4

Units—Work, Power and Energy

Introduction

Engineering is an applied science dealing with a very large number of *physical quantities like distance, time, speed, temperature, force, voltage, resistance *etc*. Although it is possible to assign a standard unit for each quantity, it is rarely necessary to do so because many of the quantities are functionally related through experiment, derivation or definition. In the study of mechanics, for example, the units of only three quantities *viz. mass, length* and *time* need to be selected. All other quantities (*e.g*. area, volume, velocity, force *etc*.) can be expressed in terms of the units of these three quantities by means of experimental, derived and defined **relationship between the physical quantities. The units selected for these three quantities are called *fundamental units.* In order to cover the entire subject of engineering, three more fundamental quantities have been selected *viz*. †*electric current, temperature* and *luminous intensity*. *Thus there are in all six fundamental quantities (viz, mass, length, time, current, temperature and luminous intensity) which need to be assigned proper and standard units.* The units of all other physical quantities can be derived from the units of these six fundamental quantities. In this chapter, we shall focus our attention on the mechanical, electrical and thermal units of work, power and energy.

4.1. International System of Units

Although several systems were evolved to assign units to the above mentioned six fundamental quantities, only international system of units (abbreviated as SI) has been universally accepted. The units assigned to these six fundamental quantities in this system are given below.

Quantity	Symbol	Unit name	Unit symbol
Length	*l, L*	metre	m
Mass	*m*	kilogram	kg
Time	*t*	second	s
Electric Current	*I*	ampere	A
Temperature	*T*	degree kelvin	K
Luminous Intensity	*I*	candela	Cd

It may be noted that the units of all other physical quantities in science and engineering (*i.e*. other than six fundamental or basic quantities above) can be derived from the above basic units and are called *derived units*. Thus unit of velocity (= 1 m/s) results when the unit of length (= 1 m) is divided by the unit of time (= 1 s). Similarly, the unit of force (= 1 newton) results when unit of mass (= 1 kg) is multiplied by the unit of acceleration (= 1 m/s^2). Therefore, units of velocity and force are the derived units.

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* A physical quantity is one which can be measured.

** For example, by definition, speed is the distance travelled per second. Therefore, speed is related to distance (*i.e.* length) and time.

† For practical reasons, electric current and *not* charge has been taken as the fundamental quantity, though one is derivable from the other. The important consideration which led to the selection of current as the fundamental quantity is that it serves as the link between electric, magnetic and mechanical quantities and can be readily measured.

4.2. Important Physical Quantities

It is profitable to give a brief description of the following physical quantities much used in science and engineering :

(*i*) **Mass.** It is the quantity of matter possessed by a body. The SI unit of mass is kilogram (kg). The mass of a body is a constant quantity and is independent of place and position of the body. Thus the mass of a body is the same whether it is on Earth's surface, the Moon's surface, on the top of a mountain or down a deep well.

$$1 \text{ quintal} = 100 \text{ kg} \ ; \quad 1 \text{ tonne} = 10 \text{ quintals} = 1000 \text{ kg}$$

(*ii*) **Force.** It is the product of mass (kg) and acceleration (m/s^2). The unit of force is newton (N) ; being the force required to accelerate a mass of 1 kg through an acceleration of 1 m/s^2.

$$\therefore \qquad F = m\,a \text{ newtons}$$

where m = mass of the body in kg

a = acceleration in m/s^2

(*iii*) **Weight.** The force with which a body is attracted towards the centre of Earth is called the weight of the body. Now, force = mass × acceleration. If m is the mass of a body in kg and g is the acceleration due to gravity in m/s^2, then,

$$\text{Weight, } W = m\,g \text{ newtons}$$

As the value of g* varies from place to place on earth's surface, therefore, the weight of the body varies accordingly. However, for practical purposes, we take $g = 9{\cdot}81$ m/s^2 so that weight of the body = $9{\cdot}81\ m$ newtons. Thus if a mass of 1 kg rests on a table, the downward force on the table *i.e.*, weight of the body is $W = 9{\cdot}81 \times 1 = 9{\cdot}81$ newtons.

The following points may be noted carefully :

(*a*) *The mass of a body is a constant quantity whereas its weight depends upon the place or position of the body.* However, it is reasonably accurate to express weight $W = 9{\cdot}81\ m$ newtons where m is the mass of the body in kg.

(*b*) Sometimes weight is given in kg. wt. units. One kg-wt means weight of mass of 1 kg *i.e.* $9{\cdot}81 \times 1 = 9{\cdot}81$ newtons.

$$\therefore \qquad 1 \text{ kg. wt.} = 9{\cdot}81 \text{ newtons}$$

Thus, when we say that a body has a weight of 100 kg, it means that it has a mass of 100 kg and that it exerts a downward force of $100 \times 9{\cdot}81$ newtons.

4.3. Units of Work or Energy

Work is said to be done on a body when a force acts on it and the body moves through some distance. This work done is stored in the body in the form of energy. Therefore, work and energy are measured in the same units. The SI unit of work or energy is *joule* and is defined as under :

The work done on a body is **one joule** *if a force of one newton moves the body through 1 m in the direction of force.*

It may be noted that work done or energy possessed in an electrical circuit or mechanical system or thermal system is measured in the same units viz. joules. This is expected because mechanical, electrical and thermal energies are interchangeable. For example, when mechanical work is transferred into heat or heat into work, the quantity of work in joules is equal to the quantity of **heat in joules.

* The value of g is about 9.81 m/s^2 at sea level whereas at equator, it is about 9.78 m/s^2 and at each pole it is about 9.832 m/s^2.

** Although heat energy was assigned a separate unit *viz.* calorie but the reader remembers that 1 calorie = 4.186 joules. In fact, the thermal unit calorie is obsolete and now-a-days heat is expressed in joules.

Note. To gain some appreciation for the magnitude of a joule of heat energy, it would require about 90,000 J to heat a cup of water from room temperature to boiling.

4.4. Some Cases of Mechanical Work or Energy

It may be helpful to give a few important cases of work done or energy possessed in a mechanical system :

(*i*) When a force of F newtons is exerted on a body through a distance 'd' metres in the direction of force, then,
Work done = $F \times d$ joules or Nm

(*ii*) Suppose a force of F newtons in maintained tangentially at a radius r metres from O as shown in Fig. 4.1. In one revolution, the point of application of force travels through a distance of $2\pi\, r$ metres.

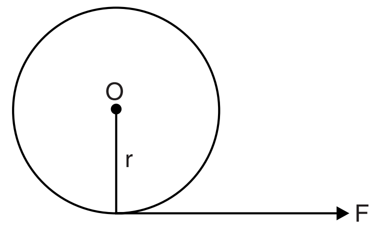

Fig. 4.1

$\therefore$ Work done in one revolution

$$= \text{Force} \times \text{Distance moved in 1 revolution}$$
$$= F \times 2\,\pi\, r$$
$$= 2\pi \times T \text{ joules or Nm}$$

where $T = F\,r$ is the torque. Clearly, the SI unit of torque will be joules or Nm. If the body makes N revolutions per minute, then,

$$\text{Work done/minute} = 2\pi\, N\, \text{T joules}$$

(*iii*) If a body of mass m kg is moving with a speed of v m/s, then kinetic energy possessed by the body is given by ;

$$\text{K.E. of the body} = \frac{1}{2} mv^2 \text{ joules}$$

(*iv*) If a body having a mass of m kg is lifted vertically through a height of h metres and g is acceleration due to gravity in m/s^2, then,

$$\text{Potential energy of body} = \text{Work done in lifting the body} = \text{Force required} \times \text{height}$$
$$= \text{Weight of body} \times \text{height} = m\, g \times h$$
$$= m\, g\, h \text{ joules}$$

4.5. Electrical Energy

The SI unit of electrical work done or electrical energy expended in a circuit is also joule—exactly the same as for mechanical energy. It is defined as under :

One joule *of energy is expended electrically when one coulomb is moved through a p.d. of 1 volt.*

Suppose a charge of Q coulomb moves through a p.d. of V volts in time t in part AB of a circuit as shown in Fig. 4.2. Then electrical energy expended is given by ;

Electrical energy expended

$$= V\,Q \text{ joules}$$
$$= V\,I\,t \text{ joules} \qquad (\because\ Q = I\,t)$$
$$= I^2 R\, t \text{ joules} \qquad (\because\ V = I\,R)$$
$$= \frac{V^2 t}{R} \text{ joules} \qquad \left(\because I = \frac{V}{R}\right)$$

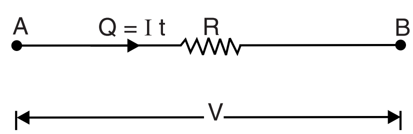

Fig. 4.2

It may be mentioned here that joule is also known as watt-second *i.e.* 1 joule = 1 watt-sec. When we are dealing with large amount of electrical energy, it is often convenient to express it in *kilowatt hours* (kWh).

$$1 \text{ kWh} = 1000 \text{ watt-hours} = 1000 \times 3600 \text{ watt-sec or joules}$$

$$\therefore \qquad 1 \text{ kWh} = 36 \times 10^5 \text{ joules or watt-sec}$$

Although practical unit of electrical energy is kWh, yet it is easy to see that this unit is readily convertible to joules with the help of above relation.

