Ch.4: User input and error handling

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Programs until now hardcode input data

$$y = v_0 t - 0.5gt^2$$

```
v0 = 5
g = 9.81
t = 0.6
y = v0*t - 0.5*g*t**2
print y
```

Note:

- Input data are explicitly set ("hardcoded")
- To change input data, we need to *edit* the program
- This is considered bad programming ¡linebreak¿ (because editing programs may easily introduce errors!)
- Rule: read input from user - do not edit a correct program

How do professional programs get their input?

- Consider a web browser: how do you specify a web address? How do you change the font?
- You don't need to go into the program and edit it...

How can we specify input data in programs?

- Until now: hardcoded initialization of variables
- From now on: ask the user questions and read answers
- More convenient: read command-line arguments

```
Terminal> python myprog.py arg1 arg2 arg3 ...
Terminal> rm -i -r temp projects univ
```

Unix programs (`rm`, `ls`, `cp`, ...) make heavy use of command-line arguments, (see e.g. `man ls`). We shall do the same.

Getting input from questions and anwsers

Consider

```
C = 21
F = (9.0/5)*C + 32
print F
```

Idea: let the program ask the user a question "C=?", read the user's answer, assign that answer to the variable C.

```
C = raw_input('C=? ')    # C becomes a string
C = float(C)             # convert to float so we can compute
F = (9./5)*C + 32
print F
```

Testing:

```
Terminal> python c2f_qa.py
C=? 21
69.8
```

Another example: print the n first even numbers

```
n = int(raw_input('n=? '))

for i in range(2, 2*n+1, 2):
    print i

# or:
print range(2, 2*n+1, 2)

# or:
for i in range(1, n+1):
    print 2*i
```

Reading from the command line

```
C = 21; F = (9.0/5)*C + 32; print F
```

The user wants to specify `C` as a *command-line argument* after the name of the program when we run the program:

```
Terminal> python c2f_cml_v1.py 21
69.8
```

Command-line arguments are the "words" after the program name, and they are stored in the list `sys.argv`:

```
import sys
print 'program name:', sys.argv[0]
print '1st command-line argument:', sys.argv[1] # string
print '2nd command-line argument:', sys.argv[2] # string
print '3rd command-line argument:', sys.argv[3] # string
etc.
```

This is how we use 'sys.argv':

```
import sys
C = float(sys.argv[1])   # read 1st command-line argument
F = 9.0*C/5 + 32
print F
```

Command-line arguments separated?

Command-line arguments are separated by blanks - use quotes to override this rule!
Test program:

```
import sys; print sys.argv[1:]
```

Demonstrations:

```
Terminal> python print_cml.py 21 string with blanks 1.3
['21', 'string', 'with', 'blanks', '1.3']

Terminal> python print_cml.py 21 "string with blanks" 1.3
['21', 'string with blanks', '1.3']
```

Note that all list elements are surrounded by quotes, demonstrating that command-line arguments are strings.

Example on reading 3 parameters from the command line

Compute the current location of an object,

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

when s_0 (initial location), v_0 (initial velocity), a (constant acceleration) and t (time) are given on the command line.
Test with $t = 3$ s, $s_0 = 1$ m and $v_0 = 1$ m/s at $t = 0$, and $a = 0.5$ m/s^2:

```
Terminal> python location_cml.py 1 1 0.5 3
6.25
```

Program:

```
import sys
s0 = float(sys.argv[1])
v0 = float(sys.argv[2])
a  = float(sys.argv[3])
t  = float(sys.argv[4])
s  = s0 + v0*t + 0.5*a*t*t
print s
```

The magic eval function

`eval(s)` evaluates a string object `s` as if the string had been written directly into the program

```
>>> s = '1+2'
>>> r = eval(s)
>>> r
3
>>> type(r)
<type 'int'>

>>> r = eval('[1, 6, 7.5] + [1, 2]')
>>> r
[1, 6, 7.5, 1, 2]
>>> type(r)
<type 'list'>
```

Be careful with eval and string values

We want `r = 'math programming'`. Writing just

```
r = eval('math programming')
```

is the same as writing

```
r = math programming
```

which is an invalid expression and illegal syntax.
Remedy: must put the string inside quotes:

```
s = "'math programming'"
r = eval(s)                # r becomes 'math programming'
```

