0.16 MATHS 103 2010/2011

1. Evaluate $\tan 15^0$ to get
(a) $1+\sqrt{2}$ (b) $1-\sqrt{3}$ (c) $2-\sqrt{3}$
(d) $\sqrt{3}-3$ (e)$2\sqrt{3}$
Solution
$\tan 15$ is $2-\sqrt{3}(C)$ using calculator

2. Find the value of
$\tan(120^0-\theta)+\tan(60^0+0)$
(a) 20 (b) 0 (c) 1 (d) 2 (e) 1.5
Solution
$tan(120-\theta)+tan(60+\theta)$
N.B $tan(2\alpha-\theta)+tan(\alpha+\theta$ is 0
$\therefore tan(120-\theta)+tan(60+\theta)$ is 0(B)

3. A straight line makes equal intercept with the coordinate axes and passes through the point $(1,1/2)$,what is its equation
(a) $2x+2y=3$ (b) $x+y=10$ (c) x+y=8
(d) $2x-3y=10$ (e) $x-y=5$
Solution
The formulae for a straight line with equal intercept is
$\frac{x}{a}+\frac{y}{b}=1$ where $(1,\frac{1}{2})=(a,b)$
$\frac{x}{1}+\frac{y}{\frac{1}{2}}=1$
$x+2y=1$ No correct option.

Given the circle $x^2+y^2-4x+6y=12$ answer question 4-10

4. The center of the circle is the point
(a) (3,2) (b) $(-4,6)$ (c) $(2,-3)$
(d) $(1,-2)$ (e) $(1,3)$
Solution
$x^2+y^2-4x+6y=12$
$x^2+(-2)^2+y^2+(3)^2=12+(-2)+(3)^2$
$(x-2)^2+(y+3)^2=12+4+9$
$(x-2)^2+(y+3)^2=25$
$\therefore (x-a)+(y-b)^2=r^2$
$(a=2,b=-3)=(2,3)(C)$

5. The radius of the circle is
(a) 8 (b) 10 (c) 15 (d) 12 (e) 4
Solution
Radius $r=\sqrt{25}=\pm5(C)$

6. The tangent drawn from point $(-3,3)$ to the circle is ...... units
(a) 8 (b) 13 (c) 4 (d) 12 (e) -3
Solution
Recall,$l^2=(x+y)^2+(y+f)^2-r^2$
$(x,y)=(-3,3); r=5; (g,f)=(-2,+3)$
$l^2=(-3-2)^2+(3-3)^2-25$
$=25+36-25$
$l=\sqrt{36}$
$l=6(E)$

7. The tangent line to the circle at the point $(-2,0)$ has slope :
(a) 4/3 (b) 2/3 (c) 2 (d) 1/3 (e) -3
Solution
Let m,be the gradient between the centre $(2,-3)$ and the given point $(-2,0)$
$M_1=\frac{0-(-3)}{-2-2}=\frac{-3}{4}$
for tangent $m_2=\frac{-1}{m_1}=\frac{4}{3}(A)$

8. The equation to the tangent line to the circle at the point (6,0) is
(a) $4x+2y-23=0$ (b) $2x+4y-21=0$
(c) $3x-4y+1=0$ (d) $3x+4y-18=0$
(e) None of the above
Solution
Recall equation of tangent is
$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$
$(x_1,y_1)=(6,0)$,g=-2 f=3 c=-12
$6x-2(x+6)+3(y)-12=0$
$6x-2x-12+3y-12+0$
$4x+3y-24=0(E)$

9. The circle touches $x^2+y^2-2x-4y=36$ at the point :
(a) (7,3) (b) $(-3,-3)$ (c) $(7,-3)$
(d) (1,2) (e) (4,3)
Solution
Given $x^2+y^2-2x-4y-36=0$
By inspection i.e substituting all the option into the above equation only option (B) (-3,-3) will give 0
i.e $(-3)^3+(-3)^2-2(-3)-4(-3)-36=0$
or from
$x^2+y^2-4x+6y=12----i$ *and*
$x^2+y^2-2x-4y=36---ii$,
$eqni-eqnii=-2x-10y=-24$
$x=5y+12$
sub $x=5y+12$ *into eqni*
$(5y+12)^2+y^2-4(5y+12)+6y=12$
$25y^2+106y+84=0$
$y=-1.1$; *or* $y=-3$
when $y=-3$

27. The value of $\cos 15^0$ is
(a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (c) $\frac{\sqrt{6}}{2}$
(d) $\frac{\sqrt{2}-\sqrt{3}}{4}$ (e) $-\frac{\sqrt{2}}{2}$
Solution
$\cos 15^0 = \cos(60-45)$ using
$\cos(A-B) = \cos A \cos B + \sin A \sin B$
$\therefore \cos(60-45) = \cos 60 \cos 45 + \sin 60 \sin 45$
$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$
$= \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}(A)$

28. The length of a perpendicular drawn from the origin to the line l is $\sqrt{48}$ *units* .The perpendicular makes an angle of 60^0 with the x-axis.The equation of l is
(a) $x - \sqrt{3}y = 6$ (b) $x + y = \sqrt{48}$
(c) $2x + \sqrt{3}y = 2$ (d) $x + \sqrt{3}y = 8\sqrt{3}$
(e) $\sqrt{3}x + y = 10$
Solution
Recall that if a perpendicular make an angle of θ with x-axis the equation of the length of the perpendicular is
$l = x\cos\theta + y\sin\theta$ i.e
$x\cos 60 + y \sin 60 = \sqrt{48}$
$\frac{x}{2} + \frac{\sqrt{3}}{2}y = \sqrt{48}$
$x + \sqrt{3}y = 8\sqrt{3}(D)$

29. $\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta}$ simplifies to
(a) $\tan\theta$ (b) $-\cot\theta$ (c) $\cos 2\theta$ (d) $-\tan 2\theta$
((e) $\sin\frac{1}{2}\theta$
Solution
Using $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
I.E
$\sin 5\theta - \sin 3\theta = 2\cos(4\theta)\sin\theta$
$\therefore \frac{\sin 5\theta - \sin 3\theta}{\sin 5\theta + \cos 3\theta} = \frac{2\cos 4\theta \sin\theta}{2\cos 4\theta\cos\theta}$
$= \tan\theta(A)$

30. if $\tan(A+60) = 2$ then $\cot A$ is
(a) $\frac{1}{3}$ (b)$\frac{1}{2}$ (c) 3 (d) $\frac{3}{4}$
(e) none of the above
Solution
$\tan(A+60) = 2$
$A + 60 = \tan^{-1} 2$
$A + 60 = 63.43$
$A = 663.43 - 60 = 3.34 \approx 3(C)$

