Fundamentals of Operations Research

HW #1: Modeling

Shiraz University – CSE Department

Problem #1:

Suppose the minimum number of bus required at the i_{th} time of the day (i = 1,2,...,24) is equal to $b_{i.}$ Each bus works for six consecutive hours. If the number of buses at i_{th} time of the day, become more than the minimum required b_{i} , the extra cost of the C_{i} per hour is consider for each additional bus. The objective is to minimize the total extra cost.

Formulate your problem as a linear programming model.

Problem #2:

The forecast of a company for the amount of requests received for one product in the next four seasons is as follows:

Spring	Summer	Fall	Winter
20	30	50	60

The company can consider two production policies to meet all the needs.

- 1) In the first policy, no warehousing is permissible and the amount of production is regulated in terms of demand.
- 2) In the second policy, however, the product is produced at a fixed rate of 40 units in each season. And warehousing can also be used.

Since the warehousing and the variability of production are costly, it is expected that a combination of these two policies will be low cost for the company.

Changing number of production from one season to another costs 60 Tomans per unit of product. That is called "hire and firing" cost. (no matter the reduction or addition of production) Also, the cost of warehousing per unit of product stock at the end of each season is 70 Tomans.

The number of products available in warehouse is initially zero and the initial production level in 40 units per season. At the end of the winter, these two factors need to have the same situation.

- **A)** Assuming each of the existing policies separately, specify the cost of production.
- **B)** Provide the right model for this problem with the aim of designing a low cost production program by combining the two policies.

Problem #3:

A final product is obtained by mixing three basic products:

- A liter of a pure product, with 20 \$/liter price. This product requires no treatment.
- x₁ liters of a P1 product with 23% impurity, with 4 \$/liter price. Each liter requires 4 minutes of treatment.
- x₂ liters of a P2 product with 13% impurity, with 11 \$/liter price. Each liter requires 5 minutes of treatment.

The final product, that is to say "the mixture", must have at most 15% impurity and will be sold on the market at a price of 10 \$/liter. In addition, the total time allocates to treatment should not exceed 36 minutes. We want to determine how much quantities of x_1 and x_2 should we buy.

- **A)** Formulate a linear programming model for this problem.
- **B)** Solve the model using a MiniZinc Program.

Problem #4:

Securities and Exchange Organization of Iran developed a list of seven investment instruments with corresponding financial factors. The "Term" of the investment is the expected number of years required for the annual rate of return to be realized. The "Return" is the expected rate of return over the 10-year of investment. The "Risk" column describes the estimate that represents the relative safety of each alternative. The growth column gives a

subjective appraisal of the potential (percentage) increase in the value of the investment for the ten-year period.

You always assume that a percentage of your money is available in one of these instruments Also, Securities and Exchange Organization suggest following restrictions, and you want to satisfy all of them:

- 1- The average length of the investment for the portfolio should not exceed 7 years.
- 2- The average risk should not exceed 5.
- 3- The average growth potential should be at least 10%.
- 4- At least 10% of all available funds must be in cash.
- 5- The proportion of funds invested must sum to one.

The table below presents these investments and their financial factors:

Investment	Term	Return(%)	Risk	Growth(%)
Iran Khodro Stock	4	3	1	0
Saipa Stock	7	12	5	18
Folad Stock	8	9	4	10
Zobahan Stock	6	20	8	32
Golgohar Stock	10	15	6	20
South Pars Stock	5	6	3	7
Cash (Your Pocket)	0	0	0	0

- A) Formulate a linear programming model for this problem.
- **B)** Solve the model using a MiniZinc Program.

Problem #5:

Hogwarts university maintains a powerful Goblet of Fire to accept student's name and selecting students for competition. During all working hours, an operator must be able to maintain the goblet, as well as to cleaning and maintaining the curse for not allowing under 17 students to put their name in the goblet. Albus Dumbledore, the director of the Hogwarts, oversees the operation.

It is now the beginning of the new semester, and Albus is confronted with the problem of assigning different working hours to his operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the following table.

Name	Wage / hour	Monday	Tuesday	Wednesday	Thursday	Friday
Harry	10.00	6	0	6	0	6
Hermione	10.10	0	6	0	6	0
Ron	9.90	4	8	4	0	4
Ginny	9.80	5	5	5	0	5
Severus	10.80	3	0	3	8	0
Minerva	11.30	0	0	0	6	2

There are six operators (four undergraduate students and two professors), they all have different wage rates because of difference in their experience in magic. The above table shows their wage rates, along with the maximum number of hours that each can work each day.

Each operator is guaranteed a certain minimum number of hours per week that will maintain an adequate knowledge of the magic. This level is set arbitrarily at 8 hours per week for the undergraduate students (Harry, Hermione, Ron, Ginny) and 7 hours per week for professors (Severus, Minerva).

The Hogwartz and Goblet of Fire is to be open for accepting names from 8 a.m to 10 p.m monday through friday with exactly one operator on duty during these hourse. On saturdays and sundays, the Goblet of Fire is used by other professors. Because of a tight budget, Albus has to minimize cost. He wishes to determine the number of hours he should assign to each operator on each day.

- A) Formulate a linear programming model for this problem.
- B) Solve the model using a MiniZinc Program.

Problem #6:

IranDookht is a Sewing Company and manufacturer of four types of clothing (A,B,C,D). The company has recently signed a contract with a store that has pledged to deliver the required clothes for up to 72 hours. So the factory has to operate full-time.

The clothing manufactured will be shipped to the store with a truck. The total capacity of the truck is 1200 jeans of type C or D. Each jeans of type A and B require three times bigger space than type C and D.

The total production budget of the company is 25 million tomans, all of them can use in clothing production.

The manufacturing company has two warehouses whose number one is dedicated to Type A and B clothing, and warehouse number two is the capacity of each of the warehouses of five hundred denim clots. The required resources, production costs and profits of each type of clothing are given in the following table:

Jeans Type	Time to product (Hour)	Cost (Tomans)	Profit(Tomans)
Α	0.1	36000	90000
В	0.25	48000	125000
С	0.08	25000	45000
D	0.21	35000	65000

- **A)** Formulate the problem that the company's profit is maximized.
- B) Solve the model using a MiniZinc Program.

Problem #7:

Two Habits are preparing to go on a mission that involves dispose of a certain ring. They are packing in a hurry and are limited by their bags, but they need to make sure they have enough food to survive the trip. Help them figure out what they should pack.

They will need food for about 60 days and require 1800 calories per person per day (they are both very short). At least 35% of their calories must come from carbs, at least 25% from protein, and at least 15% from fat. Furthermore, no more than 25% of the calories can come

from fat. (Those percentages pertain to the minimum calorie threshold, and not total calories eaten if they decide to pack extra.) The foods available, their contents, and the amount of each available (in servings), are detailed below. Decide how many servings of each food they should bring so they can satisfy their dietary requirements while keeping their backpacks as low weight as possible. As a final constraint, they have eaten a lot of turnips and onions recently and are not willing to take more than 45 cumulative turnips and onions.

Food	Calories from			Max	Unit
	Carbohydrates	Proteins	Fats	Available	Weight(g)
Loaf of High Protein bread	800	1500	200	80	160
Tater	680	40	10	10	80
Rabbit	0	1400	400	20	400
Turnip	45	5	0	30	40
Carrot	18	4	0	30	20
Onion	35	5	40	30	40
Foraged Nuts	25	25	150	100	10

- A) Formulate an integer programming model for this problem
- **B)** Model and solve this problem using MiniZinc.