



Supplementary Materials for

Inferring change points in the spread of COVID-19 reveals the effectiveness of interventions

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(available at science.sciencemag.org/cgi/content/full/science.abb9789/DC1)

MDAR Reproducibility Checklist (PDF)

Table S1.

Table S1: Overview of model parameters.

Variable	Parameter
λ	Spreading rate
μ	Recovery rate
$\lambda^* = \lambda - \mu$	Effective spreading rate
λ_i	Spreading rate after i -th intervention
t_i	Time of i -th intervention
f_w	Amplitude of weekend corrections
Φ_w	Phase shift of weekend correction
σ	Scale factor of the width of Student's t-distribution
N	Population size (83.000.000)
S_t	Susceptible at time t
I_t	Infected at time t
R_t	Recovered at time t
Δt	Time step
$R_t^{\text{new}} = \mu \Delta t I_{t-1}$	New recoveries at time t
$I_t^{\text{new}} = \lambda \Delta t \frac{S_{t-1}}{N} I_{t-1}$	New infections at time t
$C_t = I_{t-D}^{\text{new}}$	New reported cases at time t
D	Delay of case detection

Table S2.

Table S2: Model comparison: Using leave-one-out (LOO) cross-validation, we compare our original “SIR main” model that features a weekend modulation (to account for fewer reported cases during weekends) with other model variants. Remarkably, the median inferred effective growth rate λ^* after the last change-point is very similar for all model variants. For full details on the model variants, see the figure captions in the SI: (i) SEIR-like with explicit incubation time, Fig. S3. (ii) SIR excluding the weekend modulation, Fig. S4. (iii) Sensitivity analysis by applying wider priors to different parameters, Figs. S5, S6, S7. Lower LOO-scores represent a better match between model and data (pLOO is the effective number of parameters).

[†]For the SEIR-like model, the magnitude of the effective growth rate is not directly comparable because of the explicit incubation period.

Model	# c-pts.	λ_0^*	λ_1^*	λ_2^*	λ_3^*	LOO-score	pLOO
SIR main	0	0.03				927 ± 9	8.31
SIR main	1	0.21	-0.03			819 ± 16	13.46
SIR main	2	0.30	0.11	-0.03		796 ± 17	12.53
SIR main	3	0.30	0.12	0.02	-0.03	787 ± 17	13.42
SIR without weekend modulation	3	0.31	0.13	0.04	-0.03	807 ± 17	9.65
SIR with wider delay prior	3	0.30	0.12	0.02	-0.04	787 ± 17	14.15
SIR with wider change point priors	3	0.30	0.12	0.02	-0.04	787 ± 17	14.29
SIR with wider transient length prior	3	0.29	0.12	0.02	-0.04	785 ± 17	13.92
SEIR-like [†]	3	1.95	0.45	0.05	-0.12	782 ± 17	10.71

Fig S1.

