EPITA

Mathematics

Final exam (S3)

December 2017

| Name: | | |
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| First name: | | |
| Class: | | |
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Exercise 1 (6 points)

Let
$$A = \begin{pmatrix} 3 & -3 & 2 \\ -1 & 5 & -2 \\ -1 & 3 & 0 \end{pmatrix}$$
 and $B = \begin{pmatrix} 0 & 2 & -1 \\ 3 & -2 & 0 \\ -2 & 2 & 1 \end{pmatrix}$.

Are A and B diagonalizable in $\mathcal{M}_3(\mathbb{R})$? If they are, determine D and P.

N.B.: the bases of the eigenspaces must be deduced from a clear reasoning, and not by randomly picking particular values.

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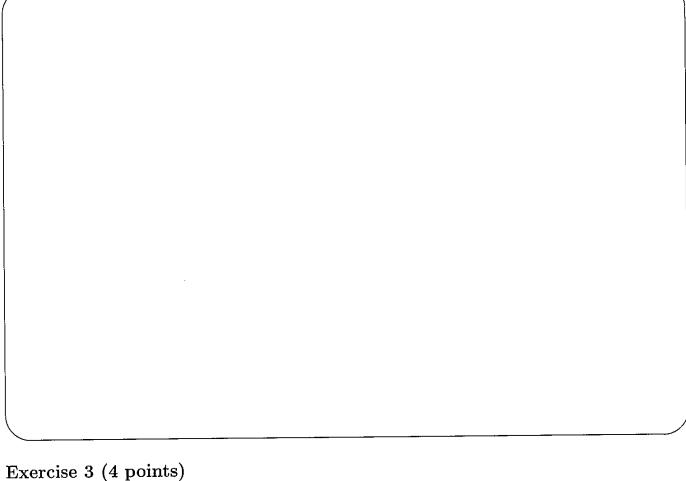
Exercise 2 (4 points)

Let $a \in \mathbb{R}$ and $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ a^2 - a & -a - 1 & a^2 + 1 \end{pmatrix}$.

Study the diagonalizability of A in $\mathcal{M}_3(\mathbb{R})$ depending on the value of a.

N.B. : when A is diagonalizable, the eigenbasis is not required.

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Exercise 3 (4 points)

1. Let $n \in \mathbb{N}$ and $f : \left\{ \begin{array}{cc} \mathbb{R}_n[X] & \longrightarrow & \mathbb{R} \\ P & \longmapsto & \int_0^1 P(t) \mathrm{d}t \end{array} \right.$

Determine the matrix of f in the standard bases of the input and output spaces.



2. Let $E = \mathbb{R}_3[X]$ and $f : \begin{cases} E \longrightarrow E \\ P(X) \longmapsto (X^2 - 1)P''(X) + 2XP'(X) \end{cases}$.

Determine the matrix of f in the standard basis $(1, X, X^2, X^3)$ of $\mathbb{R}_3[X]$.

| DAGICIOC A (I POLITO) | Exercise | 4 | (4 | points) |) |
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Let E be a finite dimensional vector space over \mathbb{R} , F and G be two supplementary linear subspaces of E. Let $\mathscr{B} = (e_1, \ldots, e_p)$ be a basis of F and $\mathscr{B}' = (f_1, \ldots, f_q)$ be a basis of G. Prove, WITHOUT referring to the property $\dim(E) = \dim(F) + \dim(G)$, that the concatenation of \mathscr{B} and \mathscr{B}' is a basis of E.

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