

**- local\_chern\_grid:**

When I was plotting the positions of zero crossings I found that the path through another direction was much more complicated so I can't tell the local chern number (CL) merely from the zero crossings. Then I realized that I had been focusing on the behavior of small eigenvalues from the start and I forgot the definition of CL should be  $1/2 * \text{signature}(\text{localizer})$ . Thus I plot CL on a quarter of the system at different  $t_c$  values (sorry it should be  $h$  I'll correct all the notations next time).

**-local\_chern\_adaptive:**

Similar to **local\_chern\_grid**, this method helps grasp the whole picture because it is faster than the previous method to find the boundaries between different CL. However it'll ignore some discrete points.

**-signature\_change\_origin....:**

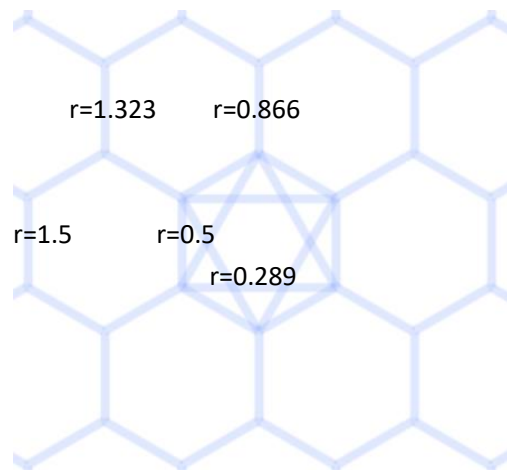
The change of CL at origin, just to make sure we have the same critical values obtained from **sigma\_change\_origin**.

**-energy\_distribution:**

This is the spectrum of Hamiltonian. Based on the histogram, I set the upper energy bound for the edge states as 0.1.

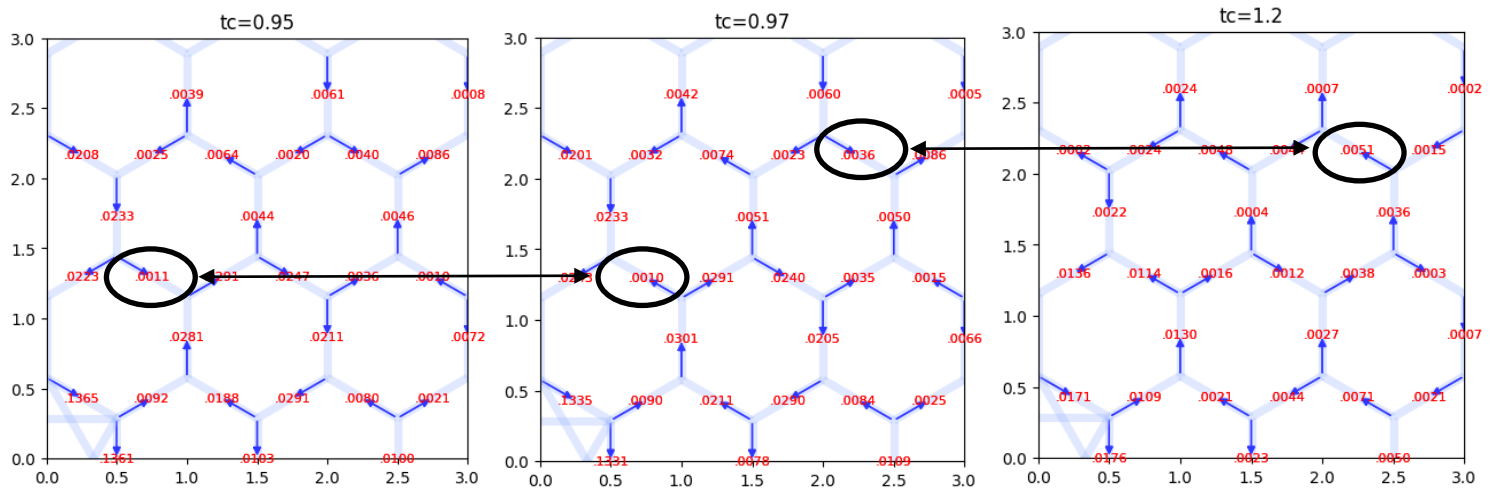
**-current\_Jr:**

This shows  $J(r)$  at different  $t_c$  values. Here  $J(r) = \text{current} * (\text{unit vector of hopping} \cdot \hat{\theta})$  and  $r$  is the distance from the midpoint of hopping to origin. For example:



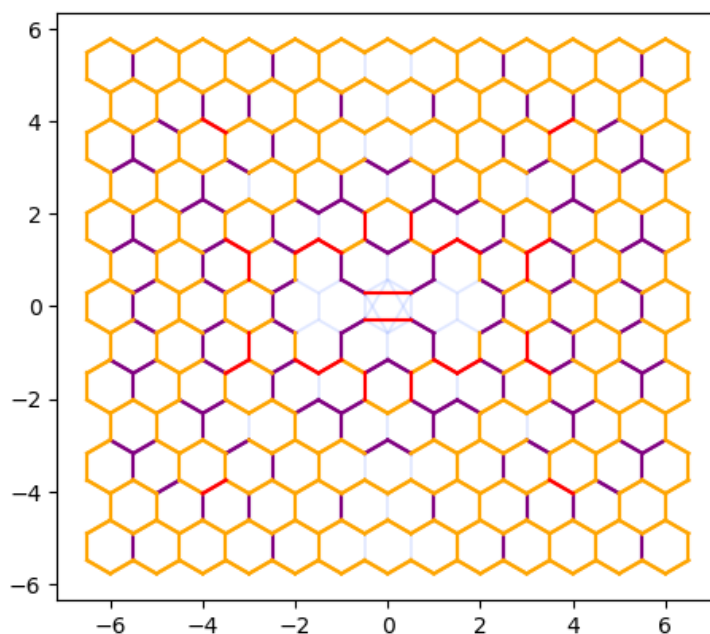
-how to find the physical representation of critical points (0.960 1.195)?

I take a look at the current for each edge and found that its direction changes with  $t_c$ . For example:



I think one clear observation is the circulation (even though  $J(r)$  for completely discrete  $r$  doesn't specifically express circulation) shifts from predominantly clockwise to increasingly counterclockwise, reaching zero eventually.

And in this sketch: the purple edge indicates a change in current direction from  $t_c=0.3$  to  $t_c=0.96$ , orange from  $t_c=0.96$  to  $t_c=1.2$ , red from  $t_c=1.2$  to  $t_c=3$ . But it doesn't show the magnitude of these changes.



My intuition is that when the central  $CL=-1$  (between  $t_c=0.96$  and  $t_c=1.2$ ), the change in circulation might be greater than at other  $t_c$  values but I'm unsure how to quantify the change in "current circulation" within the closed system.