AE-512 Molecular Gas Dynamics

Final Project - Part 1

Abhyudaya Singh

Supersonic LTE flow inside a converging-diverging nozzle was solved using the indirect method [1]. A MATLAB code was developed to:

1) Read the thermodynamic and area data.
2) Create Radial Basis Function (RBF) interpolators of thermodynamic variables: Pressure, Temperature, Density, and Speed of sound as a function of enthalpy and entropy.
3) Create interpolators of enthalpy and entropy as a function of thermodynamic variables: Pressure and Temperature.
4) Calculate reservoir and throat conditions using the interpolators.
5) Solve the flow in the nozzle using the indirect method.

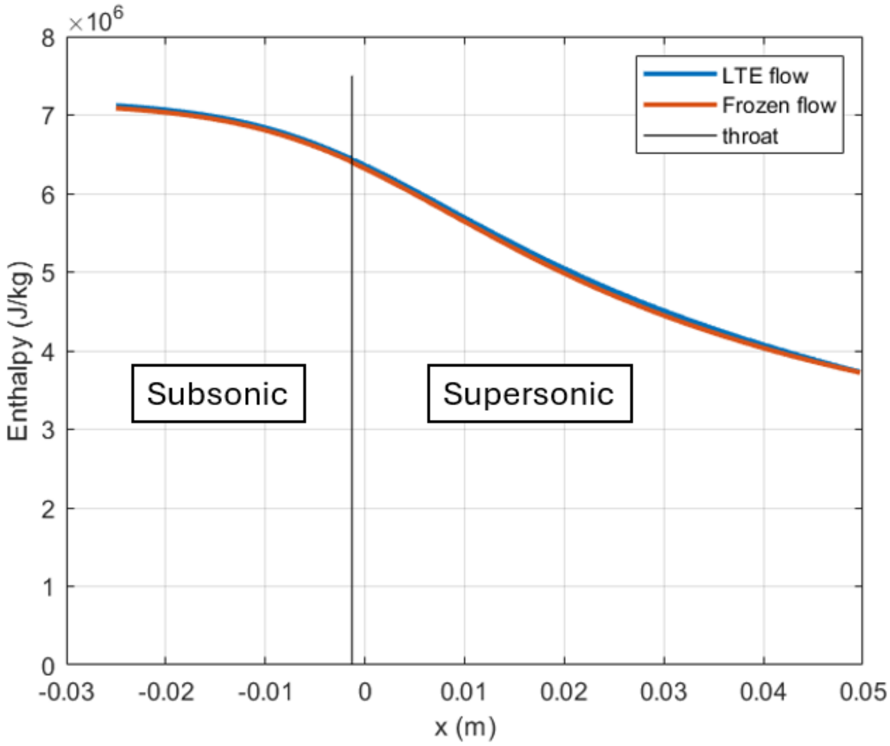
Brief description of the MATLAB subroutines

