Ch.1: Computing with formulas

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Everybody in this country should learn how to program a computer... because it teaches you how to think. Steve Jobs, 1955-2011.

- Present a case (example)
- Present the complete program
- Dissect and discuss every line
- Simulate programs by hand (be the computer!)

- Study and try to understand examples
- Program a lot!
- This course has many compulsory exercises
- The course curriculum is defined through exercises

Why?

- Everybody understands the problem
- Many fundamental concepts are introduced
  - variables
  - arithmetic expressions
  - objects
  - printing text and numbers

A physicist, a biologist and a mathematician were at a cafe when across the street two people entered a house. Moments later three people came out. The physicist said, "Hmm, that must be a measurement error." The biologist wondered, "It must be reproduction!" And the mathematician said, "If someone goes into the house, it will be empty again."

Height of a ball in vertical motion

$$y(t) = v_0 t - \frac{1}{2}gt^2$$

where

- y is the height (position) as function of time t
- v_0 is the initial velocity at $t = 0$
- g is the acceleration of gravity

Task: given v_0, g and t, compute y.

What is a program?

A sequence of instructions to the computer, written in a programming language, somewhat like English, but very much simpler - and very much stricter.

This course teaches the Python language.

Our first example program:

Evaluate $y(t) = v_0 t - \frac{1}{2}gt^2$ for $v_0 = 5$, $g = 9.81$ and $t = 0.6$:

$$y = 5 \cdot 0.6 - \frac{1}{2} \cdot 9.81 \cdot 0.6^2$$

The complete Python program:

```
hilsen = 'Kjære Åsmund!'  # er æ og Å lov i en streng?
print hilsen
```

leads to an error:

```
 SyntaxError: Non-ASCII character ...
```

The printf syntax gives great flexibility in formatting text with numbers

Output from calculations often contain text and numbers, e.g.,

```
At t=0.6 s, y is 1.23 m.
```

We want to control the formatting of numbers: no of decimals, style: 0.6 vs 6E-01 or 6.0e-01. So-called *printf formatting* is useful for this purpose:

```
t = 0.6;   y = 1.2342
print 'At t=%g s, y is %.2f m.' % (t, y)
```

The printf format has "slots" where the variables listed at the end are put: %g $\leftarrow$ t, %.2f $\leftarrow$ y

```
%g           most compact formatting of a real number
%f           decimal notation (-34.674)
%10.3f       decimal notation, 3 decimals, field width 10
%.3f         decimal notation, 3 decimals, minimum width
%e or %E     scientific notation (1.42e-02 or 1.42E-02)
%9.2e        scientific notation, 2 decimals, field width 9
%d           integer
%5d          integer in a field of width 5 characters
%s           string (text)
%-20s        string, field width 20, left-adjusted
%%           the percentage sign % itself
```

(See the the book for more explanation and overview)

Using printf formatting in our program

Triple-quoted strings (""") can be used for multi-line output, and here we combine such a string with printf formatting:

```
v0 = 5
g = 9.81
t = 0.6
y = v0*t - 0.5*g*t**2

print """
At t=%f s, a ball with
initial velocity v0=%.3E m/s
is located at the height %.2f m.
""" % (t, v0, y)
```

Running the program:

```
Terminal> python ball_print2.py

At t=0.600000 s, a ball with
initial velocity v0=5.000E+00 m/s
is located at the height 1.23 m.
```

- Program or code or application
- Source code (program text)
- Code/program snippet
- Execute or run a program
- Algorithm (recipe for a program)
- Implementation (writing the program)
- Verification (does the program work correctly?)
- Bugs (errors) and debugging

Computer science meaning of terms is often different from the human language meaning

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```
a = 1          # 1st statement (assignment statement)
b = 2          # 2nd statement (assignment statement)
c = a + b      # 3rd statement (assignment statement)
print c        # 4th statement (print statement)
```

Normal rule: one statement per line, but multiple statements per line is possible with a semicolon in between the statements:

```
a = 1;  b = 2;  c = a + b;  print c
```

Assignment statements evaluate right-hand side and assign the result to the variable on the left-hand side

```
myvar = 10
myvar = 3*myvar    # = 30
```

Syntax is the exact specification of instructions to the computer

Programs must have correct syntax, i.e., correct use of the computer language grammar rules, and no misprints!

This is a program with two syntax errors:

```
myvar = 5.2
prinnt Myvar

    prinnt Myvar
                ^
SyntaxError: invalid syntax
```

Only the first encountered error is reported and the program is stopped (correct the error and continue with next error)

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Blanks (whitespace) can be used to nicely format the program text

Blanks may or may not be important in Python programs. These statements are equivalent (blanks do not matter):

```
v0=3
v0   =   3
v0=    3
v0 = 3
```

Here blanks do matter:

```
counter = 1
while counter <= 4:
    counter = counter + 1     # correct (4 leading blanks)

while counter <= 4:
counter = counter + 1         # invalid syntax
```

(more about this in Ch. 2)

A program takes some known *input* data and computes some *output* data

