Introduction to Operations Research

Koorush Ziarati

General Introduction

Slides: *Institute of Applied Optimization*
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- we want to get a rough feeling about what *optimization* is.

Let us start by looking at some examples for optimization problems.

Section Outline

1. Introductory Example
2. Optimization Problems
3. Optimization Algorithms
4. Runtime vs. Solution Quality
5. Other Algorithm Features
6. Summary

Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]

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- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
- What does this mean?

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- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
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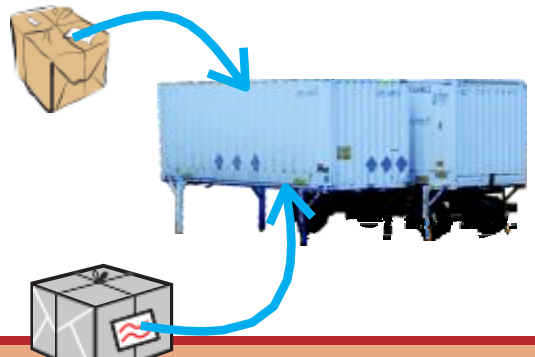

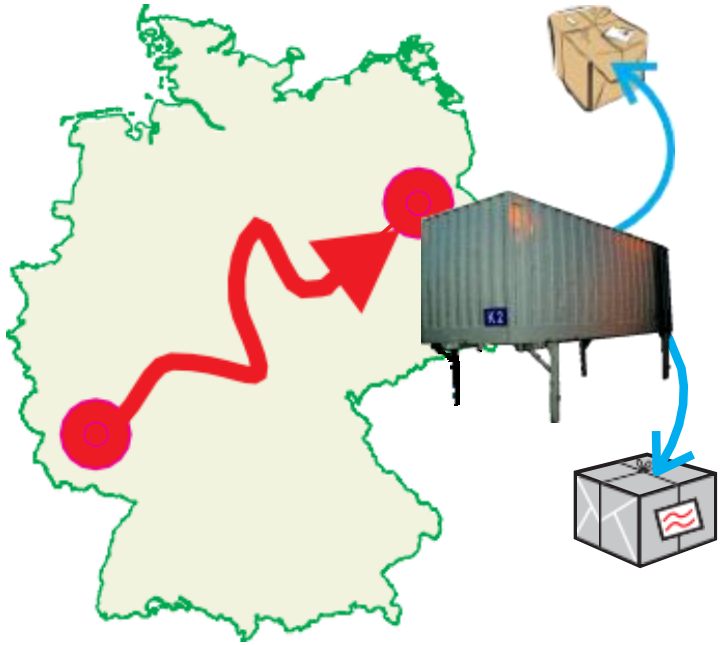

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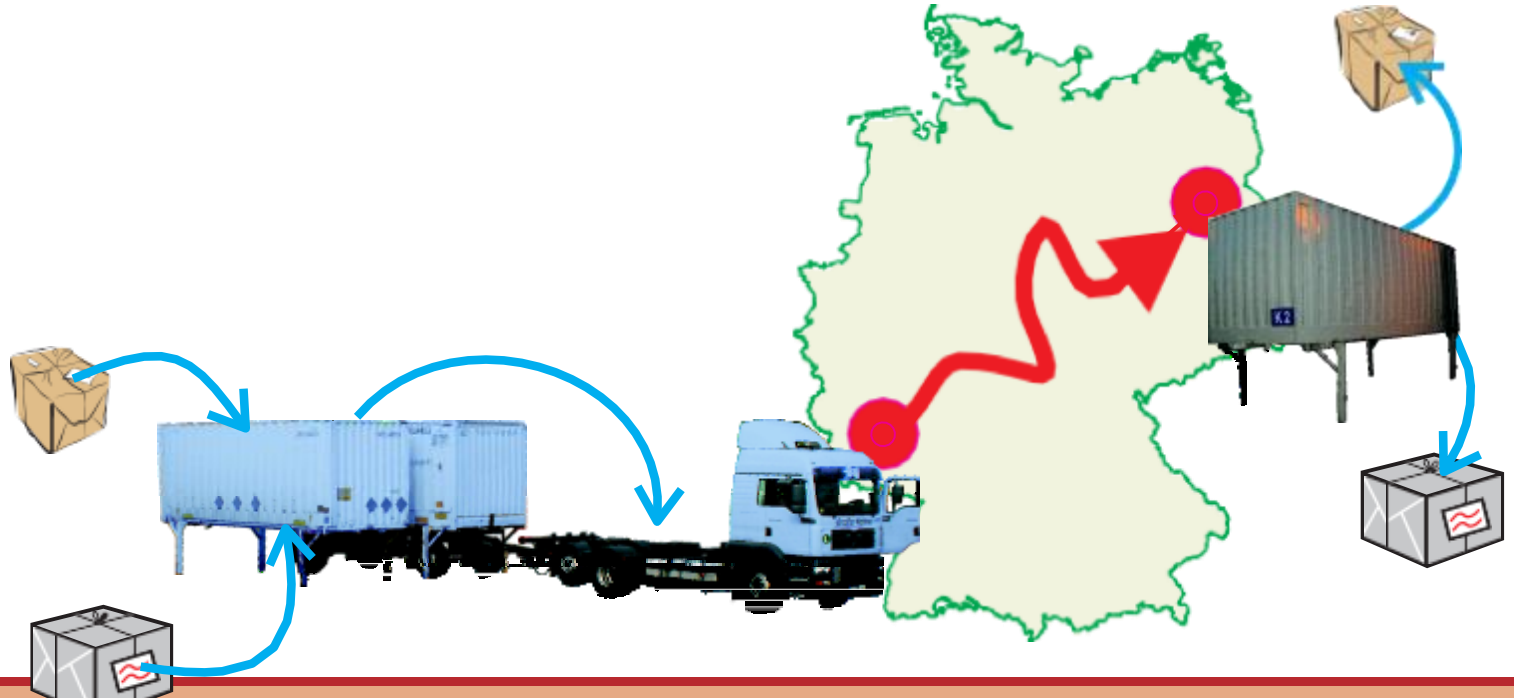
- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
  1. Find routes on the map and assignments of orders to containers and containers to trucks



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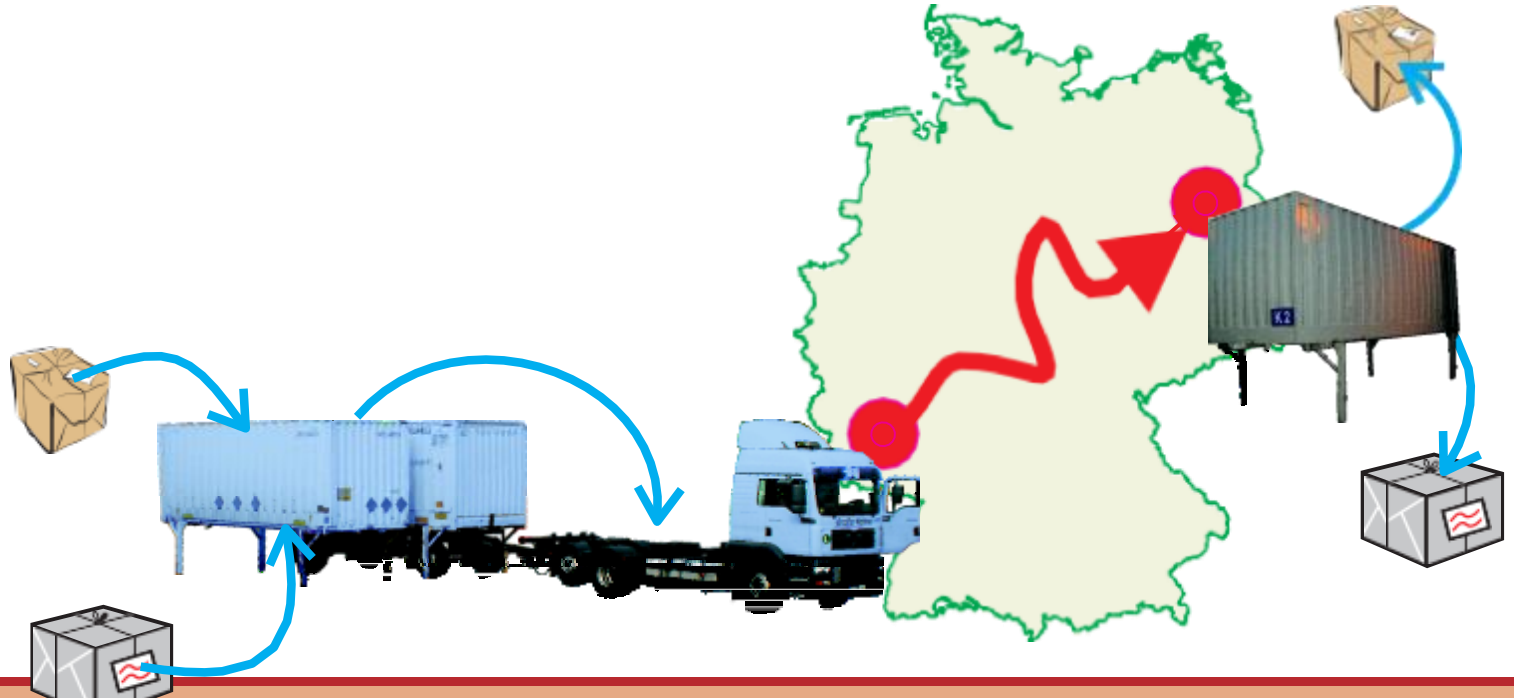

