EE24BTECH11019 - DWARAK A

Question:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Solution:

Variable	Description	Value
A	First point	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$
В	Second point	$\begin{pmatrix} -3\\4 \end{pmatrix}$
C	Mid-point of A and B	$\left(\frac{A+B}{2}\right)$
X	Set of points equidistant from A and B	$\begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 0: Variables Used

If X is equidistant form the points A and B

$$||A - X|| = ||B - X|| \tag{0.1}$$

$$||A - X||^2 = ||B - X||^2 (0.2)$$

$$||A||^2 - 2A^{\mathsf{T}}X + ||X||^2 = ||B||^2 - 2B^{\mathsf{T}}X + ||X||^2 \tag{0.3}$$

$$(A - B)^{\mathsf{T}} X = \frac{\|A\|^2 - \|B\|^2}{2} \tag{0.4}$$

The above equation is the general expression for the perpendicular bisecting plane between any points A and B.

Substituting the A and B values in the derived equation.

$${\binom{6}{2}}^{\mathsf{T}} X = \frac{{\left\| {\binom{3}{6}} \right\|}^2 - {\left\| {\binom{-3}{4}} \right\|}^2}{2} = \frac{20}{2} = 10$$
 (0.5)

Comparing with $n^{\mathsf{T}}x = c$

$$n = \begin{pmatrix} 6\\2 \end{pmatrix} \tag{0.6}$$

$$c = 10 \tag{0.7}$$

Line equation:

$$6x + 2y = 10 (0.8)$$

Final line equation:

$$3x + y = 5 \tag{0.9}$$

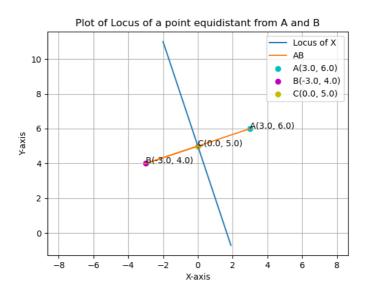


Fig. 0.1: Plot of point X, equidistant from A and B