The electricity bills are made on the basis of total electrical energy consumed by the consumer. *The unit for billing of electrical energy is 1 kWh*. Thus when we say that a consumer has consumed 100 units, it means the electrical energy consumption is 100 kWh. Note that 1 kWh is also called *Board of Trade Unit* (*B.O.T.U.*) or unit of electricity.

4.6. Thermal Energy

Heat is a particularly important form of energy in the study of electricity, not only because it affects the electrical properties of the materials but also because it is *liberated* whenever electric current flows. This liberation of heat is infact the conversion of electrical energy to heat energy.

The thermal energy was originally assigned the unit 'calorie'. One calorie is the amount of heat required to raise the temperature of 1 gm of water through 1ºC. If S is the specific heat of a body, then amount of heat required to raise the temperature of m gm of body through θºC is given by ;

$$\text{Heat gained} = (m\, S\, \theta) \text{ calories}$$

It has been found experimentally that 1 calorie = 4·186 joules so that heat energy in calories can be expressed in joules as under :

$$\text{Heat gained} = (m\, S\, \theta) \times 4{\cdot}186 \text{ joules}$$

The reader may note that SI unit of heat is also joule. In fact, the thermal unit calorie is obsolete and unit joule is preferred these days.

4.7. Units of Power

Power is the *rate* at which energy is expended or the rate at which work is performed. Since energy and work both have the units of joules, it follows that power, being rate, has the units joule/second. Now Joule/second is also called **watt.** In general,

$$\text{Power} = \frac{W}{t} \text{ watts}$$

where W is the total number of joules of work performed or total joules of energy expended in t seconds.

Suppose a charge of Q coulomb moves through a p.d. of V volts in time t in part AB of a circuit as shown in Fig. 4.2. Then,

$$\text{Electrical energy expended} = VQ = VIt = I^2Rt = \frac{V^2t}{R}$$

$$\therefore \qquad \text{Power of circuit, } P = \frac{VIt}{t} = \frac{I^2Rt}{t} = \frac{V^2t}{Rt}$$

$$\text{or} \qquad P = VI = I^2R = \frac{V^2}{R}$$

In practice, watt is often found to be inconveniently small, consequently the unit kilowatt (kW) is used. One kW is equal to 1000 watts *i.e.*

$$1 \text{ kW} = 1000 \text{ watts}$$

For larger powers, the unit megawatt (MW) is used. One megawatt is equal to 1000 kW *i.e.*

$$1 \text{ MW} = 1000 \text{ kW} = 1000 \times 1000 \text{ watts}$$

$$\therefore \qquad 1 \text{ MW} = 10^6 \text{ watts}$$

It may be noted that power of an electrical system or mechanical system or thermal system is measured in the same units viz joules/sec. or watts.

Important points. The following points are worth noting :

(*i*) Sometimes power is measured in *horse power (h.p.).

$$1 \text{ h.p.} = 746 \text{ watts}$$

(*ii*) If a body makes N r.p.m. and the torque acting is T newton-metre, then,

$$\text{Work done/minute} = 2\pi\, N\, T \text{ joules} \qquad \text{[See Art. 4·4]}$$

$$\text{Work done/sec} = \frac{2\pi NT}{60} \text{ joules/sec or watts}$$

i.e.,

$$\text{Power} = \frac{2\pi NT}{60} \text{ watts}$$

$$\text{Since 746 watts} = 1 \text{ h.p., we have,}$$

$$\text{Power} = \frac{2\pi NT}{60 \times 746} \text{ h.p.}$$

where T is in newton-m and N is in r.p.m.

(*iii*) Power can also be expressed in terms of force and velocity.

$$\text{Power} = \text{Work done/sec} = \text{Force} \times \text{distance/sec}$$

$$\therefore \quad \text{Power} = \text{Force} \times \text{velocity}$$

4.8. Efficiency of Electric Device

The efficiency of an electric device is the ratio of useful output power to the input power, i.e.

$$\text{Efficiency, } \eta = \frac{\text{Useful output power}}{\text{Input power}}$$

$$= \frac{\text{Useful output Energy}}{\text{Input Energy}}$$

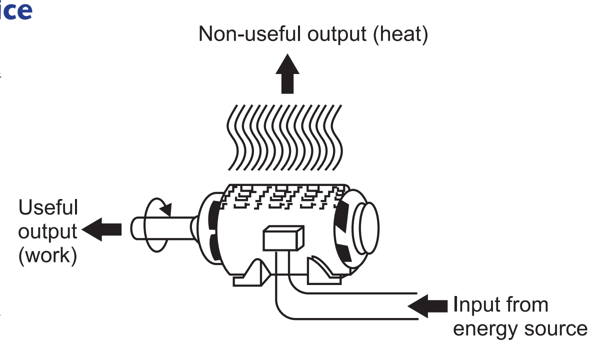

Fig. 4.3

The law of conservation of energy states that "energy cannot be created or destroyed but can be converted from one form to another." Some of the input energy to an electric device may be converted into a form that is not useful. For example, consider an electric motor shown in Fig. 4.3. The purpose of the motor is to convert electric energy into mechanical energy. It does this but it also converts a part of input energy into heat. The heat produced is not useful. Therefore, the useful output energy is less than the input energy. In other words, the efficiency of motor is less than 100%. While selecting an electric device, its efficiency is an important consideration because the operating cost of the device depends upon this factor.

Some electric devices are nearly 100% efficient. An electric heater is an example. In a heater, the heat is useful output energy and practically all the input electric energy is converted into heat energy.

4.9. Harmful Effects of Poor Efficiency

The poor (or low) efficiency of a device or of a circuit has the following harmful effects :

(*i*) Poor efficiency means waste of energy on non-useful output.

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* This unit for power was conceived by James Watt, a Scottish scientist who invented the steam engine. In his experiments, he compared the output of his engine with the power a horse could put out. He found that an "average" horse could do work at the rate of 746 joules/sec. Although power can be expressed in watts or kW, the unit h.p. is still used.

(*ii*) Non-useful output of a device or circuit usually appears in the form of heat. Therefore, poor efficiency means a significant temperature rise. High temperature is one of the major limiting factors in producing reliable electric and electronic devices. Circuits and devices that run hot are more likely to fail.

(*iii*) The heat produced as a result of poor efficiency has to be dissipated *i.e.*, heat has to be transferred to the atmosphere or some other mass. Heat removal can become quite difficult in high power circuits and adds to the cost and size of the equipment.

Example 4.1. *An electrically driven pump lifts 80 m^3 of water per minute through a height of 12 m. Allowing an overall efficiency of 70% for the motor and pump, calculate the input power to motor. If the pump is in operation for an average of 2 hours per day for 30 days, calculate the energy consumption in kWh and the cost of energy at the rate of Rs 2 per kWh. Assume 1 m^3 of water has a mass of 1000 kg and g = 9·81 m/s^2.*

Solution. Mass of 80 m^3 of water, $m = 80 \times 1000 = 8 \times 10^4$ kg

Weight of water lifted, $W = m\,g = 8 \times 10^4 \times 9{\cdot}81$ N

Height through which water lifted, $h = 12\ m$

$$\text{W.D. by motor/minute} = m\,g\,h = 8 \times 10^4 \times 9{\cdot}81 \times 12 \text{ joules}$$

$$\text{W.D. by motor/second} = \frac{8\times10^4\times9.81\times12}{60} = 156960 \text{ watts}$$

$$\therefore \quad \text{Output power of motor} = 156960 \text{ watts}$$

$$\text{Input power to motor} = \frac{\text{Motor output}}{\text{Efficiency}} = \frac{156960}{0.7} = 2{,}24{,}228 \text{ W} = \mathbf{224{\cdot}228\ kW}$$

Total energy consumption = Input power × Time of operation

$$= (224{\cdot}228) \times (2 \times 30) \text{ kWh} = \mathbf{13453\ kWh}$$

Total cost of energy =Rs 2 × 13453 = **Rs. 26906**

Example 4.2. *Fig. 4.4 shows an electric motor driving an electric generator. The 2 h.p. motor draws 14·6 A from a 120 V source and the generator supplies 56 A at 24 V.*

(*i*) *Find the motor efficiency and generator efficiency*

(*ii*) *Find the overall efficiency.*

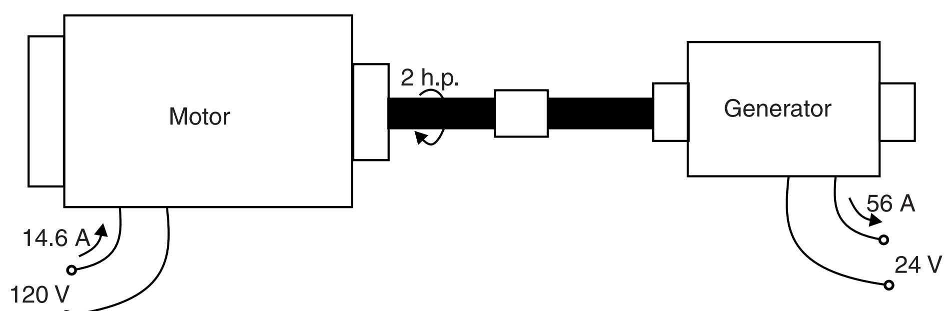

Fig. 4.4

Solution. Efficiency of a machine is output power (P_o) divided by input power (P_i).

