Total questions:

Figure ***Error! No text of specified style in document.****-1, Total Question page 1, 1 to 5*

1. Fit $y = ab^x$ by the method of least squares to the given data

x	0	1	2	3	4	5	6	7
y	10	21	35	59	92	200	400	610

2. By the methods of moments fit a straight line to the following data.

x	1	2	3	4	5	6
y	4	8	10	12	16	20

3. Solve this reciprocal eq

$$x^5 + 8x^4 + 21x^3 + 21x^2 + 8x + 1 = 0$$

4. State the theorem for method of false position.

5. Solve the system of equations by Gauss law.

$32 = 28x + 4y - z$, $x + 3y + 10z = 24$ and $2x + 17y + 4z = 35$

Total question page 2, 6 to 10.

Q6 Solve $y_{n+2} - 4y_{n+1} + 3y_n = 3^n + 1$

Q7 In the table, one value of y is incorrect and that y is a cubic polynomial in x.

x	0	1	2	3	4	5	6	7
y	25	21	18	18	27	45	76	123

Q8 For any equation proves the bisection method.

Q9 Solve the eq by synthetic divi[...] make sure the roots are in form $\pm a, \pm b, c$

$x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 = 0$

Q10 Remove the second term of transformed eq $x^4 - 8x^3 - x^2 + 68x + 6$[...] and hence solve it.

Question no.1 answer below:

Example 1.14 Fit $y = ab^x$ by the method of least squares to the data given below.

x	0	1	2	3	4	5	6	7
y	10	21	35	59	92	200	400	610

Solution The curve to be fitted is $y = ab^x$ or $Y = A + Bx$, where $A = \log_{10} a$, $B = \log_{10} b$ and $Y = \log_{10} y$.

$\therefore$ The normal equations are

$$\Sigma Y = 8A + B\Sigma x \quad \text{and} \quad \Sigma xY = A\Sigma x + B\Sigma x^2$$

x	y	$Y = \log_{10} y$	x^2	xY
0	10	1.0000	0	0
1	21	1.3222	1	1.3222
2	35	1.5441	4	3.0882
3	59	1.7708	9	5.3124
4	92	1.9638	16	7.8552
5	200	2.3010	25	11.5050
6	400	2.6021	36	15.6126
7	610	2.7853	49	19.4971
$\Sigma x = 28$		$\Sigma Y = 15.2893$	$\Sigma x^2 = 140$	$\Sigma xY = 64.1927$

Question no.2 answer below:

Not clear solution present

Question no.3 answer below:

Example 2.19 Solve $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$

Solution Here, sum of all the coefficients is 0.

$\therefore x = 1$ is a root of the given equation.

$x = 1$	1	−5	9	−9	5	−1
	0	1	−4	5	−4	1
	1	−4	5	−4	1	$\boxed{0}$

Therefore, the equation can be written as

$$(x-1)(x^4 - 4x^3 + 5x^2 - 4x + 1) = 0$$

Consider $x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

Dividing by x^2 on both sides and adjusting the terms, we get

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + 5 = 0$$

put $x + \frac{1}{x} = t$ so that $x^2 + \frac{1}{x^2} = t^2 - 2$

$\therefore \quad t^2 - 2 - 4(t) + 5 = 0 \quad$ or $t^2 - 4t + 3 = 0$

$\therefore \quad t = 1$ or 3

Now $t = 1 \Rightarrow x + 1/x = 1 \quad$ or $x^2 - x + 1 = 0$

$$\therefore \quad x = \frac{1 \pm \sqrt{3}i}{2}$$

This question is similar to this, but solution not found.

Theory of Equations 2.21

Again, $t = 3 \Rightarrow x + 1/x = 3 \quad$ or $x^2 - 3x + 1 = 0$

$$x = \frac{3 \pm \sqrt{5}}{2}.$$

$\therefore$

Hence, the roots of the given equation are

$$x = 1, \frac{1 \pm \sqrt{3}i}{2}, \frac{3 \pm \sqrt{5}}{2}.$$

Question no.4 answer below:

3.4 METHOD OF FALSE POSITION

This method, also known as *regula falsi method*, is the oldest method of finding the real root of an equation $f(x) = 0$ and is somewhat similar to the bisection method.

