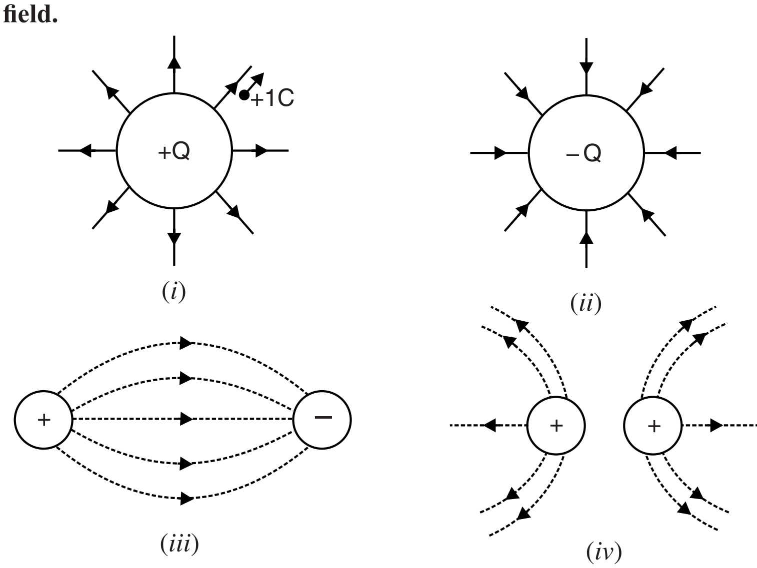

Fig. 5.10

The electric field around a charged body is represented by imaginary lines, called *electric lines of *force*. By convention, the direction of these lines of force at any point is the direction along which a unit positive charge (*i.e.,* positive charge of 1C) placed at that point would move or tend to move. The unit positive charge is sometimes called a *test charge* because it is used as an indicator to find the direction of electric field. Following this convention, it is clear that electric lines of force would always originate from a positive charge and end on a negative charge. The electric lines of force leave or enter the charged surface **normally.

Fig. 5.10 shows typical field distribution. Fig. 5.10 (*i*) shows electric field due to an isolated positively charged sphere. A unit positive charge placed near it will experience a force directed radially away from the sphere. Therefore, the direction of electric field will be radially outward as shown in Fig. 5.10 (*i*). For the negatively charged sphere [See Fig. 5.10 (*ii*)], the force acting on the unit positive charge would be directed radially towards the sphere. Fig. 5.10 (*iii*) shows the electric field between a positive charge and a negative charge while Fig. 5.10 (*iv*) shows electric field between two similarly charged (*i.e.* + vely charged) bodies.

5.9. Properties of Electric Lines of Force

(*i*) The electric field lines are directed away from a positive charge and towards a negative charge so that at any point, the tangent to a field line gives the direction of electric field at that point.

(*ii*) Electric lines of force start from a positive charge and end on a negative charge.

(*iii*) Electric lines of force leave or enter the charged surface normally.

(*iv*) Electric lines of force cannot pass through a ***conductor. This means that electric field inside a conductor is zero.

(*v*) Electric lines of force can never intersect each other. In case the two electric lines of force intersect each other at a point, then two tangents can be drawn at that point. This would mean two directions of electric field at that point which is impossible.

(*vi*) Electric lines of force have the tendency to contract in length. This explains attraction between oppositely charged bodies.

(*vii*) Electric lines of force have the tendency to expand laterally *i.e.* they tend to separate from each other in the direction perpendicular to their lengths. This explains repulsion between two like charges.

5.10. Electric Intensity or Field Strength (E)

To describe an electric field, we must specify its intensity or strength. The intensity of electric field at any point is determined by the force acting on a unit positive charge placed at that point.

Electric intensity (*or* **field strength**) *at a point in an electric field is the force acting on a unit positive charge placed at that point. Its direction is the direction along which the force acts.*

Electric intensity at a point, $E = \dfrac{F}{+Q}$ N/C

where Q = Charge in coulombs placed at that point

F = Force in newtons acting on Q coulombs

---

* So called because forces are experienced by charges in this region.

** If a line of force is at an angle other than 90°, it will have a tangential component. This tangential component would cause redistribution (*i.e.* movement) of charge. By definition, electrostatic charge is static and hence tangential component cannot exist.

*** However, electric lines of force can pass through an insulator.

Thus, if a charge of 2 coulombs placed at a point in an electric field experiences a force of 10N, then electric intensity at that point will be 10/2 = 5N/C. The following points may be noted carefully:

(i) Since electric intensity is a force, it is a vector quantity possessing both magnitude and direction.

(ii) Electric intensity can also be *described in terms of lines of force. Where the lines of force are close together, the intensity is high and where the lines of force are widely separated, intensity will be low.

(iii) Electric intensity can also be expressed in V/m.

$$1 \text{ V/m} = 1 \text{ N/C}$$ (See foot note on page 284)

Electric intensity due to a point charge. The value of electric intensity at any point in an electric field due to a point charge can be calculated by Coulomb's laws. Suppose it is required to find the electric intensity at point *P* situated at a distance *d* metres from a charge of + *Q* coulomb (See Fig. 5.11). Imagine a unit positive charge (*i.e.* + 1C) is placed at point *P*. Then, by definition, electric intensity at *P* is the force acting on + 1C placed at *P i.e.*

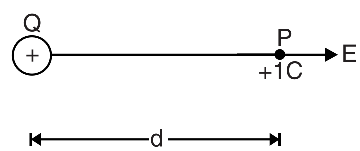

Fig. 5.11

$$\text{Electric intensity at } P,\ E = \text{Force on + 1C placed at } P$$

$$= 9\times10^9 \frac{Q\times1}{\varepsilon_r d^2}$$

$$\therefore \quad E = 9\times10^9 \frac{Q}{\varepsilon_r d^2} \quad \text{...in a medium}$$

$$= 9\times10^9 \frac{Q}{d^2} \quad \text{...in air}$$

Note the direction of electric intensity. It is acting radially away from + *Q*. For a negative charge (*i.e.* − *Q*), its direction would have been radially towards the charge.

The electric field intensity in vector form is given as :

$$\vec{E} = 9\times10^9 \frac{Q}{d^2}\hat{d} \quad \text{... in air}$$

$$= 9\times10^9 \frac{Q}{\varepsilon_r d^2}\hat{d} \quad \text{... in a medium}$$

where $\hat{d}$ is a unit vector directed from + *Q* to + 1C.

Electric field intensity due to a group of point charges. The resultant (or net) electric field intensity at a point due to a group of point charges can be found by applying **superposition principle. Thus electric field intensity at a point *P* due to *n* point charges ($q_1, q_2, q_3 \ldots q_n$) is equal to the vector sum of electric field intensities due to $q_1, q_2, q_3 \ldots q_n$ at point *P i.e.*

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n$$

where $\vec{E}$ = Net or resultant electric field intensity at *P*

$\vec{E}_1$ = Electric field intensity at *P* due to q_1

$\vec{E}_2$ = Electric field intensity at *P* due to q_2 and so on.

* It may be noted that electric lines of force do not actually exist. It is only a way of representing an electric field. However, it is a useful method of representation. It is a usual practice to indicate high field strength by drawing lines of force close together and low field strength by widely spaced lines.

** Since the electric force obeys the superposition principle, so does the electric field intensity—the force per unit charge.

Example 5.9. *Two equal and opposite charges of magnitude 2×10^{-7} C are placed 15cm apart. (i) What is the magnitude and direction of electric intensity (E) at a point mid-way between the charges? (ii) What force would act on a proton (charge = + $1{\cdot}6 \times 10^{-19}$ C) placed there ?*

Solution. Fig. 5.12 shows two equal and opposite charges separated by a distance of 15cm *i.e.* 0·15 m. Let M be the mid-point *i.e.* $AM = MB = 0{\cdot}075$ m.

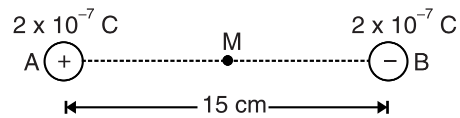

Fig. 5.12

(*i*) Imagine a charge of + 1 C placed at M.

∴ Electric intensity at M due to charge + 2×10^{-7} C is

$$E_1 = 9 \times 10^9 \times \frac{2 \times 10^{-7}}{(0.075)^2} = 0.32 \times 10^6 \text{ N/C } along\ AM$$

Electric intensity at M due to charge -2×10^{-7}C is

$$E_2 = 9 \times 10^9 \times \frac{2 \times 10^{-7}}{(0.075)^2} = 0.32 \times 10^6 \text{ N/C } along\ MB$$

Since electric intensities are acting in the same direction, the resultant intensity E is the sum of E_1 and E_2.

∴ Resultant intensity at point M is

$$E = 0{\cdot}32 \times 10^6 + 0{\cdot}32 \times 10^6 = \mathbf{0{\cdot}64 \times 10^6 \text{ N/C}}\ along\ AB$$

(*ii*) Electric intensity E at M is $0{\cdot}64 \times 10^6$ N/C. Therefore, force F acting on a proton (charge, $Q = +1{\cdot}6 \times 10^{-19}$ C) placed at M is

$$F = E\,Q = (0{\cdot}64 \times 10^6) \times (1{\cdot}6 \times 10^{-19}) = \mathbf{1{\cdot}024 \times 10^{-13} \text{ N}}\ along\ AB$$

Example 5.10. *A charged oil drop remains stationary when situated between two parallel plates 25mm apart. A p.d. of 1000V is applied to the plates. If the mass of the drop is 5×10^{-15} kg, find the charge on the drop (take $g = 10ms^{-2}$).*

Solution. Let Q coulomb be the charge on the oil drop. Since the drop is stationary,

Upward force on drop = Weight of drop [See Fig. 5.13]

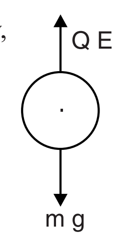

Fig. 5.13

or $$Q\,E = m\,g$$

Here $$E = \frac{V}{d} = \frac{1000}{25 \times 10^{-3}} = 4 \times 10^4 \text{ V/m}$$

∴ $$Q = \frac{mg}{E} = \frac{(5 \times 10^{-15}) \times 10}{4 \times 10^4} = \mathbf{1.25 \times 10^{-18} \text{ C}}$$

Example 5.11. *The diameter of a hollow metallic sphere is 60cm and the sphere carries a charge of 500μC. Find the electric field intensity (i) at a distance of 100cm from the centre of the sphere and (ii) at the surface of sphere.*

Solution. The electric field due to a charged sphere has spherical symmetry. Therefore, a charged sphere behaves for external points as if the whole charge is placed at its centre. [See Fig. 5.14]

(*i*) $d = OP = 100$cm = 1m ; $Q = 500$ μC = 500×10^{-6}C

∴ $$E = 9 \times 10^9 \frac{Q}{d^2} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{1} = \mathbf{4.5 \times 10^6 \text{ N/C}}$$

(*ii*) $d = OP' = 30$ cm = 0·3 m ; $Q = 500$μC = 500×10^{-6}C

∴ $$E = 9 \times 10^9 \frac{Q}{d^2} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{(0.3)^2} = \mathbf{5 \times 10^7 \text{ N/C}}$$

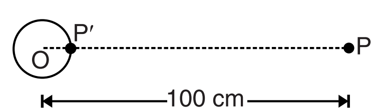

Fig. 5.14

Example 5.12. *Three point charges of +8 × 10^{-9} C, +32 × 10^{-9} C and +24 × 10^{-9} C are placed at the corners A, B and C of a square ABCD having each side 4 cm. Find the electric field intensity at the corner D. Assume that the medium is air.*

Solution. The conditions of the problem are represented in Fig. 5.15. It is clear that $BD = \sqrt{2} \times 0.04\,\text{m}$.