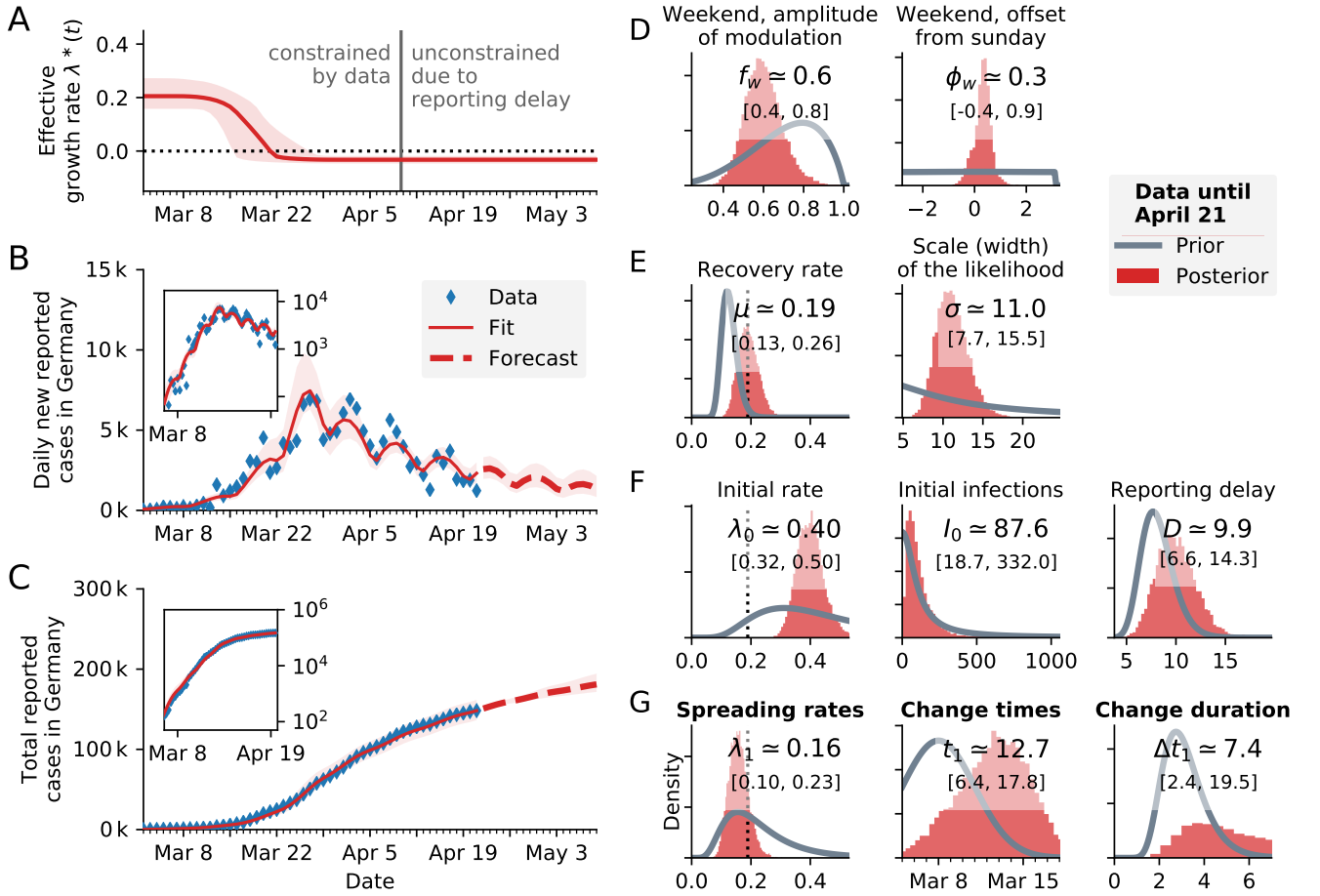


Figure S1: **Model comparison:** Change-point detection as in Fig. 3 (three change points, main text) with the **same model** “SIR main” but only **one change point**. With only one change point, the model cannot describe the data well after April 1. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Forecast and comparison of daily new reported cases with the model fit, **inset:** Same on log-lin scale. **C:** Same as B but for cumulative (total) cases. **D–G:** Posterior distributions from the change point detection (red) compared to prior distributions (gray). Please refer to Fig. 1 (main text) for a more detailed description of the distributions.

Fig S2.

