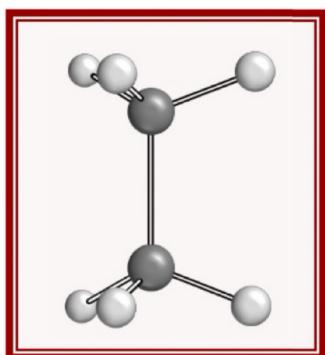


PGF5261/IFUSP: Teoria de Grupos Aplicada a Sólidos e Moléculas.
Prof.: Lucy Assali

Questão 1. Considere as simetrias da molécula de etano eclipsado C_2H_6 , pertencente ao grupo D_{3h} e responda os seguintes ítems:



- (1.1) Identifique as operações de simetria da molécula.
- (1.2) Apresente a **tabela de multiplicação**.
- (1.3) Esse grupo é **cíclico**? Justifique.
- (1.4) O grupo é **abeliano**? Justifique.
- (1.5) Quais são as **classes** deste grupo?
- (1.6) Este grupo possui **subgrupos**?

Figure 1: Molécula do etano eclipsado

Resposta para o Ítem 1.1 - Simetrias da Molécula

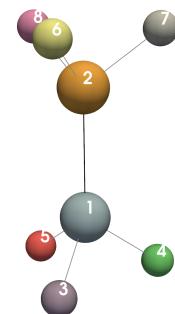


Figure 2: Simetria E

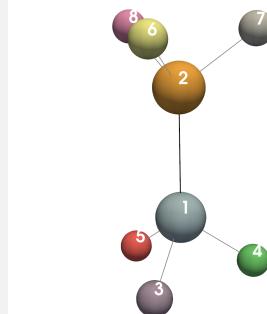


Figure 3: Operação E aplicada

$$: \quad E(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 3, 4, 5, 6, 7, 8)$$

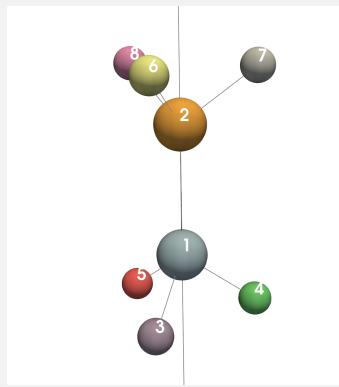


Figure 4: Eixo de rotação C_3

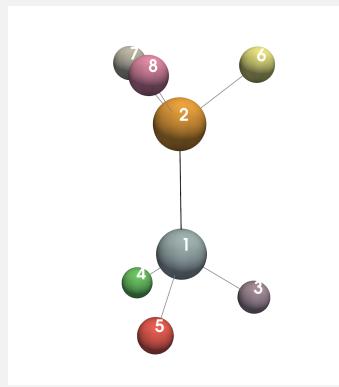


Figure 5: Operação C_3 aplicada

$$: C_3(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 5, 3, 4, 8, 6, 7)$$

$$C_3^2(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 4, 5, 3, 7, 8, 6)$$

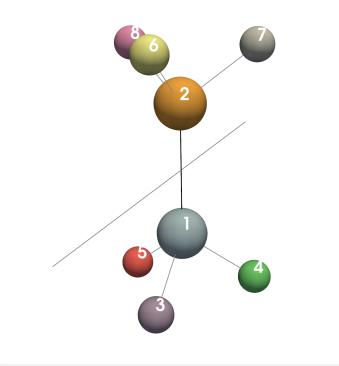


Figure 6: Eixo de rotação C_2

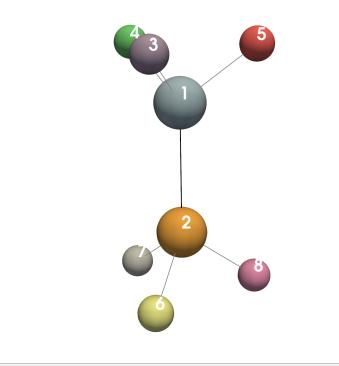


Figure 7: Operação $C_2^{(a)}$ aplicada

$$: C_2^{(a)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 6, 8, 7, 3, 5, 4)$$

$$C_2^{(b)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 8, 7, 6, 5, 4, 3)$$

$$C_2^{(c)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 7, 6, 8, 4, 3, 5)$$

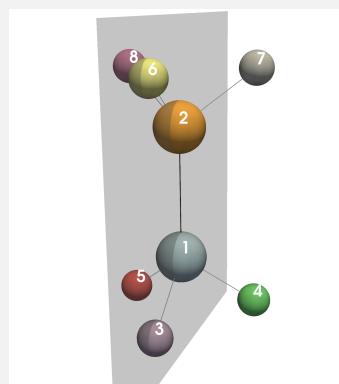


Figure 8: Plano de reflexão σ_v

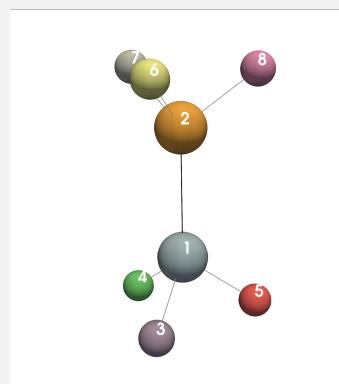


Figure 9: Operação σ_{v1} aplicada

$$: \sigma_{v1}(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 4, 5, 3, 7, 8, 6)$$

$$\sigma_{v2}(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 5, 3, 4, 8, 6, 7)$$

$$\sigma_{v3}(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 4, 3, 5, 7, 6, 8)$$

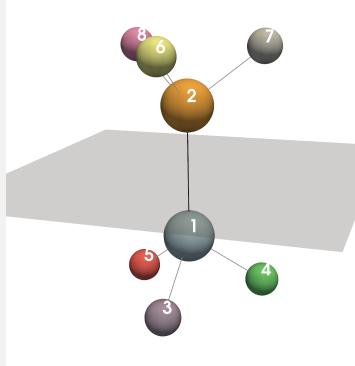


Figure 10: Plano de reflexão σ_h

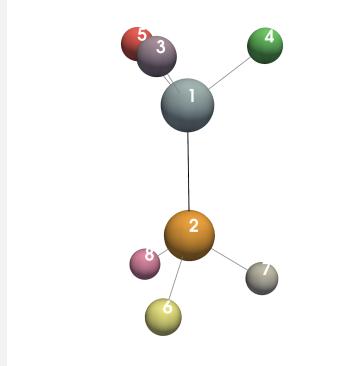


Figure 11: Operação σ_h aplicada

$$: \quad \sigma_h(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 6, 7, 8, 3, 4, 5)$$