A circle passes through the points $(3, 1-)$, $(6, 0)$ & $(0, 8)$ use the information to answer question 31-38

31. The equation of the circle is
(a) $x^2+y^2-3x-4y=4$ (b) $x^2+y^2=25$
(c) $x^2+y^2-6x-8y=0$
(d) $x^2+y^2-2x-5y=0$
Solution
$(3,-1) = (x_1, y_1)$ $(0,8) = (x_3, y_3)$
$(6,0) = (x_2, y_2)$
Using the formula
$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$
@ $(3,-1)$
$9 + 1 + 6g - 2f + c = 0$
$6g - 2f + c = -10 - - - - - - - - (i)$
at $(0,8)$
$0 + 64 + 0 + 16f + c = 0$
$16f + c = -64 - - - - - - - - (ii)$
at $(6,0)$
$36 + 12g + c = 0$
$12g + c = 0 - - - - - - - - (iii)$
Solving the 3 equation simultaneously $g = -3, f = -4, c = 0$
i.e $x^2 + y^2 + 2gx + 2fy + c = 0$
$x^2 + y^2 - 6x - 8y = 0(C)$

32. The coordinate of the centre is
(a) (4,3) (b) (2,4) (c) (0,4)
(d) (3,4) (e) (4,2)
Solution
$x^2 + y^2 - 6x - 8y = 0$
$x^2 + (-3)^2 + y^2 + (-4)^2 = 0 + 9 + 16$
$(x-3)^2 + (y-4)^2 = 25$
compare with $(x-a)^2 + (y-b)^2 = r^2$
$a = 3, b = 4, r = 5$
center (3,4) (D)

33. The equation of its tangent at the point (3,9) is
(a) $x+y=3$ (b) $x-y=3$ (c) $x+9=0$
(d) $y=9$ (e) $x+y=9$
Solution
Using the formula
$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$
$(x_1, y_1) = (3,9)$, $g = -3$, $c = 0, f = -4$
$3x + 9y - 3(x+3) - 4(y+9) + 0 = 0$
$5y = 45$
$y = 9(D)$

34. The units of length of tangent drawn from the point $(8,-6)$ is
(a) 12 (b) 6 (c) 8 (d) 14 (e)10

Solution

$\cos 300 \sin 390 + \cos 660 \sin 570$

using calculator $= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{-1}{2}$

$\frac{1}{4} - \frac{1}{4} = 0(E)$

33. The points $(-1,-1)$ and $(1,1)$ are ends of a diameter of a circle whose equation is
(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 + 2x = 0$ (c) $x^2 + y^2 - 4x = 0$ (d) $x^2 + y^2 = 16$ (e) $x^2 - y^2 = 0$

Solution

The midpoint of the diameter is $(-1,-1) and (1,1)$

and $r = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$

Using $(x-a)^2 + (y-b)^2 = r^2$

$(x-0)^2 + (y-0)^2 = (\sqrt{2})^2$

$x^2 + y^2 = 2(A)$

Given that $\sin x = \frac{4}{5}, \tan y = \frac{5}{12}$ x and y are acute angles.Answer question 34-40

34. The value of $\cos 2x$ is
(a)$-\frac{1}{2}$ (b) $\frac{4}{7}$ (c) $-\frac{7}{25}$ (d) $\frac{9}{25}$ (e) -2

Solution

$\sin x = \frac{4}{5}, \tan y = \frac{5}{12}$

using Pythagoras theorem

$cos x = \frac{3}{5}, sin y = \frac{5}{13},$

$tan x = \frac{4}{3}, cos y = \frac{12}{13}$

Recall,$cos 2x = 1 - 2 sin^2 x = 1 - 2(\frac{4}{5})^2$

$= 1 - \frac{32}{25} = \frac{-7}{25}(C)$

35. The value of $\cos(x-y)$ is
(a) $-\frac{3}{13}$ (b) $\frac{7}{13}$ (c) $\frac{4}{65}$ (d) $\frac{21}{25}$ (e) $\frac{56}{65}$

Solution

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} \frac{36}{65} + \frac{20}{65} = \frac{56}{65}(E)$

36. The value of $\tan \frac{1}{2}$x is
(a) $\frac{1}{2}$ (b) -1 (c) $-\frac{1}{5}$ (d) $\frac{2}{3}$ (e) 0

Solution

$\tan \frac{1}{2} x = \frac{\sin \frac{1}{2} x}{cos \frac{1}{2} x}$

But $sin \frac{1}{2} x = \sqrt{\frac{1 - cos x}{2}}$

Also $\cos \frac{1}{2} x = \sqrt{\frac{1 + cos\theta}{2}}$

$\therefore \tan \frac{1}{2} x = \frac{1 - \cos x}{\sin x} = \frac{1}{\sin x} - \cot x$

$= \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}(A)$

37. The value of $\sec \frac{1}{2} y$ is :
(a) $\frac{\sqrt{13}}{5}$ (b) $\frac{5\sqrt{26}}{26}$ (c) $\frac{25}{26}$ (d) $\frac{\sqrt{26}}{5}$ (e) -2

Solution

Recall,$\sec^2 x = 1 + \tan^2 x$

$\therefore \sec^2 \frac{1}{2} y = 1 + \tan^2 \frac{1}{2} y$

$\sec \frac{1}{2} y = \sqrt{1 + \tan^2 \frac{1}{2} y}$

But $\tan \frac{1}{2} y = \frac{1}{\sin y} - \cot y = \frac{13}{5} - \frac{12}{5} = \frac{1}{5}$

$\sec \frac{1}{2} y = \sqrt{1 + \frac{1}{25}} = \sqrt{\frac{26}{25}}$

$= \frac{\sqrt{26}}{5}(D)$

38. The value of $\frac{\sin 2y}{\sin 2x}$ is
(a) $\frac{8}{65}$ (b) $\frac{4}{13}$ (c) $\frac{120}{169}$ (d) $\frac{24}{25}$ (e) $\frac{125}{169}$

Solution

$\frac{\sin 2y}{\sin 2x} = \frac{2 \sin y \cos y}{2 \sin x \cos x} = \frac{\frac{5}{13} \times \frac{12}{13}}{\frac{4}{5} \times \frac{3}{5}}$

$= \frac{60}{169} \times \frac{25}{12} = \frac{125}{169}(E)$

39. The value of $\tan(x+y)$ is
(a) $\frac{23}{4}$ (b) $\frac{7}{4}$ (c) $\frac{63}{16}$ (d) $\frac{9}{4}$ (e) $\frac{21}{16}$

