

# ASSIGNMENT-3

## GATE XE-2014

EE24BTECH11019 - DWARAK A

A : ENGINEERING MATHEMATICS

*Q.1 to Q.7 carry one mark each.*

- 1) Ten chocolates are distributed randomly among three children standing in a row. The probability that the first child receives exactly three chocolates is

- a)  $\frac{5 \times 2^{11}}{3^9}$
- b)  $\frac{5 \times 2^{10}}{3^9}$
- c)  $\frac{1}{3^9}$
- d)  $\frac{1}{3}$

- 2) Let the function  $f : [5, 0] \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} 2x + 5, & 0 \leq x < 1 \\ 2x^2 + 5, & 1 \leq x < 2 \\ \frac{2}{3}x^3 + \frac{23}{3}, & 2 \leq x \leq 5 \end{cases}$$

The number of points where  $f$  is not differentiable in  $(0, 5)$ , is \_\_\_\_\_.

- 3) An integrating factor of the differential equation  $(3x^2y^3e^y + y^3 + y^2)dx + (x^3y^3e^y - xy)dy = 0$  is

- a)  $\frac{1}{y}$
- b)  $\frac{1}{y^2}$
- c)  $\frac{1}{y^3}$
- d)  $\ln y$

- 4) If a cubic polynomial passes through the points  $(0, 1)$ ,  $(1, 0)$ ,  $(2, 1)$  and  $(3, 10)$ , then it also passes through the point

- a)  $(-2, -11)$
- b)  $(-1, -2)$
- c)  $(-1, -4)$
- d)  $(-2, -23)$

*Q.8 to Q.10 carry two marks each.*

- 5) Let the function  $f : [0, \infty) \rightarrow \mathbb{R}$  be such that  $f'(x) = \frac{8}{x^2 + 3x + 4}$  for  $x > 0$  and  $f(0) = 1$ . Then  $f(1)$  lies in the interval

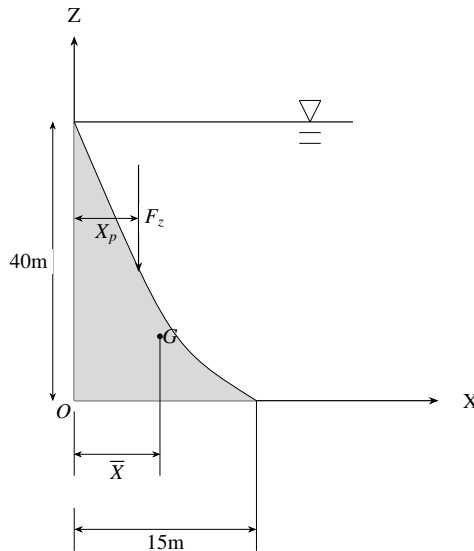
- a)  $[0, 1]$
- b)  $[2, 3]$
- c)  $[4, 5]$
- d)  $[6, 7]$

- 6) The perimeter of a rectangle having the largest area that can be inscribed in the ellipse  $\frac{x^2}{8} + \frac{y^2}{32} = 1$ , is \_\_\_\_\_.
- 7) If the work done in moving a particle once around a circle  $x^2 + y^2 = 4$  under the force field  $\mathbf{F}(x, y) = (2x - ay)\hat{i} + (2y + ax)\hat{j}$  is  $16\pi$ , then  $|a|$  is equal to \_\_\_\_\_.
- 8) Let  $r$  and  $s$  be real numbers. If  $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ r & s & 0 \end{pmatrix}$  and  $b = \begin{pmatrix} 1 \\ 1 \\ s - 1 \end{pmatrix}$ , then the system of linear equations  $AX = b$  has
- a) no solutions for  $s \neq 2r$ .
  - b) infinitely many solutions for  $s = 2r \neq 2$ .
  - c) a unique solution for  $s = 2r = 2$ .
  - d) infinitely many solutions for  $s = 2r = 2$ .

## B : FLUID MECHANICS

*Q.1 to Q.9 carry one mark each.*

- 9) A dam with a curved shape is shown in the figure. The cross sectional area of the dam (shaded portion) is  $100\text{m}^2$  and its centroid is at  $\bar{x} = 10\text{m}$ . The vertical component of the hydrostatic force,  $F_z$ , is acting at a distance  $x_p$ . The value of  $x_p$  is \_\_\_\_\_m.



- 10) For an unsteady incompressible fluid flow, the velocity field is  $\mathbf{V} = (3x^2 + 3)t\hat{i} - 6xyt\hat{j}$ , where  $x, y$  are in meters and  $t$  is in seconds. Acceleration in  $\text{m/s}^2$  at the point  $x = 10\text{m}$  and  $y = 0$ , as measured by a stationary observer is

- a) 303
- b) 162
- c) 43
- d) 13

11) For an incompressible flow, the existence of components of acceleration for different types of flow is described in the table below.

Type of Flow	Components of Acceleration
P: Steady and uniform	1: Local exists, convective does not exist
Q: Steady and non-uniform	2: Both exist
R: Unsteady and uniform	3: Both do not exist
S: Unsteady and non-uniform	4: Local does not exist, convective exists

Which one of the following options connecting the left column with the right column is correct?

- a) P-1; Q-4; R-3; S-2
- b) P-4; Q-1; R-2; S-3
- c) P-3; Q-2; R-1; S-4
- d) P-3; Q-4; R-1; S-2

12) Velocity in a two-dimensional flow field is specified as  $u = x^2y$ ;  $v = -y^2x$ . The magnitude of the rate of angular deformation at a location ( $x = 2m$  and  $y = 1m$ ) is \_\_\_\_\_  $s^{-1}$ .

13) For a plane irrotational flow, equi-potential lines and streamlines are

- a) parallel to each other.
- b) at an angle of  $90^\circ$  to each other.
- c) at an angle of  $45^\circ$  to each other.
- d) at an angle of  $60^\circ$  to each other.