With eval, a little program can do much...

```
i1 = eval(raw_input('Give input: '))
i2 = eval(raw_input('Give input: '))
r = i1 + i2
print '%s + %s becomes %s\nwith value %s' % \
      (type(i1), type(i2), type(r), r)
```

We can add integer and float:

```
Terminal> python add_input.py
operand 1: 1
operand 2: 3.0
<type 'int'> + <type 'float'> becomes <type 'float'>
with value 4
```

or two lists:

```
Terminal> python add_input.py
operand 1: [1,2]
operand 2: [-1,0,1]
<type 'list'> + <type 'list'> becomes <type 'list'>
with value [1, 2, -1, 0, 1]
```

This great flexibility also quickly breaks programs...

```
Terminal> python add_input.py
operand 1: (1,2)
operand 2: [3,4]
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: can only concatenate tuple (not "list") to tuple

Terminal> python add_input.py
operand 1: one
Traceback (most recent call last):
  File "add_input.py", line 1, in <module>
    i1 = eval(raw_input('operand 1: '))
  File "<string>", line 1, in <module>
NameError: name 'one' is not defined

Terminal> python add_input.py
operand 1: 4
operand 2: 'Hello, World!'
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

A similar magic function: exec

- `eval(s)` evaluates an *expression* `s`
- `eval('r = 1+1')` is illegal because this is a statement, not only an expression (assignment statement: variable = expression)
- ...but we can use `exec` for complete statements:

```python
statement = 'r = 1+1'   # store statement in a string
exec(statement)
print r
```

will print 2

- For longer code we can use multi-line strings:

```python
somecode = '''
def f(t):
    term1 = exp(-a*t)*sin(w1*x)
    term2 = 2*sin(w2*x)
    return term1 + term2
'''
exec(somecode)  # execute the string as Python code
```

What can exec be used for?

- Build code at run-time, e.g., a function:

```python
formula = raw_input('Write a formula involving x: ')
code = """
def f(x):
    return %s
""" % formula
exec(code)

x = 0
while x is not None:
    x = eval(raw_input('Give x (None to quit): '))
    if x is not None:
        y = f(x)
        print 'f(%g)=%g' % (x, y)
```

- While the program is running, the user types a formula, which becomes a function, the user gives `x` values until the answer is `None`, and the program evaluates the function `f(x)`
- Note: the programmer knows nothing about `f(x)`!

StringFunction: string formulas → functions

- It is common for programs to read formulas and turn them into functions so we have made a special tool for this purpose:

```python
>>> from scitools.StringFunction import StringFunction
>>> formula = 'exp(x)*sin(x)'
>>> f = StringFunction(formula)
>>> f(0)
0.0
>>> f(pi)
2.8338239229952166e-15
```

- The function can have parameters: $g(t) = Ae^{-at}\sin(\omega x)$

```python
g = StringFunction('A*exp(-a*t)*sin(omega*x)',
    independent_variable='t', A=1, a=0.1, omega=pi, x=5)
print g(1.2)
g.set_parameters(A=2, x=10)
print g(1.2)
```

Command-line arguments with options

Many programs, especially on Unix systems, take a set of command-line arguments of the form `--option value`

```
Terminal> python location.py --v0 1 --t 3 --s0 1 --a 0.5
Terminal> python location.py --t 3
```

The latter run relies on default values for `v0`, `s0`, and `a`: we provide only the values we want to change.
Such option-value pairs make it easier to understand what the input is (cf. keyword arguments).

Programming option-value pairs with the argparse module

```python
import argparse
parser = argparse.ArgumentParser()

# Define command-line arguments
parser.add_argument('--v0', '--initial_velocity', type=float,
                    default=0.0, help='initial velocity')

parser.add_argument('--s0', '--initial_position', type=float,
                    default=0.0, help='initial position')

parser.add_argument('--a', '--acceleration', type=float,
                    default=1.0, help='acceleration')

parser.add_argument('--t', '--time', type=float,
                    default=1.0, help='time')

