Ch.4: User input and error handling

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Programs until now hardcode input data

$$y = v_0 t - 0.5 g t^2$$

```
v0 = 5
g = 9.81
t = 0.6
y = v0*t - 0.5*g*t**2
print y
```

Note:

- Input data are explicitly set ("hardcoded")
- To change input data, we need to *edit* the program
- This is considered bad programming <linebreak> (because editing programs may easily introduce errors!)
- Rule: read input from user - do not edit a correct program

How do professional programs get their input?

- Consider a web browser: how do you specify a web address? How do you change the font?
- You don't need to go into the program and edit it...

How can we specify input data in programs?

- Until now: hardcoded initialization of variables
- From now on: ask the user questions and read answers
- More convenient: read command-line arguments

```
Terminal> python myprog.py arg1 arg2 arg3 ...
Terminal> rm -i -r temp projects univ
```

Unix programs (`rm`, `ls`, `cp`, ...) make heavy use of command-line arguments, (see e.g. `man ls`). We shall do the same.

Getting input from questions and anwsers

Consider

```
C = 21
F = (9.0/5)*C + 32
print F
```

Idea: let the program ask the user a question "C=?", read the user's answer, assign that answer to the variable `C`.

```
C = raw_input('C=? ')    # C becomes a string
C = float(C)             # convert to float so we can compute
F = (9./5)*C + 32
print F
```

Testing:

```
Terminal> python c2f_qa.py
C=? 21
69.8
```

Another example: print the n first even numbers

```
n = int(raw_input('n=? '))

for i in range(2, 2*n+1, 2):
    print i

# or:
print range(2, 2*n+1, 2)

# or:
for i in range(1, n+1):
    print 2*i
```

Reading from the command line

```
C = 21; F = (9.0/5)*C + 32; print F
```

The user wants to specify `C` as a *command-line argument* after the name of the program when we run the program:

```
Terminal> python c2f_cml_v1.py 21
69.8
```

Command-line arguments are the "words" after the program name, and they are stored in the list `sys.argv`:

```
import sys
print 'program name:', sys.argv[0]
print '1st command-line argument:', sys.argv[1] # string
print '2nd command-line argument:', sys.argv[2] # string
print '3rd command-line argument:', sys.argv[3] # string
etc.
```

This is how we use 'sys.argv':

```
import sys
C = float(sys.argv[1])   # read 1st command-line argument
F = 9.0*C/5 + 32
print F
```

Command-line arguments separated?

Command-line arguments are separated by blanks - use quotes to override this rule!

Test program:

```
import sys; print sys.argv[1:]
```

Demonstrations:

```
Terminal> python print_cml.py 21 string with blanks 1.3
['21', 'string', 'with', 'blanks', '1.3']

Terminal> python print_cml.py 21 "string with blanks" 1.3
['21', 'string with blanks', '1.3']
```

Note that all list elements are surrounded by quotes, demonstrating that command-line arguments are strings.

Example on reading 3 parameters from the command line

Compute the current location of an object,

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

when s_0 (initial location), v_0 (initial velocity), a (constant acceleration) and t (time) are given on the command line.

Test with $t = 3$ s, $s_0 = 1$ m and $v_0 = 1$ m/s at $t = 0$, and $a = 0.5$ m/s^2:

```
Terminal> python location_cml.py 1 1 0.5 3
6.25
```

Program:

```
import sys
s0 = float(sys.argv[1])
v0 = float(sys.argv[2])
a  = float(sys.argv[3])
t  = float(sys.argv[4])
s  = s0 + v0*t + 0.5*a*t*t
print s
```

The magic eval function

`eval(s)` evaluates a string object `s` as if the string had been written directly into the program

```
>>> s = '1+2'
>>> r = eval(s)
>>> r
3
>>> type(r)
<type 'int'>

>>> r = eval('[1, 6, 7.5] + [1, 2]')
>>> r
[1, 6, 7.5, 1, 2]
>>> type(r)
<type 'list'>
```

Be careful with eval and string values

We want `r = 'math programming'`. Writing just

```
r = eval('math programming')
```

is the same as writing

```
r = math programming
```

which is an invalid expression and illegal syntax.