Solution

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} =$

$\frac{16+5}{12} \div \frac{16}{36} = \frac{21}{12} \times \frac{36}{16} = \frac{63}{16}(C)$

40. The value of $\sin 3x + \sin x$ is
(a) $\frac{36}{5}$ (b) $\frac{20}{125}$ (c) $\frac{9}{125}$ (d) $-\frac{36}{25}$ (e) $\frac{16}{25}$

Solution

$\sin 3x + \sin x = 2 \sin 2x \cos x$

$= 2 \cos x (2 \sin x \cos x) = 4 \sin x \cos^2 x$

$= 4 \times \frac{4}{5} \times \frac{9}{25} = \frac{144}{125}$

No correct option

$x = 5(-3) + 12 = -3$
$= (-3, -3)(B)$

10. The sum of squares of the radii of the given circle and $x^2 + y^2 - 2x - 4y = 36$ is
(a) 36 (b) 66 (c) 41 (d) 46 (e) 54
Solution
From $x^2 + y^2 - 4x + 6y = 12$; $r_1 = 25$
from $x^2 + y^2 - 2x - 4y = 36$
$(x-1)^2 + (y-2)^2 = 41 \quad r_2 = 41$
$r_1 + r_2 = 41 + 25 = 66(B)$

11. The angle between the lines $3x - 3xy + 17 = 0$ and $x - y - 3 = 0$ is
(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$ (e) π
Solution
$3x - 3y + 17 = 0$
$3y = 3x + 17$
$y = x + \frac{17}{3} \quad m_1 = 1$
$x - y - 3 = 0$
$y = x - 3 \quad m_2 = 1$
Recall Angle between two line is
$tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$
$tan\theta = \frac{1-1}{2} = 0$
$\theta = tan^{-1}0 = 0(A)$

12. The length of a perpendicular drawn from the origin to the line l is $5\sqrt{2}$ unit.The perpendicular makes an angle of 45^0 with the axis.The equation of l is
(a) $x-y = 6$ (b) $x+y = 50$ (c) $x+y = \sqrt{2}$
(d) $2x + y = 1$ (e) x+y= 10
Solution
The formulae is $xcos\theta + ysin\theta = l$
$xcos45 + ysin45 = 5\sqrt{2}$
$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$
$x + y = 10$

13. The area in square units of a triangle whose vertices $(1, 3), (3, -1), (-3, -3)$ *is*
(a) 36 (b) 19 (c) 11 (d) 14 (e) 13
Solution
Given$(x_1, y_1) = (1, 3)$
$(x_2, y_2) = (3, -1)(x_3, y_3) = (-3, -3)$
Area of a triangle is given by $area = \frac{1}{2}(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)) Or$

$Area = \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$
$area = \frac{1}{2}(1(-1+3) + 3(-3-3) + (-3)(3+1))$
$= \frac{1}{2}(-2 - 18 - 12) = \frac{-32}{2} = 16.$
No correct option

14. The value of $\cos 75^0 - \cos 15^0$ is
(a) $-\frac{2\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{\frac{3}{2}}$
(d) $-\frac{\sqrt{2}}{3}$ (e) $-\frac{\sqrt{2}}{2}$
Solution
$cos75^0 - cos15$
using $cosA - cosB = -2sin\frac{A+B}{2}sin\frac{A-B}{2}$
$cos75 - cos15 = -2sin45sin30$
$= -2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$= \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}(E)$

15. What is the value of the angle between the pair of lines $x^2 + xy - 6y^2 = 0$?
(a) $\frac{\pi}{4}$ ((b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π (e) 0
Solution

16. Find the value of x if,
$\tan^{-1}(1) + \tan^{-1}3x = \frac{\pi}{2}$
(a) 1 (b) $-1/6$ (c) 0
(d) 1/3 (e) Not defined
Solution
$Tan^{-1}1 + tan^{-1}3x = \frac{\pi}{2}$
Let $\alpha = tan^{-1}1 \quad \therefore \tan\alpha = 1$
and $\beta = tan^{-1}3x$
$tan\beta = 3x$
$tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha tan\beta} = \frac{\pi}{2}$
$\frac{1+3x}{1-3x} = \frac{\pi}{2}$
$\frac{1+3x}{1-3x} = 90$
$1 + 3x = 90 - 270x$
$273x = 89$
$x = \frac{273}{89} = \frac{1}{3}(D)$

17. if $\tan(A + 60^0) = \sqrt{3}$ then $\cot A$ is
(a) $-\sqrt{3}$ (b) 2/3 (c) 4/3
(d) $\sqrt{3}$ (e) Undefined

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Given that $\cot x = 2$, and $\tan y = \frac{1}{3}$, x,y are acute angles.Answer question 1-12

1. The value of $\cos 2x$ is
   (a) $\frac{3}{5}$ (b) $\frac{4}{7}$ (c) $-\frac{1}{5}$ (d) $\frac{9}{25}$ (e) -2
   Solution
   To answer $1-12$
   $\cot x = 2 \quad \& \quad \tan y = \frac{1}{3}$
   $\tan x = \frac{1}{2}$
   Applying both SOH CAH TOA and Pythagoras theorem
   $\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{Adj}{Hyp}, \tan\theta = \frac{opp}{Adj}$
   For x Hyp $= \sqrt{4+1} = \sqrt{5}$
   For y hYP $= \sqrt{3+1} = 2 \therefore$
   $\sin x = \frac{1}{\sqrt{5}} \quad \& \quad \sin y = \frac{1}{2}$
   $\cos x = \frac{2}{\sqrt{5}} \quad \& \quad \cos y = \frac{3}{2}$
   $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$
   $= 1 - 2\sin^2 x$
   Any of the above equation will work
   $\cos 2x = 1 - 2\sin^2 x = 1 - 2\times(\frac{1}{\sqrt{5}})^2$
   $= 1 - \frac{2}{5} = \frac{3}{5}(A)$

2. The value of $\tan(x-y)$ is
   (a) $-\frac{2}{7}$ (b) $\frac{1}{7}$ (c) $\frac{4}{3}$ (d) -1 (e) $\frac{7}{6}$
   Solution
   $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
   $\tan(x-y) = \left(\frac{1}{2} - \frac{1}{3}\right) \div \left(1 + \frac{1}{6}\right)$
   $= \frac{1}{6} \div \frac{7}{6} = \frac{1}{6} \times \frac{6}{7} = \frac{1}{7}(B)$