# Read the command line and interpret the arguments
args = parser.parse_args()

# Extract values
s = args.s0 + args.v0*t + 0.5*args.a*args.t**2
# or
s0 = args.s0; v0 = args.v0; a = args.a; t = args.t
s = s0 + v0*t + 0.5*a*t**2
```

The program has long and short command-line arguments

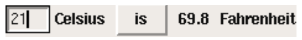
Can use short or long options:

```
Terminal> python location.py --v0 1.2 --t 0.2
Terminal> python location.py --initial_velocity 1.2 --time 0.2
```

Graphical user interfaces

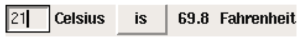
- Most programs today fetch input data from *graphical user interfaces* (GUI), consisting of windows and graphical elements on the screen: buttons, menus, text fields, etc.
- Why don't we learn to make such type of programs?
  - GUI demands much extra complicated programming
  - GUI is an advantage for novice users
  - Experienced users often prefer command-line input ¡linebreak¿ (it's much quicker and can be automated)
  - The authors of a program are very experienced users...
  - Programs with command-line or file input can easily be combined with each other, this is difficult with GUI-based programs
- Assertion: command-line input will probably fill all your needs in university courses
- But let's have a look at GUI programming!

A graphical Celsius-Fahrenheit conversion program



- The Celsius degrees can be filled in as a number in a field
- Clicking the "is" button computes the corresponding Fahrenheit temperature

The GUI code



```python
from Tkinter import *
root = Tk()
C_entry = Entry(root, width=4)
C_entry.pack(side='left')
Cunit_label = Label(root, text='Celsius')
Cunit_label.pack(side='left')

def compute():
    C = float(C_entry.get())
    F = (9./5)*C + 32
    F_label.configure(text='%g' % F)

compute = Button(root, text=' is ', command=compute)
compute.pack(side='left', padx=4)

F_label = Label(root, width=4)
F_label.pack(side='left')
Funit_label = Label(root, text='Fahrenheit')
Funit_label.pack(side='left')

root.mainloop()
```

Handling errors in input

A user can easily use our program in a wrong way, e.g.,

```
Terminal> python c2f_cml_v1.py
Traceback (most recent call last):
  File "c2f_cml_v1.py", line 2, in ?
    C = float(sys.argv[1])
IndexError: list index out of range
```

(the user forgot to provide a command-line argument...)
How can *we* take control, explain what was wrong with the input, and stop the program without strange Python error messages?

```python
if len(sys.argv) < 2:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)  # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

```
Terminal> python c2f_cml_v2.py
You failed to provide a command-line arg.!
```

Exceptions instead of if tests

- Rather than test "f something is wrong, recover from error, else do what we indended to do", it is common in Python (and many other languages) to *try* to do what we indend to, and if it fails, we recover from the error
- This principle makes use of a `try-except` block

```python
try:
    <statements we indend to do>
except:
    <statements for handling errors>
```

If something goes wrong in the `try` block, Python raises an *exception* and the execution jumps immediately to the `except` block.

Temperature conversion example with try-except

Try to read C from the command-line, if it fails, tell the user, and abort execution:

```
import sys
try:
    C = float(sys.argv[1])
except:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)  # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

Execution:

```
Terminal> python c2f_cml_v3.py
You failed to provide a command-line arg.!

Terminal> python c2f_cml_v4.py 21C
You failed to provide a command-line arg.!
```

It is best to test for specific exceptions

Here we jump to the `except` block for *any* exception raised when executing the `try` block:

```
try:
    <statements>
except:
    <statements>
```

It is good programming style to test for specific exceptions:

```
try:
    C = float(sys.argv[1])
except IndexError:
    ...
```

If we have an index out of bounds in `sys.argv`, an `IndexError` exception is raised, and we jump to the `except` block.
If any other exception arises, Python aborts the execution:

```
Terminal>> python c2f_cml_tmp.py 21C
Traceback (most recent call last):
  File "tmp.py", line 3, in <module>
    C = float(sys.argv[1])
ValueError: invalid literal for float(): 21C
```

Branching into different except blocks

We can test for different exceptions:

```
import sys
try:
    C = float(sys.argv[1])
except IndexError:
    print 'No command-line argument for C!'
    sys.exit(1)  # abort execution
except ValueError:
    print 'C must be a pure number, not "%s"' % sys.argv[1]
    sys.exit(1)