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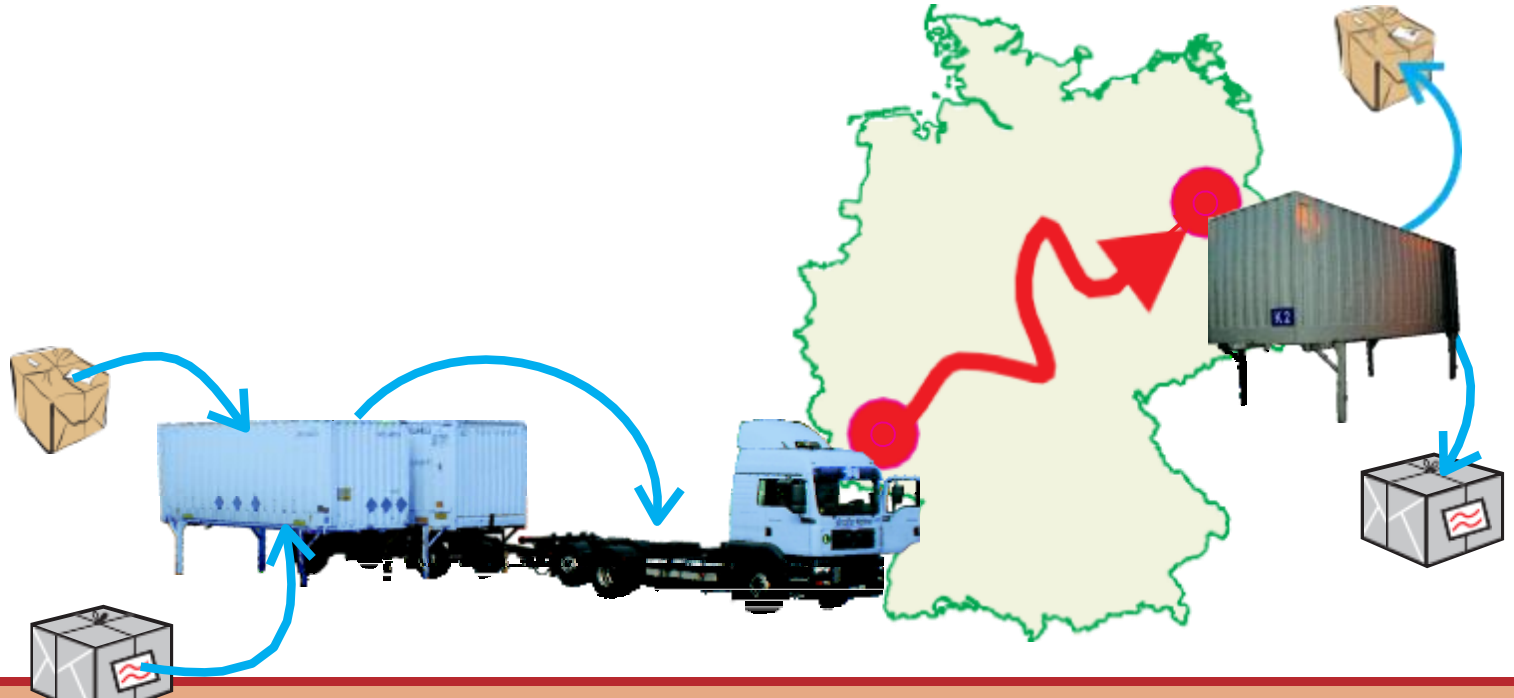

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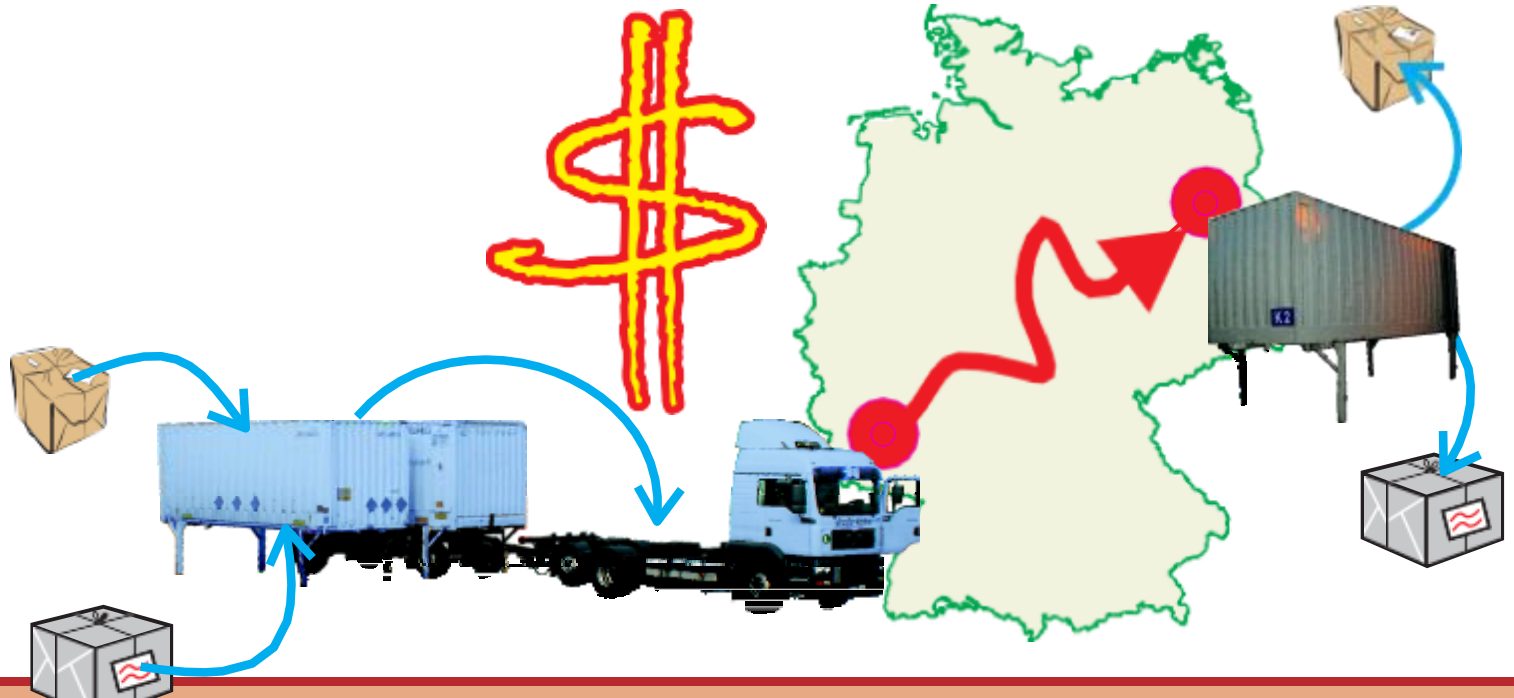

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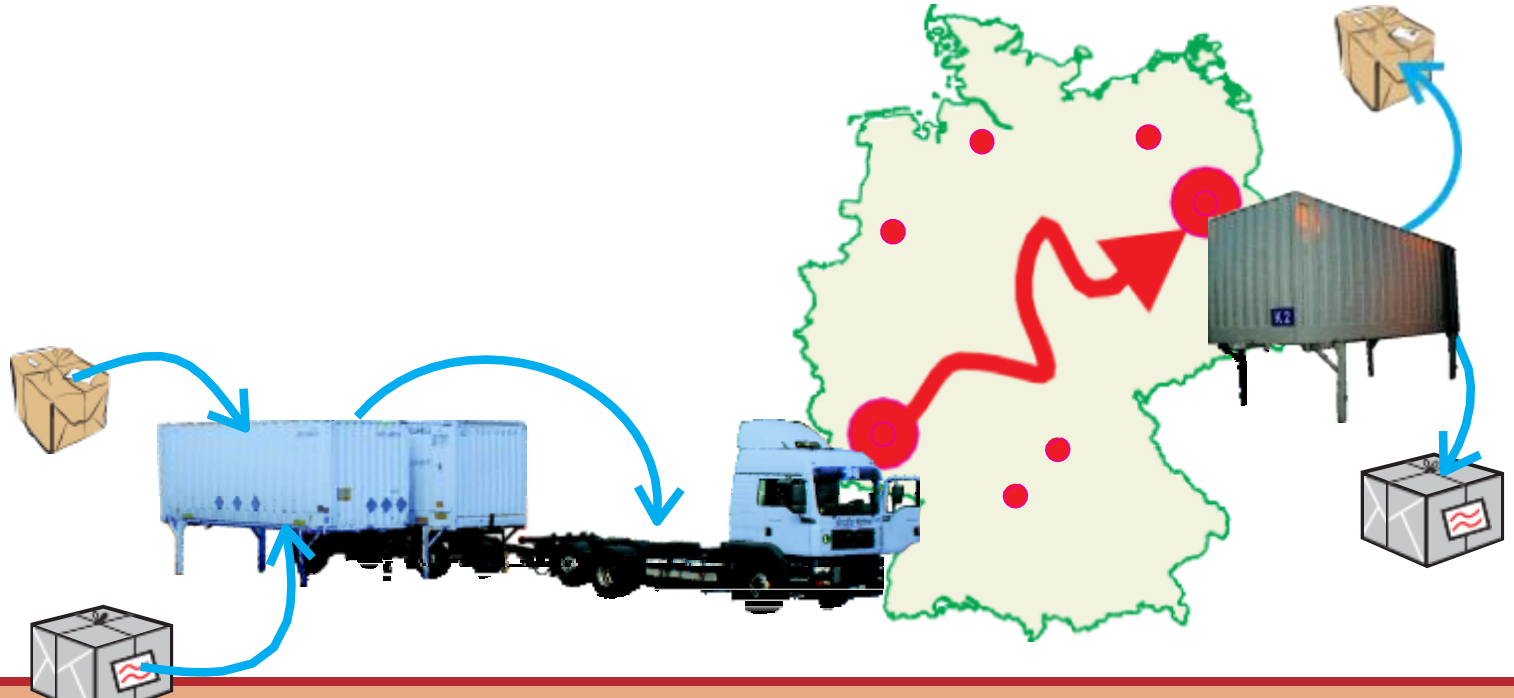

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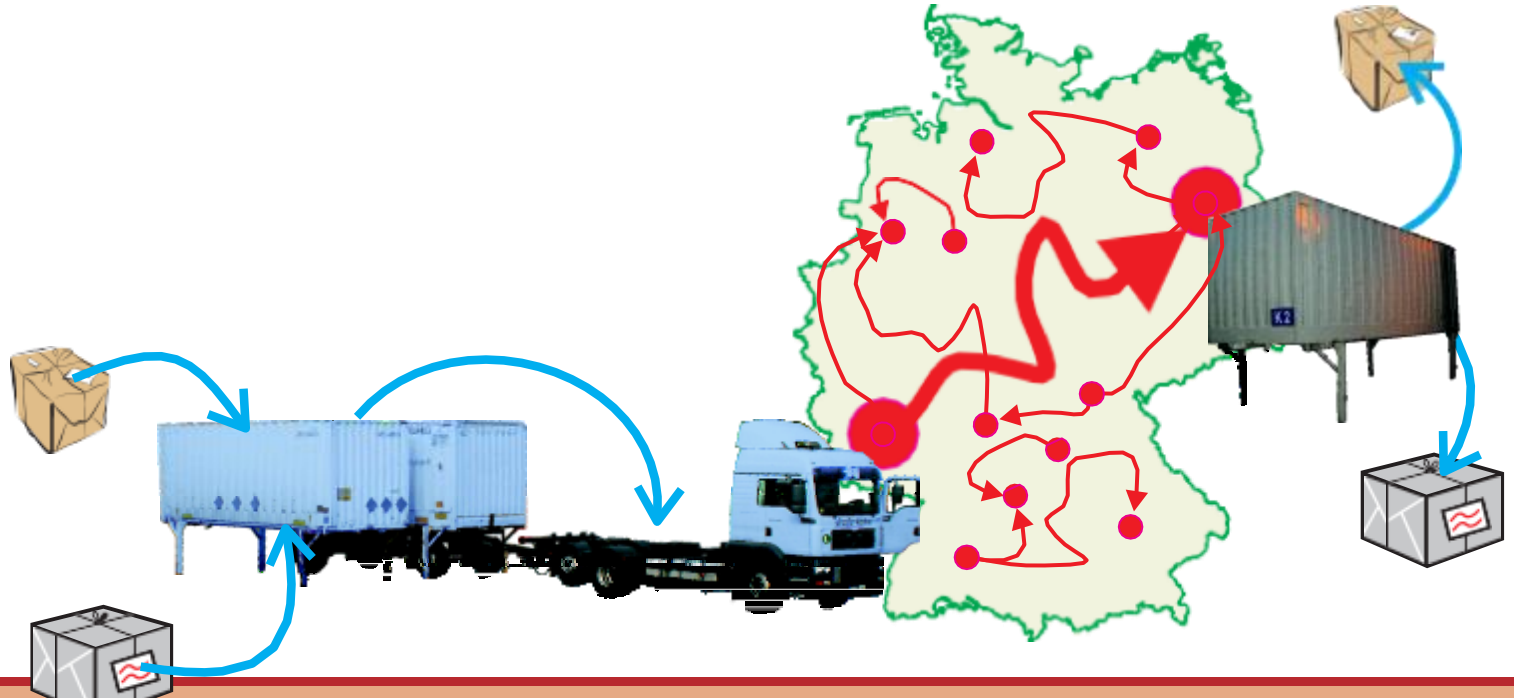

