ASSIGNMENT-1
GATE XE-2007

EE24BTECH11019 - Dwarak A

Q.1-Q.6 CARRY ONE MARK EACH.

1) Let $M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Then the maximum number of linearly independent eigenvectors of M is
   a) 0
   b) 1
   c) 2
   d) 3
2) Let $L = \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 2x}{\left(x - \frac{\pi}{2}\right)^2}$. Then L is equal to
   a) -4
   b) 0
   c) 2
   d) 4
3) Let $f(z) = \frac{1}{1-z^2}$. The coefficient $\frac{1}{z-1}$ in the Laurent expansion of $f(z)$ about $z = 1$ is
   a) -1
   b) $-\frac{1}{2}$
   c) $\frac{1}{2}$
   d) 1
4) Let $u(x,t)$ be the solution of the initial value problem
   $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, t > 0, -\infty < x < \infty,$
   $u(x,0) = x + 5,$
   $\frac{\partial u}{\partial t}(x,0) = 0.$
   Then $u(2,2)$ is
   a) 7
   b) 13
   c) 14
   d) 26
5) Two students take a test consisting of five TRUE/FALSE questions. To pass the test the students the students have to answer at least three questions correctly. Both of them know the correct answers to two questions and guess the answers to the remaining three. The probability that only one student passes the test is equal to
   a) $\frac{6}{32}$
   b) $\frac{7}{32}$
   c) $\frac{1}{4}$
   d) $\frac{3}{4}$
6) The equation $g(x) = x$ is solved by Newton-Raphson iteration method, starting with an initial approximation x_0 near the simple root α. If x_{n+1} is the approximation to α at the $(n+1)^{th}$ iteration, then
   a) $x_{n+1} = \frac{x_n g'(x_n) - g(x_n)}{1 - g'(x_n)}$
   b) $x_{n+1} = \frac{x_n g'(x_n) - g(x_n)}{g'(x_n) - 1}$
   c) $x_{n+1} = g(x_n)$
   d) $x_{n+1} = \frac{x_n g'(x_n) - g(x_n) + 2x_n}{g'(x_n) + 1}$

Q.7-Q.24 CARRY TWO MARKS EACH.

7) Let $Ax = b$ be a system of m linear equations in n unknowns with $m < n$ and $b \neq 0$. Then the system has
   a) $n - m$ solutions
   b) either zero or infinitely many solutions
   c) exactly one solution
   d) n solutions
8) Let R be an $n \times n$ nonsingular matrix. Let P and Q be two $n \times n$ matrices that $Q = R^{-1}PR$. If x is an eigenvector of P corresponding to a nonzero eigenvalue λ of P, then
   a) Rx is an eigenvector of Q corresponding to the eigenvalue λ of Q
   b) Rx is an eigenvector of Q corresponding to the eigenvalue $\frac{1}{\lambda}$ of Q
   c) $R^{-1}x$ is an eigenvector of Q corresponding to the eigenvalue $\frac{1}{\lambda}$ of Q
   d) $R^{-1}x$ is an eigenvector of Q corresponding to the eigenvalue λ of Q
9) Let M be a 2×2 matrix with eigenvalues 1 and 2. Then M^{-1} is

a) $\frac{M-3I}{2}$
b) $\frac{3I+M}{2}$
c) $\frac{3I-M}{2}$
d) $\frac{-M-3I}{2}$

10) The number $n \times n$ matrices that are simultaneously Hermitian, unitary and diagonal is
a) 2^n
b) n^2
c) $2n$
d) 2

11) Let $M = \begin{pmatrix} 1 & b & a \\ 0 & 2 & c \\ 0 & 0 & 1 \end{pmatrix}$, where a, b, c are real numbers. Then M is diagonalizable
a) for all values of a, b, c
b) only when $bc \neq a$
c) only when $b + c = a$
d) only when $bc = a$

12) The maximum value of the function $2x+3y+4z$ on the ellipsoid $2x^2 + 3y^2 + 4z^2 = 1$ is
a) 2
b) 3
c) 6
d) 9

13) Let $f : \mathcal{R} \to \mathcal{R}$ be a twice differentiable real valued function such that $f\left(\frac{1}{n}\right) = 1$ for $n = 1, 2, 3 \ldots.$ Then
a) $f'(0) = 0$
b) $f'(0) = 1$
c) $0 < f'(0) < 1$
d) $f'(0) > 1$

14) Let $f(x) = \int_0^{x^2} \sin \sqrt{t} dt$ for $x \geq 0$. Then $f'\left(\frac{\pi}{2}\right)$ is equal to
a) 0
b) π
c) 1
d) $\frac{\pi}{2}$

15) The value of the contour integral $\oint_{|z|=1} \frac{\cosh z}{4z^2+1} dz$ is equal to
a) $2\pi \cosh\left(\frac{i}{2}\right)$
b) $\pi \cosh\left(\frac{i}{2}\right)$
c) 0
d) $2\pi i$

16) Let $f(x + iy) = u(x, y) + iv(x, y)$ be an analytic function defined on the complex plane satisfying $2u^2 + 3v^2 = 1$. Then
a) f is a constant
b) $f(z) = kz$ for some nonzero real number k
c) $u(x, y) = \frac{\cos(x+y)}{\sqrt{2}}$
d) $v(x, y) = \frac{\sin(x-y)}{\sqrt{3}}$

17) The value of $\oint_C \left(xy^2 + 2x\right) dx + \left(x^2 y + 4x\right) dy$ along the circle $C : x^2 + y^2 = 4$ in the anticlockwise direction is
a) -16π
b) -4π
c) 4π
d) 16π