3. The value of $\tan\frac{1}{2}x$ is
   (a) 1 (b) $-1+\sqrt{2}$ (c) $-\frac{1}{\sqrt{2}}$
   (d) $\frac{1}{\sqrt{5}}$ (e) $\sqrt{5}-2$
   Solution
   $\tan\frac{1}{2}x = \frac{\sin\frac{1}{2}x}{\cos\frac{1}{2}x}$
   From $\cos 2\theta = 1 - 2\sin^2\theta$
   $\therefore \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$
   Let $\theta = \frac{1}{2}\theta$
   $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$
   Similarly $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$
   Therefore $\tan\frac{x}{2} = \sin\frac{x}{2} \div \cos\frac{x}{2}$
   $= \sqrt{\frac{1-\cos x}{2}} \div \sqrt{\frac{1-\cos x}{2}}$
   $= \sqrt{\frac{1-\cos x}{1+\cos x} \times \frac{(1-\cos x)}{1-\cos x}}$
   $= \sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}}$
   $= \frac{\sqrt{(1-\cos x)^2}}{\sqrt{\sin^2 x}}$
   $= \frac{1-\cos x}{\sin x} = \frac{1}{\sin x} - \cot x$
   $\tan\frac{x}{2} = \sqrt{5} - 2(E)$

4. The value of $\tan\frac{1}{2}y$ is
   (a) $2\sqrt{10}-3$ (b) $\frac{\sqrt{10}}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$ (e) -2
   Solution
   $\tan\frac{1}{2}y = \frac{1-\cos y}{\sin y} = \frac{1}{\sin y} - \cot y$
   $= 2 - 3 = -1$ No correct option

5. The value of $\frac{\sin 2y}{\sin 2x}$ is
   (a) $\frac{5}{6}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{5}$ (e) $\frac{2}{5}$
   Solution
   $\frac{\sin 2y}{\sin 2x} = \frac{2\sin y\cos y}{\sin x\cos x}$
   $= \frac{1}{2} \times \frac{3}{2} \div \left(\frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}\right) = \frac{3}{4} \div \frac{2}{5}$
   $= \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$
   No correct option

6. The value of $\tan(x+y)$ is
   (a) 1 (b) $-\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) 2
   Solution
   $\tan(x+y) = \frac{\tan x - \tan y}{1 - \tan x \tan y}$
   $= \frac{1}{2} + \frac{1}{3} \div \left(1 - \frac{1}{6}\right)$
   $= \frac{5}{6} \div \frac{5}{6}$
   $= \frac{5}{6} \times \frac{6}{5} = 1(A)$

7. The value of $\sin 3x - \sin x$ is
   (a) $\frac{3}{5}$ (b) $-\frac{\sqrt{10}}{25}$ (c) $\frac{9}{125}$ (d) $-\frac{36}{25}$ (e) $-\frac{6}{5}$
   Solution

Solution

The line AC at $(1,5)(-1,-4)$ is

$$\frac{-4-5}{-1-1}=\frac{y-5}{x-1}$$

$$2y-9x-1=0------(1)$$

The line BD at (4,1) $(-4,3)$ is

$$\frac{3-1}{-4-4}=\frac{y-1}{x-4}$$

$$\frac{2}{8}=\frac{y-1}{x-4}$$

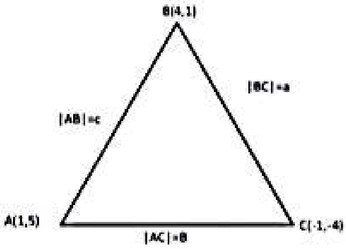
$$4y+x-8=0-------(2)$$

solving equ (2) and equ (1) simultaneously to get the point of intersection

$$y=\frac{73}{38} \quad x=\frac{6}{19}$$

$$(x,y)=\left(\frac{6}{19},\frac{73}{38}\right)(D)$$

21. The value of $7\tan A$ is
(a) 14 (b) 24 (c) 26 (d) 13 (e) 12
Solution From the $\triangle\ ABC$



$$|AB|=\sqrt{(5-1)^2+(1-4)^2}=\sqrt{25}=c$$

$$|BC|=\sqrt{25+25}=\sqrt{50}=a$$

$$|AC|=\sqrt{4+81}=\sqrt{85}=b$$

Using Chain rule

$$\text{A}=\cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$=\cos\left(\frac{85+25-50}{2\times\sqrt{85}\times\sqrt{25}}\right)$$

$$=\cos^{-1}\left(\tfrac{60}{92.195}\right)=49.40^0$$

$\therefore\quad 7\tan A=7\times\tan 49.40=8.167$

No correct option

22. The value of $5\cot C$
(a) -2 (b) 2 (c) 3 (d) 4 (e)-4
Solution

$$5\cot C=\frac{5}{\tan C}$$

Using Sine Rule

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$

$$\sin C=\frac{C\sin A}{a}=\frac{5\times\sin 49.40}{\sqrt{50}}$$

$$c=\sin^{-1}(0.54)=32.47^0$$

$5\cot c=\dfrac{5}{\tan 32.47}=7.86\approx 8$ No correct option

23. The least non-zero value for $0\leq x\leq 180^0$ in the solution of $\cos 3x+\cos x=0$ is
(a) 180^0 (b) 45^0 (c) 22.5^0 (d) 30^0 (e) 90^0
Solution

Recall $\cos 3x=4\cos^3 x-3\cos x$ then substitute into the question

i.e $4\cos^3 x-3\cos x+\cos x=0$

$$4\cos^3 x-2\cos x=0$$

$$4\cos^3 x=2\cos x$$

$$\cos=\frac{1}{\sqrt{2}}$$

$$x=\cos^{-1}\left(\tfrac{1}{\sqrt{2}}\right)=45^0(B)$$

24. $\dfrac{\cos(A+B)}{\sin A\cos B}+\tan B$ simplifies to
(a) $\sin A$ (b) $\tan B$ (c) $\cot B$
(d) $\cos B$ (e) $\tan B$
Solution

$$\frac{\cos(A+B)}{\sin A\cos B}+\tan B$$

$$=\frac{\cos A\cos B-\sin A\sin B}{\sin A\cos B}+\frac{\sin B}{\cos B}$$

$$=\frac{\cos A\cos B}{\sin A\cos B}-\frac{\sin A\sin B}{\sin A\cos B}+\frac{\sin B}{\cos B}$$

$$=\frac{\cos A}{\sin A}-\frac{\sin B}{\cos B}+\frac{\sin B}{\cos B}$$

$$=\frac{\cos A}{\sin A}=\cot A(C)$$

25. $\cot x+\dfrac{\sin x}{1+\cos x}$ simplifies to
(a) $\cot x$ (b) $\sin 2x$ (c) $\sec x$ (d) $cosecx$
(e) $\cos 2x$
Solution

$$\cot x+\frac{\sin x}{1+\cos x}=\frac{\cos x}{\sin x}+\frac{\sin x}{1+\cos x}$$

$$=\frac{\cos x(1+\sin x)+\sin^2 x}{\sin x(1+\cos x)}$$

$$=\frac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$$

$$=\frac{\cos x+1}{\sin x(1+\cos x)}=\frac{1}{\sin x}=cosecx(D)$$

26. The angle of depression of a boat 120m from the top of a mast is 30^0.if the mast is standing on the cliff of 50m high,the height of the mast is
(a) 12m (b) 5m (c) 10m (d) 20m (e) 15m
Solution