```
v0 = 3;  g = 9.81;  t = 0.6
position = v0*t - 0.5*g*t*t
velocity = v0 - g*t
print 'position:', position, 'velocity:', velocity
```

- Input: v0, g, and t
- Output: `position` and `velocity`

- Linux, Unix (Ubuntu, RedHat, Suse, Solaris)
- Windows (95, 98, NT, ME, 2000, XP, Vista, 7, 8)
- Macintosh (old Mac OS, Mac OS X)
- Mac OS X $\approx$ Unix $\approx$ Linux $\neq$ Windows
- Typical OS commands are quite similar:
  - Linux/Unix: `mkdir folder; cd folder; ls`
  - Windows: `mkdir folder; cd folder; dir`
- Python supports cross-platform programming, i.e., a program is independent of which OS we run the program on

Given C as a temperature in Celsius degrees, compute the corresponding Fahrenheit degrees F:

$$F = \frac{9}{5}C + 32$$

Program:

```
C = 21
F = (9/5)*C + 32
print F
```

Execution:

```
Terminal> python c2f_v1.py
53
```

We must always check that a new program calculates the right answer

Using a calculator:

9/5 times 21 plus 32 is 69.8, not 53.

The error is caused by (unintended) integer division

- 9/5 is not 1.8 but 1 in most computer languages (!)
- If a and b are integers, a/b implies integer division: the largest integer c such that $cb \le a$
- Examples: $1/5 = 0$, $2/5 = 0$, $7/5 = 1$, $12/5 = 2$
- In mathematics, 9/5 is a real number (1.8) - this is called float division in Python and is the division we want
- One of the operands (a or b) in a/b must be a real number ("float") to get float division
- A float in Python has a dot (or decimals): 9.0 or 9. is float
- No dot implies integer: 9 is an integer
- 9.0/5 yields 1.8, 9/5. yields 1.8, 9/5 yields 1

Corrected program (with correct output 69.8):

```
C = 21
F = (9.0/5)*C + 32
print F
```

Variables refer to objects:

```python
a = 5        # a refers to an integer (int) object
b = 9        # b refers to an integer (int) object
c = 9.0      # c refers to a real number (float) object
d = b/a      # d refers to an int/int => int object
e = c/a      # e refers to float/int => float object
s = 'b/a=%g' % (b/a)   # s is a string/text (str) object
```

We can convert between object types:

```python
a = 3              # a is int
b = float(a)       # b is float 3.0
c = 3.9            # c is float
d = int(c)         # d is int 3
d = round(c)       # d is float 4.0
d = int(round(c))  # d is int 4
d = str(c)         # d is str '3.9'
e = '-4.2'         # e is str
f = float(e)       # f is float -4.2
```

Arithmetic expressions are evaluated as you have learned in mathematics

- Example: $\frac{5}{9} + 2a^4/2$, in Python written as 5/9 + 2*a**4/2
- Same rules as in mathematics: proceed term by term (additions/subtractions) from the left, compute powers first, then multiplication and division, in each term
- r1 = 5/9 (=0)
- r2 = a**4
- r3 = 2*r2
- r4 = r3/2
- r5 = r1 + r4
- Use parenthesis to override these default rules - or use parenthesis to explicitly tell how the rules work:
  (5/9) + (2*(a**4))/2

- What if we need to compute $\sin x$, $\cos x$, $\ln x$, etc. in a program?
- Such functions are available in Python's `math` module
- In general: lots of useful functionality in Python is available in modules - but modules must be *imported* in our programs

Compute $\sqrt{2}$ using the `sqrt` function in the `math` module:

```python
import math
r = math.sqrt(2)
# or
from math import sqrt
r = sqrt(2)
# or
from math import *    # import everything in math
r = sqrt(2)
```

Evaluate

$$Q = \sin x \cos x + 4 \ln x$$

for $x = 1.2$.

```python
from math import sin, cos, log
x = 1.2
Q = sin(x)*cos(x) + 4*log(x)    # log is ln (base e)
print Q
```

Computers have inexact arithmetics because of rounding errors

Let us compute $1/49 \cdot 49$ and $1/51 \cdot 51$:

```
v1 = 1/49.0*49
v2 = 1/51.0*51
print '%.16f %.16f' % (v1, v2)
```

Output with 16 decimals becomes

```
0.999999999999999  1.000000000000000
```

- Most real numbers are represented inexactly on a computer (17 digits)
- Neither 1/49 nor 1/51 is represented exactly, the error is typically 10^{-16}
- Sometimes such small errors propagate to the final answer, sometimes not, and somtimes the small errors accumulate through many mathematical operations
- Lesson learned: real numbers on a computer and the results of mathematical computations are only approximate

What is printed?

```
a = 1; b = 2;
computed = a + b
expected = 3
correct = computed == expected
print 'Correct:', correct
```

Change to a = 0.1 and b = 0.2 (expected = 0.3). What is now printed? Why? How can the comparison be performed?

Answer to exercise on previous slide: use equality test with tolerance!