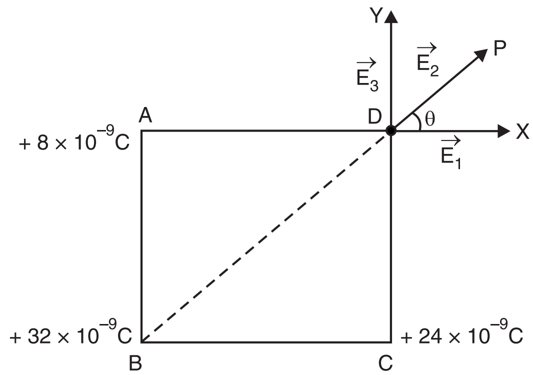

Fig. 5.15

Magnitude of electric field intensity at D due to charge +8 × 10^{-9} C is

$$E_1 = 9 \times 10^9 \times \frac{8 \times 10^{-9}}{(0.04)^2} = 4{\cdot}5 \times 10^4 \text{ N/C} \quad \text{along } DX$$

Magnitude of electric field intensity at D due to charge +32 × 10^{-9} C is

$$E_2 = 9 \times 10^9 \times \frac{32 \times 10^{-9}}{\left(\sqrt{2} \times 0.04\right)^2} = 9 \times 10^4 \text{ N/C along } DP$$

Magnitude of electric field intensity at D due to charge +24 × 10^{-9} C is

$$E_3 = 9 \times 10^9 \times \frac{24 \times 10^{-9}}{(0.04)^2} = 13{\cdot}5 \times 10^4 \text{ N/C along } DY$$

It is easy to see that $\theta = 45°$.

Resolving electric field intensities along X-axis and Y-axis, we have,

$$\text{Total } X\text{-component} = E_1 + E_2 \cos\theta + 0$$
$$= 4{\cdot}5 \times 10^4 + 9 \times 10^4 \times \cos 45° = 10{\cdot}86 \times 10^4 \text{ N/C}$$

$$\text{Total } Y\text{-component} = 0 + E_2 \sin 45° + E_3$$
$$= 0 + 9 \times 10^4 \sin 45° + 13{\cdot}5 \times 10^4 = 19{\cdot}86 \times 10^4 \text{ N/C}$$

∴ Magnitude of resultant electric intensity at D

$$= \sqrt{(10.86 \times 10^4)^2 + (19.86 \times 10^4)^2} = \mathbf{22{\cdot}63 \times 10^4 \text{ N/C}}$$

Let the resultant intensity make an angle ϕ with DX.

$$\therefore \quad \tan\phi = \frac{Y - \text{component}}{X - \text{component}} = \frac{19.86 \times 10^4}{10.86 \times 10^4} = 1{\cdot}828$$

or $$\phi = \tan^{-1} 1{\cdot}828 = \mathbf{61{\cdot}32°}$$

Tutorial Problems

1. What is the magnitude of a point charge chosen so that electric field 20 cm away from it has a magnitude of 18 × 10^6 N/C ? **[80μC]**

2. Two point charge s of $0{\cdot}12$ μC and $-0{\cdot}06$ μC are situated 3m apart in air. Calculate the electric field strength at a point midway between them on the line joining their centres.

[720 N/C towards −ve charge]

3. An oil drop of 12 excess electrons is held stationary in a uniform electric field of $2{\cdot}55 \times 10^4$ N/C. If the density of oil is 12600 kg/m^3, find (*i*) mass of the drop (*ii*) radius of the drop.

[(*i*) $1{\cdot}5 \times 10^{-15}$ kg (*ii*) $9{\cdot}8 \times 10^{-7}$ m]

4. A point charge of $0{\cdot}33 \times 10^{-8}$C is placed in a medium of relative permittivity of 5. Calculate electric field intensity at a point 10cm from the charge. **[525 N/C]**

5. Three point charges of $+0{\cdot}33 \times 10^{-8}$C, $+ 0{\cdot}33 \times 10^{-8}$C and $0{\cdot}165 \times 10^{-8}$C are at the points *A*, *B* and *C* respectively of a square *ABCD*. Find the electric field intensity at the corner *D*. **[$1{\cdot}63 \times 10^4$ N/C]**

5.11. Electric Flux (ψ)

Fig. 5.16 shows electric field between two equal and oppositely charged parallel plates. The electric field is considered to be filled with electric flux and each unit of charge is assumed to give rise to one unit of electric flux. The symbol for electric flux is the Greek letter ψ(psi) and it is measured in coulombs. Thus in Fig. 5.16, the charge on each plate is *Q* coulombs so that electric flux between the plates is

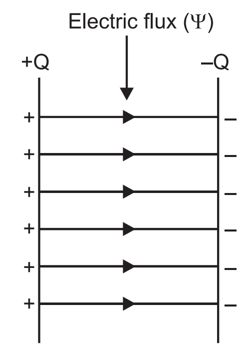

Fig. 5.16

$$\text{Electric flux, } \psi = Q \text{ coulombs}$$

Electric flux is a measure of electric lines of force. The greater the electric flux passing through an area, the greater is the number of electric lines of force passing through that area and *vice-versa*. Suppose there is a charge of *Q* coulombs in a medium of absolute permittivity ε ($= \varepsilon_0 \varepsilon_r$) where ε_r is the relative permittivity of the medium. Then number of electric lines of force *N* produced by this charge is

$$N = \frac{Q}{\varepsilon} = \frac{Q}{\varepsilon_0 \varepsilon_r}$$

(*i*) The electric flux through a surface area has maximum value when the surface is perpendicular to the electric field.

(*ii*) The electric flux through the surface is zero when the surface is parallel to the electric field.

5.12. Electric Flux Density (D)

The **electric flux density** *at any section in an electric field is the electric flux crossing normally per unit area of that section i.e.*

$$\text{Electric flux density, } D = \frac{\Psi}{A}$$

The SI unit of electric flux density is *C/m^2.

For example, when we say that electric flux density in an electric field is 4C/m^2, it means that 4C of electric flux passes normally through an area of 1m^2. Electric flux density is a vector quantity; possessing both magnitude and direction. Its direction is the same as the direction of electric intensity.

Relation between D and E. Consider a charge of $+Q$ coulombs placed in a medium of relative permittivity ε_r as shown in Fig. 5.17. The electric flux density at *P* at a distance *d* metres from the charge can be found as follows. With centre at the charge and radius *d* metres, an imaginary sphere can be considered. The electric flux of *Q* coulombs will pass normally through this imaginary sphere. Now area of sphere = $4\pi\, d^2$.

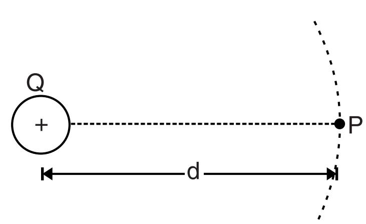

Fig. 5.17

* $D = \varepsilon_0 \varepsilon_r E = [\text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}]\ [\text{N/C}] = \text{Cm}^{-2} = \text{C/m}^2$

$\therefore$ Flux density at P, $D = \dfrac{\text{Flux}}{\text{Area}} = \dfrac{Q}{4\pi d^2}$

Also, Electric intensity at P, $E = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r d^2} = \dfrac{Q}{4\pi d^2} \times \dfrac{1}{\varepsilon_0\,\varepsilon_r}$

$$= \frac{D}{\varepsilon_r\varepsilon_0} \qquad \left[\because D = \frac{Q}{4\pi d^2}\right]$$

$\therefore$ $D = \varepsilon_0\,\varepsilon_r\,E$

Hence flux density at any point in an electric field is $\varepsilon_0\,\varepsilon_r$ times the electric intensity at that point.

The electric flux density (D) is also called **electric displacement.**

It may be noted that D and E are vector quantities having magnitude and direction. Therefore, in vector form,

$$\vec{D} = \varepsilon_0\varepsilon_r\,\vec{E}$$

Also $\vec{D} = \dfrac{Q}{4\pi d^2}\hat{d}$

The direction of $\vec{D}$ at every point is the same as that of $\vec{E}$ but its magnitude is $D = \varepsilon_0\varepsilon_r E$.

(*i*) *The value of E depends upon the permittivity $\varepsilon(=\varepsilon_0\varepsilon_r)$ of the surrounding medium, that of D is independent of it.*

(*ii*) Electric flux density (D) is directly related to electric field intensity (E); permittivity $\varepsilon(=\varepsilon_0\varepsilon_r)$ of the medium being the factor by which one quantity differs from the other.

(*iii*) The importance of relation $D = \varepsilon_0\varepsilon_r E$ lies in the fact that it relates density concept to intensity concept.

(*iv*) Electric intensity at a point is also defined as equal to the electric lines of force passing normally through a unit cross-sectional area at that point. If Q coulombs is the charge, then number of electric lines of force produced by it is Q/ε. If these lines fall normally on area A m^2 surrounding the point, then electric intensity E at the point is

$$E = \frac{Q/\varepsilon}{A} = \frac{Q}{\varepsilon A}$$

But $\dfrac{Q}{A} = D$ = Electric flux density over the area.

$$\therefore \quad E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0\varepsilon_r} \quad \text{... in a medium}$$

$$= \frac{D}{\varepsilon_0} \quad \text{... in air}$$

Example 5.13. *Calculate the dielectric flux between two parallel flat metal plates each 35 cm square with an air gap of 1·5 mm between; the potential difference being 3000 V. A sheet of insulating material 1·5 mm thick is inserted between the plates and the potential difference raised to 7400V. What is the relative permittivity of this material if the charge is now 32 μC ?*

Solution. $E = V/d \;;\; D = \varepsilon_0\varepsilon_r E = \dfrac{\varepsilon_0\varepsilon_r V}{d} \;;\; \psi = DA$

$$\therefore \quad \psi = \left(\frac{\varepsilon_0\varepsilon_r V}{d}\right)\times A$$

When medium is air ($\varepsilon_r = 1$)

$$\psi = \frac{\varepsilon_0 V}{d}\times A = \frac{(8.85\times10^{-12})\times3000\times(35\times35\times10^{-4})}{1.5\times10^{-3}}$$

$$= 21{\cdot}6 \times 10^{-7}\text{C} = \mathbf{2{\cdot}16\ \mu C}$$

When medium is insulating material

$$\psi = \frac{\varepsilon_0 \varepsilon_r V}{d} \times A$$

Here $\psi = Q = 32\ \mu\text{C} = 32 \times 10^{-6}$ C ; $V = 7400$ volts ; $d = 1{\cdot}5 \times 10^{-3}$ m

$$\therefore \qquad \varepsilon_r = \frac{\psi \times d}{\varepsilon_0 VA} = \frac{32 \times 10^{-6} \times 1.5 \times 10^{-3}}{8.85 \times 10^{-12} \times 7400 \times (35)^2 \times 10^{-4}} = \mathbf{6}$$

Tutorial Problems

1. What is the total flux passing through a 10 cm × 6 cm surface in a region where the electric flux density is 2700 μC/m^2 ? **[$1{\cdot}62 \times 10^{-5}$ C]**
2. At a certain point in a material, the flux density is 0·09 C/m^2 and electric field intensity is $1{\cdot}2 \times 10^9$ V/m. What is the absolute permittivity of the material ? **[$7{\cdot}5 \times 10^{-11}$ C^2 N^{-1} m^{-2}]**

5.13. Gauss's Theorem

This theorem was first expressed by a German scientist Karl Fredrich Gauss (1777–1855) and may be stated as under :

The electric flux passing through a closed surface surrounding a number of charges is equal to the algebraic sum of the charges inside the closed surface.

