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1o. semestre, 2025
Exercício 02 - 22/04

PGF5261/IFUSP: Teoria de Grupos Aplicada a Sólidos e Moléculas. Prof.: Lucy Assali

Questão 1. Considere as simetrias da molécula de etano eclipsado C_2H_6, pertencente ao grupo D_{3h} e responda os seguintes ítems:

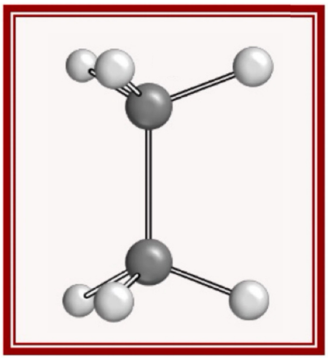


Figure 1: Molécula do etano eclipsado

(1.1) Identifique as operações de simetria da molécula.

(1.2) Apresente a **tabela de multiplicação**.

(1.3) Esse grupo é **cíclico**? Justifique.

(1.4) O grupo é **abeliano**? Justifique.

(1.5) Quais são as **classes** deste grupo?

(1.6) Este grupo possui **subgrupos**?

Resposta para o Ítem 1.1 - Simetrias da Molécula

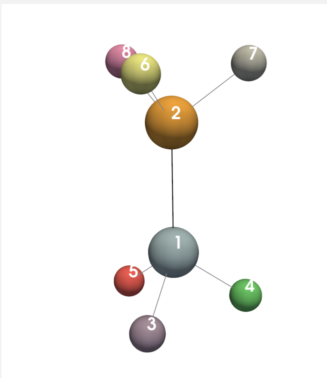


Figure 2: Simetria E

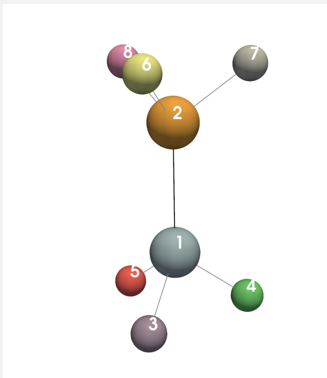


Figure 3: Operação E aplicada

$$: \quad E(1,2,3,4,5,6,7,8) = (1,2,3,4,5,6,7,8)$$

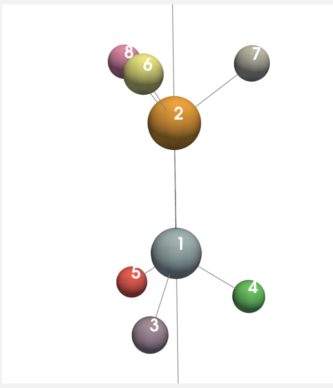


Figure 4: Eixo de rotação C_3

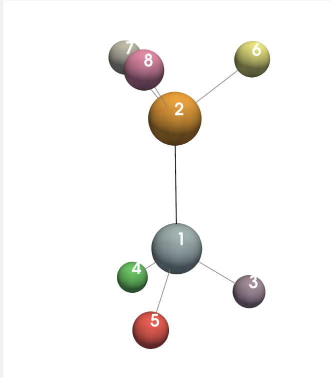


Figure 5: Operação C_3 aplicada

$$
\textbf{:}\quad \begin{aligned} C_3(1,2,3,4,5,6,7,8) &= (1,2,5,3,4,8,6,7) \\ C_3^2(1,2,3,4,5,6,7,8) &= (1,2,4,5,3,7,8,6) \end{aligned}
$$

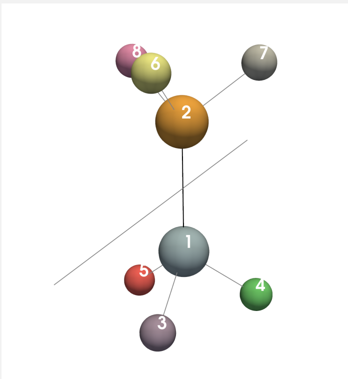


Figure 6: Eixo de rotação C_2

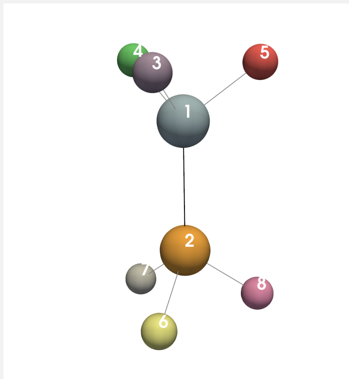


Figure 7: Operação $C_2^{(a)}$ aplicada

$$
\textbf{:}\quad \begin{aligned} C_2^{(a)}(1,2,3,4,5,6,7,8) &= (2,1,6,8,7,3,5,4) \\ C_2^{(b)}(1,2,3,4,5,6,7,8) &= (2,1,8,7,6,5,4,3) \\ C_2^{(c)}(1,2,3,4,5,6,7,8) &= (2,1,7,6,8,4,3,5) \end{aligned}
$$

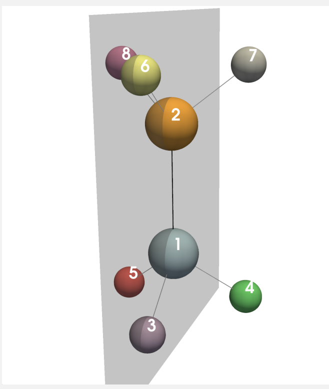


Figure 8: Plano de reflexão σ_v

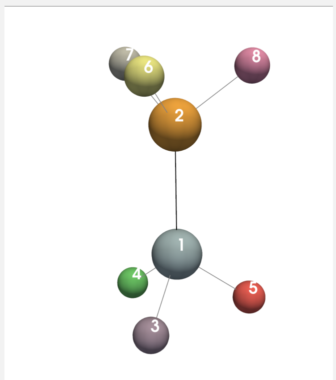


Figure 9: Operação σ_{v1} aplicada

$$
\textbf{:}\quad \begin{aligned} \sigma_{v1}(1,2,3,4,5,6,7,8) &= (1,2,4,5,3,7,8,6) \\ \sigma_{v2}(1,2,3,4,5,6,7,8) &= (1,2,5,3,4,8,6,7) \\ \sigma_{v3}(1,2,3,4,5,6,7,8) &= (1,2,4,3,5,7,6,8) \end{aligned}
$$

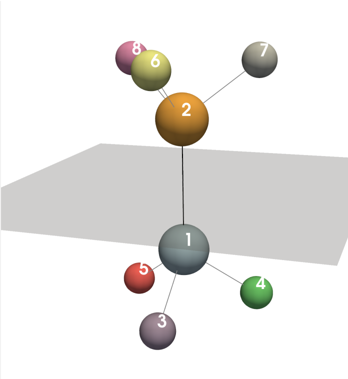


Figure 10: Plano de reflexão σ_h

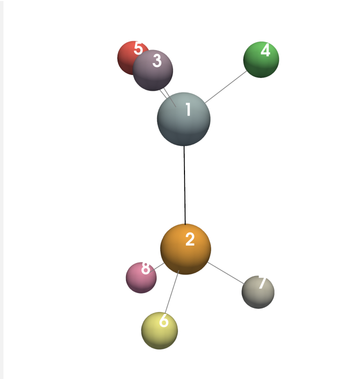


Figure 11: Operação σ_h aplicada

$$: \quad \sigma_h(1,2,3,4,5,6,7,8) = (2,1,6,7,8,3,4,5)$$

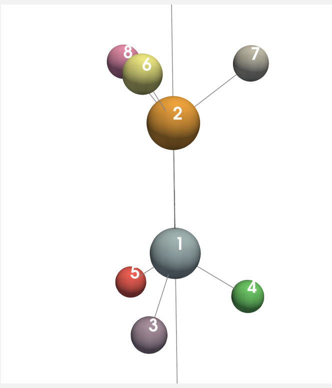


Figure 12: Eixo de rotação C_3

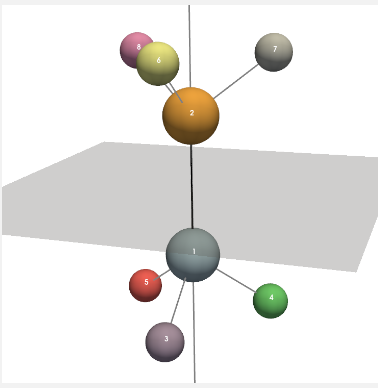


Figure 13: Plano de reflexão σ_h

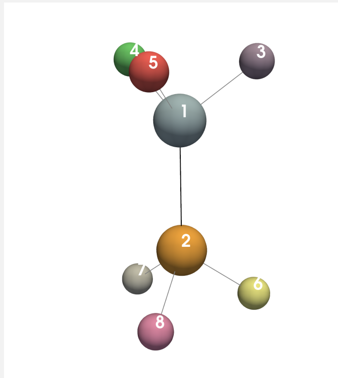


Figure 14: Operação S_3 aplicada

$$\begin{aligned} : \quad & S_3(1,2,3,4,5,6,7,8) = (2,1,8,6,7,5,3,4) \\ & S_3^5(1,2,3,4,5,6,7,8) = (2,1,7,8,6,4,5,3) \end{aligned}$$

Resposta para o Ítem 1.2 - Tabela de Multiplicação

Operações de Multiplicação:

$$\begin{aligned}
\mathrm{E} \circ \mathrm{E} &= \mathrm{E} \circ [1,2,3,4,5,6,7,8] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{E} \circ \mathrm{C}_3 &= \mathrm{E} \circ [1,2,4,5,3,7,8,6] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{E} \circ \mathrm{C}_3^2 &= \mathrm{E} \circ [1,2,5,3,4,8,6,7] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{E} \circ \mathrm{C}_2^{(a)} &= \mathrm{E} \circ [2,1,6,8,7,3,5,4] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{E} \circ \mathrm{C}_2^{(b)} &= \mathrm{E} \circ [2,1,8,7,6,5,4,3] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{E} \circ \mathrm{C}_2^{(c)} &= \mathrm{E} \circ [2,1,7,6,8,4,3,5] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{E} \circ \sigma_{\mathrm{v1}} &= \mathrm{E} \circ [1,2,3,5,4,6,8,7] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{E} \circ \sigma_{\mathrm{v2}} &= \mathrm{E} \circ [1,2,5,4,3,8,7,6] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{E} \circ \sigma_{\mathrm{v3}} &= \mathrm{E} \circ [1,2,4,3,5,7,6,8] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{E} \circ \sigma_{\mathrm{h}} &= \mathrm{E} \circ [2,1,6,7,8,3,4,5] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{E} \circ \mathrm{S}_3 &= \mathrm{E} \circ [2,1,7,8,6,4,5,3] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{E} \circ \mathrm{S}_3^5 &= \mathrm{E} \circ [2,1,8,6,7,5,3,4] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_3 \circ \mathrm{E} &= \mathrm{C}_3 \circ [1,2,3,4,5,6,7,8] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_3 \circ \mathrm{C}_3 &= \mathrm{C}_3 \circ [1,2,4,5,3,7,8,6] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_3 \circ \mathrm{C}_3^2 &= \mathrm{C}_3 \circ [1,2,5,3,4,8,6,7] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_3 \circ \mathrm{C}_2^{(a)} &= \mathrm{C}_3 \circ [2,1,6,8,7,3,5,4] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{C}_3 \circ \mathrm{C}_2^{(b)} &= \mathrm{C}_3 \circ [2,1,8,7,6,5,4,3] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{C}_3 \circ \mathrm{C}_2^{(c)} &= \mathrm{C}_3 \circ [2,1,7,6,8,4,3,5] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{C}_3 \circ \sigma_{\mathrm{v1}} &= \mathrm{C}_3 \circ [1,2,3,5,4,6,8,7] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{C}_3 \circ \sigma_{\mathrm{v2}} &= \mathrm{C}_3 \circ [1,2,5,4,3,8,7,6] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}}
\end{aligned}$$

