9

Electromagnetic Induction

Introduction

In the beginning of nineteenth century, Oersted discovered that a magnetic field exists around a current-carrying conductor. In other words, magnetism can be created by means of an electric current. Can a magnetic field create an electric current in a conductor ? In 1831, Michael Faraday, the famous English scientist, discovered that this could be done. He demonstrated that when the magnetic flux linking a conductor changes, an e.m.f. is induced in the conductor. This phenomenon is known as *electromagnetic induction*. The great discovery of electromagnetic induction by Faraday through a series of brilliant experiments has brought a revolution in the engineering world. Most of the electrical devices (*e.g.* electric generator, transformer, telephones *etc*.) are based on this principle. In this chapter, we shall confine our attention to the various aspects of eletromagnetic induction.

9.1. Electromagnetic Induction

When the magnetic flux *linking a conductor changes, an e.m.f. is induced in the conductor. If the conductor forms a complete loop or circuit, a current will flow in it. This phenomenon is known as **electromagnetic induction.

The phenomenon of production of e.m.f. and hence current in a conductor or coil when the magnetic flux linking the conductor or coil changes is called **electromagnetic induction.**

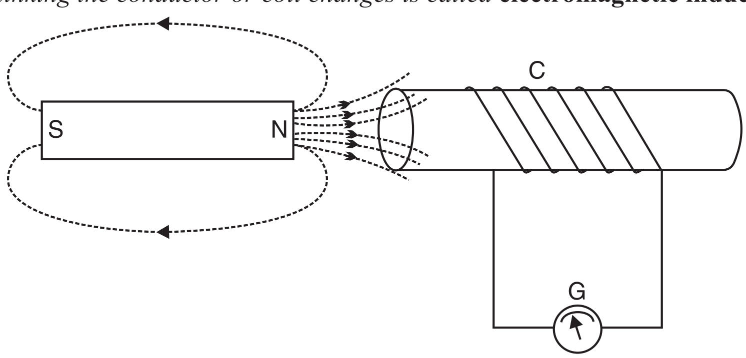


Fig. 9.1

To demonstrate the phenomenon of electromagnetic induction, consider a coil *C* of several turns connected to a centre zero galvanometer *G* as shown in Fig. 9.1. If a permanent magnet is moved towards the coil, it will be observed that the galvanometer shows deflection in one direction. If the magnet is moved away from the coil, the galvanometer again shows deflection but in the opposite direction. In either case, *the deflection will persist so long as the magnet is in motion*. The production of e.m.f. and hence current in the coil *C* is due to the fact that when the magnet is in motion (towards or away from the coil), the amount of flux linking the coil changes—the basic requirement for inducing e.m.f. in the coil. If the movement of the magnet is stopped, though the flux is linking the

* Magnetic lines of force form closed loops. Flux linking the conductor means that the flux embraces it *i.e.* it encircles the conductor.

** So called because electricity is produced from magnetism (*i.e. electromagnetic*) and that there is no physical connection (*induction*) between the magnetic field and the conductor.

coil, there is *no change in flux* and hence no e.m.f. is induced in the coil. Consequently, the deflection of the galvanometer reduces to zero.

The following points may be noted carefully :

(*i*) *The basic requirement for inducing e.m.f. in a coil is not the magnetic flux linking the coil but the change in flux linking the coil. No change in flux, no e.m.f. induced in the coil.*

(*ii*) The change in flux linking the coil can be brought about in two ways. First, the conductors (or coils) are moved through a stationary magnetic field as is the case with d.c. generators. Secondly, the conductors are stationary and the magnetic field is moving as is the case with a.c. generators. In either case, the basic principle is the same *i.e.* the amount of flux linking the conductors (or coils) is changed.

(*iii*) *The e.m.f. and hence current in the conductors (or coils) will persist so long as the magnetic flux linking them is changing.*

Note. We have seen that when magnetic flux linking a conductor changes, an e.m.f. is induced in it. An equivalent statement is like this : *When a conductor cuts magnetic field lines , an e.m.f. is induced in it.* If the conductor moves parallel to the magnetic field lines, no e.m.f. is induced. This terminology is very helpful in visualising the concept of production of e.m.f.

9.2. Flux Linkages

The product of number of turns (N) of the coil and the magnetic flux (φ) linking the coil is called **flux linkages** *i.e.*

$$\text{Flux linkages} = N\phi$$

Experiments show that the magnitude of e.m.f. induced in a coil is directly proportional to the rate of change of flux linkages. If N is the number of turns of the coil and the magnetic flux linking the coil changes (say increases) from ϕ_1 to ϕ_2 in t seconds, then,

$$\text{Induced e.m.f., } e \propto \text{Rate of change of flux linkages}$$

or $$e \propto \frac{N\phi_2 - N\phi_1}{t}$$

9.3. Faraday's Laws of Electromagnetic Induction

Faraday performed a series of experiments to demonstrate the phenomenon of electromagnetic induction. He summed up his conclusions into two laws, known as Faraday's laws of electromagnetic induction.

First Law. It tells us about the condition under which an e.m.f. is induced in a conductor or coil and may be stated as under :

When the magnetic flux linking a conductor or coil changes, an e.m.f. is induced in it.

It does not matter how the change in magnetic flux is brought about. The essence of the first law is that the induced e.m.f. appears in a circuit subjected to a changing magnetic field.

Second Law. It gives the magnitude of the induced e.m.f. in a conductor or coil and may be stated as under :

The magnitude of the e.m.f. induced in a conductor or coil is directly proportional to the rate of change of flux linkages i.e.

$$\text{Induced e.m.f., } e \propto \frac{N\phi_2 - N\phi_1}{t}$$

or $$e = k\frac{N\phi_2 - N\phi_1}{t}$$

where the value of k is *unity in SI units.

$$\therefore \qquad e = \frac{N\phi_2 - N\phi_1}{t}$$

In differential form, we have, $e = N\dfrac{d\phi}{dt}$

The direction of induced e.m.f. (and hence of induced current if the circuit is closed) is given by **Lenz's law.** The magnitude and direction of induced e.m.f. should be written as :

$$e = -N\frac{d\phi}{dt} \qquad ...(i)$$

The minus sign on the R.H.S. represents Lenz's law mathematically. In SI units, e is measured in volts, ϕ in webers and t in seconds.

9.4. Direction of Induced E.M.F. and Current

The direction of induced e.m.f. and hence current (if the circuit is closed) can be determined by one of the following two methods :

(*i*) Lenz's Law (*ii*) Fleming's right-hand rule

(*i*) Lenz's law. Emil Lenz, a German scientist, gave the following simple rule (known as Lenz's law) to find the direction of the induced current :

The induced current will flow in such a direction so as to oppose the cause that produces it i.e. the induced current will set up magnetic flux to oppose the change in flux.

Note that Lenz's law is reflected mathematically in the minus sign on the R.H.S. of Faraday's second law viz. $e = -N\,d\phi/dt$.

The negative sign simply reminds us that the induced current *opposes* the changing magnetic field that caused the induced current. The negative sign has no other meaning.

Let us apply Lenz's law to Fig. 9.2. Here the N-pole of the magnet is approaching a coil of several turns. As the N-pole of the magnet moves towards the coil, the magnetic flux linking the coil increases. Therefore an e.m.f. and hence current is induced in the coil according to Faraday's laws of electromagnetic induction. According to Lenz's law, the direction of the induced current will be such so as to oppose the cause that produces it. In the present case, the cause of the induced current is the increasing magnetic flux linking the coil. Therefore, the induced current will set up magnetic flux that opposes the increase in flux through the coil. This is possible only if the left hand face of the coil becomes N-pole. Once we know the magnetic polarity of the coil face, the direction of the induced current can be easily determined by applying right-hand rule for the coil. If the magnet is moved away from the coil, then by Lenz's law, the left hand face of the coil will become S-pole. Therefore, by right-hand rule for the coil, the direction of induced current in the coil will be opposite to that in the first case.

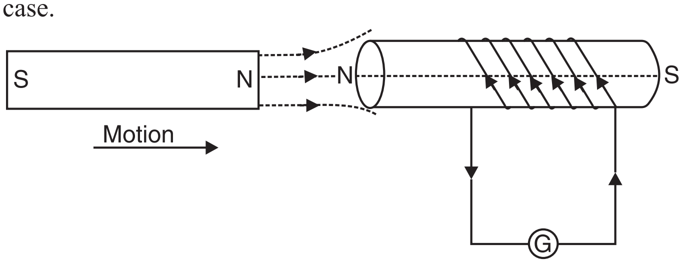


Fig. 9.2

* One volt (SI unit of e.m.f.) has been so defined that the value of k becomes unity. Thus 1V is said to be induced in a coil if the flux linkages change by 1 Wb-turn in 1 second.

Here, $N\phi_2 - N\phi_1 = 1$ Wb-turn, $t = 1$ s and $e = 1$ volt $\therefore \; 1 = k \times \frac{1}{1}$ or $k = 1$.

It may be noted here that Lenz's law directly follows from the law of conservation of energy *i.e.* in order to set up induced current, some energy must be expended. In the above case, for example, when the *N*-pole of the magnet is approaching the coil, the induced current will flow in the coil in such a direction that the left-hand face of the coil becomes *N*-pole. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this opposition is converted into electrical energy which appears in the coil. Thus Lenz's law is consistent with the law of conservation of energy.

(*ii*) Fleming's Right-Hand Rule. This law is particularly suitable to find the direction of the induced e.m.f. and hence current when the conductor moves at right angles to a stationary magnetic field. It may be stated as under :

Stretch out the **forefinger, middle finger** *and* **thumb** *of your right hand so that they are at right angles to one another. If the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current.*

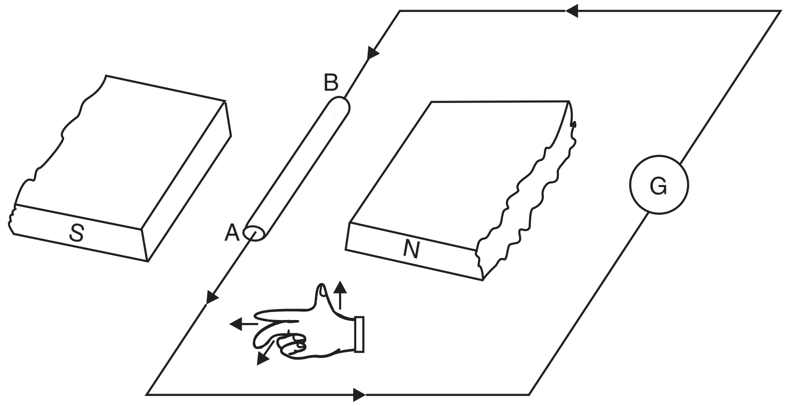


Fig. 9.3

Consider a conductor *AB* moving upwards at right angles to a uniform magnetic field as shown in Fig. 9.3. Applying Fleming's right-hand rule, it is clear that the direction of induced current is from *B* to *A*. If the motion of the conductor is downward, keeping the direction of magnetic field unchanged, then the direction of induced current will be from *A* to *B*.

Example 9.1. *A coil of 200 turns of wire is wound on a magnetic circuit of reluctance 2000 AT/Wb. If a current of 1A flowing in the coil is reversed in 10 ms, find the average e.m.f. induced in the coil.*

Solution. Flux in the coil $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{200 \times 1}{2000} = 0 \cdot 1 \text{ Wb}$

When the current (*i.e.* 1A) in the coil is reversed, flux through the coil is also reversed.

$$e = N\frac{d\phi}{dt}$$

Here, $N = 200\ ;\ d\phi = 0{\cdot}1 - (-0{\cdot}1) = 0{\cdot}2 \text{ mWb}\ ;\ dt = 10 \times 10^{-3}\text{s}$

$$\therefore \quad e = 200 \times \frac{0 \cdot 2 \times 10^{-3}}{10 \times 10^{-3}} = \mathbf{4\ V}$$

Example 9.2. *The field winding of a 4-pole d.c. generator consists of 4 coils connected in series, each coil being wound with 1200 turns. When the field is excited, there is a magnetic flux of 0·04 Wb/pole. If the field switch is opened at such a speed that the flux falls to the residual value of 0·004 Wb/pole in 0·1 second, calculate the average value of e.m.f. induced across the field winding terminals.*

Solution. Total no. of turns, $N = 1200 \times 4 = 4800$

Total initial flux $= 4 \times 0{\cdot}04 = 0{\cdot}16$ Wb

$$\text{Total residual flux} = 4 \times 0{\cdot}004 = 0{\cdot}016 \text{ Wb}$$

$$\text{Change in flux, } d\phi = 0{\cdot}16 - 0{\cdot}016 = 0{\cdot}144 \text{ Wb}$$

$$\text{Time taken, } dt = 0{\cdot}1 \text{ second}$$

$$\therefore \quad \text{Induced e.m.f., } e = N\frac{d\phi}{dt} = 4800 \times \frac{0.144}{0.1} = \mathbf{6912\ V}$$

Example 9.3. *A fan blade of length 0·5m rotates perpendicular to a magnetic field of 5×10^{-5}T. If the e.m.f. induced between the centre and end of the blade is 10^{-2}V, find the rate of rotation of the blade.*

Solution. Let n be the required number of rotations in one second. The magnitude of induced e.m.f. is given by ;

$$e = N\frac{d\phi}{dt} = N\frac{d}{dt}(BA) = B\frac{dA}{dt} \qquad (\because N = 1)$$

Here dA is the area swept by the blade in one revolution and dt is the time taken to complete one revolution.

Now $\quad e = 10^{-2}$ V ; $B = 5 \times 10^{-5}$T ; $dA = \pi r^2 = \pi \times (0{\cdot}5)^2$ m^2 ; $\quad dt = \dfrac{1}{n}$s

$$\therefore \quad 10^{-2} = 5\times10^{-5} \times \frac{\pi\times(0\cdot5)^2}{1/n}$$

or

$$n = \frac{10^{-2}}{\left(5\times10^{-5}\right)\times\pi\times(0\cdot5)^2} = \mathbf{254.7\ rev\ /\ second}$$

Doubling the speed of rotation of the blade would double the value of dA/dt. Hence, the e.m.f. induced would be doubled.

Example 9.4. *A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through 180° in 1/10 second. Calculate the average induced e.m.f.*

Solution. $\qquad \phi = NBA \cos\theta$

When the plane of the coil is perpendicular to the field, θ = 0°. When the coil is turned through 180°, θ = 180°. Therefore, initial flux linked with the coil is

$$\phi_1 = NBA \cos 0° = NBA$$

Flux linked with coil when turned through 180° is

$$\phi_2 = NBA \cos 180° = -NBA$$

Change in flux linking the coil is

$$\Delta\phi = \phi_2 - \phi_1 = (-NBA) - (NBA) = -2\,NBA$$

$$\therefore \quad \text{Average induced e.m.f., } e = -\frac{\Delta\phi}{\Delta t} = \frac{2NBA}{\Delta t}$$

Here $N = 1000$; $B = 0.4$ gauss $= 0.4 \times 10^{-4}$ T ; $A = 500 \times 10^{-4}$ m^2 ; $\Delta t = 0.1$ s

$$\therefore \quad e = \frac{2\times1000\times(0.4\times10^{-4})\times500\times10^{-4}}{0.1} = \mathbf{0.04\ V}$$

Example 9.5. *The magnetic flux passing perpendicular to the plane of the coil and directed into the paper (See Fig. 9.4) is varying according to the relation :*

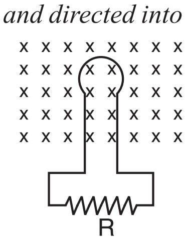


Fig. 9.4

$$\phi_B = 6t^2 + 7t + 1$$

where ϕ_B is in mWb and t in seconds.

(i) What is the magnitude of induced e.m.f. in the loop when t = 2 seconds?

(ii) What is the direction of current through the resistor R ?

Solution. $\qquad \phi_B = (6t^2 + 7t + 1)$ mWb $= (6t^2 + 7t + 1) \times 10^{-3}$ Wb

(i) Magnitude of induced e.m.f. is

$$e = \frac{d\phi_B}{dt} = \frac{d}{dt}(6t^2 + 7t + 1) \times 10^{-3} = (12t + 7) \times 10^{-3} \text{ V}$$

$$\text{At } t = 2 \text{ sec, } e = (12 \times 2 + 7) \times 10^{-3} = 31 \times 10^{-3} \text{ V} = \mathbf{31 \text{ mV}}$$

(*ii*) According to Lenz's law, the direction of induced current will be such so as to oppose the change in flux. This means that direction of current in the loop will be such as to produce magnetic field opposite to the given field. For this (*i.e.*, upward field), the current induced in the loop will be anticlockwise. Therefore, **current in resistor *R* will be from left to right.**

Tutorial Problems

1. A square coil of side 5 cm contains 100 loops and is positioned perpendicular to a uniform magnetic field of 0.6 T. It is quickly removed from the field (moving perpendicular to the field) to a region where magnetic field is zero. It takes 0.1 s for the whole coil to reach field-free region. If resistance of the coil is 100 Ω, how much energy is dissipated in the coil ? **[2·3×10^{-3}J]**
2. A flat search coil containing 50 turns each of area 2×10^{-4} m^2 is connected to a galvanometer; the total resistance of the circuit is 100Ω. The coil is placed so that its plane is normal to a magnetic field of flux density 0·25 T.
   (*i*) What is the change in magnetic flux linking the circuit when the coil is moved to a region of negligible magnetic field ?
   (*ii*) What charge passes through the galvanometer ? **[(*i*) 2·5×10^{-3} Wb (*ii*) 25 μC]**
3. The magnetic flux passing perpendicular to the plane of a coil and directed into the plane of the paper is varying according to the following equation :
   $$\phi = 5t^2 + 6t + 2$$
   where ϕ is in mWb and t in seconds. Find the e.m.f. induced in the coil at $t = 1$ s. **[16mV]**
4. A coil has an area of 0.04 m^2 and has 1000 turns. It is suspended in a magnetic field of 5×10^{-5} Wb/m^2 perpendicular to the field. The coil is rotated through 90º in 0·2s. Calculate the average e.m.f. induced in the coil due to rotation. **[0·01V]**
5. A gramophone disc of brass of diameter 30 cm rotates horizontally at the rate of 100/3 revolutions per minute. If the vertical component of earth's field is 0·01 T, calculate the e.m.f. induced between the centre and the rim of the disc. **[3·9 × 10^{-4} V]**

9.5. Induced E.M.F.

When the magnetic flux linking a conductor (or coil) changes, an e.m.f. is induced in it. This change in flux linkages can be brought about in the following two ways :

(*i*) The conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. The e.m.f. induced in this way is called **dynamically induced e.m.f.** (as in a d.c. generator). It is so called because e.m.f. is induced in the conductor which is in motion.

(*ii*) The conductor is stationary and the magnetic field is moving or changing. The e.m.f. induced in this way is called **statically induced e.m.f.** (as in a transformer). It is so called because the e.m.f. is induced in a conductor which is stationary.

It may be noted that in either case, the magnitude of induced e.m.f. is given by N$d\phi/dt$ or derivable from this relation.

9.6. Dynamically Induced E.M.F.

Consider a single conductor of length l metres moving at *right angles to a uniform magnetic field of B Wb/m^2 with a velocity of v m/s [See Fig. 9.5 (*i*)]. Suppose the conductor moves through a small distance dx in dt seconds. Then area swept by the conductor is = $l \times dx$.

* If the conductor is moved parallel to the magnetic field, there would be no change in flux and hence no e.m.f. would be induced.

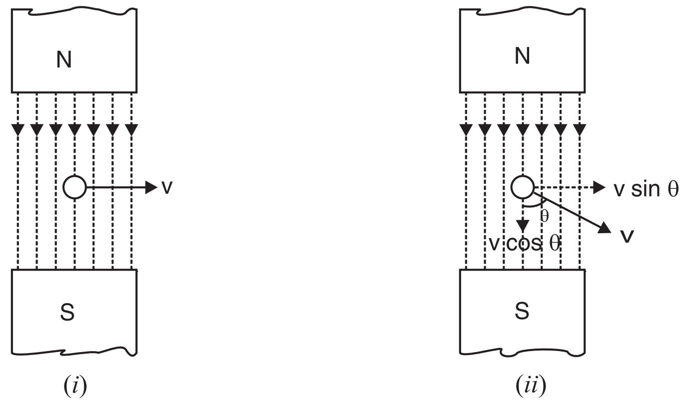


Fig. 9.5

$\therefore$ Flux cut, $d\phi$ = Flux density × Area swept = $B\,l\,dx$ Wb

According to Faraday's laws of electromagnetic induction, the magnitude of e.m.f. e induced in the conductor is given by ;

$$e = N\frac{d\phi}{dt} = \frac{B\,l\,dx}{dt} \qquad (\because N = 1)$$

$$\therefore \quad e = B\,l\,v \text{ volts} \qquad (\because dx/dt = v)$$

Special case. If the conductor moves at angle θ to the magnetic field [See Fig. 9.5 (*ii*)], then the velocity at which the conductor moves across the field is *v sin θ.

$$\therefore \quad e = B\,l\,v \sin\theta$$

The direction of the induced e.m.f. can be determined by Fleming's right-hand rule.