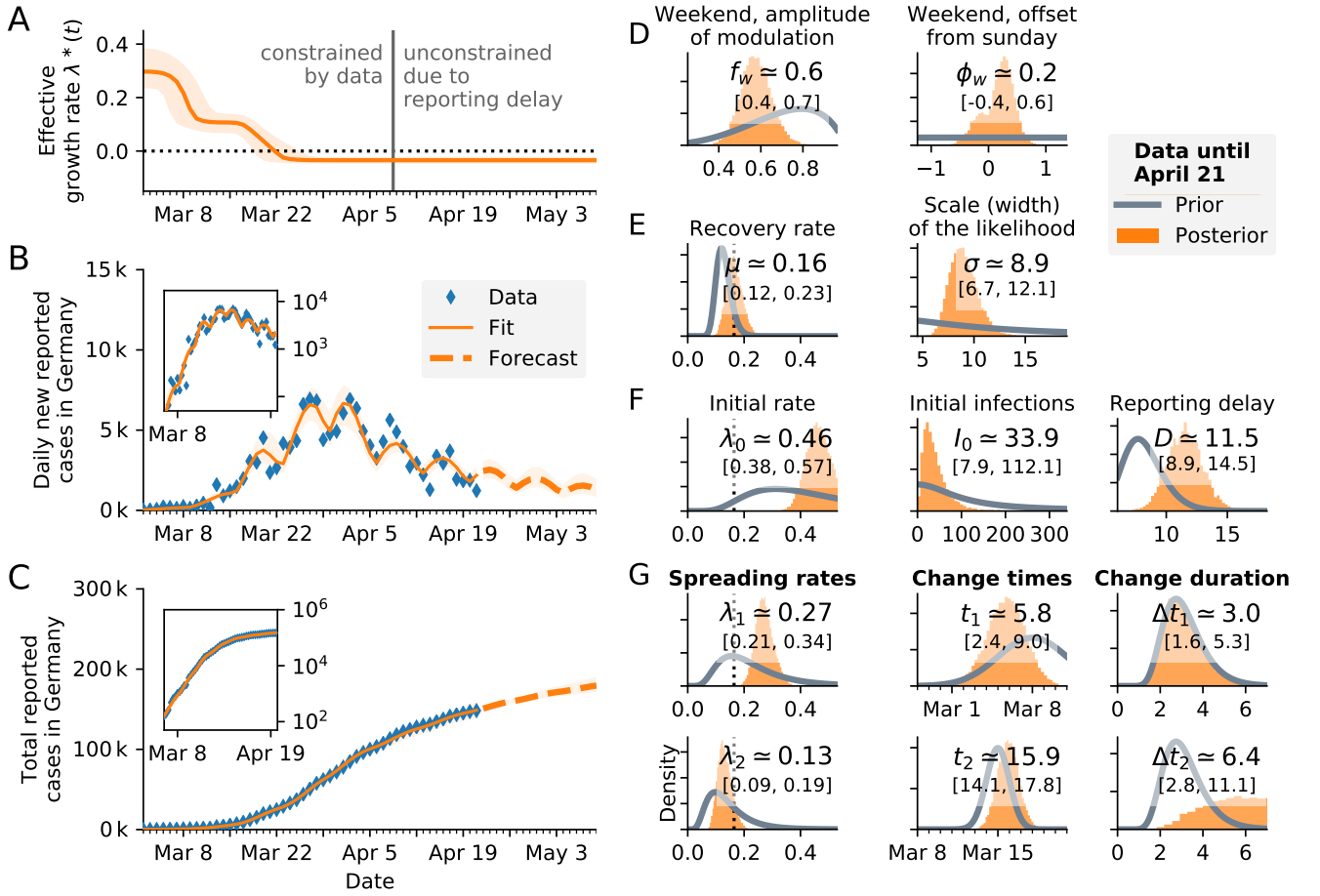


Figure S2: **Model comparison:** Change-point detection as in Fig. 3 (three change points, main text) with the **same model** “SIR main” but with **two change points**. With two change points, the onset time of the second change point is close to the (middle) one inferred when using the 3-c.p. model. However, the effective growth rate λ^* after the respective last c.p. is the same in both cases, $\lambda^* = -0.03$ (see also Table S2). **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. Two change points are clearly visible. **B:** Forecast and comparison of daily new reported cases with the model fit, **inset:** Same on log-lin scale. **C:** Same as B but for cumulative (total) cases. **D–G:** Posterior distributions from the change point detection (orange) compared to prior distributions (gray). Please refer to Fig. 1 (main text) for a more detailed description of the distributions.

Fig S3.

