

Latihan 2.3

1. Selesaikan integrasi berikut:

d.

$$\int \frac{1}{x^2 \sqrt{9-x^2}} dx = \int \frac{1}{x^2 \sqrt{3^2-x^2}} dx$$

substitusi trigonometri:

$$\text{misal: } x = 3 \sin u \rightarrow u = \sin^{-1} \frac{x}{3}$$

$$\frac{dx}{du} = 3 \cos u$$

$$\int \frac{1}{x^2 \sqrt{3^2-x^2}} dx = \int \frac{1}{(3 \sin u)^2 \sqrt{3^2-(3 \sin u)^2}} (3 \cos u \, du)$$

$$= \int \frac{3 \cos u}{3^2 \sin^2 u \sqrt{3^2-3^2 \sin^2 u}} du$$

$$= \int \frac{3 \cos u}{3^2 \sin^2 u \sqrt{3^2 \cos^2 u}} du$$

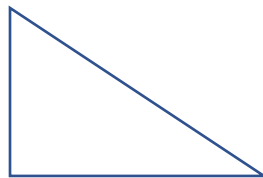
$$= \int \frac{3 \cos u}{3^2 \sin^2 u (3 \cos u)} du$$

$$= \int \frac{1}{9 \sin^2 u} du$$

$$= \frac{1}{9} \int \csc^2 u \, du$$

$$= \frac{1}{9} (-\cot u) + C$$

$$= -\frac{1}{9} \cot \left(\sin^{-1} \left(\frac{x}{3} \right) \right)$$



$$= -\frac{\sqrt{9-x^2}}{9x}$$

5. Selesaikan integrasi yang diberikan berikut:

f.

$$\int \frac{x}{x^2 + 6x + 3} dx$$

menyempurnakan kuadrat sempurna:

$$\int \frac{x}{x^2 + 6x + 3 + 6 - 6} dx$$

$$\int \frac{x}{(x^2 + 6x + 9) - 6} dx$$

$$\int \frac{x}{(x+3)^2 - 6} dx$$

$$\text{misal: } u = x + 3 \rightarrow x = u - 3$$

$$\frac{du}{dx} = 1$$

sehingga:

$$\int \frac{u-3}{u^2-6} \frac{du}{1}$$

$$\int \frac{u-3}{(u+3)(u-3)} du$$

$$\int \frac{1}{u+3} du$$

$$= \ln(u+3) + C$$

$$= \ln((x+3)+3) + C$$

$$= \ln(x+6) + C$$

w.

$$\int \frac{1}{x\sqrt{x^3+1}} dx$$

$$\text{misal: } u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$\int \frac{1}{x\sqrt{x^3+1}} dx = \int \frac{1}{x\sqrt{u+1}} \frac{du}{3x^2}$$

$$= \int \frac{1}{3x^3\sqrt{u+1}} du$$

$$= \int \frac{1}{3u\sqrt{u+1}} du$$

$$\text{misal: } v = \sqrt{u+1} \rightarrow u = v^2 - 1$$

$$\frac{dv}{du} = \frac{1}{2\sqrt{u+1}} = \frac{1}{2v}$$

sehingga:

$$\int \frac{1}{3u\sqrt{u+1}} du = \int \frac{1}{3(v^2-1)(v)} (2v dv)$$

$$= \frac{2}{3} \int \frac{1}{(v^2-1)} dv$$

$$= \frac{2}{3} \int \frac{1}{(v+1)(v-1)} dv$$

pecahan parsial:

$$\frac{1}{(v+1)(v-1)} = \frac{A}{v+1} + \frac{B}{v-1}$$

$$1 = A(v-1) + B(v+1)$$

$$1 = (A+B)v + (B-A)$$

didapatkan:

$$A+B=0$$

$$B-A=1$$

$$\text{substitusi: } A = -B$$

$$B - (-B) = 1$$

$$B = \frac{1}{2}$$

$$A = -\frac{1}{2}$$

sehingga:

$$\begin{aligned} \frac{2}{3} \int \frac{1}{(v+1)(v-1)} dv &= \frac{2}{3} \int \left(-\frac{1}{2(v+1)} + \frac{1}{2(v-1)} \right) dv \\ &= \frac{2}{3} \left(-\frac{\ln(v+1)}{2} + \frac{\ln(v-1)}{2} \right) \\ &= \frac{1}{3} (-\ln(\sqrt{u+1}+1) + \ln(\sqrt{u+1}-1)) \\ &= \frac{1}{3} (-\ln(\sqrt{x^3+1}+1) + \ln(\sqrt{x^3+1}-1)) \end{aligned}$$