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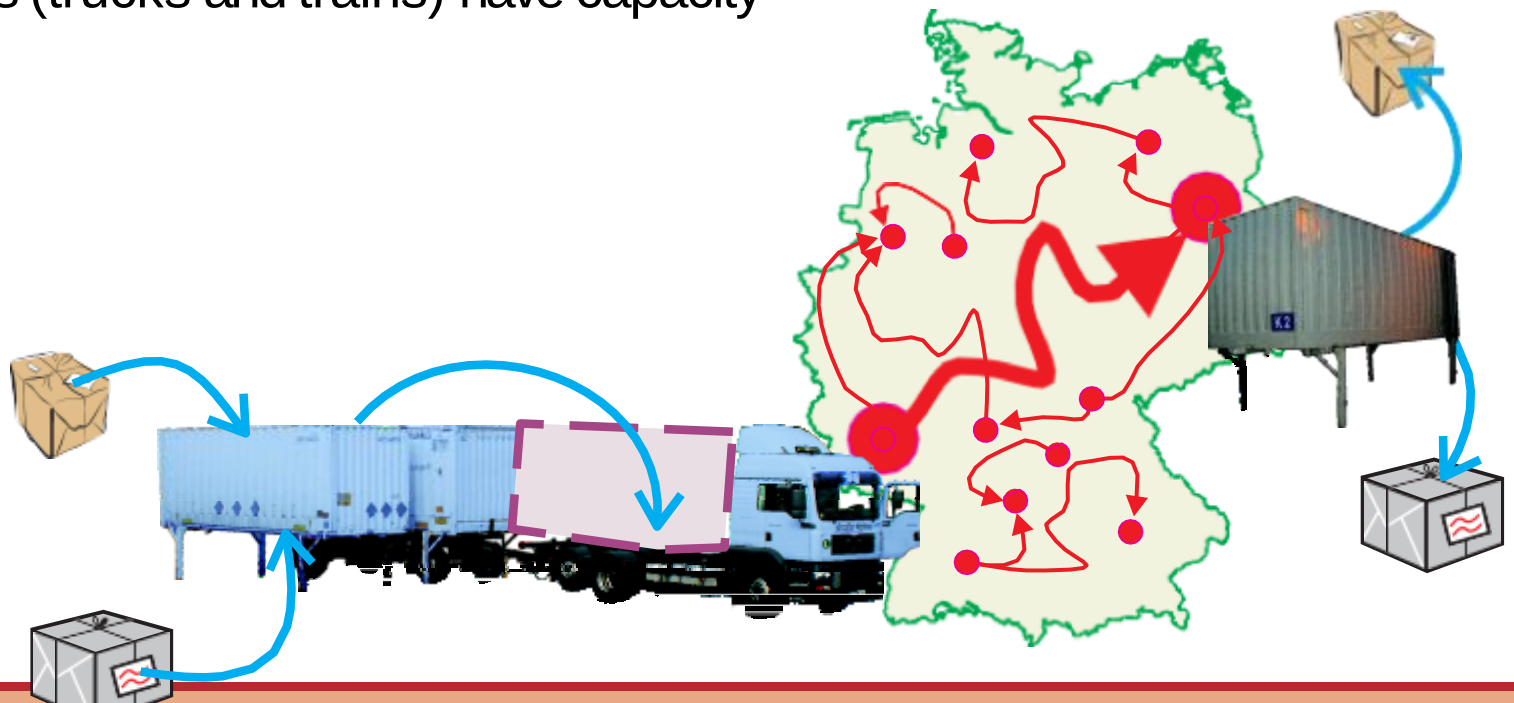

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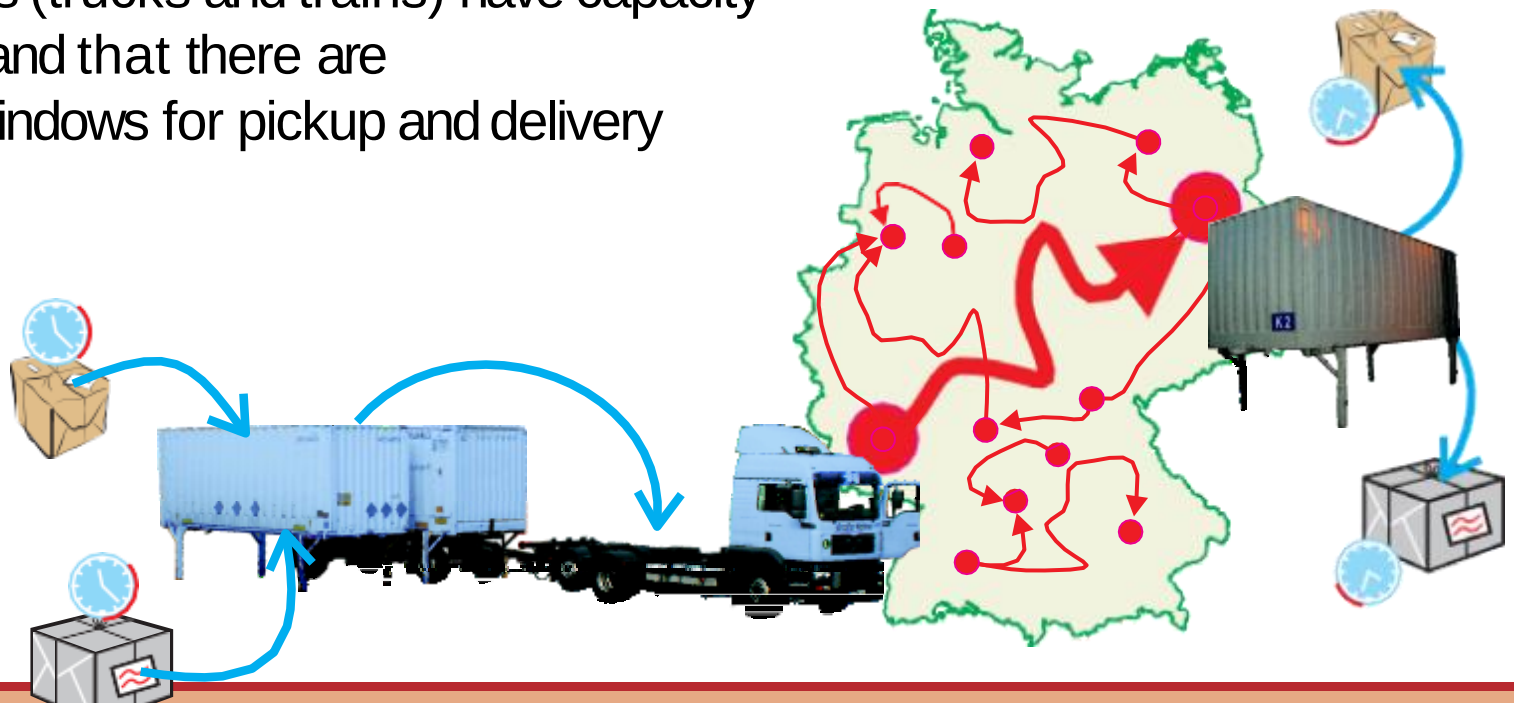

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  2. multiple depots and pickup and delivery locations, while considering that
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  4. time windows for pickup and delivery
  5. and constraints and laws.
  6. Time limit for optimization: 1 day

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- With an optimization algorithm, we can get better solutions than that.
- In this course, you will learn how we can do such a thing.

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What is optimization?

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Definition (Optimization)

Optimization is the process of solving an optimization problem, i.e., finding suitable solutions for it.

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- Ok, so what is an optimization problem?

Definition (Optimization Problem: Economical View)

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined/required benefit at minimal costs.

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An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined/required benefit at minimal costs.

Definition (Optimization Problem: Simplified Mathematical View)

Solving an optimization problem requires finding an input value x for which a mathematical function f takes on the smallest possible value (while usually obeying to some restrictions on the possible values of x).*

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- Then, we want to find approximate solutions: solutions which are "as good as possible" within a feasible time
- This is what metaheuristic optimization algorithms do. This is what we will learn in this course.

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  - Find the *shortest* tour for a salesman to visit a certain set of cities in China and return to Hefei!

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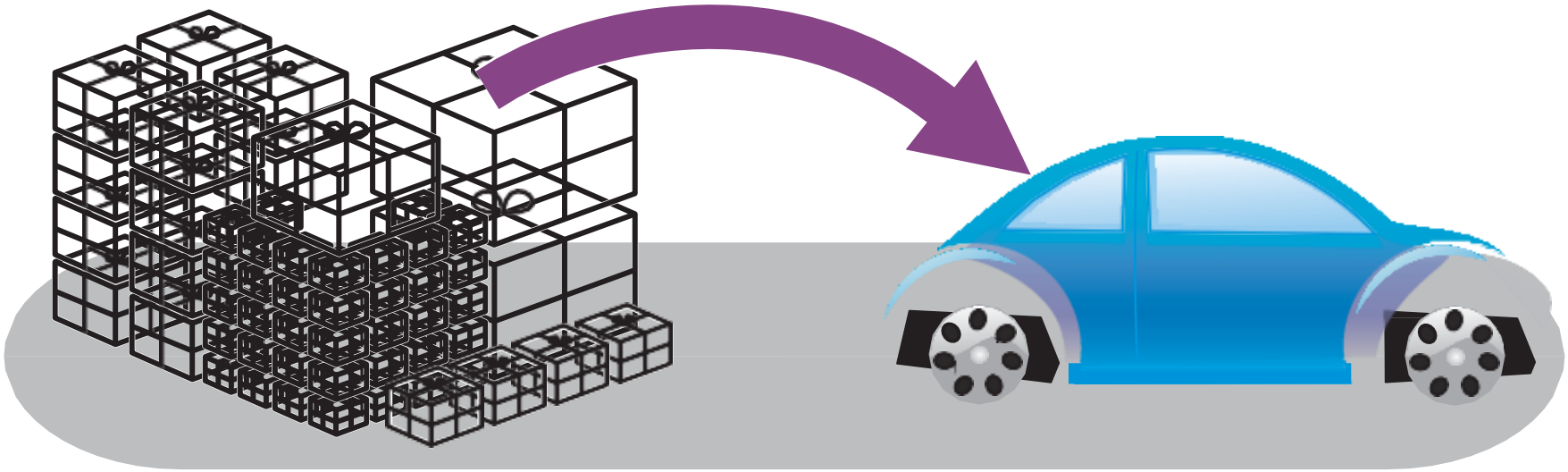
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city but they are too big to transport them all at once. How can I load them best into my car so that I have to travel back and forth the least times?



