ASSIGNMENT-3
GATE XE-2014

EE24BTECH11019 - DWARAK A

A : ENGINEERING MATHEMATICS

Q.1 to Q.7 carry one mark each.

1) Ten chocolates are distributed randomly among three children standing in a row. The probability that the first child receives exactly three chocolates is
   a) $\frac{5\times2^{11}}{3^9}$
   b) $\frac{5\times2^{10}}{3^9}$
   c) $\frac{1}{3^9}$
   d) $\frac{1}{3}$
2) Let the function $f : [5, 0] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2x + 5, & 0 \le x < 1 \\ 2x^2 + 5, & 1 \le x < 2 \\ \frac{2}{3}x^3 + \frac{23}{3}, & 2 \le x \le 5 \end{cases}$$

   The number of points where f is not differentiable in $(0, 5)$, is _____.
3) An integrating factor of the differential equation $\left(3x^2y^3e^y + y^3 + y^2\right)dx + \left(x^3y^3e^y - xy\right)dy = 0$ is
   a) $\frac{1}{y}$
   b) $\frac{1}{y^2}$
   c) $\frac{1}{y^3}$
   d) $\ln y$
4) If a cubic polynomial passes through the points $(0, 1)$, $(1, 0)$, $(2, 1)$ and $(3, 10)$, then it also passes through the point
   a) $(-2, -11)$
   b) $(-1, -2)$
   c) $(-1, -4)$
   d) $(-2, -23)$

Q.8 to Q.11 carry two marks each.

5) Let the function $f : [0, \infty) \to \mathbb{R}$ be such that $f'(x) = \frac{8}{x^2+3x+4}$ for $x > 0$ and $f(0) = 1$. Then $f(1)$ lies in the interval

a) $[0, 1]$
b) $[2, 3]$
c) $[4, 5]$
d) $[6, 7]$

6) The perimeter of a rectangle having the largest area that can be inscribed in the ellipse $\frac{x^2}{8} + \frac{y^2}{32} = 1$, is _____.

7) If the work done in moving a particle once around a circle $x^2 + y^2 = 4$ under the force field $\mathbf{F}(x, y) = (2x - ay)\hat{i} + (2y + ax)\hat{j}$ is 16π, then $|a|$ is equal to _____.

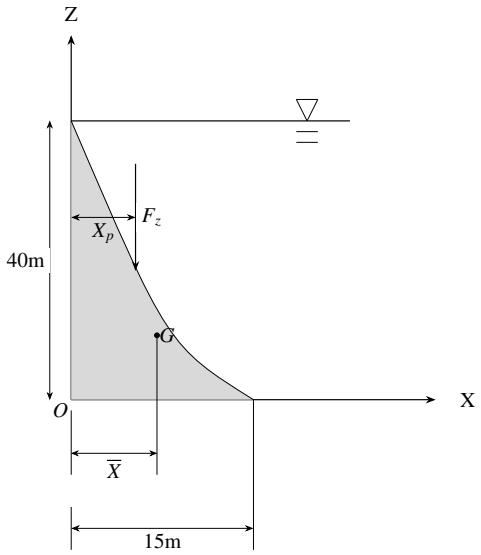
8) Let r and s be real numbers. If $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ r & s & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ s-1 \end{pmatrix}$, then the system of linear equations $AX = b$ has

a) no solutions for $s \neq 2r$.
b) infinitely many solutions for $s = 2r \neq 2$.
c) a unique solution for $s = 2r = 2$.
d) infinitely many solutions for $s = 2r = 2$.

B : FLUID MECHANICS

Q.1 to Q.9 carry one mark each.

9) A dam with a curved shape is shown in the figure. The cross sectional area of the dam (shaded portion) is $100m^2$ and its centroid is at $\bar{x} = 10m$. The vertical component of the hydrostatic force, F_z, is acting at a distance x_p. The value of x_p is _____m.



10) For an unsteady incompressible fluid flow, the velocity field is $\mathbf{V} = \left(3x^2 + 3\right)t\,\hat{i} - 6xyt\,\hat{j}$, where x, y are in meters and t is in seconds. Acceleration in m/s^2 at the point $x = 10m$ and $y = 0$, as measured by a stationary observer is

a) 303
b) 162
c) 43
d) 13

11) For an incompressible flow, the existence of components of acceleration for different types of flow is described in the table below.

Type of Flow	Components of Acceleration
P: Steady and uniform	1: Local exists, convective does not exist
Q: Steady and non-uniform	2: Both exist
R: Unsteady and uniform	3: Both do not exist
S: Unsteady and non-uniform	4: Local does not exist, convective exists

Which one of the following options connecting the left column with the right column is correct?

a) P-1; Q-4; R-3; S-2
b) P-4; Q-1; R-2; S-3
c) P-3; Q-2; R-1; S-4
d) P-3; Q-4; R-1; S-2

12) Velocity in a two-dimensional flow field is specified as $u = x^2y; v = -y^2x$. The magnitude of the rate of angular deformation at a location ($x = 2m$ and $y = 1m$) is _____s^{-1}.

13) For a plane irrotational flow, equi-potential lines and streamlines are

a) parallel to each other.
b) at an angle of 90° to each other.
c) at an angle of 45° to each other.
d) at an angle of 60° to each other.