Remedy: must put the string inside quotes:

```
s = "'math programming'"
r = eval(s)                # r becomes 'math programming'
```

With eval, a little program can do much...

```
i1 = eval(raw_input('Give input: '))
i2 = eval(raw_input('Give input: '))
r = i1 + i2
print '%s + %s becomes %s\nwith value %s' % \
      (type(i1), type(i2), type(r), r)
```

We can add integer and float:

```
Terminal> python add_input.py
operand 1: 1
operand 2: 3.0
<type 'int'> + <type 'float'> becomes <type 'float'>
with value 4
```

or two lists:

```
Terminal> python add_input.py
operand 1: [1,2]
operand 2: [-1,0,1]
<type 'list'> + <type 'list'> becomes <type 'list'>
with value [1, 2, -1, 0, 1]
```

This great flexibility also quickly breaks programs...

```
Terminal> python add_input.py
operand 1: (1,2)
operand 2: [3,4]
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: can only concatenate tuple (not "list") to tuple

Terminal> python add_input.py
operand 1: one
Traceback (most recent call last):
  File "add_input.py", line 1, in <module>
    i1 = eval(raw_input('operand 1: '))
  File "<string>", line 1, in <module>
NameError: name 'one' is not defined

Terminal> python add_input.py
operand 1: 4
operand 2: 'Hello, World!'
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

A similar magic function: exec

- `eval(s)` evaluates an *expression* `s`
- `eval('r = 1+1')` is illegal because this is a statement, not only an expression (assignment statement: variable = expression)
- ...but we can use `exec` for complete statements:

```
statement = 'r = 1+1'    # store statement in a string
exec(statement)
print r
```

will print 2

- For longer code we can use multi-line strings:

```
somecode = '''
def f(t):
    term1 = exp(-a*t)*sin(w1*x)
    term2 = 2*sin(w2*x)
    return term1 + term2
'''
exec(somecode)  # execute the string as Python code
```

What can exec be used for?

- Build code at run-time, e.g., a function:

```

formula = raw_input('Write a formula involving x: ')
code = """
def f(x):
    return %s
""" % formula
exec(code)

x = 0
while x is not None:
    x = eval(raw_input('Give x (None to quit): '))
    if x is not None:
        y = f(x)
        print 'f(%g)=%g' % (x, y)
```

- While the program is running, the user types a formula, which becomes a function, the user gives `x` values until the answer is `None`, and the program evaluates the function `f(x)`
- Note: the programmer knows nothing about `f(x)`!

StringFunction: string formulas → functions

- It is common for programs to read formulas and turn them into functions so we have made a special tool for this purpose:

```
>> from scitools.StringFunction import StringFunction
>> formula = 'exp(x)*sin(x)'
>> f = StringFunction(formula)
>> f(0)
0.0
>> f(pi)
2.8338239229952166e-15
```

- The function can have parameters: $g(t) = Ae^{-at}\sin(\omega x)$

```
g = StringFunction('A*exp(-a*t)*sin(omega*x)',
    independent_variable='t', A=1, a=0.1, omega=pi, x=5)
print g(1.2)
g.set_parameters(A=2, x=10)
print g(1.2)
```

Command-line arguments with options

Many programs, especially on Unix systems, take a set of command-line arguments of the form `--option value`

```
Terminal> python location.py --v0 1 --t 3 --s0 1 --a 0.5
Terminal> python location.py --t 3
```

The latter run relies on default values for `v0`, `s0`, and `a`: we provide only the values we want to change.

Such option-value pairs make it easier to understand what the input is (cf. keyword arguments).

Programming option-value pairs with the argparse module

```
import argparse
parser = argparse.ArgumentParser()

# Define command-line arguments
parser.add_argument('--v0', '--initial_velocity', type=float,
                    default=0.0, help='initial velocity')

parser.add_argument('--s0', '--initial_position', type=float,
                    default=0.0, help='initial position')

parser.add_argument('--a', '--acceleration', type=float,
                    default=1.0, help='acceleration')

parser.add_argument('--t', '--time', type=float,
                    default=1.0, help='time')

# Read the command line and interpret the arguments
args = parser.parse_args()
```

```python
# Extract values
s = args.s0 + args.v0*t + 0.5*args.a*args.t**2
# or
s0 = args.s0; v0 = args.v0; a = args.a; t = args.t
s = s0 + v0*t + 0.5*a*t**2
```

The program has long and short command-line arguments

Can use short or long options:

```
Terminal> python location.py --v0 1.2 --t 0.2
Terminal> python location.py --initial_velocity 1.2 --time 0.2
```

Graphical user interfaces

- Most programs today fetch input data from *graphical user interfaces* (GUI), consisting of windows and graphical elements on the screen: buttons, menus, text fields, etc.
- Why don't we learn to make such type of programs?
  - GUI demands much extra complicated programming
  - GUI is an advantage for novice users
  - Experienced users often prefer command-line input <linebreak> (it's much quicker and can be automated)
  - The authors of a program are very experienced users...
  - Programs with command-line or file input can easily be combined with each other, this is difficult with GUI-based programs
- Assertion: command-line input will probably fill all your needs in university courses
- But let's have a look at GUI programming!

A graphical Celsius-Fahrenheit conversion program

- The Celsius degrees can be filled in as a number in a field
- Clicking the "is" button computes the corresponding Fahrenheit temperature

The GUI code