F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

Executions:

```
Terminal> python c2f_cml_v3.py
No command-line argument for C!

Terminal> python c2f_cml_v3.py 21C
Celsius degrees must be a pure number, not "21C"
```

The programmer can raise exceptions

- Instead of just letting Python raise exceptions, we can raise our own and tailor the message to the problem at hand
- We provide two examples on this:
  - catching an exception, but raising a new one with an improved (tailored) error message
  - raising an exception because of wrong input data
- Syntax: `raise ExceptionType(message)`

Examples on raising exceptions

```
def read_C():
    try:
        C = float(sys.argv[1])
    except IndexError:
        # re-raise, but with specific explanation:
        raise IndexError(
          'Celsius degrees must be supplied on the command line')
    except ValueError:
        # re-raise, but with specific explanation:
        raise ValueError(
         'Degrees must be number, not "%s"' % sys.argv[1])

    # C is read correctly as a number, but can have wrong value:
    if C < -273.15:
        raise ValueError('C=%g is a non-physical value!' % C)
    return C
```

Calling the previous function and running the program

```
try:
    C = read_C()
except (IndexError, ValueError), e:
    # print exception message and stop the program
    print e
    sys.exit(1)
```

Executions:

```
Terminal> c2f_cml.py
Celsius degrees must be supplied on the command line

Terminal> c2f_cml.py 21C
Celsius degrees must be a pure number, not "21C"

Terminal> c2f_cml.py -500
C=-500 is a non-physical value!

Terminal> c2f_cml.py 21
21C is 69.8F
```

Reading data from files

Scientific data are often available in files. We want to read the data into objects in a program to compute with the data.

Example on a data file.

```
21.8
18.1
19
23
26
17.8
```

One number on each line. How can we read these numbers

Reading a file line by line

Basic file reading:

```
infile = open('data.txt', 'r')    # open file
for line in infile:
    # do something with line
infile.close()                    # close file
```

Compute the mean values of the numbers in the file:

```
infile = open('data.txt', 'r')    # open file
mean = 0
for number in infile:
    mean = mean + float(number)   # number is string!
mean = mean/len(lines)
```

Alternative ways to read a file

Read all lines at ones into a list of strings (lines):

```
lines = infile.readlines()
for line in lines:
    # process line
```

The modern `with` statement:

```
with open('data.txt', 'r') as infile:
    for line in infile:
        # process line
```

The old-fashioned `while` construction:

```
while True:
    line = infile.readline()
    if not line:
        break
    # process line
```

Reading the whole file into a string:

```
text = infile.read()
# process the string text
```

Most data files contain text mixed with numbers

Data about rainfall:

```
Average rainfall (in mm) in Rome: 1188 months between 1782 and 1970
Jan  81.2
Feb  63.2
Mar  70.3
Apr  55.7
May  53.0
Jun  36.4
Jul  17.5
Aug  27.5
Sep  60.9
Oct  117.7
Nov  111.0
Dec  97.9
Year 792.9
```

How do we read such a file?

Reading a mixture of text and numbers

The key idea to process each line is to split the line into words:

```
months = []
values = []
for line in infile:
    words = line.split()  # split into words
    if words[0] != 'Year':
        months.append(words[0])
        values.append(float(words[1]))
```

Can split with respect to any string `s`: `line.split(s)`

```
>>> line = 'Values: 1.2, 1.4, 2.7'
>>> line.split()
['Values:', '1.2,', '1.4,', '2.7']
>>> line.split()[':']
['Values', ' 1.2, 1.4, 2.7']
>>> text, values = line.split()[':']
>>> values.split(',')
[' 1.2', ' 1.4', ' 2.7']
>>> values = [float(v) for v in values.split(',')]
>>> values
[1.2, 1.4, 2.7]
```

Complete program for reading rainfall data

```
def extract_data(filename):
    infile = open(filename, 'r')
    infile.readline() # skip the first line
    months = []
    rainfall = []
    for line in infile:
        words = line.split()
        # words[0]: month, words[1]: rainfall
        months.append(words[0])
        rainfall.append(float(words[1]))
    infile.close()
    months = months[:-1]      # Drop the "Year" entry
    annual_avg = rainfall[-1] # Store the annual average
    rainfall = rainfall[:-1]  # Redefine to contain monthly data
    return months, rainfall, annual_avg

months, values, avg = extract_data('rainfall.dat')
print 'The average rainfall for the months:'
for month, value in zip(months, values):
    print month, value
print 'The average rainfall for the year:', avg
```

Writing data to file

Basic pattern:

```
outfile = open(filename, 'w')  # 'w' for writing

for data in somelist:
    outfile.write(sometext + '\n')

outfile.close()
```

Can append text to a file with `open(filename, 'a')`.