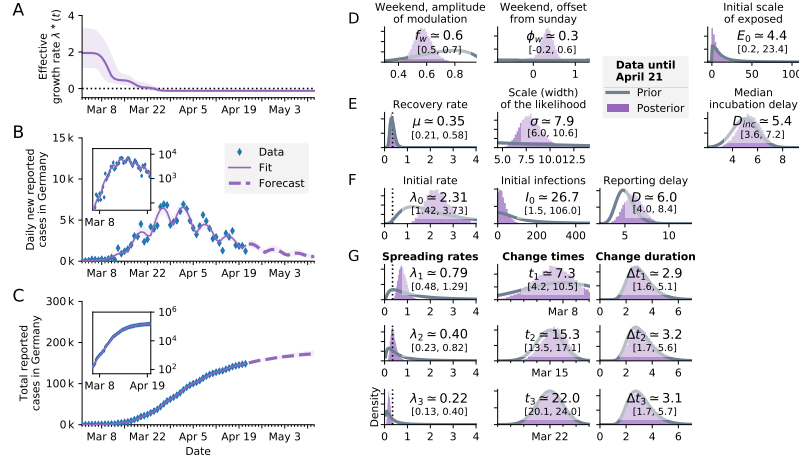


Figure S3: Model comparison: Change-point detection as in Fig. 3 (three change points, main text) but with a more involved **SEIR-like** model and **three change points**, code available online (31). Arguably, the SEIR-like model provides a more realistic (but also more complex) description of virus propagation. It yields a slightly better (lower) LOO-score than our “SIR main” model in the cross-validation, Table S2. However, inferred parameters are compatible with the simpler model and the inferred dynamics are similar. **Model details:** The SEIR-like model builds on our “SIR main” model (which includes a weekend correction, see Methods). The SEIR-like model features an explicit log-normal incubation period and a lognormal reporting delay. The incubation period is implemented as a discrete convolution of multiple exposed pools with a lognormal kernel. The discrete lognormal kernel is parameterized as follows (to match the characteristic incubation time of COVID-19 (32)): median D_{inc} , scale parameter 0.418 and normalized to 1. The median is a free parameter with prior $\text{Normal}(5, 1)$ (days) (32). The reporting delay is implemented in a similar manner: as a convolution of the number of new cases with a lognormal kernel and the scale parameter is fixed to 0.3. In order to match the total delay of the main model (between the infection and the observation), the median D is a free parameter with prior $\text{LogNormal}(5, 0.2)$ (days). Note that because of the lognormal-distributed incubation period in the SEIR-like model, the spreading and recovery rates are not directly comparable to the SIR models. We correspondingly adapted the respective priors (17): We adapted the prior median of the recovery rate μ to $1/3$ with a scale factor of 0.3, and the prior median for λ_0 , λ_1 , λ_2 , and λ_3 to 2.0, 1.0, 0.5 and 0.25, respectively, with a scale parameter of 1 each. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Comparison of daily new reported cases and the model (inset: log-lin scale). **C:** As B but for total (cumulative) reported cases. **D–G:** Prior and posterior distributions of all free parameters.

Fig S4.

