ASSIGNMENT - 2

EE24BTECH11019 - DWARAK A

SECTION-A — JEE ADVANCED / IIT-JEE

E - SUBJECTIVE PROBLEMS

1) Let $a > 0$, $d > 0$. Find the value of the determinant

(1996 - 5 Marks)

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

2) Prove that for all values of θ ,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

(2000 - 3 Marks)

3) If matrix $\mathbf{A} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $\mathbf{A}^\top\mathbf{A} = \mathbf{I}$, then find the value of $a^3 + b^3 + c^3$.

(2003 - 2 Marks)

4) If $\mathbf{M}$ is a 3×3 matrix, where det $\mathbf{M} = 1$ and $\mathbf{M}\mathbf{M}^\top = \mathbf{I}$, where '$\mathbf{I}$' is an identity matrix, prove that $\det(\mathbf{M} - \mathbf{I}) = 0$.

(2004 - 2 Marks)

5) If $\mathbf{A} = \begin{pmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{pmatrix}$, $\mathbf{U} = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$, $\mathbf{V} = \begin{pmatrix} a^2 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\mathbf{AX} = \mathbf{U}$ has infinitely many solutions, prove that $\mathbf{BX} = \mathbf{V}$ has no unique solution. Also show that if $afd \neq 0$, then $\mathbf{BX} = \mathbf{V}$ has no solution.

F - MATCH THE FOLLOWING

1) Consider the lines given by $L_1 : x + 3y - 5 = 0; L_2 : 3x - ky - 1 = 0; L_3 : 5x + 2y - 12 = 0$
Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

(2008)

Column I	Column II
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = 9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = \frac{-6}{5}$
(C) L_1, L_2, L_2 from a triangle, if	(r) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

2) Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

(2008)

Column I	Column II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let **A** and B be 3×3 matrices of real numbers, where **A** is symmetric, **B** is skew-symmetric and $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = (\mathbf{A}-\mathbf{B})(\mathbf{A}+\mathbf{B})$. If $(\mathbf{AB})^\top = (-1)^k \mathbf{AB}$, where $(\mathbf{AB})^\top$ is the transpose of the matrix **AB**, then the possible values of k are	(q) 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{\left(-k+3^{-a}\right)} < 2$, must be less than	(r) 2
(D) If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) are$	(s) 3

G - COMPREHENSION BASED QUESTIONS

PASSAGE - 1

Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ and $\mathbf{U_1}$, $\mathbf{U_2}$ and $\mathbf{U_3}$ are columns of a 3×3 matrix **U**. If column matrices $\mathbf{U_1}$, $\mathbf{U_2}$ and $\mathbf{U_3}$ satisfying $\mathbf{AU_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{AU_2} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$, $\mathbf{AU_3} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ evaluate as directed in the following questions.

1) The value $|\mathbf{U}|$ is

(2006 - 5M, −2)

a) 3
b) −3
c) $\frac{3}{2}$
d) 2

2) The sum of the elements of the matrix $\mathbf{U^{-1}}$ is

(2006 - 5M, −2)

a) −1
b) 0
c) 1
d) 3

3) The value of $\begin{pmatrix} 3 & 2 & 0 \end{pmatrix} \mathbf{U} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ is

(2006 - 5M, −2)

a) 5
b) $\frac{5}{2}$
c) 4
d) $\frac{3}{2}$

PASSAGE - 2

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

1) The number of matrices in $\mathcal{A}$ is

(2009)

a) less than 4
b) at least 4 but less than 7
c) at least 7 but less than 10
d) at least 10

2) The number of matrices **A** in $\mathcal{A}$ for which the system of linear equations

$$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

is inconsistent, is

(2009)

a) 0
b) more than 2
c) 2
d) 1

PASSAGE - 3

Let p be an odd prime number and $\mathbf{T_p}$ be the following set of 2×2 matrices :

$$\mathbf{T_p} = \left\{ \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$

(2010)

1) The number of **A** in $\mathbf{T_p}$ such that **A** is either symmetric or skew-symmetric or both, and det(**A**) divisibly by p is

a) $(p-1)^2$
b) $2(p-1)$
c) $(p-1)^2 + 1$
d) $2p - 1$

2) The number of **A** in $\mathbf{T_p}$ such that the trace of **A** is not divisible by p but det(**A**) is divisible by p is
[**Note:** The trace of a matrix is the sum of its diagonal entries.]
   a) $(p-1)\left(p^2-p+1\right)$
   b) $p^3-(p-1)^2$
   c) $(p-1)^2$
   d) $(p-1)\left(p^2-2\right)$

3) The number of **A** in $\mathbf{T_p}$ such that det(**A**) is not divisible by p is
   a) $2p^2$
   b) p^3-5p
   c) p^3-3p
   d) p^3-p^2