Example: Writing a table to file

Problem:

We have a nested list (rows and columns):

```
data = \
[[ 0.75,        0.29619813, -0.29619813, -0.75      ],
 [ 0.29619813,  0.11697778, -0.11697778, -0.29619813],
 [-0.29619813, -0.11697778,  0.11697778,  0.29619813],
 [-0.75,       -0.29619813,  0.29619813,  0.75      ]]
```

Write these data to file in tabular form

Solution:

```
outfile = open('tmp_table.dat', 'w')
for row in data:
    for column in row:
        outfile.write('%14.8f' % column)
    outfile.write('\n')
outfile.close()
```

Resulting file:

```
    0.75000000     0.29619813    -0.29619813    -0.75000000
    0.29619813     0.11697778    -0.11697778    -0.29619813
   -0.29619813    -0.11697778     0.11697778     0.29619813
   -0.75000000    -0.29619813     0.29619813     0.75000000
```

Making your own modules

We have frequently used modules:

```
from math import log
r = log(6)   # call log function in math module

import sys
x = eval(sys.argv[1])  # access list argv in sys module
```

Characteristics of modules:

- Collection of useful data and functions ¡linebreak¿ (later also classes)
- Functions in a module can be reused in many different programs
- If you have some general functions that can be handy in more than one program, make a module with these functions
- It's easy: just collect the functions you want in a file, and that's a module!

Case on making our own module

Here are formulas for computing with interest rates:

$$A_0 = A\left(1 + \frac{p}{360\cdot 100}\right)^{-n}, \tag{1}$$

$$n = \frac{\ln \frac{A}{A_0}}{\ln\left(1 + \frac{p}{360\cdot 100}\right)}, \tag{2}$$

$$p = 360\cdot 100\left(\left(\frac{A}{A_0}\right)^{1/n} - 1\right). \tag{3}$$

A_0: initial amount, p: percentage, n: days, A: final amount

We want to make a module with these four functions.

First we make Python functions for the formuluas

```
from math import log as ln

def present_amount(A0, p, n):
    return A0*(1 + p/(360.0*100))**n

def initial_amount(A, p, n):
    return A*(1 + p/(360.0*100))**(-n)

def days(A0, A, p):
    return ln(A/A0)/ln(1 + p/(360.0*100))

def annual_rate(A0, A, n):
    return 360*100*((A/A0)**(1.0/n) - 1)
```

Then we can make the module file

- Collect the 4 functions in a file `interest.py`
- Now `interest.py` is actually a module `interest` (!)

Example on use:

```
# How long does it take to double an amount of money?

from interest import days
A0 = 1; A = 2; p = 5
n = days(A0, 2, p)
years = n/365.0
print 'Money has doubled after %.1f years' % years
```

Adding a test block in a module file

- Module files can have an if test at the end containing a *test block* for testing or demonstrating the module
- The test block is not executed when the file is imported as a module in another program
- The test block is executed *only* when the file is run as a program

```
if __name__ == '__main__': # this test defineds the test block
    <block of statements>
```

In our case:

```
if __name__ == '__main__':
    A = 2.2133983053266699
    A0 = 2.0
    p = 5
    n = 730
    print 'A=%g (%g) A0=%g (%.1f) n=%d (%d) p=%g (%.1f)' % \
          (present_amount(A0, p, n), A,
           initial_amount(A, p, n), A0,
           days(A0, A, p), n,
           annual_rate(A0, A, n), p)
```