To illustrate Gauss's theorem, consider Fig. 5.18 where charges Q_1, Q_2, Q_3 and $-Q_4$ coulombs are placed inside a closed surface. According to Gauss, the total electric flux ψ passing through this closed surface is given by the algebraic sum of the charges inside the closed surface *i.e.*

ψ = Algebraic sum of the charges inside the closed surface

$$= (Q_1) + (Q_2) + (Q_3) + (-Q_4)$$

$$= Q_1 + Q_2 + Q_3 - Q_4 \text{ coulombs}$$

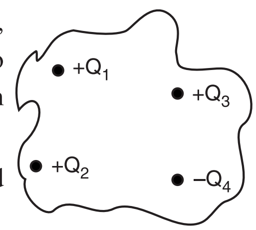

Fig. 5.18

The following points may be noted :

(*a*) The location of charge/charges inside the closed surface does not matter.

(*b*) The shape of the surface does not matter provided it is a closed surface enclosing the charge/charges.

Explanation. (*i*) Consider a charge of +Q coulomb placed at the centre of sphere of radius r as shown in Fig. 5.19 (*i*). Since the charge is at the centre of the sphere, electric flux density (D) is uniform over all the surface and perpendicular to the surface at every point.

$$D = \frac{\text{Charge}}{\text{Area of sphere}} = \frac{Q}{4\pi r^2}$$

Therefore, the electric flux ψ passing outward through the sphere is

$$\psi = D \times \text{Area} = \frac{Q}{4\pi r^2} \times 4\pi r^2 = Q \text{ coulomb}$$

The number of electric lines of force passing through the closed surface normally is Q/ε_0.

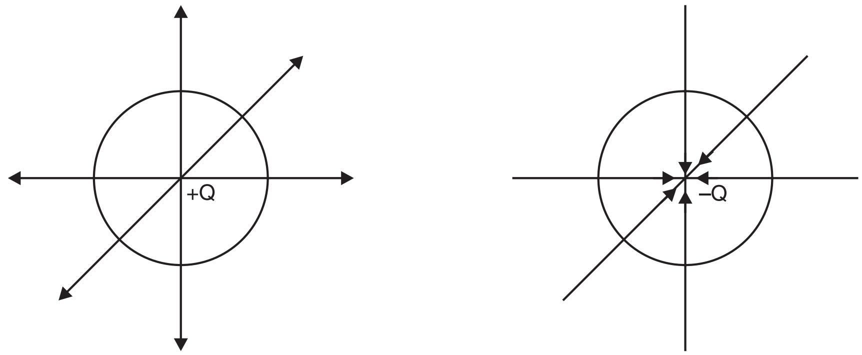

Fig. 5.19

Thus the electric flux passing through the surface of sphere is equal to Q, the charge enclosed in the sphere. This establishes Gauss's theorem.

If the sphere were enclosing a charge $-Q$ placed at the centre [See Fig. 5.19 (*ii*)], then electric flux $\psi = Q$ coulomb would pass inward through the surface and terminate at the charge.

(*ii*) Now consider that the charge $+Q$ coulomb is placed at any other point (other than centre O) inside the sphere as shown in Fig. 5.20. The electric lines of force flow outward but not normal to the surface. However, at any point on the sphere (such as point P), electric flux can be resolved into two rectangular components *viz*

(*a*) Component normal to the surface *i.e.*, cos θ component.

(*b*) Component perpendicular to the normal to the surface *i.e.* sin θ component.

If we add all the sin θ components of electric flux over the whole surface, the result will be zero. It is because various sin θ components cancel each other. However, all cos θ components of flux are normal to the sphere surface and meet at the centre if produced backward. Hence the resultant of all cos θ components over the surface of sphere is equal to Q coulomb *i.e.*

$$\psi = Q \text{ coulomb}$$

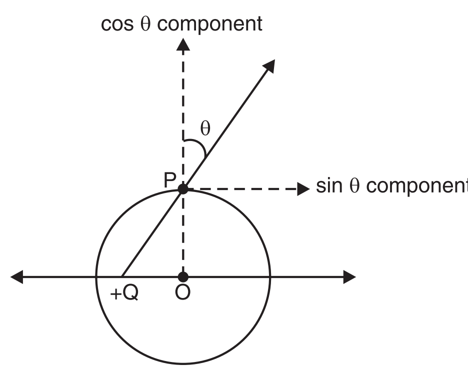

Fig. 5.20

The number of electric lines of force passing through the closed surface normally is Q/ε_0.

Thus irrespective of the position of charge Q within the sphere, the flux passing through the sphere surface is Q coulomb. This establishes Gauss's theorem. Similarly, it can be shown that if a surface encloses a number of charges, the electric flux passing through the surface is equal to the algebraic sum of charges inside the closed surface.

Gauss's law can also be expressed *mathematically*.

We know that : $\psi = \oint \vec{E} \cdot \vec{dS}$

where $\oint \vec{E} \cdot \vec{dS}$ is the surface integral of electric field $(\vec{E})$ over the entire closed surface enclosing the charge Q.

$$\therefore \qquad \psi = \oint \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon_0}$$

Hence, Gauss's law may be stated as under :

If a closed surface encloses a net charge (Q), then surface integral of electric field $(\vec{E})$ *over the closed surface is equal to* $1/\varepsilon_0$ *times the charge enclosed.*

5.14. Proof of Gauss's Law

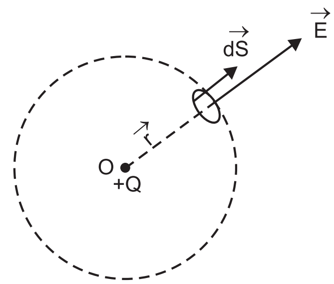

Fig. 5.21

Consider a positive charge + Q located at point O as shown in Fig. 5.21. We draw a sphere of radius r with charge + Q as its centre. We now show that total electric flux (*i.e.* total number of electric lines of force) passing through the closed surface is Q/ε_0. The magnitude of electric field at any point on the spherical surface is given by ;

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The electric field is directed radially outward from + Q. The spherical surface is only imaginary and is called *Gaussian surface*.

Consider a small elementary area $\vec{dS}$ on the surface of sphere as shown in Fig. 5.21. It is clear that $\vec{E}$ is * parallel to $\vec{dS}$ *i.e.* angle between $\vec{E}$ and $\vec{dS}$ is zero. Therefore, electric flux through the entire closed spherical surface is

$$\psi = \oint \vec{E} \cdot \vec{dS} = \oint E\, dS \cos 0° = \oint E\, dS$$

Since E (magnitude of $\vec{E}$) is constant over the considered closed surface, it can be taken out of integral.

$$\therefore \qquad \psi = E \oint dS$$

Now $\qquad E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ and $\oint dS$ = Surface area of sphere = $4\pi r^2$

$$\therefore \qquad \psi = \frac{Q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\text{Hence,} \qquad \psi = \oint \vec{E}.\vec{dS} = \frac{Q}{\varepsilon_0}$$

Note. We know : $\psi = \oint \vec{E}.\vec{dS} = \dfrac{Q}{\varepsilon_0}$

$$= \oint \varepsilon_0 \vec{E}.\vec{dS} = Q$$

$$\therefore \qquad \psi = \oint \vec{D}.\vec{dS} = Q \qquad (\because\ \varepsilon_0 \vec{E} = \vec{D})$$

Note that ψ can be expressed in Q or Q/ε_0.

Hence Gauss's law may be stated in terms of flux density $(\vec{D})$ as under :

If a closed surface encloses a net charge (Q), then surface integral of $\vec{D}$ *(electric flux density) over the closed surface is equal to the charge enclosed by the closed surface.*

Example 5.14. *A spherical surface 50 cm in diameter is penetrated by an inward flux uniformly distributed over the surface, the electric flux density being* $2{\cdot}5 \times 10^{-7}$ C/m^2. *What is the magnitude and sign of the charge enclosed by this surface ?*

Solution. Area of spherical surface is

$$A = 4\pi r^2 = 4\pi \times (25 \times 10^{-2})^2 = 0{\cdot}785 \text{ m}^2$$

* This is true for every elementary area on the surface.

$$\text{Electric flux, } \psi = D \times A = (2\cdot5 \times 10^{-7}) \times (0\cdot785) = 0\cdot1962 \times 10^{-6} \text{ C}$$

$$\therefore \quad \text{Charge enclosed} = 0\cdot1962 \times 10^{-6} \text{ C} = \mathbf{0\cdot1962\ \mu C}$$

Since the electric flux is passing inward through the sphere, the charge enclosed is **negative**.

5.15. Electric Potential Energy

We know that earth has gravitational field which attracts the bodies towards earth. When a body is raised above the ground level, it possesses mechanical potential energy which is equal to the amount of work done in raising the body to that point. The greater the height to which the body is raised, the greater will be its potential energy. Thus, the potential energy of the body depends upon its position in the gravitational field; being zero on earth's surface. Strictly speaking, sea level is chosen as the place of zero potential energy.

Like earth's gravitational field, every charge ($+ Q$) has electric field which theoretically extends upto infinity. If a small positive test charge $+ q_0$ is placed in this electric field, the test charge will experience a force of repulsion. If test charge $+ q_0$ is moved towards $+ Q$, work will have to be done against the force of repulsion. This work done is stored in $+ q_0$ in the form of potential energy. We say the charge $+ q_0$ has electric potential energy. The electric potential energy of $+ q_0$ depends upon its position in the electric field ; being zero if q_0 is situated at infinity.

From the above discussion, it follows that just as a mass has mechanical potential energy in the gravitational field, similarly a charge has electric potential energy in the electric field. The electric potential energy of a charge is positive or negative depending upon the kind of charge.

5.16. Electric Potential

Just as we define electric field intensity as the force per unit charge, similarly *electric potential is defined as the electric potential energy per unit charge*.

Consider an isolated charge $+Q$ fixed in space as shown in Fig. 5.22. If a unit positive charge (*i.e.* + 1C) is placed at infinity, the force on it due to charge $+Q$ is *zero. If the unit positive charge at infinity is moved towards $+Q$, a force of repulsion acts on it (like charges repel) and hence work is required to be done to bring it to a point like A. Hence when the unit positive charge is at A, it has some amount of electric potential energy which is a measure of electric potential. The closer the point to the charge, the higher will be the electric potential energy and hence the electric potential at that point. Therefore, electric potential at a point due to a charge depends upon the position of the point; being zero if the point is situated at infinity. Obviously, in electric field, infinity is chosen as the point of **zero potential.

