

## **RATIONAL INFLATIONARY BUBBLES\***

**Behzad T. DIBA**

*Federal Reserve Bank of Philadelphia, Philadelphia, PA 19106, USA*

**Herschel I. GROSSMAN**

*Brown University, Providence, RI 02912, USA*

Received February 1987, final version received February 1987

This paper shows that if a rational inflationary bubble exists, then it must have started on the date of initial issuance of the fiat money. Moreover, the existence of a rational inflationary bubble would imply that agents who anticipated the initial issuance of the fiat money expected a rational inflationary bubble to occur. The analysis also implies that once a rational inflationary bubble bursts it cannot restart. The limitations on the inception and existence of rational inflationary bubbles also apply to rational exchange-rate bubbles.

### **1. Introduction**

Flood and Garber (1980) – henceforth F&G – utilize the rational-expectations version of Cagan's inflation model to analyze the theoretical and empirical implications of rational inflationary bubbles. This model implies that the logarithm of the market-clearing price level satisfies a first-order linear expectational difference equation with a stochastic forcing term that consists of the variables shifting the demand and supply of money. F&G define the market-fundamentals component of the price level to be the particular solution to this expectational difference equation that is obtained by setting the solution to the associated homogeneous equation equal to zero. They define other solutions to the homogeneous equation to be the rational-bubbles component of the price level.

Defined in this way, the market-fundamentals component relates the current price level uniquely to the parameters of the money demand and supply functions and, except in extreme cases of the forcing processes, to the current and expected future values of the stochastic forcing variables. The existence of a rational-bubbles component would reflect a self-confirming belief that the price level depends on other variables, which are intrinsically irrelevant (that

\*The views expressed are solely those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

is, not part of market fundamentals), or on truly relevant variables in a way that involves parameters that are not part of market fundamentals.

In the F&G model, the assumption that the demand for real money balances depends negatively on the expected rate of inflation implies that the eigenvalue of the expectational difference equation governing price fluctuations is greater than unity. This property of the difference equation has two important consequences. First, it guarantees the existence of an economically meaningful (i.e., forward-looking) market-fundamentals solution except in extreme cases of the forcing processes. Second, it implies that rational bubbles have explosive conditional expectations. Specifically, the expected value of a rational-bubbles component of the price level either would increase or would decrease geometrically into the infinite future.

As Kingston (1982) and Gray (1984) illustrate, the Cagan money demand schedule used by F&G can be derived from models based explicitly on utility maximization by individuals with infinite planning horizons. Because a rational bubble would have an explosive conditional expectation, rational *deflationary* bubbles (and, more generally, positive rational bubbles in the value of any asset) cannot exist if individuals have infinite planning horizons – see Brock (1974), Gray (1984), Obstfeld and Rogoff (1983, 1986), and Tirole (1982). Specifically, adopting the proof of Benveniste and Scheinkman (1982) to the discrete-time rational expectations framework, Brock (1982) shows that a necessary condition for the optimization problem of agents with infinite planning horizons is a ‘transversality condition’ that requires the discounted expected value of their wealth to tend to zero. The existence of a rational deflationary bubble would imply that the discounted expected value of the wealth of individuals holding money balances does not tend to zero and, therefore, would imply that these individuals would expect to gain utility from reducing their money holdings permanently.

Interestingly, however, utility maximization does not preclude the existence of rational *inflationary* bubbles unless money is essential to the economy in the sense that no finite amount of extra consumption could compensate agents for reducing their holdings of inconvertible fiat money to zero – see Brock (1978), Scheinkman (1980), and Obstfeld and Rogoff (1983, 1986). Kingston (1982) shows that the Cagan money-demand schedule used by F&G implies that money is not essential to the economy in this sense.

Although utility maximization does not rule out the existence of rational inflationary bubbles, the inception of a rational bubble after the date of initial issuance of a fiat money would involve an innovation in the price level. This observation, together with the observation that rational deflationary bubbles are not possible, has important implications for the possible inception of rational inflationary bubbles. The present paper explores these implications.