$$
\begin{aligned}
\mathrm{C}_3 \circ \sigma_{\mathrm{v3}} &= \mathrm{C}_3 \circ [1,2,4,3,5,7,6,8] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{C}_3 \circ \sigma_{\mathrm{h}} &= \mathrm{C}_3 \circ [2,1,6,7,8,3,4,5] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{C}_3 \circ \mathrm{S}_3 &= \mathrm{C}_3 \circ [2,1,7,8,6,4,5,3] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_3 \circ \mathrm{S}_3^5 &= \mathrm{C}_3 \circ [2,1,8,6,7,5,3,4] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{C}_3^2 \circ \mathrm{E} &= \mathrm{C}_3^2 \circ [1,2,3,4,5,6,7,8] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_3^2 \circ \mathrm{C}_3 &= \mathrm{C}_3^2 \circ [1,2,4,5,3,7,8,6] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_3^2 \circ \mathrm{C}_3^2 &= \mathrm{C}_3^2 \circ [1,2,5,3,4,8,6,7] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(a)} &= \mathrm{C}_3^2 \circ [2,1,6,8,7,3,5,4] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(b)} &= \mathrm{C}_3^2 \circ [2,1,8,7,6,5,4,3] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(c)} &= \mathrm{C}_3^2 \circ [2,1,7,6,8,4,3,5] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{v1}} &= \mathrm{C}_3^2 \circ [1,2,3,5,4,6,8,7] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{v2}} &= \mathrm{C}_3^2 \circ [1,2,5,4,3,8,7,6] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{v3}} &= \mathrm{C}_3^2 \circ [1,2,4,3,5,7,6,8] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{h}} &= \mathrm{C}_3^2 \circ [2,1,6,7,8,3,4,5] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_3^2 \circ \mathrm{S}_3 &= \mathrm{C}_3^2 \circ [2,1,7,8,6,4,5,3] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{C}_3^2 \circ \mathrm{S}_3^5 &= \mathrm{C}_3^2 \circ [2,1,8,6,7,5,3,4] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{C}_2^{(a)} \circ \mathrm{E} &= \mathrm{C}_2^{(a)} \circ [1,2,3,4,5,6,7,8] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{C}_2^{(a)} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(a)} \circ [1,2,4,5,3,7,8,6] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{C}_2^{(a)} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(a)} \circ [1,2,5,3,4,8,6,7] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(a)} &= \mathrm{C}_2^{(a)} \circ [2,1,6,8,7,3,5,4] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(b)} &= \mathrm{C}_2^{(a)} \circ [2,1,8,7,6,5,4,3] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(c)} &= \mathrm{C}_2^{(a)} \circ [2,1,7,6,8,4,3,5] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{v1}} &= \mathrm{C}_2^{(a)} \circ [1,2,3,5,4,6,8,7] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{v2}} &= \mathrm{C}_2^{(a)} \circ [1,2,5,4,3,8,7,6] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{v3}} &= \mathrm{C}_2^{(a)} \circ [1,2,4,3,5,7,6,8] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{h}} &= \mathrm{C}_2^{(a)} \circ [2,1,6,7,8,3,4,5] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{C}_2^{(a)} \circ \mathrm{S}_3 &= \mathrm{C}_2^{(a)} \circ [2,1,7,8,6,4,5,3] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{C}_2^{(a)} \circ \mathrm{S}_3^5 &= \mathrm{C}_2^{(a)} \circ [2,1,8,6,7,5,3,4] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{C}_2^{(b)} \circ \mathrm{E} &= \mathrm{C}_2^{(b)} \circ [1,2,3,4,5,6,7,8] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{C}_2^{(b)} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(b)} \circ [1,2,4,5,3,7,8,6] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{C}_2^{(b)} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(b)} \circ [1,2,5,3,4,8,6,7] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{C}_2^{(b)} \circ \mathrm{C}_2^{(a)} &= \mathrm{C}_2^{(b)} \circ [2,1,6,8,7,3,5,4] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(b)} \circ \mathrm{C}_2^{(b)} &= \mathrm{C}_2^{(b)} \circ [2,1,8,7,6,5,4,3] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(b)} \circ \mathrm{C}_2^{(c)} &= \mathrm{C}_2^{(b)} \circ [2,1,7,6,8,4,3,5] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(b)} \circ \sigma_{\mathrm{v1}} &= \mathrm{C}_2^{(b)} \circ [1,2,3,5,4,6,8,7] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{C}_2^{(b)} \circ \sigma_{\mathrm{v2}} &= \mathrm{C}_2^{(b)} \circ [1,2,5,4,3,8,7,6] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{C}_2^{(b)} \circ \sigma_{\mathrm{v3}} &= \mathrm{C}_2^{(b)} \circ [1,2,4,3,5,7,6,8] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_2^{(b)} \circ \sigma_{\mathrm{h}} &= \mathrm{C}_2^{(b)} \circ [2,1,6,7,8,3,4,5] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{C}_2^{(b)} \circ \mathrm{S}_3 &= \mathrm{C}_2^{(b)} \circ [2,1,7,8,6,4,5,3] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{C}_2^{(b)} \circ \mathrm{S}_3^5 &= \mathrm{C}_2^{(b)} \circ [2,1,8,6,7,5,3,4] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{C}_2^{(c)} \circ \mathrm{E} &= \mathrm{C}_2^{(c)} \circ [1,2,3,4,5,6,7,8] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(c)} \\
\mathrm{C}_2^{(c)} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(c)} \circ [1,2,4,5,3,7,8,6] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(b)} \\
\mathrm{C}_2^{(c)} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(c)} \circ [1,2,5,3,4,8,6,7] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(a)} \\
\mathrm{C}_2^{(c)} \circ \mathrm{C}_2^{(a)} &= \mathrm{C}_2^{(c)} \circ [2,1,6,8,7,3,5,4] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(c)} \circ \mathrm{C}_2^{(b)} &= \mathrm{C}_2^{(c)} \circ [2,1,8,7,6,5,4,3] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(c)} \circ \mathrm{C}_2^{(c)} &= \mathrm{C}_2^{(c)} \circ [2,1,7,6,8,4,3,5] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(c)} \circ \sigma_{\mathrm{v1}} &= \mathrm{C}_2^{(c)} \circ [1,2,3,5,4,6,8,7] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{C}_2^{(c)} \circ \sigma_{\mathrm{v2}} &= \mathrm{C}_2^{(c)} \circ [1,2,5,4,3,8,7,6] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{C}_2^{(c)} \circ \sigma_{\mathrm{v3}} &= \mathrm{C}_2^{(c)} \circ [1,2,4,3,5,7,6,8] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{C}_2^{(c)} \circ \sigma_{\mathrm{h}} &= \mathrm{C}_2^{(c)} \circ [2,1,6,7,8,3,4,5] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{C}_2^{(c)} \circ \mathrm{S}_3 &= \mathrm{C}_2^{(c)} \circ [2,1,7,8,6,4,5,3] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{C}_2^{(c)} \circ \mathrm{S}_3^5 &= \mathrm{C}_2^{(c)} \circ [2,1,8,6,7,5,3,4] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\sigma_{\mathrm{v1}} \circ \mathrm{E} &= \sigma_{\mathrm{v1}} \circ [1,2,3,4,5,6,7,8] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\sigma_{\mathrm{v1}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{v1}} \circ [1,2,4,5,3,7,8,6] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\sigma_{\mathrm{v1}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{v1}} \circ [1,2,5,3,4,8,6,7] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{\mathrm{v1}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{v1}} \circ [2,1,6,8,7,3,5,4] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\sigma_{\mathrm{v1}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{v1}} \circ [2,1,8,7,6,5,4,3] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\sigma_{\mathrm{v1}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{v1}} \circ [2,1,7,6,8,4,3,5] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\sigma_{\mathrm{v1}} \circ \sigma_{\mathrm{v1}} &= \sigma_{\mathrm{v1}} \circ [1,2,3,5,4,6,8,7] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{v1}} \circ \sigma_{\mathrm{v2}} &= \sigma_{\mathrm{v1}} \circ [1,2,5,4,3,8,7,6] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{v1}} \circ \sigma_{\mathrm{v3}} &= \sigma_{\mathrm{v1}} \circ [1,2,4,3,5,7,6,8] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{v1}} \circ \sigma_{\mathrm{h}} &= \sigma_{\mathrm{v1}} \circ [2,1,6,7,8,3,4,5] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{v1}} \circ \mathrm{S}_3 &= \sigma_{\mathrm{v1}} \circ [2,1,7,8,6,4,5,3] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{v1}} \circ \mathrm{S}_3^5 &= \sigma_{\mathrm{v1}} \circ [2,1,8,6,7,5,3,4] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{v2}} \circ \mathrm{E} &= \sigma_{\mathrm{v2}} \circ [1,2,3,4,5,6,7,8] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\sigma_{\mathrm{v2}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{v2}} \circ [1,2,4,5,3,7,8,6] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\sigma_{\mathrm{v2}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{v2}} \circ [1,2,5,3,4,8,6,7] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\sigma_{\mathrm{v2}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{v2}} \circ [2,1,6,8,7,3,5,4] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\sigma_{\mathrm{v2}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{v2}} \circ [2,1,8,7,6,5,4,3] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\sigma_{\mathrm{v2}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{v2}} \circ [2,1,7,6,8,4,3,5] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\sigma_{\mathrm{v2}} \circ \sigma_{\mathrm{v1}} &= \sigma_{\mathrm{v2}} \circ [1,2,3,5,4,6,8,7] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{v2}} \circ \sigma_{\mathrm{v2}} &= \sigma_{\mathrm{v2}} \circ [1,2,5,4,3,8,7,6] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{v2}} \circ \sigma_{\mathrm{v3}} &= \sigma_{\mathrm{v2}} \circ [1,2,4,3,5,7,6,8] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{v2}} \circ \sigma_{\mathrm{h}} &= \sigma_{\mathrm{v2}} \circ [2,1,6,7,8,3,4,5] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{v2}} \circ \mathrm{S}_3 &= \sigma_{\mathrm{v2}} \circ [2,1,7,8,6,4,5,3] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{v2}} \circ \mathrm{S}_3^5 &= \sigma_{\mathrm{v2}} \circ [2,1,8,6,7,5,3,4] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{v3}} \circ \mathrm{E} &= \sigma_{\mathrm{v3}} \circ [1,2,3,4,5,6,7,8] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\sigma_{\mathrm{v3}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{v3}} \circ [1,2,4,5,3,7,8,6] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\sigma_{\mathrm{v3}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{v3}} \circ [1,2,5,3,4,8,6,7] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\sigma_{\mathrm{v3}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{v3}} \circ [2,1,6,8,7,3,5,4] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\sigma_{\mathrm{v3}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{v3}} \circ [2,1,8,7,6,5,4,3] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\sigma_{\mathrm{v3}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{v3}} \circ [2,1,7,6,8,4,3,5] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\sigma_{\mathrm{v3}} \circ \sigma_{\mathrm{v1}} &= \sigma_{\mathrm{v3}} \circ [1,2,3,5,4,6,8,7] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{v3}} \circ \sigma_{\mathrm{v2}} &= \sigma_{\mathrm{v3}} \circ [1,2,5,4,3,8,7,6] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{v3}} \circ \sigma_{\mathrm{v3}} &= \sigma_{\mathrm{v3}} \circ [1,2,4,3,5,7,6,8] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{v3}} \circ \sigma_{\mathrm{h}} &= \sigma_{\mathrm{v3}} \circ [2,1,6,7,8,3,4,5] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{v3}} \circ \mathrm{S}_3 &= \sigma_{\mathrm{v3}} \circ [2,1,7,8,6,4,5,3] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{v3}} \circ \mathrm{S}_3^5 &= \sigma_{\mathrm{v3}} \circ [2,1,8,6,7,5,3,4] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{h}} \circ \mathrm{E} &= \sigma_{\mathrm{h}} \circ [1,2,3,4,5,6,7,8] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\sigma_{\mathrm{h}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{h}} \circ [1,2,4,5,3,7,8,6] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\sigma_{\mathrm{h}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{h}} \circ [1,2,5,3,4,8,6,7] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\sigma_{\mathrm{h}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{h}} \circ [2,1,6,8,7,3,5,4] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\sigma_{\mathrm{h}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{h}} \circ [2,1,8,7,6,5,4,3] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\sigma_{\mathrm{h}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{h}} \circ [2,1,7,6,8,4,3,5] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\sigma_{\mathrm{h}} \circ \sigma_{\mathrm{v1}} &= \sigma_{\mathrm{h}} \circ [1,2,3,5,4,6,8,7] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{h}} \circ \sigma_{\mathrm{v2}} &= \sigma_{\mathrm{h}} \circ [1,2,5,4,3,8,7,6] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{h}} \circ \sigma_{\mathrm{v3}} &= \sigma_{\mathrm{h}} \circ [1,2,4,3,5,7,6,8] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{h}} \circ \sigma_{\mathrm{h}} &= \sigma_{\mathrm{h}} \circ [2,1,6,7,8,3,4,5] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{h}} \circ \mathrm{S}_3 &= \sigma_{\mathrm{h}} \circ [2,1,7,8,6,4,5,3] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{h}} \circ \mathrm{S}_3^5 &= \sigma_{\mathrm{h}} \circ [2,1,8,6,7,5,3,4] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{S}_3 \circ \mathrm{E} &= \mathrm{S}_3 \circ [1,2,3,4,5,6,7,8] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S}_3 \\
\mathrm{S}_3 \circ \mathrm{C}_3 &= \mathrm{S}_3 \circ [1,2,4,5,3,7,8,6] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S}_3^5 \\
\mathrm{S}_3 \circ \mathrm{C}_3^2 &= \mathrm{S}_3 \circ [1,2,5,3,4,8,6,7] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{S}_3 \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{S}_3 \circ [2,1,6,8,7,3,5,4] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{S}_3 \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{S}_3 \circ [2,1,8,7,6,5,4,3] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{S}_3 \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{S}_3 \circ [2,1,7,6,8,4,3,5] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{S}_3 \circ \sigma_{\mathrm{v1}} &= \mathrm{S}_3 \circ [1,2,3,5,4,6,8,7] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{S}_3 \circ \sigma_{\mathrm{v2}} &= \mathrm{S}_3 \circ [1,2,5,4,3,8,7,6] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{S}_3 \circ \sigma_{\mathrm{v3}} &= \mathrm{S}_3 \circ [1,2,4,3,5,7,6,8] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{S}_3 \circ \sigma_{\mathrm{h}} &= \mathrm{S}_3 \circ [2,1,6,7,8,3,4,5] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{S}_3 \circ \mathrm{S}_3 &= \mathrm{S}_3 \circ [2,1,7,8,6,4,5,3] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{S_3} \circ \mathrm{S_3^5} &= \mathrm{S_3} \circ [2,1,8,6,7,5,3,4] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{S_3^5} \circ \mathrm{E} &= \mathrm{S_3^5} \circ [1,2,3,4,5,6,7,8] &&= [2,1,8,6,7,5,3,4] &&= \mathrm{S_3^5} \\
\mathrm{S_3^5} \circ \mathrm{C_3} &= \mathrm{S_3^5} \circ [1,2,4,5,3,7,8,6] &&= [2,1,6,7,8,3,4,5] &&= \sigma_{\mathrm{h}} \\
\mathrm{S_3^5} \circ \mathrm{C_3^2} &= \mathrm{S_3^5} \circ [1,2,5,3,4,8,6,7] &&= [2,1,7,8,6,4,5,3] &&= \mathrm{S_3} \\
\mathrm{S_3^5} \circ \mathrm{C_2^{(a)}} &= \mathrm{S_3^5} \circ [2,1,6,8,7,3,5,4] &&= [1,2,4,3,5,7,6,8] &&= \sigma_{\mathrm{v3}} \\
\mathrm{S_3^5} \circ \mathrm{C_2^{(b)}} &= \mathrm{S_3^5} \circ [2,1,8,7,6,5,4,3] &&= [1,2,3,5,4,6,8,7] &&= \sigma_{\mathrm{v1}} \\
\mathrm{S_3^5} \circ \mathrm{C_2^{(c)}} &= \mathrm{S_3^5} \circ [2,1,7,6,8,4,3,5] &&= [1,2,5,4,3,8,7,6] &&= \sigma_{\mathrm{v2}} \\
\mathrm{S_3^5} \circ \sigma_{\mathrm{v1}} &= \mathrm{S_3^5} \circ [1,2,3,5,4,6,8,7] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C_2^{(c)}} \\
\mathrm{S_3^5} \circ \sigma_{\mathrm{v2}} &= \mathrm{S_3^5} \circ [1,2,5,4,3,8,7,6] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C_2^{(a)}} \\
\mathrm{S_3^5} \circ \sigma_{\mathrm{v3}} &= \mathrm{S_3^5} \circ [1,2,4,3,5,7,6,8] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C_2^{(b)}} \\
\mathrm{S_3^5} \circ \sigma_{\mathrm{h}} &= \mathrm{S_3^5} \circ [2,1,6,7,8,3,4,5] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C_3^2} \\
\mathrm{S_3^5} \circ \mathrm{S_3} &= \mathrm{S_3^5} \circ [2,1,7,8,6,4,5,3] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{S_3^5} \circ \mathrm{S_3^5} &= \mathrm{S_3^5} \circ [2,1,8,6,7,5,3,4] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C_3}
\end{aligned}
$$

Tabela de Multiplicação Resultante:

	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	S_3	S_3^5
E	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	S_3	S_3^5
C_3	C_3	C_3^2	E	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$	S_3	σ_{v2}	σ_{v3}	σ_{v1}	S_3^5	σ_h
C_3^2	C_3^2	E	C_3	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	S_3^5	σ_{v3}	σ_{v1}	σ_{v2}	σ_h	S_3
$C_2^{(a)}$	$C_2^{(a)}$	$C_2^{(c)}$	$C_2^{(b)}$	E	C_3	C_3^2	σ_{v1}	σ_h	S_3	S_3^5	σ_{v3}	σ_{v2}
$C_2^{(b)}$	$C_2^{(b)}$	$C_2^{(a)}$	$C_2^{(c)}$	C_3^2	E	C_3	σ_{v2}	S_3^5	σ_h	S_3	σ_{v1}	σ_{v3}
$C_2^{(c)}$	$C_2^{(c)}$	$C_2^{(b)}$	$C_2^{(a)}$	C_3	C_3^2	E	σ_{v3}	S_3	S_3^5	σ_h	σ_{v2}	σ_{v1}
σ_h	σ_h	S_3	S_3^5	σ_{v1}	σ_{v2}	σ_{v3}	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$
σ_{v1}	σ_{v1}	σ_{v3}	σ_{v2}	σ_h	S_3	S_3^5	$C_2^{(a)}$	E	C_3	C_3^2	$C_2^{(b)}$	$C_2^{(c)}$
σ_{v2}	σ_{v2}	σ_{v1}	σ_{v3}	S_3^5	σ_h	S_3	$C_2^{(b)}$	C_3^2	E	C_3	$C_2^{(c)}$	$C_2^{(a)}$
σ_{v3}	σ_{v3}	σ_{v2}	σ_{v1}	S_3	S_3^5	σ_h	$C_2^{(c)}$	C_3	C_3^2	E	$C_2^{(a)}$	$C_2^{(b)}$
S_3	S_3	S_3^5	σ_h	σ_{v3}	σ_{v1}	σ_{v2}	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$	C_3	C_3^2	E
S_3^5	S_3^5	σ_h	S_3	σ_{v2}	σ_{v3}	σ_{v1}	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	C_3^2	E	C_3

Resposta para o Ítem 1.3 - Cíclico

Um grupo é considerado cíclico quando todos os seus elementos podem ser obtidos repetindo a composição de um único elemento com ele mesmo. No caso do grupo D_{3h}, isso não é possível, pois há operações como reflexões e rotações impróprias que não surgem apenas a partir das potências de C_3, ou de qualquer outro elemento isolado. Dessa forma, o D_{3h} não é um grupo cíclico.

Resposta para o Ítem 1.4 - Abeliano

O grupo D_{3h} não é abeliano, pois a ordem das operações afeta o resultado final. Em um grupo abeliano, qualquer par de elementos pode ser composto em qualquer ordem sem alterar o produto. No entanto, no D_{3h}, isso não acontece. Por exemplo, $C_3 \circ \sigma_{v1} \neq \sigma_{v1} \circ C_3$. Essa assimetria pode ser verificada diretamente na tabela de multiplicação, onde a ausência de simetria em relação à diagonal evidencia que as composições não são comutativas. Logo, concluímos que D_{3h} é um grupo não abeliano.

Resposta para o Ítem 1.5 - Classes

Conjugações do elemento E

$\mathrm{E} \circ \mathrm{E} \circ \mathrm{E}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_3} \circ \mathrm{E} \circ \mathrm{C_3}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_3^2} \circ \mathrm{E} \circ \mathrm{C_3^2}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(a)}} \circ \mathrm{E} \circ \mathrm{C_2^{(a)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(b)}} \circ \mathrm{E} \circ \mathrm{C_2^{(b)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(c)}} \circ \mathrm{E} \circ \mathrm{C_2^{(c)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$

$\sigma_{\mathrm{d1}} \circ \mathrm{E} \circ \sigma_{\mathrm{d1}}{}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\sigma_{\mathrm{d2}} \circ \mathrm{E} \circ \sigma_{\mathrm{d2}}{}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\sigma_{\mathrm{d3}} \circ \mathrm{E} \circ \sigma_{\mathrm{d3}}{}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{S}_6 \circ \mathrm{E} \circ \mathrm{S}_6{}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{S}_6^5 \circ \mathrm{E} \circ \mathrm{S}_6^{5\,-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{i} \circ \mathrm{E} \circ \mathrm{i}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$

Conjugações de C_3

$\mathrm{E} \circ \mathrm{C}_3 \circ \mathrm{E}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$
$\mathrm{C}_3 \circ \mathrm{C}_3 \circ \mathrm{C}_3{}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$
$\mathrm{C}_3^2 \circ \mathrm{C}_3 \circ \mathrm{C}_3^{2\,-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$
$\mathrm{C}_2^{(a)} \circ \mathrm{C}_3 \circ \mathrm{C}_2^{(a)\,-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\mathrm{C}_2^{(b)} \circ \mathrm{C}_3 \circ \mathrm{C}_2^{(b)\,-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\mathrm{C}_2^{(c)} \circ \mathrm{C}_3 \circ \mathrm{C}_2^{(c)\,-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\sigma_{\mathrm{d1}} \circ \mathrm{C}_3 \circ \sigma_{\mathrm{d1}}{}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\sigma_{\mathrm{d2}} \circ \mathrm{C}_3 \circ \sigma_{\mathrm{d2}}{}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\sigma_{\mathrm{d3}} \circ \mathrm{C}_3 \circ \sigma_{\mathrm{d3}}{}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C}_3^2$
$\mathrm{S}_6 \circ \mathrm{C}_3 \circ \mathrm{S}_6{}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$
$\mathrm{S}_6^5 \circ \mathrm{C}_3 \circ \mathrm{S}_6^{5\,-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$
$\mathrm{i} \circ \mathrm{C}_3 \circ \mathrm{i}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C}_3$

Conjugações de $\mathrm{C}_2^{(a)}$

$\mathrm{E} \circ \mathrm{C}_2^{(a)} \circ \mathrm{E}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C}_2^{(a)}$
$\mathrm{C}_3 \circ \mathrm{C}_2^{(a)} \circ \mathrm{C}_3{}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C}_2^{(b)}$
$\mathrm{C}_3^2 \circ \mathrm{C}_2^{(a)} \circ \mathrm{C}_3^{2\,-1} = [2,1,7,6,8,4,3,5] = \mathrm{C}_2^{(c)}$
$\mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(a)\,-1} = [2,1,8,7,6,5,4,3] = \mathrm{C}_2^{(a)}$
$\mathrm{C}_2^{(b)} \circ \mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(b)\,-1} = [2,1,7,6,8,4,3,5] = \mathrm{C}_2^{(c)}$
$\mathrm{C}_2^{(c)} \circ \mathrm{C}_2^{(a)} \circ \mathrm{C}_2^{(c)\,-1} = [2,1,6,8,7,3,5,4] = \mathrm{C}_2^{(b)}$
$\sigma_{\mathrm{d1}} \circ \mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{d1}}{}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C}_2^{(b)}$
$\sigma_{\mathrm{d2}} \circ \mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{d2}}{}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C}_2^{(c)}$
$\sigma_{\mathrm{d3}} \circ \mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{d3}}{}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C}_2^{(a)}$
$\mathrm{S}_6 \circ \mathrm{C}_2^{(a)} \circ \mathrm{S}_6{}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C}_2^{(c)}$
$\mathrm{S}_6^5 \circ \mathrm{C}_2^{(a)} \circ \mathrm{S}_6^{5\,-1} = [2,1,6,8,7,3,5,4] = \mathrm{C}_2^{(b)}$
$\mathrm{i} \circ \mathrm{C}_2^{(a)} \circ \mathrm{i}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C}_2^{(a)}$

Conjugações de σ_{d1}

$\mathrm{E} \circ \sigma_{\mathrm{d1}} \circ \mathrm{E}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d1}}$
$\mathrm{C}_3 \circ \sigma_{\mathrm{d1}} \circ \mathrm{C}_3{}^{-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d3}}$
$\mathrm{C}_3^2 \circ \sigma_{\mathrm{d1}} \circ \mathrm{C}_3^{2\,-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d2}}$
$\mathrm{C}_2^{(a)} \circ \sigma_{\mathrm{d1}} \circ \mathrm{C}_2^{(a)\,-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d2}}$
$\mathrm{C}_2^{(b)} \circ \sigma_{\mathrm{d1}} \circ \mathrm{C}_2^{(b)\,-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d3}}$
$\mathrm{C}_2^{(c)} \circ \sigma_{\mathrm{d1}} \circ \mathrm{C}_2^{(c)\,-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d1}}$
$\sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d1}}{}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d1}}$
$\sigma_{\mathrm{d2}} \circ \sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d2}}{}^{-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d3}}$
$\sigma_{\mathrm{d3}} \circ \sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d3}}{}^{-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d2}}$
$\mathrm{S}_6 \circ \sigma_{\mathrm{d1}} \circ \mathrm{S}_6{}^{-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d2}}$
$\mathrm{S}_6^5 \circ \sigma_{\mathrm{d1}} \circ \mathrm{S}_6^{5\,-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d3}}$
$\mathrm{i} \circ \sigma_{\mathrm{d1}} \circ \mathrm{i}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d1}}$

Conjugações de S_6

$\mathrm{E} \circ \mathrm{S}_6 \circ \mathrm{E}^{-1} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$

$C_3 \circ S_6 \circ {C_3}^{-1} = [2,1,6,7,8,4,5,3] = S_6$
$C_3^2 \circ S_6 \circ {C_3^2}^{-1} = [2,1,6,7,8,4,5,3] = S_6$
$C_2^{(a)} \circ S_6 \circ {C_2^{(a)}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$C_2^{(b)} \circ S_6 \circ {C_2^{(b)}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$C_2^{(c)} \circ S_6 \circ {C_2^{(c)}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$\sigma_{d1} \circ S_6 \circ {\sigma_{d1}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$\sigma_{d2} \circ S_6 \circ {\sigma_{d2}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$\sigma_{d3} \circ S_6 \circ {\sigma_{d3}}^{-1} = [2,1,8,6,7,3,4,5] = S_6^5$
$S_6 \circ S_6 \circ {S_6}^{-1} = [2,1,6,7,8,4,5,3] = S_6$
$S_6^5 \circ S_6 \circ {S_6^5}^{-1} = [2,1,6,7,8,4,5,3] = S_6$
$i \circ S_6 \circ i^{-1} = [2,1,6,7,8,4,5,3] = S_6$

Conjugações de i

$E \circ i \circ E^{-1} = [2,1,7,8,6,5,3,4] = i$
$C_3 \circ i \circ {C_3}^{-1} = [2,1,7,8,6,5,3,4] = i$
$C_3^2 \circ i \circ {C_3^2}^{-1} = [2,1,7,8,6,5,3,4] = i$
$C_2^{(a)} \circ i \circ {C_2^{(a)}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$C_2^{(b)} \circ i \circ {C_2^{(b)}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$C_2^{(c)} \circ i \circ {C_2^{(c)}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$\sigma_{d1} \circ i \circ {\sigma_{d1}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$\sigma_{d2} \circ i \circ {\sigma_{d2}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$\sigma_{d3} \circ i \circ {\sigma_{d3}}^{-1} = [2,1,7,8,6,5,3,4] = i$
$S_6 \circ i \circ {S_6}^{-1} = [2,1,7,8,6,5,3,4] = i$
$S_6^5 \circ i \circ {S_6^5}^{-1} = [2,1,7,8,6,5,3,4] = i$
$i \circ i \circ i^{-1} = [2,1,7,8,6,5,3,4] = i$

Portanto o grupo de simetria D_{3d} possui as seguintes classes de conjugação:

$$\Big[\{E\},\ \{C_3,\ C_3^2\},\ \{C_2^{(a)},\ C_2^{(b)},\ C_2^{(c)}\},\ \{i\},\ \{\sigma_{d1},\ \sigma_{d2},\ \sigma_{d3}\},\ \{S_6,\ S_6^5\}\Big]$$

Resposta para o Ítem 1.6 - Subgrupos

Um subconjunto $H \subseteq G$ é um **subgrupo** de G se ele for um grupo sob a mesma operação que define o grupo G. Isso equivale a exigir para H todos os axiomas que definem um grupo:

- $e \in H$ — a identidade de G pertence a H;
- $\forall a, b \in H, \quad a \circ b \in H$ — H é fechado sob a mesma operação que define G;
- $\forall a \in H, \quad a^{-1} \in H$ — cada elemento de H possui um elemento inverso, que também pertence a G;
- $\forall a, b, c \in H, \quad (a \circ b) \circ c = a \circ (b \circ c)$ — a operação herdada de G que define H é associativa em H.

Ou seja, um subgrupo é um subconjunto que também satisfaz os axiomas de grupo, sob a mesma operação do grupo de origem. Portanto, qualquer subconjunto de G que contenha a identidade, seja fechado sob a mesma operação que define G, que seja associativa e contenha os inversos dos seus elementos é um subgrupo G.

Os subgrupos próprios do grupo D_{3h} podem ser obtidos por meio de duas abordagens complementares:

- A **análise do fluxograma de simetria** permite visualizar os subgrupos que preservam subconjuntos das operações de simetria do grupo original. A cada bifurcação do fluxograma, são descartadas operações, sendo que o grupo final contém todas as simetrias consideradas em grupos anteriores, que são subgrupos deste.
- A **geração cíclica a partir de elementos e seus inversos** identifica subgrupos abelianos de ordem 2 e 3, neste caso, formados por repetições de operações como rotações ou reflexões. É uma forma simples de obter

conjuntos de operações que satisfazem todos axiomas de definição de grupo, por exemplo: $\langle C_3 \rangle = \{E, C_3, C_3^2\}$ ou $\langle \sigma_{v1} \rangle = \{E, \sigma_{v1}\}$.

Essas duas técnicas permitiram encontrar todos os subgrupos próprios do grupo D_{3h}, excetuando-se os subgrupos $\{E\}$ e o grupo total D_{3h}, que são considerados subgrupos impróprios e são deduzidos trivialmente.

