1.9.19

EE24BTECH11019 - DWARAK A

Question: Find the values of x for which the distance between the points $\mathbf{A}(x, 2)$ and $\mathbf{B}(9, 8)$ is 10 units.

Solution: Distance formula :

Variable	Description	Value
A	First point	$\begin{pmatrix} x \\ 2 \end{pmatrix}$
B	Second point	$\begin{pmatrix} 9 \\ 8 \end{pmatrix}$
d	Distance between **A** and **B**	10

TABLE 0: Variables Used

$$d = \|A - B\| \tag{0.1}$$

$$d = \sqrt{\|A\|^2 - 2A^\top B + \|B\|^2} \tag{0.2}$$

$$d^2 = \|A\|^2 - 2A^\top B + \|B\|^2 \tag{0.3}$$

Substituting values,

$$10^2 = \begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} - 2\begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} 9 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 & 8 \end{pmatrix}\begin{pmatrix} 9 \\ 8 \end{pmatrix} \tag{0.4}$$

$$100 = (x^2 + 4) - 2(9x + 16) + (81 + 64) \tag{0.5}$$

$$\tag{0.6}$$

Rearranging terms,

$$x^2 - 18x + 17 = 0 \tag{0.7}$$

$$(x - 17)(x - 1) = 0 \tag{0.8}$$

So,

$$x_1 = 17, x_2 = 1 \tag{0.9}$$

Finally,

$$A_1 = \begin{pmatrix} 17 \\ 2 \end{pmatrix}, A_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.10}$$

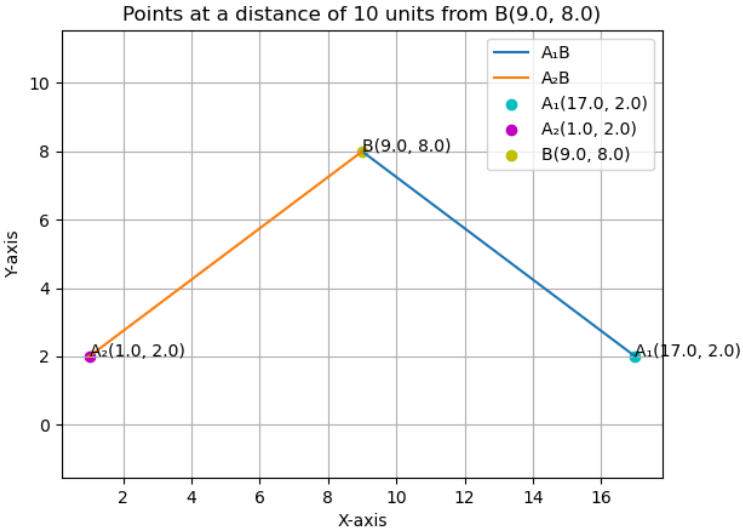

Fig. 0.1: Plot of points $\mathbf{A_1}$ and $\mathbf{A_2}$ at a distance of 10 units from $\mathbf{B}$