Example 9.6. *An aircraft has a wing span of 56 m. It is flying horizontally at a speed of 810 km/hr and the vertical component of earth's magnetic field is 4×10^{-4} Wb/m^2. Calculate the potential difference between the wing tips of the aircraft.*

Solution. Induced e.m.f. = $B\,lv$

Here $B = 4 \times 10^{-4}$ Wb/m^2 ; l = 56 m ; $v = \dfrac{810 \times 1000}{3600} = 225$ m/s

$\therefore$ Induced e.m.f. = $(4 \times 10^{-4}) \times 56 \times (225) = 5{\cdot}04$ V

or Potential difference = **5·04 V**

Example 9.7. *A d.c. generator consists of conductors lying in a radius of 10 cm and the effective length of a conductor in a constant radial field of strength 0·9 Wb/m^2 is 12 cm. The armature rotates at 1400 r.p.m. Given that the generator has 152 conductors in series, calculate the voltage being generated.*

Solution. Since the magnetic field is radial, the conductors cut the magnetic lines of force at right angles.

$$\text{Velocity, } v = \omega \times r = \frac{2\pi N}{60} \times r = \frac{2\pi \times 1400}{60} \times 0{\cdot}1 = 14{\cdot}66 \text{ m/s}$$

Voltage generated in each conductor = $B\,lv$ = 0·9 × 0·12 × 14·66 = 1·583 V

Voltage generated in 152 conductors in series

= 1·583 × 152 = **240·6 V**

Note that effective length (l) is that portion of the conductor which takes part in the actual cutting of magnetic flux lines.

* The component v cos θ is parallel to magnetic field and hence no e.m.f. is induced in the conductor due to this component.

Example 9.8. *A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2\ Wb/m^2$ as shown in Fig. 9.6. The magnetic field lines are perpendicular to the plane of the loop directed into the paper. The loop is connected to a network of resistors each of value 3 Ω. The resistances of lead wires OS and PQ are negligible. What should be the speed v_0 of the loop so as to have a steady current of 1 mA in the loop ? Also indicate the direction of current in the loop.*

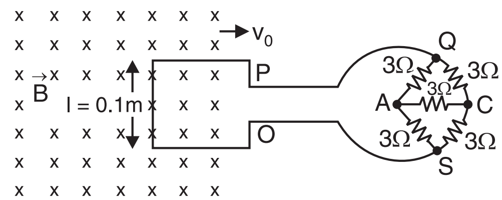


Fig. 9.6

Solution. We shall first find the equivalent resistance of the network. It is clear that network is a balanced Wheatstone bridge. Therefore, the resistance in the branch AC is ineffective. The equivalent resistance R' of the network is given by ;

$$\frac{1}{R'} = \frac{1}{6}+\frac{1}{6}=\frac{1}{3} \quad \text{or} \quad R' = 3\ \Omega$$

The resistance of the loop is 1 Ω.

∴ Effective resistance of the circuit, $R = R' + 1 = 3 + 1 = 4\ \Omega$

E.M.F. induced in the loop, $e = Bl\, v_0$

$$\text{Current in the loop, } i = \frac{e}{R}=\frac{Blv_0}{R} \quad \therefore \quad \text{Speed of the loop,} \quad v_0 = \frac{iR}{Bl}$$

Here $i = 1\text{ mA} = 10^{-3}\text{ A}$; $R = 4\ \Omega$; $B = 2\ \text{Wb/m}^2$; $l = 0.1\text{ m}$

$$\therefore \quad v_0 = \frac{10^{-3}\times 4}{2\times 0.1} = 2\times 10^{-2}\ \text{ms}^{-1} = \textbf{2 cm/second}$$

According to Fleming's right-hand rule, direction of induced current is **clockwise from O to P**.

Example 9.9. *A wheel with 10 metal spokes each 0.5 m long is rotated with a speed of 120 r.p.m. in a plane normal to earth's magnetic field at a place. If the magnitude of the field is 0.4 G, what is the magnitude of induced e.m.f. between the axle and rim of the wheel ?*

Solution. Length of spoke, l = radius $r = 0.5$ m

Frequency of rotation, n = 120 r.p.m. = 2 r.p.s.

Magnetic flux density, $B = 0.4\ G = 0.4 \times 10^{-4}$ T

Angular frequency, $\omega = 2\pi\, n = 2\pi \times 2 = 4\pi$ rad s^{-1}

As the wheel rotates, the linear velocity of spoke end at the rim = $\omega\, r$ and linear velocity of spoke end at the axle = 0.

$$\therefore \quad \text{Average linear velocity, } v = \frac{0+\omega r}{2}=\frac{1}{2}\omega r$$

Induced e.m.f. across the ends of each spoke is

$$e = Bl\, v = (B)\,(r)\left(\frac{1}{2}\omega\, r\right)=\frac{1}{2}B\,r^2\,\omega$$

or

$$e = \frac{1}{2}B\,r^2\,\omega=\frac{1}{2}(0.4\times 10^{-4})\times(0.5)^2\times 4\pi = \mathbf{6.28\times 10^{-5}\ V}$$

One end of all 10 spokes is connected to the rim and the other end to the axle. Therefore, the spokes are connected in parallel. As a result, e.m.f. between rim and axle is equal to the e.m.f. across the ends of each spoke.

Example 9.10. *A conductor 10 cm long and carrying a current of 50 A lies perpendicular to a field of strength 1000 A/m. Calculate :*

(i) the force acting on the conductor.

(*ii*) *the mechanical power to move this conductor against the force with a speed of 1 m/s.*

(*iii*) *e.m.f. induced in the conductor.*

Solution. (*i*) $F = BIl$. Now $H = 1000$ A/m

$$\therefore \quad B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 = 4\pi \times 10^{-4} \text{ Wb/m}^2$$

$$\therefore \quad F = (4\pi \times 10^{-4}) \times 50 \times 0.1 = \mathbf{6.28 \times 10^{-3} \text{ N}}$$

(*ii*) Mechanical power required is given by ;

$$P = F \times v = 6.28 \times 10^{-3} \times 1 = \mathbf{6.28 \times 10^{-3} \text{ W}}$$

(*iii*) E.M.F. induced in the conductor is given by ;

$$e = Blv = (4\pi \times 10^{-4}) \times 0.1 \times 1 = \mathbf{4\pi \times 10^{-5} \text{ V}}$$

Note that electric power developed = $eI = (4\pi \times 10^{-5}) \times 50 = 6.28 \times 10^{-3}$ W. This is equal to the mechanical input power. Therefore, law of conservation of energy is obeyed.

Tutorial Problems

1. A copper disc 40 cm in diameter is rotated at 3000 r.p.m. on a horizontal axis perpendicular to and through the centre of the disc, the axis lying in the magnetic meridian. Two brushes make contact with the disc, one at the edge and the other at the centre. If the horizontal component of earth's field be 0.02 m Wb/m^2, calculate the e.m.f. induced between the brushes. **[0.12 mV]**
2. A meter driving motor consists of a horizontal disc of aluminium 20 cm in diameter, pivoted on a vertical spindle and lying in a permanent magnetic field of density 0.3 Wb/m^2. The current flow is radial from the spindle to the circumference of the disc. The circuit resistance is 0.225 Ω and a p.d. of 2.3 V is required to pass a current of 10 A through the motor. Calculate the rotational speed of the disc and the power lost in friction. **[319 r.p.m. ; 0.5 W]**
3. If the vertical component of earth's magnetic field be 4×10^{-5} Wb/m^2, then what will be the induced potential difference produced between the rails of a metre-gauge when a train is running on them with a speed of 36 km/hr ? **[4×10^{-4} V]**

9.7. Statically Induced E.M.F.

When the conductor is stationary and the field is moving or changing, the e.m.f. induced in the conductor is called statically induced e.m.f. A statically induced e.m.f. can be further sub-divided into :

1. Self-induced e.m.f. 2. Mutually induced e.m.f.

1. Self-induced e.m.f. *The e.m.f. induced in a coil due to the change of its own flux linked with it is called* **self-induced e.m.f.**

When a coil is carrying current (See Fig. 9.7), a magnetic field is established through the coil. If current in the coil changes, then the flux linking the coil also changes. Hence an e.m.f. (= $N\, d\phi/dt$) is induced in the coil. This is known as self-induced e.m.f. The direction of this e.m.f. (by Lenz's law) is such so as to oppose the cause producing it, namely the change of current (and hence field) in the coil. The self-induced e.m.f. will persist so long as the current in the coil is changing. The following points are worth noting :

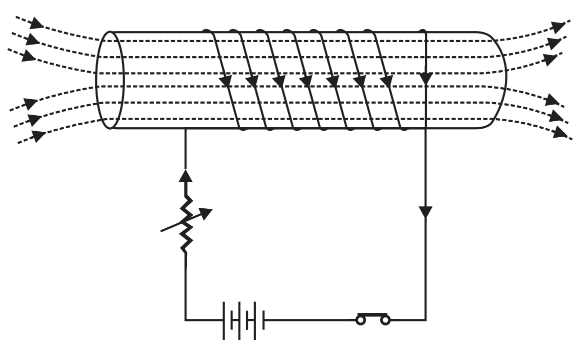


Fig. 9.7

(*i*) When current in a coil changes, the self-induced e.m.f. opposes the change of current in the coil. This property of the coil is known as its *self-inductance* or *inductance.*

(*ii*) *The self-induced e.m.f. (and hence inductance) does not prevent the current from changing ; it serves only to delay the change.* Thus after the switch is closed (See Fig. 9.7), the current will rise from zero ampere to its final steady value in some time (a fraction of a second). This delay is due to the self-induced e.m.f. of the coil.

2. Mutually induced e.m.f. *The e.m.f. induced in a coil due to the changing current in the neighbouring coil is called* **mutually induced e.m.f.**

Consider two coils *A* and *B* placed adjacent to each other as shown in Fig. 9.8. A part of the magnetic flux produced by coil *A* passes through or links with coil *B*. This flux which is common to both the coils *A* and *B* is called *mutual flux* (ϕ_m). If current in coil *A* is varied, the mutual flux also varies and hence e.m.f. is induced in both the coils. The e.m.f. induced in coil *A* is called self-induced e.m.f. as already discussed. The e.m.f. induced in coil *B* is known as *mutually induced e.m.f.*

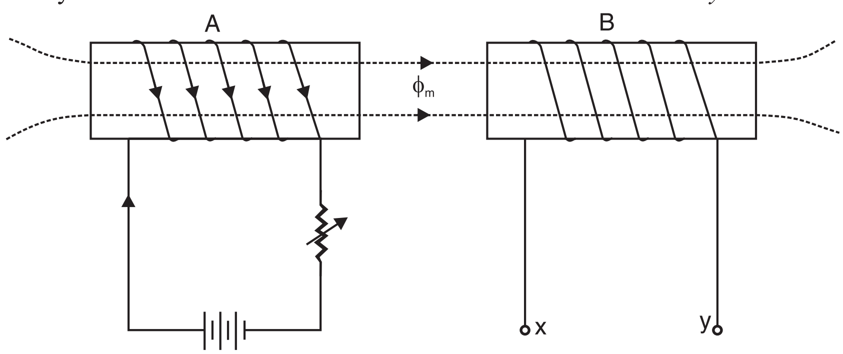


Fig. 9.8

The magnitude of mutually induced e.m.f. is given by Faraday's laws *i.e.* $e_M = N_B\, d\phi_m/dt$ where N_B is the number of turns of coil *B* and $d\phi_m/dt$ is the rate of change of mutual flux *i.e.* flux common to both the coils. The direction of mutually induced e.m.f. (by Lenz's law) is always such so as to oppose the very cause producing it. The cause producing the mutually induced e.m.f. in coil *B* is the changing mutual flux produced by coil *A*. Hence the direction of induced current (when the circuit is completed) in coil *B* will be such that the flux set up by it will oppose the changing mutual flux produced by coil *A*.

The following points may be noted carefully :

(*i*) The mutually induced e.m.f. in coil *B* persists so long as the current in coil *A* is changing. If current in coil *A* becomes steady, the mutual flux also becomes steady and mutually induced e.m.f. drops to zero.

(*ii*) The property of two neighbouring coils to induce voltatge in one coil due to the change of current in the other is called *mutual inductance*.

9.8. Self-inductance (L)

The property of a coil that opposes any change in the amount of current flowing through it is called its **self-inductance** or **inductance.**

This property (*i.e. inductance*) is due to the self-induced e.m.f. in the coil itself by the changing current. If the current in the coil is increasing, the self-induced e.m.f. is set up in such a direction so as to oppose the rise of current *i.e.* direction of self-induced e.m.f. is opposite to that of the applied voltage. Similarly, if the current in the coil is decreasing, self-induced voltage will be such so as to oppose the decrease in current *i.e.* self-induced e.m.f. will be in the same direction as the applied voltage. It may be noted that self-inductance does not prevent the current from changing ; it serves only to delay the change.

Factors affecting inductance. The greater the self-induced voltage, the greater the self-inductance of the coil and hence larger is the opposition to the changing current. According to Faraday's laws of electromagnetic induction, induced voltage in a coil depends upon the number of turns (N) and the rate of change of flux ($d\phi/dt$) linking the coil. Hence, the inductance of a coil depends upon these factors, *viz* :

(*i*) Shape and number of turns.

(*ii*) Relative permeability of the material surrounding the coil.

(*iii*) The speed with which the magnetic field changes.

In fact, anything that affects magnetic field also affects the inductance of the coil. Thus, increasing the number of turns of a coil increases its inductance. Similarly, substituting an iron core for air core increases its inductance.

It may be noted carefully that inductance makes itself felt in a circuit (or coil) only when there is a changing current. Thus, although a circuit element may have inductance by virtue of its geometrical and magnetic properties, its presence in the circuit is not exhibited unless there is a change of current in the circuit. For example, if a steady direct current (d.c.) is flowing in a circuit, there will be no inductance. However, when alternating current is flowing in the same circuit, the current is constantly changing and hence the circuit exhibits inductance.

Note. The self-inductance of a coil opposes the change of current (increase or decrease) through the coil. This opposition occurs because a changing current produces self-induced e.m.f. (e) which opposes the change of current. For this reason, *self-inductance of a coil is called* **electrical inertia** *of the coil*.

9.9. Magnitude of Self-induced E.M.F.

Consider a coil of N turns carrying a current of I amperes. If current in the coil changes, the flux linkages of the coil will also change. This will set up a self-induced e.m.f. e in the coil given by ;

$$e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$$

Since flux is due to current in the coil, it follows that flux linkages (= $N\phi$) will be proportional to I.

$$\therefore \quad e = \frac{d}{dt}(N\phi) \propto \frac{dI}{dt}$$

$$\therefore \quad e = \text{Constant} \times \frac{dI}{dt}$$

$$\text{or} \quad e = L\frac{dI}{dt} \quad (\textit{in magnitude}) \quad ...(i)$$

where L is a constant called **self-inductance** or **inductance** of the coil. The unit of inductance is henry (H). If in eq. (i) above, e = 1 volt, dI/dt = 1 A/second, then L = 1 H.

Hence a coil (or circuit) has an inductance of **1 henry** *if an e.m.f. of 1 volt is induced in it when current through it changes at the rate of* 1 *ampere per second.*

Note. The magnitude of self-induced e.m.f. is $e = LdI/dt$. However, the magnitude and direction of self-induced e.m.f. should be written as :

$$e = -L\frac{dI}{dt}$$

The minus sign is because the self-induced e.m.f. tends to send current in the coil in such a direction so as to produce magnetic flux which opposes the change in flux produced by the change in current in the coil. In fact, minus sign represents Lenz's law mathematically.

9.10. Expressions for Self-inductance

The self-inductance (L) of a circuit or coil can be determined by one of the following three ways :

***(i)* First Method.** If the magnitude of self-induced e.m.f. (e) and the rate of change of current (dI/dt) are known, then inductance can be determined from the following relation :

$$e = L\frac{dI}{dt}$$

$$\therefore \qquad L = \frac{e}{(dI/dt)} \qquad ...(i)$$

***(ii)* Second Method.** If the flux linkages of the coil and current are known, then inductance can be determined as under :

$$e = L\frac{dI}{dt} = \frac{d}{dt}(LI)$$

Also $$e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$$

From the two expressions, we have,

$$LI = N\phi$$

$$\therefore \qquad L = \frac{N\phi}{I} \qquad ...(ii)$$

Thus, inductance is the flux linkages of the coil per ampere. If $N\phi = 1$ Wb-turn and $I = 1$ A, then $L = 1$H.

Hence a coil has an inductance of **1 henry** *if a current of 1 A in the coil sets up flux linkages of 1 Wb-turn.*

Note. Relation (*ii*) above reveals that inductance depends upon the ratio ϕ/I. Therefore, inductance is constant only when the flux changes uniformly with current. This condition is met only when the flux path is entirely composed of non-magnetic material *e.g.* air. But when the flux path is through a magnetic material (*e.g.* coil wound over iron bar), inductance of the coil will be constant only over the linear portion of the magnetisation curve.

***(iii)* Third Method.** The inductance of a magnetic circuit can be found in terms of its physical dimensions. Consider an iron-cored *solenoid of dimensions as shown in Fig. 9.9. Inductance of the solenoid is given by [from exp. (*ii*) above] ;

$$L = N\frac{d\phi}{dI}$$

Now $$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l/a\mu_0\mu_r}$$

Differentiating ϕ *w.r.t.* I, we get,

$$\frac{d\phi}{dI} = \frac{Na\mu_0\mu_r}{l}$$

$$\therefore \qquad L = N\frac{(Na\mu_0\mu_r)}{l}$$

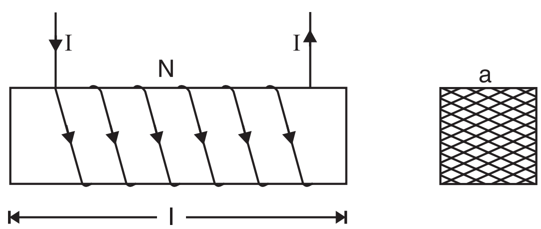


Fig. 9.9

or $$L = \frac{N^2 a\mu_0\mu_r}{l} \qquad ...(iii)$$

$$= \frac{N^2}{l/a\mu_0\mu_r} = \frac{N^2}{\text{Reluctance}\,(S)} \qquad ...(iv)$$

Thus, inductance can be determined by using the relation (*iii*) or (*iv*). *It is important to note [See relation (iv)] that inductance is directly proportional to turns squared and inversely proportional*

* Solenoid is an important winding arrangement, being simple to manufacture, it is found in relays, inductors, small transformers in the form considered.

to the reluctance of the magnetic path. The smaller the reluctance of the magnetic path, the larger the inductance and *vice-versa.* For this reason, an iron-cored coil has more inductance than the equivalent air-cored coil.

Example 9.11. *A coil wound on an iron core of permeability 400 has 150 turns and a cross-sectional area of 5 cm². Calculate the inductance of the coil. Given that a steady current of 3 mA produces a magnetic field of 10 lines/cm² when air is present as the medium.*

Solution.
$$\mu_i = \frac{\text{Flux density in iron}}{\text{Flux density in air}} = \frac{B_i}{10}$$

$$\therefore \quad B_i = 10 \times \mu_i = 10 \times 400 = 4000 \text{ lines/cm}^2$$

Flux produced by 3 mA current in the iron core is

$$\phi = B_i \times a = 4000 \times 5 = 20{,}000 \text{ lines} = 2 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad L = \frac{N\,\phi}{I} = \frac{150 \times 2 \times 10^{-4}}{3 \times 10^{-3}} = \mathbf{10\ H}$$

Example 9.12. *A solenoid with 900 turns has a total flux of 1.33 × 10⁻⁷ Wb through its air core when the coil current is 100 mA. If the flux takes 75 ms to grow from zero to its maximum level, calculate the inductance of the coil. Also, calculate the induced e.m.f. in the coil during the flux growth.*

Solution. The magnitude of induced e.m.f. is given by the following two expressions :

$$e = L\frac{dI}{dt} \;;\; e = N\frac{d\phi}{dt}$$

$$\therefore \quad L\frac{dI}{dt} = N\frac{d\phi}{dt} \quad \text{or} \quad L = N\frac{d\phi}{dI}$$

Here $N = 900$; $d\phi = 1.33 \times 10^{-7}$ Wb ; $dt = 75$ ms $= 75 \times 10^{-3}$ s ;

$dI = 100$ mA $= 100 \times 10^{-3}$ A

$$\therefore \quad L = 900 \times \frac{1.33 \times 10^{-7}}{100 \times 10^{-3}} = 1.2 \times 10^{-3} \text{ H} = \mathbf{1.2\ mH}$$

$$\text{Induced e.m.f., } e = N\frac{d\phi}{dt} = 900 \times \frac{1.33 \times 10^{-7}}{75 \times 10^{-3}} = 1.6 \times 10^{-3} \text{ V} = \mathbf{1.6\ mV}$$

Example 9.13. *An air-cored choke is designed to have an inductance of 20H when operating at a flux density of 1 Wb/m² ; the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding ; given that the flux path has a mean length of 22 cm in the iron core and 1 mm in air gap and that its cross-section is 10 cm².*

Solution.
$$L = N^2/S_T$$

where S_T is the total reluctance of the magnetic path.

$$S_T = S_{iron} + S_{air} = \frac{l_{iron}}{a\,\mu_0\,\mu_r} + \frac{l_{air}}{a\,\mu_0\,\mu_r}$$

$$= \frac{0 \cdot 22}{\left(10 \times 10^{-4}\right) \times 4\pi \times 10^{-7} \times 4000} + \frac{0 \cdot 001}{\left(10 \times 10^{-4}\right) \times 4\pi \times 10^{-7} \times 1}$$

$$= 43767 + 795774 = 839541 \text{ AT/Wb}$$

Now
$$L = N^2/S_T$$

$$\therefore \quad N = \sqrt{L\,S_T} = \sqrt{20 \times 839541} = \mathbf{4097\ turns}$$

