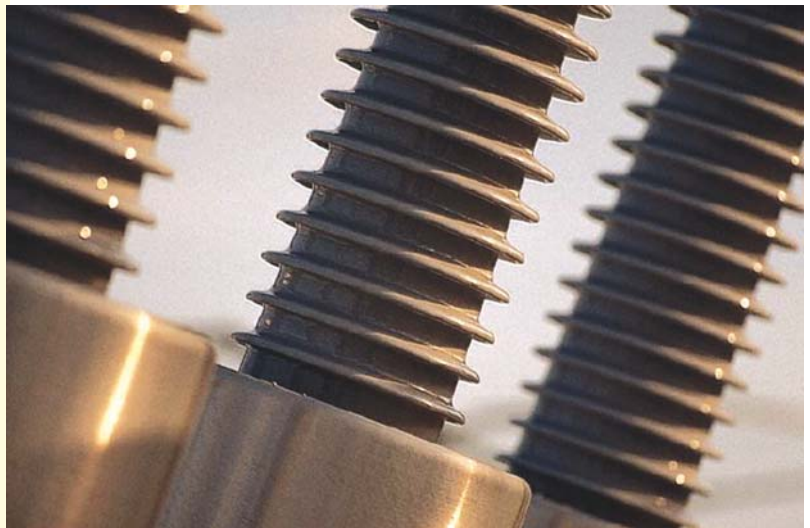


# CHAPTER 8

## Learning Objectives

- Magnetic Hysteresis
- Area of Hysteresis Loop
- Properties and Application of Ferromagnetic Materials
- Permanent Magnet Materials
- Steinmetz Hysteresis Law
- Energy Stored in Magnetic Field
- Rate of Change of Stored Energy
- Energy Stored per Unit Volume
- Lifting Power of Magnet
- Rise of Current in Inductive Circuit
- Decay of Current in Inductive Circuit
- Details of Transient Current Rise in  $R$ - $L$  Circuit
- Details of Transient Current Decay in  $R$ - $L$  Circuit
- Automobile Ignition System

# MAGNETIC HYSTERESIS



Magnetic hysteresis is one of the important considerations in choosing and designing the cores of transformers and other electric machines

### 8.1. Magnetic Hysteresis

It may be defined as the lagging of magnetisation or induction flux density ( $B$ ) behind the magnetising force ( $H$ ). Alternatively, it may be defined as that quality of a magnetic substance, due to which energy is dissipated in it, on the reversal of its magnetism.

Let us take an unmagnetised bar of iron  $AB$  and magnetise it by placing it within the field of a solenoid (Fig. 8.1). The field  $H (= NI/l)$  produced by the solenoid is called the magnetising force. The value of  $H$  can be increased or decreased by increasing or decreasing current through the coil. Let  $H$  be increased in steps from zero up to a certain maximum value and the corresponding values of flux density ( $B$ ) be noted. If we plot the relation between  $H$  and  $B$ , a curve like  $OA$ , as shown in Fig. 8.2, is obtained. The material becomes magnetically saturated for  $H = OM$  and has at that time a maximum flux density of  $B_{max}$  established through it.

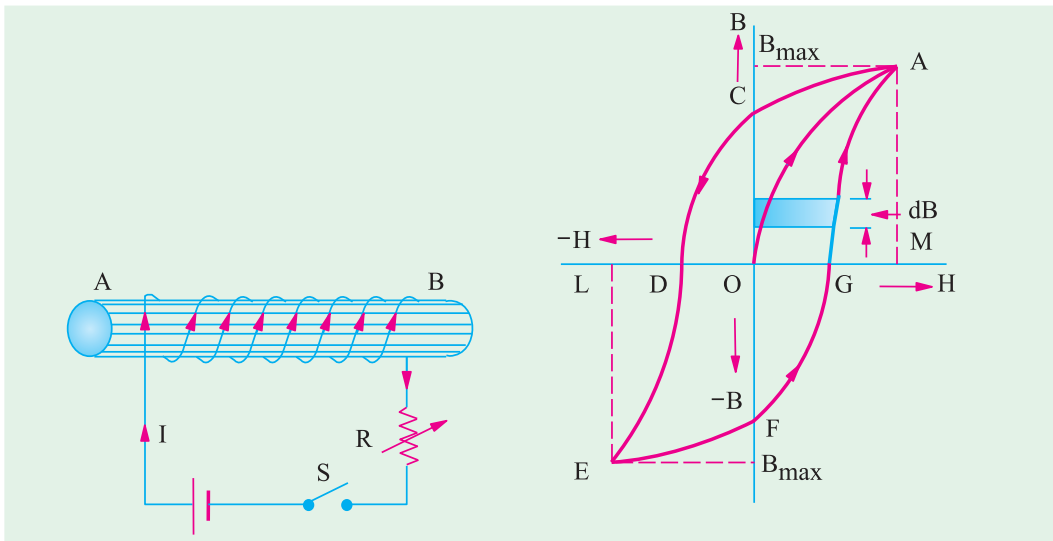


Fig. 8.1

Fig. 8.2

If  $H$  is now decreased gradually (by decreasing solenoid current), flux density  $B$  will not decrease along  $AO$ , as might be expected, but will decrease less rapidly along  $AC$ . When  $H$  is zero,  $B$  is not but has a definite value  $B_r = OC$ . It means that on removing the magnetising force  $H$ , the iron bar is not completely demagnetised. This value of  $B (= OC)$  measures the **retentivity or remanence** of the material and is called the remanent or residual flux density  $B_r$ .

To demagnetise the iron bar, we have to apply the magnetising force in the reverse direction. When  $H$  is reversed (by reversing current through the solenoid), then  $B$  is reduced to zero at point  $D$  where  $H = OD$ . This value of  $H$  required to wipe off residual magnetism is known as **coercive force** ( $H_c$ ) and is a measure of the **coercivity** of the material *i.e.* its 'tenacity' with which it holds on to its magnetism.

If, after the magnetisation has been reduced to zero, value of  $H$  is further increased in the 'negative' *i.e.* reversed direction, the iron bar again reaches a state of magnetic saturation, represented by point  $L$ . By taking  $H$  back from its value corresponding to negative saturation, ( $= OL$ ) to its value for positive saturation ( $= OM$ ), a similar curve  $EFGA$  is obtained. If we again start from  $G$ , the same curve  $GACDEFG$  is obtained once again.\*

\* In fact, when  $H$  is varied a number of times between fixed positive and negative maxima, the size of the loop becomes smaller and smaller till the material is cyclically magnetised. A material is said to be cyclically magnetised when for each increasing (or decreasing) value of  $H$ ,  $B$  has the same value in successive cycles.

It is seen that  $B$  always lag behind  $H$ . The two never attain zero value simultaneously. This lagging of  $B$  behind  $H$  is given the name 'hysteresis' which literally means '*to lag behind*'. The closed loop  $ACDEFGA$  which is obtained when iron bar is taken through one complete cycle of magnetisation is known as 'hypothesis loop'.

By one cycle of magnetisation of a magnetic material is meant its being carried through one reversal of magnetisation, as shown in Fig. 8.3.

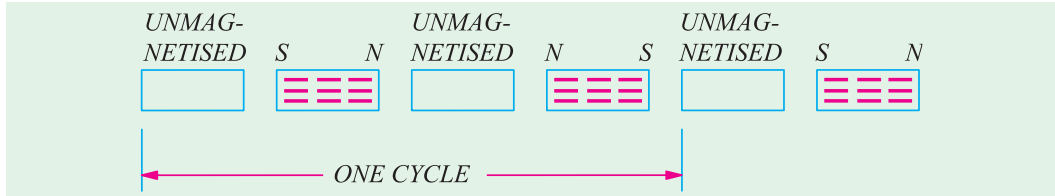


Fig. 8.3

## 8.2. Area of Hysteresis Loop

Just as the area of an indicator diagram measures the energy made available in a machine, when taken through one cycle of operation, so also the area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

According to **Weber's Molecular Theory** of magnetism, when a magnetic material is magnetised, its molecules are forced along a straight line. So, energy is spent in this process. Now, if iron has no retentivity, then energy spent in straightening the molecules could be recovered by reducing  $H$  to zero in the same way as the energy stored up in a spring can be recovered by allowing the spring to release its energy by driving some kind of load. Hence, in the case of magnetisation of a material of **high retentivity**, all the energy put into it originally for straightening the molecules is not recovered when  $H$  is reduced to zero. We will now proceed to find this loss of energy per cycle of magnetisation.

Let  $l$  = mean length of the iron bar ;  $A$  = its area of cross-section;  $N$  = No. of turns of wire of the solenoid.

If  $B$  is the flux density at any instant, then  $\Phi = BA$ .

When current through the solenoid changes, then flux also changes and so produces an induced e.m.f. whose value is

$$e = N \frac{d\Phi}{dt} \text{ volt} = N \frac{d}{dt} (BA) = NA \frac{dB}{dt} \text{ volt} \quad (\text{neglecting } -\text{ve sign})$$

$$\text{Now} \quad H = \frac{NI}{l} \quad \text{or} \quad I = \frac{Hl}{N}$$

The power or rate of expenditure of energy in maintaining the current ' $I$ ' against induced e.m.f. ' $e$ ' is

$$= e I \text{ watt} = \frac{Hl}{N} \times NA \frac{dB}{dt} = AlH \frac{dB}{dt} \text{ watt}$$

$$\text{Energy spent in time } 'dt' = AlH \frac{dB}{dt} \times dt = Al.H.dB \text{ joule}$$

$$\text{Total net work done for one cycle of magnetisation is } W = Al \oint H dB \text{ joule}$$

where  $\oint$  stands for integration over the whole cycle. Now, ' $H dB$ ' represents the shaded area in Fig. 8.2. Hence,  $\oint H dB$  = area of the loop i.e. the area between the  $B/H$  curve and the  $B$ -axis

$$\therefore \text{work done/cycle} = A_l \times (\text{area of the loop}) \text{ joule. Now } Al = \text{volume of the material}$$

$$\therefore \text{net work done/cycle/m}^3 = (\text{loop area}) \text{ joule, or } W_h = (\text{Area of } B/H \text{ loop}) \text{ joule m}^3/\text{cycle}$$

### Precaution

**Scale of  $B$  and  $H$  should be taken into consideration while calculating the actual loop area.**

For example, if the scales are,  $1 \text{ cm} = x \text{ AT/m}$  –for  $H$  and  $1 \text{ cm} = y \text{ Wb/m}^2$  –for  $B$   
then  $W_h = xy (\text{area of } B/H \text{ loop}) \text{ joule/m}^3/\text{cycle}$

In the above expression, loop area has to be in  $\text{cm}^2$ .

As seen from above, hysteresis loop measures the energy dissipated due to hysteresis which appears in the form of heat and so raises the temperature of that portion of the magnetic circuit which is subjected to magnetic reversal. The shape of the hysteresis loop depends on the nature of the magnetic material (Fig. 8.4).

Loop 1 is for hard steel. Due to its high retentivity and collectivity, it is well suited for making permanent magnets. But due to large hysteresis loss (as shown by large loop area) it is not suitable for rapid reversals of magnetisation. Certain alloys of aluminium, nickel and steel called Alnico alloys have been found extremely suitable for making permanent magnets.

Loop 2 is for wrought iron and cast steel. It shows that these materials have high permeability and fairly good coercivity, hence making them suitable for cores of electromagnets.

Loop 3 is for alloyed sheet steel and it shows high permeability and low hysteresis loss. Hence, such materials are most suited for making armature and transformer cores which are subjected to rapid reversals of magnetisation.