(*i*) P_i (motor) = 120 × 14·6 = 1752 W

P_o (motor) = 2 h.p. = 2 × 746 = 1492 W

$$\therefore \quad \eta \text{ (motor)} = \frac{1492}{1752} = \mathbf{0.8516} \text{ or } \mathbf{85{\cdot}16\%}$$

$$P_i \text{ (generator)} = 2 \text{ h.p.} = 1492 \text{ W}$$

$$P_o \text{ (generator)} = 24 \times 56 = 1344 \text{ W}$$

$$\therefore \quad \eta \text{ (generator)} = \frac{1344}{1492} = \mathbf{0{\cdot}90} \quad \text{or} \quad \mathbf{90\%}$$

(ii)

$$\eta \text{ (overall)} = \frac{P_o\text{(generator)}}{P_i\text{(motor)}} = \frac{1344}{1752} = \mathbf{0{\cdot}767} \text{ or } \mathbf{76{\cdot}7\%}$$

Note that overall η is the product of efficiencies of the individual machines.

$$\eta \text{ (overall)} = \eta \text{ (motor)} \times \eta \text{ (generator)} = 0{\cdot}8516 \times 0{\cdot}90 = 0{\cdot}767.$$

Example 4.3. *Neglecting losses, at what horse power rate could energy be obtained from Bhakra dam which has an average height of 225 m and water flows at a rate of 500,000 kg/minute ? If the overall efficiency of conversion were 25%, how many 100 watt light bulbs could Bhakra dam supply ?*

Solution. Wt. of water flowing/minute

$$= m\,g = 500{,}000 \times 9{\cdot}81 \text{ N}$$

$$\text{Work done/minute} = m\,g\,h = 500{,}000 \times 9{\cdot}81 \times 225 \text{ joules}$$

$$\text{Work done/second} = \frac{500{,}000 \times 9.81 \times 225}{60} = 18394 \times 10^3 \text{ watts}$$

$$\therefore \quad \text{Gross power obtained} = 18394 \times 10^3 \text{ watts} = 18394 \text{ kW}$$

$$\text{Useful output power} = 18394 \times 0{\cdot}25 = 4598{\cdot}5 \text{ kW}$$

$$= \frac{4598.5 \times 10^3}{746} \text{ h.p.} = \mathbf{6164 \text{ h.p.}}$$

No. of 100-watt bulbs that could be lighted

$$= \frac{4598.5 \times 10^3}{100} = \mathbf{45985}$$

Example 4.4. *A 100 MW hydro-electric station is supplying full-load for 10 hours a day. Calculate the volume of water which has been used. Assume effective head of station as 200 m and overall efficiency of the station as 80%.*

Solution. Energy supplied by the station in 10 hours

$$= (100 \times 10^3) \times 10 = 10^6 \text{ kWh}$$

$$= 36 \times 10^5 \times 10^6 = 36 \times 10^{11} \text{ joules}$$

$$\text{Energy input of station} = 36 \times 10^{11}/0{\cdot}8 = 45 \times 10^{11} \text{ joules}$$

Suppose m kg is the mass of water used in 10 hours.

Then, $$m\,g\,h = 45 \times 10^{11}$$

or $$m = \frac{45 \times 10^{11}}{9.81 \times 200} = 22.93 \times 10^8 \text{ kg}$$

Since 1 m^3 of water has a mass of 1000 kg,

$\therefore$ Volume of water used $= 22{\cdot}93 \times 10^8/10^3 = \mathbf{22{\cdot}93 \times 10^5\ m^3}$

Example 4.5. *Two coils are connected in parallel and a voltage of 200 V is applied to the terminals. The total current taken is 15A and the power dissipated in one of the coils is 1500 W. What is the resistance of each coil ?*

Solution. Let R_1 and R_2 be the resistances of the coils and I_1 and I_2 be the current drawn from the supply. Since the coils are connected in parallel, voltage across each coil is the same *i.e.* 200 V.

$$VI_1 = W \quad \text{or} \quad I_1 = W/V = 1500/200 = 7{\cdot}5\text{A}$$

$$\therefore \quad R_1 = V/I_1 = 200/7{\cdot}5 = \mathbf{26.7\ \Omega}$$

$$I_1 + I_2 = 15 \quad \text{..... given} \quad \therefore \quad I_2 = 15 - I_1 = 15 - 7{\cdot}5 = 7{\cdot}5\text{A}$$

$$\therefore \quad R_2 = \frac{V}{I_2} = \frac{200}{7.5} = \mathbf{26.7\ \Omega}$$

Although not technically correct usage, it is convenient to say that resistance "dissipates power", meaning that it dissipates (liberates) heat at a certain rate.

Example 4.6. *A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.*

Solution. Angular speed, $\omega = 2\pi N/60 \text{ rad/s} = 2\pi \times 75/60 = 7{\cdot}85 \text{ rad/s}$

$$\text{Power required per start, } P = \frac{\text{Torque} \times \text{Angular speed}}{\text{Efficiency of motor}} = \frac{60 \times 7.85}{0.25} = 1884 \text{ W}$$

$$\text{Energy required/start} = P \times \text{Time for start}$$
$$= 1884 \times 8 = 15072 \text{ Ws} = 15072 \text{ J}$$
$$= 15072/3600 = 4{\cdot}187 \text{ Wh}$$

$\therefore$ No. of starts with a fully-charged battery

$$= 100/4{\cdot}187 \simeq \mathbf{24}$$

Example 4.7. *A hydro-electric power station has a reservoir of area 2·4 square kilometres and capacity 5×10^6 m^3. The effective head of water is 100 m. The penstock, turbine and generator efficiencies are 95%, 90% and 85% respectively.*

(*i*) *Calculate the total energy in kWh which can be generated from the power station.*

(*ii*) *If a load of 15,000 kW has been supplied for 3 hours, find the fall in reservoir level.*

Solution.

(***i***) Wt. of water available, W = Volume of reservoir × 1000 × 9·81 N

$$= (5 \times 10^6) \times (1000) \times (9{\cdot}81) = 49{\cdot}05 \times 10^9 \text{ N}$$

Overall efficiency, $\eta_{overall} = 0{\cdot}95 \times 0{\cdot}90 \times 0{\cdot}85 = 0{\cdot}726$

Electrical energy that can be generated from the station

$$= W \times \text{Effective head} \times \eta_{overall}$$
$$= (49{\cdot}05 \times 10^9) \times (100) \times (0{\cdot}726) = 35{\cdot}61 \times 10^{11} \text{ watt-sec.}$$
$$= \frac{35.61 \times 10^{11}}{1000 \times 3600} \text{ kWh} = \mathbf{9{,}89{,}116 \text{ kWh}}$$

(***ii***)
$$\text{Level of reservoir} = \frac{\text{Volume of reservoir}}{\text{Area of reservoir}} = \frac{5 \times 10^6}{2.4 \times 10^6} = 2{\cdot}083 \text{ m}$$

$$\text{kWh generated in 3 hrs} = 15000 \times 3 = 45{,}000 \text{ kWh}$$

Using unitary method, we get,

$$\text{Fall in reservoir level} = \frac{2.083}{9{,}89{,}166} \times 45{,}000 = 0{\cdot}0947 \text{ m} = \mathbf{9{\cdot}47 \text{ cm}}$$

Example 4.8. *A large hydel power station has a head of 324 m and an average flow of 1370 m^3/sec. The reservoir is a lake covering an area of 6400 sq. km. Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate (i) the available electric power and (ii) the number of days this power could be supplied for a drop in water level by 1 metre.*

Solution. Water discharge = 1370 m^3/sec ; Water head, h = 324 m ; $\eta_{overall} = 0.9 \times 0.95$

(*i*) As mass of 1 m^3 of water is 1000 kg,

∴ Mass of water flowing/sec, $m = 1370 \times 1000$ kg $= 137 \times 10^4$ kg

Weight of water flowing/sec, $W = mg = 137 \times 10^4 \times 9.81$ N

Energy or work available per second (*i.e.* power) is

$$\text{Power available, } P = Wh \times \eta_{overall}$$
$$= (137 \times 10^4 \times 9.81) \times 324 \times (0.9 \times 0.95)$$
$$= 3723 \times 10^6 \text{ W} = \mathbf{3723\ MW}$$