```
from Tkinter import *
root = Tk()
C_entry = Entry(root, width=4)
C_entry.pack(side='left')
Cunit_label = Label(root, text='Celsius')
Cunit_label.pack(side='left')

def compute():
    C = float(C_entry.get())
    F = (9./5)*C + 32
    F_label.configure(text='%g' % F)

compute = Button(root, text=' is ', command=compute)
compute.pack(side='left', padx=4)

F_label = Label(root, width=4)
F_label.pack(side='left')
Funit_label = Label(root, text='Fahrenheit')
Funit_label.pack(side='left')

root.mainloop()
```

Handling errors in input

A user can easily use our program in a wrong way, e.g.,

```
Terminal> python c2f_cml_v1.py
Traceback (most recent call last):
  File "c2f_cml_v1.py", line 2, in ?
    C = float(sys.argv[1])
IndexError: list index out of range
```

(the user forgot to provide a command-line argument...)

How can *we* take control, explain what was wrong with the input, and stop the program without strange Python error messages?

```
if len(sys.argv) < 2:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)   # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

```
Terminal> python c2f_cml_v2.py
You failed to provide a command-line arg.!
```

Exceptions instead of if tests

- Rather than test “f something is wrong, recover from error, else do what we indended to do”, it is common in Python (and many other languages) to *try* to do what we indend to, and if it fails, we recover from the error
- This principle makes use of a `try-except` block

```
try:
    <statements we indend to do>
except:
    <statements for handling errors>
```

If something goes wrong in the `try` block, Python raises an *exception* and the execution jumps immediately to the `except` block.

Temperature conversion example with try-except

Try to read `C` from the command-line, if it fails, tell the user, and abort execution:

```
import sys
try:
    C = float(sys.argv[1])
except:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)  # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

Execution:

```
Terminal> python c2f_cml_v3.py
You failed to provide a command-line arg.!

Terminal> python c2f_cml_v4.py 21C
You failed to provide a command-line arg.!
```

It is best to test for specific exceptions

Here we jump to the `except` block for *any* exception raised when executing the `try` block:

```
try:
    <statements>
except:
    <statements>
```

It is good programming style to test for specific exceptions:

```
try:
    C = float(sys.argv[1])
except IndexError:
    ...
```

If we have an index out of bounds in `sys.argv`, an `IndexError` exception is raised, and we jump to the `except` block.

If any other exception arises, Python aborts the execution:

```
Terminal>> python c2f_cml_tmp.py 21C
Traceback (most recent call last):
  File "tmp.py", line 3, in <module>
    C = float(sys.argv[1])
ValueError: invalid literal for float(): 21C
```

Branching into different except blocks

We can test for different exceptions:

```
import sys
try:
    C = float(sys.argv[1])
except IndexError:
    print 'No command-line argument for C!'
    sys.exit(1)  # abort execution
except ValueError:
    print 'C must be a pure number, not "%s"' % sys.argv[1]
    sys.exit(1)

F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

Executions:

```
Terminal> python c2f_cml_v3.py
No command-line argument for C!

Terminal> python c2f_cml_v3.py 21C
Celsius degrees must be a pure number, not "21C"
```

The programmer can raise exceptions

- Instead of just letting Python raise exceptions, we can raise our own and tailor the message to the problem at hand
- We provide two examples on this:
  - catching an exception, but raising a new one with an improved (tailored) error message
  - raising an exception because of wrong input data
- Syntax: `raise ExceptionType(message)`

Examples on raising exceptions

```
def read_C():
    try:
        C = float(sys.argv[1])
    except IndexError:
```

```
        # re-raise, but with specific explanation:
        raise IndexError(
          'Celsius degrees must be supplied on the command line')
    except ValueError:
        # re-raise, but with specific explanation:
        raise ValueError(
         'Degrees must be number, not "%s"' % sys.argv[1])

    # C is read correctly as a number, but can have wrong value:
    if C < -273.15:
        raise ValueError('C=%g is a non-physical value!' % C)
    return C
```

Calling the previous function and running the program

```
try:
    C = read_C()
except (IndexError, ValueError), e:
    # print exception message and stop the program
    print e
    sys.exit(1)
```

Executions:

```
Terminal> c2f_cml.py
Celsius degrees must be supplied on the command line

Terminal> c2f_cml.py 21C
Celsius degrees must be a pure number, not "21C"

Terminal> c2f_cml.py -500
C=-500 is a non-physical value!

Terminal> c2f_cml.py 21
21C is 69.8F
```

Reading data from files

Scientific data are often available in files. We want to read the data into objects in a program to compute with the data.