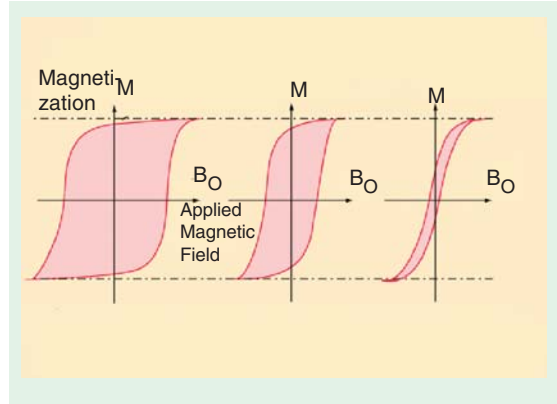
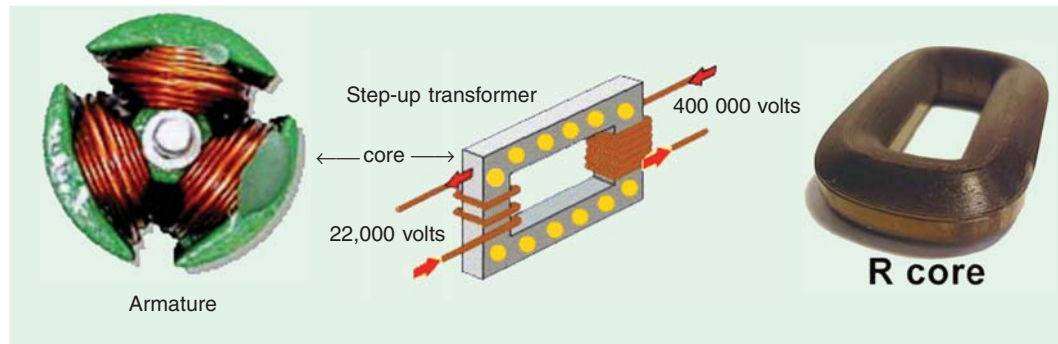


Fig. 8.4

### 8.3. Properties and Applications of Ferromagnetic Materials

Ferromagnetic materials having low retentivities are widely used in power and communication apparatus. Since silicon iron has high permeability and saturation flux density, it is extensively used



in the magnetic circuits of electrical machines and heavy current apparatus where a high flux density is desirable in order to limit the cross-sectional area and, therefore, the weight and cost. Thin silicon-iron laminations (clamped together but insulated from each other by varnish, paper or their own surface scale) are used in the construction of transformer and armature cores where it is essential to minimize hysteresis and eddy-current losses.

In field systems (where flux remains constant), a little residual magnetism is desirable. For such systems, high permeability and high saturation flux density are the only important requirements which are adequately met by fabricated rolled steel or cast or forged steel.

Frequencies used in line communication extend up to 10 MHz whereas those used in radio vary from about 100 kHz to 10 GHz. Hence, such material which have high permeability and low losses are very desirable. For these applications, nickel-iron alloys containing up to 80 per cent of nickel and a small percentage of molybdenum or copper, cold rolled and annealed are very suitable.

### 8.4. Permanent Magnet Materials

Permanent magnets find wide application in electrical measuring instruments, magnetos, mag-

netic chucks and moving-coil loudspeakers etc. In permanent magnets, high retentivity as well as high coercivity are most desirable in order to resist demagnetisation. In fact, the product  $B_r H_c$  is the best criterion for the merit of a permanent magnet. The material commonly used for such purposes are carbon-free iron-nickel-aluminium copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses  $B_r H_c$  value of about  $40,000 \text{ J/m}^3$  as compared with  $2,500 \text{ J/m}^3$  for chromium-steel.

**Example 8.1.** The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of  $1.3 \text{ Wb/m}^2$  has an area of  $93 \text{ cm}^2$ , the scales being  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$  and  $1 \text{ cm} = 50 \text{ AT/m}$ . Calculate the hysteresis loss in watts when  $1500 \text{ cm}^3$  of the same material is subjected to an alternating flux density of  $1.3 \text{ Wb/m}^2$  peak value of a frequency of  $65 \text{ Hz}$ .

(Electromechanics, Allahabad Univ, 1992)

**Solution.**

$$\begin{aligned} \text{Loss} &= xy \text{ (area of } B/H \text{ loop) } \text{J/m}^3/\text{cycle} \\ &= 0.1 \times 50 \times 93 = 465 \text{ J/m}^3/\text{cycle} \\ \text{Volume} &= 1500 \text{ cm}^3 = 15 \times 10^{-4} \text{ m}^3; \text{ No. of reversals/second} = 65 \\ \therefore W_h &= 465 \times 15 \times 10^{-4} \times 65 \text{ J/s} = \mathbf{45.3 \text{ W}} \end{aligned}$$

**Note.** The given value of  $B_{\max} = 1.3 \text{ Wb/m}^2$  is not required for solution.

**Example 8.2.** Calculate the hourly loss of energy in kWh in a specimen of iron, the hysteresis loop of which is equivalent in area to  $250 \text{ J/m}^3$ . Frequency  $50 \text{ Hz}$ ; specific gravity of iron  $7.5$ ; weight of specimen  $10 \text{ kg}$ .

(Electrical Engg. Materials, Nagpur Univ. 1991)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= 250 \text{ J/m}^3/\text{cycle}, \text{ Mass of iron} = 10 \text{ kg} \\ \text{Volume of iron specimen} &= 10/7.5 \times 10^3 \text{ m}^3 = 10^2/7.5 \text{ m}^3 \\ \text{No. of cycles of reversals/hr} &= 60 \times 50 = 3000 \\ \therefore \text{loss/hour} &= 250 \times (10^2/7.5) \times 3000 = 1000 \text{ J} = 1000/36 \times 10^5 = \mathbf{27.8 \times 10^5 \text{ kWh}} \end{aligned}$$

**Example 8.3.** The hysteresis loop for a certain magnetic material is drawn to the following scales:  $1 \text{ cm} = 200 \text{ AT/m}$  and  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$ . The area of the loop is  $48 \text{ cm}^2$ . Assuming the density of the material to be  $7.8 \times 10^3 \text{ kg/m}^3$ , calculate the hysteresis loss in watt/kg at  $50 \text{ Hz}$ .

(Elect. Circuits & Fields, Gujarat Univ.)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= xy \text{ (area of } B/H \text{ loop) } \text{J/m}^3/\text{cycle} \\ \text{Now, } 1 \text{ cm} &= 200 \text{ AT/m}; 1 \text{ cm} = 0.1 \text{ Wb/m}^2 \\ \therefore x &= 200, y = 0.1, \text{ area of loop} = 48 \text{ cm}^2 \\ \therefore \text{loss} &= 200 \times 0.1 \times 48 = 960 \text{ J/m}^3/\text{cycles}, \text{ Density} = 7.8 \times 10^3 \text{ kg/m}^3 \\ \text{Volume of 1 kg of material} &= \text{mass/density} = 1/7.8 \times 10^3 \text{ m}^3 \\ \therefore \text{loss} &= 960 \times 1/7.8 \times 10^3 \text{ J/cycle} \text{ No. of reversals/second} = 50 \\ \therefore \text{loss} &= 960 \times 50 \times 10^3/7.8 = 6.15 \text{ J/s or watt} \\ \therefore \text{hysteresis loss} &= \mathbf{6.15 \text{ watt/kg.}} \end{aligned}$$

**Example 8.4.** Determine the hysteresis loss in an iron core weighing  $50 \text{ kg}$  having a density of  $7.8 \times 10^3 \text{ kg/m}^3$  when the area of the hysteresis loop is  $150 \text{ cm}^2$ , frequency is  $50 \text{ Hz}$  and scales on X and Y axes are:  $1 \text{ cm} = 30 \text{ AT/cm}$  and  $1 \text{ cm} = 0.2 \text{ Wb/m}^2$  respectively.

(Elements of Elect. Engg-1, Bangalore Univ.)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= xy \text{ (area of } B/H \text{ loop) } \text{J/m}^3/\text{cycle} \\ 1 \text{ cm} &= 30 \text{ AT/cm} = 3000 \text{ AT/m}; 1 \text{ cm} = 0.2 \text{ Wb/m}^2 \\ x &= 3000, y = 0.2, A = 150 \text{ cm}^2 \\ \therefore \text{loss} &= 3000 \times 0.2 \times 150 = 90,000 \text{ J/m}^3/\text{cycle} \\ \text{Volume of 50 kg of iron} &= m/\rho = 50/7.8 \times 10^3 = 6.4 \times 10^{-3} \text{ m}^3 \\ \therefore \text{loss} &= 90,000 \times 6.4 \times 10^{-3} \times 50 = 28,800 \text{ J/s or watts} = \mathbf{28.8 \text{ kW}} \end{aligned}$$

**Example 8.5.** In a transformer core of volume  $0.16 \text{ m}^3$ , the total iron loss was found to be  $2,170 \text{ W}$  at  $50 \text{ Hz}$ . The hysteresis loop of the core material, taken to the same maximum flux density, had an area of  $9.0 \text{ cm}^2$  when drawn to scales of  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$  and  $1 \text{ cm} = 250 \text{ AT/m}$ . Calculate the total iron loss in the transformer core if it is energised to the same maximum flux density but at a frequency of  $60 \text{ Hz}$ .

**Solution.** $W_h = xy \times (\text{area of hysteresis loop})$  where  $x$  and  $y$  are the scale factors.

$$W_h = 9 \times 0.1 \times 250 = 225 \text{ J/m}^3/\text{cycle}$$

**At 50 Hz**Hysteresis loss =  $225 \times 0.16 \times 50 = 1,800 \text{ W}$  ; Eddy-current loss =  $2,170 - 1800 = 370 \text{ W}$ **At 60 Hz**Hysteresis loss =  $1800 \times 60/50 = 2,160 \text{ W}$  ; Eddy-current loss =  $370 \times (60/50)^2 = 533 \text{ W}$ Total iron loss =  $2,160 + 533 = \mathbf{2,693 \text{ W}}$ **Tutorial Problems No. 8.1**

1. The area of a hysteresis loop of a material is  $30 \text{ cm}^2$ . The scales of the co-ordinates are :  $1 \text{ cm} = 0.4 \text{ Wb/m}^2$  and  $1 \text{ cm} = 400 \text{ AT/m}$ . Determine the hysteresis power loss if  $1.2 \times 10^{-3} \text{ m}^3$  of the material is subjected to alternating flux density at 50 Hz. **[288 W] (Elect. Engg., Aligarh Univ.)**
2. Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of the hysteresis loop represents  $240 \text{ joules/m}^3$  and the density of iron is  $7800 \text{ kg/m}^3$ . **[138,240] (Principles of Elect. Engg. I, Jadavpur Univ.)**
3. The hysteresis loop of a specimen weighing 12 kg is equivalent to  $300 \text{ joules/m}^3$ . Find the loss of energy per hour at 50 Hz. Density of iron is  $7500 \text{ kg/m}^3$ . **[86,400] (Electrotechnics – I, Gawahati Univ.)**
4. The area of the hysteresis loop for a steel specimen is  $3.84 \text{ cm}^2$ . If the ordinates are to the scales :  $1 \text{ cm} = 400 \text{ AT/m}$  and  $1 \text{ cm} = 0.5 \text{ Wb/m}^2$ , determine the power loss due to hysteresis in  $1,200 \text{ cm}^3$  of the steel if it is magnetised from a supply having a frequency of 50 Hz. **[46.08 W]**
5. The armature of a 4-pole d.c. motor has a volume of  $0.012 \text{ m}^3$ . In a test on the steel iron used in the armature carried out to the same value of maximum flux density as exists in the armature, the area of the hysteresis loop obtained represented a loss of  $200 \text{ J/m}^3$ . Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m. **[72 W]**
6. In a magnetisation test on a sample of iron, the following values were obtained.

