JEE MAIN 2020
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EE24BTECH11019 - Dwarak A

SECTION-A

1) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} - (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :
   a) $\frac{1}{3}$
   b) $\frac{1}{9}$
   c) $\frac{1}{81}$
   d) 3

2) The locus of mid points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :
   a) $3x - 2y = 0$
   b) $2x - 3y = 0$
   c) $7x - 5y = 0$
   d) $5x - 7y = 0$

3) Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(\mathbf{AB})^2$ is equal to:
   a) $\frac{32}{5}$
   b) $\frac{52}{5}$
   c) $\frac{56}{5}$
   d) $\frac{64}{5}$

4) Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :
   a) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \subseteq D$
   b) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \not\subseteq D$
   c) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
   d) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

5) Let $y = y(x)$ be the solution curve of the differential equation $\left(y^2 - x\right)\frac{dy}{dx} = 1$ satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is :
   a) 2
   b) $2 + e$
   c) $2 - e$
   d) $-e$

6) If θ_1 and θ_2 be respectively the smallest and largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to :
   a) $\frac{\pi}{3} + \frac{1}{6}$
   b) $\frac{\pi}{9}$
   c) $\frac{\pi}{3}$
   d) $\frac{2\pi}{3}$

7) If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots$ is $(102)m$, then m is equal to :
   a) 25
   b) 20
   c) 10
   d) 5

8) The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :
   a) 6
   b) 4
   c) 3
   d) 2

9) The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|} \, dx = 5$ is:
   a) $\log_e\left(\frac{4}{3}\right)$
   b) $\log_e 2$
   c) $\log_e \sqrt{2}$
   d) $\log_e\left(\frac{3}{2}\right)$

10) Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$ then which of the following is not true ?
   a) f is an odd function
   b) $x = 1$ is a point of maxima and $x = -1$ is a point of minima of f.
   c) $f(1) - 4f(-1) = 4$
   d) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

11) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three unit vectors such that $\mathbf{a} +$

$\mathbf{b}+\mathbf{c}=\mathbf{0}$. If $\lambda=\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}$ and $\mathbf{d}=\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$, then the ordered pair $(\lambda,\mathbf{d})$ is equal to:

a) $\left(-\frac{3}{2},3\mathbf{a}\times\mathbf{b}\right)$
b) $\left(\frac{3}{2},3\mathbf{a}\times\mathbf{c}\right)$
c) $\left(-\frac{3}{2},3\mathbf{c}\times\mathbf{b}\right)$
d) $\left(\frac{3}{2},3\mathbf{b}\times\mathbf{c}\right)$

12) The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+\cdots+x^{10}$ is :

a) 120
b) 210
c) 330
d) 420

13) Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k, k\geq 1$, then which one of the following statements is not true ?

a) $p_5=11$
b) $p_3=p_5-p_4$
c) $(p_1+p_2+p_3+p_4+p_5)=26$
d) $p_5=p_2\cdot p_3$

14) The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, when $x\in[0,1]$ is:

a) $\frac{4-\sqrt{7}}{3}$
b) $\frac{\sqrt{7}-2}{3}$
c) $\frac{4-\sqrt{5}}{3}$
d) $\frac{2}{3}$

15) The area (in sq. units) of the region $\left\{(x,y)\in\mathbb{R}^2|4x^2\leq y\leq 8x+12\right\}$ is :

a) $\frac{124}{3}$
b) $\frac{125}{3}$
c) $\frac{127}{3}$
d) $\frac{128}{3}$