2
D.C. Circuits

Introduction

It is well known that electric current flows in a closed path. The closed path followed by electric current is called an electric circuit. The essential parts of an electric circuit are (*i*) the source of power (*e.g.* battery, generator etc.), (*ii*) the conductors used to carry current and (*iii*) the load* (*e.g.* lamp, heater, motor etc.). The source supplies electrical energy to the load which converts it into heat or other forms of energy. Thus, conversion of electrical energy into other forms of energy is possible only with suitable circuits. For instance, conversion of electrical energy into mechanical energy is achieved by devising a suitable motor circuit. In fact, the innumerable uses of electricity have been possible only due to the proper use and application of electric circuits. In this chapter, we shall confine our discussion to d.c. circuits only *i.e.* circuits carrying direct current.

2.1. D.C. Circuit

The closed path followed by direct current (d.c.) is called a **d.c. circuit.**

A d.c. circuit essentially consists of a source of d.c. power (*e.g.* battery, d.c. generator etc.), the conductors used to carry current and the load. Fig. 2.1 shows a torch bulb connected to a battery through conducting wires. The direct current **starts from the positive terminal of the battery and comes back to the starting point *via* the load. The direct current follows the path *ABCDA* and *ABCDA* is a d.c. circuit. The load for a d.c. circuit is usually a *** resistance. In a d.c. circuit, loads (*i.e.* resistances) may be connected in series or parallel or series-parallel. Accordingly, d.c. circuits can be classified as :

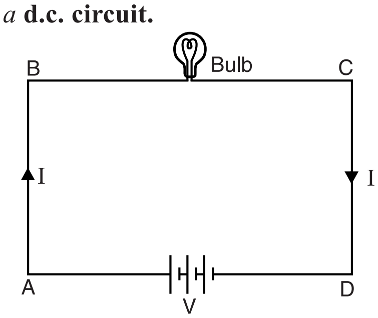

Fig. 2.1

(*i*) Series circuits

(*ii*) Parallel circuits

(*iii*) Series-parallel circuits.

2.2. D.C. Series Circuit

The d.c. circuit in which resistances are connected end to end so that there is only one path for current to flow is called a **d.c. series circuit.**

Consider three resistances R_1, R_2 and R_3 ohms connected in series across a battery of V volts as shown in Fig. 2.2 (*i*). Obviously, there is only one path for current I *i.e.* current is same throughout the circuit. By Ohm's law, voltage across the various resistances is

$$V_1 = I R_1 \; ; \; V_2 = I R_2 \; ; \; V_3 = I R_3$$

Now $$V = V_1 + V_2 + V_3$$
$$= I R_1 + I R_2 + I R_3$$

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* The device which utilises electrical energy is called load. For instance, heater converts electrical energy supplied to it into heat. Therefore, heater is the load.

** This is the direction of conventional current. However, the electron flow will be in the opposite direction.

*** Other passive elements *viz.* inductance and capacitance are relevant only in a.c. circuits.

$$= I(R_1 + R_2 + R_3)$$

or $$\frac{V}{I} = R_1 + R_2 + R_3$$

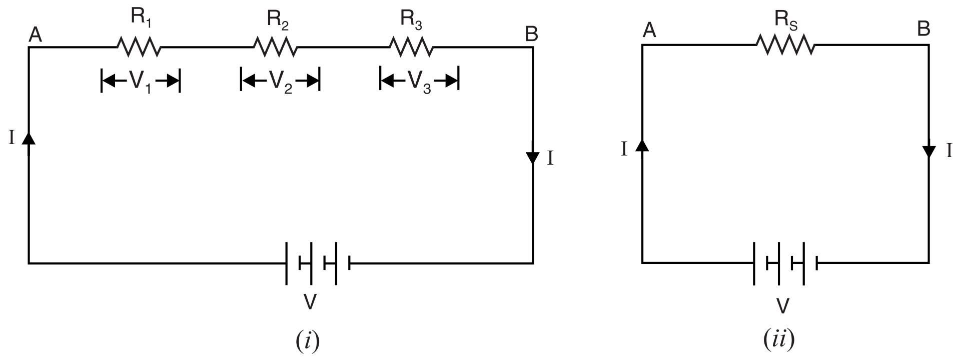


Fig. 2.2

But V/I is the total resistance R_S between points A and B. Note that R_S is called the *total or equivalent resistance of the three resistances.

$\therefore$ $$R_S = R_1 + R_2 + R_3$$

Hence when a number of resistances are connected in series, the total resistance is equal to the sum of the individual resistances.

The total conductance G_S of the circuit is given by ;

$$G_S = \frac{1}{R_S} = \frac{1}{R_1 + R_2 + R_3}$$

Also $$\frac{1}{G_S} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

The main characteristics of a series circuit are :

(i) The current in each resistor is the same.

(ii) The total resistance in the circuit is equal to the sum of individual resistances.

(iii) The total power dissipated in the circuit is equal to the sum of powers dissipated in individual resistances. Thus referring to Fig. 2.2 (*i*),

$$R_S = R_1 + R_2 + R_3$$

or $$I^2R_S = I^2R_1 + I^2R_2 + I^2R_3$$

or $$P_S = P_1 + P_2 + P_3$$

Thus total power dissipated in a series circuit is equal to the sum of powers dissipated in individual resistances. As we shall see, this is also true for parallel and series-parallel d.c. circuits.

Note. A series resistor circuit [See Fig. 2.2 (*i*)] can be considered to be a *voltage divider circuit* because the potential difference across any one resistor is a fraction of the total voltage applied across the series combination; the fraction being determined by the values of the resistances.

Example 2.1. *Two filament lamps A and B take 0.8 A and 0.9 A respectively when connected across 110 V supply. Calculate the value of current when they are connected in series across a 220-V supply, assuming the filament resistances to remain unaltered. Also find the voltage across each lamp.*

* Total or equivalent resistance is the single resistance, which if substituted for the series resistances, would provide the same current in the circuit.

Solution. For lamp A, $R_A = 110/0.8 = 137.5\ \Omega$

For lamp B, $R_B = 110/0.9 = 122.2\ \Omega$

When the lamps are connected in series, total resistance is

$$R_S = 137.5 + 122.2 = 259.7\ \Omega$$

$\therefore$ Circuit current, $I = V/R_S = 220/259.7 =$ **0.847 A**

Voltage across lamp A $= I R_A = 0.847 \times 137.5 =$ **116.5 V**

Voltage across lamp B $= I R_B = 0.847 \times 122.2 =$ **103.5 V**

Example 2.2. *A 100 watt, 250 V lamp is connected in series with a 100 watt, 200 V lamp across 250 V supply. Calculate (i) circuit current and (ii) voltage across each lamp. Assume the lamp resistances to remain unaltered.*

Solution. (*i*) Resistance, $R = \dfrac{V^2}{P}$

Resistance of 100 watt, 250 V lamp, $R_1 = (250)^2/100 = 625\ \Omega$

Resistance of 100 watt, 200 V lamp, $R_2 = (200)^2/100 = 400\ \Omega$

When the lamps are connected in series, total resistance is

$$R_S = 625 + 400 = 1025\ \Omega$$

$\therefore$ Circuit current, $I = V/R_S = 250/1025 =$ **0.244 A**

(*ii*) Voltage across 100 W, 250 V lamp $= I R_1 = 0.244 \times 625 =$ **152.5 V**

Voltage across 100 W, 200 V lamp $= I R_2 = 0.244 \times 400 =$ **97.6 V**

Example 2.3. *The element of 500 watt electric iron is designed for use on a 200 V supply. What value of resistance is needed to be connected in series in order that the iron can be operated from 240 V supply?*

Solution. Current rating of iron, $I = \dfrac{\text{Wattage}}{\text{Voltage}} = \dfrac{500}{200} = 2.5$ A

If R ohms is the required value of resistance to be connected in series, then voltage to be dropped across this resistance = 240 − 200 = 40 V.

$\therefore$ $R = 40 / 2.5 =$ **16 Ω**

Example 2.4. *Determine the resistance and the power dissipation of a resistor that must be placed in series with a 75 - ohm resistor across 120 V source in order to limit the power dissipation in the 75 - ohm resistor to 90 watts.*

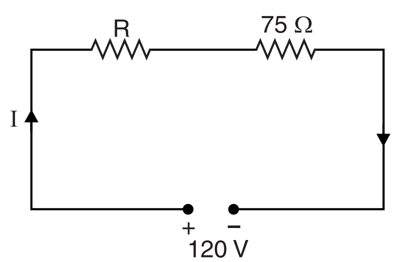


Fig. 2.3

Solution. Fig. 2.3 represents the conditions of the problem.

$$I^2 \times 75 = 90$$

$$\therefore \quad I = \sqrt{90/75} = 1.095 \text{ A}$$

$$\text{Now,} \quad I = \frac{120}{R + 75}$$

$$\text{or} \quad 1.095 = \frac{120}{R + 75}$$

$\therefore$ $R =$ **34.6 Ω**

Power dissipation in $R = I^2 R = (1.095)^2 \times 34.6 =$ **41.5 watts**

Example 2.5. *A generator of e.m.f. E volts and internal resistance r ohms supplies current to a water heater. Calculate the resistance R of the heater so that three-quarter of the total energy developed by the generator is absorbed by the water.*

Solution. Current supplied by generator, $I = \dfrac{E}{R+r}$

Power developed by generator $= E\,I = \dfrac{E^2}{R+r}$

Power dissipated by heater $= I^2R = R \times \dfrac{E^2}{(R+r)^2} = \dfrac{E^2 R}{(R+r)^2}$

As per the conditions of the problem, we have,

$$\frac{E^2 R}{(R+r)^2} = \frac{3}{4}\times\frac{E^2}{R+r} \quad \text{or} \quad \frac{R}{R+r} = \frac{4}{3} \quad \therefore \quad R = \mathbf{3\,r}$$

Example 2.6. *A direct current arc has a voltage/current relation expressed as :*

$$V = 44 + \frac{30}{I} \text{ volts}$$

It is connected in series with a resistor across 100 V supply. If voltages across the arc and resistor are equal, find the ohmic value of the resistor.

Solution. Let R ohms be resistance of the resistor. The voltage across the arc as well as resistor = 50 volts.

Now $\qquad 50 = 44 + \dfrac{30}{I} \qquad \therefore \quad I = 5$ A

$\therefore \qquad R = \dfrac{V}{I} = \dfrac{50}{5} = \mathbf{10\ \Omega}$

Tutorial Problems

1. If the resistance of a circuit having 12 V source is increased by 4 Ω, the current drops by 0.5 A. What is the original resistance of the circuit ? **[8 Ω]**
2. A searchlight takes 100 A at 80 V. It is to be operated from a 220 V supply. Find the value of the resistor to be connected in series. **[1.4 Ω]**
3. The maximum resistance of a rheostat is 4.8 Ω and the minimum resistance is 0.5 Ω. Find for each condition the voltage across the rheostat when current is 1.2 A. **[5.76V ; 0.6V]**
4. What is the drop across the 150 Ω resistor in Fig. 2.4 ? **[5.33 V]**

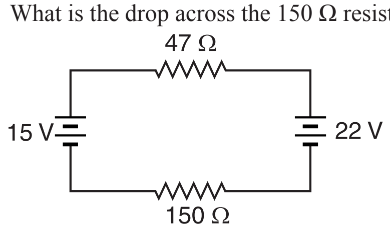


Fig. 2.4

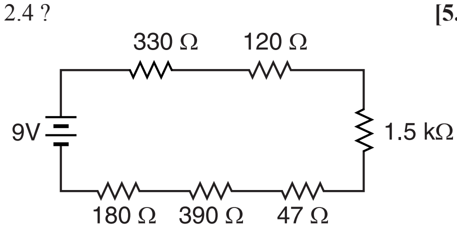


Fig. 2.5

5. Calculate the current flow for Fig. 2.5. **[3.51 mA]**

2.3. D.C. Parallel Circuit

When one end of each resistance is joined to a common point and the other end of each resistance is joined to another common point so that there are as many paths for current flow as the number of resistances, it is called a **parallel circuit.**

Consider three resistances R_1, R_2 and R_3 ohms connected in parallel across a battery of V volts as shown in Fig. 2.6 (*i*). The total current I divides into three parts : I_1 flowing through R_1, I_2 flowing through R_2 and I_3 flowing through R_3. Obviously, the voltage across each resistance is the same (*i.e.* V volts in this case) and there are as many current paths as the number of resistances. By Ohm's law, current through each resistance is

$$I_1 = V/R_1 \; ; \; I_2 = V/R_2 \; ; \; I_3 = V/R_3$$

Now,
$$I = I_1 + I_2 + I_3$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

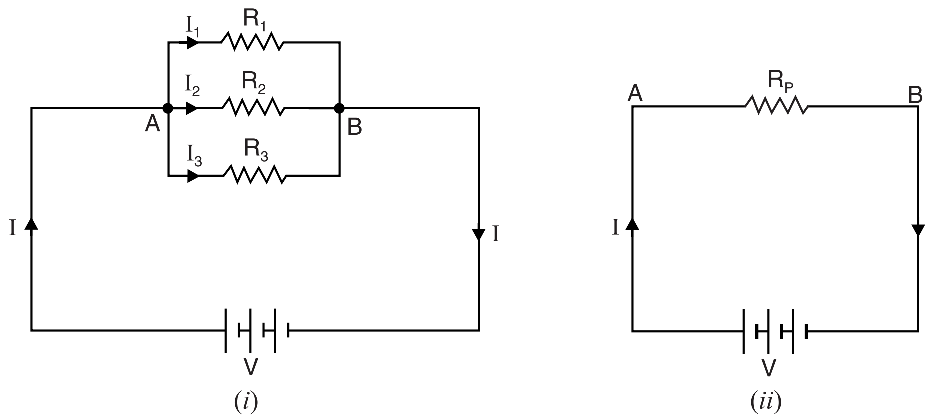


Fig. 2.6

or
$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But V/I is equivalent resistance R_P of the parallel resistances [See Fig. 2.6 (*ii*)] so that $I/V = 1/R_P$.

∴
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence when a number of resistances are connected in parallel, the reciprocal of total resistance is equal to the sum of the reciprocals of the individual resistances.

Also
$$G_P = G_1 + G_2 + G_3$$

Hence total conductance G_P of resistors in parallel is equal to the sum of their individual conductances.

We can also express currents I_1, I_2 and I_3 in terms of conductances.

$$I_1 = \frac{V}{R_1} = VG_1 = \frac{I}{G_P}G_1 = I \times \frac{G_1}{G_P} = I \times \frac{G_1}{G_1 + G_2 + G_3}$$

Similarly,
$$I_2 = I \times \frac{G_2}{G_1 + G_2 + G_3} \; ; \; I_3 = I \times \frac{G_3}{G_1 + G_2 + G_3}$$

2.4. Main Features of Parallel Circuits

The following are the characteristics of a parallel circuit :

(*i*) The voltage across each resistor is the same.

(*ii*) The current through any resistor is inversely proportional to its resistance.

(*iii*) The total current in the circuit is equal to the sum of currents in its parallel branches.

(*iv*) The reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

(*v*) As the number of parallel branches is increased, the total resistance of the circuit is decreased.

(*vi*) The total resistance of the circuit is always less than the smallest of the resistances.

(*vii*) If n resistors, each of resistance R, are connected in parallel, then total resistance $R_P = R/n$.

(*viii*) The conductances are additive.

(*ix*) The total power dissipated in the circuit is equal to the sum of powers dissipated in the individual resistances. Thus referring to Fig. 2.6 (*i*),

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or
$$\frac{V^2}{R_P} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or
$$P_P = P_1 + P_2 + P_3$$

Like a series circuit, *the total power dissipated in a parallel circuit is equal to the sum of powers dissipated in the individual resistances.*

Note. A parallel resistor circuit [See Fig. 2.6 (*i*)] can be considered to be a *current divider circuit* because the current through any one resistor is a fraction of the total circuit current; the fraction depending on the values of the resistors.

2.5. Two Resistances in Parallel

A frequent special case of parallel resistors is a circuit that contains two resistances in parallel. Fig. 2.7 shows two resistances R_1 and R_2 connected in parallel across a battery of V volts. The total current I divides into two parts ; I_1 flowing through R_1 and I_2 flowing through R_2.

(*i*) Total resistance R_P.
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\therefore \quad R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{Product}}{\text{Sum}}$$

Hence the total value of two resistors connected in parallel is equal to the product divided by the sum of the two resistors.

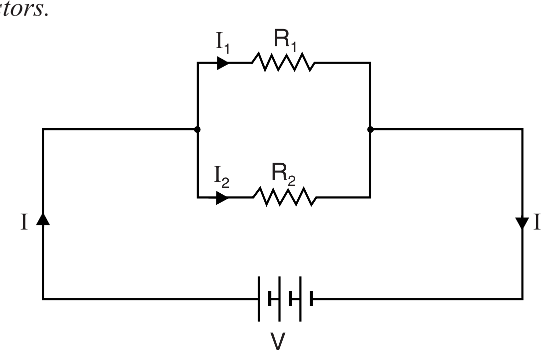


Fig. 2.7

(*ii*) Branch Currents.
$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = I R_P = I \frac{R_1 R_2}{R_1 + R_2}$$

Current through R_1, $$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2} \qquad \left[\text{Putting } V = I \frac{R_1 R_2}{R_1 + R_2}\right]$$

$$\text{Current through } R_2,\ I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$$

Hence in a parallel circuit of two resistors, the current in one resistor is the line current (i.e. total current) times the opposite resistor divided by the sum of the two resistors.

We can also express currents in terms of conductances.

$$G_P = G_1 + G_2$$

$$I_1 = \frac{V}{R_1} = VG_1 = \frac{I}{G_P} \times G_1 = I \times \frac{G_1}{G_P} = I \times \frac{G_1}{G_1 + G_2}$$

$$I_2 = \frac{V}{R_2} = VG_2 = \frac{I}{G_P} \times G_2 = I \times \frac{G_2}{G_P} = I \times \frac{G_2}{G_1 + G_2}$$

Note. When two resistances are connected in parallel and one resistance is much greater than the other, then the total resistance of the combination is very nearly equal to the smaller of the two resistances. For example, if $R_1 = 10\ \Omega$ and $R_2 = 10\ \text{k}\Omega$ and they are connected in parallel, then total resistance R_P of the combination is given by ;

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 10^4}{10 + 10^4} = \frac{10^5}{10{,}010} = 9.99\ \Omega \simeq R_1$$

In general, if R_2 is 10 times (or more) greater than R_1, then their combined resistance in parallel is nearly equal to R_1.

2.6. Advantages of Parallel Circuits

The most useful property of a parallel circuit is the fact that potential difference has the same value between the terminals of each branch of parallel circuit. This feature of the parallel circuit offers the following advantages :

(*i*) The appliances rated for the same voltage but different powers can be connected in parallel without disturbing each other's performance. Thus a 230 V, 230 W TV receiver can be operated independently in parallel with a 230 V, 40 W lamp.

(*ii*) If a break occurs in any one of the branch circuits, it will have no effect on other branch circuits.

Due to above advantages, electrical appliances in homes are connected in parallel. We can switch on or off any light or appliance without affecting other lights or appliances.

2.7. Applications of Parallel Circuits

Parallel circuits find many applications in electrical and electronic circuits. We shall give two applications by way of illustration.

(*i*) Identical voltage sources may be connected in parallel to provide a greater current capacity. Fig. 2.8 shows two 12 V automobile storage batteries in parallel. If the starter motor draws 400 A at starting, then each battery will supply half the current *i.e.* 200 A. A single battery might not be able to provide a load current of 400 A. Another benefit is that two batteries in parallel will supply a given load current for twice the time when compared to a single battery before discharge is reached.

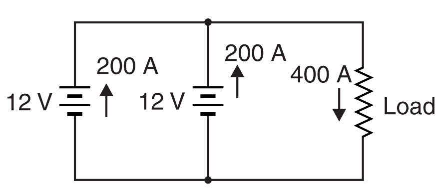


Fig. 2.8

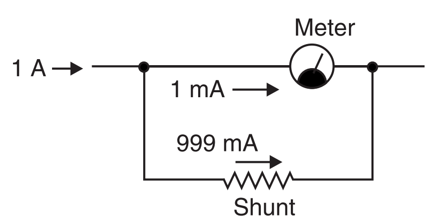


Fig. 2.9

(*ii*) Fig. 2.9 shows another application for parallel connection. A low resistor, called a *shunt*, is connected in parallel with an ammeter to increase the current range of the meter. If shunt is not used, the ammeter is able to measure currents up to 1 mA. However, the use of shunt permits to measure currents up to 1 A. Thus shunt increases the range of the ammeter.

Example 2.7. *Two coils connected in series have a resistance of 18 Ω and when connected in parallel have a resistance of 4 Ω. Find the value of resistances.*

Solution. Let R_1 and R_2 be the resistances of the coils. When resistances are connected in series, $R_S = 18\ \Omega$.

$$\therefore \quad R_1 + R_2 = 18 \qquad \text{...}(i)$$

When resistances are connected in parallel, $R_P = 4\ \Omega$.

$$\therefore \quad 4 = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{....}(ii)$$

Multiplying Eqns. (*i*) and (*ii*), we get, $R_1 R_2 = 18 \times 4 = 72$

$$\text{Now} \quad R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4R_1R_2} = \sqrt{(18)^2 - 4 \times 72}$$

$$\therefore \quad R_1 - R_2 = \pm 6 \qquad \text{...}(iii)$$

Solving Eqns. (*i*) and (*iii*), we get, $R_1 =$ **12 Ω** or **6 Ω** ; $R_2 =$ **6 Ω** or **12 Ω**

Example 2.8. *A 100 watt, 250 V lamp is connected in parallel with an unknown resistance R across a 250 V supply. The total power dissipated in the circuit is 1100 watts. Find the value of unknown resistance. Assume the resistance of lamp remains unaltered.*

Solution. The total power dissipated in the circuit is equal to the sum of the powers consumed by the lamp and unknown resistance R.

$$\therefore \quad \text{Power consumed by } R = 1100 - 100 = 1000 \text{ watts}$$

$$\therefore \quad \text{Value of resistance, } R = \frac{V^2}{\text{Power consumed}} = \frac{(250)^2}{1000} = \mathbf{62.5\ \Omega}$$

Example 2.9. *A coil has a resistance of 5.2 ohms; the resistance has to be reduced to 5 Ω by connecting a shunt across the coil. If this shunt is made of manganin wire of diameter 0.025 cm, find the length of wire required. Specific resistance for manganin is 47 × 10^{-8} Ω m.*

Solution. Let R ohms be the required resistance of the shunt.

$$R_P = \frac{R \times 5.2}{R + 5.2} \quad \text{or} \quad 5 = \frac{5.2R}{R + 5.2} \qquad \therefore \quad R = 130\ \Omega$$

$$a = \frac{\pi}{4}\left(0.025 \times 10^{-2}\right)^2 = 490 \times 10^{-10} \text{ m}^2 \text{ ; } \rho = 47 \times 10^{-8}\ \Omega\text{-}m$$

$$\text{Now} \quad R = \rho \frac{l}{a}$$

$$\therefore \quad l = \frac{Ra}{\rho} = \frac{130 \times (490 \times 10^{-10})}{47 \times 10^{-8}} = \mathbf{13.55\ m}$$

Example 2.10. *Three equal resistors are connected as shown in Fig 2.10. Find the equivalent resistance between points A and B.*

Solution. The reader may observe that one end of each resistor is connected to point A and the other end of each resistor is connected to point B. Hence the three resistors are in parallel.

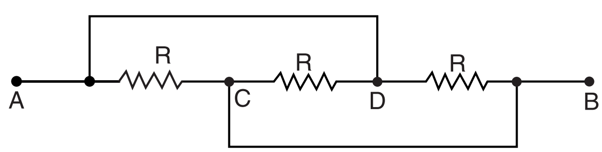


Fig. 2.10

$$\therefore \quad \frac{1}{R_{AB}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \quad \text{or} \quad R_{AB} = \frac{\boldsymbol{R}}{\mathbf{3}}$$

Example 2.11. *Find the branch currents for Fig. 2.11 using the current divider rule for parallel conductances.*

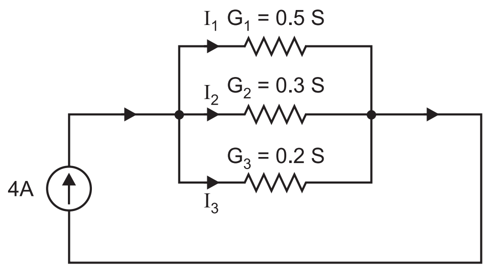


Fig. 2.11

Solution.

$$G_P = G_1 + G_2 + G_3 = 0.5 + 0.3 + 0.2 = 1 \text{ S}$$

$$\therefore \quad I_1 = I\frac{G_1}{G_P} = 4 \times \frac{0.5}{1} = \mathbf{2\ A}$$

$$I_2 = I\frac{G_2}{G_P} = 4 \times \frac{0.3}{1} = \mathbf{1.2\ A}$$

$$I_3 = I\frac{G_3}{G_P} = 4 \times \frac{0.2}{1} = \mathbf{0.8\ A}$$

Example 2.12. *Find the three branch currents in the circuit shown in Fig. 2.12. What is the potential difference between points A and B?*

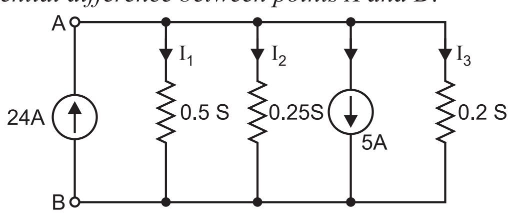


Fig. 2.12

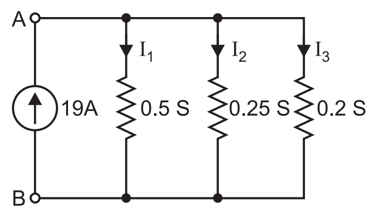


Fig. 2.13

Solution. Current sources in parallel add *algebraically*. Therefore, the two current sources can be combined to give the resultant current source of current $I = 24 - 5 = 19$ A as shown in Fig. 2.13. Referring to Fig. 2.13,

$$G_P = G_1 + G_2 + G_3 = 0.5 + 0.25 + 0.2 = 0.95 \text{ S}$$

$$\therefore \quad I_1 = I \times \frac{G_1}{G_P} = 19 \times \frac{0.5}{0.95} = \mathbf{10\ A}$$

$$I_2 = I \times \frac{G_2}{G_P} = 19 \times \frac{0.25}{0.95} = \mathbf{5\ A}$$

$$I_3 = I \times \frac{G_3}{G_P} = 19 \times \frac{0.2}{0.95} = \mathbf{4\ A}$$

The voltage across each conductance is the same.

$$\therefore \quad V_{AB} = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3}$$

or

$$V_{AB} = \frac{I_1}{G_1} = \frac{10 \text{ A}}{0.5 \text{ S}} = \mathbf{20\ V}$$

Example 2.13. *A current of 90 A is shared by three resistances in parallel. The wires are of the same material and have their lengths in the ratio 2 : 3 : 4 and their cross-sectional areas in the ratio 1 : 2 : 3. Determine current in each resistance.*

Solution. Conductance, $G = \sigma \frac{a}{l}$ so that $G \propto \frac{a}{l}$ $(\because \sigma$ is same)

$$\therefore \quad G_1 : G_2 : G_3 :: \frac{a_1}{l_1} : \frac{a_2}{l_2} : \frac{a_3}{l_3}$$

or $$G_1 : G_2 : G_3 :: \frac{1}{2} : \frac{2}{3} : \frac{3}{4}$$

or $$G_1 : G_2 : G_3 :: 6 : 8 : 9$$

$$\therefore \quad I_1 = I \times \frac{G_1}{G_1 + G_2 + G_3} = 90 \times \frac{6}{6+8+9} = \textbf{23.48 A}$$

$$I_2 = I \times \frac{G_2}{G_1 + G_2 + G_3} = 90 \times \frac{8}{6+8+9} = \textbf{31.30 A}$$

$$I_3 = I \times \frac{G_3}{G_1 + G_2 + G_3} = 90 \times \frac{9}{6+8+9} = \textbf{35.22 A}$$

Example 2.14. *An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that current in the aluminium wire is 3 A. The diameter of aluminium wire is 1 mm. Determine the diameter of copper wire. Resistivity of copper is 0.017 μΩ m and that of aluminium is 0.028 μΩ m.*

Solution. Assign suffix A to aluminium and C to copper. Then,

$$I_A = 3 \text{ A} \quad \text{and} \quad I_C = 5 - I_A = 5 - 3 = 2 \text{ A}$$

In a parallel circuit, the current in any branch is directly proportional to conductance (G) of that branch ($\because I = VG$).

$$\therefore \quad I_A \propto G_A \quad \text{and} \quad I_C \propto G_C$$

$$\therefore \quad \frac{G_C}{G_A} = \frac{I_C}{I_A} = \frac{2}{3}$$

Now, $$G_C = \frac{a_C}{\rho_C l_C} \quad \text{and} \quad G_A = \frac{a_A}{\rho_A l_A}$$

$$\therefore \quad \frac{G_C}{G_A} = \frac{a_C}{\rho_C l_C} \times \frac{\rho_A l_A}{a_A}$$

or $$\frac{2}{3} = \frac{a_C}{0.017 \times 6} \times \frac{0.028 \times 7.5}{a_A}$$

or $$\frac{a_C}{a_A} = \frac{2}{3} \times \frac{0.017 \times 6}{0.028 \times 7.5} = 0.3238$$

$$\therefore \quad a_C = 0.3238 \times a_A = 0.3238 \times \frac{\pi}{4}(1 \text{ mm})^2$$

or $$\frac{\pi}{4}(d_C)^2 = 0.3238 \times \frac{\pi}{4}$$

$$\therefore \quad d_C = \sqrt{0.3238} = \textbf{0.57 mm}$$

Example 2.15. *A voltage of 200 V is applied to a tapped resistor of 500 Ω. Find the resistance between two tapping points connected to a circuit needing 0.1 A at 25 V. Calculate the total power consumed.*

Solution. Fig. 2.14 shows the conditions of the problem.

$$\text{Current in } AB = 0.1 + \frac{25}{R}$$

$$\text{Also current in } AB = \frac{200 - 25}{500 - R} = \frac{175}{500 - R}$$

$$\therefore \qquad 0.1 + \frac{25}{R} = \frac{175}{500 - R}$$

$$\text{or} \qquad \frac{0.1R + 25}{R} = \frac{175}{500 - R}$$

$$\text{or} \qquad (500 - R)(0.1R + 25) = 175\,R$$

$$\text{or} \qquad 0.1\,R^2 + 150\,R - 12500 = 0$$

On solving and taking the positive value, R = **79 Ω**.

$$\text{Total current, } I = \text{Current in } AB$$

$$= 0.1 + \frac{25}{79} = 0.4165 \text{ A}$$

$$\therefore \qquad \text{Total power} = 200 \times I = 200 \times 0.4165 = \mathbf{83.5\ W}$$

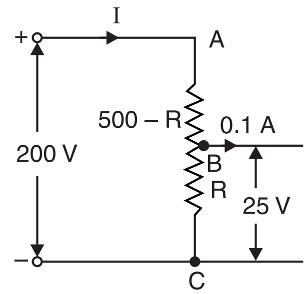


Fig. 2.14

Example 2.16. *A heater has two similar elements controlled by a 3-heat switch. Draw a connection diagram of each position of the switch. What is the ratio of heat developed for each position of the switch?*

Solution. Fig. 2.15 shows the connections of 3-heat switch controlling two similar elements. Suppose the supply voltage is V.