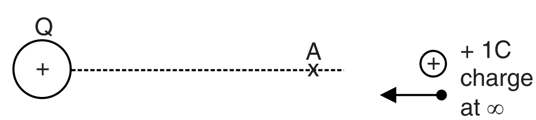

Fig. 5.22

Hence **electric potential** *at a point in an electric field is the amount of work done in bringing a unit positive charge* (*i.e.* +1 C) *from infinity to that point i.e.*

$$\text{Electric potential} = \frac{\text{Work}}{\text{Charge}} = \frac{W}{Q}$$

where W is the work done to bring a charge of Q coulombs from infinity to the point under consideration.

---

* $F = 9 \times 10^9 \times \frac{Q \times 1}{d^2}$; As $d \to \infty$, $F \to 0$

** In practice, earth is chosen to be at zero electric potential. It is because earth is such a huge conductor that its electric potential practically remains constant.

Unit. The *SI* unit of electric potential is *volt and may be defined as under :

The electric potential at a point in an electric field is **1 volt** *if 1 joule of work is done in bringing a unit positive charge* (*i.e.* + *1* C) *from infinity to that point* **against the electric field.*

Thus when we say that potential at a point in an electric field is +5V, it simply means that 5 joules of work has been done in bringing a unit positive charge from infinity to that point.

5.17. Electric Potential Difference

In practice, we are more concerned with potential difference between two points rather than their †absolute potentials. The potential difference (p.d.) between two points may be defined as under :

The potential difference between two points is the amount of work done in moving a unit positive charge (*i.e.* + *1*C) *from the point of lower potential to the point of higher potential.*

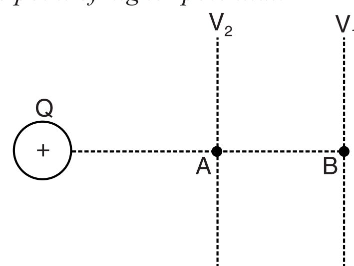

Fig. 5.23

Consider two points A and B in the electric field of a charge $+Q$ as shown in Fig. 5.23. Let V_2 and V_1 be the absolute potentials at A and B respectively. Clearly, $V_2 > V_1$. The potential V_1 at B means that V_1 joules of work has been done in bringing a unit positive charge from infinity to point B. Let the extra work done to bring the unit positive charge from B to A be W joules.

$\therefore$ Potential at $A = V_1 + W$

$\therefore$ P.D. between A and $B = (V_1 + W) - V_1$

or $V_2 - V_1 = W = W.D.$ to move + 1C from B to A

The SI unit of potential difference is volt and may be defined as under :

The p.d. between two points is **1 V** *if 1 joule of work is done in bringing a unit positive charge* (*i.e.* + *1 C*) *from the point of lower potential to the point of higher potential.*

Thus when we say that p.d. between two points is 5 volts, it simply means that 5 joules of work will have to be done to bring +1C of charge from the point of lower potential to the point of higher potential. Conversely, 5 joules of work or energy will be released if + 1 C charge moves from the point of higher potential to the point of lower potential.

5.18. Potential at a Point Due to a Point Charge

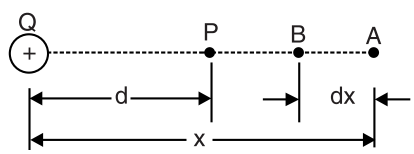

Fig. 5.24

Consider an isolated positive charge of Q coulombs placed in a medium of relative permittivity ε_r. It is desired to find the electric potential at point P due to this charge. Let P be at a distance d metres from the charge. Imagine a unit positive charge (*i.e.* + 1 C) placed at A and situated x metres from the charge. Then the force acting on this unit charge (*i.e.* electric intensity) is given by [See Fig. 5.24] ;

$$F = E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}$$

If this unit positive charge at A is moved through a small distance dx towards the charge $+Q$, then work done is given by ;

$$dW = \frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2} \times (-\dagger\dagger\, dx) = -\frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}dx$$

* Electric potential = W/Q = joules/coulomb. Now joule/coulomb has been given a special name *viz* volt.

** Note if the field is due to a positive charge (as is in this case), work will be done against the electric field. However, if the field is due to a negative charge, work is done by the electric field.

† The potential at a point with infinity as reference is termed as absolute potential.

†† The negative sign is taken because dx is considered in the negative direction of distance (x).

Total work done in bringing a unit positive charge from infinity to point P is

$$\text{Total work done, } W = \int_{\infty}^{d} -\frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}\,dx = -\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\int_{\infty}^{d}\frac{1}{x^2}\,dx$$

$$= -\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[-\frac{1}{x}\right]_{\infty}^{d} = \frac{-Q}{4\pi\varepsilon_0\varepsilon_r}\left[-\frac{1}{d}-\left(-\frac{1}{\infty}\right)\right]$$

$$= \frac{Q}{4\pi\varepsilon_0\varepsilon_r d}$$

$$= 9\times10^9\,\frac{Q}{\varepsilon_r d}\text{ joules} \qquad \left[\because \frac{1}{4\pi\varepsilon_0} = 9\times10^9\right]$$

By definition, the work done in joules to bring a unit positive charge from infinity to point P is equal to potential at P in volts.

$$\therefore \quad V_P = 9\times10^9\,\frac{Q}{\varepsilon_r d}\text{ volts} \qquad \textit{...in a medium}$$

$$= 9\times10^9\,\frac{Q}{d}\text{ volts} \qquad \textit{...in air}$$

The following points may be noted carefully :

(*i*) The potential varies inversely with the distance d from the point charge Q. If the distance is increased three times, the potential is reduced one-third of its value and so on.

(*ii*) Electric potential is a scalar quantity.

(*iii*) At $d = \infty$ in air/vacuum, $V_P = 9\times10^9\,\frac{q}{\infty} = 0$.

(*iv*) If Q is positive, then potential at P is *positive. On the other hand, if Q is negative, then potential at P is negative.

5.19. Potential at a Point Due to Group of Point Charges

Electric potential obeys superposition principle. Therefore, electric potential at any point P due to a group of point charges Q_1, Q_2, Q_3 .... Q_n is equal to the algebraic sum of potentials due to Q_1, Q_2, Q_3 ... Q_n at point P. Note that an algebraic sum is one in which sign of the physical quantity (potential in this case) is taken into account.

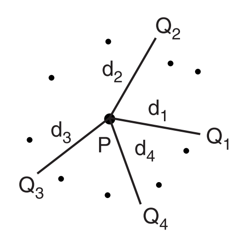

Fig. 5.25

Let the distances of Q_1, Q_2, Q_3, ... Q_n be d_1, d_2, d_3 ... d_n respectively from point P as shown in Fig. 5.25. Further, let V_1, V_2, V_3 ... V_n be the potentials at P due to Q_1, Q_2, Q_3 .... Q_n respectively. Assuming the medium to be free space/air,

$$\text{Total potential at } P,\ V_P = V_1 + V_2 + V_3 + \ldots.. + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{d_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{d_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{d_3} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{Q_n}{d_n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \ldots + \frac{Q_n}{d_n}\right]$$

$$\therefore \quad V_P = 9\times10^9\left[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \ldots + \frac{Q_n}{d_n}\right]$$

* The potential near an isolated positive charge is positive because work is done by an external agency to push a test charge (positive) from infinity to that point. The potential near an isolated negative charge is negative because outside agent must exert a restraining force as test charge comes in from infinity.

If the system of charges is placed in a medium of relative permittivity ε_r, then,

$$V_P = \frac{9\times10^9}{\varepsilon_r}\left[\frac{Q_1}{d_1}+\frac{Q_2}{d_2}+\frac{Q_3}{d_3}+\ldots+\frac{Q_n}{d_n}\right]$$

5.20. Behaviour of Metallic Conductors in Electric Field

When a metallic conductor (solid or hollow) is placed in an electric field, there is a momentary flow of charges (*i.e.*, free electrons). Once the flow of charges ceases, the conductor is said to be in *electrostatic equilibrium*. It has been seen experimentally that under the conditions of electrostatic equilibrium, a conductor (solid or hollow) shows the following properties [See Fig. 5.26] :

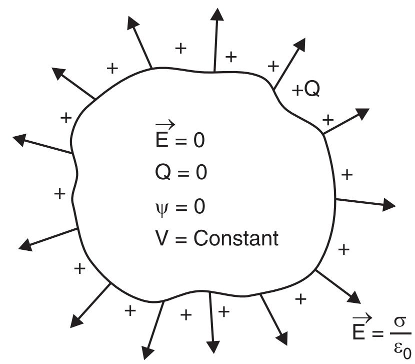

Fig. 5.26

(*i*) *The net electric field inside a charged conductor is zero i.e., no electric lines of force exist inside the conductor.*

(*ii*) *The net charge inside a charged conductor is zero.*

(*iii*) *The electric field (i.e., electric lines of force) on the surface of a charged conductor is perpendicular to the surface of the conductor at every point.*

(*iv*) *The magnitude of electric field just outside a charged conductor is σ/ε_0 where σ is the surface charge density.*

(*v*) *The electric potential is the same (i.e., constant) at the surface and inside a charged conductor.*

Inside a charged conductor, $E = 0$

Now $\quad E = -\frac{dV}{dS} \quad$ or $\quad 0 = -\frac{dV}{dS}$

This means that V is constant.

5.21. Potential of a Charged Conducting Sphere

Consider an isolated conducting sphere of radius r metres placed in air and charged uniformly with Q coulombs. The field has spherical symmetry *i.e.* lines of force spread out normally from the surface and meet at the centre of the sphere if produced backward. *Outside the sphere,* the field is exactly the same as though the charge Q on sphere were concentrated at its centre.

(*i*) Potential at the sphere surface. Due to spherical symmetry of the field, we can imagine the charge Q on the sphere as concentrated at its centre O [See Fig. 5.27 (*i*)]. The problem then reduces to find the potential at a point r metres from a charge Q.

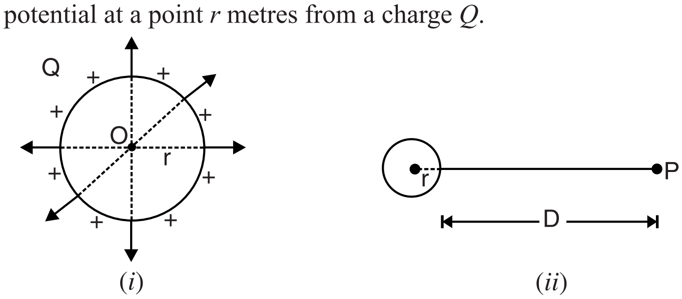

Fig. 5.27

∴ Potential at the surface of sphere

$$= \frac{Q}{4\pi\varepsilon_0 r} \text{ volts} \qquad \text{[See Art. 5·18]}$$

$$= 9\times10^9\frac{Q^*}{r} \text{ volts}$$

(*ii*) Potential outside the sphere. Consider a point *P* outside the sphere as shown in Fig. 5.27 (*ii*). Let this point be at a distance of *D* metres from the surface of sphere.

$$\text{Then potential at } P = 9\times10^9\frac{Q}{(D+r)} \text{ volts}$$

(*iii*) Potential inside the sphere. Since there is no electric flux inside the sphere, electric intensity inside the sphere is zero.

$$\text{Now, electric intensity} = \frac{\text{Change in potential}}{r}$$

or $\quad 0 = \text{Change in potential}$

Hence, all the points inside the sphere are at the same potential as the points on the surface.

