Research Proposal of Machine Learning for Quantitative Finance

1. Introduction.

Partial integro-differential equations (PIDEs) have emerged as powerful mathematical tools for modeling complex phenomena in numerous fields, including finance and option pricing. These equations involve both differential and integral terms, making them challenging to solve using traditional numerical methods due to their high computational complexity. Machine learning (ML) techniques, particularly neural networks (NN) and reinforcement learning (RL), offer promising approaches to estimate these integro-differential processes accurately and efficiently. This research proposal aims to reproduce and build upon the NN-based approach proposed by Lu et al. [1], with the objective of enhancing the accuracy and efficiency of option pricing models.

2. Literature review

A general format of PIDEs consists of an integro-differential function, initial conditions, and boundary functions. In comparison, partial differential equations (PDEs) are a special case of PIDEs, where the integral term is not considered. Due to their relative simplicity, there have been numerous machine learning-based approaches for solving PDEs based on a physics-driven scheme as Figure 1, a.k.a AI4Science method recently. Typically, NNs are employed to estimate the differential function, while the boundary functions are incorporated as constraints.

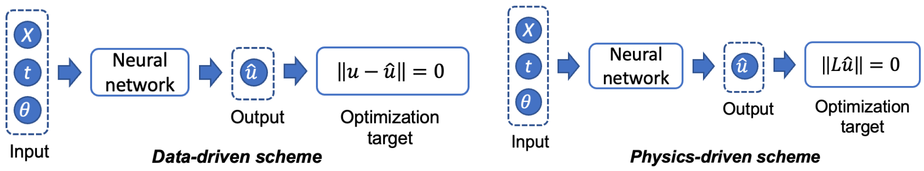

Figure 1. The data-driven and physics-driven scheme for neural networks

For instance, in [2], the authors used artificial neural networks (ANNs) to estimate PDEs. Various approaches can be employed to handle the constraint optimization problem. These include using constrained optimization procedures, Lagrange multipliers, or penalty formulations to incorporate the constraints into the optimization process. Another approach involves designing the ansatz in such a way that the constraints are automatically fulfilled, transforming the constrained optimization problem into an unconstrained one.

The existing literature on approaches for PIDEs is relatively limited. This research aims to fill this gap by reproducing and extending the work of Lu et al. [1] that utilizes NNs and RL techniques to solve PIDEs and apply them to option analysis. By leveraging the advancements in deep learning and reinforcement learning, we can overcome the challenges posed by high computational complexity and improve the accuracy and efficiency of option pricing models.

3. Research plan

In this research, we focus on the PIED in the format of:

$$\begin{cases} \frac{\partial u}{\partial t} + b\nabla u + \frac{1}{2}Tr\left(\sigma\sigma^T \mathrm{H}(u)\right) + \mathcal{A}u + f = 0 \\ u(T,\cdot) = g(\cdot) \end{cases}$$

where $\mathcal{A}u$ contains the integral term.

In [1], the authors primarily concentrated on estimating the integral term and improving efficiency by avoiding the simulation of entire trajectories. To better understand the functionality of different components of the model and enable a better comparison with other ANN-based PDE solvers, we have divided the project into three subtasks:

a. Construction of the backbone for PDE. In this task, we will construct the backbone for solving PDEs, where the integral term is omitted. The integral term will be set as 0 in the PIDEs, and the simulation of the Levy-process and corresponding loss functions will be performed. The neural network will have only one output. We will verify the model's capability to solve PDEs using the neural network and RL agents, as well as assess the efficiency improvement gained by avoiding the simulation of entire trajectories.

b. Introducing the integral term. In this task, we will consider the integral term by incorporating the integral process in the PDEs. An additional scalar output will be added to the neural network. The corresponding loss introduced by the integral term will be optimized using appropriate loss functions.

c. Option analysis. After reproducing the PIDEs using the neural network, we will apply it as an option pricing model for quantitative analysis. Based on recent studies [3,4], which relax the complete market hypothesis of the classical linear Black–Scholes model and assume a Lévy stochastic process dynamics for the underlying stock price, we can obtain a model for pricing options through PIDEs. In the application, we will consider different scenarios, including Asian options, barrier options, and up-and-out put options.

4. Summary

In summary, this research plan outlines the approach to be taken for solving PIDEs using neural networks and reinforcement learning. The project has been divided into three subtasks: constructing the backbone for PDEs, introducing the integral term, and applying the model to option analysis. By following this plan, we aim to develop an efficient and accurate option pricing model that considers the integral term and provides valuable insights for quantitative analysis in finance.

[1] Lu L, Guo H, Yang X, et al. Temporal difference learning for high-dimensional PIDEs with jumps[J]. arXiv preprint arXiv:2307.02766, 2023.

[2] Berg J, Nyström K. A unified deep artificial neural network approach to partial differential equations in complex geometries[J]. Neurocomputing, 2018, 317: 28-41.

[3] Cruz JMTS, Ševčovič D. On solutions of a partial integro-differential equation in Bessel potential spaces with applications in option pricing models[J]. Japan Journal of Industrial and Applied Mathematics, 2020, 37: 697-721.

[4] Chen Y, Hu R. L^∞-norm convergence rates of an IMEX scheme for solving a partial integro-differential equation system arising from regime-switching jump-diffusion Asian option pricing[J]. International Journal of Computer Mathematics, 2023, 100(6): 1373-1394.