Elemento	E	C_6	C_3	C_2	C_3^2	C_6^5	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(d)}$	$C_2^{(e)}$	$C_2^{(f)}$
Inverso	E	C_6^5	C_3^2	C_2	C_3	C_6	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(d)}$	$C_2^{(e)}$	$C_2^{(f)}$

Elemento	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}	σ_h	i	S_6	S_6^5	S_3	S_3^2
Inverso	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}	σ_h	i	S_6^5	S_6	S_3^2	S_3

Ordem 2 (cíclicos e abelianos):

- $\langle C_2 \rangle = \{E, C_2\}$
- $\langle C_2^{(a)} \rangle = \{E, C_2^{(a)}\}$
- $\langle C_2^{(b)} \rangle = \{E, C_2^{(b)}\}$
- $\langle C_2^{(c)} \rangle = \{E, C_2^{(c)}\}$
- $\langle \sigma_{v1} \rangle = \{E, \sigma_{v1}\}$
- $\langle \sigma_{v2} \rangle = \{E, \sigma_{v2}\}$
- $\langle \sigma_{v3} \rangle = \{E, \sigma_{v3}\}$
- $\langle \sigma_h \rangle = \{E, \sigma_h\}$

Ordem 3 (cíclicos e abelianos):

- $\langle C_3 \rangle = \{E, C_3, C_3^2\}$
- $\langle S_3 \rangle = \{E, S_3, S_3^2\}$

Ordem 6 (fluxograma):

- $C_{3v} = \{E, C_3, C_3^2, \sigma_{v1}, \sigma_{v2}, \sigma_{v3}\}$
- $C_{3h} = \{E, C_3, C_3^2, \sigma_h, S_3, S_3^2\}$
- $D_3 = \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$

Ordem	**Subgrupos**
1	$\{E\}$
2	$\{E, C_2\},\ \{E, C_2^{(a)}\},\ \{E, C_2^{(b)}\},\ \{E, C_2^{(c)}\},\ \{E, \sigma_{v1}\},\ \{E, \sigma_{v2}\},\ \{E, \sigma_{v3}\},\ \{E, \sigma_h\}$
3	$\{E, C_3, C_3^2\},\ \{E, S_3, S_3^2\}$
6	$\{E, C_3, C_3^2, \sigma_{v1}, \sigma_{v2}, \sigma_{v3}\},\ \{E, C_3, C_3^2, \sigma_h, S_3, S_3^2\},\ \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$
12	$\{E,\ C_3,\ C_3^2,\ C_2^{(a)},\ C_2^{(b)},\ C_2^{(c)},\ \sigma_{v1},\ \sigma_{v2},\ \sigma_{v3},\ \sigma_h,\ S_3,\ S_3^2\}$

Questão 2. Considere as simetrias da molécula de etano estrelado C_2H_6 e responda os seguintes ítems:

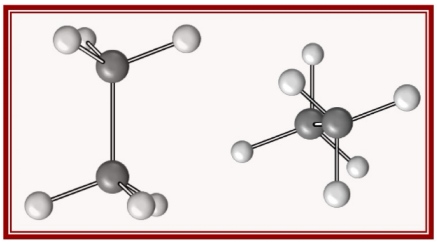


Figure 15: Molécula do etano estrelado

(2.1) Identifique o **grupo** de simetria da molécula.

(2.2) Apresente a **tabela de multiplicação**.

(2.3) Esse grupo é **cíclico**? Justifique.

(2.4) O grupo é **abeliano**? Justifique.

(2.5) Quais são as **classes** deste grupo?

(2.6) Este grupo possui **subgrupos**?

Resposta para o Ítem 2.1 - Grupo

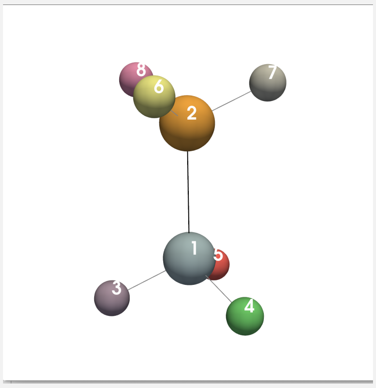


Figure 16: Simetria E

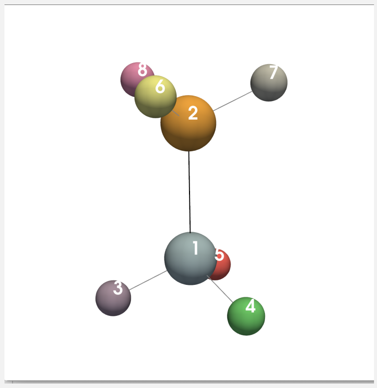


Figure 17: Operação E aplicada

: $E(1,2,3,4,5,6,7,8) = (1,2,3,4,5,6,7,8)$

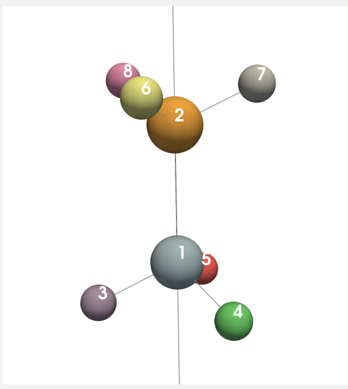


Figure 18: Eixo de rotação C_3

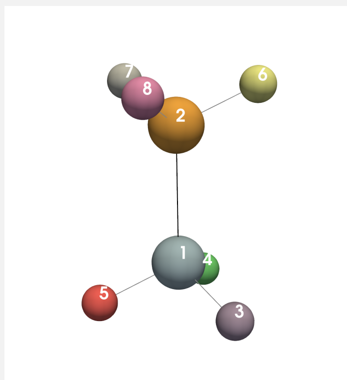


Figure 19: Operação C_3 aplicada

: $C_3(1,2,3,4,5,6,7,8) = (1,2,5,3,4,8,6,7)$

$C_3^2(1,2,3,4,5,6,7,8) = (1,2,4,5,3,7,8,6)$

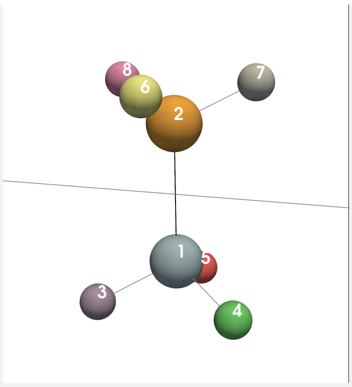


Figure 20: Eixo de rotação C_2

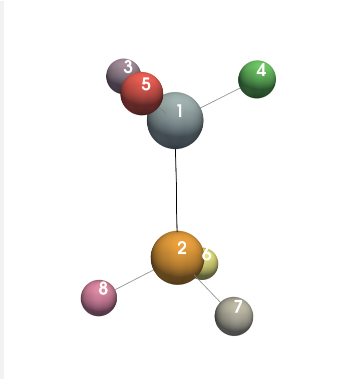


Figure 21: Operação $C_2^{(a)}$ aplicada

$$
\begin{aligned}
: \quad & C_2^{(a)}(1,2,3,4,5,6,7,8) = (2,1,8,7,6,5,4,3) \\
& C_2^{(b)}(1,2,3,4,5,6,7,8) = (2,1,6,7,8,3,4,5) \\
& C_2^{(c)}(1,2,3,4,5,6,7,8) = (2,1,7,8,6,4,5,3)
\end{aligned}
$$

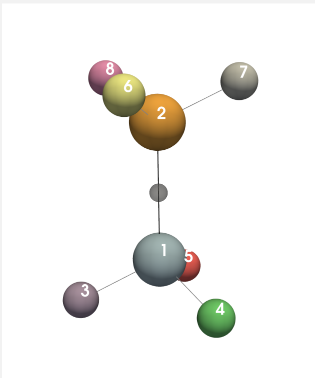


Figure 22: Origem da inversão i

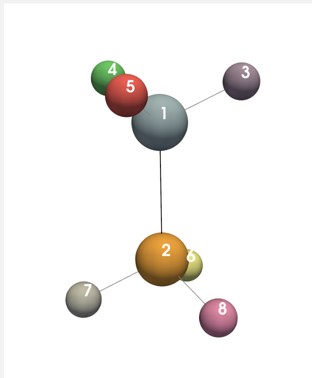


Figure 23: Operação i aplicada

$$
: \quad i(1,2,3,4,5,6,7,8) = (2,1,7,8,6,5,3,4)
$$

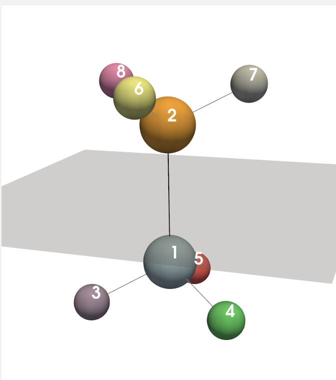


Figure 24: Plano de reflexão σ_h

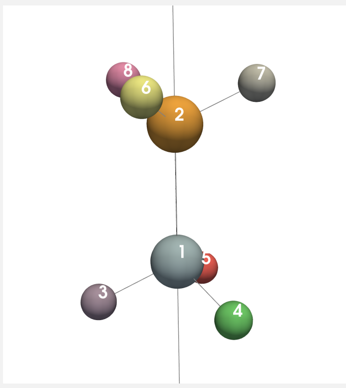


Figure 25: Eixo de rotação C_6

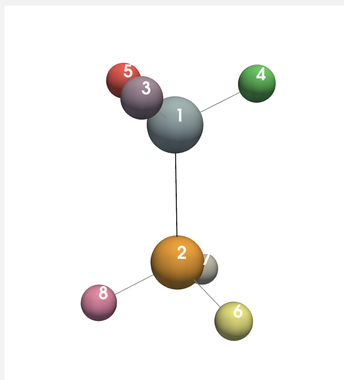


Figure 26: Operação S_6 aplicada

$$
\begin{aligned}
: \quad & S_6(1,2,3,4,5,6,7,8) = (2,1,8,6,7,3,4,5) \\
& S_6^5(1,2,3,4,5,6,7,8) = (2,1,6,7,8,4,5,3)
\end{aligned}
$$

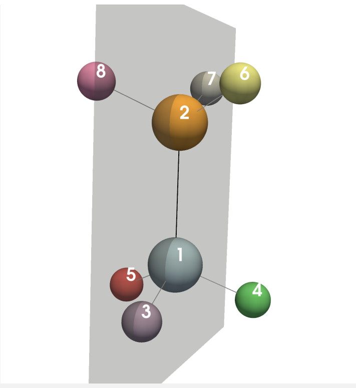


Figure 27: Plano de reflexão σ_d

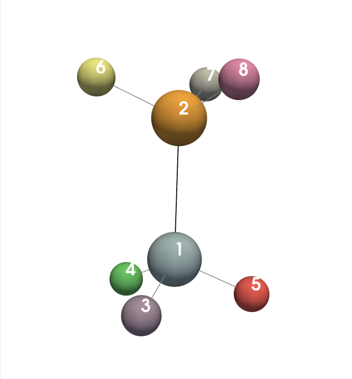


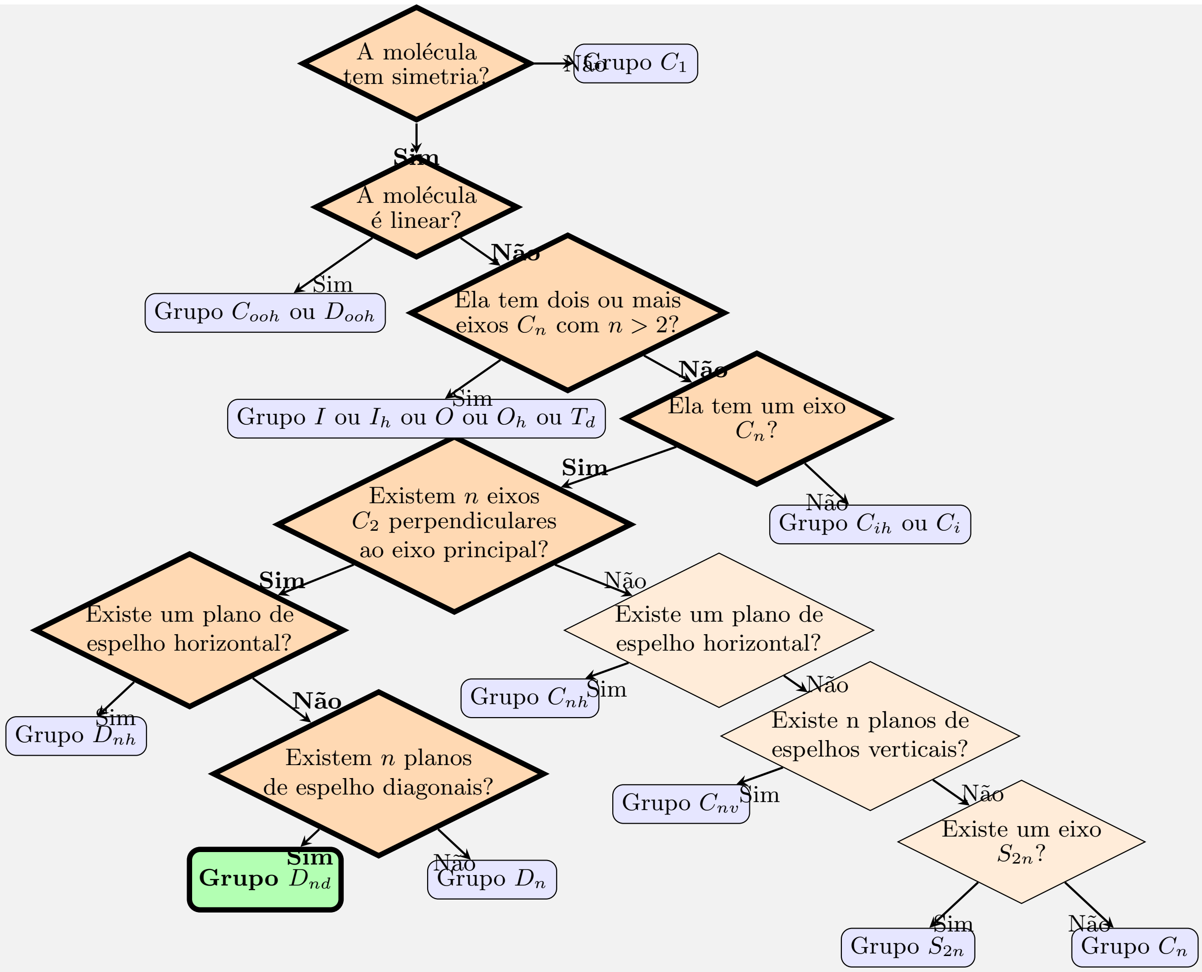
Figure 28: Operação σ_d aplicada

$$
\begin{aligned}
: \quad & \sigma_d(1,2,3,4,5,6,7,8) = (1,2,3,5,4,8,7,6) \\
& \sigma_d(1,2,3,4,5,6,7,8) = (1,2,5,4,3,7,6,8) \\
& \sigma_d(1,2,3,4,5,6,7,8) = (1,2,4,3,5,6,8,7)
\end{aligned}
$$

Conforme ilustrado nas figuras, a molécula de etano na **conformação estrelada** apresenta as seguintes simetrias:

- **Um eixo de rotação** C_3, ao longo do eixo C–C, passando pelo centro dos triângulos formados pelos três hidrogênios de cada carbono. Com rotações de 120° e 240°.
- **Três eixos de rotação** C_2, todos perpendiculares ao eixo C_3, localizados em planos que contêm pares de hidrogênios opostos (um de cada carbono). Estes eixos estão espaçados de 120° entre si ao redor do eixo principal. Com rotacões de 180° cada.
- **Três planos de reflexão diagonais** σ_d, cada um contendo o eixo C_3 e passando entre pares de hidrogênios de carbonos opostos. Estes planos cortam os triângulos de hidrogênios ao meio.
- **Uma operação de inversão** i, situada no ponto médio da ligação C–C, que transforma cada átomo em sua imagem oposta através do centro de massa da molécula.
- **Duas rotações impróprias** S_6, correspondentes à combinação de uma reflexão por um plano σ_h perpendicular ao eixo seguida de rotação C_6. Rotações impróprias de 60° e 300°.