(*ii*) Area of reservoir, $A = 6400 \text{ km}^2 = 6400 \times 10^6 \text{ m}^2$

Rate of water discharge, $Q = 1370 \text{ m}^3/\text{sec}$

Fall of reservoir level, $h' = 1$ m

Volume of water used $= A \times h'$

$$\therefore \quad \text{Required time, } t = \frac{A \times h'}{Q} = \frac{6400 \times 10^6 \times 1}{1370}$$
$$= 4.67 \times 10^6 \text{ sec.} = \mathbf{54.07\ days}$$

Example 4.9. *Calculate the current required by a 500 V d.c. locomotive when drawing 100 tonne load at 25 km/hr with a tractive resistance of 7 kg/tonne along* (*i*) *level road and* (*ii*) *a gradient 1 in 100. Given that the efficiency of motor and gearing is 70%.*

Solution. Weight of locomotive, W = 100 tonne = 100,000 kg

Tractive resistance, F = 7 × 100 = 700 kg-wt = 700 × 9·81 = 6867 N

(*i*) **Level Track.** In this case, the force required is equal to the tractive resistance F [See Fig. 4.5 (*i*)].

$$\text{Distance travelled/sec} = \frac{25 \times 1000}{3600} = 6{\cdot}94 \text{ m}$$

Work done/sec = Force × Distance/sec

or Motor output = 6867 × 6·94 = 47,657 watts

Motor input = 47,657/0·7 = 68,081 watts

∴ Current drawn = 68,081/500 = **136·16A**

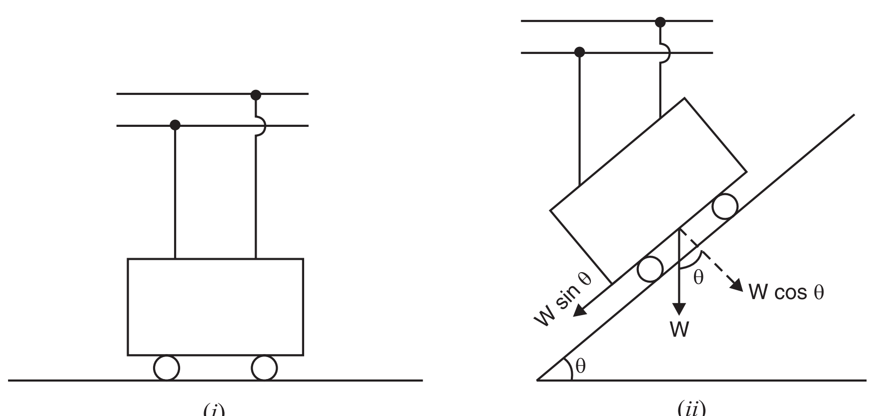

Fig. 4.5

***(ii)* Inclined plane.** In this case, the total force required is the sum of tractive resistance F and component $W \sin\theta$ of locomotive weight [See Fig. 4.5 (*ii*)]. Clearly, $\sin\theta = 1/100 = 0{\cdot}01$.

$$\therefore \quad \text{Force required} = W \sin\theta + F$$
$$= (100{,}000 \times 0{\cdot}01 + 700)\ 9{\cdot}81\ \text{N} = 16{,}677\ \text{N}$$
$$\text{Work done/sec} = \text{Force} \times \text{distance travelled/sec}$$
$$= 16{,}677 \times 6{\cdot}94 = 1{,}15{,}738\ \text{watts}$$
$$\therefore \quad \text{Motor output} = 1{,}15{,}738\ \text{watts}$$
$$\text{Motor input} = 1{,}15{,}738/0{\cdot}7 = 1{,}65{,}340\ \text{watts}$$
$$\therefore \quad \text{Current drawn} = 1{,}65{,}340/500 = \mathbf{330{\cdot}68A}$$

Example 4.10. *A diesel-electric generator set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35%, calculate (i) the mass of oil required per hour (ii) the electric energy generated per tonne of the fuel.*

Solution. Output power of set = 25 kW ; $\eta_{overall} = 35\% = 0.35$

$\therefore$ Input power to set = 25/0.35 = 71.4 kW

(i) Input energy/hour = 71.4 kW × 1h = 71.4 kWh = 71.4 × 860 kcal

As 1 kg of fuel oil produces 12,500 kcal,

$$\therefore \quad \text{Mass of fuel oil required/hour} = \frac{71.4 \times 860}{12{,}500} = \mathbf{4.91\ kg}$$

(ii) Heat content in 1 tonne fuel oil (= 1000 kg) = 1000 × 12,500 = 12.5×10^6 kcal

$$= \frac{12.5 \times 10^6}{860}\ \text{kWh} = 14{,}534\ \text{kWh}$$

$$\therefore \quad \text{Energy generated/tonne} = 14{,}534 \times 0.35 = \mathbf{5087\ kWh}$$

Example 4.11. *The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is 45×10^{10} kg. What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85% ? Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.*

Solution. Actual available head, $h = 230 - 30 = 200$ m

Energy available at turbine house is given by ;

$$E = mgh = 45 \times 10^{10} \times 9{\cdot}81 \times 200 = 8{\cdot}829 \times 10^{14}\ \text{J}$$
$$= \frac{8.829 \times 10^{14}}{36 \times 10^5}\ \text{kWh} = 24{\cdot}52 \times 10^7\ \text{kWh}$$

$$\text{Energy available at bus-bars} = E \times \eta = 24{\cdot}52 \times 10^7 \times 0.85 = \mathbf{20{\cdot}84 \times 10^7\ kWh}$$

$$\text{K.E. of water} = \text{Loss of potential energy of water}$$

$$\text{or} \quad \frac{1}{2} m v^2 = mgh \quad \therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 200} = 62{\cdot}65\ \text{m/s}$$

Power available from m kg of water is

$$P = \frac{1}{2} m v^2 = \frac{1}{2} \times m \times (62.65)^2\ \text{W}$$

This power is equal to 45 MW (= 45×10^6 W).

$\therefore \quad P = 45 \times 10^6$ W

or $\quad \frac{1}{2} \times m \times (62.65)^2 = 45 \times 10^6 \quad \therefore \quad m = 22930$ kg/s

If A is the total area of two pipes in m^2, then flow of water is Av m^3/s.

$\therefore$ Mass of water flowing/second = $Av \times 10^3$ kg ($\because$ 1 m^3 of water = 1000 kg)

$$\therefore \quad Av \times 10^3 = 22930 \quad \text{or} \quad A = \frac{22930}{62.65 \times 10^3} = 0{\cdot}366 \text{ m}^2$$

Area of each pipe = $0{\cdot}366/2 = 0{\cdot}183$ m^2

If d is the diameter of each pipe, then,

$$\frac{\pi}{4}d^2 = 0.183 \quad \text{or} \quad d = \sqrt{\frac{0.183 \times 4}{\pi}} = \mathbf{0{\cdot}4826\ m}$$

Example 4.12. *A proposed hydro-electric station has an available head of 30 m, catchment area of 50 × 10^6 m^2, the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies: Penstock 95%, Turbine 80% and Generator 85%.*

Solution. Available head, h = 30 m ; $\eta_{overall} = 0.95 \times 0.8 \times 0.85 = 0.646$

Volume of water *available/annum = $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7$ m^3

Mass of water available/annum = $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10}$ kg

Mass of water available/sec; $m = \dfrac{4.2 \times 10^{10}}{365 \times 24 \times 3600} = 1.33 \times 10^3$ kg

Potential energy available/sec = $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3$ J/s

$\therefore$ Power that could be generated = $\eta_{overall} \times 391 \times 10^3$ W

$= 0.646 \times 391 \times 10^3 = 253 \times 10^3$ W = **253 kW**

Example 4·13. *A current of 20A flows for one hour in a resistance across which there is a voltage of 8V. Determine the velocity in metres per second with which a weight of one tonne must move in order that kinetic energy shall be equal in amount to the energy dissipated in the resistance.*

Solution. Energy dissipated in resistance

$= V\,It = 8 \times 20 \times 3600 = 576 \times 10^3$ J

Mass of body, m = 1 tonne = 1000 kg

Let v m/s be the required velocity of the weight.