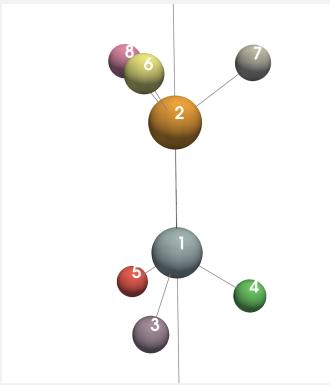


Figure 12: Eixo de rotação C_3

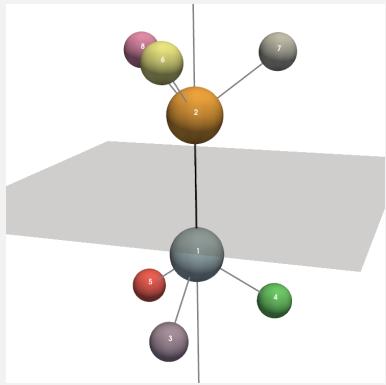


Figure 13: Plano de reflexão σ_h

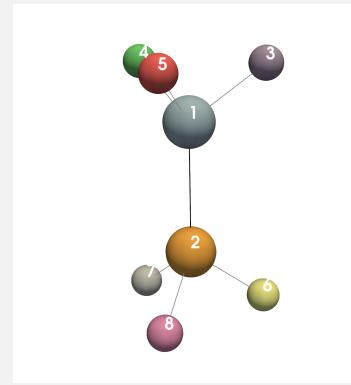


Figure 14: Operação S_3 aplicada

$$: \quad S_3(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 8, 6, 7, 5, 3, 4)$$

$$S_3^5(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 7, 8, 6, 4, 5, 3)$$

Resposta para o Ítem 1.2 - Tabela de Multiplicação

Operações de Multiplicação:

$E \circ E$	$= E \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$E \circ C_3$	$= E \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$= [1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$E \circ C_3^2$	$= E \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$= [1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$E \circ C_2^{(a)}$	$= E \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$= [2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$E \circ C_2^{(b)}$	$= E \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$= [2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$E \circ C_2^{(c)}$	$= E \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$= [2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$E \circ \sigma_{v1}$	$= E \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$= [1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$E \circ \sigma_{v2}$	$= E \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$= [1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$E \circ \sigma_{v3}$	$= E \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$= [1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$E \circ \sigma_h$	$= E \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$= [2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$E \circ S_3$	$= E \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$= [2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$E \circ S_3^5$	$= E \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$= [2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_3 \circ E$	$= C_3 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$= [1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_3 \circ C_3$	$= C_3 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$= [1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_3 \circ C_3^2$	$= C_3 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_3 \circ C_2^{(a)}$	$= C_3 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$= [2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$C_3 \circ C_2^{(b)}$	$= C_3 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$= [2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_3 \circ C_2^{(c)}$	$= C_3 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$= [2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$C_3 \circ \sigma_{v1}$	$= C_3 \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$= [1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$C_3 \circ \sigma_{v2}$	$= C_3 \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$= [1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$

$C_3 \circ \sigma_{v3}$	$= C_3 \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$C_3 \circ \sigma_h$	$= C_3 \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$C_3 \circ S_3$	$= C_3 \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_3 \circ S_3^5$	$= C_3 \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$C_3^2 \circ E$	$= C_3^2 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_3^2 \circ C_3$	$= C_3^2 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_3^2 \circ C_3^2$	$= C_3^2 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_3^2 \circ C_2^{(a)}$	$= C_3^2 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_3^2 \circ C_2^{(b)}$	$= C_3^2 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$C_3^2 \circ C_2^{(c)}$	$= C_3^2 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$C_3^2 \circ \sigma_{v1}$	$= C_3^2 \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$C_3^2 \circ \sigma_{v2}$	$= C_3^2 \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$C_3^2 \circ \sigma_{v3}$	$= C_3^2 \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$C_3^2 \circ \sigma_h$	$= C_3^2 \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_3^2 \circ S_3$	$= C_3^2 \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$C_3^2 \circ S_3^5$	$= C_3^2 \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$C_2^{(a)} \circ E$	$= C_2^{(a)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$C_2^{(a)} \circ C_3$	$= C_2^{(a)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(a)} \circ C_3^2$	$= C_2^{(a)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$C_2^{(a)} \circ C_2^{(a)}$	$= C_2^{(a)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(a)} \circ C_2^{(b)}$	$= C_2^{(a)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(a)} \circ C_2^{(c)}$	$= C_2^{(a)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(a)} \circ \sigma_{v1}$	$= C_2^{(a)} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$C_2^{(a)} \circ \sigma_{v2}$	$= C_2^{(a)} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_2^{(a)} \circ \sigma_{v3}$	$= C_2^{(a)} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$C_2^{(a)} \circ \sigma_h$	$= C_2^{(a)} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$C_2^{(a)} \circ S_3$	$= C_2^{(a)} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$C_2^{(a)} \circ S_3^5$	$= C_2^{(a)} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$C_2^{(b)} \circ E$	$= C_2^{(b)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$C_2^{(b)} \circ C_3$	$= C_2^{(b)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$C_2^{(b)} \circ C_3^2$	$= C_2^{(b)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(b)} \circ C_2^{(a)}$	$= C_2^{(b)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(b)} \circ C_2^{(b)}$	$= C_2^{(b)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(b)} \circ C_2^{(c)}$	$= C_2^{(b)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(b)} \circ \sigma_{v1}$	$= C_2^{(b)} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$C_2^{(b)} \circ \sigma_{v2}$	$= C_2^{(b)} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$C_2^{(b)} \circ \sigma_{v3}$	$= C_2^{(b)} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_2^{(b)} \circ \sigma_h$	$= C_2^{(b)} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$C_2^{(b)} \circ S_3$	$= C_2^{(b)} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$C_2^{(b)} \circ S_3^5$	$= C_2^{(b)} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$C_2^{(c)} \circ E$	$= C_2^{(c)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(c)} \circ C_3$	$= C_2^{(c)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$C_2^{(c)} \circ C_3^2$	$= C_2^{(c)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$C_2^{(c)} \circ C_2^{(a)}$	$= C_2^{(c)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(c)} \circ C_2^{(b)}$	$= C_2^{(c)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(c)} \circ C_2^{(c)}$	$= C_2^{(c)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(c)} \circ \sigma_{v1}$	$= C_2^{(c)} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$C_2^{(c)} \circ \sigma_{v2}$	$= C_2^{(c)} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$C_2^{(c)} \circ \sigma_{v3}$	$= C_2^{(c)} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$C_2^{(c)} \circ \sigma_h$	$= C_2^{(c)} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$C_2^{(c)} \circ S_3$	$= C_2^{(c)} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$C_2^{(c)} \circ S_3^5$	$= C_2^{(c)} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$\sigma_{v1} \circ E$	$= \sigma_{v1} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$\sigma_{v1} \circ C_3$	$= \sigma_{v1} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$\sigma_{v1} \circ C_3^2$	$= \sigma_{v1} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$