In what follows, section 2 reviews the basic properties of rational bubbles in the F&G model. Section 3 derives the main result that, given the impossibility

of rational deflationary bubbles, a rational inflationary bubble can start only on the date of initial issuance of the fiat money. Section 4 derives the further result that rational bubbles cannot burst and simultaneously restart and discusses the possibility of a rational inflationary bubble that shrinks periodically, but never bursts. Section 5 discusses the relevance of the arguments developed in this paper for recent claims – see, for example, Meese (1986) – that rational bubbles exist in foreign exchange rates. Section 6 generalizes the analysis for a non-linear model of the demand for money based explicitly on utility maximization. Section 7 provides a summary.

## 2. Properties of rational bubbles

F&G analyze the familiar Cagan model of inflation with rational expectations of future inflation replacing Cagan's adaptive expectations. In this model, the current price level satisfies a condition of equality between the real money stock, given by the left-hand side of eq. (1), and the demand for real money balances, given by the right-hand side of (1):

$$M_t - P_t = \alpha_t - \beta(E_t P_{t+1} - P_t), \quad \beta > 0, \quad (1)$$

where

$M_t$  = the logarithm of the nominal money stock at date  $t$ ,

$P_t$  = the logarithm of the price level at date  $t$ ,

$\alpha_t$  = all of the variables that influence demand other than expected inflation, and

$\beta$  = the semi-elasticity of real money demand with respect to expected inflation.

The conditional expectations operator  $E_t$  is based on an information set that contains the current and lagged values of  $M_t$ ,  $P_t$ , and  $\alpha_t$ . Eq. (1) applies for any date  $t$  such that  $t \geq 0$ , where the fiat money was initially issued at date zero.

Rearranging terms in eq. (1) leads to the following linear first-order expectational difference equation:

$$E_t P_{t+1} - (1 + \beta^{-1})P_t = \beta^{-1}(\alpha_t - M_t). \quad (2)$$

Because the eigenvalue,  $1 + \beta^{-1}$ , is greater than unity, the forward-looking solution for  $P_t$  involves a convergent sum, as long as the sequence  $\{E_t(M_{t+i} - \alpha_{t+i})\}_{i=1}^{\infty}$  does not grow at a geometric rate equal to or greater than  $1 + \beta^{-1}$ . The forward-looking solution, denoted by  $F_t$  and referred to as the market-

fundamentals component of the price level, is

$$F_t = (1 + \beta)^{-1} \left[ (M_t - \alpha_t) + \sum_{i=1}^{\infty} (1 + \beta^{-1})^{-i} E_t (M_{t+i} - \alpha_{t+i}) \right]. \quad (3)$$

Eq. (3) says that  $F_t$  is proportionate to a weighted sum of current and expected future realizations of the money supply and the variables that shift money demand.

The general solution to eq. (2) for  $P_t$  is the sum of the market-fundamentals component,  $F_t$ , and the rational-bubbles component,  $B_t$  – that is,

$$P_t = B_t + F_t, \quad (4)$$

where  $B_t$  is the solution to the homogeneous expectational difference equation,

$$E_t B_{t+1} - (1 + \beta^{-1}) B_t = 0. \quad (5)$$

A non-zero value of  $B_t$  would reflect the existence of a rational bubble at date  $t$  – that is, a self-confirming belief that the price level does not conform to the market-fundamentals component,  $F_t$ . A positive value of  $B_t$  would represent a rational inflationary bubble and would imply that the fiat money is undervalued (relative to market fundamentals) at date  $t$ . A negative value of  $B_t$  would represent a rational deflationary bubble and would imply that the fiat money is overvalued at date  $t$ .

Solutions to eq. (5) satisfy the stochastic difference equation

$$B_{t+1} - (1 + \beta^{-1}) B_t = z_{t+1}, \quad (6)$$

where  $z_{t+1}$  is a random variable (or combination of random variables) generated by a stochastic process that satisfies

$$E_{t-j} z_{t+1} = 0 \quad \text{for all } j \geq 0. \quad (7)$$

The key to the relevance of eq. (6) for the general solution for  $P_t$  is that eq. (5) relates  $B_t$  to  $E_t B_{t+1}$ , rather than to  $B_{t+1}$  itself<sup>6</sup> as would be the case in a perfect-foresight model.

The random variable  $z_{t+1}$  is an innovation, comprising new information available at date  $t+1$ . This information can be intrinsically irrelevant – that is, unrelated to  $F_{t+1}$  – or it can be related to truly relevant variables, like  $\alpha_{t+1}$  and  $M_{t+1}$ , through parameters that are not present in  $F_{t+1}$ . The critical property of  $z_{t+1}$ , given by eq. (7), is that its expected future values are always zero.

