JEE MAIN 2023
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EE24BTECH11019 - Dwarak A

SECTION-A

1) The statement $B \implies ((\sim A) \vee B)$ is equivalent to :
   a) $B \implies (A \implies B)$
   b) $A \implies (A \iff B)$
   c) $A \implies ((\sim A) \implies B)$
   d) $B \implies ((\sim A) \implies B)$
2) The shortest distance between the lines
$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \text{ and } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$
is
   a) $2\sqrt{3}$
   b) $4\sqrt{3}$
   c) $3\sqrt{3}$
   d) $5\sqrt{3}$
3) If $\mathbf{a} = \hat{i} + 2\hat{k}, \mathbf{b} = \hat{i} + \hat{j} + \hat{k}, \mathbf{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}, \mathbf{r} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} = \mathbf{0}$ and $\mathbf{r} \cdot \mathbf{a} = 0$ then $\mathbf{r} \cdot \mathbf{c}$ is equal to :
   a) 34
   b) 12
   c) 36
   d) 30
4) Let $S = \{w_1, w_2, \dots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2$. Let $A = \{2k + 3l; k, l \in \mathbb{N}\}$ and $B = \{w_n; n \in A\}$. Then $P(B)$ is equal to
   a) $\frac{3}{32}$
   b) $\frac{3}{64}$
   c) $\frac{1}{16}$
   d) $\frac{1}{32}$
5) The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is :
   a) $\tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$
   b) $\tan^{-1} 2 - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
   c) $\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$
   d) $\tan^{-1}\frac{1}{2} - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
6) Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1 + x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99}K}{a} = \frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to :
   a) $(50, 51)$
   b) $(51, 99)$
   c) $(50, 101)$
   d) $(51, 101)$
7) Let f and g be twice differentiable functions on $\mathbb{R}$ such that
$$f''(x) = g''(x) + 6x$$
$$f'(1) = 4g'(1) - 3 = 9$$
$$f(2) = 3g(2) = 12$$
Then which of the following is NOT true ?
   a) $g(-2) - f(-2) = 20$
   b) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$
   c) $|f'(x) - g'(x)| < 6 \implies -1 < x < 1$
   d) There exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = g(x_0)$
8) The set of all values of $t \in \mathbb{R}$, for which the matrix
$$\begin{pmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{pmatrix}$$
is invertible, is :
   a) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$
   b) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$
   c) $\{k\pi, k \in \mathbb{Z}\}$
   d) $\mathbb{R}$
9) The area of the region
$$A = \left\{(x, y) : |\cos x - \sin x| \leq y \leq \sin x, 0 \leq x \leq \frac{\pi}{2}\right\}$$
   a) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$
   b) $\sqrt{5} + 2\sqrt{2} - 4.5$

c) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$
d) $\sqrt{5} - 2\sqrt{2} + 1$

10) The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$
   a) $[-2, -1]$
   b) $\left[-2, -\frac{3}{2}\right]$
   c) $\left[-1, -\frac{1}{2}\right]$
   d) $\left[-\frac{3}{2}, -1\right]$

11) The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is :
   a) 89
   b) 84
   c) 86
   d) 79

12) The plane $2x - y + z = 4$ intersects the line segment joining the points $\mathbf{A}(a, -2, 4)$ and $\mathbf{B}(2, b, -3)$ at the point $\mathbf{C}$ in the ratio $2 : 1$ and the distance of the point $\mathbf{C}$ from the origin is $\sqrt{5}$. If $ab < 0$ and $\mathbf{P}$ is the point $(a-b, b, 2b-a)$ then $\mathbf{CP}^2$ is equal to :
   a) $\frac{17}{3}$
   b) $\frac{16}{3}$
   c) $\frac{73}{3}$
   d) $\frac{97}{3}$

13) Let $\mathbf{a} = 4\hat{i} + 3\hat{j}$ and $\mathbf{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\mathbf{c}$ is a vector such that $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) + 25 = 0, \quad \mathbf{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ and projection of $\mathbf{c}$ on $\mathbf{a}$ is 1, then the projection of $\mathbf{c}$ on $\mathbf{b}$ equals:
   a) $\frac{5}{\sqrt{2}}$
   b) $\frac{1}{5}$
   c) $\frac{1}{\sqrt{2}}$
   d) $\frac{3}{\sqrt{2}}$

14) If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point $\mathbf{P}$, then the distance of the point $\mathbf{P}$ from the plane $z = a$ is :
   a) 16
   b) 28
   c) 10
   d) 22

15) The value of the integral $\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to :
   a) $\pi \log_e 2$
   b) $\frac{1}{2} \log_e 2$
   c) $\frac{\pi}{4} \log_e 2$
   d) $\frac{\pi}{2} \log_e 2$