$H \text{ (AT/m)}$	1,900	2,000	3,000	4,000	4,500	3,000	1,000	0	-1,000	-1,900
$B \text{ (Wb/m}^2\text{)}$	0	0.2	0.58	0.7	0.73	0.72	0.63	0.54	0.38	0

Draw the hysteresis loop and find the loss in watts if the volume of iron is  $0.1 \text{ m}^3$  and frequency is 50 Hz. **[22 kW]**

**8.5. Steinmetz Hysteresis Law**

It was experimentally found by Steinmetz that hysteresis loss per  $\text{m}^3$  per cycle of magnetisation of a magnetic material depends on (i) the maximum flux density established in it i.e.  $B_{\max}$  and (ii) the magnetic quality of the material.

$$\therefore \text{Hysteresis loss } W_h \propto B_{\max}^{1.6} \text{ joule/m}^3/\text{cycle} = \eta B_{\max}^{1.6} \text{ joule/m}^3 \text{ cycle}$$

where  $\eta$  is a constant depending on the nature of the magnetic material and is known as **Steinmetz hysteresis coefficient**. The index 1.6 is empirical and holds good if the value of  $B_{\max}$  lies between  $0.1$  and  $1.2 \text{ Wb/m}^2$ . If  $B_{\max}$  is either lesser than  $0.1 \text{ Wb/m}^2$  or greater than  $1.2 \text{ Wb/m}^2$ , the index is greater than 1.6.

$$\therefore W_h = \eta B_{\max}^{1.6} fV \text{ J/s or watt}$$

where  $f$  is frequency of reversals of magnetisation and  $V$  is the volume of the magnetic material.

The armatures of electric motors and generators and transformer cores etc. which are subjected to rapid reversals of magnetisation should, obviously, be made of substances having low hysteresis coefficient in order to reduce the hysteresis loss.

**Example 8.6.** A cylinder of iron of volume  $8 \times 10^{-3} \text{ m}^3$  revolves for 20 minutes at a speed of 3,000 r.p.m in a two-pole field of flux density  $0.8 \text{ Wb/m}^2$ . If the hysteresis coefficient of iron is  $753.6 \text{ joule/m}^3$ , specific heat of iron is  $0.11$ , the loss due to eddy current is equal to that due to hysteresis and 25% of the heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as  $7.8 \times 10^3 \text{ kg/m}^3$ . **(Elect. Engineering-I, Osmania Univ.)**



**Solution.** An armature revolving in a multipolar field undergoes one magnetic reversal after passing under a pair of poles. In other words, number of magnetic reversals in the same as the number of pair of poles. If  $P$  is the number of poles, the magnetic reversals in one revolution are  $P/2$ . If speed of armature rotation is  $N$  r.p.m., then number of revolutions/second =  $N/60$ .

$$\begin{aligned}\text{No. of reversals/second} &= \text{reversals in one revolutions} \times \text{No. of revolutions/second} \\ &= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ reversals/second}\end{aligned}$$

Here  $N = 3,000$  r.p.m ;  $P = 2 \therefore f = \frac{3,000 \times 2}{120} = 50$  reversals/second

According to Steinmetz's hysteresis law,  $W_h = \eta B_{\max}^{1.6} f V$  watt

Note that  $f$  here stands for magnetic reversals/second and not for mechanical frequency armature rotation.

$$W_h = 753.6 \times (0.8)^{1.6} \times 50 \times 8 \times 10^{-3} = 211 \text{ J/s}$$

$$\text{Loss in 20 minutes} = 211 \times 1,200 = 253.2 \times 10^3 \text{ J}$$

$$\text{Eddy current loss} = 253.2 \times 10^3 \text{ J}; \text{ Total loss} = 506.4 \times 10^3 \text{ J}$$

$$\text{Heat produced} = 506.4 \times 10^3 / 4200 = 120.57 \text{ kcal}; \text{ Heat utilized} = 120.57 \times 0.75 = 90.43 \text{ kcal}$$

$$\text{Heat absorbed by iron} = (8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 t \text{ kcal}$$

$$\therefore (8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 \times t = 90.43 \therefore t = 13.17^\circ\text{C}$$

**Example 8.7.** The area of the hysteresis loop obtained with a certain specimen of iron was  $9.3 \text{ cm}^2$ . The coordinates were such that  $1 \text{ cm} = 1,000 \text{ AT/m}$  and  $1 \text{ cm} = 0.2 \text{ Wb/m}^2$ . Calculate (a) the hysteresis loss per  $\text{m}^3$  per cycle and (b) the hysteresis loss per  $\text{m}^3$  at a frequency of 50 Hz if the maximum flux density were  $1.5 \text{ Wb/m}^2$  (c) calculate the hysteresis loss per  $\text{m}^3$  for a maximum flux density of  $1.2 \text{ Wb/m}^2$  and a frequency of 30 Hz, assuming the loss to be proportional to  $B_{\max}^{1.6}$ .

(Elect. Technology, Allahabad Univ. 1991)

**Solution.** (a)  $W_h = xy \times (\text{area of } B/H \text{ loop})$

$$= 1,000 \times 0.2 \times 9.3 = 1860 \text{ J/m}^2/\text{cycle}$$

(b)  $W_h = 1,860 \times 50 \text{ J/s/m}^3 = 93,000 \text{ W/m}^3$

(c)  $W_h = B_{\max}^{1.8} f V W$  For a given specimen,  $W_h \propto B_{\max}^{1.8} f$

In (b) above,  $93,000 \propto 1.5^{1.8} \times 50$  and  $W_h \propto 1.2^{1.8} \times 30$

$$\therefore \frac{W_h}{93,000} = \left( \frac{1.2}{1.5} \right)^{1.8} \times \frac{30}{50}; W_h = 93,000 \times 0.669 \times 0.6 = 37.360$$

**Example 8.8.** Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak density reached is  $1.3 \text{ Wb/m}^2$  and the frequency is 25 Hz. Assume Steinmetz coefficient as 628  $\text{J/m}^3$  and density of iron as  $7.8 \times 10^3 \text{ kg/m}^3$ .

What will be the area of  $B/H$  curve of this specimen if  $1 \text{ cm} = 12.4 \text{ AT/m}$  and  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$ .

(Elect. Engg. ; Madras Univ.)

**Solution.**  $W_h = \eta B_{\max}^{1.6} f V$  watt ; volume  $V = \frac{50}{7.8 \times 10^3} = 6.41 \times 10^{-3} \text{ m}^3$

$$\therefore W_h = 628 \times 1.3^{1.6} \times 25 \times 6.41 \times 10^{-3} = 152 \text{ J/s}$$

$$\text{Loss in one hour} = 152 \times 3,600 = 551,300 \text{ J}$$

As per Steinmetz law, hysteresis loss =  $\eta B_{\max}^{1.6} \text{ J/m}^3/\text{cycle}$

Also, hysteresis loss =  $xy$  (area of  $B/H$  loop)

Equating the two, we get

$$628 \times 1.3^{1.6} = 12.5 \times 0.1 \times \text{loop area}$$

$$\therefore \text{loop area} = 628 \times 1.3^{1.6} / 1.25 = 764.3 \text{ cm}^2$$

## Tutorial Problems No. 8.2

1. In a certain transformer, the hysteresis loss is 300 W when the maximum flux density is  $0.9 \text{ Wb/m}^2$  and the frequency 50 Hz. What would be the hysteresis loss if the maximum flux density were increased to  $1.1 \text{ Wb/m}^2$  and the frequency reduced to 40 Hz. Assume the hysteresis loss over this range to be proportional to  $B_{\max}^{1.7}$  [337 W]
2. In a transformer, the hysteresis loss is 160 W when the value of  $B_{\max} = 1.1 \text{ Wb/m}^2$  and when supply frequency is 60 Hz. What would be the loss when the value of  $B_{\max}$  is reduced to  $0.9 \text{ Wb/m}^2$  and the supply frequency is reduced to 50 Hz. [97 W] (Elect. Engg. II, Bangalore Univ.)

## 8.6. Energy Stored in a Magnetic Field

For establishing a magnetic field, energy must be spent, though no energy is required to *maintain* it. Take the example of the exciting coils of an electromagnet. The energy supplied to it is spent in two ways (i) part of it goes to meet  $I^2R$  loss and is lost once for all (ii) part of it goes to create flux and is stored in the magnetic field as potential energy and is similar to the potential energy of a raised weight. When a weight  $W$  is raised through a height of  $h$ , the potential energy stored in it is  $W_h$ . Work is done in *raising* this weight but once raised to a certain height, no further expenditure of energy is required to *maintain* it at that position. This mechanical potential energy can be recovered, so can be the electrical energy stored in the magnetic field.

When current through an inductive coil is gradually changed from zero to maximum value  $I$ , then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this stored energy may be found in the following two ways :

(i) **First Method.** Let, at any instant,

$i$  = instantaneous value of current ;  $e$  = induced e.m.f. at that instant  $= L \cdot di/dt$

Then, work done in time  $dt$  in overcoming this opposition is

$$dW = e i dt = L \cdot \frac{di}{dt} \times i \times dt = Li di$$

Total work done in establishing the maximum steady current of  $I$  is

$$\int_0^W dW = \int_0^I Li di = LI^2 \text{ or } W = \frac{1}{2} LI^2$$

This work is stored as the energy of the magnetic field  $\therefore E = \frac{1}{2} LI^2$  joules

(ii) **Second Method**

If current grows uniformly from zero value to its maximum steady value  $I$ , then average current is  $I/2$ . If  $L$  is the inductance of the circuit, then self-induced e.m.f. is  $e = LI/t$  where 't' is the time for the current change from zero to  $I$ .

$\therefore$  Average power absorbed = induced e.m.f.  $\times$  average current

$$= L \frac{1}{t} \times \frac{1}{2} I = \frac{1}{2} \frac{LI^2}{t}$$

$$\text{Total energy absorbed} = \text{power} \times \text{time} = \frac{1}{2} \frac{LI^2}{t} \times t = \frac{1}{2} LI^2$$

$$\therefore \text{energy stored } E = \frac{1}{2} LI^2 \text{ joule}$$

It may be noted that in the case of series-aiding coils, energy stored is

$$E = \frac{1}{2} (L_1 + L_2 + 2M) I^2 = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 + M I^2$$

Similarly, for series-opposing coils,  $E = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 - M I^2$



**Example 8.9.** Reluctance of a magnetic circuit is known to be  $10^5$  AT/Wb and excitation coil has 200 turns. Current in the coil is changing uniformly at 200 A/s. Calculate (a) inductance of the coil (b) voltage induced across the coil and (c) energy stored in the coil when instantaneous current at  $t = 1$  second is 1 A. Neglect resistance of the coil. (Elect. Technology, Univ. of Indore, 1987)

**Solution.** (a)  $L = N^2/S = 200^2/10^5 = 0.4 \text{ H}$   
 (b)  $e_L = L \, dI/dt = 0.4 \times 200 = 80 \text{ V}$   
 (c)  $E = \frac{1}{2} L I^2 = 0.5 \times 0.4 \times 1^2 = 0.2 \text{ J}$

**Example 8.10.** An iron ring of 20 cm mean diameter having a cross-section of  $100 \text{ cm}^2$  is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of  $1 \text{ Wb/m}^2$  if the relative permeability of iron is 1000. What is the value of energy stored ?