With points 1 and 3 linked and supply connected across 1 and 3, the two elements will be in parallel.

$$\therefore \qquad \text{Power dissipated, } P_1 = \frac{V^2}{R/2} = \frac{2V^2}{R}$$

With voltage across 1 and 2 or 2 and 3, only one element is in the circuit.

$$\therefore \qquad \text{Power dissipated, } P_2 = \frac{V^2}{R}$$

With voltage across 1 and 3, the two elements are in series.

$$\therefore \qquad \text{Power dissipated, } P_3 = \frac{V^2}{2R}$$

$$\therefore \qquad P_1 : P_2 : P_3 = \frac{2V^2}{R} : \frac{V^2}{R} : \frac{V^2}{2R} = 2 : 1 : \frac{1}{2} = \mathbf{4 : 2 : 1}$$

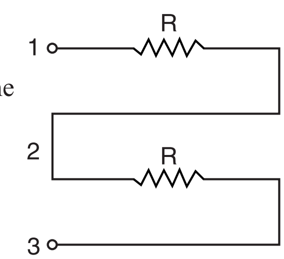


Fig. 2.15

Example 2.17. *The frame of an electric motor is connected to three earthing plates having resistance to earth of 10 Ω, 20 Ω and 30 Ω respectively. Due to a fault, the frame becomes live. What proportion of total fault energy is dissipated at each earth connection ?*

Solution. The three resistances are in parallel. During the fault, suppose voltage to ground is V. Then ratios of energy dissipated are :

$$\frac{V^2}{10} : \frac{V^2}{20} : \frac{V^2}{30} = \frac{1}{10} : \frac{1}{20} : \frac{1}{30} = 6 : 3 : 2$$

$$\text{\% of fault energy dissipated in } 10\ \Omega = \frac{6}{6+3+2} \times 100 = \mathbf{54.5\%}$$

$$\text{\% of fault energy dissipated in } 20\ \Omega = \frac{3}{6+3+2} \times 100 = \mathbf{27.3\%}$$

$$\text{\% of fault energy dissipated in } 30\ \Omega = \frac{2}{6+3+2} \times 100 = \mathbf{18.2\%}$$

Example 2.18. *A 50 Ω resistor is in parallel with 100 Ω resistor. Current in 50 Ω resistor is 7.2 A. How will you add a third resistor and what will be its value if the line current is to be 12.1 A?*

Solution. $\qquad$ Source voltage = 50 × 7.2 = 360 V

$$\therefore \qquad \text{Current in } 100\ \Omega \text{ resistor} = \frac{360}{100} = 3.6 \text{ A}$$

Total current drawn by 50 Ω and 100 Ω resistors = 7.2 + 3.6 = 10.8 A

In order that line current is 12.1 A, **some resistance *R* must be added in parallel**. The current in *R* is to be = 12.1 – 10.8 = 1.3 A.

∴ Value of $R = \dfrac{360}{1.3} = \mathbf{277\ \Omega}$

Tutorial Problems

1. Two resistors of 4 Ω and 6 Ω are connected in parallel. If the total current is 30 A, find the current through each resistor. **[18 A ; 12 A]**
2. Four resistors are in parallel. The currents in the first three resistors are 4 mA, 5 mA and 6 mA respectively. The voltage drop across the fourth resistor is 200 volts. The total power dissipated is 5 watts. Determine the values of the resistances of the branches and the total resistance. **[50 k Ω, 40 k Ω, 33.33 k Ω, 8 k Ω, 5 k Ω]**
3. Four resistors of 2 Ω, 3 Ω, 4 Ω and 5 Ω respectively are connected in parallel. What potential difference must be applied to the group in order that total power of 100 watts may be absorbed ? **[8.826 volts]**
4. Three resistors are in parallel. The current in the first resistor is 0.1 A. The power dissipated in the second is 3 watts. The voltage drop across the third is 100 volts. Determine the ohmic values of resistors and the total resistance if total current is 0.2 A. **[1000 Ω, 3333.3 Ω, 1428.5 Ω, 500 Ω]**
5. Two coils each of 250 Ω resistance are connected in series across a constant voltage mains. Calculate the value of resistance to be connected in parallel with one of the coils to reduce the p.d. across its terminals by 1%. **[12,375 Ω]**
6. When a resistor is placed across a 230 volt supply, the current is 12 A. What is the value of resistor that must be placed in parallel to increase the load to16 A ? **[57.5 Ω]**
7. A 50-ohm resistor is in parallel with a 100-ohm resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistance to be added in parallel to make the line current 12.1 A ? **[276.9 Ω]**
8. Five equal resistors each of 2 Ω are connected in a network as shown in Fig. 2.16. Find the equivalent resistance between points *A* and *B*. **[2 Ω]**

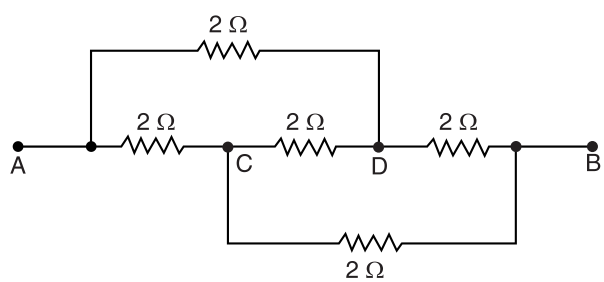


Fig. 2.16

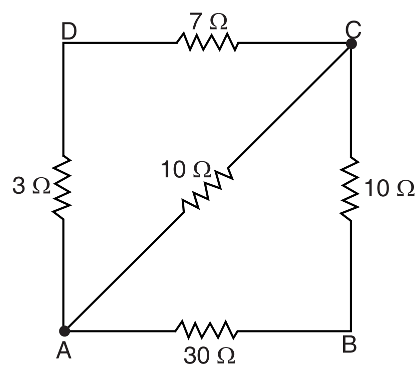


Fig. 2.17

9. Find the equivalent resistance between points *A* and *B* in the circuit shown in Fig. 2.17. **[10 Ω]**
10. Fig. 2.18 shows a 50 V source connected to three resistances : R_1 = 5 kΩ; R_2 = 25 kΩ and R_3 = 10 kΩ. Calculate (*i*) branch currents (*ii*) total current for the given figure. **[(*i*) I_1 = 10 mA ; I_2 = 2 mA; I_3 = 5 mA (*ii*) I = 17 mA]**
11. A parallel circuit consists of four parallel-connected 480 Ω resistors in parallel with six 360 Ω resistors. What is the total resistance and total conductance of the circuit? **[40 Ω ; 0.025 S]**

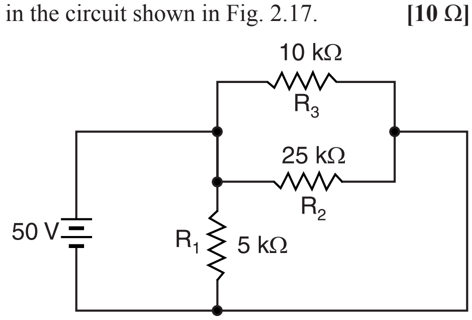


Fig. 2.18

2.8. D.C. Series-Parallel Circuit

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Fig. 2.19. Note that R_2 and R_3 are connected in parallel with each other and that both together are connected in series with R_1. One simple rule to solve such circuits is to first reduce the parallel branches to an equivalent series branch and then solve the circuit as a simple series circuit.

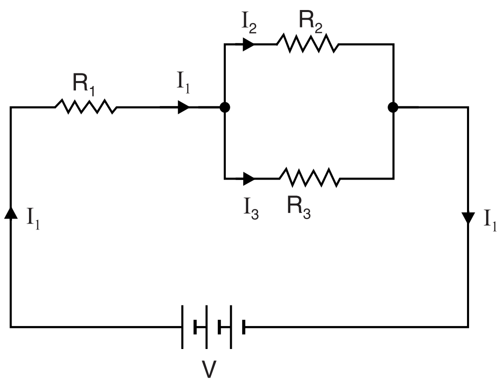


Fig. 2.19

Referring to the series-parallel circuit shown in Fig. 2.19,

$$R_P \text{ for parallel combination } = \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{Total circuit resistance } = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{Voltage across parallel combination } = I_1 \times \frac{R_2 R_3}{R_2 + R_3}$$

The reader can now readily find the values of I_1, I_2, I_3.

Like series and parallel circuits, the total power dissipated in the circuit is equal to the sum of powers dissipated in the individual resistances *i.e.*,

$$\text{Total power dissipated, } P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

2.9. Applications of Series-Parallel Circuits

Series-parallel circuits combine the advantages of both series and parallel circuits and minimise their disadvantages. Generally, less copper is required and a smaller size wire can be used. Such circuits are used whenever various types of circuits must be fed from the same power supply. A few common applications of series-parallel circuits are given below :

(*i*) In an automobile, the starting, lighting and ignition circuits are all individual circuits joined to make a series-parallel circuit drawing its power from one battery.

(*ii*) Radio and television receivers contain a number of separate circuits such as tuning circuits, r.f. amplifiers, oscillator, detector and picture tube circuits. Individually, they may be simple series or parallel circuits. However, when the receiver is considered as a whole, the result is a series-parallel circuit.

(*iii*) Power supplies are connected in series to get a higher voltage and in parallel to get a higher current.

2.10. Internal Resistance of a Supply

All supplies (*e.g.* a cell) must have some internal resistance, however small. This is shown as a series resistor external to the supply. Fig 2.20 shows a cell of *e.m.f.* E volts and internal resistance r. When the cell is delivering no current (*i.e.* on no load), the p.d. across the terminals will be equal to *e.m.f.* E of the cell as shown in Fig. 2.20 (*i*).

When some load resistance R is connected across the terminals of the cell, the current I starts flowing in the circuit. This current causes a voltage drop across internal resistance r of the cell so that terminal voltage V available will be less than E. The relationship between E and V can be easily established [See Fig. 2.20 (*ii*)].

$$I = \frac{E}{R+r}$$

or $\quad IR = E - Ir$

But $\quad IR = V$, the terminal voltage of the cell.

$\therefore \quad V = E - Ir$

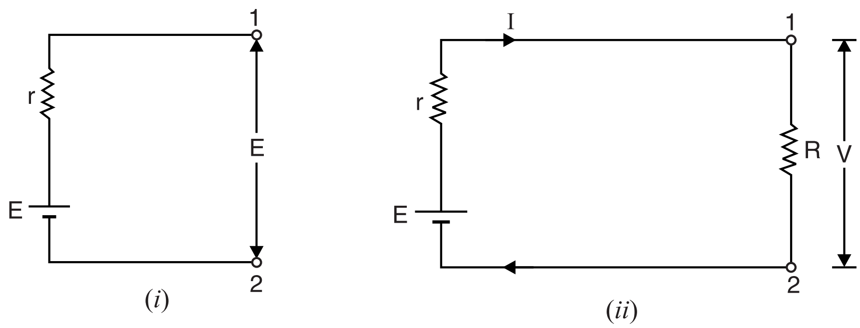


Fig. 2.20

Internal resistance of cell, $r = \dfrac{E-V}{I} = \dfrac{(E-V)}{V}R \qquad \left(\because I = \dfrac{V}{R}\right)$

2.11. Equivalent Resistance

Sometimes we come across a complicated circuit consisting of many resistances. The resistance between the two desired points (or terminals) of such a circuit can be replaced by a single resistance between these points using laws of series and parallel resistances. Then this single resistance is called equivalent resistance of the circuit between these points.

The **equivalent resistance** *of a circuit or network between its any two points* (*or terminals*) *is that single resistance which can replace the entire circuit between these points* (*or terminals*).

Once equivalent resistance is found, we can use Ohm's law to solve the circuit. It is important to note that resistance between two points of a circuit is different for different point-pairs. This is illustrated in Fig. 2.21.

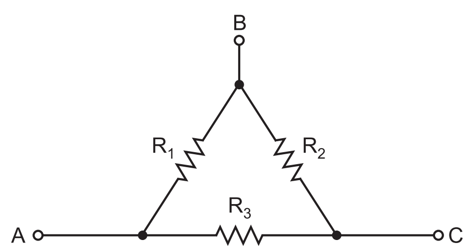


Fig. 2.21

(*i*) Between points A and B, R_1 is in parallel with the series combination of R_2 and R_3 *i.e.*

$$R_{AB} = R_1 \parallel (R_2 + R_3) = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

(*ii*) Between points A and C, R_3 is in parallel with the series combination of R_1 and R_2 *i.e.*

$$R_{AC} = R_3 \parallel (R_1 + R_2) = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

(*iii*) Between points B and C, R_2 is in parallel with the series combination of R_1 and R_3 *i.e.*

$$R_{BC} = R_2 \parallel (R_1 + R_3) = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$$

Example 2.19. *A battery having an e.m.f. of E volts and internal resistance 0.1 Ω is connected across terminals A and B of the circuit shown in Fig. 2.22. Calculate the value of E in order that power dissipated in 2 Ω resistor shall be 2 W.*

Solution. Resistance between E and F is given by ;

$$\frac{1}{R_{EF}} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{6}{6}$$

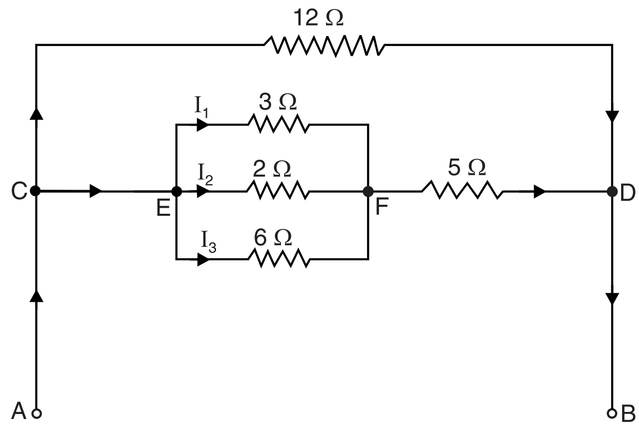


Fig. 2.22

$$\therefore \quad R_{EF} = 6/6 = 1\ \Omega$$

$$\text{Resistance of branch } CEFD = 1 + 5 = 6\ \Omega$$

$$\text{Current through } 2\ \Omega = \sqrt{\frac{\text{Power loss}}{\text{Resistance}}} = \sqrt{\frac{2}{2}} = 1\text{A}$$

$$\text{P.D. across } EF = 1 \times 2 = 2\text{V}$$

$$\text{Current through } 3\ \Omega = 2/3 = 0.67\ \text{A}$$

$$\text{Current through } 6\ \Omega = 2/6 = 0.33\ \text{A}$$

$$\text{Current in branch } CED = 1 + 0.67 + 0.33 = 2\ \text{A}$$

$$\text{P.D. across } CD = 6 \times 2 = 12\ \text{V}$$

$$\text{Current through } 12\ \Omega = 12/12 = 1\ \text{A}$$

$$\text{Current supplied by battery} = 2 + 1 = 3\ \text{A}$$

$$\therefore \quad E = \text{P.D. across } AB \text{ or } CD + \text{Drop in battery resistance}$$

$$= 12 + 0.1 \times 3 = \mathbf{12.3\ V}$$

Example 2.20. *Calculate the values of various currents in the circuit shown in Fig. 2.23. What is total circuit conductance and total resistance?*

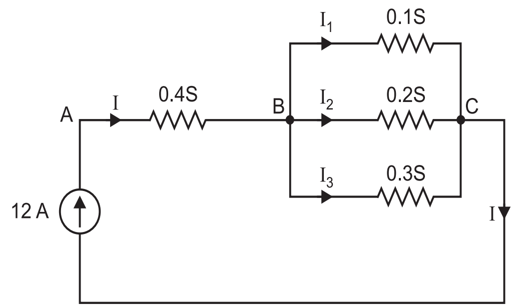


Fig. 2.23

Solution. $I = 12$ A ; $G_{BC} = 0.1 + 0.2 + 0.3 = 0.6$ S

$$\therefore \quad I_1 = I \times \frac{0.1}{G_{BC}} = 12 \times \frac{0.1}{0.6} = \mathbf{2\ A}\ ;\ I_2 = I \times \frac{0.2}{0.6} = 12 \times \frac{0.2}{0.6} = \mathbf{4\ A}\ ;$$

$$I_3 = I \times \frac{0.3}{0.6} = \mathbf{6\ A}\ ;\ I = \mathbf{12\ A}$$

Now, $G_{AB} = 0.4$ S and $G_{BC} = 0.6$ S are in series.

$$\therefore \quad \frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6} \quad \therefore \quad G_{AC} = \mathbf{\frac{6}{25}\ S}$$

$$\text{Total circuit resistance, } R_{AC} = \frac{1}{G_{AC}} = \frac{1}{6/25} = \mathbf{\frac{25}{6}\ \Omega}$$

Example 2.21. *Six resistors are connected as shown in Fig. 2.24. If a battery having an e.m.f. of 24 volts and internal resistance of 1 Ω is connected to the terminals A and B, find (i) the current from the battery, (ii) p.d. across 8 Ω and 4 Ω resistors and (iii) the current taken from the battery if a conductor of negligible resistance is connected in parallel with 8 Ω resistor.*

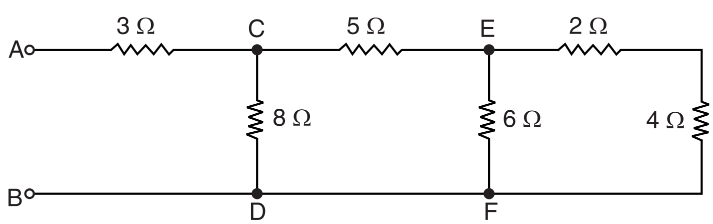


Fig. 2.24

Solution.

$$\text{Resistance between } E \text{ and } F,\ R_{EF} = \frac{(4+2) \times 6}{(4+2)+6} = 3\ \Omega$$

$$\text{Resistance between } C \text{ and } D,\ R_{CD} = \frac{(5+3) \times 8}{(5+3)+8} = 4\ \Omega$$

Resistance between A and B, $R_{AB} = 3 + 4 = 7\ \Omega$

Total circuit resistance, $R_T = R_{AB}$ + Supply resistance $= 7 + 1 = 8\ \Omega$

(i) Current from battery, $I = E/R_T = 24/8 = \mathbf{3\ A}$

(ii) P.D. across 8 Ω $= E - I(3 + 1) = 24 - 3(4) = \mathbf{12\ V}$

Current through 8 Ω $= 12/8 = 1.5$ A

Current through 5 Ω $= 3 - 1.5 = 1.5$A

P.D. across EF $= 12 - 1.5 \times 5 = 4.5$ V

Current through 6Ω $= 4.5/6 = 0.75$A

$\therefore$ Current through 4 Ω = 1.5 − 0.75 = 0.75 A

$\therefore$ Voltage across 4Ω = 0.75 × 4 = **3 V**

(iii) When a conductor of negligible resistance is connected across 8 Ω, then resistance between *C* and *D* is zero. Therefore, total resistance in the circuit is now 3 Ω resistor in series with 1 Ω internal resistance of battery.

$$\therefore \quad \text{Current from battery} = \frac{24}{3+1} = \mathbf{6\ A}$$

Example 2.22. *Two resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100 V supply. The voltage drops across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50000 Ω. Find the sum of two readings.*

Solution. When voltmeter is connected across resistor R_1 [See Fig. 2.25 (*i*)], it becomes a series-parallel circuit and total circuit resistance decreases.

$$\text{Total circuit resistance} = 4000 + \frac{2500 \times 50000}{2500 + 50000} = 4000 + 2381 = 6381\ \Omega$$

$$\text{Circuit current, } I = \frac{100}{6381}\text{A}$$

$$\text{Voltmeter reading, } V_1 = I \times 2381 = \frac{100}{6381} \times 2381 = 37.3\ \text{V}$$

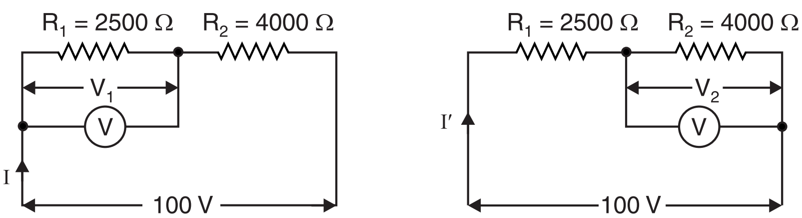


Fig. 2.25

When voltmeter is connected across R_2 [See Fig. 2.25 (*ii*)], it becomes a series-parallel circuit.

$$\text{Total circuit resistance} = 2500 + \frac{4000 \times 50000}{4000 + 50000} = 2500 + 3703.7 = 6203.7\ \Omega$$

$$\text{Circuit current, } I' = \frac{100}{6203.7}\text{A}$$

$$\text{Voltmeter reading, } V_2 = I' \times 3703.7 = \frac{100}{6203.7} \times 3703.7 = 59.7\ \text{V}$$

$$\therefore \quad \text{Sum of two readings} = V_1 + V_2 = 37.3 + 59.7 = \mathbf{97\ V}$$

Example 2.23. *A battery of unknown e.m.f. is connected across resistances as shown in Fig. 2.26. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter? What is the e.m.f. of the battery?*

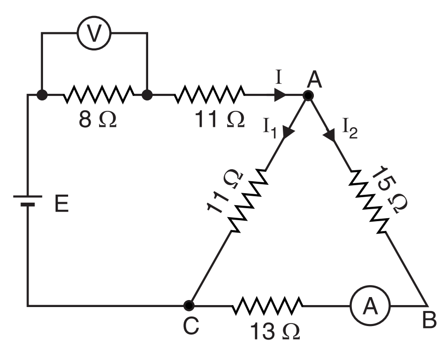


Fig. 2.26

Solution. The current through 8 Ω resistance is I = 20/8 = 2.5 A. At point *A* in Fig. 2.26, the current I is divided into two paths *viz* I_2 flowing in path *ABC* of 15 + 13 = 28 Ω resistance and current I_1 flowing in path *AC* of 11 Ω resistor. By current divider rule, the value of I_2 is given by ;

$$I_2 = I \times \frac{11}{11+28} = 2.5 \times \frac{11}{39} = 0.7 \text{ A}$$

Therefore, ammeter reads **0.7 A**.

Resistance between A and C = $(28 \times 11)/39 = 308/39\ \Omega$

Total circuit resistance, $R_T = 8 + 11 + (308/39) = 1049/39\ \Omega$

$\therefore \quad E = I \times R_T = 2.5 \times (1049/39) =$ **67.3 V**

Example 2.24. *Find the voltage V_{AB} in the circuit shown in Fig. 2.27.*

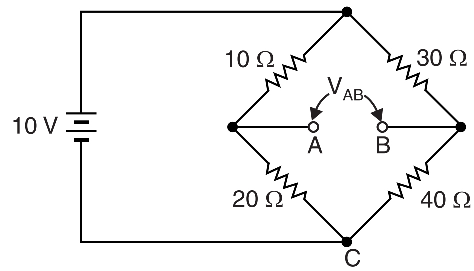


Fig. 2.27

Solution. The resistors 10 Ω and 20 Ω are in series and voltage across this combination is 10 V.

$$\therefore \quad V_{AC} = \frac{20}{10+20} \times 10 = 6.667 \text{ V}$$

The resistors 30 Ω and 40 Ω are in series and voltage across this combination is 10 V.

$$\therefore \quad V_{BC} = \frac{40}{30+40} \times 10 = 5.714 \text{ V}$$

The point A is positive *w.r.t.* point B.

$\therefore \quad V_{AB} = V_{AC} - V_{BC} = 6.667 - 5.714 =$ **0.953 V**

Example 2.25. *A circuit consists of four 100 W lamps connected in parallel across a 230 V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter ?*

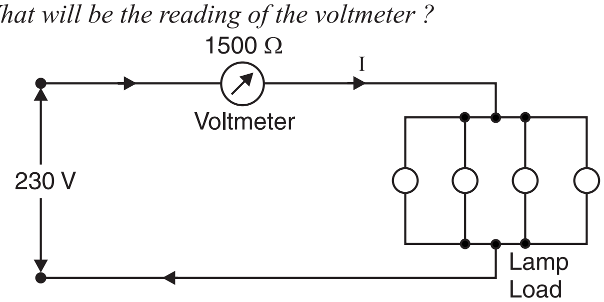


Fig. 2.28

Solution. Fig. 2.28 shows the conditions of the problem. When burning normally, the resistance of each lamp is $R = V^2/P = (230)^2/100 = 529\ \Omega$. Under the conditions shown in Fig. 2.28, resistance of each lamp = $6 \times 529 = 3174\ \Omega$.

$\therefore$ Equivalent resistance of 4 lamps under stated conditions is $R_P = 3174/4 = 793\ \Omega$

Total circuit resistance $= 1500 + R_P$

$= 1500 + 793.5 = 2293.5\ \Omega$

$\therefore$ Circuit current, $I = \dfrac{230}{2293.5}$ A

$\therefore$ Voltage drop across voltmeter $= I \times 1500 = \dfrac{230}{2293.5} \times 1500 \simeq$ **150 V**

Example 2.26. *Find the current supplied by the d.c. source in the circuit shown in Fig. 2.29.*

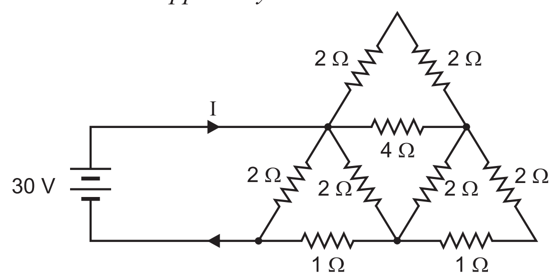


Fig. 2.29

Solution. In the circuit shown in Fig. 2.29, the resistances in series can be combined and the circuit reduces to the one shown in Fig. 2.30 (*i*). In Fig. 2.30 (*i*), the resistances in parallel can be combined using the formula product divided by sum and the circuit reduces to the one shown in Fig. 2.30 (*ii*).

In Fig. 2.30 (*ii*), the resistances in series can be combined and the circuit reduces to the one shown in Fig. 2.30 (*iii*). In Fig. 2.30 (*iii*), 3.2 Ω and 2 Ω are in parallel and their combined resistance is 16/13 Ω. Now 16/13 Ω and 1 Ω are in series and this series combination is in parallel with 2 Ω.

