

# 4.4.33

EE24BTECH11019 - DWARAK A

**Question:** Find the value of  $x$  such that the four points with position vectors  $\mathbf{A}(3\hat{i} + 2\hat{j} + \hat{k})$ ,  $\mathbf{B}(4\hat{i} + x\hat{j} + 5\hat{k})$ ,  $\mathbf{C}(4\hat{i} + 2\hat{j} - 2\hat{k})$ , and  $\mathbf{D}(6\hat{i} + 5\hat{j} - \hat{k})$  are coplanar.

**Solution:**

Symbol	Description	Value
<b>A</b>	Coordinates of Point <b>A</b>	$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$
<b>B</b>	Coordinates of Point <b>B</b>	$\begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix}$
<b>C</b>	Coordinates of Point <b>C</b>	$\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$
<b>D</b>	Coordinates of Point <b>D</b>	$\begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix}$

TABLE 0: Variables Used

Plane Equation,

$$\mathbf{n}^\top \mathbf{x} = 1 \quad (0.1)$$

If **A**, **C**, **D** are coplanar

$$(\mathbf{A} \quad \mathbf{C} \quad \mathbf{D})^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (0.2)$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 2 & -2 \\ 6 & 5 & -1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (0.3)$$

Augmented Matrix,

$$\begin{pmatrix} 3 & 2 & 1 & 1 \\ 4 & 2 & -2 & 1 \\ 6 & 5 & -1 & 1 \end{pmatrix} \xleftrightarrow{R_1 \rightarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 4 & 2 & -2 & 1 \\ 6 & 5 & -1 & 1 \end{pmatrix} \quad (0.4)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 - 6R_1} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 4 & 2 & -2 & 1 \\ 0 & 1 & -3 & -1 \end{pmatrix} \quad (0.5)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - 4R_1} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{2}{3} & -\frac{10}{3} & -\frac{1}{3} \\ 0 & 1 & -3 & -1 \end{pmatrix} \quad (0.6)$$

$$\xleftrightarrow{R_2 \rightarrow -\frac{3R_2}{2}} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 1 & -3 & -1 \end{pmatrix} \quad (0.7)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 - R_2} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & -8 & -\frac{3}{2} \end{pmatrix} \quad (0.8)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 - \frac{2R_2}{3}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & -8 & -\frac{3}{2} \end{pmatrix} \quad (0.9)$$

$$\xleftrightarrow{R_3 \rightarrow -\frac{R_3}{8}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \quad (0.10)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 + 3R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{16} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \quad (0.11)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - 5R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{16} \\ 0 & 1 & 0 & -\frac{7}{16} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \quad (0.12)$$

$$\mathbf{n} = \begin{pmatrix} \frac{9}{16} \\ -\frac{7}{16} \\ \frac{3}{16} \end{pmatrix} \quad (0.13)$$

$$\mathbf{n}^\top B = 1 \quad (0.14)$$

$$(9 \quad -7 \quad 3) \begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix} = 16 \quad (0.15)$$

$$36 - 7x + 15 = 16 \quad (0.16)$$

$$7x = 35 \quad (0.17)$$

$$x = 5 \quad (0.18)$$

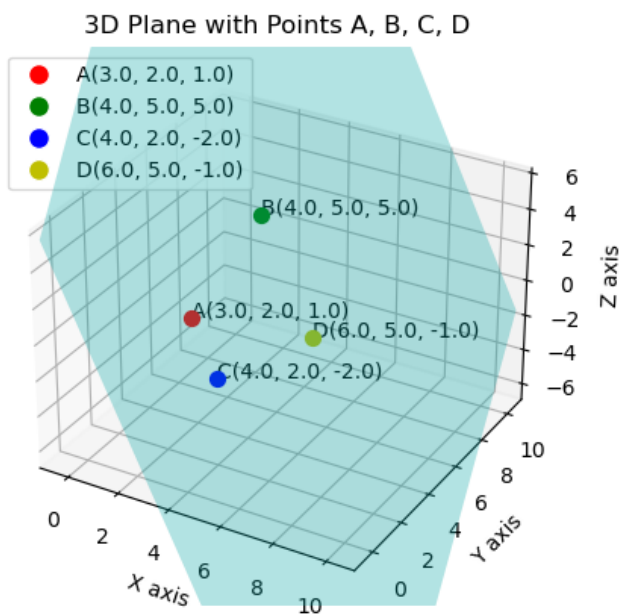


Fig. 0.1: Plot of the plane with points A, B, C and D