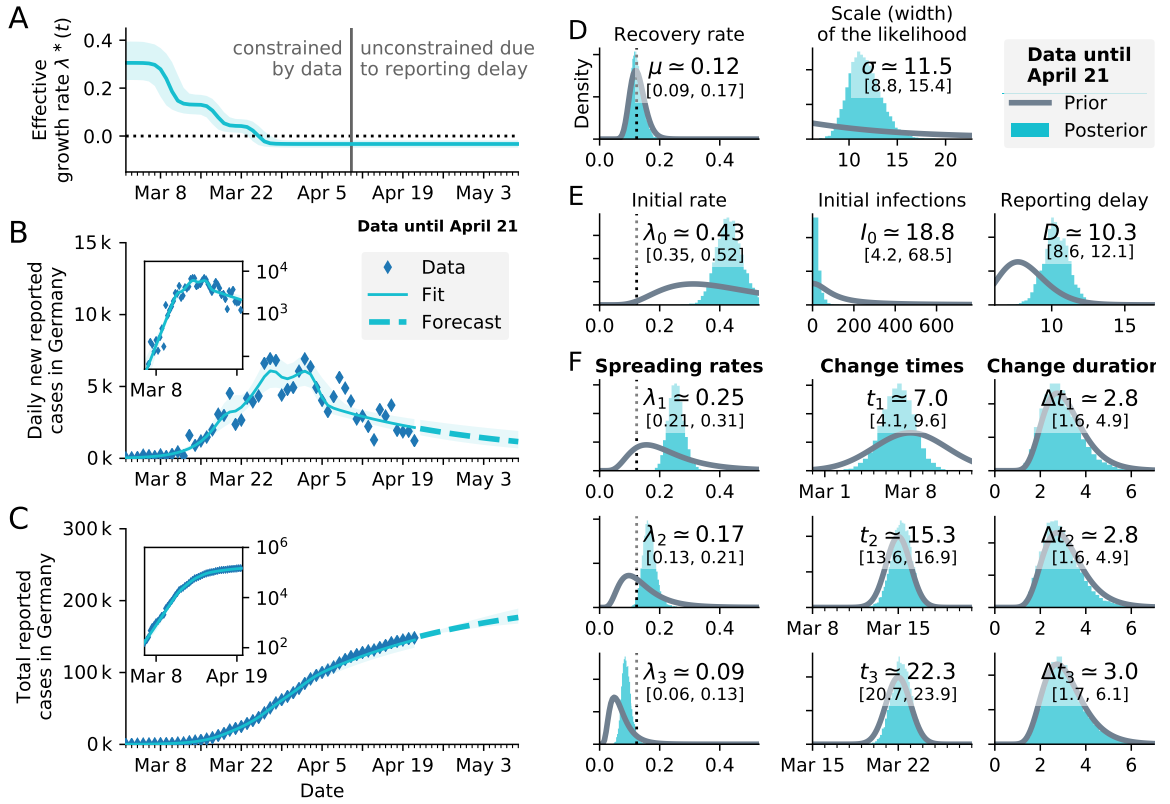


Figure S4: Model comparison: Change-point detection as in Fig. 3 (three change points, main text) with an **SIR model that excludes the weekend modulation** but features the **three change points**. In this version of the model, we excluded the assumption that daily new reported cases depend on the weekday (which is modeled as an absolute sine with an amplitude and a phase shift as inferred parameters in the main model). While the inferred parameters from the model that excludes the weekend modulation (in particular rates and onset times of change points) match the model that includes the modulation, the LOO-scores of the cross-validation are worse, Table S2. Especially in panel B it becomes clear that the (empirically motivated) dependence of reported on cases on the weekday is justified. Without the modulation, the model fit does not capture the periodic changes in the data. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Comparison of daily new reported cases and the model (inset: log-lin scale). **C:** As B but for total (cumulative) reported cases. **D–F:** Prior and posterior distributions of all free parameters.

Fig S5.