Kinetic energy = $\frac{1}{2}mv^2$ joules

In order that K.E. of weight is equal to energy dissipated in resistance,

$$\frac{1}{2}mv^2 = 576 \times 10^3 \quad \therefore\ v = \sqrt{\frac{2 \times 576 \times 10^3}{1000}} = \mathbf{33{\cdot}9\ m/s}$$

Example 4·14. *What must be the horse-power of an engine to drive by means of a belt a generator supplying 7000 lamps each taking 0·5 A at 250 V ? The line drop is 5V and the efficiency of the generator is 95%. There is a 2·5% loss in the belt drive.*

Solution. Total current supplied by generator, $I = 0{\cdot}5 \times 7000 = 3500$ A

Generated voltage, E = Load voltage + Line drop = 250 + 5 = 255 V

Generator output = $EI = 255 \times 3500$ W

* 0.7 × (Catchment area in m^2 × Rainfall in m)

$\therefore$ Engine output $= \dfrac{255 \times 3500}{0.95 \times 0.975} = 963562 \text{ W} = \dfrac{963562}{746}$ h.p. = **1292 h.p.**

Example 4.15. *Find the head in metres of a hydroelectric generating station in which the reservoir of area 4000 m² falls by 30 cm when 75 kWh is developed in the turbine. The efficiency of the turbine is 70%.*

Solution. Hydroelectric generating stations are generally built in hilly areas.

$$\text{Volume of water used, } V = 4000 \times 0{\cdot}3 = 1200 \text{ m}^3$$

$$\text{Mass of water used, } m = 1200 \times 10^3 = 1{\cdot}2 \times 10^6 \text{ kg}$$

$$\text{Useful energy developed in turbine} = mgh \times \eta = 1{\cdot}2 \times 10^6 \times 9.81 \times h \times 0{\cdot}7$$

$$\text{But useful energy developed in turbine} = 75 \text{ kWh} = 75 \times 3{\cdot}6 \times 10^6 \text{ J}$$

$$\therefore \quad 1{\cdot}2 \times 10^6 \times 9{\cdot}81 \times h \times 0{\cdot}7 = 75 \times 3{\cdot}6 \times 10^6$$

or $h =$ **32·76 m**

Example 4.16. *A room measures 3m × 4m × 4·75m and air in it has to be always kept 10°C higher than that of the incoming air. The air inside has to be renewed every 30 minutes. Neglecting radiation losses, find the necessary rating of electric heater for this purpose. Take specific heat of air as 0·24 and density as 1·28 kg/m³.*

Solution. It is desired to find the power of the electric heater.

$$\text{Volume of air to be changed/second} = \frac{3 \times 4 \times 4.75}{30 \times 60} = 0{\cdot}032 \text{ m}^3$$

$$\text{Mass of air to be changed/second} = 0{\cdot}032 \times 1{\cdot}28 = 0{\cdot}041 \text{ kg}$$

$$\text{Heat required/second} = \text{Mass/second} \times \text{Specific heat} \times \text{Rise in temp.}$$

$$= 0{\cdot}041 \times 0{\cdot}24 \times 10 \text{ kcal}$$

$$= 0{\cdot}041 \times 0{\cdot}24 \times 10 \times 4186 \text{ W} = \mathbf{411\ W}$$

Here, we have neglected radiation losses. However, in practice, radiation losses do occur so that heater power required would be greater than the calculated value. $\left(\because \dfrac{1 \text{ kcal}}{\text{sec.}} = 4186 \text{ W}\right)$

Example 4.17. *An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWh supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230V d.c. Assume the efficiency of the motor at 90%.*

Solution. Work done by lift $= mgh = (5 \times 10^3) \times 9.8 \times 30 = 1.47 \times 10^6$ J

$$\text{Input energy to lift} = \frac{1.47 \times 10^6}{\eta_{lift}^*} = \frac{1.47 \times 10^6}{0.75} = 1.96 \times 10^6 \text{ J}$$

$$= \frac{1.96 \times 10^6}{36 \times 10^5} \text{ kWh} = \mathbf{0.545\ kWh}$$

$$\text{Motor energy output} = \text{Input energy to lift} = 1.96 \times 10^6 \text{ J}$$

$$\text{Motor energy input} = \frac{1.96 \times 10^6}{\eta_{motor}} = \frac{1.96 \times 10^6}{0.9} = 2.18 \times 10^6 \text{ J}$$

* Since 25% energy is *wasted* in the motor and gearing, the efficiency of the lift is 75%.

$$\text{Power rating of motor} = \frac{\text{Work done}}{\text{Time taken}} = \frac{2.18\times10^6}{27\times60} = \textbf{1346 W}$$

$$\text{Current taken by motor} = \frac{1346}{230} = \textbf{5.85 A}$$

Example 4.18. *An electric hoist makes 10 double journeys per hour. In each journey, a load of 6000 kg is raised to a height of 60 m in 90 seconds and the hoist returns empty in 75 seconds. The hoist cage weighs 500 kg and has a balance weight of 3000 kg. The efficiency of the hoist is 80% and that of the driving motor 88%. Calculate* (*i*) *the electrical energy absorbed per double journey* (*ii*) *the hourly consumption in kWh* (*iii*) *the horse-power of the motor (iv) the cost of electric energy if hoist works for 4 hours per day for 30 days. Cost per kWh is Rs 4.50.*

Solution. When the hoist cage goes up, the balance weight goes down and when the cage goes down, the balance weight goes up.

$$\begin{aligned}
\text{Total mass lifted on upward journey} &= \text{Load + mass of cage} - \text{mass of balance weight} \\
&= 6000 + 500 - 3000 = 3500 \text{ kg} \\
\text{Work done during upward journey} &= mgh = 3500 \times 9{\cdot}8 \times 60 \text{ J} \\
\text{Total mass moved on downward journey} &= \text{Mass of balance wt.} - \text{Mass of cage} \\
&= 3000 - 500 = 2500 \text{ kg} \\
\text{Work done during downward journey} &= mgh = 250 \times 9{\cdot}8 \times 60 \text{ J} \\
\text{Work done during each double journey} &= 9{\cdot}8 \times 60\,(3500 + 2500)\text{J} = 353 \times 10^4 \text{ J} \\
\text{Overall } \eta &= 0{\cdot}8 \times 0{\cdot}88 = 0{\cdot}704
\end{aligned}$$

(*i*) Input energy per double journey $= 353 \times 10^4/0{\cdot}704 = 501 \times 10^4$ J

$$= \frac{501\times10^4}{3.6\times10^6} \text{ kWh} = \textbf{1·4 kWh}$$

(*ii*) Hourly consumption $= 1{\cdot}4 \times$ No. of double journeys/hr

$$= 1{\cdot}4 \times 10 = \textbf{14 kWh}$$

(*iii*) The maximum rate of working is during upward journey.

$$\therefore \quad \text{h.p. rating of motor} = \frac{\text{Work done in upward journey}}{\text{Hoist efficiency} \times \text{time for up journey} \times 746}$$

$$= \frac{3500\times9.8\times60}{0.8\times90\times746} = \textbf{38·4 h.p.}$$

(*iv*) Energy consumption for 30 days = Hourly consumption × 4 × 30 = 14 × 4 × 30 = 1680 kWh

Total cost of energy = Rs. 1680 × 4·5 = **Rs. 7560**

Example 4.19. *A generator supplies power to a factory through cables of total resistance 20 ohms. The potential difference at the generator is 5000 V and power output is 50 kW. Calculate* (*i*) *power suppled by the generator,* (*ii*) *potential difference at the factory*.

Solution. Fig. 4.6 shows the conditions of the problem.

Output power of generator is given by ;

$$P = 50 \text{ kW} = 50 \times 10^3 \text{ W}$$

P.D. at the generator, $E = 5000$ V

$\therefore$ Current in cables is given by ;

$$I = \frac{P}{E} = \frac{50\times10^3}{5000} = 10 \text{ A}$$

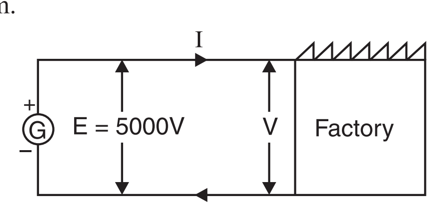

Fig. 4.6

(*i*) Power loss in cables $= I^2 R = (10)^2 \times 20 = 2000$ W

$\therefore$ Power supplied at the factory $= 50 \times 10^3 - 2000 =$ **48,000 W**

(*ii*) Voltage drop in cables $= I R = 10 \times 20 = 200$ V

$\therefore$ P.D. at the factory, $V = E - I R = 5000 - 200 =$ **4800 V**

Tutorial Problems

1. The power required to drive a certain machine at 350 r.p.m. is 600 kW. Calculate the driving torque. **[16370 Nm]**
2. An electrically driven pump lifts 1500 litres of water per minute through a height of 25 m. Allowing an overall efficiency of 75%, calculate the input power to the motor. If the pump is in operation for an average of 8 hours per day for 30 days, calculate the energy consumed in kWh and the cost of energy at the rate of 50 P/kWh. Assume 1 litre of water has a mass of 1000 kg and $g = 9{\cdot}81$ m/s^2. **[8·167 kW, 1960 kWh, Rs. 980]**
3. A 440-volt motor is used to drive an irrigation pump. The efficiency of motor is 85% and the efficiency of pump is 66%. The pump is required to lift 240 tonne of water per hour to a height of 30 metres. Calculate the current taken by the motor. **[79·48 A]**
4. A hydro-electric generating plant is supplied from a reservoir of capacity 2×10^7 m^3 with a head of 200 m. The hydraulic efficiency of the plant is 0·8 and electric efficiency is 0·9. What is the total available energy ? **[7·85 × 10^9 watt-hours]**
5. A 460-V d.c. motor drives a hoist which raises a load of 100 kg with a velocity of 15 m/s. Calculate :
   (*i*) The power output of the motor assuming the hoist gearing to have an efficiency of 0·8.
   (*ii*) The motor current, assuming the motor efficiency to be 0·75. **[(*i*) 18·4 kW (*ii*) 53·2 A]**
6. When a certain electric motor is operated for 30 minutes, it consumes 0·75 kWh of energy. During that time, its total energy loss is 3×10^5 J.
   (*i*) What is the efficiency of the motor ?
   (*ii*) How many joules of work does it perform in 30 minutes ? **[(*i*) 88·8% (*ii*) 2·4 × 10^6J]**
7. The total power supplied to an engine that drives an electric generator is 40·25 kW. If the generator delivers 15A to a 100 Ω load, what is the efficiency of the system ? **[55·9%]**
8. A certain system consists of three identical devices in cascade, each having efficiency 0·85. The first device draws 3A from a 20V source. How much current does the third device deliver to a 50Ω load ? **[0·027 A]**