(Elect. Engg-I, Nagpur Univ. 1992)

**Solution.**  $B = \mu_0 \mu_r NI/l \text{ Wb/m}^2$   
 $\therefore 1 = 4\pi \times 10^{-7} \times 1000 \times 400 I/0.2\pi \text{ or } I = 1.25 \text{ A}$   
 Now,  $L = \mu_0 \mu_r AN^2/l = 4\pi \times 10^{-7} \times 10^3 \times (100 \times 10^{-4}) \times (400)^2/0.2\pi = 32 \text{ H}$   
 $E = \frac{1}{2} LI^2 = \frac{1}{2} \times 32 \times 1.25^2 = 2.5 \text{ J}$

## 8.7. Rate of Change of Stored Energy

As seen from Art. 8.6,  $E = \frac{1}{2} L I^2$ . The rate of change of energy can be found by differentiating the above equation

$$\frac{dE}{dt} = \frac{1}{2} \left[ L \cdot 2I \cdot \frac{dI}{dt} + I^2 \frac{dL}{dt} \right] = LI \cdot \frac{dI}{dt} + \frac{1}{2} I^2 \frac{dL}{dt}$$

**Example 8.11.** A relay (Fig. 8.5) has a coil of 1000 turns and an air-gap of area  $10 \text{ cm}^2$  and length 1.0 mm. Calculate the rate of change of stored energy in the air-gap of the relay when

- (i) armature is stationary at 1.0 mm from the core and current is 10 mA but is increasing at the rate of 25 A/s.
- (ii) current is constant at 20 mA but inductance is changing at the rate of 100 H/s.

**Solution.**  $L = \frac{\mu_0 N^2 A}{l_g}$   
 $= \frac{4\pi \times 10^{-7} \times (10^3)^2 \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 1.26 \text{ H}$

(i) Here,  $dI/dt = 25 \text{ A/s}$ ,  $dL/dt = 0$  because armature is stationary.

$$\therefore \frac{dE}{dt} = L I \frac{dI}{dt} = 1.26 \times 10 \times 10^{-3} \times 25 = 0.315 \text{ W}$$

(ii) Here,  $dL/dt = 100 \text{ H/s}$ ;  $dI/dt = 0$  because current is constant.

$$\therefore \frac{dE}{dt} = \frac{1}{2} I^2 \frac{dL}{dt} = \frac{1}{2} (20 \times 10^{-3})^2 \times 100 = 0.02 \text{ W}$$

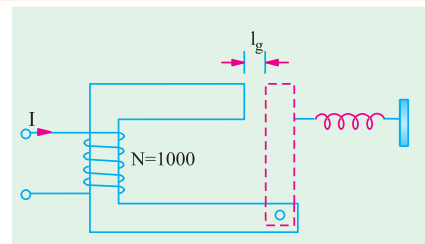


Fig. 8.5

## 8.8. Energy Stored Per Unit Volume

It has already been shown that the energy stored in a magnetic field of length  $l$  metre and of cross-section  $A \text{ m}^2$  is  $E = \frac{1}{2} L I^2$  joule or  $E = \frac{1}{2} \times \frac{\mu_0 \mu_r AN^2}{l} \cdot I^2$  joule

$$\text{Now } H = \frac{NI}{l} \therefore E = \left( \frac{NI}{l} \right)^2 \times \frac{1}{2} \mu_0 \mu_r Al = \frac{1}{2} \mu_0 \mu_r H^2 \times Al \text{ joule}$$

Now,  $Al$  = volume of the magnetic field in  $\text{m}^3$

$$\therefore \text{energy stored/m}^3 = \frac{1}{2} \mu_0 \mu_r H^2 = \frac{1}{2} BH \text{ joule} \quad (\because \mu_0 \mu_r H = B)$$

$$= \frac{B^2}{2 \mu_0 \mu_r} \text{ joule} \quad \text{—in a medium}$$

$$\text{or} \quad = \frac{B^2}{2 \mu_0} \text{ joule} \quad \text{—in air}$$

### 8.9. Lifting Power of a Magnet

In Fig. 8.6 let,  $P$  = pulling force in newtons between two poles and  $A$  = pole area in  $\text{m}^2$

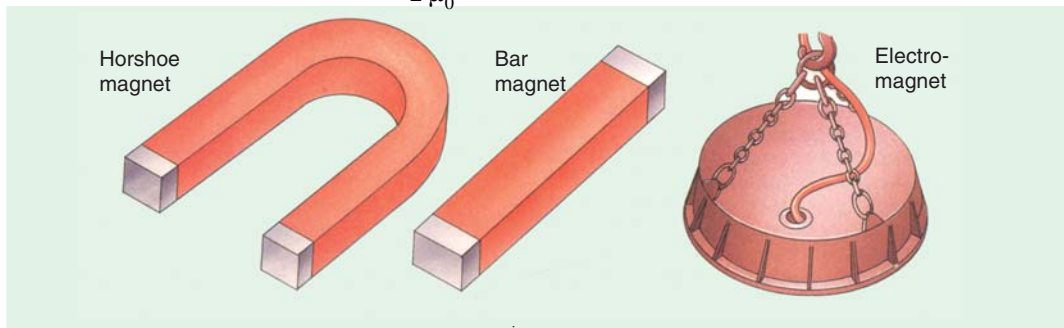
If one of the poles (say, upper one) is pulled apart against this attractive force through a distance of  $dx$  metres, then work done =  $P \times dx$  joule ...(i)

This work goes to provide energy for the additional volume of the magnetic field so created.

Additional volume of the magnetic field created is

$$= A \times dx \text{ m}^3$$

$$\text{Rate of energy requirement is } = \frac{B^2}{2 \mu_0} \text{ joule/m}^3$$



$$\therefore \text{energy required for the new volume} = \frac{B^2}{2 \mu_0} \times A dx \quad \text{...(ii)}$$

Equating (i) and (ii), we get,

$$P \cdot dx = \frac{B^2 \times A \cdot dx}{2 \mu_0}$$

$$\therefore P = \frac{B^2 A}{2 \mu_0} \text{ N} = 4,00,000 \frac{B^2}{A} \text{ N}$$

$$\text{or} \quad P = \frac{B^2 A}{2 \mu_0} \text{ N/m}^2 = 4,00,000 B^2 \text{ N/m}^2$$

$$\text{Also} \quad P = \frac{B^2 A}{9.81 \times 2 \mu_0} = \frac{B^2 A}{19.62 \mu_0} \text{ kg-wt}$$

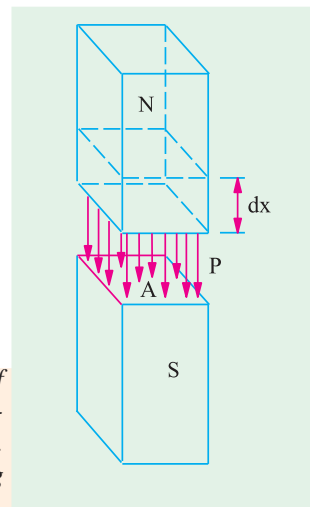


Fig. 8.6

**Example 8.12.** A horse-shoe magnet is formed out of a bar of wrought iron 45.7 cm long, having a cross-section of  $6.45 \text{ cm}^2$ . Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.

(Elect. Engg. A.M.Ae. S.I., June, 1992)

**Solution.** Horse-shoe magnet is shown in Fig. 8.7.

Force of attraction of each pole =  $68/2 = 34 \text{ kg} = 34 \times 9.81 = 333.5 \text{ N}$

$$A = 6.45 \text{ cm}^2 = 6.45 \times 10^{-4} \text{ m}^2$$

$$\text{Since } F = \frac{B^2 A}{2 \mu_0} \text{ N}$$

$$\therefore 333.5 = \frac{B^2 \cdot 6.45 \cdot 10^{-4}}{2 \cdot 4 \cdot 10^{-7}}$$

$$B = \sqrt{1.3} = 1.14 \text{ Wb/m}^2$$

$$\text{and } H = B/\mu_0 \mu_r = 1.14/4\pi \times 10^{-7} \times 700 = 1296 \text{ AT/m}$$

$$\text{Length of the plate} = 45.7 \text{ cm} = 0.457 \text{ m}$$

$$\therefore \text{AT required} = 1296 \times 0.457 = 592.6$$

$$\text{No. of turns} = 500 \times 2 = 1000 \quad \therefore \text{current required} = 592.6/1000 = \mathbf{0.593 \text{ A}}$$

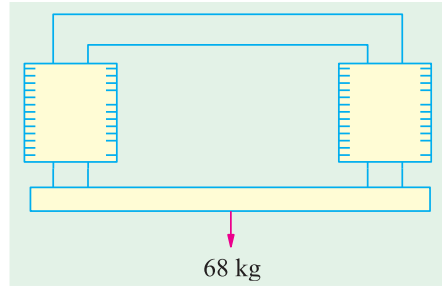


Fig. 8.7

**Example 8.13.** The pole face area of an electromagnet is  $0.5 \text{ m}^2/\text{pole}$ . It has to lift an iron ingot weighing  $1000 \text{ kg}$ . If the pole faces are parallel to the surface of the ingot at a distance of  $1 \text{ millimetre}$ , determine the coil m.m.f. required. Assume permeability of iron to be infinity at the permeability of free space is  $4\pi \times 10^{-7} \text{ H/m}$ . (Elect. Technology, Univ. of Indore)

**Solution.** Since iron has a permeability of infinity, it offers zero reluctance to the magnetic flux.

$$\text{Force at two poles} = 2 \times B^2 A / 2\mu_0 = B^2 A / \mu_0$$

$$\therefore B^2 \times 0.5/4\pi \times 10^{-7} = 1000 \times 9.8, B = 0.157 \text{ Wb/m}^2$$

$$\therefore H = 0.157/4\pi \times 10^{-7} = 125 \times 10^3 \text{ AT/m}, l = 2 \times 1 = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\therefore \text{AT required} = 125 \times 10^3 \times 2 \times 10^{-3} = \mathbf{250}.$$

**Example 8.14.** A soft iron ring having a mean circumference of  $40 \text{ cm}$  and cross-sectional area of  $3 \text{ cm}^2$  has two radial saw cuts made at diametrically opposite points. A brass plate  $0.5 \text{ mm}$  thick is inserted in each gap. The ring is wound with  $800$  turns. Calculate the magnetic leakage and fringing. Assume the following data for soft iron :

$B \text{ (Wb/m}^2\text{)} :$	0.76	1.13	1.31	1.41	1.5
$H \text{ (AT/m)} :$	50	100	150	200	250

(Elect. Engineering-I, Delhi Univ.)

**Solution.** It should be noted that brass is a non-magnetic material.

$$\text{Force at one separation} = B^2 A / 2\mu_0 \text{ newton.}$$

$$\text{Force at both separations} = B^2 A / \mu_0 \text{ newton.}$$

$$\text{Now } F = 12 \text{ kg wt} = 12 \times 9.81 = 117.7 \text{ N}$$

$$\therefore 117.7 = B^2 \times 3 \times 10^{-4} / 4\pi \times 10^{-7}; B = 0.7 \text{ Wb/m}^2$$

If  $B/H$  curve is drawn, it will be found that for  $B = 0.7 \text{ Wb/m}^2$ , value of  $H = 45 \text{ AT/m}$ .

$$\text{Now, length of iron path} = 40 \text{ cm} = 0.4 \text{ m. AT required for iron path} = 45 \times 0.4 = 18$$

$$\text{Value of } H \text{ in the non-magnetic brass plates} = B/\mu_0 = 0.7/4\pi \times 10^{-7} = 557,042 \text{ AT/m}$$

$$\text{Total thickness of brass plates} = 0.5 \times 2 = 1 \text{ mm}$$

$$\text{AT required} = 557,042 \times 1 \times 10^{-3} = 557, \text{ Total AT needed} = 18 + 557 = 575$$

$$\therefore \text{magnetising current required} = 557/800 = \mathbf{0.72 \text{ A}}$$

**Example 8.15.** The arm of a d.c. shunt motor starter is held in the 'ON' position by an electro-magnet having a pole face area of  $4 \text{ cm}^2$  and air gap of  $0.6 \text{ mm}$ . The torque exerted by the spring is  $12 \text{ N-m}$  and effective radius at which the force is exerted is  $15 \text{ cm}$ . What is the minimum number of AT required to keep the arm in the 'ON' position ?

