1.8.11

EE24BTECH11019 - DWARAK A

Question:
Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.
Solution:

Variable	**Description**	**Value**
$\mathbf{A}$	First point	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$
$\mathbf{B}$	Second point	$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$
$\mathbf{C}$	Mid-point of $\mathbf{A}$ and $\mathbf{B}$	$\left(\frac{\mathbf{A}+\mathbf{B}}{2}\right)$
$\mathbf{X}$	Set of points equidistant from $\mathbf{A}$ and $\mathbf{B}$	$\begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 0: Variables Used

If $\mathbf{X}$ is equidistant form the points $\mathbf{A}$ and $\mathbf{B}$

$$\|\mathbf{A} - \mathbf{X}\| = \|\mathbf{B} - \mathbf{X}\| \tag{0.1}$$

$$\|\mathbf{A} - \mathbf{X}\|^2 = \|\mathbf{B} - \mathbf{X}\|^2 \tag{0.2}$$

$$\|\mathbf{A}\|^2 - 2\mathbf{A}^\top\mathbf{X} + \|\mathbf{X}\|^2 = \|\mathbf{B}\|^2 - 2\mathbf{B}^\top\mathbf{X} + \|\mathbf{X}\|^2 \tag{0.3}$$

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{X} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{0.4}$$

The above equation is the general expression for the perpendicular bisecting plane between any points $\mathbf{A}$ and $\mathbf{B}$.
Substituting the $\mathbf{A}$ and $\mathbf{B}$ values in the derived equation.

$$\begin{pmatrix} 6 \\ 2 \end{pmatrix}^\top \mathbf{X} = \frac{\left\|\begin{pmatrix} 3 \\ 6 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} -3 \\ 4 \end{pmatrix}\right\|^2}{2} = \frac{20}{2} = 10 \tag{0.5}$$

Comparing with $n^\top x = c$

$$n = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \tag{0.6}$$

$$c = 10 \tag{0.7}$$

Line equation :

$$6x + 2y = 10 \tag{0.8}$$

Final line equation :

$$3x + y = 5 \tag{0.9}$$

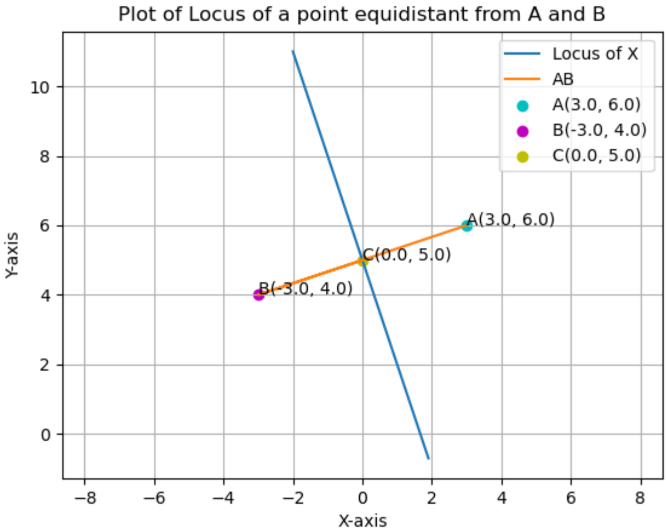

Fig. 0.1: Plot of point **X**, equidistant from **A** and **B**