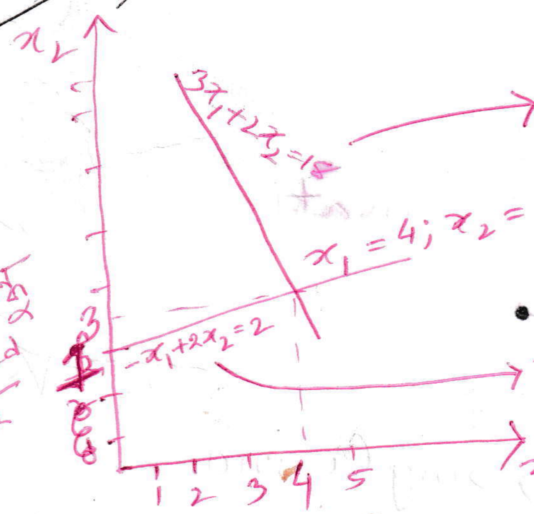
pg-233 Sheet-1

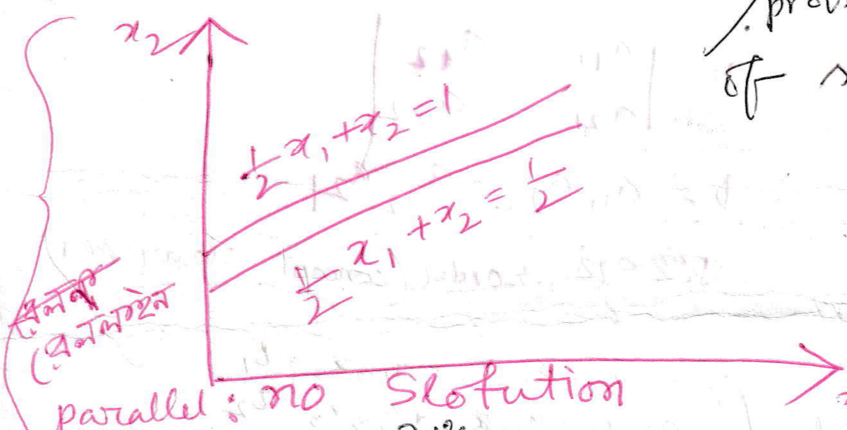
Lecture-1
~~1st class~~



$x_2 = \left(-\frac{3}{2}\right)x_1 + 9$ — Slope, intercept

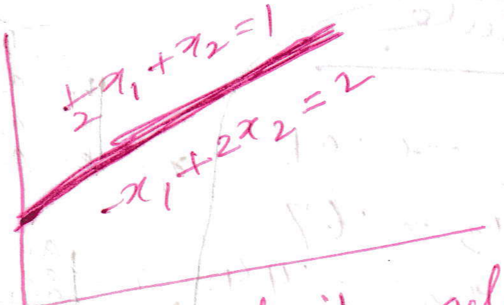
$x_2 = \frac{1}{2}x_1 + 1$

For 3 simultaneous equ^{ns}, each eq^n would be represented by a plane in 3 dimensional co-ordinate system. The point where the 3 planes intersect would represent the sol^n. Beyond 3 equations, graphical methods break down and, consequently, have little practical value for solving simultaneous equations. However, they sometimes prove useful in visualizing properties of solutions. e.g.; Fig 9.2

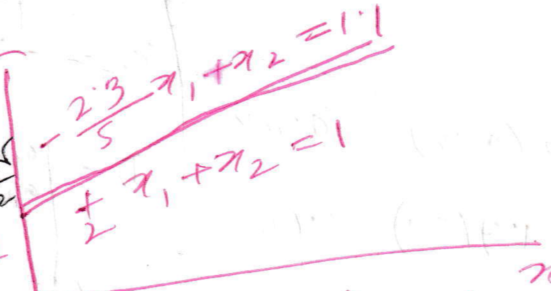


parallel: no solution
9.2a

Singular



coincident: infinite solution



ill-conditioned

Slope very close: point of intersection is difficult to detect

Cramer's Rule এর আগে
Determinant জেনে নেই

For a set of 3 equations
$\{A\}\{X\} = \{B\}$

$$\{A\} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow \text{matrix}$$

সম্পূর্ণ ভিন্ন ([illegible])

a mathematical concept

"Bracket হলে matrix
straight line হলে determinant"

$$D = \begin{vmatrix} & \text{same} & \end{vmatrix} \rightarrow \text{Single no.}$$