Example on a data file.

```
21.8
18.1
19
23
26
17.8
```

One number on each line. How can we read these numbers

Reading a file line by line

Basic file reading:

```
infile = open('data.txt', 'r')     # open file
for line in infile:
    # do something with line
infile.close()                     # close file
```

Compute the mean values of the numbers in the file:

```
infile = open('data.txt', 'r')     # open file
mean = 0
for number in infile:
    mean = mean + float(number)    # number is string!
mean = mean/len(lines)
```

Alternative ways to read a file

Read all lines at ones into a list of strings (lines):

```
lines = infile.readlines()
for line in lines:
    # process line
```

The modern `with` statement:

```
with open('data.txt', 'r') as infile:
    for line in infile:
        # process line
```

The old-fashioned `while` construction:

```
while True:
    line = infile.readline()
    if not line:
        break
    # process line
```

Reading the whole file into a string:

```
text = infile.read()
# process the string text
```

Most data files contain text mixed with numbers

Data about rainfall:

```
Average rainfall (in mm) in Rome: 1188 months between 1782 and 1970
Jan  81.2
Feb  63.2
Mar  70.3
Apr  55.7
May  53.0
Jun  36.4
Jul  17.5
Aug  27.5
Sep  60.9
```

```
Oct  117.7
Nov  111.0
Dec  97.9
Year 792.9
```

How do we read such a file?

Reading a mixture of text and numbers

The key idea to process each line is to split the line into words:

```
months = []
values = []
for line in infile:
    words = line.split()  # split into words
    if words[0] != 'Year':
        months.append(words[0])
        values.append(float(words[1]))
```

Can split with respect to any string `s`: `line.split(s)`

```
>> line = 'Values: 1.2, 1.4, 2.7'
>> line.split()
['Values:', '1.2,', '1.4,', '2.7']
>> line.split()[':']
['Values', ' 1.2, 1.4, 2.7']
>> text, values = line.split()[':']
>> values.split(',')
[' 1.2', ' 1.4', ' 2.7']
>> values = [float(v) for v in values.split(',')]
>> values
[1.2, 1.4, 2.7]
```

Complete program for reading rainfall data

```
def extract_data(filename):
    infile = open(filename, 'r')
    infile.readline() # skip the first line
    months = []
    rainfall = []
    for line in infile:
        words = line.split()
        # words[0]: month, words[1]: rainfall
        months.append(words[0])
        rainfall.append(float(words[1]))
    infile.close()
    months = months[:-1]      # Drop the "Year" entry
    annual_avg = rainfall[-1] # Store the annual average
    rainfall = rainfall[:-1]  # Redefine to contain monthly data
    return months, rainfall, annual_avg

months, values, avg = extract_data('rainfall.dat')
print 'The average rainfall for the months:'
for month, value in zip(months, values):
    print month, value
print 'The average rainfall for the year:', avg
```

Writing data to file

Basic pattern:

```
outfile = open(filename, 'w')  # 'w' for writing

for data in somelist:
    outfile.write(sometext + '\n')

outfile.close()
```

Can append text to a file with `open(filename, 'a')`.

Example: Writing a table to file

Problem: We have a nested list (rows and columns):

```
data = \
[[ 0.75,        0.29619813, -0.29619813, -0.75      ],
 [ 0.29619813,  0.11697778, -0.11697778, -0.29619813],
 [-0.29619813, -0.11697778,  0.11697778,  0.29619813],
 [-0.75,       -0.29619813,  0.29619813,  0.75      ]]
```

Write these data to file in tabular form

Solution:

```
outfile = open('tmp_table.dat', 'w')
for row in data:
    for column in row:
        outfile.write('%14.8f' % column)
    outfile.write('\n')
outfile.close()
```

Resulting file:

```
    0.75000000    0.29619813   -0.29619813   -0.75000000
    0.29619813    0.11697778   -0.11697778   -0.29619813
   -0.29619813   -0.11697778    0.11697778    0.29619813
   -0.75000000   -0.29619813    0.29619813    0.75000000
```

Making your own modules

We have frequently used modules:

```
from math import log
r = log(6)   # call log function in math module

import sys
x = eval(sys.argv[1])  # access list argv in sys module
```

Characteristics of modules:

- Collection of useful data and functions <linebreak> (later also classes)
- Functions in a module can be reused in many different programs

- If you have some general functions that can be handy in more than one program, make a module with these functions
- It's easy: just collect the functions you want in a file, and that's a module!

Case on making our own module

Here are formulas for computing with interest rates:

$$A_0 = A\left(1 + \frac{p}{360\cdot 100}\right)^{-n}, \tag{1}$$

$$n = \frac{\ln\frac{A}{A_0}}{\ln\left(1 + \frac{p}{360\cdot 100}\right)}, \tag{2}$$

$$p = 360\cdot 100\left(\left(\frac{A}{A_0}\right)^{1/n} - 1\right). \tag{3}$$

A_0: initial amount, p: percentage, n: days, A: final amount

We want to make a module with these four functions.