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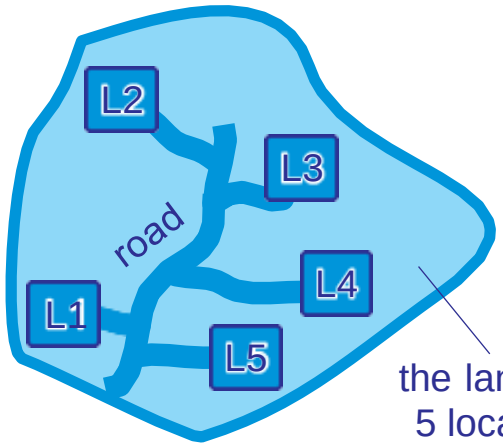
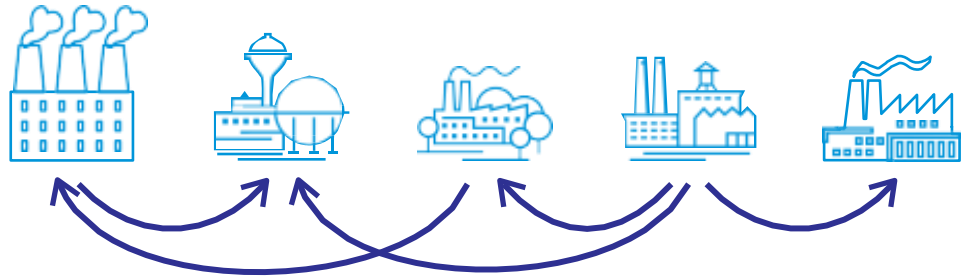
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
  - I want to build a large factory with n workshops. I know the flow of material between each two workshops and now need to choose the locations of the workshops such that the overall running cost incurred by material transportation is *minimized*.



the land with
5 locations



5 workshops and goods flows betwee them
which need to be assigned to locations

Optimization

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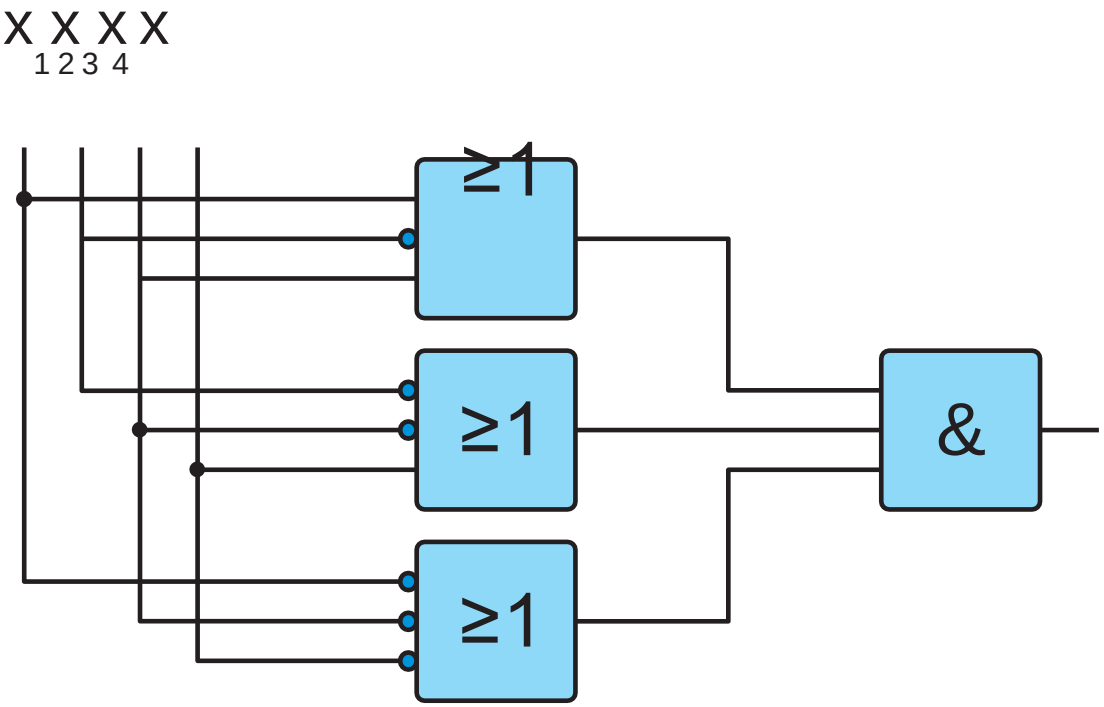
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
  - Assign workshops to locations
  - Which setting of x_1, x_2, x_3, and x_4 can make $(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_4)$ become true? (or, at least, as *many* of its terms as possible)?



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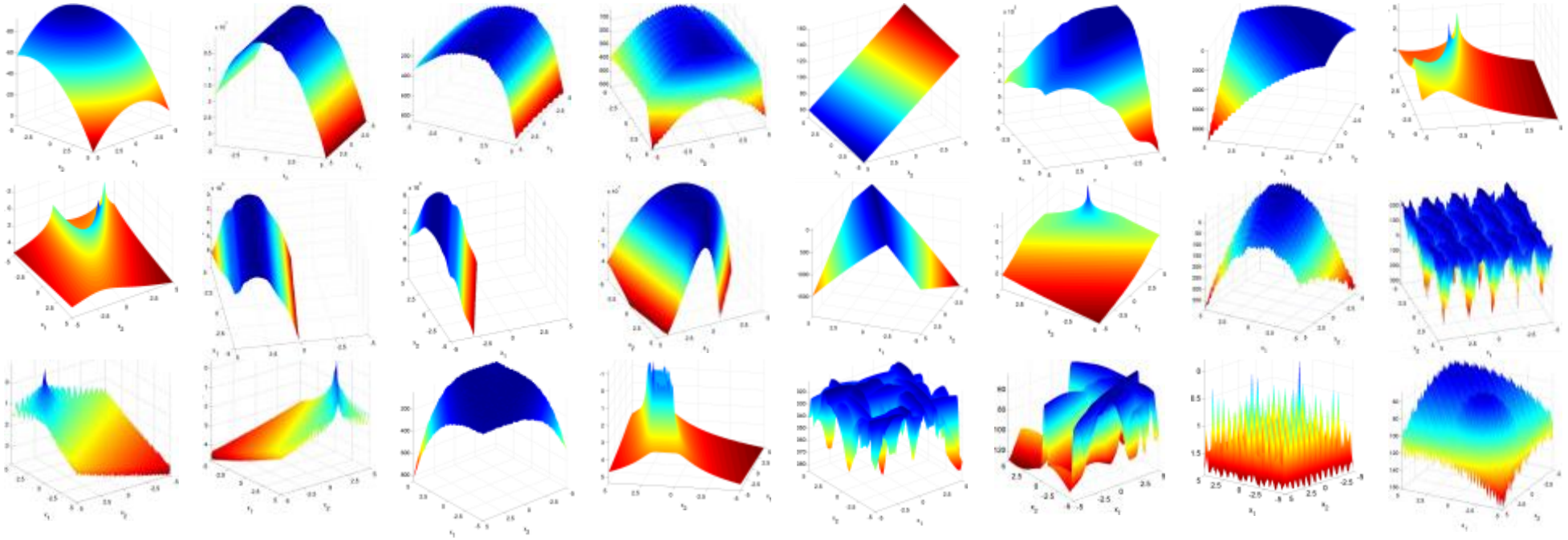
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
  - Assign workshops to locations
  - Satisfy Boolean formula
  - Find the minima of complex, multi-dimensional mathematical formulas



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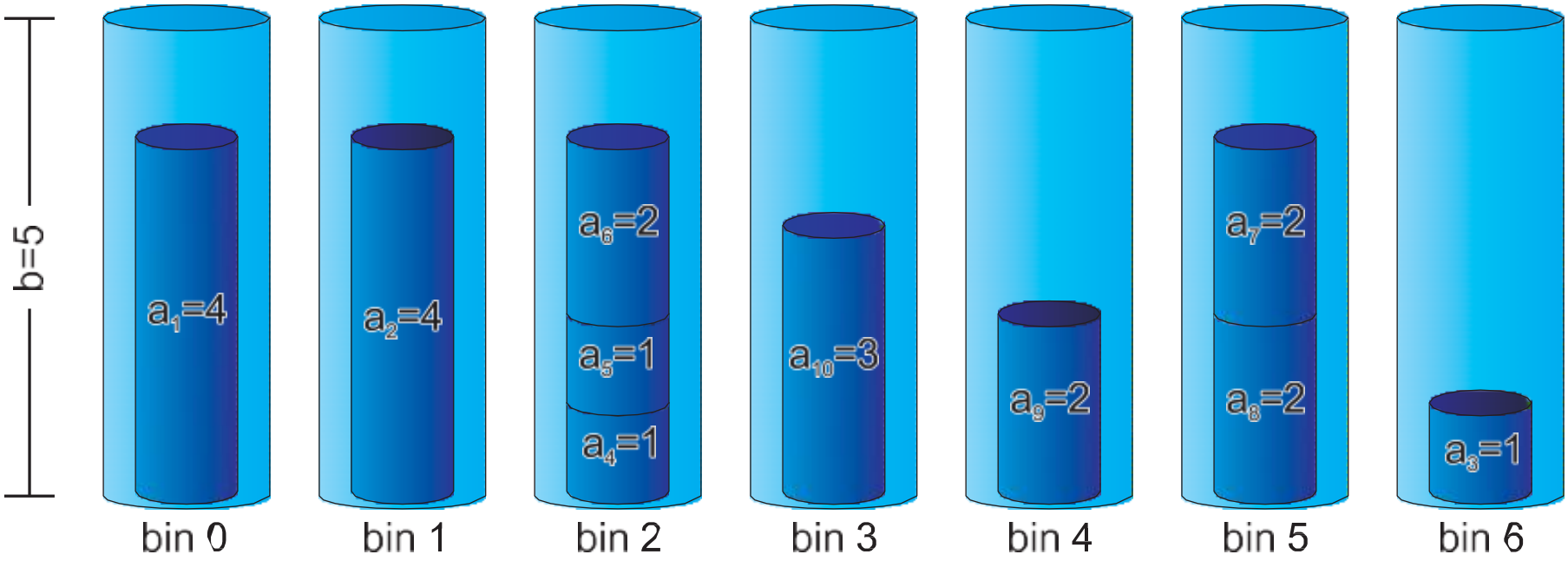