$\sigma_{v1} \circ C_2^{(a)}$	$= \sigma_{v1} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$\sigma_{v1} \circ C_2^{(b)}$	$= \sigma_{v1} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$\sigma_{v1} \circ C_2^{(c)}$	$= \sigma_{v1} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$\sigma_{v1} \circ \sigma_{v1}$	$= \sigma_{v1} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{v1} \circ \sigma_{v2}$	$= \sigma_{v1} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{v1} \circ \sigma_{v3}$	$= \sigma_{v1} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{v1} \circ \sigma_h$	$= \sigma_{v1} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$\sigma_{v1} \circ S_3$	$= \sigma_{v1} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{v1} \circ S_3^5$	$= \sigma_{v1} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$\sigma_{v2} \circ E$	$= \sigma_{v2} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$\sigma_{v2} \circ C_3$	$= \sigma_{v2} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$\sigma_{v2} \circ C_3^2$	$= \sigma_{v2} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$\sigma_{v2} \circ C_2^{(a)}$	$= \sigma_{v2} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$\sigma_{v2} \circ C_2^{(b)}$	$= \sigma_{v2} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$\sigma_{v2} \circ C_2^{(c)}$	$= \sigma_{v2} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$\sigma_{v2} \circ \sigma_{v1}$	$= \sigma_{v2} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{v2} \circ \sigma_{v2}$	$= \sigma_{v2} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{v2} \circ \sigma_{v3}$	$= \sigma_{v2} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{v2} \circ \sigma_h$	$= \sigma_{v2} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$\sigma_{v2} \circ S_3$	$= \sigma_{v2} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$\sigma_{v2} \circ S_3^5$	$= \sigma_{v2} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{v3} \circ E$	$= \sigma_{v3} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$\sigma_{v3} \circ C_3$	$= \sigma_{v3} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$\sigma_{v3} \circ C_3^2$	$= \sigma_{v3} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$\sigma_{v3} \circ C_2^{(a)}$	$= \sigma_{v3} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$\sigma_{v3} \circ C_2^{(b)}$	$= \sigma_{v3} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$\sigma_{v3} \circ C_2^{(c)}$	$= \sigma_{v3} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$\sigma_{v3} \circ \sigma_{v1}$	$= \sigma_{v3} \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{v3} \circ \sigma_{v2}$	$= \sigma_{v3} \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{v3} \circ \sigma_{v3}$	$= \sigma_{v3} \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{v3} \circ \sigma_h$	$= \sigma_{v3} \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{v3} \circ S_3$	$= \sigma_{v3} \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$\sigma_{v3} \circ S_3^5$	$= \sigma_{v3} \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$\sigma_h \circ E$	$= \sigma_h \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$\sigma_h \circ C_3$	$= \sigma_h \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$\sigma_h \circ C_3^2$	$= \sigma_h \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$\sigma_h \circ C_2^{(a)}$	$= \sigma_h \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$\sigma_h \circ C_2^{(b)}$	$= \sigma_h \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$\sigma_h \circ C_2^{(c)}$	$= \sigma_h \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$\sigma_h \circ \sigma_{v1}$	$= \sigma_h \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$\sigma_h \circ \sigma_{v2}$	$= \sigma_h \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$\sigma_h \circ \sigma_{v3}$	$= \sigma_h \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_h \circ \sigma_h$	$= \sigma_h \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_h \circ S_3$	$= \sigma_h \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_h \circ S_3^5$	$= \sigma_h \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$S_3 \circ E$	$= S_3 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$S_3 \circ C_3$	$= S_3 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$S_3 \circ C_3^2$	$= S_3 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$S_3 \circ C_2^{(a)}$	$= S_3 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$S_3 \circ C_2^{(b)}$	$= S_3 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$S_3 \circ C_2^{(c)}$	$= S_3 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$S_3 \circ \sigma_{v1}$	$= S_3 \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$S_3 \circ \sigma_{v2}$	$= S_3 \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$S_3 \circ \sigma_{v3}$	$= S_3 \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$S_3 \circ \sigma_h$	$= S_3 \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$S_3 \circ S_3$	$= S_3 \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$

$S_3 \circ S_3^5$	$= S_3 \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$S_3^5 \circ E$	$= S_3^5 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$= [2, 1, 8, 6, 7, 5, 3, 4]$	$= S_3^5$
$S_3^5 \circ C_3$	$= S_3^5 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$= [2, 1, 6, 7, 8, 3, 4, 5]$	$= \sigma_h$
$S_3^5 \circ C_3^2$	$= S_3^5 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$= [2, 1, 7, 8, 6, 4, 5, 3]$	$= S_3$
$S_3^5 \circ C_2^{(a)}$	$= S_3^5 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$= [1, 2, 4, 3, 5, 7, 6, 8]$	$= \sigma_{v3}$
$S_3^5 \circ C_2^{(b)}$	$= S_3^5 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$= [1, 2, 3, 5, 4, 6, 8, 7]$	$= \sigma_{v1}$
$S_3^5 \circ C_2^{(c)}$	$= S_3^5 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$= [1, 2, 5, 4, 3, 8, 7, 6]$	$= \sigma_{v2}$
$S_3^5 \circ \sigma_{v1}$	$= S_3^5 \circ [1, 2, 3, 5, 4, 6, 8, 7]$	$= [2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$S_3^5 \circ \sigma_{v2}$	$= S_3^5 \circ [1, 2, 5, 4, 3, 8, 7, 6]$	$= [2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(a)}$
$S_3^5 \circ \sigma_{v3}$	$= S_3^5 \circ [1, 2, 4, 3, 5, 7, 6, 8]$	$= [2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(b)}$
$S_3^5 \circ \sigma_h$	$= S_3^5 \circ [2, 1, 6, 7, 8, 3, 4, 5]$	$= [1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$S_3^5 \circ S_3$	$= S_3^5 \circ [2, 1, 7, 8, 6, 4, 5, 3]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$S_3^5 \circ S_3^5$	$= S_3^5 \circ [2, 1, 8, 6, 7, 5, 3, 4]$	$= [1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$

Tabela de Multiplicação Resultante:

	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	S_3	S_3^5
E	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	S_3	S_3^5
C_3	C_3	C_3^2	E	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$	S_3	σ_{v2}	σ_{v3}	σ_{v1}	S_3^5	σ_h
C_3^2	C_3^2	E	C_3	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	S_3^5	σ_{v3}	σ_{v1}	σ_{v2}	σ_h	S_3
$C_2^{(a)}$	$C_2^{(a)}$	$C_2^{(c)}$	$C_2^{(b)}$	E	C_3	C_2^2	σ_{v1}	σ_h	S_3	S_3^5	σ_{v3}	σ_{v2}
$C_2^{(b)}$	$C_2^{(b)}$	$C_2^{(a)}$	$C_2^{(c)}$	C_3^2	E	C_3	σ_{v2}	S_3^5	σ_h	S_3	σ_{v1}	σ_{v3}
$C_2^{(c)}$	$C_2^{(c)}$	$C_2^{(b)}$	$C_2^{(a)}$	C_3	C_2^2	E	σ_{v3}	S_3	S_3^5	σ_h	σ_{v2}	σ_{v1}
σ_h	σ_h	S_3	S_3^5	σ_{v1}	σ_{v2}	σ_{v3}	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$
σ_{v1}	σ_{v1}	σ_{v3}	σ_{v2}	σ_h	S_3	S_3^5	$C_2^{(a)}$	E	C_3	C_3^2	$C_2^{(b)}$	$C_2^{(c)}$
σ_{v2}	σ_{v2}	σ_{v1}	σ_{v3}	S_3^5	σ_h	S_3	$C_2^{(b)}$	C_3^2	E	C_3	$C_2^{(c)}$	$C_2^{(a)}$
σ_{v3}	σ_{v3}	σ_{v2}	σ_{v1}	S_3	S_3^5	σ_h	$C_2^{(c)}$	C_3	C_3^2	E	C_2^2	$C_2^{(b)}$
S_3	S_3	S_3^5	σ_h	σ_{v3}	σ_{v1}	σ_{v2}	C_2^2	$C_2^{(c)}$	$C_2^{(a)}$	C_3	C_3^2	E
S_3^5	S_3^5	σ_h	S_3	σ_{v2}	σ_{v3}	σ_{v1}	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	C_3^2	E	C_3

Resposta para o Ítem 1.3 - Cíclico

Um grupo é considerado cíclico quando todos os seus elementos podem ser obtidos repetindo a composição de um único elemento com ele mesmo. No caso do grupo D_{3h} , isso não é possível, pois há operações como reflexões e rotações impróprias que não surgem apenas a partir das potências de C_3 , ou de qualquer outro elemento isolado. Dessa forma, o D_{3h} não é um grupo cíclico.

