

## Midterm exam of physics

*Calculators and documents are not allowed. The number of points per question is indicative.*

**Answers to be written on this document only. Useful formulas are given in the annex**

### **Exercise 1** (5 points)

The questions a and b are independent.

- a- Let consider  $T(x, y, z)$  a temperature scalar function.  $T(x, y, z)$  is a total differential, prove that:

$$\overrightarrow{\text{curl}}(\overrightarrow{\text{grad}}(T)) = \vec{0}$$

- b- A spherical distribution of electric charges creates an electric potential  $V(r)$ . Its expression at a point M of space is given by:  $V(r) = K.r \exp(-\alpha.r)$  ; where  $\alpha$  and K are constant.

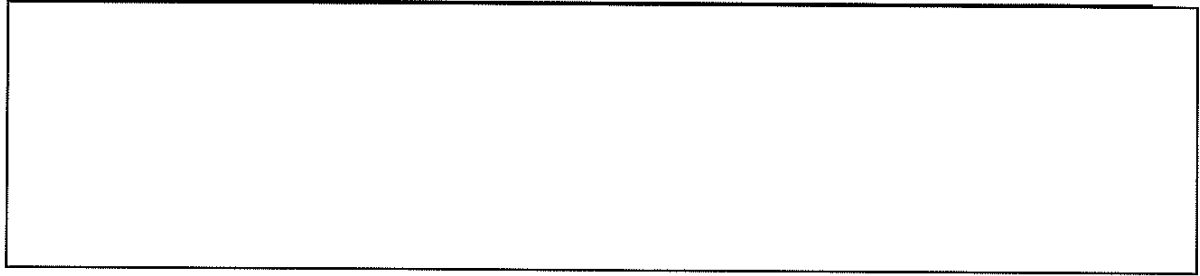
Using spherical coordinates:

$$\overrightarrow{\text{grad}}(f(r, \theta, \varphi)) = \frac{\partial f}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{e}_\theta + \frac{1}{r} \sin \theta \frac{\partial f}{\partial \varphi} \vec{e}_\varphi$$

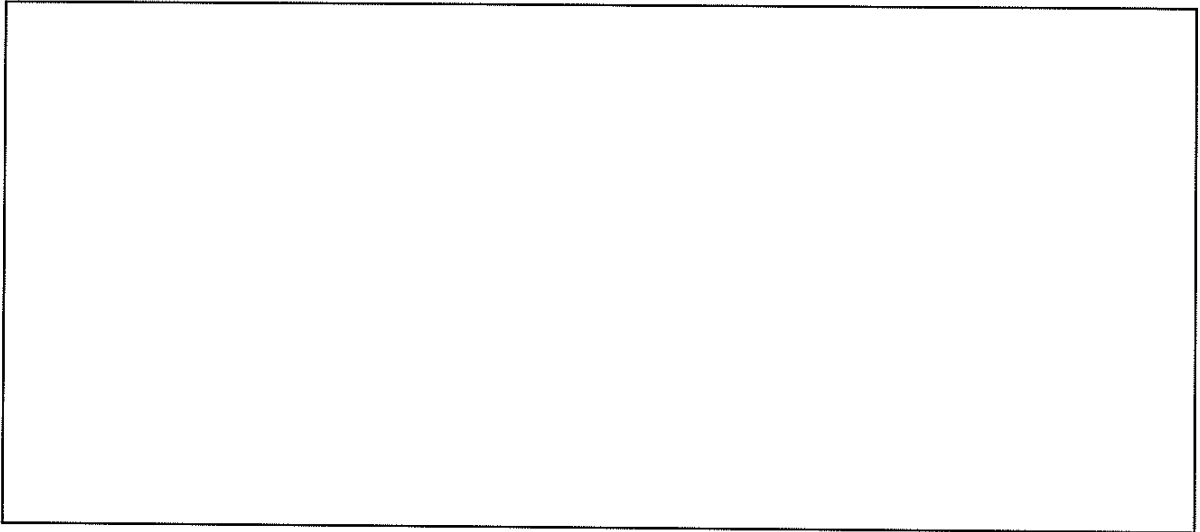
Where  $f(r, \theta, \varphi)$  is a given function.

