Fundamentals of Operations Research

HW #2: Solving Linear Programming

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Problem #1:

Solve the following model using Big-M simplex method.

Max $z = x_1 + x_2$

s.t

$3x_1 + 2x_2 \leq 20$

$2x_1 + 3x_2 \leq 20$

$x_1 + 2x_2 \geq 2$

$x_1 , x_2 \geq 0$

Problem #2:

Solve the following model using two-phase simplex method.

Min $z = -x_1 - 2x_2 - 3x_3$

s.t

$x_1 + x_2 + x_3 = 6$

$-x_1 + x_2 + 2x_3 = 4$

$2x_2 + 3x_3 = 10$

$x_3 \leq 2$

$x_1 , x_2 , x_3 \geq 0$

Problem #3:

Consider the following LP model:

$$\text{Max } z = 2x_1 + 3x_2$$

s.t

$$x_1 + x_2 \geq 3$$

$$x_1 - 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

A) Find the optimal solution using graphical (geometry) method.

B) Find the optimal solution using Big-M method.

C) Showing the path of solving solution by simplex (part B) on the part A graph.

D) Find the optimal solution using two-phase method.

Problem #4:

Consider the following problem and assume that b_1 and b_2 are constants.

$$\text{Max } Z = 5x_1 + 2x_2 + 3x_3$$

s.t

$$x_1 + 5x_2 + 2x_3 \leq b_1$$

$$x_1 - 5x_2 - 6x_3 \leq b_2$$

$$x_1, x_2, x_3 \geq 0$$

For a value of b_1 and b_2 the optimal tableau is as follow:

	z	x_1	x_2	x_3	S_1	S_2	RHS
z	1	0	a	7	d	e	150
x_1	0	1	b	2	1	0	30
S_2	0	0	c	-8	-1	1	10

A) Determine the value of b_1 and b_2.

B) What is the optimal solutoin for dual problem.

C) Determine the value of a, b and c.

D) For increasing the Z, we have to increase the value of b_1 or b_2? Why? How much we can increase?

Problem #5:

Consider the following LP model:

$$\text{Max } Z = 3x_1 + 2x_2$$

s.t

$$2x_1 + x_2 \leq 4$$

$$-2x_1 + x_2 \leq 2$$

$$x_1 - x_2 \leq 1$$

$$x_1 , x_2 \geq 0$$

A) Find the optimal solution with graphical method.

B) Rewrite the model in the standard from.

C) Write the sequence of meeting each basic solution in simplex algorithm to reach the optimal solution.

Problem #6:

Find the optimal value of objective function without using graphical or simplex method.

$$\text{Min } Z = 10x_1 + 4x_2 + 5x_3$$

s.t

$$5x_1 - 7x_2 + 3x_3 \geq 50$$

$$x_1 , x_2 , x_3 \geq 0$$

Problem #7:

The final tableau of simplex is as below. Determine the shadow price. If the value of RHS incereas from 4, 12, 18 to 5, 14, 18 respectively, What is the effect of the Z function value?

	z	x_1	x_2	S_1	S_2	S_3	RHS
z	1	0	0	0	3.2	1	36
S_1	0	0	0	1	1.3	-1.3	2
x_2	0	0	1	0	1.2	0	6
x_1	0	1	0	0	-1.3	1.3	2

Problem #8:

Consider the following model:

$$\text{Max } Z = 2x_1 + x_2$$

s.t

$$x_1 + 2x_2 \leq 14$$
$$2x_1 - x_2 \leq 10$$
$$x_1 - x_2 \leq 3$$
$$x_1 , x_2 \geq 0$$

A) Determine the dual form of this model.

B) Is the x=(20.3, 11.3) feasible?

Problem #9:

The following LP model describes a problem of allocating three resources to the annual production of three commodities by a manufacturing firm. The amounts of the three products to be produced are denoted by x_1, x_2, and x_3. The objective function reflects the dollar contribution to profit of these products.

$$\text{Max } Z = 10x_1 + 15x_2 + 5x_3$$

s.t

$$2x_1 + x_2 \le 6000$$
$$3x_1 + 3x_2 + x_3 \le 9000$$
$$x_1 + 2x_2 + 2x_3 \le 4000$$
$$x_1,\ x_2,\ x_3 \ge 0$$

A) Without using the simplex method, verify that the optimal basis consists of the slack variable of the first constraint, x_1, and x_2.

B) Make use of the information in Part (a) to write the optimal tableau.

C) The Research and Development Department proposes a new product whose production coefficients are represented by <2,4,2>. If the contribution to profit is $15 per unit of this new product, should this product be produced? If so, what is the new optimal solution?

D) What is the minimal contribution to profit that should be expected before production of this new product would actually increase the value of the objective function?

Problem #10:

Find the optimal solution of following model by converting to dual form:

$$\text{Max } Z = 3x_1 + 3x_2 + 21x_3$$

s.t

$$6x_1 + 9x_2 + 25x_3 \le 15$$
$$3x_1 + 2x_2 + 25x_3 \le 20$$
$$x_1,\ x_2,\ x_3 \ge 0$$