Example 9.14. *An iron rod, 1 cm diameter and 50 cm long is formed into a closed ring and uniformly wound with 400 turns of wire. A direct current of 0·5 A is passed through the winding and produces a flux density of 0·75 Wb/m². If all the flux links with every turn of the winding, calculate* *(i) the relative permeability of iron (ii) the inductance of the coil (iii) the average value of e.m.f. induced when the interruption of current causes the flux in the iron to decay to 20% of its original value in 0·01 second.*

Solution. (*i*)
$$H = \frac{NI}{l} = \frac{400 \times 0 \cdot 5}{0 \cdot 5} = 400 \text{ AT/m}$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.75}{4\pi \times 10^{-7} \times 400} = \mathbf{1492}$$

(*ii*)
$$\phi = B \times a = 0 \cdot 75 \times \frac{\pi}{4}\left(1 \times 10^{-2}\right)^2 = 0 \cdot 589 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad L = \frac{N\phi}{I} = \frac{(400) \times 0 \cdot 589 \times 10^{-4}}{0 \cdot 5} = \mathbf{0.0471\ H}$$

(*iii*) Change in flux, $d\phi$ = 80% of original flux = $0 \cdot 8 \times 0 \cdot 589 \times 10^{-4} = 0 \cdot 47 \times 10^{-4}$ Wb

$$\therefore \quad e = N\frac{d\phi}{dt} = 400 \times \frac{0.47 \times 10^{-4}}{0.01} = \mathbf{1.88\ V}$$

Example 9.15. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm^2 in section. With 4A, the flux density is 1 Wb/m^2 and with 9A, it is 1.4 Wb/m^2. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9A to 4A in 0·05 seconds.*

Solution.
$$L = N\frac{d\phi}{dI} = N\frac{d}{dI}(BA) = NA\frac{dB}{dI}$$

Here $N = 1000$; $dB = 1 \cdot 4 - 1 = 0 \cdot 4$ Wb/m^2 ; $dI = 9 - 4 = 5$A

$$\therefore \quad L = (1000) \times (20 \times 10^{-4}) \times \frac{0.4}{5} = \mathbf{0.16\ H}$$

Also
$$e = L\frac{dI}{dt} = 0 \cdot 16 \times \frac{5}{0 \cdot 05} = \mathbf{16\ V}$$

Example 9.16. *A single element has the current and voltage functions graphed in Fig. 9.10 (i) and (ii). Determine the element.*

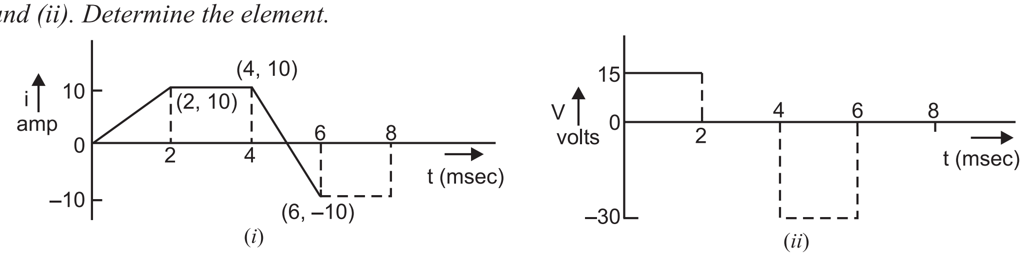


Fig. 9.10

Solution. From $i - t$ and $V - t$ graph of the element, we observe that :

Between 0 – 2 ms ; $di = 10$A ; $dt = 2$ ms ; $V = 15$ volts

$$\therefore \quad \frac{di}{dt} = \frac{10\text{ A}}{2 \times 10^{-3}\text{ s}} = 5000 \text{ A/s. Now, } L = \frac{V}{di/dt} = \frac{15}{5000} = 3 \times 10^{-3} \text{ H} = 3 \text{ mH}$$

Between 4 – 6 ms ; $di = -20$A ; $dt = 2$ ms ; $V = -30$ volts

$$\therefore \quad \frac{di}{dt} = \frac{-20\text{A}}{2 \times 10^{-3}\text{ s}} = -10{,}000 \text{ A/s. Now, } L = \frac{V}{di/dt} = \frac{-30}{-10{,}000} = 3 \times 10^{-3} \text{ H} = 3 \text{ mH}$$

Note that when current is constant, $di/dt = 0$ so that voltage across L is zero. **Hence, the element is 3 mH inductor.**

Example 9.17. *A 300-turn coil has a resistance of 6 Ω and an inductance of 0·5 H. Determine the new resistance and new inductance if one-third of the turns are removed. Assume all the turns have the same circumference.*

Solution. As the resistance of a coil is directly proportional to its length,

$$\therefore \quad R_1/R_2 = N_1/N_2 \quad \text{or} \quad 6/R_2 = 300/200$$

$$\therefore \quad R_2 = 6 \times \frac{200}{300} = \mathbf{4\ \Omega}$$

$$\text{Also} \quad \frac{L_1}{L_2} = \frac{N_1^2/S}{N_2^2/S} \quad \text{or} \quad \frac{0.5}{L_2} = \frac{(300)^2}{(200)^2}$$

$$\therefore \quad L_2 = 0.5 \times \frac{(200)^2}{(300)^2} = \mathbf{0{\cdot}22\ H}$$

Example 9.18. *A battery of 24 V is connected to the primary (coil 1) of a two-winding transformer as shown in Fig. 9.11 and the secondary (coil 2) is open-circuited. The coil parameters are :*

$$R_1 = 10\ \Omega \qquad R_2 = 30\ \Omega$$
$$N_1 = 100 \text{ turns} \qquad N_2 = 160 \text{ turns}$$
$$\phi_1 = 0{\cdot}01 \text{ Wb} \qquad \phi_2 = 0{\cdot}008 \text{ Wb}$$

Calculate (i) the self-inductance of coil 1 (ii) the mutual inductance (iii) the coefficient of coupling and (iv) the self-inductance of coil 2.

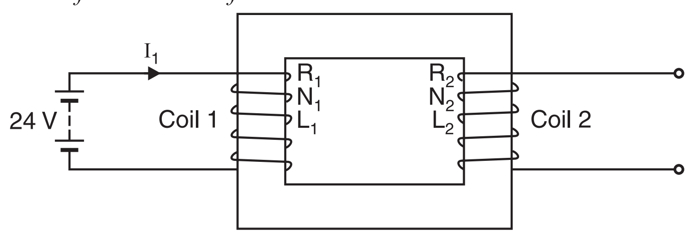


Fig. 9.11

Solution. ***(i)*** $\quad I_1 = V/R_1 = 24/10 = 2{\cdot}4\text{A}$

$$\therefore \quad L_1 = \frac{N_1\,\phi_1}{I_1} = \frac{100 \times 0.01}{2{\cdot}4} = \mathbf{0.417\ H}$$

(ii)
$$M = \frac{N_2\,\phi_2}{I_1} = \frac{160 \times 0.008}{2{\cdot}4} = \mathbf{0.533\ H}$$

(iii) $\quad k = 0{\cdot}008/0{\cdot}01 = \mathbf{0{\cdot}8}$

(iv) $\quad M = k\sqrt{L_1 L_2} \quad \text{or} \quad 0{\cdot}533 = 0{\cdot}8\sqrt{0{\cdot}417 \times L_2} \quad \therefore \quad L_2 = \mathbf{1{\cdot}064\ H}$

Example 9.19. *A coil of 1000 turns is wound on a laminated core of steel having a cross-section of 5 cm*^{2}*. The core has an air gap of 2 mm cut at right angle. What value of current is required to have an air gap flux density of 0.5 T? Permeability of steel may be taken as infinity. Determine the coil inductance.*

Solution. $B_g = 0.5$ T ; $a = 5 \times 10^{-4}$ m^2 ; $N = 1000$ turns ; $l_g = 2 \times 10^{-3}$ m ; $\mu_r = \infty$

$$\text{Total } AT \text{ required} = H_i l_i + H_g l_g = \frac{B_g}{\mu_0 \mu_r} l_i + \frac{B_g}{\mu_0} l_g$$

$$= 0 + \frac{B_g}{\mu_0} l_g = 0 + \frac{0.5}{4\pi \times 10^{-7}} \times 2 \times 10^{-3} = 796 \text{ AT} \qquad (\because \mu_r = \infty)$$

Now $NI = 796 \quad \therefore I = \frac{796}{N} = \frac{796}{1000} = \mathbf{0.796\ A}$

$$\text{Inductance of coil, } L = \frac{N\phi}{I} = \frac{N \times (B_g \times a)}{I} = \frac{1000 \times (0.5 \times 5 \times 10^{-4})}{0.796} = \mathbf{0.314\ H}$$

Tutorial Problems

1. A current of 2·5 A flows through a 1000-turn coil that is air-cored. The coil inductance is 0·6 H. What magnetic flux is set up ? **[1·5 m Wb]**
2. A 2000-turn coil is uniformly wound on an ebonite ring of mean diameter 320 mm and cross-sectional area 400 mm^2. Calculate the inductance of the toroid so formed. **[2 mH]**
3. A coil has self-inductance of 10 H. If a current of 200 mA is reduced to zero in a time of 1 ms, find the average value of induced e.m.f. across the terminals of the coil. **[2000 V]**
4. A coil consists of 750 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1200 μWb. Calculate the inductance of the coil and determine the average e.m.f. induced in the coil when this current is reversed in 0·01 second. **[0·09 H ; 180 V]**
5. Calculate the inductance of a solenoid of 2000 turns wound uniformly over a length of 50 cm on a cylindrical paper tube 4 cm in diameter. The medium is air. **[12·62 mH]**
6. A circular iron ring of mean diameter 100 mm and cross-sectional area 500 mm^2 has 200 turns of wire uniformly wound around the circumference. If the relative permeability of iron is assumed to be 1200, find the self-inductance of the coil. **[96 mH]**
7. A certain 40-turn coil has an inductance of 6 H. Determine the new inductance if 10 turns are added to the coil. **[9·38 H]**
8. The e.m.f. induced in a coil is 100V when current through it changes from 1A to 10 A in 0·1s. Calculate the inductance of the coil. **[1·11 H]**
9. A 6-pole, 500 V d.c. generator has a flux/pole of 50 mWb produced by a field current of 10 A. Each pole is wound with 600 turns. The resistance of entire field circuit is 50 Ω. If the field circuit is broken in 0·02s, calculate (*i*) the inductance of the field coils (*ii*) the induced e.m.f. and (*iii*) the value of discharge resistance so that the induced e.m.f. should not exceed 1000V. **[(*i*) 18 H (*ii*) 1500 V (*iii*) 50 Ω]**
10. What is the inductance of a single layer 10-turn air-cored coil that is 1 cm long and 0·5 cm in diameter ? **[0·214 μH]**

9.11. Magnitude of Mutually Induced E.M.F.

Consider two coils A and B placed adjacent to each other as shown in Fig. 9.12. If a current I_1 flows in the coil A, a flux is set up and a part ϕ_{12} (*mutual flux*) of this flux links the coil B. If current in coil A is varied, the mutual flux also varies and hence an e.m.f. is induced in the coil B. The e.m.f. induced in coil B is termed as mutually induced e.m.f. Note that coil B is not electrically connected to coil A ; the two coils being magnetically linked.

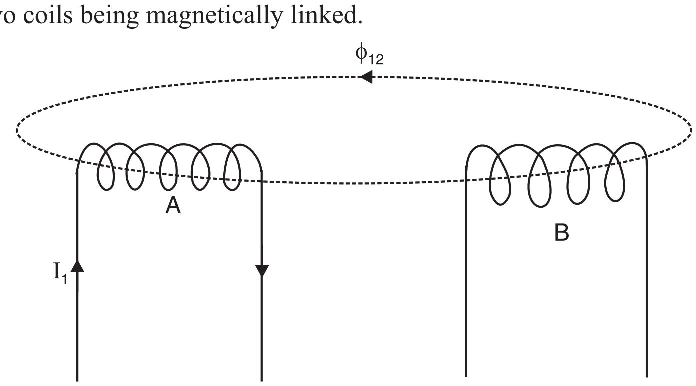


Fig. 9.12

The larger the rate of change of current in coil A, the greater is the e.m.f. induced in coil B. In other words, mutually induced e.m.f. in coil B is directly proportional to the rate of change of current in coil A *i.e.,*

Mutually induced e.m.f. in coil $B \propto$ Rate of change of current in coil A

or $$e_M \propto \frac{dI_1}{dt}$$

or $$e_M = M\frac{dI_1}{dt} \qquad (\textit{in magnitude}) \qquad ...(i)$$

where M is a constant called **mutual inductance** between the two coils. The unit of mutual inductance is henry (H). If in exp. (i), $e_M = 1$ volt, $dI_1/dt = 1$ A/sec, then, $M = 1$ H.

Hence mutual inductance between two coils is **1 henry** *if current changing at the rate of 1 A/sec in one coil induces an e.m.f. of 1 V in the other coil.*

Mutual inductance comes into picture when two coils are placed close together in such a way that flux produced by one links the other. We say then that the two coils are coupled. Each coil has its own inductance but in addition, there is further inductance due to the induced voltage produced by coupling between the coils. We call this further inductance as mutual inductance. We say the two coils are coupled together by mutual inductance. The terms **magnetic** or **inductive coupling** are sometimes used.

Note. The magnitude of mutually induced e.m.f. in coil B (secondary) is $e_M = M\, dI_1/dt$ where dI_1 is the change of current in coil A (primary). However, the magnitude and direction of mutually induced e.m.f. in coil B should be written as :

$$e_M = -M\frac{dI_1}{dt}$$

The minus sign is because the mutually induced e.m.f. sends current in coil B in such a direction so as to produce magnetic flux which opposes the change in flux produced by change in current in coil A. In fact, minus sign represents Lenz's law mathematically.

9.12. Expressions for Mutual Inductance

The mutual inductance between two coils can be determined by one of the following three methods :

(*i*) First Method. If the magnitude of mutually induced e.m.f. (e_M) in one coil for the given rate of change of current in the other is known, then M between the two coils can be determined from the following relation :

$$e_M = M\frac{dI_1}{dt}$$

or $$M = \frac{e_M}{dI_1/dt} \qquad ...(i)$$

(*ii*) Second Method. Let there be two magnetically coupled coils A and B having N_1 and N_2 turns respectively (See Fig. 9.13). Suppose a current I_1 flowing in coil A produces a mutual flux ϕ_{12}. Note that mutual flux ϕ_{12} is that part of the flux created by coil A which links the coil B.

$$e_M = M\frac{dI_1}{dt} = \frac{d}{dt}(M\,I_1)$$

Also $$e_M = N_2\frac{d\phi_{12}}{dt} = \frac{d}{dt}(N_2\phi_{12})$$

From these two expressions, we have,

$$MI_1 = N_2\,\phi_{12}$$

or $$M = \frac{N_2\phi_{12}}{I_1} \qquad ...(ii)$$

Thus, mutual inductance between two coils is equal to the flux linkages of one coil ($N_2\phi_{12}$) due to one ampere in the other coil. If $N_2\,\phi_{12} = 1$ Wb-turn and $I_1 = 1$ A, then, $M = 1$ H.

Hence mutual inductance between two coils is **1 henry** *if a current of 1 A flowing in one coil produces flux linkages of 1 Wb-turn in the other.*

(*iii*) Third Method. The mutual inductance between the two coils can be determined in terms of physical dimensions of the magnetic circuit. Fig. 9.13 shows two magnetically coupled coils *A* and *B* having N_1 and N_2 turns respectively. Suppose l and 'a' are the length and area of cross-section of the magnetic circuit respectively. Let μ_r be the relative permeability of the material of which the magnetic circuit is composed.

$$\text{Mutual flux, } \phi_{12} = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{N_1 I_1}{l / a\,\mu_0\,\mu_r}$$

or $$\frac{\phi_{12}}{I_1} = \frac{N_1\,a\,\mu_0\,\mu_r}{l}$$

Now $$M = \frac{N_2\,\phi_{12}}{I_1}$$

$\therefore$ $$M = \frac{N_1 N_2\,a\,\mu_0\,\mu_r}{l} \quad \text{...}(iii)$$

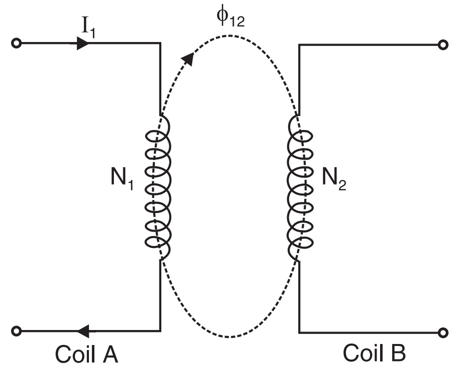


Fig. 9.13

$$= \frac{N_1 N_2}{l / a\,\mu_0\,\mu_r}$$

$$= \frac{N_1\,N_2}{\text{Reluctance}\,(S)} \quad \text{...}(iv)$$

The mutual inductance can be found by using relation (*iii*) or (*iv*). Note that mutual inductance is inversely proportional to the reluctance of the magnetic circuit. The smaller the reluctance of the magnetic circuit, the greater is the mutual inductance and *vice-versa*.

9.13. Coefficient of Coupling

The **coefficient of coupling (*k*)** *between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other.*

When the entire flux of one coil links the other, coefficient of coupling is 1 (*i.e.*, 100%). If only half the flux set up in one coil links the other, then coefficient of coupling is 0·5 (or 50%). If two coils have self-inductances L_1 and L_2, then mutual inductance M between them is given by ;

$$M = k\sqrt{L_1\,L_2}$$

where k = coefficient of coupling. Clearly, the mutual inductance M between the coils will be maximum when $k = 1$. If flux of one coil does not at all link with the other coil, then $k = 0$. Under such condition, mutual inductance (M) between the coils will be zero.

Proof. Consider two magnetically coupled coils 1 and 2 having N_1 and N_2 turns respectively (See Fig. 9.14). The current I_1 flowing in coil 1 produces a magnetic flux ϕ_1. Suppose the coefficient of coupling between the two coils is k. It means that flux $k\phi_1$ links with coil 2. Then, by definition,

$$L_1 = \frac{N_1\phi_1}{I_1}$$

and $$M_{12} = \frac{k\phi_1 N_2}{I_1} \quad \text{...}(i)$$

where M_{12} represents mutual inductance of coil 1 to coil 2.

The current I_2 flowing in coil 2 will produce flux ϕ_2. Since the coefficient of coupling between the coils is k, it means that flux $k\phi_2$ will link with coil 1. Then,

$$L_2 = \frac{\phi_2 N_2}{I_2}$$

and $$M_{21} = \frac{k\phi_2 N_1}{I_2} \quad ...(ii)$$

where M_{21} represents mutual inductance of coil 2 to coil 1.

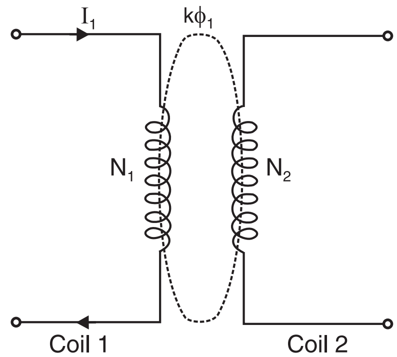


Fig. 9.14

Mutual inductance between the two coils is exactly the same *i.e.*, $M_{12} = M_{21} = M$.

$$\therefore \quad M_{12} \times M_{21} = \frac{(k\phi_1 N_2)}{I_1} \times \frac{(k\phi_2 N_1)}{I_2}$$

or $$M^2 = k^2 \frac{\phi_1 N_1}{I_1} \times \frac{\phi_2 N_2}{I_2} = k^2 L_1 L_2$$

$$\therefore \quad M = k\sqrt{L_1 L_2} \quad ...(iii)$$

Expression (*iii*) gives the relation between the mutual inductance of the two coils and their self-inductances. The reader may note that mutual inductance between the two coils will be maximum when $k = 1$. Obviously, the maximum value of mutual inductance between the two coils is $= \sqrt{L_1 L_2}$.

$$\therefore \quad k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\text{Actual mutual inductance}}{\text{Max. possible mutual inductance}}$$

Hence, coefficient of coupling can also be defined *as the ratio of the actual mutual inductance* (M) *between the two coils to the maximum possible value* ($\sqrt{L_1 L_2}$).