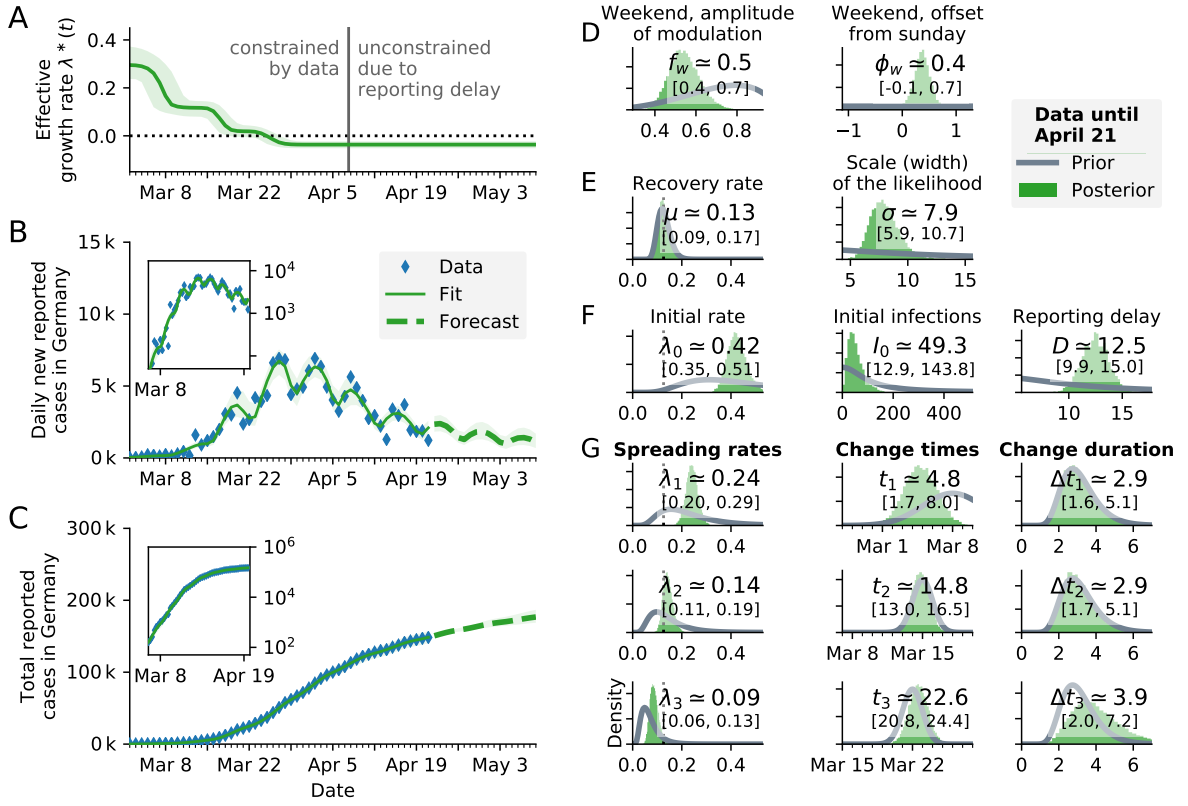


Figure S5: **Sensitivity analysis:** Change-point detection as in Fig. 3 (main text, three change points, same model) but with **a prior for the reporting delay that is 4 times wider** (panel F, third column). All parameters and change points are constrained by data. In particular, the posterior distribution of the reporting delay is slightly wider than with the original prior, but it is well constrained by data. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Comparison of daily new reported cases and the model (inset: log-lin scale). **C:** As B but for total (cumulative) reported cases. **D–G:** Prior and posterior distributions of all free parameters.

Fig S6.

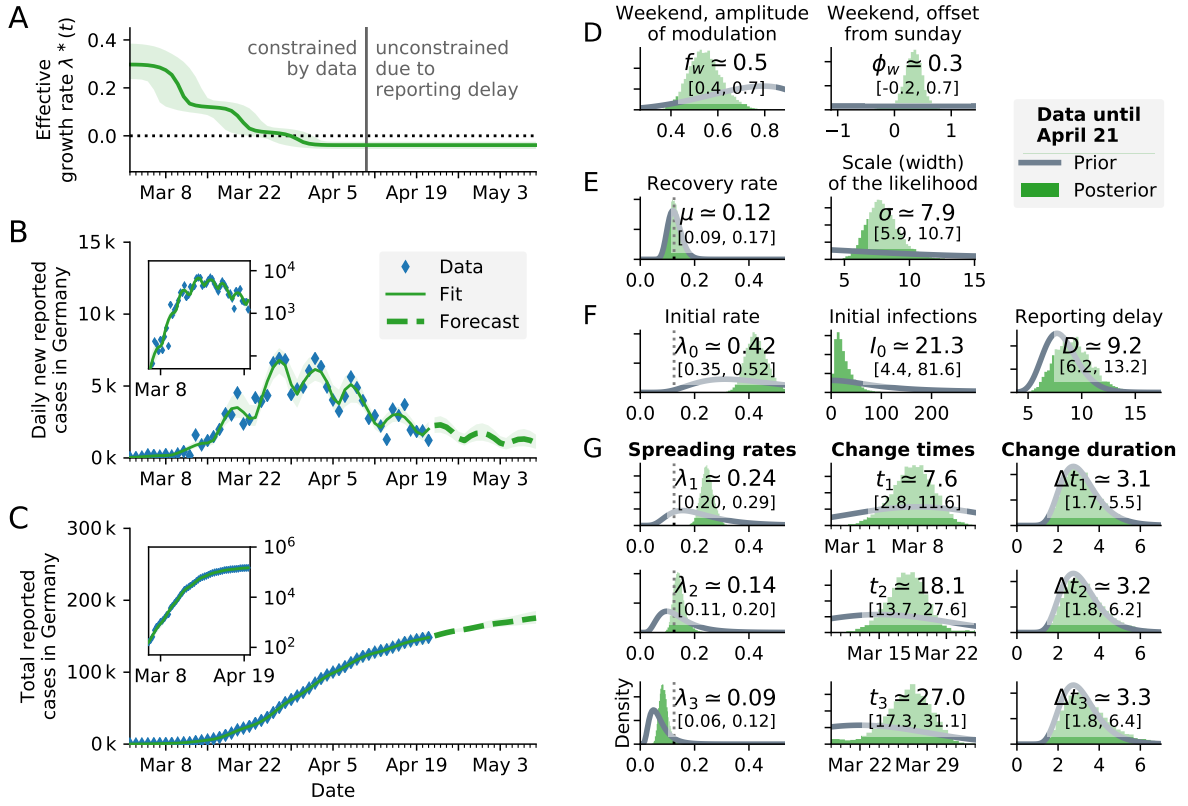


Figure S6: **Sensitivity analysis:** Change-point detection as in Fig. 3 (main text, three change points, same model) but with **a prior for the change times that is 14 days wide**, instead of ~ 2 days, (panel G, second column). All parameters and change points are constrained by data but the change points occur at later times compared to the original priors. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Comparison of daily new reported cases and the model (inset: log-lin scale). **C:** As B but for total (cumulative) reported cases. **D–G:** Prior and posterior distributions of all free parameters.