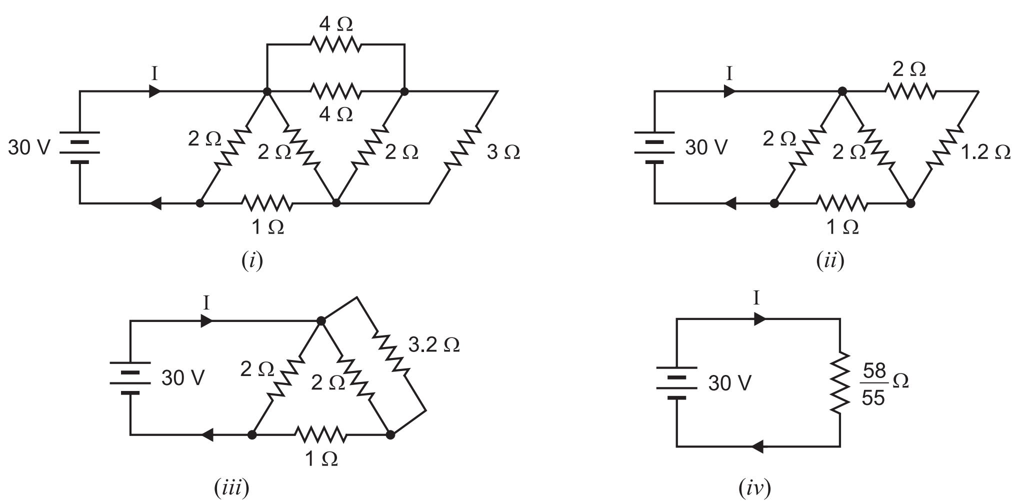


Fig. 2.30

$\therefore$ Effective resistance of the circuit is

$$R_{eff} = \left(\frac{16}{13}+1\right)\Omega \,||\, 2\,\Omega = \frac{58}{55}\,\Omega \qquad \text{[See Fig. 2.30 (}iv\text{)]}$$

$\therefore$ Current supplied by source $= \dfrac{30}{R_{eff}} = \dfrac{30}{58/55} =$ **28.45 A**

Example 2.27. *Determine the current drawn by a 12 V battery with internal resistance 0.5 Ω by the following infinite network (See Fig. 2.31).*

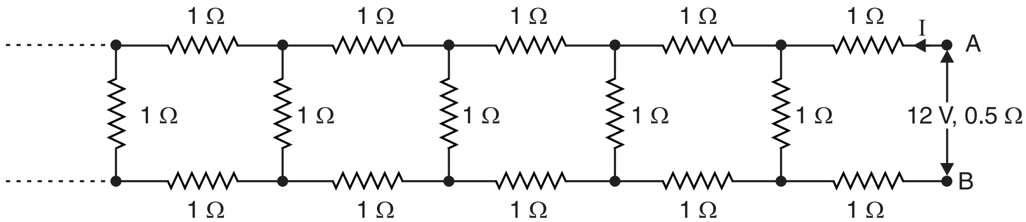


Fig. 2.31

Solution. Let x be the equivalent resistance of the network. Since the network is infinite, the addition of one set of three resistances, each of 1 **Ω**, will not change the total resistance, *i.e.*, it will remain x. The network would then become as shown in Fig. 2.32. The resistances x and 1 **Ω** are in parallel and their total resistance is R_P given by ;

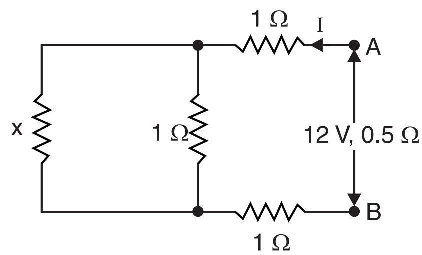


Fig. 2.32

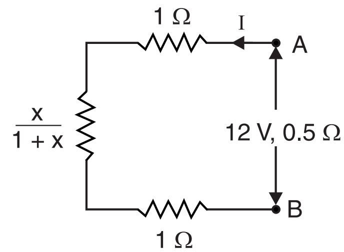


Fig. 2.33

$$R_P = \frac{x \times 1}{x+1} = \frac{x}{1+x}$$

The circuit then reduces to the one shown in Fig. 2.33. Referring to Fig. 2.33,

Total resistance of the network $= 1 + 1 + \frac{x}{1+x} = 2 + \frac{x}{1+x}$

But total resistance of the network is x as mentioned above.

$$\therefore \quad x = 2 + \frac{x}{1+x}$$

or $$x + x^2 = 2 + 2x + x$$

or $$x^2 - 2x - 2 = 0$$

$$\therefore \quad x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

or $$x = 1 \pm \sqrt{3}$$

As the value of the resistance cannot be negative,

$$\therefore \quad x = 1 + \sqrt{3} = 1 + 1.732 = 2.732\ \Omega$$

Total circuit resistance, $R_T = x$ + internal resistance of the supply

$= 2.732 + 0.5 = 3.232\ \Omega$

$\therefore$ Current drawn by the network is

$$I = \frac{E}{R_T} = \frac{12}{3.232} = \mathbf{3.71\ A}$$

Example 2.28. *Find R_{AB} in the circuit shown in Fig. 2.34.*

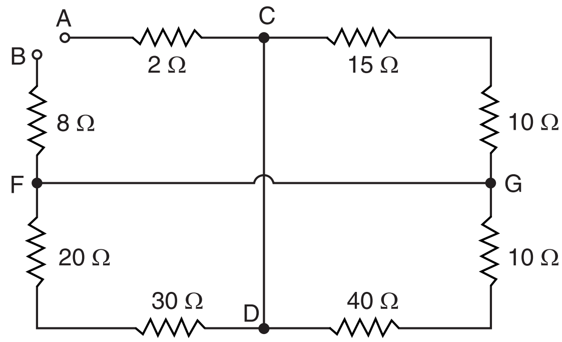
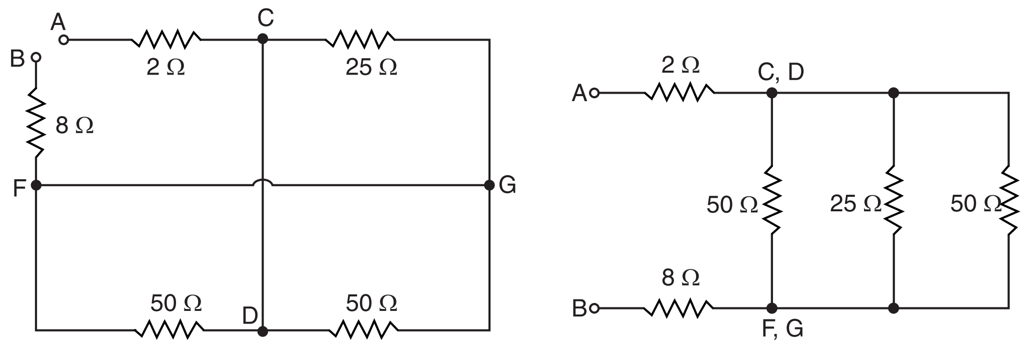


Fig. 2.34

Solution. The circuit shown in Fig. 2.34 reduces to the one shown in Fig. 2.35 (*i*). This circuit further reduces to the circuit shown in Fig. 2.35 (*ii*).



(*i*) (*ii*)

Fig. 2.35

Referring to Fig. 2.35 (*ii*), we have,

$$R_{AB} = 2 + (50 \parallel 25 \parallel 50) + 8$$
$$= 2 + (25 \parallel 25) + 8$$
$$= 2 + 12.5 + 8 = \mathbf{22.5\ \Omega}$$

Example 2.29. *What is the equivalent resistance between the terminals A and B in Fig. 2.36?*

Solution. The network shown in Fig. 2.36 can be redrawn as shown in Fig. 2.37 (*i*). It is a balanced Wheatstone bridge. Therefore, points *C* and *D* are at the same potential. Since no current flows in the branch *CD*, this branch is ineffective in determining the equivalent resistance between terminals *A* and *B* and can be removed. The circuit then reduces to that shown in Fig. 2.37 (*ii*).

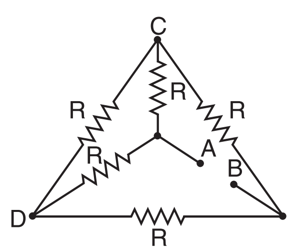
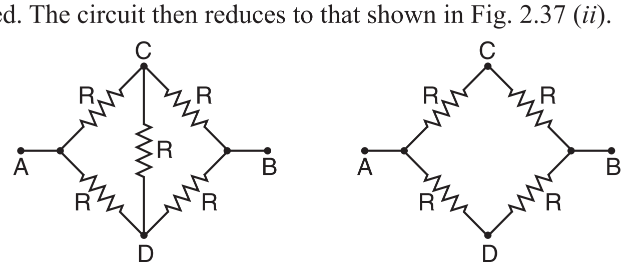


Fig. 2.36



(*i*) (*ii*)

Fig. 2.37

The branch *ACB* (= $R + R = 2R$) is in parallel with branch *ADB* (= $R + R = 2R$).

$$\therefore \quad R_{AB} = \frac{(2R)\times(2R)}{2R+2R} = \mathbf{R}$$

Example 2.30. *An electrical network is arranged as shown in Fig. 2.38. Find the value of current in the branch AF.*

Solution. Resistance between E and C,

$$R_{EC} = \frac{(5+9)\times 14}{(5+9)+14} = 7\ \Omega$$

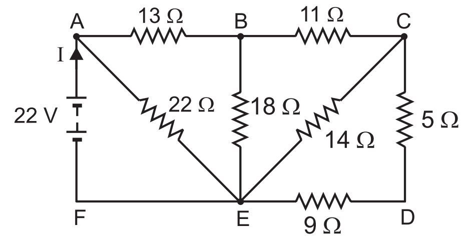


Fig. 2.38

Resistance between B and E,

$$R_{BE} = \frac{(11+7)\times 18}{(11+7)+18} = 9\ \Omega$$

Resistance between A and E,

$$R_{AE} = \frac{(13+9)\times 22}{(13+9)+22} = 11\ \Omega$$

i.e.,Total circuit resistance, $R_T = 11\ \Omega$

$\therefore$ Current in branch AF, $I = V/R_T = 22/11 =$ **2 A**

Example 2.31. *A resistor of 5 Ω is connected in series with a parallel combination of a number of resistors each of 5 Ω. If the total resistance of the combination is 6 Ω, how many resistors are in parallel?*

Solution. Let n be the required number of 5 Ω resistors to be connected in parallel. The equivalent resistance of this parallel combination is

$$\frac{1}{R_P} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5}\ldots n \text{ times} = \frac{n}{5}$$

Therefore, $R_P = 5/n$

Now R_P (= $5/n$) in series with 5 Ω is equal to 6 Ω *i.e.*,

$$\frac{5}{n}+5 = 6 \quad \therefore \quad n = \mathbf{5}$$

Example 2.32. *A letter A consists of a uniform wire of resistance 1 Ω per cm. The sides of the letter are each 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.*

Solution. Fig. 2.39 shows the conditions of the problem. Point B is the mid-point of AC, point D is the mid-point of EC and $BD = 10$ cm.

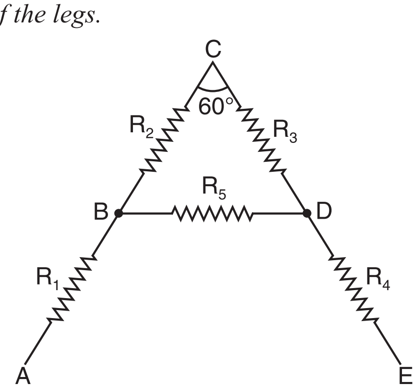


Fig. 2.39

$\therefore$ $AB = BC = CD = DE = BD = 10$ cm

or $R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega (\because 1 \text{ cm} = 1\ \Omega)$

Now R_2 and R_3 are in series and their total resistance $= 10 + 10 = 20\ \Omega$. This $20\ \Omega$ resistance is in parallel with R_5.

$$\therefore \quad R_{BD} = 20\ \Omega \parallel R_5 = 20\ \Omega \parallel 10\ \Omega$$

$$= \frac{20\times 10}{20+10} = \frac{20}{3}\ \Omega$$

Now, R_1, R_{BD} and R_4 are in series so that :

$$R_{AE} = R_1 + R_{BD} + R_4$$

$$= 10 + \frac{20}{3} + 10 = \mathbf{26.67\ \Omega}$$

Example 2.33. *All the resistances in Fig. 2.40 are in ohms. Find the effective resistance between the points A and B.*

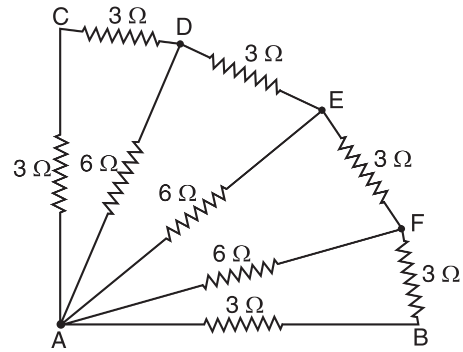


Fig. 2.40

Solution. Resistance between points *A* and *D* is

$$R_{AD} = (3+3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

$$R_{AE} = (R_{AD}+3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

$$R_{AF} = (R_{AE}+3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

∴ Resistance between points *A* and *B* is

$$R_{AB} = (R_{AF}+3)\ \Omega \parallel 3\ \Omega$$

$$= \frac{6\times 3}{6+3} = \frac{18}{9} = \mathbf{2\ \Omega}$$

Example 2.34. *What is the equivalent resistance of the ladder network shown in Fig. 2.41?*

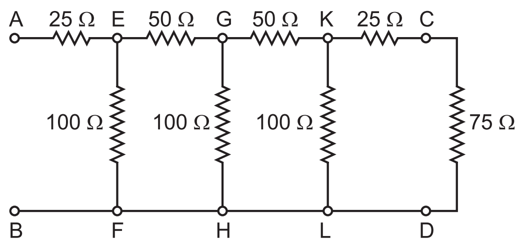


Fig. 2.41

Solution. Referring to Fig. 2.41, the resistance between points *K* and *L* is

$$R_{KL} = (25+75)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

The circuit of Fig. 2.41 then reduces to the one shown in Fig. 2.42 (*i*). Referring to Fig. 2.42 (*i*),

$$R_{GH} = (50+50)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

The circuit of Fig. 2.42 (*i*) then reduces to the one shown in Fig. 2.42 (*ii*). Referring to Fig. 2.42 (*ii*),

$$R_{EF} = (50+50)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

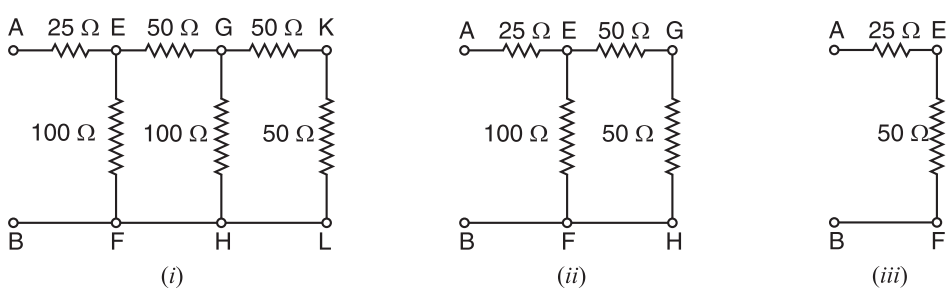


Fig. 2.42

The circuit of Fig. 2.42 (*ii*) then reduces to the one shown in Fig. 2.42 (*iii*). Referring to Fig. 2.42 (*iii*),

Equivalent resistance of the ladder network

$$= 25 + 50 = \mathbf{75\ \Omega}$$

Tutorial Problems

1. A resistor of 3.6 Ω is connected in series with another of 4.56 Ω. What resistance must be placed across 3.6 Ω so that the total resistance of the circuit shall be 6 Ω ? **[2.4 Ω]**
2. A circuit consists of three resistors of 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of e.m.f. 12 V and internal resistance 6 Ω is connected across the circuit. Find the total current in the circuit and terminal voltage across the battery. **[1.059 A, 5.65 V]**
3. A resistance *R* is connected in series with a parallel circuit comprising two resistors of 12 Ω and 8 Ω respectively. The total power dissipated in the circuit is 70 W when the applied voltage is 22 volts. Calculate the value of *R*. **[0.91 Ω]**
4. Two resistors R_1 and R_2 of 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance R_3 and a battery which has an internal resistance of 0.25 Ω. Determine the e.m.f. of the battery. **[12.6 V]**
5. Find the voltage across and current through 4 kΩ resistor in the circuit shown in Fig. 2.43. **[4 V ; 1 mA]**

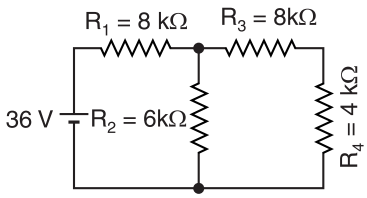


Fig. 2.43

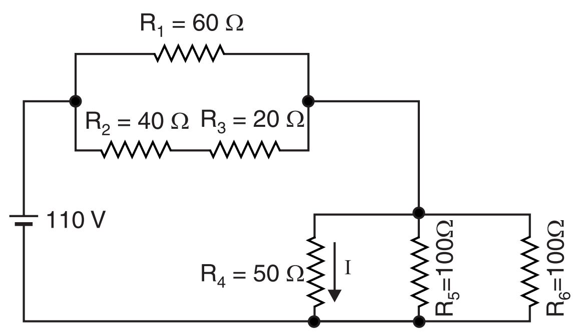


Fig. 2.44

6. Find the current *I* in the 50 Ω resistor in the circuit shown in Fig. 2.44. **[1 A]**
7. Find the current in the 1 kΩ resistor in Fig. 2.45. **[6.72 mA]**

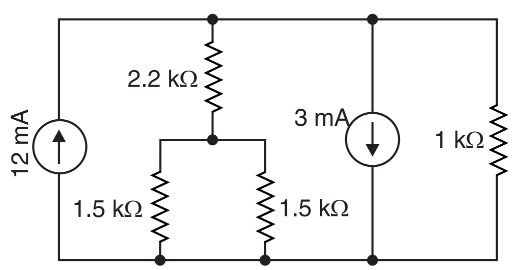


Fig. 2.45

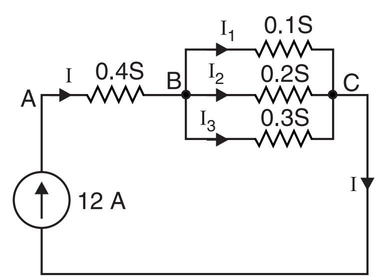


Fig. 2.46

8. Calculate the value of different currents for the circuit shown in Fig. 2.46. What is the total circuit conductance and resistance ? **[$I = 12$ A ; $I_1 = 2$ A ; $I_2 = 4$ A ; $I_3 = 6$ A ; $G_{AC} = 6/25$ S ; $R_{AC} = 25/6$ Ω]**
9. For the parallel circuit of Fig. 2.47, calculate (*i*) *V* (*ii*) I_1 (*iii*) I_2. **[(*i*) 20 V (*ii*) 5 A (*iii*) –5 A]**

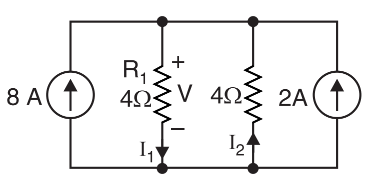


Fig. 2.47

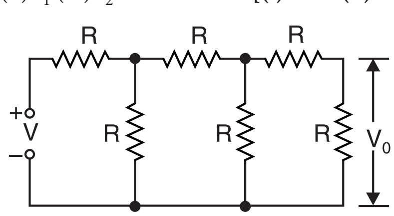


Fig. 2.48

10. Prove that output voltage V_0 in the circuit of Fig. 2.48 is $V/13$.

11. Find the current I supplied by the 50 V source in Fig. 2.49. **[I = 13.7 A]**

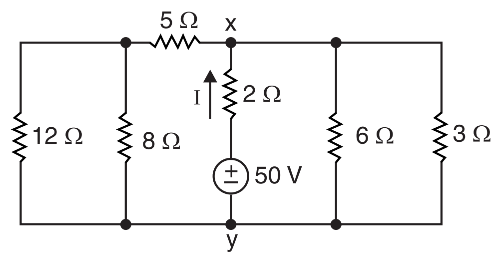


Fig. 2.49

12. An electric heating pad rated at 110 V and 55 W is to be used at a 220 V source. It is proposed to connect the heating pad in series with a series-parallel combination of light bulbs, each rated at 100 V ; bulbs are having ratings of 25 W, 60 W, 75 W and 100 W. Obtain a possible scheme of the pad-bulbs combination. At what rate will heat be produced by the pad with this modification ?

[100 W bulb in series with a parallel combination of two 60 W bulbs ; 54.54 W]

2.12. Open Circuits

As the name implies, an *open* is a gap or break or interruption in a circuit path.

When there is a break in any part of a circuit, that part is said to be **open-circuited.**

No current can flow through an open. Since no current can flow through an open, according to Ohm's law, an open has infinite resistance ($R = V/I = V/0 = \infty$). An open circuit may be as a result of component failure or disintegration of a conducting path such as the breaking of a wire.

1. Open circuit in a series circuit. Fig. 2.50 shows an open circuit fault in a series circuit. Here resistor R_4 is burnt out and an open develops. Because of the open, no current can flow in the circuit.

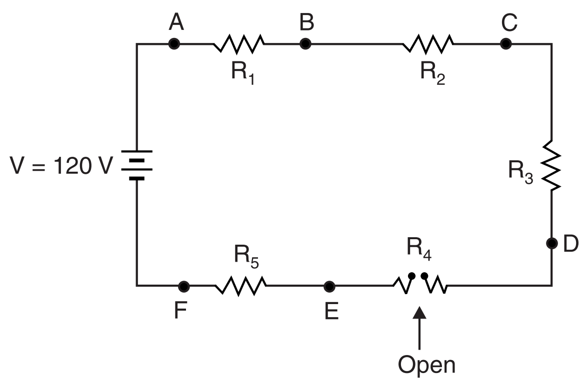


Fig. 2.50

When an open occurs in a series circuit, the following symptoms can be observed :

(*i*) The circuit current becomes zero.

(*ii*) There will be no voltage drop across the resistors that are normal.

(*iii*) *The entire voltage drop appears across the open.* This can be readily proved. Applying Kirchhoff's voltage law to the loop *ABCDEFA*, we have,

$$-0 \times R_1 - 0 \times R_2 - 0 \times R_3 - V_{DE} - 0 \times R_5 + 120 = 0$$

$$\therefore \qquad V_{DE} = 120 \text{ V}$$

(*iv*) Since the circuit current is zero, there is no voltage drop in the internal resistance of the source. Therefore, terminal voltage may appear higher than the normal.

2. Open circuit in a parallel circuit. One or more branches of a parallel circuit may develop an open. Fig. 2.51 shows a parallel circuit with an open. Here resistor R_3 is burnt out and now has infinite resistance.

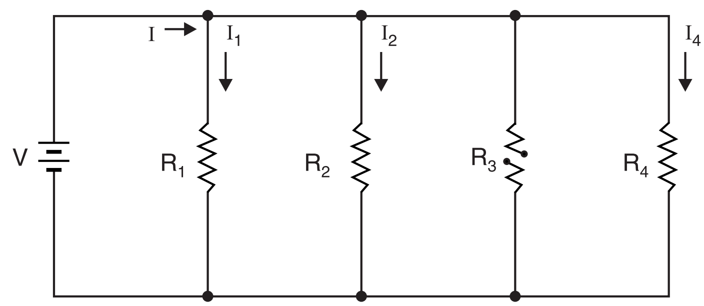


Fig. 2.51

The following symptoms can be observed :

(*i*) Branch current I_3 will be zero because R_3 is open.

(*ii*) The total current I will be less than the normal.

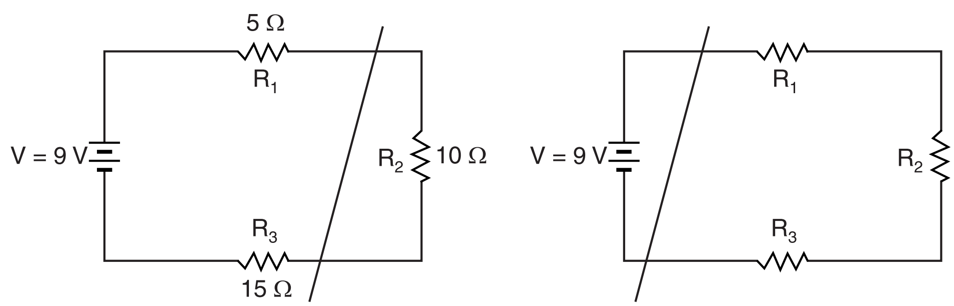
(*iii*) The operation of the branches without opens will be normal.

(*iv*) The open device will not operate. If R_3 is a lamp, it will be out. If it is a motor, it will not run.

2.13. Short Circuits

A short circuit or short is a path of low resistance. *A* **short circuit** *is an unwanted path of low resistance.* When a short circuit occurs, the resistance of the circuit becomes low. As a result, current greater than the normal flows which can cause damage to circuit components. The short circuit may be due to insulation failure, components get shorted etc.

1. Partial short in a series circuit. Fig. 2.52 (*i*) shows a **series circuit** with a **partial short.** An unwanted path has connected R_1 to R_3 and has eliminated R_2 from the circuit. Therefore, the circuit resistance decreases and the circuit current becomes greater than normal. The voltage drop across components that are not shorted will be higher than normal. Since current is increased, the power dissipation in the components that are not shorted will be greater than the normal. A partial short may cause healthy component to burn out due to abnormally high dissipation.



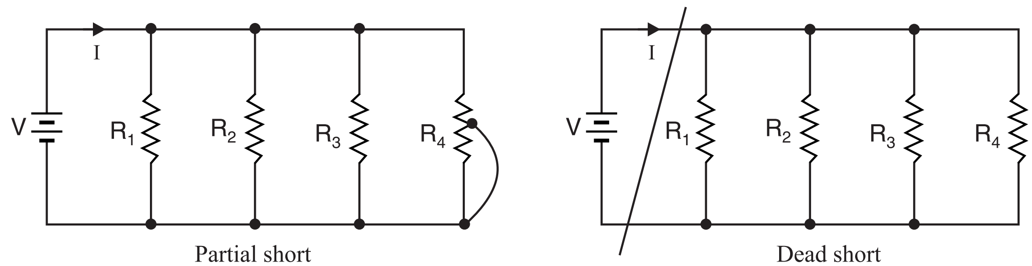
Partial short
(*i*)

Dead short
(*ii*)

Fig. 2.52

2. Dead short in a series circuit. Fig. 2.52 (*ii*) shows a **series circuit** with a **dead short.** Here all the loads (*i.e.* resistors in this case) have been removed by the unwanted path. Therefore, the circuit resistance is almost zero and the circuit current becomes extremely high. If there are no protective devices (fuse, circuit breaker etc.) in the circuit, drastic results (smoke, fire, explosion etc.) may occur.

3. Partial short in a parallel circuit. Fig. 2.53 (*i*) shows a **parallel circuit** with a **partial short.** The circuit resistance will decrease and total current becomes greater than the normal. Further, the current flow in the healthy branches will be less than the normal. Therefore, healthy branches may operate but not as they are supposed to.



Partial short
(*i*)

Dead short
(*ii*)

Fig. 2.53

4. Dead short in a parallel circuit. Fig. 2.53 (*ii*) shows a **parallel circuit** with a **dead short.** Note that all the loads are eliminated by the short circuit so that the circuit resistance is almost zero. As a result, the circuit current becomes abnormally high and may cause extensive damage unless it has protective devices (*e.g.* fuse, circuit breaker etc.).

2.14. Duality Between Series and Parallel Circuits

Two physical systems or circuits are called dual if they are described by equations of the same mathematical form.

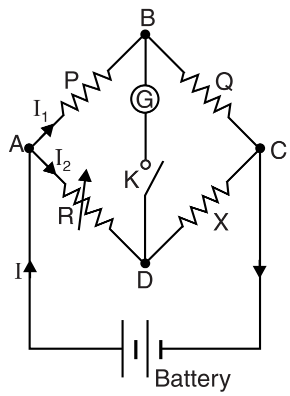
This peculiar pattern of relationship exists between series and parallel circuits. For example, consider the following table for d.c. series circuit and d.c. parallel circuit.

D.C. series circuit	D.C. parallel circuit
$I_1 = I_2 = I_3 = \ldots$	$V_1 = V_2 = V_3 = \ldots$
$V = V_1 + V_2 + V_3 + \ldots$	$I = I_1 + I_2 + I_3 + \ldots$
$R_S = R_1 + R_2 + R_3 + \ldots$	$G_P = G_1 + G_2 + G_3 + \ldots$
$I = \dfrac{V_1}{R_1} = \dfrac{V_2}{R_2} = \dfrac{V_3}{R_3} = \ldots$	$V = \dfrac{I_1}{G_1} = \dfrac{I_2}{G_2} = \dfrac{I_3}{G_3} = \ldots$
$V_1 = V\dfrac{R_1}{R_S}$; $V_2 = V\dfrac{R_2}{R_S}$	$I_1 = I\dfrac{G_1}{G_P}$; $I_2 = I\dfrac{G_2}{G_P}$

Note that the relations for parallel circuit can be obtained from the series circuit by replacing voltage by current, current by voltage and resistance by conductance. In like manner, relations for series circuit can be obtained from the parallel circuit by replacing current by voltage, voltage by current and conductance by resistance. Such a pattern is known as *duality* and the two circuits are said to be dual of each other. Thus series and parallel circuits are dual of each other. Other examples of duals are : short circuits and open circuits are duals and nodes and meshes are duals.

2.15. Wheatstone Bridge

This bridge was first proposed by Wheatstone (an English telegraph engineer) for measuring accurately the value of an unknown resistance. It consists of four resistors (two fixed known resistances P and Q, a known variable resistance R and the unknown resistance X whose value is to be found) connected to form a diamond-shaped circuit $ABCDA$ as shown in Fig.2.54 (*i*). Across one pair of opposite junctions (A and C), battery is connected and across the other opposite pair of



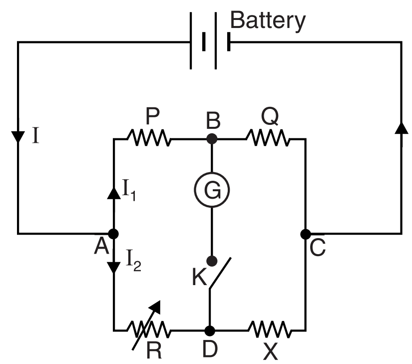


Fig. 2.54

junctions (B and D), a galvanometer is connected through the key K. The circuit is called a bridge because galvanometer bridges the opposite junctions B and D. Fig. 2.54 (*ii*) shows another* way of drawing the Wheatstone bridge.

Working. The values of P and Q are properly fixed. The value of R is varied such that on closing the key K, there is no current through the galvanometer. Under such conditions, the bridge is said to be *balanced*. The point at which the bridge is balanced is called the *null point*. Let I_1 and I_2 be the currents through P and R respectively when the bridge is balanced. Since there is no current through the galvanometer, the currents in Q and X are also I_1 and I_2 respectively. As the galvanometer reads zero, points B and D are at the same potential. This means that voltage drops from A to B and A to D must be equal. Also voltage drops from B to C and D to C must be equal. Hence,

$$I_1 P = I_2 R \quad ...(i)$$

and

$$I_1 Q = I_2 X \quad ...(ii)$$

Dividing exp. (*i*) by (*ii*), we get,

$$P/Q = R/X$$

or

$$P\,X = Q\,R$$

i.e. *Product of opposite arms* = *Product of opposite arms*

$$\text{Unknown resistance, } X = \frac{Q}{P} \times R \quad ...(iii)$$

Since the **values of Q, P and R are known, the value of unknown resistance X can be calculated. It should be noted that exp. (*iii*) is true only under the balanced conditions of Wheatstone bridge.

Note. When the bridge is balanced, $V_B = V_D$ so the voltage across galvanometer is zero *i.e.* $V_{BD} = V_B - V_D = 0$. When there is zero voltage across the galvanometer, there is also zero current though the galvanometer. Consequently, **in a balanced Wheatstone bridge, galvanometer can be replaced by either a short circuit or an open circuit without affecting the voltages and currents anywhere else in the circuit.**

Example 2.35. *Verify that the Wheatstone bridge shown in Fig. 2.55 is balanced. Then find the voltage V_T across the 0.2 A current source by* (*i*) *replacing the 200 Ω resistor with a short.* (*ii*) *replacing the 200 Ω resistor with an open.*

* Note the four points A,B, C and D, each lying at the junction between two resistors. A galvanometer should bridge a pair of opposite points such as B and D and the battery the other pair A and C.

