6
Capacitance and Capacitors

Introduction

It is well known that different bodies hold different charge when given the same potential. This charge holding property of a body is called *capacitance* or *capacity* of the body. In order to store sufficient charge, a device called capacitor is purposely constructed. A capacitor essentially consists of two conducting surfaces (say metal plates) separated by an insulating material (*e.g.*, air, mica, paper etc.). It has the property to store electrical energy in the form of electrostatic charge. The capacitor can be connected in a circuit so that this stored energy can be made to flow in a desired circuit to perform a useful function. Capacitance plays an important role in d.c. as well as a.c. circuits. In many circuits (*e.g.*, radio and television circuits), capacitors are intentionally inserted to introduce the desired capacitance. In this chapter, we shall confine our attention to the role of capacitance in d.c. circuits only.

6.1. Capacitor

Any two conducting surfaces separated by an insulating material is called a ***capacitor** *or* **condenser.** Its purpose is to store charge in a small space.

The conducting surfaces are called the *plates* of the capacitor and the insulating material is called the ***dielectric.* The most commonly used dielectrics are air, mica, waxed paper, ceramics *etc.* The following points may be noted carefully :

(*i*) The ability of a capacitor to store charge (*i.e.* its capacitance) depends upon the area of plates, distance between plates and the nature of insulating material (or dielectric).

(*ii*) A capacitor is generally named after the dielectric used *e.g.* air capacitor, paper capacitor, mica capacitor *etc.*

(*iii*) The capacitor may be in the form of parallel plates, concentric cylinder or other arrangement.

6.2. How does a Capacitor Store Charge ?

Fig. 6.1 shows how a capacitor stores charge when connected to a d.c. supply. The parallel plate capacitor having plates A and B is connected across a battery of V volts as shown in Fig. 6.1 (*i*). When the switch S is open as shown in Fig. 6.1 (*i*), the capacitor plates are neutral *i.e.* there is no charge on the plates. When the switch is closed as shown in Fig. 6.1 (*ii*), the electrons from plate A will be attracted by the +ve terminal of the battery and these electrons start ***accumulating on plate B. The result is that plate A attains more and more positive charge and plate B gets more and more negative charge. This action is referred to as charging a capacitor because the capacitor plates are becoming charged. This process of electron flow or charging (*i.e.* detaching electrons from plate A and accumulating on B) continues till p.d. across capacitor plates becomes equal to battery voltage V. When the capacitor is charged to battery voltage V, the current flow ceases as shown in Fig. 6.1

* The name is derived from the fact that this arrangement has the capacity to store charge. The name condenser is given to the device due to the fact that when p.d. is applied across it, the electric lines of force are condensed in the small space between the plates.

** A steady current cannot pass through an insulator but an electric field can. For this reason, an insulator is often referred to as a dielectric.

*** The electrons cannot flow from plate B to A as there is insulating material between the plates. Hence electrons detached from plate A start piling up on plate B.

(*iii*). If now the switch is opened as shown in Fig. 6.1 (*iv*), the capacitor plates will retain the charges. Thus the capacitor plates which were neutral to start with now have charges on them. This shows that a capacitor stores charge. The following points may be noted about the action of a capacitor :

(*i*) When a d.c. potential difference is applied across a capacitor, a charging current will flow until the capacitor is fully charged when the current will cease. This whole charging process takes place in a very short time, a fraction of a second. *Thus a capacitor once charged, prevents the flow of direct current*.

(*ii*) *The current does not flow through the capacitor i.e. between the plates*. There is only transference of electrons from one plate to the other.

(*iii*) When a capacitor is charged, the two plates carry equal and opposite charges (say $+Q$ and $-Q$). This is expected because one plate loses as many electrons as the other plate gains. *Thus charge on a capacitor means charge on either plate.*

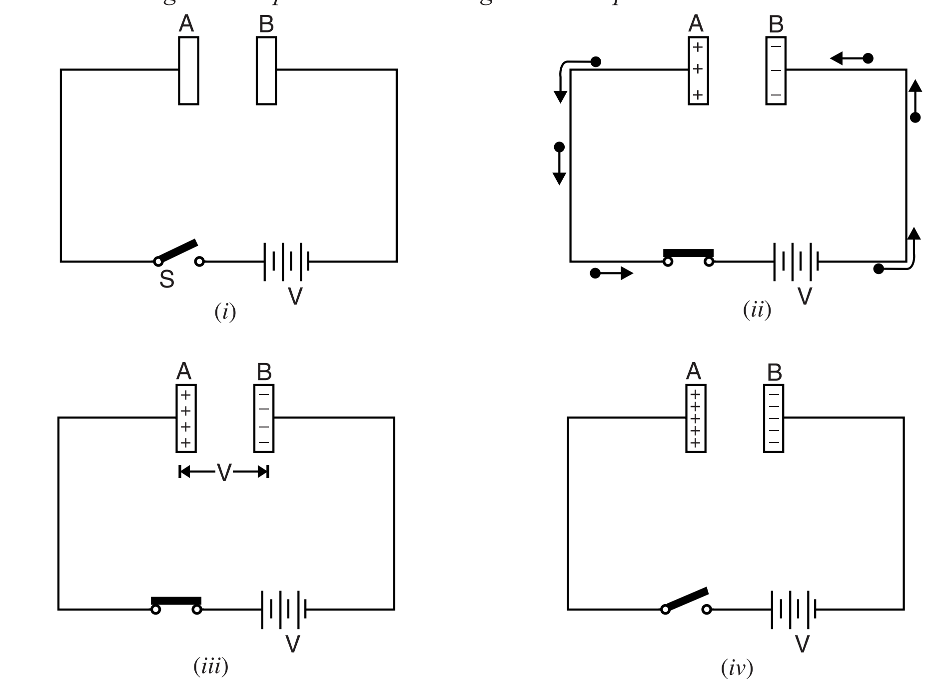


Fig. 6.1

(*iv*) The energy required to charge the capacitor (*i.e.* transfer of electrons from one plate to the other) is supplied by the battery.

6.3. Capacitance

The ability of a capacitor to store charge is known as its capacitance. It has been found experimentally that charge Q stored in a capacitor is directly proportional to the p.d. V across it *i.e.*

$$Q \propto V$$

or
$$\frac{Q}{V} = \text{Constant} = C$$

The constant C is called the capacitance of the capacitor. Hence capacitance of a capacitor can be defined as under :

The ratio of charge on capacitor plates to the p.d. across the plates is called **capacitance** *of the capacitor*.

Unit of capacitance

We know that : $C = Q/V$

The SI unit of charge is 1 coulomb and that of voltage is 1 volt. Therefore, the SI unit of capacitance is one coulomb/volt which is also called *farad* (Symbol F) in honour of Michael Faraday.

$$1 \text{ farad} = 1 \text{ coulomb/volt}$$

A capacitor is said to have a capacitance of **1 farad** *if a charge of 1 coulomb accumulates on each plate when a p.d. of 1 volt is applied across the plates.*

Thus if a charge of 0·1C accumulates on each plate of a capacitor when a p.d. of 10V is applied across its plates, then capacitance of the capacitor = 0·1/10 = 0·01 F. The farad is an extremely large unit of capacitance. Practical capacitors have capacitances of the order of microfarad (μF) and micro-microfarad (μμF) or picofarad (pF).

$$1 \ \mu F = 10^{-6} F \quad ; \quad 1 \mu\mu F \text{ (or 1 pF)} = 10^{-12} F$$

6.4. Factors Affecting Capacitance

The ability of a capacitor to store charge (*i.e.* its capacitance) depends upon the following factors :

(*i*) **Area of plate.** The greater the area of capacitor plates, the larger is the capacitance of the capacitor and *vice-versa.* It is because larger the plates, the greater the charge they can hold for a given p.d. and hence greater will be the capacitance.

(*ii*) **Thickness of dielectric.** The capacitance of a capacitor is inversely proportional to the thickness (*i.e.* distance between plates) of the dielectric. The smaller the thickness of dielectric, the greater the capacitance and *vice-versa.* When the plates are brought closer, the electrostatic field is intensified and hence capacitance increases.

(*iii*) **Relative permittivity of dielectric.** The greater the relative permittivity of the insulating material (*i.e.*, dielectric), the greater will be the capacitance of the capacitor and *vice-versa.* It is because the nature of dielectric affects the electrostatic field between the plates and hence the charge that accumulates on the plates.

6.5. Dielectric Constant or Relative Permittivity

The insulating material between the plates of a capacitor is called dielectric. When the capacitor is charged, the electrostatic field extends across the dielectric. The presence of dielectric* increases the concentration of electric lines of force between the plates and hence the charge on each plate. The degree of concentration of electric lines of force between the plates depends upon the nature of dielectric.

The ability of a dielectric material to concentrate electric lines of force between the plates of a capacitor is called **dielectric constant** *or* **relative permittivity** *of the material.*

Air has been assigned a reference value of dielectric constant (or relative permittivity) as 1. The dielectric constant of all other insulating materials is greater than unity. The dielectric constants of materials commonly used in capacitors range from 1 to 10. For example, dielectric constant of mica is 6. It means that if mica is used as a dielectric between the plates of a capacitor, the charge on each plate will be 6 times the value when air is used; other things remaining equal. In other words, with mica as dielectric, the capacitance of the capacitor becomes 6 times as great as when air is used.

* Normally, the electrons of the atoms of the dielectric revolve around their nuclei in their regular orbits. When the capacitor is charged, electrostatic field causes distortion of the orbits of the electrons of the dielectric. This distortion of orbits causes an additional electrostatic field within the dielectric which causes more electrons to be transferred from one plate to the other. Hence, the presence of dielectric increases the charge on the capacitor plates and hence the capacitance.

Let V = Potential difference between capacitor plates

Q = Charge on capacitor when air is dielectric

Then, $C_{air} = Q/V$

When mica is used as a dielectric in the capacitor and the same p.d. is applied, the capacitor will now hold a charge of $6Q$.

$$\therefore \quad C_{mica} = \frac{6Q}{V} = 6\frac{Q}{V} = 6\,C_{air}$$

or $$\frac{C_{mica}}{C_{air}} = 6 = \text{Dielectric constant of mica}$$

Hence **dielectric constant** (*or* **relative permittivity**) *of a dielectric material is the ratio of capacitance of a capacitor with that material as a dielectric to the capacitance of the same capacitor with air as dielectric.*

6.6. Capacitance of an Isolated Conducting Sphere

We can find the capacitance of an isolated spherical conductor by assuming that "missing" plate is earth (zero potential). Suppose an isolated conducting sphere of radius r is placed in a medium of relative permittivity ε_r as shown in Fig. 6.2. Let charge $+Q$ be given to this spherical conductor. The charge is spread *uniformly over the surface of the sphere. Therefore, in order to find the potential at any point on the surface of sphere (or outside the sphere), we can assume that entire charge $+Q$ is concentrated at the centre O of the sphere.

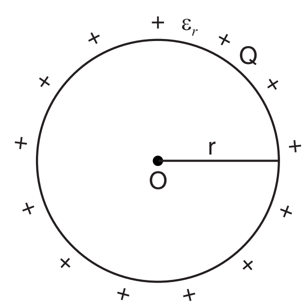


Fig. 6.2

Potential at the surface of the sphere, $V = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r}$

$\therefore$ Capacitance of isolated sphere, $C = \dfrac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$

$\therefore$ $**C = 4\pi\,\varepsilon_0\,\varepsilon_r\,r$ ... *in a medium*

$= 4\pi\,\varepsilon_0\,r$ ... *in air*

The following points may be noted :

(*i*) *The capacitance of an isolated spherical conductor is directly proportional to its radius.* Therefore, for a given potential, a large spherical conductor (more r) will hold more charge Q (= CV) than the smaller one.

(*ii*) Unit of $\varepsilon_0 = C/4\pi r$ = F/m. Hence, the SI unit of ε_0 is F/m.

Example 6.1. *Twenty seven spherical drops, each of radius 3 mm and carrying 10^{-12} C of charge are combined to form a single drop. Find the capacitance and potential of the bigger drop.*

Solution. Let r and R be the radii of smaller and bigger drops respectively.

Volume of bigger drop = 27 × Volume of smaller drop

or $$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

or $$R = 3r = 3 \times 3 = 9 \text{ mm} = 9 \times 10^{-3} \text{ m}$$

Capacitance of bigger drop, $C = 4\pi\,\varepsilon_0\,R = \dfrac{1}{9\times10^9}\times9\times10^{-3} = 10^{-12}$ F = **1 pF**

* Note that a charged conductor is an equipotential surface. Therefore, electric lines of force emerging from the sphere are everywhere normal to the sphere.

** Note that values of Q and V do not occur in the expression for capacitance. This again reminds us that capacitance is a property of physical construction of a capacitor.

Since charge is conserved, the charge on the bigger drop is 27×10^{-12} C.

$\therefore$ Potential of bigger drop, $V = \dfrac{Q}{C} = \dfrac{27 \times 10^{-12}}{10^{-12}} = \mathbf{27\ V}$

6.7. Capacitance of Spherical Capacitor

We shall discuss two cases.

(*i*) When outer sphere is earthed. A spherical capacitor consists of two concentric hollow metallic spheres *A* and *B* which do not touch each other as shown in Fig. 6.3. The outer sphere *B* is earthed while charge is given to the inner sphere *A*. Suppose the medium between the two spheres has relative permittivity ε_r.

Let r_A = radius of inner sphere *A*

r_B = radius of outer sphere *B*

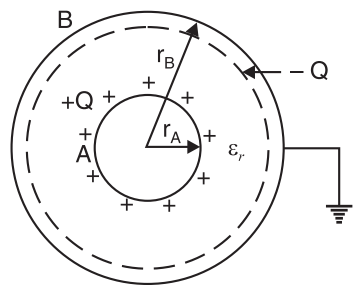


Fig. 6.3

When a charge $+Q$ is given to the inner sphere *A*, it induces a charge $-Q$ on the inner surface of outer sphere *B* and $+Q$ on the outer surface of *B*. Since sphere *B* is earthed, $+Q$ charge on its outer surface is neutralised by earth.

*Potential at inner sphere *A*, $V_A = \left(\dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r_A}\right) + \left(\dfrac{-Q}{4\pi\varepsilon_0\varepsilon_r r_B}\right)$

$$= \frac{Q}{4\pi\,\varepsilon_0\varepsilon_r}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{Q(r_B - r_A)}{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}$$

Since sphere *B* is earthed, its potential is zero (*i.e.,* $V_B = 0$).

$\therefore$ P.D. between *A* and *B*, $V_{AB} = V_A - V_B = V_A - 0 = V_A$

$\therefore$ Capacitance of spherical capacitor, $C = \dfrac{Q}{V_A} = \dfrac{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}{(r_B - r_A)}$

$$\therefore \qquad C = \frac{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}{(r_B - r_A)} \qquad \text{... in a medium}$$

$$= \frac{4\pi\,\varepsilon_0\, r_A\, r_B}{(r_B - r_A)} \qquad \text{... in air}$$

(*ii*) When inner sphere is earthed. Fig. 6.4 shows the situation. The system constitutes two capacitors in parallel.

(*a*) One capacitor (C_{BA}) consists of the inner surface of *B* and outer surface of *A*. Its capacitance as found above is

$$C_{BA} = \frac{4\pi\,\varepsilon_0\,\varepsilon_r\, r_A\, r_B}{r_B - r_A}$$

(*b*) The second capacitor (C_{BG}) consists of outer surface of *B* and earth. Its capacitance is that of an isolated sphere.

$\therefore \qquad C_{BG} = 4\pi\,\varepsilon_0\, r_B$ ... if surrounding medium is air

$\therefore$ Total capacitance $= C_{BA} + C_{BG}$

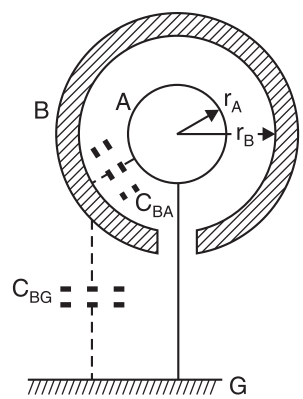


Fig. 6.4

Note. Unless stated otherwise, the outer sphere of a spherical capacitor is assumed to be earthed.

* Potential on sphere *A* = (Potential on sphere *A* due to its own charge $+Q$) + (Potential on sphere *A* due to charge $-Q$ on sphere *B*) $= \left(\dfrac{Q}{4\pi\,\varepsilon_0\varepsilon_r\, r_A}\right) + \left(\dfrac{-Q}{4\pi\,\varepsilon_0\varepsilon_r\, r_B}\right)$

Example 6.2. *The thickness of air layer between two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the capacitance of sphere of 1.2 m diameter. Find the radii of its surfaces.*

Solution. Given : $\dfrac{4\pi\,\varepsilon_0\, r_A\, r_B}{r_B - r_A} = 4\pi\,\varepsilon_0\, R \quad \therefore \quad \dfrac{r_A\, r_B}{r_B - r_A} = R$

Here, $r_B - r_A = 2$ cm and $R = 1{\cdot}2/2 = 0{\cdot}6$ m $= 60$ cm

$\therefore \quad \dfrac{r_A\, r_B}{2} = 60$ or $r_A\, r_B = 120$

Now $(r_B + r_A)^2 = (r_B - r_A)^2 + 4r_A\, r_B = (2)^2 + 4 \times 120 = 484$

$\therefore \quad r_B + r_A = \sqrt{484} = 22$ cm

Since $r_B - r_A = 2$ cm and $r_B + r_A = 22$ cm, $r_B =$ **12 cm** ; $r_A =$ **10 cm**

Example 6.3. *A capacitor has two concentric thin spherical shells of radii 8 cm and 10 cm. The outer shell is earthed and a charge is given to the inner shell. Calculate (i) the capacitance of this capacitor and (ii) the final potential acquired by the inner shell if the outer shell is removed after the inner shell has acquired a potential of 200 V.*

Solution. It is assumed that medium between the two spherical shells is air so that $\varepsilon_r = 1$.

(i) Radius of inner sphere, $r_A = 8$ cm $= 0.08$ m; Radius of outer sphere, $r_B = 10$ cm $= 0.1$ m

The capacitance C of the spherical capacitor is

$$C = \frac{4\pi\varepsilon_0\varepsilon_r r_A r_B}{r_B - r_A} = \frac{4\pi \times 8.854 \times 10^{-12} \times 0.08 \times 0.1}{0.1 - 0.08} = \mathbf{44.44 \times 10^{-12}\ F}$$

(ii) Charge on the capacitor when the inner sphere acquires a potential of 200 V is

$$Q = CV = 44.44 \times 10^{-12} \times 200 = 8888 \times 10^{-12}\ \text{C}$$

When the outer shell is removed, the capacitance C' of the resulting isolated sphere is

$$C' = 4\pi\varepsilon_0\varepsilon_r r_A = \frac{1}{9 \times 10^9} \times 1 \times 0.08 = 8.88 \times 10^{-12}\ \text{F}$$

$\therefore$ Potential V' acquired by the inner shell when outer shell is removed is

$$V' = \frac{Q}{C'} = \frac{8888 \times 10^{-12}}{8.88 \times 10^{-12}} = \mathbf{1000\ V}$$

Tutorial Problems

1. Calculate the capacitance of a conducting sphere of radius 10 cm situated in air. How much charge is required to raise it to a potential of 1000 V? **[11 pF; 1.1 × 10^{-8} C]**
2. When 1.0×10^{12} electrons are transferred from one conductor to another of a capacitor, a potential difference of 10V develops between the two conductors. Calculate the capacitance of the capacitor. **[1.6 × 10^{-8} F]**
3. Calculate the capacitance of a spherical capacitor if the diameter of inner sphere is 0.2 m and that of the outer sphere is 0.3 m, the space between them being filled with a liquid having dielectric constant 12. **[4 × 10^{-10} F]**
4. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of the earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km. **[0.092 F]**
5. A spherical capacitor has an outer sphere of radius 0.15 m and the inner sphere of radius 0.1m. The outer sphere is earthed and inner sphere is given a charge of 6μC. The space between the concentric spheres is filled with a material of dielectric constant 18. Calculate the capacitance and potential of the inner sphere. **[6 × 10^{-10} F; 10^4 V]**

6.8. Capacitance of Parallel-Plate Capacitor with Uniform Medium

We have seen that the capacitance of a capacitor can be determined from its electrical properties using the relation $C = Q/V$. However, it is often desirable to determine the capacitance of a capacitor in terms of its dimensions and relative permittivity of the dielectric. Although there are many forms of capacitors, the most important arrangement is the parallel-plate capacitor.

Consider a parallel plate capacitor consisting of two plates, each of area A square metres and separated by a *uniform dielectric* of thickness d metres and relative permittivity ε_r as shown in Fig. 6.5. Let a p.d. of V volts applied between the plates place a charge of $+Q$ and $-Q$ on the plates. With reasonable accuracy, it can be assumed that electric field between the plates is uniform.

Electric flux density between plates is

$$D = Q/A \text{ coulomb/m}^2$$

Electric intensity between plates is

$$E = V/d$$

But $\quad D = \varepsilon_0 \varepsilon_r E \quad$ ...See Art. 5.12

or $$\frac{Q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d}$$

or $$\frac{Q}{V} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

The ratio Q/V is the capacitance C of the capacitor.

$$\therefore \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad \text{...in a medium}$$

$$= \frac{\varepsilon_0 A}{d} \quad \text{...in air}$$

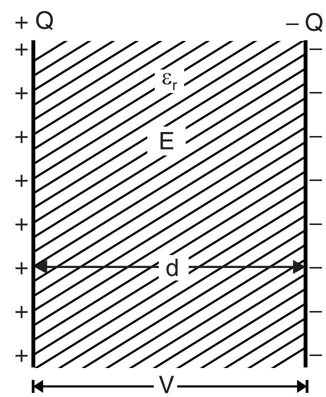


Fig. 6.5

The following points may be noted carefully :

(*i*) Capacitance is directly proportional to ε_r and A and inversely proportional to d.

(*ii*) $\dfrac{C_{med}}{C_{air}} = \varepsilon_r$ = Relative permittivity of medium

(*iii*) Re-arranging the relation for C in air

$$\varepsilon_0 = \frac{Cd}{A} = \frac{\text{farad} \times \text{m}}{\text{m}^2} = \text{F/m}$$

Obviously, permittivity can also be measured in F/m.

