

# 3.2.19

EE24BTECH11019 - DWARAK A

**Question:** Construct a triangle  $\triangle ABC$  if its perimeter is 10.4 and two angles are  $45^\circ$  and  $120^\circ$ , and give justification.

**Solution:**

Variable	Description	Value
$\angle B$	Angle at vertex <b>B</b>	$45^\circ$
$\angle C$	Angle at vertex <b>C</b>	$120^\circ$
$K = a + b + c$	Perimeter of $\triangle ABC$	$10.4cm$

TABLE 0: Variables Used

$$a + b + c = K \quad (0.1)$$

$$b \cos C + c \cos B - a = 0 \quad (0.2)$$

$$b \sin C - c \sin B = 0 \quad (0.3)$$

Resulting in the matrix equation,

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos C & \cos B \\ 0 & \sin C & -\sin B \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (0.4)$$

Augmented matrix,

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos C & \cos B & 0 \\ 0 & \sin C & -\sin B & 0 \end{pmatrix} \quad (0.5)$$

Row-Reduction,

$$\begin{pmatrix} 1 & 0 & 0 & \frac{\sin(B+C)}{\sin B + \sin C + \sin(B+C)} \\ 0 & 1 & 0 & \frac{\sin B}{\sin B + \sin C + \sin(B+C)} \\ 0 & 0 & 1 & \frac{\sin C}{\sin B + \sin C + \sin(B+C)} \end{pmatrix} \quad (0.6)$$

Substituting values,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = k \begin{pmatrix} \frac{\sin(B+C)}{\sin B + \sin C + \sin(B+C)} \\ \frac{\sin B}{\sin B + \sin C + \sin(B+C)} \\ \frac{\sin C}{\sin B + \sin C + \sin(B+C)} \end{pmatrix} \quad (0.7)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 10.4 \begin{pmatrix} \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \\ \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \\ \frac{\sqrt{3}-1}{\sqrt{6}+\sqrt{3}+1} \end{pmatrix} \quad (0.8)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.4693 \\ 4.0142 \\ 4.9164 \end{pmatrix} \quad (0.9)$$

Sides of  $\triangle ABC$ ,

$$a = 1.4693cm, b = 4.0142cm, c = 4.9164cm \quad (0.10)$$

Co-ordinates of  $\triangle ABC$ ,

$$A = \begin{pmatrix} a \\ b \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (0.11)$$

$$\Rightarrow A = \begin{pmatrix} 1.4693 \\ 4.0142 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 1.4693 \\ 0 \end{pmatrix} \quad (0.12)$$

Variable	Description	Value
$\angle A$	Angle at vertex <b>A</b>	$15^\circ$
$\angle B$	Angle at vertex <b>B</b>	$45^\circ$
$\angle C$	Angle at vertex <b>C</b>	$120^\circ$
$a$	Length of Side <b>BC</b>	$1.4693cm$
$b$	Length of Side <b>CA</b>	$4.0142cm$
$c$	Length of Side <b>AB</b>	$4.9164cm$
<b>A</b>	Coordinates of Point <b>A</b>	$\begin{pmatrix} 1.4693 \\ 4.0142 \end{pmatrix}$
<b>B</b>	Coordinates of Point <b>B</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
<b>C</b>	Coordinates of Point <b>C</b>	$\begin{pmatrix} 1.4693 \\ 0 \end{pmatrix}$

TABLE 0: Triangle Properties

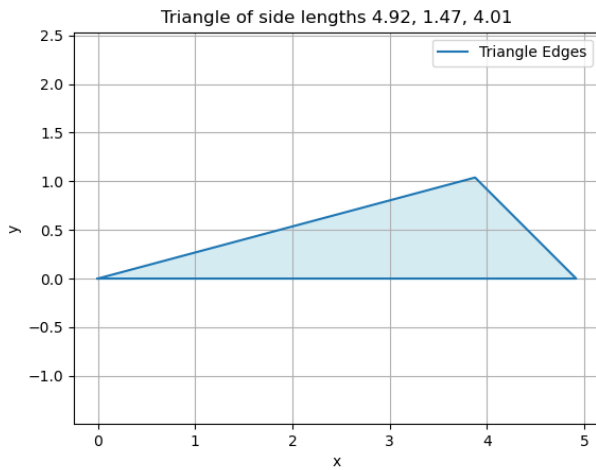


Fig. 0.1: Triangle with  $\angle B = 45^\circ$ ,  $\angle C = 120^\circ$  and Perimeter = 10.4cm