Portanto, conforme demonstrado no fluxograma de classificação, o grupo de simetria da molécula de etano na **conformação estrelada** é o D_{3d}.



Resposta para o Ítem 2.2 - Tabela de Multiplicação

Operações de Multiplicação:

$$\begin{aligned}
\mathrm{E}\circ\mathrm{E} &= \mathrm{E}\circ[1,2,3,4,5,6,7,8] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E}\\
\mathrm{E}\circ\mathrm{C}_3 &= \mathrm{E}\circ[1,2,4,5,3,7,8,6] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3\\
\mathrm{E}\circ\mathrm{C}_3^2 &= \mathrm{E}\circ[1,2,5,3,4,8,6,7] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2\\
\mathrm{E}\circ\mathrm{C}_2^{(\mathrm{a})} &= \mathrm{E}\circ[2,1,8,7,6,5,4,3] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})}\\
\mathrm{E}\circ\mathrm{C}_2^{(\mathrm{b})} &= \mathrm{E}\circ[2,1,6,8,7,3,5,4] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})}\\
\mathrm{E}\circ\mathrm{C}_2^{(\mathrm{c})} &= \mathrm{E}\circ[2,1,7,6,8,4,3,5] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})}\\
\mathrm{E}\circ\sigma_{\mathrm{d1}} &= \mathrm{E}\circ[1,2,3,5,4,8,7,6] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}}\\
\mathrm{E}\circ\sigma_{\mathrm{d2}} &= \mathrm{E}\circ[1,2,5,4,3,7,6,8] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}}\\
\mathrm{E}\circ\sigma_{\mathrm{d3}} &= \mathrm{E}\circ[1,2,4,3,5,6,8,7] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}}\\
\mathrm{E}\circ\mathrm{S}_6 &= \mathrm{E}\circ[2,1,6,7,8,4,5,3] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6\\
\mathrm{E}\circ\mathrm{S}_6^5 &= \mathrm{E}\circ[2,1,8,6,7,3,4,5] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5\\
\mathrm{E}\circ\mathrm{i} &= \mathrm{E}\circ[2,1,7,8,6,5,3,4] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i}\\
\mathrm{C}_3\circ\mathrm{E} &= \mathrm{C}_3\circ[1,2,3,4,5,6,7,8] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3\\
\mathrm{C}_3\circ\mathrm{C}_3 &= \mathrm{C}_3\circ[1,2,4,5,3,7,8,6] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2\\
\mathrm{C}_3\circ\mathrm{C}_3^2 &= \mathrm{C}_3\circ[1,2,5,3,4,8,6,7] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E}\\
\mathrm{C}_3\circ\mathrm{C}_2^{(\mathrm{a})} &= \mathrm{C}_3\circ[2,1,8,7,6,5,4,3] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})}\\
\mathrm{C}_3\circ\mathrm{C}_2^{(\mathrm{b})} &= \mathrm{C}_3\circ[2,1,6,8,7,3,5,4] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})}\\
\mathrm{C}_3\circ\mathrm{C}_2^{(\mathrm{c})} &= \mathrm{C}_3\circ[2,1,7,6,8,4,3,5] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})}\\
\mathrm{C}_3\circ\sigma_{\mathrm{d1}} &= \mathrm{C}_3\circ[1,2,3,5,4,8,7,6] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}}\\
\mathrm{C}_3\circ\sigma_{\mathrm{d2}} &= \mathrm{C}_3\circ[1,2,5,4,3,7,6,8] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}}\\
\mathrm{C}_3\circ\sigma_{\mathrm{d3}} &= \mathrm{C}_3\circ[1,2,4,3,5,6,8,7] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}}\\
\mathrm{C}_3\circ\mathrm{S}_6 &= \mathrm{C}_3\circ[2,1,6,7,8,4,5,3] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i}\\
\mathrm{C}_3\circ\mathrm{S}_6^5 &= \mathrm{C}_3\circ[2,1,8,6,7,3,4,5] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6\\
\mathrm{C}_3\circ\mathrm{i} &= \mathrm{C}_3\circ[2,1,7,8,6,5,3,4] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5
\end{aligned}$$

$$\begin{aligned}
\mathrm{C}_3^2 \circ \mathrm{E} &= \mathrm{C}_3^2 \circ [1,2,3,4,5,6,7,8] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_3^2 \circ \mathrm{C}_3 &= \mathrm{C}_3^2 \circ [1,2,4,5,3,7,8,6] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_3^2 \circ \mathrm{C}_3^2 &= \mathrm{C}_3^2 \circ [1,2,5,3,4,8,6,7] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{C}_3^2 \circ [2,1,8,7,6,5,4,3] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{C}_3^2 \circ [2,1,6,8,7,3,5,4] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{C}_3^2 \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{C}_3^2 \circ [2,1,7,6,8,4,3,5] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{d}1} &= \mathrm{C}_3^2 \circ [1,2,3,5,4,8,7,6] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{d}2} &= \mathrm{C}_3^2 \circ [1,2,5,4,3,7,6,8] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1} \\
\mathrm{C}_3^2 \circ \sigma_{\mathrm{d}3} &= \mathrm{C}_3^2 \circ [1,2,4,3,5,6,8,7] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2} \\
\mathrm{C}_3^2 \circ \mathrm{S}_6 &= \mathrm{C}_3^2 \circ [2,1,6,7,8,4,5,3] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{C}_3^2 \circ \mathrm{S}_6^5 &= \mathrm{C}_3^2 \circ [2,1,8,6,7,3,4,5] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{C}_3^2 \circ \mathrm{i} &= \mathrm{C}_3^2 \circ [2,1,7,8,6,5,3,4] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{E} &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,3,4,5,6,7,8] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,4,5,3,7,8,6] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,5,3,4,8,6,7] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,8,7,6,5,4,3] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,6,8,7,3,5,4] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,7,6,8,4,3,5] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(\mathrm{a})} \circ \sigma_{\mathrm{d}1} &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,3,5,4,8,7,6] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{C}_2^{(\mathrm{a})} \circ \sigma_{\mathrm{d}2} &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,5,4,3,7,6,8] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{C}_2^{(\mathrm{a})} \circ \sigma_{\mathrm{d}3} &= \mathrm{C}_2^{(\mathrm{a})} \circ [1,2,4,3,5,6,8,7] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{S}_6 &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,6,7,8,4,5,3] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{S}_6^5 &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,8,6,7,3,4,5] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2} \\
\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{i} &= \mathrm{C}_2^{(\mathrm{a})} \circ [2,1,7,8,6,5,3,4] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{E} &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,3,4,5,6,7,8] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,4,5,3,7,8,6] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,5,3,4,8,6,7] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,8,7,6,5,4,3] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,6,8,7,3,5,4] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,7,6,8,4,3,5] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(\mathrm{b})} \circ \sigma_{\mathrm{d}1} &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,3,5,4,8,7,6] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{C}_2^{(\mathrm{b})} \circ \sigma_{\mathrm{d}2} &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,5,4,3,7,6,8] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \sigma_{\mathrm{d}3} &= \mathrm{C}_2^{(\mathrm{b})} \circ [1,2,4,3,5,6,8,7] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{S}_6 &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,6,7,8,4,5,3] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{S}_6^5 &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,8,6,7,3,4,5] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1} \\
\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{i} &= \mathrm{C}_2^{(\mathrm{b})} \circ [2,1,7,8,6,5,3,4] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{E} &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,3,4,5,6,7,8] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{C}_3 &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,4,5,3,7,8,6] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{C}_3^2 &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,5,3,4,8,6,7] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,8,7,6,5,4,3] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,6,8,7,3,5,4] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,7,6,8,4,3,5] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \sigma_{\mathrm{d}1} &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,3,5,4,8,7,6] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \sigma_{\mathrm{d}2} &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,5,4,3,7,6,8] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{C}_2^{(\mathrm{c})} \circ \sigma_{\mathrm{d}3} &= \mathrm{C}_2^{(\mathrm{c})} \circ [1,2,4,3,5,6,8,7] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{S}_6 &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,6,7,8,4,5,3] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{S}_6^5 &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,8,6,7,3,4,5] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3} \\
\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{i} &= \mathrm{C}_2^{(\mathrm{c})} \circ [2,1,7,8,6,5,3,4] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1} \\
\sigma_{\mathrm{d}1} \circ \mathrm{E} &= \sigma_{\mathrm{d}1} \circ [1,2,3,4,5,6,7,8] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1} \\
\sigma_{\mathrm{d}1} \circ \mathrm{C}_3 &= \sigma_{\mathrm{d}1} \circ [1,2,4,5,3,7,8,6] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3} \\
\sigma_{\mathrm{d}1} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{d}1} \circ [1,2,5,3,4,8,6,7] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2} \\
\sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{d}1} \circ [2,1,8,7,6,5,4,3] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{d}1} \circ [2,1,6,8,7,3,5,4] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{d}1} \circ [2,1,7,6,8,4,3,5] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i}
\end{aligned}$$