Test blocks are often collected in functions

Let's make a real *test function*:

```
def test_all_functions():
    # Define compatible values
    A = 2.2133983053266699; A0 = 2.0; p = 5; n = 730
    # Given three of these, compute the remaining one
    # and compare with the correct value (in parenthesis)
    A_computed = present_amount(A0, p, n)
    A0_computed = initial_amount(A, p, n)
    n_computed = days(A0, A, p)
    p_computed = annual_rate(A0, A, n)
    def float_eq(a, b, tolerance=1E-14):
        """Return True if a == b within the tolerance."""
        return abs(a - b) < tolerance

    success = float_eq(A_computed,  A)  and \
              float_eq(A0_computed, A0) and \
              float_eq(p_computed,  p)  and \
              float_eq(n_computed,  n)
    assert success  # could add message here if desired

if __name__ == '__main__':
    test_all_functions()
```

How can Python find our new module?

- If the module is in the same folder as the main program, everything is simple and ok
- Home-made modules are normally collected in a common folder, say `/Users/hpl/lib/python/mymods`
- In that case Python must be notified that our module is in that folder

Technique 1: add folder to `PYTHONPATH` in `.bashrc`:

```
export PYTHONPATH=$PYTHONPATH:/Users/hpl/lib/python/mymods
```

Technique 2: add folder to `sys.path` in the program:

```
sys.path.insert(0, '/Users/hpl/lib/python/mymods')
```

Technique 3: move the module file in a directory that Python already searches for libraries.

Summary of reading from the keyboard and command line

Question and answer input:

```
var = raw_input('Give value: ')   # var is string!

# if var needs to be a number:
var = float(var)
# or in general:
var = eval(var)
```

Command-line input:

```
import sys
parameter1 = eval(sys.argv[1])
parameter3 = sys.argv[3]          # string is ok
parameter2 = eval(sys.argv[2])
```

Recall: `sys.argv[0]` is the program name

Summary of reading options-value pairs

`--option` value pairs with the aid of `argparse`:

```
import argparse
parser = argparse.ArgumentParser()
parser.add_argument('--p1', '--parameter_1', type=float,
                    default=0.0, help='1st parameter')
parser.add_argument('--p2', type=float,
                    default=0.0, help='2nd parameter')

args = parser.parse_args()
p1 = args.p1
p2 = args.p2
```

On the command line we can provide any or all of these options:

```
Terminal> program prog.py --parameter_1 2.1 --p2 -9
```

Summary of eval and exec

Evaluating string expressions with `eval`:

```
>>> x = 20
>>> r = eval('x + 1.1')
>>> r
21.1
>>> type(r)
<type 'float'>
```

Executing strings with Python code, using `exec`:

```
exec("""
def f(x):
    return %s
""" % sys.argv[1])
```

Summary of exceptions

Handling exceptions:

```
try:
    <statements>
except ExceptionType1:
    <provide a remedy for ExceptionType1 errors>
except ExceptionType2, ExceptionType3, ExceptionType4:
    <provide a remedy for three other types of errors>
except:
    <provide a remedy for any other errors>
...
```

Raising exceptions:

```
if z < 0:
    raise ValueError(
        'z=%s is negative - cannot do log(z)' % z)
```

Summary of file reading and writing

```
infile  = open(filename, 'r')   # read
outfile = open(filename, 'w')   # write
outfile = open(filename, 'a')   # append

# Reading
line    = infile.readline()   # read the next line
filestr = infile.read()       # read rest of file into string
lines   = infile.readlines()  # read rest of file into list
for line in infile:           # read rest of file line by line

# Writing
outfile.write(s)   # add \n if you need it

# Closing
infile.close()
outfile.close()
```

A Summarizing example: solving $f(x) = 0$

Nonlinear algebraic equations like

$$x = 1 + \sin x$$
$$\tan x + \cos x = \sin 8x$$
$$x^5 - 3x^3 = 10$$

are usually impossible to solve by pen and paper, but can be solved by numerical methods. To this end, rewrite any equation

$$f(x) = 0$$

For the above we have (put everything on the left-hand side)

$$f(x) = x - 1 - \sin x$$
$$f(x) = \tan x + \cos x - \sin 8x$$
$$f(x) = x^5 - 3x^3 - 10$$

We shall learn about a method for solving $f(x) = 0$

A solution x of $f(x) = 0$ is called a *root* of $f(x)$

Outline of the the next slides:

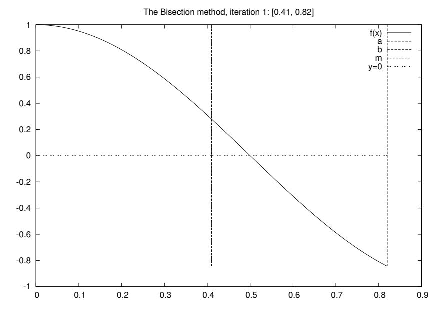
- Formulate a method for finding a root
- Translate the method to a precise algorithm
- Implement the algorithm in Python
- Test the implementation

The Bisection method

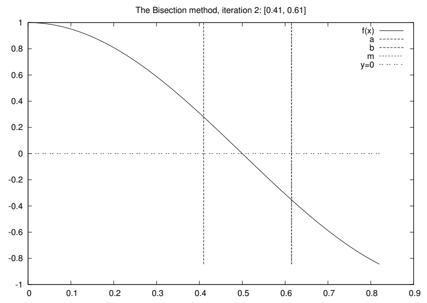
- Start with an interval $[a, b]$ in which $f(x)$ changes sign
- Then there must be (at least) one root in $[a, b]$
- Halve the interval:
  - $m = (a + b)/2$; does f change sign in left half $[a, m]$?
  - Yes: continue with left interval $[a, m]$ (set $b = m$)
  - No: continue with right interval $[m, b]$ (set $a = m$)
- Repeat the procedure

- After halving the initial interval $[p, q]$ n times, we know that $f(x)$ must have a root inside a (small) interval $2^{-n}(q - p)$
- The method is slow, but very safe
- Other methods (like Newton's method) can be faster, but may also fail to locate a root - bisection does not fail

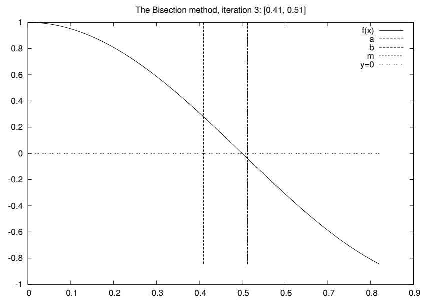
Solving $\cos \pi x = 0$: iteration no. 1



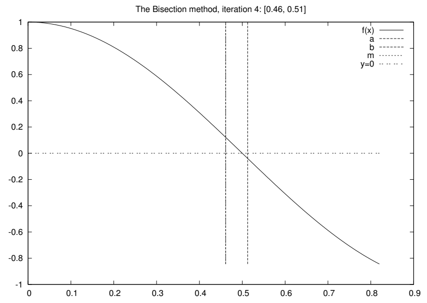
Solving $\cos \pi x = 0$: iteration no. 2



Solving $\cos \pi x = 0$: iteration no. 3



Solving $\cos \pi x = 0$: iteration no. 4



From method description to a precise algorithm

- We need to translate the mathematical description of the Bisection method to a Python program
- An important intermediate step is to formulate a precise algorithm
- Algorithm = detailed, code-like formulation of the method

```
for i = 0,1,2, ..., n:
    m = (a + b)/2
    if f(a)*f(m) <= 0:
        b = m  # root is in left half
    else:
        a = m  # root is in right half

# f(x) has a root in [a,b]
```

The algorithm can be made more efficient

- $f(a)$ is recomputed in each if test
- This is not necessary if a has not changed since last pass in the loop
- On modern computers and simple formulas for $f(x)$ these extra computations do not matter
- However, in science and engineering one meets f functions that take hours or days to evaluate at a point, and saving some $f(a)$ evaluations matters!
- Rule of thumb: remove redundant computations ¡linebreak¿ (unless the code becomes much more complicated, and harder to verify)

New, more efficient version of the algorithm

Idea: save $f(x)$ evaluations in variables

```
f_a = f(a)
for i = 0,1,2, ..., n:
    m = (a + b)/2
    f_m = f(m)
    if f_a*f_m <= 0:
        b = m   # root is in left half
    else:
        a = m   # root is in right half
        f_a = f_m

# f(x) has a root in [a,b]
```

