L3 – Da Vinci Xi Inverse Kinematics

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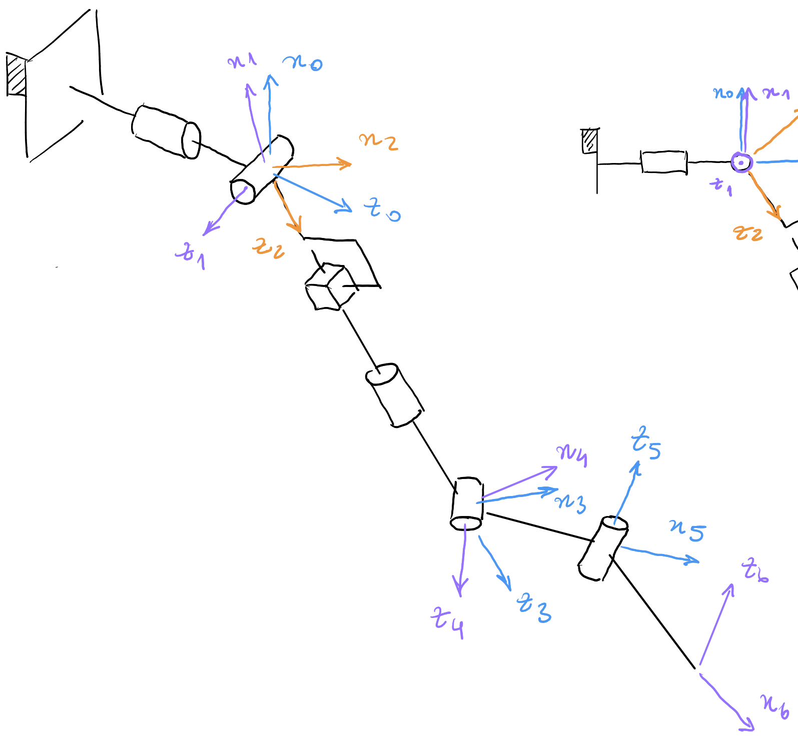
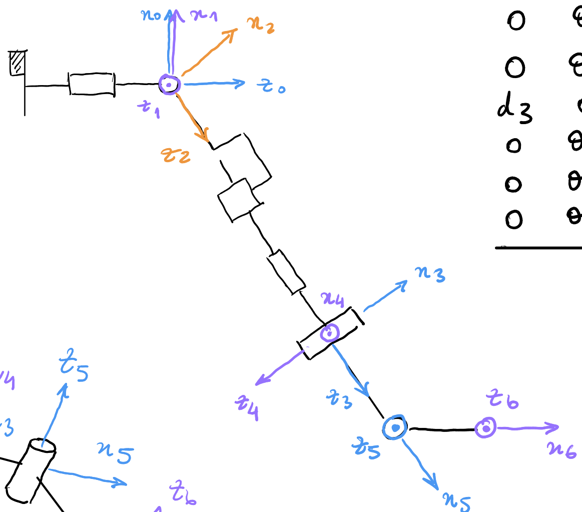
Kinematic Model (Denavit-Hartenberg)





d_i	θ_i	a_i	α_i
0	θ_1	0	$-\pi/2$
0	θ_2	0	$\pi/2$
d_3	0	0	0
0	θ_4	0	$-\pi/2$
0	θ_5	a_5	$-\pi/2$
0	θ_6	a_6	0

Joint Offsets

You may choose to add joint offsets to your Kinematic model. These values will also offset the solution of the inverse kinematics.

Note that the offsets create a new set of coordinates q', such that

$$q = q' + \text{offset} .$$

One possible choice may be

$$\text{offset} = \left(0 \ \ -\frac{\pi}{2} \ \ 0 \ \ \frac{\pi}{2} \ \ -\frac{\pi}{2} \ \ 0\right)^T,$$

as indicated in the Table on the right.

d_i	θ_i	a_i	α_i
0	θ_1'	0	$-\pi/2$
0	$\theta_2' - \pi/2$	0	$\pi/2$
d_3'	0	0	0
0	$\theta_4' + \pi/2$	0	$-\pi/2$
0	$\theta_5' - \pi/2$	a_5	$-\pi/2$
0	θ_6'	a_6	0

The ensuing inverse kinematics solution determines the q coordinates. The q' coordinates are determined at the end

Inverse Kinematics

The inverse kinematics solution determines the joint coordinates corresponding to a given desired position and orientation

