Bill Amount	How many people are eating	How many servers serve	Tips
X_1	X_2	X_3	Y
100	5	2	50
200	2	1	20

Intercept

$$\begin{bmatrix} 1 & 100 & 5 & 2 \\ 1 & 200 & 2 & 1 \\ 1 & 50 & 1 & 1 \\ 1 & & & \end{bmatrix} \begin{bmatrix} b_1 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} \$50 \\ 20 \\ 10 \end{bmatrix}$$

Linear Regression

Machine Learning

Historical data (Insurance premium)

Pattern → Model

Predict (new) future data

ML

- Supervised learning
- Unsupervised learning
- Reinforcement learning

Supervised Learning ✓

Target Data is available

||

which you need to find or predict

X	Y
Input	Output

Experience	Salary

Y

- Continuous → Regression ✓
  (Sales)
- Categorical → Classification ✓
  (spam)

Unsupervised

No Target Data

X_1	X_2	X_3

Segmentation - Clustering

Recommendation -

Reinforcement -

Reward

Iterative do job → based on the

Reward state doing to get better

Robotics,

Linear Regression

X	Y

$$Y = 3X + 2$$

$3 \rightarrow$ Slope ✓

$2 \rightarrow$ Intercept ✓

$Y \Rightarrow$ Model ✓

$X \rightarrow$ Input ✓

Slopeee

Stepness

Intercept

Intersection point on y when $a=0$

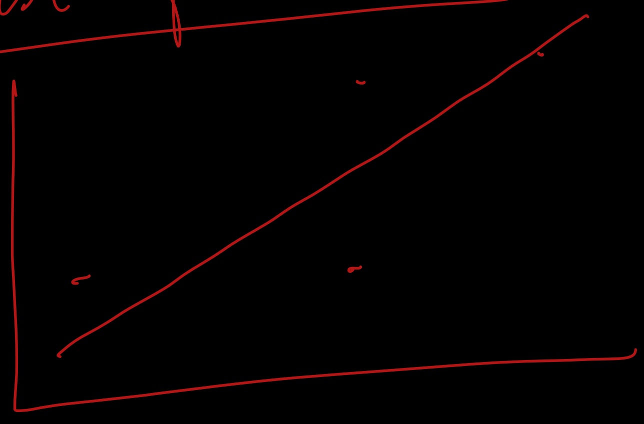
If you chang the slope
angle change → rotate

If you chang the intercept
It will form a mirror line
parallel

$$y = \underline{\acute{m}x + \acute{b}} + \underline{\varepsilon}\downarrow$$

ε → error → As minimal as possible

Best fit line ✓

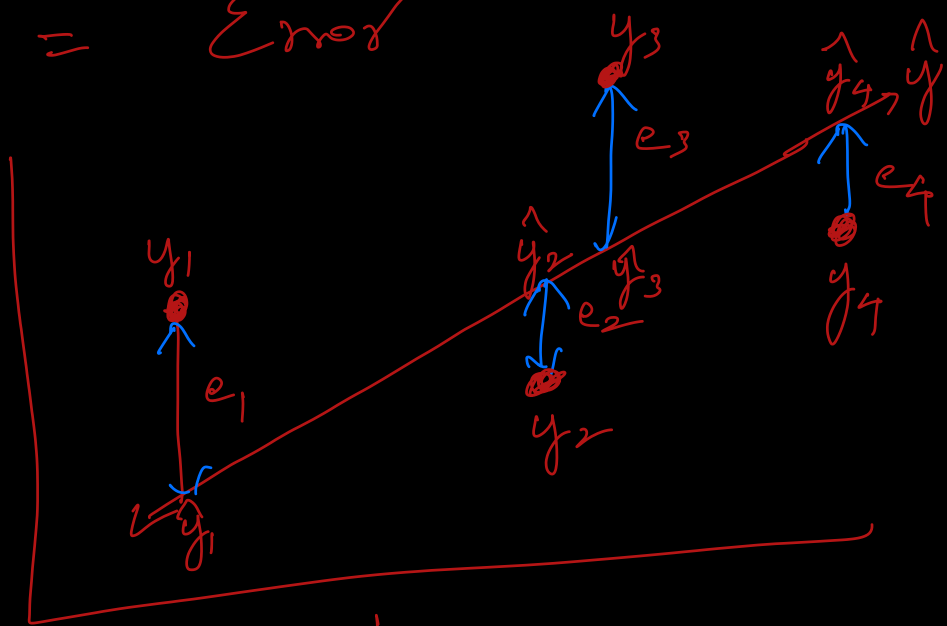


Passes through the Centroid

$$\left(\frac{x_1 + x_2 + x_3}{3} , \frac{y_1 + y_2 + y_3}{3} \right)$$

$\Sigma \Rightarrow$ minimum

Σ = Error



$e_1 = y_1 - \hat{y}_1$

$e_2 = y_2 - \hat{y}_2$

$e_3 = y_3 - \hat{y}_3$

$e_4 = y_4 - \hat{y}_4$

$e_1 + e_2 + e_3 + e_4$

$$\sum_{i=1}^{4} (e)_i$$

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)$$

end up in zero

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \Rightarrow \text{Sum of squared error (SSE)}$$

$$\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n} = \text{Mean Squared error (MSE)}$$