Example 5.15. *Two positive point charges of 16 × 10^{-10} C and 12 × 10^{-10} C are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.*

Solution. Suppose the charge 16 × 10^{-10} C to be fixed.

Potential of a point 10 cm from the charge 16 × 10^{-10} C

$$= 9\times10^9\frac{16\times10^{-10}}{0.1} = 144 \text{ V}$$

Potential of a point 6 cm from the charge 16 × 10^{-10} C

$$= 9\times10^9\frac{16\times10^{-10}}{0.06} = 240 \text{ V}$$

∴ Potential difference = 240 − 144 = 96 V

Work done = Charge × p.d. = 12 × 10^{-10} × 96 = **11·52 × 10^{-8} joules**

Example 5.16. *A square ABCD has each side of 1 m. Four point charges of +0·01 μC, − 0·02 μC, +0·03 μC and +0·02 μC are placed at A, B, C and D respectively. Find the potential at the centre of the square.*

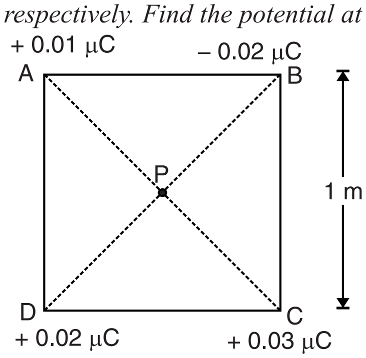

Fig. 5.28

Solution. Fig. 5.28 shows the square *ABCD* with charges placed at its corners. The diagonals of the square intersect at point *P*. Clearly, point *P* is the centre of the square. The distance of each charge from point *P* (*i.e.* centre of square) is

$$= \frac{1}{2}\sqrt{1^2+1^2} = 0\cdot707 \text{ m}$$

The potential at point *P* due to all charges is equal to the algebraic sum of potentials due to each charge.

∴ Potential at *P* due to all charges

$$= 9\times10^9\left[\frac{Q_1}{0.707}+\frac{Q_2}{0.707}+\frac{Q_3}{0.707}+\frac{Q_4}{0.707}\right]$$

$$= \frac{9\times10^9}{0.707}\left[(0.01-0.02+0.03+0.02)10^{-6}\right]$$

$$= \frac{9\times10^9}{0.707}\times0.04\times10^{-6} = \mathbf{509.2V}$$

* If the sphere is placed in a medium (ε_r), then potential is

$$= 9\times10^9\frac{Q}{\varepsilon_r r}$$

Example 5.17. *A hollow sphere is charged to 12μC. Find the potential (i) at its surface (ii) inside the sphere (iii) at a distance of 0·3m from the surface. The radius of the sphere is 0·1m.*

Solution. (*i*) The potential at the surface of the sphere in air is

$$V = \frac{Q}{4\pi\varepsilon_0 d} = 9\times10^9 \times \frac{Q}{d}$$

Here $Q = 12\ \mu C = 12 \times 10^{-6}\ C \ ;\ d = 0{\cdot}1 m$

$$\therefore \quad V = 9\times10^9 \times \frac{12\times10^{-6}}{0.1} = \mathbf{108 \times 10^4\ volts}$$

(*ii*) Potential inside the sphere is the same as at the surface *i.e.* **108×10^4 volts.**

(*iii*) Distance of the point from the centre = 0·3 + 0·1 = 0·4m

$$\therefore \quad \text{Potential} = 9\times10^9 \times \frac{12\times10^{-6}}{0.4} = \mathbf{27 \times 10^4\ volts}$$

Example 5.18. *If 300 J of work is done in carrying a charge of 3 C from a place where the potential is −10 V to another place where potential is V, calculate the value of V.*

Solution. $$V_B - V_A = \frac{W}{Q}$$

Here $V_B = V \ ;\ V_A = -10V \ ;\ W = 300\ J \ ;\ Q = 3\ C$

$$\therefore \quad V - (-10) = 300/3 \quad \text{or} \quad V + 10 = 100$$

$$\therefore \quad V = 100 - 10 = \mathbf{90\ volts}$$

Example 5.19. *The electric field at a point due to a point charge is 30 N/C and the electric potential at that point is 15 J/C. Calculate the distance of the point from the charge and magnitude of charge.*

Solution. Suppose q coulomb is the magnitude of charge and its distance from the point is r metres.

Now, $$E = \frac{kq}{r^2} = 30 \ ; \quad V = \frac{kq}{r} = 15$$

$$\therefore \quad \frac{E}{V} = \frac{1}{r} \quad \text{or} \quad r = \frac{V}{E} = \frac{15}{30} = \mathbf{0{\cdot}5\ m}$$

Now $$kq = 15\,r = 15 \times 0{\cdot}5 = 7{\cdot}5$$

$$\therefore \quad q = \frac{7.5}{k} = \frac{7.5}{9\times10^9} = \mathbf{0{\cdot}83 \times 10^{-9}\ C}$$

Example 5.20. *Two point charges of +4 μC and −6 μC are separated by a distance of 20 cm in air. At what point on the line joining the two charges is the electric potential zero ?*

Solution. Fig. 5.29 shows the conditions of the problem. Suppose C is the point of zero potential. Potential at point C is given by ;

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{4\times10^{-6}}{d_1} - \frac{6\times10^{-6}}{d_2}\right]$$

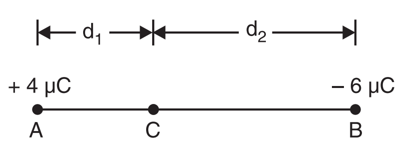

Fig. 5.29

or $$0 = \frac{10^{-6}}{4\pi\varepsilon_0}\left[\frac{4}{d_1} - \frac{6}{d_2}\right]$$

or $$\frac{4}{d_1} - \frac{6}{d_2} = 0 \quad \text{or} \quad d_1 = \frac{2}{3}d_2 \qquad ...(i)$$

Also $$d_1 + d_2 = 20\ cm \qquad ...(ii)$$

Solving eqs. (*i*) and (*ii*), we get, $d_1 = 8$ cm ; $d_2 = 12$ cm.

Therefore, the point of zero potential lies **8 cm from the charge of +4 μC** or at 12 cm from the charge of −6 μC.

Tutorial Problems

1. A charge of $-4{\cdot}5 \times 10^{-7}$ C is carried from a distant point upto a charged metal sphere. What is the electrical potential of the body if the work done is $1{\cdot}8 \times 10^{-3}$ joule ? **[4×10^3 V]**
2. The difference of potentials between two points in an electric field is 6 volts. How much work is required to move a charge of 300 μC between these points ? **[$1{\cdot}8 \times 10^{-3}$ joule]**
3. A force of 0·032 N is required to move a charge of 42 μC in an electric field between two points 25 cm apart. What potential difference exists between the two points ? **[$1{\cdot}9 \times 10^2$ V]**
4. What is the magnitude of an isolated positive charge to give an electric potential of 100V at 10 cm from the charge ? **[$1{\cdot}11 \times 10^{-9}$ C]**
5. A square *ABCD* has each side of 1m. Four charges of +0·02 μC, +0·04 μC, +0·06 μC and +0·02 μC are placed at *A*, *B*, *C* and *D* respectively. Find the potential at the centre of the square. **[1000V]**
6. A sphere of radius 0·1 m has a charge of 5×10^{-8} C. Determine the potential (*i*) at the surface of sphere, (*ii*) inside the sphere and (*iii*) at a distance of 1m from the surface of the sphere. Assume air as the medium. [(*i*) **4500 V** (*ii*) **4500 V** (*iii*) **409 V**]

5.22. Potential Gradient

The change of potential per unit distance is called **potential gradient** *i.e.*

$$\text{Potential gradient} = \frac{V_2 - V_1}{S}$$

where $V_2 - V_1$ is the change in potential (or p.d.) between two points S metres apart. Obviously, the unit of potential gradient will be volts/m.

Consider a charge $+Q$ and let there be two points A and B situated S metres apart in its electric field as shown in Fig. 5.30. Clearly, potential at point A is more than the potential at point B. If distance S is small, then the electric intensity will be approximately the same in this small distance. Let it be E newtons/coulomb. It means that a force of E newtons will act on a unit positive charge (*i.e.* + 1C) placed anywhere between A and B. If a unit positive charge is moved from B to A, then work done to do so is given by ;

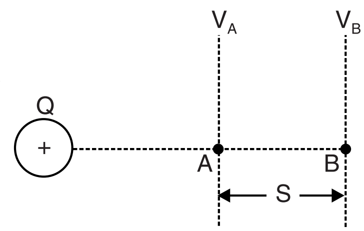

Fig. 5.30

$$\text{Work done} = E \times S \text{ joules}$$

But work done in bringing a unit positive charge from B to A is the potential difference $(V_A - V_B)$ between A and B.

$$\therefore \quad E \times S = V_A - V_B$$

or
$$E = \frac{V_A - V_B}{S} = \text{Potential gradient}$$

In differential form,
$$E = -\frac{{}^*dV}{dS}$$

Hence electric intensity at a point is numerically equal to the potential gradient at that point.

Since electric intensity is numerically equal to potential gradient at any point, both must be measured in the same units. Clearly, electric intensity can also be measured in V/m. For example, when we say that potential gradient at a point is 1000 V/m, it means that electric intensity at that point is also **1000 V/m or 1000 N/C.

* Since work done in moving +1C from B to A is against electric field, a negative sign must be used to make the equation technically correct.

** It can be shown that 1 V/m = 1 N/C.

$$1 \text{ V/m} = \frac{\text{joule/coulomb}}{\text{metre}} = \frac{\text{newton} \times \text{metre}}{\text{metre} \times \text{coulomb}} = 1 \text{ N/C}$$

5.23. Breakdown Voltage or Dielectric Strength

In an insulator or dielectric, the valence electrons are tightly bound so that no free electrons are available for current conduction. However, when voltage applied to a dielectric is gradually increased, a point is reached when these electrons are torn away, a large current (much larger than the usual leakage current) flows through the dielectric and the material loses its insulating properties. Usually, a *spark or arc occurs which burns up the material. The minimum voltage required to break down a dielectric is called breakdown voltage or dielectric strength.

The maximum voltage which a unit thickness of a dielectric can withstand without being punctured by a spark discharge is called ****dielectric strength*** *of the material.*

The dielectric strength (or breakdown voltage) is generally measured in kV/cm or kV/mm. For example, air has a dielectric strength of 30kV/cm. It means that maximum p.d. which 1 cm thickness of air can withstand across it without breaking down is 30kV. If p.d. exceeds this value, the break-down of air insulation will occur; allowing a large current to flow through it. Below is given the table showing dielectric constant and dielectric strength of some common insulators or dielectrics :

S.No.	Dielectric	Dielectric Constant (ε_r)	Dielectric strength (kV/cm)
1	Air	1	30
2	Paper (oiled)	2	400
3	Paraffin	2·25	350
4	Mica	6	500
5	Glass	8	1000

The following points may be noted :

(*i*) The value of dielectric strength of an insulator (or dielectric) depends upon temperature, moisture content, shape *etc*.