When two coils are wound on a single ferromagnetic core as shown in Fig. 9.15 (*i*), effectively all of the magnetic flux produced by one coil links with the other. The coils are then said to be **tightly coupled.** Another way to ensure tight coupling is shown in Fig. 9.15 (*ii*) where each turn of the secondary winding is side by side with one turn of primary winding. Coils wound in this fashion are said to be bifilar and it is called **bifilar winding.**

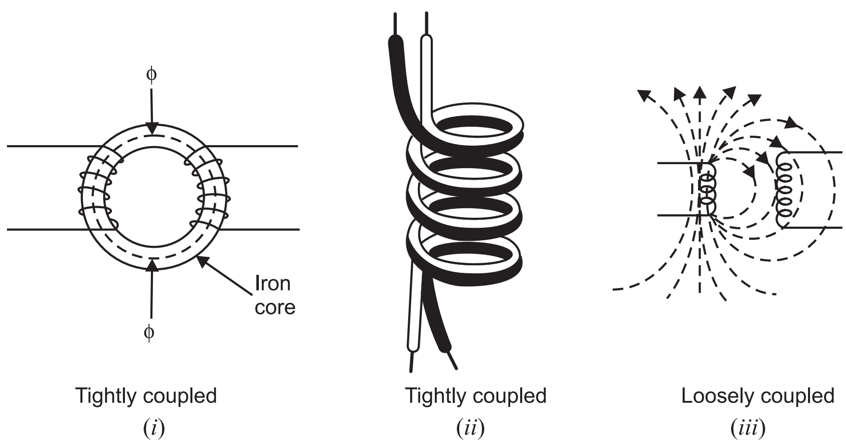


Fig. 9.15

When the two coils are air-cored as shown in Fig. 9.15 (*iii*), then only a fraction of magnetic flux produced by one coil may link with the other coil. The coils are then said to be **loosely coupled.**

Example 9.20. *Two identical coils A and B of 1000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. A current of 5 A flowing in coil A produces a flux of 0·05 mWb in it. If the current in coil A changes from + 12A to −12A in 0·02 second, calculate (i) the mutual inductance and (ii) the e.m.f. induced in coil B.*

Solution. (*i*) $$M = \frac{N_2\phi_{12}}{I_1}$$

Here $N_2 = 1000$; $I_1 = 5$ A ; $*\phi_{12} = 0{\cdot}8 \times 0{\cdot}05 \times 10^{-3} = 0{\cdot}4 \times 10^{-4}$ Wb

$$\therefore \quad M = \frac{1000 \times 0\cdot4 \times 10^{-4}}{5} = \mathbf{0.008\ H}$$

(*ii*) E.M.F. in coil B, $$e_B = M\frac{dI_1}{dt}$$

Here $M = 0{\cdot}008$ H ; $dI_1 = 12 - (-12) = 24$ A ; $dt = 0{\cdot}02$ s

$$\therefore \quad e_B = 0\cdot008 \times \frac{24}{0\cdot02} = \mathbf{9.6\ V}$$

Example 9.21. *Coils A and B in a magnetic circuit have 600 and 500 turns respectively. A current of 8 A in coil A produces a flux of 0·04 Wb. If the coefficient of coupling is 0·2, calculate :*

(*i*) *Self-inductance of coil A, with B open-circuited.*
(*ii*) *Flux linking with coil B.*
(*iii*) *The average e.m.f. induced in coil B when the flux with it changes from zero to full value in 0·02 second.*
(*iv*) *Mutual inductance.*
(*v*) *Average e.m.f. in B when current in A changes from 0 to 8 A in 0·05 second.*

Solution. (*i*) Inductance of coil A, $$L_A = \frac{N_A\,\phi_A}{I_A} = \frac{600 \times 0\cdot04}{8} = \mathbf{3\ H}$$

(*ii*) Flux linking coil B, $\phi_B = k \times \phi_A = 0{\cdot}2 \times 0{\cdot}04 = \mathbf{0{\cdot}008\ Wb}$

(*iii*) e.m.f. in coil B, $$e_B = N_B\frac{\phi_B - 0}{t} = 500\frac{0\cdot008}{0\cdot02} = \mathbf{200\ V}$$

(*iv*) Mutual inductance, $$M = \frac{k\,\phi_A\,N_B}{I_A} = \frac{0\cdot2 \times 0\cdot04 \times 500}{8} = \mathbf{0.5\ H}$$

(*v*) e.m.f. in coil B $$= M\frac{dI_A}{dt} = 0.5 \times \frac{8-0}{0.05} = \mathbf{80\ V}$$

Example 9.22. *Two identical coils are wound on a ring-shaped iron core that has a relative permeability of 500. Each coil has 100 turns and the core dimensions are : area, a = 3 cm² and magnetic path length, l = 20 cm. Calculate the inductance of each coil and the mutual inductance between the coils.*

Solution. $N = 100$ turns ; $\mu_r = 500$; $a = 3 \times 10^{-4}$ m^2 ; $l = 20 \times 10^{-2}$ m

The statement of the problem suggests that each coil has the same inductance.

$$\therefore \quad L_1 = L_2 = \mu_0\mu_r N^2\frac{a}{l}$$

$$= 4\pi \times 10^{-7} \times 500 \times (100)^2 \times \frac{3 \times 10^{-4}}{20 \times 10^{-2}} = 9.42 \times 10^{-3}\ \text{H} = \mathbf{9.42mH}$$

Since the coils are wound on the same iron core, coefficient of coupling $k = 1$.

$$\therefore \quad M = k\sqrt{L_1L_2} = 1\sqrt{9.42 \times 9.42} = \mathbf{9.42\ mH}$$

* Note that 80% of flux produced in coil A links with coil B. Therefore, mutual flux (ϕ_{12}) is 80% of 0·05 mWb.

Example 9.23. *Two identical 750-turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in coil A induces an e.m.f. of 11·25 V in coil B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of coil B.*

Solution. Induced e.m.f. in coil B, $e_B = M\dfrac{dI_A}{dt}$

or $\quad 11{\cdot}25 = M \times 1500 \quad \therefore \quad M = 7{\cdot}5 \times 10^{-3}\text{ H} = \mathbf{7{\cdot}5\ mH}$

Now $\quad L_1 = \dfrac{N_1\phi_1}{I_1} \quad \therefore \quad \dfrac{\phi_1}{I_1} = \dfrac{L_1}{N_1} = \dfrac{15\times10^{-3}}{750} = \mathbf{2\times10^{-5}\ Wb/A}$

Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{M}{\sqrt{L^2}} = \dfrac{7{\cdot}5\times10^{-3}}{15\times10^{-3}} = \mathbf{0{\cdot}5}$ or **50%**

Example 9.24. *Two coils A and B of 500 and 750 turns respectively are connected in series on the same magnetic circuit of reluctance $1{\cdot}55 \times 10^6$ AT/Wb. Assuming that there is no flux leakage, calculate (i) self-inductance of each coil and (ii) mutual inductance between coils.*

Solution. (*i*) $\quad L_A = \dfrac{N_A^{\,2}}{\text{Reluctance}} = \dfrac{(500)^2}{1{\cdot}55\times10^6} = \mathbf{0.16\ H}$

$$L_B = \frac{N_B^{\,2}}{\text{Reluctance}} = \frac{(750)^2}{1{\cdot}55\times10^6} = \mathbf{0.36\ H}$$

(*ii*) $\quad M = \dfrac{N_AN_B}{\text{Reluctance}} = \dfrac{500\times750}{1{\cdot}55\times10^6} = \mathbf{0.24\ H}$

Alternatively. $\quad M = k\sqrt{L_1L_2} = 1\sqrt{0{\cdot}16\times0{\cdot}36} = \mathbf{0.24\ H}$

Example 9.25. *Two coils A and B are wound side by side on a paper tube former. An e.m.f. of 0·25 V is induced in coil A when the flux linking it changes at the rate of 10^{-3} Wb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance between the coils?*

Solution. Induced e.m.f. in coil $A = N_1\dfrac{d\phi}{dt}$ $\quad$ or $\quad 0{\cdot}25 = N_1 \times 10^{-3}$

$\therefore \quad N_1 = 0{\cdot}25 / 10^{-3} = 250$ turns

Flux linkages in coil A due to 2 A in coil $B = 250 \times 10^{-5}$ Wb-turns.

$\therefore \quad M = \dfrac{\text{Flux linkages in coil } A}{\text{Current in coil } B}$

$$= 250 \times 10^{-5}/2 = \mathbf{1{\cdot}25\times10^{-3}\ H}$$

Example 9.26. *The coefficient of coupling between two coils is 0·6 or 60%. The excited coil produces 0·1 Wb of magnetic flux. How much flux is coupled to the other coil ? What is the value of the leakage flux ?*

Solution. The coefficient of coupling is given by ;

$$k = \frac{\phi_m}{\phi_t}$$

where $\quad \phi_t$ = flux of the coil receiving current $\quad ; \quad \phi_m$ = flux that links with the other coil

$\therefore \quad 0{\cdot}6 = \phi_m/\phi_t$

or $\quad \phi_m = 0{\cdot}6 \times \phi_t = 0{\cdot}6 \times 0{\cdot}1 = \mathbf{0{\cdot}06\ Wb}$

The difference between ϕ_t and ϕ_m is the leakage flux.

$\therefore$ Leakage flux, $\phi_l = \phi_t - \phi_m = 0{\cdot}1 - 0{\cdot}06 = \mathbf{0{\cdot}04\ Wb}$

Example 9.27. *Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60% of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0·6 mWb while the same current in B produces 0·8 mWb. Determine (i) mutual inductance and (ii) coupling coefficient.*

Solution. (*i*) Mutual inductance, $M = \dfrac{k\phi_A N_B}{I_A}$

Here $k = 0{\cdot}6$; $\phi_A = 0{\cdot}6$ mWb $= 0{\cdot}6 \times 10^{-3}$ Wb ; $N_B = 16000$; $I_A = 5$A

$$\therefore \quad M = \frac{0.6 \times 0.6 \times 10^{-3} \times 16000}{5} = \mathbf{1{\cdot}15\ H}$$

(*ii*) Now
$$L_A = \frac{N_A\phi_A}{I_A} = \frac{12500 \times 0.6 \times 10^{-3}}{5} = 1500 \times 10^{-3}\ \text{H} = 1500\ \text{mH}$$

and
$$L_B = \frac{N_B\phi_B}{I_B} = \frac{16000 \times 0{\cdot}8 \times 10^{-3}}{5} = 2560 \times 10^{-3}\ \text{H} = 2560\ \text{mH}$$

$$\therefore \quad \text{Coefficient of coupling, } k = \frac{M}{\sqrt{L_A L_B}} = \frac{1{\cdot}15 \times 10^3}{\sqrt{1500 \times 2560}} = \mathbf{0{\cdot}586}$$

The coefficient of coupling is a measure of how tightly the two coils are coupled. It is a pure number (no units) that varies from 0 to 1. The only way to closely approach $k = 1$ is to wind both coils on the same high-permeability core. This couples them tightly.

Example 9.28. *The coefficient of coupling between two coils is 0.85. Coil 1 has 250 turns. When the current in coil 1 is 2A, the total flux of this coil is 3×10^{-4} Wb. When I_1 is changed from 2A to zer linearly in 2 ms, the voltage induced in the coil 2 is 63.75 V. Find L_1, L_2, M and N_2.*

Solution. Inductance of coil 1, $L_1 = \dfrac{N_1\phi_1}{I_1} = \dfrac{250 \times 3 \times 10^{-4}}{2} = \mathbf{37.5 \times 10^{-3}\ H}$

e.m.f. induced in coil 2, $e_2 = M\dfrac{dI_1}{dt}$

Here, $e_2 = 63.75$ V ; $dI_1 = 2 - 0 = 2$A ; $dt = 2$ms $= 2 \times 10^{-3}$ s

$$\therefore \quad 63.75 = M \times \frac{2}{2 \times 10^{-3}} \quad \text{or} \quad M = \mathbf{63.75 \times 10^{-3}\ H}$$

Now, $M = k\sqrt{L_1 L_2}$

Here, $M = 63.75 \times 10^{-3}$ H ; $k = 0.85$; $L_1 = 37.5 \times 10^{-3}$ H

$$\therefore \quad 63.75 \times 10^{-3} = 0.85 \times \sqrt{37.5 \times 10^{-3} \times L_2} \quad \text{or} \quad L_2 = \mathbf{150 \times 10^{-3}\ H}$$

Now
$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} \quad \text{or} \quad \frac{37.5 \times 10^{-3}}{150 \times 10^{-3}} = \frac{(250)^2}{N_2^2}$$

$$\therefore \quad 0.25 = \frac{(250)^2}{N_2^2} \quad \text{or} \quad N_2 = \mathbf{500}$$

Example 9.29. *The dimensions of the magnetic core shown in Fig. 9.16 are :*

Cross-sectional area, a = 3 cm^2 ; magnetic path length, l = 10 cm and the relative permeability is 250.

The primary coil has N_P = 100 turns and the secondary coil has N_S = 75 turns. If the current is increased from 0 to 5A in 0.1s, determine the e.m.f. induced in the secondary.

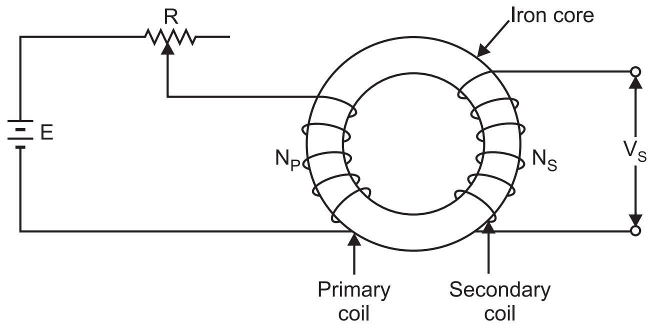


Fig. 9.16

Solution. m.m.f. $= N_P I = 100 \times 5 = 500$ AT

$$\text{Magnetising force, } H = \frac{\text{m.m.f.}}{l} = \frac{500}{10 \times 10^{-2}} = 5000 \text{ AT/m}$$

$$\text{Flux density in core, } B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 250 \times 5000 = 1.57 \text{ Wb/m}^2$$

$$\text{Total flux in core, } \phi = B \times a = 1.57 \times 3 \times 10^{-4} = 471 \times 10^{-6} \text{ Wb}$$

∴ Induced e.m.f. in the secondary is given by ;

$$e_S = N_S \frac{d\phi}{dt} = 75 \times \frac{471 \times 10^{-6}}{0.1} = \mathbf{0.35\ V}$$

Example 9.30. *A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 T/m. There is a small concenterated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils if the concentrated coil has 120 turns on an effective diameter of (i) 8 cm and (ii) 12 cm.*

Solution. Let I_1 be the current flowing through the solenoid.

(*i*) Fig. 9.17 (*i*) shows the conditions of the problem when the effective diameter of concentrated search coil is 8 cm (less than that of the solenoid).

Magnetising force H inside the solenoid is

$$H = \frac{NI_1}{l} = \frac{N}{l} I_1 = 2500\, I_1 \qquad \left(\because \frac{N}{l} = 2500\right)$$

∴ Flux density at the centre of the solenoid is

$$B = \mu_0 H = 2500\, \mu_0 I_1 \text{ Wb/m}^2$$

$$\text{Area of search coil, } a_S = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.08)^2 = 0.005 \text{ m}^2$$

Flux linking with search coil is given by ;

$$\phi_2 = B\, a_S = 2500\, \mu_0 I_1 \times 0.005 = 15.79 \times 10^{-6} I_1 \text{ Wb}$$

$$\therefore \quad M = \frac{N_2 \phi_2}{I_1} = \frac{120 \times 15.79 \times 10^{-6} I_1}{I_1} = \mathbf{1.895 \times 10^{-3}\ H}$$

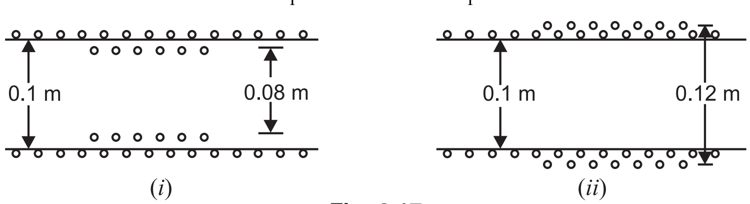


Fig. 9.17

(*ii*) Fig. 9.17 (*ii*) shows the conditions of the problem when the effective diameter of concentrated search coil is 12 cm (*i.e.* more than that of the solenoid). Since the field strength outside the solenoid is negligible, the effective area of search coil will be equal to the area of solenoid *i.e.*

$$a'_S = \frac{\pi}{4}(0.1)^2$$

$\therefore$ Flux linking with the search coil is given by ;

$$\phi'_2 = B\,a'_S = 2500\,\mu_0\,I_1 \times \frac{\pi}{4}\times(0.1)^2$$

$$\therefore \quad M = \frac{N_2\phi'_2}{I_1} = 120\times\frac{2500\mu_0 I_1\times(\pi/4)\times(0.1)^2}{I_1} = \mathbf{2.962\times10^{-3}\ H}$$

Tutorial Problems

1. A solenoid 70 cm in length and of 2100 turns has a radius of 4·5 cm. A second coil of 750 turns is wound upon the middle part of the solenoid. Find the mutual inductance between the two coils. **[18·2 mH]**
2. Two coils having 150 and 200 turns respectively are wound side by side on a closed iron circuit of section 150 cm^2 and mean length of 300 cm. Determine the mutual inductance between the coils and e.m.f. induced in the second coil if current changes from zero to 10A in the first coil in 0·02 second. Relative permeability of iron = 2000. **[0·377 H; 188·5 V]**
3. The self-inductance of a coil of 500 turns is 0·25H. If 60% of the flux is linked with a second coil of 10,000 turns, calculate the mutual inductance between the two coils. **[3 H]**
4. The windings of a transformer has an inductance of L_1 = 6H; L_2 = 0.06 H and a coefficient of coupling k = 0·9. Find the e.m.f. in both the windings when current in primary increases at the rate of 1000 A/s. **[6000 V; 540 V]**
5. An air-cored solenoid with length 30 cm, area of X-section 25 cm^2 and number of turns 500 carries a current of 2·5 A. The current is suddenly switched off in a brief time of 10^{-4} second. How much average e.m.f. is induced across the ends of the open switch in the circuit ? Ignore the variation of magnetic field near the ends of the solenoid. **[6·5 V]**

9.14. Inductors in Series

Consider two coils connected in series as shown in Fig. 9.18.

Let L_1 = inductance of first coil

L_2 = inductance of second coil

M = mutual inductance between the two coils

(*i*) Series-aiding. This is the case when the coils are so arranged that their fluxes *aid each other *i.e.* in the same direction as shown in Fig. 9.18 (*i*). Suppose the current is changing at the rate di/dt. The total induced e.m.f. in the circuit will be equal to the sum of e.m.f.s induced in L_1 and L_2 *plus* the mutually induced e.m.f.s, *i.e.*

$$e = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} \qquad \text{... in magnitude}$$

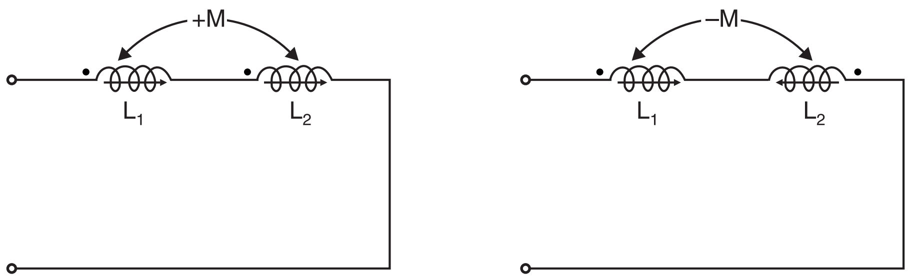
$$= (L_1 + L_2 + 2M)\,di/dt$$

If L_T is the total inductance of the circuit, then,

$$e = L_T\frac{di}{dt}$$

$$\therefore \quad L_T = L_1 + L_2 + 2M \qquad \text{... fluxes additive}$$

* **Dot notation.** It is generally not possible to state from the figure whether the fluxes of the two coils are additive or in opposition. Dot notation removes this confusion. The end of the coil through which the current enters is indicated by placing a dot behind it. If the current after leaving the dotted end of coil L_1 enters the dotted end of coil L_2, it means the fluxes of the two coils are additive otherwise in opposition.



(*i*) (*ii*)

Fig. 9.18

***(ii)* Series-opposing.** Fig. 9.18 (*ii*) shows the series-opposing connection *i.e.* the fluxes of the two coils oppose each other. Suppose the current is changing at the rate di/dt. The total induced e.m.f. in the circuit will be equal to sum of e.m.f.s induced in L_1 and L_2 *minus* the mutually induced e.m.f.s.

$$e = L_1\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$$

If L_T is the total inductance of the circuit, then,

$$e = L_T\frac{di}{dt}$$

$\therefore \quad L_T = L_1 + L_2 - 2M$ ...*fluxes subtractive*

In general, $L_T = L_1 + L_2 \pm 2M$

Use + sign if fluxes are additive and –ve sign if fluxes are subtractive.

If the two coils are so positioned that $^*M = 0$, then, $L_T = L_1 + L_2$.

9.15. Inductors in Parallel with no Mutual Inductance

Consider three inductances L_1, L_2 and L_3 in parallel as shown in Fig. 9.19. Assume that mutual inductance between the coils is zero. Referring to Fig. 9.19, we have,

$$i_T = i_1 + i_2 + i_3$$

or $$\frac{di_T}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

But $$e = L\frac{di}{dt} \quad or \quad \frac{di}{dt} = \frac{e}{L}$$

$\therefore$ $$\frac{e}{L_T} = \frac{e}{L_1} + \frac{e}{L_2} + \frac{e}{L_3}$$

or $$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \quad ...(i)$$

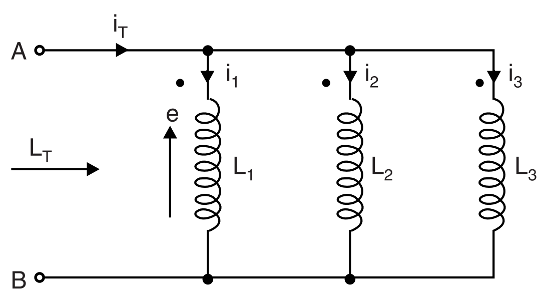


Fig. 9.19

If only two inductors L_1 and L_2 are in parallel, then,

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{L_1 + L_2}{L_1 L_2}$$

or $$L_T = \frac{L_1 L_2}{L_1 + L_2} \quad i.e. \quad \frac{\text{Product}}{\text{Sum}}$$

* If the coils are so placed that fluxes produced by them are at right angles to each other, then mutual flux will be zero and hence $M = 0$.