** Resistances P and Q are called the ratio arms of bridge and are usually made equal to definite ratio such as 1 to 1, 10 to 1 or 100 to 1. The resistance R is ca]lled the rheostat arm and is made continuously variable from 1 to 1000 ohms or from 1 to 10,000 ohms.

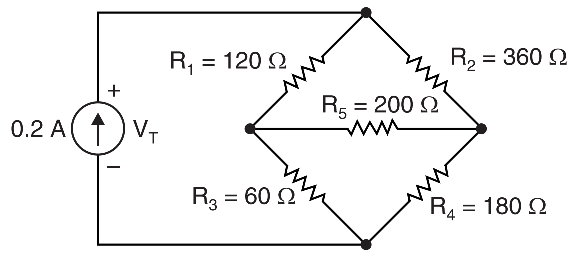


Fig. 2.55

Solution. The Wheatstone bridge is balanced if the products of the resistances of the opposite arms of the bridge are equal. An inspection of Fig. 2.55 shows that $R_1R_4 = R_2R_3$. Therefore, the bridge is balanced.

(*i*) When 200 Ω resistor is shorted. Fig. 2.56 (*i*) shows the bridge when the 200 Ω resistor (R_5) is replaced by a short. In this case, the circuit is equivalent to a series-parallel circuit as shown in Fig. 2.56 (*ii*). Referring to Fig. 2.56 (*ii*), the circuit is equivalent to parallel combination of R_1 and R_2 in series with the parallel combination of R_3 and R_4.

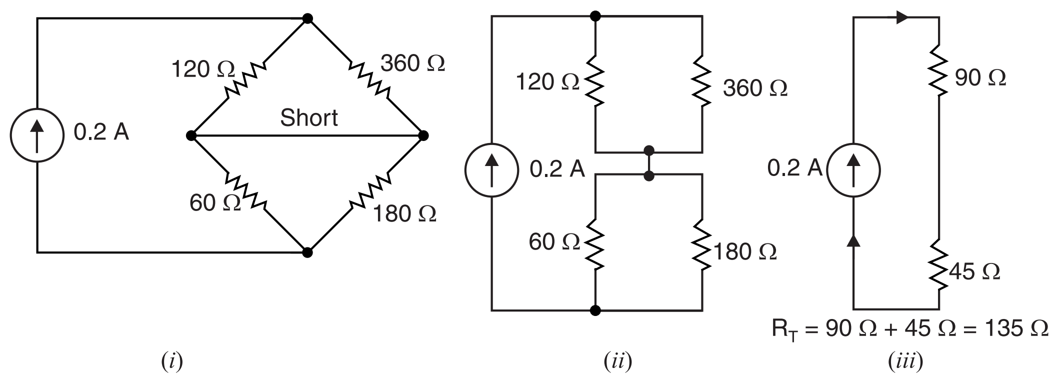


Fig. 2.56

The circuit shown in Fig. 2.56 (*ii*) further reduces to the one shown in Fig. 2.57 (*iii*). Therefore, total circuit resistance, $R_T = 90 + 45 = 135\ \Omega$.

∴ Voltage across 0.2 A current source is

$$V_T = I\,R_T = 0.2 \times 135 = \mathbf{27\ V}$$

(*ii*) When 200 Ω resistor is open-circuited. Fig. 2.57 (*i*) shows the bridge when 200 Ω resistor is replaced by an open. In this case, the circuit is equivalent to a series-parallel circuit in which series combination of R_1 and R_3 is in parallel with the series combination of R_2 and R_4. This is shown in Fig. 2.57 (*ii*).

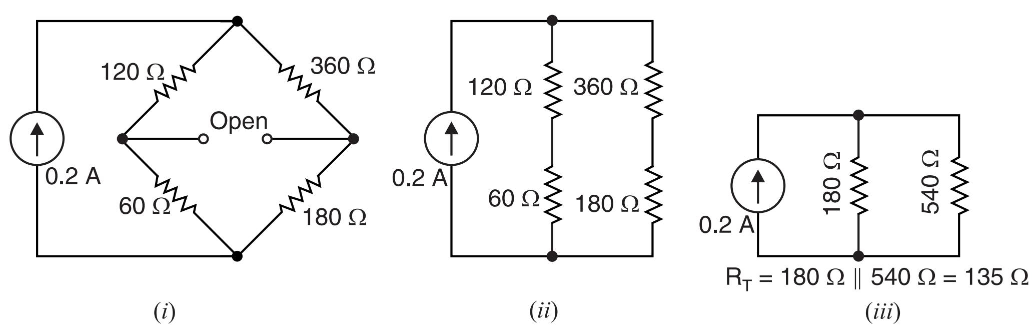


Fig. 2.57

The circuit shown in Fig. 2.57 (*ii*) further reduces to the one shown in Fig. 2.57 (*iii*). Referring to Fig. 2.57 (*iii*), the total circuit resistance R_T is given by ;

$$R_T = \frac{180 \times 540}{180 + 540} = 135\ \Omega$$

$\therefore$ Voltage across 0.2 A current source, $V_T = I\,R_T = 0.2 \times 135 =$ **27 V**

Note that the voltage across current source is unaffected whether 200 Ω resistor is replaced by a short or an open.

2.16. Complex Circuits

Sometimes we encounter circuits where simplification by series and parallel combinations is impossible. Consequently, Ohm's law cannot be applied to solve such circuits. This happens when there is more than one e.m.f. in the circuit or when resistors are connected in a complicated manner. Such circuits are called *complex circuits*. We shall discuss two such circuits by way of illustration.

(*i*) Fig. 2.58 shows a circuit containing two sources of *e.m.f.* E_1 and E_2 and three resistors. This circuit cannot be solved by series-parallel combinations. Are resistors R_1 and R_3 in parallel? Not quite, because there is an *e.m.f.* source E_1 between them. Are they in series? Not quite, because same current does not flow between them.

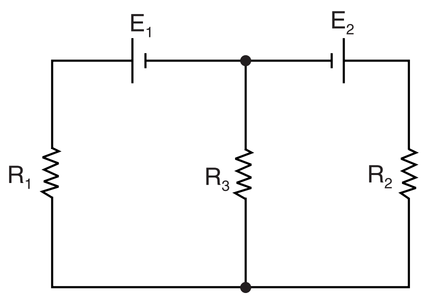


Fig. 2.58

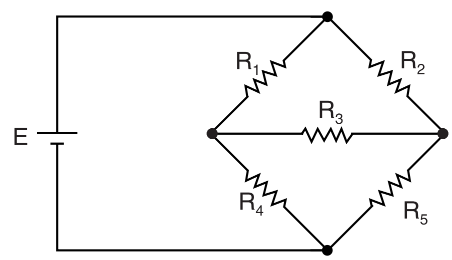


Fig. 2.59

(*ii*) Fig. 2.59 shows another circuit where we cannot solve the circuit by series-parallel combinations. Though this circuit has one source of *e.m.f.* (*E*), it cannot be solved by using series and parallel combinations. Thus resistors R_1 and R_2 are neither in series nor in parallel; the same is true for other pair of resistors.

In order to solve such complex circuits, Gustav Kirchhoff gave two laws, known as Kirchhoff's laws.

2.17. Kirchhoff's Laws

Kirchhoff gave two laws to solve complex circuits, namely ;

1. Kirchhoff's Current Law (KCL)
2. Kirchhoff's Voltage Law (KVL)

1. KIRCHHOFF'S CURRENT LAW (KCL)

This law relates to the currents at the *junctions of an electric circuit and may be stated as under :

The algebraic sum of the currents meeting at a junction in an electrical circuit is zero.

An algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents I_1, I_2, I_3 and I_4 and meeting at point O as shown in Fig. 2.60.

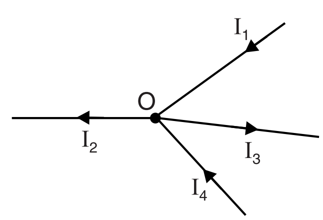


Fig. 2.60

* A junction is that point in an electrical circuit where *three or more* circuit elements meet.

If we take the signs of currents flowing towards point O as positive, then currents flowing away from point O will be assigned negative sign. Thus, applying Kirchhoff's current law to the junction O in Fig. 2.60, we have,

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

or $$I_1 + I_4 = I_2 + I_3$$

i.e., Sum of incoming currents = Sum of outgoing currents

Hence, Kirchhoff's current law may also be stated as under :

The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction. Kirchhoff's current law is also called junction rule.

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence, Kirchhoff's current law is based on the law of conservation of charge.

2. KIRCHHOFF'S VOLTAGE LAW (KVL)

This law relates to *e.m.fs* and voltage drops in a closed circuit or loop and may be stated as under :

In any closed electrical circuit or mesh, the algebraic sum of all the electromotive forces (e.m.fs) and voltage drops in resistors is equal to zero, i.e.,

In any closed circuit or mesh,

Algebraic sum of e.m.fs + Algebraic sum of voltage drops = 0

The validity of Kirchhoff's voltage law can be easily established by referring to the closed loop *ABCDA* shown in Fig. 2.61. If we start from any point (say point *A*) in this closed circuit and go back to this point (*i.e.,* point *A*) after going around the circuit, then there is no increase or decrease in potential. This means that algebraic sum of the *e.m.fs* of all the sources (here only one *e.m.f.* source is considered) met on the way *plus* the algebraic sum of the voltage drops in the resistances must be zero. Kirchhoff's voltage law is based on the law of *conservation of energy, *i.e.*, net change in the energy of a charge after completing the closed path is zero.

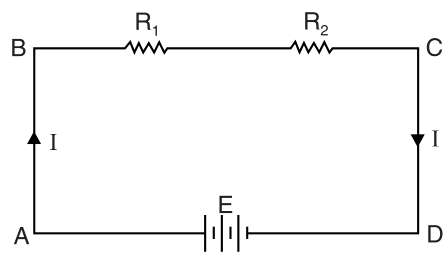


Fig. 2.61

Note. Kirchhoff's voltage law is also called *loop rule.*

2.18. Sign Convention

While applying Kirchhoff's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to *e.m.fs* and voltage drops in the closed circuit. The following convention may be followed :

A **rise in potential should be considered positive and fall in potential should be considered negative.

(*i*) Thus if we go from the positive terminal of the battery to the negative terminal, there is fall of potential and the *e.m.f.* should be assigned negative sign. Thus in Fig. 2.62 (*i*), as we go from *A* to *B*, there is a fall in potential and the *e.m.f.* of the cell will be assigned negative

* As a charge traverses a loop and returns to the starting point, the sum of rises of potential energy associated with *e.m.fs* in the loop must be equal to the sum of the drops of potential energy associated with resistors.

** The reverse convention is equally valid *i.e.* rise in potential may be considered negative and fall in potential as positive.

sign. On the other hand, if we go from the negative terminal to the positive terminal of the battery or source, there is a rise in potential and the *e.m.f* should be assigned positive sign. Thus in Fig. 2.62 (*ii*) as we go from *A* to *B*, there is a rise in potential and the *e.m.f.* of the cell will be assigned positive sign. *It may be noted that the sign of e.m.f. is independent of the direction of current through the branch under consideration.*

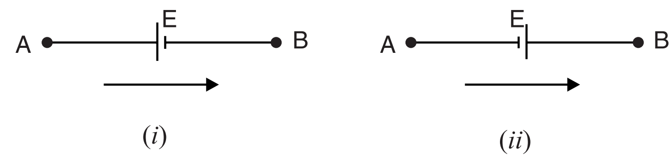


Fig. 2.62

(*ii*) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in potential because current flows from higher potential to lower potential. Hence this voltage drop should be assigned negative sign. In Fig. 2.63 (*i*), as we go from *A* to *B*, there is a fall in potential and the voltage drop across the resistor will be assigned negative sign. On the other hand, if we go through the resistor against the current flow, there is a rise in potential and the voltage drop should be given positive sign. Thus referring to Fig. 2.63 (*ii*), as we go from *A* to *B*, there is a rise in potential and this voltage drop will be given positive sign. *It may be noted that sign of voltage drop depends on the direction of current and is independent of the polarity of the e.m.f. of source in the circuit under consideration.*

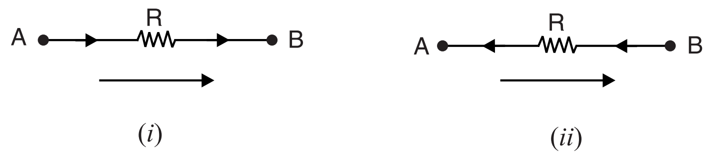


Fig. 2.63

2.19. Illustration of Kirchhoff's Laws

Kirchhoff's Laws can be beautifully explained by referring to Fig. 2.64. Mark the directions of currents as indicated. The direction in which currents are assumed to flow is unimportant, since if wrong direction is chosen, it will be indicated by a negative sign in the result.

(*i*) The magnitude of current in any branch of the circuit can be found by applying Kirchhoff's current law. Thus at junction *C* in Fig. 2.64, the incoming currents to the junction are I_1 and I_2. Obviously, the current in branch *CF* will be I_1+I_2.

(*ii*) There are three closed circuits in Fig 2.64 *viz*. *ABCFA, CDEFC* and *ABCDEFA*. Kirchhoff's voltage law can be applied to these closed circuits to get the desired equations.

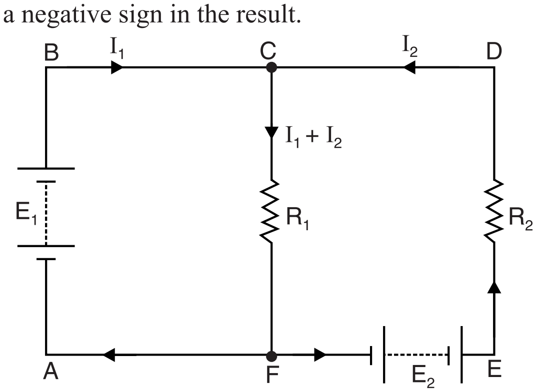


Fig. 2.64

Loop ABCFA. In this loop, *e.m.f.* E_1 will be given *positive* sign. It is because as we consider the loop in the order *ABCFA*, we go from −ve terminal to the positive terminal of the battery in the branch *AB* and hence there is a rise in potential. The voltage drop in branch *CF* is $(I_1 + I_2)\,R_1$ and shall bear *negative* sign. It is because as we consider the loop in the order *ABCFA*, we go with current in branch *CF* and there is a fall in potential. Applying Kirchhoff's voltage law to the loop *ABCFA*,

$$-(I_1 + I_2)\,R_1 + E_1 = 0$$

or $$E_1 = (I_1 + I_2)\,R_1 \qquad ...(i)$$

Loop CDEFC. As we go around the loop in the order *CDEFC*, drop I_2R_2 is *positive,* e.m.f. E_2 is *negative* and drop $(I_1 + I_2)\,R_1$ is *positive*. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2\,R_2 + (I_1 + I_2)\,R_1 - E_2 = 0$$

or $$I_2\,R_2 + (I_1 + I_2)\,R_1 = E_2 \qquad ...(ii)$$

Since E_1, E_2, R_1 and R_2 are known, we can find the values of I_1 and I_2 from the above two equations. Hence currents in all branches can be determined.

2.20. Method to Solve Circuits by Kirchhoff's Laws

(*i*) Assume unknown currents in the given circuit and show their direction by arrows.

(*ii*) Choose any closed circuit and find the algebraic sum of voltage drops *plus* the algebraic sum of e.m.fs in that loop.

(*iii*) Put the algebraic sum of voltage drops plus the algebraic sum of e.m.fs equal to zero.

(*iv*) Write equations for as many closed circuits as the number of unknown quantities. Solve equations to find unknown currents.

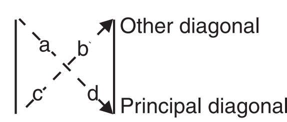
(*v*) If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.

Note. It may be noted that Kirchhoff's laws are also applicable to a.c. circuits. The only thing to be done is that **I**, **V** and **Z** are substituted for *I*, *V* and *R*. Here **I**, **V** and **Z** are phasor quantities.

2.21. Matrix Algebra

The solution of two or three simultaneous equations can be achieved by a method that uses *determinants.* A determinant is a numerical value assigned to a square arrangement of numbers called a *matrix.* The advantage of determinant method is that it is less difficult for three unknowns and there is less chance of error. The theory behind this method is not presented here but is available in any number of mathematics books.

Second-order determinant. A 2 × 2 matrix has four numbers arranged in two rows and two columns. The value of such a matrix is called a *second-order determinant* and is *equal to the product of the principal diagonal minus the product of the other diagonal.* For example, value of the matrix = *ad* – *cb*.



Second-order determinant can be used to solve simultaneous equations with two unknowns. Consider the following equations :

$$a_1\,x + b_1\,y = c_1$$
$$a_2\,x + b_2\,y = c_2$$

The unknowns are x and y in these equations. The numbers associated with the unknowns are called *coefficients.* The coefficients in these equations are a_1, a_2, b_1 and b_2. The right hand number (c_1 or c_2) of each equation is called a *constant.* The coefficients and constants can be arranged as a *numerator matrix* and as a *denominator matrix.* The matrix for the numerator is formed by replacing the coefficients of the unknown by the constants. The denominator matrix is called *characteristic matrix* and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Note that the characteristic determinant (denominator) is the same in both cases and needs to be evaluated only once. Also note that the coefficients for x are replaced by the constants when solving for x and that the coefficients for y are replaced by the constants when solving for y.

Third-order determinant. A third-order determinant has 9 numbers arranged in 3 rows and 3 columns. Simultaneous equations with three unknowns can be solved with third-order determinants. Consider the following equations :

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The characteristic matrix forms the denominator and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$\text{Denominator} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The matrix for each numerator is formed by replacing the coefficient of the unknown with the constant.

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\text{Denominator}}$$

Example 2.36. *In the network shown in Fig. 2.65, the different currents and voltages are as under :*

$i_2 = 5e^{-2t}$; $i_4 = 3 \sin t$; $v_3 = 4e^{-2t}$

Using KCL, find voltage v_1.

Solution. Current through capacitor is

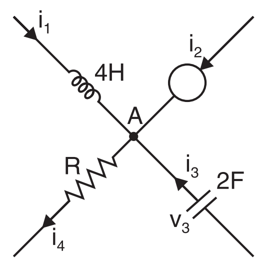


Fig. 2.65

$$i_3 = C\frac{dv_3}{dt} = C\frac{d}{dt}(v_3) = \frac{2d}{dt}(4e^{-2t})$$
$$= -16e^{-2t}$$

Applying *KCL* to junction *A* in Fig. 2.65,

$$i_1 + i_2 + i_3 + (-i_4) = 0$$

or $\quad i_1 + 5e^{-2t} - 16e^{-2t} - 3 \sin t = 0$

or $\quad i_1 = 3 \sin t + 11e^{-2t}$

$\therefore$ Voltage developed across 4H coil is

$$v_1 = L\frac{di_1}{dt} = L\frac{d}{dt}(i_1) = 4\frac{d}{dt}(3 \sin t + 11e^{-2t})$$
$$= 4(3 \cos t - 22e^{-2t}) = \mathbf{12 \cos t - 88e^{-2t}}$$

Example 2.37. *For the circuit shown in Fig. 2.66, find the currents flowing in all branches.*

Solution. Mark the currents in various branches as shown in Fig. 2.66. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

Loop *ABCFA*. Applying *KVL*,

$$30 - 2I_1 - 10 + 5I_2 = 0$$

or $$2I_1 - 5I_2 = 20 \quad ...(i)$$

Loop *FCDEF*. Applying *KVL*,

$$-5I_2 + 10 - 3(I_1 + I_2) - 5 - 4(I_1 + I_2) = 0$$

or $$7I_1 + 12I_2 = 5 \quad ...(ii)$$

Multiplying eq. *(i)* by 7 and eq. *(ii)* by 2, we get,

$$14I_1 - 35I_2 = 140 \quad ...(iii)$$

$$14I_1 + 24I_2 = 10 \quad ...(iv)$$

Subtracting eq. (*iv*) from eq. (*iii*), we get,

$$-59I_2 = 130$$

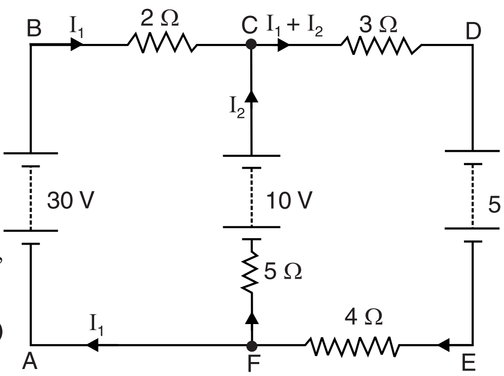


Fig. 2.66

$\therefore$ $$I_2 = -130/59 = -2.2\text{A} = \textbf{2.2 A} \textit{ from C to F}$$

Substituting the value of $I_2 = -2.2$ A in eq. (*i*), we get, I_1 = **4.5 A**

Current in branch *CDEF* = $I_1 + I_2 = (4.5) + (-2.2)$ = **2.3 A**

Example 2.38. *A Wheatstone bridge ABCD has the following details ; AB = 1000 Ω ; BC = 100 Ω; CD = 450 Ω ; DA = 5000 Ω.*

A galvanometer of resistance 500 Ω is connected between B and D. A 4.5-volt battery of negligible resistance is connected between A and C with A positive. Find the magnitude and direction of galvanometer current.

Solution. Fig. 2.67 shows the Wheatstone bridge *ABCD*. Mark the currents in the various sections as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_g), three loops will be considered.

Loop *ABDA*. Applying *KVL*,

$$-1000I_1 - 500I_g + 5000I_2 = 0$$

or $$2I_1 + I_g - 10I_2 = 0 \quad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-100(I_1 - I_g) + 450(I_2 + I_g) + 500I_g = 0$$

or $$2I_1 - 21I_g - 9I_2 = 0 \quad ...(ii)$$

Loop *EABCFE*. Applying *KVL*,

$$-1000I_1 - 100(I_1 - I_g) + 4.5 = 0$$

or $$1100I_1 - 100I_g = 4.5 \quad ...(iii)$$

Subtracting eq. (*ii*) from eq. (*i*), we get,

$$22I_g - I_2 = 0 \quad ...(iv)$$

Multiplying eq. (*i*) by 550 and subtracting eq. (*iii*) from it, we get,

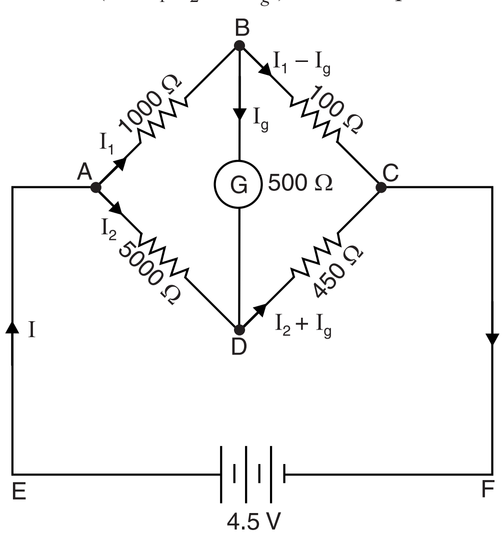


Fig. 2.67

$$650I_g - 5500I_2 = -4.5 \quad ...(v)$$

Multiplying eq. (*iv*) by 5500 and subtracting eq. (*v*) from it, we get,

$$120350I_g = 4.5$$

$\therefore$ $$I_g = \frac{4.5}{120350} = 37.4 \times 10^{-6}\text{ A} = \textbf{37.4 μA} \textit{ from B to D}$$

Example 2.39. *A Wheatstone bridge ABCD is arranged as follows : AB = 1 Ω ; BC = 2 Ω ; CD = 3 Ω ; DA = 4 Ω. A resistance of 5 Ω is connected between B and D. A 4-volt battery of internal resistance 1 Ω is connected between A and C. Calculate (i) the magnitude and direction of current in 5 Ω resistor and (ii) the resistance between A and C.*

Solution. ***(i)*** Fig. 2.68 shows the Wheatstone bridge *ABCD*. Mark the currents in the various branches as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered.

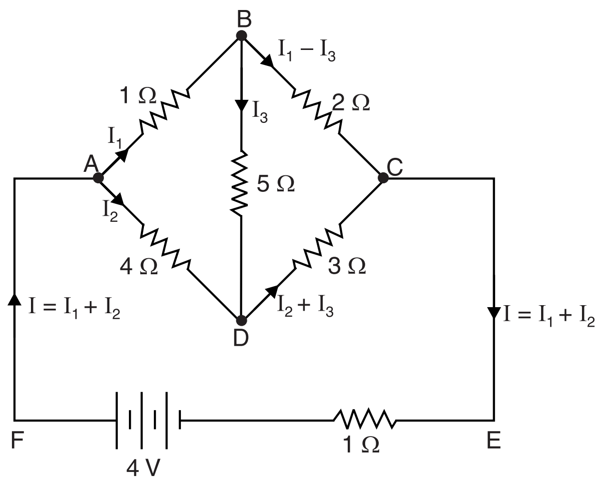


Fig. 2.68

Loop *ABDA*. Applying *KVL*,

$$-1 \times I_1 - 5\,I_3 + 4\,I_2 = 0$$

or $$I_1 + 5\,I_3 - 4\,I_2 = 0 \qquad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-2\,(I_1 - I_3) + 3\,(I_2 + I_3) + 5I_3 = 0$$

or $$2\,I_1 - 10\,I_3 - 3\,I_2 = 0 \qquad ...(ii)$$

Loop *FABCEF*. Applying *KVL*,

$$-I_1 \times 1 - 2\,(I_1 - I_3) - 1\,(I_1 + I_2) + 4 = 0$$

or $$4\,I_1 - 2\,I_3 + I_2 = 4 \qquad ...(iii)$$

Multiplying eq.(*i*) by 2 and subtracting *eq.* (*ii*) from it, we get,

$$20\,I_3 - 5\,I_2 = 0 \qquad ...(iv)$$

Multiplying *eq.* (*i*) by 4 and subtracting *eq.* (*iii*) from it, we get,

$$22\,I_3 - 17\,I_2 = -4 \qquad ...(v)$$

Multiplying *eq.* (*iv*) by 17 and eq. (*v*) by 5, we get,

$$340\,I_3 - 85\,I_2 = 0 \qquad ...(vi)$$

$$110\,I_3 - 85\,I_2 = -20 \qquad ...(vii)$$

Subtracting *eq.* (*vii*) from *eq.* (*vi*), we get,

$$230\,I_3 = 20$$

$\therefore$ $I_3 = 20/230 = 0.087$ A

i.e Current in 5 Ω, I_3 = **0.087 A** *from B* to *D*

(ii) Substituting the value of I_3 = 0.087 A in eq. (*iv*), we get, $I_2 = 0.348$ A.

Substituting values of I_3 = 0.087 A and I_2 = 0.348 A in *eq. (ii)*, $I_1 = 0.957$ A.

Current supplied by battery, $I = I_1 + I_2 = 0.957 + 0.348 = 1.305$ A

P.D. between *A* and *C* = E.M.F. of battery − Drop in battery = 4 − 1.305 × 1 = 2.695 V

$\therefore$ Resistance between *A* and *C* $= \dfrac{P.D.\text{ across } AC}{\text{Battery current}} = \dfrac{2.695}{1.305} = \mathbf{2.065\ \Omega}$

Example 2.40. *Determine the current in 4 Ω resistance of the circuit shown in Fig. 2.69.*

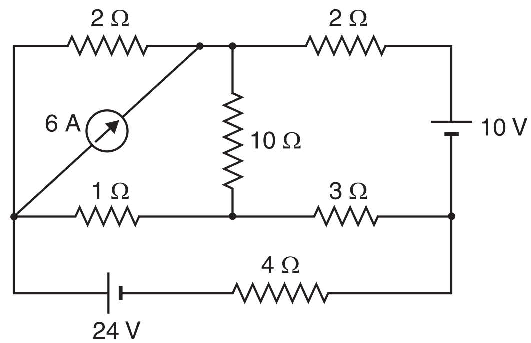


Fig. 2.69

Solution. The given circuit is redrawn as shown in Fig. 2.70. Mark the currents in the various branches of the circuit using *KCL*. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered. While applying *KVL* to any loop, rise in potential is considered positive while fall in potential is considered negative. This convention is followed throughout the book.

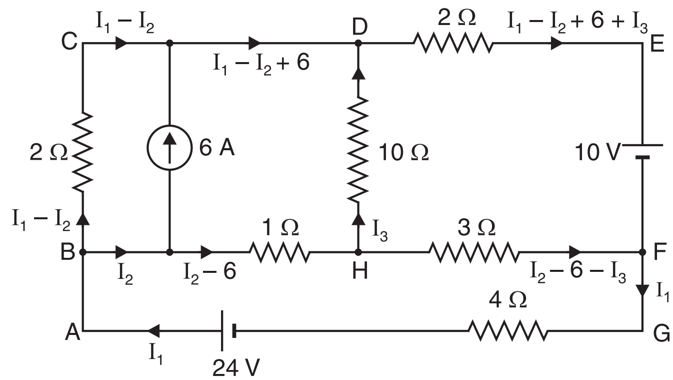


Fig. 2.70

Loop *BCDHB*. Applying *KVL*, we have,

$$-2(I_1 - I_2) + 10I_3 + 1 \times (I_2 - 6) = 0$$

or $$2I_1 - 3I_2 - 10I_3 = -6 \qquad ...(i)$$

Loop *DEFHD*. Applying K*VL*, we have,

$$-2(I_1 - I_2 + 6 + I_3) - 10 + 3(I_2 - 6 - I_3) - 10I_3 = 0$$

or $$2I_1 - 5I_2 + 15I_3 = -40 \qquad ...(ii)$$

Loop *BHFGAB*. Applying *KVL*, we have,

$$-1(I_2 - 6) - 3(I_2 - 6 - I_3) - 4I_1 + 24 = 0$$

or $$4I_1 + 4I_2 - 3I_3 = 48 \qquad ...(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get, I_1 = 4.1 A.