উদা:

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$D = a_{11}a_{22} - a_{12}a_{21}$$

এইভাবে 3 order (concept clear হবে)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$
$$\vdots$$
$$b_n$$

Cramer's Rule

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{D}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{D}$$

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{D}$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Ex: 9.3 : Cramer's Rule

$$.3x_1 + .52x_2 + x_3 = -0.01$$
$$.5x_1 + x_2 + 1.9x_3 = .67$$
$$.1x_1 + .3x_2 + .5x_3 = -.44$$

Determinant:

$$D = \begin{vmatrix} .3 & .52 & 1 \\ .5 & 1 & 1.9 \\ .1 & .3 & .5 \end{vmatrix}$$

minors

$$A_1 = \begin{vmatrix} 1 & 1.9 \\ .3 & .5 \end{vmatrix} = 1(0.5) - 1.9(0.3) = -0.07$$

$$A_2 = \begin{vmatrix} .5 & 1.9 \\ .1 & 0.5 \end{vmatrix} = 0.5(0.5) - 1.9(0.1) = 0.06$$

$$A_3 = \begin{vmatrix} .5 & 1 \\ .1 & .3 \end{vmatrix} = .5 \times .3 - 1 \times .1 = 0.05$$

[illegible] $.05 = .0022$ [illegible] $= .04356$

more than 3 Cramer's rule becomes impractical because as the number of equations increases, the determinants are time consuming by hand (or by computer)

Gauss Elimination

$$3x_1 - .1x_2 - .2x_3 = 7.85 \quad \text{--- (1)}$$

a[0][0]

a[1][0]

$$.1x_1 + 7x_2 - .3x_3 = -19.3 \quad \text{--- (2)}$$

$$.3x_1 - .2x_2 + 10x_3 = 71.4 \quad \text{--- (3)}$$

Step 1: Forward Elimination

Step 2: Back Substitution

Step 1

$(2) - (1) \times \frac{.1}{3} \Rightarrow$

$7.00333x_2 - .29333x_3 = -19.5617$ --- (4)

$\left(+7 + \frac{.01}{3}\right)x_2 = 7.00333x_2$

$(-.3 + .00667)x_3 = -.29333x_3$

$-19.3 - \frac{7.85 \times .1}{3} = -19.5617$

$(3) - (1) \times \frac{.3}{3} \Rightarrow$

$-.19x_2 + 10.02x_3 = 70.615$ --- (5)

$(5) - (4) \times \frac{-.19}{7.00333} \Rightarrow$

$10.02x_3 = 70.0843$

Overall $\Rightarrow$

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \quad \text{--- (6)}$$

$$7.00333x_2 - .293333x_3 = -19.5617 \quad \text{--- (7)}$$

$$10.0200x_3 = 70.0843 \quad \text{--- (8)}$$

Step 2

(8) $\Rightarrow x_3 = \dfrac{70.0843}{10.02} = 7.00003$

(7) $\Rightarrow 7.00333x_2 - .29333(7.00003) = -19.5617$

$\Rightarrow x_2 = -2.5$

(6) $\Rightarrow 3x_1 - 0.1(-2.5) - 0.2(7.00003) = 7.85$

$\Rightarrow x_1 = 3$

[illegible]
Assignment
class এ দুইবার Roll call

Gauss - Jordan

Variation of Gauss Elimination

Difference Bet^n Gauss - Jordan & Elimantion (i) Unknown is eliminated from all other equations rather than just the subsequent ones

