3.2.19

EE24BTECH11019 - DWARAK A

Question: Construct a triangle $\triangle$**ABC** if its perimeter is $10.4cm$ and two angles are $45°$ and $120°$, and give justification.

Solution:

Variable	Description	Value
$\angle B$	Angle B	$45°$
$\angle C$	Angle C	$120°$
$K = a + b + c$	Perimeter of $\triangle$**ABC**	$10.4cm$

TABLE 0: Variables Used

$$a + b + c = K \tag{0.1}$$

$$b\cos C + c\cos B - a = 0 \tag{0.2}$$

$$b\sin C - c\sin B = 0 \tag{0.3}$$

Resulting in the matrix equation,

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos C & \cos B \\ 0 & \sin C & -\sin B \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{0.4}$$

Augmented matrix,

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos C & \cos B & 0 \\ 0 & \sin C & -\sin B & 0 \end{pmatrix} \tag{0.5}$$

Row-Reduction,

$$\begin{pmatrix} 1 & 0 & 0 & \frac{\sin(B+C)}{\sin B+\sin C+\sin(B+C)} \\ 0 & 1 & 0 & \frac{\sin B}{\sin B+\sin C+\sin(B+C)} \\ 0 & 0 & 1 & \frac{\sin C}{\sin B+\sin C+\sin(B+C)} \end{pmatrix} \tag{0.6}$$

Substituting values,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = k \begin{pmatrix} \frac{\sin(B+C)}{\sin B+\sin C+\sin(B+C)} \\ \frac{\sin B}{\sin B+\sin C+\sin(B+C)} \\ \frac{\sin C}{\sin B+\sin C+\sin(B+C)} \end{pmatrix} \tag{0.7}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 10.4 \begin{pmatrix} \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \\ \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \\ \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \end{pmatrix} \tag{0.8}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.4693 \\ 4.0142 \\ 4.9164 \end{pmatrix} \tag{0.9}$$

Sides of $\triangle\mathbf{ABC}$,

$$\implies a = 1.4693cm, b = 4.0142cm, c = 4.9164cm \tag{0.10}$$

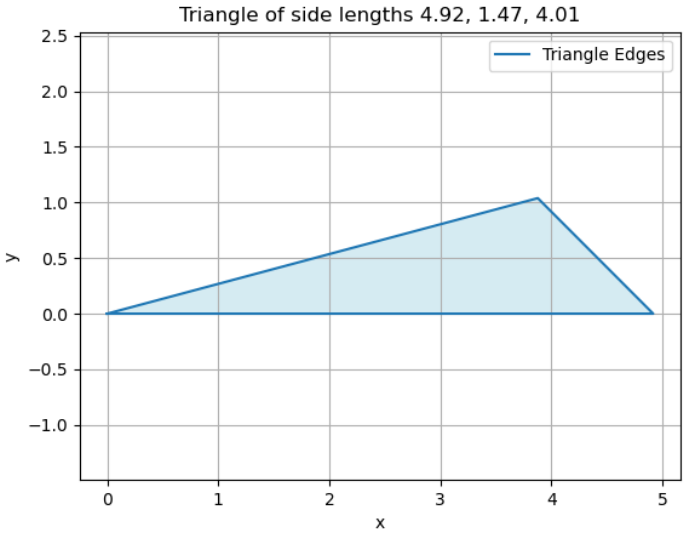

Fig. 0.1: Triangle with $\angle B = 45°$, $\angle C = 120°$ and Perimeter $= 10.4cm$