**Solution.** The arm is shown in Fig. 8.8

Let  $F$  be the force in newtons exerted by the two poles of the electromagnet.

$$\text{Torque} = \text{Force} \times \text{radius}$$

$$\therefore 12 = F \times 0.15; \quad F = 80 \text{ N}$$

$$\text{Force per pole} = 80/2 = 40 \text{ N}$$

$$\text{Now } F = \frac{B^2 A}{2\mu_0} \text{ N} \therefore 40 = \frac{B^2 \times 4 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}}$$

$$\therefore B = 0.5 \text{ Wb/m}^2 \therefore H = 0.5/\pi \times 10^{-7} \text{ AT/m}$$

$$\text{Total air-gap} = 2 \times 0.6 \times 10^{-3} = 1.2 \times 10^{-3} \text{ m}$$

$$\therefore \text{AT reqd.} = Hl = \frac{0.5 \times 1.2 \times 10^{-3}}{4\pi \times 10^{-7}} = 477$$

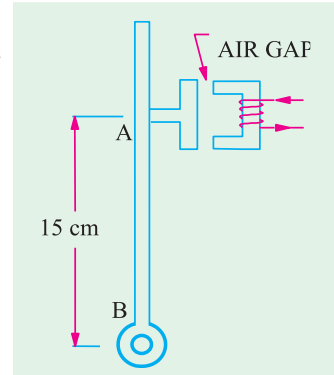


Fig. 8.8

**Example 8.16.** The following particulars are taken from the magnetic circuit of a relay; Mean length of iron circuit = 20 cm; length of air gap = 2 mm, number of turns on core = 8000, current through coil = 50 mA, relative permeability of iron = 500. Neglecting leakage, what is the flux density in the air-gap? If the area of the core is  $0.5 \text{ cm}^2$ , what is the pull exerted on the armature?

**Solution.** Flux  $\Phi = \frac{NI}{\sum l/\mu_0 \mu_r A}$

Now, m.m.f =  $NI = 8000 \times 50 \times 10^{-3} = 400 \text{ AT}$

$$\begin{aligned} \text{Total circuit reluctance} &= \frac{l}{\sum \mu_0 \mu_r A} \text{ AT/Wb or } H^{-1} \\ &= \frac{0.2}{500 \times 4\pi \times 10^{-7} \times 0.5 \times 10^{-4}} + \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.5 \times 10^{-4}} \\ \Phi &= \frac{400}{12 \times 10^{-7} / \phi} \text{ Wb ; Flux density } B = \frac{\Phi}{A} = \frac{400\pi}{12 \times 10^{-7} \times 0.5 \times 10^{-4}} = 0.21 \text{ Wb/m}^2 \end{aligned}$$

$$\text{The pull on the armature} = \frac{B^2 A}{2\mu_0} \text{ N} = \frac{0.21^2 \times 0.5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 0.87 \text{ N}$$

### Tutorial Problems No. 8.2

1. An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1,000 turns and also find the energy stored in it if the current rises from zero to 5 A.  
[0.7 mH; 8.7 mJ] (Elect. Engg. and Electronic Bangalore Univ. 1998)
2. An air-cored solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calculate (i) the self inductance (ii) the energy stored in the magnetic field when a current of 2 A flows in the solenoid.  
[(i) 0.2468 H (ii) 0.4936 J] (F.E. Pune Univ.)
3. Determine the force required to separate two magnetic surfaces with contact area of  $100 \text{ cm}^2$  if the magnetic flux density across the surface is  $0.1 \text{ Wb/m}^2$ . Derive formula used, if any.  
[39.8 N] (Elect. Engg. A.M.Ae.S.I.)
4. In a telephone receiver, the size of each of the two poles is  $1.2 \text{ cm} \times 0.2 \text{ cm}$  and the flux between each pole and the diaphragm is  $3 \times 10^{-6} \text{ Wb}$ ; with what force is the diaphragm attracted to the poles?  
[0.125 N] (Elect. Engg. A.M.Ae.S.I. June 1991)
5. A lifting magnet is required to raise a load of 1,000 kg with a factor of safety of 1.5. If the flux density across the pole faces is  $0.8 \text{ Wb/m}^2$ , calculate the area of each pole.  
[577  $\text{cm}^2$ ]
6. Magnetic material having a surface of  $100 \text{ cm}^2$  are in contact with each other. They are in a magnetic circuit of flux. 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces.  
[3,978 N] (Elect. Engg. Kerala Univ.)

7. A steel ring having a mean diameter of 35 cm and a cross-sectional area of  $2.4 \text{ cm}^2$  is broken by a parallel-sided air-gap of length 1.2 cm. Short pole pieces of negligible reluctance extend the effective cross-sectional area of the air-gap to  $12 \text{ cm}^2$ . Taking the relative permeability of steel as 700 and neglecting leakage, determine (a) the current necessary in 300 turns of wire wound on the ring to produce a flux density in the air-gap of  $0.25 \text{ Wb/m}^2$  (b) the tractive force between the poles.

[ (a) 13.16 A (b) 29.9 N ]

8. A cast iron ring having a mean circumference of 40 cm and a cross-sectional area of  $3 \text{ cm}^2$  has two radial saw-cuts at diametrically opposite points. A brass plate is inserted in each gap (thickness 0.5 mm). If the ring is wound with 800 turns, calculate the magnetising current to exert a total pull of 3 kg between the two halves. Neglect any magnetic leakage and fringing and assume the magnetic data for the cast iron to be :

$B \text{ (Wb/m}^2\text{)} :$	0.2	0.3	0.4	0.5
$H \text{ (AT)} :$	850	1150	1500	2000

[1.04 A]

9. A magnetic circuit in the form of an inverted U has an air-gap between each pole and the armature of 0.05 cm. The cross-section of the magnetic circuit is  $5 \text{ cm}^2$ . Neglecting magnetic leakage and fringing, calculate the necessary exciting ampere-turns in order that the armature may exert a pull of 15 kg. The ampere-turns for the iron portion of the magnetic circuit may be taken as 20 percent of those required for the double air-gap.

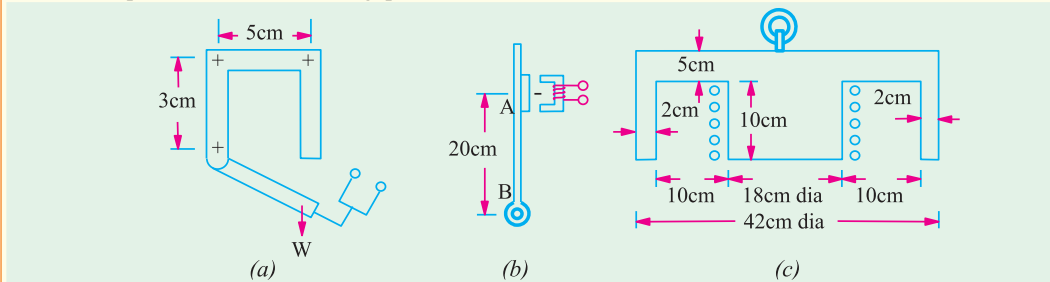


Fig. 8.9

10. In Fig. 8.9 (a) is shown the overload trip for a shunt motor starter. The force required to lift the armature is equivalent to a weight of  $W = 0.8165 \text{ kg}$  positioned as shown. The air-gaps in the magnetic circuit are equivalent to a single gap of 0.5 cm. The cross-sectional area of the circuit is  $1.5 \text{ cm}^2$  throughout and the magnetisation curve is as follows :

$H \text{ (AT/m)} :$	1000	2000	3000	4000
$B \text{ (Wb/m}^2\text{)} :$	0.3	0.5	0.62	0.68

Calculate the number of turns required if the trip is to operate when 80 A passes through the coil.

[21 turns]

11. The armature of a d.c. motor starter is held in the 'ON' position by means of an electromagnet [Fig 8.9. (b)]. A spiral spring exerts a mean counter torque of 8 N-m on the armature in this position after making allowance for the weight of the starter arm. The length between the centre of the armature and the pivot on the starter arms is 20 cm and cross sectional area of each pole face of the electromagnet  $3.5 \text{ cm}^2$ .

Find the minimum number of AT required on the electromagnet to keep the arm in the 'ON' position when the air-gap between the armature and the electromagnet is 0.5 mm. (Neglect the AT needed for the iron of the electromagnet).

[301 AT]

12. A cylindrical lifting magnet of the form shown in Fig. 8.9 (c) has a winding of 200 turns which carries a current of 5 A. Calculate the maximum lifting force which could be exerted by the magnet on a flat iron sheet 5 cm thick. Why would this value not be realized in practice ? The relative permeability of the iron can be taken as 500.

[698 N]

### 8.10. Rise of current in an Inductive Circuit

In Fig. 8.10 is shown a resistance of  $R$  in series with a coil of self-inductance  $L$  henry, the two being put across a battery of  $V$  volt. The  $R$ - $L$  combination becomes connected to battery when switch

$S$  is connected to terminal 'a' and is short-circuited when  $S$  is connected to 'b'. The inductive coil is assumed to be resistanceless, its actual small resistance being included in  $R$ .

When  $S$  is connected to 'a' the  $R$ - $L$  combination is suddenly put across the voltage of  $V$  volt. Let us take the instant of closing  $S$  as the starting zero time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

We will now investigate the growth of current  $i$  through such an inductive circuit.

The applied voltage  $V$  must, at any instant, supply not only the ohmic drop  $iR$  over the resistance  $R$  but must also overcome the e.m.f. of self inductance *i.e.*  $L di/dt$ .

$$\therefore V = v_R + v_L = iR + L \frac{di}{dt}$$

$$\text{or} \quad (V - iR) = L \frac{di}{dt} \therefore \frac{di}{V - iR} = \frac{dt}{L} \quad \dots(i)$$

Multiplying both sides by  $(-R)$ , we get  $(-R) \frac{di}{(V - iR)} = -\frac{R}{L} dt$

$$\text{Integrating both sides, we get } \int \frac{(-R) di}{(V - iR)} = \int dt \therefore \log_e^{V-iR} = -\frac{R}{L} t + K \quad \dots(ii)$$

where  $e$  is the Napierian logarithmic base = 2.718 and  $K$  is constant of integration whose value can be found from the initial known conditions.

To begin with, when  $t = 0$ ,  $i = 0$ , hence putting these values in (ii) above, we get

$$\log_e^V = K$$

Substituting this value of  $K$  in the above given equation, we have

$$\log_e^{V-iR} = \frac{R}{L} t + \log_e^V \text{ or } \log_e^{V-iR} - \log_e^V = -\frac{R}{L} t$$

$$\text{or} \quad \log_e \frac{V-iR}{V} = -\frac{R}{L} t = -\frac{1}{\lambda} t \text{ where } L/R = \lambda \text{ 'time constant'}$$

$$\therefore \frac{V-iR}{V} = e^{-t/\lambda} \text{ or } i = \frac{V}{R} (1 - e^{-t/\lambda})$$

Now,  $V/R$  represents the maximum steady value of current  $I_m$  that would eventually be established through the  $R$ - $L$  circuit.

$$\therefore i = I_m (1 - e^{-t/\lambda}) \quad \dots(iii)$$

This is an exponential equation whose graph is shown in Fig. 8.11. It is seen from it that current rise is rapid at first and then decreases until at  $t = \infty$  it becomes zero. Theoretically, current does not reach its maximum steady value  $I_m$  until infinite time. However, in practice, it reaches this value in a relatively short time of about  $5\lambda$ .