(*ii*) The electric intensity, potential gradient and dielectric strength are numerically equal *i.e.*

$$\textit{Electric intensity} = \textit{Potential gradient} = \textit{Dielectric strength}$$

(*iii*) The breakdown of solid insulating material (dielectric) usually renders it unfit for further use by puncturing, burning, cracking or otherwise damaging it. Gaseous and liquid dielectrics are self-healing and may be used repeatedly following breakdown.

(*iv*) *For reasons of safety, electric field applied to a dielectric is only 10% of the dielectric strength of the dielectric material.*

Note. To avoid electric breakdown of dielectric, capacitors are rated according to their *working voltage*, meaning the maximum safe voltage that can be applied to the capacitor.

5.24. Uses of Dielectrics

The insulating materials (or dielectrics) are widely used to provide electrical insulation to electrical and electronics apparatus. The choice of a dielectric for a particular situation will depend upon service requirements. A few cases are given below by way of illustration :

(*i*) If the dielectric is to be subjected to a great heat, as in soldering irons or toasters; mica should be used.

(*ii*) If space, flexibility and a fair dielectric strength are the deciding factors, as in the dielectric for small fixed capacitors, cellulose and animal tissue materials are used.

* This spark may burn a path through such dielectrics as paper, cloth, wood or mica. Hard materials such as porcelain or glass will crack or allow a small path to be melted through them.

** Dielectric strength should not be confused with dielectric constant (relative permittivity).

(*iii*) If a high dielectric strength is desired, as in case of high voltage transformers, glass and porcelain should be used.

(*iv*) If the insulation must remain liquid, like that used in large switches and circuit breakers to quench the arc when the circuit is opened, then various oils are used.

Example 5.21. *A parallel plate capacitor has plates 1 mm apart and a dielectric with relative permittivity of 3·39. Find (i) electric intensity and (ii) the voltage between plates if the surface charge density is 3×10^{-4} C/m^2.*

Solution. (*i*) The surface charge density is equal to electric flux density D.

Now, $$D = \varepsilon_0 \varepsilon_r E$$

$\therefore$ Electric intensity, $$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{3 \times 10^{-4}}{8.854 \times 10^{-12} \times 3.39} = \mathbf{10^7\ V/m}$$

(*ii*) P.D. between plates, $V = E \times dx = 10^7 \times (1 \times 10^{-3}) = \mathbf{10^4 V}$

Example 5.22. *The electric potential difference between the parallel deflection plates in an oscilloscope is 300V. If the potential drops uniformly when going from one plate to the other and if distance between the plates is 0·75 cm, what is the magnitude of the electric field between them and in which direction does it point?*

Solution. Let us choose the positive direction of ΔS to be in the direction of increasing potential.

$\therefore$ $$E = -\frac{\Delta V}{\Delta S}$$

Here $\Delta V = +300\text{V}$; $\Delta S = +0{\cdot}75 \text{ cm} = 0{\cdot}75 \times 10^{-2}$ m

$\therefore$ $$E = -\frac{300}{0.75 \times 10^{-2}} = \mathbf{-40,000\ V/m}$$

The negative value of E tells us that E is directed opposite to ΔS. Thus E is directed from the higher-voltage plate towards the lower-voltage one.

Example 5.23. *A uniform electric field is acting from left to right. If a + 2C charge moves from a to b, a distance of 4m, [See Fig. 5.31], find (i) electric field strength and (ii) potential energy of charge at b w.r.t. a. Given that p.d. between a and b is 50 volts.*

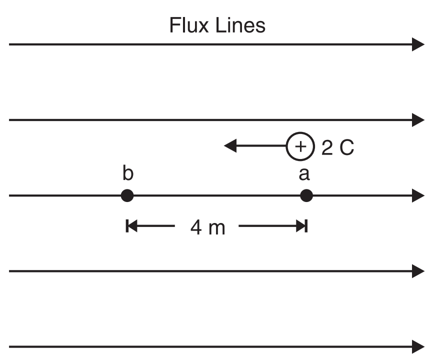

Fig. 5.31

Solution. Referring to Fig. 5.31, we have,

(*i*) Electric intenstity = Potential gradient = 50/4
= **12·5 V/m**

(*ii*) Potential energy of charge (*i.e.,* +2C) at b w.r.t. a
= Work per unit charge × Charge
= Voltage between a and b × Charge
= 50 joules/C × (2C) = **100 joules**

Example 5.24. *A sheet of glass 1·5 cm thick and of relative permittivty 7 is introduced between two parallel brass plates 2 cm apart. The remainder of the space between the plates is occupied by air. If a p.d. of 10,000 V is applied between the plates, calculate (i) electric intensity in air film between glass and plate and (ii) in the glass sheet.*

Solution. Fig. 5.32 shows the arrangement. Let V_1 and V_2 be the p.d. across air and glass respectively and E_1 and E_2 the corresponding electric intensities.

Now, $$V_1 = E_1 x_1 = E_1 \times (0{\cdot}5 \times 10^{-2})$$

and $V_2 = E_2 x_2 = E_2 \times (1 \cdot 5 \times 10^{-2})$

Now $V = V_1 + V_2$

or $10{,}000 = (0 \cdot 5\, E_1 + 1 \cdot 5\, E_2)10^{-2}$

or $E_1 + 3E_2 = 2 \times 10^6$ ...(*i*)

Now electric flux density D (= $\varepsilon_0\, \varepsilon_r\, E$) is the same in the two media because it is independent of the surrounding medium.

$\therefore$ $\varepsilon_0\, \varepsilon_{r2}\, E_1 = \varepsilon_0\, \varepsilon_{r2}\, E_2$

or $E_1 = 7\, E_2$ ...(*ii*)

From exps. (*i*) and (*ii*), we get,

(*i*) Electric intensity in air = $\mathbf{1 \cdot 4 \times 10^6}$ **V/m**

(*ii*) Electric intensity in glass = $\mathbf{0 \cdot 2 \times 10^6}$ **V/m**

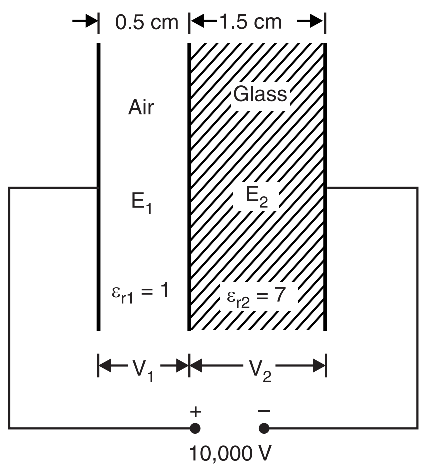

Fig. 5.32

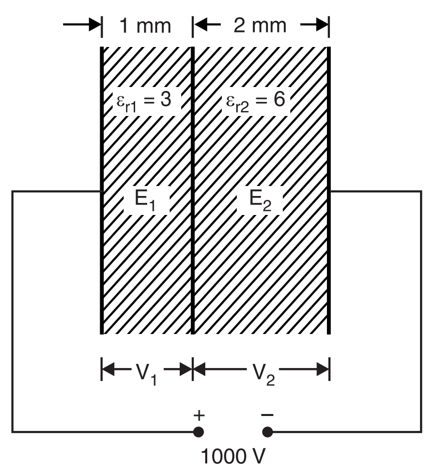

Fig. 5.33

Example 5.25. *A capacitor has two dielectrics 1 mm and 2 mm thick. The relative permittivities of these dielectrics are 3 and 6 respectively. Calculate the potential gradient along the dielectrics if a p.d. of 1000 V is applied between the plates.*

Solution. Fig. 5.33 shows the arrangement. Finding the potential gradient means to find the electric intensity (or electric stress).

$$V_1 = E_1\, x_1 = E_1 \times (1 \times 10^{-3})$$

$$V_2 = E_2\, x_2 = E_2 \times (2 \times 10^{-3})$$

Now $V = V_1 + V_2$

or $1000 = (E_1 + 2E_2)\, 10^{-3}$

or $E_1 + 2E_2 = 10^6$ ...(*i*)

Since flux density D (= $\varepsilon_0\, \varepsilon_r\, E$) is the same in the two media,

$\therefore$ $\varepsilon_0\, \varepsilon_{r1}\, E_1 = \varepsilon_0\, \varepsilon_{r2}\, E_2$

or $3\, E_1 = 6\, E_2$ ...(*ii*)

From exps. (*i*) and (*ii*), we get, $E_1 = \mathbf{0 \cdot 5 \times 10^6}$ **V/m** ; $E_2 = \mathbf{0 \cdot 25 \times 10^6}$ **V/m**

Example 5.26. *Two series connected parallel plate capacitors have plate areas of 0·2 m^2 and 0·04 m^2, plate separation of 0·5 mm and 0·125 mm and relative permittivities of 1 and 6 respectively. Calculate the total voltage across the capacitors that will produce a potential gradient of 100 kV/cm between the plates of first capacitor.*

Solution. We shall use suffix 1 for the first capacitor and suffix 2 for the second capacitor. Suppose for a potential gradient of 100 kV/cm between the plates of first capacitor, the voltages across first and second capacitors are V_1 and V_2 respectively. Then,

Total voltage V across capacitors is

$$V = V_1 + V_2$$

For the first capacitor $E_1 = 100 \text{ kV/cm} = 100 \times 10^3 \times 10^2 = 10^7 \text{ V/m}$

$$\therefore \quad V_1 = E_1 d_1 = (10^7) \times (0{\cdot}5 \times 10^{-3}) = 5 \times 10^3 \text{ V} = 5\text{kV}$$

$$D_1 = \varepsilon_0\, \varepsilon_{r1} E_1 = \varepsilon_0 \times 10^7 \qquad (\because \ \varepsilon_{r1} = 1)$$

$$Q_1 = A_1 D_1 = (0{\cdot}2) \times \varepsilon_0 \times 10^7 \text{ C}$$

For the second capacitor. Since the capacitors are connected in series, the charge on them is the same *i.e.*

$$Q_1 = Q_2 = 0{\cdot}2 \times \varepsilon_0 \times 10^7 \text{ C}$$

$$\therefore \quad D_2 = \frac{Q_2}{A_2} = \frac{0.2 \times \varepsilon_0 \times 10^7}{0.04} = 0{\cdot}5 \times 10^8 \times \varepsilon_0 \text{ C/m}^2$$

$$\therefore \quad E_2 = \frac{D_2}{\varepsilon_0 \varepsilon_{r2}} = \frac{0.5 \times 10^8 \times \varepsilon_0}{\varepsilon_0 \times 6} = \frac{10^8}{12} \text{V/m} \qquad (\because \ \varepsilon_{r2} = 6)$$

$$\therefore \quad V_2 = E_2 d_2 = \frac{10^8}{12}(0.125 \times 10^{-3}) = 1{\cdot}04 \times 10^3 \text{ V} = 1{\cdot}04 \text{ kV}$$

$\therefore$ Total voltage across the capacitors is

$$V = V_1 + V_2 = 5 + 1{\cdot}04 = \mathbf{6{\cdot}04\ kV}$$

Example 5.27. *A parallel plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon (ε_r = 2) 6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100V is applied across the capacitor, find the electric field E_a in the air, electric field E_t in Teflon, flux density D_a in air, flux density D_t in Teflon and potential difference V_t across Teflon slab.*

Solution. Electric flux density (D) in the two media is the same. However, electric field intensity (E) is inversely proportional to the relative permittivity (ε_r) of the medium. If E_a is the electric intensity in air, then electric intensity in Teflon is $E_t = E_a/2$ ($\because$ relative permittivity of Teflon = 2).