$$
\begin{aligned}
\sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d1}} &= \sigma_{\mathrm{d1}} \circ [1,2,3,5,4,8,7,6] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d2}} &= \sigma_{\mathrm{d1}} \circ [1,2,5,4,3,7,6,8] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{d1}} \circ \sigma_{\mathrm{d3}} &= \sigma_{\mathrm{d1}} \circ [1,2,4,3,5,6,8,7] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{d1}} \circ \mathrm{S}_6 &= \sigma_{\mathrm{d1}} \circ [2,1,6,7,8,4,5,3] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{d1}} \circ \mathrm{S}_6^5 &= \sigma_{\mathrm{d1}} \circ [2,1,8,6,7,3,4,5] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{d1}} \circ \mathrm{i} &= \sigma_{\mathrm{d1}} \circ [2,1,7,8,6,5,3,4] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{d2}} \circ \mathrm{E} &= \sigma_{\mathrm{d2}} \circ [1,2,3,4,5,6,7,8] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}} \\
\sigma_{\mathrm{d2}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{d2}} \circ [1,2,4,5,3,7,8,6] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}} \\
\sigma_{\mathrm{d2}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{d2}} \circ [1,2,5,3,4,8,6,7] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}} \\
\sigma_{\mathrm{d2}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{d2}} \circ [2,1,8,7,6,5,4,3] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\sigma_{\mathrm{d2}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{d2}} \circ [2,1,6,8,7,3,5,4] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\sigma_{\mathrm{d2}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{d2}} \circ [2,1,7,6,8,4,3,5] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\sigma_{\mathrm{d2}} \circ \sigma_{\mathrm{d1}} &= \sigma_{\mathrm{d2}} \circ [1,2,3,5,4,8,7,6] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{d2}} \circ \sigma_{\mathrm{d2}} &= \sigma_{\mathrm{d2}} \circ [1,2,5,4,3,7,6,8] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{d2}} \circ \sigma_{\mathrm{d3}} &= \sigma_{\mathrm{d2}} \circ [1,2,4,3,5,6,8,7] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{d2}} \circ \mathrm{S}_6 &= \sigma_{\mathrm{d2}} \circ [2,1,6,7,8,4,5,3] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\sigma_{\mathrm{d2}} \circ \mathrm{S}_6^5 &= \sigma_{\mathrm{d2}} \circ [2,1,8,6,7,3,4,5] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{d2}} \circ \mathrm{i} &= \sigma_{\mathrm{d2}} \circ [2,1,7,8,6,5,3,4] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{d3}} \circ \mathrm{E} &= \sigma_{\mathrm{d3}} \circ [1,2,3,4,5,6,7,8] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}} \\
\sigma_{\mathrm{d3}} \circ \mathrm{C}_3 &= \sigma_{\mathrm{d3}} \circ [1,2,4,5,3,7,8,6] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}} \\
\sigma_{\mathrm{d3}} \circ \mathrm{C}_3^2 &= \sigma_{\mathrm{d3}} \circ [1,2,5,3,4,8,6,7] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}} \\
\sigma_{\mathrm{d3}} \circ \mathrm{C}_2^{(\mathrm{a})} &= \sigma_{\mathrm{d3}} \circ [2,1,8,7,6,5,4,3] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\sigma_{\mathrm{d3}} \circ \mathrm{C}_2^{(\mathrm{b})} &= \sigma_{\mathrm{d3}} \circ [2,1,6,8,7,3,5,4] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\sigma_{\mathrm{d3}} \circ \mathrm{C}_2^{(\mathrm{c})} &= \sigma_{\mathrm{d3}} \circ [2,1,7,6,8,4,3,5] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\sigma_{\mathrm{d3}} \circ \sigma_{\mathrm{d1}} &= \sigma_{\mathrm{d3}} \circ [1,2,3,5,4,8,7,6] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\sigma_{\mathrm{d3}} \circ \sigma_{\mathrm{d2}} &= \sigma_{\mathrm{d3}} \circ [1,2,5,4,3,7,6,8] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\sigma_{\mathrm{d3}} \circ \sigma_{\mathrm{d3}} &= \sigma_{\mathrm{d3}} \circ [1,2,4,3,5,6,8,7] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\sigma_{\mathrm{d3}} \circ \mathrm{S}_6 &= \sigma_{\mathrm{d3}} \circ [2,1,6,7,8,4,5,3] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\sigma_{\mathrm{d3}} \circ \mathrm{S}_6^5 &= \sigma_{\mathrm{d3}} \circ [2,1,8,6,7,3,4,5] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\sigma_{\mathrm{d3}} \circ \mathrm{i} &= \sigma_{\mathrm{d3}} \circ [2,1,7,8,6,5,3,4] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{S}_6 \circ \mathrm{E} &= \mathrm{S}_6 \circ [1,2,3,4,5,6,7,8] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{S}_6 \circ \mathrm{C}_3 &= \mathrm{S}_6 \circ [1,2,4,5,3,7,8,6] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{S}_6 \circ \mathrm{C}_3^2 &= \mathrm{S}_6 \circ [1,2,5,3,4,8,6,7] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{S}_6 \circ [2,1,8,7,6,5,4,3] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}} \\
\mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{S}_6 \circ [2,1,6,8,7,3,5,4] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}} \\
\mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{S}_6 \circ [2,1,7,6,8,4,3,5] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}} \\
\mathrm{S}_6 \circ \sigma_{\mathrm{d1}} &= \mathrm{S}_6 \circ [1,2,3,5,4,8,7,6] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{S}_6 \circ \sigma_{\mathrm{d2}} &= \mathrm{S}_6 \circ [1,2,5,4,3,7,6,8] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{S}_6 \circ \sigma_{\mathrm{d3}} &= \mathrm{S}_6 \circ [1,2,4,3,5,6,8,7] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{S}_6 \circ \mathrm{S}_6 &= \mathrm{S}_6 \circ [2,1,6,7,8,4,5,3] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{S}_6 \circ \mathrm{S}_6^5 &= \mathrm{S}_6 \circ [2,1,8,6,7,3,4,5] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{S}_6 \circ \mathrm{i} &= \mathrm{S}_6 \circ [2,1,7,8,6,5,3,4] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{S}_6^5 \circ \mathrm{E} &= \mathrm{S}_6^5 \circ [1,2,3,4,5,6,7,8] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{S}_6^5 \circ \mathrm{C}_3 &= \mathrm{S}_6^5 \circ [1,2,4,5,3,7,8,6] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6 \\
\mathrm{S}_6^5 \circ \mathrm{C}_3^2 &= \mathrm{S}_6^5 \circ [1,2,5,3,4,8,6,7] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{S}_6^5 \circ \mathrm{C}_2^{(\mathrm{a})} &= \mathrm{S}_6^5 \circ [2,1,8,7,6,5,4,3] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d1}} \\
\mathrm{S}_6^5 \circ \mathrm{C}_2^{(\mathrm{b})} &= \mathrm{S}_6^5 \circ [2,1,6,8,7,3,5,4] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d3}} \\
\mathrm{S}_6^5 \circ \mathrm{C}_2^{(\mathrm{c})} &= \mathrm{S}_6^5 \circ [2,1,7,6,8,4,3,5] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d2}} \\
\mathrm{S}_6^5 \circ \sigma_{\mathrm{d1}} &= \mathrm{S}_6^5 \circ [1,2,3,5,4,8,7,6] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})} \\
\mathrm{S}_6^5 \circ \sigma_{\mathrm{d2}} &= \mathrm{S}_6^5 \circ [1,2,5,4,3,7,6,8] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})} \\
\mathrm{S}_6^5 \circ \sigma_{\mathrm{d3}} &= \mathrm{S}_6^5 \circ [1,2,4,3,5,6,8,7] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})} \\
\mathrm{S}_6^5 \circ \mathrm{S}_6 &= \mathrm{S}_6^5 \circ [2,1,6,7,8,4,5,3] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E} \\
\mathrm{S}_6^5 \circ \mathrm{S}_6^5 &= \mathrm{S}_6^5 \circ [2,1,8,6,7,3,4,5] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2 \\
\mathrm{S}_6^5 \circ \mathrm{i} &= \mathrm{S}_6^5 \circ [2,1,7,8,6,5,3,4] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3 \\
\mathrm{i} \circ \mathrm{E} &= \mathrm{i} \circ [1,2,3,4,5,6,7,8] &&= [2,1,7,8,6,5,3,4] &&= \mathrm{i} \\
\mathrm{i} \circ \mathrm{C}_3 &= \mathrm{i} \circ [1,2,4,5,3,7,8,6] &&= [2,1,8,6,7,3,4,5] &&= \mathrm{S}_6^5 \\
\mathrm{i} \circ \mathrm{C}_3^2 &= \mathrm{i} \circ [1,2,5,3,4,8,6,7] &&= [2,1,6,7,8,4,5,3] &&= \mathrm{S}_6
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{i}\circ\mathrm{C}_2^{(\mathrm{a})} &= \mathrm{i}\circ[2,1,8,7,6,5,4,3] &&= [1,2,4,3,5,6,8,7] &&= \sigma_{\mathrm{d}3}\\
\mathrm{i}\circ\mathrm{C}_2^{(\mathrm{b})} &= \mathrm{i}\circ[2,1,6,8,7,3,5,4] &&= [1,2,5,4,3,7,6,8] &&= \sigma_{\mathrm{d}2}\\
\mathrm{i}\circ\mathrm{C}_2^{(\mathrm{c})} &= \mathrm{i}\circ[2,1,7,6,8,4,3,5] &&= [1,2,3,5,4,8,7,6] &&= \sigma_{\mathrm{d}1}\\
\mathrm{i}\circ\sigma_{\mathrm{d}1} &= \mathrm{i}\circ[1,2,3,5,4,8,7,6] &&= [2,1,7,6,8,4,3,5] &&= \mathrm{C}_2^{(\mathrm{c})}\\
\mathrm{i}\circ\sigma_{\mathrm{d}2} &= \mathrm{i}\circ[1,2,5,4,3,7,6,8] &&= [2,1,6,8,7,3,5,4] &&= \mathrm{C}_2^{(\mathrm{b})}\\
\mathrm{i}\circ\sigma_{\mathrm{d}3} &= \mathrm{i}\circ[1,2,4,3,5,6,8,7] &&= [2,1,8,7,6,5,4,3] &&= \mathrm{C}_2^{(\mathrm{a})}\\
\mathrm{i}\circ\mathrm{S}_6 &= \mathrm{i}\circ[2,1,6,7,8,4,5,3] &&= [1,2,5,3,4,8,6,7] &&= \mathrm{C}_3^2\\
\mathrm{i}\circ\mathrm{S}_6^5 &= \mathrm{i}\circ[2,1,8,6,7,3,4,5] &&= [1,2,4,5,3,7,8,6] &&= \mathrm{C}_3\\
\mathrm{i}\circ\mathrm{i} &= \mathrm{i}\circ[2,1,7,8,6,5,3,4] &&= [1,2,3,4,5,6,7,8] &&= \mathrm{E}
\end{aligned}
$$

Tabela de Multiplicação Resultante:

	E	C_3	C_3^2	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	i	S_6	S_6^5	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$
E	E	C_3	C_3^2	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	i	S_6	S_6^5	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$
C_3	C_3	C_3^2	E	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	S_6^5	i	S_6	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$
C_3^2	C_3^2	E	C_3	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$	S_6	S_6^5	i	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$
$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(b)}$	E	C_3^2	C_3	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	i	S_6^5	S_6
$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(c)}$	C_3^2	C_3	E	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	S_6^5	S_6	i
$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(a)}$	C_3	E	C_3^2	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	S_6	i	S_6^5
i	i	S_6^5	S_6	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	E	C_3^2	C_3	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$
S_6	S_6	i	S_6^5	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	C_3	E	C_3^2	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$
S_6^5	S_6^5	S_6	i	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	C_3^2	C_3	E	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$
$\sigma_d^{(a)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	i	S_6^5	S_6	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	E	C_3^2	C_3
$\sigma_d^{(b)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	S_6^5	i	S_6	$\mathrm{C}_2^{(a)}$	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	C_3	E	C_3^2
$\sigma_d^{(c)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	S_6	S_6^5	i	$\mathrm{C}_2^{(b)}$	$\mathrm{C}_2^{(c)}$	$\mathrm{C}_2^{(a)}$	C_3^2	C_3	E

Resposta para o Ítem 2.3 - Cíclico

Embora o grupo D_{3d} contenha rotações C_3 e C_3^2, que geralmente formam grupos cíclicos, as demais operações de simetria, como as rotações de 180° em torno dos três eixos C_2 perpendiculares ao eixo principal, as reflexões diagonais σ_d, e também as rotações impróprias S_6 e S_6^5, não podem ser obtidas apenas por repetições de qualquer uma dessas rotações isoladamente. Por exemplo, aplicar sucessivas potências de C_3 nunca resultará em uma reflexão σ_d, nem em uma inversão i, nem nas rotações C_2.

Assim, devido à presença de múltiplos tipos de operações de simetria que não são geradas a partir de um único elemento, o grupo D_{3d} não pode ser classificado como cíclico.

Resposta para o Ítem 2.4 - Abeliano

O grupo D_{3d} não é abeliano, pelo mesmo motivo de que a ordem das operações afeta o resultado final. Em um grupo abeliano, qualquer par de elementos pode ser composto em qualquer ordem sem alterar o produto. No entanto, no D_{3d}, isso não acontece. Por exemplo, $C_3 \circ C_2^{(a)} \neq C_2^{(a)} \circ C_3$. Essa não comutatividade pode ser verificada diretamente na tabela de multiplicação, onde a ausência de simetria em relação à diagonal principal evidencia que as composições não são comutativas. Logo, concluímos que D_{3d} é um grupo não abeliano.

Resposta para o Ítem 2.5 - Classes

De acordo com a definição, dois elementos A e B pertencem à mesma classe de conjugação se existe um elemento $g \in G$ tal que:

$$B = g \circ A \circ g^{-1}$$

Portanto, para determinar as classes de conjugação do grupo D_{3d}, é necessário calcular o conjugado de cada operação por todas as demais:

$$g \circ A \circ g^{-1}, \quad \text{para todo } g \in D_{3d}$$

Após realizar esses cálculos, agrupamos os elementos $A \in D_{3d}$ que produzem o mesmo conjunto de conjugados. Esses agrupamentos formam as classes de conjugação do grupo D_{3d}, refletindo as equivalências sob a ação de conjugação interna ao grupo.

Conjugações do elemento E

$\mathrm{E} \circ \mathrm{E} \circ \mathrm{E}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_3} \circ \mathrm{E} \circ \mathrm{C_3}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_3^2} \circ \mathrm{E} \circ \mathrm{C_3^2}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(a)}} \circ \mathrm{E} \circ \mathrm{C_2^{(a)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(b)}} \circ \mathrm{E} \circ \mathrm{C_2^{(b)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{C_2^{(c)}} \circ \mathrm{E} \circ \mathrm{C_2^{(c)}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\sigma_{\mathrm{d1}} \circ \mathrm{E} \circ \sigma_{\mathrm{d1}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\sigma_{\mathrm{d2}} \circ \mathrm{E} \circ \sigma_{\mathrm{d2}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\sigma_{\mathrm{d3}} \circ \mathrm{E} \circ \sigma_{\mathrm{d3}}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{S_6} \circ \mathrm{E} \circ \mathrm{S_6}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{S_6^5} \circ \mathrm{E} \circ \mathrm{S_6^5}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$
$\mathrm{i} \circ \mathrm{E} \circ \mathrm{i}^{-1} = [1,2,3,4,5,6,7,8] = \mathrm{E}$

Conjugações de $\mathrm{C_3}$

$\mathrm{E} \circ \mathrm{C_3} \circ \mathrm{E}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$
$\mathrm{C_3} \circ \mathrm{C_3} \circ \mathrm{C_3}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$
$\mathrm{C_3^2} \circ \mathrm{C_3} \circ \mathrm{C_3^2}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$
$\mathrm{C_2^{(a)}} \circ \mathrm{C_3} \circ \mathrm{C_2^{(a)}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\mathrm{C_2^{(b)}} \circ \mathrm{C_3} \circ \mathrm{C_2^{(b)}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\mathrm{C_2^{(c)}} \circ \mathrm{C_3} \circ \mathrm{C_2^{(c)}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\sigma_{\mathrm{d1}} \circ \mathrm{C_3} \circ \sigma_{\mathrm{d1}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\sigma_{\mathrm{d2}} \circ \mathrm{C_3} \circ \sigma_{\mathrm{d2}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\sigma_{\mathrm{d3}} \circ \mathrm{C_3} \circ \sigma_{\mathrm{d3}}^{-1} = [1,2,5,3,4,8,6,7] = \mathrm{C_3^2}$
$\mathrm{S_6} \circ \mathrm{C_3} \circ \mathrm{S_6}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$
$\mathrm{S_6^5} \circ \mathrm{C_3} \circ \mathrm{S_6^5}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$
$\mathrm{i} \circ \mathrm{C_3} \circ \mathrm{i}^{-1} = [1,2,4,5,3,7,8,6] = \mathrm{C_3}$

Conjugações de $\mathrm{C}_2^{(a)}$

$\mathrm{E} \circ \mathrm{C_2^{(a)}} \circ \mathrm{E}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C_2^{(a)}}$
$\mathrm{C_3} \circ \mathrm{C_2^{(a)}} \circ \mathrm{C_3}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C_2^{(b)}}$
$\mathrm{C_3^2} \circ \mathrm{C_2^{(a)}} \circ \mathrm{C_3^2}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C_2^{(c)}}$
$\mathrm{C_2^{(a)}} \circ \mathrm{C_2^{(a)}} \circ \mathrm{C_2^{(a)}}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C_2^{(a)}}$
$\mathrm{C_2^{(b)}} \circ \mathrm{C_2^{(a)}} \circ \mathrm{C_2^{(b)}}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C_2^{(c)}}$
$\mathrm{C_2^{(c)}} \circ \mathrm{C_2^{(a)}} \circ \mathrm{C_2^{(c)}}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C_2^{(b)}}$
$\sigma_{\mathrm{d1}} \circ \mathrm{C_2^{(a)}} \circ \sigma_{\mathrm{d1}}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C_2^{(b)}}$
$\sigma_{\mathrm{d2}} \circ \mathrm{C_2^{(a)}} \circ \sigma_{\mathrm{d2}}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C_2^{(c)}}$
$\sigma_{\mathrm{d3}} \circ \mathrm{C_2^{(a)}} \circ \sigma_{\mathrm{d3}}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C_2^{(a)}}$
$\mathrm{S_6} \circ \mathrm{C_2^{(a)}} \circ \mathrm{S_6}^{-1} = [2,1,7,6,8,4,3,5] = \mathrm{C_2^{(c)}}$
$\mathrm{S_6^5} \circ \mathrm{C_2^{(a)}} \circ \mathrm{S_6^5}^{-1} = [2,1,6,8,7,3,5,4] = \mathrm{C_2^{(b)}}$
$\mathrm{i} \circ \mathrm{C_2^{(a)}} \circ \mathrm{i}^{-1} = [2,1,8,7,6,5,4,3] = \mathrm{C_2^{(a)}}$

Conjugações de σ_{d1}

$\mathrm{E} \circ \sigma_{\mathrm{d1}} \circ \mathrm{E}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d1}}$
$\mathrm{C_3} \circ \sigma_{\mathrm{d1}} \circ \mathrm{C_3}^{-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d3}}$