The rate of rise of current  $di/dt$  at any stage can be found by differentiating Eq. (ii) above w.r.t. time. However, the *initial* rate of rise of

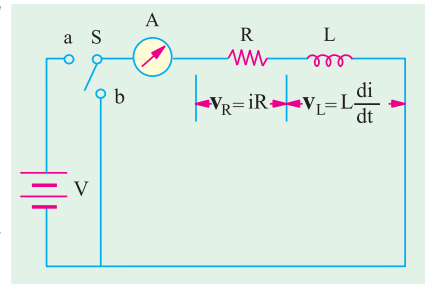


Fig. 8.10

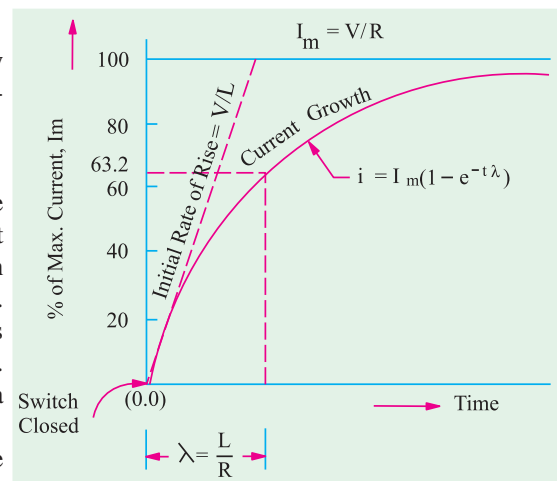


Fig. 8.11



current can be obtained by putting  $t = 0$  and  $i = 0$  in (i)\* above.

$$\therefore V = 0 \times R + L \frac{di}{dt} \text{ or } \frac{di}{dt} = \frac{V}{L}$$

The constant  $\lambda = L/R$  is known as the **time-constant** of the circuit. It can be variously defined as :

(i) It is the **time** during which current would have reached its maximum value of  $I_m (= V/R)$  had it maintained its initial rate of rise.

$$\text{Time taken} = \frac{I_m}{\text{initial rate of rise}} = \frac{V/R}{V/L} = \frac{L}{R}$$

But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0.632 of its maximum values as shown below :

Putting  $t = L/R = \lambda$  in Eq. (iii) above, we get

$$i = I_m (1 - e^{-t/\lambda}) = I_m \left(1 - \frac{1}{e}\right) = I_m \left(1 - \frac{1}{2.718}\right) = 0.632 I_m$$

(ii) Hence, the time-constant  $\lambda$  of an  $R$ - $L$  circuit may also be defined as the time during which the current **actually** rises to 0.632 of its maximum steady value (Fig. 8.11).

This delayed rise of current in an inductive circuit is utilized in providing time lag in the operation of electric relays and trip coils etc.

### 8.11. Decay of Current in an Inductive Circuit

When the switch  $S$  (Fig. 8.10) is connected to point 'b', the  $R$ - $L$  circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit.

The equation for decay of current with time is found by putting  $V = 0$  in Eq. (i) of Art. 8.10

$$0 = iR + L \frac{di}{dt} \text{ or } \frac{di}{i} = -\frac{R}{L} dt$$

Integrating both sides, we have

$$\int \frac{di}{i} = -\frac{R}{L} \int dt \therefore \log i = \frac{R}{L} t + K \quad \dots(i)$$

Now, at the instant of switching off current,  $i = I_m$  and if time is counted from this instant, then  $t = 0$

$$\therefore \log_e I_m = 0 + K$$

Putting the value of  $K$  in Eq (i) above, we get,

$$\log_e i = -\frac{t}{\lambda} = \log_e I_m$$

$$\therefore \log_e i/I_m = -\frac{t}{\lambda}$$

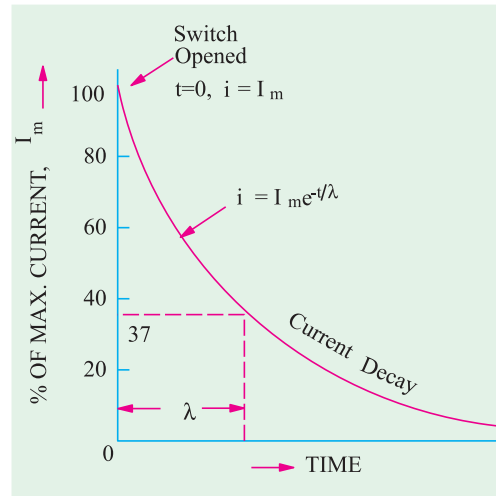


Fig. 8.12

\* Initial value of  $di/dt$  can also be found by differentiating Eq. (iii) and putting  $t = 0$  in it. In fact, the three quantities  $V$ ,  $L$ ,  $R$  give the following various combinations :

$V/R = I_m$  – the maximum final steady current.

$V/L$  = initial rate of rise of current.

$L/R$  = time constant of the circuit.

The first rule of switching is that the current flowing through an inductance cannot change instantaneously.

The second rule of switching is that the voltage across a capacitor cannot change instantaneously.

$$\therefore \frac{i}{I_m} = e^{-t/\lambda}$$

$$\text{or } i = I_m e^{-t/\lambda} \quad \dots(ii)$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about  $5\lambda$ .

Again, putting  $t = \lambda$  in Eq. (ii) above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37 I_m.$$

Hence, time constant ( $\lambda$ ) of an  $R$ - $L$  circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).

**Example 8.17.** A coil having an effective resistance of  $20 \Omega$  and an inductance of  $5 H$  is suddenly connected across a  $50\text{-V}$  dc supply. What is the rate at which energy is stored in the field of the coil when current is (a)  $0.5 A$  (b)  $1.0 A$  and (c) steady? Also find the induced EMF in the coil under the above conditions.

**Solution.** (a) Power input =  $50 \times 0.5 = 25 \text{ W}$

Power wasted as heat =  $i^2 R = 0.5^2 \times 25 = 6.25 \text{ W}$ . Hence, rate of energy storage in the coil field is  $25 - 6.25 = 18.75 \text{ W}$  or  $\text{J/s}$ .

(b) Power input =  $50 \times 1 = 50 \text{ W}$

Power lost as heat =  $1^2 \times 25 = 25 \text{ W}$ .  $\therefore$  Rate of energy storage in field =  $50 - 25 = 25 \text{ W}$  or  $\text{J/s}$ .

(c) Steady value of current =  $50/25 = 2 \text{ A}$ . Power input =  $50 \times 2 = 100 \text{ W}$

Power lost as heat =  $2^2 \times 25 = 100 \text{ W}$

Rate of energy storage in field =  $100 - 100 = 0$ ; Now,  $V = iR + e_L = V - iR$

(a)  $e_L = 50 - 0.5 \times 25 = 37.5 \text{ V}$  (b)  $e_L = 50 - 1 \times 25 = 25 \text{ V}$  (c)  $e_L = 50 - 2 \times 25 = 0 \text{ V}$ .

**Example 8.18.** A coil having a resistance of  $10 \Omega$  and an inductance of  $4 H$  is switched across a  $20\text{-V}$  dc source. Calculate (a) time required by the current to reach 50% of its final steady value and (b) value of the current after 0.5 second.

**Solution.** The rise of current through an inductive circuit is given by the equation  $i = I(1 - e^{-t/\lambda})$ . It may be written as

$$e^{-t/\lambda} = \frac{I - i}{I} \quad \text{or} \quad \frac{I}{e^{t/\lambda}} = \frac{I - i}{I} \quad \text{or} \quad e^{t/\lambda} = \frac{I}{I - i}$$

Taking logs of both sides, we have

$$\frac{t}{\lambda} \cdot \log e = \log \frac{I}{I - i} = \ln \frac{I}{I - i}$$

$$\therefore \frac{Rt}{L} = R_n \frac{I}{(I - i)} \quad \text{or} \quad t = \frac{L}{R} \ln \frac{I}{I - i}$$

(a) Now,  $I = V/R = 20/10 = 2 \text{ A}$

$$\therefore t = \frac{4}{10} \ln \frac{2}{(2 - 1)} = \frac{4}{10} \times 0.693 = 0.2777 \text{ s.}$$

(b)  $\lambda = L/R = 4/10 = 0.4 \text{ s}$  and  $t = 0.5 \text{ s}$

$$\therefore i = 2(1 - e^{-t/0.4})$$

**Example 8.19.** With reference to the circuit shown in Fig. 8.13, calculate :

(i) the current taken from the d.c. supply at the instant of closing the switch

(ii) the rate of increase of current in the coil at the instant of switch

(iii) the supply and coil currents after the switch has been closed for a long time

(iv) the maximum energy stored in the coil

(v) the e.m.f. induced in the coil when the switch is opened.

**Solution.** (i) When switch  $S$  is closed (Fig. 8.13), the supply d.c. voltage of  $120 \text{ V}$  is applied

across both arms. The current in  $R_2$  will immediately become  $120/30 = 4$  A. However, due to high inductance of the second arm, there would be no instantaneous flow of current in it. Hence current taken from the supply at the instant of switching on will be **4 A**.

(ii) Since at the *instant* of switching on, there is no current through the inductor arm, no potential drop will develop across  $R_1$ . The whole of the supply voltage will be applied across the inductor. If  $di/dt$  is the rate of increase of current through the inductor at the instant of switching on, the back e.m.f. produced in it is  $L \cdot di/dt$ . This e.m.f. is equal and opposite to the applied voltage.

$$120 = L \, di/dt \quad \text{or} \quad di/dt = 120/2 = \mathbf{60 \, A/s}$$

(iii) When switch has been closed for a sufficiently long time, current through the inductor arm reaches a steady value  $= 120/R_1 = 120/15 = \mathbf{8 \, A}$

$$\text{Current through } R_2 = 120/30 = 4 \, \text{A} ; \text{ Supply current} = 8 + 4 = \mathbf{12 \, A}$$

(iv) Maximum energy stored in the inductor arm

$$= \frac{1}{2} L I^2 = \frac{1}{2} \times 2 \times 8^2 = \mathbf{64 \, J}$$

(v) When switch is opened, current through the inductor arm cannot change immediately because of high self-inductance of the inductor. Hence, inductance current remains at 8 A. But the current through  $R_2$  can change immediately. After the switch is opened, the inductor current path lies through  $R_1$  and  $R_2$ . Hence, e.m.f. induced in the inductor at the instant of switching off is  $= 8 \times (30 + 15) = \mathbf{360 \, V}$ .

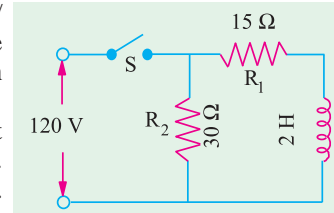


Fig. 8.13

**Example 8.20.** A coil has a time constant of 1 second and an inductance of 8 H. If the coil is connected to a 100 V d.c. source, determine :

- (i) the rate of rise of current at the instant of switching (ii) the steady value of the current and (iii) the time taken by the current to reach 60% of the steady value of the current.