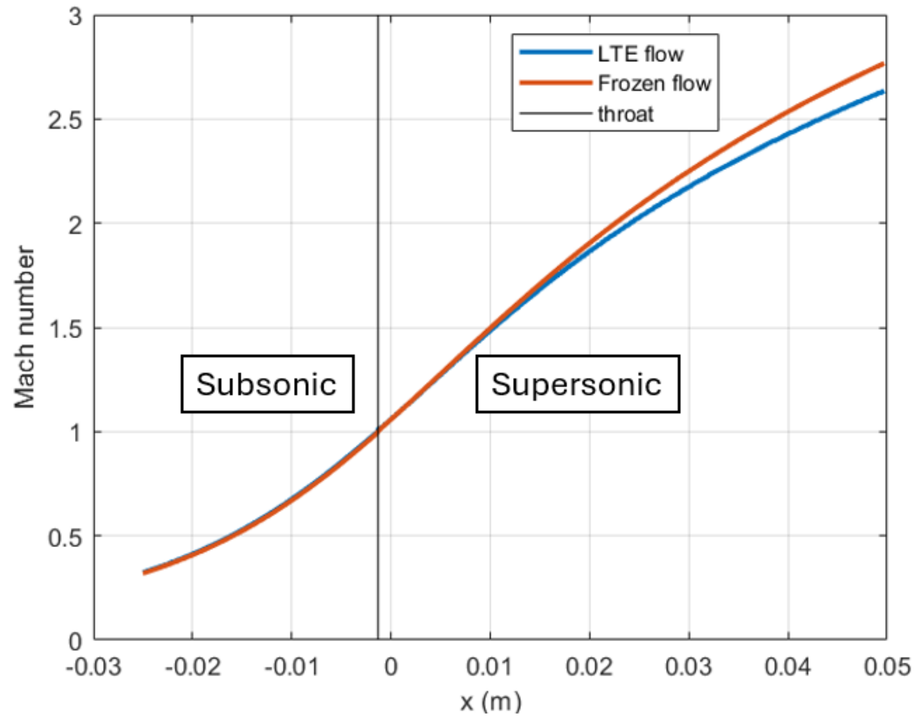
1) load_thermodynamic_data.m and Load_area_data.m: Read and store the thermodynamic data and the variation of the nozzle cross-section area with distance x.
2) construct_rbf_interpolators.m: Creates RBF interpolators for the thermodynamic data. Although multiple options for the basis function were available, for this study, the multiquadric basis function($\varphi = -(1+r^2)^{1/2}$) [2] gave the best results and hence was used in the code.
3) create_reservoir_interpolators: Creates interpolators for enthalpy and entropy as a function of Pressure and Temperature.
4) get_reservoir_h_and_s: Computes the reservoir enthalpy h_0 and entropy s_0 based on reservoir pressure and temperature.
5) find_closest_index: finds the index position of the throat.
6) compute_hstar_sstar: Finds the entropy h^* and entropy s^* at the throat using bisection method.
7) compute_rho_star_astar_Fstar: Finds density, speed of sound and f=rho*a at throat.
8) process_nozzle_indirect_method: Finds the properties along the nozzle using the indirect method.
9) process_indirect_nozzle_perfect_gas: Finds the properties along the nozzle assuming frozen flow [3].
10) main: The main script is used to call all the functions and plot the property variation.

The flow was assumed to be in Local Thermodynamic Equilibrum (LTE) and for frozen flow, constant values of ratio of specific heats and specific gas constants was used. The flow was assumed to isentropic. The flow in the reservoir was assumed to be in stagnant state hence V=0.

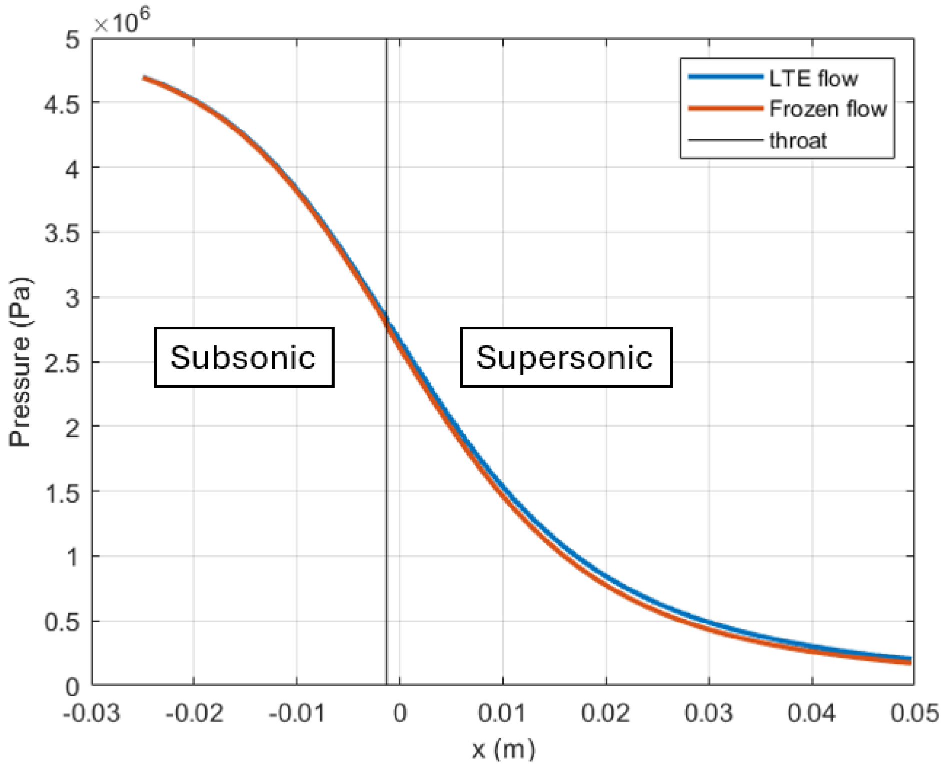
Results

The figures below show the variation of enthalpy, Mach number, Pressure, Density, Velocity and Temperature along the nozzle for both LTE and frozen flow approach.

