The "Big M" Method

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4. Introduce an **artificial variable** $\bar{x}_j \geq 0$ in each '$=$' constraint.

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4. Introduce an **artificial variable** $\bar{x}_j \geq 0$ in each '$=$' constraint.

5. For each artificial variable $\bar{x}_i$, add a **penalty term** '$-M\bar{x}_i$' to the objective function. Use the same constant M for all the artificial variables. (*In numerical software, use a very large number for M.*)

The "Big M" Method: Example

Example (Big M in Action)

Maximize $P = 2x_1 + x_2$
subject to

$$x_1 + x_2 \leq 10$$
$$-x_1 + x_2 \geq 2$$

with $\quad x_1,\ x_2 \geq 0.$

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The Big M Simplex Tableau

Eq	Z	x_1	x_2	s_1	s_2	$\bar{x}_1$	b
(0)	1	-2	-1	0	0	M	0
(1)	0	1	1	1	0	0	10
(2)	0	-1	1	0	-1	1	2

Use Maple

The "Big M" Method: Exercise

Exercise (O-Jay)

O-Jay is a mixture of orange juice and orange soda. We need to restrict the amount of sugar to 4gm/bottle and maintain at least 20mg/bottle of vitamin C. What is the least cost mixture?
Let:

- $x_1 =$ *number of ounces of orange soda in a bottle of O-Jay*
- $x_2 =$ *number of ounces of orange juice in a bottle of O-Jay*

The LP is:
Minimize $z = 2x_1 + 3x_2$
subject to

$$\begin{aligned} 0.5x_1 + 0.25x_2 &\leq 4 && \text{(sugar constraint)} \\ x_1 + 3x_2 &\geq 20 && \text{(Vitamin C constraint)} \\ x_1 + x_2 &= 10 && \text{(10 oz in per bottle)} \end{aligned}$$

with $\quad x_1, x_2 \geq 0$

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4. Add an artificial variable $\bar{x}_k$ to each ‘$=$’ constraint.

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1. If the problem is "minimize Z," change to "maximize (-Z)."
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5. Add '$-M\bar{x}_j$' to the objective function for each artificial variable $\bar{x}_j$.

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5. Add '$-M\bar{x}_j$' to the objective function for each artificial variable $\bar{x}_j$.
6. Use a row operation with each artificial variable row to eliminate M from the objective function in $\bar{x}_j$ columns.[1]

[1] Just operate on Row (0), the objective function, to reduce arithmetic.

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1. If the problem is "minimize Z," change to "maximize (-Z)."
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5. Add '$-M\bar{x}_j$' to the objective function for each artificial variable $\bar{x}_j$.
6. Use a row operation with each artificial variable row to eliminate M from the objective function in $\bar{x}_j$ columns.[1]
7. Run the simplex algorithm.

[1] Just operate on Row (0), the objective function, to reduce arithmetic.

The "Big M" Method: Big Example

Example (Big "Big M")

Maximize $\quad Z = 2x_1 + 5x_2 + 3x_3$
subject to

$$
\begin{aligned}
x_1 + 2x_2 - x_3 &\le 7 \\
-x_1 + x_2 - 2x_3 &\le -5 \\
x_1 + 4x_2 + 3x_3 &\ge 1 \\
2x_1 - x_2 + 4x_3 &= 6
\end{aligned}
$$

with $\quad x_1,\ x_2,\ x_3 \ge 0.$

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with $\quad x_1,\ x_2,\ x_3 \geq 0$.

Variables

There are:

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with $\quad x_1,\, x_2,\, x_3 \geq 0.$

Variables

There are:

- 3 decision variables x_i

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with $\quad x_1,\, x_2,\, x_3 \geq 0.$

Variables

There are:

- 3 decision variables x_i
- 1 slack variable s_1

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Example (Big "Big M")

Maximize $\quad Z = 2x_1 + 5x_2 + 3x_3$
subject to

$$\begin{aligned} x_1 + 2x_2 - x_3 &\le 7 \\ -x_1 + x_2 - 2x_3 &\le -5 \\ x_1 + 4x_2 + 3x_3 &\ge 1 \\ 2x_1 - x_2 + 4x_3 &= 6 \end{aligned}$$

with $\quad x_1,\, x_2,\, x_3 \ge 0.$

Variables

There are:

- 3 decision variables x_i
- 1 slack variable s_1
- 2 surplus variables s_j

The "Big M" Method: Big Example

Example (Big "Big M")

Maximize $\quad Z = 2x_1 + 5x_2 + 3x_3$
subject to

$$x_1 + 2x_2 - x_3 \leq 7$$
$$-x_1 + x_2 - 2x_3 \leq -5$$
$$x_1 + 4x_2 + 3x_3 \geq 1$$
$$2x_1 - x_2 + 4x_3 = 6$$

with $\quad x_1,\, x_2,\, x_3 \geq 0.$

Variables

There are:

- 3 decision variables x_i
- 1 slack variable s_1
- 2 surplus variables s_j
- 3 artifical variables $\bar{x}_j$

The "Big M" Method: Big Example

Initial Artificial Problem Tableau

BV	Eq	Z	x_1	x_2	x_3	s_1	s_2	s_3	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	b
	(0)	1	-2	-5	-3	0	0	0	M	M	M	0
s_1	(1)	0	1	2	-1	1	0	0	0	0	0	7
$\bar{x}_1$	(2)	0	1	-1	2	0	-1	0	1	0	0	5
$\bar{x}_2$	(3)	0	1	4	3	0	0	-1	0	1	0	1
$\bar{x}_3$	(4)	0	2	-1	4	0	0	0	0	0	1	6

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Initial Artificial Problem Tableau

BV	Eq	Z	x_1	x_2	x_3	s_1	s_2	s_3	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	b
	(0)	1	-2	-5	-3	0	0	0	M	M	M	0
s_1	(1)	0	1	2	-1	1	0	0	0	0	0	7
$\bar{x}_1$	(2)	0	1	-1	2	0	-1	0	1	0	0	5
$\bar{x}_2$	(3)	0	1	4	3	0	0	-1	0	1	0	1
$\bar{x}_3$	(4)	0	2	-1	4	0	0	0	0	0	1	6

Beginning Simplex Tableau

Eq	x_1	x_2	x_3	s_1	s_2	s_3	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	b
(0)	$-2-4M$	$-5-2M$	$-3-9M$	0	M	M	0	0	0	$-12M$
(1)	1	2	-1	1	0	0	0	0	0	7
(2)	1	-1	2	0	-1	0	1	0	0	5
(3)	1	4	3	0	0	-1	0	1	0	1
(4)	2	-1	4	0	0	0	0	0	1	6

Use Maple