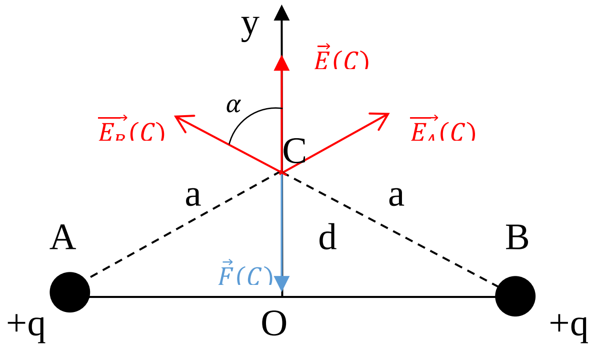
Physics exam n°1

Exercise 1 *(4 pts)*



1- As the charges at points A and B are positive the direction of the generated fields is such as above.

2- For the intensity of the fields created by A and B we just write the usual expression :

$$E_A(C) = \frac{kq}{a^2} = E_A(C)$$

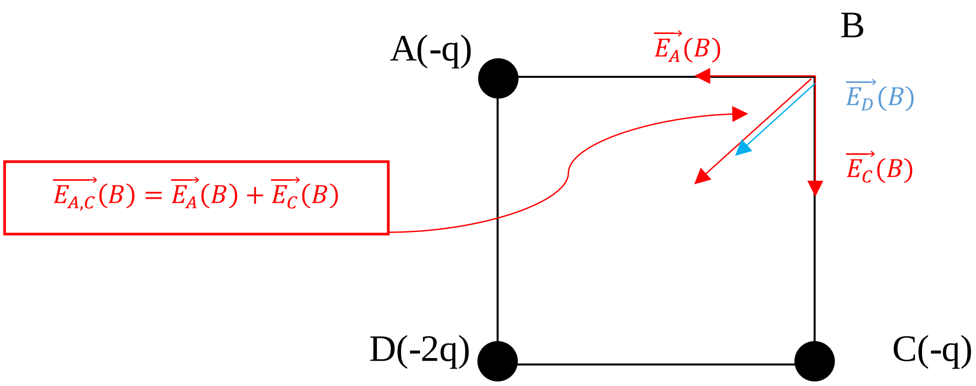
Then pay attention that we can only write $\vec{E}(C) = \overrightarrow{E_A}(C) + \overrightarrow{E_B}(C)$ using vectors. To get the norm you used different methods. Here I will use a method that I didn't often see while correcting.
First you know that

$$E(C)^2 = \left\|\overrightarrow{E_A}(C) + \overrightarrow{E_B}(C)\right\|^2 = \left\|\overrightarrow{E_A}(C)\right\|^2 + \left\|\overrightarrow{E_B}(C)\right\|^2 + 2\left\|\overrightarrow{E_A}(C)\right\|.\left\|\overrightarrow{E_B}(C)\right\|.\cos(2\,\alpha)$$
$$= 2\left\|\overrightarrow{E_A}(C)\right\|^2(1+\cos(2\,\alpha)) = 4\left\|\overrightarrow{E_A}(C)\right\|^2\cos^2(\alpha) = 4\left(\frac{kq}{a^2}\right)^2\left(\frac{d}{a}\right)^2$$

Thus the norm $E(C)$ is $\frac{2kqd}{a^3}$.

3- We can use the link between the field $E(C)$ and $F_{-q}(C)$: $F_{-q}(C) = |-q|.E(C) = \frac{2kq^2d}{a^3}$

Exercise 2 *(6 pts)*



1- Check on the picture.

2- We have the norms quite easily : $E_A(B) = \frac{qk}{a^2} = E_C(B)$ and $E_D(B) = \frac{2qk}{(a\sqrt{2})^2} = \frac{qk}{a^2}$

Obviously $\overrightarrow{E_{A,C}}(B) = \overrightarrow{E_A}(B) + \overrightarrow{E_C}(B)$ has the same direction than $\overrightarrow{E_D}(B)$ so the norm of the electric field is given by the sum of $\|\overrightarrow{E_{A,C}}(B)\| = \frac{qk}{a^2}\sqrt{2}$ and $\|\overrightarrow{E_D}(B)\| = \frac{qk}{a^2}$ i.e. $E(B) = \frac{qk}{a^2}(1+\sqrt{2})$.

