4.4.33

EE24BTECH11019 - DWARAK A

Question: Find the value of x such that the four points with position vectors $\mathbf{A}(3\hat{i}+2\hat{j}+\hat{k})$, $\mathbf{B}(4\hat{i}+x\hat{j}+5\hat{k})$, $\mathbf{C}(4\hat{i}+2\hat{j}-2\hat{k})$, and $\mathbf{D}(6\hat{i}+5\hat{j}-\hat{k})$ are coplanar.

Solution:

Symbol	Description	Value
A	Coordinates of Point **A**	$\begin{pmatrix}3\\2\\1\end{pmatrix}$
B	Coordinates of Point **B**	$\begin{pmatrix}4\\x\\5\end{pmatrix}$
C	Coordinates of Point **C**	$\begin{pmatrix}4\\2\\-2\end{pmatrix}$
D	Coordinates of Point **D**	$\begin{pmatrix}6\\5\\-1\end{pmatrix}$

TABLE 0: Variables Used

Plane Equation,

$$\mathbf{n}^\top x = 1 \tag{0.1}$$

If $\mathbf{A}, \mathbf{C}, \mathbf{D}$ are coplanar

$$\begin{pmatrix}A & C & D\end{pmatrix}^\top \mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.2}$$

$$\begin{pmatrix}3 & 2 & 1\\4 & 2 & -2\\6 & 5 & -1\end{pmatrix}\mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.3}$$

Augmented Matrix,

$$\begin{pmatrix}3 & 2 & 1 & 1\\4 & 2 & -2 & 1\\6 & 5 & -1 & 1\end{pmatrix} \xrightarrow{R_1\to\frac{1}{3}R_1} \begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\4 & 2 & -2 & 1\\6 & 5 & -1 & 1\end{pmatrix} \tag{0.4}$$

$$\xrightarrow{R_3\to R_3-6R_1} \begin{pmatrix}1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\4 & 2 & -2 & 1\\0 & 1 & -3 & -1\end{pmatrix} \tag{0.5}$$

$$\xrightarrow{R_2 \to R_2 - 4R_1} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{2}{3} & -\frac{10}{3} & -\frac{1}{3} \\ 0 & 1 & -3 & -1 \end{pmatrix} \tag{0.6}$$

$$\xrightarrow{R_2 \to -\frac{3R_2}{2}} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 1 & -3 & -1 \end{pmatrix} \tag{0.7}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & -8 & -\frac{3}{2} \end{pmatrix} \tag{0.8}$$

$$\xrightarrow{R_1 \to R_1 - \frac{2R_2}{3}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & -8 & -\frac{3}{2} \end{pmatrix} \tag{0.9}$$

$$\xrightarrow{R_3 \to -\frac{R_3}{8}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \tag{0.10}$$

$$\xrightarrow{R_1 \to R_1 + 3R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{16} \\ 0 & 1 & 5 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \tag{0.11}$$

$$\xrightarrow{R_2 \to R_2 - 5R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{16} \\ 0 & 1 & 0 & -\frac{7}{16} \\ 0 & 0 & 1 & \frac{3}{16} \end{pmatrix} \tag{0.12}$$

$$\mathbf{n} = \begin{pmatrix} \frac{9}{16} \\ -\frac{7}{16} \\ \frac{3}{16} \end{pmatrix} \tag{0.13}$$

$$\mathbf{n}^\top B = 1 \tag{0.14}$$

$$\begin{pmatrix} 9 & -7 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix} = 16 \tag{0.15}$$

$$36 - 7x + 15 = 16 \tag{0.16}$$

$$7x = 35 \tag{0.17}$$

$$x = 5 \tag{0.18}$$

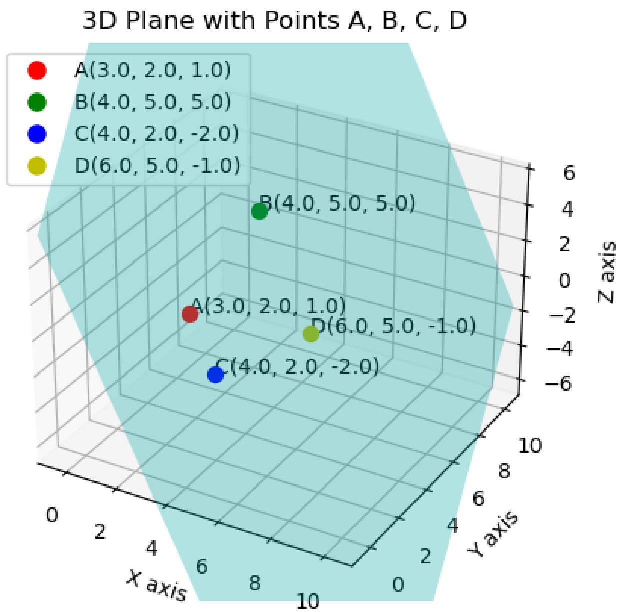

Fig. 0.1: Plot of the plane with points A, B, C and D