Gauss-Jordan Elimination Method with a Real-Life Example

Gauss-Jordan Elimination

- **Gauss-Jordan Elimination** is a step-by-step method used to solve a system of linear equations
  - by transforming the **augmented matrix** into **Reduced Row Echelon Form (RREF)** using row operations.

Steps of the Method:

- Form the augmented matrix of the system.
- Make the leading entry (pivot) in each row a **1**.
- Make all other entries in the pivot's column **0**.
- Continue until the matrix is in **RREF**, then read off the solution.

✅ Real-Life Example:

📌 Scenario:

A small business sells 3 products: A, B, and C. Their total profits from 3 different sales zones are given as:

- Zone 1: A + 2B + C = 100
- Zone 2: 2A + 3B + 3C = 200
- Zone 3: A + B + 2C = 150

Let's solve to find the individual profits per product (A, B, and C).

🔹 Step 1: Form the Augmented Matrix

$$\begin{bmatrix} 1 & 2 & 1 & | & 100 \\ 2 & 3 & 3 & | & 200 \\ 1 & 1 & 2 & | & 150 \end{bmatrix}$$

🔹 Step 2: Apply Gauss-Jordan Elimination

Perform row operations:

1. R1 stays the same
2. R2 = R2 - 2×R1 → [0, -1, 1, | 0]
3. R3 = R3 - R1 → [0, -1, 1, | 50]
4. R3 = R3 - R2 → [0, 0, 0, | 50] → ❌ contradiction!

📌 **Interpretation:**

The last row implies **0 = 50**, which is not possible.

This is an **inconsistent system** — no solution exists. This suggests there is a mistake or inconsistency in the profit data.

✅ Another Real-Life Consistent Example:

Let's solve the following:

$$\begin{aligned} x + y + z &= 6 \\ 2x + 3y + 7z &= 20 \\ x + 3y + 4z &= 13 \end{aligned}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 3 & 7 & 20 \\ 1 & 3 & 4 & 13 \end{array}\right]$$

After Gauss-Jordan elimination:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

📌 Solution:

- $x = 1$
- $y = 2$
- $z = 3$

🔍 Real-World Applications:

- **Business**: Profit analysis per product
- **Engineering**: Solving circuit systems or forces in structures
- **Chemistry**: Determining chemical mixtures
- **Economics**: Resource allocation models

Gauss-Jordan Elimination method vs Elimination

Feature	Gauss Elimination	Gauss–Jordan Elimination
Final Form	Upper Triangular Matrix	Reduced Row Echelon Form (RREF)
Requires Back Substitution?	✓ Yes	✕ No
Simpler for Programming?	✓ Yes	✕ More complex
Faster for Large Systems?	✓ Often	✕ Slower
Direct Solution?	✕ No	✓ Yes (you **don't need back-substitution**)
Ideal for Inverse Finding?	✕ No	✓ Yes (Yes — Gauss–Jordan Elimination is the ideal method for finding the inverse of a matrix.)