6.9. Parallel-Plate Capacitor with Composite Medium

Suppose the space between the plates is occupied by three dielectrics of thicknesses d_1, d_2 and d_3 metres and relative permittivities ε_{r1}, ε_{r2} and ε_{r3} respectively as shown in Fig. 6.6. *The electric flux density D in the dielectrics remains the *same and is equal to Q/A*. However, the electric intensities in the three dielectrics will be different and are given by ;

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} \; ; \quad E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r2}} \; ; \quad E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r3}}$$

If V is the total p.d. across the capacitor and V_1, V_2 and V_3 the p.d.s. across the three dielectrics respectively, then,

$$V = V_1 + V_2 + V_3$$
$$= E_1 d_1 + E_2 d_2 + E_3 d_3$$

* The total charge on each plate is Q. Hence Q coulombs is also the total electric flux through each dielectric.

$$= \frac{D}{\varepsilon_0\,\varepsilon_{r1}}d_1 + \frac{D}{\varepsilon_0\,\varepsilon_{r2}}d_2 + \frac{D}{\varepsilon_0\,\varepsilon_{r3}}d_3$$

$$= \frac{D}{\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right]$$

$$= \frac{Q}{\varepsilon_0\,A}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right] \qquad \left(\because D = \frac{Q}{A}\right)$$

or $$\frac{Q}{V} = \frac{\varepsilon_0\,A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)}$$

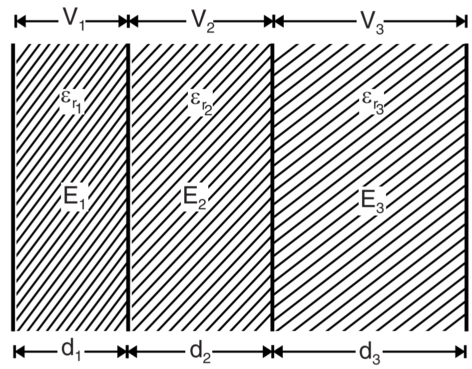


Fig. 6.6

But Q/V is the capacitance C of the capacitor.

$$\therefore \quad C = \frac{\varepsilon_0\,A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)} \text{ farad}$$

In general,

$$C = \frac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}} \text{ farad} \qquad ...(i)$$

Different cases. We shall discuss the following two cases :

(*i*) **Medium partly air.** Fig. 6.7 shows a parallel plate capacitor having plates d metres apart. Suppose the medium between the plates consists partly of air and partly of dielectric of thickness t metres and relative permittivity ε_{r2}. Then thickness of air is $d - t$. Using the relation (*i*) above, we have,

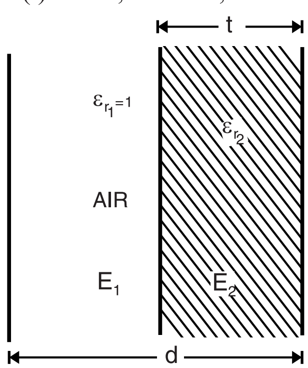


Fig. 6.7

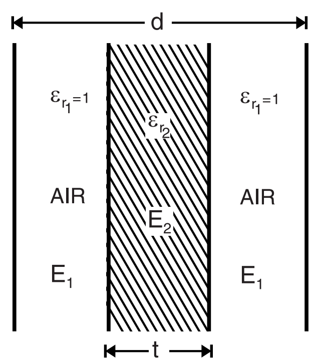


Fig. 6.8

$$C = \frac{\varepsilon_0\,A}{\frac{d-t}{1} + \frac{t}{\varepsilon_{r2}}} = \frac{\varepsilon_0\,A}{d - \left(t - \frac{t}{\varepsilon_{r2}}\right)} \text{ farad}$$

(*ii*) **When dielectric slab introduced.** Fig. 6.8 shows a parallel-plate air capacitor having plates d metres apart. Suppose a dielectric slab of thickness t metres and relative permittivity ε_{r2} is introduced between the plates of the capacitor.

Using the relation (*i*) above, we have,

$$C = \frac{\varepsilon_0\,A}{\frac{d-t}{1} + \frac{t}{\varepsilon_{r2}}} = \frac{\varepsilon_0\,A}{d - \left(t - \frac{t}{\varepsilon_{r2}}\right)} \text{ farad}$$

6.10. Special Cases of Parallel-Plate Capacitor

We have seen that capacitance of a capacitor depends upon plate area, thickness of dielectric and value of relative permittivity of the dielectric.

We consider two cases by way of illustration.

(*i*) Fig. 6.9 shows that dielectric thickness is d but plate area is divided into two parts; area A_1 having air as the dielectric and area A_2 having dielectric of relative permittivity ε_r. The arrangement is equivalent to two capacitors in parallel. Their capacitances are :

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad ; \quad C_2 = \frac{\varepsilon_0 \varepsilon_r A_2}{d}$$

The total capacitance C of this parallel-plate capacitor is

$$C = C_1 + C_2$$

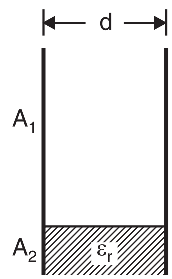


Fig. 6.9

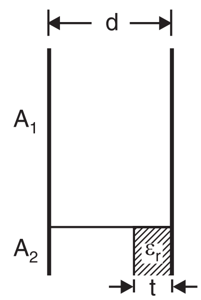


Fig. 6.10

(*ii*) Fig. 6.10 shows that plate area is divided into two parts ; area A_1 has dielectric (air) of thickness d and area A_2 has a dielectric (ε_r) of thickness t and the remaining thickness is occupied by air. The arrangement is equivalent to two capacitors connected in parallel. Their capacitances are :

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad ; \quad C_2 = \frac{\varepsilon_0 A_2}{[d-(t-t/\varepsilon_r)]}$$

The total capacitance C of this parallel plate capacitor is

$$C = C_1 + C_2$$

6.11. Multiplate Capacitor

The most *convenient way of achieving large capacitance is by using large plate area. Increasing the plate area may increase the physical size of the capacitor enormously. In order to obtain a large area of plate surface without using too bulky a capacitor, multiplate construction is employed. In this construction, the capacitor is built up of alternate sheets of metal foil (*i.e.* plates) and thin sheets of dielectric. The odd-numbered metal sheets are connected together to form one terminal T_1 and even-numbered metal sheets are connected together to form the second terminal T_2.

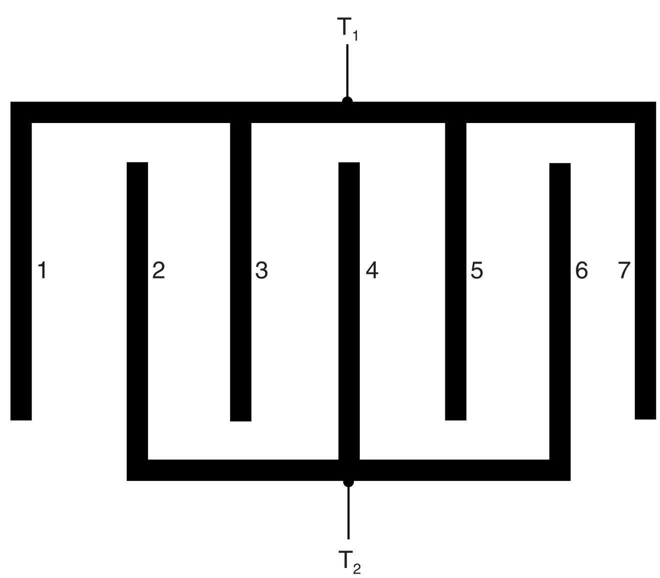


Fig. 6.11

Fig. 6.11 shows a multiplate capacitor with seven plates. A little

* The capacitance of a capacitor can also be increased by (*i*) using a dielectric of high ε_r and (*ii*) decreasing the distance between plates. High cost limits the choice of dielectric and dielectric strength of the insulating material limits the reduction in spacing between the plates..

reflection shows that this arrangement is equivalent to 6 capacitors in parallel. The total capacitance will, therefore, be 6 times the capacitance of a single capacitor (formed by say plates 1 and 2). If there are n plates, each of area A, then $(n - 1)$ capacitors will be in parallel.

$\therefore$ Capacitance of n plate capacitor is

$$C = (n-1)\frac{\varepsilon_0 \, \varepsilon_r \, A}{d}$$

where d is the distance between any two adjacent plates and ε_r is the relative permittivity of the medium. It may be seen that plate area is increased from A to $A\,(n-1)$.

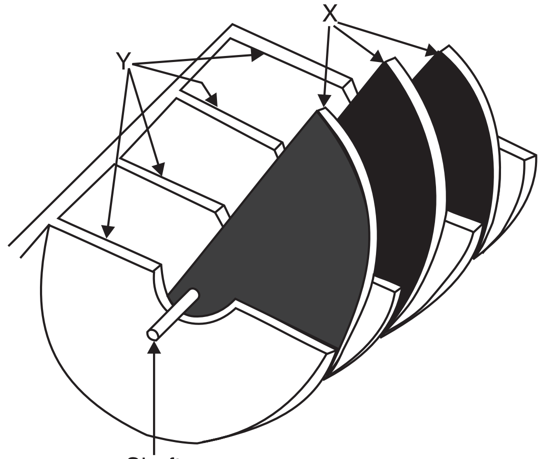


Fig. 6.12

Variable Air capacitor. It is a multiplate air capacitor whose capacitance can be varied by changing the plate area. Fig. 6.12 shows a variable air capacitor commonly used to "tune in" radio stations in the radio receiver. It consists of a set of stationary metal plates Y fixed to the frame and another set of movable metal plates X fixed to the central shaft. The two sets of plates are electrically insulated from each other. Rotation of the shaft moves the plates X into the spaces between plates Y, thus changing the *common (or effective) plate area and hence the capacitance. The capacitance of such a capacitor is given by ;

$$C = (n-1)\frac{\varepsilon_0 \, A}{d} \qquad (\because\ \varepsilon_r = 1)$$

When the movable plates X are completely rotated in (*i.e.* the two sets of plates completely overlap each other), the common plate area 'A' is maximum and so is the capacitance of the capacitor. Minimum capacitance is obtained when the movable plates X are completely rotated out of stationary plates Y. The capacitance of such variable capacitors is from zero to about 4000 pF.

Note. In all the formulas derived for capacitance, capacitance will be in farad if area is in m^2 and the distance between plates is in m.

Example 6.4. *A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two parallel plates, each having an area of 0·01 m^2 separated by a dielectric 1 mm thick. The resulting capacitance of the arrangement is 300 pF. Calculate (i) total electric flux (ii) electric flux density (iii) potential gradient and (iv) relative permittivity of the dielectric.*

Solution. $C = 300 \times 10^{-12}$ F ; $V = 10 \times 10^3 = 10^4$ volts

(i) Total electric flux, $Q = C\,V = (300 \times 10^{-12}) \times 10^4 = 3 \times 10^{-6}\ \text{C} = \mathbf{3\mu C}$

(ii) Electric flux density, $D = \dfrac{Q}{A} = \dfrac{3\times10^{-6}}{0\cdot01} = \mathbf{3 \times 10^{-4}\ C/m^2}$

(iii) Potential gradient $= \dfrac{V}{d} = \dfrac{10^4}{1\times10^{-3}} = \mathbf{10^7\ V/m}$

(iv) Now, $E = 10^7$ V/m

Since $D = \varepsilon_0 \varepsilon_r E$

$$\therefore \quad \varepsilon_r = \frac{D}{\varepsilon_0 E} = \frac{3\times10^{-4}}{(8.854\times10^{-12})\times10^{7}} = \mathbf{3\cdot39}$$

---

* Remember in the formula for capacitance, A is the common plate area *i.e.* plate area facing the opposite polarity plate area.

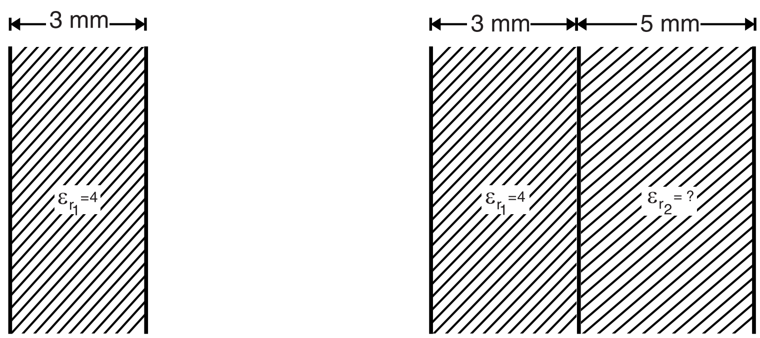
Example. 6.5. *A capacitor is composed of two plates separated by 3mm of dielectric of permittivity 4. An additional piece of insulation 5mm thick is now inserted between the plates. If the capacitor now has capacitance one-third of its original capacitance, find the relative permittivity of the additional dielectric.*

Solution. Figs. 6.13 (*i*) and 6.13 (*ii*) respectively show the two cases.

For the first case,
$$C = \frac{\varepsilon_0 \varepsilon_{r1} A}{d} = \frac{\varepsilon_0 \times 4 \times A}{3 \times 10^{-3}} \quad ...(i)$$

For the second case,
$$\frac{C}{3} = \frac{\varepsilon_0 A}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}}}$$

$$= \frac{\varepsilon_0 A}{\dfrac{3 \times 10^{-3}}{4} + \dfrac{5 \times 10^{-3}}{\varepsilon_{r2}}} \quad ...(ii)$$



(*i*) (*ii*)

Fig. 6.13

Dividing eq. (*i*) by eq. (*ii*), we get,

$$3 = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{\varepsilon_{r2}}\right)$$

or $\quad 9 = 3 + 20/\varepsilon_{r2} \quad \therefore \varepsilon_{r2} = 20/6 = \mathbf{3 \cdot 33}$

Example 6.6. *Determine the dielectric flux in microcoulombs between two parallel plates each 0.35 metre square with an air gap of 1.5 mm between them, the p. d. being 3000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the relative permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7500 V.*

Solution. $A = 0.35 \times 0.35 = 0.1225$ m^2 ; $d = 1.5$ mm $= 1.5 \times 10^{-3}$ m ; $\varepsilon_r = 1$(air).

Capacitance C of the parallel-plate air capacitor is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 1 \times 0.1225}{1.5 \times 10^{-3}} = 723 \times 10^{-12} \text{ F}$$

Dielectric flux, $\psi = Q = CV = 723 \times 10^{-12} \times 3000 = 2.17 \times 10^{-6}$ C = **2.17 μC**

Suppose the potential gradient in air is g_a. Then potential gradient in the insulating material is $g_i = g_a/\varepsilon_r = g_a/6$.

Thickness of air ; $t_a = 1.5 - 1 = 0.5$ mm $= 0.5 \times 10^{-3}$ m ; Thickness of insulating material, t_i = 1mm = 10^{-3} m.

$\therefore \quad$ Applied voltage, $V = g_a t_a + g_i t_i$

or $$7500 = g_a \times 0.5 \times 10^{-3} + \frac{g_a}{6} \times 10^{-3}$$

∴ $$g_a = \mathbf{11.25 \times 10^6\ V/m}$$

and $$g_i = \frac{g_a}{6} = \frac{11.25 \times 10^6}{6} = \mathbf{1.875 \times 10^6\ V/m}$$

Example 6.7. *An air capacitor has two parallel plates of 1500 cm*^{2} *in area and 5 mm apart. If a dielectric slab of area 1500 cm*^{2}*, thickness 2 mm and relative permittivity 3 is now introduced between the plates, what must be the new separation between the plates to bring the capacitance to the original value?*

Solution. This is a case of introduction of dielectric slab into an air capacitor. As proved in Art. 6.9, the capacitance under this condition becomes :

$$C = \frac{\varepsilon_0 A}{d - (t - t/\varepsilon_r)} \qquad ...(i)$$

If the medium were totally air, capacitance would have been

$$C_{air} = \frac{\varepsilon_0 A}{d} \qquad ...(ii)$$

Inspection of eqs. (*i*) and (*ii*) shows that with the introduction of dielectric slab between the plates of air capacitor, its capacitance increases. The distance between the plates is effectively reduced by $t - (t/\varepsilon_r)$. In order to bring the capacitance to the original value, the plates must be separated by this much distance in air.

∴ New separation between the plates

$$= d + (t - t/\varepsilon_r) = 5 + (2 - 2/3) = \mathbf{6{\cdot}33\ mm}$$

Example 6.8. *A variable air capacitor has 11 movable plates and 12 stationary plates. The area of each plate is 0·0015 m*^{2} *and separation between opposite plates is 0·001 m. Determine the maximum capacitance of this variable capacitor.*

Solution. The capacitance will be maximum when the movable plates are completely rotated in *i.e.* when the two sets of plates completely overlap each other. Under this condition, the common (or effective) area is equal to the physical area of each plate.

$$C = (n-1)\frac{\varepsilon_0 \varepsilon_r A}{d}$$

Here $$n = 11 + 12 = 23\ ; \quad \varepsilon_r = 1\ ; \quad A = 0{\cdot}0015\ \text{m}^2\ ; \quad d = 0{\cdot}001\ \text{m}$$

∴ $$C = (23-1) \times \frac{8.854 \times 10^{-12} \times 1 \times 0.0015}{0.001} = 292 \times 10^{-12}\text{F} = \mathbf{292\ pF}$$

Example 6.9. *The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0º to 180º. With dial set at 180º, the capacitor is connected to 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is tuned at 0º. What is the potential difference across the capacitor when the dial reads 0º ?*

Solution. With dial at 0º, the capacitance of the capacitor is

$$C_1 = 50\ \text{pF} = 50 \times 10^{-12}\ \text{F}$$

With dial at 180º, the capacitance of the capacitor is

$$C_2 = 950\ \text{pF} = 950 \times 10^{-12}\ \text{F}$$

P.D. across C_2, V_2 = 400 V

∴ Charge on C_2, $Q = C_2V_2 = (950 \times 10^{-12}) \times 400 = 380 \times 10^{-9}$ C

When the battery is disconnected, charge Q remains the same. Suppose V_1 is the potential difference across the capacitor when the dial reads 0º.

∴ $$Q = C_1V_1$$

or $$V_1 = \frac{Q}{C_1} = \frac{380\times10^{-9}}{50\times10^{-12}} = \mathbf{7600\ V}$$

Example 6.10. *A parallel plate capacitor has plates of area 2 m² spaced by three layers of different dielectric materials. The relative permittivities are 2, 4, 6 and thicknesses are 0.5, 1.5 and 0.3 mm respectively. Calculate the combined capacitance and the electric stress (potential gradient) in each material when applied voltage is 1000 V.*

Solution. Capacitance, $$C = \frac{\varepsilon_0 A}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}} + \dfrac{d_3}{\varepsilon_{r3}}}$$

$$= \frac{8\cdot854\times10^{-12}\times2}{\dfrac{0\cdot5\times10^{-3}}{2} + \dfrac{1\cdot5\times10^{-3}}{4} + \dfrac{0\cdot3\times10^{-3}}{6}} = \mathbf{0{\cdot}0262\times10^{-6}\ F}$$

Charge on each plate, $Q = CV = (0{\cdot}0262\times10^{-6})\times1000 = 26{\cdot}2\times10^{-6}$ C

Electric flux density, $$D = \frac{Q}{A} = \frac{26\cdot2\times10^{-6}}{2} = 13\cdot1\times10^{-6}\ \text{C/m}^2$$

Electric stress in the material with $\varepsilon_{r1} = 2$ is

$$E_1 = \frac{D}{\varepsilon_0\,\varepsilon_{r1}} = \frac{13\cdot1\times10^{-6}}{8\cdot854\times10^{-12}\times2} = \mathbf{74\times10^4\ V/m}$$

Electric stress in the material with $\varepsilon_{r2} = 4$ is

$$E_2 = \frac{13\cdot1\times10^{-6}}{8.854\times10^{-12}\times4} = \mathbf{37\times10^4\ V/m}$$

Electric stress in the material with $\varepsilon_{r3} = 6$ is

$$E_3 = \frac{13\cdot1\times10^{-6}}{8854\times10^{-12}\times6} = \mathbf{24.67\times10^4\ V/m}$$

It is clear from the above example that electric stress is greatest in the material having the least relative permittivity. Since air has the lowest relative permittivity, efforts should be made to avoid air pockets in the dielectric materials.

Example 6.11. *A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm slab is introduced between the plates in order to maintain the same potential difference, the distance between the plates is increased by 2·4 mm. Find the dielectric constant of the slab.*

Solution. The capacitance of parallel-plate capacitor in air is

$$C = \frac{\varepsilon_0 A}{d} \qquad ...(i)$$

With the introduction of slab of thickness t, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t(1 - 1/\varepsilon_r)} \qquad ...(ii)$$

Now the charge ($Q = CV$) remains the same in the two cases.

$$\therefore \quad \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t(1-1/\varepsilon_r)}$$

or $$d = d' - t(1 - 1/\varepsilon_r)$$

Here, $d' = d + 2{\cdot}4\times10^{-3}$ m ; $t = 3$ mm $= 3\times10^{-3}$ m

$$\therefore \quad d = d + 2\cdot4\times10^{-3} - 3\times10^{-3}\left(1 - \frac{1}{\varepsilon_r}\right)$$

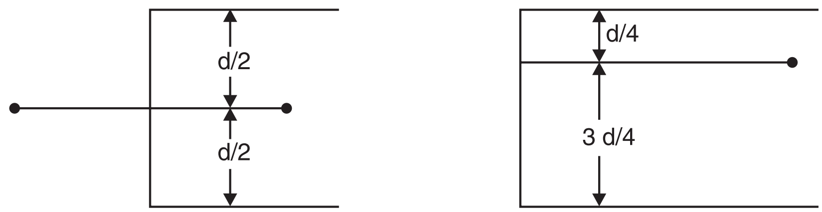
or $$2\cdot4\times10^{-3} = 3\times10^{-3}\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\therefore \quad \varepsilon_r = \mathbf{5}$$

Example 6.12. *A parallel plate capacitor has three similar parallel plates. Find the ratio of capacitance when the inner plate is mid-way between the outers to the capacitance when inner plate is three times as near one plate as the other.*

Solution. Fig. 6.14 (*i*) shows the condition when the inner plate is mid-way between the outer plates. This arrangement is equivalent to two capacitors in parallel.

$\therefore$ Capacitance of the capacitor $C_1 = \dfrac{\varepsilon_0 \varepsilon_r A}{d/2} + \dfrac{\varepsilon_0 \varepsilon_r A}{d/2} = \dfrac{4\varepsilon_0 \varepsilon_r A}{d}$



(*i*) (*ii*)

Fig. 6.14

Fig. 6.14 (*ii*) shows the condition when inner plate is three times as near as one plate as the other.

$\therefore$ Capacitance of the capacitor $C_2 = \dfrac{\varepsilon_0 \varepsilon_r A}{d/4} + \dfrac{\varepsilon_0 \varepsilon_r A}{3d/4} = \dfrac{16\varepsilon_0 \varepsilon_r A}{3d}$

$\therefore$ $C_1/C_2 = $ **0.75**

Example 6.13. *The permittivity of the dielectric material between the plates of a parallel-plate capacitor varies uniformly from ε_1 at one plate to ε_2 at other plate. Show that the capacitance is given by ;*

$$C = \frac{A}{d} \frac{\varepsilon_2 - \varepsilon_1}{\log_e \varepsilon_2/\varepsilon_1}$$

where A and d are the area of each plate and separation between the plates respectively.

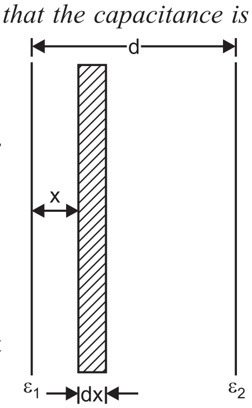


Fig. 6.15

Solution. Fig. 6.15 shows the conditions of the problem. The permittivity of the dielectric material at a distance x from the left plate is

$$\varepsilon_x = \varepsilon_1 + \frac{x}{d}(\varepsilon_2 - \varepsilon_1)$$

Consider an elementary strip of width dx at a distance x from the left plate. The capacitance C of this strip is

$$C = \frac{\varepsilon_x A}{dx}$$

or
$$\frac{1}{C} = \frac{dx}{\varepsilon_x A} = \frac{dx}{A\left[\varepsilon_1 + \frac{x}{d}(\varepsilon_2 - \varepsilon_1)\right]} = \frac{d}{A} \frac{dx}{\varepsilon_1 d + x(\varepsilon_2 - \varepsilon_1)}$$

$\therefore$ Total capacitance C_T between the plates is

$$\frac{*1}{C_T} = \int_{x=0}^{x=d} \frac{1}{C} = \frac{d}{A}\int_0^d \frac{dx}{\varepsilon_1 d + x(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{d}{A} {}^{**}\left[\frac{\log_e\{\varepsilon_1 d + (\varepsilon_2 - \varepsilon_1)x\}}{\varepsilon_2 - \varepsilon_1}\right]_0^d$$

* The arrangement constitutes capacitors in series.