(ii) Identity matrix rather than triangular matrix

(iii) No back substitution

$3x_1 - .1x_2 - .2x_3 = 7.85$
$.1x_1 + 7x_2 - .3x_3 = -19.3$
$.3x_1 - .2x_2 + 10x_3 = 71.4$

$\rightarrow$ Augmented matrix $\rightarrow$
$$\left[\begin{array}{ccc|c} 3 & -.1 & -.2 & 7.85 \\ .1 & 7 & -.3 & -19.3 \\ .3 & -.2 & 10 & 71.4 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & -.033 & -.067 & 2.616 \\ .1 & 7 & -.3 & -19.3 \\ .3 & -.2 & 10 & 71.4 \end{array}\right] r_1' = \frac{r_1}{3}$$

$$\left[\begin{array}{cccc} 1 & -.03 & -.067 & 2.616 \\ 0 & 7.0033 & -.29 & -19.5617 \\ 0 & -.19 & 10.02 & 70.615 \end{array}\right] \begin{array}{l} r_2' = r_2 - r_1' \times .1 \\ r_3' = r_3 - r_1' \times .3 \end{array}$$

$$\left[\begin{array}{cccc} & -.033 & -.067 & 2.616 \\ & 1 & -.041 & -2.79 \\ & -.19 & 10.02 & 70.615 \end{array}\right] \quad r_2' = \frac{r_2}{7.0033}$$

$$\begin{bmatrix} 1 & 0 & -.068 & 2.52 \\ 0 & 1 & -.041 & -2.79 \\ 0 & 0 & 10.012 & 70.08 \end{bmatrix} \quad \begin{array}{l} r_1' = r_1 + r_2 \times .0333 \\ r_3' = r_3 + r_2 \times \cancel{(-10.01200)} (.19) \end{array}$$

$$\begin{bmatrix} 1 & 0 & -.068 & 2.52 \\ 0 & 1 & .041 & -2.79 \\ 0 & 0 & 1 & 7.003 \end{bmatrix} \quad r_3 = \frac{r_3}{10.012}$$

$$.041 - 1 \times .041$$

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}\begin{bmatrix} 1 & 0 & 0 & 3.000 \\ 0 & 1 & 0 & -3.07 \\ 0 & 0 & 1 & 7.003 \end{bmatrix}$$

$$r_2 = r_2 - r_3 \times .041$$
$$= -2.79 - 7.003 \times .041 = -3.07$$
$$r_1 = r_1 - r_3 \times (-.068)$$
$$= 2.52 - 7.003(-.068)$$
$$=$$

Gauss-Jordan involves approximately 50% more operation than Gauss elimination. But it is still used in engineering as well as in some numerical algorithms

Triangularisation method or Factorisation process (Choleski's process)

$AX = B \Rightarrow LUX = B \Rightarrow UX = Y \Rightarrow LY = B$ এবং $UX = Y$

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B. RAO

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

Comparing $u_{11} = 1, \ u_{12} = 1, \ u_{13} = -1$

~~[illegible]~~

~~[illegible]~~

$u_{11}, u_{12}, u_{13}, l_{21}, u_{22}, u_{23}, l_{31}, l_{32}, u_{33}$

$AX = B \Rightarrow LUX = B$

i.e.;
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 7 \end{bmatrix} X = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

Let, $UX = Y$ ——— (2)

$\therefore LY = B$

i.e.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

$y_1 = 2,\ 2y_1 + y_2 = -3 \Rightarrow 2 \times 2 + y_2 = -3 \Rightarrow y_2 = -7$

$3y_1 - y_2 + y_3 = 6 \Rightarrow y_3 = 6 - 3 \times 2 + (-7) = -7$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ -7 \end{bmatrix}$$

$7x_3 = -7 \Rightarrow x_3 = -1$

$x_2 + 7x_3 = -7$ giving $x_2 = 0$

$x_1 + x_2 - x_3 = 2$

$\Rightarrow x_1 = 1$

$x_1 = 1$

$x_2 = 0$

$x_3 = -1$