∴ Current in 4 Ω resistance = I_1 = **4.1 A**

Example 2.41. *Two batteries E_1 and E_2 having e.m.fs of 6V and 2V respectively and internal resistances of 2 Ω and 3 Ω respectively are connected in parallel across a 5 Ω resistor. Calculate* (*i*) *current through each battery and* (*ii*) *terminal voltage.*

Solution. Fig. 2.71 shows the conditions of the problem. Mark the currents in the various branches. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

(*i*) **Loop *HBCDEFH*.** Applying Kirchhoff's voltage law to loop *HBCDEFH*, we get,

$$2\,I_1 - 6 + 2 - 3\,I_2 = 0$$

or $$2\,I_1 - 3\,I_2 = 4 \qquad ...(i)$$

Loop *ABHFEGA*. Applying Kirchhoff's voltage law to loop *ABHFEGA*, we get,

$$3\,I_2 - 2 + 5\,(I_1 + I_2) = 0$$

or $$5\,I_1 + 8\,I_2 = 2 \qquad ...(ii)$$

Multiplying eq. (*i*) by 8 and eq. (*ii*) by 3 and then adding them, we get,

$$31\, I_1 = 38$$

or $$I_1 = \frac{38}{31} = \mathbf{1.23\ A}$$

i.e. battery E_1 is being discharged at 1.23 A. Substituting I_1 = 1.23 A in eq. (*i*), we get, I_2 = **− 0.52A** *i.e.* battery E_2 is being charged.

(ii) Terminal voltage $= (I_1 + I_2)\, 5$

$= (1.23 - 0.52)\, 5 = \mathbf{3.55\ V}$

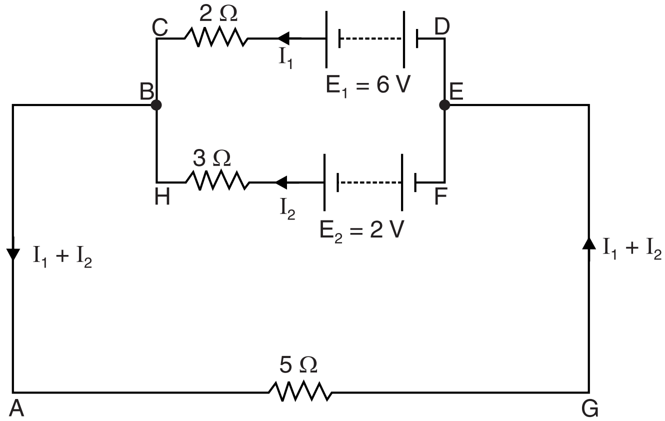


Fig. 2.71

Example 2.42. *Twelve wires, each of resistance r, are connected to form a skeleton cube. Find the equivalent resistance between the two diagonally opposite corners of the cube.*

Solution. Let *ABCDEFGH* be the skeleton cube formed by joining 12 wires, each of resistance *r* as shown in Fig. 2.72. Suppose a current of 6*I* enters the cube at the corner *A*. Since the resistance of each wire is the same, the current at corner *A* is divided into three equal parts: 2*I* flowing in *AE*, 2*I* flowing in *AB* and 2*I* flowing in *AD*. At points *B, D* and *E*, these currents are divided into equal parts, each part being equal to *I*. Applying Kirchhoff's current law, 2*I* current flows in each of the wires *CG, HG* and *FG*. These three currents add up at the corner *G* so that current flowing out of this corner is 6*I*.

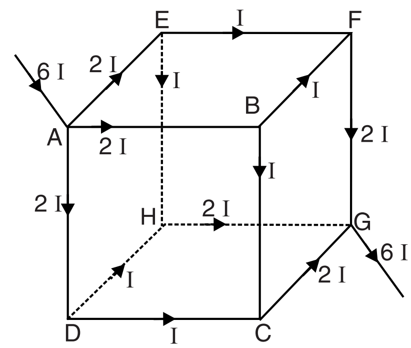


Fig. 2.72

Let *E* = e.m.f. of the battery connected to corners *A* and *G* of the cube ; corner *A* being connected to the +ve terminal. Now consider any closed circuit between corners *A* and *G*, say the closed circuit *AEFGA*. Applying Kirchhoff's voltage law to the closed circuit *AEFGA*, we have,

$$-2\,I\,r - I\,r - 2\,I\,r = -E \quad \text{or} \quad 5\,I\,r = E \qquad ...(i)$$

Let *R* be the equivalent resistance between the diagonally opposite corners *A* and *G*.

Then, $$E = 6\,I\,R \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we get, $6IR = 5I\,r$ or $R = \mathbf{(5/6)\ r}$

Example 2.43. *Determine the current supplied by the battery in the circuit shown in Fig. 2.73.*

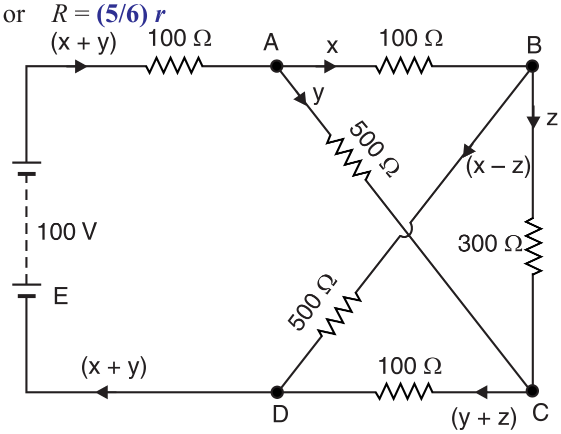


Fig. 2.73

Solution. Mark the currents in the various branches as shown in Fig. 2.73. Since there are three unknown quantities *x*, *y* and *z*, three equations must be formed by considering three loops.

Loop *ABCA*. Applying *KVL*, we have,

$$-100x - 300z + 500y = 0$$

or $$x - 5y + 3z = 0 \qquad ...(i)$$

Loop *BCDB*. Applying *KVL*, we have,

$-300x - 100(y+z) + 500(x-z) = 0$

or $\quad 5x - y - 9z = 0 \quad$ ...(*ii*)

Loop *ABDEA*. Applying *KVL*, we have,

$-100x - 500(x-z) + 100 - 100(x+y) = 0$

or $\quad 7x + y - 5z = 1 \quad$ ...(*iii*)

From eqs. (*i*), (*ii*) and (*iii*), $x = \frac{1}{5}\text{A} \quad ; \quad y = \frac{1}{10}\text{A} \quad ; \quad z = \frac{1}{10}\text{A}$

By Determinant Method. We shall now find the values of *x*, *y* and *z* by determinant method.

$$x - 5y + 3z = 0 \qquad \text{...}(i)$$

$$5x - y - 9z = 0 \qquad \text{...}(ii)$$

$$7x + y - 5z = 1 \qquad \text{...}(iii)$$

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \qquad x = \frac{\begin{vmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{0\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 0 & -9 \\ 1 & -5 \end{vmatrix} + 3\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}}{1\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 5 & -9 \\ 7 & -5 \end{vmatrix} + 3\begin{vmatrix} 5 & -1 \\ 7 & 1 \end{vmatrix}}$$

$$= \frac{0\,[(-1\times-5)-(1\times-9)]+5\,[(0\times-5)-(1\times-9)]+3\,[(0\times1)-(1\times-1)]}{1\,[(-1\times-5)-(1\times-9)]+5\,[(5\times-5)-(7\times-9)+3\,[(5\times1)-(7\times-1)]}$$

$$= \frac{0+45+3}{14+190+36} = \frac{48}{240} = \frac{1}{5}\text{A}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$$z = \frac{\begin{vmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$\therefore$ Current supplied by battery $= x + y = \frac{1}{5} + \frac{1}{10} = \mathbf{\frac{3}{10}\,A}$

Example 2.44. *Use Kirchhoff's voltage law to find the voltage V_{ab} in Fig. 2.74.*

Solution. We shall use Kirchhoff's voltage law to solve this problem, although other methods can be used.

Total circuit resistance, $R_T = 2 + 1 + 3 = 6\ \text{k}\Omega$

Circuit current, $I = \dfrac{V}{R_T} = \dfrac{24\ \text{V}}{6\ \text{k}\Omega} = 4\ \text{mA}$

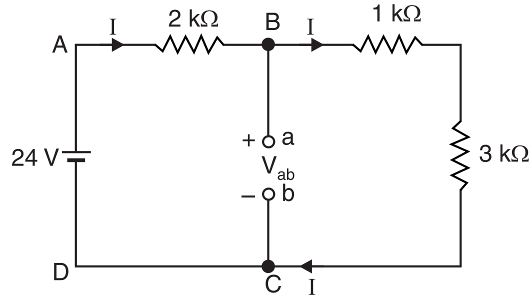


Fig. 2.74

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$$24 - 4\ \text{mA} \times 2\ \text{k}\Omega - {}^*V_{ab} = 0$$

or $\quad 24 - 8 - V_{ab} = 0 \qquad \therefore \quad V_{ab} = 24 - 8 = \mathbf{16\ V}$

Example 2.45. *For the ladder network shown in Fig. 2.75, find the source voltage V_s which results in a current of 7.5 mA in the 3 Ω resistor.*

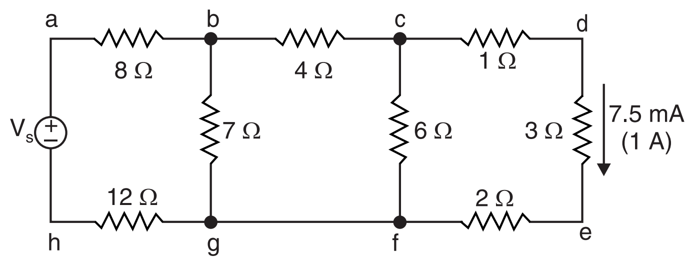


Fig. 2.75

Solution. Let us assume that current in branch *de* is 1 A.

Since the circuit is linear, the voltage necessary to produce 1 A is in the same ratio to 1 A as V_s to 7.5 mA.

Voltage between *c* and *f*, $\quad V_{cf} = 1\,(1 + 3 + 2) = 6\ \text{V}$

$\therefore$ Current in branch *cf*, $I_{cf} = 6/6 = 1\ \text{A}$

Applying *KCL* at junction *c*,

$$I_{bc} = 1 + 1 = 2\ \text{A}$$

Applying *KVL* to loop *bcfgb,* we have,

$$-4 \times 2 - 6 \times 1 + V_{bg} = 0 \quad \therefore \quad V_{bg} = 8 + 6 = 14\ \text{V}$$

$\therefore$ Current in branch *bg*, $I_{bg} = \dfrac{V_{bg}}{7} = \dfrac{14}{7} = 2\ \text{A}$

Applying *KCL* to junction *b*, we have, $I_{ab} = 2 + 2 = 4\ \text{A}$

Applying *KVL* to loop *abgha,* we have,

$$-8 \times 4 - 7 \times 2 - 12 \times 4 + V_{ah} = 0 \quad \therefore \quad V_{ah} = 94\ \text{V}$$

Now $\quad \dfrac{V_{ah}}{1\ \text{A}} = \dfrac{V_s}{7.5\ \text{mA}} \quad$ or $\quad \dfrac{94}{1\ \text{A}} = \dfrac{V_s}{7.5 \times 10^{-3}\ \text{A}} \qquad \therefore \quad V_s = \mathbf{0.705\ V}$

Example 2.46. *Determine the readings of an ideal voltmeter connected in Fig. 2.76 to (i) terminals a and b, (ii) terminals c and g. The average power dissipated in the 5 Ω resistor is equal to 20 W.*

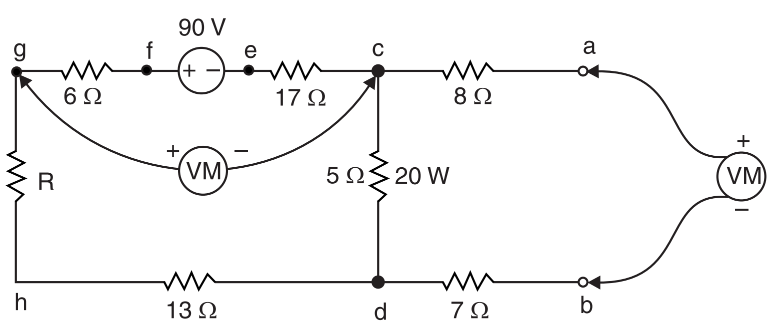


Fig. 2.76

* Note that point *a* is positive w.r.t. point *b*.

Solution. The polarity of 90 V source suggests that point d is positive w.r.t. c. Therefore, current flows from point d to c. The average power in 5 Ω resistor is 20 W so that $V_{dc}^2/5 = 20$. Therefore, $V_{dc} = 10$ V. An *ideal* voltmeter has an infinite resistance and indicates the voltage without drawing any current.

(*i*) Applying *KVL* to loop *acdba,* we have,

$$V_{ac} + V_{cd} + V_{db} + V_{ba} = 0$$

or $$0 + 10 + 0 + V_{ba} = 0 \qquad \therefore \qquad V_{ba} = \mathbf{-10\ V}$$

If the meter is of digital type, it will indicate – 10 V. For moving-coil galvanometer, the leads of voltmeter will be reversed to obtain the reading.

(*ii*) Applying *KVL* to loop *cefgc*, we have,

$$-V_{ce} + V_{ef} - V_{fg} - V_{gc} = 0$$

or $$-17 \times 2 + 90 - 6 \times 2 - V_{gc} = 0 \qquad \therefore \qquad V_{gc} = \mathbf{44\ V}$$

Example 2.47. *Using Kirchhoff's current law and Ohm's law, find the magnitude and polarity of voltage V in Fig. 2.77. Directions of the two current sources are shown.*

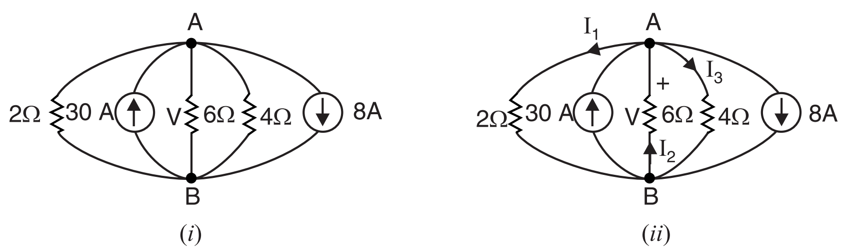


Fig. 2.77

Solution. Let us assign the directions of I_1, I_2 and I_3 and polarity of V as shown in Fig. 2.77 (*ii*). We shall see in the final result whether our assumptions are correct or not. Referring to Fig. 2.77 (*ii*) and applying *KCL* to junction *A*, we have,

$$\text{Incoming currents} = \text{Outgoing currents}$$

or $$I_2 + 30 = I_1 + I_3 + 8$$

$$\therefore \qquad I_1 - I_2 + I_3 = 22 \qquad ...(i)$$

Applying Ohm's law to Fig. 2.77 (*ii*), we have,

$$I_1 = \frac{V}{2} \quad ; \quad I_3 = \frac{V}{4} \quad ; \quad I_2 = -\frac{V}{6}$$

Putting these values of I_1, I_2 and I_3 in eq. (*i*), we have,

$$\frac{V}{2} - \left(-\frac{V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = \mathbf{24\ V}$$

Now $$I_1 = V/2 = 24/2 = 12\text{ A} \quad ; \quad I_2 = -24/6 = -4\text{ A} \quad ; \quad I_3 = 24/4 = 6\text{ A}$$

The negative sign of I_2 indicates that the direction of its flow is opposite to that shown in Fig. 2.77 (*ii*).

Example 2.48. *In the network shown in Fig. 2.78, v_1 = 4 volts ; v_4 = 4 cos 2t and $i_3 = 2e^{-t/3}$. Determine i_2.*

Solution. Voltage across 6 H coil is

$$v_3 = L\frac{di_3}{dt} = L\frac{d}{dt}(i_3)$$

$$= 6\frac{d}{dt}(2e^{-t/3}) = -4e^{-t/3}$$

Applying *KVL* to loop *ABCDA*, we have,

$$-v_1 - v_2 + v_3 + v_4 = 0$$

or $$-4 - v_2 - 4e^{-t/3} + 4\cos 2t = 0$$

$$\therefore \quad v_2 = 4\cos 2t - 4e^{-t/3} - 4$$

Current through 8 F capacitor is

$$i_2 = C\frac{dv_2}{dt} = C\frac{d}{dt}(v_2)$$

$$= 8\frac{d}{dt}(4\cos 2t - 4e^{-t/3} - 4)$$

$$= 8\left(-8\sin 2t + \frac{4}{3}e^{-t/3}\right)$$

$$= \mathbf{-64\sin 2t + \frac{32}{3}e^{-t/3}}$$

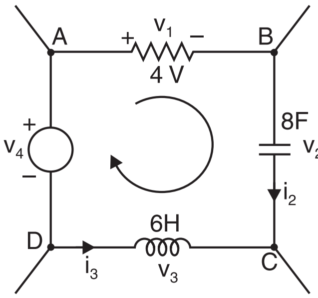


Fig. 2.78

Tutorial Problems

1. Using Kirchhoff's laws, find the current in various resistors in the circuit shown in Fig. 2.79.

[6.574 A, 3.611 A ,10.185 A]

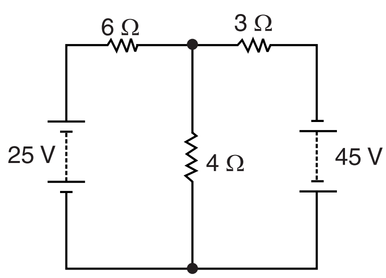


Fig. 2.79

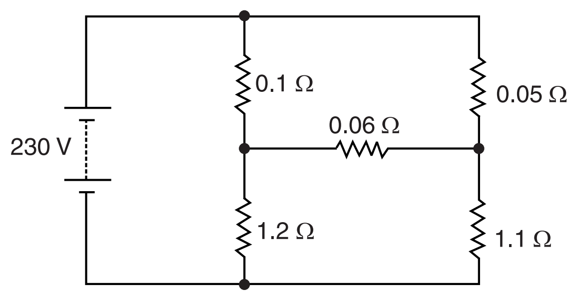


Fig. 2.80

2. For the circuit shown in Fig. 2.80, determine the branch currents using Kirchhoff's laws.

[151.35A, 224.55A, 27.7A , 179.05 A, 196.84 A]

3. Two batteries *A* and *B* having e.m.fs. 12 V and 8 V respectively and internal resistances of 2 Ω and 1 Ω respectively, are connected in parallel across 10 Ω resistor. Calculate (*i*) the current in each of the batteries and the external resistor and (*ii*) p.d. across external resistor.

[(*i*) I_A = 1.625 A *discharge* ; I_B = 0.75 A *charge*; 0.875 A (*ii*) 8.75 V]

4. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 20 Ω, *BC* = 5 Ω, *CD* = 4 Ω and *DA* = 10 Ω. A galvanometer of resistance 6Ω is connected between *B* and *D*. A 100-volt supply of negligible resistance is connected between *A* and *C* with *A* positive. Find the magnitude and direction of galvanometer current.

[0.667 A *from D to B*]

5. A network *ABCD* consists of the following resistors : *AB* = 5 kΩ, *BC* = 10 kΩ, *CD* = 15 kΩ and *DA* = 20 kΩ. A fifth resistor of 10 kΩ is connected between *A* and *C*. A dry battery of e.m.f. 120 V and internal resistance 500Ω is connected across the resistor *AD*. Calculate (*i*) the total current supplied by the battery, (*ii*) the p.d. across points *C* and *D* and (*iii*) the magnitude and direction of current through branch *AC*. **[(*i*) 11.17 mA (*ii*) 81.72 V (*iii*) 3.27 mA *from A to C*]**

6. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 10 Ω, *BC* = 30 Ω, *CD* = 15Ω and *DA* = 20Ω. A 2 volt battery of internal resistance 2Ω is connected between *A* and *C* with *A* positive. A galvanometer of resistance 40Ω is connected between *B* and *D*. Find the magnitude and direction of galvanometer current. **[11.5 mA *from B to D*]**

7. Two batteries E_1 and E_2 having *e.m.fs* 6 V and 2 V respectively and internal resistances of 2 Ω and 3 Ω respectively are connected in parallel across a 5 Ω resistor. Calculate (*i*) current through each battery and (*ii*) terminal voltage. **[(*i*) 1.23A; –0.52A (*ii*) 3.55V]**

8. Calculate the current in 20 Ω resistor in Fig. 2.81. **[26.67 mA]**

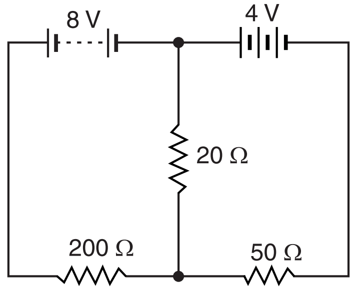


Fig. 2.81

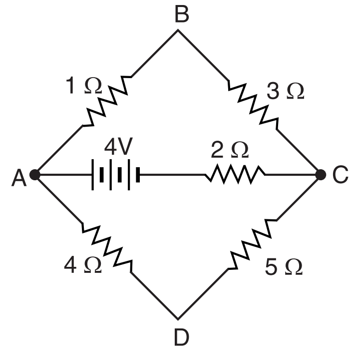


Fig. 2.82

9. In the circuit shown in Fig. 2.82, find the current in each branch and the current in the battery. What is the p.d. between *A* and *C* ?

[Branch *ABC* = 0.581 A ; Branch *ADC* = 0.258 A ; Branch AC = 0.839 A ; V_{AC} = 2.32 V]

10. Two batteries *A* and *B* having e.m.f.s of 20 V and 21 V respectively and internal resistances of 0.8 Ω and 0.2 Ω respectively, are connected in parallel across 50 Ω resistor. Calculate (*i*) the current through each battery and (*ii*) the terminal voltage. **[(*i*) Battery *A* = 0.4725 A ; Battery *B* = 0.0714 A (*ii*) 20 V]**

11. A battery having an e.m.f. of 10 V and internal resistance 0.01 Ω is connected in parallel with a second battery of e.m.f. 10 V and internal resistance 0.008 Ω. The two batteries in parallel are properly connected for charging from a d.c. supply of 20 V through a 0.9 Ω resistor. Calculate the current taken by each battery and the current from the supply. **[4.91 A, 6.14 A, 10.05 A]**

12. Find i_x and v_x in the network shown in Fig. 2.83. **[i_x = – 5 A; v_x = – 15 V]**

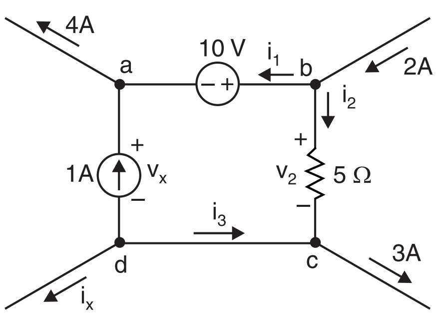


Fig. 2.83

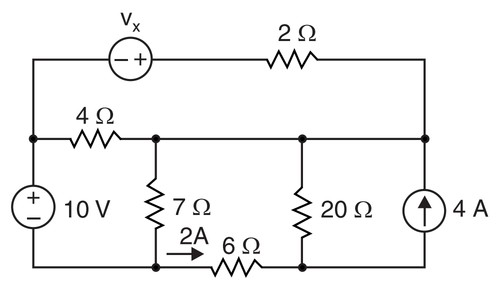


Fig. 2.84

13. Find v_x for the network shown in Fig. 2.84. **[31 V]**

14. Find *i* and v_{ab} for the network shown in Fig. 2.85. **[3 A ; 19 V]**

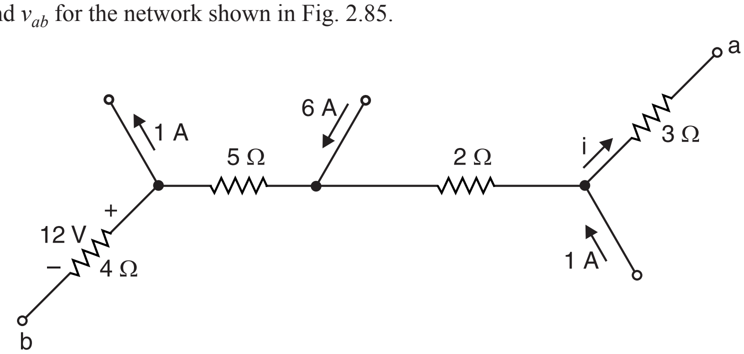


Fig. 2.85

2.22. Voltage and Current Sources

The term *voltage source* is used to describe a source of energy which establishes a potential difference across its terminals. Most of the sources encountered in everyday life are voltage sources *e.g.*, batteries, d.c. generators, alternators etc. The term *current source* is used to describe a source of energy that provides a current *e.g.*, collector circuits of transistors. Voltage and current sources are called active elements because they provide electrical energy to a circuit.

Unlike a voltage source, which we can imagine as two oppositely charged electrodes, it is difficult to visualise the structure of a current source. However, as we will learn in later sections, a real current source can always be converted into a real voltage source. In other words, we can regard a current source as a convenient fiction that aids in solving circuit problems, yet we feel secure in the knowledge that the current source can be replaced by the equivalent voltage source, if so desired.

2.23. Ideal Voltage Source or Constant-Voltage Source

An **ideal voltage source** (*also called constant-voltage source*) *is one that maintains a constant terminal voltage, no matter how much current is drawn from it.*

An ideal voltage source has zero internal resistance. Therefore, it would provide constant terminal voltage regardless of the value of load connected across its terminals. For example, an ideal 12V source would maintain 12V across its terminals when a 1 MΩ resistor is connected (so I = 12 V/1 MΩ = 12A) as well as when a 1 kΩ resistor is connected (I = 12 mA) or when a 1 Ω resistor is connected (I = 12A). This is illustrated in Fig. 2.86.

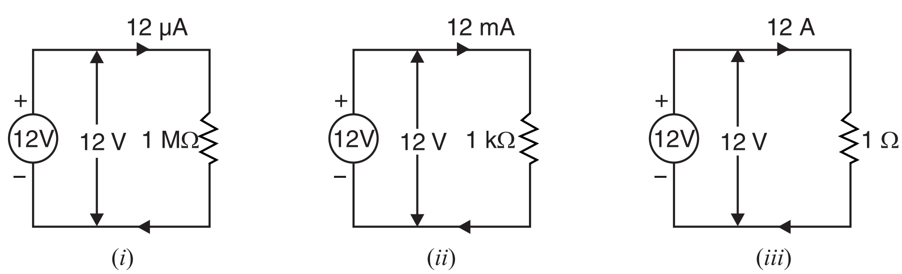


Fig. 2.86

It is not possible to construct an ideal voltage source because every voltage source has some internal resistance that causes the terminal voltage to fall due to the flow of current. *However, if the internal resistance of voltage source is very small, it can be considered as a constant voltage source.* This is illustrated in Fig. 2.87. It has a d.c. source of 6 V with an internal resistance R_i = 0.005 Ω. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i (= 0.005 Ω) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 and 5.95 volts. This can be considered constant voltage compared with wide variations in load current. The practical example of a constant voltage source is the lead-acid cell. The internal resistance of lead-acid cell is very small (about 0.01 Ω) so that it can be regarded as a constant voltage source for all practical purposes. A constant voltage source is represented by the symbol shown in Fig. 2.88.

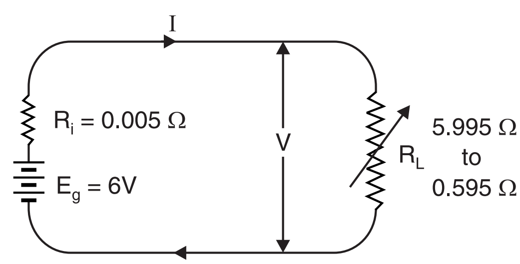


Fig. 2.87

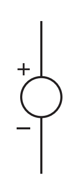


Fig. 2.88

2.24. Real Voltage Source

A real or non-ideal voltage source has low but finite internal resistance (R_{int}) that causes its terminal voltage to decrease when load current is increased and *vice-versa. A* **real voltage source** *can be represented as an ideal voltage source in series with a resistance equal to its internal resistance* (R_{int}) *as shown in Fig. 2.89.*

When load R_L is connected across the terminals of a real voltage source, a load current I_L flows through the circuit so that output voltage V_o is given by ;

$$V_o = E - I_L R_{int}$$

Here E is the voltage of the ideal voltage source *i.e.*, it is the potential difference between the terminals of the source when no current (*i.e.*, $I_L = 0$) is drawn. Fig. 2.90 shows the graph of output voltage V_o versus load current I_L of a real or non-ideal voltage source.

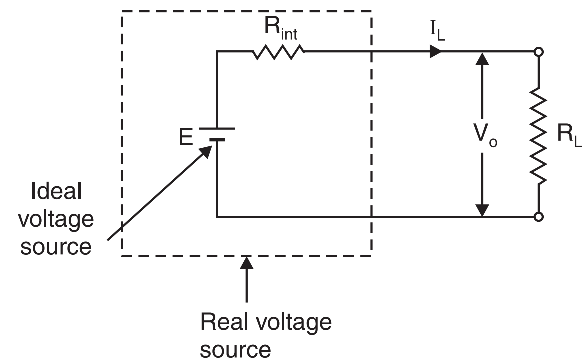


Fig. 2.89

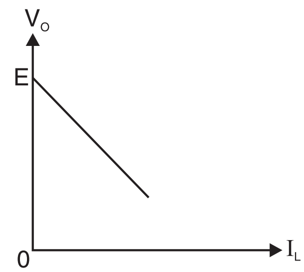


Fig. 2.90

As R_{int} becomes smaller, the real voltage source more closely approaches the ideal voltage source. Sometimes it is convenient when analysing electric circuits to assume that a real voltage source behaves like an ideal voltage source. This assumption is justified by the fact that in circuit analysis, we are not usually concerned with changing currents over a wide range of values.