** $\displaystyle\int \frac{dx}{a+bx} = \frac{\log_e(a+bx)}{b}$

$$= \frac{d}{A(\varepsilon_2 - \varepsilon_1)}[\log_e(\varepsilon_1 d + \varepsilon_2 d - \varepsilon_1 d) - \log_e \varepsilon_1 d]$$

$$= \frac{d}{A(\varepsilon_2 - \varepsilon_1)}\log_e \frac{\varepsilon_2 d}{\varepsilon_1 d} = \frac{d}{A(\varepsilon_2 - \varepsilon_1)}\log_e \frac{\varepsilon_2}{\varepsilon_1}$$

$$\therefore \quad C_T = \frac{A}{d}\frac{\varepsilon_2 - \varepsilon_1}{\log_e \frac{\varepsilon_2}{\varepsilon_1}}$$

Tutorial Problems

1. A capacitor consisting of two parallel plates 0·5 mm apart in air and each of effective area 500 cm^2 is connected to a 100V battery. Calculate (*i*) the capacitance and (*ii*) the charge. **[(*i*) 885 pF (*ii*) 0·0885 μC]**
2. A capacitor consisting of two parallel plates in air, each of effective area 50 cm^2 and 1 mm apart, carries a charge of 1770 × 10^{-12} *C*. Calculate the p.d. between the plates. If the distance between the plates is increased to 5mm, what will be the electrical effect ? **[40 V ; p.d. across plates is increased to 200 V]**
3. Two insulated parallel plates each of 600 cm^2 effective area and 5 mm apart in air are charged to a p.d. of 1000 V. Calculate (*i*) the capacitance and (*ii*) the charge on each plate.

   The source of supply is now disconnected, the plates remaining insulated. Calculate (*iii*) the p.d. between the plates when their spacing is increased to 10 mm and (*iv*) the p.d. when the plates, still 10 mm apart, are immersed in oil of relative permittivity 5. **[(*i*) 106·2 pF (*ii*) 106·2 × 10^{-12} C (*iii*) 2000 V (*iv*) 400 V]**
4. A p.d. of 500 V is applied across a parallel plate capacitor with a plate area of 0·025 m^2. The plates are separated by a dielectric of relative permittivity 2·5. If the capacitance of the capacitor is 500 μF, find (*i*) the electric flux (*ii*) electric flux density and (*iii*) the electric intensity.

   [(*i*) 0·25 μC (*ii*) 0·01 mC/m^2 (*iii*) 45.3 × 10^6 V/m]
5. A capacitor consists of two parallel metal plates, each of area 2000 cm^2 and 5 mm apart. The space between the plates is filled with a layer of paper 2 mm thick and a sheet of glass 3 mm thick. The relative permittivities of paper and glass are 2 and 8 respectively. A p.d. of 5 kV is applied across the plates. Calculate (*i*) the capacitance of the capacitor and (*ii*) the potential gradient in each dielectric.

   [(*i*) 1290 pF (*ii*) 1820 V/mm (paper); **453 V/mm** (glass)**]**
6. A parallel plate capacitor has a plate area of 20 cm^2 and the plates are separated by three dielectric layers each 1 mm thick and of relative permittivity 2, 4 and 5 respectively. Find the capacitance of the capacitor and the electric stress in each dielectric if applied voltage is 1000 V.

   [18·6 pF ; 5·26 × 10^5 V/m; 2·63 × 10^5 V/m; 2·11 × 10^5 V/m]
7. A 1μF parallel plate capacitor that can just withstand a p.d. of 6000 V uses a dielectric having a relative permittivity 5, which breaks down if the electric intensity exceeds 30 × 10^6 V/m. Find (*i*) the thickness of dielectric required and (*ii*) the effective area of each plate. **[(*i*) 0·2 mm (*ii*) 4·5 m^2]**
8. An air capacitor has two parallel plates 10 cm^2 in area and 5 mm apart. When a dielectric slab of area 10 cm^2 and thickness 5 mm was inserted between the plates, one of the plates has to be moved by 0·4 cm to restore the capacitance. What is the dielectric constant of the slab ? **[5]**
9. A multiplate parallel capacitor has 6 fixed plates connected in parallel, interleaved with 5 similar plates; each plate has effective area of 120 cm^2. The gap between the adjacent plates is 1 mm. The capacitor is immersed in oil of relative permittivity 5. Calculate the capacitance. **[5·31 pF]**
10. Calculate the number of sheets of tin foil and mica for a capacitor of 0·33 μF capacitance if area of each sheet of tin foil is 82 cm^2, the mica sheets are 0·2 mm thick and have relative permittivity 5.

    [182 sheets of mica; **183** sheets of tin foil**]**

6.12. Cylindrical Capacitor

A cylindrical capacitor consists of two co-axial cylinders separated by an insulating medium. This is an important practical case since *a single core cable is in effect a capacitor of this kind*. The conductor (or core) of the cable is the inner cylinder while the outer cylinder is represented by lead sheath which is at earth potential. The two co-axial cylinders have insulation between them.

Consider a single core cable with conductor diameter d metres and inner sheath diameter D metres (See Fig. 6.16). Let the charge per metre axial length of the cable be Q coulombs and ε_r be the relative permittivity of the insulating material. Consider a cylinder of radius x metres. According to Gauss's theorem, electric flux passing through this cylinder is Q coulombs. The surface area of this cylinder is

$$= 2\pi x \times 1 = 2\pi x \text{ m}^2$$

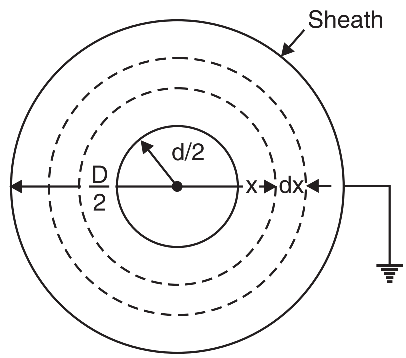


Fig. 6.16

∴ Electric flux density at any point P on the considered cylinder is given by ;

$$D_x = \frac{Q}{2\pi x} \text{ C/m}^2$$

Electric intensity at point P is given by;

$$E_x = \frac{D_x}{\varepsilon_0 \varepsilon_r} = \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} \text{ V/m}$$

The work done in moving a unit positive charge from point P through a distance dx in the direction of electric field is $E_x\, dx$. Hence the work done in moving a unit positive charge from conductor to sheath, which is the p.d. V between the conductor and sheath, is given by ;

$$V = \int_{d/2}^{D/2} E_x\, dx = \int_{d/2}^{D/2} \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} dx = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}$$

$$\therefore \quad \text{Capacitance of cable, } C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e \dfrac{D}{d}} \text{ F/m} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log_e (D/d)} \text{ F/m}$$

$$= \frac{2\pi \times 8\cdot 854 \times 10^{-12} \times \varepsilon_r}{2\cdot 303 \log_{10} (D/d)} \text{ F/m} = \frac{\varepsilon_r}{41\cdot 4 \log_{10} (D/d)} \times 10^{-9} \text{ F/m}$$

If the cable has a length of l metres, then capacitance of the cable is

$$= \frac{\varepsilon_r l}{41\cdot 4 \log_{10} (D/d)} \times 10^{-9} \text{ F} = \frac{24 \varepsilon_r l}{\log_{10} (D/d)} \text{pF}$$

Example 6.14. *In a concentric cable 20 cm long, the diameter of inner and outer cylinders are 15 cm and 15·4 cm respectively. The relative permittivity of the insulation is 5. If a p.d. of 5000 V is maintained between the two cylinders, calculate :*

(*i*) *capacitance of cylindrical capacitor*

(*ii*) *the charge*

(*iii*) *the electric flux density and electric intensity in the dielectric.*

Solution. (*i*) Capacitance of the cylindrical capacitor is

$$C = \frac{\varepsilon_r l}{41\cdot 4 \log_{10} (D/d)} \times 10^{-9} = \frac{5 \times 0\cdot 2}{41\cdot 4 \log_{10} (15\cdot 4/15)} \times 10^{-9} \text{ F} = \mathbf{2.11 \times 10^{-9} \text{ F}}$$

(*ii*) Charge on capacitor, $Q = CV = (2\cdot 11 \times 10^{-9}) \times 5000 = 10\cdot 55 \times 10^{-6}$ C = **10·55 μC**

(*iii*) To determine D and E in the dielectric, we shall consider the average radius of dielectric, *i.e.,*

$$\text{Average radius of dielectric, } x = \frac{1}{2}\left[\frac{15}{2} + \frac{15\cdot 4}{2}\right] = 7\cdot 6 \text{ cm} = 0\cdot 076 \text{ m}$$

Flux density in dielectric, $D = \dfrac{Q}{2\pi x l}\text{C/m}^2 = \dfrac{10 \cdot 55 \times 10^{-6}}{2\pi \times 0 \cdot 076 \times 0 \cdot 2}$ = **110·47 × 10⁻⁶ C/m²**

Electric intensity in dielectric, $E = \dfrac{D}{\varepsilon_0\,\varepsilon_r} = \dfrac{110 \cdot 47 \times 10^{-6}}{8 \cdot 854 \times 10^{-12} \times 5}$ = **2.5×10^6 V/m**

Example 6.15. *A 33 kV, 50 Hz, 3-phase underground cable, 4 km long uses three single core cables. Each of the conductor has a diameter of 2·5 cm and the radial thickness of insulation is 0·5 cm. Determine (i) capacitance of the cable/phase (ii) charging current/phase (iii) total charging kVAR. The relative permittivity of insulation is 3.*

Solution. (*i*) Capacitance of cable/phase, $C = \dfrac{\varepsilon_r\ l}{41 \cdot 4\ \log_{10}(D/d)} \times 10^{-9}$ F

Here $\varepsilon_r = 3$; $l = 4$ km = 4000 m

$d = 2 \cdot 5$ cm ; $D = 2.5 + 2 \times 0.5 = 3.5$ cm

Putting these values in the above expression, we get,

$$C = \frac{3 \times 4000 \times 10^{-9}}{41 \cdot 4 \times \log_{10}(3 \cdot 5 / 2 \cdot 5)} = \mathbf{1984 \times 10^{-9}\ F}$$

(*ii*) Voltage/phase, $V_{ph} = \dfrac{33 \times 10^3}{\sqrt{3}} = 19 \cdot 05 \times 10^3$ V

Charging current/phase, $I_C = \dfrac{V_{ph}}{X_C} = 2\pi f C V_{ph}$

$= 2\pi \times 50 \times 1984 \times 10^{-9} \times 19 \cdot 05 \times 10^3 =$ **11·87 A**

(*iii*) Total charging kVAR $= 3 V_{ph} I_C = 3 \times 19 \cdot 05 \times 10^3 \times 11 \cdot 87 =$ **$678 \cdot 5 \times 10^3$ kVAR**

6.13. Potential Gradient in a Cylindrical Capacitor

Under operating conditions, the insulation of a cable is subjected to electrostatic forces. This is known as dielectric stress. The dielectric stress at any point in a cable is infact the potential gradient (or *electric intensity) at that point.

Consider a single core cable with core diameter d and internal sheath diameter D. As proved in Art. 6.12, the electric intensity at a point x metres from the centre of the cable is

$$E_x = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ volts/m}$$

By definition, electric intensity is equal to potential gradient. Therefore, potential gradient g at a point x metres from the centre of the cable is

$$g = E_x$$

or $$g = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ volts/m} \quad ...(i)$$

As proved in Art. 6.12, potential difference V between conductor and sheath is

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\frac{D}{d} \text{ volts}$$

or $$Q = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e \dfrac{D}{d}} \quad ...(ii)$$

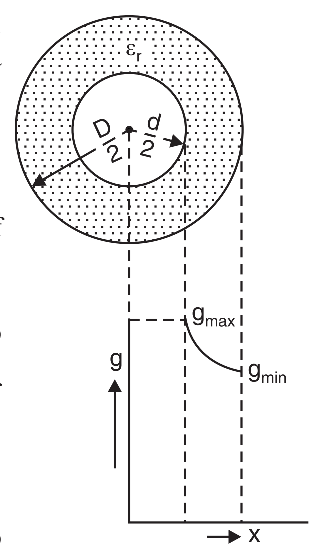


Fig. 6.17

* It may be recalled that potential gradient at any point is equal to the electric intensity at that point.

Substituting the value of Q from exp. (*ii*) in exp. (*i*), we get,

$$g = \frac{\dfrac{2\pi\varepsilon_0\varepsilon_r V}{\log_e D/d}}{2\pi\varepsilon_0\varepsilon_r x} = \frac{V}{x\log_e \dfrac{D}{d}} \text{ volts/m} \quad ...(iii)$$

It is clear from exp. (*iii*) that potential gradient varies inversely as the distance x. Therefore, potential gradient will be maximum when x is minimum *i.e.,* when $x = d/2$ or at the surface of the conductor. On the other hand, potential gradient will be minimum at $x = D/2$ or at sheath surface.

∴ Maximum potential gradient, $g_{max} = \dfrac{2V}{d\log_e \dfrac{D}{d}}$ volts/m [Putting $x = d/2$ in exp. (*iii*)]

Minimum potential gradient, $g_{min} = \dfrac{2V}{D\log_e \dfrac{D}{d}}$ volts/m [Putting $x = D/2$ in exp. (*iii*)]

$$\therefore \quad \frac{g_{max}}{g_{min}} = \frac{\dfrac{2V}{d\log_e D/d}}{\dfrac{2V}{D\log_e D/d}} = \frac{D}{d}$$

The variation of stress in the dielectric is shown in Fig. 6.17. *It is clear that dielectric stress is maximum at the conductor surface and its value goes on decreasing as we move away from the conductor*. It may be noted that maximum stress is an important consideration in the design of a cable. For instance, if a cable is to be operated at such a voltage that *maximum stress is 5 kV/mm, then the insulation used must have a dielectric strength of atleast 5 kV/mm, otherwise breakdown of the cable will become inevitable.

6.14. Most Economical Conductor Size in a Cable

It has already been shown that maximum stress in a cable occurs at the surface of the conductor. For safe working of the cable, dielectric strength of the insulation should be more than the maximum stress. Rewriting the expression for maximum stress, we get,

$$g_{max} = \frac{2V}{d\log_e \dfrac{D}{d}} \text{ volts/m} \quad ...(i)$$

The values of working voltage V and internal sheath diameter D have to be kept fixed at certain values due to design considerations. This leaves conductor diameter d to be the only variable in exp. (*i*). For given values of V and D, the most economical conductor diameter will be one for which g_{max} has a minimum value. The value of g_{max} will be minimum when $d\log_e D/d$ is maximum *i.e.*

$$\frac{d}{dd}\left[d\log_e \frac{D}{d}\right] = 0 \quad \text{or} \quad \log_e \frac{D}{d} + d.\frac{d}{D}.\frac{-D}{d^2} = 0$$

$$\therefore \quad \log_e (D/d) - 1 = 0$$

$$\text{or} \quad \log_e (D/d) = 1 \quad \text{or} \quad (D/d) = e = 2{\cdot}718$$

∴ Most economical conductor diameter, $d = \dfrac{D}{2.718}$

and the value of g_{max} under this condition is

$$g_{max} = \frac{2V}{d} \text{ volts/m} \quad [\text{Putting } \log_e D/d = 1 \text{ in exp. } (i)]$$

* Of course, it will occur at the conductor surface.

For low and medium voltage cables, the value of conductor diameter arrived at by this method (*i.e.,* $d = 2V/g_{max}$) is often too small from the point of view of current density. Therefore, the conductor diameter of such cables is determined from the consideration of safe current density. For high voltage cables, designs based on this theory give a very high value of *d,* much too large from the point of view of current carrying capacity and it is, therefore, advantageous to increase the conductor diameter to this value. There are three ways of doing this without using excessive copper :

(*i*) Using aluminium instead of copper because for the same current, diameter of aluminium will be more than that of copper.

(*ii*) Using copper wires stranded around a central core of hemp.

(*iii*) Using a central lead tube instead of hemp.

Example 6.16. *The maximum and minimum stresses in the dielectric of a single core cable are 40 kV/cm (r.m.s.) and 10 kV/cm (r.m.s.) respectively. If the conductor diameter is 2 cm, find :*

(*i*) *thickness of insulation* (*ii*) *operating voltage*

Solution. Here, g_{max} = 40 kV/cm ; g_{min} = 10 kV/cm ; d = 2 cm ; D = ?

(*i*) As proved in Art. 6.13,

$$\frac{g_{max}}{g_{min}} = \frac{D}{d} \quad \text{or} \quad D = \frac{g_{max}}{g_{min}} \times d = \frac{40}{10} \times 2 = 8 \text{ cm}$$

$$\therefore \quad \text{Insulation thickness} = \frac{D-d}{2} = \frac{8-2}{2} = \mathbf{3\ cm}$$

(*ii*)

$$g_{max} = \frac{2V}{d \log_e \frac{D}{d}}$$

$$\therefore \quad V = \frac{g_{max}\, d \log_e \frac{D}{d}}{2} = \frac{40 \times 2 \log_e 4}{2} \text{ kV} = \mathbf{55.45\ kV}\ \textit{r.m.s.}$$

Example 6.17. *A single core cable for use on 11 kV, 50 Hz system has conductor area of 0·645 cm*^{2} *and internal diameter of sheath is 2·18 cm. The permittivity of the dielectric used in the cable is 3·5. Find (i) the maximum electrostatic stress in the cable (ii) minimum electrostatic stress in the cable (iii) capacitance of the cable per km length (iv) charging current.*

Solution. Area of cross-section of conductor, $a = 0.645$ cm^2

$$\text{Diameter of the conductor, } d = \sqrt{\frac{4a}{\pi}} = \sqrt{\frac{4 \times 0 \cdot 645}{\pi}} = 0 \cdot 906 \text{ cm}$$

Internal diameter of sheath, D = 2·18 cm

(*i*) Maximum electrostatic stress in the cable is

$$g_{max} = \frac{2V}{d \log_e \frac{D}{d}} = \frac{2 \times 11}{0.906 \log_e \frac{2.18}{0.906}} \text{ kV/cm} = \mathbf{27.65\ kV/cm}\ \textit{r.m.s.}$$

(*ii*) Minimum electrostatic stress in the cable is

$$g_{min} = \frac{2\,V}{D \log_e \frac{D}{d}} = \frac{2 \times 11}{2.18 \log_e \frac{2.18}{0.906}} \text{ kV/cm} = \mathbf{11.5\ kV/cm\ r.m.s.}$$

(*iii*) Capacitance of cable,

$$C = \frac{\varepsilon_r\ l}{41.4 \log_{10} \frac{D}{d}} \times 10^{-9} \text{ F}$$

Here ε_r = 3.5 ; l = 1 km = 1000 m

$\therefore \quad C = \dfrac{3\cdot5\times1000}{41\cdot4\log_{10}\dfrac{2\cdot18}{0\cdot906}}\times10^{-9} = \mathbf{0\cdot22\times10^{-6}\ F}$

(iv) Charging current, $I_C = \dfrac{V}{X_C} = 2\pi f C V = 2\pi\times50\times0\cdot22\times10^{-6}\times11000 = \mathbf{0\cdot76\ A}$

Example 6.18. *Find the most economical size of a single-core cable working on a 132 kV, 3-phase system, if a dielectric stress of 60 kV/cm can be allowed.*

Solution. Phase voltage of cable $= 132/\sqrt{3} = 76\cdot21$ kV

Peak value of phase voltage, $V = 76\cdot21\times\sqrt{2} = 107\cdot78$ kV

Max. permissible stress, $g_{max} = 60$ kV/cm

$\therefore$ Most economical conductor diameter is

$$d = \frac{2V}{g_{max}} = \frac{2\times107\cdot78}{60} = \mathbf{3\cdot6\ cm}$$

Internal diameter of sheath, $D = 2\cdot718\, d = 2\cdot718\times3\cdot6 = \mathbf{9\cdot78\ cm}$

Therefore, the cable should have a conductor diameter of 3.6 cm and internal sheath diameter of 9·78 cm.

Example 6.19. *The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. Calculate the radius of the lead sheath which covers the rubber insulation and the cable capacitance per metre. A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3. The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18×10^6 V/m.*

Solution. As proved in Art 6.13,

$$g_{max} = \frac{2V}{d\log_e\dfrac{D}{d}}$$

Here, $g_{max} = E_{max} = 18\times10^6$ V/m ; V = Breakdown voltage × Safety factor

$= 10^4\times3 = 30{,}000$ volts ; $d = 2.25\times2 = 4.5$ mm

$$\therefore \quad 18\times10^6 = \frac{2\times30{,}000}{4.5\times10^{-3}\times\log_e\dfrac{D}{d}}$$

or $\dfrac{D}{d} = 2.1 \quad \therefore D = 2.1\times d = 2.1\times4.5 = 9.45$ mm

$\therefore$ Radius of sheath $= \dfrac{D}{2} = \dfrac{9.45}{2} = \mathbf{4.72\ mm}$

Capacitance, $C = \dfrac{\varepsilon_r l}{41.4\log_{10}\dfrac{D}{d}}\times10^{-9}\text{ F} = \dfrac{4\times1}{41.4\log_{10}\dfrac{9.45}{4.5}}\times10^{-9} = \mathbf{0.3\times10^{-9}\ F}$

6.15. Capacitance Between Parallel Wires

This case is of practical importance in overhead transmission lines. The simplest system for power transmission is 2-wire d.c. or a.c. system. Consider 2-wire transmission line consisting of two parallel conductors A and B spaced d metres apart in air. Suppose that radius of each conductor is r metres. Let their respective charges be $+Q$ and $-Q$ coulombs per metre length [See Fig. 6.18].

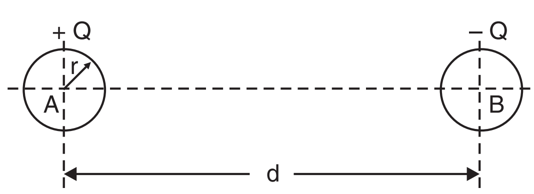


Fig. 6.18

The total p.d. between conductor A and neutral "infinite" plane is

$$V_A^* = \int_r^\infty \frac{Q}{2\pi x \varepsilon_0} dx + \int_d^\infty \frac{-Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] \text{volts} = \frac{Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Similarly, p.d. between conductor B and neutral "infinite" plane is

$$V_B = \int_r^\infty \frac{-Q}{2\pi x \varepsilon_0} dx + \int_d^\infty \frac{Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{-Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] = \frac{-Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Both these potentials are *w.r.t.* the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$$\therefore \quad \text{Capacitance, } C_{AB} = Q/V_{AB} = \frac{Q}{\dfrac{2Q}{2\pi \varepsilon_0} \log_e \dfrac{d}{r}} \text{ F/m}$$

$$\therefore \quad C_{AB} = \frac{\pi \varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F/m} \qquad ...(i)$$

The capacitance for a length l is given by ;

$$C_{AB} = \frac{\pi \varepsilon_0 l}{\log_e \dfrac{d}{r}} \text{F} \qquad \textit{... in air}$$

$$= \frac{\pi \varepsilon_0 \varepsilon_r l}{\log_e \dfrac{d}{r}} \text{F} \qquad \textit{... in a medium}$$

Example 6.20. *A 3-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2 m side. Calculate the capacitance of each line conductor per km. Given that diameter of each conductor is 1·25 cm.*

Solution. Conductor radius, r = 1·25/2 = 0·625 cm ; Spacing of conductors, d = 2 m = 200 cm

$$\text{Capacitance of each line conductor} = \frac{2\pi \varepsilon_0}{\log_e d/r} \text{F/m} = \frac{2\pi \times 8\cdot854 \times 10^{-12}}{\log_e 200/0.625} \text{F/m}$$

$$= 0\cdot0096 \times 10^{-9} \text{ F/m} = 0\cdot0096 \times 10^{-6} \text{ F/km} = \mathbf{0\cdot0096\ \mu F/km}$$

* The electric intensity E at a distance x from the centre of the conductor in air is given by ;

$$E = \frac{Q}{2\pi x \varepsilon_0} \text{ volts/m}$$

Here, Q = charge per metre length ; ε_0 = permittivity of air

As x approaches infinity, the value of E approaches zero. Therefore, the potential difference between the conductors and infinity distant neutral plane is

$$V_A = \int_r^\infty \frac{Q}{2\pi x \varepsilon_0} dx$$

6.16. Insulation Resistance of a Cable Capacitor

The cable conductor is provided with a suitable thickness of insulating material in order to prevent leakage current. The path for leakage current is radial through the insulation. The opposition offered by insulation to leakage current is known as insulation resistance of the cable. For satisfactory operation, the insulation resistance of the cable should be very high.