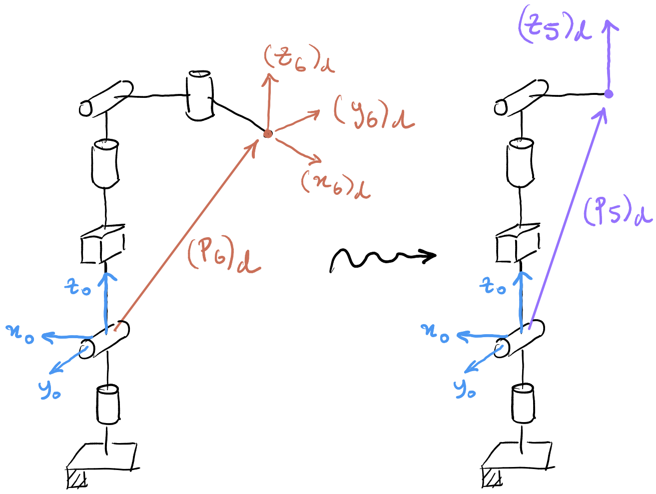
$$(P_6)_d = (e_x \ e_y \ e_z)^T, \quad (R_6)_d = (x_6 \ y_6 \ z_6)_d .$$

Decoupling: The ensuing decoupling allows to partition the solution into the arm solution $(\theta_1, \theta_2, d_3)$ and the wrist solution $(\theta_4, \theta_5, \theta_6)$. That said, given P_6 and R_6 the position of frame 5 is readily available,

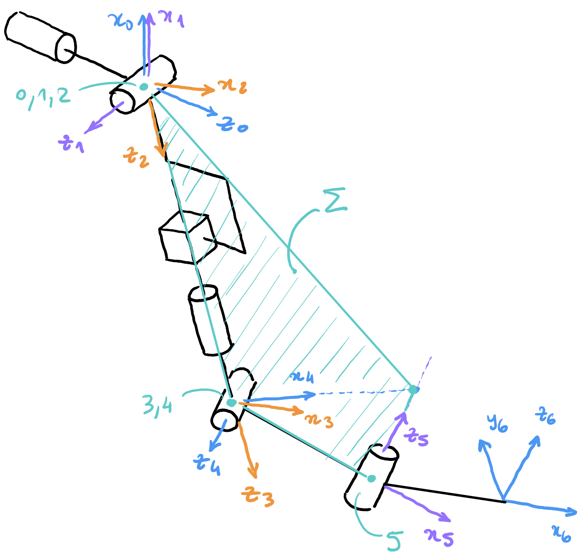
$$P_5 = P_6 - a_6 x_6 .$$

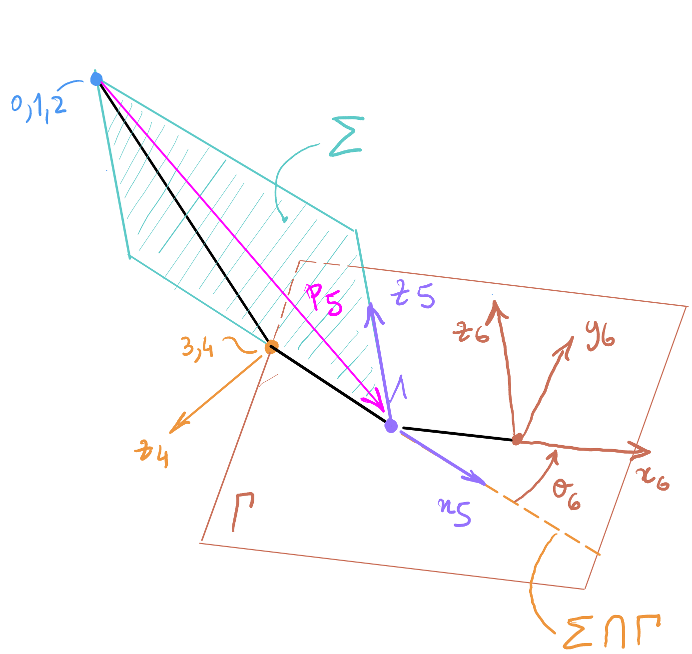
From the kinematic model, it is also true that z_6 and z_5 are parallel and with same sense, therefore

$$z_5 = z_6 .$$



The next step is less straightforward. We start by noting that points P_0, P_1, P_2, P_3, P_4 and P_5 always lie in the same plane (plane Σ). Moreover, z_5 also lies along plane Σ (coplanar), and z_4 is normal to plane Σ.