2.25. Ideal Current Source

An **ideal current source** *or* **constant current source** *is one which will supply the same current to any resistance* (*load*) *connected across its terminals.*

An ideal current source has infinite internal resistance. Therefore, it supplies the same current to any resistance connected across its terminals. This is illustrated in Fig. 2.91. The symbol for ideal current source is shown in Fig. 2.92. The arrow shows the direction of current (conventional) produced by the current source.

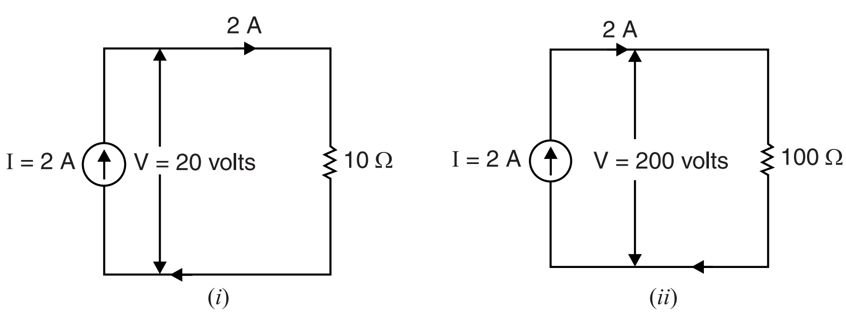


Fig. 2.91

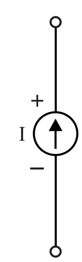


Fig. 2.92

Since an ideal current source supplies the same current regardless of the value of resistance connected across its terminals, it is clear that the terminal voltage V of the current source will depend on the value of load resistance. For example, if a 2 A current source has 10 Ω across its terminals, then terminal voltage of the source is $V = 2\text{ A} \times 10\ \Omega = 20$ volts. If load resistance is changed to 100 Ω, then terminal voltage of the current source becomes $V = 2\text{ A} \times 100\ \Omega = 200$ volts. This is illustrated in Fig. 2.91.

2.26. Real Current Source

A real or non-ideal current source has high but finite internal resistance (R_{int}). Therefore, the load current (I_L) will change as the value of load resistance (R_L) changes. *A* **real current source** *can be represented by an ideal current source* (I) *in parallel with its internal resistance* (R_{int}) *as shown in Fig. 2.93*. When load resistance R_L is connected across the terminals of the real current source, the load current I_L is equal to the current I from the ideal current source *minus* that part of the current that passes through the parallel internal resistance (R_{int}) *i.e.*,

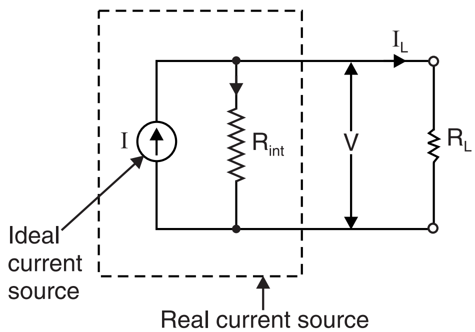


Fig. 2.93

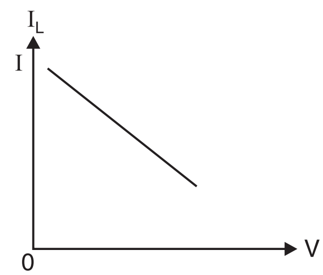


Fig. 2.94

$$I_L = I - \frac{V}{R_{int}}$$

where V = P.D. between output terminals

Fig. 2.94 shows the graph of load current I_L versus output voltage V of a real current source.

Note that load current I_L is less than it would be if the source were ideal. As the internal resistance of real current source becomes greater, the current source more closely approaches the ideal current source.

Note. Current sources in parallel add *algebraically*. If two current sources are supplying currents in the same direction, their equivalent current source supplies current equal to the sum of the individual currents. If two current sources are supplying currents in the opposite directions, their equivalent current source supplies a current equal to the difference of the currents of the two sources.

2.27. Source Conversion

A real voltage source can be converted to an *equivalent* real current source and *vice-versa*. When the conversion is made, the sources are equivalent in every sense of the word; it is impossible to make any measurement or perform any test at the external terminals that would reveal whether the source is a voltage source or its equivalent current source.

(*i*) Voltage to current source conversion. Let us see how a real voltage source can be converted to an equivalent current source. We know that a real voltage source can be represented by constant voltage E in series with its internal resistance R_{int} as shown in Fig. 2.95 (*i*).

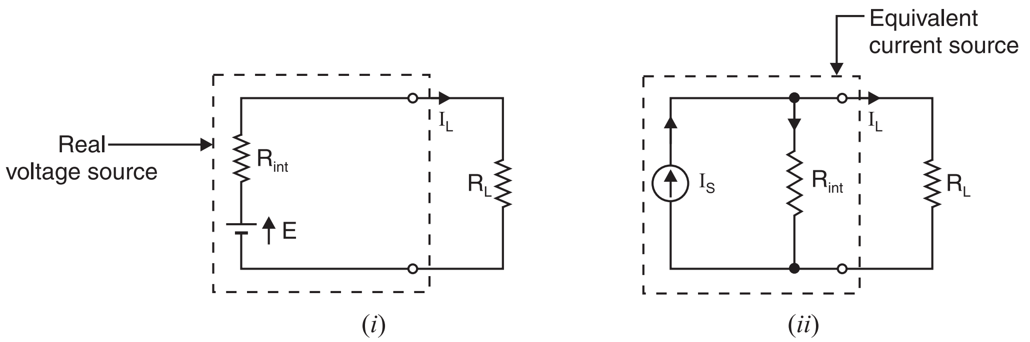


Fig. 2.95

It is clear from Fig. 2.95 (*i*) that load current I_L is given by ;

$$I_L = \frac{E}{R_{int} + R_L} = \frac{\dfrac{E}{R_{int}}}{\dfrac{R_{int} + R_L}{R_{int}}} = \frac{E}{R_{int}} \times \frac{R_{int}}{R_{int} + R_L}$$

$$\therefore \qquad I_L = I_S \times \frac{R_{int}}{R_{int} + R_L} \qquad \text{...}(i)$$

where $I_S = \dfrac{E*}{R_{int}}$

= Current which would flow in a short circuit across the output terminals of voltage source in Fig. 2.95 (*i*)

From eq. (*i*), the voltage source appears as a current source of current I_S which is dividing between the internal resistance R_{int} and load resistance R_L connected in parallel as shown in Fig. 2.95 (*ii*). Thus the current source shown in Fig. 2.95 (*ii*) (dotted box) is equivalent to the real voltage source shown in Fig. 2.95 (*i*) (dotted box).

Thus a real voltage source of constant voltage E and internal resistance R_{int} is equivalent to a current source of current $I_S = E/R_{int}$ and R_{int} in parallel with current source.

Note that internal resistance of the equivalent current source has the same value as the internal resistance of the original voltage source but is in parallel with current source. The two circuits shown in Fig. 2.95 are equivalent and either can be used for circuit analysis.

(*ii*) Current to voltage source conversion. Fig. 2.96 (*i*) shows a real current source whereas Fig. 2.96 (*ii*) shows its equivalent voltage source. Note that series resistance R_{int} of the voltage source

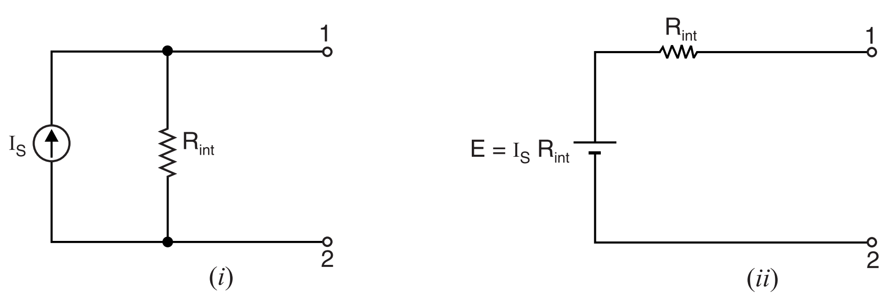


Fig. 2.96

* The source voltage is E and its internal resistance is R_{int}. Therefore, E/R_{int} is the current that would flow when source terminals in Fig. 2.95 (*i*) are shorted.

has the same value as the parallel resistance of the original current source. The value of voltage of the equivalent voltage source is $E = I_S R_{int}$ where I_S is the magnitude of current of the current source.

Note that the two circuits shown in Fig. 2.96 are equivalent and either can be used for circuit analysis.

Note. The source conversion (voltage source into equivalent current source and vice-versa) often simplifies the analysis of many circuits. Any resistance that is in series with a voltage source, whether it be internal or external resistance, can be included in its conversion to an equivalent current source. Similarly, any resistance in parallel with current source can be included when it is converted to an equivalent voltage source.

Example 2.49. *Show that the equivalent sources shown in Fig. 2.97 have exactly the same terminal voltage and produce exactly the same external current when the terminals (i) are shorted, (ii) are open and (iii) have a 500 Ω load connected.*

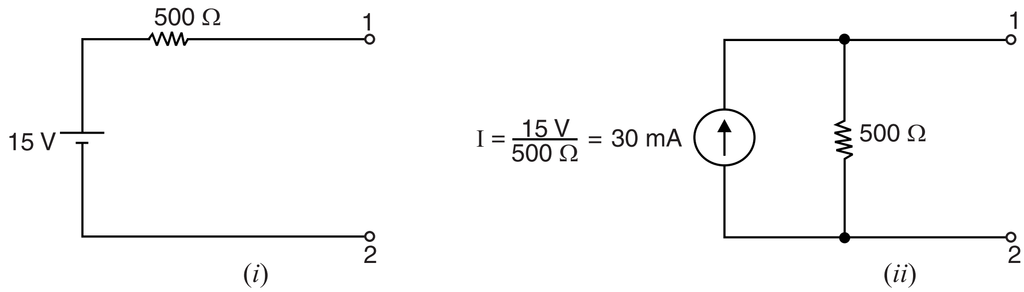


Fig. 2.97

Solution. Fig 2.97 (*i*) shows a voltage source whereas Fig. 2.97 (*ii*) shows its equivalent current source.

(*i*) When terminals are shorted. Referring to Fig. 2.98, the terminal voltage is 0 V in both circuits because the terminals are shorted.

$$I_L = \frac{15 \text{ V}}{500 \ \Omega} = 30 \text{ mA ... voltage source}$$

$$I_L = 30 \text{ mA ...current source}$$

Note that in case of current source, 30 mA flows in the shorted terminals because the short diverts all of the source current around the 500 Ω resistor.

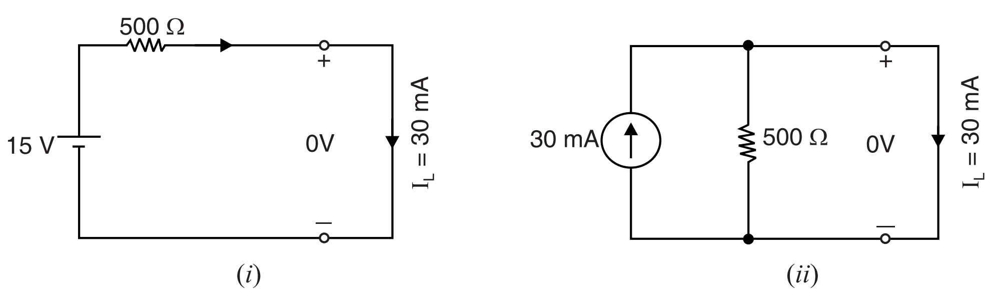


Fig. 2.98

(*ii*) When the terminals are open. Referring to Fig. 2.99 (*i*), the voltage across the open terminals of voltage source is 15 V because no current flows and there is no voltage drop across 500 Ω resistor. Referring to Fig. 2.99 (*ii*), the voltage across the open terminals of the current source is also 15 V ; $V = 30 \text{ mA} \times 500 \ \Omega = 15$ V. The current flowing from one terminal into the other is zero in both cases because the terminals are open.

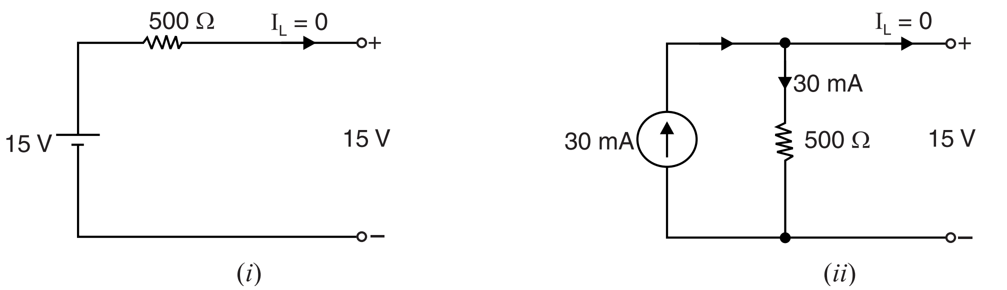


Fig. 2.99

(*iii*) Terminals have a 500 Ω load connected.

(*a*) Voltage source. Referring to Fig. 2.100 (*i*),

$$\text{Current in } R_L,\ I_L = \frac{15\ \text{V}}{(500+500)\ \Omega} = 15\ \text{mA}$$

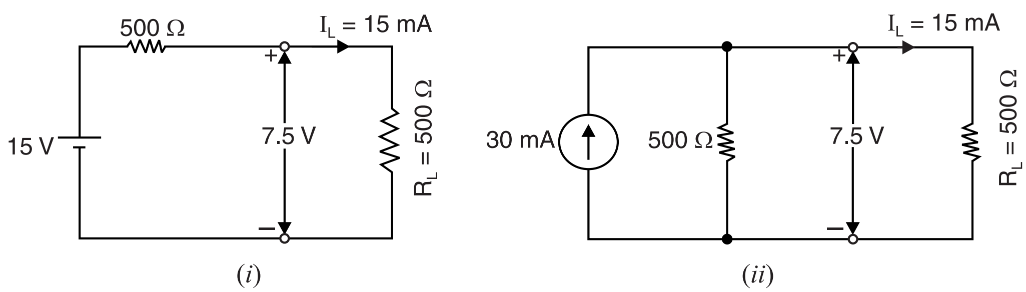


Fig. 2.100

Terminal voltage of source, $V = I_L R_L = 15\ \text{mA} \times 500\ \Omega = 7.5\ \text{V}$

(*b*) Current source. Referring to Fig. 2.100 (*ii*),

$$\text{Current in } R_L,\ I_L = 30 \times \frac{500}{500+500} = 15\ \text{mA}$$

$$\text{Terminal voltage of source} = I_L R_L = 15\ \text{mA} \times 500\ \Omega = 7.5\ \text{V}$$

We conclude that equivalent sources produce exactly the same voltages and currents at their external terminals, no matter what the load and that they are therefore indistinguishable.

Example 2.50. *Find the current in 6 kΩ resistor in Fig. 2.101* (*i*) *by converting the current source to a voltage source.*

Solution. Since we want to find the current in 6 kΩ resistor, we use 3 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.101 (*ii*), the equivalent voltage is

$$E = 15\ \text{mA} \times 3\ \text{k}\Omega = 45\ \text{V}$$

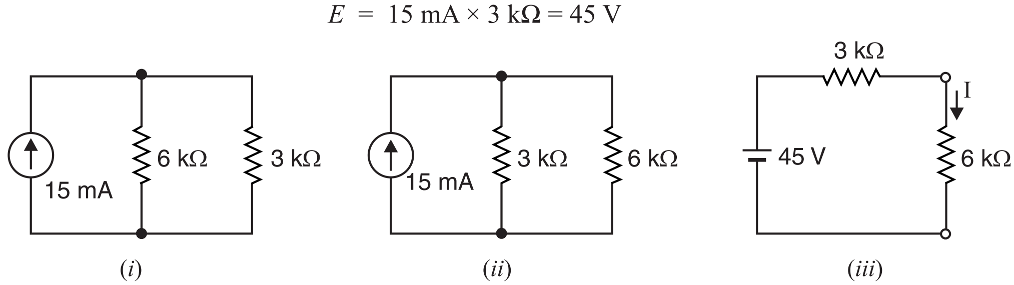


Fig. 2.101

The circuit then becomes as shown in Fig. 2.101 (*iii*). Note that polarity of the equivalent voltage source is such that it produces current in the same direction as the original current source.

Referring to Fig. 2.101 (*iii*), the current in 6 kΩ is

$$I = \frac{45\text{ V}}{(3+6)\text{ k}\Omega} = \mathbf{5\text{ mA}}$$

In the series circuit shown in Fig. 2.101 (*iii*), it would appear that current in 3 kΩ resistor is also 5 mA. However, 3 kΩ resistor was involved in source conversion, so we *cannot* conclude that there is 5 mA in the 3 kΩ resistor of the original circuit [See Fig. 2.101 (*i*)]. Verify that the current in the 3 kΩ resistor in that circuit is, in fact, 10 mA.

Example 2.51. *Find the current in the 3 kΩ resistor in Fig. 2.101* (*i*) *above by converting the current source to a voltage source.*

Solution. The circuit shown in Fig. 2.101 (*i*) is redrawn in Fig. 2.102 (*i*). Since we want to find the current in 3 kΩ resistor, we use 6 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.102 (*i*), the equivalent voltage is

$$E = 15\text{ mA} \times 6\text{ k}\Omega = 90\text{ V}$$

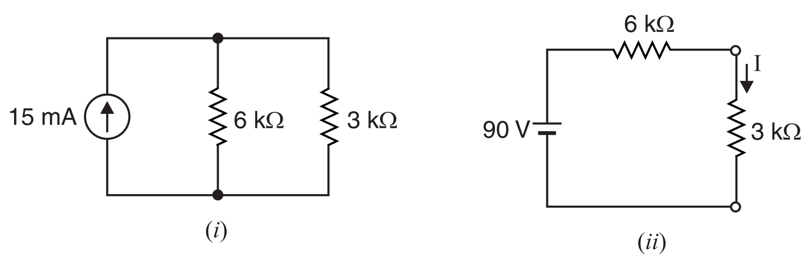


Fig. 2.102

The circuit then reduces to that shown in Fig. 2.102 (*ii*). The current in 3 kΩ resistor is

$$I = \frac{90\text{ V}}{(6+3)\text{k}\Omega} = \frac{90\text{ V}}{9\text{ k}\Omega} = \mathbf{10\text{ mA}}$$

Example 2.52. *Find the current in various resistors in the circuit shown in Fig. 2.103* (*i*) *by converting voltage sources into current sources.*

Solution. Referring to Fig. 2.103 (*i*), the 100 Ω resistor can be considered as the internal resistance of 15 V battery. The equivalent current is

$$I = \frac{15\text{ V}}{100\ \Omega} = 0.15\text{ A}$$

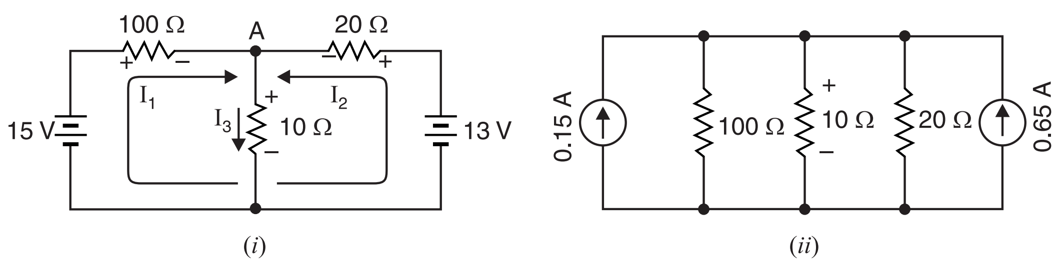


Fig. 2.103

Similarly, 20 Ω resistor can be considered as the internal resistance of 13 V battery. The equivalent current is

$$I = \frac{13\text{ V}}{20\ \Omega} = 0.65\text{ A}$$

Replacing the voltage sources with current sources, the circuit becomes as shown in Fig. 2.103 (*ii*). The current sources are parallel-aiding for a total flow = 0.15 + 0.65 = 0.8 A. The parallel resistors can be combined.

$$100\ \Omega \parallel 10\ \Omega \parallel 20\ \Omega = 6.25\ \Omega$$

The total current flowing through this resistance produces the drop :

$$0.8\text{ A} \times 6.25\ \Omega = 5\text{ V}$$

This 5 V drop can now be "transported" back to the original circuit. It appears across 10 Ω resistor [See Fig. 2.104]. Its polarity is negative at the bottom and positive at the top. Applying Kirchhoff's voltage law (KVL), the voltage drop across 100 Ω resistor = 15 – 5 = 10 V and drop across 20 Ω resistor = 13 – 5 = 8 V.

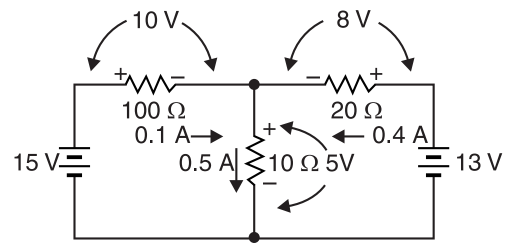


Fig. 2.104

$$\therefore \quad \text{Current in 100 Ω resistor} = \frac{10}{100} = \mathbf{0.1\ A}$$

$$\text{Current in 10 Ω resistor} = \frac{5}{10} = \mathbf{0.5\ A}$$

$$\text{Current in 20 Ω resistor} = \frac{8}{20} = \mathbf{0.4\ A}$$

Example 2.53. *Find the current in and voltage across 2 Ω resistor in Fig. 2.105.*

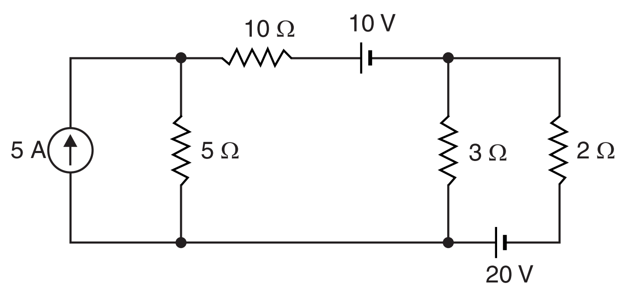


Fig. 2.105

Solution. We use 5 Ω resistor to convert the current source to an equivalent voltage source. The equivalent voltage is

$$E = 5\text{ A} \times 5\ \Omega = 25\text{ V}$$

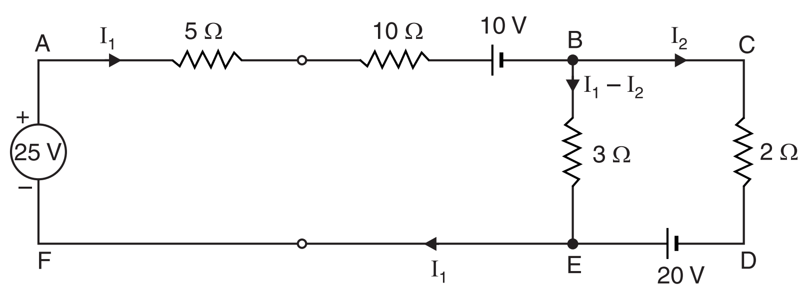


Fig. 2.106

The circuit shown in Fig. 2.105 then becomes as shown in Fig. 2.106.

Loop *ABEFA*. Applying Kirchhoff's voltage law to loop *ABEFA*, we have,

$$-5\,I_1 - 10\,I_1 - 10 - 3\,(I_1 - I_2) + 25 = 0$$

or $$-18\,I_1 + 3\,I_2 = -15 \qquad ...(i)$$

Loop *BCDEB*. Applying Kirchhoff's voltage law to loop *BCDEB*, we have,

$$-2\,I_2 + 20 + 3\,(I_1 - I_2) = 0$$

or $$3\,I_1 - 5\,I_2 = -20 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get, $I_2 = 5$ A.

∴ Current through 2 Ω resistor = I_2 = **5 A**

Voltage across 2 Ω resistor = $I_2 \times 2 = 5 \times 2$ = **10 V**

Example 2.54. *Find the current in 28 Ω resistor in the circuit shown in Fig. 2.107.*

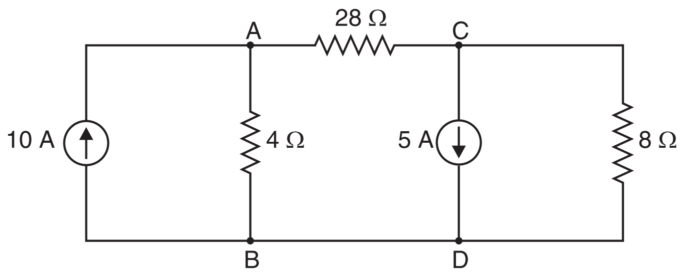


Fig. 2.107

Solution. The two current sources cannot be combined together because 28 Ω resistor is present between points *A* and *C*. However, this difficulty is overcome by converting current sources into equivalent voltage sources. Now 10 A current source in parallel with 4 Ω resistor can be converted into equivalent voltage source of voltage = 10 A × 4 Ω = 40 V in series with 4 Ω resistor as shown in Fig. 2.108 (*i*). Note that polarity of the equivalent voltage source is such that it provides current in the same direction as the original current source.

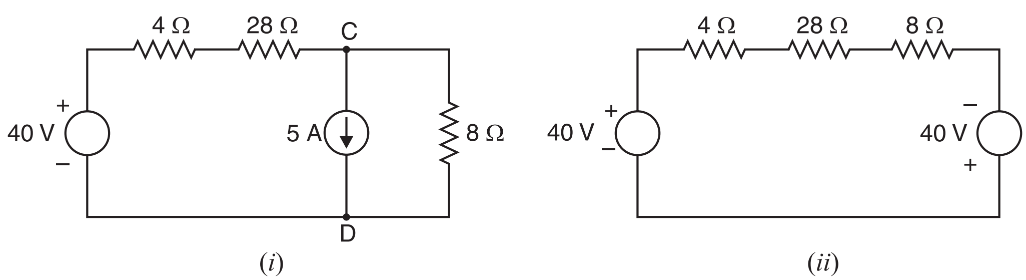


Fig. 2.108

Similarly, 5 A current source in parallel with 8 Ω resistor can be converted into equivalent voltage source of voltage = 5 A × 8 Ω = 40 V in series with 8 Ω resistor. The circuit then becomes as shown in Fig. 2.108 (*ii*). Note that polarity of the voltage source is such that it provides current in the same direction as the original current source. Referring to Fig. 2.108 (*ii*),

$$\text{Total circuit resistance} = 4 + 28 + 8 = 40\ \Omega$$

$$\text{Total voltage} = 40 + 40 = 80\ \text{V}$$

$$\therefore \quad \text{Current in 28 Ω resistor} = \frac{80}{40} = \mathbf{2\ A}$$

Example 2.55. *Using source conversion technique, find the load current I_L in the circuit shown in Fig. 2.109 (i).*

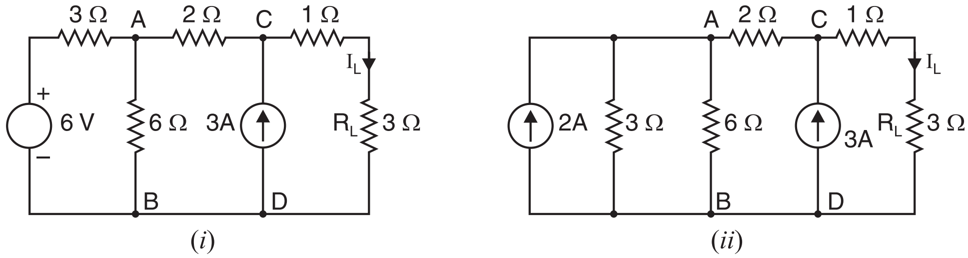


Fig. 2.109

Solution. We first convert 6 V source in series with 3 Ω resistor into equivalent current source of current = 6 V/3 Ω = 2 A in parallel with 3 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*ii*). Note that polarity of current source is such that it provides current in the same direction as the original voltage source. In Fig. 2.109 (*ii*), 3 Ω and 6 Ω resistors are in parallel and their equivalent resistance = (3 × 6)/3 + 6 = 2 Ω. Therefore, circuit of Fig. 2.109 (*ii*) reduces to the one shown in Fig. 2.109 (*iii*).

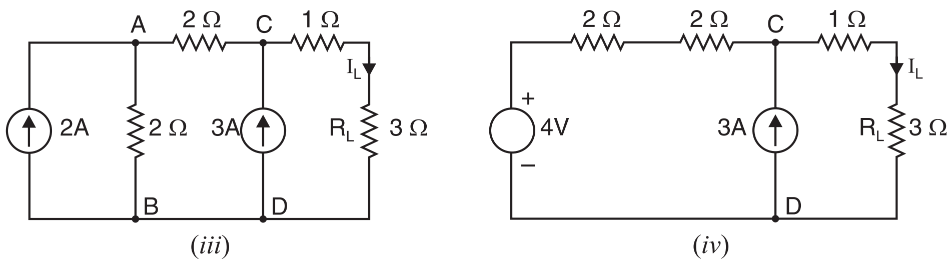


Fig. 2.109

In Fig. 2.109 (*iii*), we now convert 2 A current source in parallel with 2 Ω resistor into equivalent voltage source of voltage = 2 A × 2 Ω = 4 V in series with 2 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*iv*). The polarity of voltage source is marked correctly. In Fig. 2.109 (*iv*), we convert 4 V source in series with 2 + 2 = 4 Ω resistor into equivalent current source of current = 4 V/4 Ω = 1 A in parallel with 4 Ω resistor as shown in Fig. 2.109 (*v*). Note that direction of current of current source is shown correctly.

