1.9.19

EE24BTECH11019 - DWARAK A

Question: Find the values of x for which the distance between the points $\mathbf{A}(x, 2)$ and $\mathbf{B}(9, 8)$ is 10 units.

Solution:

Variable	Description	Value
A	First point	$\begin{pmatrix} x \\ 2 \end{pmatrix}$
B	Second point	$\begin{pmatrix} 9 \\ 8 \end{pmatrix}$
d	Distance between **A** and **B**	10

TABLE 0: Variables Used

Distance formula :

$$\|A - B\| = d \tag{0.1}$$

$$\sqrt{\|A\|^2 - 2A^\top B + \|B\|^2} = d \tag{0.2}$$

$$\|A\|^2 - 2A^\top B + \|B\|^2 = d^2 \tag{0.3}$$

Substituting values,

$$\begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} - 2\begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} 9 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 & 8 \end{pmatrix}\begin{pmatrix} 9 \\ 8 \end{pmatrix} = 10^2 \tag{0.4}$$

$$(x^2 + 4) - 2(9x + 16) + (81 + 64) = 100 \tag{0.5}$$

$$x^2 - 18x + 17 = 0 \tag{0.6}$$

$$(x - 17)(x - 1) = 0 \tag{0.7}$$

$$x_1 = 17, x_2 = 1 \tag{0.8}$$

$$\implies A_1 = \begin{pmatrix} 17 \\ 2 \end{pmatrix}, A_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.9}$$

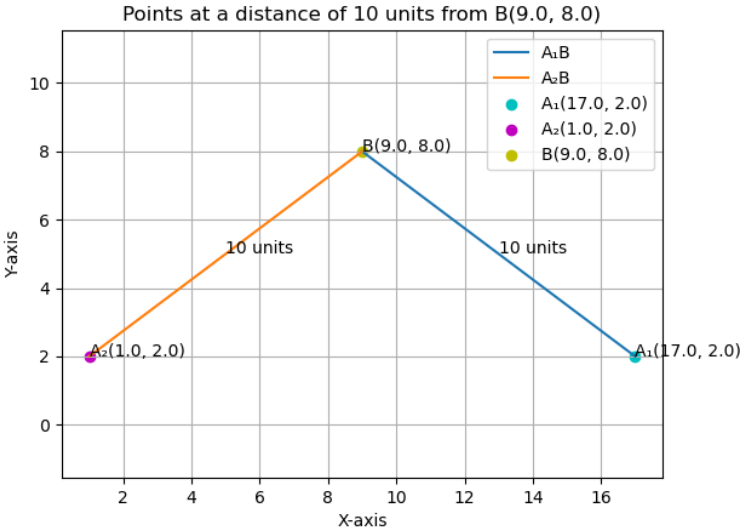

Fig. 0.1: Plot of points $\mathbf{A_1}$ and $\mathbf{A_2}$ at a distance of 10 units from $\mathbf{B}$