$\mathrm{C}_3^2 \circ \sigma_{\mathrm{d}1} \circ \mathrm{C}_3^{2^{-1}} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d}2}$
$\mathrm{C}_2^{(\mathrm{a})} \circ \sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{a})^{-1}} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d}2}$
$\mathrm{C}_2^{(\mathrm{b})} \circ \sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{b})^{-1}} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d}3}$
$\mathrm{C}_2^{(\mathrm{c})} \circ \sigma_{\mathrm{d}1} \circ \mathrm{C}_2^{(\mathrm{c})^{-1}} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d}1}$
$\sigma_{\mathrm{d}1} \circ \sigma_{\mathrm{d}1} \circ \sigma_{\mathrm{d}1}{}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d}1}$
$\sigma_{\mathrm{d}2} \circ \sigma_{\mathrm{d}1} \circ \sigma_{\mathrm{d}2}{}^{-1} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d}3}$
$\sigma_{\mathrm{d}3} \circ \sigma_{\mathrm{d}1} \circ \sigma_{\mathrm{d}3}{}^{-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d}2}$
$\mathrm{S}_6 \circ \sigma_{\mathrm{d}1} \circ \mathrm{S}_6{}^{-1} = [1,2,5,4,3,7,6,8] = \sigma_{\mathrm{d}2}$
$\mathrm{S}_6^5 \circ \sigma_{\mathrm{d}1} \circ \mathrm{S}_6^{5^{-1}} = [1,2,4,3,5,6,8,7] = \sigma_{\mathrm{d}3}$
$\mathrm{i} \circ \sigma_{\mathrm{d}1} \circ \mathrm{i}^{-1} = [1,2,3,5,4,8,7,6] = \sigma_{\mathrm{d}1}$

Conjugações de S_6

$\mathrm{E} \circ \mathrm{S}_6 \circ \mathrm{E}^{-1} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$
$\mathrm{C}_3 \circ \mathrm{S}_6 \circ \mathrm{C}_3{}^{-1} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$
$\mathrm{C}_3^2 \circ \mathrm{S}_6 \circ \mathrm{C}_3^{2^{-1}} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$
$\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{a})^{-1}} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{b})^{-1}} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{S}_6 \circ \mathrm{C}_2^{(\mathrm{c})^{-1}} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\sigma_{\mathrm{d}1} \circ \mathrm{S}_6 \circ \sigma_{\mathrm{d}1}{}^{-1} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\sigma_{\mathrm{d}2} \circ \mathrm{S}_6 \circ \sigma_{\mathrm{d}2}{}^{-1} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\sigma_{\mathrm{d}3} \circ \mathrm{S}_6 \circ \sigma_{\mathrm{d}3}{}^{-1} = [2,1,8,6,7,3,4,5] = \mathrm{S}_6^5$
$\mathrm{S}_6 \circ \mathrm{S}_6 \circ \mathrm{S}_6{}^{-1} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$
$\mathrm{S}_6^5 \circ \mathrm{S}_6 \circ \mathrm{S}_6^{5^{-1}} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$
$\mathrm{i} \circ \mathrm{S}_6 \circ \mathrm{i}^{-1} = [2,1,6,7,8,4,5,3] = \mathrm{S}_6$

Conjugações de i

$\mathrm{E} \circ \mathrm{i} \circ \mathrm{E}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{C}_3 \circ \mathrm{i} \circ \mathrm{C}_3{}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{C}_3^2 \circ \mathrm{i} \circ \mathrm{C}_3^{2^{-1}} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{C}_2^{(\mathrm{a})} \circ \mathrm{i} \circ \mathrm{C}_2^{(\mathrm{a})^{-1}} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{C}_2^{(\mathrm{b})} \circ \mathrm{i} \circ \mathrm{C}_2^{(\mathrm{b})^{-1}} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{C}_2^{(\mathrm{c})} \circ \mathrm{i} \circ \mathrm{C}_2^{(\mathrm{c})^{-1}} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\sigma_{\mathrm{d}1} \circ \mathrm{i} \circ \sigma_{\mathrm{d}1}{}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\sigma_{\mathrm{d}2} \circ \mathrm{i} \circ \sigma_{\mathrm{d}2}{}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\sigma_{\mathrm{d}3} \circ \mathrm{i} \circ \sigma_{\mathrm{d}3}{}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{S}_6 \circ \mathrm{i} \circ \mathrm{S}_6{}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{S}_6^5 \circ \mathrm{i} \circ \mathrm{S}_6^{5^{-1}} = [2,1,7,8,6,5,3,4] = \mathrm{i}$
$\mathrm{i} \circ \mathrm{i} \circ \mathrm{i}^{-1} = [2,1,7,8,6,5,3,4] = \mathrm{i}$

Portanto o grupo de simetria D_{3d} possui as seguintes classes de conjugação:

$$\Big[\{\mathrm{E}\},\ \{\mathrm{C}_3,\ \mathrm{C}_3^2\},\ \{\mathrm{C}_2^{(a)},\ \mathrm{C}_2^{(b)},\ \mathrm{C}_2^{(c)}\},\ \{i\},\ \{\sigma_{d1},\ \sigma_{d2},\ \sigma_{d3}\},\ \{\mathrm{S}_6,\ \mathrm{S}_6^5\}\Big]$$

Resposta para o Ítem 2.6 - Subgrupos

Apenas lembrando, um subconjunto $H \subseteq G$ é um **subgrupo** de G se ele for um grupo sob a mesma operação que define o grupo G.
Isso equivale a exigir para H todos os axiomas que definem um grupo:

- $e \in H$ — a identidade de G pertence a H;
- $\forall a, b \in H, \quad a \circ b \in H$ — H é fechado sob a mesma operação que define G;
- $\forall a \in H, \quad a^{-1} \in H$ — cada elemento de H possui um elemento inverso, que também pertence a G;

- $\forall a, b, c \in H, \quad (a \circ b) \circ c = a \circ (b \circ c)$ — a operação herdada de G que define H é associativa em H.

Ou seja, um subgrupo é um subconjunto que também satisfaz os axiomas de grupo, sob a mesma operação do grupo de origem. Portanto, qualquer subconjunto de G que contenha a identidade, seja fechado sob a mesma operação que define G, que seja associativa e contenha os inversos dos seus elementos é um subgrupo G.

Para identificar os subgrupos de D_{3d} podemos utlizar:

1. **Teorema de Lagrange**: os subgrupos devem ter ordem divisora de 12, ou seja: $1, 2, 3, 4, 6, 12$, embora como verificado não existe nenhum subconjunto de 4 elementos do grupo D_{3d} que seja fechado sob composição e contenha a identidade.
2. **Elementos geradores similares**: concentração em rotações, reflexões, inversão ou improvisas.
3. **Classes de conjugação**: se um elemento está num subconjunto e um elemento conjugado pertence ao grupo, então todo seu conjunto conjugado também precisa estar.
4. **Fechamento**: verificar tabela de multiplicação para confirmar fechamento, presença da identidade e inversos.

Elemento	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6	S_6^5	σ_{d1}	σ_{d2}	σ_{d3}
Inverso	E	C_3^2	C_3	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6^5	S_6	σ_{d1}	σ_{d2}	σ_{d3}

Essas técnicas permitiram encontrar todos os subgrupos próprios do grupo D_{3d}, excetuando-se os subgrupos triviais $\{E\}$ e o grupo total D_{3d}, que são considerados subgrupos impróprios e são deduzidos trivialmente.

Ordem 2 (cíclicos e abelianos):

- $\langle C_2^{(a)} \rangle = \{E, C_2^{(a)}\}$
- $\langle C_2^{(b)} \rangle = \{E, C_2^{(b)}\}$
- $\langle C_2^{(c)} \rangle = \{E, C_2^{(c)}\}$
- $\langle i \rangle = \{E, i\}$
- $\langle \sigma_{d1} \rangle = \{E, \sigma_{d1}\}$
- $\langle \sigma_{d2} \rangle = \{E, \sigma_{d2}\}$
- $\langle \sigma_{d3} \rangle = \{E, \sigma_{d3}\}$

Ordem 3 (cíclicos e abelianos):

- $\langle C_3 \rangle = \{E, C_3, C_3^2\}$
- $\langle S_6 \rangle = \{E, S_6, S_6^5\}$

Ordem 6 (não abelianos):

- $D_3 = \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$
- $C_{3v} = \{E, C_3, C_3^2, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$

Ordem	Subgrupos
1	$\{E\}$
2	$\{E, C_2^{(a)}\}$, $\{E, C_2^{(b)}\}$, $\{E, C_2^{(c)}\}$, $\{E, i\}$, $\{E, \sigma_{d1}\}$, $\{E, \sigma_{d2}\}$, $\{E, \sigma_{d3}\}$
3	$\{E, C_3, C_3^2\}$, $\{E, S_6, S_6^5\}$
6	$\{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$, $\{E, C_3, C_3^2, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$
12	$\{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}, i, S_6, S_6^5, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$

Todas as ordens aparecem como divisores de 12, mostrando a conformidade ao teorema de Lagrange. A presença de rotações, reflexões e inversão agrupadas em cada subgrupo respeita fechamento e classes de conjugação.

Questão 3. Considere as simetrias da molécula do hexafluoreto de enxofre SF_6, pertencente ao grupo O_h. Liste os 48 elementos de simetria desta molécula.

Resposta para o Ítem 3 - Lista de simetrias

Considerando cada elemento de fluor alinhado aos eixos cartesianos x, y e z, podemos numerar os fluors que compõe a molécula da seguinte forma: 1: z, 2: x, 3: y, 4: -z, 5: -x, 6: -y. As operações de simetria são as que seguem:

- A simetria identidade E:

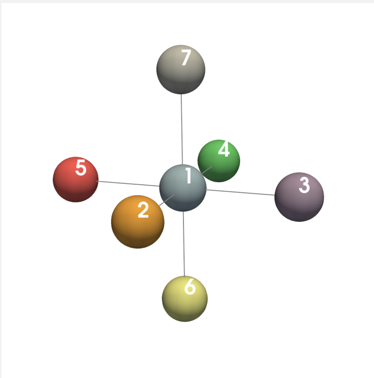


Figure 29: Simetria E

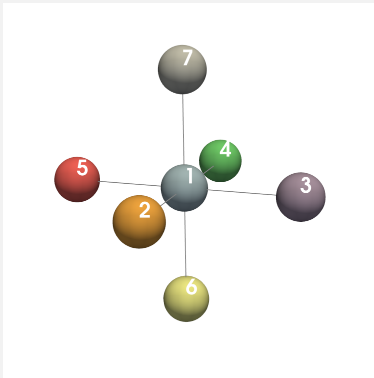


Figure 30: Operação E aplicada

$$\textbf{Identidade:} \quad E(1,2,3,4,5,6,7) = (1,2,3,4,5,6,7)$$

- Duas rotações próprias C_4 não triviais em torno de cada eixos x, y e z:

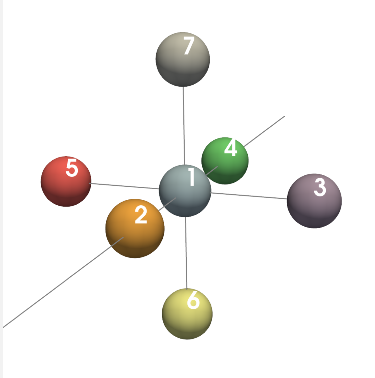


Figure 31: Eixo de simetria C_{4x}

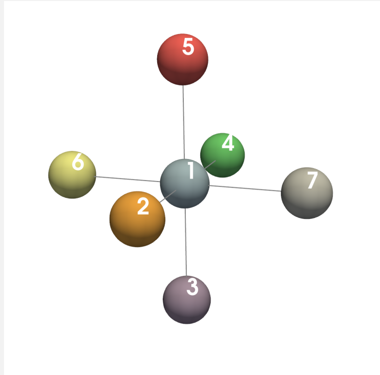


Figure 32: Operação C_{4x}^{+} aplicada

$$\textbf{Rotações } C_4\textbf{:}\quad \begin{aligned} C_{4x}^{+}(1,2,3,4,5,6,7) &= (1,2,7,4,6,3,5) \\ C_{4x}^{-}(1,2,3,4,5,6,7) &= (1,2,6,4,7,5,3) \\ C_{4y}^{+}(1,2,3,4,5,6,7) &= (1,6,3,7,5,4,2) \\ C_{4y}^{-}(1,2,3,4,5,6,7) &= (1,7,3,6,5,2,4) \\ C_{4z}^{+}(1,2,3,4,5,6,7) &= (1,3,4,5,2,6,7) \\ C_{4z}^{-}(1,2,3,4,5,6,7) &= (1,5,2,3,4,6,7) \end{aligned}$$

- Duas rotações próprias C_3 não triviais em torno de cada um dos 4 eixos diagonais que ligam vértices opostos do cubo em que a molécula se inscreve em configuração de face centrada:

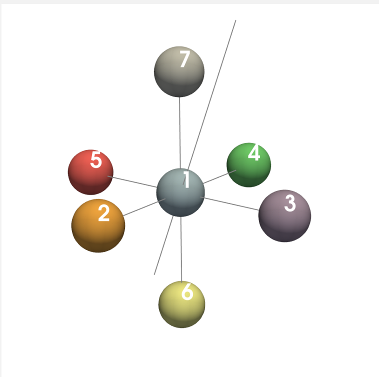


Figure 33: Eixo de simetria $C_{3<111>}$

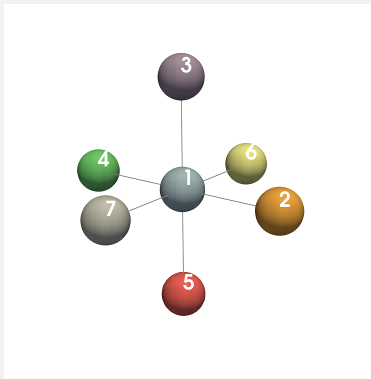


Figure 34: Operação $C_{3\langle 111\rangle}^{+}$ aplicada

$$\textbf{Rotações } C_3 :\quad \begin{aligned} C_{3\langle 111\rangle}^{+}(1,2,3,4,5,6,7) &= (1,7,2,6,4,5,3) \\ C_{3\langle 111\rangle}^{-}(1,2,3,4,5,6,7) &= (1,3,7,5,6,4,2) \\ C_{3\langle 1\bar{1}1\rangle}^{+}(1,2,3,4,5,6,7) &= (1,6,4,7,2,5,3) \\ C_{3\langle 1\bar{1}1\rangle}^{-}(1,2,3,4,5,6,7) &= (1,5,7,3,6,2,4) \\ C_{3\langle \bar{1}1\bar{1}\rangle}^{+}(1,2,3,4,5,6,7) &= (1,7,4,6,2,3,5) \\ C_{3\langle \bar{1}1\bar{1}\rangle}^{-}(1,2,3,4,5,6,7) &= (1,5,6,3,7,4,2) \\ C_{3\langle \bar{1}\bar{1}1\rangle}^{+}(1,2,3,4,5,6,7) &= (1,6,2,7,4,3,5) \\ C_{3\langle \bar{1}\bar{1}1\rangle}^{-}(1,2,3,4,5,6,7) &= (1,3,6,5,7,2,4) \end{aligned}$$