First we make Python functions for the formuluas

```
from math import log as ln

def present_amount(A0, p, n):
    return A0*(1 + p/(360.0*100))**n

def initial_amount(A, p, n):
    return A*(1 + p/(360.0*100))**(-n)

def days(A0, A, p):
    return ln(A/A0)/ln(1 + p/(360.0*100))

def annual_rate(A0, A, n):
    return 360*100*((A/A0)**(1.0/n) - 1)
```

Then we can make the module file

- Collect the 4 functions in a file `interest.py`
- Now `interest.py` is actually a module `interest` (!)

Example on use:

```
# How long does it take to double an amount of money?

from interest import days
A0 = 1; A = 2; p = 5
n = days(A0, 2, p)
years = n/365.0
print 'Money has doubled after %.1f years' % years
```

Adding a test block in a module file

- Module files can have an if test at the end containing a *test block* for testing or demonstrating the module
- The test block is not executed when the file is imported as a module in another program
- The test block is executed *only* when the file is run as a program

```
if __name__ == '__main__': # this test defineds the test block
    <block of statements>
```

In our case:

```
if __name__ == '__main__':
    A = 2.2133983053266699
    A0 = 2.0
    p = 5
    n = 730
    print 'A=%g (%g) A0=%g (%.1f) n=%d (%d) p=%g (%.1f)' % \
          (present_amount(A0, p, n), A,
           initial_amount(A, p, n), A0,
           days(A0, A, p), n,
           annual_rate(A0, A, n), p)
```

Test blocks are often collected in functions

Let's make a real *test function*:

```
def test_all_functions():
    # Define compatible values
    A = 2.2133983053266699; A0 = 2.0; p = 5; n = 730
    # Given three of these, compute the remaining one
    # and compare with the correct value (in parenthesis)
    A_computed = present_amount(A0, p, n)
    A0_computed = initial_amount(A, p, n)
    n_computed = days(A0, A, p)
    p_computed = annual_rate(A0, A, n)
    def float_eq(a, b, tolerance=1E-14):
        """Return True if a == b within the tolerance."""
        return abs(a - b) < tolerance
```

```
    success = float_eq(A_computed,  A)  and \
              float_eq(A0_computed, A0) and \
              float_eq(p_computed,  p)  and \
              float_eq(n_computed,  n)
    assert success  # could add message here if desired

if __name__ == '__main__':
    test_all_functions()
```

How can Python find our new module?

- If the module is in the same folder as the main program, everything is simple and ok
- Home-made modules are normally collected in a common folder, say `/Users/hpl/lib/python/mymods`
- In that case Python must be notified that our module is in that folder

Technique 1: add folder to `PYTHONPATH` in `.bashrc`:

```
export PYTHONPATH=$PYTHONPATH:/Users/hpl/lib/python/mymods
```

Technique 2: add folder to `sys.path` in the program:

```
sys.path.insert(0, '/Users/hpl/lib/python/mymods')
```

Technique 3: move the module file in a directory that Python already searches for libraries.

Summary of reading from the keyboard and command line

Question and answer input:

```
var = raw_input('Give value: ')   # var is string!

# if var needs to be a number:
var = float(var)
# or in general:
var = eval(var)
```

Command-line input:

```
import sys
parameter1 = eval(sys.argv[1])
parameter3 = sys.argv[3]          # string is ok
parameter2 = eval(sys.argv[2])
```

Recall: `sys.argv[0]` is the program name

Summary of reading options-value pairs

`--option value` pairs with the aid of `argparse`:

```
import argparse
parser = argparse.ArgumentParser()
parser.add_argument('--p1', '--parameter_1', type=float,
                    default=0.0, help='1st parameter')
parser.add_argument('--p2', type=float,
                    default=0.0, help='2nd parameter')

args = parser.parse_args()
p1 = args.p1
p2 = args.p2
```

On the command line we can provide any or all of these options:

```
Terminal> program prog.py --parameter_1 2.1 --p2 -9
```

Summary of eval and exec

Evaluating string expressions with `eval`:

```
>>> x = 20
>>> r = eval('x + 1.1')
>>> r
21.1
>>> type(r)
<type 'float'>
```

Executing strings with Python code, using `exec`:

```

exec("""
def f(x):
    return %s
""" % sys.argv[1])
```

Summary of exceptions

Handling exceptions:

```
try:
    <statements>
except ExceptionType1:
    <provide a remedy for ExceptionType1 errors>
except ExceptionType2, ExceptionType3, ExceptionType4:
    <provide a remedy for three other types of errors>
except:
    <provide a remedy for any other errors>
...
```

