JEE MAIN 2021
AUGUST 26, SHIFT-1

EE24BTECH11019 - Dwarak A

SECTION-A

1) The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$
   a) $-\frac{11\pi}{30}$
   b) $\frac{\pi}{10}$
   c) $-\frac{7\pi}{30}$
   d) $-\frac{\pi}{15}$

2) The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If a and b are the mean and standard deviation respectivelyfor correct data, then (a, b) is :
   a) $(11, 26)$
   b) $(10.5, 25)$
   c) $(11, 25)$
   d) $(10.5, 26)$

3) On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at **P** to the ellipse is perpendicular to the line $x+2y = 0$. Let **S** and **S′** be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle **SPS′** then, the value of $\left(5 - e^2\right) \cdot A$ is :
   a) 6
   b) 12
   c) 14
   d) 24

4) Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0, x \in \left(0, \frac{\pi}{2}\right)$. If $\lim\limits_{x\to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :
   a) $-\frac{\pi}{4}$
   b) $\frac{\pi}{4} - 1$
   c) $\frac{\pi}{4} + 1$
   d) $\frac{\pi}{4}$

5) Let A and B be independent events such that $P(A) = p, P(B) = 2p$. The largest value of p, for which P(exactly one of A, B occurs) $= \frac{5}{9}$, is :
   a) $\frac{1}{3}$
   b) $\frac{2}{9}$
   c) $\frac{4}{9}$
   d) $\frac{5}{12}$

6) Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations
$$\left(1 + \cos^2\theta\right)x + \sin^2\theta y + 4\sin 3\theta z = 0$$
$$\cos^2\theta x + \left(1 + \sin^2\theta\right)y + 4\sin 3\theta z = 0$$
$$\cos^2\theta x + \sin^2\theta y + (1 + 4\sin 3\theta)\,z = 0$$
has a non-trivial solution, then the value of θ is :
   a) $\frac{4\pi}{9}$
   b) $\frac{7\pi}{18}$
   c) $\frac{\pi}{18}$
   d) $\frac{5\pi}{18}$

7) Let $f(x) = \cos\left(2\tan^{-}1\sin\left(\cot^{-}1\sqrt{\frac{1-x}{x}}\right)\right), 0 < x < 1$. Then :
   a) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$
   b) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$
   c) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$
   d) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

8) The sum of the series
$$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \cdots + \frac{2^{100}}{x^{2^{100}}+1}$$
when $x = 2$ is :
   a) $1 + \frac{2^{101}}{4^{101}-1}$
   b) $1 + \frac{2^{100}}{4^{101}-1}$
   c) $1 - \frac{2^{100}}{4^{100}-1}$
   d) $1 - \frac{2^{101}}{4^{101}-1}$

9) If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum\limits_{r=0}^{20} r^2\,{}^{20}C_r$ is equal to :
   a) 420×2^{19}

b) 380×2^{19}
c) 380×2^{18}
d) 420×2^{18}

10) Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If $K\%$ of them are suffering from both ailments, then K can not belong to the set :
a) $\{80, 83, 86, 89\}$
b) $\{84, 86, 88, 90\}$
c) $\{79, 81, 83, 85\}$
d) $\{84, 87, 90, 93\}$

11) The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with
a) centre at $(0, -1)$ and radius $\sqrt{2}$
b) centre at $(0, 1)$ and radius $\sqrt{2}$
c) centre at $(0, 0)$ and radius $\sqrt{2}$
d) centre at $(0, 1)$ and radius 2

12) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = \hat{j} - \hat{k}$. If $\mathbf{c}$ is a vector such that $\mathbf{a} \times \mathbf{c} = \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c} = 3$, then $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is equal to :
a) -2
b) -6
c) 6
d) 2

13) If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$ then the length of this chord is :
a) 5
b) 7
c) $5\sqrt{3}$
d) $3\sqrt{5}$

14) The value of $\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{\frac{1}{2}} dx$ is :
a) $\log_e 4$
b) $\log_e 16$
c) $2\log_e 16$
d) $4\log_e\left(3 + 2\sqrt{2}\right)$

15) A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P ?
a) $(-1, 1, 2)$
b) $(0, 1, 1)$
c) $(1, 0, 1)$
d) $(2, -1, 1)$