(Electrotechnics-I, M.S. Univ. Baroda)

**Solution.**  $\lambda = L/R$  ;  $R = L/\lambda = 8/1 = 8 \, \text{ohm}$

(i) Initial  $di/dt = V/L = 100/8 = \mathbf{12.5 \, A/s}$

(ii)  $I_M = V/R = 100/8 = \mathbf{12.5 \, A}$

(iii) Here,  $i = 60\% \text{ of } 12.5 = 7.5 \, \text{A}$

Now,  $i = I_m (1 - e^{-t/\lambda}) \therefore 7.5 = 12.5 (1 - e^{-t/1})$ ;  $t = \mathbf{0.915 \, \text{second}}$

**Example 8.21.** A d.c. voltage of 80 V is applied to a circuit containing a resistance of  $80 \, \Omega$  in series with an inductance of  $20 \, \text{H}$ . Calculate the growth of current at the instant (i) of completing the circuit (ii) when the current is  $0.5 \, \text{A}$  and (iii) when the current is  $1 \, \text{A}$ .

(Circuit Theory, Jadavpur Univ.)

**Solution.** The voltage equation for an R-L circuit is

$$V = iR + L \frac{di}{dt} \quad \text{or} \quad L \frac{di}{dt} = V - iR \quad \text{or} \quad \frac{di}{dt} = \frac{1}{L} (V - iR)$$

(i) when  $i = 0$ ;  $\frac{di}{dt} = \frac{1}{L} (V - 0 \times R) = \frac{V}{L} = \frac{80}{20} = \mathbf{4 \, A/s}$

(ii) when  $i = 0.5 \, \text{A}$ ;  $\frac{di}{dt} = \frac{80 - 0.5 \times 80}{20} = \mathbf{2 \, A/s}$

(iii) when  $i = 1 \, \text{A}$ ;  $\frac{di}{dt} = \frac{80 - 80 \times 1}{20} = \mathbf{0}$ .

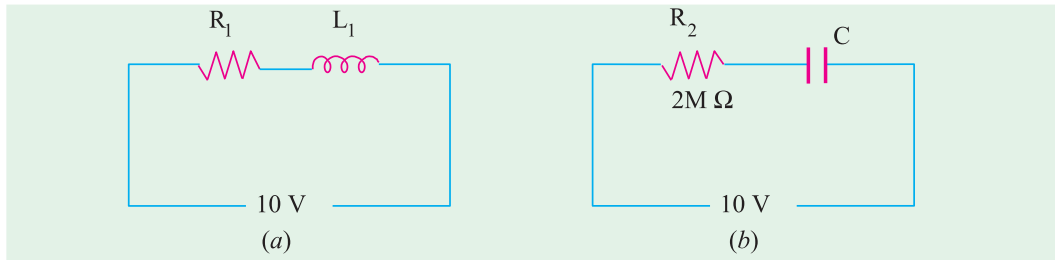
In other words, the current has become steady at 1 ampere.

**Example 8.22.** The two circuits of Fig. 8.14 have the same time constant of 0.005 second. With the same d.c. voltage applied to the two circuits, it is found that the steady state current of circuit (a) is 2000 times the initial current of circuit (b). Find  $R_p$ ,  $L_1$  and  $C$ .

(Elect. Engg.-I, Bombay Univ.)

**Solution.** The time constant of circuit 8.14 (a) is  $\lambda = L_1/R_1$  second, and that of circuit 8.14 (b) is  $\lambda = CR_2$  second.

$$\therefore \quad \begin{aligned} L_1/R_1 &= 0.005 \\ C \times 2 \times 10^6 &= 0.005, \quad C = 0.0025 \times 10^{-6} = \mathbf{0.0025 \mu F} \end{aligned}$$



**Fig. 8.14**

Steady-state current of circuit 8.14 (a) is  $= V/R_1 = 10/R_1$  amperes.

Initial current of circuit 8.14 (b)  $= V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6} A^*$

Now  $10/R_1 = 2000 \times 5 \times 10^{-6} \therefore R_1 = 1000 \Omega$

Also  $L_1/R_1 = 0.005 \therefore L_1 = 1000 \times 0.005 = \mathbf{5 H}$

**Example 8.23.** A constant voltage is applied to a series R-L circuit at  $t = 0$  by closing a switch. The voltage across  $L$  is 25 V at  $t = 0$  and drops to 5 V at  $t = 0.025$  second. If  $L = 2 H$ , what must be the value of  $R$  ?  
(Elect. Engg.-I Bombay Univ.)

**Solution.** At  $t = 0, i = 0$ , hence there is no  $iR$  drop and the applied voltage must equal the back e.m.f. in the coil. Hence, the voltage across  $L$  at  $t = 0$  represents the applied voltage.

At  $t = 0.025$  second, voltage across  $L$  is 5 V, hence voltage across

$R = 25 - 5 = 20 V \therefore iR = 20 V$  at  $t = 0.025$  second. Now  $i = I_m (1 - e^{-t/\lambda})$

Here  $I_m = 25/R$  ampere,  $t = 0.025$  second  $\therefore i = \frac{25}{R} (1 - e^{-0.025/\lambda})$

$$R \times \frac{25}{R} (1 - e^{-0.025/\lambda}) = 20 \quad \text{or} \quad e^{0.025/\lambda} = 5 \quad \therefore 0.025/\lambda = 2.3 \log_{10} 5 = 1.6077$$

$\therefore \lambda = 0.025/1.6077$  Now  $\lambda = L/R = 2/R \therefore 2/R = 0.025/1.6077 \therefore \mathbf{R = 128.56 \Omega}$

**Example 8.24.** A circuit of resistance  $R$  ohms and inductance  $L$  henries has a direct voltage of 230 V applied to it. 0.3 second after switching on, the current in the circuit was found to be 5 A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5 A at 0.3 second after short-circuiting the coil. Find the value of  $R$  and  $L$ .  
(Basic Electricity, Bombay Univ.)

**Solution.** For growth ;  $5 = I_m (1 - e^{-0.3/\lambda})$  ... (i)

For decay;  $5 = I_m e^{-0.3/\lambda}$

Equating the two, we get,  $I_m e^{-0.3/\lambda} = (1 - e^{-0.3/\lambda}) I_m$   
or  $2 e^{-0.3/\lambda} = 1 \therefore e^{-0.3/\lambda} = 0.5$  or  $\lambda = 0.4328$

Putting this value in (i), we get,

$$5 = I_m = e^{0.3/0.4328} \quad \text{or} \quad I_m = 5 e^{0.3/0.4328} = 5 \times 2 = 10 A.$$

Now,  $I_m = V/R \therefore 10 = 230/R$  or  $R = 230/10 = 23 \Omega$  (approx.)

As  $\lambda = L/R = 0.4328 ; L = 0.4328 \times 23 = \mathbf{9.95 H}$

\* Because just at the time of starting the current, there is no potential drop across  $C$  so that the applied voltage is dropped across  $R_2$ . Hence, the initial charging current  $= V/R_2$ .

**Example 8.25.** A relay has a coil resistance of  $20\ \Omega$  and an inductance of  $0.5\text{ H}$ . It is energized by a direct voltage pulse which rises from  $0$ – $10\text{ V}$  instantaneously, remains constant for  $0.25\text{ second}$  and then falls instantaneously to zero. If the relay contacts close when the current is  $200\text{ mA}$  (increasing) and open when it is  $100\text{ mA}$  (decreasing), find the total time during which the contacts are closed.

**Solution.** The time constant of the relay coil is

$$\lambda = L/R = 0.5/20 = 0.025\text{ second}$$

Now, the voltage pulse remains constant at  $10\text{ V}$  for  $0.25\text{ second}$  which is long enough for the relay coil current to reach its steady value of  $V/R = 10/20 = 0.5\text{ A}$

Let us now find the value of time required by the relay coil current to reach a value of  $200\text{ mA} = 0.2\text{ A}$ . Now  $i = I_m (1 - e^{-t/\lambda})$   $\therefore 0.2 = 0.5 (1 - e^{-t/0.025})$   $\therefore e^{-40t} = 5/3$

$$\therefore t = 0.01276\text{ second}$$

Hence, relay contacts close at  $t = 0.01276\text{ second}$  and will remain closed till current falls to  $100\text{ mA}$ . Let us find the time required by the current to fall from  $0.5\text{ A}$  to  $0.1\text{ A}$ .

At the end of the voltage pulse, the relay current decays according to the relation

$$i = I_m e^{-t/\lambda} \quad \therefore 0.1 = 0.5 e^{-t/0.025} \quad \therefore e^{40t} = 5$$

$$\therefore t = 0.04025\text{ second after the end of the voltage pulse.}$$

Hence, the time for which contacts remain closed is

$$= (0.25 - 0.01276) + 0.04025\text{ second} = \mathbf{277.5\text{ milli-second (approx)}}$$

## 8.12. Details of Transient Current Rise in an R-L Circuit

As shown in Fig. 8.15 (a), when switch  $S$  is shifted to position  $a$ , the  $R$ - $L$  circuit is suddenly energised by  $V$ . Since a coil opposes any change in current, the initial value of current is zero at  $t = 0$  and but then it rises exponentially, although its rate of rise keeps decreasing. After some time, it reaches a maximum value of  $I_m$  when it becomes constant i.e. its rate of rise becomes zero. Hence, just at the start of the transient

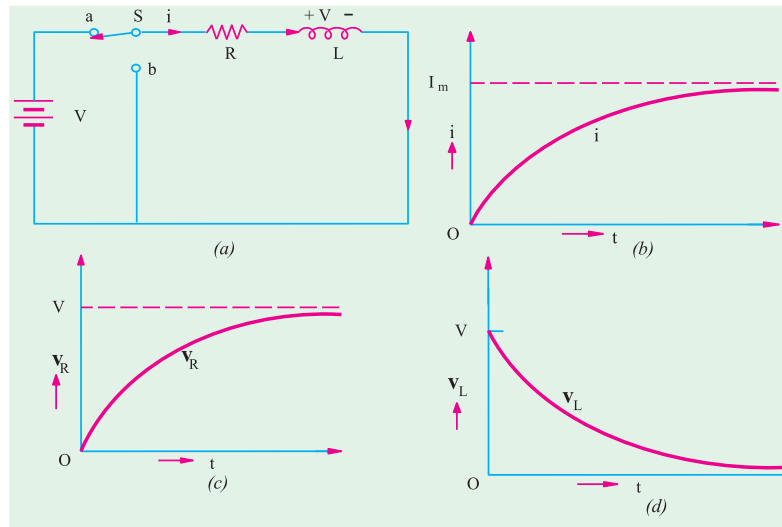


Fig. 8.15

state,  $i = 0$ ,  $V_R = 0$  and  $V_L = V$  with its polarity opposite to that of battery voltage as shown in Fig. 8.15 (a). Both  $i$  and  $V_R$  rise exponentially during the transient state, as shown in Fig. 8.15 (b) and (c) respectively. However  $V_L$  decreases exponentially to zero from its initial maximum value of  $V = I_m R$ . It does not become negative during the transient rise of current through the circuit.

Hence, during the transient rise of current, the following equations hold good :

$$i = \frac{V}{R} (1 - e^{-t/\lambda}) = I_m (1 - e^{-t/\lambda}); V_R = iR = V (1 - e^{-t/\lambda}) = I_m R (1 - e^{-t/\lambda}); V_L = V e^{-t/\lambda}$$

If  $S$  remains at 'a' long enough,  $i$  reaches a steady value of  $I_m$  and  $V_R$  equals  $I_m R$  but since  $di/dt = 0$ ,  $v_L = 0$ .

**Example 8.26.** A voltage as shown in Fig. 8.16 is applied to an inductor of  $0.2\text{ H}$ , find the current in the inductor at  $t = 2\text{ sec}$ .