9.16. Inductors in Parallel with Mutual Inductance

Consider two coils A and B of inductances L_1 and L_2 connected in parallel as shown in Fig. 9.20. Let the mutual inductance between the two coils be M. The supply current i divides into two branch currents i_1 and i_2.

By KCL, $\quad i = i_1 + i_2$

$$\therefore \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad ...(i)$$

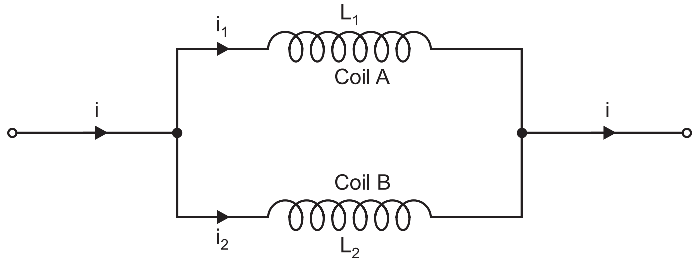


Fig. 9.20

$$\text{Self-induced e.m.f. in coil } A = -L_1\frac{di_1}{dt}$$

$$\text{Mutually induced e.m.f. in coil } A = -M\frac{di_2}{dt}$$

$$\text{Total e.m.f. induced in coil } A = -\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$$

$$\text{Similarly, total e.m.f. induced in coil } B = -\left(L_2\frac{di_2}{dt} + M\frac{di_1}{dt}\right)$$

Since the two coils are in parallel, these e.m.f.s are equal *i.e.*

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

or
$$\frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M)$$

$$\therefore \quad \frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \qquad ...(ii)$$

Putting this value of di_1/dt in eq. (i), we have,

$$\frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt} \qquad ...(iii)$$

If L_T is the equivalent inductance of the parallel combination, then,

$$\text{Induced e.m.f.} = -L_T\frac{di}{dt}$$

Since induced e.m.f. in the parallel combination is equal to the e.m.f. induced in any one coil (say coil A),

$$\therefore \quad L_T\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

or
$$\frac{di}{dt} = \frac{1}{L_T}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$$

Putting the value of di_1/dt from eq. (ii), we have,

$$\frac{di}{dt} = \frac{1}{L_T}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), we have,

$$\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L_T}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$$

or
$$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L_T}\left(\frac{L_1 L_2 - M^2}{L_1 - M}\right)$$

$\therefore$
$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \quad \text{... when mutual flux aids the individual fluxes}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \quad \text{... when mutual flux opposes the individual fluxes}$$

If there is no mutual inductance between the two coils (*i.e.* $M = 0$), then,

$$L_T = \frac{L_1 L_2 - (0)^2}{L_1 + L_2 \pm 2(0)} = \frac{L_1 L_2}{L_1 + L_2}$$

Example 9.31. *When two coils are connected in series, their effective inductance is found to be 10 H. When the connections of one coil are reversed, the effective inductance is 6 H. If the co-efficient of coupling is 0·6, calculate the self-inductance of each coil and the mutual inductance.*

Solution.
$$10 = L_1 + L_2 + 2M \qquad ...(i)$$
$$6 = L_1 + L_2 - 2M \qquad ...(ii)$$

Subtracting (*ii*) from (*i*), we get, $4 = 4M$ or $M =$ **1 H**

Putting $M = 1$ H in eq. (*i*), we have, $L_1 + L_2 = 8$ ...(*iii*)

Also
$$L_1 L_2 = \frac{M^2}{k^2} = \frac{(1)^2}{(0.6)^2} = 2{\cdot}78 \qquad ...(iv)$$

Now
$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2 = (8)^2 - 4 \times 2{\cdot}78 = 52{\cdot}88$$

$\therefore$
$$L_1 - L_2 = \sqrt{52.88} = 7{\cdot}27 \qquad ...(v)$$

Solving eqs. (*iii*) and (*v*), $L_1 =$ **7·635 H** and $L_2 =$ **0·365 H**

Example 9.32. *The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2H, depending upon the relative direction of the currents in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate (i) the mutual inductance between the two coils, (ii) the self-inductance of coil B, (iii) the coupling factor between the coils, and (iv) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A/s in the series circuit.*

Solution. **(*i*)** Combined inductance of two coils, $L = L_1 + L_2 \pm 2M$

For series-aiding : $L_1 + L_2 + 2M = 0.5$ ...(*i*)

For series-opposing : $L_1 + L_2 - 2M = 0.2$ ...(*ii*)

Subtracting eq. (*ii*) from eq. (*i*), we have,

$$4M = 0.3 \quad \therefore M = \mathbf{0.075\ H}$$

(*ii*) Adding eq. (*i*) and eq. (*ii*), we have,

$$2(L_1 + L_2) = 0.7 \quad \text{or} \quad 2(0.2 + L_2) = 0.7 \quad \therefore L_2 = \mathbf{0.15\ H}$$

(*iii*) Coefficient of coupling is given by ;

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.075}{\sqrt{0.2 \times 0.15}} = \mathbf{0.433\ or\ 43.3\%}$$

(*iv*)
$$e_1 = L_1\frac{di}{dt} \pm M\frac{di}{dt}$$

$$\therefore \qquad e_1 = L_1\frac{di}{dt} + M\frac{di}{dt} = 0.2 \times 1000 + 0.075 \times 1000 = \mathbf{275\ V}$$

or $$e_1 = L_1\frac{di}{dt} - M\frac{di}{dt} = 0.2 \times 1000 - 0.075 \times 1000 = \mathbf{125\ V}$$

Example 9.33. *Two mutually coupled coils, A and B, are connected in series to a 360 V d.c. supply. Coil A has a resistance of 6 Ω and inductance 4 H. Coil B has resistance of 11 Ω and inductance 9 H. At a certain instant after the circuit is energised, the current is 10 A and is increasing at the rate of 10 A/s. Calculate (i) the mutual inductance between the coils and (ii) the coefficient of coupling.*

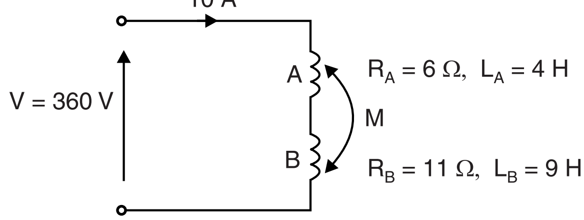


Fig. 9.21

Solution. Fig. 9.21 shows the conditions of the problem.

(*i*) Total circuit resistance, $R_T = R_A + R_B$

$= 6 + 11 = 17\ \Omega$

Total circuit inductance, $L_T = L_A + L_B + 2M$

$= 4 + 9 + 2M = 13 + 2M$

Now $$V = iR_T + L_T\frac{di}{dt}$$

or $$360 = 10 \times 17 + (13 + 2M)\,10 \quad \therefore \quad M = \mathbf{3\ H}$$

(*ii*) Coefficient of coupling, $$k = \frac{M}{\sqrt{L_A L_B}} = \frac{3}{\sqrt{4 \times 9}} = \mathbf{0{\cdot}5}$$

Example 9.34. *Two identical coils with terminals, T_1T_2 and T_3T_4 respectively are placed side by side. The inductances measured under different sets of connections are as follows :*

When T_2 is connected to T_3 and inductance measured between T_1 and T_4, it is 4H.

When T_2 is connected to T_4 and inductance measured between T_1 and T_3, it is 0.8 H.

Determine the self inductance of each coil, the mutual inductance between the coils and the coefficient of coupling.

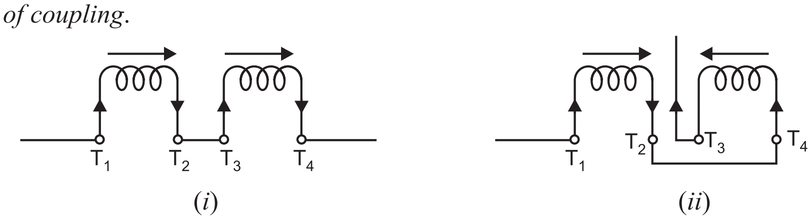


Fig. 9.22

Solution. Since the two coils are identical, each has inductance L (say).

When T_2 is connected to T_3 as shown in Fig. 9.22 (*i*), it is a series-aiding connection so that :

$$L + L + 2M = 4 \quad \text{or} \quad L + M = 2 \qquad ...(i)$$

When T_2 is connected to T_4 as shown in Fig. 9.22 (*ii*), it is a series-opposing connection so that:

$$L + L - 2M = 0.8 \quad \text{or} \quad L - M = 0.4 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), L = **1.2 H** ; M = **0.8 H**

Coefficient of coupling, $$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.8}{\sqrt{1.2 \times 1.2}} = \mathbf{0.667} \text{ or } \mathbf{66.7\ \%}$$

Example 9.35. *Find the total inductance of the circuit shown in Fig. 9.23.*

$L_1 = 10\ H \qquad M_{12} = 5\ H$

$L_2 = 15\ H \qquad M_{23} = 3\ H$

$L_3 = 12\ H \qquad M_{13} = 1\ H$

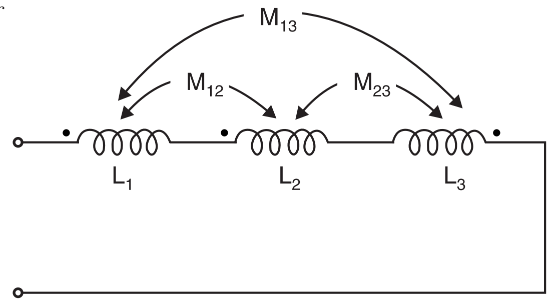


Fig. 9.23

Solution. The fluxes of L_1 and L_2 add to each other and hence M_{12} is positive. The fluxes of L_1 and L_3 are in opposition so M_{13} is negative. Similarly, it can be seen that M_{23} is negative.

$$\therefore \quad L_T = (L_1 + M_{12} - M_{13}) + (L_2 - M_{23} + M_{12}) + (L_3 - M_{23} - M_{13})$$
$$= (10 + 5 - 1) + (15 - 3 + 5) + (12 - 3 - 1)$$
$$= 14 + 17 + 8 = \mathbf{39\ H}$$

Example 9.36. *Fig. 9.24 shows three inductances in series. Find the total inductance of the circuit from the following data :*

$L_1 = 12\ H \qquad k_1 = 0{\cdot}33$

$L_2 = 14\ H \qquad k_2 = 0{\cdot}37$

$L_3 = 14\ H \qquad k_3 = 0{\cdot}65$

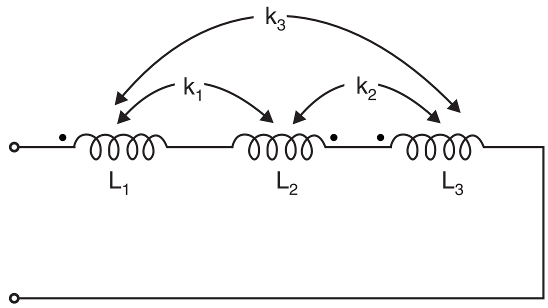


Fig. 9.24

Solution.

$$M_{12} = k_1\sqrt{L_1L_2} = 0.33\sqrt{12\times 14} = 4{\cdot}28\ \text{H}$$

$$M_{23} = k_2\sqrt{L_2L_3} = 0\,37\sqrt{14\times 14} = 5{\cdot}18\ \text{H}$$

$$M_{13} = k_3\sqrt{L_1L_3} = 0\,65\sqrt{12\times 14} = 8{\cdot}42\ \text{H}$$

$$\therefore \quad L_T = (L_1 - M_{12} + M_{13}) + (L_2 - M_{12} - M_{23}) + (L_3 + M_{13} - M_{23})$$
$$= (12 - 4.28 + 8.42) + (14 - 4.28 - 5.18) + (14 + 8.42 - 5.18)$$
$$= 16{\cdot}14 + 4{\cdot}54 + 17{\cdot}24 = \mathbf{37{\cdot}92\ H}$$

Example 9.37. *Two coils of self-inductances 150 mH and 250 mH and of mutual inductance 120 mH are connected in parallel. Determine the equivalent inductance of the combination if (i) mutual flux helps the individual flux and (ii) mutual flux opposes the individual flux.*

Solution. Here, $L_1 = 0.15$ H ; $L_2 = 0.25$ H ; $M = 0.12$ H

(i) Equivalent inductance L_T of the parallel combination when mutual flux helps the individual flux is

$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{0.15\times 0.25 - (0.12)^2}{0.15 + 0.25 - 2\times 0.12} = \mathbf{0.144\ H}$$

(ii) Equivalent inductance L_T of the parallel combination when the mutual flux opposes the individual flux is

$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.15\times 0.25 - (0.12)^2}{0.15 + 0.25 + 2\times 0.12} = \mathbf{0.036\ H}$$

Example 9.38. *Two coils of inductances 0.3 H and 0.8 H are connected in parallel. If the coefficient of coupling is 0.7, calculate the equivalent inductance of the combination if mutual inductance assists the self-inductance.*

Solution. Here, $L_1 = 0.3$ H ; $L_2 = 0.8$ H ; $k = 0.7$

Mutual inductance M between the two coils is

$$M = k\sqrt{L_1 L_2} = 0.7\sqrt{0.3\times 0.8} = 0.343 \text{ H}$$

$\therefore$ Equivalent inductance L_T of the combination when mutual inductance assists the self-inductance is

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{0.3\times 0.8 - (0.343)^2}{0.3 + 0.8 - 2\times 0.343} = \mathbf{0.2955\ H}$$

Example 9.39. *Find the equivalent inductance L_{AB} in Fig. 9.25.*

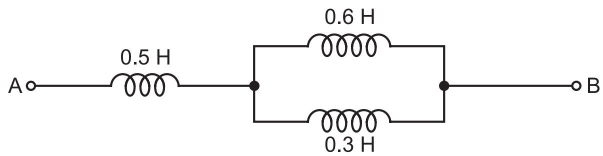


Fig. 9.25

Solution. It is understood that there is no mutual coupling between the coils because it is not given in the problem.

$$\text{Here, } L_1 = 0.5 \text{ H} ; L_2 = 0.6\text{H} ; L_3 = 0.3 \text{ H}$$

$$\therefore \quad L_{AB} = L_1 + \frac{L_2 L_3}{L_2 + L_3} = 0.5 + \frac{0.6\times 0.3}{0.6 + 0.3} = \mathbf{0.7\ H}$$

Tutorial Problems

1. The mutual inductance between two coils in a radio receiver is 100 mH. One coil has 100 mH of self-inductance. What is the self-inductance of the other if coefficient of coupling between the coils is 0·5 ? **[400 mH]**
2. The self-inductances of two coils are $L_1 = 150$ mH, $L_2 = 250$ mH. When they are connected in series with their fluxes aiding, their total inductance is 620 mH. When the connection to one of the coils is reversed (they are still in series), their total inductance is 180 mH. How much mutual inductance exists between them ? **[110 mH]**
3. Two coils of self-inductances 5 H and 8 H are connected in series with their fluxes aiding. If the co-efficient of coupling between the coils is 0·45, find the total inductance of the circuit. **[18·06 H]**
4. The self-inductances of three coils are $L_A = 20$ H, $L_B = 30$ H and $L_C = 40$ H. The coils are connected in series in such a way that fluxes of L_A and L_B add, fluxes of L_A and L_C are in opposition and fluxes of L_B and L_C are in opposition. If $M_{AB} = 8$ H, $M_{BC} = 12$ H and $M_{AC} = 10$ H, find the total inductance of the circuit. **[62 H]**

9.17. Energy Stored in a Magnetic Field

In order to establish a magnetic field around a coil, energy is *required, though no energy is needed to ***maintain* it. This energy is stored in the magnetic field and is not used up. When the current is decreased, the flux surrounding the coil is decreased, causing the stored energy to be returned to the circuit. Consider an inductor connected to a d.c. source as shown in Fig. 9.26 (*i*). The inductor is equivalent to inductance L in series with a small resistance R as shown in Fig. 9.26 (*ii*). The energy supplied to the circuit is spent in two ways :

* When the coil is connected to supply, current increases from zero gradually and reaches the final value I (= V/R) after some time. During this change of current, an e.m.f. is induced in L due to the change in flux linkages. This induced e.m.f. opposes the rise of current. Electrical energy must be supplied to meet this opposition. This supplied energy is stored in the magnetic field.

** To impart a kinetic energy of $\frac{1}{2}mv^2$ to a body, energy is required but no energy is required to maintain it at that energy level.

(*i*) A part of supplied energy is spent to meet I^2R losses and cannot be recovered.

(*ii*) The remaining part is spent to create flux around the coil (or inductor) and is stored in the magnetic field. When the field collapses, the stored energy is returned to the circuit.

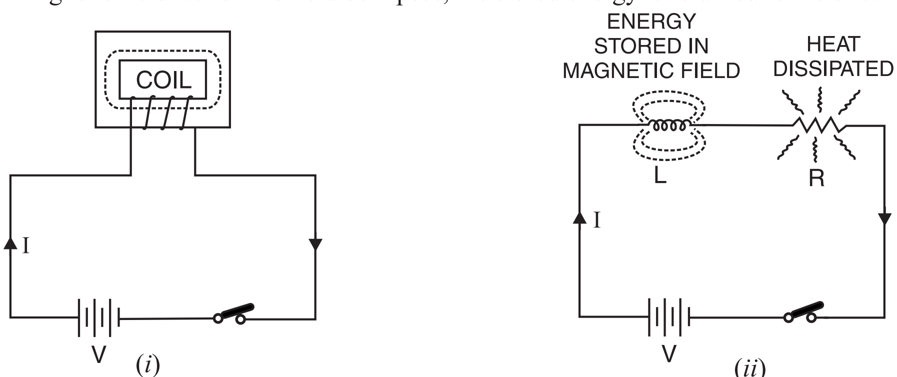


Fig. 9.26

Mathematical Expression. Suppose at any instant the current in the coil is *i* and is increasing at the rate of *di/dt*. The e.m.f. *e* across *L* is given by ;

$$e = L\frac{di}{dt}$$

$\therefore$ Instantaneous power, $p = ei = Li\frac{di}{dt}$

During a short interval of time *dt*, the energy *dw* put into the magnetic field is equal to power multiplied by time *i.e.*

$$dw = p.dt = \left(Li\frac{di}{dt}\right)dt = Li\,di$$

The total energy put into the magnetic field from the time current is zero until it has attained the final steady value *I* is given by ;

$$W = \int_0^I Li\,di = \frac{1}{2}LI^2$$

$\therefore$ Energy stored in magnetic field, $E = \frac{1}{2}LI^2$ joules

It is clear that energy stored in an inductor depends upon inductance and current through the inductor. For a given inductor, the amount of energy stored is determined by the maximum current through the inductor. Note that energy stored will be in joules if inductance (*L*) and current (*I*) are in henry and amperes respectively.

Note. If current in an inductor varies, the stored energy rises and falls in step with the current. Thus, whenever current increases, the coil absorbs energy and whenever current falls, energy is returned to the circuit.

Alternate method. In order to determine the amount of energy an inductor stores, we need to determine inductor's current and voltage during the time it is storing energy. Since the inductor stores energy only during the time the current is increasing, we must determine the average current during the time the current is rising. This can be done by referring to Fig. 9.27 which shows the current in an inductor increasing at a constant rate until it reaches the maximum value I_m. Since the current rises linearly from 0 to I_m, the average value of current is

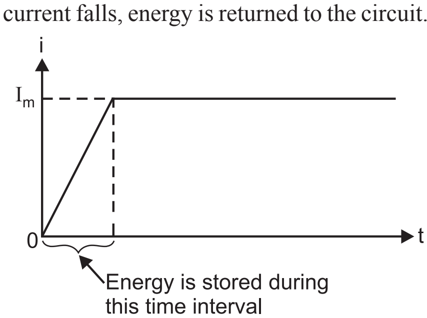


Fig. 9.27

$$I_{av} = \frac{0 + I_m}{2} = 0.5\,I_m$$

The voltage V_L across the inductor during the time it is storing energy is

$$V_L = L\frac{dI}{dt}$$

Since current rises from 0 to I_m linearly, dI/dt remains constant. Therefore, V_L remains constant during the time the current in inductor is increasing. As a result, expression for V_L reduces to :

$$V_L = \frac{LI_m}{t} \qquad (\because dI = I_m \text{ and } dt = t)$$

$\therefore$ Energy stored in the inductor during time t is

$$E = V_L I_{av}\, t = \frac{LI_m}{t}\times 0.5 I_m \times t = 0.5\, I_m^2 L$$

or

$$E = \frac{1}{2}LI_m^2$$

The subscript m is usually dropped so that :

$$E = \frac{1}{2}LI^2$$

Note that I is the final steady current through the inductor. It may be kept in mind that an inductor stores energy in its magnetic field when the current is rising and returns energy to the circuit when the current is falling.

Note. In case of inductors connected in series, the energy stored is given by ;

$$E = \frac{1}{2}(L_1 + L_2 + 2M)I^2 \qquad \textit{... series-aiding}$$

$$E = \frac{1}{2}(L_1 + L_2 - 2M)I^2 \qquad \textit{... series - opposing}$$

Example 9.40. *A current of 20 mA is passed through a coil of self-inductance 500 mH. Find the magnetic energy stored. If the current is halved, find the new value of energy stored and the energy released back to the electrical circuit.*

Solution. Magnetic energy stored when current is 20 mA is

$$E_1 = \frac{1}{2}LI^2 = \frac{1}{2}(500\times10^{-3})\times(20\times10^{-3})^2 = \mathbf{100 \times 10^{-6}\ J}$$

Magnetic energy stored when current becomes 10 mA is

$$E_2 = \frac{1}{2}LI^2 = \frac{1}{2}(500\times10^{-3})\,(10\times10^{-3})^2 = \mathbf{25 \times 10^{-6}\ J}$$

Magnetic energy released back to the circuit

$$= E_1 - E_2 = (100 - 25)\times 10^{-6} = \mathbf{75 \times 10^{-6}\ J}$$

Example 9.41. *The field winding of a machine consists of 8 coils in series, each containing 1200 turns. When the current is 3A, flux linked with each coil is 20 mWb. Calculate (i) the inductance of the circuit, (ii) the energy stored in the circuit and (iii) the average value of induced e.m.f. if the circuit is broken in 0·1 s.*

Solution.