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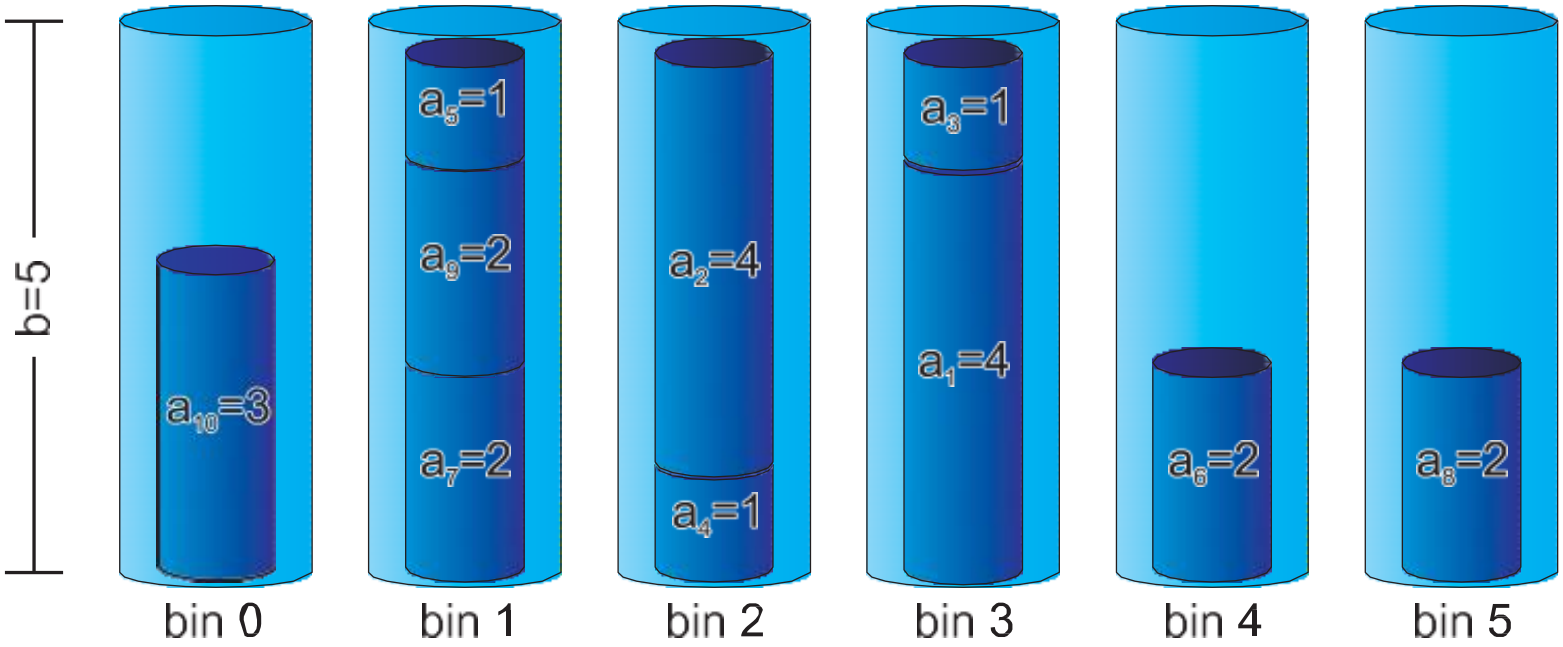

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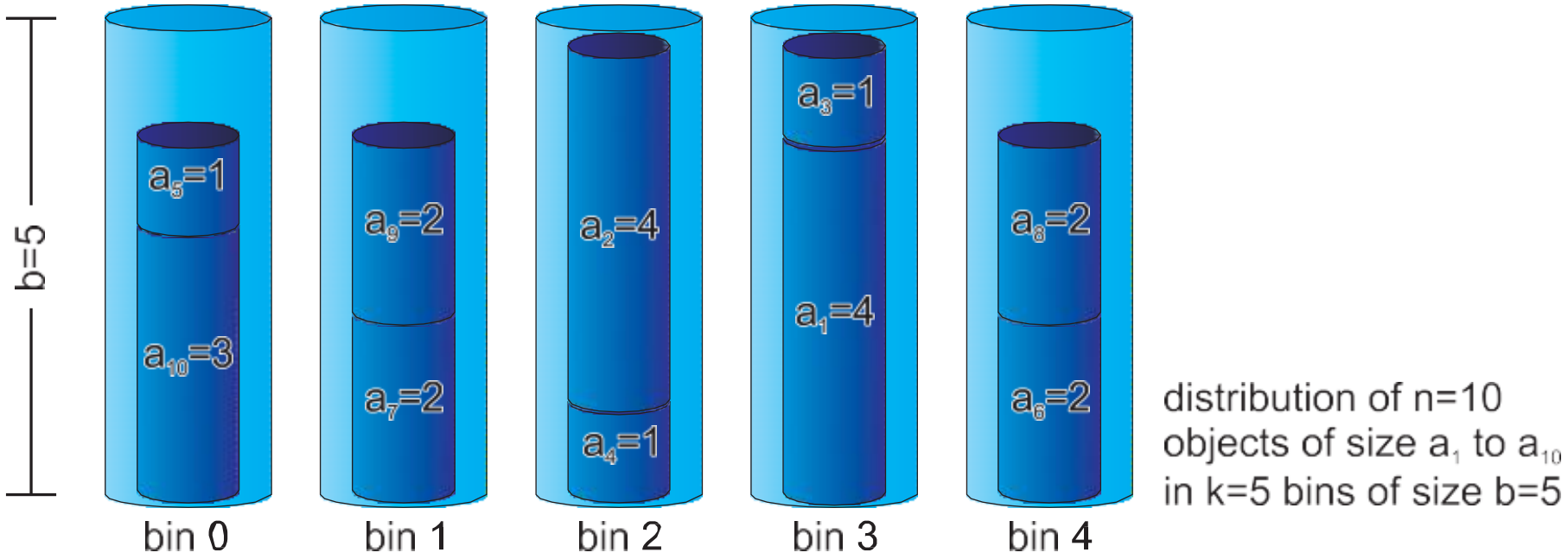

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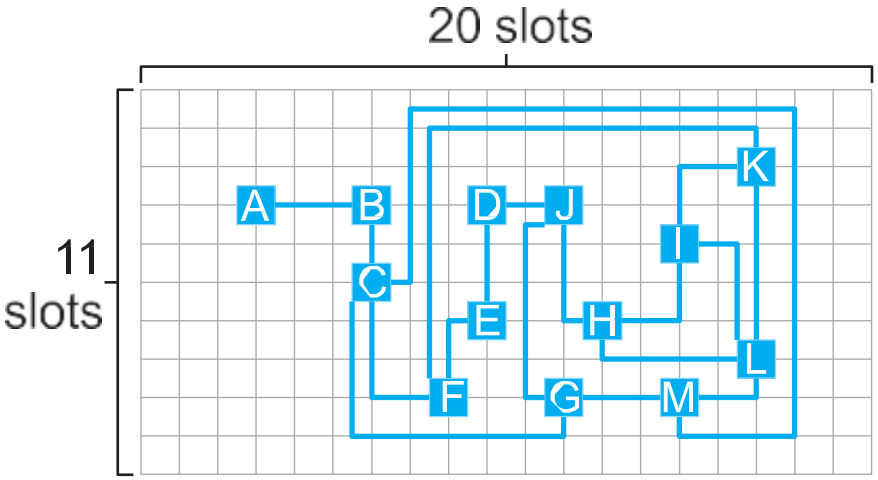

S=20×11

C={A, B, C, D, E, F, G, H, I, J, K, L, M}

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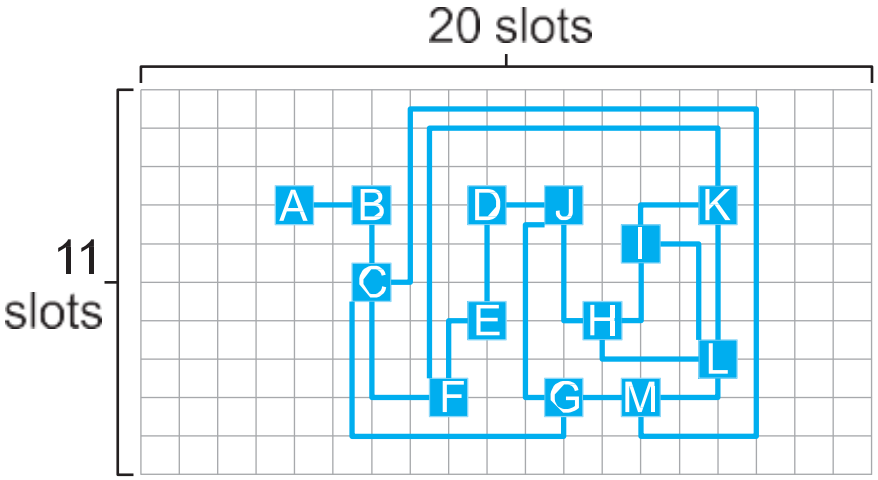

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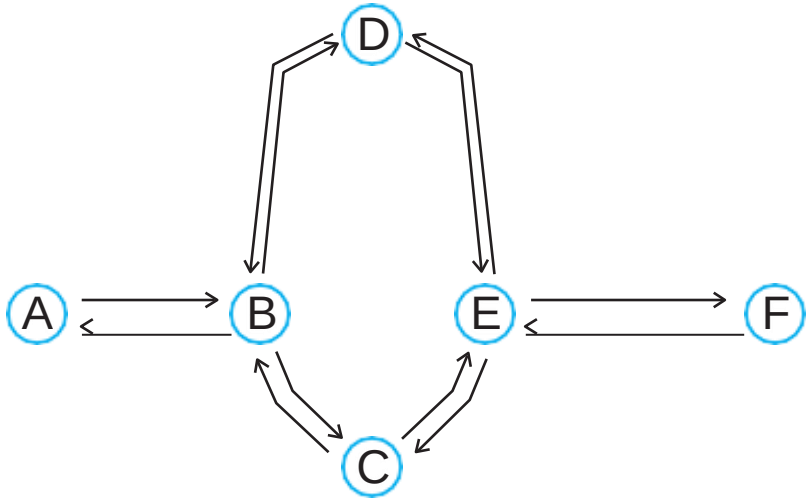
   $g(x) = 5 + \ln \cdot \sin \cdot x^2 + e^{\frac{x}{\tan x}} + \sum + \cos x \cdot - \tan^{-1} |x + 3|!$
4. Find the shortest path between two nodes in a network! [14–16]



