Milne's Predictor Corrector Method Explained Easily / Milne's Method

Milne's Predictor-Corrector Method Explained

Q. Given $\frac{dy}{dx} = \frac{1}{2}(x+y)$, $y(0) = 2$,
$y(0.5) = 2.636$, $y(1.0) = 3.595$, $y(1.5) = 4.968$
Now, find $y(2)$ by Milne's Method.

Solution:- Milne's Predictor Formula is:

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right] \quad \text{---(1)}$$

Putting $n = 3$, we get

$$y_{4p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right] \quad \text{---(2)}$$

We're given that,

$x_0 = 0$, $x_1 = 0.5$, $x_2 = 1.0$, $x_3 = 1.5$

$y_0 = 2$, $y_1 = 2.636$, $y_2 = 3.595$, $y_3 = 4.968$

The given differential equation is $y' = \frac{1}{2}(x+$

$y_0 = 2$, $y_1 =$ [illegible]

The given differential equation is $y' = \frac{1}{2}(x+y)$

From the above equation, we'll calculate y_1', y_2', y_3'

$y_1' = \frac{1}{2}(x_1 + y_1) = \frac{1}{2}(0.5 + 2.636) = 1.568$

$y_2' = \frac{1}{2}(x_2 + y_2) = \frac{1}{2}(1.0 +$ [illegible] $) = 2.2975$

$y_2 =$ [illegible]

$y_3' = \frac{1}{2}(x_3 + y_3) = \frac{1}{2}(1.5 + 4.968) = 3.324$

Substituting values in (2), we get

$y_{4p} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$

$= 2 + \frac{4 \times 0.5}{3}\left[2 \times 1.568 - 2.2975 + 2 \times 3.3\right.$ [illegible]

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6.871

"If four values of y are not given you have to use Taylor's Method to find remaining values of y.

So, our predicted value is 6.871.

Now, we'll correct it to get actual value by Milne's Corrector Formula, which is

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

By putting $n = 3$, we get

$$y_{4c} = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right] \quad \text{---(4)}$$

Now, $y'_4 = \frac{1}{2}(x_4 + y_4) = \frac{1}{2}(2 + 6.871)$

$= 4.4355$

Now, by putting required values in (4), we get

$$y_{4c} = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 3.595 + \frac{0.5}{3}\left[2.2 + 4 \times 3.234 + 4.4355\right]$$

$$= 6.8731$$

$\therefore y(2) = 6.8731$ Ans