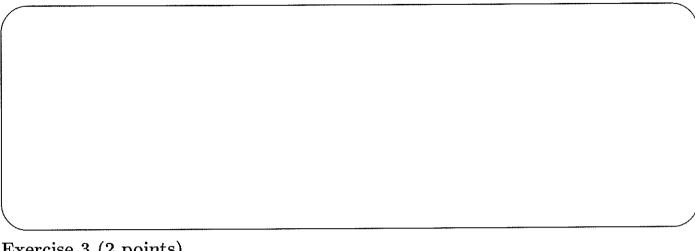
Final exam n°1

Name:	First name :
nstructions:	
no sheets other than the stapled ones pro	
answers written using lead penils shall not be	corrected.
xercise 1 (4 points)	
rite the negation of the following sentences:	
1. « No graduate of EPITA will have a first g	ross annual salary below 40 k€».
2. « If I join the research lab of EPITA, I'll b	e in a position to work in the medical imaging sector ».
3. « Some MiMo are complicated ».	
4. « All your movements on IONISx are analy	yzed ».
exercise 2 (2 points)	
et $x \in \mathbb{R}_+^*$. Prove by induction that for all $n \in$	$\mathbb{N}^*, (1+x)^n \geqslant 1 + nx.$



Exercise 3 (2 points)

Write in mathematic language (using quantifiers) the following sentences (disregard about the validity of the sentences, they may be true or false):

1. « Any real number is the cube of a real number ».

2.	"There exists a real number which is the cube of all the real numbers ".

3.	« Any natural number is even or odd ».

« Between two distinct real numbers, one can always find a rational number ».

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Exercise 4 (2 points)

For each of the following questions, CIRCLE the correct answers.

1. Let
$$f: \left\{ \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2 \end{array} \right.$$
 Then,

- a. f is injective
- b. f is not injective
- c. f is surjective
- d. f is not surjective

2. Let
$$f:\left\{ egin{array}{ll} \mathbb{R} & \longrightarrow \ \mathbb{R} \\ x & \longmapsto \ x^2 \end{array}
ight.$$
 . Then,

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4. Let $f: \left\{ \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ x & \longmapsto & x^2 \end{array} \right.$. Then,

- a. f is injective
- b. f is not injective
- c. f is surjective
- d. f is not surjective

Exercise 5 (3 points)

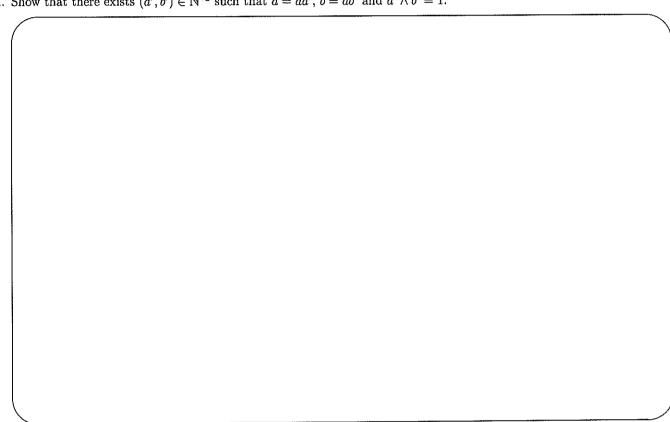
1. Using Euclid's algorithm, determine a particular solution of the equation 524x + 144y = 4.

Jsing im	peratively Gauss's	theorem, determine	the set of all the o	rdered pairs (x, y)	$\in \mathbb{Z}^2$ such that 52	4x + 144y =

Exercise 6 (2 points)

Let a and b be two non-zero natural numbers and $d = a \wedge b$.

1. Show that there exists $(a',b') \in \mathbb{N}^{*2}$ such that a = da', b = db' and $a' \wedge b' = 1$.



2. Using the previous question and Bézout's theorem, show that there exists $(u, v) \in \mathbb{Z}^2$ such that au + bv = d.

Exercise 7 (2 points)

Determine the order of multiplicity of the root 1 of the polynomial $P(X) = X^4 - X^3 - 3X^2 + 5X - 2$.

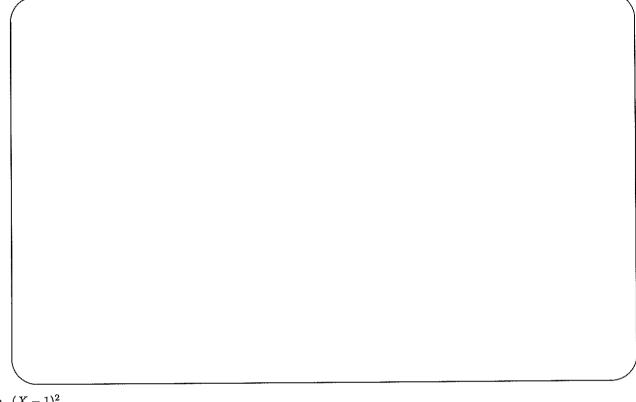
Exercise 8 (3 points)

Let $n \geqslant 2$.

1. Show that the polynomial $P(X) = (X-2)^{2n} + (X-1)^n - 1$ is divisible by $X^2 - 3X + 2$.

2. Determine the remainder of the euclidean division of $Q(X) = (X-2)^{2n} + (X-1)^n - 2$ by :

a. (X-2)(X-1)



Exercise 9 (2 points) For which value(s) of $a \in \mathbb{R}$ does the polynomial $Q(X) = (X+1)^7 - X^7 - a$ have a real root which is at least of order two?						