**Solution.**  $L(di/dt) = 2\text{ volts}$ , for  $t$  between 0 and 1 sec, and corresponding value of  $di/dt = 2/0.2 = 10\text{ amp/sec}$ , uniform during this period.

After  $t > 1$  voltage is zero, hence  $di/dt = 0$

Current variation is marked on the same diagram.

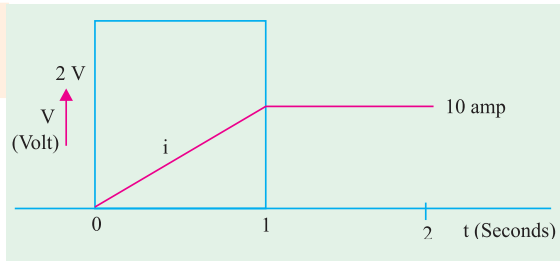


Fig. 8.16

### 8.13. Details of Transient Current Decay in an R-L Circuit

Now, let us consider the conditions during the transient decay of current when  $S$  is shifted to point 'b'. Just at the start of the decay condition, the following values exist in the circuit.

$i = I_m = V/R$ ,  $v_R = I_m R = V$  and since initial  $di/dt$  is maximum,  $v_L = -V = -I_m R$ .

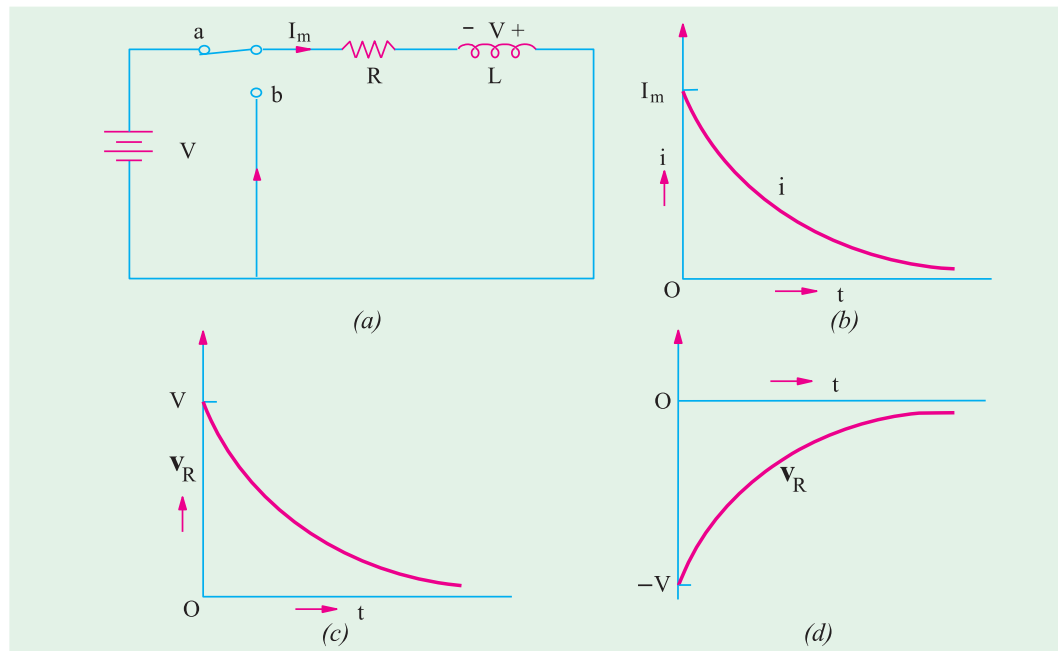


Fig. 8.17

The change in the polarity of voltage across the coil in Fig. 8.17 (a) is worth noting. Due to its property of self-induction, the coil will not allow the circuit current to die immediately, but only gradually. In fact, by reversing the sign of its voltage, the coil tends to maintain the flow of current in the original direction. Hence, as the decay continues  $i$  decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (b). Similarly,  $v_R$  decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (c). However,  $v_L$  is reversed in polarity and decreases exponentially from its initial value of  $-V$  to zero as shown in Fig. 8.17 (d).

During the transient decay of current and voltage, the following relations hold good :

$$i = i_L = I_m e^{-t/\lambda} = \frac{V}{R} e^{-t/\lambda}$$

$$v_R = V e^{-t/\lambda} = I_m R e^{-t/\lambda}$$

$$v_L = -V e^{-t/\lambda} = -I_m R e^{-t/\lambda}$$



### 8.14. Automobile Ignition System

Practical application of mutual induction is found in the single-spark petrol-engine ignition system extensively employed in automobiles and air-engines. Fig. 8.18 shows the circuit diagram of such a system as applied to a 4-cylinder automobile engine.

It has a spark coil (or induction coil) which consists of a primary winding (of a few turns) and a secondary winding (of a large number of turns)

wound on a common iron core (for increasing mutual induction). The primary circuit (containing battery  $B$ ) includes a 'make and break contact' actuated by a timer cam. The secondary circuit includes the rotating blade of the distributor and the spark gap in the spark plug as shown in Fig. 8.17. The timer cam and the distributor are mounted on the same shaft and are geared to rotate at exactly half the speed of the engine shaft. It means that in the case of automobile engines (which are four-cycle engines) each cylinder is fired only once for every two revolutions of the engine shaft.

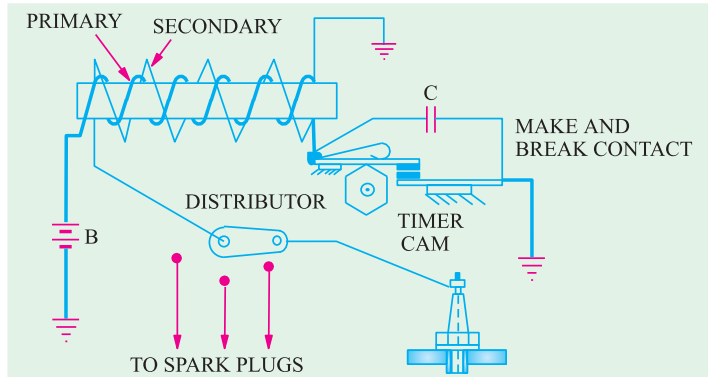


Fig. 8.18

#### Working

When timer cam rotates, it alternately closes and opens the primary circuit. During the time primary circuit is closed, current through it rises exponentially after the manner shown in Fig. 8.11 and so does the magnetic field of the primary winding. When the cam suddenly opens the primary circuit, the magnetic field collapses rapidly thereby producing a very large e.m.f. in secondary by mutual induction. During the time this large e.m.f. exists, the distributor blade rotates and connects the secondary winding across the proper plug and so the secondary circuit is completed except for the spark gap in the spark plug. However, the induced e.m.f. is large enough to make the current jump across the gap thus producing a spark which ignites the explosive mixture in the engine cylinder.

The function of capacitor  $C$  connected across the 'make and break' contact is two-fold :

- (i) to make the break rapid so that large e.m.f. is induced in secondary and
- (ii) to reduce sparking and burning at the 'make-and-break' contact thereby prolonging their life.

#### Tutorial Problems No. 8.3

- A relay has a resistance of  $300\ \Omega$  and is switched on to a  $110\ \text{V d.c.}$  supply. If the current reaches 63.2 per cent of its final steady value in 0.002 second, determine
  - the time-constant of the circuit
  - the inductance of the circuit
  - the final steady value of the circuit
  - the initial rate of rise of current.

[(a) 0.002 second (b) 0.6 H (c) 0.366 A (d) 183 A/second]
- A coil with a self-inductance of  $2.4\ \text{H}$  and resistance  $12\ \Omega$  is suddenly switched across a  $120\text{-V d.c.}$  supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.1 second, the final steady value of the current and the time taken for the current to reach 5 A.
 

[(a) 0.2 second; 3.94 A; 10 A; 0.139 second]
- A circuit whose resistance is  $20\ \Omega$  and inductance  $10\ \text{H}$  has a steady voltage of  $100\ \text{V}$  suddenly applied to it. For the instant 0.5 second after the voltage is applied, determine (a) the total power input to the circuit (b) the power dissipated in the resistance. Explain the reason for the difference between (a) and (b).
 

[(a) 316 W (b) 200 W]
- A lighting circuit is operated by a relay of which the coil has a resistance of  $5\ \Omega$  and an inductance of  $0.5\ \text{H}$ . The relay coil is supplied from a  $6\text{-V d.c.}$  source through a push-button switch. The relay operates when the current in the relay coil attains a value of  $500\ \text{mA}$ . Find the time interval between the pressing of the push-button and the closing of lighting circuit.
 

[53.8 ms]

5. The field winding of a separately-excited d.c. generator has an inductance of 60 H and a resistance of 30  $\Omega$ . A discharge resistance of 50  $\Omega$  is permanently connected in parallel with the winding which is excited from a 200-V supply. Find the value of decay current 0.6 second after the supply has been switched off. **[3.0 A]**
6. The field winding of a dynamo may be taken to have a constant inductance of 120 H and an effective resistance of 30  $\Omega$ . When it is carrying a current of 5 A, the supply is interrupted and a resistance of 50  $\Omega$  is connected across the winding. How long will it take for the current to fall to 1-0 ? **[2.415 s]**
7. A 200-V d.c. supply is suddenly switched to a relay coil which has a time constant of 3 milli-second. If the current in the coil reaches 0.2 A after 3 milli-second, determine the final steady value of the current and the resistance of the coil. **[0.316 A; 632  $\Omega$  1.896 H]**
8. Explain the terms related to magnetic circuits :  
(i) coercive force (ii) residual flux. **(Nagpur University, Summer 2002)**
9. The B-H characteristic of cast iron may be drawn from the following :-

B(Wb/m <sup>2</sup> )	H(AT/m)
0.1	280
0.2	620
0.3	990
0.4	1400
0.5	2000
0.6	2800

**(Nagpur University, Winter 2003)**

10. Derive an expression for the energy stored in the magnetic field of a coil of an inductance L henry. **(Gujrat University, June/July 2003)**
11. A cast steel ring has a circular cross-section of 3 cm in diameter and a mean circumference of 80 cm. A 1 mm air gap is cut out in the ring which is wound with a coil of 600 turns. Estimate the current required to establish a flux of 0.75 mWb in the air gap. Neglect fringing and leakage. The magnetization data of the material is as under :

**(R.G.P.V. Bhopal University, June 2004)**

H (At/m)	B (T)
200	0.10
400	0.32
600	0.60
800	0.90
1000	1.08
1200	1.18
1400	1.27
1600	1.32
1800	1.36

12. What is the difference between B.H. curve and hysteresis loop? **(Anna University, April 2002)**
13. Explain with neat diagram how can you obtain B.H. curve and hysteresis loop of ring specimens. **(Anna University, April 2002)**
14. Derive an expression for energy stored in an inductance. **(V.T.U., Belgaum Karnataka University, Summer 2002)**
15. Derive an expression for the energy stored in a magnetic circuit. **(V.T.U., Belgaum Karnataka University, January/February 2003)**
16. What is hysteresis and eddy current losses? What are the undesirable effects of eddy currents and hysteresis loss? How can they be minimised? Mention some applications of eddy currents. **(U.P. Technical University, 2002) (RGPV Bhopal June 2002)**

### OBJECTIVE TESTS – 8

1. Permanent magnets are normally made of  
(a) aluminium (b) wrought iron  
(c) cast iron (d) alnico alloys
2. A coil of 1000 turns is wound on a core. A current of 1 A flowing through the coil creates a core flux of 1 mWb. The energy stored in the magnetic field is  
(a) 0.25 J (b) 0.5 J  
(c) 1 J (d) 2 J

**(ESE 2003)**