EPITA / S3 December 2016

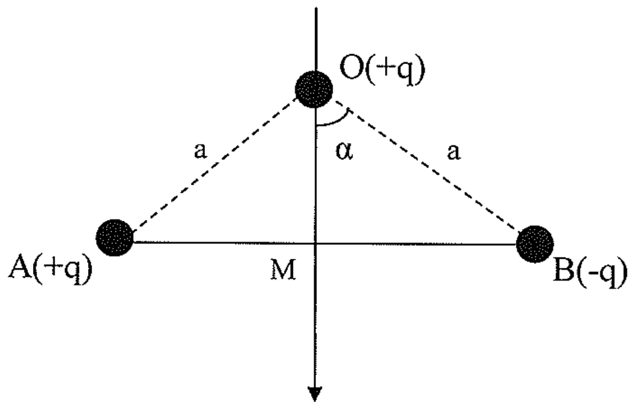
NAME : .... FIRSTNAME:.. GROUP :.........

Physics Midterm 1 (Duration:1h30)

Calculator and documents not allowed.

Exercise 1 ***Discrete distribution*** (4 points)

We consider three pointlike charges +q, +q and –q respectively located at points O, A and B. Point M belongs to AB bisector. Let's denote OA = OB = a.



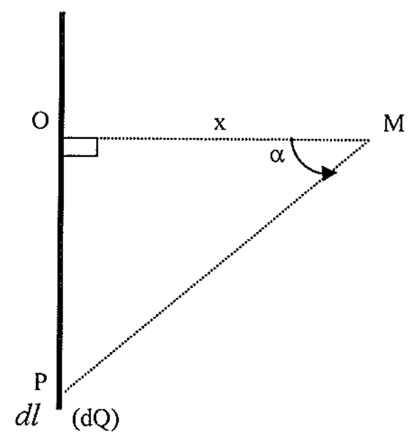
1-a) Sketch in picture above the vectors describing the electrostatic fields generated at point M by the three charges and the total field $\vec{E}(M)$.

b) Write the norms $E_O(M)$, $E_A(M)$ and $E_B(M)$ as function of k, q, a and α. Deduce then the norm of the total field : $E(M)$.

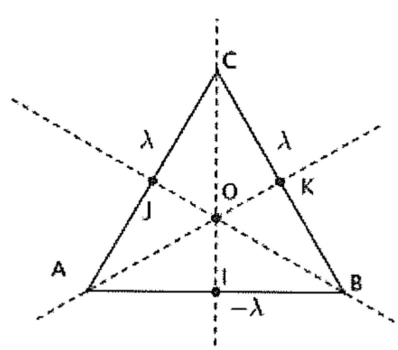
2- Write the electric potential V(M) created at point M as function of k, q, a and α.

Exercise 2 *Continuous distribution* (4 points)

We remind you that a linear element with charge dQ located at point P, which belongs to a wire with constant lineic charge λ, generates an elementary electric field $dE_x(M) = \frac{k.\lambda}{x}\cos(\alpha)d\alpha$ where α is defined as follows :



1-By using this result compute the norms of vectors $\overrightarrow{E_{AC}}(O)$, $\overrightarrow{E_{CB}}(O)$ and $\overrightarrow{E_{BA}}(O)$ respectively created at point O by the following continuous charge distributions. Sketch those vectors.



where ABC is an equilateral triangle whose edge length equals to 2a. Lines [AC] and [BC] are carrying a lineic charge density λ and [AB] a negative density $-\lambda$.

2) Deduce from it the expression of the total field created at point O as function of k, λ and a.

Exercise 3 ***Gauss' theorem*** (6 points)

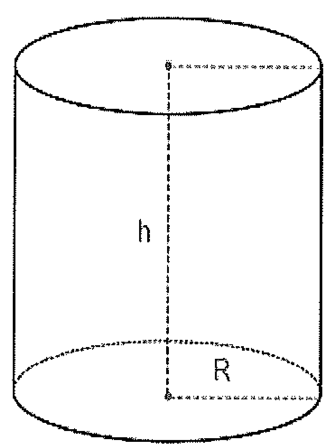
A wire **of infinite length h** is carrying a uniformly distributed positive charge Q of constant density.

1- Use symmetries and invariances to find the direction of the electric field vector which is generated by the wire at any exterior point M. One assumes that the wire is along (Oz)-axis.

2- By using Gauss' theorem write the electric field E(r) generated outside the wire at point M.

3- From now on this wire with charge Q is surrounded by **a hollow cylinder** of same (Oz)-axis, of length h, radius R and such that its lateral surface is charged with a constant positive density σ.

a) Give the expression of the electric field E(r) in domains $r < R$ and $r > R$.



b) Deduce from it the expression of the electric potential V(r) in domains r < R and r > R.

Exercise 4 ***Electrokinetics*** ***Part A*** (3 points)

Let's consider a cylindrical conductor of axis Oz and radius R which is crossed by a current I with variable density $J(r) = J_0 \frac{r^2}{R^2}$, where J_0 and R are constants.

1- Write the total current I crossing the conductor as function of R and J_0. Compute it explicitly for $J_0 = 10^6$ A/m² and R = 3mm. Use the approximation $\pi \approx 3$.

2- Write the current I' which is crossing a section of radius r < R as function of r.

Part B (3 points)

A conducting copper wire with conductivity $\gamma = 10^8\ \Omega^{-1}.m^{-1}$, length L = 1m and radius R=1 mm is crossed by a current I with uniform density $\vec{J}$ equals to $J = 2.10^7$ A/m². You will use the approximation $\pi \approx 3$.

Compute :

1- The current I crossing the conductor.

2- The electric field inside the conductor. Draw the quantities I, $\vec{J}$ and $\vec{E}$.

3- The potential difference U between terminals of the conductor.

4- The conductor resistance R.

5- The electronic density n_{e-}. We have assumed that the mean velocity of charges is :
$V_{mean} = 0{,}2 m.s^{-1}$. Given data : $q_{e-} = -1.6.10^{-19} C$.

Useful formulas

1-Gauss' theorem : $\Phi(\vec{E}) = \oiint_{Sg} \vec{E}.d\vec{S} = \dfrac{Q_{\text{int}}}{\varepsilon_0}$

2- Lateral surface element of a cylinder of radius r and height h : $dS_{lat} = rd\theta.dz$

3- Gradient components in cylindrical coordinates

$$gr\vec{a}d = \left(\frac{\partial}{\partial r}, \frac{1}{r}.\frac{\partial}{\partial \theta}, \frac{\partial}{\partial z}\right)$$