Thickness of air, t_a = 4 mm ; Thickness of Teflon, t_t = 6 mm

Voltage between two plates, $V = E_a t_a + E_t t_t$

$$\text{or} \quad 100 = E_a \times 4 + \frac{E_a}{2} \times 6 \qquad \left[\because E_t = \frac{E_a}{2}\right]$$

$$\therefore \quad E_a = \frac{100}{7} \text{ volts/mm} = \mathbf{14.286\ kV/m}$$

$$\text{Electric field in Teflon, } E_t = \frac{14.286}{2} = \mathbf{7.143\ kV/m}$$

As electric flux density is the same in the two media,

$$\therefore \quad D_a = D_t = \varepsilon_0 \varepsilon_r E_a = 8.854 \times 10^{-12} \times 1 \times 14.286 \times 1000$$

$$= \mathbf{1.265 \times 10^{-7}\ C/m^2}$$

P.D. across Teflon slab, $V_t = E_t \times t_t = 7.143 \times 1000 \times 6 \times 10^{-3} = \mathbf{42.86\ V}$

Tutorial Problems

1. An electron (charge = $1{\cdot}6 \times 10^{-19}$ C; mass = $9{\cdot}1 \times 10^{-31}$ kg) is released in a vacuum between two flat, parallel metal plates that are 10cm apart and are maintained at a constant electric potential difference of 750V. If the electron is released at the negative plate, what is the speed just before it strikes the positive plate ? **[$1{\cdot}6 \times 10^7$ ms^{-1}]**

2. To move a charged particle through an electric potential difference of 10^{-3}V requires 2×10^{-6} J of work. What is the magnitude of charge ? **[2×10^{-3} C]**

3. A proton of mass $1{\cdot}67 \times 10^{-27}$ kg and charge = $1{\cdot}6 \times 10^{-19}$ C is accelerated from rest through an electric potential of 400 kV. What is its final speed ? **[$8{\cdot}8 \times 10^{6}$ ms^{-1}]**

5.25. Refraction of Electric Flux

When electric flux passes from one uniform dielectric medium to another of different permittivities, the electric flux gets refracted at the boundary of the two dielectric media. Under this condition, the following two conditions exist at the boundary (called **boundary conditions**) :

(*i*) The normal components of electric flux density are equal *i.e.*

$$D_{1n} = D_{2n}$$

(*ii*) The tangential components of electric field intensities are equal *i.e.*

$$E_{1t} = E_{2t}$$

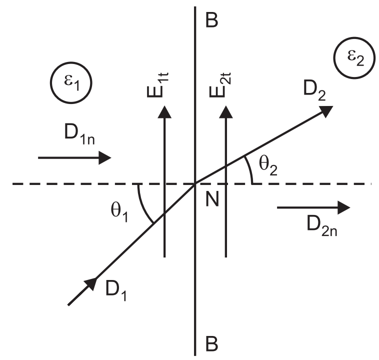

Fig. 5.34

Fig. 5.34 shows the refraction of electric flux at the boundary *BB* of two dielectric media of permittivities ε_1 and ε_2. As shown, the electric flux in the first medium (ε_1) approaches the boundary *BB* at an angle θ_1 and leaves it at θ_2. D_{1n} and D_{2n} are the normal components of D_1 and D_2 while E_{1t} and E_{2t} are the tangential components of E_1 and E_2. Referring to Fig. 5.34,

$$D_{1n} = D_1 \cos\theta_1 \text{ and } D_{2n} = D_2 \cos\theta_2$$

Also $\quad E_1 = D_1/\varepsilon_1$ and $E_{1t} = D_1 \sin\theta_1/\varepsilon_1$

Similarly, $\quad E_2 = D_2/\varepsilon_2$ and $E_{2t} = D_2 \sin\theta_2/\varepsilon_2$

$$\therefore \quad \frac{D_{1n}}{E_{1t}} = \frac{\varepsilon_1}{\tan\theta_1} \text{ and } \frac{D_{2n}}{E_{2t}} = \frac{\varepsilon_2}{\tan\theta_2}$$

Since $D_{1n} = D_{2n}$ and $E_{1t} = E_{2t}$,

$$\therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \quad \text{... (i)}$$

Eq. (*i*) gives the law of refraction of electric flux at the boundary of two dielectric media whose permittivities are different.

It is clear that if $\varepsilon_2 > \varepsilon_1$, then $\theta_2 > \theta_1$.

Note. When electric flux passes from one of the commonly used dielectrics (ε being 2 to 8) into another or air, there is hardly any refraction of electric flux.

Example 5.28. *An electric field in a medium with relative permittivity 7 passes into a medium of relative permittivity 2. If E makes an angle of 60° with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric ?*

Solution. As proved in Art 5.25,

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Here $\theta_1 = 60°$; $\varepsilon_1 = 7$; $\varepsilon_2 = 2$; $\theta_2 = ?$

$$\therefore \quad \frac{\tan 60°}{\tan\theta_2} = \frac{7}{2} \quad \text{or} \quad \tan\theta_2 = \sqrt{3} \times \frac{2}{7} = 0.495$$

$$\therefore \quad \theta_2 = \tan^{-1} 0.495 = \mathbf{26.33°}$$

5.26. Equipotential Surface

Any surface over which the potential is constant is called an **equipotential surface.**

In other words, the potential difference between any two points on an equipotential surface is zero. For example, consider two points A and B on an equipotential surface as shown in Fig. 5.35.

$$V_B - V_A = 0 \quad \therefore V_B = V_A$$

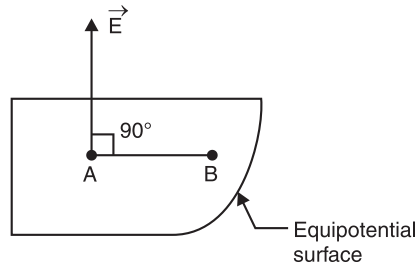

Fig. 5.35

The two important properties of equipotential surfaces are :

(*a*) *Work done in moving a charge over an equipotential surface is zero.*

$$\text{Work done} = \text{Charge} \times \text{P.D.}$$

Since potential difference (P.D.) over an equipotential surface is zero, work done is zero.

(*b*) *The electric field (or electric lines of force) are *perpendicular to an equipotential surface.*

Some cases of Equipotential surfaces. The fact that the electric field lines and equipotential surfaces are mutually perpendicular helps us to locate the equipotential surfaces when the electric field lines are known.

(*i*) **Isolated point charge.** The potential at a point P at a distance r from a point charge $+q$ is given by ;

$$V_P = k\frac{q}{r} \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

It is clear that potential at various points equidistant from the point charge is the same. Hence, in case of an isolated point charge, the spheres concentric with the charge will be the equipotential surfaces as shown in Fig. 5.36. Note that in drawing the equipotential surfaces, the potential difference is kept the same, *i.e.,* 10 V in this case. It may be seen that distance between charge and equipotential surface I is small so that E (= dV/dr = $10/dr$) is high. However, the distance between charge and equipotential surfaces II and III is large so that E (= dV/dr = $10/dr$) is small. It follows, therefore, that equipotential surfaces near the charge are crowded (*i.e.,* more E) and become widely spaced as we move away from the charge.

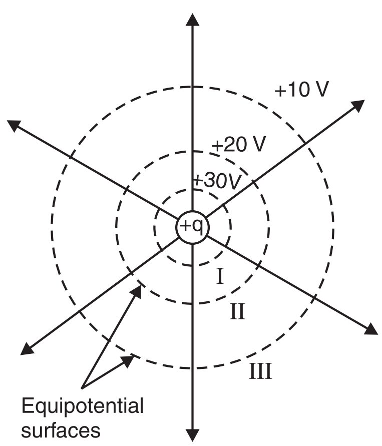

Fig. 5.36

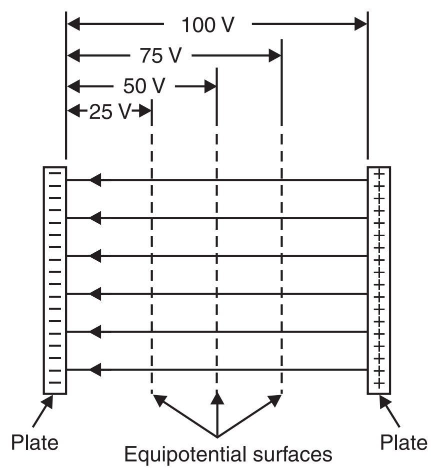

Fig. 5.37

* If this were not so that is if there were a component of $\vec{E}$ parallel to the surface — it would require work to move the charge along the surface against this component of $\vec{E}$; and this would contradict that it is an equipotential surface.

***(ii)* Uniform electric field.** In case of uniform electric field (*e.g.*, electric field between the plates of a charged parallel-plate capacitor), the field lines are straight and equally spaced. Therefore, equipotential surfaces will be parallel planes at right angles to the field lines as shown in Fig. 5.37.

5.27. Motion of a Charged Particle in Uniform Electric Field

Consider that a charged particle of charge $+q$ and mass m enters at right angles to a uniform electric field of strength E with velocity v along OX-axis as shown in Fig. 5.38. The electric field is along OY-axis and acts over a horizontal distance x.

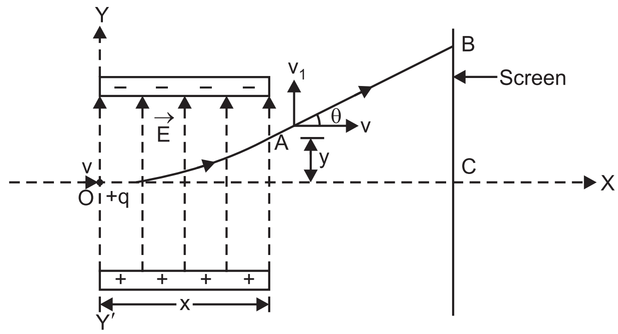

Fig. 5.38

Since the electric field is along OY-axis, no horizontal force acts on the charged particle entering the field. Therefore, the horizontal velocity v of the charged particle remains the same throughout the journey. *The electric field accelerates the charged particle along OY-axis only*.

Force on the charged particle, $F = qE$ ... along OY

Acceleration of the charged particle, $a = \dfrac{qE}{m}$ ...along OY

Time taken to traverse the field, $t = \dfrac{x}{v}$

If y is the displacement of the charged particle along OY direction in the electric field during the time t, then,

$$y = {}^*(0)t + \frac{1}{2}at^2$$

or $$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2$$

or $$y = \frac{qE}{2mv^2}x^2$$

or $$y = kx^2 \qquad \left(\because \frac{qE}{2mv^2} = \text{Constant} = k\right)$$

This is the equation of a parabola. *Therefore, inside the electric field, the charged particle follows a parabolic path OA*. As the charged particle leaves the electric field at A, it follows a straight line path AB tangent to path OA at A.