The general solution to eq. (6) for any date  $t$ ,  $t \geq 0$ , is

$$B_t = (1 + \beta^{-1})^t B_0 + \sum_{\tau=1}^t (1 + \beta^{-1})^{t-\tau} z_{\tau}. \quad (8)$$

Eq. (8) expresses the rational-bubbles component at date  $t$  as composed of two terms. The first term is the product of the eigenvalue raised to the power  $t$  and the value of the rational-bubbles component at date zero. The second term is a weighted sum of realizations of  $z_{\tau}$  from  $\tau = 1$  to  $\tau = t$ . The weights are powers of the eigenvalues such that the contribution of  $z_{\tau}$  to  $B_t$  increases exponentially with the difference between  $t$  and  $\tau$ . For example, a past realization  $z_{\tau}$ ,  $1 \leq \tau < t$ , contributed only the amount  $z_{\tau}$  to  $B_{\tau}$ , but contributes  $(1 + \beta^{-1})^{t-\tau} z_{\tau}$  to  $B_t$ . Blanchard and Watson (1982) suggest, as an empirically interesting specification for a rational-bubbles component, a process in which the analog to  $z_{\tau}$  is not covariance stationary and in which rational bubbles can burst and restart repeatedly.

The assumption of rational expectations implies that in forming  $E_t B_{t+j}$ , for all  $j > 0$ , agents behave as if they know that any rational-bubbles component of the price level would conform to eq. (5) in all future periods. Accordingly, eq. (5) implies that the expectations would have the property

$$E_t B_{t+j} = (1 + \beta^{-1})^j B_t \quad \text{for all } j > 0. \quad (9)$$

Eq. (9) says that the existence of a non-zero rational-bubbles component at date  $t$  would imply that the expected value of the rational-bubbles component at date  $t+j$  either increases or decreases with  $j$  at the geometric rate  $1 + \beta^{-1}$ . Therefore, the existence of a rational bubble would imply that the expected value of the logarithm of the price level,  $\{E_t P_{t+j}\}_{j=1}^{\infty}$ , either increases or decreases without bound at approximately the geometric rate  $1 + \beta^{-1}$ . In particular, the existence of a rational deflationary bubble at date  $t$  would imply that the expected future value of a unit of fiat money (in units of consumption goods) grows without bound at this increasing proportionate rate. Accordingly, if, as discussed above, agents cannot rationally expect the value of a unit of fiat money to grow in this way, then rational deflationary bubbles cannot exist.

### 3. The inception of rational inflationary bubbles

Given that rational deflationary bubbles are not possible, the rational-bubbles component of the price level as given by eq. (6) also satisfies  $B_{t+1} \geq 0$ .

Consequently, the realization of  $z_{t+1}$  must satisfy

$$z_{t+1} \geq -(1 + \beta^{-1})B_t \quad \text{for all } t \geq 0. \quad (10)$$

Eq. (10) says that the realization  $z_{t+1}$  must be large enough to ensure that eq. (6) implies a non-negative value for  $B_{t+1}$ .

Suppose that  $B_t$  equals zero. In that case, eq. (10) implies that  $z_{t+1}$  must be non-negative. But, eq. (7) says that the expected value of  $z_{t+1}$  is zero. Thus, if  $B_t$  equals zero, then  $z_{t+1}$  equals zero with probability one.

This result says that if a rational bubble does not exist at date  $t$ ,  $t \geq 0$ , a rational bubble cannot get started at date  $t+1$ , nor, by extension, at any subsequent date. Therefore, if a rational bubble exists, it must have started at date zero, the date of initial issuance of the fiat money, and hence, this fiat money must have always been undervalued relative to market fundamentals. The essential idea underlying this line of argument is that, because the inception of a rational bubble at any date after the introduction of the fiat money would involve an innovation in the price level, the expected initial values of a rational inflationary bubble and a rational deflationary bubble would have to be equal. Accordingly, if a deflationary rational-bubbles component cannot exist, then an inflationary rational-bubbles component also cannot start after the date of initial issuance of a fiat money.

