Biomedical Engineering

Masters Degree

Project #1 - Stochastic Optimal Control [EN]

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1 Introduction

In the following report, the group implemented a stochastic optimal control system to simulate the reaching movement of a patient. The goal was to understand how the human internal model reacts to different external inputs and biological delays.

This patient was modelled after his 2 limbs - humerus and the forearm (radius + ulna). The group modelled a one joint and two joint systems, performing experiments on both of them.

2 Model Formulation

The One Joint Model came from the following equations

$$I\ddot{\theta} = -G\dot{\theta} + T \tag{1}$$

$$\tau\dot{T} = u - T \tag{2}$$

which yields,

$$\dot{x}^T = [\dot{\theta}, \ddot{\theta}, \dot{T}, \dot{\theta}_d, \ddot{\theta}_d, \dot{T}_d],, \ d := \text{desired} \tag{3}$$

For,

$$\dot{x} = A_c x + B_c u \tag{4}$$

The group then discretized both matrices using the Euler method for a step time of 1ms. In the above equations G and T are torques and $\tau = 0.06s$ is a 1^{st} order filter. The state variables of $\dot{x}$ represent speed, acceleration and torque of the limb - $\dot{\theta}$, $\ddot{\theta}$, $\dot{T}$.

For the Two Joint Model the group adapted the previous equations doing a state space augmentation such that,

$$\ddot{\theta} = M(\theta)^{-1}(\tau - C(\theta, \dot{\theta}) - B\dot{\theta}) \iff \ddot{\theta} = \underbrace{-M(\theta)^{-1}B}_{A^*}\dot{\theta} + \underbrace{M(\theta)^{-1}}_{B^*}\tau, \ C(\theta, \dot{\theta}) = 0 \tag{5}$$

which yields,

$$\dot{x}^T = [\dot{\theta}_1, \dot{\theta}_2, \ddot{\theta}_1, \ddot{\theta}_2, \dot{T}_1, \dot{T}_2, \dot{\theta}_{1d}, \dot{\theta}_{2d}, \ddot{\theta}_{1d}, \ddot{\theta}_{2d}, \dot{T}_{1d}, \dot{T}_{2d}] \tag{6}$$

where τ is the vector of torques and the state space variables are defined for each limb.

When it comes to **Mechanical Pertubations** the group perfomed a state space augmentation of the model 5, including two external forces that add to the torque vector - $M(\theta)^{-1}(\tau + F_{\text{ext}})$. As for the **delay**, the group performed a state space augmentation to the model 5, such that for, $x_t^T = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, T_1, T_2]$ there exists $z_t^T = [x_t^T, x_{t-1}^T, ..., x_{t-h}^T]$ which yields,

$$z_{t+1} = Az_t + Bu_t \tag{7}$$

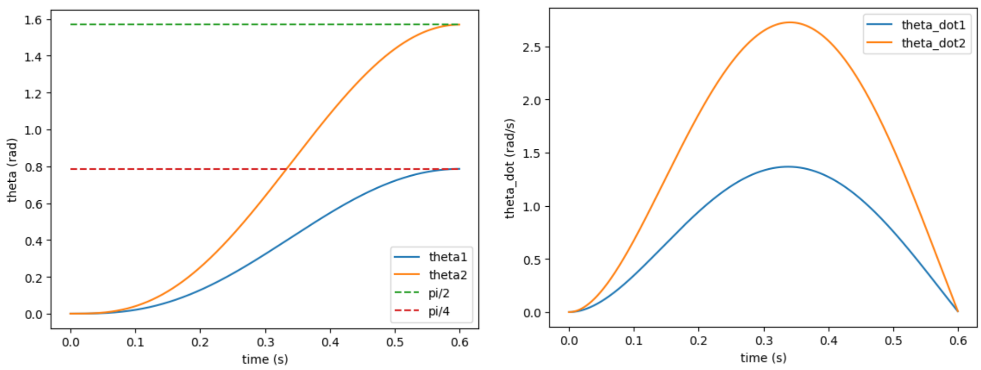
$$y_t = Hz_t + \omega_t \tag{8}$$

where h represents the delay steps, $H = [0_n, 0_n, ..., I_n]$ and the A matrix is equal to the augmented 5 A^*,

$$A = \begin{bmatrix} A & 0 & ... & 0 \\ \mathbb{I} & 0 & ... & 0 \\ 0 & \mathbb{I} & ... & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & ... & \mathbb{I} & 0 \end{bmatrix} \tag{9}$$