Raising exceptions:

```
if z < 0:
    raise ValueError(
        'z=%s is negative - cannot do log(z)' % z)
```

Summary of file reading and writing

```
infile  = open(filename, 'r')   # read
outfile = open(filename, 'w')   # write
outfile = open(filename, 'a')   # append

# Reading
line    = infile.readline()   # read the next line
filestr = infile.read()       # read rest of file into string
lines   = infile.readlines()  # read rest of file into list
for line in infile:           # read rest of file line by line

# Writing
outfile.write(s)   # add \n if you need it

# Closing
infile.close()
outfile.close()
```

A Summarizing example: solving $f(x) = 0$

Nonlinear algebraic equations like

$$
\begin{aligned}
x &= 1 + \sin x \\
\tan x + \cos x &= \sin 8x \\
x^5 - 3x^3 &= 10
\end{aligned}
$$

are usually impossible to solve by pen and paper, but can be solved by numerical methods. To this end, rewrite any equation

$$f(x) = 0$$

For the above we have (put everything on the left-hand side)

$$
\begin{aligned}
f(x) &= x - 1 - \sin x \\
f(x) &= \tan x + \cos x - \sin 8x \\
f(x) &= x^5 - 3x^3 - 10
\end{aligned}
$$

We shall learn about a method for solving $f(x) = 0$

A solution x of $f(x) = 0$ is called a *root* of $f(x)$

Outline of the the next slides:

- Formulate a method for finding a root

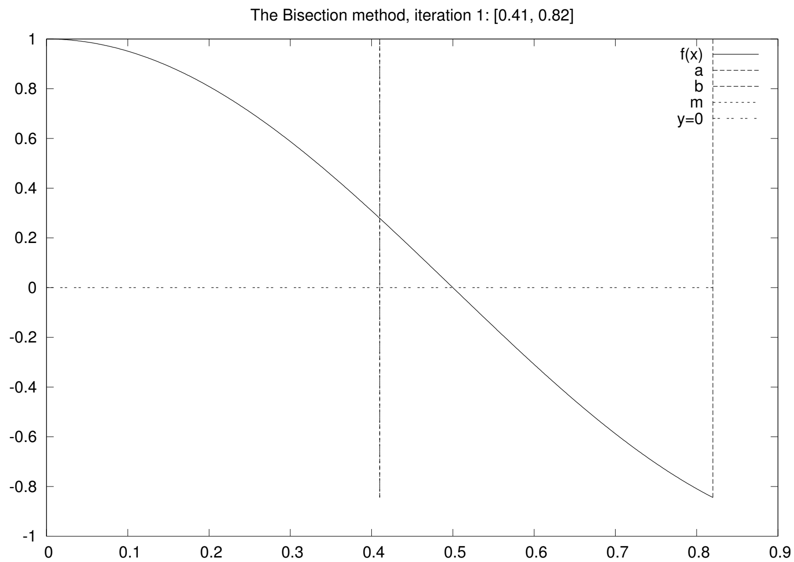
- Translate the method to a precise algorithm
- Implement the algorithm in Python
- Test the implementation

The Bisection method

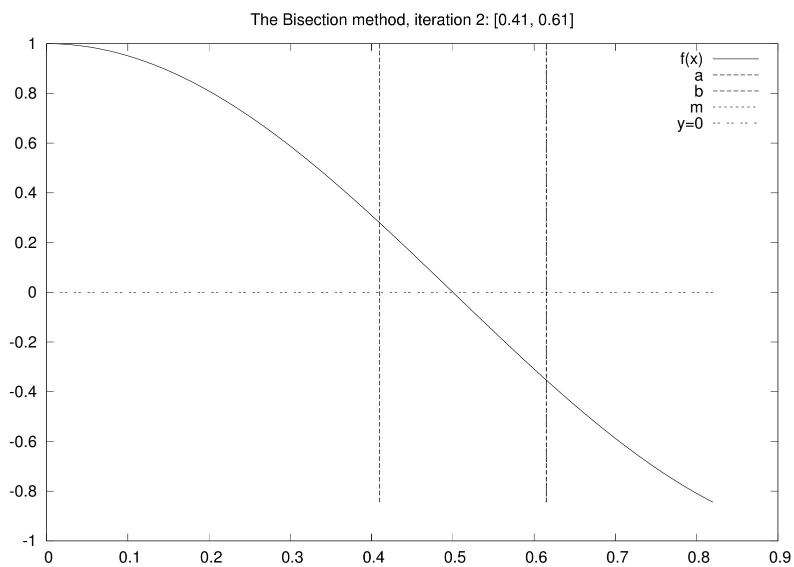
- Start with an interval $[a, b]$ in which $f(x)$ changes sign
- Then there must be (at least) one root in $[a, b]$
- Halve the interval:
  - $m = (a + b)/2$; does f change sign in left half $[a, m]$?
  - Yes: continue with left interval $[a, m]$ (set $b = m$)
  - No: continue with right interval $[m, b]$ (set $a = m$)
- Repeat the procedure