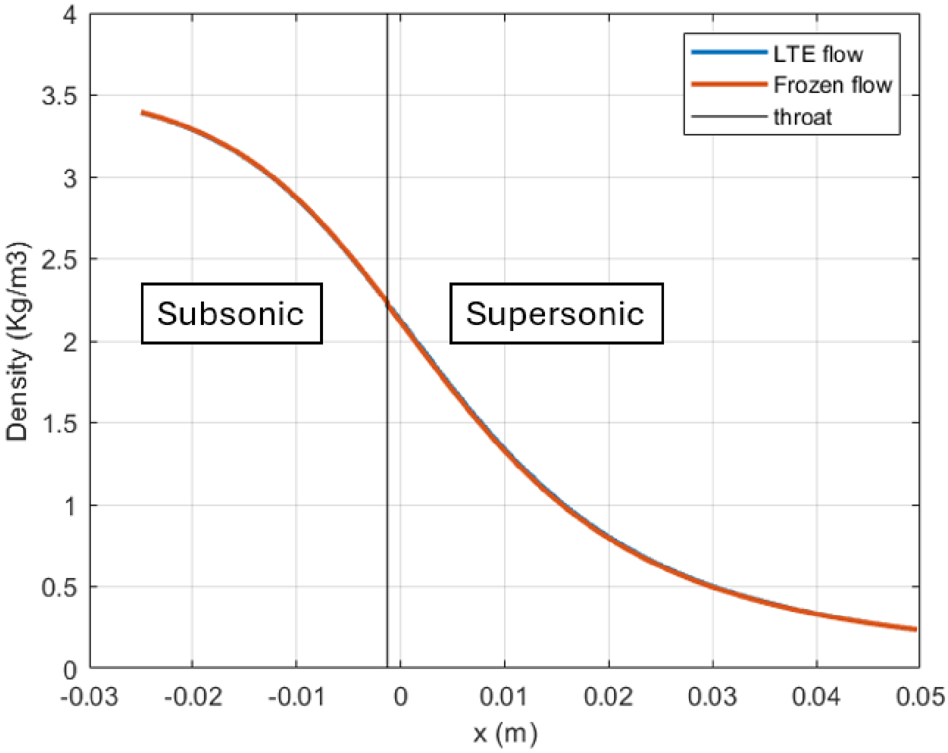




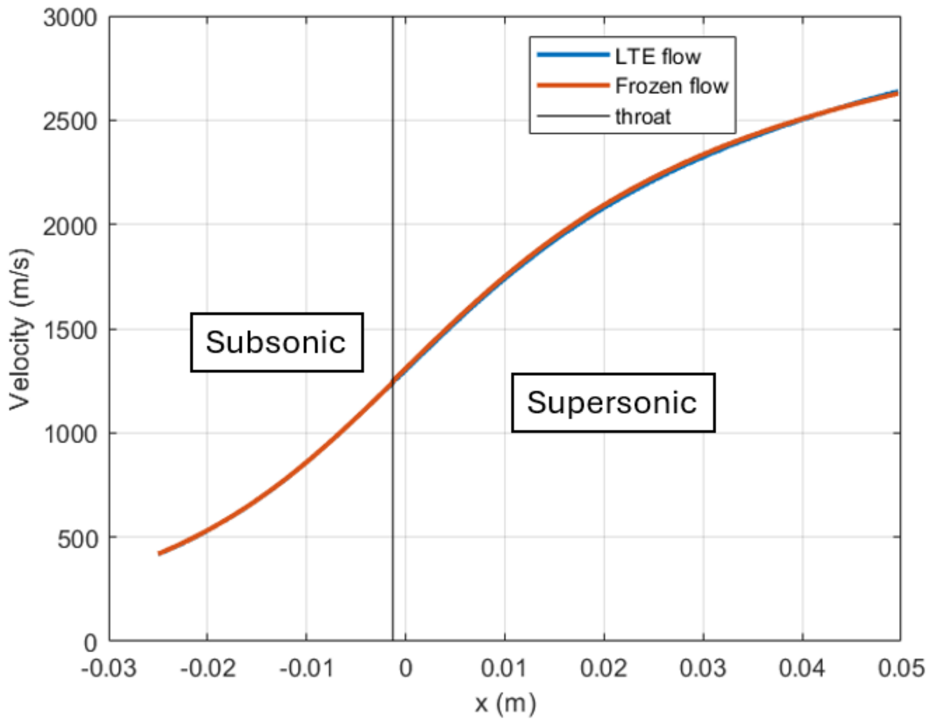
Mach number
x (m)
LTE flow
Frozen flow
throat
Subsonic
Supersonic