- 1- Without doing calculation, give the direction of the electric field vector  $\vec{E}$ , knowing that:

$$\vec{E} = -\overrightarrow{\text{grad}}(V).$$



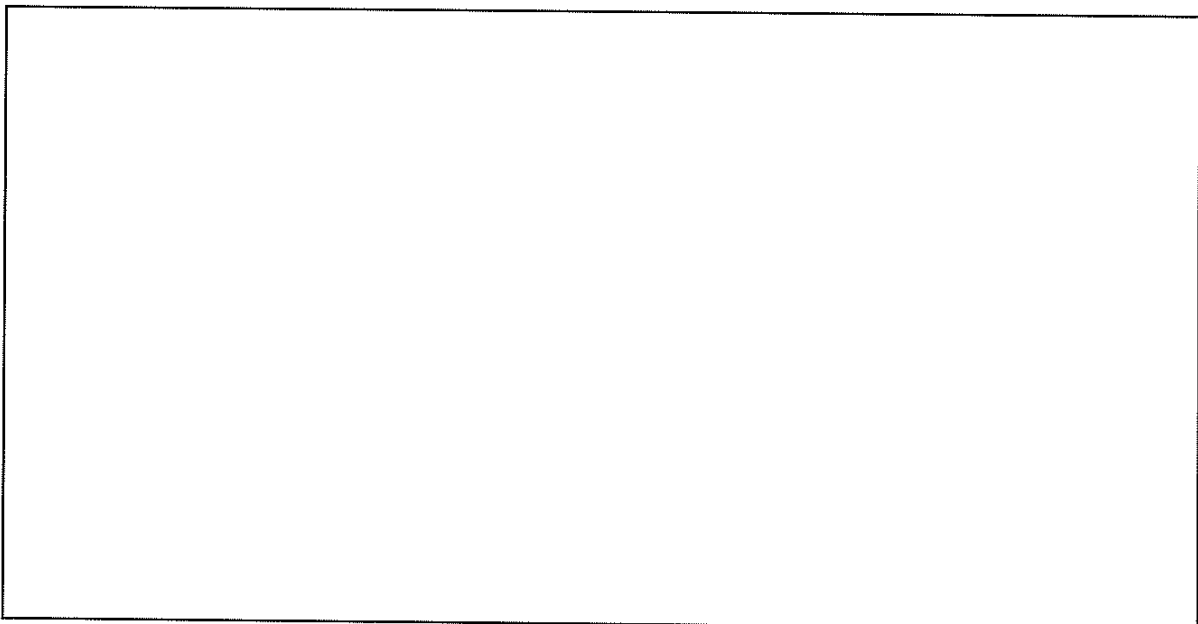
- 2- Use the previous formula to express the components of the electric field vector  $\vec{E}$  created by the spherical distribution.



**Exercise 2** (8 points)

The questions I, II and III are independent.

- I-a) Do the demonstration of the second Maxwell's equation.



I- b) Give the meaning of the second Maxwell's equation.

II) Do the demonstration of the first Maxwell's equation.

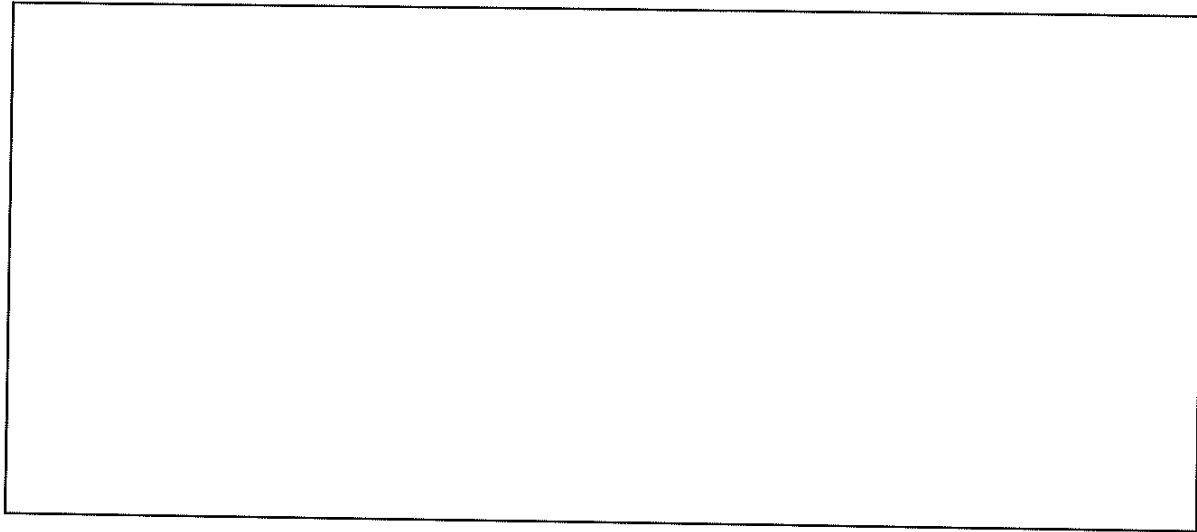
III- a) Using the Maxwell equations, find the propagation equation of the magnetic field in any material medium, given by:

$$\Delta \vec{B} - \mu \cdot \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu \cdot \overrightarrow{\text{curl}}(\vec{J}).$$

