1 Solving the following equation using implicit methods

$$\frac{\partial u}{\partial t} = \nabla^2 u + f(u)$$

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \frac{u_{i+1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{(\Delta y)^2} + \lambda u_{i,j}^{n} - (u_{i,j}^{n})^3$$

Only the Laplacian terms are expected to be solved implicitly hence why we look at the next time step for the Laplacian terms only

$$u_{i,j}^{n+1} - \frac{(\Delta t)(u_{i+1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i-1,j}^{n+1})}{(\Delta x)^2} - \frac{(\Delta t)(u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1})}{(\Delta y)^2} = (\Delta t)\lambda u_{i,j}^{n} - (\Delta t)(u_{i,j}^{n})^3 + u_{i,j}^{n}$$

$$(1 + \frac{2\Delta t}{(\Delta x)^2} + \frac{2\Delta t}{(\Delta y)^2})u_{i,j}^{n+1} - \frac{\Delta t}{(\Delta x)^2}u_{i+1,j}^{n+1} - \frac{\Delta t}{(\Delta x)^2}u_{i-1,j}^{n+1} - \frac{\Delta t}{(\Delta y)^2}u_{i,j+1}^{n+1} - \frac{\Delta t}{(\Delta y)^2}u_{i,j-1}^{n+1} = (\Delta t)\lambda u_{i,j}^{n} - (\Delta t)(u_{i,j}^{n})^3 + u_{i,j}^{n}$$

$$\mathbf{A}\mathbf{U}^{n+1} = (\Delta t\lambda + 1)u_{i,j}^{n} - (\Delta t)(u_{i,j}^{n})^3$$

$$\mathbf{U}^{n+1} = \mathbf{A}^{-1}(\Delta t\lambda + 1)u_{i,j}^{n} - (\Delta t)(u_{i,j}^{n})^3$$