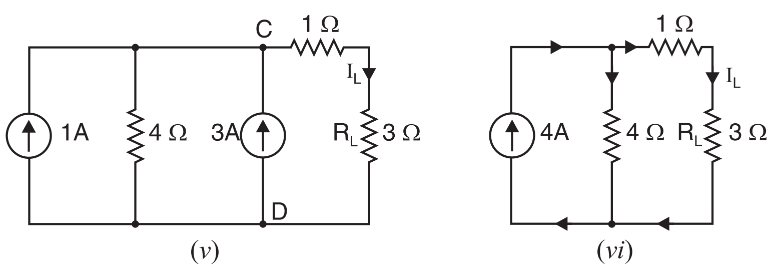


Fig. 2.109

In Fig. 2.109 (*v*), the two current sources can be combined together to give resultant current source of 3 + 1 = 4 A. The circuit then becomes as shown in Fig. 2.109 (*vi*). Referring to Fig. 2.109 (*vi*) and applying current-divider rule,

$$\text{Load current, } I_L = 4 \times \frac{4}{(3+1)+4} = \mathbf{2\ A}$$

Tutorial Problems

1. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.110. **[20 V]**

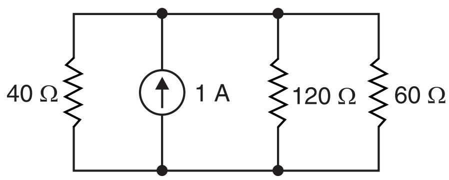


Fig. 2.110

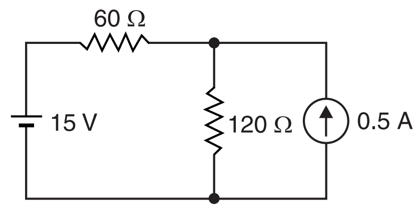


Fig. 2.111

2. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.111. **[30 V]**

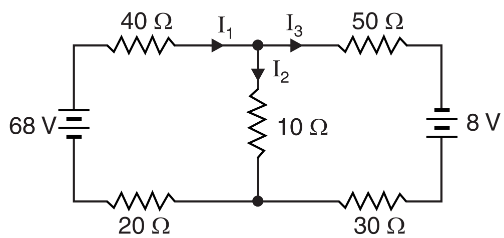


Fig. 2.112

3. By performing an appropriate source conversion, find the currents I_1, I_2 and I_3 in the circuit shown in Fig. 2.112. **[I_1 = 1 A; I_2 = 0.2 A; I_3 = 0.8 A]**

2.28. Independent Voltage and Current Sources

So far we have been dealing with independent voltage and current sources. We now give brief description about these two active elements.

(*i*) **Independent voltage source.** *An independent voltage source is a two-terminal element (e.g. a battery, a generator etc.) that maintains a specified voltage between its terminals.*

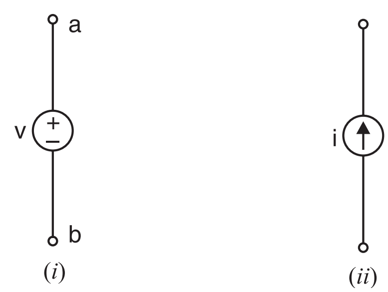


Fig. 2.113

An independent voltage source provides a voltage independent of any other voltage or current. The symbol for independent voltage source having *v* volts across its terminals is shown in Fig. 2.113. (*i*). As shown, the terminal *a* is *v* volts above terminal *b*. If *v* is greater than zero, then terminal *a* is at a higher potential than terminal *b*. In Fig. 2.113 (*i*), the voltage *v* may be time varying or it may be constant in which case we label it *V*.

(*ii*) **Independent current source.** *An independent current source is a two-terminal element through which a specified current flows.*

An independent current source provides a current that is completely independent of the voltage across the source. The symbol for an independent current source is shown in Fig. 2.113 (*ii*) where *i* is the specified current. The direction of the current is indicated by the arrow. In Fig. 2.113 (*ii*), the current *i* may be time varying or it may be constant in which case we label it *I*.

2.29. Dependent Voltage and Current Sources

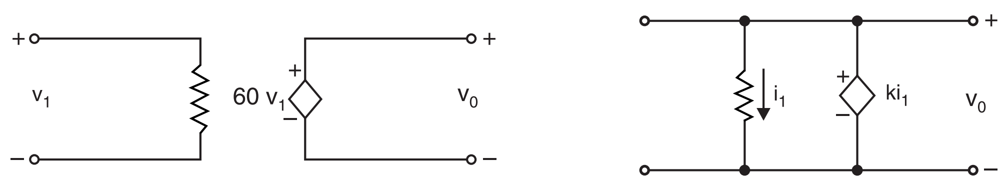
A dependent source provides a voltage or current between its output terminals which depends upon another variable such as voltage or current.

For example, a voltage amplifier can be considered to be a dependent voltage source. It is because the output voltage of the amplifier depends upon another voltage *i.e.* the input voltage to the amplifier. A dependent source is represented by a *diamond-shaped symbol as shown in the figures below. There are four possible dependent sources :

(*i*) Voltage-dependent voltage source (*ii*) Current-dependent voltage source

(*iii*) Voltage-dependent current source (*iv*) Current-dependent current source

(*i*) Voltage-dependent voltage source. *A voltage-dependent voltage source is one whose output voltage (v_0) depends upon or is controlled by an input voltage (v_1).* Fig. 2.114 (*i*) shows a voltage-dependent voltage source. Thus if in Fig. 2.114 (*i*), v_1 = 20 mV, then v_0 = 60 × 20 mV = 1.2 V. If v_1 changes to 30 mV, then v_0 changes to 60 × 30 mV = 1.8 V. Note that the constant (60) that multiplies v_1 is dimensionless.

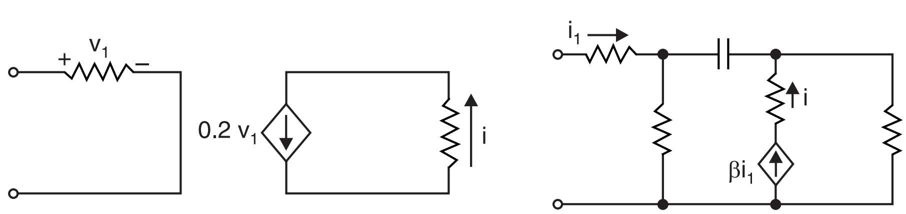


A voltage-dependent voltage source (*i*) A current-dependent voltage source (*ii*)

Fig. 2.114

(*ii*) Current-dependent voltage source. *A current-dependent voltage source is one whose output voltage (v_0) depends on or is controlled by an input current (i_1).* Fig. 2.114 (*ii*) shows a current-dependent voltage source. Note that the controlling current i_1 is in the same circuit as the controlled source itself. The constant that multiplies the value of voltage produced by the controlled source is sometimes designated by a letter k or β. Note that the constant k has the dimensions of V/A or ohm. Thus if i_1 = 50 μA and constant k is 0.5 V/A, then $v_0 = 50 \times 10^{-6} \times 0.5 = 25$ μV.

(*iii*) Voltage-dependent current source. *A voltage-dependent current source is one whose output current (i) depends upon or is controlled by an input voltage (v_1).* Fig. 2.115 (*i*) shows a voltage-dependent current source. The constant that multiplies the value of voltage v_1 has the dimensions of *A/V i.e.* mho or siemen. For example, in Fig. 2.115. (*i*), if the constant is 0.2 siemen and if input voltage v_1 is 10 mV, then the output current i = 0.2 S × 10 mV = 2 mA.



A voltage-dependent current source (*i*) A current-dependent current source (*ii*)

Fig. 2.115

* So as not to confuse with the symbol of independent source.

(*iv*) **Current-dependent current source.** *A current-dependent current source is one whose output current* (*i*) *depends upon or is controlled by an input current* (i_1). Fig. 2.115 (*ii*) shows a current-dependent current source. Note that controlling current i_1 is in the same circuit as the controlled source itself. The constant (β) that multiplies the value of current produced by the controlled source is dimensionless. Thus in Fig. 2.115 (*ii*), if $i_1 = 50$ μA and if constant β equals 100, then the current produced by the controlled current source is $i = 100 \times 50$ μA = 5 mA. If i_1 changes to 20 μA, then i changes to $i = 100 \times 20$ μA = 2 mA.

2.30. Circuits With Dependent-Sources

Fig. 2.116 shows the circuit that has an independent source, a dependent-source and two resistors. The dependent-source is a voltage source controlled by the current i_1. The constant for the dependent-source is 0.5 V/A. Dependent sources are essential components in amplifier circuits. Circuits containing dependent-sources are analysed in the same manner as those without dependent-sources. That is, Ohm's law for resistors and Kirchhoff's voltage and current laws apply, as well as the concepts of equivalent resistance and voltage and current division. We shall solve a few examples by way of illustration.

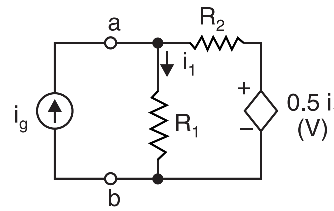


Fig. 2.116

Example 2.56. *Find the value of v in the circuit shown in Fig. 2.117. What is the value of dependent-current source ?*

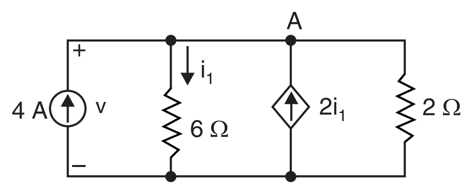


Fig. 2.117

Solution. By applying *KCL* to node* *A* in Fig. 2.117, we get,

$$4 - i_1 + 2i_1 = \frac{v}{2} \qquad ...(i)$$

By Ohm's law, $\quad i_1 = \dfrac{v}{6}$

Putting $i_1 = v/6$ in eq. (*i*), we get,

$$4 - \frac{v}{6} + \frac{2v}{6} = \frac{v}{2} \qquad \therefore \quad v = \mathbf{12\ V}$$

Value of dependent-current source = $=2\, i_1 = \dfrac{2v}{6} = \dfrac{2\times 12}{6} = \mathbf{4\ A}$

Example 2.57. *Find the values of v, i_1 and i_2 in the circuit shown in Fig. 2.118 (i) which contains a voltage-dependent current source. Resistance values are in ohms.*

Solution. Applying *KCL* to node *A* in Fig. 2.118 (*i*), we get,

$$2 - i_1 + 4v = i_2 \qquad ...(i)$$

Now $\quad$ By Ohm's law, $i_1 = \dfrac{v}{3}$ and $i_2 = \dfrac{v}{6}$

Putting $\quad i_1 = \dfrac{v}{3}$ and $i_2 = \dfrac{v}{6}$ in eq. (*i*), we get,

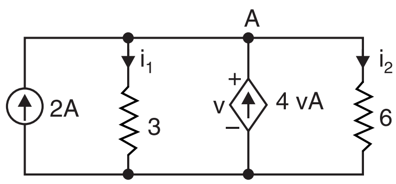
$$2 - \frac{v}{3} + 4v = \frac{v}{6} \quad \therefore \quad v = \mathbf{\frac{-4}{7}\ V}$$

$$\therefore \qquad i_1 = \frac{v}{3} = \frac{1}{3}\times v = \frac{1}{3}\times\frac{-4}{7} = \mathbf{\frac{-4}{21}\ A}$$

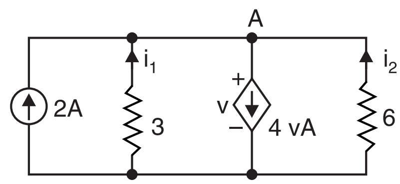
* A node of a network is an equipotential surface at which *two or more* circuit elements are joined.

$$\therefore \qquad i_2 = \frac{v}{6} = \frac{1}{6} \times v = \frac{1}{6} \times \frac{-4}{7} = \mathbf{\frac{-2}{21} \text{ A}}$$

Value of dependent current source $= 4v = 4 \times \frac{-4}{7} = \frac{-16}{7}$ A



(*i*)



(*ii*)

Fig. 2.118

Since the value of i_1, i_2 comes out to be negative, it means that directions of flow of currents are opposite to that assigned in Fig. 2.118. (*i*). The same is the case for current source. The actual directions are shown in Fig. 2.118 (*ii*).

Example 2.58. *Find the value of i in the circuit shown in Fig. 2.119 if R = 10* Ω.

Solution. Applying *KVL* to the loop *ABEFA*, we have,

$$5 - 10\, i_1 + 5\, i_1 = 0 \qquad \therefore \quad i_1 = 1 \text{ A}$$

Applying *KVL* to the loop *BCDEB*, we have,

$$10\, i - 25 - 5\, i_1 = 0$$

or $$10\, i - 25 - 5 = 0 \qquad \therefore \quad i = \mathbf{3\ A}$$

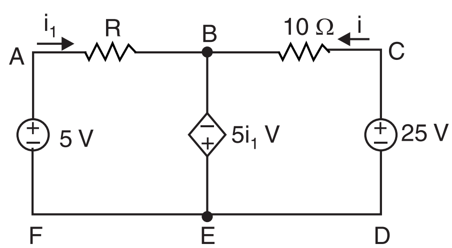


Fig. 2.119

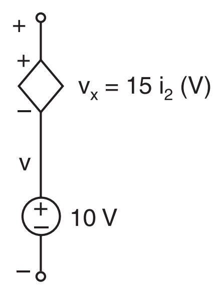


Fig. 2.120

Example 2.59. *Find the voltage v in the branch shown in Fig. 2.120. for (i) $i_2 = 1$ A, (ii) $i_2 = -2$ A and (iii) $i_2 = 0$A.*

Solution. The voltage v is the sum of the current-independent 10 V source and the current-dependent voltage source v_x. Note the factor 15 multiplying the control current carries the units of ohm.

(*i*) $v = 10 + v_x = 10 + 15\,(1) = \mathbf{25\ V}$

(*ii*) $v = 10 + v_x = 10 + 15\,(-2) = \mathbf{-20\ V}$

(*iii*) $v = 10 + v_x = 10 + 15\,(0) = \mathbf{10\ V}$

Example 2.60. *Find the values of current i and voltage drops v_1 and v_2 in the circuit of Fig. 2.121 which contains a current-dependent voltage source. What is the voltage of the dependent-source? All resistance values are in ohms.*

Solution. Note that the factor 4 multiplying the control current carries the units of ohms. Applying *KVL* to the loop *ABCDA* in Fig. 2.121, we have,

$$-v_1 + 4\, i - v_2 + 6 = 0$$

or $$v_1 - 4\, i + v_2 = 6 \qquad \ldots(i)$$

By Ohm's law, $v_1 = 2\,i$ and $v_2 = 4\,i$.

Putting the values of $v_1 = 2\,i$ and $v_2 = 4\,i$ in eq. (i), we have,

$$2\,i - 4\,i + 4\,i = 6 \qquad \therefore \quad i = \mathbf{3\ A}$$

$$\therefore \qquad v_1 = 2i = 2 \times 3 = \mathbf{6\ V}\ ;\ v_2 = 4\,i = 4 \times 3 = \mathbf{12\ V}$$

Voltage of the dependent source = $4\,i = 4 \times 3 =$ **12 V**

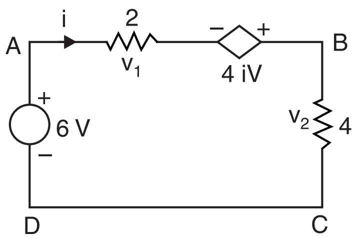


Fig. 2.121

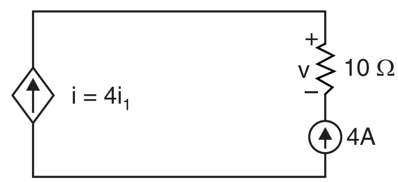


Fig. 2.122

Example 2.61. *Find the voltage v across the 10 Ω resistor in Fig. 2.122, if the control current i_1 in the dependent current-source is (i) 2A (ii) – 1A.*

Solution.

(*i*) $v = (i - 4)10 = [4\,(2) - 4]10 =$ **40 V**

(*ii*) $v = (i - 4)10 = [4(-1) - 4]\ 10 =$ **– 80 V**

Example 2.62. *Calculate the power delivered by the dependent-source in Fig. 2.123.*

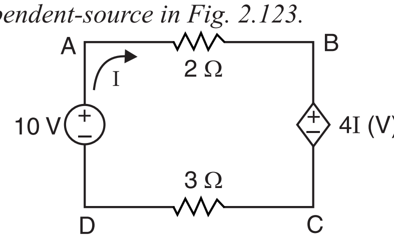


Fig. 2.123

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$-2\,I - 4\,I - 3\,I + 10 = 0$$

$$\therefore \qquad I = 10/9 = 1.11\ \text{A}$$

The current I enters the positive terminal of dependent-source. Therefore, power absorbed = 1.11 × 4 (1.11) = 4.93 watts. Hence power delivered is **– 4.93 W.**

Example 2.63. *In the circuit of Fig. 2.124, find the values of i and v. All resistances are in ohms.*

Solution. Referring to Fig. 2.124, it is clear that $v_a = 12 + v$.

Therefore, $\qquad v = v_a - 12$

Voltage drop across left 2 Ω resistor = $0 - v_a$

Voltage drop across top 2 Ω resistor = $v_a - 12$

Applying *KCL* to the node a, we have,

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \qquad \text{or} \quad v_a = 4\ \text{V}$$

$$\therefore \qquad v = v_a - 12 = 4 - 12 = \mathbf{-8V}$$

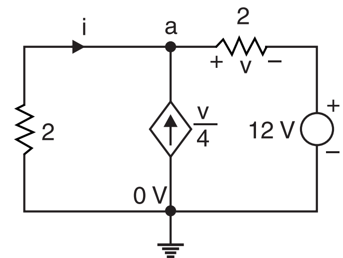


Fig. 2.124

The negative sign shows that the polarity of v is opposite to that shown in Fig. 2.124. The current that flows from point a to ground = 4/2 = 2 A. Hence $i =$ **– 2 A**.

Example 2.64. *In Fig. 2.125, both independent and dependent-current sources drive current through resistor R. Is the value of R uniquely determined ?*

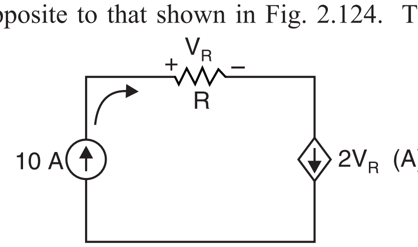


Fig. 2.125

Solution. By definition of an independent source, the current I must be 10 A.

$\therefore \quad I = 10 \text{ A} = 2\, V_R$

or $\quad V_R = 10/2 = 5 \text{ V}$

Now $\quad 5 \text{ V} = (10)(R) \quad \therefore R = 5/10 =$ **0.5 Ω**

No other value of R is possible.

Example 2.65. *Find the value of current i_2 supplied by the voltage-controlled current source (VCCS) shown in Fig. 2.126.*

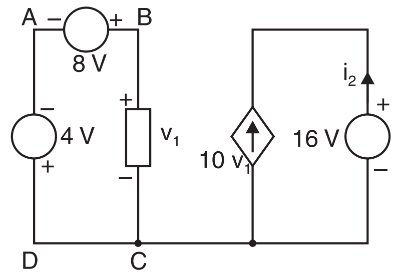


Fig. 2.126

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$8 - v_1 - 4 = 0 \quad \therefore v_1 = 4\text{V}$$

The current supplied by VCCS = $10\, v_1 = 10 \times 4 = 40$A

As i_2 flows in opposite direction to this current, therefore, $i_2 =$ **– 40A.**

Example 2.66. *By using voltage divider rule, calculate the voltages v_x and v_y in the circuit shown in Fig. 2.127.*

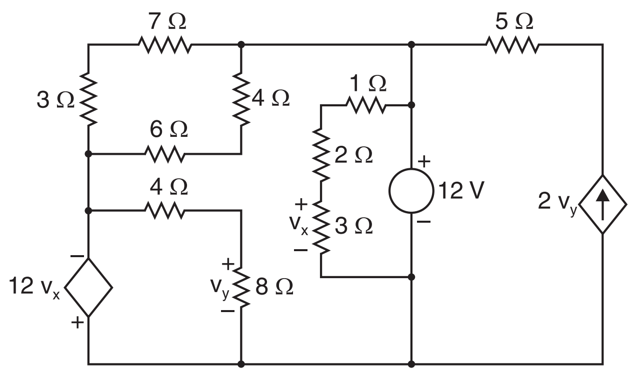


Fig. 2.127

Solution. As can be seen from Fig. 2.127, 12 V drop is over the series combination of 1Ω, 2Ω and 3Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 3Ω, $v_x = 12 \times \dfrac{3}{1+2+3} =$ **6V**

$\therefore$ Voltage of dependent source = $12v_x = 12 \times 6 = 72$ V

As seen 72 V drop is over series combination of 4Ω and 8Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 8Ω, $v_y = 72 \times \dfrac{8}{4+8} = 48$ V

The actual sign of polarities of v_y is opposite to that shown in Fig. 2.127. Hence $v_y =$ **– 48 V.**

Example 2.67. *Find the values of i_1, v_1, v_x and v_{ab} in the network shown in Fig. 2.128 with its terminals a and b open.*

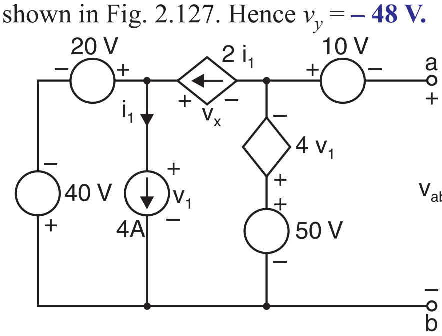


Fig. 2.128

Solution. It is clear from the circuit that i_1 = **4A.**

Applying *KVL* to the left-hand loop, we have,

$$20 - v_1 - 40 = 0 \quad \therefore v_1 = -\textbf{20 V}$$

Applying *KVL* to the second loop from left, we have,

$$-v_x + 4v_1 - 50 + v_1 = 0$$

$$\therefore \quad v_x = 5v_1 - 50 = 5(-20) - 50 = \mathbf{-150\ V}$$

Applying *KVL* to the third loop containing v_{ab}, we have,

$$-10 - v_{ab} + 50 - 4v_1 = 0$$

$$\therefore \quad v_{ab} = -10 + 50 - 4v_1 = -10 + 50 - 4(-20) = \mathbf{120\ V}$$

Tutorial Problems

1. The circuit of Fig. 2.129 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. **[8A; 1920 W]**

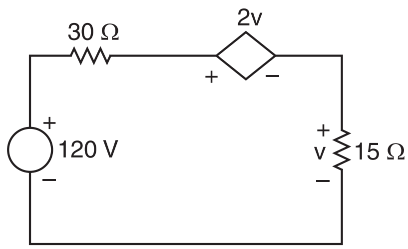


Fig. 2.129

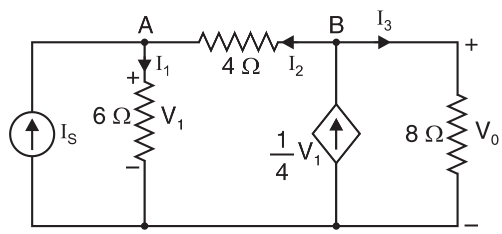


Fig. 2.130

2. Applying Kirchhoff's current law, determine current I_S in the electric circuit of Fig. 2.130. Take V_0 = 16V. **[1A]**

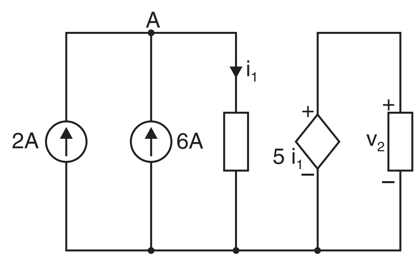


Fig. 2.131

3. Find the voltage drop v_2 across the current-controlled voltage source shown in Fig. 2.131. **[40 V]**

2.31. Ground

Voltage is relative. That is, the voltage at one point in a circuit is always measured relative to another point in the circuit. For example, if we say that voltage at a point in a circuit is + 100V, we mean that the point is 100V more positive than some reference point in the circuit. This reference point in a circuit is usually called the *ground point*. Thus ground is used as reference point for specifying voltages. The ground may be used as common connection (*common ground*) or as a zero reference point (*earth ground*). There are different symbols for chassis ground, common ground and earth ground as shown in Fig. 2.132. *However, earth ground symbol is often used in place of chassis ground or common ground.*

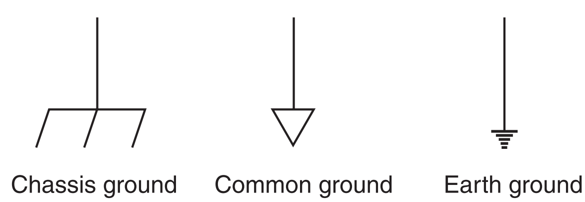


Fig. 2.132

(*i*) Ground as a common connection. It is a usual practice to mount the electronic and electrical components on a metal base called *chassis* (See Fig. 2.133). Since chassis is good conductor, it provides a conducting return path as shown in Fig. 2.134. It may be seen that all points connected to chassis are shown as grounded and represent the same potential. The adoption of this scheme (*i.e.* showing points of same potential as grounded) often simplifies the electrical and electronic circuits.

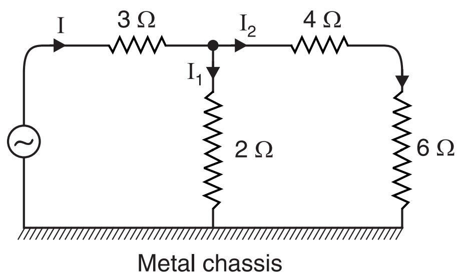


Fig. 2.133

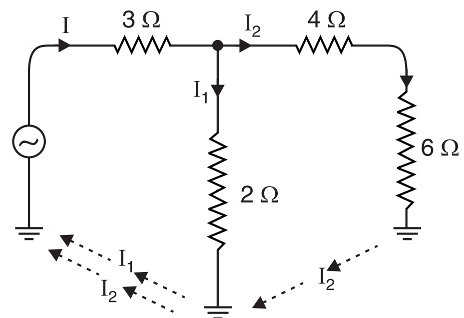
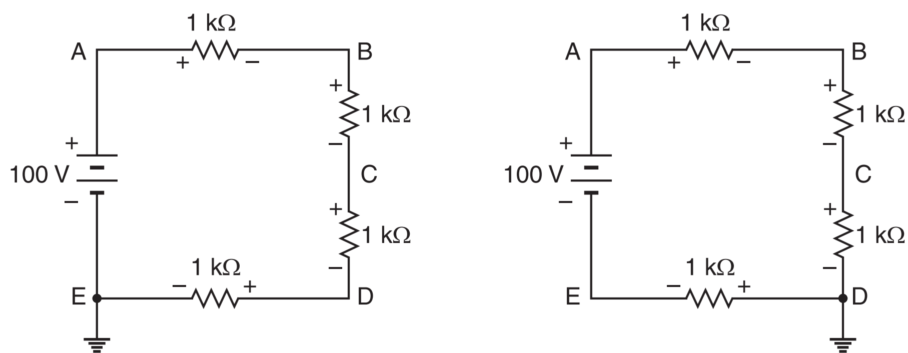


Fig. 2.134

(*ii*) **Ground as a zero reference point.** Many times connection is made to earth which acts as a reference point. The earth (ground) has a potential of zero volt (0V) with respect to all other points in the circuit. Thus in Fig. 2.135(*i*), point *E* is grounded (*i.e.,* point *E* is connected to earth) and has zero potential. The voltage across each resistor is 25 volts. The voltages of the various points with respect to ground or earth (*i.e.,* point *E*) are :

$$V_E = 0\text{V} \;;\; V_D = +25\text{ V} \;;\; V_C = +50\text{ V} \;;\; V_B = +75\text{ V} \;;\; V_A = +100\text{V}$$



(*i*) (*ii*)

Fig. 2.135

If instead of point *E*, the point *D* is grounded as shown in Fig. 2.135 (*ii*), then potentials of various points with respect to ground (*i.e.,* point *D*) will be :

$$V_E = -25\text{ V} \;;\; V_D = 0\text{ V} \;;\; V_C = +25\text{ V} \;;\; V_B = +50\text{ V} \;;\; V_A = +75\text{ V}$$

Example 2.68. *In Fig. 2.136, find the relative potentials of points A, B, C, D and E when point A is grounded.*

Solution. Net circuit voltage, $V = 34 - 10 = 24$ V

Total circuit resistance, $R_T = 6 + 4 + 2 = 12\ \Omega$

Circuit current, $I = V/R_T = 24/12 = 2$ A

Drop across 2 Ω resistor $= 2 \times 2 = 4$ V

Drop across 4 Ω resistor $= 2 \times 4 = 8$ V

Drop across 6 Ω resistor $= 2 \times 6 = 12$ V

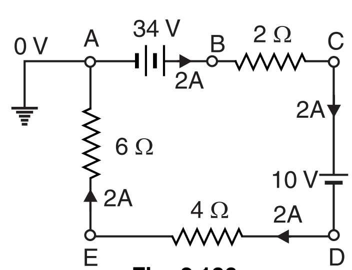


Fig. 2.136

$\therefore$ Potential at point B, V_B = $34 - 0 =$ **34 V**

Potential at point C, V_C = 34 – drop in $2\ \Omega$

= $34 - 2 \times 2 =$ **30 V**

Potential at point D, V_D = $V_C - 10 = 30 - 10 =$ **20 V**

Potential at point E, V_E = V_D – drop in $4\Omega = 20 - 2 \times 4 =$ **12 V**

Potential at point A, V_A = V_E – drop in $6\ \Omega$

= $12 - 6 \times 2 =$ **0 V**

Example 2.69. *Fig. 2.137 shows the circuit with common ground symbols. Find the total current I drawn from the 25 V source.*

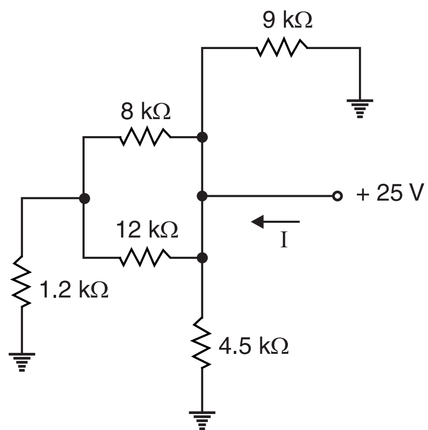


Fig. 2.137

Solution. The circuit shown in Fig. 2.137 is redrawn by eliminating the common ground symbols. The equivalent circuit then becomes as shown in Fig. 2.138. (*i*). We see that 8 kΩ and 12 kΩ resistors are in parallel as are the 9 kΩ and 4.5 kΩ resistors. Fig. 2.138 (*ii*) shows the circuit when these parallel combinations are replaced by their equivalent resistances :

$$\frac{8 \times 12}{8 + 12} = 4.8\ \text{k}\Omega \quad \text{and} \quad \frac{9 \times 4.5}{9 + 4.5} = 3\ \text{k}\Omega$$

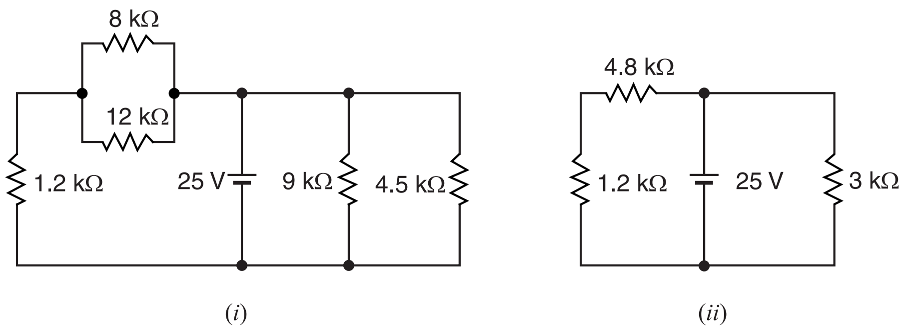


Fig. 2.138

Referring to Fig. 2.138 (*ii*), it is clear that 4.8 kΩ resistance is in series with 1.2 kΩ resistance, giving an equivalent resistance of $4.8 + 1.2 = 6$ kΩ.