Consider a single-core cable of conductor radius r_1 and internal sheath radius r_2 as shown in Fig. 6.19. Let l be the length of the cable and ρ be the resistivity of the insulation.

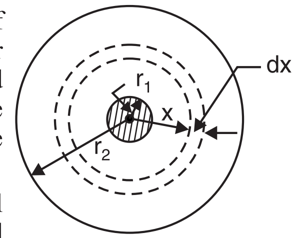


Fig. 6.19

Consider a very small layer of insulation of thickness dx at a radius x. The length through which leakage current tends to flow is dx and the area of X-section offered to this flow is $2\pi\, x\, l$.

$\therefore$ Insulation resistance of considered layer

$$= \rho \frac{dx}{2\pi\, x\, l}$$

Insulation resistance of the whole cable is

$$R = \int_{r_1}^{r_2} \rho \frac{dx}{2\pi\, x\, l} = \frac{\rho}{2\pi l}\int_{r_1}^{r_2} \frac{1}{x} dx$$

$$\therefore \quad R = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

This shows that insulation resistance of a cable is inversely proportional to its length. In other words, if the cable length increases, its insulation resistance decreases and *vice-versa.*

Example 6.21. *Two underground cables having conductor resistances of 0.7Ω and 0.5Ω and insulation resistances of 300 MΩ and 600 MΩ respectively are joined (i) in series (ii) in parallel. Find the resultant conductor and insulation resistance.*

Solution. (*i*) Series connection. In this case, conductor resistances are added like resistances in series. However, insulation resistances are given by reciprocal relation.

$\therefore$ Total conductor resistance = 0.7 + 0.5 = **1.2Ω**

The total insulation resistance R is given by ;

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{600} \quad \therefore R = \mathbf{200\ M\Omega}$$

(*ii*) Parallel connection. In this case, conductor resistances are governed by reciprocal relation while insulation resistances are added.

$$\therefore \quad \text{Total conductor resistance} = \frac{0.7 \times 0.5}{0.7 + 0.5} = \mathbf{0.3\ \Omega}$$

Total insulation resistance = 300 + 600 = **900 MΩ**

Example 6.22. *The insulation resistance of a single-core cable is 495 MΩ per km. If the core diameter is 2·5 cm and resistivity of insulation is 4·5 × 10¹⁴ Ω-cm, find the insulation thickness.*

Solution. Length of cable, l = 1 km = 1000 m

Cable insulation resistance, R = 495 MΩ = $495 \times 10^6 \Omega$

Conductor radius, r_1 = 2·5/2 = 1·25 cm

Resistivity of insulation, ρ = $4{\cdot}5 \times 10^{14}$ Ω-cm = $4{\cdot}5 \times 10^{12}$ Ωm

Let r_2 cm be the internal sheath radius.

$$\text{Now,} \quad R = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

$$\text{or} \qquad \log_e \frac{r_2}{r_1} = \frac{2\pi l R}{\rho} = \frac{2\pi \times 1000 \times 495 \times 10^6}{4 \cdot 5 \times 10^{12}} = 0 \cdot 69$$

$$\text{or} \qquad 2 \cdot 3 \log_{10} r_2/r_1 = 0 \cdot 69$$

$$\text{or} \qquad r_2/r_1 = \text{Antilog } 0 \cdot 69/2 \cdot 3 = 2$$

$$\text{or} \qquad r_2 = 2\, r_1 = 2 \times 1 \cdot 25 = 2 \cdot 5 \text{ cm}$$

$$\therefore \qquad \text{Insulation thickness} = r_2 - r_1 = 2 \cdot 5 - 1 \cdot 25 = \mathbf{1 \cdot 25\ cm}$$

Example 6.23. *The insulation resistance of a kilometre of the cable having a conductor diameter of 1.5 cm and an insulation thickness of 1.5 cm is 500 MΩ. What would be the insulation resistance if the thickness of the insulation were increased to 2.5 cm?*

Solution. $R_1 = 500$ MΩ ; $l = 100$ m ; $R_2 = ?$

$$\text{For first case :} \qquad R_1 = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

$$\text{For second case:} \qquad R_2 = \frac{\rho}{2\pi l} \log_e \frac{r_2'}{r_1'}$$

$$\therefore \qquad \frac{R_2}{R_1} = \frac{\log_e (r_2'/r_1')}{\log_e (r_2/r_1)}$$

Now, $r_1 = 1.5/2 = 0.75$ cm ; $r_2 = 0.75 + 1.5 = 2.25$ cm $\therefore r_2/r_1 = 3$

$r'_1 = 0.75$ cm ; $r'_2 = 0.75 + 2.5 = 3.25$ cm ; $\therefore r'_2/r'_1 = 4.333$

$$\therefore \qquad \frac{R_2}{500} = \frac{\log_e (4.333)}{\log_e (3)} = 1.334$$

$$\text{or} \qquad R_2 = 500 \times 1.334 = \mathbf{667.3\ M\Omega}$$

Tutorial Problems

1. A single-core cable has a conductor diameter of 2·5 cm and insulation thickness of 1·2 cm. If the specific resistance of insulation is $4 \cdot 5 \times 10^{14}$ Ω cm, calculate the insulation resistance per kilometre length of the cable. **[305·5 MΩ]**
2. A single core cable 3 km long has an insulation resistance of 1820 MΩ. If the conductor diameter is 1·5 cm and sheath diameter is 5 cm, calculate the resistivity of the dielectric in the cable. **[$28 \cdot 57 \times 10^{12}$ Ωm]**
3. Determine the insulation resistance of a single-core cable of length 3 km and having conductor radius 12·5 mm, insulation thickness 10 mm and specific resistance of insulation of 5×10^{12}Ωm. **[156 MΩ]**

6.17. Leakage Resistance of a Capacitor

The resistance of the dielectric of the capacitor is called **leakage resistance**. The dielectric in an *ideal* capacitor is a perfect insulator (*i.e.,* it has infinite resistance) and zero current flows through it when a voltage is applied across its terminals. The dielectric in a *real* capacitor has a large but finite resistance so a very small current flows between the capacitor plates when a voltage is applied.

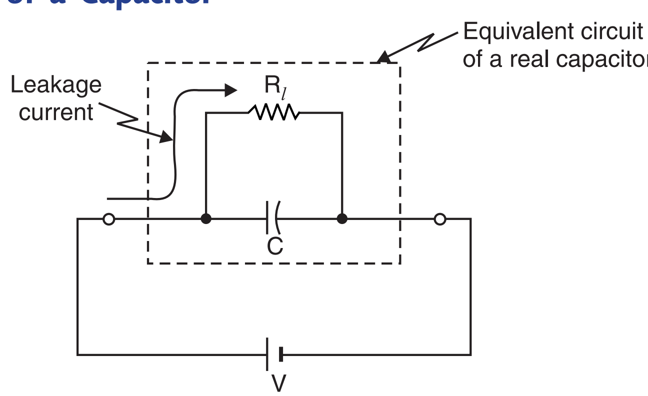


Fig. 6.20

Fig. 6.20 shows the equivalent circuit of a real capacitor consisting of an ideal capacitor in parallel with leakage resistance R_l. Typical values of leakage resistance may range from about 1 MΩ (considered a very "leaky" capacitor) to greater than 100,000 MΩ. A well designed capacitor has very high leakage resistance (> 10^4 MΩ) so that very little power is dissipated even when high voltage is applied across it.

6.18. Voltage Rating of a Capacitor

The maximum voltage that may be safely applied to a capacitor is usually expressed in terms of its d.c. working voltage.

The maximum d.c. voltage that can be applied to a capacitor without breakdown of its dielectric is called **voltage rating** *of the capacitor.*

If the voltage rating of a capacitor is exceeded, the dielectric may break down and conduct current, causing permanent damage to the capacitor. Both capacitance and voltage rating must be taken into consideration before a capacitor is used in a circuit application.

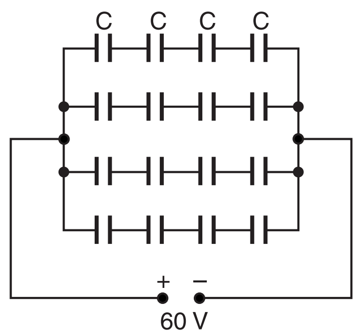


Fig. 6.21

Example 6.24. *Given some capacitors of 0·1 μF capable of withstanding 15 V. Calculate the number of capacitors needed if it is desired to obtain a capacitance of 0·1 μF for use in a circuit involving 60 V.*

Solution. Fig. 6.21 shows the conditions of the problem.

Capacitance of each capacitor, C = 0.1 μF

Voltage rating of each capacitor, V_C = 15 V

Supply voltage, V = 60 V

Since each capacitor can withstand 15 V, the number of capacitors to be connected in series = 60/15 = 4.

Capacitance of 4 series-connected capacitors, $C_T = C/4 = 0{\cdot}1/4 = 0{\cdot}025$ μF. Since it is desired to have a total capacitance of 0·1 μF, number of such rows in parallel = $C/C_T = 0{\cdot}1/0{\cdot}025 = 4$.

∴ Total number of capacitors = 4 × 4 = **16**

Fig. 6.21 shows the arrangement of capacitors.

Example 6.25. *A capacitor of capacitance C_1 = 1 μF withstands the maximum voltage V_1 = 6 kV while another capacitance C_2 = 2 μF withstands the maximum voltage V_2 = 4 kV. What maximum voltage will the system of these two capacitors withstand if they are connected in series ?*

Solution. The maximum charges Q_1 and Q_2 that can be placed on C_1 and C_2 are :

$$Q_1 = C_1V_1 = (1 \times 10^{-6}) \times (6 \times 10^3) = 6 \times 10^{-3}\ \text{C}$$

$$Q_2 = C_2V_2 = (2 \times 10^{-6}) \times (4 \times 10^3) = 8 \times 10^{-3}\ \text{C}$$

The charge on capacitor C_1 should not exceed 6×10^{-3} C. Therefore, when capacitors are connected in series, the maximum charge that can be placed on the capacitors is 6×10^{-3} C (= Q_1).

$$\therefore \quad V_{max} = \frac{Q_1}{C_1} + \frac{Q_1}{C_2} = \frac{6\times10^{-3}}{1\times10^{-6}} + \frac{6\times10^{-3}}{2\times10^{-6}}$$

$$= 6 \times 10^3 + 3 \times 10^3 = 10^3\,(6+3) = 9 \times 10^3\ \text{V} = \mathbf{9\ kV}$$

Example 6.26. *A parallel plate capacitor has plates of dimensions 2 cm × 3 cm. The plates are separated by a 1 mm thickness of paper.*

(*i*) *Find the capacitance of the paper capacitor. The dielectric constant of paper is 3·7.*

(*ii*) *What is the maximum charge that can be placed on the capacitor ? The dielectric strength of paper is 16×16^6 V/m.*

Solution. (*i*) $$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Here $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ F/m ; $\varepsilon_r = 3{\cdot}7$; $A = 6 \times 10^{-4}$ m^2; $d = 1 \times 10^{-3}$ m

$$\therefore \quad C = \frac{(8{\cdot}85 \times 10^{-12}) \times (3{\cdot}7) \times (6 \times 10^{-4})}{1 \times 10^{-3}} = \mathbf{19{\cdot}6 \times 10^{-12}\ F}$$

(*ii*) Since the thickness of the paper is 1 mm, the maximum voltage that can be applied before breakdown occurs is

$$V_{max} = E_{max} \times d$$

Here $E_{max} = 16 \times 10^6$ V/m ; $d = 1$ mm $= 1 \times 10^{-3}$ m

$$\therefore \quad V_{max} = (16 \times 10^6) \times (1 \times 10^{-3}) = 16 \times 10^3 \text{ V}$$

$\therefore$ Maximum charge that can be placed on capacitor is

$$Q_{max} = CV_{max} = (19{\cdot}6 \times 10^{-12}) \times (16 \times 10^3) = 0.31 \times 10^{-6} \text{ C} = \mathbf{0{\cdot}31\ \mu C}$$

6.19. Capacitors in Series

Consider three capacitors, having capacitances C_1, C_2 and C_3 farad respectively, connected in series across a p.d. of V volts [See Fig. 6.22 (*i*)]. In series connection, charge on each capacitor is the *same (*i.e.* $+Q$ on one plate and $-Q$ on the other) but p.d. across each is different.

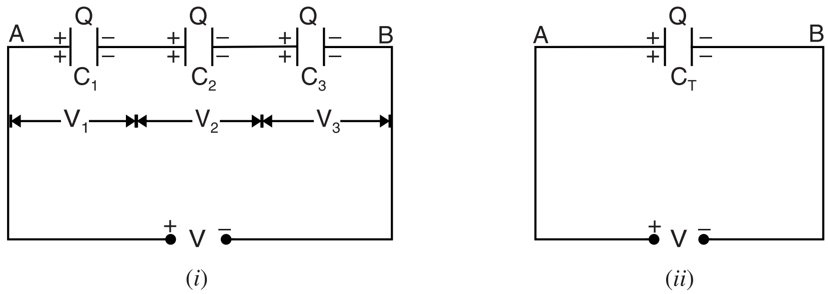


Fig. 6.22

Now, $$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

But Q/V is the **total capacitance C_T between points A and B so that $V/Q = 1/C_T$ [See Fig. 6.22 (*ii*)].

$$\therefore \quad \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus capacitors in series are treated in the same manner as are resistors in parallel.

Special Case. Frequently we come across two capacitors in series. The total capacitance in such a case is given by ;

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

* When voltage V is applied, a similar electron movement occurs on each plate. Hence the same charge is stored by each capacitor. Alternatively, current (charging) in a series circuit is the same. Since $Q = It$ and both I and t are the same for each capacitor, the charge on each capacitor is the same.

** Total or equivalent capacitance is the single capacitance which if substituted for the series capacitances, would provide the same charge for the same applied voltage.

or $$C_T = \frac{C_1 C_2}{C_1 + C_2} \quad \textit{i.e.} \quad \frac{\text{Product}}{\text{Sum}}$$

Note. The capacitors are connected in series when the circuit voltage exceeds the voltage rating of individual units. In using the series connection, it is important to keep in mind that the voltages across capacitors in series are not the same unless the capacitances are equal. The greater voltage will be across the smaller capacitance which may result in its failure if the capacitances differ very much.

6.20. Capacitors in Parallel

Consider three capacitors, having capacitances C_1, C_2 and C_3 farad respectively, connected in parallel across a p.d. of V volts [See Fig. 6.23 (*i*)]. In parallel connection, p.d. across each capacitor is the same but charge on each is different.

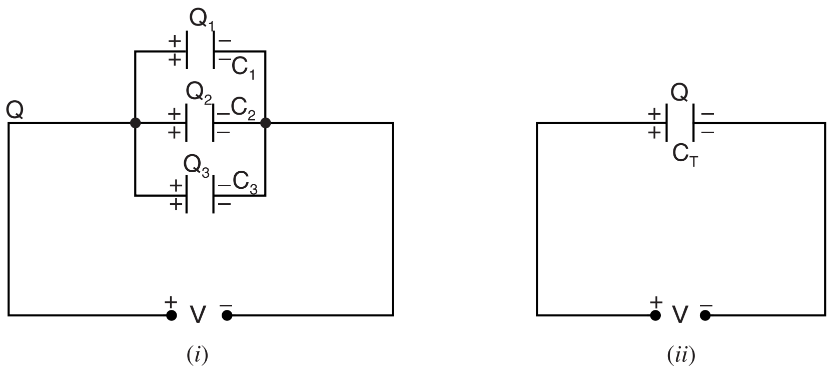


Fig. 6.23

Now, $$Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V$$
$$= V(C_1 + C_2 + C_3)$$

or $$Q/V = C_1 + C_2 + C_3$$

But Q/V is the total capacitance C_T of the parallel combination [See Fig. 6.23 (*ii*)].

$\therefore$ $$C_T = C_1 + C_2 + C_3$$

Thus capacitors in parallel are treated in the same manner as are resistors in series.

Note. Capacitors may be connected in parallel to obtain larger values of capacitance than are available from individual units.

Example 6.27. *In the circuit shown in Fig. 6.24, the total charge is 750 μC. Determine the values of V_1, V and C_2.*

Solution. $$V_1 = \frac{Q}{C_1} = \frac{750 \times 10^{-6}}{15 \times 10^{-6}} = \mathbf{50\ V}$$

$$V = V_1 + V_2 = 50 + 20 = \mathbf{70\ V}$$

Charge on $C_3 = C_3 \times V_2$
$= (8 \times 10^{-6}) \times 20$
$= 160 \times 10^{-6}\ \text{C} = 160\ \mu\text{C}$

$\therefore$ Charge on $C_2 = 750 - 160 = 590\ \mu\text{C}$

$\therefore$ Capacitance of $C_2 = \dfrac{590 \times 10^{-6}}{20}$
$= 29{\cdot}5 \times 10^{-6}\ \text{F} = \mathbf{29{\cdot}5\ \mu F}$

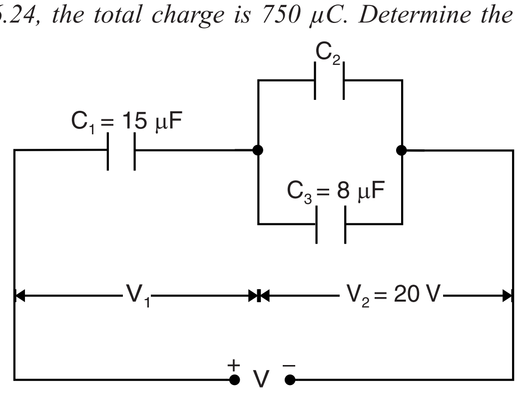
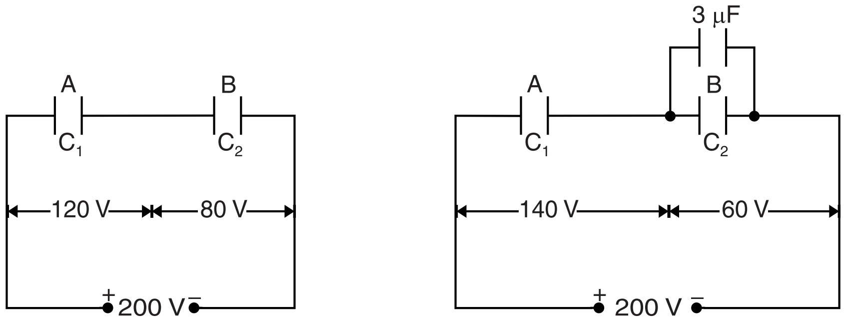


Fig. 6.24

Example 6.28. *Two capacitors A and B are connected in series across a 200 V d.c. supply. The p.d. across A is 120 V. This p.d. is increased to 140 V when a 3μF capacitor is connected in parallel with B. Calculate the capacitances of A and B.*

Solution. Let C_1 and C_2 μF be the capacitances of capacitors A and B respectively. When the capacitors are connected in series [See Fig. 6.25 (*i*)], charge on each capacitor is the same.



(*i*) (*ii*)

Fig. 6.25

$$\therefore \quad C_1 \times 120 = C_2 \times 80 \quad \text{or} \quad C_2 = 1{\cdot}5\ C_1 \qquad ...(i)$$

When a 3μF capacitor is connected in parallel with B [See Fig. 6.25 (*ii*)], the combined capacitance of this parallel branch is $(C_2 + 3)$. Thus the circuit shown in Fig. 6.25 (*ii*) can be thought as a series circuit consisting of capacitances C_1 and $(C_2 + 3)$ connected in series.

$$\therefore \quad C_1 \times 140 = (C_2 + 3)\ 60$$

$$\text{or} \quad 7C_1 - 3\ C_2 = 9 \qquad ...(ii)$$

Solving eqs. (*i*) and (*ii*), we get, C_1 = **3.6 μF ;** C_2 = **5·4 μF**

Example 6.29. *Obtain the equivalent capacitance for the network shown in Fig. 6.26. For 300 V d.c. supply, determine the charge and voltage across each capacitor.*

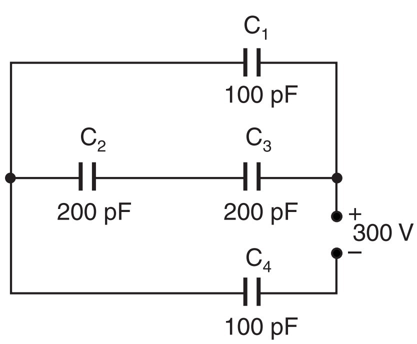


Fig. 6.26

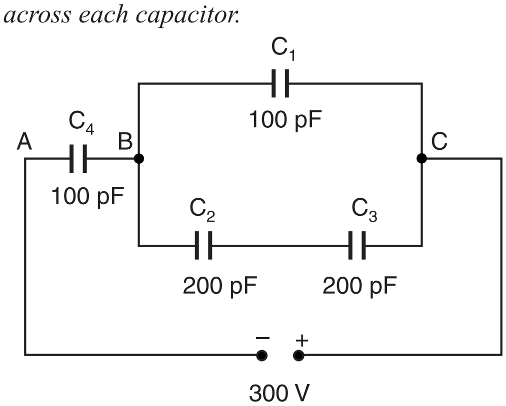


Fig. 6.27

Solution. Equivalent Capacitance. The above network can be redrawn as shown in Fig. 6.27. The equivalent capacitance C' of series-connected capacitors C_2 and C_3 is

$$C' = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

The equivalent capacitance of parallel combination C' (= 100 pF) and C_1 is

$$C_{BC} = C' + C_1 = 100 + 100 = 200 \text{ pF}$$

The entire circuit now reduces to two capacitors C_4 and C_{BC} (= 200 pF) in series.