(*i*) Inductance of each coil, $L = \frac{N\phi}{I} = \frac{1200\times 20\times10^{-3}}{3} = 8\text{H}$

$\therefore$ Total inductance, $L_T = 8\,L = 8 \times 8 = \mathbf{64\ H}$

(*ii*) Magnetic energy stored $= \frac{1}{2}L_T\, I^2 = \frac{1}{2}\times 64\times 3^2 = \mathbf{288\ J}$

(*iii*) Average e.m.f. induced, $e = L_T\frac{di}{dt} = 64\times\frac{3-0}{0\cdot1} = \mathbf{1920\ V}$

Example 9.42. *A coil of inductance 5 H and resistance 100 Ω carries a steady current of 2 A. Calculate the initial rate of fall of current in the coil after a short-circuiting switch connected across its terminals has been suddenly closed. What was the energy stored in the coil and in what form is it dissipated ?*

Solution. The conditions of the problem are represented in Fig. 9.28.

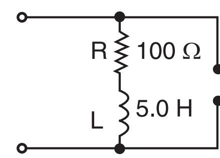


Fig. 9.28

$$V = iR + L\frac{di}{dt}$$

or $$0 = 2\times 100 + 5\frac{di}{dt}$$

∴ $$\frac{di}{dt} = \frac{-200}{5} = \mathbf{-40\ A/s}$$

Magnetic energy stored in coil $= \frac{1}{2}LI^2 = \frac{1}{2}\times 5\times (2)^2 = \mathbf{10\ J}$

The stored magnetic energy is dissipated in the form of **heat.**

Example 9.43. *(a) A coil of 100 turns is wound on a toroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5A and is increasing at the rate of 200 A/s, determine (i) energy stored in the magnetic circuit and (ii) voltage applied across the coil. Assume coil resistance as zero.*

(b) How are your answers affected if the coil resistance is 2Ω ?

Solution. N = 100 turns ; Reluctance of core, $S = 10^4$ AT/Wb

(*a*) Inductance of coil, $L = \frac{N^2}{S} = \frac{(100)^2}{10^4} = 1$ H

(*i*) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\times 1\times (5)^2 = \mathbf{12.5\ J}$

(*ii*) Voltage applied across coil = Self-induced e.m.f. in the coil

$$= L\frac{dI}{dt} = 1 \times 200 = \mathbf{200\ V}$$

(*b*) If the coil resistance is 2Ω, the energy stored will remain the same *i.e.,* **12.5 J.**

Voltage across coil $= IR + L\frac{dI}{dt} = 5 \times 2 + 1 \times 200 = \mathbf{210\ V}$

However, there will be a loss of energy $= I^2R = (5)^2 \times 2 = 50$W

Example 9.44. *An iron ring 15 cm in diameter and 10 cm^2 in cross-section is wound with 200 turns of wire. For a flux density of 1 Wb/m^2 and a relative permeability of 500, find the exciting current, the inductance and the stored energy. Find the corresponding quantities when there is a 2 mm air gap.*

Solution. Magnetic flux, $\phi = B \times a = 1 \times (10 \times 10^{-4}) = 10^{-3}$ Wb

Magnetic length, $l = 0{\cdot}15 \times \pi$ m

Now Flux density, $B = \mu_0\,\mu_r\,H$

∴ Magnetising force, $H = \frac{B}{\mu_0\mu_r} = \frac{1}{(4\pi\times 10^{-7})\times 500} = 1590$ AT/m

Total ampere-turns $= H \times l = 1590 \times (0{\cdot}15 \times \pi)$ AT

∴ Exciting current, $I = \frac{\text{Total AT}}{N} = \frac{1590\times(0{\cdot}15\times\pi)}{200} = \mathbf{3{\cdot}75\ A}$

Inductance, $L = \frac{N\phi}{I} = \frac{200\times 10^{-3}}{3{\cdot}75} = 53{\cdot}4 \times 10^{-3}$ H = **53·4 mH**

$$\text{Magnetic energy stored} = \frac{1}{2}LI^2 = \frac{1}{2}\times 53{\cdot}4\times 10^{-3}\times(3{\cdot}75)^2 = \mathbf{0{\cdot}375\ J}$$

With 2 mm air gap. The length of air gap, $l_g = 2$ mm $= 2 \times 10^{-3}$ m

$$\text{Air gap } AT = H \times l_g = \frac{B}{\mu_0}\times l_g = \frac{1}{4\pi\times 10^{-7}}\times 2\times 10^{-3} = 1590 \text{ AT}$$

$$\text{Additional current required} = 1590/200 = 7{\cdot}95 \text{ A}$$

$\therefore$ Total exciting current, $I_T = 3{\cdot}75 + 7{\cdot}95 =$ **11·7 A**

$$\text{Inductance, } L = \frac{N\phi}{I_T} = \frac{200\times 10^{-3}}{11{\cdot}7} = 17{\cdot}1 \times 10^{-3} \text{ H} = \mathbf{17{\cdot}1\ mH}$$

$$\text{Magnetic energy stored} = \frac{1}{2}LI_T^2 = \frac{1}{2}\times(17{\cdot}1\times 10^{-3})\times(11{\cdot}7)^2 = \mathbf{1{\cdot}17\ J}$$

Example 9.45. *An inductor with 10 Ω resistance and 200 mH inductance is connected across 24 V d.c. source. Calculate (i) energy stored in the inductance, (ii) power dissipated by the resistance and (iii) power dissipated by the inductance.*

Solution. $V = 24$ volts ; $R = 10\ \Omega$; $L = 200$ mH $= 0.2$ H

(*i*) Final current in inductor, $I = \dfrac{V}{R} = \dfrac{24}{10} = 2.4$ A

Energy stored in inductance $= \dfrac{1}{2}LI^2 = \dfrac{1}{2}\times 0.2\times(2.4)^2 =$ **0.576 J**

(*ii*) Power dissipated by resistor $= I^2R = (2.4)^2 \times 10 =$ **57.6 W**

(*iii*) Power dissipated by inductance = **0 W**

Example 9.46. *A coil of inductance 0.25 H and negligible resistance is connected to a source of supply represented by v = 4 t volts. If the voltage is applied at t = 0 and switched off at t = 5 sec., find (i) the maximum value of current, (ii) r.m.s. value of current and (iii) the energy stored during this period.*

Solution. (*i*) $v = 4t$ or $L\dfrac{di}{dt} = 4t$ or $0.25\dfrac{di}{dt} = 4t$

$$\therefore \quad 0.25\int_0^I di = \int_0^5 4t\,dt$$

$$\text{or} \quad 0.25\,I = \left|\frac{4t^2}{2}\right|_0^5 = 50$$

$\therefore$ Max. value of current, $I = 50/0{\cdot}25 =$ **200 A**

(*ii*) Suppose i is the current at any time t. Then,

$$0{\cdot}25\,i = \int_0^t 4t\,dt = 2t^2$$

$$\therefore \quad i = 8t^2$$

The sum of squares of current from 0 to 5 sec.

$$= \int_0^5 64t^4 dt = \frac{64\times 5^5}{5} = 64 \times 5^4$$

$$\therefore \quad \text{Mean square value} = \frac{64\times 5^4}{5} = 64 \times 5^3$$

$\therefore$ R.M.S. value $= \sqrt{64 \times 5^3} = \mathbf{89{\cdot}5\ A}$

(*iii*) Energy stored $= \int_0^5 vi\,dt = \int_0^5 (4t \times 8t^2)\,dt$

$$= \left| \frac{32t^4}{4} \right|_0^5 = \frac{32 \times 5^4}{4} = \mathbf{5000\ J}$$

Example 9.47. *A direct current of 1 A is passed through a coil of 5000 turns and produces a flux of 0·1 mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil ? What would be the voltage developed across the coil if the current were interrupted in 10^{-3} second ? What would be the maximum voltage developed across the coil if a capacitor of 10 µF were connected across the switch breaking the d.c. supply ?*

Solution. Inductance of coil, $L = \frac{N\phi}{I} = \frac{5000 \times 0{\cdot}1 \times 10^{-3}}{1} = \mathbf{0{\cdot}5\ H}$

E.M.F. induced in coil, $e = L\frac{dI}{dt} = 0{\cdot}5 \times \frac{1-0}{10^{-3}} = \mathbf{500\ V}$

When capacitor is connected, the voltage developed will be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of voltage developed, then,

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

or $\frac{1}{2} \times (10 \times 10^{-6})\,V^2 = \frac{1}{2} \times 0{\cdot}5 \times (1)^2$

$\therefore$ $V = \mathbf{2{\cdot}24\ volts}$

Tutorial Problems

1. The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω. The exciting voltage is 230 V and the magnetic flux linking the coil is 5 mWb. Find (*i*) self-inductance of the coil and (*ii*) the energy stored in the magnetic field. **[(*i*) 1.043H (*ii*) 11.04 J]**
2. An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 Wb/m^2 if the relative permeability of iron is 1000. What is the value of energy stored? **[1.25 A ; 2.5 J]**
3. The inductance of a coil is 0.15H. The coil has 100 turns. Find (*i*) total magnetic flux through the coil when the current is 4A (*ii*) energy stored in the magnetic field (*iii*) voltage induced in the coil when current is reduced to zero in 0.01 second. **[(*i*) 6 mWb (*ii*) 1.2 J (*iii*) 60 V]**
4. An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1000 turns and also find the energy stored in it if the current rises from zero to 5A. **[0.7 mH ; 8.7 mJ]**

9.18. Magnetic Energy Stored Per Unit Volume

Consider a coil of N turns wound over a magnetic circuit of length l metres and uniform cross-sectional area of 'a' m^2.

$$\text{Magnetic energy stored} = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{N^2 a\,\mu_0\,\mu_r}{l}\right)I^2 = \frac{1}{2}(a\,\mu_0\,\mu_r)\,l\left(\frac{N\,I}{l}\right)^2$$

$$= \frac{1}{2}(\mu_0\,\mu_r)(a\,l)\,H^2 \qquad (\because H = NI/l)$$

Now, al = volume of magnetic field in m^3

$$\therefore \quad \text{Magnetic energy stored/m}^3 = \frac{1}{2}\mu_0\mu_r H^2$$

$$= \frac{1}{2}\mu_0\,\mu_r\left(\frac{B}{\mu_0\,\mu_r}\right)^2 \qquad \left[\because H = \frac{B}{\mu_0\mu_r}\right]$$

$$= \frac{B^2}{2\,\mu_0\,\mu_r}\text{ joules} \qquad \textit{...in a medium}$$

$$= \frac{B^2}{2\,\mu_0}\text{ joules} \qquad \textit{...in air}$$

Note that magnetic energy stored will be in joules if the flux density B is in Wb/m^2.

9.19. Lifting Power of a Magnet

When two opposite polarity magnetic poles are separated by a short distance in air, there is a force of attraction tending to pull the two poles together. The magnitude of this force can be calculated in terms of flux density in the air gap and cross-sectional area of the pole.

Consider two poles, north and south, each of area 'a' square metres separated by a short distance in air as shown in Fig. 9.29. Let P newtons be the force of attraction between the two poles. If one of the poles, say S-pole, is pulled apart through a small distance dx, then work will have to be done against the force of attraction.

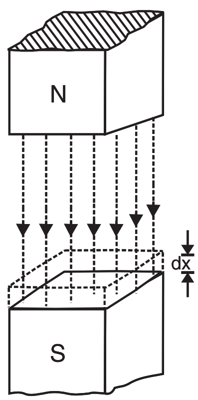


Fig. 9.29

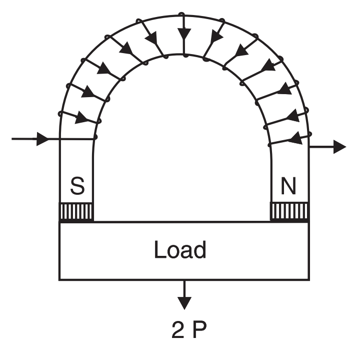


Fig. 9.30

$$\text{Work done} = P \times dx \text{ joules}$$

This work done is stored in the additional volume of the magnetic field created.

Additional volume of magnetic field created

$$= a \times dx \text{ m}^3$$

$$\therefore \quad \text{Increase in stored energy} = \frac{B^2}{2\mu_0} \times a\,dx$$

$$\text{But increase in stored energy} = \text{Work done}$$

$$\text{or} \quad \frac{B^2}{2\mu_0} \times a\,dx = P \times dx$$

$$\therefore \quad P = \frac{B^2 a}{2\mu_0}\text{ newtons}$$

It may be noted that P will be in newtons if B is in Wb/m^2 and 'a' in m^2.

Note that P is the force of attraction at each pole. In a practical magnet, there are two poles (See Fig. 9.30) so that total force of attraction is $2P$. Electromagnets are widely used for commercial lifting jobs such as loading scrap iron into a truck or raising an armature to a higher position.

Example 9.48. *A lifting magnet of inverted U-shape is formed out of an iron bar 60 cm long and 10 cm² in cross-sectional area [See Fig. 9.31]. Exciting coils of 750 turns each are wound on the two side limbs and are connected in series. Calculate the exciting current necessary for the magnet to lift a load of 60 kg, assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of the material of magnet is 600.*

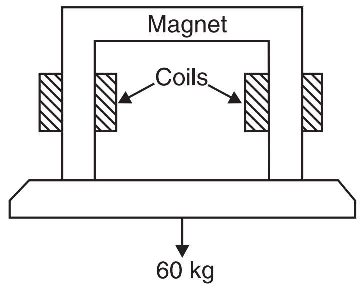


Fig. 9.31

Solution. Attractive force at eact pole is

$$P = \frac{60 \times 9.81}{2} = 294.3 \text{ N}$$

Now, $$P = \frac{B^2 a}{2\mu_0}$$

or $$B^2 = \frac{2\mu_0 P}{a}$$

$$= \frac{2 \times (4\pi \times 10^{-7}) \times 294.3}{10 \times 10^{-4}} = 0.74$$

$\therefore$ $$B = \sqrt{0.74} = 0.86 \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{B}{\mu_0 \mu_r} = \frac{0.86}{4\pi \times 10^{-7} \times 600} = 1141 \text{ AT/m}$$

Length of magnetic path, l = 60 cm = 0·6 m

Total AT required = 0·6 × 1141 = 684·6 AT

Total number of turns = 2 × 750 = 1500

$\therefore$ Exciting current required = 684·6/1500 = **0.456 A**

Example 9.49. *A smooth core armature working in a 4-pole field magnet has a gap (iron to iron) of 0·5 cm. The area of the surface of each pole is 0·1 m². The ampere-turns absorbed by each pole are 3000. Calculate (i) the mechanical force exerted by each pole on the armature and (ii) energy stored in the four air gaps.*

Solution. (*i*) AT per gap = Flux × Reluctance of air gap $= (B \times a) \times \left(\frac{l_g}{a\mu_0}\right) = \frac{B\, l_g}{\mu_0}$

or $$\frac{B}{\mu_0} = \frac{AT \text{ per gap}}{l_g} = \frac{3000}{0 \cdot 5 \times 10^{-2}} = 6 \times 10^5$$

or $$B = \mu_0 \times 6 \times 10^5 = (4\pi \times 10^{-7}) \times 6 \times 10^5 = 0{\cdot}75 \text{ Wb/m}^2$$

Mechanical force exerted by each pole is

$$P = \frac{B^2 a}{2\mu_0} = \frac{(0.75)^2 \times 0.1}{2 \times 4\pi \times 10^{-7}} = \mathbf{22381 \text{ N}}$$

(*ii*) Volume of 4 air gaps = $4\, a\, l_g = 4 \times 0{\cdot}1 \times 0{\cdot}5 \times 10^{-2} = 0{\cdot}002 \text{ m}^3$

$$\text{Energy stored in air gaps} = \frac{B^2}{2\mu_0} \times \text{Volume of 4 airgaps}$$

$$= \frac{(0.75)^2}{2 \times 4\pi \times 10^{-7}} \times 0.002 = \mathbf{448 \text{ J}}$$

Example 9.50. *An iron ring having a mean circumference of 30 cm and a cross-sectional area of 1 cm^2 has two radial saw cuts at diameterically opposite points. A brass plate is inserted in each gap (thickness of each gap being 0·1 mm). If the ring is wound with 200 turns, calculate the magnetising current to exert a pull of 5 kg between the two halves. Assume the magnetic data for the iron to be :*

B (Wb/m^2)	*0·79*	*1·0*	*1·3*
H (AT/m)	*250*	*350*	*520*

Solution. The total force of attraction at the two separations is = 5 × 9·80 = 49 N. Therefore, force of attraction at each separation, P = 49/2 = 24·5 N.

Now, $$P = \frac{B^2 a}{2\mu_0} \quad \therefore \quad B = \sqrt{\frac{2\times 4\pi\times 10^{-7}\times 24{\cdot}5}{1\times 10^{-4}}} = 0{\cdot}79 \text{ Wb/m}^2$$

Corresponding to B = 0·79 Wb/m^2, we have, H = 250 AT/m.

Length of iron path = 30 cm = 0·3 m

AT for iron path = 250 × 0·3 = 75 AT

H for brass = $B/\mu_0 = 0{\cdot}79/4\,\pi \times 10^{-7}$ = 628662 AT/m

Thickness of brass plates = 0·1 + 0·1 = 0·2 mm = 0·2 × 10^{-3} m

AT for brass paths = 628662 × 0·2 × 10^{-3} = 125·73 AT

Total AT required = 75 + 125·73 = 200·73 AT

∴ Magnetising current required = 200·73/200 = **1 A**

Example 9.51. *The arm of a starter is held in the "ON" position by means of an electromagnet. The torque exerted by the spring is 5 Nm and the effective radius at which the force is exerted is 10 cm. Area of each pole face is 2·5 cm^2 and each air gap is 0·4 mm. Find the minimum number of AT required to keep the arm in the "ON" position.*

Solution. Fig. 9.32 shows the whole arrangement. Let F newtons be the force exerted by the electromagnet.

Torque = Force × radius

or 5 = F × 0.1 ∴ F = 5/0.1 = 50 N

The force exerted at each pole of the magnet, P = 50/2 = 25 N

Now $$P = \frac{B^2 a}{2\mu_0}$$

$$\therefore \quad B = \sqrt{\frac{25\times 2\times 4\pi\times 10^{-7}}{2{\cdot}5\times 10^{-4}}} = 0.5 \text{ Wb/m}^2$$

The AT for iron path may be neglected ; being very small.

H in air gap = $B/\mu_0 = 0.5/4\pi \times 10^{-7}$ = 397887 AT/m

Total air gap length = 2 × 0.4 × 10^{-3} = 0.8 × 10^{-3} m

AT required = 397887 × 0.8 × 10^{-3} = **318.3 AT**

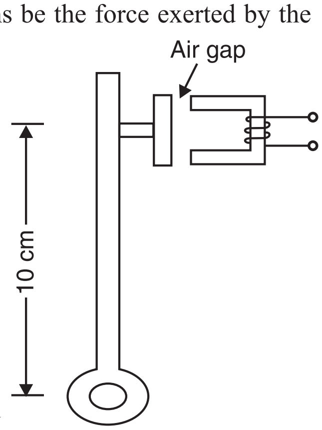


Fig. 9.32

Example 9.52. *The electromagnet shown in Fig. 9.33 has pole pieces each having a cross-sectional area of 25 cm^2. The total flux crossing each pole is 250 μWb. Determine the maximum weight of iron plate that can be lifted by the magnet. Neglect magnetic leakage and fringing.*

Solution. Flux density in the air gap is

$$B = \frac{\phi}{a} = \frac{250\times 10^{-6}}{25\times 10^{-4}} = 0.1 \text{ Wb/m}^2$$

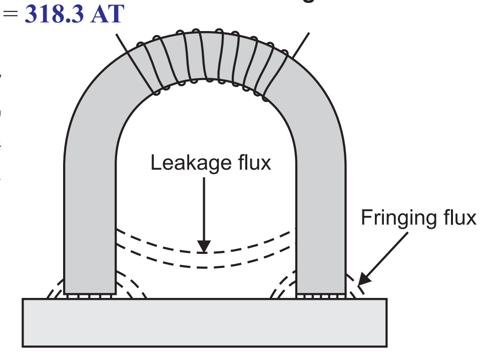


Fig. 9.33

Attractive force at each pole is

$$P = \frac{B^2 a}{2\mu_0} = \frac{(0.1)^2 \times 25 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 9.95 \text{ N}$$

Total force due to two poles $= 2P = 2 \times 9.95 = 19.9$ N

Let m be the maximum mass of the plate that can be lifted.