$diameter = 2r$
$2 \times 3 = 6(C)$

24. Given that $\tan\theta = \frac{7}{24}$, θ is in the first quadrant then $5\cos\theta - 10\sin\theta =$
(a) -3 (b) 2 (c) -2 (d) 4 (e) 3
Solution
If $\tan\theta = \frac{7}{24}$ then $\cos\theta = \frac{24}{25}$
also $\sin\theta = \frac{7}{25}$
$5\cos\theta - 10\sin\theta = 5 \times \frac{24}{25} - 10 \times \frac{7}{25}$
$= \frac{120}{25} - \frac{70}{25} = \frac{50}{25} = 2(B)$

25. The area in squares units of the quadrilateral whose angular points are $(0,1), (1,3)(2,-3)$ and (3,1) is
(a) 6 (b) 9 (c) 10 (d) 7 (e)8
Solution
A(0,1)=(x_1, y_1), B(1,3)=(x_2, y_2)
C(2,-3)=(x_3, y_3) D(3,1)=(x_4, y_4)
Area of quadrilateral is given by:
$\frac{1}{2}[y_1(x_3 - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_1)]$
$+\frac{1}{2}[y_1(x_3 - x_4) + y_3(x_4 - x_1) + y_4(x_1 - x_y)]$
$AREA = \frac{1}{2}[1(2-1) + 3(0-2) + (-3)(1-0)]$
$+\frac{1}{2}[1(2-3) + (-3)(3-0) + 1(0-2)]$
$= \frac{1}{2}[1-6-3] + \frac{1}{2}[-1-9-2]$
$= \frac{8}{2} + \frac{12}{2} = 10(C)$

26. The equation of a straight passing through (1,1) and $(-1,-1)$ is
(a) $x - y = 0$ (b) x+y=0 (c) $x - 2y = 0$ (d) 2x+2y=1 (e) $x + y + 1 = 0$
Solution
Using the formulae $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
$\frac{y-1}{x-1} = \frac{-1-1}{-1-1}$
$\frac{y-1}{x-1} = 1$
$y - 1 = x - 1$
$x - y = 0(A)$

27. $\frac{\sin(B-A)}{\sin A \cos B} + \tan A$ simplifies to
(a) $\sin A$ (b) $\tan B$ (c) $\cot B$ (d) $\cos B$ (e) $\tan A$

Solution

28. The least non-zero value for $0 \le x \le 180$ in the solution of $\sin 3x - \sin x = 0$ is
(a) 180^0 (b) 45^0 (c) 22.5^0 (d) 30^0 (e) 90^0
Solution
Recall $sin3x = 3sinx - 4sin^3x$
From question $sin3x - sinx = 0$
$\therefore\ 3sinx - 4sin^3x - sinx = 0$
$2sinx - 4sin^3x = 0$
$2sin = 4sin^3x$
$sin^2x = \frac{1}{2}$
$sinx = \pm\frac{1}{\sqrt{2}}$
$x = sin^{-1}(\frac{1}{\sqrt{2}}) = 45^0(B)$

29. The perpendicular distance from the point $(5,-2)$ to the line $24x + 7y + 19 = 0$ is
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5
Solution
Using the formulae $d = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$
From the point$(5,-2) = (x_1, y_1)$
$d = \frac{24(5) + 7(-2) + 19}{\sqrt{24^2 + 7^2}}$
$= \frac{125}{25} = 5(E)$

30. An equation to a line bisecting the angle between $2x - 3y + 2 = 0$ & $3x - 2y + 5 = 0$ is
(a) $x+y = -3$ (b) $x-y = 5$ (c) $5x+2y = 1$ (d) $2x - 5y = 0$ (e) $x - y = 0$
Solution

31. $\frac{\sin 6\alpha}{\sin 2\alpha} - \frac{\cos 6\alpha}{\cos 2\alpha}$ simplifies to
(a) 4α (b) 2 (c) $\tan 2\alpha$ (d) 4 (e) 1/2
Solution
$\frac{\sin 6\alpha}{\sin 2\alpha} - \frac{\cos 6\alpha}{\cos 2\alpha}$
$= \frac{\sin 6\alpha \cos 2\alpha - \cos 6\alpha \sin 2\alpha}{\sin 2\alpha \cos 2\alpha}$
$= \frac{\sin 4\alpha}{\sin 2\alpha \cos 2\alpha} = \frac{2\sin 2\alpha \cos 2\alpha}{\sin 2\alpha \cos 2\alpha}$
$= 2(B)$

32. The value of $\cos 300^0 \sin 390 + \cos 660 \sin 570$ is
(a) 2 (b) 3/2 (c) 1 (d) $\frac{1}{2}$ (e) 0

Let $\theta = \frac{1}{2}\theta$

Solution

$\sin 3x - \sin x$ using
$\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$
$A = 3x \quad B = x$
$\sin 3x - \sin x = 2\cos(\frac{4x}{2})\sin(\frac{2x}{2})$
$= 2\cos 2x \sin x = 2\sin x(1 - 2\sin^2 x)$
$= 2\sin x - 4\sin^3 x = \frac{2}{\sqrt{5}} - 4(\frac{1}{\sqrt{5}})^3$
$= \frac{2}{\sqrt{5}} - \frac{4}{25\sqrt{5}} = \frac{50-4}{25\sqrt{5}} = \frac{46}{25\sqrt{5}}$
$= \frac{46\sqrt{5}}{125}$
No correct option