Consider the equation $f(x) = 0$. Let a and b $(a < b)$ be two values of x such that $f(a)$ and $f(b)$ are of opposite signs. Then the graph of $y = f(x)$ crosses the x-axis at some point between a and b (see Fig.3.3).

Therefore, the equation of the chord joining the two points $A\ [a, f(a)]$ and $B\ [b, f(b)]$ is

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a) \tag{3.6}$$

Now in the interval, (a, b), the graph of the function can be considered as a straight line. So the intersection of the line given by Eqn (3.6) with the x-axis will give an approximate value of the root. Putting $y = 0$ in Eqn (3.6), we get

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

or

$$x = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

Hence, the first approximation to the root is given by

$$x_1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} \tag{3.7}$$

Now, if $f(x_1)$ and $f(a)$ are of opposite sign then the root lies in between a and x_1. So we replace b by x_1 in Eqn (3.7) and get the next approximation x_2.

But if $f(x_1)$ and $f(a)$ are of the same sign then $f(x_1)$ and $f(b)$ will be of opposite signs and therefore, the root lies in between x_1 and b. Hence, we replace a by x_1 in Eqn (3.7) and get the next approximation x_2. The process is to be repeated till the root is found to the desired accuracy.

Question no.5 answer below:

Question 5 part 1 1

Example 4.2 Solve the system of equations, $28x + 4y - z = 32$, $x + 3y + 10z = 24$ and $2x + 17y + 4z = 35$ by Gauss elimination method. (*M.U. 1990,199*)

Solution The given system is equivalent to

$$\begin{bmatrix} 28 & 4 & -1 \\ 1 & 3 & 10 \\ 2 & 17 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 24 \\ 35 \end{bmatrix}$$

$$A \qquad X = B$$

The argumented matrix is

$$[A/B] = \left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 1 & 3 & 10 & 24 \\ 2 & 17 & 4 & 35 \end{array}\right]$$

Now we will make A as upper triangular choosing 28 as pivot.

$$\begin{array}{l} R_2 \to R_2 - \dfrac{R_1}{28} \\ R_3 \to R_3 - \dfrac{R_1}{14} \end{array} \sim \left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 0 & \frac{80}{28} & \frac{281}{28} & \frac{640}{28} \\ 0 & \frac{234}{14} & \frac{57}{14} & \frac{458}{14} \end{array}\right]$$

Question 5 part 1 2

Now the pivot is $\dfrac{234}{14}$

$$\therefore\ R(2,3) \sim \left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 0 & \frac{234}{14} & \frac{57}{14} & \frac{458}{14} \\ 0 & \frac{80}{28} & \frac{281}{28} & \frac{640}{28} \end{array}\right]$$

$$R_3 \to R_3 - \frac{20}{117}R_2 \sim \left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 0 & \frac{234}{14} & \frac{57}{14} & \frac{458}{14} \\ 0 & 0 & \frac{30597}{1638} & \frac{56560}{1638} \end{array}\right]$$

From this, we get

$$28x + 4y - z = 32$$
$$234y + 57z = 458$$

and

$$30597z = 56560$$

Now by back substitution, we get

$$\therefore \quad z = \frac{56560}{30597} = 1.8485472$$

$$y = \frac{458 - 57z}{234} = 1.5069778$$

and

$$x = \frac{-4y + z + 32}{28} = 0.9935941$$

Question no.6 answer below:

Example 10.9 Solve $y_{n+2} - 4y_{n+1} + 3y_n = 3^n + 1$.

