JEE MAINS 2020

EE24BTECH11019 - Dwarak A

JANUARY 7 SHIFT-2

1) If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{9}\right)=1$ for some $a \in \mathbb{R}$, then the distance between the foci of the ellipse is:
   a) $2\sqrt{5}$
   b) $2\sqrt{7}$
   c) $2\sqrt{2}$
   d) 4

2) Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If A $\nsubseteq$ B and B $\nsubseteq$ D then A $\nsubseteq$ C" is :
   a) If A $\subseteq$ C, then B $\subset$ A or D $\subset$ B
   b) If A $\nsubseteq$ C, then A $\subseteq$ B and B $\subseteq$ D
   c) If A $\nsubseteq$ C, then A $\nsubseteq$ B and B $\subseteq$ D
   d) If A $\nsubseteq$ C, then A $\nsubseteq$ B or B $\nsubseteq$ D

3) The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+\cdots+x^{10}$ is :
   a) 420
   b) 330
   c) 210
   d) 120

4) In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to:
   a) $\frac{17}{2}$
   b) 4
   c) $\frac{17}{4}$
   d) $\frac{17}{8}$

5) The locus of mid points of the perpendiculars drawn from points on the line $x = 2y$ to the line $x = y$ is:
   a) $2x - 3y = 0$
   b) $3x - 2y = 0$
   c) $5x - 7y = 0$
   d) $7x - 5y = 0$

6) The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx = 5$ is:
   a) $\log_e 2$
   b) $\log_e \sqrt{2}$
   c) $\log_e\left(\frac{4}{3}\right)$
   d) $\log_e\left(\frac{3}{2}\right)$

7) If the sum of the first 40 terms of the series, $3+4+8+9+13+14+18+19+\ldots$ is $(102)m$, then m is equal to :
   a) 10
   b) 25
   c) 5
   d) 20

8) If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta \in [0, 2\pi]$ is a real number, then the argument of $\sin\theta + i\cos\theta$ is :
   a) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
   b) $\tan^{-1}\left(\frac{4}{3}\right)$
   c) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
   d) $\tan^{-1}\left(\frac{3}{4}\right)$

9) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} - (3)^{(i+j-2)}a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :
   a) $\frac{1}{9}$
   b) $\frac{1}{81}$
   c) $\frac{1}{3}$
   d) 3

10) Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$ then which of the following is not true ?
   a) $f(1) - 4f(-1) = 4$
   b) $x = 1$ is a point of maxima and $x = -1$ is a point of minima of f.
   c) f is an odd function
   d) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

11) The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :
   a) 4
   b) 6
   c) 2

d) 3

12) Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$ then λ is equal to :

a) 171
b) $\frac{511}{3}$
c) -171
d) -513

13) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$. If $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair $\left(\lambda, \vec{d}\right)$ is equal to:

a) $\left(\frac{3}{2}, 3\mathbf{a} \times \mathbf{c}\right)$
b) $\left(\frac{-3}{2}, 3\mathbf{c} \times \mathbf{b}\right)$
c) $\left(\frac{-3}{2}, 3\mathbf{a} \times \mathbf{b}\right)$
d) $\left(\frac{3}{2}, 3\mathbf{b} \times \mathbf{c}\right)$

14) Let $y = y(x)$ be the solution curve of the differential equation $(y^2 - x)\left(\frac{dy}{dx}\right) = 1$ satisfying $y(0) = 1$ This curve intersects the x-axis at a point whose abscissa is :

a) $2 + e$
b) 2
c) $2 - e$
d) $-e$

15) If θ_1 and θ_2 be respectively the smallest and largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to:

a) $\frac{2\pi}{3}$
b) $\frac{\pi}{3}$
c) $\left(\frac{\pi}{3}\right) + \left(\frac{1}{6}\right)$
d) $\frac{\pi}{9}$