MSE is Minimal

Objective

Find Slope and Intercept

Criteria

MSE $\downarrow$

Linear Algebra

$$X\underline{\theta} = Y$$

$$X^T X \theta = X^T Y$$

$$\theta = \frac{1}{X X^T} X^T Y$$

$$\underline{\theta} = (X X^T)^{-1} X^T Y$$

$$\begin{bmatrix} b \checkmark \\ m \checkmark \end{bmatrix} = \theta \quad X = \begin{bmatrix} 1 & 50 \\ 1 & 5 \\ 1 & 67 \end{bmatrix} \quad Y = \begin{bmatrix} 20 \\ 10 \\ 5 \end{bmatrix}$$

Gradient Descent

$$\boxed{y = 6x^2 + 2x + 10}$$ ✓

find the value of x where
y is minimum

$$\frac{d}{dx} x^{n} = n x^{n-1}$$

$$\frac{d}{dx} x^2 = 2 x^{2-1} = 2x$$

$$\frac{d(x)}{dx} = 1 x^{1-1} = 1 \cdot x^0 = 1 \cdot 1$$

$$\frac{d\,x^5}{dx} = 5\,x^{5-1} = 5x^4$$

$$\frac{d\,(5)}{dx} = \frac{d(5\cdot x^0)}{dx} = 0 \times 5\,x^{0-1}$$

$$= 0$$

$$\frac{d\,(6)}{dx} = 0$$

$$\frac{d\,(6\times x^0)}{dx} = 0 \times x^{0-1} \times 6$$

$$= 0$$

Constant differentiation $= 0$

$$y = 6x^2 + 2x + 10$$

$$\frac{d(y)}{dx} = 6 \times 2 \times x^{2-1} + 2 \times 1 \times x^{1-1} + 0$$

$$= 12x^1 + 2$$

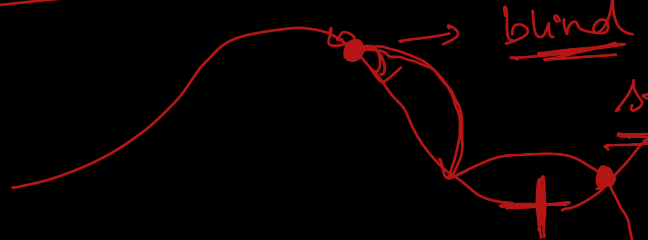
$$= 12x + 2$$

$$y = 6x^2 + 2x + 2$$

find value of x where y is minimum

$$\frac{dy}{dx} = 12x + 2 \longrightarrow \text{Direction}$$

We will start giving x with a random number



→ blind

stick → direction

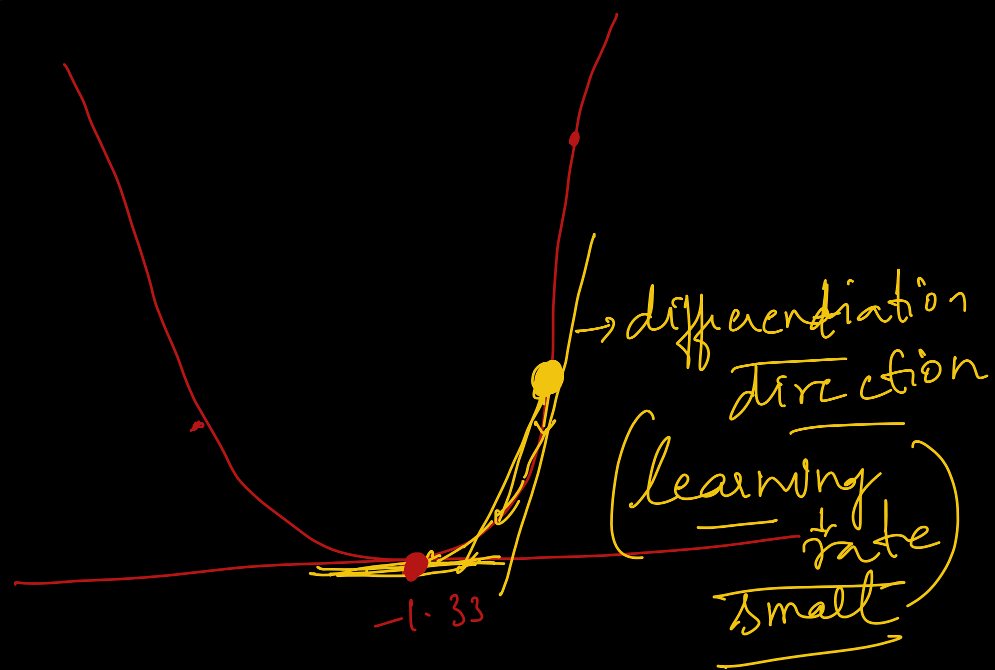
↓

step (How)

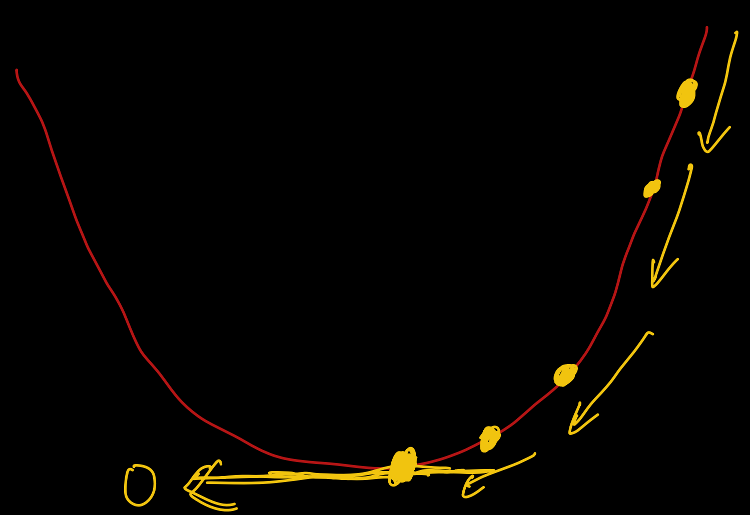
small step ⇒ better

1 → direction

2 → momentum → step



decide from you start



1. diffuentiation of equation
2. start with random number
3. find the diffention of random number choosen
4. learning rate to move to new point
5. with new point find the diffrention
6. repeat till the differentiation is ~~near~~ near to zero