Suppose that, prior to the issuance of a new fiat money, agents anticipate its introduction and form an expectation about the initial price level. Suppose further that this expectation coincides with market fundamentals – that is,

$$E_{-1}\tilde{B}_0 = E_{-1}P_0 - E_{-1}F_0 = 0. \quad (11)$$

Eq. (11) would imply that  $B_0$  is a random variable with mean zero. Accordingly, given the non-negativity condition  $B_0 \geq 0$ ,  $B_0$  would equal zero with probability one.

This observation implies that if a rational inflationary bubble exists, those agents who anticipated the introduction of the new fiat money expected it to be undervalued relative to market fundamentals. In addition, the inception of a rational inflationary bubble would require incompleteness of markets. Specifically, a rational inflationary bubble could start only if prior to the issuance of a new fiat money agents could not negotiate contingent contracts to exchange the new fiat money for goods or for other assets.

#### 4. Can rational inflationary bubbles burst and restart?

Consider the following model of the innovations  $z_{t+1}$ :

$$z_{t+1} = [\theta_{t+1} - (1 + \beta^{-1})]B_t + \varepsilon_{t+1}. \quad (12)$$

where  $\epsilon_{t+1}$  and  $\theta_{t+1}$  are mutually and serially independent random variables. If the processes generating  $\theta_{t+1}$  and  $\epsilon_{t+1}$  satisfy

$$E_{t-j}\theta_{t+1} = 1 + \beta^{-1} \quad \text{for all } j \geq 0, \quad (13)$$

$$E_{t-j}\epsilon_{t+1} = 0 \quad \text{for all } j \geq 0, \quad (14)$$

then  $z_{t+1}$  as given by eq. (12) satisfies eq. (7).

Substituting for  $z_{t+1}$  in eq. (6) from eq. (12) gives

$$B_{t+1} = \theta_{t+1}B_t + \epsilon_{t+1}. \quad (15)$$

Quah (1985) suggests the model of the rational-bubbles component given by eq. (15) as a generalization of the specification assumed by Blanchard and Watson (1982). Eq. (15) says that, with  $z_{t+1}$  given by eq. (12), an existing rational-bubbles component,  $B_t$ , will burst next period if the event  $\theta_{t+1} = 0$  occurs. If this event has positive probability, then any rational-bubbles component would burst at a random, but almost surely finite, future date. Specifically, if the probability associated with  $\theta_{t+1} = 0$  is  $\Pi$ ,  $0 < \Pi < 1$ , then the expected duration of a rational-bubbles component is  $\Pi^{-1}$  periods and the probability that  $B_t$  will not burst by date  $T$  ( $T > t$ ) is  $(1 - \Pi)^{T-t}$ , which tends to zero as  $T$  approaches infinity.

Given that realizations of  $\theta_{t+1}$  and  $\epsilon_{t+1}$  are mutually and serially independent and also independent of  $B_0$ , then  $\epsilon_{t+1}$  is independent of  $B_t$  for all  $t \geq 0$ . In this case, if the event  $\theta_{t+1} = 0$  were by chance to coincide with a positive realization of  $\epsilon_{t+1}$ , then, according to eq. (15), as an existing rational-bubbles component bursts, a new rational-bubbles component, which is independent of all existing and past rational-bubbles components, would simultaneously start.

In this model, the impossibility of rational deflationary bubbles would imply that, in addition to satisfying eq. (15), the rational-bubbles component satisfies  $B_{t+1} \geq 0$ . Therefore, the event  $\theta_{t+1} = 0$  cannot coincide with a negative realization of  $\epsilon_{t+1}$ . Accordingly, given that the event  $\theta_{t+1} = 0$  has positive probability and that the random variables  $\epsilon_{t+1}$  and  $\theta_{t+1}$  are independent,  $\epsilon_{t+1}$  must be non-negative. But, eq. (14) says that the expected value of  $\epsilon_{t+1}$  is zero. Therefore,  $\epsilon_{t+1}$  equals zero with probability one and the chance coincidence of  $\theta_{t+1} = 0$  and  $\epsilon_{t+1} > 0$  has zero probability.

This result says that the impossibility of rational deflationary bubbles, in addition to implying that an inflationary rational-bubbles component that bursts could not restart at a later date, also precludes the possibility that a new independent inflationary rational bubble could simultaneously start when an

existing inflationary rational bubble bursts. In sum, the analysis of sections 3 and 4 has shown that, given the impossibility of rational deflationary bubbles, an inflationary rational-bubbles component can start only on the date of initial issuance of the fiat money and either must continue to exist forever or, as in Blanchard's (1979) specification, must burst at a date in the finite future and never restart.

