Calculus SC-107

End Semester Examination 2022

Full marks 50, Time 2 Hours 30 Minutes

Student ID: Student Name:

Write the correct answer in the space provided for each of the questions (Q1-Q16). Each question (Q1 to Q16) carry 2 marks. In questions Q17 to Q20, you have to write full answers in the space provided.

1. The principal argument of $z = \frac{i}{1+i}$ is ______________

2. Let z be a complex number such that $\dfrac{z-i}{z-1}$ is purely imaginary. Then the minimum value of $|z-(2+2i)|$ is ______________

3. Let $f(x,y) = x^4+y^4-6(x^2+y^2)+8xy$. Then $f(x,y)$ has local maximum at ____________- and local minimum at ____________ and at ____________ .

4. The value of the integral $\displaystyle\iint_E e^{y/x}\,dx\,dy$, where E is the region bounded by the lines $y = x,\ y = 0$, and $x = 1$ is ________________ .

5. At the point $(1,2)$, the function $f(x,y)$ has derivative 2 in the direction toward $(2,2)$. At the same point $(1,2)$ the function $f(x,y)$ has derivative -2 in the direction toward $(1,1)$. Find the derivative of $f(x,y)$ at $(1,2)$ in the direction toward the point $(4,6)$.

   Correct Answer: ______________

6. Find the area of the region R bounded by one loop of the lemniscate $r^2 = a^2 \sin 2\theta$, where $a > 0$.

   Correct Answer: ________________

7. The value of the integral

$$\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2}dydx$$

is ____________________

8. Consider the function $f(z) = (1-z^3)e^{\left(\frac{1}{z}\right)}$. Then which of the following is true?

(a) $f(z)$ has an essential singularity at $z=0$ with principal part $\sum_{n=1}^{\infty}\left(\frac{1}{n!}+\frac{1}{(n+3)!}\right)\frac{1}{z^n}$

(b) $f(z)$ has a removable singularity at $z=0$

(c) $f(z)$ has an essential singularity at $z=0$ with principal part $\sum_{n=1}^{\infty}\frac{1}{n!}\frac{1}{z^n}$

(d) $f(z)$ has an essential singularity at $z=0$ with principal part $\sum_{n=1}^{\infty}\left(\frac{1}{n!}-\frac{1}{(n+3)!}\right)\frac{1}{z^n}$

Correct Answer: ________________________

9. Does the limit $\lim_{(x,y)\to(0,0)}\frac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$ exist? If yes what is the value?

Correct Answer: ____________________________

10. The radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{(2n!)}{(n!)^2}(z-3i)^n$ is ______________ .

11. The value of the integral $\int_{|z-3|=3}\frac{z^3+3z^2-5}{(z-2i)^2}\ dz$ is ____________________

12. The arc length of the graph of the curve $f(x) = 3x^{2/3} - 10$, from the point $(8,2)$ to $(27,17)$ is _______________________

13. The general solution of $y'' + 2y' = \cos 2x$ is ————————————————

14. The general solution of $\dfrac{dy}{dx} + y \tan x = \sec x + 2x \cos x$ is ————————————-

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15. Consider the region bounded above by the line $y = 1$, bounded below by the curve $y = \sqrt{\cos x}$ and on the sides by the lines $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. The volume of the solid generated by revolving the above region about x-axis is ————————————

16. If $\displaystyle\int_0^{x^2} f(t)dt = x\cos(\pi x)$, then the value of $f(4)$ is ————————————-

17. Evaluate the integrals

(i) $\displaystyle\int_{|z|=1} \frac{z+3}{z^4+az^3}dz, \quad |a|>1$

(ii) $\displaystyle\int_{|z|=5} \frac{z+5}{z^2-3z-4}dz$ [5]

18. Let $f(z) = u + iv$ be an analytic function in a domain D. If $v = u^2$ then prove that $f(z)$ is a constant function. [3]

19. Find the maximum and minimum value of the function $f(x, y) = x^3 + 3xy - y^3$ in the triangular region R with vertices $(1, 2),\ (1, -2)$ and $(-1, -2)$. [5]

20. Solve the differential equation $\dfrac{d^2y}{dx^2} + y = \sec^2 x, \quad \frac{-\pi}{2} < x < \frac{\pi}{2}.$ [5]