Resposta para o Ítem 1.4 - Abeliano

O grupo D_{3h} não é abeliano, pois a ordem das operações afeta o resultado final. Em um grupo abeliano, qualquer par de elementos pode ser composto em qualquer ordem sem alterar o produto. No entanto, no D_{3h} , isso não acontece. Por exemplo, $C_3 \circ \sigma_{v1} \neq \sigma_{v1} \circ C_3$. Essa assimetria pode ser verificada diretamente na tabela de multiplicação, onde a ausência de simetria em relação à diagonal evidencia que as composições não são comutativas. Logo, concluímos que D_{3h} é um grupo não abeliano.

Resposta para o Ítem 1.5 - Classes

Conjugações do elemento E

$$E \circ E \circ E^{-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$C_3 \circ E \circ C_3^{-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$C_3^2 \circ E \circ C_3^{2-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$C_2^{(a)} \circ E \circ C_2^{(a)-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$C_2^{(b)} \circ E \circ C_2^{(b)-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$C_2^{(c)} \circ E \circ C_2^{(c)-1} = [1, 2, 3, 4, 5, 6, 7, 8] = E$$

$$\begin{aligned}
\sigma_{d1} \circ E \circ \sigma_{d1}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\
\sigma_{d2} \circ E \circ \sigma_{d2}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\
\sigma_{d3} \circ E \circ \sigma_{d3}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\
S_6 \circ E \circ S_6^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\
S_6^5 \circ E \circ S_6^{5-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\
i \circ E \circ i^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E
\end{aligned}$$

Conjugações de C_3

$$\begin{aligned}
E \circ C_3 \circ E^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\
C_3 \circ C_3 \circ C_3^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\
C_3^2 \circ C_3 \circ C_3^{2-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\
C_2^{(a)} \circ C_3 \circ C_2^{(a)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
C_2^{(b)} \circ C_3 \circ C_2^{(b)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
C_2^{(c)} \circ C_3 \circ C_2^{(c)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
\sigma_{d1} \circ C_3 \circ \sigma_{d1}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
\sigma_{d2} \circ C_3 \circ \sigma_{d2}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
\sigma_{d3} \circ C_3 \circ \sigma_{d3}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\
S_6 \circ C_3 \circ S_6^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\
S_6^5 \circ C_3 \circ S_6^{5-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\
i \circ C_3 \circ i^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3
\end{aligned}$$

Conjugações de $C_2^{(a)}$

$$\begin{aligned}
E \circ C_2^{(a)} \circ E^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\
C_3 \circ C_2^{(a)} \circ C_3^{-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\
C_3^2 \circ C_2^{(a)} \circ C_3^{2-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\
C_2^{(a)} \circ C_2^{(a)} \circ C_2^{(a)-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\
C_2^{(b)} \circ C_2^{(a)} \circ C_2^{(b)-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\
C_2^{(c)} \circ C_2^{(a)} \circ C_2^{(c)-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\
\sigma_{d1} \circ C_2^{(a)} \circ \sigma_{d1}^{-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\
\sigma_{d2} \circ C_2^{(a)} \circ \sigma_{d2}^{-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\
\sigma_{d3} \circ C_2^{(a)} \circ \sigma_{d3}^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\
S_6 \circ C_2^{(a)} \circ S_6^{-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\
S_6^5 \circ C_2^{(a)} \circ S_6^{5-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\
i \circ C_2^{(a)} \circ i^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)}
\end{aligned}$$

Conjugações de σ_{d1}

$$\begin{aligned}
E \circ \sigma_{d1} \circ E^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\
C_3 \circ \sigma_{d1} \circ C_3^{-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
C_3^2 \circ \sigma_{d1} \circ C_3^{2-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
C_2^{(a)} \circ \sigma_{d1} \circ C_2^{(a)-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
C_2^{(b)} \circ \sigma_{d1} \circ C_2^{(b)-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
C_2^{(c)} \circ \sigma_{d1} \circ C_2^{(c)-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\
\sigma_{d1} \circ \sigma_{d1} \circ \sigma_{d1}^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\
\sigma_{d2} \circ \sigma_{d1} \circ \sigma_{d2}^{-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
\sigma_{d3} \circ \sigma_{d1} \circ \sigma_{d3}^{-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
S_6 \circ \sigma_{d1} \circ S_6^{-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
S_6^5 \circ \sigma_{d1} \circ S_6^{5-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
i \circ \sigma_{d1} \circ i^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1}
\end{aligned}$$

Conjugações de S_6

$$E \circ S_6 \circ E^{-1} = [2, 1, 6, 7, 8, 4, 5, 3] = S_6$$

$$\begin{aligned}
C_3 \circ S_6 \circ C_3^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
C_3^2 \circ S_6 \circ C_3^{2-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
C_2^{(a)} \circ S_6 \circ C_2^{(a)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
C_2^{(b)} \circ S_6 \circ C_2^{(b)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
C_2^{(c)} \circ S_6 \circ C_2^{(c)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d1} \circ S_6 \circ \sigma_{d1}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d2} \circ S_6 \circ \sigma_{d2}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d3} \circ S_6 \circ \sigma_{d3}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
S_6 \circ S_6 \circ S_6^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
S_6^5 \circ S_6 \circ S_6^{5-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
i \circ S_6 \circ i^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6
\end{aligned}$$

Conjugações de i

$$\begin{aligned}
E \circ i \circ E^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_3 \circ i \circ C_3^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_3^2 \circ i \circ C_3^{2-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(a)} \circ i \circ C_2^{(a)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(b)} \circ i \circ C_2^{(b)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(c)} \circ i \circ C_2^{(c)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d1} \circ i \circ \sigma_{d1}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d2} \circ i \circ \sigma_{d2}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d3} \circ i \circ \sigma_{d3}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
S_6 \circ i \circ S_6^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
S_6^5 \circ i \circ S_6^{5-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
i \circ i \circ i^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i
\end{aligned}$$

Portanto o grupo de simetria D_{3d} possui as seguintes classes de conjugação:

$$[\{E\}, \{C_3, C_3^2\}, \{C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}, \{i\}, \{\sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}, \{S_6, S_6^5\}]$$

Resposta para o Ítem 1.6 - Subgrupos

Um subconjunto $H \subseteq G$ é um **subgrupo** de G se ele for um grupo sob a mesma operação que define o grupo G . Isso equivale a exigir para H todos os axiomas que definem um grupo:

- $e \in H$ — a identidade de G pertence a H ;
- $\forall a, b \in H, a \circ b \in H$ — H é fechado sob a mesma operação que define G ;
- $\forall a \in H, a^{-1} \in H$ — cada elemento de H possui um elemento inverso, que também pertence a G ;
- $\forall a, b, c \in H, (a \circ b) \circ c = a \circ (b \circ c)$ — a operação herdada de G que define H é associativa em H .

