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ON THE INCEPTION OF RATIONAL BUBBLES

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I. A GENERIC MODEL OF ASSET PRICES

The recent literature includes several empirical and theoretical analyses of rational asset-price bubbles in the context of linear (or log-linear) rational expectations models of the form,

$$E_t y_{t+1} = \lambda y_t - x_t. \tag{1}$$

Equation (1) is a first-order expectational difference equation in y with a forcing variable x. The conditional expectations operator E_t is based on an information set that includes the current and past realizations of x and y. In this class of models, the assumption that the demand for the asset depends positively on its expected rate of return implies that the eigenvalue λ exceeds unity.

In one application of this model, the variable y represents the product of the marginal utility of consumption and the price of a stock, and the variable x reflects movements in the product of the marginal utility of consumption and the dividends paid to stockholders (see, for example, Blanchard and Watson [1982], Quah [1985], Diba and Grossman [1987a], and West [1986]). In another application, y represents the logarithm of the price level, as in Flood and Garber [1980] and Diba and Grossman [1987b], or of the exchange rate, as in Meese [1986], and x captures the determinants of money supply and money demand.

Given that the eigenvalue λ is greater than unity, the forward-looking solution for y_t involves a convergent sum, so long as $E_t x_{t+j}$ does not grow with j at a geometric rate equal to or greater than λ. The forward-looking solution, denoted by F_t and referred to as the market-fundamentals component of y_t, is

$$y_t = E_t \sum_{i=0}^{\infty} \lambda^{-(i+1)} x_{t+1}. \tag{2}$$

The general solution to equation (1) is the sum of the market-fundamentals component F_t and a potential rational-bubbles component B_t—that is,

$$y_t = B_t + F_t, \tag{3}$$

where B_t is the solution to the homogeneous expectational differ-

The Quarterly Journal of Economics, August 1987

ence equation,

$$E_tB_{t+1} - \lambda B_t = 0. \tag{4}$$

A nonzero value of B_t would reflect a self-confirming belief that y_t does not conform to the market-fundamentals component F_t.

Solutions to equation (4) satisfy the stochastic difference equation,

$$B_{t+1} - \lambda B_t = z_{t+1}, \tag{5}$$

where z_{t+1} is a random variable with the property,

$$E_{t-j}z_{t+1} = 0 \qquad \text{for all } j \geq 0. \tag{6}$$

In other words, z_{t+1} is an innovation comprising new information available at date $t + 1$. This information can be intrinsically irrelevant—that is, unrelated to F_{t+1}—or it can be related to truly relevant variables, like x_{t+1}, through parameters that are not present in F_{t+1}. The critical property of z_{t+1}, given by equation (6), is that its expected future values are always zero.

If equation (1) constituted the entire model, then the rational bubbles generated by equation (5) could have empirically interesting properties. For example, as Shiller [1978] suggests, a rational bubble could start at any time and, as in the specification of Blanchard and Watson, could burst and restart repeatedly.

In the applications mentioned above, however, equation (1) does not constitute the entire model. For example, the models of stock-price determination cited above imply that in addition to equation (1), y must satisfy a nonnegativity constraint implied by free disposal of shares. An optimizing model of demand for a real asset like gold that does not pay dividends but is intrinsically useful would also imply a nonnegativity constraint. The following section explores the implications of such a nonnegativity constraint for the inception of rational bubbles.

II. The Inception of Rational Bubbles in Stock Prices

Equation (4) implies that any rational-bubbles component would have explosive conditional expectations; that is,

$$E_tB_{t+j} = \lambda^jB_t \qquad \text{for all } j > 0. \tag{7}$$

Accordingly, a negative rational-bubbles component cannot exist in the price of a stock because the existence of such a bubble would imply that $\{E_t B_{t+j}\}_{j=1}^{\infty}$ decreases without bound at the geometric rate λ and, hence, that $E_t y_{t+j}$ becomes negative for some finite j.

The impossibility of negative rational bubbles in stock prices, in turn, implies that, in addition to satisfying equation (5), the rational-bubbles component of y_{t+1} satisfies $B_{t+1} \geq 0$. Taken together, equation (5) and this nonnegativity condition imply that realizations of z_{t+1} must satisfy

$$(8) \qquad z_{t+1} \geq -\lambda B_t \qquad \text{for all } t \geq 0.$$

Suppose that B_t equals zero. In that case, equation (8) implies that z_{t+1} must be nonnegative. But, equation (6) says that the expected value of z_{t+1} is zero. Thus, if B_t equals zero, then z_{t+1} equals zero with probability one. The essential idea is that, because the inception of a rational bubble at any date after the first date of trading would involve an innovation in the stock price, the expected initial values of a positive rational bubble and a negative rational bubble would have to be equal to each other, and, accordingly, equal to zero.

This result says that, if a rational bubble does not exist at date t, $t \geq 0$, then a rational bubble cannot get started at date $t + 1$, nor at any subsequent date. In Diba and Grossman [1987a], we extend this analysis to show that, if an existing rational bubble bursts, a new independent rational bubble cannot simultaneously start. In sum, this analysis shows that, if a rational bubble exists at present, then it must have started at date zero, the first date of trading of the stock, and this stock must have been continuously overvalued relative to market fundamentals.

III. Rational Bubbles in the Value of a Fiat Money

Unlike the case of real assets, free disposal does not preclude the possibility of a negative rational bubble in the value of a fiat money (see Brock [1978], Scheinkman [1980], and Obstfeld and Rogoff [1983]). A positive rational bubble in the value of a fiat money—that is, a rational deflationary bubble—cannot, however, arise if agents have infinite planning horizons or if, in an economy with a succession of overlapping generations of asset holders, the wealth of the young generation does not grow rapidly enough to sustain such a rational bubble.

Interpreting y in equation (1) as the logarithm of the price level or of the exchange rate, the impossibility of a positive rational bubble in the value of a fiat money would imply that $z_{t+1} \geq -\lambda B_t$. A symmetric argument to that of the preceding section, then, would restrict the possible inception of a negative rational bubble to the date of initial issuance of the fiat money. Moreover, as shown in Diba and Grossman [1987b], this analysis generalizes to impose the same restriction on the inception of a rational inflationary bubble in a general nonlinear model of money demand based explicitly on utility maximization.

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