These restrictions do not preclude a rational inflationary bubble that began on the date of initial issuance and, while never bursting, periodically shrinks. An example of such a rational-bubbles component would be

$$B_{t+1} = \begin{cases} \delta B_t + \varepsilon_{t+1} & \text{with probability } \Pi, \\ (1 - \Pi)^{-1}(1 + \beta^{-1} - \delta\Pi)B_t + \varepsilon_{t+1} & \text{with probability } 1 - \Pi, \end{cases} \quad (16)$$

where  $\delta$  is a small positive constant and where  $E_t \varepsilon_{t+1} = 0$  and  $B_0 > 0$ . This specification corresponds to setting  $\theta_{t+1}$  in eq. (15) equal to  $\delta$  with probability  $\Pi$  and equal to  $(1 - \Pi)^{-1}(1 + \beta^{-1} - \delta\Pi)$  with probability  $1 - \Pi$  and allowing  $\varepsilon_{t+1}$  to depend on  $B_t$  and  $\theta_{t+1}$  in such a way that  $B_{t+1}$  remains non-negative with probability one. In particular, given  $\theta_{t+1} = \delta$ , realizations of  $\varepsilon_{t+1}$  must satisfy  $\varepsilon_{t+1} \geq -\delta B_t$ . Eq. (16) specifies an inflationary rational-bubbles component that starts on the date of initial issuance, that collapses with probability  $\Pi$  in any period, but that, given  $\delta$  greater than zero and the appropriate restriction on the realizations of  $\varepsilon_{t+1}$ , always remains positive.

## 5. Rational bubbles in exchange rates?

Although the analysis in this paper focuses on the determination of the value of a fiat money in units of goods and services, it also has implications for the determination of this value in units of foreign currency. Utilizing a model that is formally identical to the model discussed in section 2, Meese (1986) suggests that rational bubbles that burst and restart occurred in foreign exchange rates during the 1970's and 1980's. Woc (1985) also suggests that in this period episodes during which exchange rates conformed to market fundamentals alternated with episodes during which rational bubbles were present.

As Singleton (1987) points out, any rational bubble in an exchange rate would have to be reflected either in a rational bubble in the price level at home or abroad or in a rational bubble in the deviation from purchasing power parity. But a rational bubble in the deviation from purchasing power parity cannot exist, because agents cannot expect unexploited potential profits from commodity arbitrage to grow geometrically without bound. Accordingly, given



the impossibility of rational deflationary bubbles, any rational bubble in exchange rates would have to coincide with a rational inflationary bubble in the depreciating currency.

The analysis in the preceding sections thus implies that the inception of a rational exchange-rate bubble can only occur at the first date of circulation of a fiat money. In particular, the rational-bubbles component of the value of a currency could neither burst and restart repeatedly – as in Meese's specification – nor only exist during certain periods – as in Woo's specification. As Hamilton and Whiteman (1985) demonstrate, the existence of rational bubbles is empirically indistinguishable from misspecification of market fundamentals. Accordingly, the correct interpretation of the econometric findings of Meese and Woo would seem to be that the models they study misspecify the market-fundamentals component of the exchange rate.

## 6. A non-linear model based explicitly on utility maximization

As Kingston (1982) points out, the F&G model can be derived as a special case of the model of Brock (1974, 1975), in which agents with infinite planning horizons choose the paths of consumption and holdings of money balances to maximize their utility. The specification of the utility function that leads to the F&G model is restrictive, but it has the advantage of yielding a linear difference equation for price dynamics, eq. (2) above. The linearity of this equation makes explicit characterization of the market-fundamentals and rational-bubbles components of the price level possible without assuming that the money supply and other forcing variables are constant over time or grow at a constant rate. The linearity of the difference equation (2) was also convenient for developing the analysis of the inception of a rational inflationary bubble. Specifically, eq. (2) implied that a rational-bubbles component in the F&G model would have to satisfy the linear stochastic difference equation (6). Setting  $B_t$  equal to zero, then, shows that the inception of a rational-bubbles component at any date after the introduction of a new fiat money must involve an innovation in the price level.

