1.8.11

EE24BTECH11019 - DWARAK A

Question:

Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Solution:

Variable	Description	Value
A	First point	$\begin{pmatrix}3\\6\end{pmatrix}$
B	Second point	$\begin{pmatrix}-3\\4\end{pmatrix}$
C	Mid-point of **A** and **B**	$\left(\frac{\mathbf{A+B}}{2}\right)$
X	Set of points equidistant from **A** and **B**	$\begin{pmatrix}x\\y\end{pmatrix}$

TABLE 0: Variables Used

If **X** is equidistant form the points **A** and **B**

$$\|A - X\| = \|B - X\| \tag{0.1}$$

$$\|A - X\|^2 = \|B - X\|^2 \tag{0.2}$$

$$\|A\|^2 - 2A^\top X + \|X\|^2 = \|B\|^2 - 2B^\top X + \|X\|^2 \tag{0.3}$$

$$(A - B)^\top X = \frac{\|A\|^2 - \|B\|^2}{2} \tag{0.4}$$

The above equation is the general expression for the perpendicular bisecting plane between any points **A** and **B**.

Substituting the **A** and **B** values in the derived equation.

$$\begin{pmatrix}6\\2\end{pmatrix}^\top X = \frac{\left\|\begin{pmatrix}3\\6\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}-3\\4\end{pmatrix}\right\|^2}{2} = \frac{20}{2} = 10 \tag{0.5}$$

Comparing with $n^\top x = c$

$$n = \begin{pmatrix}6\\2\end{pmatrix} \tag{0.6}$$

$$c = 10 \tag{0.7}$$

Line equation :

$$6x + 2y = 10 \tag{0.8}$$

Final line equation :

$$3x + y = 5 \tag{0.9}$$

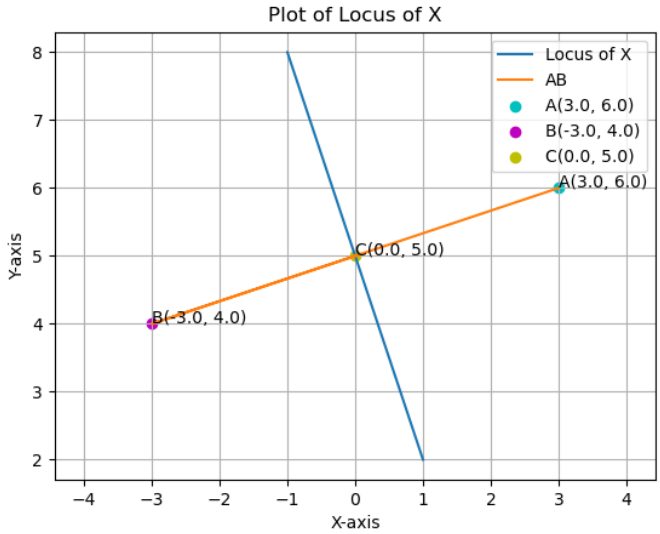

Fig. 0.1: Plot of point **X**, equidistant from **A** and **B**