8. The value of $\cos 3x + \cos x$
(a)$\frac{3\sqrt{10}}{5}$ (b) $\frac{12\sqrt{5}}{25}$ (c) $\frac{-2\sqrt{5}}{5}$ (d) $\frac{2\sqrt{5}}{5}$
(e) $-\frac{6\sqrt{5}}{25}$
Solution
$\cos 3x + \cos x$ Using
$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$= 2\cos 2x\cos x = 2\cos x(2\cos^2 x - 1)$
$= 4\cos^3 x - 2\cos x$
$= 4\left(\frac{2}{\sqrt{3}}\right)^3 - 2\times\frac{2}{\sqrt{5}}$
$= \frac{32}{25\sqrt{5}} - \frac{4}{\sqrt{5}} = -\frac{68}{25\sqrt{5}}$
$= -\frac{68\sqrt{5}}{125}$ No correct option

9. The value of $\sin 4x$ is
(a) $-\frac{4}{25}$ (b) $-\frac{7}{25}$ (c) $\frac{3\sqrt{10}}{5}$
(d) $-\frac{2}{5}$ (e) $-\frac{6}{5}$
Solution
$\sin 4x = \sin(2x + 2x)$
$= \sin 2x\cos 2x + \cos 2x \sin 2x = 2\sin 2x\cos 2x$
$= 2\times\frac{4}{5}\times\frac{3}{5} = \frac{24}{25}$ No correct option

10. The value of $\cos 4x$ is
(a) $\frac{4}{25}$ (b) $\frac{7}{25}$ (c) $\frac{3\sqrt{10}}{10}$ (d) $-\frac{2}{5}$ (e) $\frac{6}{25}$
Solution
$\cos 4x = \cos(2x + 2x) = \cos^2 2x - \sin^2 2x$
$= (\cos 2x)^2 - (\sin 2x)^2$
$= (\frac{3}{5})^2 - (\frac{4}{5})^2 = \frac{9}{25} - \frac{16}{25}$
$= -\frac{7}{25}$ No correct option

11. The value of $\frac{\cos 4y}{\sin 2x}$
(a) $\frac{4}{25}$ (b) $\frac{7}{25}$ (c) $\frac{3\sqrt{10}}{10}$ (d)$\frac{3}{2}$ (e) $\frac{11}{2}$

Solution
$\frac{\cos 4y}{\sin 2x} = \frac{(\cos 2x)^2 - (\sin 2x)^2}{2\sin x\cos x}$
$= 1 - 2(\sin 2y)^2 \div 2\sin x\cos x$
$= 1 - 2(\frac{3}{2})^2 \div (2\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}})$
$= 1 - 2(\frac{3}{2})^2 \div (2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}})$
$1 - \frac{9}{2}\div\frac{4}{5} = -\frac{7}{2}\times\frac{4}{5} = -\frac{14}{5}$
No correct option

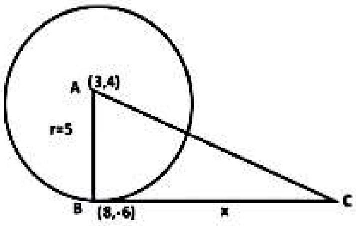
12. The value of $\cot(2y - x)$ is
(a) $\frac{11}{7}$ (b) $\frac{33}{7}$ (c) $\frac{22}{3}$ (d) $\frac{3}{2}$ (e) $\frac{11}{2}$
Solution
$\cot(2y-x) = \frac{1}{\tan(2y-x)} = \frac{\tan 2y - \tan x}{1 + \tan 2y\tan x}$
$\tan 2y = \frac{2\tan y}{1 - \tan^2 y} = 2\times\frac{1}{3}\div\left(1 - \frac{1}{9}\right)$
$= \frac{2}{3}\div\frac{8}{9} = \frac{3}{4}$
$\tan(2y - x) = \frac{3}{4} - \frac{1}{2}\div\left(1 - \frac{3}{4}\times\frac{1}{2}\right)$
$= \frac{6-4}{8}\div(1 - \frac{3}{8}) = \frac{2}{8}\div\frac{5}{8} = \frac{2}{5}$
No correct option

The angular point of a quadrilateral ABCD are $(1,5), (4,1), (-1,-4), (-4,3)$ respectively.Use this to answer question 13-22

13. The area of the quadrilateral in square units is :
(a) 28 (b) 36 (c) 38 (d) 72 (e) 34
Solution
Area of Quadrilateral is given by
$\text{Area} = \frac{1}{2}[y_1(x_3 - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_1)]$
$[y_1(x_3 - x_4) + y_3(x_4 - x_1) + y_4(x_1 - x_3)]$ OR
$\text{Area} = \frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_4 & y_4 \\ 1 & x_3 & y_3 \end{bmatrix}$
Given $A(\overset{x_1}{1}, \overset{y_1}{5})$, $A(\overset{x_2}{4}, \overset{y_2}{1})$,
$A(\overset{x_3}{-1}, \overset{y_3}{-4})$, $A(\overset{x_4}{-4}, \overset{y_4}{3})$
$\text{Area} = \frac{1}{2}[5(-1-4) + 1(1-(-1)) + (-4)(4-1)]$
$+\frac{1}{2}[5(-1+4) + (-4)(-4-1) + 3(1+1)]$
$= \frac{1}{2}[-25 + 2 + (-12)] + \frac{1}{2}[15 + 20 + 6] =$
$\frac{|-35|}{2} + \frac{|41|}{2} = 38(B)$

14. The area of the triangle ABD in squares

Solution



$AC = \sqrt{10^2+5^2} = \sqrt{125}$
By Pythagoras $x = \sqrt{(\sqrt{25})^2+25} = 12.25 \approx 12(A)$

35. The unit length of radius of the circle is
(a) 4 (b)3 (c) 5 (d) 7 (e) 10
Solution
From question 32 solution
$r = \pm 5$

36. The circle touches $x^2+y^2-4x+6y=12$ at the point
(a) (6,7) (b) $(-1,4)$ (c) (5,7) (d) $(-1,7)$ (e) (0,6)
Solution
from question 31 solution the equation of the circle is
$x^2+y^2-6x-8y=0------(i)$
the given circle is
$x^2+y^2-4x+6y=12---(ii)$
equ (i)-equ (ii) $\quad 2x+14y=12$
$\therefore \quad x = 6-7y$ substitute into equ (ii)
$(6-7y)^2+y^2-4(6-7y)-6y=12$
$36-84y+49y^2+y^2-24-28y+6y=12$
$50y^2-50y=0$
$\therefore \quad y = 0 \; or \; 1$
at $y=0$;x= $6-0=6$ i.e (6,0)
at y=1 ; $6-7=-1$ i.e $(-1,1)$ No correct option