3 One Joint Model

The group developed the state space model evolution (4) with the proper backwards and forwards optimal control recursion methods, as provided in the notice, for the implementation of the LQG. In this way the group obtained the following state space evolutions for the shoulder (θ_1) and the elbow (θ_2) acting independently,

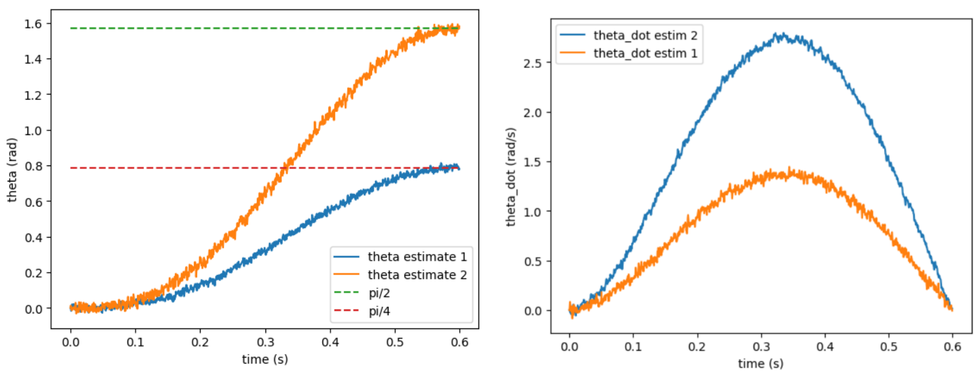


(a) θ Evolution **(b)** $\dot{\theta}$ Evolution

Figure 1: One Joint State Space Evolution

It is possible to see that both limbs reach the desired end positions of 45 and 90 degrees, respectively, while beginning at 0. Moreover, the velocities for both angles begin at 0 rad/s, then progressively increase reaching a maximum at half of the movement, and eventually slowly decrease. This is due to the fact that the final condition imposed on the velocities is to be null, so the subject needs to slow down progressively to reach the target and stop on top of it.

It was then computed the state estimation for the problem. The measurements y_k of the position of the hand, representing a noisy feedback were implemented. A noise term ω_k is present in y_k, due to the uncertainty of the measurement as in, for example, the accuracy of the measurement system. From this noisy measurement it is possible to reconstruct the estimation of the state,



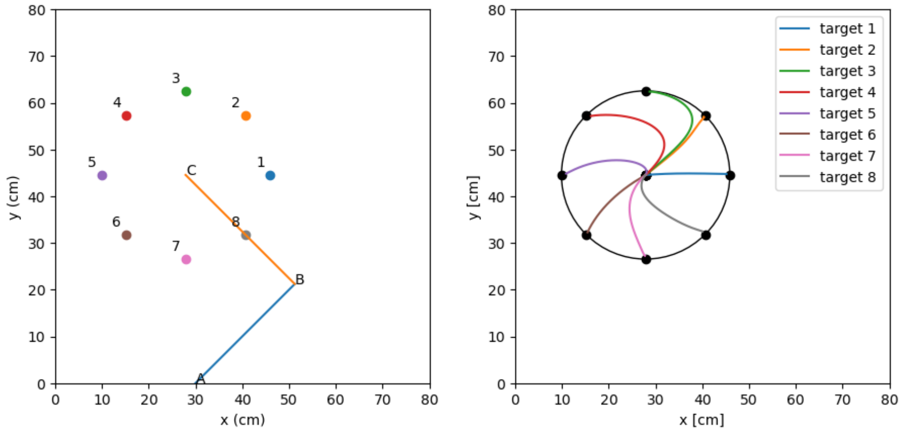
(a) θ Evolution **(b)** $\dot{\theta}$ Evolution

Figure 2: One Joint State Space Evolution with Feedback

This estimation resembles the trend of the true state, but shows also the level of uncertainty of the feedback, as expected.