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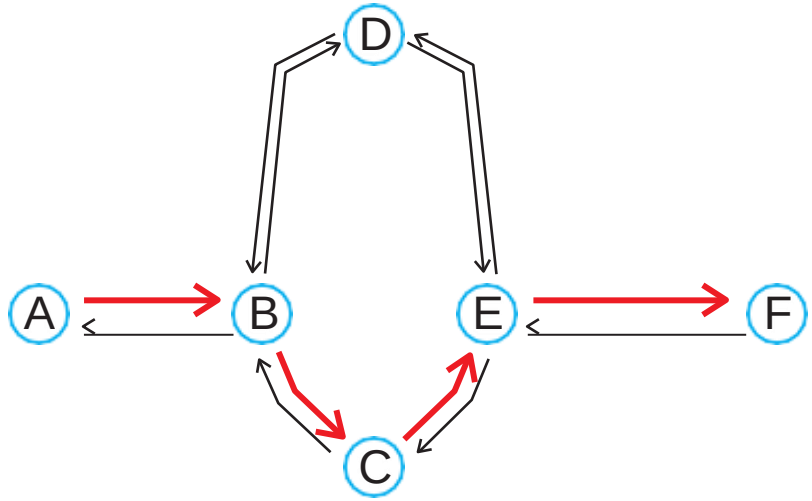

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5. Assign sub-jobs $j_i \in J$ of jobs J to machines $m \in M$ under order and deadline constraints! [17–19]

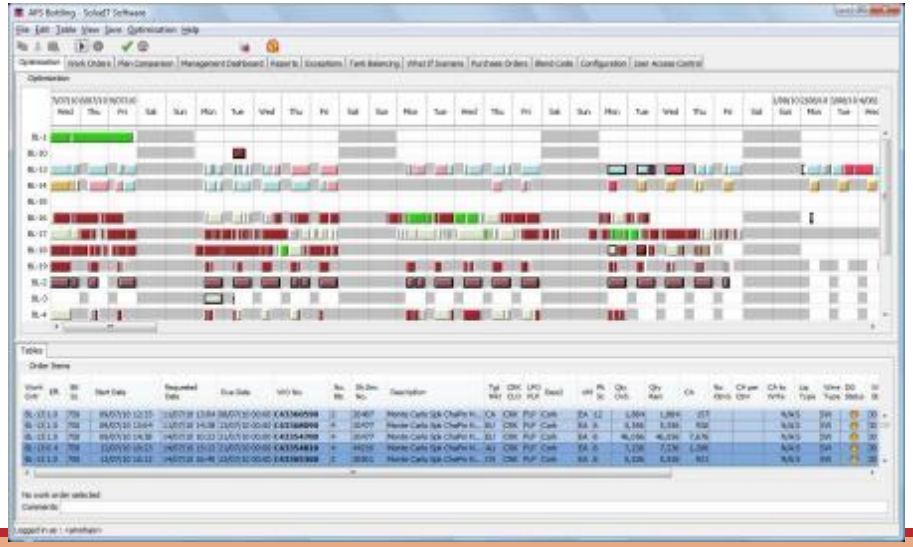

Image Source: [17]

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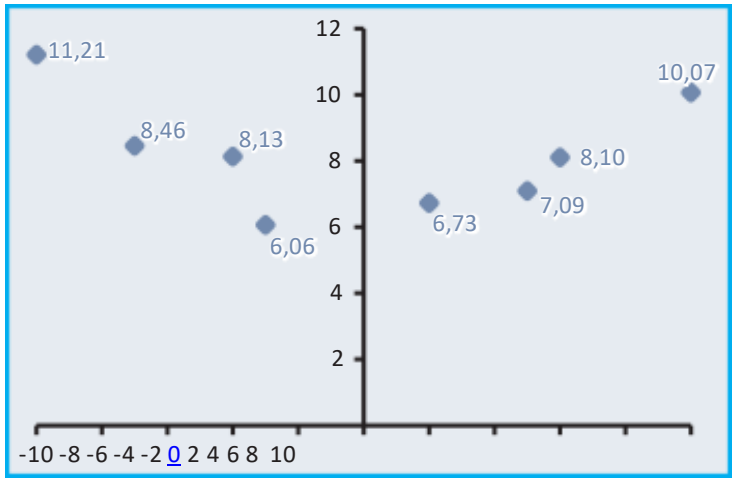
- Let us look at some more examples for optimization (problems)



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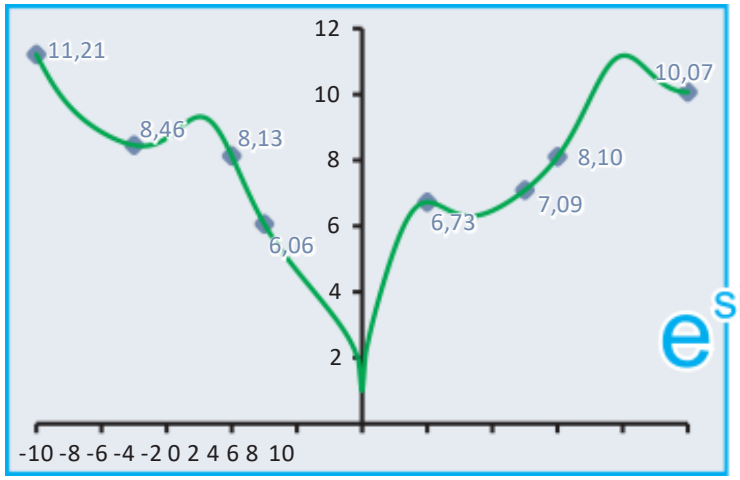

$$e^{\sin x} + 3\sqrt{|x|}$$

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More Examples for Optimization Problems

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6. Find a formula which fits to given measurements! [18, 20, 21]
7. Based on their past prices, which stocks are promising? [22–25]

Classification of Problem Types

- Are there different classes of optimization problems?

Classification of Problem Types

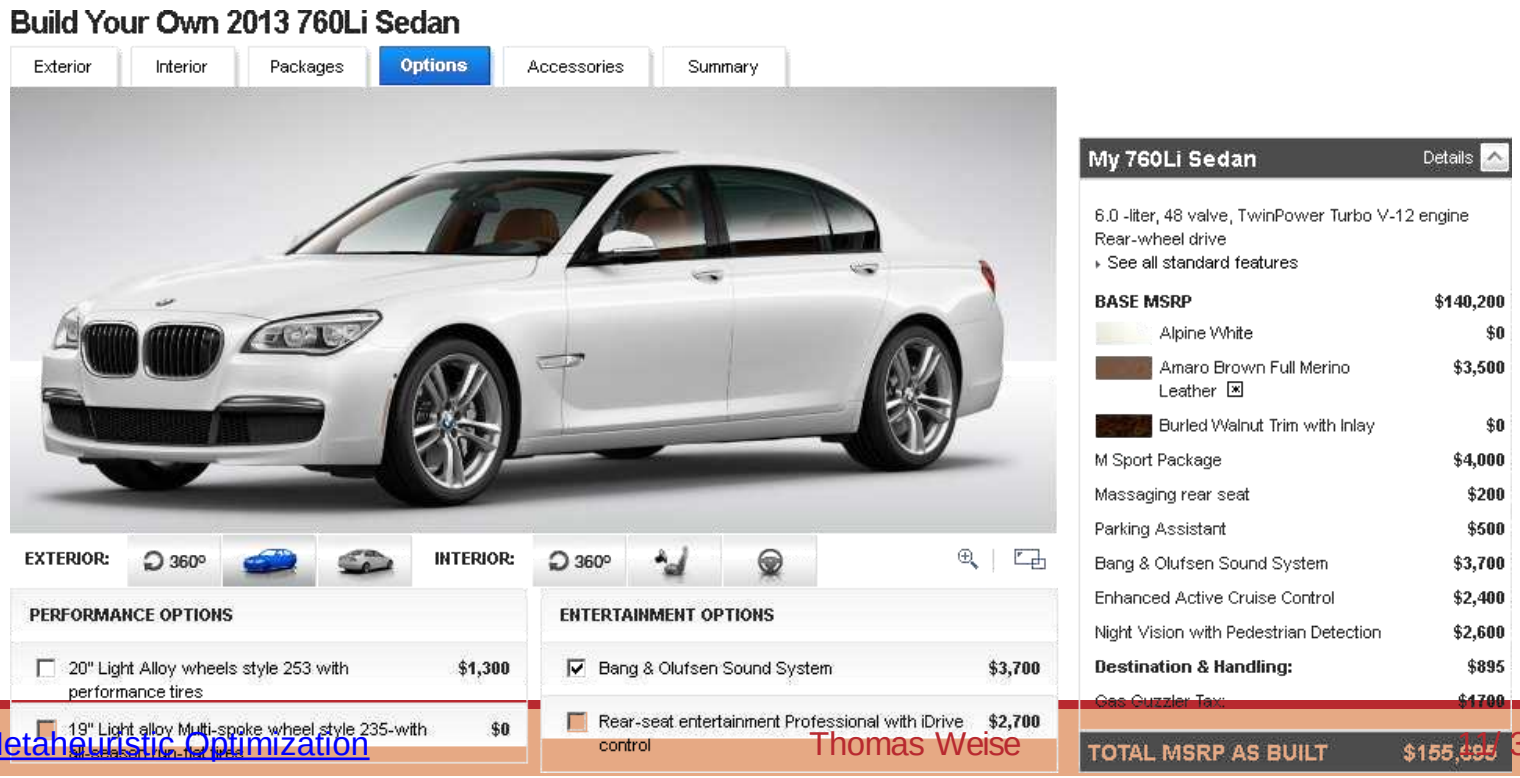
- Many problems are either combinatorial or numerical:

Classification of Problem Types

- Many problems are either combinatorial or numerical:

Definition (Combinatorial Optimization Problem)

Combinatorial optimization problems [32–38] are problems which have finitely many and discrete solutions.