Finally, note that z_5 is perpendicular to another plane (plane Γ) which is coplanar with x_6 and y_6. Therefore x_5 lies along the intersection between planes Σ and Γ. x_5 can be determined as follows,

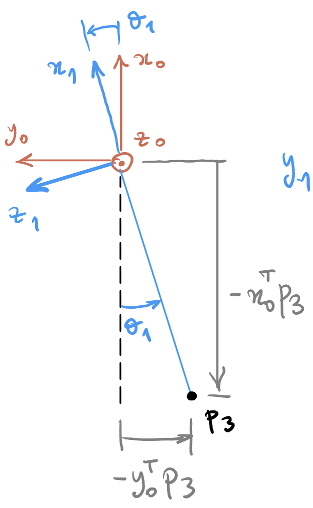
$$z_4 = p_5 \times z_5 \,/\, \|p_5 \times z_5\|,$$

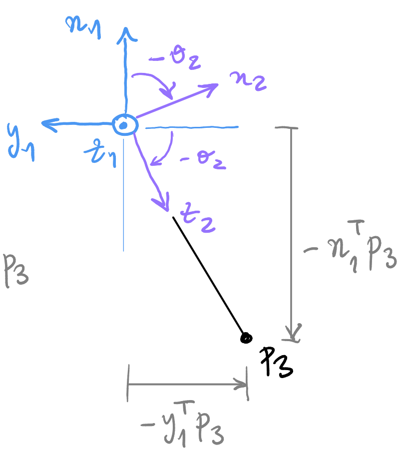
$$x_5 = z_5 \times z_4 \, .$$

The decoupling is complete upon calculation of p_3,

$$p_3 = p_5 - a_5 \, x_5 \, .$$

<u>Arm Solution</u>:





The arm solution follows from projection of p_3 onto planes $\langle x, y \rangle_0$ and $\langle x, y \rangle_1$,

$$\theta_1 = \arctan(-y_0^T p_3, -x_0^T p_3),$$

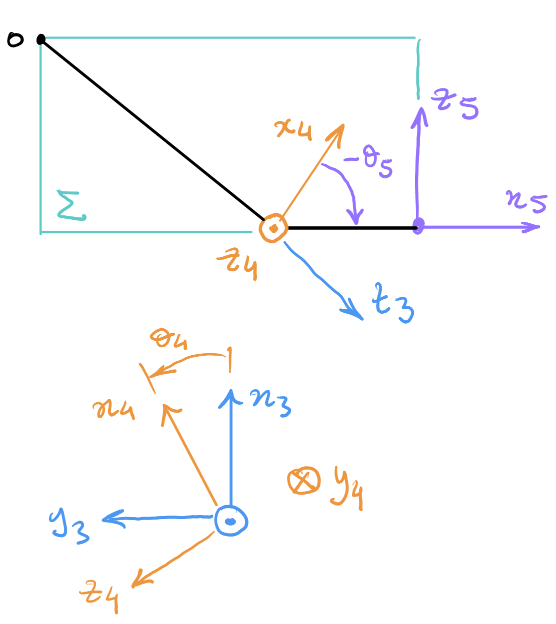
$$\theta_2 = -\arctan(-x_1^T p_3, -y_1^T p_3),$$

$$d_3 = \|p_3\| \, .$$

Note that $(\hat{i}\ \hat{j}\ \hat{k}) = (x_0\ y_0\ z_0)$ and $x_1 = R_1 \hat{i}$, $y_1 = R_1 \hat{j}$, where R_1 is computed from known θ_1,

$$R_1 = \begin{pmatrix} c_1 & 0 & -s_1 \\ s_1 & 0 & c_1 \\ 0 & -1 & 0 \end{pmatrix} .$$

Wrist Solution:



The wrist solution follows along planes Σ and Γ,

$$z_3 = p_3 / \| p_3 \| ,$$

$$x_4 = z_4 \times z_3 ,$$

$$\theta_5 = -\arctan(z_5^T x_4 , x_5^T x_4).$$

$$y_5 = z_5 \times x_5 ,$$

$$\theta_6 = \arctan(y_5^T x_6 , x_5^T x_6) .$$

$$\theta_4 = \arctan(y_3^T x_4 , x_3^T x_4) .$$

Note that $x_3 = R_3 \hat{i}$, $y_3 = R_3 \hat{j}$ where R_3 is computed from known θ_1 and θ_2,

$$R_3^0 = R_1^0 R_2^1 R_3^2 = \begin{pmatrix} c_1 & 0 & -s_1 \\ s_1 & 0 & c_1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ s_2 & 0 & -c_2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_1 c_2 & -s_1 & c_1 s_2 \\ s_1 c_2 & c_1 & s_1 s_2 \\ -s_2 & 0 & c_2 \end{pmatrix} .$$

Joint offsets

The offseted joint coordinates, q', are finally computed from the previous solution, q,

$$q' = q - \text{offset} .$$

Simulink implementation and validation example

```
function q = daVinciIK(R,p)

q=zeros(6,1);
a5=.01;
a6=.02;
ii=[1 0 0].';
jj=[0 1 0].';
kk=[0 0 1].';

p5=p-a6*R*ii;
z5=R*kk;
```