- Uma rotação própria (C_2) não trivial por eixo x, y e z:

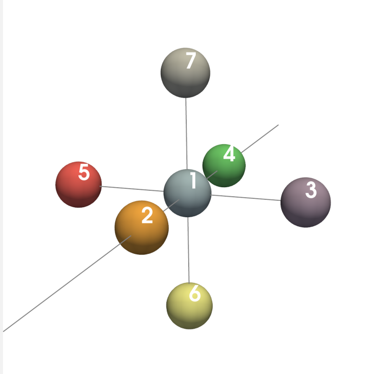


Figure 35: Eixo de simetria C_{2z}

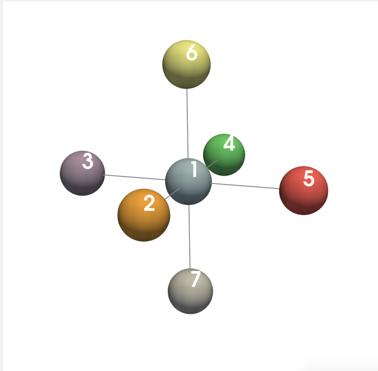


Figure 36: Operação C_{2z} aplicada

Rotações C_2**:**

$$C_{2z}^{\pm}(1,2,3,4,5,6,7) = (4,5,6,1,2,3)$$
$$C_{2x}^{\pm}(1,2,3,4,5,6,7) = (4,5,6,1,2,3)$$
$$C_{2y}^{\pm}(1,2,3,4,5,6,7) = (4,2,6,1,5,3)$$

- Uma rotação própria C_2 não trivial por cada eixo formado pelo ponto médio de arestas opostas no cubo inscrito pela molécula na configuração de face centrada:

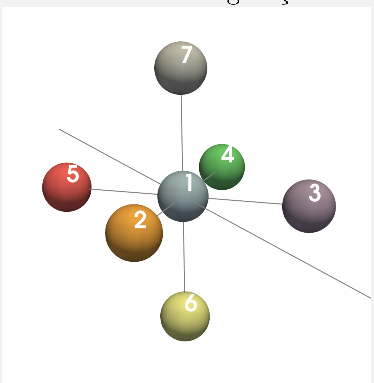


Figure 37: Eixo de simetria $C_{2<110>}$

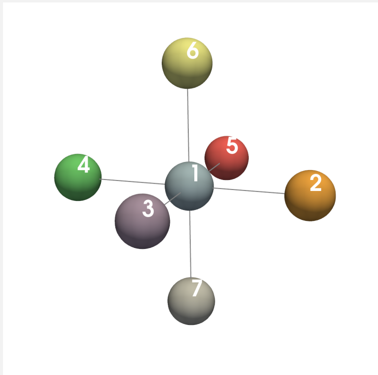


Figure 38: Operação $C_{2<110>}$ aplicada

Rotações C_2**:**

$$C_{2<110>}^{\pm}(1,2,3,4,5,6,7) = (1,3,2,5,4,7,6)$$
$$C_{2<1\bar{1}0>}^{\pm}(1,2,3,4,5,6,7) = (1,5,4,3,2,7,6)$$
$$C_{2<101>}^{\pm}(1,2,3,4,5,6,7) = (1,4,7,2,6,5,3)$$
$$C_{2<10\bar{1}>}^{\pm}(1,2,3,4,5,6,7) = (1,4,6,2,7,3,5)$$
$$C_{2<011>}^{\pm}(1,2,3,4,5,6,7) = (1,7,5,6,3,4,2)$$
$$C_{2<01\bar{1}>}^{\pm}(1,2,3,4,5,6,7) = (1,6,5,7,3,2,4)$$

- Uma reflexão σ_h horizontal em cada plano xy, zy e xz:

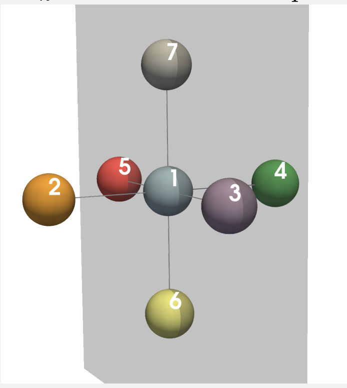


Figure 39: Plano de reflexão σ_{yz}

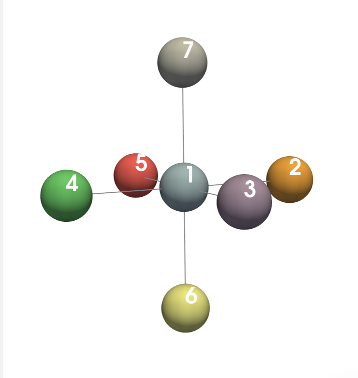


Figure 40: Operação σ_{yz} aplicada

$$\textbf{Reflexão } \sigma_h\textbf{:} \quad \begin{aligned} \sigma_{yz}(1,2,3,4,5,6,7) &= (1,4,3,2,5,6,7) \\ \sigma_{xy}(1,2,3,4,5,6,7) &= (1,2,3,4,5,7,6) \\ \sigma_{xz}(1,2,3,4,5,6,7) &= (1,2,5,4,3,6,7) \end{aligned}$$

- Uma reflexão para cada um dos planos de simetria diagonais σ_d (diagonais do cubo inscrito pela molécula na configuracão de face centrada)

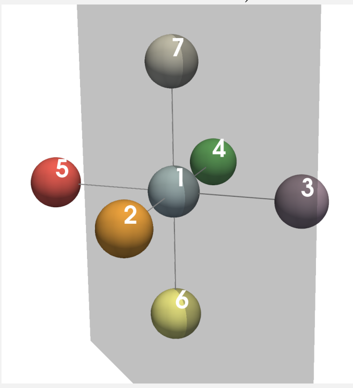


Figure 41: Plano de reflexão σ_d^{xz}

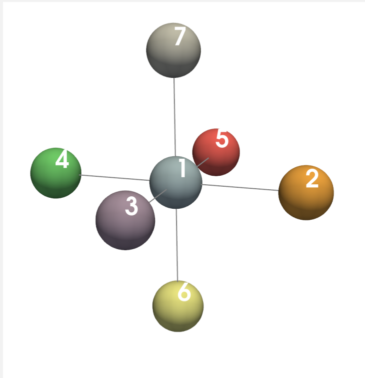


Figure 42: Operação σ_d^{xz} aplicada

$$\textbf{Reflexões } \sigma_d\textbf{:} \quad \begin{aligned} \sigma_d^{xz}(1,2,3,4,5,6,7) &= (1,3,2,5,4,6,7) \\ \sigma_d^{xy}(1,2,3,4,5,6,7) &= (1,5,4,3,2,6,7) \\ \sigma_d^{yz}(1,2,3,4,5,6,7) &= (1,2,6,4,7,3,5) \\ \sigma_d^{x\bar{y}}(1,2,3,4,5,6,7) &= (1,2,7,4,6,5,3) \\ \sigma_d^{x\bar{z}}(1,2,3,4,5,6,7) &= (1,6,3,7,5,2,4) \\ \sigma_d^{y\bar{z}}(1,2,3,4,5,6,7) &= (1,7,3,6,5,4,2) \end{aligned}$$

- Um ponto de inversão i:

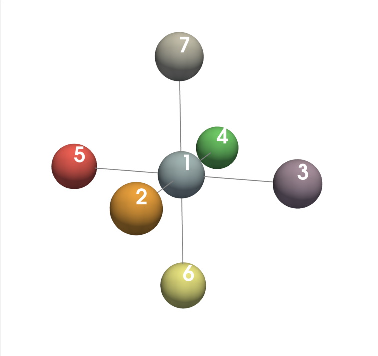


Figure 43: Ponto de inversão i na origem

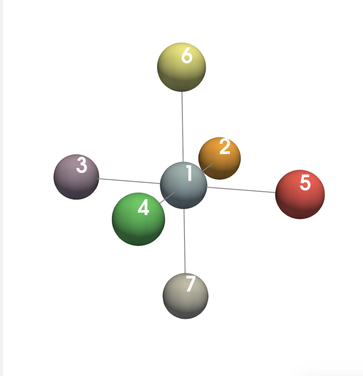


Figure 44: Operação i aplicada

Inversão: $i(1, 2, 3, 4, 5, 6, 7) = (1, 4, 5, 2, 3, 7, 6)$

- Duas rotações impróprias S_4 (rotação 90° e 270° [−90°] seguida de reflexão no plano perpendicular a este eixo) por cada eixo C_4

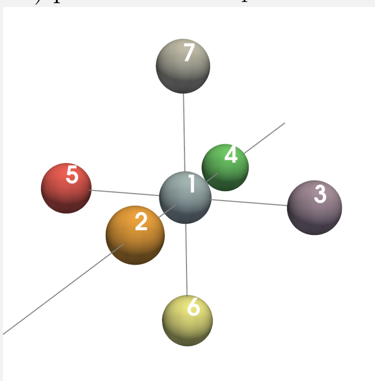


Figure 45: Eixo de rotação $C4$

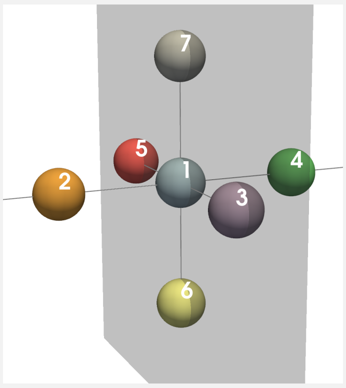


Figure 46: Plano de reflexão σ_{yz}

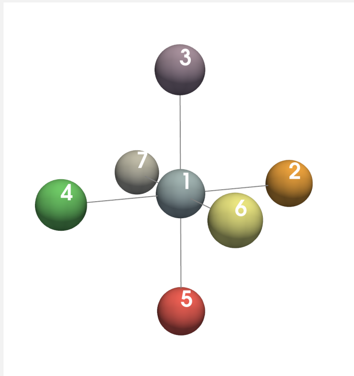


Figure 47: Operação S_4^x aplicada

Rotações impróprias S_4:

$$S_4^{x+}(1, 2, 3, 4, 5, 6, 7) = (1, 4, 6, 2, 7, 5, 3)$$
$$S_4^{x-}(1, 2, 3, 4, 5, 6, 7) = (1, 4, 7, 2, 6, 3, 5)$$
$$S_4^{y+}(1, 2, 3, 4, 5, 6, 7) = (1, 7, 5, 6, 3, 2, 4)$$
$$S_4^{y-}(1, 2, 3, 4, 5, 6, 7) = (1, 6, 5, 7, 3, 4, 2)$$
$$S_4^{z+}(1, 2, 3, 4, 5, 6, 7) = (1, 5, 2, 3, 4, 7, 6)$$
$$S_4^{z-}(1, 2, 3, 4, 5, 6, 7) = (1, 3, 4, 5, 2, 7, 6)$$

- Duas rotações impróprias S_6 (rotação 60° e 300° [−60°] seguida de reflexão no plano perpendicular a este eixo) para cada eixo $C_{6<110>}$ e equivalentes simétricos:

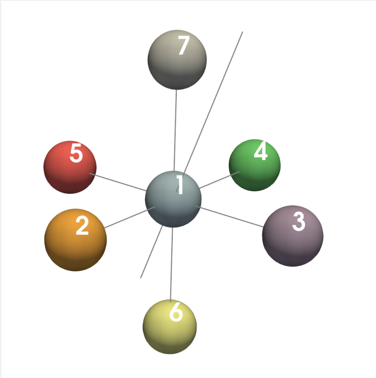


Figure 48: Eixo de rotação $C_{6<111>}$

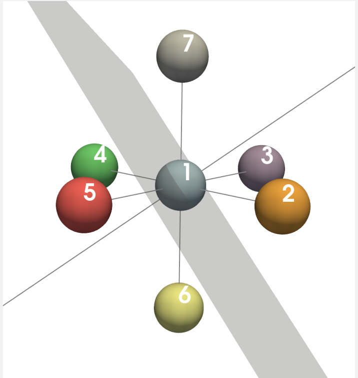


Figure 49: Plano de inversão σ_d

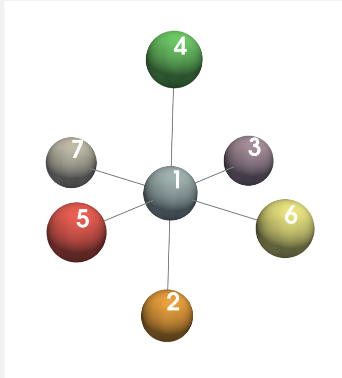


Figure 50: Operação $S_6^{\langle 111 \rangle}$ aplicada

Rotações Impróprias S_6:

$$S_6^{\langle 111 \rangle +}(1,2,3,4,5,6,7) = (1,6,4,7,2,3,5)$$
$$S_6^{\langle 111 \rangle -}(1,2,3,4,5,6,7) = (1,5,6,3,7,2,4)$$
$$S_6^{\langle 1\bar{1}1 \rangle +}(1,2,3,4,5,6,7) = (1,3,6,5,7,4,2)$$
$$S_6^{\langle 1\bar{1}1 \rangle -}(1,2,3,4,5,6,7) = (1,7,2,6,4,3,5)$$
$$S_6^{\langle \bar{1}11 \rangle +}(1,2,3,4,5,6,7) = (1,3,7,5,6,2,4)$$
$$S_6^{\langle \bar{1}11 \rangle -}(1,2,3,4,5,6,7) = (1,6,2,7,4,5,3)$$
$$S_6^{\langle \bar{1}\bar{1}1 \rangle +}(1,2,3,4,5,6,7) = (1,5,7,3,6,4,2)$$
$$S_6^{\langle \bar{1}\bar{1}1 \rangle -}(1,2,3,4,5,6,7) = (1,7,4,6,2,5,3)$$

[1] Todas as simetrias e imagens apresentadas neste trabalho foram geradas utilizando o programa desenvolvido no contexto desta disciplina. O código-fonte está disponível publicamente no repositório GitHub: `https://github.com/naavilam/simetria-molecular`. Além disso, uma versão online da ferramenta pode ser acessada em: `https://naavilam.github.io/simetria-molecular/` ou pelo qrcode, permitindo a reprodução de todos os cálculos apresentados.