The circuit then becomes as shown in Fig. 2.139 (*i*).

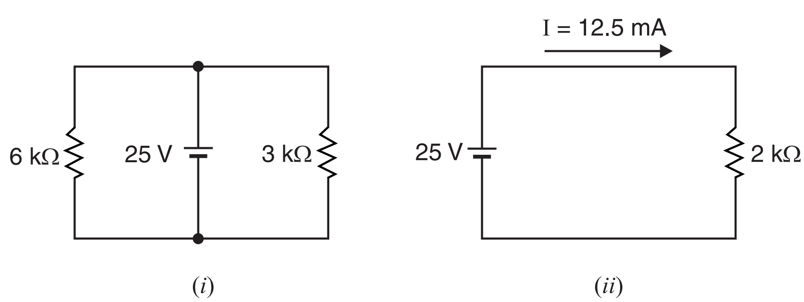


Fig. 2.139

Referring to Fig. 2.139 (*i*), 6 kΩ is in parallel with 3 kΩ giving the total resistance R_T as :

$$R_T = \frac{6 \times 3}{6+3} = 2\ \text{k}\Omega$$

The circuit then reduces to the one shown in Fig. 2.139 (*ii*).

∴ Total current *I* drawn from 25 V source is

$$I = \frac{25\ \text{V}}{R_T} = \frac{25\ \text{V}}{2\ \text{k}\Omega} = \mathbf{12.5\ mA}$$

Example 2.70. *What is the potential difference between X and Y in the network shown in Fig. 2.140 ?*

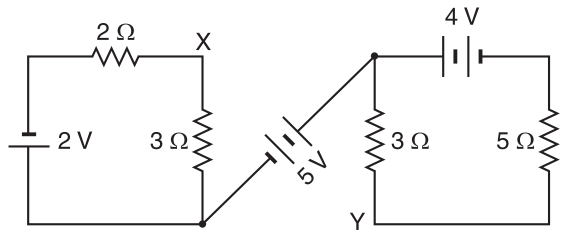


Fig. 2.140

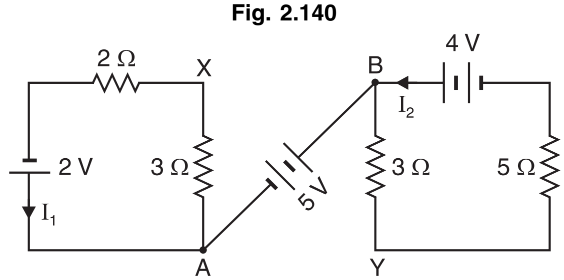


Fig. 2.141

Solution. Fig. 2.140 is reproduced as Fig 2.141 with required labeling. Consider the two battery circuits separately. Referring to Fig. 2.141,

Current flowing in 2Ω and 3Ω resistors is

$$I_1 = \frac{2}{2+3} = 0.4\text{A}$$

Current flowing in 3Ω and 5Ω resistors is

$$I_2 = \frac{4}{3+5} = 0.5\ \text{A}$$

∴ Potential difference between *X* and *Y* is

$$V_{XY} = V_{XA} + V_{AB} - V_{BY} \qquad \text{[See Fig. 2.141]}$$
$$= 3I_1 + 5 - 3I_2$$
$$= 3 \times 0.4 + 5 - 3 \times 0.5 = \mathbf{4.7\ V}$$

2.32. Voltage Divider Circuit

A **voltage divider** *(or potential divider) is a series circuit that is used to provide two or more reduced voltages from a single input voltage source.*

Fig. 2.142 shows a simple voltage divider circuit which provides two reduced voltages V_1 and V_2 from a single input voltage *V*. Since no load is connected to the circuit, it is called **unloaded voltage divider**. The values of V_1 and V_2 can be found as under :

$$\text{Circuit current, } I = \frac{V}{R_1 + R_2} = \frac{V}{R_T}$$

where R_T = Total resistance of the voltage divider

$$\therefore \quad V_1 = IR_1 = V \times \frac{R_1}{R_T}$$

and $$V_2 = IR_2 = V \times \frac{R_2}{R_T}$$

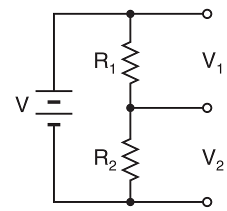


Fig. 2.142

Therefore, voltage drop across any resistor in an unloaded voltage divider is equal to the ratio of that resistance value to the total resistance multiplied by the source voltage.

Loaded voltage divider. When load R_L is connected to the output terminals of the voltage divider as shown in Fig. 2.143, the output voltage (V_2) is reduced by an amount depending on the value of R_L. It is because load resistor R_L is in parallel with R_2 and reduces the resistance from point A to point B. As a result, the output voltage is reduced. The larger the value of R_L, the less the output voltage is reduced from the unloaded value. Loading a voltage divider has the following effects :

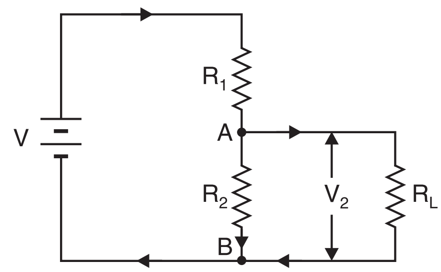


Fig. 2.143

(*i*) The output voltage is reduced depending upon the value of load resistance R_L.

(*ii*) The current drawn from the source is increased because total resistance of the circuit is reduced. The decrease in total resistance is due to the fact that loaded voltage divider becomes series-parallel circuit.

Example 2.71. *Design a voltage divider circuit that will operate the following loads from a 20 V source :*

5 V at 5 mA ; 12 V at 10 mA ; 15 V at 5 mA

The bleeder current is 4 mA.

Solution. A voltage divider that produces a *bleeder current requires $N + 1$ resistors where N is the number of loads. In this example, the number of loads is three. Therefore, four resistors are required for this voltage divider. The required circuit is shown in Fig. 2.144. Here R_1 is the bleeder resistor. The loads are arranged in ascending order of their voltage requirements, starting at the bottom of the divider network.

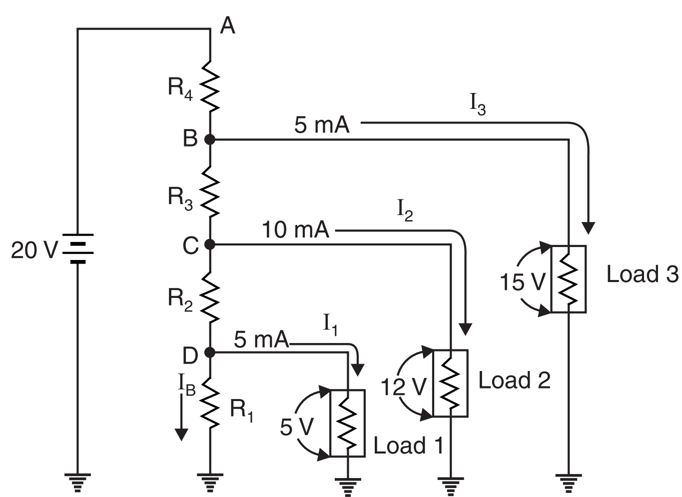


Fig. 2.144

Voltage across bleeder resistor R_1 = 5 V ; Current through R_1, I_B = 4 mA .

$$\therefore \quad \text{Value of } R_1 = \frac{5\text{V}}{4\text{mA}} = \mathbf{1.25\ k\Omega}$$

Next we shall find the value of resistor R_2. For this purpose, we find the current through R_2 and voltage across R_2.

* The current drawn continuously from a power supply by the resistive voltage divider circuit is called bleeder current. Without a bleeder current, the voltage divider outputs go up to full value of supply voltage if all the loads are disconnected.

Current through $R_2 = I_B + 5$ mA = 4 mA + 5 mA = 9 mA

Voltage across $R_2 = V_C - V_D = 12 - 5 = 7$ V

$$\therefore \quad \text{Value of } R_2 = \frac{7\,\text{V}}{9\,\text{mA}} = \mathbf{778\ \Omega}$$

Now we shall find the value of resistor R_3.

Current through R_3 = Current in R_2 + 10 mA = 9 mA + 10 mA = 19 mA

Voltage across $R_3 = V_B - V_C = 15 - 12 = 3$ V

$$\therefore \quad \text{Value of } R_3 = \frac{3\,\text{V}}{19\,\text{mA}} = \mathbf{158\ \Omega}$$

Finally, we shall determine the value of resistor R_4.

Current through R_4 = Current through R_3 + 5 mA = 19 mA + 5 mA = 24 mA

Voltage across $R_4 = V_A - V_B = 20 - 15 = 5$ V

$$\therefore \quad \text{Value of } R_4 = \frac{5\,\text{V}}{24\,\text{mA}} = \mathbf{208\ \Omega}$$

The design of voltage divider circuit means finding the values of R_1, R_2, R_3 and R_4. Therefore, the design of voltage divider circuit stands completed.

Example 2.72. *Fig. 2.145 shows the voltage divider circuit. Find (i) the unloaded output voltage, (ii) the loaded output voltage for R_L = 10 kΩ and R_L = 100 kΩ.*

Solution. **(*i*)** When load R_L is removed, the voltage across R_2 is the unloaded output voltage of the voltage divider.

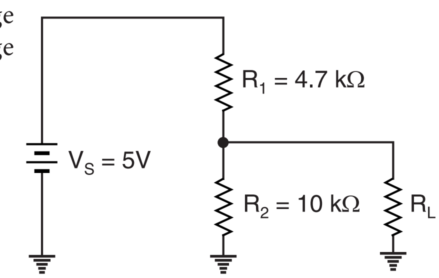


Fig. 2.145

$$\therefore \quad \text{Unloaded output voltage} = \frac{R_2}{R_1 + R_2} \times V_S = \frac{10}{4.7 + 10} \times 5 = \mathbf{3.4\ V}$$

(*ii*) When R_L = 10 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is

$$R_T = \frac{R_2 R_L}{R_2 + R_L} = \frac{10 \times 10}{10 + 10} = 5\ \text{k}\Omega$$

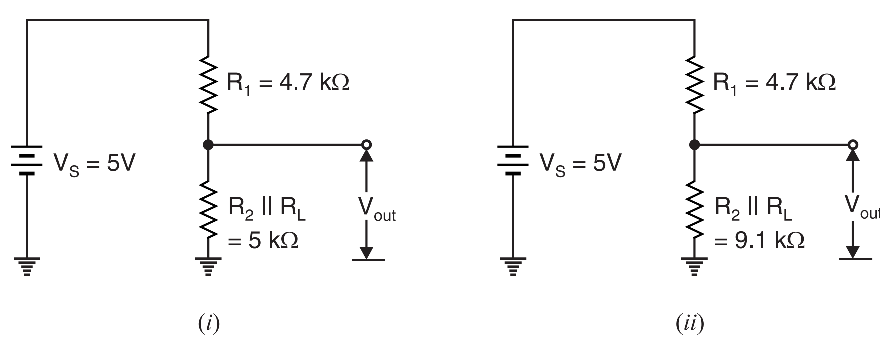


Fig. 2.146

The circuit then becomes as shown in Fig. 2.146 (*i*).

$$\therefore \quad \text{Loaded output voltage} = \frac{R_T}{R_1 + R_T} \times V_S = \frac{5}{4.7 + 5} \times 5 = \mathbf{2.58\ V}$$

When R_L = 100 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is given by ; $R'_T = \frac{R_2 R_L}{R_2 + R_L} = \frac{10 \times 100}{10 + 100} = 9.1$ kΩ

The circuit then becomes as shown in Fig. 2.146 (*ii*).

$$\therefore \quad \text{Loaded output voltage} = \frac{R'_T}{R_1 + R'_T} \times V_S = \frac{9.1}{4.7 + 9.1} \times 5 = \mathbf{3.3\ V}$$

Example 2.73. *Find the values of different voltages that can be obtained from 25V source with the help of voltage divider circuit of Fig. 2.147.*

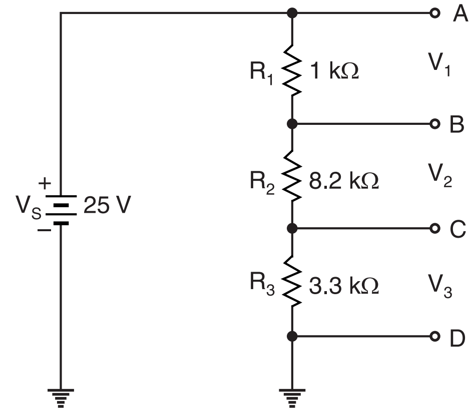


Fig. 2.147

Solution. Total circuit resistance, $R_T = R_1 + R_2 + R_3 = 1 + 8.2 + 3.3 = 12.5$ kΩ

$$\text{Voltage drop across } R_1,\ V_1 = \frac{R_1}{R_T} \times V_S = \frac{1}{12.5} \times 25 = 2 \text{ V}$$

$$\therefore \quad \text{Voltage at point } B,\ V_B = 25 - 2 = 23 \text{ V}$$

$$\text{Voltage drop across } R_2,\ V_2 = \frac{R_2}{R_T} \times V_S = \frac{8.2}{12.5} \times 25 = 16.4 \text{ V}$$

$$\therefore \quad \text{Voltage at point } C,\ V_C = V_B - V_2 = 23 - 16.4 = 6.6 \text{ V}$$

The different available load voltages are :

$V_{AB} = V_A - V_B = 25 - 23 =$ **2 V** ; $V_{AC} = V_A - V_C = 25 - 6.6 =$ **18.4 V**

$V_{BC} = V_B - V_C = 23 - 6.6 =$ **16.4 V** ; $V_{AD} =$ **25 V** ; $V_{CD} = V_C - V_D = 6.6 - 0 =$ **6.6 V**

$V_{BD} = V_B - V_D = 23 - 0 =$ **23 V**

Example 2.74. *Fig. 2.148 shows a 10 kΩ potentiometer connected in a series circuit as an adjustable voltage divider. What total range of voltage V_1 can be obtained by adjusting the potentiometer through its entire range ?*

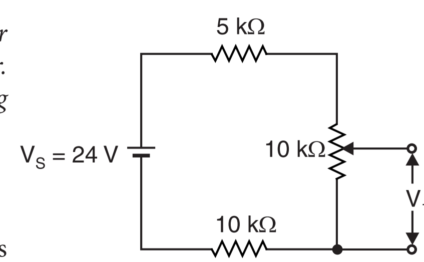


Fig. 2.148

Solution. Total circuit resistance is

$$R_T = 5 + 10 + 10 = 25 \text{ kΩ}$$

The total voltage *E* that appears across the end terminals of potentiometer is

$$E = \frac{10}{R_T} \times V_S = \frac{10}{25} \times 24 = 9.6 \text{ V}$$

When the wiper arm is at the top of the potentiometer,

$$V_1 = \frac{10}{10} \times E = \frac{10}{10} \times 9.6 = 9.6 \text{ V}$$

When the wiper arm is at the bottom of the potentiometer,

$$V_1 = \frac{0}{10} \times E = \frac{0}{10} \times 9.6 = 0 \text{ V}$$

Therefore, V_1 can be adjusted between **0 and 9.6 V.**

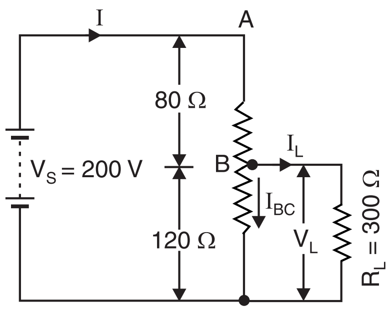


Fig. 2.149

Example 2.75. *Fig. 2.149 shows the voltage divider circuit. Find (i) the current drawn from the supply, (ii) voltage across the load R_L, (iii) the current fed to R_L and (iv) the current in the tapped portion of the divider.*

Solution. It is a loaded voltage divider.

(*i*) $R_{BC} = 120\ \Omega \parallel 300\ \Omega = \dfrac{120 \times 300}{120 + 300} = 85.71\ \Omega$

$$V_{AB} = \frac{R_{AB}}{R_{AB} + R_{BC}} \times V_S = \frac{80}{80 + 85.71} \times 200 = 96.55 \text{ V}$$

$\therefore$ The current I drawn from the supply is

$$I = \frac{V_{AB}}{R_{AB}} = \frac{96.55}{80} = \mathbf{1.21\ A}$$

(*ii*)

$$V_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} \times V_S = \frac{85.71}{80 + 85.71} \times 200 = \mathbf{103.45\ V}$$

(*iii*) $\therefore$ Current fed to load, $I_L = \dfrac{V_{BC}}{R_L} = \dfrac{103.45}{300} = \mathbf{0.35\ A}$

(*iv*) Current in the tapped portion of the divider is

$$I_{BC} = I - I_L = 1.21 - 0.35 = \mathbf{0.86 A}$$

Tutorial Problems

1. Redraw the circuit shown in Fig. 2.150 using the common ground symbol.

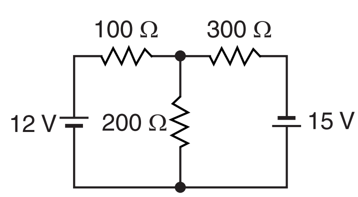
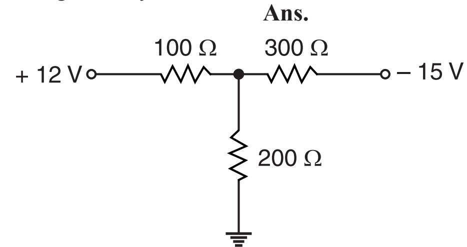


Fig. 2.150

Ans.



2. Redraw the circuit shown in Fig. 2.151 using the common ground symbol.

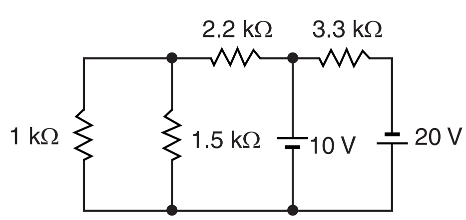
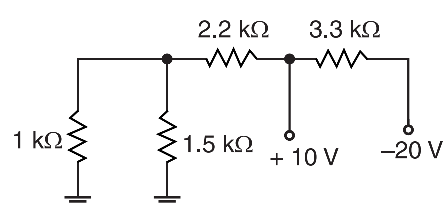


Fig. 2.151

Ans.



3. Draw the circuit shown in Fig. 2.152 by eliminating the common ground symbols.

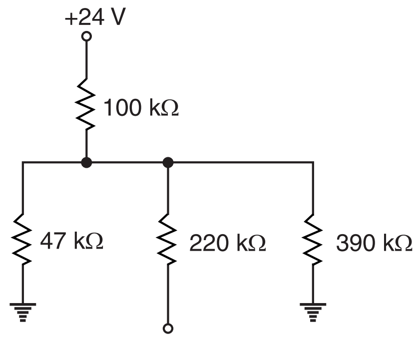
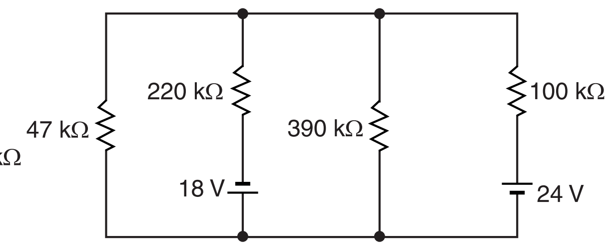


Fig. 2.152

Ans.



4. A voltage of 200 V is applied to a tapped resistor of 500 Ω. Find the resistance between the tapped points connected to a circuit reading 0.1 A at 25 V. Also calculate the total power consumed. **[79Ω ; 83.3W]**

Objective Questions

1. Two resistances are joined in parallel whose resultant resistance is 6/5 ohms. One of the resistance wire is broken and the effective resistance becomes 2 ohms. Then the resistance of the wire that got broken is

   (*i*) 6/5 ohms (*ii*) 3 ohms
   (*iii*) 2 ohms (*iv*) 3/5 ohms

2. The smallest resistance obtained by connecting 50 resistances of 1/4 ohm each is

   (*i*) 50/4 Ω (*ii*) 4/50 Ω
   (*iii*) 200 Ω (*iv*) 1/200 Ω

3. Five resistances are connected as shown in Fig. 2.153. The effective resistance between points *A* and *B* is

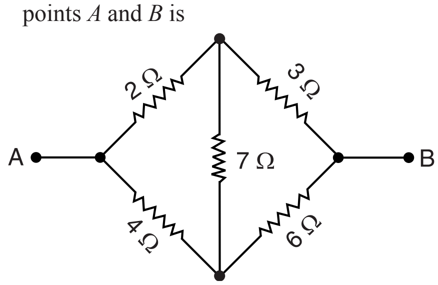


Fig. 2.153

   (*i*) 10/3 Ω (*ii*) 20/3 Ω
   (*iii*) 15 Ω (*iv*) 6 Ω

4. A 200 W and a 100 W bulb both meant for operation at 220 V are connected in series. When connected to a 220 V supply, the power consumed by them will be

   (*i*) 33 W (*ii*) 100 W
   (*iii*) 66 W (*iv*) 300 W

5. A wire has a resistance of 12 ohms. It is bent in the form of a circle. The effective resistance between two points on any diameter is

   (*i*) 6 Ω (*ii*) 24 Ω
   (*iii*) 16 Ω (*iv*) 3 Ω

6. A primary cell has an e.m.f. of 1.5 V. When short-circuited, it gives a current of 3 A. The internal resistance of the cell is

   (*i*) 4.5 Ω (*ii*) 2 Ω
   (*iii*) 0.5 Ω (*iv*) 1/4.5 Ω

7. Fig. 2.154 shows a part of a closed electrical circuit. Then $V_A - V_B$ is

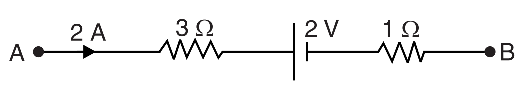


Fig. 2.154

   (*i*) – 8 V (*ii*) 6 V
   (*iii*) 10 V (*iv*) 3 V

8. The current *I* in the electric circuit shown in Fig. 2.155 is

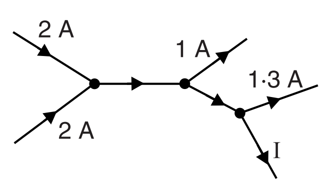


Fig. 2.155

   (*i*) 1.3 A (*ii*) 3.7 A
   (*iii*) 1A (*iv*) 1.7 A

9. Three 2 ohm resistors are connected to form a triangle. The resistance between any two corners is

(*i*) 6Ω (*ii*) 2Ω
(*iii*) 3/4Ω (*iv*) 4/3Ω

10. A current of 2 A flows in a system of conductors shown in Fig. 2.156. The potential difference $V_A - V_B$ will be

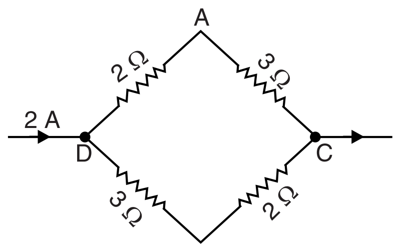


Fig. 2.156

(*i*) +2 V (*ii*) +1 V
(*iii*) –1 V (*iv*) –2 V

11. A uniform wire of resistance *R* is divided into 10 equal parts and all of them are connected in parallel. The equivalent resistance will be

(*i*) 0.01 *R* (*ii*) 0.1 *R*
(*iii*) 10 *R* (*iv*) 100 *R*

12. A cell of negligible resistance and e.m.f. 2 volts is connected to series combination of 2, 3 and 5 ohms. The potential difference in volts between the terminals of 3-ohm resistance will be

(*i*) 0.6 V (*ii*) $\frac{2}{3}$ V
(*iii*) 3 V (*iv*) 6 V

13. The equivalent resistance between points *X* and *Y* in Fig. 2.157 is

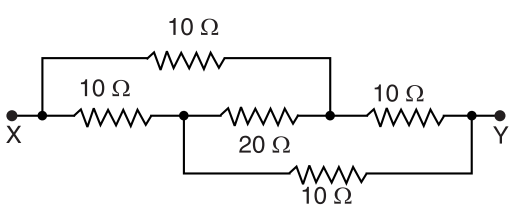


Fig. 2.157

(*i*) 10 Ω (*ii*) 22 Ω
(*iii*) 20 Ω (*iv*) 50 Ω

14. If each resistance in the network shown in Fig. 2.158 is *R*, what is the equivalent resistance between terminals *A* and *B* ?

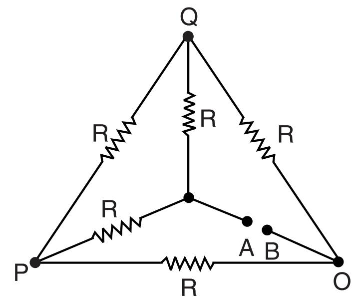


Fig. 2.158

(*i*) 5 *R* (*ii*) 3 *R*
(*iii*) 6 *R* (*iv*) *R*

15. Fig. 2.159 represents a part of a closed circuit. The potential difference between *A* and *B* (*i.e.* $V_A - V_B$) is

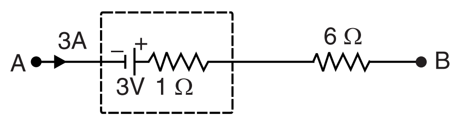


Fig. 2.159

(*i*) 24 V (*ii*) 0 V
(*iii*) 18 V (*iv*) 6 V

16. In the arrangement shown in Fig. 2.160, the potential difference between *B* and *D* will be zero if the unknown resistance *X* is

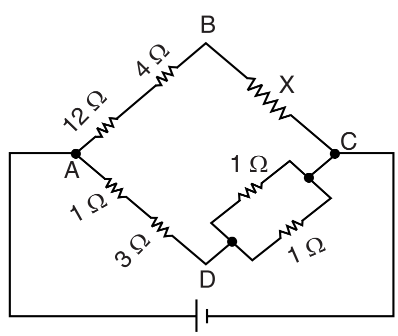


Fig. 2.160

(*i*) 4 Ω (*ii*) 2 Ω
(*iii*) 20 Ω (*iv*) 3 Ω

17. Resistances of 6 Ω each are connected in a manner shown in Fig. 2.161. With the current 0.5A as shown in the figure, the potential difference $V_P - V_Q$ is

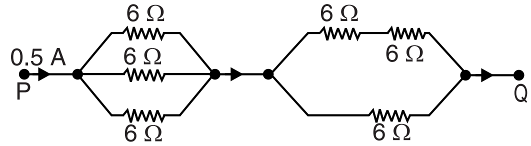


Fig. 2.161

(*i*) 3.6 V (*ii*) 6 V
(*iii*) 3 V (*iv*) 7.2 V

18. An electric fan and a heater are marked 100 W, 220 V and 1000 W, 220 V respectively. The resistance of the heater is

(*i*) zero
(*ii*) greater than that of fan
(*iii*) less than that of fan
(*iv*) equal to that of fan

19. In the circuit shown in Fig. 2.162, the final voltage drop across the capacitor *C* is

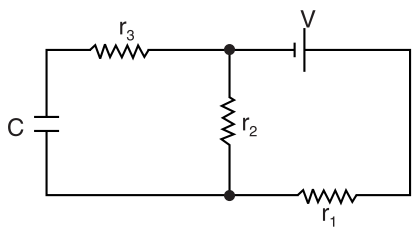


Fig. 2.162

(*i*) $\dfrac{V r_1}{r_1 + r_2}$ (*ii*) $\dfrac{V r_2}{r_1 + r_2}$

(*iii*) $\dfrac{V(r_1 + r_2)}{r_2}$ (*iv*) $\dfrac{V(r_2 + r_1)}{r_1 + r_2 + r_3}$

20. A primary cell has an e.m.f. of 1.5 V. When short circuited, it gives a current of 3 A. The internal resistance of the cell is

(*i*) 4.5 Ω (*ii*) 2 Ω
(*iii*) 0.5 Ω (*iv*) (1/4.5) Ω

Answers

1. (*ii*)	**2.** (*iv*)	**3.** (*i*)	**4.** (*iii*)	**5.** (*iv*)
6. (*iii*)	**7.** (*iii*)	**8.** (*iv*)	**9.** (*iv*)	**10.** (*ii*)
11. (*i*)	**12.** (*i*)	**13.** (*i*)	**14.** (*iv*)	**15** (*iii*)
16. (*ii*)	**17.** (*iii*)	**18.** (*iii*)	**19.** (*ii*)	**20.** (*iii*)