$\therefore$ Equivalent capacitance of the network is

$$C = \frac{C_4 \times C_{BC}}{C_4 + C_{BC}} = \frac{100 \times 200}{100 + 200} = \mathbf{\frac{200}{3}\ pF}$$

Charges and p.d. on various capacitors

$$\text{Total charge, } Q = CV = \left(\frac{200}{3}\times 10^{-12}\right)\times 300 = 2\times 10^{-8}\text{ C}$$

$$\therefore \quad \text{Charge on } C_4 = \mathbf{2\times 10^{-8}\ C}$$

$$\therefore \quad \text{P.D. across } C_4, V_4 = \frac{Q}{C_4} = \frac{2\times 10^{-8}}{100\times 10^{-12}} = \mathbf{200\ V}$$

$$\text{P.D. between } B \text{ and } C, V_{BC} = 300 - 200 = 100\text{ V}$$

$$\text{Charge on } C_1, Q_1 = C_1 V_{BC} = (100\times 10^{-12})\times 100 = \mathbf{10^{-8}\ C}$$

$$\text{P.D. across } C_1, V_1 = V_{BC} = \mathbf{100\ V}$$

$$\text{P.D. across } C_2 = \text{P.D. across } C_3 = 100/2 = \mathbf{50\ V}$$

$$\text{Charge on } C_2 = \text{Charge on } C_3 = \text{Total charge} - \text{Charge on } C_1$$

$$= (2\times 10^{-8}) - (10^{-8}) = \mathbf{10^{-8}\ C}$$

Example 6.30. *Two perfect insulated capacitors are connected in series. One is an air capacitor with a plate area of 0.01 m², the plates being 1 mm apart, the other has a plate area of 0.001 m², the plates separated by a solid dielectric of 0.1 mm thickness with a dielectric constant of 5. Determine the voltage across the combination if the potential gradient in the air capacitor is 200 V/mm.*

Solution. Capacitance C_1 of air capacitor is

$$C_1 = \frac{\varepsilon_0\varepsilon_{r1}A_1}{t_1} = \frac{8.854\times 10^{-12}\times 1\times 0.01}{1\times 10^{-3}} = 88.54\times 10^{-12}\text{ F}$$

Capacitance C_2 of the capacitor with dielectric of $\varepsilon_{r_2} = 5$ is

$$C_2 = \frac{\varepsilon_0\varepsilon_{r2}A_2}{t_2} = \frac{8.854\times 10^{-12}\times 5\times 0.001}{0.1\times 10^{-3}} = 442.7\times 10^{-12}\text{ F}$$

$$\text{Voltage across } C_1, V_1 = g_1\times t_1 = 200\text{V/mm}\times 1\text{ mm} = 200\text{ V}$$

$$\text{Charge on } C_1, Q_1 = C_1V_1 = 88.54\times 10^{-12}\times 200 = 177.08\times 10^{-10}\text{ C}$$

As the capacitors are in series, the charge on each capacitor is the same *i.e.* $Q_2 = Q_1 = 177.08\times 10^{-10}$ C.

$$\therefore \quad \text{Voltage across } C_2, V_2 = \frac{Q_2}{C_2} = \frac{177.08\times 10^{-10}}{442.7\times 10^{-12}} = 40\text{ V}$$

$$\therefore \quad \text{Voltage across combination, } V = V_1 + V_2 = 200 + 40 = \mathbf{240\ volts}$$

Example 6.31. *In the network shown in Fig. 6.28 (i), $C_1 = C_2 = C_3 = C_4 = 8\ \mu F$ and $C_5 = 10\ \mu F$. Find the equivalent capacitance between points A and B.*

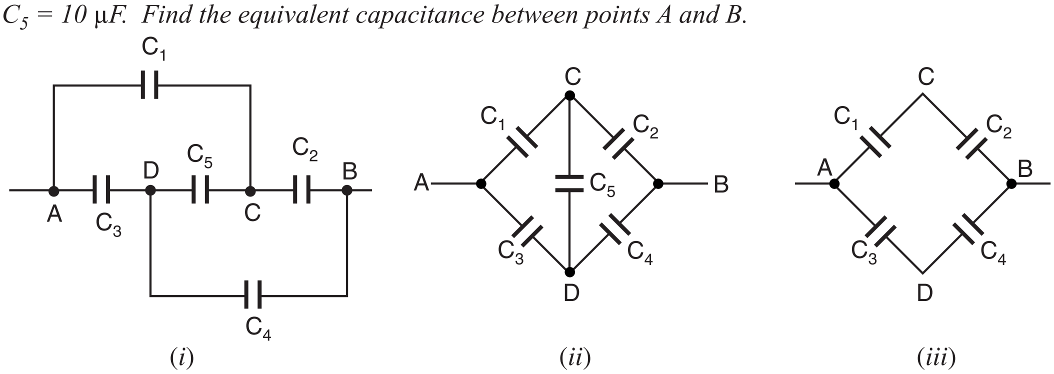


Fig. 6.28

Solution. A little reflection shows that circuit of Fig. 6.28 (*i*) can be redrawn as shown in Fig. 6.28 (*ii*). We find that the circuit is a Wheatstone bridge. Since the product of opposite arms of the bridge are equal ($C_1C_4 = C_2C_3$ because $C_1 = C_2 = C_3 = C_4$), the bridge is balanced. It means that points C and D are at the same potential. Therefore, there will be no charge on capacitor C_5. Hence, this capacitor is ineffective and can be removed from the circuit as shown in Fig. 6.28 (*iii*). Referring to Fig. 6.28 (*iii*), the equivalent capacitance C' of the series connected capacitors C_1 and C_2 is

$$C' = \frac{C_1C_2}{C_1+C_2} = \frac{8\times 8}{8+8} = 4\ \mu\text{F}$$

The equivalent capacitance C'' of series connected capacitors C_3 and C_4 [See Fig. 6.28 (*iii*)] is

$$C'' = \frac{C_3C_4}{C_3+C_4} = \frac{8\times 8}{8+8} = 4\ \mu\text{F}$$

Now $\quad C_{AB} = C' \parallel C'' = 4 \parallel 4 = 4 + 4 = \mathbf{8\ \mu F}$

Example 6.32. *Find the charge on 5 μF capacitor in the circuit shown in Fig. 6.29.*

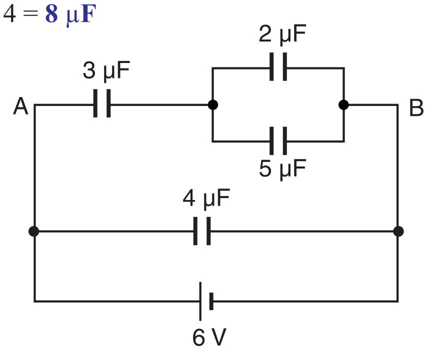


Fig. 6.29

Solution. The p.d. between A and B is 6 V. Considering the branch AB, the capacitors 2 μF and 5 μF are in parallel and their equivalent capacitance = 2 + 5 = 7 μF. The branch AB then has 7 μF and 3 μF in series. Therefore, the effective capacitance of branch AB is

$$C_{AB} = \frac{7\times 3}{7+3} = \frac{21}{10}\ \mu\text{F}$$

Total charge in branch AB is

$$Q = C_{AB}V = \frac{21}{10} \times 6 = \frac{63}{5}\ \mu\text{C}$$

$$\text{P.D. across 3 μF capacitor} = \frac{Q}{3} = \frac{63}{5}\times\frac{1}{3} = \frac{21}{5}\ \text{volts}$$

$$\therefore \quad \text{P.D. across parallel combination} = 6 - \frac{21}{5} = \frac{9}{5}\ \text{volts}$$

$$\text{Charge on 5 μF capacitor} = (5 \times 10^{-6}) \times \frac{9}{5} = 9 \times 10^{-6}\ \text{C} = \mathbf{9\ \mu C}$$

Example 6.33. *Two parallel plate capacitors A and B having capacitances of 1 μF and 5 μF are charged separately to the same potential of 100 V. Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive plate of B. Find the final charge on each capacitor.*

Solution. Initial charge on A, $Q_1 = C_1V = (1 \times 10^{-6}) \times 100 = 100\ \mu\text{C}$

Initial charge on B, $Q_2 = C_2V = (5 \times 10^{-6}) \times 100 = 500\ \mu\text{C}$

When the oppositely charged plates of A and B are connected together, the net charge is

$$Q = Q_2 - Q_1 = 500 - 100 = 400\ \mu\text{C}$$

$$\text{Final potential difference} = \frac{\text{Net charge}}{\text{Net capacitance}} = \frac{400\times 10^{-6}}{(1+5)10^{-6}} = \frac{200}{3}\ \text{V}$$

$$\text{Final charge on } A = C_1 \times \frac{200}{3} = (1\times 10^{-6})\times\frac{200}{3} = \mathbf{\frac{200}{3}\ \mu C}$$

$$\text{Final charge on } B = C_2 \times \frac{200}{3} = (5\times 10^{-6})\times\frac{200}{3} = \mathbf{\frac{1000}{3}\ \mu C}$$

Example 6.34. *A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants K_1 and K_2 respectively. Find the capacitances in two possible arrangements.*

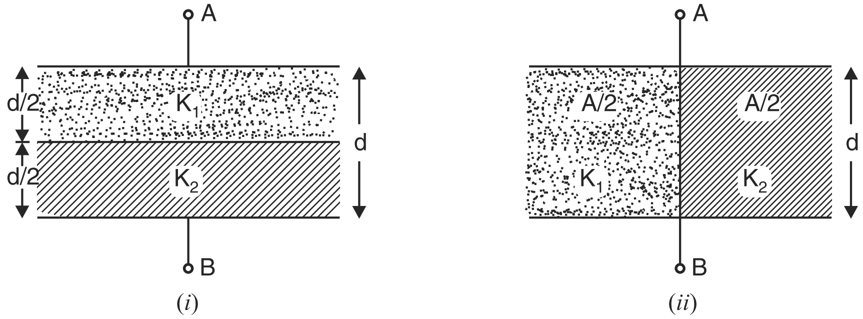


Fig. 6.30

Solution. The two possible arrangements are shown in Fig. 6.30.

(i) The arrangement shown in Fig. 6.30 (*i*) is equivalent to two capacitors in series, each with plate area A and plate separation $d/2$ *i.e.,*

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d} \;;\quad C_2 = \frac{K_2 \varepsilon_0 A}{d/2} = \frac{2K_2 \varepsilon_0 A}{d}$$

The equivalent capacitance C' is given by ;

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \quad C' = \boldsymbol{\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)}$$

(ii) The arrangement shown in Fig. 6.30 (*ii*) is equivalent to two capacitors in parallel, each with plate area $A/2$ and plate separation d *i.e.,*

$$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{d} = \frac{K_1 \varepsilon_0 A}{2d} \;;\quad C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$

The equivalent capacitance C'' is given by ;

$$C'' = C_1 + C_2 = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$\therefore \quad C'' = \boldsymbol{\frac{\varepsilon_0 A}{2d}(K_1 + K_2)}$$

Example 6.35. *Determine the capacitance between terminals A and B of the network shown in Fig. 6.31. The values shown are capacitances in μF.*

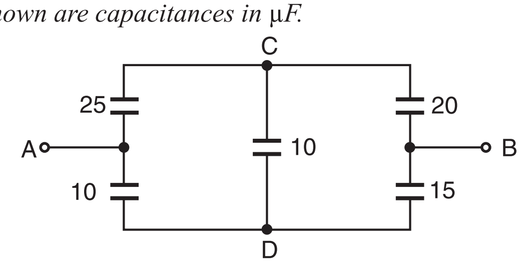


Fig. 6.31

Solution. The circuit shown in Fig. 6.31 is equivalent to the circuit shown in Fig. 6.32.

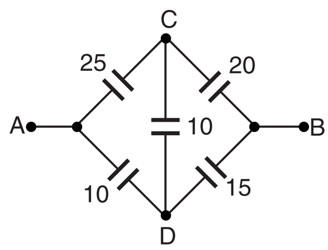


Fig. 6.32

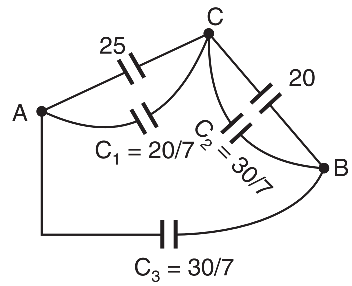


Fig. 6.33

Replacing the star network at D (consisting of capacitances 10, 10 and 15) by equivalent delta, we have,

$$C_1 = \frac{10\times10}{10+10+15} = \frac{20}{7} \qquad \text{(between } A \text{ and } C\text{)}$$

$$C_2 = \frac{10\times15}{10+10+15} = \frac{30}{7} \qquad \text{(between } B \text{ and } C\text{)}$$

$$C_3 = \frac{10\times15}{10+10+15} = \frac{30}{7} \qquad \text{(between } A \text{ and } B\text{)}$$

The circuit then reduces to the circuit shown in Fig. 6.33. Referring to Fig. 6.33,

$$C_{AC} = 25+\frac{20}{7} = \frac{195}{7} = 27{\cdot}86\ ;\ C_{BC} = 20+\frac{30}{7} = \frac{170}{7} = 24{\cdot}29$$

The circuit then reduces to the circuit shown in Fig. 6.34.

$$\therefore \qquad C_{AB} = \frac{C_{AC}\times C_{BC}}{C_{AC}+C_{BC}} + C_3$$

$$= \frac{27{\cdot}86\times24{\cdot}29}{27{\cdot}86+24{\cdot}29} + 4{\cdot}28$$

$$= 12{\cdot}98 + 4{\cdot}28 = \mathbf{17{\cdot}3\ \mu F}$$

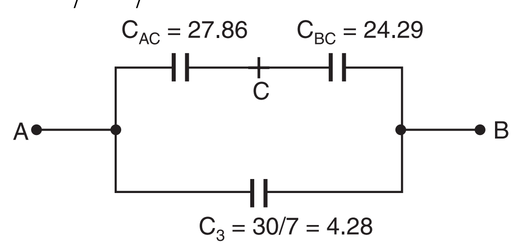


Fig. 6.34

Example 6.36. *In the network shown in Fig. 6.35, the capacitances are in μF. If the capacitance between terminals P and Q is 5 μF, find the value of C.*

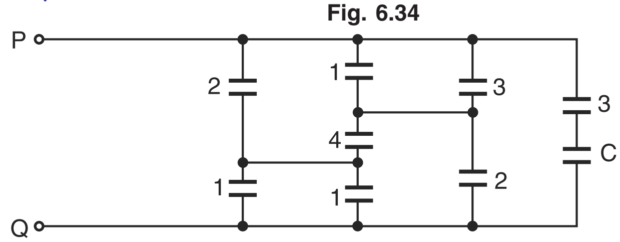


Fig. 6.35

Solution. The capacitances 1 and 1 are in parallel and their equivalent capacitance = 1 + 1 = 2. Likewise, the capacitances 1 and 3 are in parallel and their equivalent capacitance = 1 + 3 = 4. Therefore, the original circuit reduces to the circuit shown in Fig. 6.36.

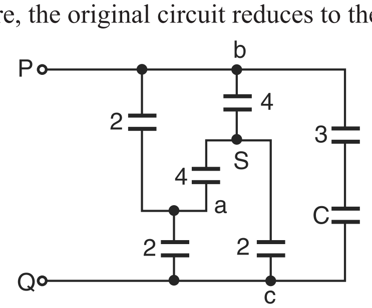


Fig. 6.36

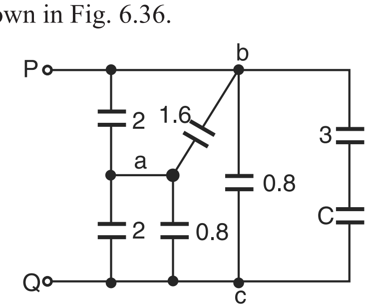


Fig. 6.37

Replacing the star network at S (consisting of capacitances 4, 4 and 2) in Fig. 6.36 by its equivalent delta network,

$$C_{ab} = \frac{4\times4}{4+4+2} = 1\cdot6 \quad ; \quad C_{bc} = \frac{4\times2}{4+4+2} = 0\cdot8 \quad ; \quad C_{ca} = \frac{4\times2}{4+4+2} = 0\cdot8$$

The circuit in Fig. 6.36 then reduces to the one shown in Fig. 6.37. Referring to Fig. 6.37, capacitances 2 and 1·6 are in parallel and their equivalent capacitance = 2 + 1·6 = 3·6. Likewise, the capacitances 2 and 0·8 are in parallel and their equivalent capacitance = 2 + 0·8 = 2·8. Therefore, the circuit shown in Fig. 6.37 reduces to that shown in Fig. 6.38.

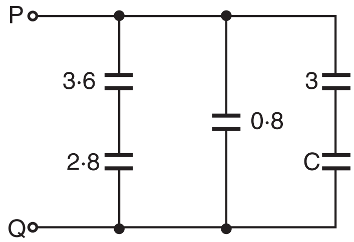


Fig. 6.38

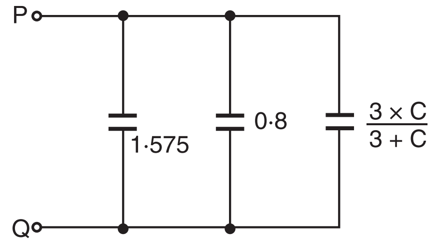


Fig. 6.39

Referring to Fig. 6.38, capacitances 3·6 and 2·8 are in series and their equivalent capacitance = 3·6 × 2·8/(3·6 + 2·8) = 1·575. Likewise, capacitances 3 and C are in series and their equivalent capacitance = 3 × C/(3 + C). The circuit shown in Fig. 6.38 reduces to that shown in Fig. 6.39. Referring to Fig. 6.39,

$$C_{PQ} = 1\cdot575 + 0\cdot8 + \frac{3\times C}{3+C}$$

or

$$5 = 1\cdot575 + 0\cdot8 + \frac{3C}{3+C} \qquad [\text{Given } C_{PQ} = 5\ \mu\text{F}]$$

$$\therefore \quad C = \mathbf{21\ \mu F}$$

Tutorial Problems

1. Three capacitors have capacitances of 2, 3 and 4μF respectively. Calculate the total capacitance when they are connected (*i*) in series (*ii*) in parallel. **[(*i*) 0·923μF (*ii*) 9μF]**
2. Three capacitors of values 8μF, 12 μF and 16μF respectively are connected in series across a 240 V d.c. supply. Calculate (*i*) the resultant capacitance and (*ii*) p.d. across each capacitor. **[(*i*) 3·7μF (*ii*) V_1 = 111V, V_2 = 74 V, V_3 = 55 V]**
3. How can three capacitors of capacitances 3μF, 6μF and 9μF respectively be arranged to give a capacitance of 11μF ? **[3μF and 6μF in series, with 9μF in parallel with both]**
4. Two capacitors of capacitances 0·5μF and 0·3μF are joined in series. What value of capacitance joined in parallel with this combination would give a capacitance of 0·5μF ? **[0·31μF]**
5. Three capacitors *A*, *B* and *C* are connected in series across a 200 V d.c. supply. The p.d.s. across the capacitors are 40 V, 70V and 90V respectively. If the capacitance of *A* is 8μF, what are the capacitances of *B* and *C* ? **[4·57 μF, 3·56 μF]**
6. A capacitor of 4μF capacitance is charged to a p.d. of 400V and then connected in parallel with an uncharged capacitor of 2μF capacitance. Calculate the p.d. across the parallel capacitors. **[267 V]**
7. Circuit *ABC* is made up as follows : *AB* consists of a 3μF capacitor, *BC* consists of a 3μF capacitor in parallel with 5μF capacitor. If a d.c. supply of 100 V is connected between *A* and *C*, determine the charge on each capacitor. **[160 μC (AB); 60 μC (3μF in BC); 100 μC]**
8. Two capacitors, *A* and *B*, having capacitances of 20μF and 30μF respectively, are connected in series to a 600 V d.c. supply. If a third capacitor *C* is connected in parallel with *A*, it is found that p.d. across *B* is 400 V. Determine the capacitance of capacitor *C*. **[40μF]**

6.21. Joining Two Charged Capacitors

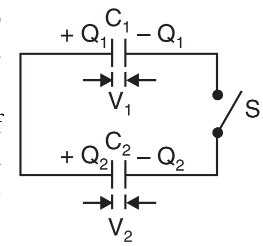


Fig. 6.40

Consider two charged capacitors of capacitances C_1 and C_2 charged to potentials V_1 and V_2 respectively as shown in Fig. 6.40. With switch S open,

$$Q_1 = C_1 V_1 \quad \text{and} \quad Q_2 = C_2 V_2$$

When switch S is closed, positive charge will flow from the capacitor of higher potential to the capacitor of lower potential. This flow of charge will continue till p.d. across each capacitor is the same. This is called *common potential* (V).

$$\text{Common potential, } V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$\therefore \qquad V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \qquad ...(i)$$

The following points may be noted :

(i) Although there is a redistribution of charge on connecting the capacitors (*i.e.,* closing switch S), the total charge before and after the connection remains the same (Remember charge is a conserved quantity). This means that charge lost by one capacitor is *equal to the charge gained by the other capacitor.

(ii) When switch S is closed, the capacitors are in parallel.

(iii) Since the two capacitors acquire the same common potential V,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

Therefore, the charges acquired by the capacitors are in the ratio of their capacitances.

(iv) In this process of charge sharing, the total stored energy of the capacitors decreases. It is because energy is dissipated as heat in the connecting wires when charge flows from one capacitor to the other.

Example 6.37. *Two capacitors of capacitances 4 μF and 6 μF respectively are connected in series across a p.d. of 250 V. The capacitors are disconnected from the supply and are reconnected in parallel with each other. Calculate the new p.d. and charge on each capacitor.*

Solution. In series-connected capacitors, p.d.s across the capacitors are in the inverse ratio of their capacitances.

$$\therefore \text{ P.D. across 4 μF capacitor} = 250 \times \frac{6}{4+6} = 150 \text{ V}$$

$$\text{Charge on 4 μF capacitor} = (4 \times 10^{-6}) \times 150 = 0{\cdot}0006 \text{ C}$$

Since the capacitors are connected in series, charge on each capacitor is the same.

$$\therefore \text{ Charge on both capacitors} = 2 \times 0{\cdot}0006 = 0{\cdot}0012 \text{ C}$$

Parallel connection. When the capacitors are connected in parallel, the total capacitance C_T = 4 + 6 = 10 μF. The total charge 0·0012 C is distributed between the capacitors to have a common p.d.

$$\therefore \qquad \text{P.D. across capacitors} = \frac{\text{Total charge}}{C_T} = \frac{0{\cdot}0012}{10 \times 10^{-6}} = \mathbf{120 \text{ V}}$$

$$\text{Charge on 4 μF capacitor} = (4 \times 10^{-6}) \times 120 = 480 \times 10^{-6} \text{C} = \mathbf{480 \text{ μC}}$$

$$\text{Charge on 6 μF capacitor} = (6 \times 10^{-6}) \times 120 = 720 \times 10^{-6} \text{C} = \mathbf{720 \text{ μC}}$$

---

* Thus referring to exp. (i), $V(C_1 + C_2) = C_1V_1 + C_2V_2$ or $C_1V_1 - C_1V = C_2V - C_2V_2$

$\therefore$ Charge lost by one = Charge gained by the other

6.22. Energy Stored in a Capacitor

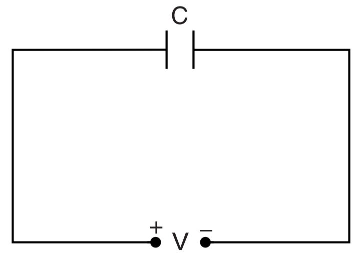


Fig. 6.41

Charging a capacitor means transferring electrons from one plate of the capacitor to the other. This involves expenditure of energy because electrons have to be moved against the *opposing forces. This energy is stored in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.

Consider a capacitor of C farad being charged from a d.c. source of V volts as shown in Fig. 6.41. Suppose at any stage of charging, the charge on the capacitor is q coulomb and p.d. across the plates is v volts.

