10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
Solution:

Variable	Description
$x(0)$	First term of the AP
d	Common difference of the AP
$y(n)$	Sum of $n+1$ terms of the AP
$x(n)$	General term

TABLE 0: Variables Used

$$y(n) = \frac{n+1}{2}\left(2x(0) + nd\right) u(n) \tag{0.1}$$

$$y(6) = 49 \tag{0.2}$$

$$y(16) = 289 \tag{0.3}$$

Then,

$$x(0) + 3d = 7 \tag{0.4}$$

$$x(0) + 8d = 17 \tag{0.5}$$

From equations 0.4 and 0.5, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{0.6}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \tag{0.7}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{0.8}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.9}$$

$$x(n) = (1 + 2n)\, u(n) \tag{0.10}$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{\left(1 - z^{-1}\right)^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{0.11}$$

$$y(n) = x(n) * u(n) \tag{0.12}$$

$$Y(z) = X(z)\ U(z) \tag{0.13}$$

$$\implies Y(z) = \left(\frac{1}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{0.14}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \tag{0.15}$$

$$(n+1)\,u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})^2}\ \{z \in \mathbb{C} : |z| > 1\} \tag{0.16}$$

$$n\,((n+1)\,u(n)) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-1}}{(1-z^{-1})^3}\ \{z \in \mathbb{C} : |z| > 1\} \tag{0.17}$$

From equations (**??**) and (**??**), taking the inverse Z Transform,

$$y(n) = (n+1)\,u(n) + n\,((n+1)\,u(n)) \tag{0.18}$$

$$\implies y(n) = (n+1)^2\ u(n) \tag{0.19}$$

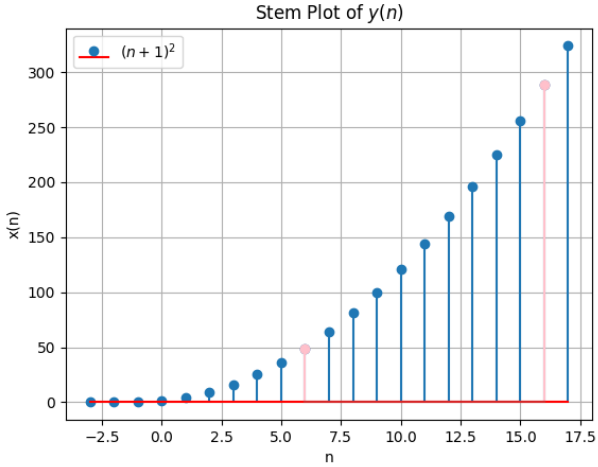

Fig. 0.1: Stem Plot of y(n)