$y = 6x^2 + 22x + 2$

$\frac{dy}{dx} = 12x + 2$

learning rate = 0.01

random number = 0

$$\left.\frac{dy}{dx}\right|_{(x=0)} = 12(0) + 2$$

$$= 2$$

current_x = 0 ✓

$$\boxed{\text{current\_x} - \text{der}(x) * \text{learning rate}}$$

$$0 - 2 * (0.01)$$

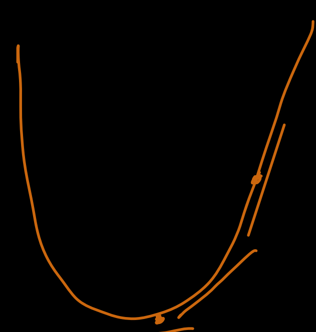
$$0 - 0.02$$

new_x = −0.02 ✓

$$\left.\frac{dy}{dx}\right|_{x=-0.02} = 12x + 2$$

$$= 12(-0.02) + 2$$

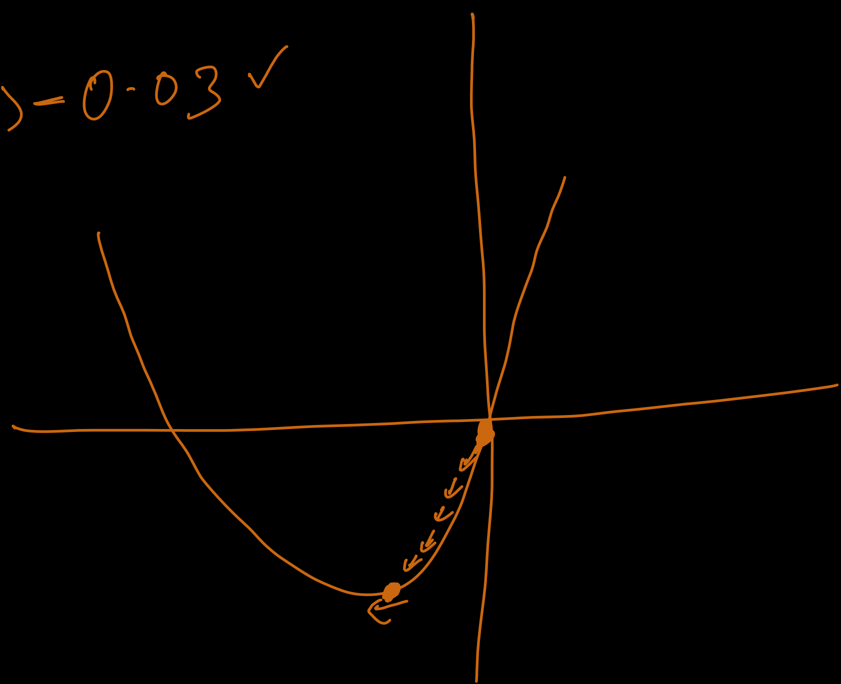
$$= 1.76$$



$$\boxed{\text{current\_x} - \underline{\text{der}(x)} * \text{learning rate}}$$

$$-0.02 - \underline{(1.76)} * 0.01$$

$$\Rightarrow -0.03 \checkmark$$



$$\text{curr\_x} = (-0.1)$$

$$\left.\frac{dy}{dx}\right|_{x=-0.1} = 12x + 2$$

$$= 12(-0.1) + 2$$

$$= -1.2 + 2 = 0.8$$

$$= -0.1 - (0.8) \times 0.01$$

$$= -0.108$$

$$-0.1$$

Given a equation you can find any variable where the equation is minimum

$$y = 6x^2 + 2x + 10$$

Found the value of x where y is mimum

MSE → Mean Squared Error

$$MSE = \frac{\sum (y - \hat{y})^2}{n}$$

$$\frac{\sum (y - (mx + b))^2}{n}$$

$$MSE = \frac{1}{n} \Sigma \left(y_i^{\nearrow \text{output}} - \left(\underset{?}{m} \underset{\text{Input}}{x_i} + \underset{?}{b} \right) \right)^2$$

Can you find the value of m and b where MSE is minimum ?

2 variable

partial derivative

$$\frac{d}{dx} \qquad \frac{\partial}{\partial x}$$

$$y = 6x^2 + 2x + 7z + 10 \qquad \text{Partial}$$