Note. When an electron (or a charged particle) at rest is accelerated through a potential difference (P.D.) of V volts, then,

Energy imparted to electron = Charge × P.D. = $e \times V$

K.E. gained by electron = $\dfrac{1}{2}mv^2$

$$\therefore \quad \frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

Here e is the charge on electron and m is the mass of electron. The velocity acquired by the electron is v.

* At the time the charged particle enters the electric field, its velocity along OY-axis is zero.

Example 5.29. *An electron moving with a velocity of 10^7 ms^{-1} enters mid-way between two horizontal plates P, Q in a direction parallel to the plates as shown in Fig. 5.39. The length of the plates is 5 cm and their separation is 2 cm. If a p.d. of 90 V is applied between the plates, calculate the transverse deflection produced by the electric field when the electron just passes the field. Assume e/m = 1.8×10^{11} C kg^{-1}.*

Solution. Fig. 5.39 shows the conditions of the problem.

$$\text{Electric field, } E = \frac{V}{d} = \frac{90}{2\times10^{-2}} = 45 \times 10^2 \text{ Vm}^{-1}$$

Downward force on the electron = eE

Downward acceleration of the electron is

$$a = \frac{eE}{m} = (1.8 \times 10^{11}) \times (45 \times 10^2) = 81 \times 10^{13} \text{ ms}^{-2}$$

$$\text{Time taken to cross the field, } t = \frac{x}{v} = \frac{5\times10^{-2}}{10^7} = 5 \times 10^{-9} \text{ s}$$

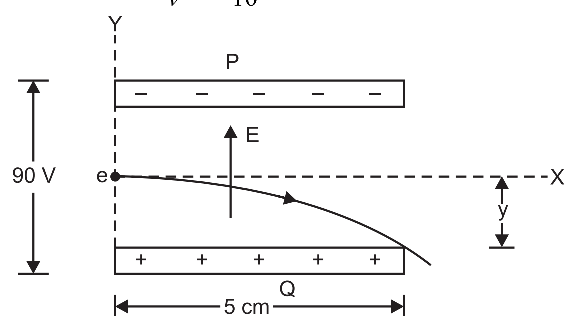

Fig. 5.39

$$\therefore \quad \text{Transverse deflection, } y = \frac{1}{2}at^2 = \frac{1}{2}(81 \times 10^{13}) \times (5 \times 10^{-9})^2 = 0.01 \text{ m} = \mathbf{1cm}$$

Example 5.30. *A potential gradient of 3×10^6 V/m is maintained between two horizontal parallel plates 1 cm apart. An electron starts from rest at the negative plate, travels under the influence of potential gradient to the positive plate. Given the mass of electron = 9.1×10^{-31} kg and the charge on electron = 1.603×10^{-19} C. Calculate (i) the force acting on the electron (ii) the ratio of electric force to gravitational force (iii) acceleration (iv) time taken to reach the positive plate.*

Solution. $E = 3 \times 10^6$ V/m ; $e = 1.603 \times 10^{-19}$ C ; $m = 9.1 \times 10^{-31}$ kg ; $S = 1 \times 10^{-2}$ m

(i) Force on electron, $F = eE = 1.603 \times 10^{-19} \times 3 \times 10^6 = \mathbf{4.81 \times 10^{-13}\ N}$

(ii) Ratio of electric force to gravitational force

$$= \frac{F}{mg} = \frac{4.81\times10^{-13}}{9.1\times10^{-31}\times9.81} = \mathbf{5.39 \times 10^{16}}$$

Note that electric force is very large compared to the gravitational force.

(iii) Acceleration of electron, $a = \dfrac{F}{m} = \dfrac{4.81\times10^{-13}}{9.1\times10^{-31}} = \mathbf{51.66 \times 10^{16}\ m/s^2}$

(iv) Distance travelled, $S = \dfrac{1}{2}at^2$

$$\therefore \quad \text{Time taken to reach + ve plate, } t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2\times1\times10^{-2}}{51.66\times10^{16}}} = \mathbf{1.968 \times 10^{-10}\ s}$$

Example 5.31. *An electron of charge 1.6 × 10^{-19} C can move freely for a distance of 2 cm in a field of 1000 V/cm. The mass of the electron is 9.1 × 10^{-28} g. If the electron starts with an initial velocity of zero, what velocity will it attain, what will be the time taken and what will be its kinetic energy?*

Solution. $e = 1.6 \times 10^{-19}$ C ; $m = 9.1 \times 10^{-31}$ kg ; $E = 1000$ V/cm $= 10^5$ V/m

Distance of free movement, $d = 2$ cm $= 0.02$ m

∴ Potential difference applied, $V = E \times d = 10^5 \times 0.02$ volts

Energy imparted to electron = Charge × P.D. = $e \times V$

$$= 1.6 \times 10^{-19} \times 10^5 \times 0.02 = 3.2 \times 10^{-16} \text{ J}$$

Now, Energy imparted = K.E. of electron = **3.2 × 10^{-16} J**

Also
$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 3.2 \times 10^{-16}}{9.1 \times 10^{-31}}} = \mathbf{2.652 \times 10^7 \text{ m/s}}$$

$$\text{Force on electron, } F = eE = 1.6 \times 10^{-19} \times 10^5 = 1.6 \times 10^{-14} \text{ N}$$

$$\text{Acceleration of electron, } a = \frac{F}{m} = \frac{1.6 \times 10^{-14}}{9.1 \times 10^{-31}} = 1.758 \times 10^{16} \text{ m/s}^2$$

$$\text{Distance travelled, } d = \frac{1}{2} a t^2$$

$$\therefore \quad \text{Time taken, } t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.02}{1.758 \times 10^{16}}} = \mathbf{1.51 \times 10^{-9} \text{ s}}$$

Objective Questions

1. The force between two electrons separated by a distance r varies as
   (*i*) r^2 (*ii*) r
   (*iii*) r^{-1} (*iv*) r^{-2}
2. Two charges are placed at a certain distance apart. A brass sheet is placed between them. The force between them will
   (*i*) increase (*ii*) decrease
   (*iii*) remain unchanged
   (*iv*) none of the above
3. Which of the following appliance will be studied under electrostatics ?
   (*i*) incandescent lamp
   (*ii*) electric iron
   (*iii*) lightning rod (*iv*) electric motor
4. The relative permittivity of air is
   (*i*) 0 (*ii*) 1
   (*iii*) 8·854 × 10^{-12} (*iv*) none of the above
5. The relative permittivity of a material is 10. Its absolute permittivity will be
   (*i*) 8·854 × 10^{-11} F/m
   (*ii*) 9 × 10^8 F/m
   (*iii*) 5 × 10^{-5} F/m (*iv*) 9 × 10^5 F/m
6. Another name for relative permittivity is
   (*i*) dielectric constant
   (*ii*) dielectric strength
   (*iii*) potential gradient
   (*iv*) none of the above
7. The relative permittivity of most materials lies between
   (*i*) 20 and 100 (*ii*) 10 and 20
   (*iii*) 100 and 200 (*iv*) 1 and 10
8. When the relative permittivity of the medium is increased, the force between two charges placed at a given distance apart
   (*i*) increases (*ii*) decreases
   (*iii*) remains the same
   (*iv*) none of the above
9. Two charges are placed at a distance apart. If a glass slab is placed between them, the force between the charges will
   (*i*) be zero (*ii*) increase
   (*iii*) decrease (*iv*) remain the same

10. There are two charges of $+1\ \mu C$ and $+5\ \mu C$. The ratio of the forces acting on them will be

(*i*) 1 : 1 (*ii*) 1 : 5
(*iii*) 5 : 1 (*iv*) 1 : 25

11. A soap bubble is given a negative charge. Its radius

(*i*) decreases (*ii*) increases
(*iii*) remains unchanged
(*iv*) information is incomplete to say anything

12. The ratio of force between two small spheres with constant charge in air and in a medium of relative permittivity K is

(*i*) $K^2 : 1$ (*ii*) $1 : K$
(*iii*) $1 : K^2$ (*iv*) $K : 1$

13. An electric field can deflect

(*i*) x-rays (*ii*) neutrons
(*iii*) α-particles (*iv*) γ-rays

14. Electric lines of force enter or leave a charged surface at an angle

(*i*) of 90º (*ii*) of 30º
(*iii*) of 60º
(*iv*) depending upon surface conditions

15. The relation between absolute permittivity of vacuum (ε_0), absolute permeability of vacuum (μ_0) and velocity of light (c) in vacuum is

(*i*) $\mu_0\varepsilon_0 = c^2$ (*ii*) $\mu_0/\varepsilon_0 = c$
(*iii*) $\varepsilon_0/\mu_0 = c$ (*iv*) $\dfrac{1}{\mu_0\varepsilon_0} = c^2$

16. As one penetrates a uniformly charged sphere, the electric field strength E

(*i*) increases (*ii*) decreases
(*iii*) is zero at all points
(*iv*) remains the same as at the surface

17. If the relative permittivity of the medium increases, the electric intensity at a point due to a given charge

(*i*) decreases (*ii*) increases
(*iii*) remains the same
(*iv*) none of the above

18. Electric lines of force about a negative point charge are

(*i*) circular, anticlockwise
(*ii*) circular, clockwise
(*iii*) radial, inward (*iv*) radial, outward

19. A hollow sphere of charge does not produce an electric field at any

(*i*) outer point (*ii*) interior point
(*iii*) beyond 2 m (*iv*) beyond 10 m

20. Two charged spheres of radii 10 cm and 15 cm are connected by a thin wire. No current will flow if they have

(*i*) the same charge (*ii*) the same energy
(*iii*) the same field on their surface
(*iv*) the same potential

21. Electric potential is a

(*i*) scalar quantity (*ii*) vector quantity
(*iii*) dimensionless
(*iv*) nothing can be said

22. A charge Q_1 exerts some force on a second charge Q_2. A third charge Q_3 is brought near. The force of Q_1 exerted on Q_2

(*i*) decreases (*ii*) increases
(*iii*) remains unchanged
(*iv*) increases if Q_3 is of the same sign as Q_1 and decreases if Q_3 is of opposite sign

23. The potential at a point due to a charge is 9 V. If the distance is increased three times, the potential at that point will be

(*i*) 27 V (*ii*) 3 V
(*iii*) 12 V (*iv*) 18 V

24. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is

(*i*) 10 V (*ii*) 0 V
(*iii*) same as at point 5 cm away from the surface
(*iv*) same as at point 25 cm away from the surface

25. If a unit charge is taken from one point to another over an equipotential surface, then,

(*i*) work is done on the charge
(*ii*) work is done by the charge
(*iii*) work on the charge is constant
(*iv*) no work is done

Answers

1. (*iv*)	**2.** (*ii*)	**3.** (*iii*)	**4.** (*ii*)	**5.** (*i*)
6. (*i*)	**7.** (*iv*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*i*)
11. (*ii*)	**12.** (*iv*)	**13.** (*iii*)	**14.** (*i*)	**15.** (*iv*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*ii*)	**20.** (*iv*)
21. (*i*)	**22.** (*iii*)	**23.** (*ii*)	**24.** (*i*)	**25.** (*iv*)