Given:  $\Delta \vec{U} = \overrightarrow{\text{grad}}(\text{div}(\vec{U})) - \overrightarrow{\text{curl}}(\overrightarrow{\text{curl}}(\vec{U}))$  for a given vector  $\vec{U}$ .

III-b) Give the propagation equation of the magnetic field in the air medium (or vacuum). Give the meaning of the constant  $\mu_0 \cdot \epsilon_0$ . Deduce the velocity in vacuum (or air) of the electromagnetic wave.

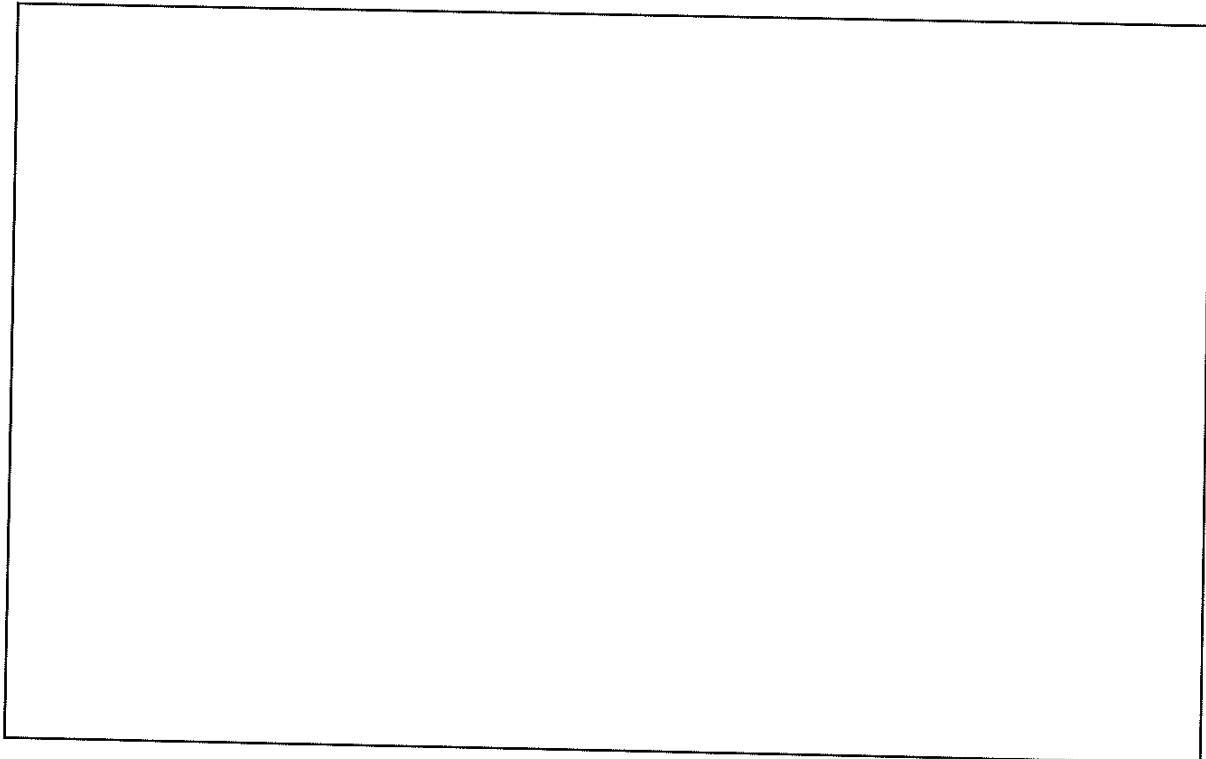
III-c) We know that the magnetic field given by :  $\vec{B}(y,t) = B_0 \cos(k \cdot y - \omega t) \vec{e}_z$  is a solution of the propagation equation in the air medium, find a relationship between  $\omega$  and  $k$ .