4 Two-Joint Model

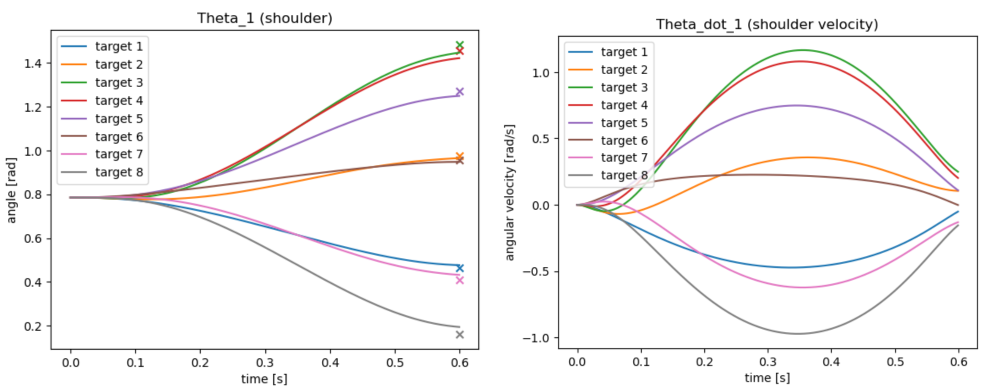
In terms of spatial design, the patient's Two-Joint Model is going to be evaluated in the following manner,



(a) Design for 8 Equidistant Targets **(b)** Hand Trajectories reaching the Targets

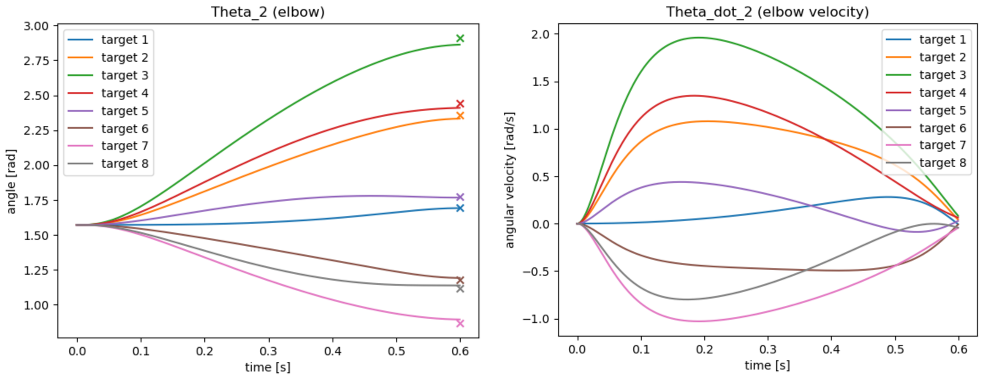
Figure 3: Spatial Display and Reaching Movement of the Patient

The Two-Joint system assumes the above state (3a) as the resting state. In this position, the hand is at the centre of the circle, the forearm begins at an angle of 90 degrees in respect to the humerus, while the shoulder begins at 45 degrees in respect to the horizontal. The hand comes back to the initial position after every reaching movement, as shown in 3b.
The objective angles θ_1 and θ_2 were calculated in every situation where the hand reached a target, using basic trigonometry. These angles are the targets ranging from 1 to 8.



(a) θ_1 Evolution **(b)** $\dot{\theta}_1$ Evolution

Figure 4: θ_1 and $\dot{\theta}_1$ for every reaching movement



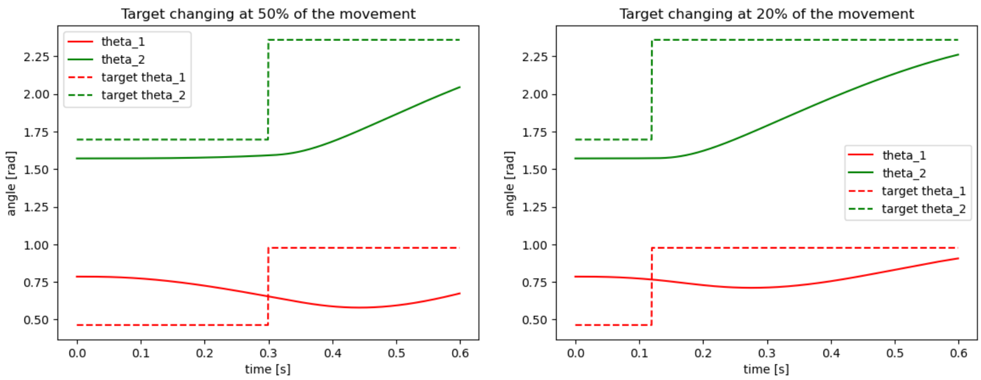
(a) θ_2 Evolution **(b)** $\dot{\theta_2}$ Evolution

Figure 5: θ_2 and $\dot{\theta_2}$ for every reaching movement

All the above plots show that the patient is able to smoothly reach the targets as he starts at 0 rad/s on both limbs and ends at 0 rad/s when the targets are reached. This implies the state space representation to be correctly built.