- After halving the initial interval $[p, q]$ n times, we know that $f(x)$ must have a root inside a (small) interval $2^{-n}(q - p)$
- The method is slow, but very safe
- Other methods (like Newton's method) can be faster, but may also fail to locate a root - bisection does not fail

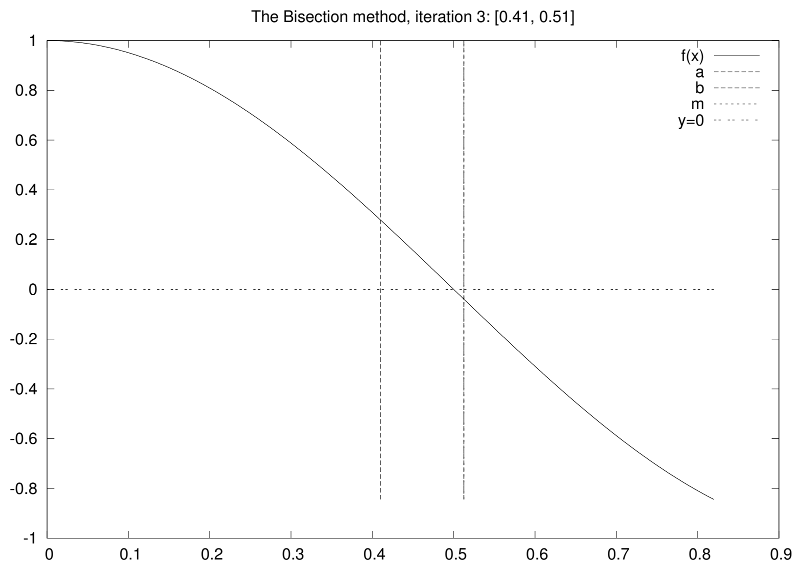
Solving $\cos \pi x = 0$: iteration no. 1



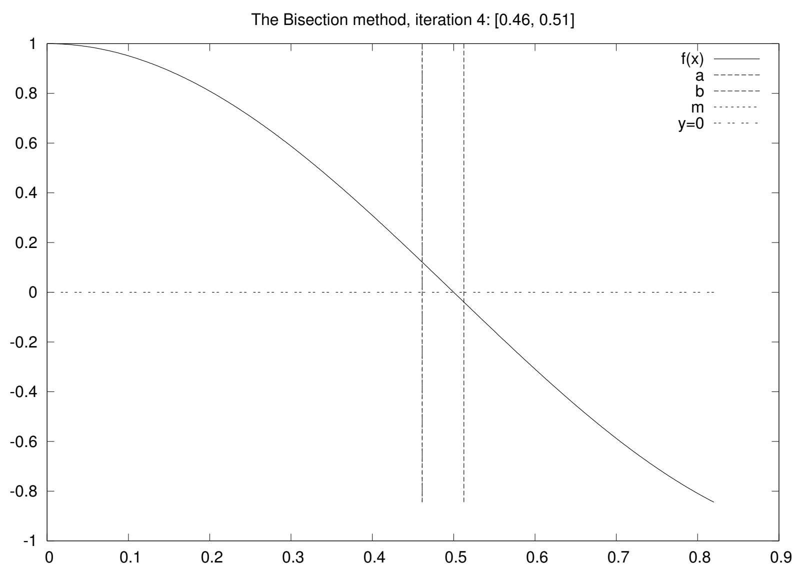
Solving $\cos \pi x = 0$: iteration no. 2



Solving $\cos \pi x = 0$: iteration no. 3



Solving $\cos \pi x = 0$: iteration no. 4



From method description to a precise algorithm

- We need to translate the mathematical description of the Bisection method to a Python program

- An important intermediate step is to formulate a precise algorithm
- Algorithm = detailed, code-like formulation of the method

```
for i = 0,1,2, ..., n:
    m = (a + b)/2
    if f(a)*f(m) <= 0:
        b = m  # root is in left half
    else:
        a = m  # root is in right half

# f(x) has a root in [a,b]
```

The algorithm can be made more efficient

- $f(a)$ is recomputed in each if test
- This is not necessary if a has not changed since last pass in the loop
- On modern computers and simple formulas for $f(x)$ these extra computations do not matter
- However, in science and engineering one meets f functions that take hours or days to evaluate at a point, and saving some $f(a)$ evaluations matters!
- Rule of thumb: remove redundant computations <linebreak> (unless the code becomes much more complicated, and harder to verify)