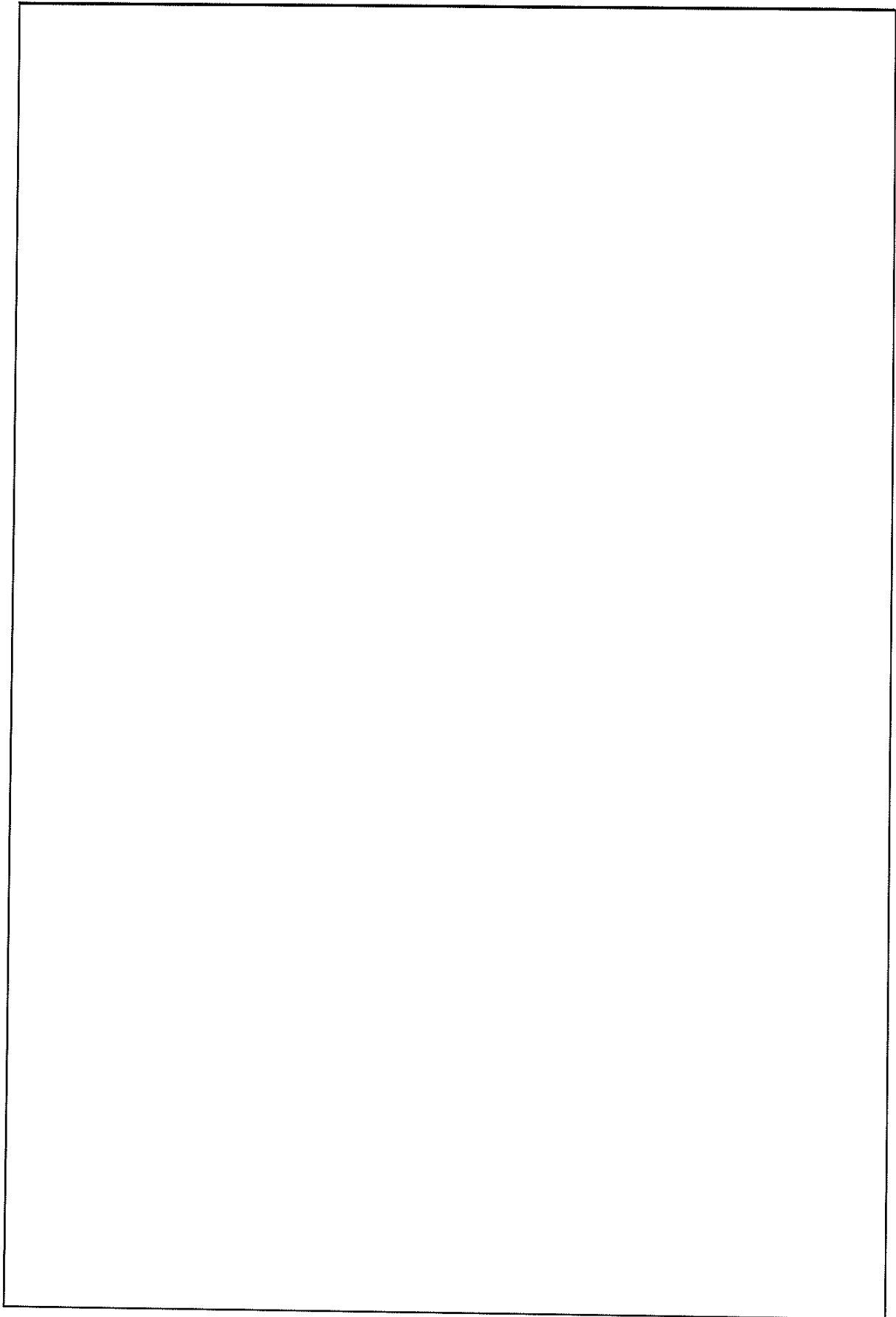
**Exercise 3** (7 points)