4.10. Heating Effect of Electric Current

When electric current is passed through a conductor, heat is produced in the conductor. This effect is called **heating effect of electric current.**

It is a matter of common experience that when electric current is passed through the element of an electric heater, the element becomes red hot. It is because electrical energy is converted into heat energy. This is called heating effect of electric current and is utilised in the manufacture of many heating appliances, *e.g.,* electric iron, electric kettle, etc. The basic principle of all these devices is the same. Electric current is passed through a high resistance (called *heating element*), thus producing the required heat.

Cause. Let us discuss the cause of heating effect of electric current. When potential difference is applied across the ends of a conductor, the free electrons move with drift velocity and current is established in the conductor. As the free electrons move through the conductor, they collide with positive ions of the conductor. On collision, the kinetic energy of an electron is transferred to the ion with which it has collided. As a result, the kinetic energy of vibration of the positive ion increases, *i.e.,* temperature of the conductor increases. Therefore, as current flows through a conductor, the free electrons lose energy which is converted into heat. Since the source of e.m.f. (*e.g.,* a battery) is maintaining current in the conductor, it is clear that electrical energy supplied by the battery is converted into heat in the conductor.

Applications. The heating effect of electric current is utilised in the manufacture of many heating appliances such as electric heater, electric toaster, electric kettle, soldering iron etc. The basic principle of all these appliances is the same. Electric current is passed through a high resistance (called heating element), thus producing the required heat. There are a number of substances used for making a heating element. One that is commonly used is an alloy of nickel and chromium, called **nichrome.** This alloy has a resistance more than 50 times that of copper. The heating element may be either nichrome wire or ribbon wound on some insulating material that is able to withstand heat.

4.11. Heat Produced in a Conductor by Electric Current

On the basis of his experimental results, Joule found that the amount of heat produced (H) when current I amperes flows through a conductor of resistance R ohms for time t seconds is $H = I^2Rt$ joules. This equation is known as Joule's law of heating.

Suppose a battery maintains a potential difference of V volts across the ends of a conductor AB of resistance R ohms as shown in Fig. 4.7. Let the steady current that passes from A to B be I amperes. If this current flows for t seconds, then charge transferred from A to B in t seconds is

$$q = It$$

The electric potential energy lost (W) by the charge q as it moves from A to B is given by ;

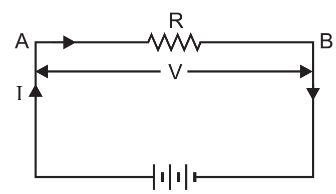

Fig. 4.7

$$W = \text{Charge} \times \text{P.D. between } A \text{ and } B$$

$$= qV = (It)\,V = I^2Rt \quad (\because V = IR)$$

or $$W = I^2Rt$$

This loss of electric potential energy of charge is converted into heat (H) because the conductor AB has resistance only.

$$\therefore \quad H = W = I^2Rt \text{ joules} = \frac{I^2Rt}{4.18} \text{ calories} \qquad ...(i)$$

It is found experimentally that 1 cal = 4.18 J.

Eq. (*i*) is known as **Joule's law of heating.** It is because Joule was the first scientist who studied the heating effect of electric current through a resistor. Thus according to Joule, heat produced in a conductor is directly proportional to

(*i*) square of current through the conductor

(*ii*) resistance of the conductor

(*iii*) time for which current is passed through the conductor.

Note.
$$H = VIt = I^2Rt = \frac{V^2}{R}t \text{ joules}$$

$$= \frac{VIt}{4.18} = \frac{I^2Rt}{4.18} = \frac{V^2t}{R \times 4.18} \text{ calories}$$

Important points. While dealing with problems on heating effect of electric current, the following points may be kept in mind :

(*i*) The electrical energy in kWh can be converted into joules by the following relation :

$$1 \text{ kWh} = 36 \times 10^5 \text{ joules}$$

(*ii*) The heat energy in calories can be converted into joules by the following relation :

$$1 \text{ calorie} = 4{\cdot}186 \text{ joules}$$
$$1 \text{ kcal} = 4186 \text{ joules}$$

(*iii*) The electrical energy in kWh can be converted into calories (or kilocalories) by the following relation :

$$1 \text{ kWh} = 36 \times 10^5 \text{ joules} = \frac{36\times10^5}{4.186} \text{ calories} = 860 \times 10^3 \text{ calories}$$

$$\therefore \quad 1 \text{ kWh} = 860 \text{ kcal}$$

(*iv*) The electrical energy supplied to the heating appliance forms the *input energy.* The heat obtained from the device is the *output energy.* The difference between the two, if any, represents the loss of energy during conversion from electrical into heat energy.

4.12. Mechanical Equivalent of Heat (J)

Joule performed a series of experiments to establish the relationship between the mechanical work done and heat produced. He found that heat produced (H) is directly proportional to the amount of mechanical work done (W) *i.e.*,

$$H \propto W \quad \text{or} \quad W = JH$$

where J is a constant of proportionality and is called *mechanical equivalent of heat*. The experimentally found value of J is

$$J = 4{\cdot}2 \text{ J/cal}$$

Note that J is a numerical factor relating mechanical units to heat units. Let us interpret the meaning of J. It takes 4·2 J of mechanical work to raise the temperature of 1g of water by 1°C. In other words, 4·2J of mechanical energy is equivalent to 1 calorie of heat energy.

Example 4.20. *In Fig. 4.8, the heat produced in 5 Ω resistor due to current flowing through it is 10 calories per second. Calculate the heat generated in 4 Ω resistor.*

Solution. Let I_1 and I_2 be the currents in the two parallel branches as shown in Fig. 4.8. The p.d. across the parallel branches is the same *i.e.*

$$I_1(4+6) = 5\,I_2 \quad \therefore \quad I_2 = 2\,I_1$$

Heat produced per second in 5Ω resistor is

$$H_1 = \frac{I_2^2 \times 5}{4.2}$$

or
$$10 = \frac{(2I_1)^2 \times 5}{4.2}$$

$$\therefore \quad I_1^2 = 2{\cdot}1$$

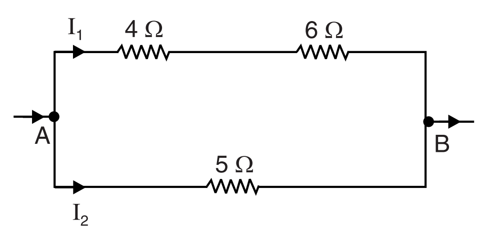

Fig. 4.8

Heat produced in 4Ω resistor per second

$$= \frac{I_1^2 \times 4}{4.2} = \frac{2.1\times4}{4.2} = \textbf{2 cal/sec}$$

Example 4.21. *An electric heater contains 4 litres of water initially at a mean temperature of 15ºC. 0·25 kWh is supplied to the water by the heater. Assuming no heat losses, what is the final temperature of the water ?*

Solution. Let tºC be the final temperature of water.