New, more efficient version of the algorithm

Idea: save $f(x)$ evaluations in variables

```
f_a = f(a)
for i = 0,1,2, ..., n:
    m = (a + b)/2
    f_m = f(m)
    if f_a*f_m <= 0:
        b = m   # root is in left half
    else:
        a = m   # root is in right half
        f_a = f_m

# f(x) has a root in [a,b]
```

How to choose n? That is, when to stop the iteration

- We want the error in the root to be ϵ or smaller
- After n iterations, the initial interval $[a, b]$ is halved n times and the current interval has length $2^{-n}(b-a)$. This is sufficiently small if $2^{-n}(b-a) = \epsilon \quad \Rightarrow \quad n = -\frac{\ln \epsilon - \ln(b-a)}{\ln 2}$
- A simpler alternative: just repeat halving until the length of the current interval is $\leq \epsilon$
- This is easiest done with a while loop: <linebreak> `while b-a <= epsilon`:
- We also add a test to check if f really changes sign in the initial inverval $[a, b]$

Final version of the Bisection algorithm

```
f_a=f(a)
if f_a*f(b) > 0:
    # error: f does not change sign in [a,b]

i = 0
while b-a > epsilon:
    i = i + 1
    m = (a + b)/2
    f_m = f(m)
    if f_a*f_m <= 0:
        b = m  # root is in left half
    else:
        a = m  # root is in right half
        f_a = f_m

# if x is the real root, |x-m| < epsilon
```

Python implementation of the Bisection algorithm

```
def f(x):
    return 2*x - 3   # one root x=1.5

eps = 1E-5
a, b = 0, 10

fa = f(a)
if fa*f(b) > 0:
    print 'f(x) does not change sign in [%g,%g].' % (a, b)
    sys.exit(1)

i = 0   # iteration counter
```

```
while b-a > eps:
    i += 1
    m = (a + b)/2.0
    fm = f(m)
    if fa*fm <= 0:
        b = m  # root is in left half of [a,b]
    else:
        a = m  # root is in right half of [a,b]
        fa = fm
x = m          # this is the approximate root
```

Implementation as a function (more reusable!)

```
def bisection(f, a, b, eps):
    fa = f(a)
    if fa*f(b) > 0:
        return None, 0

    i = 0   # iteration counter
    while b-a < eps:
        i += 1
        m = (a + b)/2.0
        fm = f(m)
        if fa*fm <= 0:
            b = m  # root is in left half of [a,b]
        else:
            a = m  # root is in right half of [a,b]
            fa = fm
    return m, i
```

Make a module of this function

- If we put the `bisection` function in a file `bisection.py`, we automatically have a module, and the `bisection` function can easily be imported in other programs to solve $f(x) = 0$
- We should make a test function too

```
def test_bisection():
    def f(x):
        return 2*x - 3   # only one root x=1.5

    x, iter = bisection(f, a=0, b=10, eps=1E-5)
    success = abs(x - 1.5) < 1E-5  # test within eps tolerance
    assert success, 'found x=%g != 1.5' % x

if __name__ == '__main__':
    test_bisection()
```

To the point of this lecture: get input!

We want to provide an $f(x)$ formula at the command line along with a and b (3 command-line args)

Usage:

```
Terminal> python bisection.py 'sin(pi*x**3)-x**2' -1 3.5
```

Reading input:

```python
def get_input():
    """Get f, a, b, eps from the command line."""
    from scitools.std import StringFunction
    f = StringFunction(sys.argv[1])
    a = float(sys.argv[2])
    b = float(sys.argv[3])
    eps = float(sys.argv[4])
    return f, a, b, eps

# Usage:
f, a, b, eps = get_input()
x, iter = bisection(f, a, b, eps)
print 'Found root x=%g in %d iterations' % (x, iter)
```

Improvements: error handling

```python
def get_input():
    """Get f, a, b, eps from the command line."""
    from scitools.std import StringFunction
    try:
        f = StringFunction(sys.argv[1])
        a = float(sys.argv[2])
        b = float(sys.argv[3])
        eps = float(sys.argv[4])
    except IndexError:
        print 'Usage %s: f a b eps' % sys.argv[0]
        sys.exit(1)
    return f, a, b, eps
```

Applications of the Bisection method

Two examples: $\tanh x = x$ and $\tanh x^5 = x^5$:

```
Terminal> python bisection_plot.py "x-tanh(x)" -1 1
Terminal> python bisection_plot.py "x**5-tanh(x**5)" -1 1
```

The first equation is easy to treat, but the second leads to much less accurate results. Why??