How to choose n? That is, when to stop the iteration

- We want the error in the root to be ϵ or smaller
- After n iterations, the initial interval $[a,b]$ is halved n times and the current interval has length $2^{-n}(b-a)$. This is sufficiently small if $2^{-n}(b-a)=\epsilon \quad \Rightarrow \quad n = -\frac{\ln\epsilon - \ln(b-a)}{\ln 2}$
- A simpler alternative: just repeat halving until the length of the current interval is $\leq \epsilon$
- This is easiest done with a while loop: ¡linebreak¿
  `while b-a <= epsilon:`
- We also add a test to check if f really changes sign in the initial inverval $[a,b]$

Final version of the Bisection algorithm

```
f_a=f(a)
if f_a*f(b) > 0:
    # error: f does not change sign in [a,b]

i = 0
while b-a > epsilon:
    i = i + 1
    m = (a + b)/2
    f_m = f(m)
    if f_a*f_m <= 0:
        b = m  # root is in left half
    else:
        a = m  # root is in right half
        f_a = f_m

# if x is the real root, |x-m| < epsilon
```

Python implementation of the Bisection algorithm

```
def f(x):
    return 2*x - 3   # one root x=1.5

eps = 1E-5
a, b = 0, 10

fa = f(a)
if fa*f(b) > 0:
    print 'f(x) does not change sign in [%g,%g].' % (a, b)
    sys.exit(1)

i = 0   # iteration counter
while b-a > eps:
    i += 1
    m = (a + b)/2.0
    fm = f(m)
    if fa*fm <= 0:
        b = m  # root is in left half of [a,b]
    else:
        a = m  # root is in right half of [a,b]
        fa = fm
x = m          # this is the approximate root
```

Implementation as a function (more reusable!)

```
def bisection(f, a, b, eps):
    fa = f(a)
    if fa*f(b) > 0:
        return None, 0

    i = 0   # iteration counter
    while b-a < eps:
        i += 1
        m = (a + b)/2.0
        fm = f(m)
        if fa*fm <= 0:
            b = m  # root is in left half of [a,b]
        else:
            a = m  # root is in right half of [a,b]
            fa = fm
    return m, i
```

Make a module of this function

- If we put the `bisection` function in a file `bisection.py`, we automatically have a module, and the `bisection` function can easily be imported in other programs to solve $f(x)=0$
- We should make a test function too

```
def test_bisection():
    def f(x):
        return 2*x - 3   # only one root x=1.5

    x, iter = bisection(f, a=0, b=10, eps=1E-5)
    success = abs(x - 1.5) < 1E-5  # test within eps tolerance
    assert success, 'found x=%g != 1.5' % x

if __name__ == '__main__':
    test_bisection()
```

To the point of this lecture: get input!

We want to provide an $f(x)$ formula at the command line along with a and b (3 command-line args)
Usage:

```
Terminal> python bisection.py 'sin(pi*x**3)-x**2' -1 3.5
```

Reading input:

```
def get_input():
    """Get f, a, b, eps from the command line."""
    from scitools.std import StringFunction
    f = StringFunction(sys.argv[1])
    a = float(sys.argv[2])
    b = float(sys.argv[3])
    eps = float(sys.argv[4])
    return f, a, b, eps

# Usage:
f, a, b, eps = get_input()
x, iter = bisection(f, a, b, eps)
print 'Found root x=%g in %d iterations' % (x, iter)
```

Improvements: error handling

```
def get_input():
    """Get f, a, b, eps from the command line."""
    from scitools.std import StringFunction
    try:
        f = StringFunction(sys.argv[1])
        a = float(sys.argv[2])
        b = float(sys.argv[3])
        eps = float(sys.argv[4])
    except IndexError:
        print 'Usage %s: f a b eps' % sys.argv[0]
        sys.exit(1)
    return f, a, b, eps
```

Applications of the Bisection method

Two examples: $\tanh x = x$ and $\tanh x^5 = x^5$:

```
Terminal> python bisection_plot.py "x-tanh(x)" -1 1
Terminal> python bisection_plot.py "x**5-tanh(x**5)" -1 1
```

The first equation is easy to treat, but the second leads to much less accurate results. Why??