Heat received by water (*i.e.* output energy)

$$= \text{mass} \times \text{sp. heat} \times \text{rise in temp.} = 4 \times 1 \times (t - 15) \text{ kcal}$$

Electrical energy supplied to heater (*i.e.* input energy)

$$= 0{\cdot}25 \text{ kWh} = 0{\cdot}25 \times 860 \text{ kcal} \qquad (\because 1 \text{ kWh} = 860 \text{ kcal})$$

As there are no losses, output energy is equal to the input energy *i.e.*

$$4 \times 1 \times (t - 15) = 0{\cdot}25 \times 860 \quad \text{or} \quad t = \mathbf{68{\cdot}8°C}$$

Example 4.22. *An immersion heater takes 1 hour to heat 50 kg of water from 20ºC to boiling point. Calculate the power rating of the heater, assuming the heating equipment to have an efficiency of 90%.*

Solution. Heat received by water (*i.e.* output energy)

$$= \text{mass} \times \text{specific heat} \times \text{rise in temperature}$$

$$= 50 \times 1 \times 80 = 4000 \text{ kcal} = 4000/860 = 4{\cdot}65 \text{ kWh}$$

Electrical energy supplied to heater (*i.e.* input energy)

$$= 4{\cdot}65/0{\cdot}9 = 5{\cdot}167 \text{ kWh}$$

$$\therefore \qquad \text{Power rating} = \frac{\text{Energy}}{\text{Time}} = \frac{5.167}{1 \text{ hour}} = \mathbf{5.167 \text{ kW}}$$

Example 4.23. *The cost of boiling 2 kg of water in an electric kettle is 12 paise. The kettle takes 6 minutes to boil water from an ambient temperature of 20ºC. Calculate (i) the efficiency of kettle and (ii) the wattage of kettle if cost of 1 kWh is 40 paise.*

Solution. ***(i)*** Heat received by water (*i.e.* output energy)

$$= 2 \times 1 \times 80 = 160 \text{ kcal}$$

Electrical energy supplied (*i.e.* input energy)

$$= 12/40 \text{ kWh} = 860 \times 12/40 = 258 \text{ kcal}$$

$$\therefore \qquad \text{Kettle efficiency} = \frac{160}{258} \times 100 = \mathbf{62\%}$$

(ii) Let W kilowatt be the power rating of the kettle.

$$\text{Input energy} = W \times \text{time in hours}$$

$$\text{or} \qquad 12/40 = W \times 6/60$$

$$\therefore \qquad \text{Wattage of kettle, } W = \frac{12}{40} \times \frac{60}{6} = \mathbf{3 \text{ kW}}$$

Example 4.24. *How long will it take to raise the temperature of 880 gm of water from 16ºC to boiling point ? The heater takes 2 amperes at 220 V and its efficiency is 90%.*

Solution. Heat received by water (*i.e.* output energy)

$$= 0{\cdot}88 \times 1 \times (100 - 16) = 73{\cdot}92 \text{ kcal} = 73{\cdot}92/860 = 0{\cdot}086 \text{ kWh}$$

Electrical energy supplied to the heater (*i.e.* input energy)

$$= 0{\cdot}086/0{\cdot}9 = 0{\cdot}096 \text{ kWh}$$

The heater is supplying a power of 220 × 2 = 440 watts = 0·44 kW. Let t hours be the required time.

$$\text{Input energy} = \text{wattage} \times \text{time} \quad \text{or} \quad 0{\cdot}096 = 0{\cdot}44 \times t$$

$$\therefore \qquad t = 0{\cdot}096/0{\cdot}44 = 0{\cdot}218 \text{ hours} = 0{\cdot}218 \times 60 = \mathbf{13{\cdot}08 \text{ minutes}}$$

Example 4.25. *An electric kettle is required to raise the temperature of 2 kg of water from 20ºC to 100ºC in 15 minutes. Calculate the resistance of the heating element if the kettle is to be used on a 240 volts supply. Assume the efficiency of the kettle to be 80%.*

Solution. Heat received by water (*i.e.* output energy)

$$= 2 \times 1 \times (100 - 20) = 160 \text{ kcal} = 160/860 = 0{\cdot}186 \text{ kWh}$$

Electrical energy supplied to the kettle

$$= 0{\cdot}186/0{\cdot}8 = 0{\cdot}232 \text{ kWh}$$

The electrical energy of 0·232 kWh is supplied in 15/60 = 0·25 hours.

$\therefore$ Power rating of kettle $= 0{\cdot}232/0{\cdot}25 = 0{\cdot}928$ kW = 928 watts

Let R ohms be the resistance of the heating element.

$$\therefore \quad V^2/R = 928 \quad \text{or} \quad R = \frac{240 \times 240}{928} = \mathbf{62\ \Omega}$$

Example 4.26. *The heater element of an electric kettle has a constant resistance of 100 Ω and the applied voltage is 250 V. Calculate the time taken to raise the temperature of one litre of water from 15°C to 90°C assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100g, find how long will it take to raise a second litre of water through the same temperature range immediately after the first.*

Solution. Mass of water, m = 1 litre = 1 kg ; $\theta = 90 - 15 = 75$°C ; $S = 1$

Heat taken by water = $mS\theta = 1 \times 1 \times 75 = 75$ kcal

Heat taken by kettle = water equivalent of kettle × $\theta = 0.1 \times 75 = 7.5$ kcal

Heat taken by both = 75 + 7.5 = 82.5 kcal

Now, $I = \dfrac{250}{100} = 2.5$ A ; $J = 4200$ J/kcal

Heat produced electrically = $\dfrac{I^2Rt}{J}$ kcal ... t in seconds

Heat available for heating = $0.85 \times \dfrac{I^2Rt}{J}$ kcal

$$\text{or} \quad 0.85 \times \frac{I^2Rt}{J} = 82.5$$

$$\text{or} \quad 0.85 \times \frac{(2.5)^2 \times 100 \times t}{4200} = 82.5$$

$$\therefore \quad t = 652 \text{ s} = \textbf{10 min. 52 seconds}$$

In the second case, heat would be required to heat water only because kettle would be already hot.

$$\therefore \quad \frac{0.85 \times (2.5)^2 \times 100 \times t}{4200} = 75 \quad \text{or} \quad t = \textbf{9 min. 53 seconds}$$

As expected, the time required for heating in the second case is less than the first case.

Example 4.27. *The heaters A and B are in parallel across the supply voltage V. Heater A produces 500 kcal in 20 minutes and B produces 1000 kcal in 10 minutes. The resistance of heater A is 10 Ω. What is the resistance of heater B ? If the same heaters are connected in series, how much heat will be produced in 5 minutes ?*

Solution. Heat produced $= \dfrac{V^2t}{R \times J}$ kcal

$$\text{For heater } A,\ 500 = \frac{V^2 \times (20 \times 60)}{10 \times J} \quad \text{...}(i)$$

$$\text{For heater } B,\ 1000 = \frac{V^2 \times (10 \times 60)}{R \times J} \quad \text{....}(ii)$$

Dividing eq. (*i*) by eq. (*ii*), we get,

$$\frac{500}{1000} = \frac{20\times 60}{10\times 60}\times\frac{R}{10} \quad \therefore \quad R = \mathbf{2{\cdot}5\ \Omega}$$

When the heaters are connected in series, the total resistance becomes $R_T = 10 + 2{\cdot}5 = 12{\cdot}5\ \Omega$.

∴ Heat produced in 5 minutes

$$= \frac{V^2 t}{R_T \times J} = \frac{V^2}{J}\times\frac{t}{R_T}$$

$$= \frac{5{,}000}{20\times 60}\times\frac{5\times 60}{12.5} = \mathbf{100\ kcal} \qquad \left\{\begin{matrix}\text{From eq. }(i)\\ \dfrac{V^2}{J} = \dfrac{5000}{20\times 60}\end{matrix}\right\}$$

Example 4.28. *A soldering iron is rated at 50 watts when connected to a 250 V supply. If the soldering iron takes 5 minutes to heat to a working temperature of 190°C from 20°C, find its mass, assuming it to be made of copper. Given specific heat capacity of copper is 390 J/kg°C.*

Solution. Let m kg be the mass of soldering iron.

Heat gained by the soldering iron = $mS\theta = m \times 390 \times (190 - 20) = 66{,}300\ m$ joules

Heat released by the heating element = power × time = (50) × (5 × 60) = 15,000 joules

Assuming all the heat released by the element is absorbed by the copper *i.e.* soldering iron is 100% efficient,

$$15{,}000 = 66{,}300\ m \quad \therefore \quad m = 15{,}000/66{,}300 = \mathbf{0.226\ kg}$$

Example 4.29. *A cubic water tank has surface area of 6 m² and is filled to 90% capacity 6 times daily. The water is heated from 20°C to 65°C. The losses per square metre of tank surface per 1°C temperature difference are 6.3 W. Find the loading in kW and the efficiency of the tank. Assume specific heat of water = 4200 J/kg/°C and 1 kWh = 3.6 MJ.*

Solution. Rise in temp, $\theta = 65 - 20 = 45$°C ; $S = 4200$ J/kg/°C. If l metres is one side of the tank, then surface area of the tank is $6l^2$.

$$\therefore \qquad 6l^2 = 6\text{m}^2 \quad \text{or} \quad l = 1\text{m}$$

Volume of tank = $l^3 = (1)^3 = 1\text{m}^3$

Volume of water to be heated daily = 6 × 0.9 = 5.4 m³. As the mass of 1 m³ of water is 1000 kg,

