4.4.33

EE24BTECH11019 - DWARAK A

Question: Find the value of x such that the four points with position vectors $\mathbf{A}(3\hat{i} + 2\hat{j} + \hat{k})$, $\mathbf{B}(4\hat{i} + x\hat{j} + 5\hat{k})$, $\mathbf{C}(4\hat{i} + 2\hat{j} - 2\hat{k})$, and $\mathbf{D}(6\hat{i} + 5\hat{j} - \hat{k})$ are coplanar.

Solution:

Symbol	Description	Value
A	Coordinates of Point **A**	$\begin{pmatrix}3\\2\\1\end{pmatrix}$
B	Coordinates of Point **B**	$\begin{pmatrix}4\\x\\5\end{pmatrix}$
C	Coordinates of Point **C**	$\begin{pmatrix}4\\2\\-2\end{pmatrix}$
D	Coordinates of Point **D**	$\begin{pmatrix}6\\5\\-1\end{pmatrix}$

TABLE 0: Variables Used

Plane Equation,

$$\mathbf{n}^\top x = 1 \tag{0.1}$$

If $\mathbf{A}, \mathbf{C}, \mathbf{D}$ are coplanar

$$\begin{pmatrix} A & C & D \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.2}$$

$$\begin{pmatrix}3 & 2 & 1\\4 & 2 & -2\\6 & 5 & -1\end{pmatrix}\mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.3}$$

Augmented Matrix,

$$\begin{pmatrix}3 & 2 & 1 & 1\\4 & 2 & -2 & 1\\6 & 5 & -1 & 1\end{pmatrix} \overset{R_1 \to \frac{1}{3}R_1}{\longleftrightarrow} \begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\4 & 2 & -2 & 1\\6 & 5 & -1 & 1\end{pmatrix} \tag{0.4}$$

$$\overset{R_3 \to R_3 - 6R_1}{\longleftrightarrow} \begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\4 & 2 & -2 & 1\\0 & 1 & -3 & -1\end{pmatrix} \tag{0.5}$$

$$\overset{R_2\to R_2-4R_1}{\longleftrightarrow}\begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & -\frac{2}{3} & -\frac{10}{3} & -\frac{1}{3}\\ 0 & 1 & -3 & -1\end{pmatrix} \tag{0.6}$$

$$\overset{R_2\to -\frac{3R_2}{2}}{\longleftrightarrow}\begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & 1 & 5 & \frac{1}{2}\\ 0 & 1 & -3 & -1\end{pmatrix} \tag{0.7}$$

$$\overset{R_3\to R_3-R_2}{\longleftrightarrow}\begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & 1 & 5 & \frac{1}{2}\\ 0 & 0 & -8 & -\frac{3}{2}\end{pmatrix} \tag{0.8}$$

$$\overset{R_1\to R_1-\frac{2R_2}{3}}{\longleftrightarrow}\begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 1 & 5 & \frac{1}{2}\\ 0 & 0 & -8 & -\frac{3}{2}\end{pmatrix} \tag{0.9}$$

$$\overset{R_3\to -\frac{R_3}{8}}{\longleftrightarrow}\begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 1 & 5 & \frac{1}{2}\\ 0 & 0 & 1 & \frac{3}{16}\end{pmatrix} \tag{0.10}$$

$$\overset{R_1\to R_1+3R_3}{\longleftrightarrow}\begin{pmatrix}1 & 0 & 0 & \frac{9}{16}\\ 0 & 1 & 5 & \frac{1}{2}\\ 0 & 0 & 1 & \frac{3}{16}\end{pmatrix} \tag{0.11}$$

$$\overset{R_2\to R_2-5R_3}{\longleftrightarrow}\begin{pmatrix}1 & 0 & 0 & \frac{9}{16}\\ 0 & 1 & 0 & -\frac{7}{16}\\ 0 & 0 & 1 & \frac{3}{16}\end{pmatrix} \tag{0.12}$$

$$\mathbf{n}=\begin{pmatrix}\frac{9}{16}\\ -\frac{7}{16}\\ \frac{3}{16}\end{pmatrix} \tag{0.13}$$

$$\mathbf{n}^{\top}B=1 \tag{0.14}$$

$$\begin{pmatrix}9 & -7 & 3\end{pmatrix}\begin{pmatrix}4\\ x\\ 5\end{pmatrix}=16 \tag{0.15}$$

$$36-7x+15=16 \tag{0.16}$$

$$7x=35 \tag{0.17}$$

$$x=5 \tag{0.18}$$

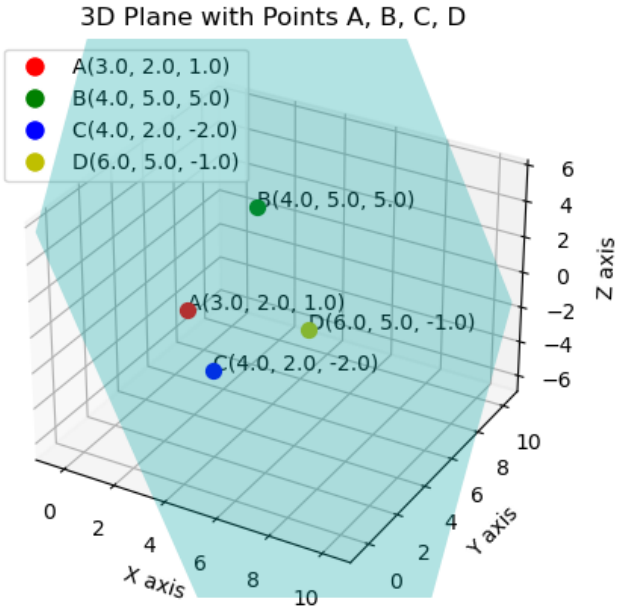

Fig. 0.1: Plot of the plane with points A, B, C and D