Then, $$C = \frac{q}{v}$$

At this instant, v joules (by definition of v) of work will be done in transferring 1 C of charge from one plate to the other. If further small charge dq is transferred, then work done is

$$dW = v\,dq$$

$$= C\,v\,dv \qquad \left[\begin{array}{l} \because q = C\,v \\ \therefore dq = C\,dv \end{array}\right]$$

∴ Total work done in raising the potential of uncharged capacitor to V volts is

$$W = \int_0^V C\,v\,dv = C\left[\frac{v^2}{2}\right]_0^V$$

or $$W = \frac{1}{2}C\,V^2 \text{ joules}$$

This work done is stored in the electrostatic field set up in the dielectric.

∴ Energy stored in the capacitor is

$$E = \frac{1}{2}C\,V^2 = \frac{1}{2}{**}Q\,V = \frac{{\dagger}Q^2}{2\,C} \text{ joules}$$

Note that an ideal (or pure) capacitor does not dissipate or consume energy; instead, it *stores* energy.

6.23. Energy Density of Electric Field

The energy stored per unit volume of the electric field is called **energy density** *of the electric field*

∴ $$\text{Energy density, } u = \frac{\text{Total energy stored}\,(U)}{\text{Volume of electric field}}$$

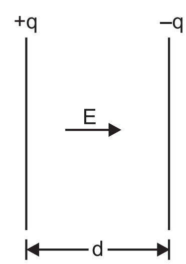


Fig. 6.42

We have seen that energy is stored in the electric field of a capacitor. In fact, wherever electric field exists, there is stored energy. While dealing with electric fields, we are generally interested in energy density (u) *i.e.* energy stored per unit volume. Consider a charged parallel plate capacitor of plate area A and plate separation d as shown in Fig. 6.42.

$$\text{Energy stored} = \frac{1}{2}CV^2$$

Volume of space between plates = $A\,d$

∴ $$\text{Energy density, } u = \frac{\text{Energy stored}}{\text{Volume}} = \frac{CV^2}{2Ad}$$

---

* Electrons are being pushed to the negative plate which tends to repel them. Similarly, electrons are removed from the positive plate which tends to attract them. In either case, forces oppose the transfer of electrons from one plate to the other. This opposition increases as the charge on the plates increases.

** Putting $C = Q/V$ in the exp., $E = \frac{1}{2}QV$

† Putting $V = Q/C$ in the exp., $E = Q^2/2C$

We know that capacitance of a parallel plate capacitor is $C = \varepsilon_0 A/d$.

$$\therefore \qquad u = \frac{\varepsilon_0 A}{d} \times \frac{V^2}{2Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

But V/d is the electric field intensity (E) between the plates.

$$\therefore \qquad \text{Energy density, } u = \frac{1}{2}\varepsilon_0 E^2 \qquad \textit{... in air} \qquad ...(i)$$

$$= \frac{1}{2}\varepsilon_0 \varepsilon_r E^2 \qquad \textit{... in a medium} \qquad ...(ii)$$

Obviously, the unit of energy density will be joules/m^3.

Therefore, energy density (i.e., electric field energy stored per unit volume) in any region of space is directly proportional to the square of the electric field intensity in that region.

Note that we derived exps. (i) and (ii) for the special case of a parallel plate capacitor. But it can be shown to be true for any region of space where electric field exists.

Note. We can also express energy density of electric field in terms of electric flux density D ($= \varepsilon_0 \varepsilon_r E$).

$$u = \frac{1}{2}DE = \frac{D^2}{2\varepsilon_0\varepsilon_r}$$

Example 6.38. *A 16 µF capacitor is charged to 100 V. After being disconnected, it is immediately connected in parallel with an uncharged capacitor of capacitance 4µF. Determine (i) the p.d. across the combination, (ii) the electrostatic energies before and after the capacitors are connected in parallel and (iii) loss of energy.*

Solution. $C_1 = 16\ \mu\text{F}\ ; \quad C_2 = 4\ \mu\text{F}$

Before joining

Charge on 16 µF capacitor, $Q = C_1 V_1 = (16 \times 10^{-6}) \times 100 = 1{\cdot}6 \times 10^{-3}$ C

$$\text{Energy stored, } E_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(16 \times 10^{-6}) \times 100^2 = \mathbf{0{\cdot}08\ J}$$

After joining. When the capacitors are connected in parallel, the total capacitance $C_T = C_1 + C_2 = 16 + 4 = 20$ µF. The charge $1{\cdot}6 \times 10^{-3}$ C distributes between the two capacitors to have a common p.d. of V volts.

$$\text{P.D. across parallel combination, } V = \frac{Q}{C_T} = \frac{1{\cdot}6\times 10^{-3}}{20\times 10^{-6}} = \mathbf{80\ V}$$

$$\text{Energy stored, } E_2 = \frac{1}{2}C_T V^2 = \frac{1}{2}(20\times 10^{-6})\times(80)^2 = \mathbf{0.064\ J}$$

$$\text{Loss of energy} = E_1 - E_2 = 0{\cdot}08 - 0{\cdot}064 = \mathbf{0{\cdot}016\ J}$$

It may be noted that there is a loss of energy. This is due to the heat dissipated in the conductor connecting the capacitors.

Example 6.39. *A capacitor-type stored-energy welder is to deliver the same heat to a single weld as a conventional weld that draws 20 kVA at 0·8 p.f. for 0·0625 second/weld. If C = 2000 µF, find the voltage to which it is charged.*

Solution. The energy supplied per weld in a conventional welder is

$$W = VA \times \cos\phi \times \text{time} = (20 \times 10^3) \times (0{\cdot}8) \times 0{\cdot}0625 = 1000 \text{ J}$$

The stored energy in the capacitor should be 1000 J.

$$\therefore \qquad 1000 = \frac{1}{2}CV^2$$

$$\text{or} \qquad V = \sqrt{\frac{2\times 1000}{C}} = \sqrt{\frac{2\times 1000}{2000\times 10^{-6}}} = \mathbf{1000\ V}$$

Example 6.40. *A parallel plate 100 μF capacitor is charged to 500 V. If the distance between the plates is halved, what will be the new potential difference between the plates and what will be the new stored energy ?*

Solution. $C = 100\ \mu F = 100 \times 10^{-6}\ F = 10^{-4}\ F\ ;\ V = 500$ volts

When plate separation is decreased to half, the new capacitance C' becomes twice *i.e.,* $C' = 2C$. Since the capacitor is not connected to the battery, the charge on the capacitor remains the same. The potential difference between the plates must decrease to maintain the same charge.

$$\therefore \quad Q = CV = C'V' \quad \text{or} \quad V' = \frac{CV}{C'} = \frac{CV}{2C} = \frac{V}{2} = \frac{500}{2} = \mathbf{250\ volts}$$

$$\text{New stored energy} = \frac{1}{2}C'V'^2 = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$$

$$= \frac{1}{2}\frac{CV^2}{2} = \frac{1}{2}\left(\frac{1}{2}CV^2\right)$$

$$= \frac{1}{2}\left[\frac{1}{2}\times 10^{-4}\times(500)^2\right] = \mathbf{6{\cdot}25\ J}$$

Example 6.41. *A parallel-plate capacitor is charged with a battery to a charge q_0 as shown in Fig. 6.43 (i). The battery is then removed and the space between the plates is filled with a dielectric of dielectric constant K. Find the energy stored in the capacitor before and after the dielectric is inserted.*

Solution. Energy stored in the capacitor in the absence of dielectric is

$$^*E_0 = \frac{1}{2}C_0V_0^2$$

Since $V_0 = q_0/C_0$, this can be expressed as :

$$E_0 = \frac{q_0^2}{2C_0} \quad \ldots(i)$$

Eq. (*i*) gives the energy stored in the capacitor in the absence of dielectric.

After the battery is removed and the dielectric is inserted between the plates, *charge on the capacitor remains the same.* But the capacitance of the capacitor is increased K times *i.e.,* new capacitance is $C' = K\,C_0$ [See Fig. 6.43 (*ii*)].

$\therefore$ Energy stored in the capacitor after insertion of dielectric is

$$E = \frac{q_0^2}{2C'} = \frac{q_0^2}{2K\,C_0} = \frac{E_0}{K}$$

or

$$E = \frac{E_0}{K} \quad \ldots(ii)$$

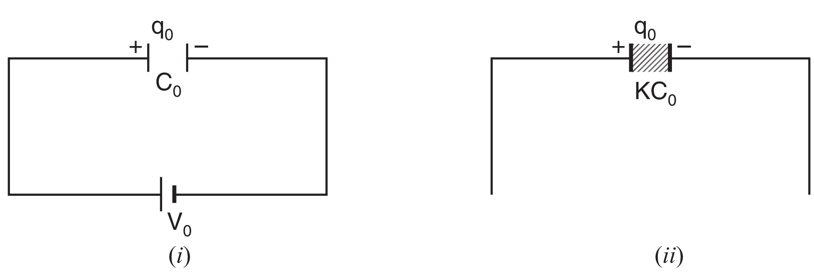


Fig. 6.43

* The subscript 0 indicates the conditions when the medium is air.

Since $K > 1$, we find that final energy is *less* than the initial energy by the factor $1/K$. How will you account for "missing energy" ? When the dielectric is inserted into the capacitor, it gets pulled into the device. The external agent must do negative work to keep the dielectric from accelerating. This work is simply $= E_0 - E$. Alternately, the positive work done by the system $= E_0 - E$.

Example 6.42. *Suppose in the above problem, the capacitor is kept connected with the battery and then dielectric is inserted between the plates. What will be the change in charge, the capacitance, the potential difference, the electric field and the stored energy ?*

Solution. Since the battery remains connected, the potential difference V_0 will **remain unchanged.**

As a result, electric field $(= V_0/d)$ will also **remain unchanged.**

The capacitance C_0 will increase to $C = \boldsymbol{K\,C_0}$.

The charge will also increase to $q = K\,q_0$ as explained below.

$$q_0 = C_0 V_0 \;;\quad q = CV_0 = KC_0 V_0 = \boldsymbol{K\,q_0}$$

$$\text{Initial stored energy, } E_0 = \frac{1}{2}C_0V_0^2$$

$$\text{Final stored energy, } E = \frac{1}{2}CV_0^2 = \frac{1}{2}K\,C_0V_0^2 = KE_0$$

$$\therefore \quad E = \boldsymbol{KE_0}$$

Note that stored energy is increased K times. Will any work be done in inserting the dielectric? The answer is yes. In this case, the work will be done by the battery. The battery not only gives the increased energy to the capacitor but also provides the necessary energy for inserting the dielectric.

Example 6.43. *An air-capacitor of capacitance 0·005 μF connected to a direct voltage of 500 V is disconnected and then immersed in oil with a relative permittivity of 2·5. Find the energy stored in the capacitor before and after immersion.*

Solution. Energy before immersion, $E_1 = \frac{1}{2}CV^2 = \frac{1}{2}\times 0\cdot 005\times 10^{-6}\times(500)^2 = \mathbf{625 \times 10^{-6}\ J}$

When the capacitor is immersed in oil, its capacitance becomes $C' = \varepsilon_r C = 2\cdot 5 \times 0\cdot 005 = 0\cdot 0125$ μF. Since charge remains the same $(V = Q/C)$, new voltage is decreased and becomes $V' = V/\varepsilon_r = 500/2\cdot 5 = 200$ V.

$\therefore$ Energy after immersion, $E_2 = \frac{1}{2}C'V'^2 = \frac{1}{2}\times 0\cdot 0125\times 10^{-6}\times(200)^2 = \mathbf{250 \times 10^{-6}\ J}$

Example 6.44. *In the circuit shown in Fig. 6.44, the battery e.m.f. is 100 V and the capacitor has a capacitance of 1 μF. The switch is operated 100 times every second. Calculate (i) the average current through the switch between switching operations and (ii) the average power dissipated in the resistor. It may be assumed that the capacitor is ideal and that the capacitor is fully charged or discharged before the subsequent switching.*

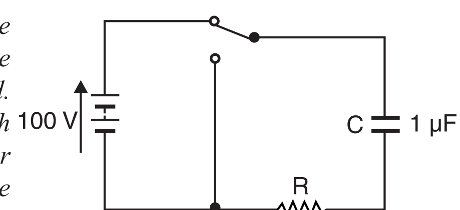


Fig. 6.44

Solution. **(*i*)** Maximum charge on capacitor, $Q = CV = (1 \times 10^{-6}) \times (100) = 10^{-4}$ C

The time taken to acquire this charge (or to lose it) is

$$T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s}$$

$$\therefore \quad \text{Average current, } I_{av} = \frac{\Delta Q}{\Delta T} = \frac{10^{-4}}{0\cdot 01} = 0\cdot 01 \text{ A} = \mathbf{10\ mA}$$

(*ii*) The maximum energy stored during charging is

$$E_m = \frac{1}{2}CV^2 = \frac{1}{2}\times 10^{-6}\times(100)^2 = 0{\cdot}005 \text{ J}$$

During the charging period, a similar quantity of energy must be dissipated in the resistor. In the subsequent discharging period, the stored energy in the capacitor is dissipated in the resistor. Hence for every switching action, 0·005 J is dissipated in the resistor. For 100 switching operations, the energy E dissipated is

$$E = 100 \times 0{\cdot}005 = 0{\cdot}5 \text{ J}$$

$$\text{Average power taken} = \frac{\Delta E}{\Delta T} = \frac{0{\cdot}5}{1} = \mathbf{0{\cdot}5\ W}$$

Note that amount of energy stored in a capacitor is very small because the value of C is very small.

6.24. Force on Charged Plates

Consider two parallel conducting plates x metres apart and carrying constant charges of $+Q$ and $-Q$ coulombs respectively as shown in Fig. 6.45. Let the force of attraction between the two plates be F newtons. If one of the plates is moved away from the other by a small distance dx, then work done is

$$\text{Work done} = F \times dx \text{ joules} \qquad \ldots(i)$$

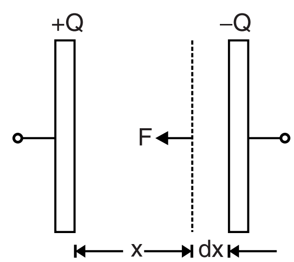


Fig. 6.45

Since the charges on the plates remain constant, no electrical energy can enter or leave the system during the movement dx.

$$\therefore \quad \text{Work done} = \text{Change in stored energy}$$

$$\text{Initial stored energy} = \frac{1}{2}\frac{Q^2}{C} \text{ joules}$$

Since the separation of the plates has increased, the capacitance will decrease by dC. The final capacitance is, therefore, $(C - dC)$.

$$\text{Final stored energy} = \frac{1}{2}\frac{Q^2}{(C-dC)} = \frac{Q^2(C+dC)^*}{2[C^2-(dC)^2]}$$

Since dC is small compared to C, $(dC)^2$ can be neglected compared to C^2.

$$\therefore \quad \text{Final stored energy} = \frac{Q^2(C+dC)}{2C^2} = \frac{Q^2}{2C} + \frac{Q^2}{2C^2}dC$$

$$\therefore \quad \text{Change in stored energy} = \left(\frac{Q^2}{2C} + \frac{Q^2}{2C^2}dC\right) - \frac{Q^2}{2C} = \frac{Q^2}{2C^2}dC \qquad \ldots(ii)$$

Equating eqs. (*i*) and (*ii*), we get,

$$F \times dx = \frac{Q^2}{2C^2}dC$$

or

$$F = \frac{Q^2}{2C^2}\frac{dC}{dx}$$

$$= \frac{1}{2}V^2\frac{dC}{dx} \qquad \ldots(iii) \quad (\because V = Q/C)$$

Now

$$C = \frac{\varepsilon_0\,\varepsilon_r\,A}{x}$$

* Note this exp. Multiply the numerator and denominator by $(C + dC)$.

$$\therefore \quad \frac{dC}{dx} = -\frac{\varepsilon_0\,\varepsilon_r A}{x^2}$$

$\therefore$ Substituting the value of dC/dx in eq. (*iii*), we get,

$$F = -\frac{1}{2}V^2\,\frac{\varepsilon_0\,\varepsilon_r\,A}{x^2} = -\frac{1}{2}\varepsilon_0\,\varepsilon_r A\left(\frac{V}{x}\right)^2$$

$$= -\frac{1}{2}\varepsilon_0\,\varepsilon_r\,A\,E^2 \qquad \text{...in a medium}$$

$$= -\frac{1}{2}\varepsilon_0\,A\,E^2 \qquad \text{...in air}$$

This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of V volts. The negative sign shows that it is a force of attraction.

Note. The force of attraction between charged plates may be utilised as a means of measuring potential difference. An instrument of this kind is known as an **electrostatic voltmeter**.

Example 6.45. *A parallel plate capacitor has its plates separated by 0·5 mm of air. The area of plates is 2 m*^{2} *and they are charged to a p.d. of 100 V. The plates are pulled apart until they are separated by 1 mm of air. Assuming the p.d. to remain unchanged, what is the mechanical force experienced in separating the plates ?*

Solution. Here, $A = 2\text{m}^2$; $d = 0.5$ mm $= 0.5 \times 10^{-3}$ m ; $V = 100$ volts

$$\text{Initial capacitance, } C_1 = \frac{\varepsilon_0\,A}{d} = \frac{8\cdot85\times10^{-12}\times2}{0\cdot5\times10^{-3}} = 35\cdot4\times10^{-9}\,\text{F}$$

$$\text{Initial stored energy, } E_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}\times(35\cdot4\times10^{-9})\times100^2 = 17\cdot7\times10^{-5}\ \text{J}$$

$$\text{Final capacitance, } C_2 = \frac{1}{2}C_1 = \frac{1}{2}(35.4\times10^{-9}) = 17.7\times10^{-9}\ \text{F}$$

$$\text{Final stored energy, } E_2 = \frac{1}{2}C_2V^2 = \frac{1}{2}(17\cdot7\times10^{-9})\times100^2 = 8\cdot85\times10^{-5}\ \text{J}$$

$$\text{Change in stored energy} = (17\cdot7 - 8\cdot85)\times10^{-5} = 8\cdot85\times10^{-5}\ \text{J}$$

Suppose F newtons is the average mechanical force between the plates. The plates are separated by a distance $dx = 1 - 0{\cdot}5 = 0{\cdot}5$ mm.

$$\therefore \quad F \times dx = \text{Change in stored energy}$$

or
$$F = \frac{8\cdot85\times10^{-5}}{0\cdot5\times10^{-3}} = \mathbf{17.7\times10^{-2}\ N}$$

Note that small low-voltage capacitors store microjoules of energy.

6.25. Behaviour of Capacitor in a D.C. Circuit

When d.c. voltage is applied to an uncharged capacitor, there is transfer of electrons from one plate (connected to +ve terminal of source) to the other plate (connected to –ve terminal of source). This is called *charging current* because the capacitor is being charged. The capacitor is *quickly* charged to the applied voltage and charging current becomes zero. Under this condition, the capacitor is said to be fully charged. When a wire is connected across the charged capacitor, the excess electrons on the negative plate move through connecting wire to the positive plate. The energy stored in the capacitor is dissipated in the resistance of the wire. The charge is neutralised when the number of free electrons on both plates are again equal. At this time, the voltage across the capacitor is zero and the capacitor is fully discharged. The behaviour of a capacitor in a d.c. circuit is summed up below :

(*i*) When d.c. voltage is applied to an uncharged capacitor, the capacitor is quickly (*not instantaneously*) charged to the applied voltage.

Charging current, $i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$

When the capacitor is fully charged, capacitor voltage becomes constant and is equal to the applied voltage. Therefore, $dV/dt = 0$ and so is the charging current. Note that dV/dt is the slope of v–t graph of a capacitor.

(*ii*) A capacitor can have voltage across it even when there is no current flowing.

(*iii*) The voltage across a capacitor ($Q = CV$) is proportional to *charge* and not the current.

(*iv*) *There is no current through the dielectric of the capacitor during charging or discharging because the dielectric is an insulating material.* There is merely transfer of electrons from one plate to the other through the connecting wires.

(*v*) When the capacitor is fully charged, there is no circuit current. *Therefore, a fully charged capacitor appears as an open to d.c.*

(*vi*) *An uncharged capacitor is equivalent to a *short circuit as far as d.c. voltage is concerned.* Therefore, a capacitor must be charged or discharged by connecting a resistance in series with it to limit the charging or discharging current.

(*vii*) When the circuit containing capacitor is disconnected from the supply, the capacitor remains charged for a long period. *If the capacitor is charged to a high value, it can be dangerous to someone working on the circuit.*

Example 6.46. *A certain voltage source causes the current to an initially discharged 1000 μF capacitor to increase at a constant rate of 0·06 A/s. Find the voltage across the capacitor after t = 10 s.*

Solution. Charging current, $i_C = 0{\cdot}06t$

$\therefore$ Voltage across the capacitor after $t = 10$ s is

$$**v_C = \frac{1}{C}\int_0^{10} i_C\,dt = \frac{1}{1000\times10^{-6}}\int_0^{10} 0\cdot06t$$

$$= 10^3\times0\cdot06\int_0^{10} t\,dt = 60\left[\frac{t^2}{2}\right]_0^{10}$$

$$= 60\times\frac{10^2}{2} = \mathbf{3000\ V}$$

Example 6.47. *A voltage across a 100 μF capacitor varies as follows : (i) uniform increase from 0 V to 700 V in 10 sec (ii) a uniform decrease from 700 V to 400 V in 2 sec (iii) a steady value of 400 V (iv) an instantaneous drop from 400 V to zero. Find the circuit current during each period.*

Solution. $i = C\frac{dv}{dt} = 100\times10^{-6}\frac{dv}{dt} = 10^{-4}\frac{dv}{dt}$ A

(*i*) $dv = 700$ V ; $dt = 10$ sec

$\therefore$ $i = 10^{-4}\times\frac{700}{10} = 7\times10^{-3}$ A $=$ **7 mA**

(*ii*) $dv = 700 - 400 = 300$ V ; $dt = 2$ sec

$\therefore$ $i = 10^{-4}\times\frac{300}{2} = 15\times10^{-3}$ A $=$ **15 mA**

* When d.c. voltage is applied to an uncharged capacitor, the charging current is limited only by the small resistance of source and any wiring resistance present. The surge current that flows when no resistor is present may be great enough to damage the capacitor, the source or both.

** $i = C\frac{dv}{dt}$ or $\frac{dv}{dt} = \frac{i}{C}$ $\therefore$ Integrating, $v = \frac{1}{C}\int_0^t i\,dt$

(*iii*) $dv/dt = 0$. Therefore, current is **zero**.

(*iv*) $dv = 400 - 0 = 400$ V ; $dt = 0$

$$\therefore \quad i = 10^{-4} \times \frac{400}{0} = \textbf{infinite}$$

Note that in this period, the current is extremely high.

6.26. Charging of a Capacitor

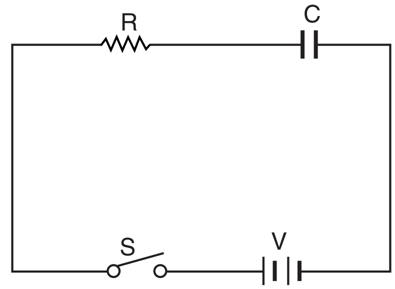


Fig. 6.46

Consider an uncharged capacitor of capacitance C farad connected in series with a resistor R to a d.c. supply of V volts as shown in Fig. 6.46. When the switch is closed, the capacitor starts charging up and charging current flows in the circuit. The charging current is maximum at the instant of switching and decreases gradually as the voltage across the capacitor increases. When the capacitor is charged to applied voltage V, the charging current becomes zero.