Ou seja, um subgrupo é um subconjunto que também satisfaz os axiomas de grupo, sob a mesma operação do grupo de origem. Portanto, qualquer subconjunto de G que contenha a identidade, seja fechado sob a mesma operação que define G , que seja associativa e contenha os inversos dos seus elementos é um subgrupo G .

Os subgrupos próprios do grupo D_{3h} podem ser obtidos por meio de duas abordagens complementares:

- A **análise do fluxograma de simetria** permite visualizar os subgrupos que preservam subconjuntos das operações de simetria do grupo original. A cada bifurcação do fluxograma, são descartadas operações, sendo que o grupo final contém todas as simetrias consideradas em grupos anteriores, que são subgrupos deste.
- A **geração cíclica a partir de elementos e seus inversos** identifica subgrupos abelianos de ordem 2 e 3, neste caso, formados por repetições de operações como rotações ou reflexões. É uma forma simples de obter

conjuntos de operações que satisfazem todos axiomas de definição de grupo, por exemplo: $\langle C_3 \rangle = \{E, C_3, C_3^2\}$ ou $\langle \sigma_{v1} \rangle = \{E, \sigma_{v1}\}$.

Essas duas técnicas permitiram encontrar todos os subgrupos próprios do grupo D_{3h} , excetuando-se os subgrupos $\{E\}$ e o grupo total D_{3h} , que são considerados subgrupos impróprios e são deduzidos trivialmente.

Elemento	E	C_6	C_3	C_2	C_3^2	C_6^5	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(d)}$	$C_2^{(e)}$	$C_2^{(f)}$
Inverso	E	C_6^5	C_3^2	C_2	C_3	C_6	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(d)}$	$C_2^{(e)}$	$C_2^{(f)}$
Elemento	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}	σ_h	i	S_6	S_6^5	S_3	S_3^2
Inverso	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}	σ_h	i	S_6^5	S_6	S_3^2	S_3

Ordem 2 (cíclicos e abelianos):

- $\langle C_2 \rangle = \{E, C_2\}$
- $\langle C_2^{(a)} \rangle = \{E, C_2^{(a)}\}$
- $\langle C_2^{(b)} \rangle = \{E, C_2^{(b)}\}$
- $\langle C_2^{(c)} \rangle = \{E, C_2^{(c)}\}$
- $\langle \sigma_{v1} \rangle = \{E, \sigma_{v1}\}$
- $\langle \sigma_{v2} \rangle = \{E, \sigma_{v2}\}$
- $\langle \sigma_{v3} \rangle = \{E, \sigma_{v3}\}$
- $\langle \sigma_h \rangle = \{E, \sigma_h\}$

Ordem 3 (cíclicos e abelianos):

- $\langle C_3 \rangle = \{E, C_3, C_3^2\}$
- $\langle S_3 \rangle = \{E, S_3, S_3^2\}$

Ordem 6 (fluxograma):

- $C_{3v} = \{E, C_3, C_3^2, \sigma_{v1}, \sigma_{v2}, \sigma_{v3}\}$
- $C_{3h} = \{E, C_3, C_3^2, \sigma_h, S_3, S_3^2\}$
- $D_3 = \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$

Ordem	Subgrupos
1	$\{E\}$
2	$\{E, C_2\}, \{E, C_2^{(a)}\}, \{E, C_2^{(b)}\}, \{E, C_2^{(c)}\}, \{E, \sigma_{v1}\}, \{E, \sigma_{v2}\}, \{E, \sigma_{v3}\}, \{E, \sigma_h\}$
3	$\{E, C_3, C_3^2\}, \{E, S_3, S_3^2\}$
6	$\{E, C_3, C_3^2, \sigma_{v1}, \sigma_{v2}, \sigma_{v3}\}, \{E, C_3, C_3^2, \sigma_h, S_3, S_3^2\}, \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$
12	$\{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}, \sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_h, S_3, S_3^2\}$

Questão 2. Considere as simetrias da molécula de etano estrelado C_2H_6 e responda os seguintes ítems:

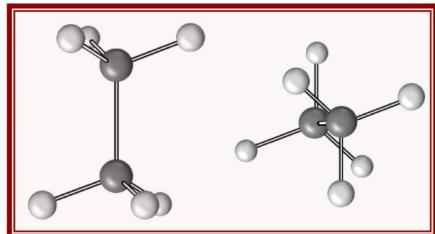


Figure 15: Molécula do etano estrelado

- (2.1) Identifique o **grupo** de simetria da molécula.
- (2.2) Apresente a **tabela de multiplicação**.
- (2.3) Esse grupo é **cíclico**? Justifique.
- (2.4) O grupo é **abeliano**? Justifique.
- (2.5) Quais são as **classes** deste grupo?
- (2.6) Este grupo possui **subgrupos**?

Resposta para o Ítem 2.1 - Grupo

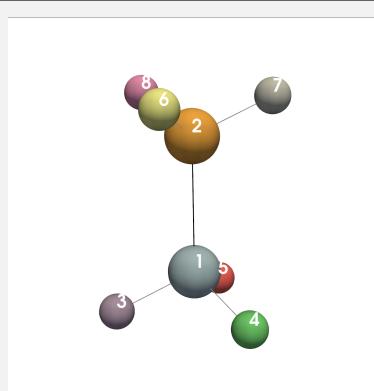


Figure 16: Simetria E

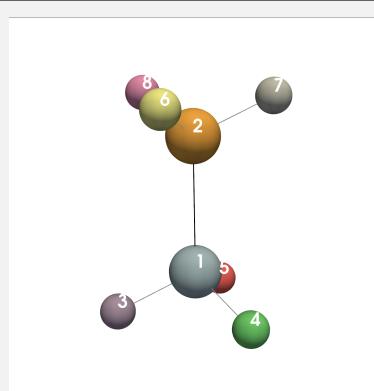


Figure 17: Operação E aplicada

$$: \quad E(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 3, 4, 5, 6, 7, 8)$$

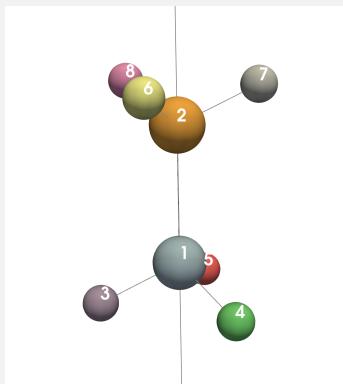


Figure 18: Eixo de rotação C_3

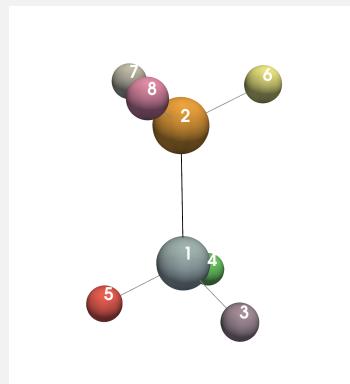


Figure 19: Operação C_3 aplicada

$$: \quad C_3(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 5, 3, 4, 8, 6, 7) \\ C_3^2(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 4, 5, 3, 7, 8, 6)$$

