WPG

Practical $\LaTeX$ Tutorial # 24

Mathematics Formulas and Algebraic Equations

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1 Mathematics Formulas and Algebraic Equations

1.1 Function Relationships

$$\begin{aligned}
\sin\theta &= \frac{1}{\csc\theta} \\
\cos\theta &= \frac{1}{\sec\theta} \\
\tan\theta &= \frac{1}{\cot\theta} \\
\csc\theta &= \frac{\sin\theta}{\cos\theta} \\
\sec\theta &= \frac{1}{\cos\theta} \\
\cot\theta &= \frac{1}{\tan\theta} \\
\cot\theta &= \frac{\cos\theta}{\sin\theta}
\end{aligned}$$

1.2 Opposite Angle Formulas

$$\begin{aligned}
\sin(-\theta) &= -\sin(-\theta) \\
\cos(-\theta) &= -\cos(\theta) \\
\tan(-\theta) &= -\tan(\theta) \\
\cot(-\theta) &= -\cot(\theta) \\
\sec(-\theta) &= -\sin(\theta) \\
\csc(-\theta) &= -\csc(\theta)
\end{aligned}$$

1.3 Confusion Formulas (in Quadrant I)

$$\begin{aligned}
\sin\theta &= \cos\left(\frac{\pi}{2}-\theta\right) \\
\tan\theta &= \cot\left(\frac{\pi}{2}-\theta\right) \\
\sec\theta &= \csc\left(\frac{\pi}{2}-\theta\right) \\
\cos\theta &= \sin\left(\frac{\pi}{2}-\theta\right) \\
\cot\theta &= \tan\left(\frac{\pi}{2}-\theta\right) \\
\csc\theta &= \sec\left(\frac{\pi}{2}-\theta\right)
\end{aligned}$$

1.4 Pythagorean Identities

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\tan^2\theta + 1 &= \sec^2\theta \\
\cot^2\theta + 1 &= \csc^2\theta
\end{aligned}$$

1.5 Half Angle Formulas

$$\begin{aligned}
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{2}} \\
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1+\cos\theta}{2}} \\
\tan\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \\
&= \frac{1-\cos\theta}{\sin\theta} \\
&= \frac{\sin\theta}{1+\cos\theta}
\end{aligned}$$

1.6 Double Angle Formulas

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
&= 1 - 2\sin^2\theta \\
&= 2\cos^2\theta - 1 \\
\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}
\end{aligned}$$

1.7 Triple Angle Formulas

$$\begin{aligned}
\sin 3\theta &= 3\sin\theta - 4\sin^3\theta \\
\cos 3\theta &= 4\cos^3\theta - 3\cos\theta \\
\tan 3\theta &= \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}
\end{aligned}$$

1.8 Angle Addition Formulas

$$\begin{aligned}
\sin(A+B) &= \sin A\cos B + \cos A\sin B \\
\sin(A-B) &= \sin A\cos B - \cos A\sin B \\
\cos(A+B) &= \cos A\cos B - \sin A\sin B \\
\cos(A-B) &= \sin A\cos B + \sin A\sin B \\
\tan(A+B) &= \frac{\tan A + \tan B}{1-\tan A\tan B} \\
\tan(A-B) &= \frac{\tan A - \tan B}{1+\tan A\tan B}
\end{aligned}$$

1.9 Power Reducing Formulas

$$\begin{aligned}
\sin^2\theta &= \frac{1-\cos 2\theta}{2} \\
\cos^2\theta &= \frac{1+\cos 2\theta}{2} \\
\tan^2\theta &= \frac{1-\cos 2\theta}{1+\cos 2\theta}
\end{aligned}$$

1.10 Product-to-Sum Formulas

$$\begin{aligned}
\sin A \cdot \sin B &= \frac{1}{2}\left[\cos\left(A-B\right) - \cos A + B\right] \\
\cos A \cdot \cos B &= \frac{1}{2}\left[\cos\left(A-B\right) + \cos A + B\right] \\
\sin A \cdot \cos B &= \frac{1}{2}\left[\sin\left(A-B\right) + \sin A + B\right] \\
\cos A \cdot \sin B &= \frac{1}{2}\left[\sin\left(A+B\right) - \sin A - B\right]
\end{aligned}$$

1.11 Sum-to-Product Formulas

$$\begin{aligned}
\sin A + \sin B &= 2 \cdot \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\
\sin A - \sin B &= 2 \cdot \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right) \\
\cos A + \cos B &= 2 \cdot \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right) \\
\cos A - \cos B &= 2 \cdot \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)
\end{aligned}$$

1.12 Arc Length

$S = r\theta$

1.13 Law of Sines

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

1.14 Law of Cosines

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
b^2 &= a^2 + c^2 - 2ac\cos B \\
c^2 &= a^2 + b^2 - 2ab\cos C
\end{aligned}$$

1.15 Law of Tangents

$\dfrac{a-b}{a+b} = \dfrac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$

1.16 Euler's Formula

$e^{i\theta} = \cos\theta + i\sin\theta$

1.17 Polar Multiplication and Division

Let: $a = r_1 cis\theta$ $b = r_2 cis\rho$

$a \cdot b = r_1 r_2 cis(\theta + \rho)$ $\dfrac{a}{b} = \dfrac{r_1}{r_2} cis(\theta - \rho)$

1.18 Mollweide's Formulas

$$\frac{a-b}{c} = \frac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\left(\frac{1}{2}C\right)}$$

$$\frac{a-b}{c} = \frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}$$