1. **At switching instant.** At the instant the switch is closed, the voltage across capacitor is zero since we started with an uncharged capacitor. The entire voltage V is dropped across resistance R and charging current is maximum (call it I_m).

$\therefore$ Initial charging current, $I_m = V/R$

Voltage across capacitor $= 0$

Charge on capacitor $= 0$

2. **At any instant.** After having closed the switch, the charging current starts decreasing and the voltage across capacitor gradually increases. Let at any time t during charging :

i = Charging current

v = P.D. across C

q = Charge on capacitor = $C\,v$

(*i*) Voltage across capacitor

According to Kirchhoff's voltage law, the applied voltage V is equal to the sum of voltage drops across resistor and capacitor.

$$\therefore \quad V = v + iR \quad \ldots(i)$$

$$\text{or} \quad V = v + CR^* \frac{dv}{dt}$$

$$\text{or} \quad -\frac{dv}{V - v} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\int -\frac{dv}{V - v} = \int -\frac{dt}{RC}$$

$$\text{or} \quad \log_e (V - v) = -\frac{t}{RC} + K \quad \ldots(ii)$$

where K is a constant whose value can be determined from the initial conditions. At the instant of closing the switch S, $t = 0$ and $v = 0$.

Substituting these values in eq. (*ii*), we get, $\log_e V = K$.

Putting the value of $K = \log_e V$ in eq. (*ii*), we get,

$$\log_e (V - v) = -\frac{t}{RC} + \log_e V$$

* $i = \dfrac{dq}{dt} = \dfrac{d}{dt}(q) = \dfrac{d}{dt}(Cv) = C\dfrac{dv}{dt}$

or $$\log_e \frac{V-v}{V} = -\frac{t}{RC}$$

or $$\frac{V-v}{V} = e^{-t/RC}$$

$\therefore$ $$v = V\left[1 - e^{-t/RC}\right] \qquad \ldots(iii)$$

This is the expression for variation of voltage across the capacitor (v) w.r.t. time (t) and is represented graphically in Fig. 6.47 (i). *Note that growth of voltage across the capacitor follows an exponential law*. An inspection of eq. (iii) reveals that as t increases, the term $e^{-t/RC}$ gets smaller and voltage v across capacitor gets larger.

(*ii*) Charge on Capacitor

$$q = \text{Charge at any time } t$$

$$Q = \text{Final charge}$$

Since $v = q/C$ and $V = Q/C$, the exp. (iii) becomes :

$$\frac{q}{C} = \frac{Q}{C}\left[1 - e^{-t/RC}\right]$$

or $$q = Q\left(1 - e^{-t/RC}\right) \qquad \ldots(iv)$$

Again the increase of charge on capacitor plates follows exponential law.

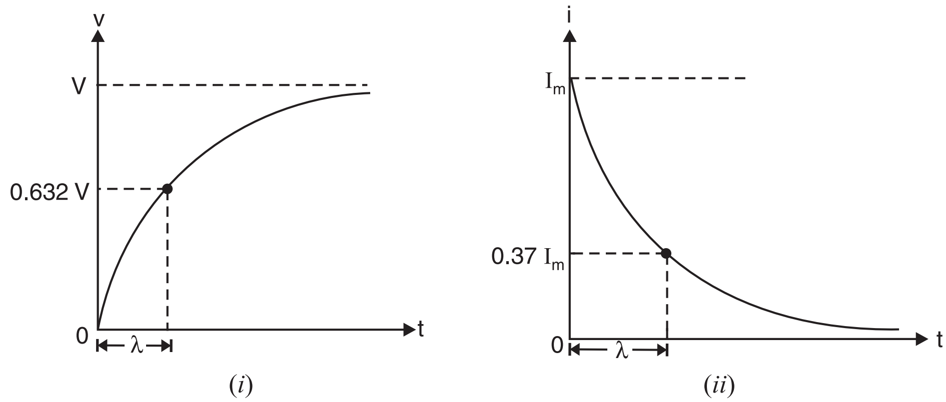


Fig. 6.47

(*iii*) Charging current

From exp. (i), $$V - v = iR$$

From exp. (iii), $$V - v = V e^{-t/RC}$$

$\therefore$ $$iR = V e^{-t/RC}$$

or $$i = \frac{V}{R} e^{-t/RC}$$

$\therefore$ $$i = I_m e^{-t/RC}$$

where I_m (= V/R) is the initial charging current. *Again the charging current decreases following exponential law.* This is also represented graphically in Fig. 6.47 (ii).

(*iv*) Rate of rise of voltage across capacitor

We have seen above that : $$V = v + CR\frac{dv}{dt}$$

At the instant the switch is closed, $v = 0$.

$\therefore$ $$V = CR\frac{dv}{dt}$$

or Initial rate of rise of voltage across capacitor is given by ;

$$\frac{dv}{dt} = \frac{V}{CR} \text{ volts/sec} \quad \ldots(iv)$$

Note. *The capacitor is almost fully charged in a time equal to 5 RC i.e., 5 time constants.*

6.27. Time Constant

Consider the eq. (*iii*) above showing the rise of voltage across the capacitor :

$$v = V(1 - e^{-t/RC})$$

The exponent of e is t/RC. The quantity RC has the *dimensions of time so that exponent of e is a number. The quantity RC in called the *time constant* of the circuit and affects the charging (or discharging) time. It is represented by λ (or T or τ).

$\therefore$ Time constant, $\lambda = RC$ seconds

Time constant may be defined in one of the following ways :

(*i*) At the instant of closing the switch, p.d. across capacitor is zero. Therefore, putting $v = 0$ in the expression $V = v + CR\frac{dv}{dt}$, we have,

$$V = CR\frac{dv}{dt}$$

or $$\frac{dv}{dt} = \frac{V}{CR}$$

If this rate of rise of voltage could continue, the capacitor voltage will reach the final value V in time = $V \div V/CR = RC$ seconds = time constant λ.

Hence **time constant** *may be defined as the time required for the capacitor voltage to rise to its final steady value V if it continued rising at its initial rate* (*i.e., V/CR*).

(*ii*) If the time interval $t = \lambda$ (or RC), then,

$$v = V(1 - e^{-t/t}) = V(1 - e^{-1}) = 0{\cdot}632\, V$$

Hence **time constant** *can also be defined as the time required for the capacitor voltage to reach 0·632 of its final steady value V.*

(*iii*) If the time interval $t = \lambda$ (or RC), then,

$$i = I_m e^{-t/t} = I_m e^{-1} = 0{\cdot}37\, I_m$$

Hence **time constant** *can also be defined as the time required for the charging current to fall to 0·37 of its initial maximum value* I_m.

Fig. 6.48 as well as adjoining table shows the percentage of final voltage (V) after each time constant interval during voltage buildup (v) across the capacitor. An uncharged capacitor charges to about 63% of its fully charged voltage (V) in first time constant. A 5 time-constant interval is accepted as the time to fully charge (or discharge) a capacitor and is called the *transient time*.

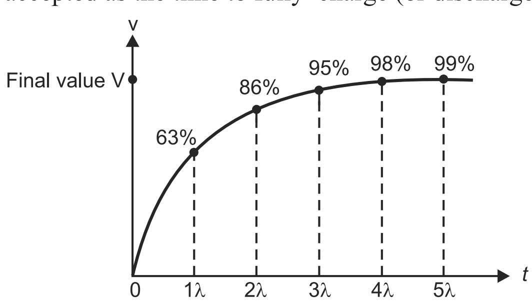


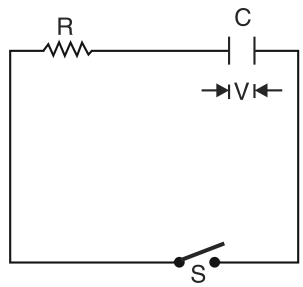
Fig. 6.48

Number of time constants	% of final value
1	63
2	86
3	95
4	98
5	99 considered 100 %

* $RC = \left(\frac{\text{Volt}}{\text{Ampere}}\right) \times \left(\frac{\text{Coulomb}}{\text{Volt}}\right) = \frac{\text{Volt}}{(\text{Coulomb/sec})} \times \left(\frac{\text{Coulomb}}{\text{Volt}}\right) = \text{seconds}$

6.28. Discharging of a Capacitor

Consider a capacitor of C farad charged to a p.d. of V volts and connected in series with a resistance R through a switch S as shown in Fig. 6.49 (*i*). When the switch is open, the voltage across the capacitor is V volts. When the switch is closed, the voltage across capacitor starts decreasing. The discharge current rises instantaneously to a value of V/R (= I_m) and then decays gradually to zero.



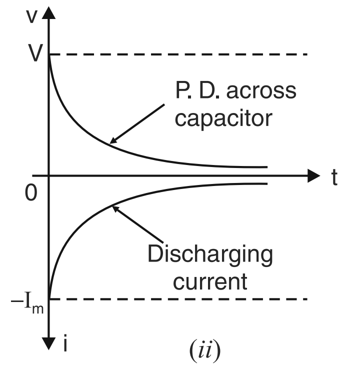


Fig. 6.49

Let at any time t during discharging,

$$v = \text{p.d. across the capacitor}$$

$$i = \text{discharging current}$$

$$q = \text{charge on capacitor}$$

By Kirchhoff's voltage law, we have,

$$0 = v + RC\frac{dv}{dt}$$

or
$$\frac{dv}{v} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\int\frac{dv}{v} = -\frac{1}{RC}\int dt$$

$\therefore$
$$\log_e v = -\frac{t}{RC} + K \quad \ldots(i)$$

At the instant of closing the switch, $t = 0$ and $v = V$. Putting these values in eq. (*i*), we get,

$$\log_e V = K$$

$\therefore$ Equation (*i*) becomes : $\log_e v = (-t/RC) + \log_e V$

or
$$\log_e \frac{v}{V} = -\frac{t}{RC}$$

or
$$\frac{v}{V} = e^{-t/RC}$$

$\therefore$
$$v = V\,e^{-t/\lambda} \quad \ldots(ii)$$

Again λ (= RC) is the time constant and has the dimensions of time.

Similarly,
$$q = Q\,e^{-t/RC}$$

and
$$i = -I_m\,e^{-t/RC}$$

Note that negative sign is attached to I_m. This is because the discharging current flows in the opposite direction to that in which the charging current flows.

Fig. 6.50 as well as adjoining table shows the percentage of initial voltage (V) after each time constant interval during discharging of capacitor. A fully charged capacitor discharges to about 37% of its initial fully charged value in first time constant. The capacitor is fully discharged in a 5 time-constant interval.

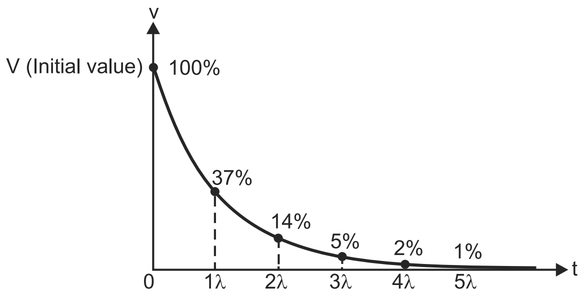


Fig. 6.50

Number of time constants	% of Initial value
1	37
2	14
3	5
4	2
5	1 considered 0

Example 6.48. *A 2 μF capacitor is connected, by closing a switch, to a supply of 100 volts through a 1 MΩ series resistance. Calculate (i) the time constant (ii) initial charging current (iii) the initial rate of rise of p.d. across capacitor (iv) voltage across the capacitor 6 seconds after the switch has been closed and (v) the time taken for the capacitor to be fully charged.*

Solution. ***(i)*** Time constant, $\lambda = RC = (10^6) \times (2 \times 10^{-6}) = $ **2 seconds**

(ii) Initial charging current, $I_m = \dfrac{V}{R} = \dfrac{100}{10^6} \times 10^6 = $ **100 μA**

(iii) Initial rate of rise of voltage across capacitor is

$$\frac{dv}{dt} = \frac{V}{CR} = \frac{100}{(2 \times 10^{-6}) \times 10^6} = \mathbf{50\ V/s}$$

(iv) $v = V(1 - e^{-t/RC})$

Here $V = 100$ volts ; $t = 6$ seconds ; $RC = 2$ seconds

$\therefore$ $v = 100\,(1 - e^{-6/2}) = 100\,(1 - e^{-3}) = $ **95.1 V**

(v) Time taken for the capacitor to be fully charged

$= 5\,RC = 5 \times 2 = $ **10 seconds**

Example 6.49. *A capacitor of 8 μF capacitance is connected to a d.c. source through a resistance of 1 megaohm. Calculate the time taken by the capacitor to receive 95% of its final charge. How long will it take the capacitor to be fully charged ?*

Solution. $q = Q\,(1 - e^{-t/RC})$

Here $RC = (10)^6 \times 8 \times 10^{-6} = 8$ seconds ; $q/Q = 0{\cdot}95$

$\therefore$ $0{\cdot}95 = 1 - e^{-t/8}$ or $e^{-t/8} = 0{\cdot}05$

$\therefore$ $e^{t/8} = 1/0{\cdot}05 = 20$

or $(t/8)\log_e e = \log_e 20$

$\therefore$ $t = 8 \log_e 20 = $ **23·96 seconds**

Time taken for the capacitor to be fully charged

$= 5\,RC = 5 \times 8 = $ **40 seconds**

Alternatively. $t = \lambda \log_e \dfrac{V - V_0}{V - v_C}$ ... See Art. 6.30

or $t = \lambda \log_e \dfrac{Q - q_0}{Q - q}$

Here, $\lambda = 8s$; $q_0 = 0$; $q = 95\%$ of $Q = 0.95$ Q

$$\therefore \quad t = 8 \times \log_e \frac{Q-0}{Q-0.95Q} = 8 \times \log_e \frac{Q}{0.05Q} = \textbf{23.96 seconds}$$

Example 6.50. *A resistance R and a 4 μF capacitor are connected in series across a 200 V d.c. supply. Across the capacitor is connected a neon lamp that strikes at 120 V. Calculate the value of R to make the lamp strike after 5 seconds.*

Solution. The voltage across the neon lamp has to rise to 120 V in 5 seconds.

Now, $\quad v = V(1 - e^{-t/\lambda}) \quad$ or $\quad 120 = 200\,(1 - e^{-5/\lambda})$

or $\quad e^{-5/\lambda} = 1 - (120/200) = 0{\cdot}4 \quad$ or $\quad e^{5/\lambda} = 1/0{\cdot}4 = 2{\cdot}5$

$\therefore \quad (5/\lambda) \log_e e = \log_e 2{\cdot}5$

or $$\lambda = \frac{5}{\log_e 2{\cdot}5} = 5{\cdot}457 \text{ seconds}$$

or $$RC = 5{\cdot}457 \quad \therefore \quad R = \frac{5{\cdot}457}{4 \times 10^{-6}} = 1{\cdot}364 \times 10^6\ \Omega = \mathbf{1{\cdot}364\ M\Omega}$$

Alternatively. $$t = \lambda \log_e \frac{V - V_0}{V - v_C}$$

Here, $t = 5s$; $V = 200$ volts ; $V_0 = 0$; $v_C = 120$ volts

Putting these values in the above expression, we get, $\lambda = 5.457s$.

Now $\lambda = RC$ or $R = \dfrac{\lambda}{C} = \dfrac{5.457}{4 \times 10^{-6}} = 1.364 \times 10^6\Omega = \mathbf{1.364\ M\Omega}$

Example 6.51. *A capacitor of 1 μF and resistance 82 kΩ are connected in series with an e.m.f. of 100 V. Calculate the magnitude of energy and the time in which energy stored in the capacitor will reach half of its equilibrium value.*

Solution. Equilibrium value of energy $= \dfrac{1}{2} CV^2$

$\therefore$ Energy stored $\propto V^2$

Half energy of the equilibrium value will be stored when voltage across capacitor is $v = 100/\sqrt{2}$ $= 70{\cdot}7$ volts.

$$\therefore \quad \text{Energy stored} = \frac{1}{2}Cv^2 = \frac{1}{2}(1 \times 10^{-6}) \times (70{\cdot}7) = \mathbf{0{\cdot}0025\ J}$$

Now, $\quad v = V(1 - e^{-t/RC})$

Here, $RC = (82 \times 10^3) \times (1 \times 10^{-6}) = 0{\cdot}082$ s ; $\quad v = 70{\cdot}7$ V ; $\quad V = 100$ V

$\therefore \quad 70{\cdot}7 = 100\,(1 - e^{-t/0{\cdot}082}) \quad$ or $\quad e^{-t/0{\cdot}082} = 1 - (70{\cdot}7/100) = 0{\cdot}293$

$\therefore \quad e^{t/0{\cdot}082} = 1/0{\cdot}293 = 3{\cdot}413$

or $\quad (t/0{\cdot}082) \log_e e = \log_e 3{\cdot}413$

$\therefore \quad t = 0{\cdot}082 \times \log_e 3{\cdot}413 = \mathbf{0{\cdot}1\ second}$

Example 6.52. *When a capacitor C charges through a resistor R from a d.c. source voltage E, determine the energy appearing as heat.*

Solution. When $R - C$ series circuit is switched on to d.c. source of voltage E, the charging current i decreases at exponential rate given by ;

$$i = I\,e^{-t/\lambda}$$

$$\text{where } I = E/R \ ; \ \lambda = RC$$

Energy appearing as heat in small time Δt is

$$\Delta W_R = i^2 R\, \Delta t$$

Total energy appearing as heat in the entire process of charging is

$$W_R = \int_0^\infty i^2 R\, dt = R\int_0^\infty (I e^{-t/\lambda})^2 dt = R\int_0^\infty I^2 e^{-2t/\lambda} dt$$

$$= R \times I^2 \int_0^\infty e^{-2t/\lambda} dt = RI^2 \left[\frac{e^{-2t/\lambda}}{-2/\lambda}\right]_0^\infty$$

$$= R \times (E/R)^2 \left(\frac{-\lambda}{2}\right)[e^{-\infty} - e^0] = \frac{E^2}{R} \times \left(\frac{-RC}{2}\right) \times (-1)$$

$$\therefore \quad W_R = \frac{1}{2} CE^2$$

Although energy stored in a capacitor is very small, it can provide a large current (and hence large power) for a short period of time.

Note. Energy stored in the capacitor at the end of charging process is $CE^2/2$. Also energy appearing as heat in the entire process of charging the capacitor is $CE^2/2$.

$\therefore$ Total energy received from the source $= \frac{1}{2}CE^2 + \frac{1}{2}CE^2 = CE^2$

Thus during charging of capacitor, the total energy received from the source is CE^2 ; half is converted into heat and the rest half stored in the capacitor.

Example 6.53. *Referring to the circuit shown in Fig. 6.51,*

(*i*) *Write the mathematical expression for charging current i and voltage v across capacitor when the switch is placed in position 1.*

(*ii*) *Write the mathematical expression for the discharging current and voltage across capacitor when switch is placed in position 2 after having been in position 1 for 1 s.*

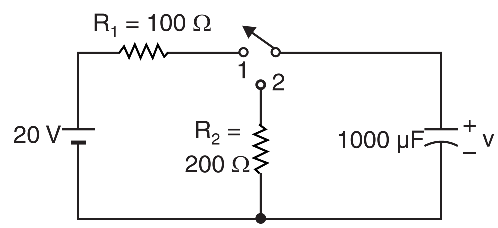


Fig. 6.51

Solution.

(*i*) When the switch is placed in position 1, the capacitor charges through R_1 only. Therefore, time constant during charging is

Time constant, $\lambda = R_1 C = (100) \times (1000 \times 10^{-6}) = 0{\cdot}1$ s

Initial charging current, $I_m = V/R_1 = 20/100 = 0{\cdot}2$ A

The charging current at any time t is given by ;

$$i = I_m e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{i = 0{\cdot}2\, e^{-t/0{\cdot}1}} \textbf{ A}$$

The voltage v across the capacitor at any time t is given by ;

$$v = V(1 - e^{-t/\lambda}) \quad \text{or} \quad \boldsymbol{v = 20\,(1 - e^{-t/0{\cdot}1})} \textbf{ volts}$$

(*ii*) Since the switch remains in position 1 for 1 s or 10 time constants, the capacitor charges fully to 20 V. When the switch is placed in position 2, the capacitor discharges through R_2 only. Therefore, time constant during discharge is

Time constant, $\lambda = R_2 C = (200) \times (1000 \times 10^{-6}) = 0{\cdot}2$ s

Initial discharging current, $I_m = V/R_2 = 20/200 = 0{\cdot}1$ A

The discharging current at any time t is given by ;

$$i = -I_m e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{i = -0{\cdot}1\, e^{-t/0{\cdot}2}} \textbf{ A}$$

The voltage v across the capacitor at any time t is given by ;

$$v = V e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{v = 20\, e^{-t/0{\cdot}2}} \textbf{ volts}$$

Example 6.54. *A cable 10 km long and of capacitance 2.5μF discharges through its insulation resistance of 50 MΩ. By what percentage the voltage would have fallen 1, 2 and 5 minutes respectively after disconnection from its bus-bars?*

Solution. Capacitance of cable capacitor, $C = 2.5 \times 10^{-6}$ F; Insulation resistance of cable, $R = 50\ M\Omega = 50 \times 10^6\ \Omega$

Time constant, $\lambda = RC = (50 \times 10^6) \times (2.5 \times 10^{-6}) = 125$ seconds

During discharging, decreasing voltage v across the capacitor is given by ;

$$v = Ve^{-t/\lambda} = Ve^{-t/125}$$

At $t = 1$ min. = 60 seconds, $v_1 = Ve^{-60/125} = 0.618$ V

At $t = 2$ min. = 120 seconds, $v_2 = Ve^{-120/125} = 0.383$ V

At $t = 5$ min. = 300 seconds, $v_3 = Ve^{-300/125} = 0.09$ V

$\therefore$ At $t = 1$ min, the % age fall in voltage across capacitor

$$= \frac{V - 0.618V}{V} \times 100 = \mathbf{38.2\%}$$

At $t = 2$ min; the % age fall in voltage across capacitor

$$= \frac{V - 0.383V}{V} \times 100 = \mathbf{61.7\%}$$

At $t = 5$ min; the % age fall in voltage across capacitor

$$= \frac{V - 0.09V}{V} \times 100 = \mathbf{91\%}$$

Tutorial Problems

1. A capacitor is being charged from a d.c. source through a resistance of 2MΩ. If it takes 0·2 second for the charge to reach 75% of its final value, what is the capacitance of the capacitor ? **[18×10^{-4} F]**
2. A 8 μF capacitor is connected is series with 0·5 MΩ resistance across 200 V supply. Calculate (*i*) initial charging current (*ii*) the current and p.d. across capacitor 4 seconds after it is connected to the supply. [(*i*) **400 μA** (*ii*) **147 μA**; **126·4 V**]
3. What resistance connected in series with a capacitance of 4μF will give the circuit a time constant of 2 seconds ? [**500 kΩ**]
4. A series *RC* circuit is to have an initial charging current of 4 mA and a time constant of 3·6 seconds when connected to 120 V d.c. supply. Calculate the values of *R* and *C*. What will be the energy stored in the capacitor ? [**30 kΩ** ; **120 μF** ; **0·864 J**]
5. A 20μF capacitor initially charged to a p.d. of 500V is discharged through an unknown resistance. After one minute, the p.d. at the terminals of the capacitor is 200 V. What is the value of the resistance ? [**3·274 MΩ**]

6.29. Transients in D.C. Circuits

When a circuit goes from one steady-state condition to another steady-state condition, it passes through a transient state which is of short duration. The word transient means temporary or short-lived. When a d.c. voltage source is first connected to a series *RC* network, the charging current flows only until the capacitor is fully charged. This charging current is called a **transient current**. In connection with d.c. circuits, a transient is a voltage or current that *changes* with time for a short duration of time and remains constant thereafter. As a capacitor charges, its voltage builds up (*i.e.,* changes) until the capacitor is fully charged and its voltage equals the source voltage. After that time, there is no further change in capacitor voltage. Thus the voltage across a capacitor during the time it is being charged is an example of a **transient voltage**.