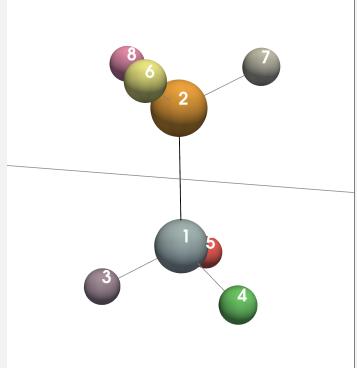


Figure 20: Eixo de rotação C_2

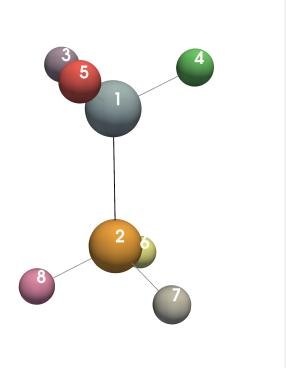


Figure 21: Operação $C_2^{(a)}$ aplicada

$$: \quad C_2^{(a)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 8, 7, 6, 5, 4, 3)$$

$$C_2^{(b)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 6, 7, 8, 3, 4, 5)$$

$$C_2^{(c)}(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 7, 8, 6, 4, 5, 3)$$

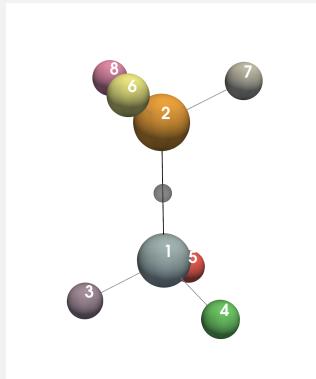


Figure 22: Origem da inversão i

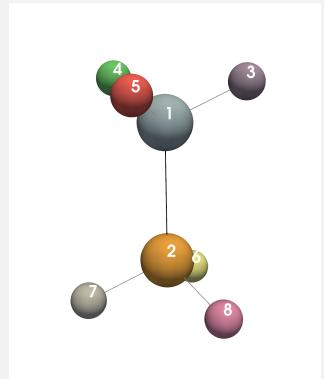


Figure 23: Operação i aplicada

$$: \quad i(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 7, 8, 6, 5, 3, 4)$$

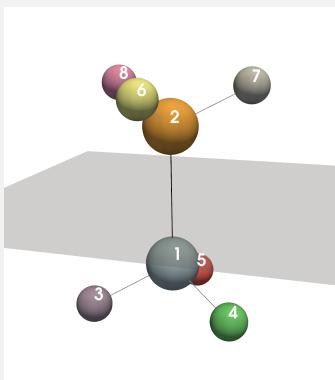


Figure 24: Plano de reflexão σ_h

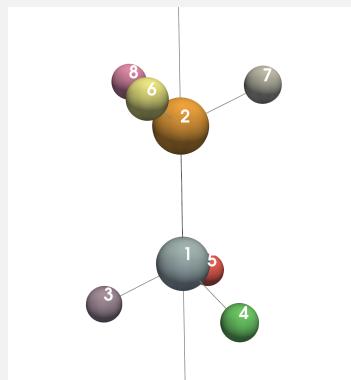


Figure 25: Eixo de rotação C_6

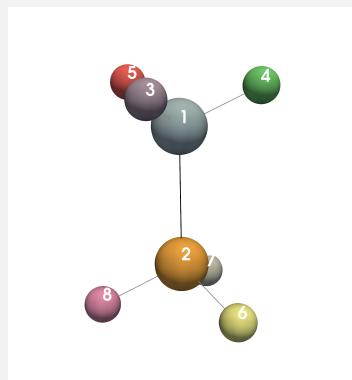


Figure 26: Operação S_6 aplicada

$$: \quad S_6(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 8, 6, 7, 3, 4, 5)$$