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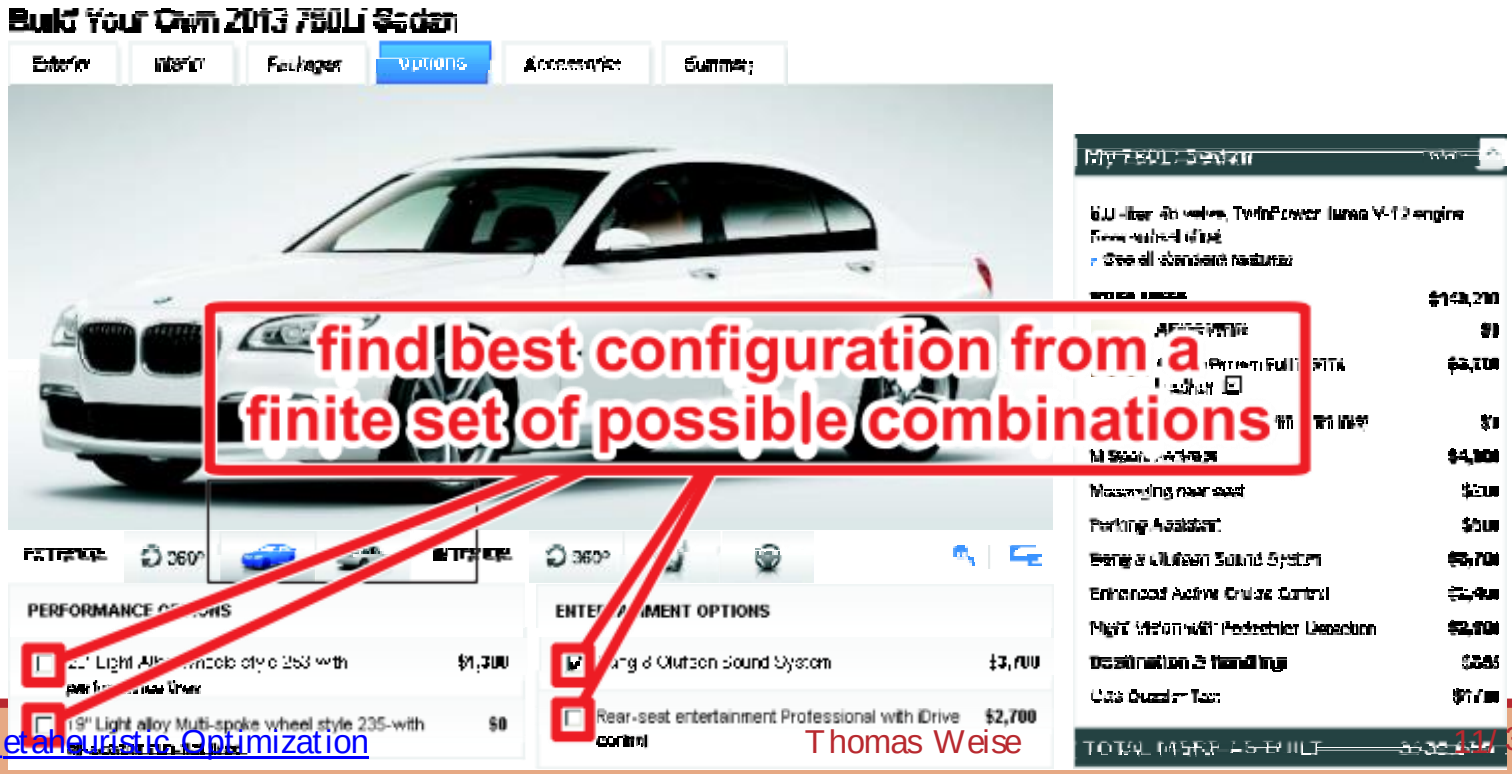

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Find the visiting sequence of cities corresponding to the shortest round-trip tour.

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1. bit strings (i.e., many yes-no decisions) or
2. permutation of elements (order something).

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Numerical optimization problems are problems that are defined over numerical domains or involve real-valued decision variables. [10, 11, 40–43]

image source: [44]

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Ingredients

125g butter, softened
100g light brown soft **sugar**
125g caster **sugar**
1 **egg**, lightly beaten
1 tsp **vanilla extract**
225g self-raising **flour**
Â½ tsp salt
200g chocolate chips

image source: [45]

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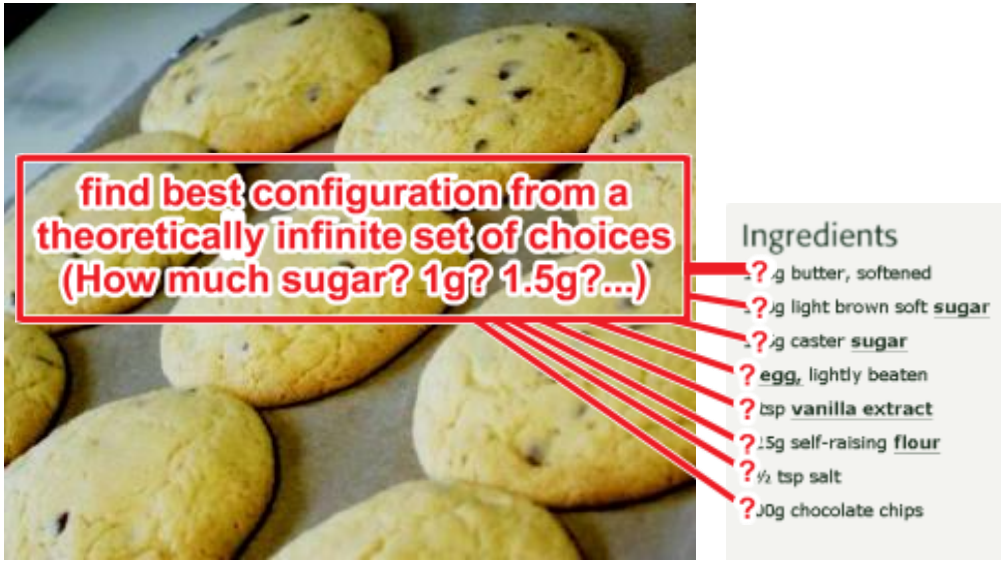

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Problem Types: Examples

- Are these problems combinatorial or numerical?

1) Traveling Salesman
2) Bin Packing
3) MAX-SAT
4) Minimize Function
5) Circuit Layout
6) Job Shop Scheduling
7) Routing
8) Find the roots of $g(x)$
9) Find mathematical formula
10) Stock Prediction
11) Medical Classification
12) Airplane Wing Design
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2)	Quadratic Assignment	combinatorial
3)	MAX-SAT	combinatorial
4)	Minimize Function	numerical
5)	Circuit Layout	combinatorial
6)	Job Shop Scheduling	combinatorial
7)	Routing	combinatorial
8)	Find the roots of $g(x)$	numerical
9)	Find mathematical formula	(usually need real numbers)
10)	Stock Prediction	(usually need real numbers)
11)	Medical Classification	(usually need real numbers)
12)	Airplane Wing Design	numerical
13)	Course Selection	combinatorial

Section Outline

1. Introductory Example
2. Optimization Problems
3. Optimization Algorithms
4. Runtime vs. Solution Quality
5. Other Algorithm Features
6. Summary

What is an optimization algorithm?

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An algorithm is a finite set of well-defined instructions for accomplishing some task. It starts in some initial state and usually terminates in a final state.

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Algorithms are the very basic of computer science. An algorithm tells us what we can do to solve a given task.

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Optimization algorithms tell us how to find solutions which are *rated best* (or at least well) from a set of possible solutions, for a general class of problems.

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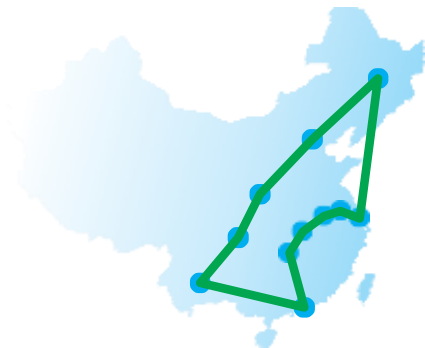

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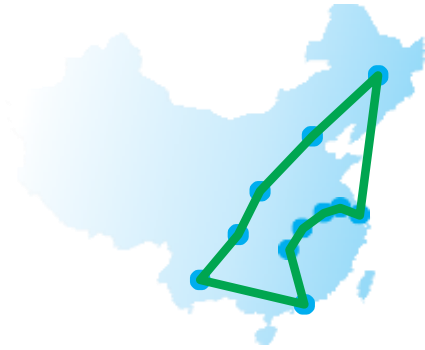
  - Theory proofs that the time to find this tour may grow exponentially with the number of cities we want to visit.



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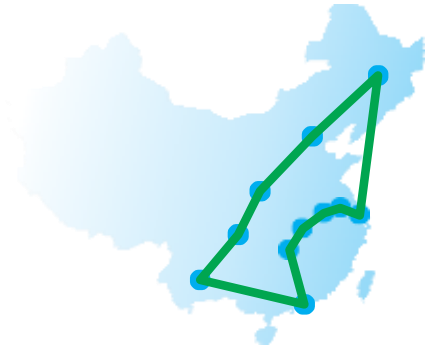

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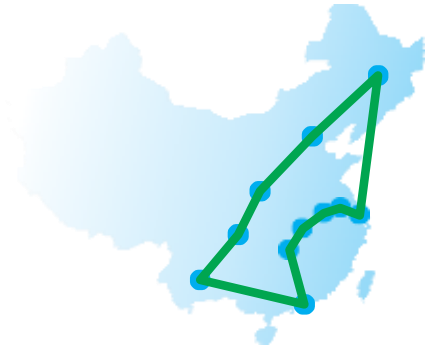

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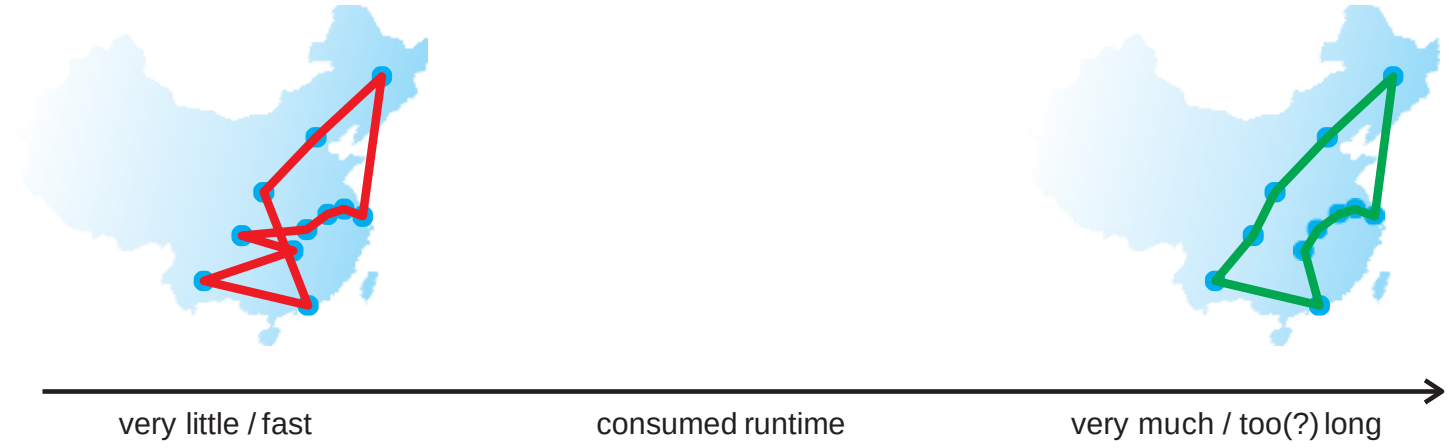

