

Archimedes' Principle

Objective: Use Archimedes' principle to measure the densities of a given solid and a provided liquid.

Theory: When an object floats in a liquid there is an upward force produced by the liquid on the object.

This force exists because of pressure in the surrounding liquid increases with depth below the surface. The upward force or buoyant force, \vec{F}_b , is in the opposite direction of the gravitational force \vec{F}_g . This action is summarized in Archimede's Principle: When a body is fully (or partially) submerged in a fluid, a buoyant force from the surrounding fluid acts on the body. The force is directed upward and has the magnitude equal to the weight $m_f g$ ($\rho_f V g$ in terms of density) of the fluid that has been displaced by the body. $m_f g = (\rho_f V g)$

$$F_b = m_f g = \rho_f V g$$

where m_f (ρ_f) is the mass (density) of the fluid that is displaced.

When a body floats in a fluid, the magnitude of the buoyant force \vec{F}_b , on the body is equal to the magnitude of the gravitational force, \vec{F}_g , on the body. Thus,

$$F_b = F_g = m_f g = \rho_f V g$$

Part I: For a solid object of volume V , heavier than water, when completely submerged in water, the buoyant force is $F_b = \rho_w V g$, where ρ_w is the density of water. The mass of the object, M , when weighed in air is $M = \rho V$ where ρ is the density of the object, therefore the volume of the object is $V = \frac{M}{\rho}$. Apparent weight is the difference between the actual weight of an object and the magnitude of the buoyant force.

$$weight_{app} = weight_{actual} - F_b$$

The apparent weight of the object is $M_w g$, and M_w is the mass of the object when totally submerged in the water. Thus the density of the object is calculated by

$$M_w g = M g - \rho_w V g$$

$$M_w = M - \rho_w V$$

$$M_w = M - \rho_w \frac{M}{\rho}$$

$$\rho = \frac{M \rho_w}{M - M_w}$$

Part II: If liquid is used instead of water, then the apparent weight of the object is $M_l g$ and M_l is the mass of the object when totally submerged in the liquid. Then, with the density of the object, ρ , and the density of the liquid represented by ρ_l , we have,

$$\rho = \frac{M \rho_l}{(M - M_l)}$$

Comparing the two expressions for ρ , the density of the liquid is found to be given by

$$\rho_l = \frac{(M - M_l) \rho_w}{(M - M_w)}$$

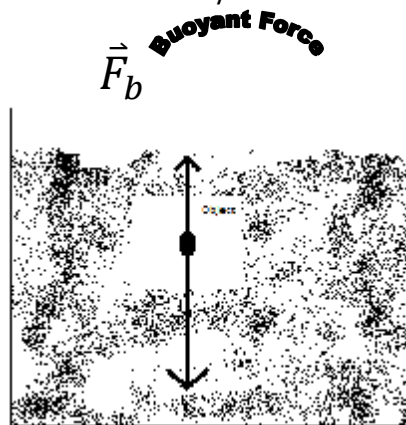
Apparatus: Solid (metal) and liquid (liquid and water) samples, graduated cylinder, beaker, thread, balance, weights, stand, micrometer, calipers

Procedure:

- Determine the density of water and liquid. Weigh 100 ml of water and 100 ml of liquid. $\rho = \frac{M}{V}$
- Weigh the cylinders of aluminum, copper, and iron. Measure the length using the vernier caliper.
- Calculate the volume. $V = \pi * L * r^2$
- Calculate the density of the three cylinders.
- Suspend the same solid cylinders from the balance, and weigh it in air and when fully submerged in water.
- Now weigh the same object when it is fully submerged in liquid.

Calculation:

- Compute the density of the solid object. Compare your answer with Aluminum ($2.7 \frac{\text{g}}{\text{cm}^3}$), Brass ($8.48 \frac{\text{g}}{\text{cm}^3}$), Copper ($8.79 \frac{\text{g}}{\text{cm}^3}$), and Steel ($7.82 \frac{\text{g}}{\text{cm}^3}$).
- Calculate the density of the liquid, ρ_l , and compare it with the value measured earlier.
- Calculate the percent of error for each of your measurements.



$$weight_{app} = weight_{actual} - F_b$$

\vec{F}_w **Weight (actual)**