$\therefore \quad m \times g = 19.9$

or $\quad m = \frac{19.9}{g} = \frac{19.9}{9.8} = 2.03$ kg

Therefore, maximum weight of the plate that can be lifted is **2.03 kg.**

Tutorial Problems

1. Find the pull exerted on the plunger of an electromagnet when the total flux uniformly distributed is 500 μWb. Diameter of the plunger is 2·54 cm. **[196·3 N]**
2. A horse shoe magnet has two poles, each of area 5 cm². Find the pull between the poles and the keeper when the flux density at the contact surface is 1 Wb/m². **[398 N]**
3. The core material for use in an electromagnet should not have a flux density more than 1·5 Wb/m². How much area each of the two poles should have if the magnet is to lift 200 kg ? **[10·95 cm²]**
4. A circular crane magnet has an iron cross-section of 200 cm² and a mean magnetic path of 80 cm. Assuming the total length of each air gap to be 1·5 mm, calculate (*i*) the *AT* to produce a gap flux of 0·025 Wb (*ii*) the force to separate the contact surface, assuming no leakage or fringing.

B(Wb/m²)	1·0	1·2	1·4
H (AT/m)	900	1230	2100

[(*i*) 4080 AT (*ii*) 2500 kg (force)**]**

5. In a telephone receiver, the size of each pole of the electromagnet is 1.2 cm × 0.2 cm and flux between each pole and diaphragm is 4×10^{-6} Wb. With what force is the diaphragm attracted towards the poles? **[0.532 N]**
6. Magnetic materials having a surface area of 100 cm² are in contact with each other. They are in a magnetic circuit of flux 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces. **[3978 N]**
7. Each of the two pole faces of a lifting magnet has an area of 150 cm² and this may also be taken as the cross-sectional area of the 40 cm long flux path in the magnet. Determine the *AT* needed on the magnet if it is to lift a 900 kg iron block separated by 0·5 mm from the pole faces. Assume the magnetic leakage factor to be 1·2. Neglect fringing of the gap flux and reluctance of the flux path in the iron block.

H(AT/m)	400	600	800	1200	1600
B(Wb/m²)	0·81	0·98	1·1	1·24	1·35

[955 AT]

9.20. Closing and Breaking an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.34. When switch S is closed, the current increases gradually and takes some time to reach the final value. The reason the current does not build up *instantly to its final value is that as the current increases, the self-induced e.m.f. in L opposes the change in current (Lenz's Law). Suppose at any instant, the current is i and is increasing at the rate of di/dt.

Then, $\quad V = v_R + v_L$

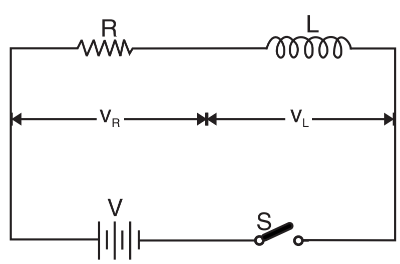


Fig. 9.34

* The current is zero at the instant the switch is closed because it must start from zero.

Now, self-induced e.m.f., $v_L = L\frac{di}{dt}$

If current change (*i.e.di*) is instant, it means $di/dt = 0$. This means that L is infinite which is impossible. So it is not possible for current in inductance to change from one value to the other in zero time.

$$= iR + L\frac{di}{dt}$$

As the current increases, v_R (= iR) increases and v_L decreases since V is constant. The decrease in v_L (= $L\,di/dt$) means that di/dt decreases because L is constant. The result is that after some time, di/dt becomes zero and so does the self- induced e.m.f. v_L (= $L\,di/dt$). At this stage, the current attains the final fixed value I given by ;

$$V = IR + 0 \quad \text{or} \quad I = \frac{V}{R}$$

Thus, when a d.c. circuit containing inductance is switched on, the current takes some time to reach the final value I (= V/R). Note that the role of inductance is to delay the change; it cannot prevent the current from attaining the final value. Similarly, when an inductive circuit is opened, the current does not jump to zero, but falls gradually. In either case, the delay in change depends upon the values of L and R as explained in the next article.

9.21. Rise of Current in an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.34. When switch S is closed, the current rises from zero to the final value I (= V/R) in a small time t. Suppose at any instant, the current is i and is increasing at the rate of di/dt. Then,

$$V = iR + L\frac{di}{dt} \quad \text{or} \quad V - iR = L\frac{di}{dt}$$

or $$\frac{di}{V - iR} = \frac{dt}{L}$$

*Multiplying both sides by $-R$, we get,

$$\frac{-R\,di}{V - iR} = \frac{-R}{L}\,dt$$

Integrating both sides, we get,

$$\int -\frac{R\,di}{V - iR} = -\frac{R}{L}\int dt$$

or $$\log_e (V - iR) = -\frac{R}{L}t + K \qquad ...(i)$$

where K is a constant whose value can be determined from the initial conditions. At $t = 0$, $i = 0$. Putting these values in exp. (i), we have, $\log_e V = K$.

$\therefore$ Equation (i) becomes :

$$\log_e (V - iR) = -\frac{R}{L}t + \log_e V$$

or $$\log_e \frac{V - iR}{V} = -\frac{R}{L}t$$

or $$\frac{V - iR}{V} = e^{-Rt/L}$$

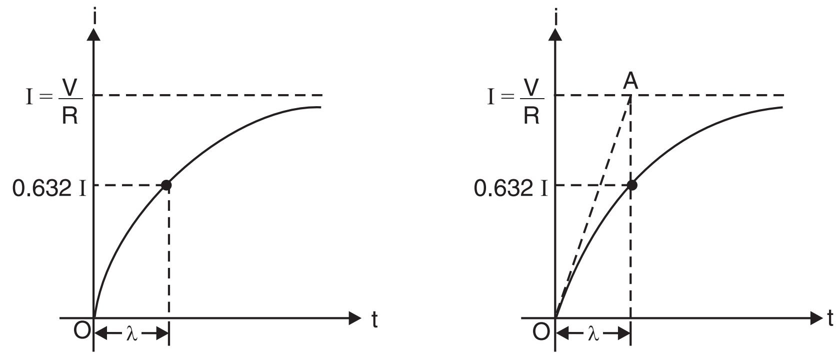
or $$V - iR = V\,e^{-Rt/L}$$

or $$i = \frac{V}{R}\,(1 - e^{-Rt/L})$$

But $V/R = I$, the final value of current attained by the circuit.

$\therefore$ $$i = I(1 - e^{-Rt/L}) \qquad ...(ii)$$

* This step makes the numerator on the L.H.S. a differential of the denominator.



(*i*) (*ii*)

Fig. 9.35

Eq. (*ii*) shows that rise of current follows an exponential law (See Fig. 9.35). As t increases, the term $e^{-Rt/L}$ gets smaller and current i in the circuit gets larger. Theoretically, the current will reach its final value I (= V/R) in an infinite time. However, practically it reaches this value in a short time.

Note. $V = iR + L\, di/dt$

At the instant the switch is closed, $i = 0$. $\therefore \quad V = L\, di/dt$

Initial rate of rise of current, $\dfrac{di}{dt} = \dfrac{V}{L}$ A/sec.

The initial rate of rise of current in an inductive circuit helps us in defining the time constant of the circuit.

9.22. Time Constant

Consider the eq. (*ii*) above showing the rise of current w.r.t. time t.

$$i = I\,(1 - e^{-Rt/L})$$

The exponent of e is Rt/L. The quantity L/R has the dimensions of time so that exponent of e (*i.e.* Rt/L) is a number. The quantity L/R is called the *time constant* of the circuit and affects the rise of current in the circuit. It is represented by λ.

$$\therefore \qquad \text{Time constant, } \lambda = L/R \text{ seconds}$$

$$\therefore \qquad i = I(1 - e^{-t/\lambda})$$

Time constant of an inductive circuit can be defined in the following ways :

(*i*) Consider the graph showing the rise of current w.r.t. time t [See Fig. 9.35 (*ii*)]. The initial rate of rise of current (*i.e.* at $t = 0$) in the circuit is

$$\frac{di}{dt} = \frac{V}{L}$$

If this rate of rise of current were maintained, the graph would be linear [*i.e.* OA in Fig. 9.35 (*ii*)] instead of exponential. If this rate of rise could continue, the circuit current will reach the final value I (= V/R) in time

$$= \frac{V}{R} \div \frac{V}{L} = \frac{L}{R} = \text{Time constant } \lambda$$

Hence **time constant** *may be defined as the time required for the current to rise to its final steady value if it continued rising at its initial rate (i.e. V/L).*

(*ii*) If time interval, $t = \lambda$ (or L/R), then,

$$i = I\,(1 - e^{-Rt/L}) = I\,(1 - e^{-1}) = 0{\cdot}632\, I$$

Hence **time constant** *can also be defined as the time required for the current to reach 0·632 of its final steady value while rising.*

Fig 9.36 as well as adjoining table shows the percentage of final current (I) after each time constant interval during current buildup (i) in the inductor. The current will increase to about 63% of its full value (I) in first time constant. A 5 time-constant time interval is accepted as the time for the current to attain its final value I.

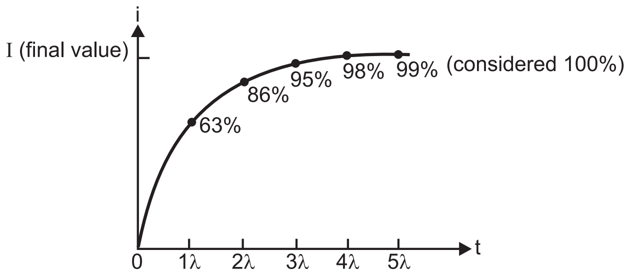


Fig. 9.36

Number of time constants	% of final value
1	63
2	86
3	95
4	98
5	99 (considered 100%)

9.23. Decay of Current in an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.37. When switch S is thrown to position 2, the current in the circuit starts rising and attains the final value I (= V/R) after some time as explained above. If now switch is thrown to position 1, it is found that current in the $R - L$ circuit does not cease immediately but gradually reduces to zero. Suppose at any instant, the current is i and is decreasing at the rate of di/dt. Then,

$$0 = iR + L\frac{di}{dt}$$

or $$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides, we get, $\log_e i = -\frac{R}{L}t + K$ ...(i)

where K is a constant whose value can be determined from the initial conditions. When $t = 0$, then $i = I$ (= V/R).

Putting these values in eq. (i), we have, $\log_e I = 0 + K$ or $K = \log_e I$

$\therefore$ Equation (i) becomes : $\log_e i = -\frac{R}{L}t + \log_e I$

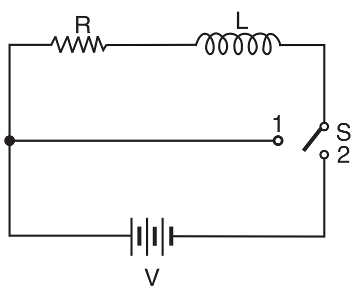


Fig. 9.37

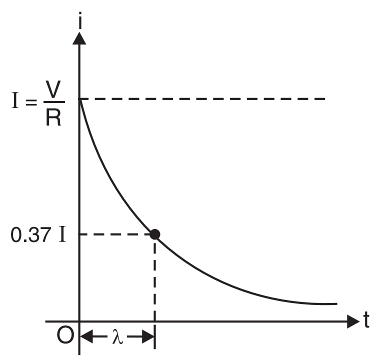


Fig. 9.38

or $$\log_e \frac{i}{I} = -\frac{R}{L}t \quad \text{or} \quad \frac{i}{I} = e^{-Rt/L}$$

$\therefore$ $$i = I\,e^{-Rt/L} \quad \text{or} \quad i = I\,e^{-t/\lambda}$$ ...(ii)

Eq. (*ii*) gives the decay of current in an $R - L$ series circuit with time t and is represented graphically in Fig. 9.38. Note that decay of current follows the exponential law.

Time constant. The quantity L/R in eq. (*ii*) is known as time constant of the circuit. When $t = \lambda\ (= L/R)$,

$$i = I e^{-1} = 0{\cdot}37\, I$$

Hence, **time constant** *may also be defined as the time taken by the current to fall to 0·37 of its final steady value I (= V/R) while decaying.*

Fig. 9.39 as well as adjoining table shows the percentage of initial current (I) after each time constant interval while the current is decreasing. During the first time constant interval, the current decreases 37% of its initial value. A 5 time-constant interval is accepted as the time for the current to reduce to zero value.

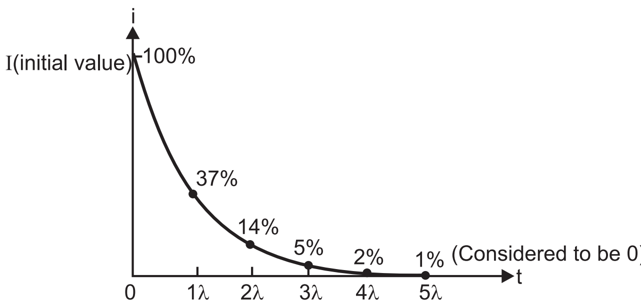


Fig. 9.39

Number of time constants	% of final value
1	37
2	14
3	5
4	2
5	1 (considered 0)

Example 9.53. *The resistance and inductance of a series circuit are 5 Ω and 20 H respectively. At the instant of closing the supply, the current increases at the rate of 4 A/s. Calculate (i) the applied voltage and (ii) the rate of growth of current when the current is 5 A.*

Solution. (*i*) The voltage equation of *R-L* series circuit is

$$V = iR + L\frac{di}{dt}$$

At the instant the switch is closed, $i = 0$.

$$\therefore \quad V = L\frac{di}{dt} = 20 \times 4 = \mathbf{80\ V}$$

(*ii*)
$$V = iR + L\frac{di}{dt}$$

Here $\quad V = 80$ volts ; $\quad i = 5$ A ; $L = 20$ H ; $\quad R = 5\ \Omega$

$$\therefore \quad 80 = 5 \times 5 + 20\frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} = \frac{80 - 25}{20} = \mathbf{2.75\ A/s}$$

An important difference between *RC* and *RL* circuits is the effect of resistance on the duration of the transient. In an *RC* circuit, a large resistance prolongs the transient because it makes the time constant λ (= *RC*) large. In an *RL* circuit, a large resistance shortens the transient because it makes time constant λ (= *L*/*R*) small.

Example 9.54. *A constant voltage is applied to a series R – L circuit at t = 0 by closing a switch. The voltage across L is 25 V at t = 0 and drops to 5 V at t = 0.025s. If L = 2H, what must be the value of R ?*

Solution. Applied voltage, $V = iR + L\dfrac{di}{dt}$

At $t = 0$, $i = 0$ and $L\dfrac{di}{dt} = 25$ volts (*given*)

$$\therefore \quad V = 0 + 25 = 25 \text{ volts}$$

At $t = 0.025$ second, $L\dfrac{di}{dt} = 5$V so that $iR = 25 - 5 = 20$ V.

Now, $i = I(1 - e^{-t/\lambda}) = \dfrac{V}{R}(1 - e^{-t/\lambda})$

or $iR = \dfrac{V}{R} \times R(1 - e^{-t/\lambda})$

$\therefore$ $iR = V(1 - e^{-t/\lambda})$

At $t = 0.025$ second, $iR = 20$ V and $V = 25$ volts.

$\therefore$ $20 = 25(1 - e^{-0.025/\lambda})$

or $1 - e^{-0.025/\lambda} = 0.8$ or $e^{+0.025/\lambda} = 5$

$\therefore$ $\dfrac{0.025}{\lambda}\log_e e = \log_e 5$ or $\lambda = \dfrac{0.025}{\log_e 5} = 0.0155$

Now, $\lambda = \dfrac{L}{R}$ or $R = \dfrac{L}{\lambda} = \dfrac{2}{0.0155} = $ **129.03Ω**

Example 9.55. *The steady current flowing in an inductor is 250 mA ; the current flowing 0.1 sec. after connecting the supply voltage is 120 mA. Calculate (i) time constant of the circuit and (ii) the time from closing the circuit at which circuit current has reached 200 mA.*

Solution. (*i*) $i = I(1 - e^{-t/\lambda})$

Here $i = 120$ mA ; $I = 250$ mA ; $t = 0.1$ sec.

$\therefore$ $120 = 250(1 - e^{-0.1/\lambda})$ or $e^{-0.1/\lambda} = 1 - (120/250) = 0{\cdot}52$

$\therefore$ $e^{0.1/\lambda} = 1/0.52 = 1.923$

or $(0.1/\lambda)\log_e e = \log_e 1.923$

$\therefore$ Time constant, $\lambda = \dfrac{0.1}{\log_e 1.923} = $ **0.153 s**

(*ii*) $i = I(1 - e^{-t/\lambda})$

Here $i = 200$ mA ; $I = 250$ mA ; $\lambda = 0.153$ sec.

$\therefore$ $200 = 250(1 - e^{-t/0.153})$ or $e^{-t/0.153} = 1 - (200/250) = 0.2$

$\therefore$ $e^{t/0.153} = 1/0{\cdot}2 = 5$

or $(t/0{\cdot}153)\log_e e = \log_e 5$

$\therefore$ $t = 0{\cdot}153 \log_e 5 = $ **0·25 s**

Example 9.56. *A coil having $L = 2{\cdot}4$ H and $R = 4\ \Omega$ is connected to a constant 100 V supply source. How long does it take the voltage across the resistance to reach 50 V ?*

Solution. $i = I(1 - e^{-t/\lambda}) = \dfrac{V}{R}(1 - e^{-t/\lambda})$

or $iR = V(1 - e^{-t/\lambda})$

Here $iR = 50$ volts ; $V = 100$ volts ; $\lambda = L/R = 2{\cdot}4/4 = 0{\cdot}6$ s

$\therefore$ $50 = 100(1 - e^{-t/0.6})$ or $e^{-t/0.6} = 1 - (50/100) = 0{\cdot}5$

$\therefore$ $e^{t/0.6} = 1/0{\cdot}5 = 2$

or $(t/0{\cdot}6)\log_e e = \log_e 2$

$\therefore$ $t = 0{\cdot}6 \log_e 2 = $ **0·416 s**

Example 9.57. *The time constant of a certain inductive coil was found to be 2·5 ms. With a resistance of 80 Ω added in series, a new time constant of 0·5ms was obtained. Find the inductance and resistance of the coil.*

Solution. Time constant, $\lambda = L/R$

For the first case, $L/R = 2{\cdot}5$; For the second case, $L/(R + 80) = 0{\cdot}5$

$$\therefore \qquad \frac{R+80}{R} = \frac{2\cdot 5}{0\cdot 5} = 5 \quad \text{or} \quad R = \mathbf{20\ \Omega}$$

Now $\qquad L/R = 2{\cdot}5 \quad \therefore \quad L = 2{\cdot}5\,R = 2{\cdot}5 \times 20 = \mathbf{50\ H}$

Example 9.58. *A coil having an effective resistance of 25 Ω and an inductance of 5 H is suddenly connected across a 50 V d.c. supply. What is the rate at which energy is stored in the field of the coil when current is (i) 0·5 A, (ii) 1A and (iii) steady ? Also find the induced EMF in the coil under the above conditions.*

Solution.

(*i*) When current is 0·5 A

Power input = 50 × 0·5 = 25 W ; Power wasted as heat = $i^2R = (0{\cdot}5)^2 \times 25 = 6{\cdot}25$ W

∴ Rate of energy storage in the field of the coil = 25 – 6·25 = **18·75 W**

(*ii*) When current is 1 A

Power input = 50 × 1 = 50 W ; Power wasted as heat = $(1)^2 \times 25 = 25$ W

∴ Rate of energy stored = 50 – 25 = **25 W**

(*iii*) When current is steady = V/R = 50/25 = 2A

Power input = 50 × 2 = 100 W ; Power wasted as heat = $(2)^2 \times 25 = 100$ W

∴ Rate of energy stored = 100 – 100 = **0 W**

Induced e.m.f.

Voltage across coil, $e_L = V - iR$

(*i*) When i = 0·5 A ; $e_L = 50 - 0{\cdot}5 \times 25 =$ **37·5 V**

(*ii*) When i = 1 A ; $e_L = 50 - 1 \times 25 =$ **25 V**

(*iii*) When i = 2 A ; $e_L = 50 - 2 \times 25 =$ **0 V**

Example 9.59. *A circuit of resistance R ohms and inductance L henries has a direct voltage of 230 V applied to it. 0.3 second after switching on, the current in the circuit was found to be 5A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5A at 0.3 second after short-circuiting the coil. Find the values of R and L.*

Solution. This is a case of growth and decay of current in $R - L$ series circuit. In both cases, i = 5A and t = 0.3 s.

For growth : $\qquad i = I(1 - e^{-t/\lambda})$

or $\qquad 5 = I(1 - e^{-0.3/\lambda}) \qquad$ ...(*i*)

For decay : $\qquad i = I\,e^{-t/\lambda}$

or $\qquad 5 = I\,e^{-0.3/\lambda} \qquad$ ...(*ii*)

From eqs. (*i*) and (*ii*), $I\,e^{-0.3/\lambda} = I(1 - e^{-0.3/\lambda})$

or $\qquad 2\,e^{-0.3/\lambda} = 1$

∴ $\qquad e^{-0.3/\lambda} = 0.5 \quad$ or $\quad \lambda = 0.4328$

Putting $\lambda = 0.4328$ in eq. (*ii*), we get,

$$5 = I\,e^{-0.3/0.4328} \quad \text{or} \quad I = 5\,e^{+0.3/0.4328} = 5 \times 2 = 10\ \text{A}$$

Now, $\qquad I = \dfrac{V}{R} \quad \therefore R = \dfrac{V}{I} = \dfrac{230}{10} = \mathbf{23\Omega}$

Also, $\qquad \lambda = \dfrac{L}{R} \quad$ or $\quad L = R\lambda = 23 \times 0.4328 = \mathbf{9.95\ H}$

Example 9.60. *Two mutually coupled coils, A and B, are connected in series to a 400 V d.c. supply. Coil A has a resistance of 14 Ω and inductance of 4 H. Coil B has a resistance of 20 Ω and inductance of 9 H. At a certain instant after the circuit is energised, the current is 5 A and is increasing at the rate of 10 A/s. Calculate (i) the mutual inductance between the coils, and (ii) the coefficient of coupling.*

Solution. Fig. 9.40 shows the conditions of the problem. When not mentioned in the problem, it is understood that the mutual fluxes of the two coils aid each other.