6.30. Transient Relations During Charging/Discharging of Capacitor

When a capacitor is charging or discharging, it goes from one steady-state condition (called *initial condition*) to another steady-state condition (called *final condition*). During this change, the voltage across and current through the capacitor change continuously. These are called **transient conditions* and exist for a short duration. *It can be shown mathematically that voltage v_C across the capacitor at any time t during charging or discharging* is given by ;

$$v_C = V - (V - V_0)e^{-t/\lambda} \quad ...(i)$$

where v_C = voltage across capacitor at any time t

V = Source voltage during charging

V_0 = Voltage across capacitor at $t = 0$

λ = Time constant (= RC)

Note that for discharging of capacitor, $V = 0$ because there is no source voltage.

1. Transient conditions during charging. When we charge an uncharged capacitor, $V_0 = 0$ so that eq. (*i*) becomes :

$$v_C = V - (V - 0)e^{-t/\lambda} = V - Ve^{-t/\lambda}$$

$$\therefore \quad v_C = V(1 - e^{-t/\lambda}) \quad ...(ii)$$

This is the same equation that we derived in Art. 6.26 for charging of a capacitor.

From eq. (*ii*), $\quad V - v_C = Ve^{-t/\lambda}$

But $V - v_C = iR$, where i is the charging current at time t.

$$\therefore \quad iR = Ve^{-t/\lambda} \quad \text{or} \quad i = \frac{V}{R}e^{-t/\lambda}$$

$$\therefore \quad i = Ie^{-t/\lambda} \quad ...(iii)$$

where I (= V/R) is the initial charging current.

Note that eq. (*iii*) is the same that we derived in Art. 6.26 for charging of a capacitor. Fig. 6.52 shows capacitor voltage (v_C) and charging current (i) waveforms for a charging capacitor. It may be seen that voltage across the capacitor is building up at an exponential rate while the charging current is decreasing at an exponential rate.

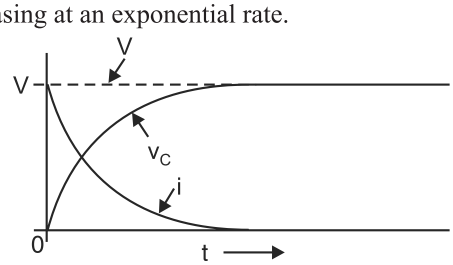


Fig. 6.52

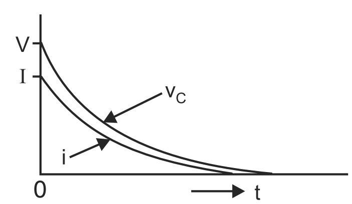


Fig. 6.53

2. Transient conditions during discharging. For discharging of a capacitor, $V = 0$ because there is no source voltage. Therefore, eq. (*i*) becomes :

$$v_C = 0 - (0 - V_0)e^{-t/\lambda}$$

$$\text{or} \quad v_C = V_0 e^{-t/\lambda} \quad ...(iv)$$

Here V_0 is, of course, the voltage to which the capacitor was originally charged. Note that this is the same expression which derived in Art. 6.28 for discharging of a capacitor.

$$\text{Now,} \quad \frac{v_C}{C} = \frac{V_0}{C}e^{-t/\lambda}$$

$$\text{or} \quad q = Q_0 e^{-t/\lambda}$$

* The word transient means temporary or short-lived.

where Q_0 is the initial charge on the capacitor and q is the charge on the capacitor at time t.

Similarly, $i = I_0 e^{-t/\lambda}$

where I is the initial discharging current and i is the discharging current at time t.

Fig. 6.53 shows the capacitor voltage and discharging current waveforms. Both decrease at exponential rate and reach zero value at the same time.

Time for charge or discharge. Sometimes it is desirable to determine how long will it take the capacitor in RC series circuit to charge or discharge to a specified voltage. This can be found as follows : From eq. (i), $v_C = V - (V - V_0)\, e^{-t/\lambda}$

or $$V - v_C = (V - V_0) e^{-t/\lambda}$$

or $$\frac{V - v_C}{V - V_0} = e^{-t/\lambda}$$

or $$\frac{V - V_0}{V - v_C} = e^{t/\lambda}$$

Taking the natural log, we have,

$$\frac{t}{\lambda}\log_e e = \log_e \frac{V - V_0}{V - v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V - V_0}{V - v_C} \quad ...(v)$$

Exp. (v) is applicable for charging as well as discharging of a capacitor.

For charging. When C is charging from 0V (*i.e.* capacitor is uncharged), $V_0 = 0$. Therefore, putting $V_0 = 0$ in exp. (v), we have,

$$t = \lambda \log_e \frac{V - 0}{V - v_C} = \lambda \log_e \frac{V}{V - v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V}{V - v_C}$$

If the capacitor has some initial charge instead of zero, then value of V_0 will be corresponding to that charge.

For discharging. In this case, $V = 0$. Therefore, putting $V = 0$ in exp. (v), we have,

$$t = \lambda \log_e \frac{0 - V_0}{0 - v_C} = \lambda \log_e \frac{V_0}{v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V_0}{v_C}$$

Example 6.55. *The uncharged capacitor in Fig. 6.54 is initially switched to position 1 of the switch for two seconds and then switched to position 2 for the next two seconds. What will be the voltage on the capacitor at the end of this period?*

Solution. When uncharged capacitor is switched to position 1, it will be instantaneously charged to 100 V because there is no resistance in the charging circuit. Therefore, after 2 seconds, the capacitor will be at 100 V. Now when switch is put to position 2, the time of discharge t is given by ;

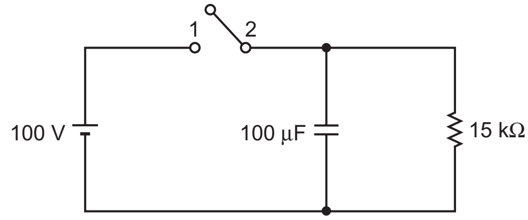


Fig. 6.54

$$t = \lambda \log_e \frac{V - V_0}{V - v_c}$$

Here $t = 2s$; $\lambda = RC = 15000 \times 100 \times 10^{-6} = 1.5s$; $V = 0$; $V_0 = 100$ volts

$$\therefore \quad 2 = 1.5 \log_e \frac{0 - 100}{0 - v_c} = 1.5 \log_e \frac{100}{v_c}$$

On solving, v_c = **26.36 V**

Example 6.56. *A 50μF capacitor and a 20 kΩ resistor are connected in series across a battery of 100 V at the instant t = 0. At instant t = 0.5s, the applied voltage is suddenly increased to 150V. Find the charge on the capacitor at t = 0.75 s.*

Solution. Time constant, $\lambda = RC = 20{,}000 \times 50 \times 10^{-6} = 1$ sec.

For first case. $$t = \lambda \log_e \frac{V - V_0}{V - v_c}$$

Here, $t = 0.5s$; $\lambda = 1s$; $V = 100$ volts ; $V_0 = 0$; $v_C = ?$

$$\therefore \quad 0.5 = 1 \times \log_e \frac{100 - 0}{100 - v_C} = \log_e \frac{100}{100 - v_C}$$

On solving, v_C = 39.4 volts

For second case. After 0.5 sec., the source voltage is increased to 150 V.

Now $$t = \lambda \log_e \frac{V - V_0}{V - v_C}$$

Here, $t = 0.75 - 0.5 = 0.25s$; $\lambda = 1s$; $V = 150$ volts ; $V_0 = 39.4$ volts ; $v'_C = ?$

$$\therefore \quad 0.25 = 1 \times \log_e \frac{150 - 39.4}{150 - v'_C} = \log_e \frac{110.6}{150 - v'_C}$$

On solving, v'_C = 63.6 volts

∴ Charge on capacitor = $C \times v'_C = 50 \times 10^{-6} \times 63.6 =$ **3.18×10^{-3} C**

Example 6.57. *Find how long it takes after the switch S is closed before the total current from the supply reaches 25 mA when V = 10 V, R_1 = 500Ω , R_2 = 700Ω and C = 100μF.*

Solution. When switch S is closed, the current in R_1 = 500Ω is set up instantaneously and its value is $= 10/R_1 = 10/500 = 0.02A = 20$ mA. In order to draw 25 mA current from the supply, current in capacitor circuit is $= 25 - 20 = 5$mA. Now when switch S is closed, the current in capacitor circuit is maximum and its value is $I = 10/R_2 = 10/700 = 0.0143A = 14.3$ mA and decreases at exponential rate. Our problem is to find the time t in which charging current in capacitor circuit decreases from 14.3 mA to 5 mA.

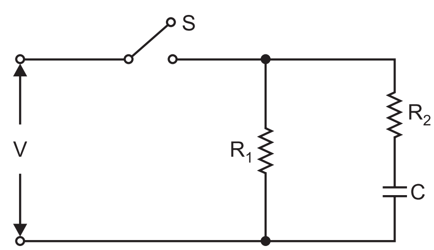


Fig. 6.55

Now, $$i = Ie^{-t/\lambda}$$

Here $i = 5$ mA ; $I = 14.3$ mA ; $\lambda = R_2C = 700 \times 100 \times 10^{-6} = 0.07$ s

$$\therefore \quad 5 = 14.3\, e^{-t/0.07}$$

On solving, t = **0.0735 s**

Example 6.58. *In an RC series circuit, R = 2MΩ, C = 5μF and applied voltage V = 100 volts. Calculate (i) initial rate of change of capacitor voltage (ii) initial rate of change of capacitor current (iii) initial rate of change of voltage across 2MΩ resistor.*

Solution. Time constant, $\lambda = RC = 2 \times 10^6 \times 5 \times 10^{-6} = 10$ seconds

(*i*)
$$v_C = V(1 - e^{-t/\lambda})$$

$$\therefore \quad \frac{dv_C}{dt} = 0 - Ve^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = \frac{V}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \quad \frac{dv_C}{dt} = \frac{V}{\lambda}e^{-0/\lambda} = \frac{V}{\lambda} = \frac{100}{10} = \mathbf{10\ V/s}$$

(*ii*)
$$i = I\,e^{-t/\lambda}$$

$$\therefore \quad \frac{di}{dt} = Ie^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = -\frac{I}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \quad \frac{di}{dt} = -\frac{I}{\lambda}e^{-0/\lambda} = -\frac{I}{\lambda} = -\frac{V/R}{\lambda} = -\frac{100/2 \times 10^6}{10} = \mathbf{-5\ \mu A/s}$$

(*iii*)
$$v_R = iR = (Ie^{-t/\lambda})R = \left(\frac{V}{R}e^{-t/\lambda}\right)R = Ve^{-t/\lambda}$$

$$\therefore \quad \frac{dv_R}{dt} = Ve^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = -\frac{V}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \quad \frac{dv_R}{dt} = -\frac{V}{\lambda}e^{-0/\lambda} = -\frac{V}{\lambda} = -\frac{100}{10} = \mathbf{-10\ V/s}$$

Example 6.59. *Calculate the values of i_2, i_3, v_2, v_3, v_C and v_L in the network shown in Fig. 6.56 at the following times :*

(*i*) *At time, $t = 0$ immediately after the switch S is closed.*

(*ii*) *At time, $t \to \infty$ i.e. in the steady state. All resistances are in ohms.*

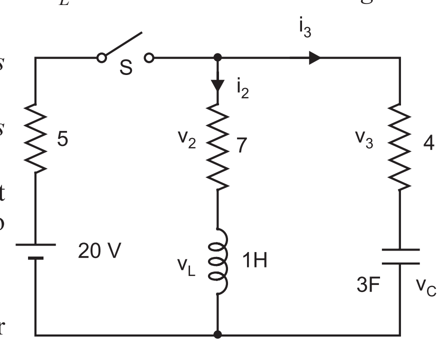


Fig. 6.56

Solution. **(*i*)** At the instant of closing the switch (*i.e.* at $t = 0$), the inductance (= 1 H) behaves as an open circuit so that no current flows in the coil.

$\therefore\ i_2 = \mathbf{0\ A}$; $v_2 = \mathbf{0\ V}$; $v_L = \mathbf{20\ V}$

At the instant of closing the switch, the capacitor behaves as a short circuit.

$$\therefore\ i_3 = \frac{20}{5+4} = \mathbf{\frac{20}{9}\,A} \;;\; v_3 = 4 \times \frac{20}{9} = \mathbf{\frac{80}{9}\,V} \;;\; v_C = \mathbf{0\ V}$$

(*ii*) Under steady state conditions (*i.e.* when the capacitor is fully charged), the capacitor behaves as an open circuit and the inductance (= 1H) as short.

$$\therefore\ i_2 = \frac{20}{5+7} = \mathbf{\frac{5}{3}\,A} \;;\; v_2 = 7 \times \frac{5}{3} = \mathbf{\frac{35}{3}\,V} \;;\; v_L = \mathbf{0\ V} \;;\; i_3 = \mathbf{0A}\;;$$

$v_3 = \mathbf{0\ V}$; $v_C = \mathbf{20\ V}$

Example 6.60. *In Fig. 6.57, the capacitor C is uncharged. Determine the final voltage on the capacitor after the switch has been in position 2 for 3s and then in position 3 for 5s.*

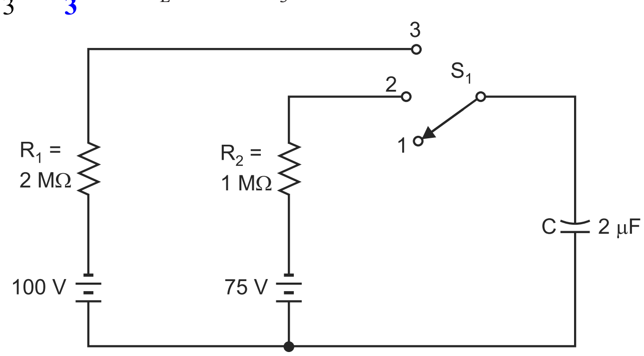


Fig. 6.57

Solution. When the switch is in position 2, the voltage v_C across the capacitor is

$$v_C = V(1 - e^{-t/\lambda})$$

Here, $V = 75$ volts ; $t = 3$s ; $\lambda = R_2C = (1 \times 10^6) \times 2 \times 10^{-6} = 2$s

$$\therefore \quad v_C = 75(1 - e^{-3/2}) = 75\ (1 - 0.223) = 58.3 \text{ V}$$

Therefore, after 2s, voltage across capacitor is 58.3 V.

When switch is in position 3, voltage v'_C across capacitor is

$$v'_C = V - (V - v_C)e^{-t/\lambda}$$

Here, $V = 100$ volts ; $t = 5$s ; $v_C = 58.3$ volts ; $\lambda = R_1C = 2 \times 10^6 \times 2 \times 10^{-6} = 4$s

$$\therefore \quad v'_C = 100 - (100 - 58.3)e^{-5/4}$$

$$= 100 - (100 - 58.3) \times 0.287 = \mathbf{88.0\ V}$$

Therefore, final voltage across the capacitor is 88.0 V.

Tutorial Problems

1. A capacitor of capacitance 12μF is allowed to discharge through its own leakage resistance and a fall of p.d. from 120 V to 100 V is recorded in a time interval of 300 seconds by an electrostatic voltmeter connected in parallel. Calculate the leakage resistance of the capacitor. **[137 MΩ]**

2. When a capacitor charged to a p.d. of 400 V is connected to a voltmeter having a resistance of 25 MΩ, the voltmeter reading is observed to have fallen to 50 V at the end of an interval of 2 minutes. Find the capacitance of the capacitor. **[2.31 μF]**

3. An 8μF capacitor is connected through a 1.5 MΩ resistance to a direct current source. After being on charge for 24 seconds, the capacitor is disconnected and discharged through a resistor. Determine what % age of the energy input from the supply is dissipated in the resistor. **[43.2%]**

4. An 8μF capacitor is connected in series with a 0.5 MΩ resistor across a 200V d.c. supply. Calculate (*i*) the time constant (*ii*) the initial charging current (*iii*) the time taken for the p.d. across the capacitor to grow to 160 V and (*iv*) the current and the p.d. across the capacitor in 4 seconds after it is connected to the supply. **[(*i*) 4s (*ii*) 0.4 mA (*iii*) 6.4s (*iv*) 0.14 mA ; 126.4 V]**

Objective Questions

1. The capacitance of a capacitor is .... relative permittivity.
(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of
(*iv*) directly proportional to square of

2. An air capacitor has the same dimensions as that of a mica capacitor. If the capacitance of mica capacitor is 6 times that of air capacitor, then relative permittivity of mica is
(*i*) 36 (*ii*) 12
(*iii*) 3 (*iv*) 6

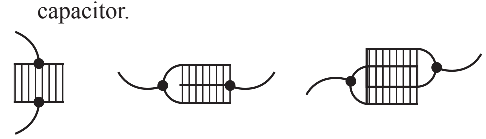
3. The most convenient way of achieving large capacitance is by using
(*i*) multiplate construction
(*ii*) decreased distance between plates
(*iii*) air as dielectric
(*iv*) dielectric of low permittivity

4. Another name for relative permittivity is
(*i*) dielectric strength
(*ii*) breakdown voltage
(*iii*) specific inductive capacity
(*iv*) potential gradient

5. A capacitor opposes
(*i*) change in current
(*ii*) change in voltage
(*iii*) both change in current and voltage
(*iv*) none of the above

6. If a multiplate capacitor has 7 plates each of area 6 cm^2, then,
(*i*) 6 capacitors will be in parallel
(*ii*) 7 capacitors will be in parallel
(*iii*) 7 capacitors will be in series
(*iv*) 6 capacitors will be in series

7. The capacitance of three-plate capacitor [See Fig. 6.58 (*ii*)] is .... that of 2-plate capacitor.



2-plate capacitor (*i*) 3-plate capacitor (*ii*) 4-plate capacitor (*iii*)

Fig. 6.58

(i) 3 times (ii) 6 times
(iii) 4 times (iv) 2 times

8. The capacitance of a 4-plate capacitor [See Fig. 6.58 (iii)] is .... that of 2-plate capacitor.
(i) 2 times (ii) 4 times
(iii) 3 times (iv) 6 times

9. Two capacitors of capacitances 3 μF and 6 μF in series will have a total capacitance of
(i) 9 μF (ii) 2 μF
(iii) 18 μF (iv) 24 μF

10. The capacitance of a parallel-plate capacitor does not depend upon
(i) area of plates
(ii) medium between plates
(iii) separation between plates
(iv) metal of plates

11. In order to increase the capacitance of a parallel-plate capacitor, one should introduce between the plates a sheet of
(i) mica (ii) tin
(iii) copper (iv) stainless steel

12. The capacitance of a parallel-plate capacitor depends upon
(i) the type of metals used
(ii) separation between plates
(iii) thickness of plates
(iv) potential difference between plates

13. The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of plates d with a potential difference V between the plates is
(i) $\dfrac{CV^2}{2d}$ (ii) $\dfrac{C^2V^2}{2d^2}$
(iii) $\dfrac{C^2V^2}{d^2}$ (iv) $\dfrac{V^2d}{C}$

14. A parallel-plate air capacitor is immersed in oil of dielectric constant 2. The electric field between the plates is
(i) increased 2 times
(ii) increased 4 times
(iii) decreased 2 times
(iv) none of above

15. Two capacitors of capacitances C_1 and C_2 are connected in parallel. A charge Q given to them is shared. The ratio of charges Q_1/Q_2 is
(i) C_2 / C_1 (ii) C_1 / C_2
(iii) $C_1C_2 / 1$ (iv) $1 / C_1C_2$

16. The dimensional formula of capacitance is
(i) $M^{-1}L^{-2}T^{-4}A^2$ (ii) $M^{-1}L^2T^4A^2$
(iii) $ML^2T^{-4}A$ (iv) $M^{-1}L^{-2}T^4A^2$

17. Four capacitors are connected as shown in Fig. 6.59. What is the equivalent capacitance between A and B ?

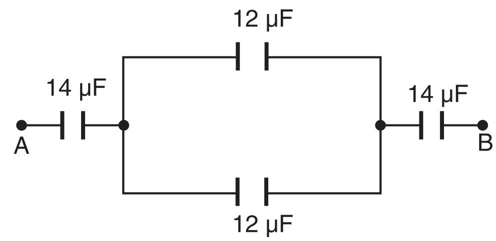


Fig. 6.59

(i) 36 μF (ii) 5·4 μF
(iii) 52 μF (iv) 11·5 μF

18. The empty space between the plates of a capacitor is filled with a liquid of dielectric constant K. The capacitance of capacitor
(i) increases by a factor K
(ii) decreases by a factor K
(iii) increases by a factor K^2
(iv) decreases by a factor K^2

19. A parallel plate capacitor is made by stacking n equally spaced plates connected alternately. If the capacitance between any two plates is C, then the resulting capacitance is
(i) C (ii) nC
(iii) $(n - 1) C$ (iv) $(n + 1) C$

20. 64 drops of radius r combine to form a bigger drop of radius R. The ratio of capacitances of bigger to smaller drop is
(i) 1 : 4 (ii) 2 : 1
(iii) 1 : 2 (iv) 4 : 1

21. Two capacitors have capacitances 25 μF when in parallel and 6 μF when in series. Their individual capacitances are
(i) 12 μF and 13 μF
(ii) 15 μF and 10 μF
(iii) 10 μF and 8 μF
(iv) none of above

22. A capacitor of 20 μF charged to 500 V is connected in parallel with another capacitor of 10 μF capacitance and charged to 200 V. The common potential is
(i) 200 V (ii) 250 V
(iii) 400 V (iv) 300 V

23. Which of the following does not change when a glass slab is introduced between the plates of a charged parallel plate capacitor?

(*i*) electric charge (*ii*) electric energy
(*iii*) capacitance
(*iv*) electric field intensity

24. A capacitor of 1 μF is charged to a potential of 50 V. It is now connected to an uncharged capacitor of capacitance 4 μF. The common potential is

(*i*) 50 V (*ii*) 20 V
(*iii*) 15 V (*iv*) 10 V

25. Three parallel plates each of area A with separation d_1 between first and second and d_2 between second and third are arranged to form a capacitor. If the dielectric constants are K_1 and K_2, the capacitance of this capacitor is

(*i*) $\dfrac{\varepsilon_0 K_1 K_2}{A(d_1 + d_2)}$ (*ii*) $\dfrac{\varepsilon_0}{A\left(\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}\right)}$

(*iii*) $\dfrac{\varepsilon_0 A K_1 K_2}{d_1 + d_2}$ (*iv*) $\dfrac{\varepsilon_0 A}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$

Answers

1. (*i*)	**2.** (*iv*)	**3.** (*i*)	**4.** (*iii*)	**5.** (*ii*)
6. (*i*)	**7.** (*iv*)	**8.** (*iii*)	**9.** (*ii*)	**10.** (*iv*)
11. (*i*)	**12.** (*ii*)	**13.** (*i*)	**14.** (*iii*)	**15.** (*ii*)
16. (*iv*)	**17.** (*ii*)	**18.** (*i*)	**19.** (*iii*)	**20.** (*iv*)
21. (*ii*)	**22.** (*iii*)	**23.** (*i*)	**24.** (*iv*)	**25.** (*iv*)