EE24BTECH11019 - DWARAK A

Question:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Solution:

Variable	Description	Value
A	First point	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$
В	Second point	$\begin{pmatrix} -3\\4 \end{pmatrix}$
C	Mid-point of A and B	$\left(\frac{\mathbf{A}+\mathbf{B}}{2}\right)$
X	Set of points equidistant from A and B	$\begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 0: Variables Used

If X is equidistant form the points A and B

$$\|\mathbf{A} - \mathbf{X}\| = \|\mathbf{B} - \mathbf{X}\| \tag{0.1}$$

$$\|\mathbf{A} - \mathbf{X}\|^2 = \|\mathbf{B} - \mathbf{X}\|^2 \tag{0.2}$$

$$\|\mathbf{A}\|^2 - 2\mathbf{A}^{\mathsf{T}}\mathbf{X} + \|\mathbf{X}\|^2 = \|\mathbf{B}\|^2 - 2\mathbf{B}^{\mathsf{T}}\mathbf{X} + \|\mathbf{X}\|^2$$
 (0.3)

$$(\mathbf{A} - \mathbf{B})^{\mathsf{T}} \mathbf{X} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}$$
 (0.4)

The above equation is the general expression for the perpendicular bisecting plane between any points A and B.

Substituting the A and B values in the derived equation.

$$\binom{6}{2}^{\mathsf{T}} \mathbf{X} = \frac{\left\| \binom{3}{6} \right\|^2 - \left\| \binom{-3}{4} \right\|^2}{2} = \frac{20}{2} = 10 \tag{0.5}$$

Comparing with $n^{\mathsf{T}}x = c$

$$n = \begin{pmatrix} 6\\2 \end{pmatrix} \tag{0.6}$$

$$c = 10 \tag{0.7}$$

Line equation:

$$6x + 2y = 10 (0.8)$$

Final line equation:

$$3x + y = 5 \tag{0.9}$$

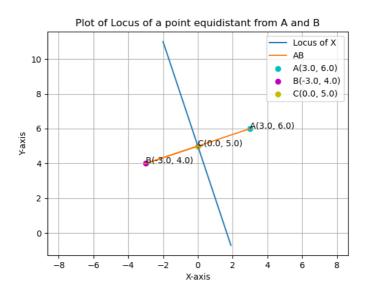


Fig. 0.1: Plot of point X, equidistant from A and B