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# Absolute, Relative and percentage errors in Numerical Analysis

Let's first know some basics about numbers used in floating-point arithmetic or in other words numerical analysis and how they are calculated.

Basically, all the numbers that we use in Numerical analysis are of two types as follows:

1. Exact numbers
   Numbers that have their exact quantity, means their value isn't going to change. For example 3, 2, 5, 7, 1/3, 4/5, or √2 etc.
2. Approximate numbers
   These numbers are represented in decimal numbers. They have some certain degrees of accuracy. Like the value of $\pi$ is 3.1416 if we want more precise value, we can write 3.14159265, but we can't write the exact value of $\pi$.
   These digits that we use in any approximate value, or in other way digits which represent the numbers are called significant digits.

**How to count significant digits in a given number:**

**For example –**

In the normal value of $\pi(3.1416)$, there are 5 significant digits and when we write more precise value of it 3.14159265 we get 9 significant digits.

Let's say we have numbers: 0.0123, 1.2300, and 0.10234. Now we have 4, 3, and 5 significant digits respectively.

**In the scientific representation of numbers –**

$2.345 \times 10^7, 8.7456 \times 10^6$ Have 4, 5 and 2 significant digits respectively.

**Absolute error:**

Let the true value of a quantity be X and the approximate value of that quantity be $X_1$. Hence absolute error has defined the difference between X and $X_1$. Absolute error is denoted by$E_A$.

Hence $E_A = X - X_1 = \delta X$.

**Absolute error example:**

For example, 24.13 is the actual value of a quantity and 25.09 is the measure or inferred value, then the absolute error will be:

Absolute error = 25.09 – 24.13=0.86

Most of the time it is sufficient to record only two decimal digits of the absolute error. Thus, it is sufficient to state that the absolute error of the approximation 4.55 to the correct value 4.538395 is 0.012.

**Relative error:**

It is defined as follow:

$$E_R = \frac{E_A}{X} = \frac{Absolute\ error}{X}$$

**Relative error example**

Three weighs are measured at 5.05g, 5.00g, and 4.95g. The absolute error is $\pm 0.05g$. The relative error is 0.05g/5.00g = 0.01 or 1%.

**Percentage error:**

It is defined as follow:

$$E_p = 100 \times E_p = 100 \times \frac{E_A}{X}$$

Let's say we have a number $\delta X = |X_1 - X|$, it is an upper limit on the magnitude of absolute error and known as absolute accuracy.

Similarly the quantity $\frac{\delta X}{|X|} or \frac{\delta X}{|X_1|} called\ relative\ accuracy$.

Now let's solve some examples as follows.

1. Example -1:
   We are given an approximate value of $\pi$ is 22/7 = 3.1428571 and true value is 3.1415926. Calculate absolute, relative and percentage errors?
   Solution:
   We have true value X = 3.1415926, and approx. value $X_1 = 3.1428571$. so now we calculate absolute error, we know that $E_A = X - X_1 = \delta X$. Hence $E_A = 3.1415926 - 3.1428571 = -0.0012645$.
   Answer is -0.0012645
   Now for relative error we have (absolute error)/ (true value of quantity).

Hence $E_R = \frac{E_A}{X} = \frac{absolute\ error}{X}, E_A = \frac{-0.0012645}{3.1415926} = -0.000402.$

Percentage error,

$$E_p = 100 \times \frac{E_A}{X} = 100 \times (-0.000402) = -0.0402$$

2. Example – 2

   Let the approximate values of a number 1/3 be 0.30, 0.33, and 0.34.

   Find out the best approximation.

   Solution:

   Our approach is to first find the value of absolute error, and any value having the least absolute will be best. So, we first calculate the absolute errors in all approx. values are given.

$$|X - X_1| = \left|\frac{1}{3} - 0.30\right| = \frac{1}{30}$$

$\left|\frac{1}{3} - 0.33\right| = 1/300.$

$$\left|\frac{1}{3} - 0.34\right| = \frac{0.02}{3} = \frac{1}{500}$$

   Hence, we can say that 0.33 is the most precise value of 1/3.

3. Example – 3

   Finding the difference

$$\sqrt{5.35} - \sqrt{4.35}$$

   Solution:

   $\sqrt{5.35} = 2.31300.$

   $\sqrt{4.35} = 2.08566.$

   Hence, $\sqrt{5.35} - \sqrt{4.35} = 2.31300 - 2.08566 = 0.22734$

   Here our answer has 5 significant digits we can modify them as per our requirements.

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## Error analysis or error types

1. Inherent errors.
   *Error that are present in the data that are input to the model are inherent errors. They are also called input error. They are classified into two-data errors and conversion errors.*
2. Round off errors.
3. Truncation errors.
   *Result from using numerical method (approximation) in place of an exact mathematical procedure to find the solution.*
4. Absolute errors.
5. Relative errors.
6. Percentage errors.

*Reference links:*


1. Relative Error Definition (Science) (thoughtco.com)
2. *Absolute, Relative and Percentage errors in Numerical Analysis - GeeksforGeeks*