JEE MAIN 2020
JANUARY 7, SHIFT-2

EE24BTECH11019

SECTION-A

1) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} - (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

   [Jan 2020]

   a) $\frac{1}{3}$
   b) $\frac{1}{9}$
   c) $\frac{1}{81}$
   d) 3

2) The locus of mid points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :

   [Jan 2020]

   a) $3x - 2y = 0$
   b) $2x - 3y = 0$
   c) $7x - 5y = 0$
   d) $5x - 7y = 0$

3) Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(\mathbf{AB})^2$ is equal to:

   [Jan 2020]

   a) $\frac{32}{5}$
   b) $\frac{52}{5}$
   c) $\frac{56}{5}$
   d) $\frac{64}{5}$

4) Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

   [Jan 2020]

   a) If $A \nsubseteq C$, then $A \nsubseteq B$ or $B \subseteq D$
   b) If $A \nsubseteq C$, then $A \nsubseteq B$ and $B \nsubseteq D$
   c) If $A \nsubseteq C$, then $A \subseteq B$ and $B \subseteq D$
   d) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

5) Let $y = y(x)$ be the solution curve of the differential equation $\left(y^2 - x\right) \frac{dy}{dx} = 1$ satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is :

   [Jan 2020]

   a) 2
   b) $2 + e$
   c) $2 - e$
   d) $-e$

6) If θ_1 and θ_2 be respectively the smallest and largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to :

   [Jan 2020]

   a) $\frac{\pi}{3} + \frac{1}{6}$
   b) $\frac{\pi}{9}$
   c) $\frac{\pi}{3}$
   d) $\frac{2\pi}{3}$

7) If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots$ is $(102)m$, then m is equal to :

   [Jan 2020]

   a) 25
   b) 20
   c) 10
   d) 5

8) The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

   [Jan 2020]

   a) 6
   b) 4
   c) 3
   d) 2

9) The value of α for which $4\alpha \int\limits_{-1}^{2} e^{-\alpha|x|} \, dx = 5$ is:

   [Jan 2020]

   a) $\log_e\left(\frac{4}{3}\right)$
   b) $\log_e 2$
   c) $\log_e \sqrt{2}$
   d) $\log_e\left(\frac{3}{2}\right)$

10) Let $f(x)$ be a polynomial of degree 5 such that

$x = \pm 1$ are its critical points. If $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$ then which of the following is not true ?

[Jan 2020]

a) f is an odd function
b) $x = 1$ is a point of maxima and $x = -1$ is a point of minima of f.
c) $f(1) - 4f(-1) = 4$
d) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

11) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. If $\lambda = \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$ and $\mathbf{d} = \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$, then the ordered pair $(\lambda, \mathbf{d})$ is equal to :

[Jan 2020]

a) $\left(-\frac{3}{2}, 3\mathbf{a} \times \mathbf{b}\right)$
b) $\left(\frac{3}{2}, 3\mathbf{a} \times \mathbf{c}\right)$
c) $\left(-\frac{3}{2}, 3\mathbf{c} \times \mathbf{b}\right)$
d) $\left(\frac{3}{2}, 3\mathbf{b} \times \mathbf{c}\right)$

12) The coefficient of x^7 in the expression $(1+x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \cdots + x^{10}$ is :

[Jan 2020]

a) 120
b) 210
c) 330
d) 420

13) Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k, k \geq 1$, then which one of the following statements is not true ?

[Jan 2020]

a) $p_5 = 11$
b) $p_3 = p_5 - p_4$
c) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
d) $p_5 = p_2 \cdot p_3$

14) The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is:

[Jan 2020]

a) $\frac{4 - \sqrt{7}}{3}$
b) $\frac{\sqrt{7} - 2}{3}$
c) $\frac{4 - \sqrt{5}}{3}$
d) $\frac{2}{3}$

15) The area (in sq. units) of the region $\left\{(x, y) \in \mathbb{R}^2 | 4x^2 \leq y \leq 8x + 12\right\}$ is :

[Jan 2020]

a) $\frac{124}{3}$
b) $\frac{125}{3}$
c) $\frac{127}{3}$
d) $\frac{128}{3}$