```
>>> a = 0.1; b = 0.2; expected = 0.3
>>> a + b == expected
False

>>> print '%.17f\n%.17f\n%.17f\n%.17f' % (0.1, 0.2, 0.1 + 0.2, 0.3)
0.10000000000000001
0.20000000000000001
0.30000000000000004
0.29999999999999999

>>> a = 0.1; b = 0.2; expected = 0.3
>>> computed = a + b
>>> diff = abs(expected - computed)
>>> tol = 1E-15
>>> diff < tol
```

The sinh x function is defined as

$$\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$$

We can evaluate this function in three ways:

1. `math.sinh`
2. combination of two `math.exp`
3. combination of two powers of `math.e`

```
from math import sinh, exp, e, pi
x = 2*pi
r1 = sinh(x)
r2 = 0.5*(exp(x) - exp(-x))
r3 = 0.5*(e**x - e**(-x))
print '%.16f %.16f %.16f' % (r1,r2,r3)
```

Output: r1 is 267.7448940410164369, r2 is
267.7448940410164369, r3 is 267.7448940410163232 (!)

- So far we have performed calculations in Python *programs*
- Python can also be used interactively in what is known as a *shell*
- Type `python`, `ipython`, or `idle` in the terminal window
- A Python shell is entered where you can write statements after »> (IPython has a different prompt)

```
Terminal> python
Python 2.7.6 (r25:409, Feb 27 2014, 19:35:40)
...
>>> C = 41
>>> F = (9.0/5)*C + 32
>>> print F
105.8
>>> F
105.8
```

Previous commands can be recalled and edited

Python has full support for complex numbers

- $2 + 3i$ in mathematics is written as 2 + 3j in Python

```
>>> a = -2
>>> b = 0.5
>>> s = complex(a, b)    # make complex from variables
>>> s
(-2+0.5j)
>>> s*w                  # complex*complex
(-10.5-3.75j)
>>> s/w                  # complex/complex
(-0.25641025641025639+0.28205128205128205j)
>>> s.real
-2.0
>>> s.imag
0.5
```

See the book for additional info

Python can also do symbolic computing

- Numerical computing: computation with numbers
- Symbolic computing: work with formulas (as in trad. math)

```python
>>> from sympy import *
>>> t, v0, g = symbols('t v0 g')
>>> y = v0*t - Rational(1,2)*g*t**2
>>> dydt = diff(y, t)                    # 1st derivative
>>> dydt
-g*t + v0
>>> print 'acceleration:', diff(y, t, t) # 2nd derivative
acceleration: -g
>>> y2 = integrate(dydt, t)
>>> y2
-g*t**2/2 + t*v0
```

```
>>> y = v0*t - Rational(1,2)*g*t**2
>>> roots = solve(y, t)      # solve y=0 wrt t
>>> roots
[0, 2*v0/g]

>>> x, y = symbols('x y')
>>> f = -sin(x)*sin(y) + cos(x)*cos(y)
>>> simplify(f)
cos(x + y)
>>> expand(sin(x+y), trig=True)    # requires a trigonometric hint
sin(x)*cos(y) + sin(y)*cos(x)
```

- Programs must be accurate!
- Variables are names for objects
- We have met different object types: `int`, `float`, `str`
- Choose variable names close to the mathematical symbols in the problem being solved
- Arithmetic operations in Python: term by term (+/-) from left to right, power before * and / - as in mathematics; use parenthesis when there is any doubt
- Watch out for unintended integer division!

Mathematical functions like $\sin x$ and $\ln x$ must be imported from the `math` module:

```
from math import sin, log
x = 5
r = sin(3*log(10*x))
```

Use printf syntax for full control of output of text and numbers!

Important terms: object, variable, algorithm, statement, assignment, implementation, verification, debugging

- *You think you know when you can learn,*
  are more sure when you can write,
  even more when you can teach,
  but certain when you can program
- *Within a computer, natural language is unnatural*
- *To understand a program you must become both the*
  machine and the program

Alan Perlis, computer scientist, 1922-1990.

We throw a ball with velocity v_0, at an angle θ with the horizontal, from the point $(x = 0, y = y_0)$. The trajectory of the ball is a parabola (we neglect air resistance):

$$y = x \tan\theta - \frac{1}{2v_0} \frac{gx^2}{\cos^2\theta} + y_0$$

- Program tasks:
  - initialize input data (v_0, g, θ, y_0)
  - import from `math`
  - compute y

- We give x, y and y_0 in m, $g = 9.81\text{m/s}^2$, v_0 in km/h and θ in degrees - this requires conversion of v_0 to m/s and θ to radians

Summarizing example: throwing a ball (solution)

Program:

```
g = 9.81      # m/s**2
v0 = 15       # km/h
theta = 60    # degrees
x = 0.5       # m
y0 = 1        # m

print """v0    = %.1f km/h
theta = %d degrees
y0    = %.1f m
x     = %.1f m""" % (v0, theta, y0, x)

# convert v0 to m/s and theta to radians:
v0 = v0/3.6
from math import pi, tan, cos
theta = theta*pi/180

y = x*tan(theta) - 1/(2*v0)*g*x**2/((cos(theta))**2) + y0

print 'y     = %.1f m' % y
```