EPITA / S3 December 2017

NAME : .... FIRSTNAME :.. GROUP :........

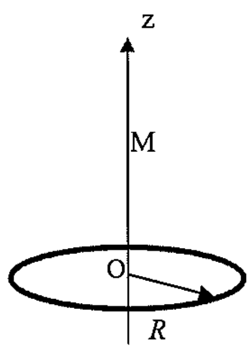
Physics Final Exam n°1 (Duration: 1h30)

Calculators and extra-documents are not allowed.

Exercise 1 (7 points)

A ring of radius R and axis (Oz) is charged with a lineic constant **positive** density.

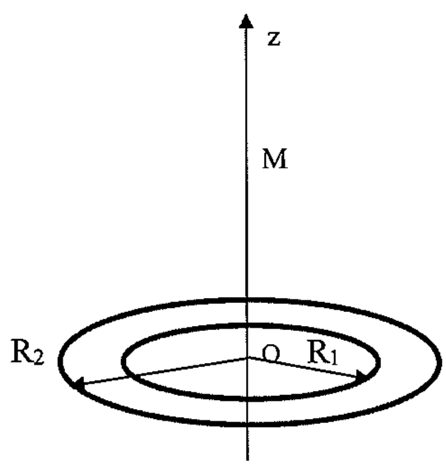
1- Use the symmetry rules to find the direction of the electric field which is generated by the ring at some point M on axis (Oz). Sketch $\vec{E}(M)$.



2- Express the elementary field dE(M) generated at point M on axis (Oz) by a charge dQ, which is located at some length element $d\ell$ of the ring. Deduce the component dE_z in terms of R, k, λ, z, θ.

3- Show that the total field at point M can be written as: $E(z) = 2\pi.k\lambda R.\dfrac{z}{(z^2+R^2)^{3/2}}$.

4- We now consider two rings, both of center O and axis (Oz), of radii R_1 and R_2. The ring of radius R_1 is charged with a constant positive density λ, whereas the ring of radius R_2 is charged with a constant negative density $-\lambda$.



a) Use the expression of question 3 to express **the norms** of the electric fields generated by the two rings at point M.
b) Sketch these vectors on the drawing above.

c) Deduce the norm of the total field at point M.

<u>Exercise 2</u>* : *Gauss's theorem (7 points)

A hollow sphere of center O and radius R carries a surfacic positive constant charge density σ.

1- Use the symmetries and the invariances to determine the direction of the electric field $\vec{E}$ and the dependence variables.

2- a) Using Gauss's theorem, express the electric field in the domains $r < R$ and $r > R$.

b) Give the curve shape as a function of r. Describe it at r = R.

3- Deduce then the potential V(r) for (r < R and r > R). (Don't determine the integration constants).

$$\vec{grad}_{Sph} = \left(\frac{\partial}{\partial r}; \frac{1}{r}\frac{\partial}{\partial \theta}; \frac{1}{r.\sin(\theta)}\frac{\partial}{\partial \varphi} \right)$$

4- Now consider a sphere of radius R, which has a **volumic** constant charge density ρ_0.

a) By using Gauss's theorem, recover the expressions of the electric field in the domains $r < R$ and $r > R$. Remember that the field is radial and depends on the radial variable r.

b) Give the shape of the curve E(r). Describe it roughly.

c) Deduce the expressions of the electric potential V(r) in the domains r < R and r > R.

Exercise 3 The questions I and II are independent (6 points)

I- One considers a cylindrical conductor of axis (Oz) and radius R. Inside it flows a current I of varying density $J(r) = J_0 \frac{r}{R}$, where J_0 and R are constant.

1- Express the total current I, which flows in the conductor, in terms of R and J_0.

2- Compute it for $J_0 = 2.10^5\ \text{A.m}^{-2}$ and R = 2mm.

II- A conducting wire of length $L = 1m$ and of section $S = 3.10^{-6}\ m^2$ is surrounded by a uniform electric field $E = 0,5\ V.m^{-1}$

Compute:

1) The voltage at conductor terminals.
2) The resistance R of the wire, by assuming that the current inside equals $I = 5\ A$.
3) The resistivity ρ and the conductivity γ of this conductor.
4) The current density J.
5) The mean velocity of the electrons. Are given: $n_e = 10^{24}\ m^{-3}$ and $|q_{e^-}| = 1{,}6.10^{-19} C$