∴ Mass of water to be heated daily, $m = 5.4 \times 1000 = 5400$ kg

Heat required to heat water to the desired temperature is

$$H_1 = mS\theta = 5400 \times 4200 \times 45 = 1020.6 \times 10^6 \text{ J}$$

$$= \frac{1020.6\times 10^6}{36\times 10^5} \text{ kWh} = 283.5 \text{ kWh}$$

$$\text{Heat losses, } H_2 = \frac{6.3\times 6\times\theta\times 24}{1000} \text{ kWh}$$

$$= \frac{6.3\times 6\times 45\times 24}{1000} = 40.82 \text{ kWh}$$

$$\text{Total energy supplied, } H = H_1 + H_2 = 283.5 + 40.82 = 324.32 \text{ kWh}$$

$$\text{Loading in kW} = \frac{H}{24 \text{ hr}} = \frac{324.32 \text{ kWh}}{24 \text{ hr}} = \mathbf{13.5\ kW}$$

$$\text{Efficiency of tank} = \frac{H_1}{H}\times 100 = \frac{283.5}{324.32}\times 100 = \mathbf{87.4\%}$$

Example 4.30. *An electric furnace is being used to melt 10 kg of aluminium. The initial temperature of aluminium is 20°C. Assume the melting point of aluminium to be 660°C, its specific heat capacity to be 950 J/kg°C and its specific latent heat of fusion to be 387000 J/kg. Calculate the power required to accomplish the conversion in 20 minutes, assuming the efficiency of conversion to be 75%. What is the cost of energy consumed if tariff is 50 paise per kWh ?*

Solution. Heat used to melt aluminium (*i.e.* output energy)

$$= 10 \times 950 \times (660 - 20) + 10 \times 387000 = 995 \times 10^4 \text{ joules}$$

$$= \frac{995 \times 10^4}{36 \times 10^5} = 2{\cdot}76 \text{ kWh}$$

Electrical energy supplied to the heating element

$$= 2{\cdot}76/0{\cdot}75 = 3{\cdot}68 \text{ kWh}$$

This much energy (*i.e.* 3·68 kWh) is to be supplied in 20/60 = 1/3 hour.

$$\therefore \quad \text{Power required} = \frac{3.68}{1/3} = 3{\cdot}68 \times 3 = \mathbf{11{\cdot}04\ kW}$$

$$\text{Cost of energy} = \text{Rs. } 0{\cdot}5 \times 3{\cdot}68 = \mathbf{Rs.\ 1{\cdot}84}$$

Example 4.31. *A transmitting valve is cooled by water circulating through its hollow electrodes. The water enters the valve at 25°C and leaves it at 85°C. Calculate the rate of flow in kg/second needed per kW of cooling. The temperature of 1 kg of water is raised to 1°C by 4178 joules.*

Solution. Heat to be taken away/sec = 1 kW × 1 sec = 1000 × 1 = 1000 joules. Let the required flow of water be m kg per second.

$$\text{Heat produced/sec} = \text{mass} \times \text{Sp. heat} \times \text{rise in temp.}$$

$$= m \times 4178 \times (85 - 25) = 250{,}680\, m \text{ joules}$$

$$\therefore \quad 250{,}680\, m = 1000 \quad \text{or} \quad m = \frac{1000}{250{,}680} = \mathbf{0{\cdot}004\ kg/sec}$$

Tutorial Problems

1. An electric kettle marked 1 kW, 230 V, takes 7·5 minutes to bring 1 kg of water at 15ºC to boiling point (100ºC). Find the efficiency of the kettle. **[79·07%]**
2. An electric kettle contains 1·5 kg of water at 15ºC. It takes 2·5 hours to raise the temperature to 90ºC. Assuming the heat losses due to radiation and heating the kettle to be 15 kcal, find (*i*) wattage of the kettle and (*ii*) current taken if supply voltage is 230 V. **[(*i*) 59·2 W (*ii*) 0·257 A]**
3. A soldering iron is rated at 50 watts when connected to a 250 *V* supply. If the soldering iron takes 5 minutes to heat to a working temperature of 190ºC from 20ºC, find its mass, assuming it to be made of copper. Given specific heat capacity of copper is 390 J/kgºC. **[0·226 kg]**
4. Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75ºC. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy ? Assume efficiencies of heating and lifting equipment to be 90% and 70% respectively **[4·36 kWh, 224 m]**
5. Calculate the time taken for a 25 kW furnace, having an overall efficiency of 80% to melt 20 kg of aluminium. Take the specific heat capacity, melting point and latent heat of fusion of aluminium as 896 J/kgºC, 657ºC and 402 kJ/kg respectively. **[16 min 13 sec]**
6. An electric boiler has two heating elements each of 230 V, 3·5 kW rating and containing 8 litres of water at 30ºC. Assuming 10% loss of heat from the boiler, find how long after switching on the heater circuit will the water boil at atmospheric pressure
   (*i*) if the two elements are in parallel
   (*ii*) if the two elements are in series ? The supply voltage is 230 V. **[(*i*) 373·3 s (*ii*) 1493·2 s]**
7. A coil of resistance 100 Ω is immersed in a vessel containing 0·5 kg of water at 16ºC and is connected to a 220 V electric supply. Calculate the time required to boil away all the water. Given J = 4200 J/kcal; latent heat of steam = 536 kcal/kg. **[44 min 50 sec]**

Objective Questions

1. A 25W, 220 V bulb and a 100 W, 220 V bulb are joined in parallel and connected to 220 V supply. Which bulb will glow more brightly ?
   (*i*) 25 W bulb
   (*ii*) 100 W bulb
   (*iii*) both will glow with same brightness
   (*iv*) neither bulb will glow
2. A 25 W, 220 V bulb and a 100 W, 220 V bulb are joined in series and connected to 220 V supply. Which bulb will glow brighter ?
   (*i*) 25 W bulb
   (*ii*) 100 W bulb
   (*iii*) both will glow with same brightness
   (*iv*) neither bulb will glow
3. You are given three bulbs of 25 W, 40 W and 60 W. Which of them has the lowest resistance ?
   (*i*) 25 W bulb
   (*ii*) 40 W bulb
   (*iii*) 60 W bulb
   (*iv*) information incomplete
4. You have the following electric appliances :
   (*a*) 1 kW, 250 V electric heater
   (*b*) 1 kW, 250 V electric kettle
   (*c*) 1 kW, 250 V electric bulb
   Which of these has the highest resistance ?
   (*i*) heater
   (*ii*) kettle
   (*iii*) all have equal resistances
   (*iv*) electric bulb
5. The time required for 1 kW electric heater to raise the temperature of 10 litres of water through 10ºC is
   (*i*) 210 sec (*ii*) 420 sec
   (*iii*) 42 sec (*iv*) 840 sec
6. Two electric bulbs rated at P_1 watt, V volt and P_2 watt, V volt are connected in series across V volt. The total power consumed is
   (*i*) $P_1 + P_2$ (*ii*) $\sqrt{P_1P_2}$
   (*iii*) $\dfrac{P_1 + P_2}{2}$ (*iv*) $\dfrac{P_1P_2}{P_1 + P_2}$
7. A tap supplies water at 22ºC. A man takes 1 litre of water per minute at 37ºC from the geyser. The power of geyser is
   (*i*) 1050 W (*ii*) 1575 W
   (*iii*) 525 W (*iv*) 2100 W
8. A 3ºC rise in temperature is observed in a conductor by passing a certain amount of current. When the current is doubled, the rise in temperature is
   (*i*) 15ºC (*ii*) 12ºC
   (*iii*) 9ºC (*iv*) 3ºC
9. How much electrical energy in kWh is consumed in operating ten 50 W bulbs for 10 hours in a day in a month of 30 days ?
   (*i*) 500 (*ii*) 15000
   (*iii*) 150 (*iv*) 15
10. Two heater wires of equal length are first connected in series and then in parallel. The ratio of heat produced in the two cases will be
    (*i*) 2 : 1 (*ii*) 1 : 2
    (*iii*) 4 : 1 (*iv*) 1 : 4
11. Two identical heaters each marked 1000 W, 250 V are placed in series and connected to 250 V supply. Their combined rate of heating is
    (*i*) 500 W (*ii*) 2000 W
    (*iii*) 1000 W (*iv*) 250 W
12. A constant voltage is applied between the ends of a uniform metallic wire. Some heat is developed in it. If both length and radius of the wire are halved, the heat developed during the same duration will become
    (*i*) half (*ii*) twice
    (*iii*) one fourth (*iv*) same
13. What is immaterial for a fuse ?
    (*i*) its specific resistance
    (*ii*) its radius
    (*iii*) its length
    (*iv*) current flowing through it
14. If the current in an electric bulb drops by 2%, then power decreases by
    (*i*) 1% (*ii*) 2%
    (*iii*) 4 % (*iv*) 16%
15. The fuse wire is made of
    (*i*) tin-lead alloy (*ii*) copper
    (*iii*) tungsten (*iv*) nichrome

Answers

1. (*ii*)	**2.** (*i*)	**3.** (*iii*)	**4.** (*iii*)	**5.** (*ii*)
6. (*iv*)	**7.** (*i*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*iv*)
11. (*i*)	**12.** (*i*)	**13.** (*iii*)	**14.** (*iii*)	**15.** (*i*)