37. The equation of the common chord of the circle and $x^2+y^2-4x+6y=12$ is
(a) $2x-y=7$ (b) $x+y=6$ (c) $x-y=6$ (d) $x+y=7$ (e) $x-y=7$
Solution
The equation to the common chord is
$x^2+y^2-6x-8y-(x^2+y^2-4x+6y-12)=0$
$2x-14y+12=0$
$x-7y=-6$ No correct option

38. Find the value of
$\tan(240^0-\theta)+\tan(120^0+\theta)$
(a) 2 (b) 1 (c) 0 (d) $-\sqrt{3}$ (e) $\sqrt{3}$
Solution
$\tan(240-\theta)+\tan(120+\theta)$
$\tan(240-\theta) = \dfrac{\tan 240 - \tan\theta}{1+\tan 240\tan\theta}$
$= \dfrac{\sqrt{3}-\tan\theta}{1+\sqrt{3}\tan\theta}$
$\tan(120+\theta) = \dfrac{\tan 120+\tan\theta}{1-\tan 120\tan\theta}$
$= \dfrac{-\sqrt{3}+\tan\theta}{1+\sqrt{3}\tan\theta}$
$\therefore \; \tan(240-\theta)+\tan(120+\theta) = \dfrac{\sqrt{3}-\tan\theta}{1+\sqrt{3}\tan\theta} + \dfrac{-\sqrt{3}+\tan\theta}{1+\sqrt{3}\tan\theta} = 0(C)$

39. The value of $\tan^{-1}(\frac{1}{3})+\sec^{-1}(\frac{\sqrt{5}}{2})$ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 2π
(e) none of the above
Solution
$\tan^{-1}(\frac{1}{3})+\sec^{-1}(\frac{\sqrt{5}}{2})$
Let $\alpha = \tan^{-1}(\frac{1}{3}) \quad \therefore \; \tan\alpha = \frac{1}{3}$
Also $\beta = \sec^{-1}(\frac{\sqrt{5}}{3}) \quad \therefore \; \sec\beta = \frac{\sqrt{5}}{3}$
Recall $\tan\beta = \sqrt{\sec^2\beta - 1}$
$= \sqrt{\frac{5}{4}-1} = \frac{1}{2}$
$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$= \frac{1}{3}+\frac{1}{2} \div (1-\frac{1}{6}) = \frac{5}{6} \div \frac{5}{6} = 1$
$\alpha+\beta = \tan^{-1}(1) = \frac{\pi}{4}(A)$

40. $\dfrac{\sin 3\alpha}{\sin\alpha} - \dfrac{\cos 3\alpha}{\cos\alpha}$ simplifies to
(a) $\frac{\alpha}{4}$ (b) $\frac{\alpha}{2}$ (c) 2 (d) 2α
(e) none of the above
Solution
$\dfrac{\sin 3\alpha}{\sin\alpha} - \dfrac{\cos 3\alpha}{\cos\alpha} = \dfrac{\sin 3\alpha\cos\alpha - \cos 3\alpha\sin\alpha}{\sin\alpha\cos\alpha}$
$= \dfrac{\sin(3\alpha-\alpha)}{\sin\alpha\cos\alpha} = \dfrac{\sin 2\alpha}{\sin\alpha\cos\alpha} = \dfrac{2\sin 2\alpha}{2\sin\alpha\cos\alpha}$
$= 2(C)$

0.18 MATHS 103 2013-2014

1. The general solution to the equation $\sin 3\theta + \sin\theta = 0$ is
(a) $\frac{n\pi}{2}$ (b) $2n\pi - \frac{\pi}{2}$ (c) $\frac{n\pi}{2}$ or $2n\pi + \frac{\pi}{2}$
(d) $2n\pi$ or $n\pi + \frac{\pi}{2}$

units is :
(a) 20 (b) 16 (c) 19 (d) 13 (e) 14

Solution

Area of triangle is given by

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Or

Area $= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$

Given $\overset{x_1}{A}(1 \; \overset{y_1}{,5})$, $\overset{x_2}{A}(4 \; \overset{y_2}{,1})$, $\overset{x_3}{A}(-1 \; \overset{y_3}{,-4})$

Area $= \frac{1}{2}[1(1-3) + 4(3-5) + (-4)(5-1)]$

$= \frac{1}{2}(-2 - 8 - 16) = \frac{-26}{2} = 13(D)$

15. The equation to the bisector of the line DC is
(a) $3x - 5y + 23 = 0$ (b) $7x + 5y + 25 = 0$
(c) $3x - 5y + 9 = 0$ (d) $7x - 2y - 1 = 0$
(e) $3x - 7y + 4 = 0$

Solution

Line DC $= (-4, 3)(-1, -4)$

Gradient of DC =M $= \frac{-4-3}{-1+4} = \frac{-7}{3}$

Gradient of the perpendicular is $m_2 = -\frac{1}{m_1} = \frac{3}{7}$

Midpoint of DC is $\left(\frac{-4-1}{2}, \frac{-4+3}{2}\right) = \left(\frac{-5}{2}, \frac{-1}{2}\right)$

The equation of the line is given by $\frac{y - y_1}{x - _1} = m_2$

$\frac{y + \frac{1}{2}}{x + \frac{5}{2}} = \frac{3}{7}$

$7y - 3x + \frac{7}{2} - \frac{15}{2} = 0$

$3x - 7y + 4 = 0(C)$

16. The coordinate of a point dividing the join BD in the ratio 3:1 is
(a) $(2, \frac{5}{2})$ (b) $(6, \frac{1}{4})$ (c) $(3, \frac{1}{2})$ (d) (1,2) (e)$(3, \frac{1}{4})$

Solution

Since $m : n = 3 : 1$ and $m > n \Rightarrow$ internally divided

$(\bar{x}, \bar{y}) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$BD = (4, 1)(-4, 3)$

$(\bar{x}, \bar{y}) = \left(\frac{3(-4) + 1(4)}{3+1}, \frac{3(3) + 1(1)}{3+1}\right)$

$= (-2, \frac{5}{2})$ No correct option

17. The equation to the line AD is
(a) $2x - 5y + 23 = 0$ (b) $x + 5y + 25 = 0$
(c) $2x - 3y + 9 = 0$ (d) $3x - 2y - 1 = 0$
(e) $x + y + 1 = 0$

Solution

AD= $(1, 5)(-4, 3)$

Using $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x1_1}$

$\frac{y - 5}{x - 1} = \frac{3 - 5}{-4 - 1}$

$2x - 5y + 23 = 0(A)$

18. The equation to the line perpendicular to AC through A is
(a) $2x - 5y + 23 = 0$ (b) $2x + 9y - 17 = 0$
(c) $2x + 3y - 9 = 0$ (d) $3x - 2y - 1 = 0$
(e) $x + y - 11 = 0$