Fig S7.

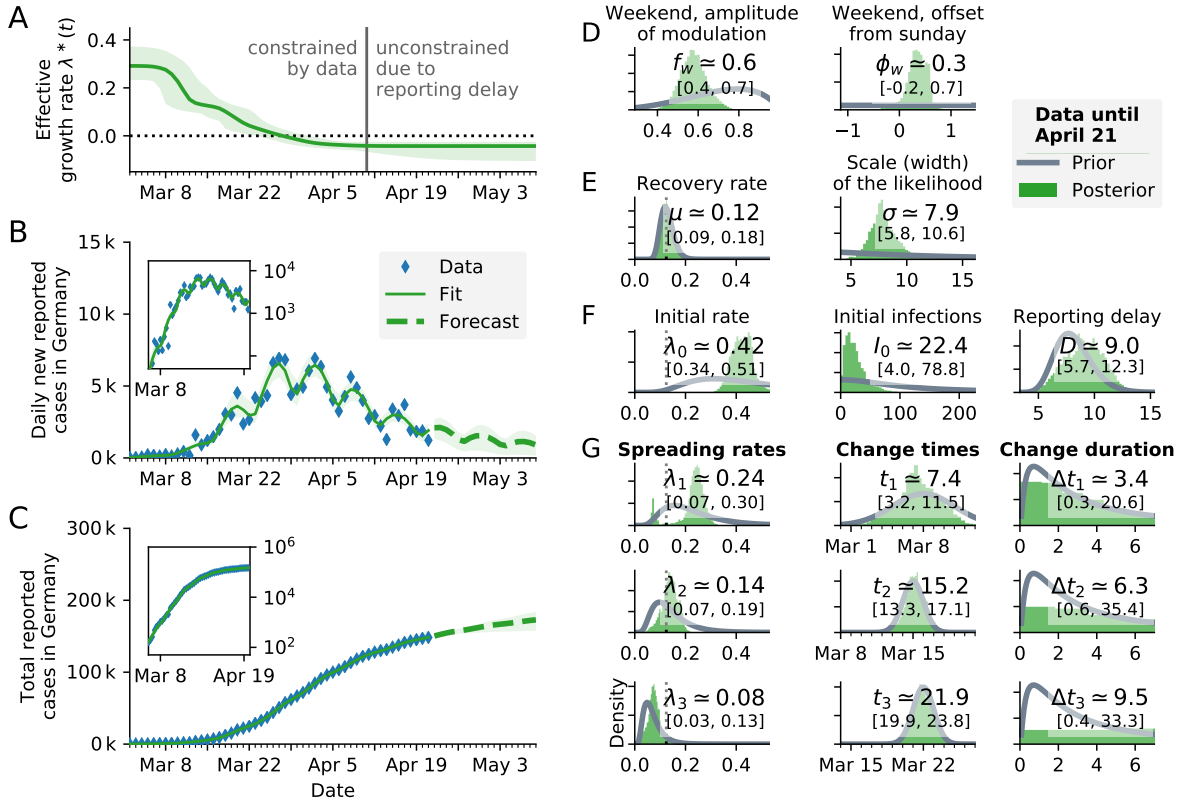


Figure S7: **Sensitivity analysis:** Change-point detection as in Fig. 3 (main text, three change points, same model) but with **a prior for the change duration that is 4 times wider** (panel G, third column). The duration of the first change point Δt_1 is robust to the wider prior; the data constrains the posterior. The durations of the second and third change point, Δt_2 and Δt_3 are not constrained by the data but they depend on our chosen priors. This is also visible in the lack of plateaus in the effective growth, panel A. However, the inferred spreading rate λ and the forecast are *not sensitive* to the wider priors. **A:** Time-dependent model estimate of the effective growth rate $\lambda^*(t)$. **B:** Comparison of daily new reported cases and the model (inset: log-lin scale). **C:** As B but for total (cumulative) reported cases. **D–G:** Prior and posterior distributions of all free parameters.

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