In a general specification of Brock's model, eq. (6) has no counterpart. The following analysis, however, shows that an argument analogous to that of the preceding sections implies similar limits on the inception of rational inflationary bubbles in the general (non-linear) model.

Assume that a representative household maximizes expected utility over an infinite horizon,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [u(c_{\tau}) + v(x_{\tau})], \quad 0 < \beta < 1, \quad (17)$$

where  $c_{\tau}$  and  $x_{\tau}$  represent, respectively, consumption of a single perishable good and holdings of real money balances at date  $\tau$ , and  $\beta$  is a discount

factor. The functions  $u(\cdot)$  and  $v(\cdot)$  are monotone increasing, strictly concave and continuously differentiable on  $(0, \infty)$ . In addition, the function  $v(\cdot)$  satisfies the Inada conditions:

$$\lim_{x \rightarrow 0} v'(x) = \infty \quad \text{and} \quad \lim_{x \rightarrow \infty} v'(x) = 0.$$

The household receives an initial endowment,  $m$ , of (nominal) money balances before date zero and a constant endowment,  $y$ , of the consumption good each period beginning at date zero. The household takes the price level,  $p_t$ , as given and chooses consumption,  $c_t$ , and nominal money balances,  $m_t$ , subject to the budget constraint

$$p_t c_t + m_t - m_{t-1} \leq p_t y. \quad (18)$$

The first-order condition for the household's utility maximization problem is

$$\beta E_t \left( \frac{u'(c_{t+1})}{p_{t+1}} \right) = \frac{u'(c_t)}{p_t} - \frac{v'(x_t)}{p_t}. \quad (19)$$

Incorporating the market-clearing conditions,  $m_t = m$  and  $c_t = y$  for all  $t \geq 0$ , and multiplying both sides of eq. (19) by  $m$  yields

$$\beta E_t x_{t+1} = \left[ 1 - \frac{v'(x_t)}{u'(y)} \right] x_t. \quad (20)$$

Eq. (20) is a non-linear first-order expectational difference equation in real money balances.

Define the market-fundamentals component,  $f_t$ , of real money balances to be the positive non-stochastic steady-state solution to eq. (20) – that is,

$$f_t = x^*, \quad (21)$$

where  $x^*$  is the unique solution to

$$v'(x^*) = (1 - \beta)u'(y). \quad (22)$$

Define the rational-bubbles component of real money balances,  $b_t$ , as any divergence from  $f_t$  – that is,

$$b_t = x_t - f_t. \quad (23)$$

In eq. (23), a positive rational-bubbles component, which would represent a rational deflationary bubble, could not exist for any plausible specification of the function  $v(\cdot)$ . Specifically, as Gray (1984) argues, a plausible model of how

money enhances utility would imply that, for a given level of consumption, the utility derived from holding real money balances is bounded from above. Boundedness of the function  $v(\cdot)$ , in turn, would imply that a transversality condition, necessary for the optimality of the household's decisions, rules out the existence of a rational deflationary bubble. In fact, weaker conditions than boundedness of the function  $v(\cdot)$  are sufficient for ruling out rational deflationary bubbles – see Benveniste and Scheinkman (1982), Brock (1974, 1982), and Obstfeld and Rogoff (1986).

Consider now the implications of the impossibility of rational deflationary bubbles in this model for the possible inception of a rational inflationary bubble. Specifically, assume that real money balances conform to market fundamentals – that is,  $x_t = f_t = x^*$  – for some  $t \geq 0$ . Eqs. (20) and (22) would then imply

$$E_t x_{t+1} = x^*. \quad (24)$$

Eq. (24), in turn, implies that, given  $b_t = 0$ , the expected value of the rational-bubbles component at date  $t + 1$ ,  $E_t b_{t+1} = E_t x_{t+1} - x^*$ , equals zero. In other words, the inception of a rational-bubbles component after date zero would involve an innovation in real money balances. Accordingly, as the analysis of the linear model demonstrated, the impossibility of rational deflationary bubbles would imply that a rational inflationary bubble cannot exist at any date unless it existed during all previous dates since the initial issuance of the fiat money. Moreover, an argument similar to that for the linear model would show that the existence of a rational inflationary bubble at any date implies that agents who anticipated the introduction of the fiat money expected a rational inflationary bubble to occur.

