

ASSIGNMENT - 2

EE24BTECH11019 - DWARAK A

SECTION-A — JEE ADVANCED / IIT-JEE

E - SUBJECTIVE PROBLEMS

- 1) Let $a > 0, d > 0$. Find the value of the determinant

(1996 - 5 Marks)

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{a+2d} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

- 2) Prove that for all values of θ ,

$$\begin{vmatrix} \sin \theta & \cos \theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

(2000 - 3 Marks)

- 3) If matrix $\mathbf{A} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $\mathbf{A}^T \mathbf{A} = \mathbf{I}$, then find the value of $a^3 + b^3 + c^3$.

(2003 - 2 Marks)

- 4) If \mathbf{M} is a 3×3 matrix, where $\det \mathbf{M} = 1$ and $\mathbf{M}\mathbf{M}^T = \mathbf{I}$, where ' \mathbf{I} ' is an identity matrix, prove that $\det(\mathbf{M} - \mathbf{I}) = 0$.

(2004 - 2 Marks)

- 5) If $\mathbf{A} = \begin{pmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{pmatrix}$, $\mathbf{U} = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$, $\mathbf{V} = \begin{pmatrix} a^2 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\mathbf{A}\mathbf{X} = \mathbf{U}$ has infinitely many solutions, prove that $\mathbf{B}\mathbf{X} = \mathbf{V}$ has no unique solution. Also show that if $afd \neq 0$, then $\mathbf{B}\mathbf{X} = \mathbf{V}$ has no solution.

F - MATCH THE FOLLOWING

- 1) Consider the lines given by $L_1 : x + 3y - 5 = 0$; $L_2 : 3x - ky - 1 = 0$; $L_3 : 5x + 2y - 12 = 0$
Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

(2008)

Column I

(A) L_1, L_2, L_3 are concurrent, if

(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if

(C) L_1, L_2, L_3 form a triangle, if

(D) L_1, L_2, L_3 do not form a triangle, if

Column II

(p) $k = 9$

(q) $k = \frac{-6}{5}$

(r) $k = \frac{5}{6}$

(s) $k = 5$

- 2) Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

(2008)

Column I

Column II

(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is

(p) 0

(B) Let \mathbf{A} and \mathbf{B} be 3×3 matrices of real numbers, where \mathbf{A} is symmetric, \mathbf{B} is skew-symmetric and $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$.

(q) 1

If $(\mathbf{AB})^T = (-1)^k \mathbf{AB}$, where $(\mathbf{AB})^T$ is the transpose of the matrix \mathbf{AB} , then the possible values of k are(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than

(r) 2

(D) If $\sin \theta = \cos \phi$, then the possible values of $\frac{1}{\pi} \left(\theta \pm \phi - \frac{\pi}{2} \right)$ are

(s) 3

G - COMPREHENSION BASED QUESTIONS

PASSAGE - 1

Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ and \mathbf{U}_1 , \mathbf{U}_2 and \mathbf{U}_3 are columns of a 3×3 matrix \mathbf{U} . If column matrices \mathbf{U}_1 , \mathbf{U}_2 and

\mathbf{U}_3 satisfying $\mathbf{AU}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{AU}_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$, $\mathbf{AU}_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ evaluate as directed in the following questions.

1) The value $|\mathbf{U}|$ is

(2006 - 5M, -2)

- a) 3
- b) -3
- c) $\frac{3}{2}$
- d) 2

2) The sum of the elements of the matrix \mathbf{U}^{-1} is

(2006 - 5M, -2)

- a) -1
- b) 0
- c) 1
- d) 3

3) The value of $\begin{pmatrix} 3 & 2 & 0 \end{pmatrix} \mathbf{U} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ is

(2006 - 5M, -2)

- a) 5
- b) $\frac{5}{2}$
- c) 4
- d) $\frac{3}{2}$

PASSAGE - 2

Let \mathcal{A} be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

1) The number of matrices in \mathcal{A} is

(2009)

- a) less than 4
- b) at least 4 but less than 7
- c) at least 7 but less than 10
- d) at least 10

2) The number of matrices \mathbf{A} in \mathcal{A} for which the system of linear equations

$$\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

has a unique solution, is

(2009)

- a) less than 4
- b) at least 4 but less than 7
- c) at least 7 but less than 10
- d) at least 10

3) The number of matrices \mathbf{A} in \mathcal{A} for which the system of linear equations

$$\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

is inconsistent, is

(2009)

- a) 0
- b) more than 2
- c) 2
- d) 1

PASSAGE - 3

Let p be an odd prime number and \mathbf{T}_p be the following set of 2×2 matrices :

$$\mathbf{T}_p = \left\{ \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

(2010)

- 1) The number of \mathbf{A} in \mathbf{T}_p such that \mathbf{A} is either symmetric or skew-symmetric or both, and $\det(\mathbf{A})$ is divisible by p is
 - a) $(p-1)^2$
 - b) $2(p-1)$
 - c) $(p-1)^2 + 1$
 - d) $2p-1$
- 2) The number of \mathbf{A} in \mathbf{T}_p such that the trace of \mathbf{A} is not divisible by p but $\det(\mathbf{A})$ is divisible by p is
 [Note: The trace of a matrix is the sum of its diagonal entries.]
 - a) $(p-1)(p^2 - p + 1)$
 - b) $p^3 - (p-1)^2$
 - c) $(p-1)^2$
 - d) $(p-1)(p^2 - 2)$
- 3) The number of \mathbf{A} in \mathbf{T}_p such that $\det(\mathbf{A})$ is not divisible by p is
 - a) $2p^2$

b) $p^3 - 5p$

c) $p^3 - 3p$

d) $p^3 - p^2$