L7 - Task-Space Trajectories

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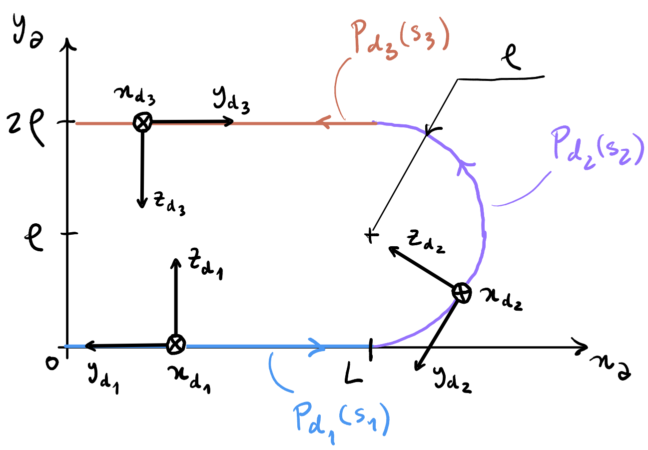
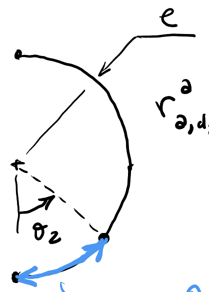
Task-Space trajectory defined in an arbitrary frame



The trajectory in "frame a" has been partitioned into sub-paths, $P_{d_1}(s_1)$, $P_{d_2}(s_2)$ and $P_{d_3}(s_3)$, where s_i is the length along sub-path i. The sub-paths are parametrized as follows.

$$
\begin{cases}
r^a_{a,d_1}(s_1) = \begin{pmatrix} s_1 \\ 0 \end{pmatrix}, & 0 \leq s_1 \leq L \\
r^a_{a,d_2}(s_2) = \begin{pmatrix} L + \rho \sin\left(\frac{s_2}{\rho}\right) \\ \rho - \rho\cos\left(\frac{s_2}{\rho}\right) \end{pmatrix}, & 0 \leq s_2 \leq \rho\pi \\
r^a_{a,d_3}(s_3) = \begin{pmatrix} L - s_3 \\ 2\rho \end{pmatrix}, & 0 \leq s_3 \leq L
\end{cases}
$$



$$r^a_{a,d_2}(\theta_2) = \begin{pmatrix} L + \rho\sin(\theta_2) \\ \rho - \rho\cos(\theta_2) \end{pmatrix}, \quad 0 \leq \theta_2 \leq \pi$$

$$s_2 = \rho\theta_2 \Rightarrow \theta_2 = \frac{s_2}{\rho}$$

The length along the trajectory is s, which is related to s_i through

$$s_i = s - \sum_{k=1}^{i-1} \ell_k ,$$

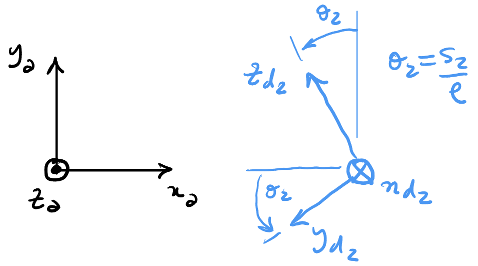
where n is the number of sub-paths and ℓ_k the total length of sub-path k.

Thus, $\{s_1 = s, \ s_2 = s - L, \ s_3 = s - L - \rho\pi\}$. Replacing, we have

$$
r^a_{a,d} = \begin{cases}
\begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix}, & 0 \leq s < L \\
\begin{pmatrix} L + \rho\sin\left(\frac{s-L}{\rho}\right) \\ \rho - \rho\cos\left(\frac{s-L}{\rho}\right) \\ 0 \end{pmatrix}, & L \leq s < L + \rho\pi \\
\begin{pmatrix} 2L + \rho\pi - s \\ 2\rho \\ 0 \end{pmatrix}, & L + \rho\pi \leq s \leq 2L + \rho\pi
\end{cases} \quad (1)
$$

For the orientation we can calculate the orientation along sub-path 2 and then compute the orientation at $s_2=0$ and $s_2=\rho\pi$ for the remaining sub-paths.

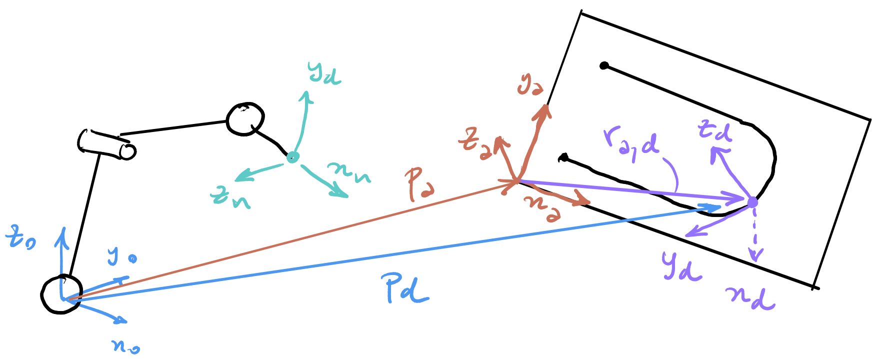
$$R^{a}_{d_2}(s_2)=\begin{pmatrix} 0 & -\cos\left(\frac{s_2}{\rho}\right) & -\sin\left(\frac{s_2}{\rho}\right) \\ 0 & -\sin\left(\frac{s_2}{\rho}\right) & \cos\left(\frac{s_2}{\rho}\right) \\ -1 & 0 & 0 \end{pmatrix}, \quad 0\le s_2\le \rho\pi .$$