(*i*) $$V = i\,(R_A + R_B) + L_T \frac{di}{dt}$$

where L_T is the total inductance of the circuit.

or $$400 = 5\,(14 + 20) + 10\,L_T \quad \therefore \quad L_T = 23 \text{ H}$$

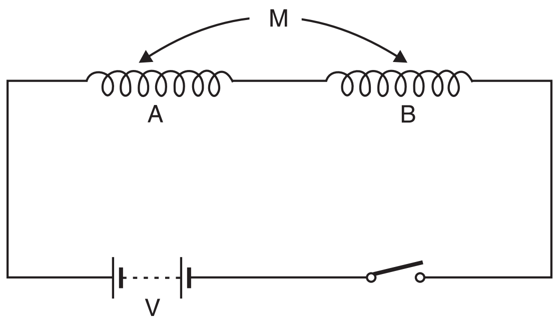


Fig. 9.40

Now, $$L_T = L_A + L_B + 2M \quad \text{or} \quad 23 = 4 + 9 + 2M \quad \therefore \quad M = \mathbf{5\ H}$$

(*ii*) Coefficient of coupling, $$k = \frac{M}{\sqrt{L_A L_B}} = \frac{5}{\sqrt{4 \times 9}} = \mathbf{0{\cdot}83}$$

Example 9.61. *The two circuits of Fig. 9.41 have the same time constant of 0·005 second. With the same d.c. voltage applied to the two circuits, it is found that steady state current of circuit (i) is 2000 times the initial current of circuit (ii). Find R_1, L and C.*

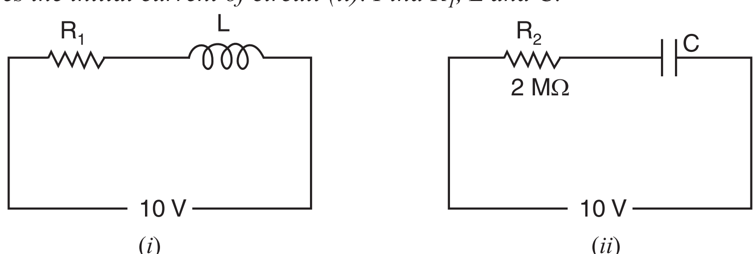


Fig. 9.41

Solution. The time constant for both the circuits is 0·005 s.

$$\therefore \quad R_2 C = 0{\cdot}005 \quad \text{or} \quad C = \frac{0.005}{R_2}$$

$$\therefore \quad C = \frac{0.005}{2 \times 10^6} = 0{\cdot}0025 \times 10^{-6} \text{ F} = \mathbf{0{\cdot}0025\ \mu F}$$

Steady state current in Fig. 9.41 (*i*) = $V/R_1 = 10/R_1$

Initial current in Fig. 9.41 (*ii*) = $V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6}$ A

As per statement of the problem, we have,

$$10/R_1 = 2000 \times (5 \times 10^{-6}) \quad \therefore \quad R_1 = \mathbf{1000\ \Omega}$$

Now $$L/R_1 = 0{\cdot}005 \quad \therefore \quad L = 1000 \times 0{\cdot}005 = \mathbf{5\ H}$$

Tutorial Problems

1. A 12 V battery is connected in series with 30 Ω resistor and a 220 mH inductor. How long will it take the current to reach half its maximum possible value ? At this instant, at what rate is energy being delivered by the battery ? **[5ms ; 2·4 W]**

2. A p.d. of 100 V is applied to a circuit consisting of a resistance of 50 Ω and an inductance of 5 H. Determine the current in the circuit 0·1 second after the application of the voltage. **[1·264 A]**
3. How many time constants one should wait for the current in an *RL* circuit to grow within 0·1% of its steady value ? **[6·9 time constants]**
4. Calculate the back e.m.f. of a 1 H, 10 Ω coil 0·1 s after 100 V d.c. supply is connected to it. **[36·8 V]**
5. The resistance and inductance of a series circuit are 50 Ω and 20 H respectively. At the instant of closing the supply, the current increases at the rate of 4 A/s. Calculate (*i*) supply voltage (*ii*) the rate of growth of current when current is 5 A. **[(*i*) 80 V (*ii*) 2·75 A/s]**

9.24. Eddy Current Loss

When a magnetic material is subjected to a changing magnetic field, in addition to the hysteresis loss, another loss that occurs in the material is the *eddy current loss*. The changing flux induces voltages in the material according to Faraday's laws of electromagnetic induction. Since the material is conducting, these induced voltages circulate currents within the body of the material. These induced currents do no useful work and are known as eddy currents. These eddy currents develop i^2R loss in the material. Like hysteresis loss, the eddy current loss also results in the rise of temperature of the material. *The hysteresis and eddy current losses in a magnetic material are sometimes called* **core losses** or **iron losses.**

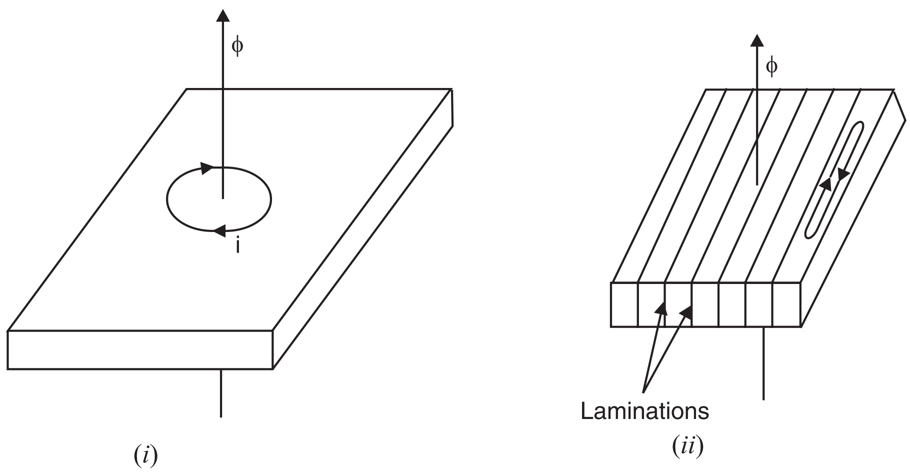


Fig. 9.42

Fig. 9.42 (*i*) shows a solid block of iron subjected to a changing magnetic field. The eddy current power loss in the block will be i^2R where i is the eddy current and R is the resistance to the eddy current path. Since the block is a continuous iron piece of large *X*-section, the magnitude of i will be very *large and hence greater eddy current loss will result. The obvious method of reducing this loss is to reduce the magnitude of eddy current. This can be achieved by splitting the solid block into thin sheets (called **laminations**) in planes parallel to the magnetic flux as shown in Fig.9·42 (*ii*). Each lamination is insulated from the other by a layer of varnish. This arrangement reduces the area of each section and hence the induced e.m.f. It also increases the resistance of eddy current paths since the area through which the currents can pass is smaller. Both effects combine to reduce the eddy current and hence eddy current loss. Further, reduction in this loss can be obtained by using a magnetic material of high resistivity (*e.g.* silicon steel).

The only drawback of laminated core is that the total cross-sectional area of the magnetic material is reduced by the total thickness of the insulation. This is generally taken into account by allowing about 10% reduction in the thickness of core when making the magnetic calculations.

* The large area of the block will have greater e.m.f. induced in it. Larger X-section also means smaller resistance to eddy current path. Both these effects increase the magnitude of eddy current to a great extent.

9.25. Formula for Eddy Current Power Loss

It is difficult to determine the eddy current power loss because the current and resistance values cannot be determined directly. Experiments have shown that eddy current power loss P_e in a magnetic material can be expressed as :

$$P_e = k_e B_m^2 t^2 f^2 V \text{ watts}$$

where k_e = eddy current coefficient and its value depends upon the nature of the material.

B_m = maximum flux density in Wb/m^2

t = thickness of lamination in m

f = frequency of flux in Hz

V = volume of material in m^3

Example 9.62. *The flux in a magnetic core is alternating sinusoidally at 50 Hz. The maximum flux density is 1·5 Wb/m^2. The eddy current loss then amounts to 140 watts. Find the eddy current loss in the core when the frequency is 75 Hz and the flux density is 1·2 Wb/m^2.*

Solution. Eddy current power loss, $P_e \propto B_m^2 f^2$

For the first case, $P_{e1} \propto (1·5)^2 \times (50)^2$; For the second case, $P_{e2} \propto (1·2)^2 \times (75)^2$

$$\therefore \quad \frac{P_{e2}}{P_{e1}} = \left(\frac{1·2}{1·5}\right)^2 \times \left(\frac{75}{50}\right)^2 = 1.44$$

$$\therefore \quad P_{e2} = 1·44\, P_{e1} = 1·44 \times 140 = \mathbf{201·6\ W}$$

Example 9.63. *Find the eddy current power loss in a 50 Hz transformer with a maximum flux density of 1 Wb/m^2. The core is of section 8 cm × 6 cm and total effective length is 50 cm constructed of laminations of thickness 0·4 mm. The eddy current coefficient is 6·58 × 10^6. Assume a space factor of 0·9.*

Solution. Total core area $= 8 \times 6 = 48 \text{ cm}^2 = 48 \times 10^{-4} \text{ m}^2$

*Useful core area $= 0·9 \times 48 \times 10^{-4} = 43·2 \times 10^{-4} \text{ m}^2$

Volume of iron in core, $V = 43·2 \times 10^{-4} \times 0·5 = 21·6 \times 10^{-4} \text{ m}^3$

Thickness of lamination, $t = 0·4 \text{ mm} = 0·4 \times 10^{-3} \text{ m}$

$$\therefore \quad P_e = k_e B_m^2 t^2 f^2 V \text{ watts}$$

$$= (6·58 \times 10^6) \times (1)^2 \times (0·4 \times 10^{-3})^2 \times (50)^2 \times 21·6 \times 10^{-4} = \mathbf{5·68\ W}$$

Example 9.64. *A transformer connected to 25 Hz supply has a core loss of 1500 watts of which 1000 watts are due to hysteresis and 500 watts due to eddy currents. If the flux density is kept constant and frequency is increased to 50 Hz, find the new value of the core loss.*

Solution.

Hysteresis power loss, $P_h \propto B_m^{1·6} f$

Eddy current power loss, $P_e \propto B_m^2 f^2$

Hysteresis loss

$$\frac{P_{h2}}{P_{h1}} = \left(\frac{B_{m2}}{B_{m1}}\right)^{1·6} \times \frac{f_2}{f_1} = (1)^{1·6} \times 50/25 = 2 \qquad (\because B_{m2} = B_{m1})$$

$$\therefore \quad P_{h2} = 2 \times 1000 = 2000 \text{ W}$$

* The core of the transformer is laminated to reduce the eddy current loss. The cross-sectional area of iron is now less than the apparent area due to the area taken up by the insulation.

$$\text{Space factor} = \frac{\text{Useful area}}{\text{Total area}}$$

Eddy current loss

$$\frac{P_{e2}}{P_{e1}} = \left(\frac{B_{m2}}{B_{m1}}\right)^2 \times \left(\frac{f_2}{f_1}\right)^2 = (1)^2 \times \left(\frac{50}{25}\right)^2 = 4$$

$\therefore$ $P_{e2} = 4\,P_{e1} = 4 \times 500 = 2000$ W

$\therefore$ New core loss $= P_{h2} + P_{e2} = 2000 + 2000 =$ **4000 W**

Example 9.65. *The core loss in a given specimen is found to be 65 W at a frequency of 30 Hz and a flux density of 1 Wb/m² and 190 W at 60 Hz and the same flux density. What are the hysteresis loss and the eddy current loss at each frequency ?*

Solution. Since the flux density, the volume of specimen and the thickness of laminations remain constant, the iron or core loss (= hysteresis loss + eddy current loss) can be written as :

$$\text{Core loss, } P_c = k'_h f + k'_e f^2 \qquad ...(i)$$

where $k'_h = k_h B_m^{1\cdot 6}\, V$ and $k'_e = k_e B_m^2 t^2\, V$

Putting the given values in eq. (*i*), we have,

$$65 = k'_h \times 30 + k'_e \times (30)^2 \qquad ...(ii)$$

$$190 = k'_h \times 60 + k'_e \times (60)^2 \qquad ...(iii)$$

Solving eqs. (*ii*) and (*iii*), $k'_h = 1\cdot167$; $k'_e = 0\cdot0333$

At 30 Hz. At 30 Hz, these losses are :

$$P_h = k'_h \times 30 = 1\cdot167 \times 30 = \mathbf{35\ W}$$

$$P_e = k'_e \times (30)^2 = 0\cdot0333 \times (30)^2 = \mathbf{30\ W}$$

At 60 Hz. At 60 Hz, these losses are :

$$P_h = k'_h \times 60 = 1\cdot167 \times 60 = \mathbf{70\ W}$$

$$P_e = k'_e \times (60)^2 = 0\cdot0333 \times (60)^2 = \mathbf{120\ W}$$

Objective Questions

1. The basic requirement for inducing e.m.f. in a coil is that ....
   (*i*) flux should link the coil
   (*ii*) there should be change in flux linking the coil
   (*iii*) coil should form a closed loop
   (*iv*) none of the above
2. The e.m.f. induced in a coil is .... the rate of change in flux linkages.
   (*i*) directly proportional to
   (*ii*) inversely proportional to
   (*iii*) independent of
   (*iv*) none of the above
3. The e.m.f. induced in a coil of *N* turns is given by ....
   (*i*) $d\phi/dt$ (*ii*) $N\,d\phi/dt$
   (*iii*) $-N\,d\phi/dt$ (*iv*) $N\,dt/d\phi$
4. The direction of induced e.m.f. in a conductor (or coil) can be determined by ....
   (*i*) work law
   (*ii*) Ampere's law
   (*iii*) Fleming's right-hand rule
   (*iv*) Fleming's left-hand rule
5. In Fig. 9.43, the conductor is moving upward. The direction of induced e.m.f. is ....

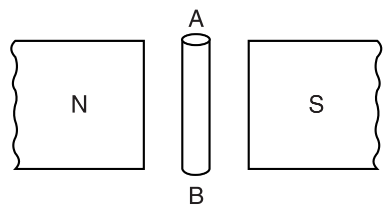


Fig. 9.43

   (*i*) from *A* to *B* (*ii*) from *B* to *A*
   (*iii*) none of the above
6. In Fig. 9.44, the direction of induced e.m.f. in the conductor *A* is ....

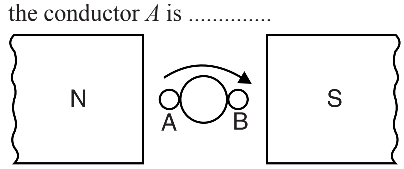


Fig. 9.44

(*i*) into the plane of paper
(*ii*) out of plane of paper
(*iii*) none of the above

7. In Fig. 9.44, the rate of change of flux linkages of conductors *A* and *B* is ....
(*i*) minimum (*ii*) maximum
(*iii*) mid-way between (*a*) and (*b*)
(*iv*) none of the above

8. The e.m.f. induced in a .... is the statically induced e.m.f.
(*i*) d.c. generator (*ii*) transformer
(*iii*) d.c. motor (*iv*) none of the above

9. The e.m.f. induced in a .... is dynamically induced e.m.f.
(*i*) alternator (*ii*) transformer
(*iii*) d.c. generator (*iv*) none of the above

10. In Fig. 9.45, 1 single conductor of length *l* metres moves at right angles to a uniform field of *B* Wb/m^2 with a velocity of *v* m/s. The e.m.f. induced is ....
(*i*) $B\,l/v$ (*ii*) Bv/l
(*iii*) Blv (*iv*) lv/B

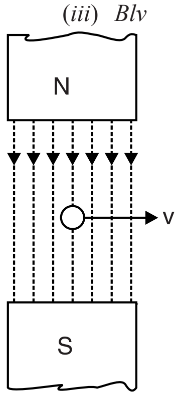


Fig. 9.45

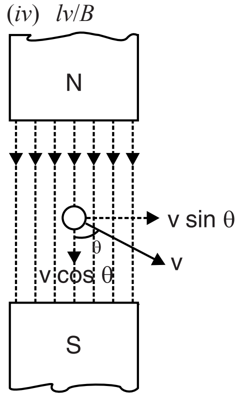


Fig. 9.46

11. In Fig. 9.46, the component of velocity that does not induce any e.m.f. in the conductor is ....
(*i*) $v \sin \theta$ (*ii*) $v \cos \theta$
(*iii*) $v \tan \theta$ (*iv*) none of the above

12. Inductance opposes .... in current in a circuit.
(*i*) only increase (*ii*) only decrease
(*iii*) change (*iv*) none of the above

13. If the number of turns of a coil is increased, its inductance ....
(*i*) remains the same (*ii*) is increased
(*iii*) is decreased (*iv*) none of the above

14. If the relative permeability of the material surrounding the coil is increased, the inductance of the coil ....
(*i*) is increased (*ii*) is decreased
(*iii*) remains unchanged
(*iv*) none of the above

15. Inductance in a circuit ....
(*i*) prevents the current from changing
(*ii*) delays the change in current
(*iii*) causes power loss
(*iv*) causes the current to lead the voltage

16. The inductance of a coil is .... the reluctance of magnetic path.
(*i*) independent of
(*ii*) directly proportional to
(*iii*) inversely proportional to
(*iv*) none of the above

17. If the number of turns of a coil is increased two times, its inductance is ....
(*i*) increased two times
(*ii*) decreased two times
(*iii*) decreased four times
(*iv*) increased four times

18. A circuit has inductance of 2H. If the circuit current changes at the rate of 10 A/second, then self-induced e.m.f. is ....
(*i*) 5 V (*ii*) 0·2 V
(*iii*) 20 V (*iv*) 10 V

19. A current of 2 A through a coil sets up flux linkages of 4 Wb-turn. The inductance of the coil is ....
(*i*) 8 H (*ii*) 0·5 H
(*iii*) 2 H (*iv*) 1 H

20. An air-cored choke is used for .... applications.
(*i*) radio frequency (*ii*) audio frequency
(*iii*) power frequency (*iv*) none of the above

21. If a 10-turn coil has a second layer of 10 turns wound over the first, then total inductance will be about .... the original inductance.
(*i*) two times (*ii*) four times
(*iii*) six times (*iv*) three times

22. An iron-cored coil of 10 turns has reluctance of 100 AT/Wb. The inductance of the coil is ....
(*i*) 1 H (*ii*) 10 H
(*iii*) 0·1 H (*iv*) 5 H

23. An iron-cored coil has an inductance of 2 H. If the reluctance of the magnetic path is 200 AT/Wb, the number of turns on the coil is ....

(*i*) 100 (*ii*) 400
(*iii*) 50 (*iv*) 20

24. The mutual inductance between two coils is .... reluctance of magnetic path.

(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of (*iv*) none of the above

25. Mutual inductance between two coils can be decreased by ....

(*i*) increasing the number of turns of either coil
(*ii*) by moving the coils closer
(*iii*) by moving the coils apart
(*iv*) none of the above

26. Mutual inductance between two coils is 4H. If current in one coil changes at the rate of 2 A/second, then e.m.f. induced in the other coil is ....

(*i*) 8 V (*ii*) 2 V
(*iii*) 0·5 V (*iv*) none of the above

27. If in Fig. 9.47, ϕ_{12} = 2 Wb, N_2 = 20 and I_2 = 20 A, then mutual inductance between the coils is ....

(*i*) 200 H (*ii*) 20 H
(*iii*) 4 H (*iv*) 2 H

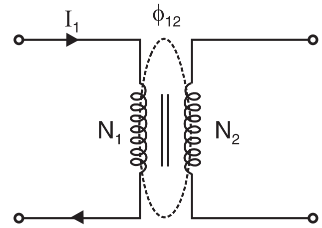


Fig. 9.47

28. If in Fig. 9.47, N_1 = 100, N_2 = 1000 and mutual inductance between the coils is 2H, the reluctance of magnetic circuit is ....

(*i*) 5×10^4 AT/Wb (*ii*) 10^5 AT/Wb
(*iii*) 20 AT/Wb (*iv*) 5 AT/Wb

29. If the coefficient of coupling between two coils is increased, mutual inductance between the coils ....

(*i*) is increased (*ii*) is decreased
(*iii*) remains unchanged
(*iv*) none of the above

30. The maximum mutual inductance between the coils shown in Fig. 9.47 is given by ....

(*i*) $L_A L_B$ (*ii*) L_A / L_B
(*iii*) $\sqrt{L_A L_B}$ (*iv*) $(L_A L_B)^2$

Answers

1. (*ii*)	**2.** (*i*)	**3.** (*iii*)	**4.** (*iv*)	**5.** (*ii*)
6. (*ii*)	**7.** (*ii*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*iii*)
11. (*ii*)	**12.** (*iii*)	**13.** (*ii*)	**14.** (*i*)	**15.** (*ii*)
16. (*iii*)	**17.** (*iv*)	**18.** (*iii*)	**19.** (*iii*)	**20.** (*i*)
21. (*ii*)	**22.** (*i*)	**23.** (*iv*)	**24.** (*ii*)	**25.** (*iii*)
26. (*i*)	**27.** (*iv*)	**28.** (*i*)	**29.** (*i*)	**30.** (*iii*)