Next, visual perturbations were implemented, representing a sudden change in the target during the movement.

Below, are represented the evolutions for θ_1 and θ_2 when the first target that the patient sees is target #1, but, at a certain point during the task (halfway (6a) and a fifth-way (6b) into the movement), the target becomes instead target #2.



(a) Target changes at 50% of the task **(b)** Target changes at 20% of the task

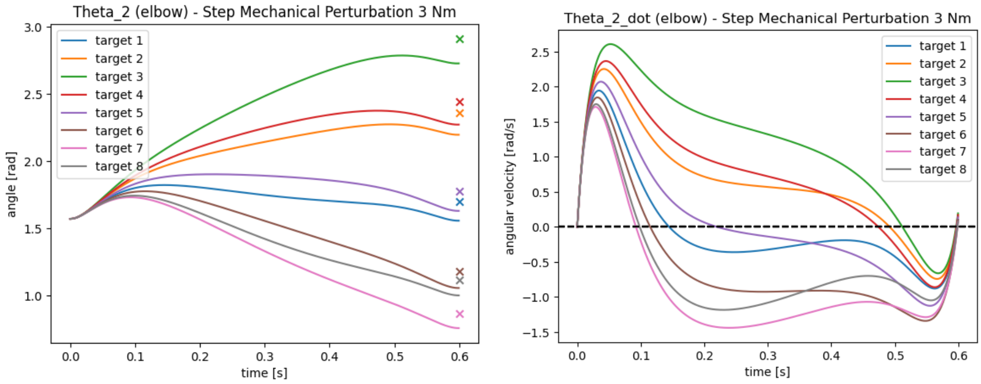
Figure 6: Two Joint State Space Evolution for Target Jump from target #1 to #2 with different timing

In the case where the change of target happens halfway through the movement (6a), the patient does not have enough time to redirect perfectly the movement, even if the second target is close to the first one, (3b), and so he's not able to reach it in 0.6 seconds.

In the second plot (6b), the change of target happens at around 0.12 seconds, and therefore, the patient has more time to redirect the movement, ending it closer to the second, correct, target.

The model behaved as expected as, in theory, the sooner we give the visual input to the patient, the more sucessful he will be.

It was always assumed that there was no internal torque associated with each limb. Therefore, the group simulated external torques being applied to the patient, specifically the patient's elbow (θ_2). In this way, it is possible to evaluate how the Two-Joint system reacts to mechanical perturbations. A note to be discussed later is the nature of these mechanical perturbations.



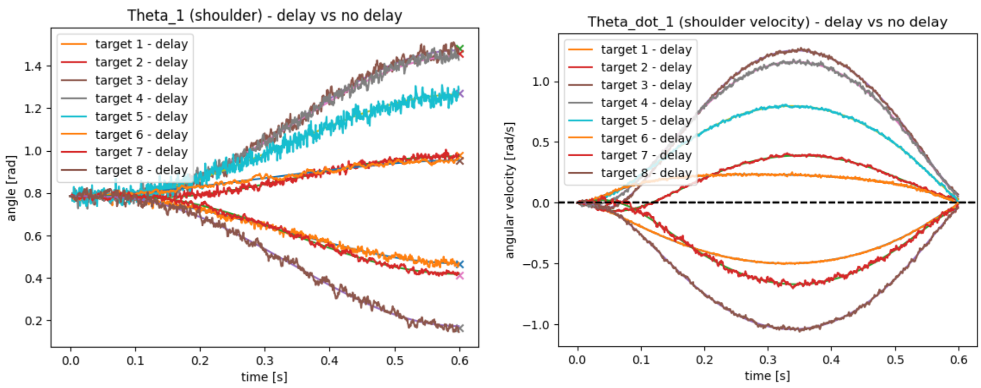
(a) θ_2 Evolution with Step Torque 3 Nm **(b)** $\dot{\theta_2}$ Evolution with Step Torque 3 Nm

Figure 7: θ_2 and $\dot{\theta_2}$ with Step Mechanical Perturbation

The group implemented a step torque as mechanical perturbation. As expected, the patient gets a sudden external force to act on his elbow which leads him to not reach the targets properly. Moreover, the patient's speed reaches higher velocities when in comparison to no outside influence - refer back to 5b where the highest velocity was 2.0 rad/s. In conclusion the patient's speed does not reach 0 rad/s and there is a slight undershoot for each target.