$$R^{a}_{d_1}=R^{a}_{d_2}(0)=\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, \quad R^{a}_{d_3}=R^{a}_{d_2}(\rho\pi)=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}.$$

$$R^{a}_{d}(s)=\begin{cases} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, & 0\le s< L \\ \begin{pmatrix} 0 & -\cos\left(\frac{s-L}{\rho}\right) & -\sin\left(\frac{s-L}{\rho}\right) \\ 0 & -\sin\left(\frac{s-L}{\rho}\right) & \cos\left(\frac{s-L}{\rho}\right) \\ -1 & 0 & 0 \end{pmatrix}, & L\le s< L+\rho\pi \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}, & L+\rho\pi\le s\le 2L+\rho\pi \end{cases} \tag{2}$$

Arbitrary placement of the trajectory



The trajectory has been defined in frame a. We are now free to displace and orient the frame however we want, through R_a and p_a.

$$R^{0}_{d}(s)=R^{0}_{a}R^{a}_{d}(s), \quad p^{0}_{d}(s)=p^{0}_{a}+R^{0}_{a}r^{a}_{a,d}(s) \tag{3}$$

Task-space velocity

The desired linear and angular velocities are a function of displacement, s, and the speed, $\dot{s}$. They are determined from the total time derivative of (3), as

$$\dot{p}_d(s,\dot{s}) = R_a\, \dot{r}^{a}_{a,d}(s) = R_a \frac{\partial}{\partial s}\left[r^{a}_{a,d}(s) \right] \cdot \dot{s} \quad , \tag{4}$$

$$\omega_d(s,\dot{s}) = \mathrm{vect}\left(\dot{R}_d(s)\, R_d^T(s) \right) = \mathrm{vect}\left(R_a \frac{\partial R^a_d(s)}{\partial s} R^a_d(s)^T R_a^T \right) \cdot \dot{s} \quad , \tag{5}$$

where $\dot{R}_a = 0$ and $\dot{p}_a = 0$ have been employed. The partial derivatives evaluate as follows,

$$\frac{\partial}{\partial s}\left[r^a_{a,d}(s) \right] = \begin{cases} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} & , 0 \leq s < L \\ \begin{pmatrix} \cos\left(\frac{s-L}{\rho}\right) \\ \sin\left(\frac{s-L}{\rho}\right) \\ 0 \end{pmatrix} & , L \leq s < L + \rho\pi \\ \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} & , L + \rho\pi \leq s \leq 2L + \rho\pi \end{cases} \tag{6}$$

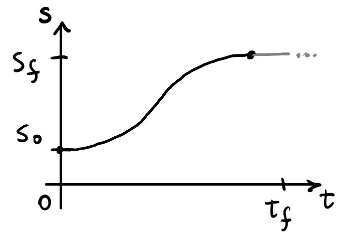
$$\frac{\partial}{\partial s}\left[R^a_d(s) \right] = \begin{cases} 0_{3\times 3} & , 0 \leq s < L \\ \begin{pmatrix} 0 & \frac{1}{\rho}\sin\left(\frac{s-L}{\rho}\right) & -\frac{1}{\rho}\cos\left(\frac{s-L}{\rho}\right) \\ 0 & -\frac{1}{\rho}\cos\left(\frac{s-L}{\rho}\right) & -\frac{1}{\rho}\sin\left(\frac{s-L}{\rho}\right) \\ 0 & 0 & 0 \end{pmatrix} & , L \leq s < L + \rho\pi \\ 0_{3\times 3} & , L + \rho\pi \leq s \leq 2L + \rho\pi \end{cases} \tag{7}$$

Recall that $S(b) = \begin{pmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{pmatrix}$, $b = \mathrm{vect}(S(b))$.

Implementation through CLIK

To span the length as a function of time, we may use

$$s(t) = \begin{cases} s_0 + \frac{3}{t_f^2}(s_f - s_0)t^2 - \frac{2}{t_f^3}(s_f - s_0)t^3, & 0 \le t \le t_f \\ s_f, & \text{otherwise} \end{cases} \quad (8)$$



$$\dot{s}(t) = \begin{cases} \frac{6}{t_f^2}(s_f - s_0)t - \frac{6}{t_f^3}(s_f - s_0)t^2, & 0 \le t \le t_f \\ 0, & \text{otherwise} \end{cases} \quad (9)$$