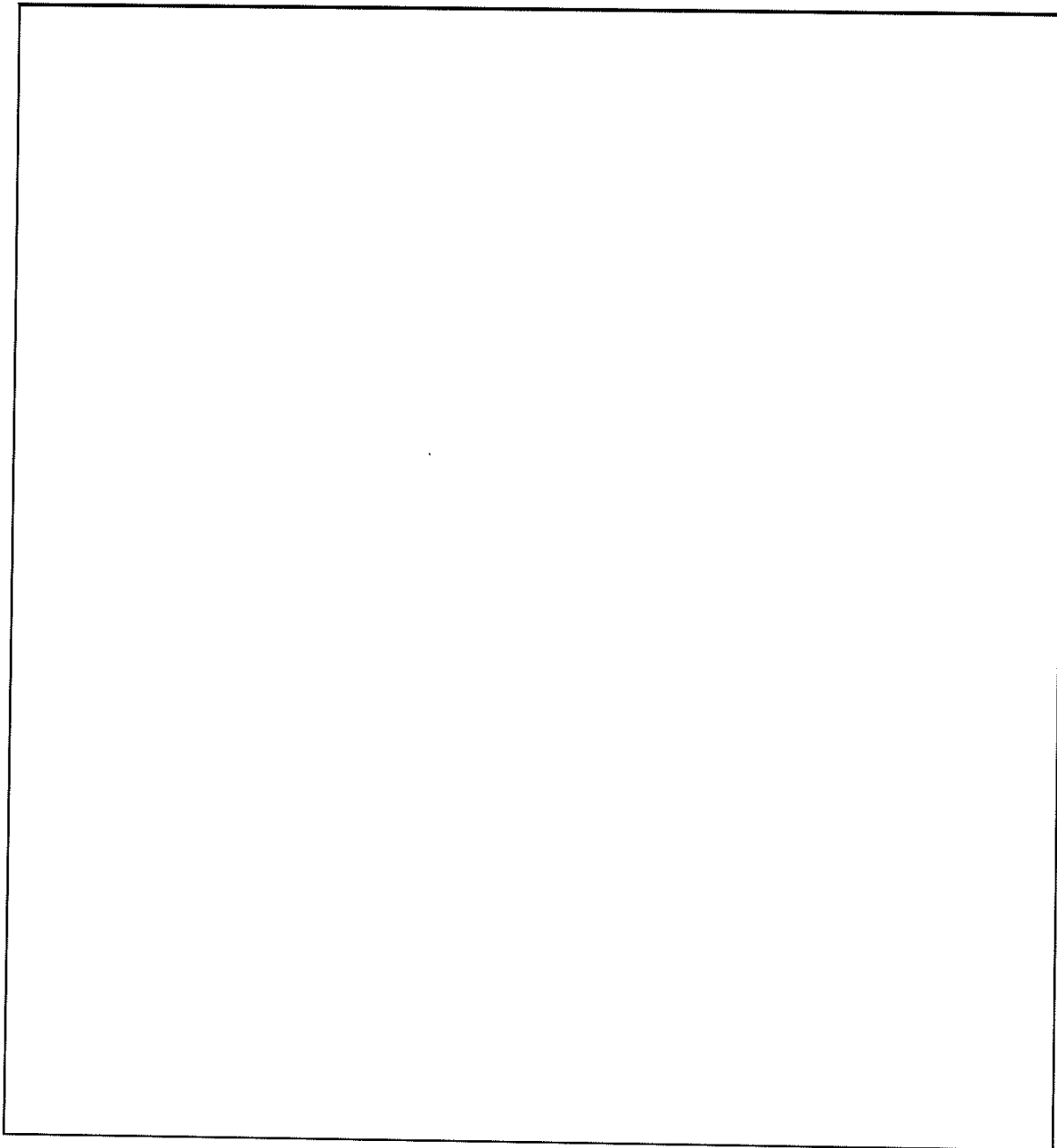
The electromagnetic field vectors of a sinusoidal progressive plane wave propagating in the air medium are given by:

$$\begin{cases} \vec{E}(x, t) = E_0 \cos(k.x - \omega.t) \vec{e}_y, \\ \vec{B}(x, t) = \frac{E_0}{c} \cos(k.x - \omega.t) \vec{e}_z \end{cases}$$

Prove that these vectors satisfy the four Maxwell equations in the air environment. We know that:  
 $\omega = k.c.$







Useful formulas

Maxwell equations in any medium:

$$1) \operatorname{div}(\vec{E}) = \frac{\rho}{\epsilon}$$

$$3) \overrightarrow{\operatorname{curl}}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$$

$$2) \operatorname{div}(\vec{B}) = 0$$

$$4) \overrightarrow{\operatorname{curl}}(\vec{B}) = \mu \cdot \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

The Green-Ostrogradski theorem:  $\oiint_S \vec{U} \cdot d\vec{S} = \iiint_{\tau} \operatorname{div}(\vec{U}) d\tau$