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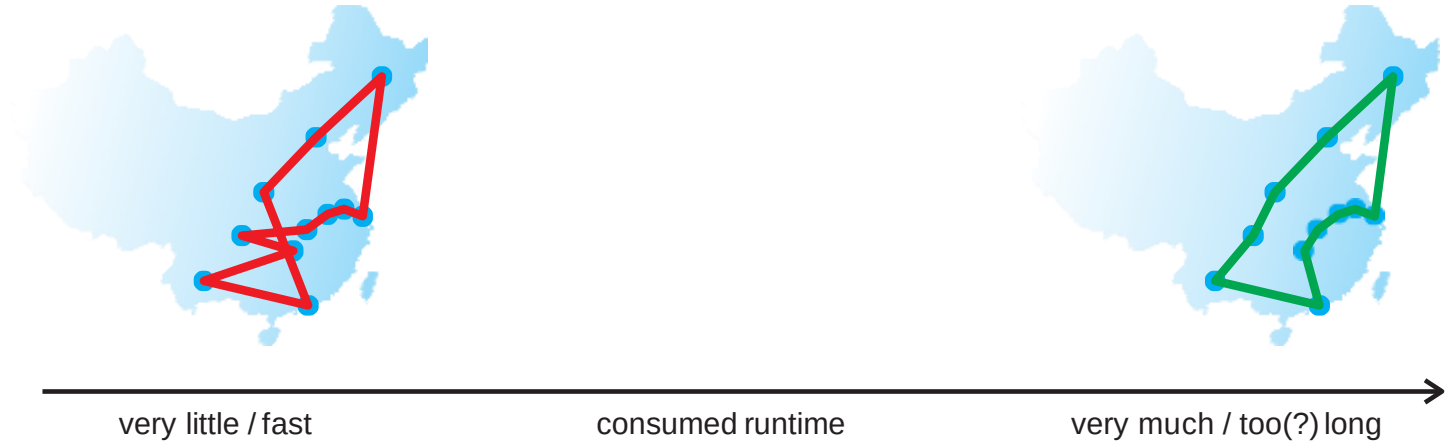

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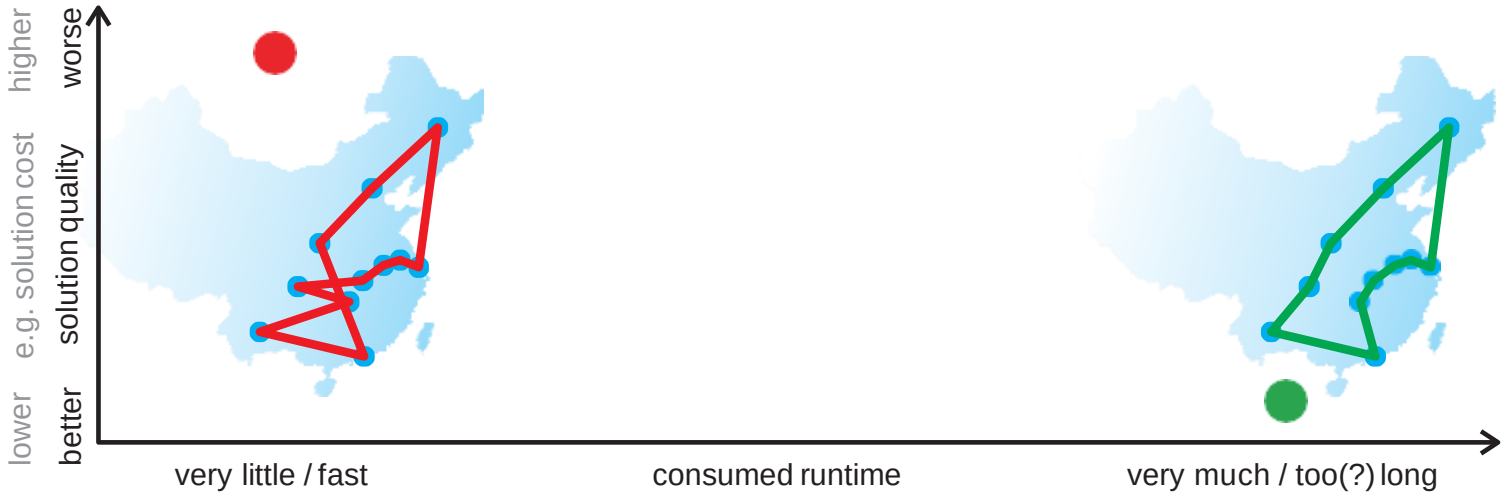

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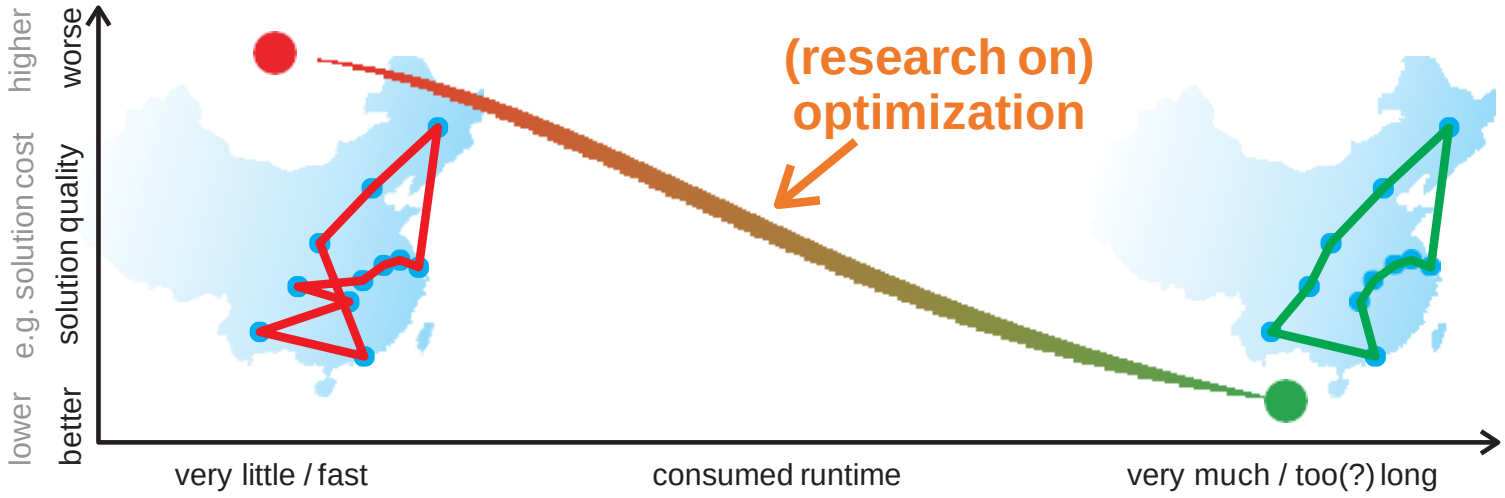

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  - Optimization often means to trade-off between solution quality and runtime and development time (the time from the definition of the problem until we have a software for producing solutions).

Metaheuristics

- Heuristics are often specialized algorithms: Say we have an approximate algorithm for the TSP, or one for the Bin Packing problem...
- However, there are may different optimization problems.
- Should we develop a completely new method for each problem?
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- Should we develop a completely new method for each problem?
- No. We want general algorithms that can be adapted to different problems. (also to reduce the development time . . . we often want a prototype quickly and can add more complex logic later)

Definition (Metaheuristic)

A metaheuristic is a method for solving very general classes of problems. It combines objective functions or heuristics in an abstract and hopefully efficient way, usually by treating them as black box-procedures. [46, 47]

Performance and Anytime Algorithms

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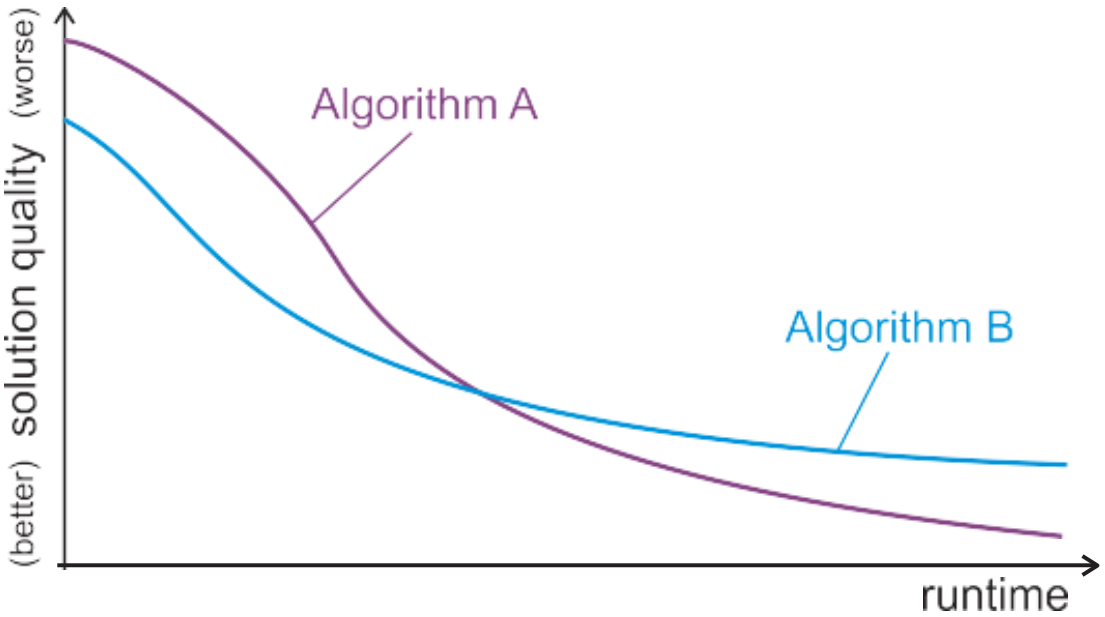

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- Several exact methods like Branch-and-Bound [51–53] are Anytime Algorithms.
- Consequence: Most optimization algorithms produce approximate solutions of different qualities at different points during their process.

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Example:

- *Exhaustive Enumeration:* Test all possible solutions one by one and, thus, find the optimum.
- *Deterministic Heuristic:* Solve the bin packing problem by putting the largest remaining element into a bin until the bin is full, then take the next bin, until all objects are packed.

Deterministic and Randomized Algorithms

- Two types of algorithms: deterministic and randomized

Definition (Randomized Algorithm)

A randomized or probabilistic algorithm includes at least one instruction that acts on the basis of random numbers. In other words, a probabilistic algorithm violates the constraint of determinism. [43, 54–57]

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- *Randomized Local Search:* Create a random solution, then try to improve it by modifying it a bit (randomly). If the new one is better, keep it. Modify again.

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- Deterministic heuristics are often simple, fast, and provide solutions with mediocre approximation quality
- Algorithms often need to make choices (Test all BMW with white color first??) from which we don't know which choice is good $\Longrightarrow$ flip a coin (i.e., avoid to always make same (potentially bad) choice) $\Longrightarrow$ randomize

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A Monte Carlo Algorithm always terminates. Its result, however, can either be correct or incorrect. [58–61]

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- . . . since we often have a limited computational budget and need something that finishes within that time

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Definition (Single-Objective Optimization)

We have a single objective function f rating solution quality and want to find the solutions where f takes on minimal (or maximal) values.

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- Optimization problems and algorithms can be divided into single-objective and multi-objective optimization.

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Definition (Constraint Optimization)

Single- or multi-objective Optimization where the solutions also need to fullful additional constraints.[67–69]

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