$$S_6^5(1, 2, 3, 4, 5, 6, 7, 8) = (2, 1, 6, 7, 8, 4, 5, 3)$$

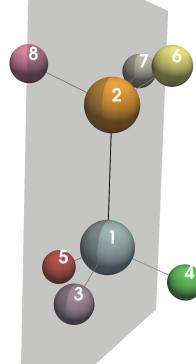


Figure 27: Plano de reflexão σ_d

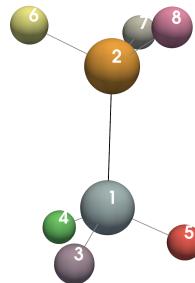


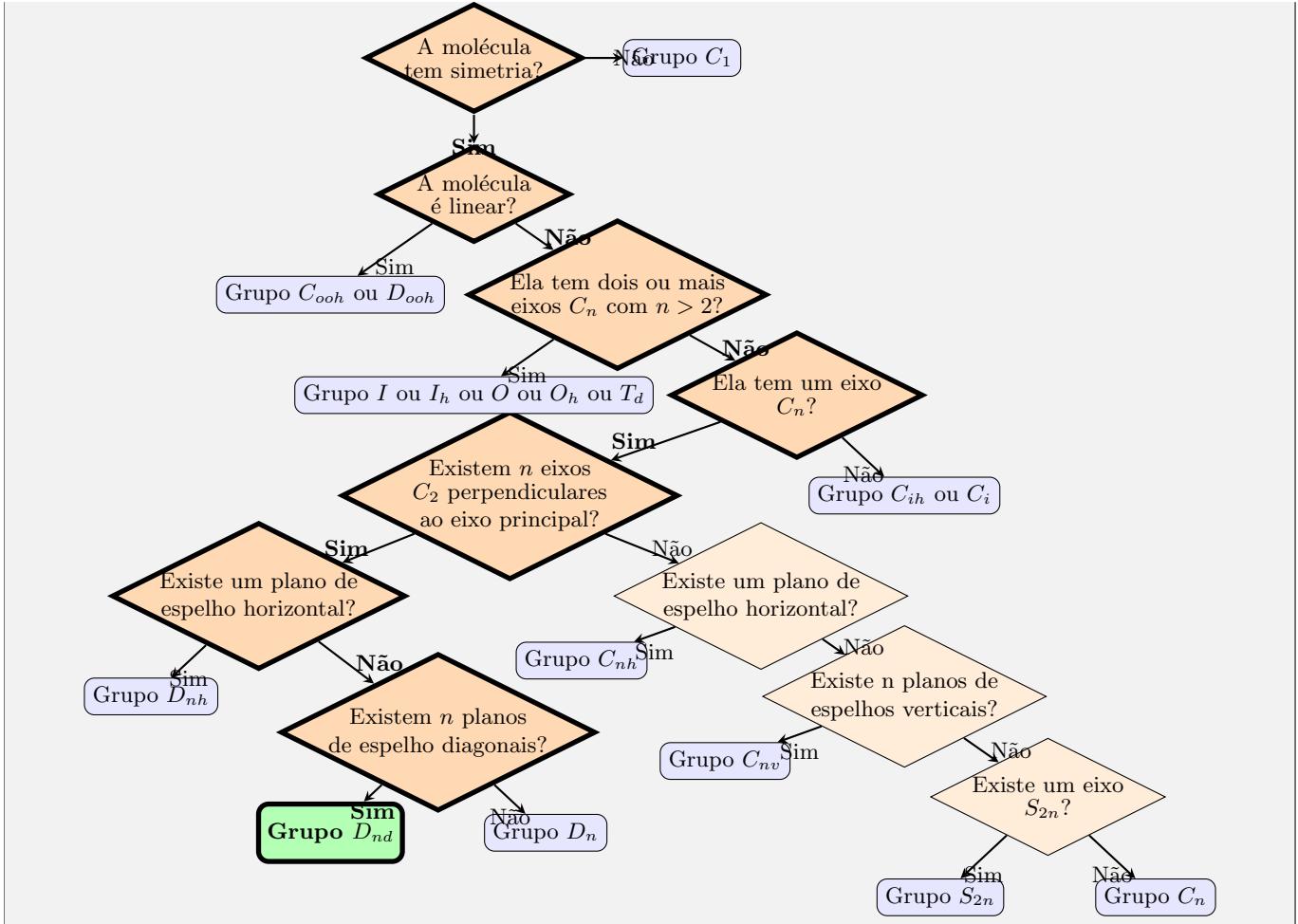
Figure 28: Operação σ_d aplicada

$$\begin{aligned} & : \quad \sigma_d(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 3, 5, 4, 8, 7, 6) \\ & \sigma_d(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 5, 4, 3, 7, 6, 8) \\ & \sigma_d(1, 2, 3, 4, 5, 6, 7, 8) = (1, 2, 4, 3, 5, 6, 8, 7) \end{aligned}$$

Conforme ilustrado nas figuras, a molécula de etano na **conformação estrelada** apresenta as seguintes simetrias:

- **Um eixo de rotação** C_3 , ao longo do eixo C–C, passando pelo centro dos triângulos formados pelos três hidrogênios de cada carbono. Com rotações de 120° e 240° .
- **Três eixos de rotação** C_2 , todos perpendiculares ao eixo C_3 , localizados em planos que contêm pares de hidrogênios opostos (um de cada carbono). Estes eixos estão espaçados de 120° entre si ao redor do eixo principal. Com rotacões de 180° cada.
- **Três planos de reflexão diagonais** σ_d , cada um contendo o eixo C_3 e passando entre pares de hidrogênios de carbonos opostos. Estes planos cortam os triângulos de hidrogênios ao meio.
- **Uma operação de inversão** i , situada no ponto médio da ligação C–C, que transforma cada átomo em sua imagem oposta através do centro de massa da molécula.
- **Duas rotações impróprias** S_6 , correspondentes à combinação de uma reflexão por um plano σ_h perpendicular ao eixo seguida de rotação C_6 . Rotações impróprias de 60° e 300° .

Portanto, conforme demonstrado no fluxograma de classificação, o grupo de simetria da molécula de etano na **conformação estrelada** é o D_{3d} .



Resposta para o Ítem 2.2 - Tabela de Multiplicação

Operações de Multiplicação:

$E \circ E$	$= E \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$E \circ C_3$	$= E \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$= [1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$E \circ C_3^2$	$= E \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$= [1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$E \circ C_2^{(a)}$	$= E \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$= [2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$E \circ C_2^{(b)}$	$= E \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$= [2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$E \circ C_2^{(c)}$	$= E \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$= [2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$E \circ \sigma_{d1}$	$= E \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$= [1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$E \circ \sigma_{d2}$	$= E \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$= [1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$E \circ \sigma_{d3}$	$= E \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$= [1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$E \circ S_6$	$= E \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$= [2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$E \circ S_6^5$	$= E \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$= [2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$E \circ i$	$= E \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$= [2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_3 \circ E$	$= C_3 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$= [1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_3 \circ C_3$	$= C_3 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$= [1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_3 \circ C_3^2$	$= C_3 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$= [1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_3 \circ C_2^{(a)}$	$= C_3 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$= [2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_3 \circ C_2^{(b)}$	$= C_3 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$= [2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$C_3 \circ C_2^{(c)}$	$= C_3 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$= [2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$C_3 \circ \sigma_{d1}$	$= C_3 \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$= [1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$C_3 \circ \sigma_{d2}$	$= C_3 \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$= [1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$C_3 \circ \sigma_{d3}$	$= C_3 \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$= [1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$C_3 \circ S_6$	$= C_3 \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$= [2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_3 \circ S_6^5$	$= C_3 \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$= [2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$C_3 \circ i$	$= C_3 \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$= [2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$

$C_3^2 \circ E$	$= C_3^2 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_3^2 \circ C_3$	$= C_3^2 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_3^2 \circ C_3^2$	$= C_3^2 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_3^2 \circ C_2^{(a)}$	$= C_3^2 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$C_3^2 \circ C_2^{(b)}$	$= C_3^2 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_3^2 \circ C_2^{(c)}$	$= C_3^2 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$C_3^2 \circ \sigma_{d1}$	$= C_3^2 \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$C_3^2 \circ \sigma_{d2}$	$= C_3^2 \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$C_3^2 \circ \sigma_{d3}$	$= C_3^2 \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$C_3^2 \circ S_6$	$= C_3^2 \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$C_3^2 \circ S_6^5$	$= C_3^2 \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_3^2 \circ i$	$= C_3^2 \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$C_2^{(a)} \circ E$	$= C_2^{(a)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$C_2^{(a)} \circ C_3$	$= C_2^{(a)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$C_2^{(a)} \circ C_3^2$	$= C_2^{(a)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(a)} \circ C_2^{(a)}$	$= C_2^{(a)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(a)} \circ C_2^{(b)}$	$= C_2^{(a)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(a)} \circ C_2^{(c)}$	$= C_2^{(a)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(a)} \circ \sigma_{d1}$	$= C_2^{(a)} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$C_2^{(a)} \circ \sigma_{d2}$	$= C_2^{(a)} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$C_2^{(a)} \circ \sigma_{d3}$	$= C_2^{(a)} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_2^{(a)} \circ S_6$	$= C_2^{(a)} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$C_2^{(a)} \circ S_6^5$	$= C_2^{(a)} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$C_2^{(a)} \circ i$	$= C_2^{(a)} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$C_2^{(b)} \circ E$	$= C_2^{(b)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$C_2^{(b)} \circ C_3$	$= C_2^{(b)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(b)} \circ C_3^2$	$= C_2^{(b)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$C_2^{(b)} \circ C_2^{(a)}$	$= C_2^{(b)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(b)} \circ C_2^{(b)}$	$= C_2^{(b)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(b)} \circ C_2^{(c)}$	$= C_2^{(b)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(b)} \circ \sigma_{d1}$	$= C_2^{(b)} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$C_2^{(b)} \circ \sigma_{d2}$	$= C_2^{(b)} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_2^{(b)} \circ \sigma_{d3}$	$= C_2^{(b)} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$C_2^{(b)} \circ S_6$	$= C_2^{(b)} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$C_2^{(b)} \circ S_6^5$	$= C_2^{(b)} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$C_2^{(b)} \circ i$	$= C_2^{(b)} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$C_2^{(c)} \circ E$	$= C_2^{(c)} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$C_2^{(c)} \circ C_3$	$= C_2^{(c)} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$C_2^{(c)} \circ C_3^2$	$= C_2^{(c)} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$C_2^{(c)} \circ C_2^{(a)}$	$= C_2^{(c)} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$C_2^{(c)} \circ C_2^{(b)}$	$= C_2^{(c)} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$C_2^{(c)} \circ C_2^{(c)}$	$= C_2^{(c)} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$C_2^{(c)} \circ \sigma_{d1}$	$= C_2^{(c)} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$C_2^{(c)} \circ \sigma_{d2}$	$= C_2^{(c)} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$C_2^{(c)} \circ \sigma_{d3}$	$= C_2^{(c)} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$C_2^{(c)} \circ S_6$	$= C_2^{(c)} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$C_2^{(c)} \circ S_6^5$	$= C_2^{(c)} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$C_2^{(c)} \circ i$	$= C_2^{(c)} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$\sigma_{d1} \circ E$	$= \sigma_{d1} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$\sigma_{d1} \circ C_3$	$= \sigma_{d1} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$\sigma_{d1} \circ C_3^2$	$= \sigma_{d1} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$\sigma_{d1} \circ C_2^{(a)}$	$= \sigma_{d1} \circ [1, 2, 8, 7, 6, 5, 4, 3]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$\sigma_{d1} \circ C_2^{(b)}$	$= \sigma_{d1} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$\sigma_{d1} \circ C_2^{(c)}$	$= \sigma_{d1} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$

