

# EPITA

## Mathematics

Final exam (S3)

December 2017

Name :

First name :

Class :

MARK :



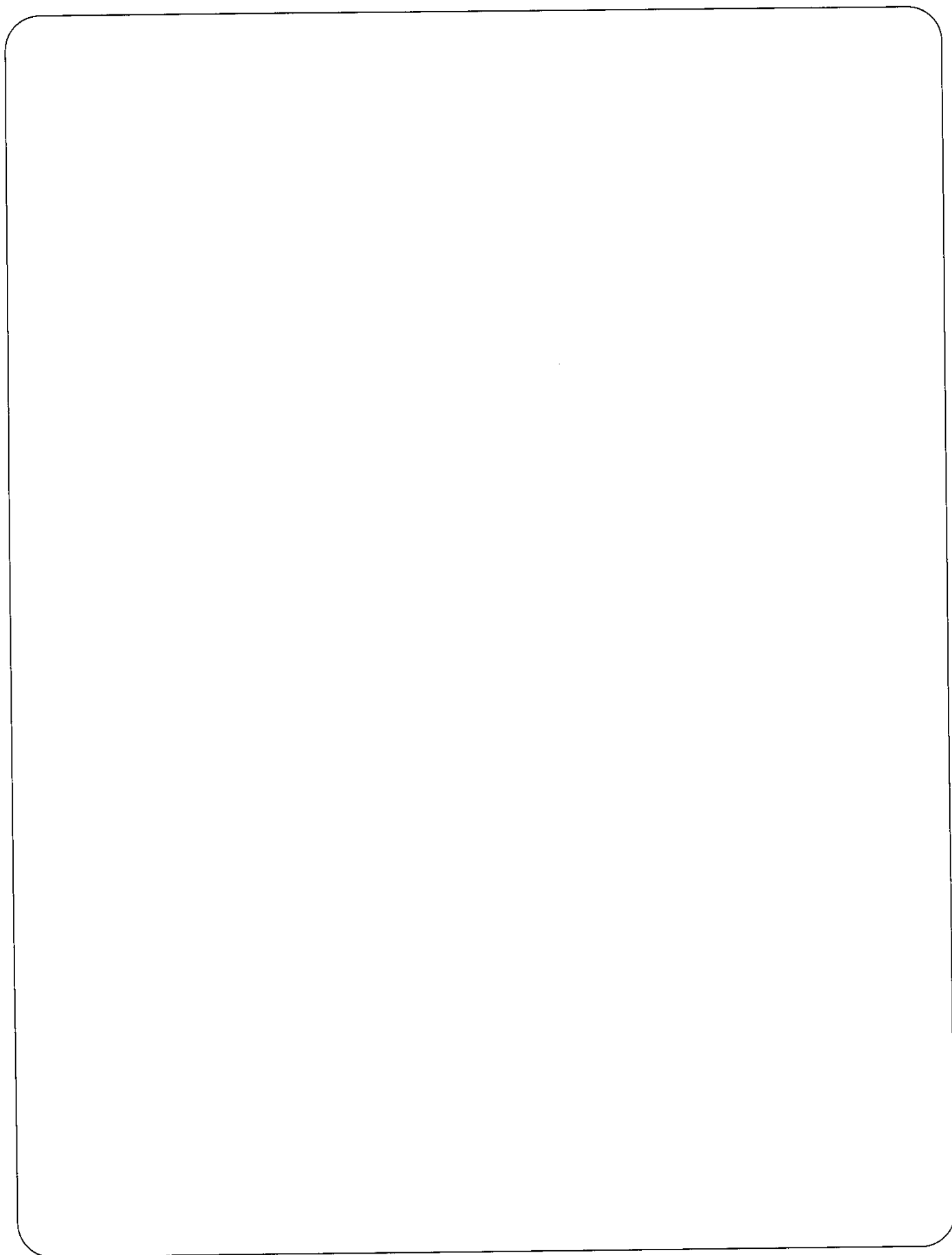
### Exercise 1 (6 points)

Let  $A = \begin{pmatrix} 3 & -3 & 2 \\ -1 & 5 & -2 \\ -1 & 3 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 2 & -1 \\ 3 & -2 & 0 \\ -2 & 2 & 1 \end{pmatrix}$ .

Are  $A$  and  $B$  diagonalizable in  $\mathcal{M}_3(\mathbb{R})$ ? If they are, determine  $D$  and  $P$ .

N.B. : the bases of the eigenspaces must be deduced from a clear reasoning, and not by randomly picking particular values.

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## Exercise 2 (4 points)

Let  $a \in \mathbb{R}$  and  $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ a^2 - a & -a - 1 & a^2 + 1 \end{pmatrix}$ .

Study the diagonalizability of  $A$  in  $\mathcal{M}_3(\mathbb{R})$  depending on the value of  $a$ .

N.B. : when  $A$  is diagonalizable, the eigenbasis is not required.

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**Exercise 3 (4 points)**

1. Let  $n \in \mathbb{N}$  and  $f : \begin{cases} \mathbb{R}_n[X] & \longrightarrow \mathbb{R} \\ P & \longmapsto \int_0^1 P(t)dt \end{cases}$ .

Determine the matrix of  $f$  in the standard bases of the input and output spaces.

2. Let  $E = \mathbb{R}_3[X]$  and  $f : \begin{cases} E & \longrightarrow E \\ P(X) & \longmapsto (X^2 - 1)P''(X) + 2XP'(X) \end{cases}$ .

Determine the matrix of  $f$  in the standard basis  $(1, X, X^2, X^3)$  of  $\mathbb{R}_3[X]$ .

### Exercise 4 (4 points)

Let  $E$  be a finite dimensional vector space over  $\mathbb{R}$ ,  $F$  and  $G$  be two supplementary linear subspaces of  $E$ . Let  $\mathcal{B} = (e_1, \dots, e_p)$  be a basis of  $F$  and  $\mathcal{B}' = (f_1, \dots, f_q)$  be a basis of  $G$ . Prove, WITHOUT referring to the property  $\dim(E) = \dim(F) + \dim(G)$ , that the concatenation of  $\mathcal{B}$  and  $\mathcal{B}'$  is a basis of  $E$ .

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**Exercise 5 (3 points)**

Calculate the determinant of the following matrix of order  $n \geq 2$  :

$$\begin{pmatrix} 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 1 & & \ddots & 1 & 0 \\ 0 & 1 & \dots & 1 & 1 \end{pmatrix}.$$