ASSIGNMENT - 2
SECTION-A — JEE ADVANCED / IIT-JEE

EE24BTECH11019 - DWARAK A

E - SUBJECTIVE PROBLEMS

1) Let $a > 0$, $d > 0$. Find the value of the determinant

(1996 - 5 Marks)

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

2) Prove that for all values of θ ,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

(2000 - 3 Marks)

3) If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$.

(2003 - 2 Marks)

4) If M is a 3×3 matrix, where det $M = 1$ and $MM^T = I$, where 'I' is an identity matrix, prove that $\det(M - I) = 0$.

(2004 - 2 Marks)

5) If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $AX = U$ has infinitely many solutions, prove that $BX = V$ has no unique solution. Also show that if afd $\neq 0$, then $BX = V$ has no solution.

(2004 - 4 Marks)

F - MATCH THE FOLLOWING

1) Consider the lines given by $L_1 : x + 3y - 5 = 0; L_2 : 3x - ky - 1 = 0; L_3 : 5x + 2y - 12 = 0$
Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.
(2008)

Column I	Column II
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = 9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = \frac{-6}{5}$
(C) L_1, L_2, L_2 from a triangle, if	(r) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

2) Match the Statements/Expressions in **Column I** with the Statements/Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.
(2008)

Column I	Column II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew-symmetric and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{\left(-k+3^{-a}\right)} < 2$, must be less than	(r) 2
(D) If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) are$	(s) 3

G - COMPREHENSION BASED QUESTIONS

PASSAGE - 1

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and U_1, U_2 and U_3 are columns of a 3×3 matrix U. If column matrices U_1, U_2 and U_3 satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ evaluate as directed in the following questions.

1) The value $|U|$ is
(2006 - 5M, -2)
   a) 3
   b) -3
   c) $\frac{3}{2}$
   d) 2
2) The sum of the elements of the matrix U^-1 is
(2006 - 5M, -2)
   a) -1
   b) 0
   c) 1
   d) 3
3) The value of $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix} \det U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is
(2006 - 5M, -2)
   a) 5
   b) $\frac{5}{2}$
   c) 4
   d) $\frac{3}{2}$

PASSAGE - 2

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

1) The number of matrices in $\mathcal{A}$ is
(2009)
   a) 12
   b) 6
   c) 9
   d) 3

2) The number of matrices in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

(2009)

a) less than 4
b) at least 4 but less than 7
c) at least 7 but less than 10
d) at least 10

3) The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

(2009)

a) 0
b) more than 2
c) 2
d) 1

PASSAGE - 3

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$

(2010)

1) The number of A in T_p such that A is either symmetric or skew-symmetric or both, and $\det(A)$ divisibly by p is
a) $(p-1)^2$
b) $2(p-1)$
c) $(p-1)^2 + 1$
d) $2p - 1$

2) The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is
[**Note:**The trace of a matrix is the sum of its diagonal e
a) $(p-1)\left(p^2 - p + 1\right)$
b) $p^3 - (p-1)^2$
c) $(p-1)^2$
d) $(p-1)\left(p^2 - 2\right)$

3) The number of A in T_p such that det(A) is not divisible by p is
a) $2p^2$
b) $p^3 - 5p$
c) $p^3 - 3p$
d) $p^3 - p^2$