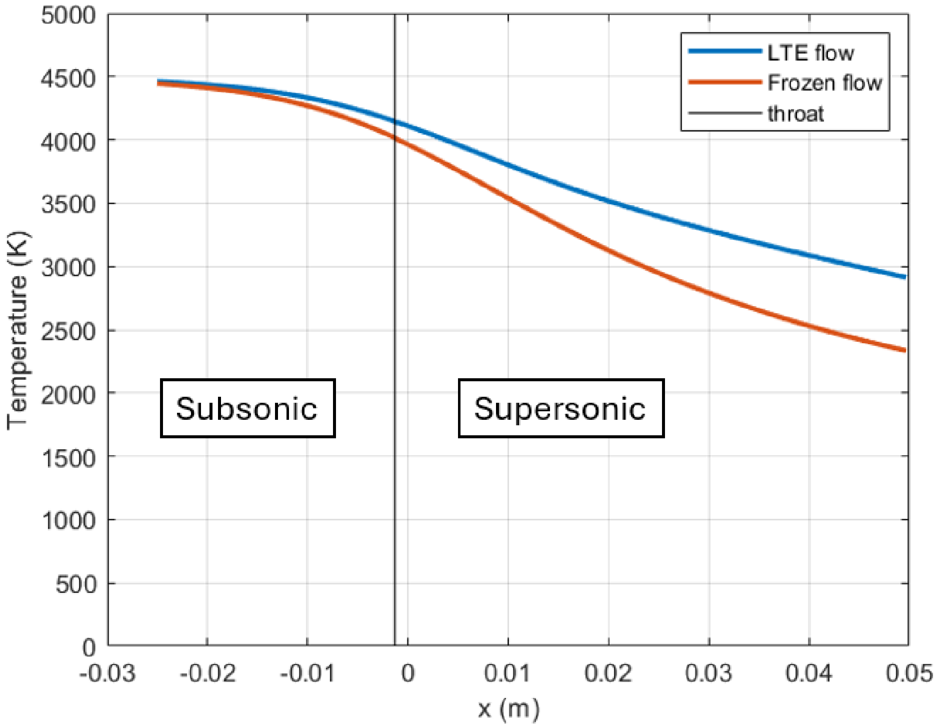
Pressure (Pa)
x (m)
$\times 10^6$
LTE flow
Frozen flow
throat
Subsonic
Supersonic



LTE flow
Frozen flow
throat
Subsonic
Supersonic
Density (Kg/m3)
x (m)
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



LTE flow
Frozen flow
throat
Subsonic
Supersonic
Velocity (m/s)
x (m)
3000
2500
2000
1500
1000
500
0
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



As the gas flows through the nozzle, it accelerates, while the temperature, pressure, density, and enthalpy decrease, and the Mach number increases. The flow remains subsonic in the converging section, reaches Mach 1 at the throat for both the LTE and frozen flow cases, and becomes supersonic in the diverging section.

The results demonstrate that the pressure distribution is largely unaffected by the choice of fluid model (LTE or frozen flow). This behavior is primarily governed by the momentum equation and the variation in the nozzle's cross-sectional area. A similar trend is observed when comparing LTE and frozen flow conditions across normal shocks, where the pressure ratio remains unaffected by the fluid model.

Because the pressure distribution remains mostly unchanged, the velocity variation is also minimally influenced by the fluid model, as pressure is the primary driving factor for velocity in the momentum equation. Consequently, the density profile is also unaffected since it depends on the velocity and the nozzle's geometry.

However, the LTE model predicts a higher temperature at the nozzle exit compared to the frozen flow model. This difference arises from species recombination in the LTE model, which releases energy. This released energy increases the translational energy of the gas, raising the static temperature near the nozzle exit.

Finally, since the flow is adiabatic, the total enthalpy remains constant throughout the nozzle.

References

[1] Vincenti, W. G., and C. H. Kruger. "Physical gas dynamics, Kriger Pub." (1975).

[2] Fasshauer, Gregory E. *Meshfree Approximation Methods with MATLAB*. World Scientific Pub Co Inc, 2007.

[3] Anderson, John David. "Modern compressible flow: with historical perspective." *(No Title)* (1990).