Fundamentals of Operations Research

HW #3: Network Flow Models/Algorithm

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Problem #1:

There are 220 bus trips that leave and arrive at a bus terminal each day between 6:00 a.m. and 12:00 p.m. In fact, each of the 220 trips starts at the terminal, goes to the end of the line and returns to the terminal. For each trip i , we know the d_i start time, the f_i end time (return to the terminal). The trips last between 20 minutes and 2 hours. The start and end times are accurate to the minute.

Drivers can only change buses at the terminal. A driver's daily schedule is therefore a sequence of trips (he may have more than two). These trip sequences are paths in a graph where the arcs are the trips and the assignments from the end of one trip to the beginning of another.

Propose aggregated networks for a), b) and c) with the least number of arcs in the worst case and give this number of arcs (do not do the aggregations that do not reduce the number of arcs in the worst case). Also give the formula for calculating the costs on each type of arcs if we know the dual variables vi to cover the trip i in the master problem.

a) Costs are $50 per day per driver plus $30 per hour during trips and $10 per hour during waiting time. Drivers are not paid before their first trip and after their last.

b) Consider that it takes at least 5 minutes from the end of and the beginning of the next trip and 15 minutes if the driver changes buses. This time is not necessary if the driver continues with the same vehicle. Obviously, we know the X_{ij}'s, the assignments of the vehicles from the end of the trips to the beginning of the following ones.

c) How should the network in (a) be modified if each driver is to take 1 hour at the terminus between 11:00 and 14:00 for dinner. No resource constraints or linear constraints that break the shortest path structure should be added.

Problem #2:

A company with 20 employees and 20 positions requiring one person each. The company has a goal to train its employees for many of its positions and change them every week. This shift change program has been in place for 5 weeks. Let $X=1^k_{ij}$ if employee i has been assigned to job j during week k, k = 1, ...,5. Let X_{ij} be the assignment variables for week 6.

Formulate an efficient model for assigning employees for week 6 if we want each employee to have a different position than the ones they held during the previous 5 weeks. Give the variables and constraints.

Let C_{ij} be the duration of the training so that employee i is ready to occupy position j. During week 6, we anticipate a fairly low workload and we want to do a lot of training. We want to add to model for the objective of maximizing the minimum amount of training an employee will receive.

Problem #3:

A house roofing contractor has a work crew that completes one house a day. He must complete 20 roofs over the next month. For each of the 20 roofs we have: d_i the first date the date from which the roof i can be installed. f_i the last date that the roof i can be installed and c_i the cost price of the roof i.

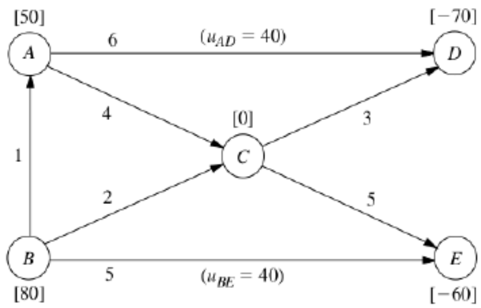
The entrepreneur receives payment for house i on the evening of date f_i even if it is completed before. He must borrow the sum c_i at the rate of 0.1% per day to finance the morning work from the beginning of the day he does the work until the evening of f_i.

a) Formulate a network model to find a schedule that minimizes interest charges if there is a feasible schedule. Consider that the 1st of the month is a Monday, that we work from Monday to Friday, that we pay interest 7 days a week and that the month has 31 days.

b) Indicate the number of nodes and an upper bound on the number of arcs according to d_i and f_i.

Problem #4:

Consider the minimum cost flow problem shown below, where the b_i values are given by the nodes, the c_{ij} values are given by the arcs, and the finite u_{ij} values are given in parentheses by the arcs. Start with an initial BFS where basic arcs are AC, BA, CD, and CE, and the non-basic arc AD being the arc that is on the upper bound, then use the network simplex method to solve this minimum cost flow problem.



Problem #5:

Consider the following linear program:

$$\text{Min } 3x_{12} + 2x_{13} + 5x_{14} + 2x_{41} + x_{23} + 2x_{24} + 6x_{42} + 4x_{34} + 4x_{43}$$

s.t

$$x_{12} + x_{13} + x_{14} - x_{41} \leq 8$$

$$x_{12} - x_{23} - x_{24} - x_{42} \leq 4$$

$$x_{34} - x_{13} - x_{23} - x_{43} \leq 4$$

$$x_{14} + x_{34} + x_{24} - x_{42} - x_{42} - x_{43} \geq 5$$

$$x_{ij} \geq 0$$

a) Show that this is a network problem, stating it in general minimum-cost flow form. Draw the associated network and give an interpretation to the flow in this network.

b) Find an initial feasible solution. (Hint. Exploit the triangular property of the basis.)