Solution

AC $(1, 5)(-1, -4)$

Gradient of AC $= m_1 = \frac{-4-5}{-1-1} = \frac{9}{2}$

Gradient of perpendicular line m_2

$m_2 = -\frac{1}{m_1} = -\frac{2}{9}$

$\frac{y - y_1}{x - x_1} = m_2$ at $B(4, 1)$

$\frac{y-1}{x-4} = -\frac{2}{9}$

$2x + 9y - 17 = 0(B)$

19. The equation to the line parallel BC through A is
(a) $2x - 5y + 2 = 0$ (b) $x + 5y + 25 = 0$
(c) $x - 3y + 9 = 0$
(d) $3x - y - 1 = 0$ (e) $x + y - 6 = 0$

Solution

Gradient of BC at $(4, 1)(-1, -4)$ is

$m_1 = \frac{-4-1}{-1-4} = 1$

m_2 is the gradient of the equation parallel to equation of m_1 i.e $m_1 = m_2 = 1$

The equation of the line through A is

$\frac{y - y_1}{x - x_1} = m$

$\frac{y - 5}{x - 1} = 1$

$x - y + 4 = 0$ No correct option

20. The lines AC and BD intersects at the point
(a) $(2, \frac{1}{17})$ (b) $(1, \frac{9}{17})$ (c) $(\frac{8}{19}, \frac{6}{19})$
(d)$(\frac{6}{19}, \frac{13}{38})$ (e) $(\frac{77}{38}, \frac{3}{19})$

Solution
$Tan(A+60)=\sqrt{3}$
$A+60=tan^{-1}\sqrt{3}$
$A+60=60$
$A=60-60=0$
$\therefore cotA=cot0$
$=\dfrac{1}{tan0}$
$cotA=\dfrac{1}{0}, undefined(E)$

18. If A(1,3),B(3, −1) and $C(-3,-3)$ are vertices of a triangle ,then tan A is
(a) 7/4 (b) 15/4 (c) 2/3 (d) −3/4 (e) 5/3
Solution
$/AB/=\sqrt{(-1-3)^2+(3-1)^2}$
$=\sqrt{16+4}=\sqrt{20}$
$/BC/=\sqrt{(-3-1)^2+(-3-3)^2}$
$=\sqrt{16+36}=\sqrt{52}$
$/AC/=\sqrt{(3+3)^2+(1+3)^2}$
$=\sqrt{36+16}=\sqrt{52}$
Using cosine Rule
$CosA=\dfrac{b^2+c^2-a^2}{2bc}$
$cosA=\dfrac{52+20-52}{2\sqrt{52}\times\sqrt{20}}$
$=\dfrac{10}{\sqrt{1040}}=\dfrac{10}{4\sqrt{65}}=\dfrac{5}{2\sqrt{65}}$
$secA=\dfrac{1}{cosA}=\dfrac{2\sqrt{65}}{5}$
$tanA=\sqrt{sec^2A-1}=\sqrt{\left(\dfrac{2\sqrt{65}}{5}\right)^2-1}$
$\sqrt{\dfrac{52}{5}-1}=\sqrt{\dfrac{47}{5}}$
No correct option.

19. $\tan x+\dfrac{\cos x}{1+\sin x}$ simplifies to
(a) $\cot x$ (b) $\sin 2x$ (c) $\sec x$
(d) $cosecx$ (e) $\cos 2x$
Solution
$\dfrac{sinx}{cosx}+\dfrac{cosx}{1+sinx}$
$\dfrac{sinx(1+sinx)+cos^2x}{cosx(1+sinx)}$
$\dfrac{sinx+sin^2x+cos^2x}{cosx(1+sinx)}$
$\dfrac{sinx+1}{cosx(1+sinx)}=\dfrac{1}{cosx}=secx(C)$

20. $\dfrac{\sin 3\theta+\sin\theta}{\cos 3\theta-\cos\theta}$ simplifies to
(a) $\tan\theta$ (b) $-\cot\theta$ (c) $-\tan 2\theta$
(d) $-\tan 2\theta$ (e) $\sin\frac{1}{2}\theta$
Solution
$\dfrac{sin3\theta+sin\theta}{cos3\theta-cos\theta}$
$using\ \ SinA{+}SinB=2Sin\dfrac{A+B}{2}cos\dfrac{A-B}{2}$
$sin3\theta+sin\theta=2sin(2\theta)cos\theta$
$similarly,$
$cos3\theta-cos\theta=-2sin(2\theta)sin\theta$
$then, \dfrac{sin3\theta+sin\theta}{cos3\theta-cos\theta}=\dfrac{-2sin2\theta cos\theta}{2sin2\theta sin\theta}=-cot\theta(B)$

21. $\dfrac{7\pi}{3}$ radians in degree is equal to
(a) $-120^0 50'$ (b) 300^0 (c)420^0
(d) 330^0 (e) $-130^0 22'$
Solution
$\dfrac{7\times 180}{3}=420^0(C)$

22. The value of $\tan^{-1}3+cosec^{-1}\dfrac{\sqrt{5}}{2}$ is
(a)π (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$
(d) $\dfrac{3\pi}{4}$ (e) none of the above
Solution
$tan^{-1}3+cosec^{-1}\dfrac{\sqrt{5}}{2}$
$\alpha=tan^{-1}3\ \ \therefore\ \ tan\alpha=3$
$\beta=cosec^{-1}\dfrac{\sqrt{5}}{2}\ \ \therefore$
$cosec\beta=\dfrac{\sqrt{5}}{2}$
$sin\beta=\dfrac{2}{\sqrt{5}}\therefore tan\beta=2$
$tan(\propto+\beta)=\dfrac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}$
$tan(\propto+\beta)=\dfrac{3+2}{1-6}=\dfrac{-6}{5}$
$\alpha+\beta=tan^{-1}\dfrac{-6}{5}=50.19(E)$

23. The length of the diameter of the circle $4x^2+4y^2-4x+8y-31=0$ is
(a) 11 (b) 8 (c) 6 (d) 9 (e) 12
Solution
$4x^2+4y^2-4x+8y=31$
$x^2+y^2-x+2y=\dfrac{31}{4}$
$(x-\frac{1}{2})^2+(y+1)^2=\frac{31}{4}+\frac{1}{4}+1$
$(x-\frac{1}{2})^2+(y+1)^2=9$
$r^2=9$
$r=\pm 3$