Solution The given equation in symbolic form is

$$(E^2 - 4E + 3)\,y_n = 3^n + 1$$

AE is $E^2 - 4E + 3 = 0$, i.e. $(E-1)(E-3) = 0$, $\therefore E = 1, 3$

$$\therefore \qquad \text{CF} = C_1 (1)^n + C_2 3^n = C_1 + C_2 3^n$$

$$\text{PI} = \frac{1}{E^2 - 4E + 3}(3^n + 1) = \frac{1}{(E-1)(E-3)}(3^n + 1)$$

$$= \frac{1}{(E-1)(E-3)} 3^n + \frac{1}{(E-1)(E-3)}(1)^n$$

$$= \frac{1}{(3-1)} \quad \frac{1}{(E-3)} 3^n + \frac{1}{(1-3)} \cdot \frac{1}{(E-1)}(1)^n$$

$$= \frac{1}{2} n\, 3^{n-1} - \frac{1}{2} n\, (1)^{n-1} = \frac{1}{2} n\, (3^{n-1} - 1)$$

Hence, the general solution to Eqn (i) is

$$y_n = C_1 + C_2 3^n + \frac{1}{2} n\, (3^{n-1} - 1)$$

Question no.7 answer below:

Example 5.9 In the following table, one value of y is incorrect and that y is a cubic polynomial in x.

x	0	1	2	3	4	5	6	7
y	25	21	18	18	27	45	76	123

Construct a difference table for y and use it to locate and correct the wrong value.

Solution Since y is a cubic polynomial in x, the difference of third order i.e. $\Delta^3 y$ must be constant.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	25			
		−4		
1	21		1	
		−3		2
2	18		3	
		0		6
3	18		9	
		9		0
4	27		9	
		18		4
5	45		13	
		31		3
6	76		16	
		47		
7	123			

The sum of the third differences = 15

$\therefore$ each entry must be $15/5 = 3$

Hence, there are errors in the first four entries. They can be written as

$$2 = 3 + (-1),\ 6 = 3 - 3(-1),\ 0 = 3 + 3(-1),\ 4 = 3 - (-1)$$

$$\therefore \varepsilon = -1$$

Thus, the correct entry corresponding to $x = 3$ in $y = 18 - \varepsilon$ $= 18 - (-1) = 19$.

Question no. 8 answer below:

For any equations proves the bisection method.

??????????????????????????????????

Question no.9 answer below:

Example 2.5 Solve the equation $x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 = 0$ given that its roots are of the form $\pm a, \pm b, c$.

Solution Sum of the roots is

$$a - a + b - b + c = 5 \;\therefore\; c = 5$$

But c is a root of the given equation. Hence, $x - 5$ is a factor of the given equation.

$x = 5$	1	−5	−5	25	4	−20
	0	5	0	−25	0	20
	1	0	−5	0	4	$\boxed{0}$

$\therefore$ The depressed equation (by synthetic division) is

$$x^4 - 5x^2 + 4 = 0 \text{ or } (x^2 - 4)(x^2 - 1) = 0$$

$$\therefore \quad x = \pm 2, \pm 1$$

Hence, the roots are $\pm 2, \pm 1$ and 5.

Question no.10 answer below:

Example 2.15 Remove the second term in the transformed equation $x^4 - 8x^3 - x^2 + 68x + 60 = 0$ and hence solve it.

Solution Here, sum of the roots is $-(-8/1) = 8$

Degree = 4 $\therefore h = 8/4 = 2$.

Now, diminishing the given equation by 2 (using synthetic division) we have

2	1	−8	−1	68	60
	0	2	−12	−26	84
2	1	−6	−13	42	$\boxed{144}$
	0	2	−8	−42	
2	1	−4	−21	$\boxed{0}$	
	0	2	−4		
2	1	−2	$\boxed{-25}$		
	0	2			
	1	$\boxed{0}$			

Therefore, the transformed equation is

$$y^4 - 25y^2 + 144 = 0 \quad (\text{where } y = x - 2)$$

or $(y^2 - 9)(y^2 - 16) = 0$, i.e. $y = \pm 3, \pm 4$

Hence, the roots of the given equation are

$$x = -1, -2, 5, 6.$$