## 7. Summary

The inception of a rational-bubbles component after the date of initial issuance of a fiat money would involve an innovation in the price level. Accordingly, any rational-bubbles component that starts after the introduction of fiat money has an expected initial value of zero. But, given the impossibility of rational deflationary bubbles, this initial value must be non-negative and therefore, in order to have a mean of zero, must equal zero with probability one.

This argument means that the impossibility of rational deflationary bubbles also rules out the inception of a rational inflationary bubble except at the date of initial issuance of a fiat money. Thus, the existence of a rational inflationary bubble at any date implies that a rational inflationary bubble has been present since the introduction of the fiat money. Moreover, the existence of a rational inflationary bubble implies that agents who anticipated the introduction of the fiat money expected a rational inflationary bubble to occur.

The analysis also implies that once a rational inflationary bubble bursts it cannot restart. In particular, rational inflationary bubbles cannot conform to the specification suggested by Blanchard and Watson (1982) in which rational bubbles start, burst, and restart repeatedly. This analysis, however, does not preclude the existence of a rational inflationary bubble that begins on the date of issuance of the fiat money and shrinks periodically, but never bursts.

Finally, because a rational bubble in exchange rates would imply the existence of a rational inflationary bubble in the depreciating currency, the same restrictions apply to the inception and existence of rational bubbles in exchange rates. Thus, results that suggest the existence of rational exchange-rate bubbles should be interpreted more correctly as symptoms of misspecification of market fundamentals.

## References

- Benveniste, L.M. and J.A. Scheinkman, 1982, Duality theory for dynamic optimization models of economics: The continuous time case, *Journal of Economic Theory* 27, 1-19.
- Blanchard, O.J., 1979, Speculative bubbles, crashes, and rational expectations, *Economics Letters* 3, 387-389.
- Blanchard, O.J. and M.W. Watson, 1982, Bubbles, rational expectations, and financial markets, in: P. Wachtel, ed., *Crises in the economic and financial structure* (Lexington Books, Lexington, MA) 295-315.
- Brock, W.A., 1974, Money and growth: The case of long-run perfect foresight, *International Economic Review* 15, 750-777.
- Brock, W.A., 1975, A simple perfect foresight monetary model, *Journal of Monetary Economics* 1, 133-150.
- Brock, W.A., 1978, A note on hyper-inflationary equilibria in long run perfect foresight monetary models: A correction, Unpublished.
- Brock, W.A., 1982, Asset prices in a production economy, in: J.J. McCall, ed., *The economics of information and uncertainty* (University of Chicago Press, Chicago, IL) 1-43.
- Flood, R. and P. Garber, 1980, Market fundamentals versus price level bubbles: The first tests, *Journal of Political Economy* 88, 745-770.
- Gray, J.A., 1984, Dynamic instability in rational expectations models: An attempt to clarify, *International Economic Review* 25, 93-122.
- Hamilton, J.D. and C.H. Whiteman, 1985, The observable implications of self-fulfilling expectations, *Journal of Monetary Economics* 16, 353-373.
- Kingston, G.H., 1982, The semi-log portfolio balance schedule is tenuous, *Journal of Monetary Economics* 9, 389-399.
- Meese, R.A., 1980, Testing for bubbles in exchange markets: A case of sparkling rates?, *Journal of Political Economy* 94, 345-373.
- Obstfeld, M. and K. Rogoff, 1983, Speculative hyperinflations in maximizing models: Can we rule them out?, *Journal of Political Economy* 91, 675-687.
- Obstfeld, M. and K. Rogoff, 1986, Ruling out divergent speculative bubbles, *Journal of Monetary Economics* 17, 349-362.
- Quah, D., 1985, Estimation of a nonfundamentals model for stock price and dividend dynamics, Unpublished.
- Scheinkman, J.A., 1980, Discussion, in: J.H. Kareken and N. Wallace, eds., *Models of monetary economics* (Federal Reserve Bank of Minneapolis, MN) 91-96.
- Singleton, K.J., 1987, Speculation and the volatility of foreign currency exchange rates, *Carnegie-Rochester Conference Series on Public Policy* 26, 9-56.
- Tirole, J., 1982, On the possibility of speculation under rational expectations, *Econometrica* 50, 1163-1181.
- Woo, W.T., 1985, Some evidence of speculative bubbles in the foreign exchange markets, Unpublished.