$$\frac{\partial y}{\partial x} = 12x + 2 + 0 + 0$$

$$\frac{\partial y}{\partial z} = 0 + 0 + 7 + 0$$

$$MSE = \frac{1}{n} \Sigma \left(y - (mx+b)\right)^2$$

$$(a-b)^2$$
$$a^2 + b^2 - 2ab$$

$$\frac{\partial E}{\partial m} = ? \qquad \frac{\partial E}{\partial b} = ?$$

$$MSE = \frac{1}{n} \Sigma \left(y^2 + (mx+b)^2 - 2y(mx+b)\right)$$

$$\frac{1}{n} \Sigma \left(y^2 + m^2x^2 + b^2 + 2mxb - 2ymx - 2yb\right)$$

$$\frac{\partial}{\partial m} = \frac{1}{n}\sum\left(0 + \underline{2mx^2} + \underline{0} + 2xb \underline{- 2yx - 0}\right)$$

$$= \frac{1}{n}\sum\left(2mx^2 + 2xb - 2yx\right)$$

$$= \frac{1}{n}\sum x\left(2mx + 2b - 2y\right)$$

$$= \frac{2}{n}\sum x\left(mx + b - y\right)$$

$$= \frac{2}{n}\sum x\left(-y + mx + b\right)$$

$$\frac{\partial}{\partial m} = \frac{2}{n}\sum\left[-x\left(y - (mx + b)\right)\right]$$

Direction with resp to m
(slope)

$$MSE = \frac{1}{n} \sum \left(y^2 + (mx+b)^2 - 2y(mx+b) \right)$$

$$\frac{1}{n} \sum \left(y^2 + m^2x^2 + b^2 + 2mxb - 2ymx - 2yb \right)$$

$$\frac{\partial}{\partial b} = \frac{1}{n} \sum \left(0 + 0 + 2b + 2mx - 0 - 2y \right)$$

$$= \frac{1}{n} \sum \left(2b + 2mx - 2y \right)$$

$$= \frac{2}{n} \sum \left(b + mx - y \right)$$

$$\frac{2}{n} \sum \left(-y + mx + b \right)$$

$$\frac{\partial}{\partial b} = \frac{2}{n} \sum -\left(y - (mx+b) \right)$$

Direction with resp to b (intercept)

$m = 0$ $\nearrow$ MSE m MSE $\downarrow$

$\searrow$