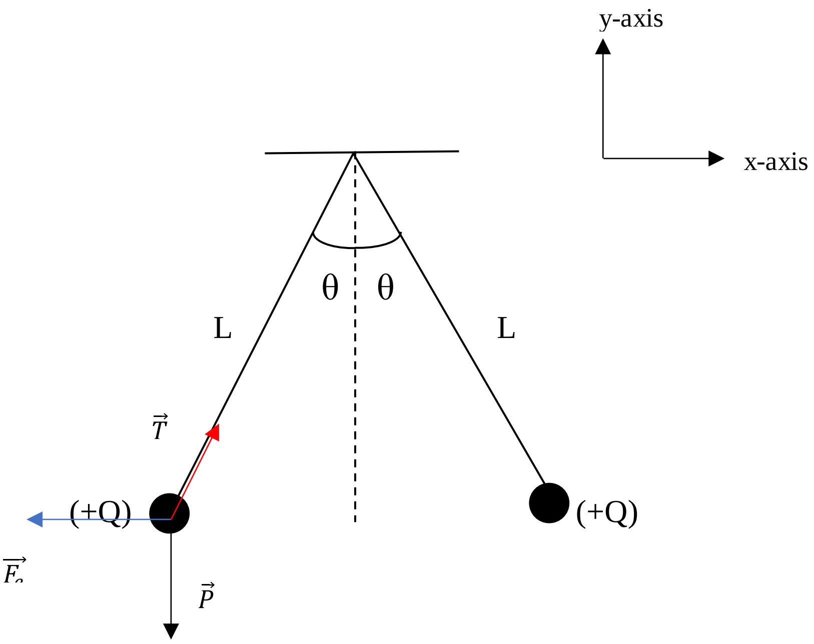
P.S : Brief remark for those who got another solution : $\sqrt{(1+\sqrt{2})^2} = \sqrt{3+2\sqrt{2}}\ldots$

3- Corners are at distance $\frac{a}{\sqrt{2}}$ from point O. So the potential can be written :

$$V(O) = V_A(O) + V_C(O) + V_D(O) = -\frac{qk\sqrt{2}}{a}(1+1+2) = -4\frac{qk\sqrt{2}}{a}$$

4- The last way of writing the potential enlightens that the potential vanishing condition reads $\frac{q_B k\sqrt{2}}{a} - 4\frac{qk\sqrt{2}}{a} = 0$ so $q_B = 4q$.

Exercise 3 *(6 pts)*



1- $\vec{T}$: tension of the string
$\vec{P}$: weight of the sphere
$\vec{F_e}$: repulsive electric force

2- The equilibrium condition reads first as a vector equality $\vec{0} = \vec{F_e} + \vec{T} + \vec{P}$ which has to be projected on frame axes. Namely here $\begin{cases} T\cos(\theta) - mg = 0 \\ T\sin(\theta) - F_e = 0 \end{cases} \Leftrightarrow \begin{cases} T\cos(\theta) = mg \\ T\sin(\theta) = \frac{kQ^2}{4L^2\sin^2(\theta)} \end{cases}$

3- a- This system implies that $\tan(\theta) = \frac{kQ^2}{4L^2\sin^2(\theta)}\frac{1}{mg}$ and then one recovers the result

$$Q = 2L\sin(\theta)\sqrt{\frac{mg\tan(\theta)}{k}}$$

b- Don't forget to convert the mass in kg and length in m ! One computes : $Q = \frac{7}{3}10^{-6}$ C

Exercise 4 *(4 pts)*

1- First I recall that the electric field is derived from potential with the following formula $\vec{E} = -\overrightarrow{grad}\, V$. Here $V(r,\theta,\varphi) = \frac{C_1}{r}\sin(\theta)e^{-C_2\varphi}$. The formula with vector notation means the following :

$$\begin{cases} E_r = -\dfrac{\partial V}{\partial r} \\ E_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial \theta} \\ E_\varphi = -\dfrac{1}{r\sin(\theta)}\dfrac{\partial V}{\partial \varphi} \end{cases} \Leftrightarrow \begin{cases} E_r = \dfrac{C_1}{r^2}\sin(\theta)e^{-C_2\varphi} \\ E_\theta = -\dfrac{C_1}{r^2}\cos(\theta)e^{-C_2\varphi} \\ E_\varphi = \dfrac{C_1 C_2}{r^2}e^{-C_2\varphi} \end{cases}$$

2- At point M whose coordinates are given one gets $\begin{cases} E_r(M) = 10 \\ E_\theta(M) = 0 \\ E_\varphi(M) = 10 \end{cases}$ so $\|\vec{E}\| = 10\sqrt{2}$ V.m^{-1}