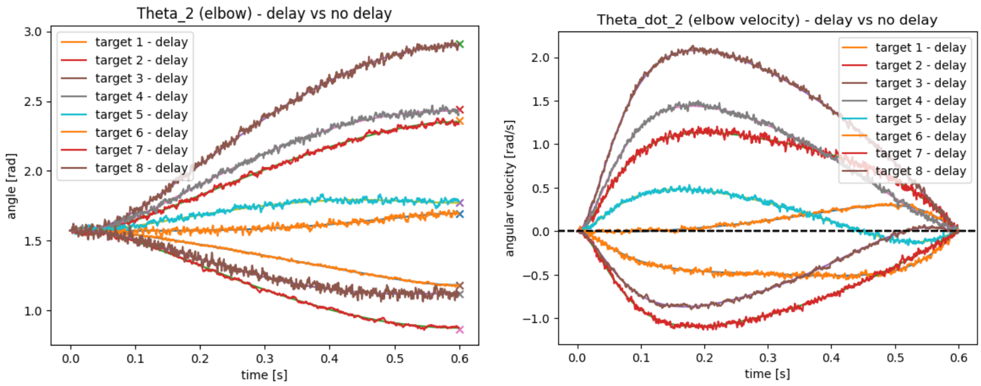
A note on step torque perturbation, is that, it is possible to implement a ramp up or ramp down perturbations, something the group tried to implement with various degress of success. The group simply stated that instead of $\dot{F}_{\text{ext}} = 0 \cdot F_{\text{ext}}$ the relationship would actually be - $\dot{F}_{\text{ext}} \neq 0 \cdot F_{\text{ext}}$. No results for this are shown.

Finally the group implemented the delay sensory feedback as explained before (2), obtaining the following plots for a delay of 4 time steps,



(a) θ_1 Evolution with 4 Delayed Steps **(b)** $\dot{\theta_1}$ Evolution with 4 Delayed Steps

Figure 8: θ_1 and $\dot{\theta_1}$ with Delay



(a) θ_2 Evolution with 4 Delayed Steps **(b)** $\dot{\theta}_2$ Evolution with 4 Delayed Steps

Figure 9: θ_2 and $\dot{\theta}_2$ with 4 Delayed Steps

With the delay implemented, it is possible to see that very much like before, 3, the patient reaches his target, but not without sensory noise. This noise represents the delay embedded in every biological system when it comes to moving a limb. It takes times for the information to travel to the brain and to come back to actuate its task.

Nonetheless, the patient reaches his targets, as expected.

5 Conclusion

With this project finalized, the group came to several conclusions.

Firstly, it is key to understand how modelling a Two-Joint versus a One-Joint system is different. In a Two-Joint system, both limbs constraint each other. The group performed a trignometric analysis, which it didn't go in depth in the report, but this analysis was crucial to implement the kinematics of the connected limbs.

Secondly, it is important to understand that every measurement comes with noise. This doesn't mean however, that it fails to show the overall movement design of the patient.

Thirdly, as it was expected, the sooner a visual perturbation is fed onto the LQG model, the faster it can react and perform the optimal control metrics to correct such movement, and allow the patient to reach the target.

Fourthly, when it comes to mechanical perturbations, which are a daily input into our own Two-Joint system, it is important for the brain to be able to course correct them and allow the patient to perform within standards. In this way it is possible to see, thorugh our LQG model, how our internal brain model reacts to these perturbations

Fifthly, when understanding the impact of biological delays, which exist on every living system, it is possible to see that while there is some noise regarding the state space evolution, the task is still fulfilled, albeist with some noise.

In conclusion, when utilizing an LQG model to understand how the human internal model of the brain acts, it is possible to understand that our brain is able to course correct at any given moment, and complete its tasks if given the time for it, due to visual/mechanical perturbations, as well as control correctly biological delays.