$\sigma_{d1} \circ \sigma_{d1}$	$= \sigma_{d1} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{d1} \circ \sigma_{d2}$	$= \sigma_{d1} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{d1} \circ \sigma_{d3}$	$= \sigma_{d1} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{d1} \circ S_6$	$= \sigma_{d1} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$\sigma_{d1} \circ S_6^5$	$= \sigma_{d1} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$\sigma_{d1} \circ i$	$= \sigma_{d1} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{d2} \circ E$	$= \sigma_{d2} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$\sigma_{d2} \circ C_3$	$= \sigma_{d2} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$\sigma_{d2} \circ C_3^2$	$= \sigma_{d2} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$\sigma_{d2} \circ C_2^{(a)}$	$= \sigma_{d2} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$\sigma_{d2} \circ C_2^{(b)}$	$= \sigma_{d2} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$\sigma_{d2} \circ C_2^{(c)}$	$= \sigma_{d2} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$\sigma_{d2} \circ \sigma_{d1}$	$= \sigma_{d2} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{d2} \circ \sigma_{d2}$	$= \sigma_{d2} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{d2} \circ \sigma_{d3}$	$= \sigma_{d2} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{d2} \circ S_6$	$= \sigma_{d2} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$\sigma_{d2} \circ S_6^5$	$= \sigma_{d2} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{d2} \circ i$	$= \sigma_{d2} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$\sigma_{d3} \circ E$	$= \sigma_{d3} \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$\sigma_{d3} \circ C_3$	$= \sigma_{d3} \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$\sigma_{d3} \circ C_3^2$	$= \sigma_{d3} \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$\sigma_{d3} \circ C_2^{(a)}$	$= \sigma_{d3} \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$\sigma_{d3} \circ C_2^{(b)}$	$= \sigma_{d3} \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$\sigma_{d3} \circ C_2^{(c)}$	$= \sigma_{d3} \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$\sigma_{d3} \circ \sigma_{d1}$	$= \sigma_{d3} \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$\sigma_{d3} \circ \sigma_{d2}$	$= \sigma_{d3} \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$\sigma_{d3} \circ \sigma_{d3}$	$= \sigma_{d3} \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$\sigma_{d3} \circ S_6$	$= \sigma_{d3} \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$\sigma_{d3} \circ S_6^5$	$= \sigma_{d3} \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$\sigma_{d3} \circ i$	$= \sigma_{d3} \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$S_6 \circ E$	$= S_6 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$S_6 \circ C_3$	$= S_6 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$S_6 \circ C_3^2$	$= S_6 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$S_6 \circ C_2^{(a)}$	$= S_6 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$S_6 \circ C_2^{(b)}$	$= S_6 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$S_6 \circ C_2^{(c)}$	$= S_6 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$S_6 \circ \sigma_{d1}$	$= S_6 \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$S_6 \circ \sigma_{d2}$	$= S_6 \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$S_6 \circ \sigma_{d3}$	$= S_6 \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$S_6 \circ S_6$	$= S_6 \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$S_6 \circ S_6^5$	$= S_6 \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$S_6 \circ i$	$= S_6 \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$S_6^5 \circ E$	$= S_6^5 \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$S_6^5 \circ C_3$	$= S_6^5 \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$
$S_6^5 \circ C_3^2$	$= S_6^5 \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$S_6^5 \circ C_2^{(a)}$	$= S_6^5 \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$S_6^5 \circ C_2^{(b)}$	$= S_6^5 \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$S_6^5 \circ C_2^{(c)}$	$= S_6^5 \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$S_6^5 \circ \sigma_{d1}$	$= S_6^5 \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$S_6^5 \circ \sigma_{d2}$	$= S_6^5 \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$S_6^5 \circ \sigma_{d3}$	$= S_6^5 \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$S_6^5 \circ S_6$	$= S_6^5 \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$
$S_6^5 \circ S_6^5$	$= S_6^5 \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$S_6^5 \circ i$	$= S_6^5 \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$i \circ E$	$= i \circ [1, 2, 3, 4, 5, 6, 7, 8]$	$[2, 1, 7, 8, 6, 5, 3, 4]$	$= i$
$i \circ C_3$	$= i \circ [1, 2, 4, 5, 3, 7, 8, 6]$	$[2, 1, 8, 6, 7, 3, 4, 5]$	$= S_6^5$
$i \circ C_3^2$	$= i \circ [1, 2, 5, 3, 4, 8, 6, 7]$	$[2, 1, 6, 7, 8, 4, 5, 3]$	$= S_6$

$i \circ C_2^{(a)}$	$= i \circ [2, 1, 8, 7, 6, 5, 4, 3]$	$[1, 2, 4, 3, 5, 6, 8, 7]$	$= \sigma_{d3}$
$i \circ C_2^{(b)}$	$= i \circ [2, 1, 6, 8, 7, 3, 5, 4]$	$[1, 2, 5, 4, 3, 7, 6, 8]$	$= \sigma_{d2}$
$i \circ C_2^{(c)}$	$= i \circ [2, 1, 7, 6, 8, 4, 3, 5]$	$[1, 2, 3, 5, 4, 8, 7, 6]$	$= \sigma_{d1}$
$i \circ \sigma_{d1}$	$= i \circ [1, 2, 3, 5, 4, 8, 7, 6]$	$[2, 1, 7, 6, 8, 4, 3, 5]$	$= C_2^{(c)}$
$i \circ \sigma_{d2}$	$= i \circ [1, 2, 5, 4, 3, 7, 6, 8]$	$[2, 1, 6, 8, 7, 3, 5, 4]$	$= C_2^{(b)}$
$i \circ \sigma_{d3}$	$= i \circ [1, 2, 4, 3, 5, 6, 8, 7]$	$[2, 1, 8, 7, 6, 5, 4, 3]$	$= C_2^{(a)}$
$i \circ S_6$	$= i \circ [2, 1, 6, 7, 8, 4, 5, 3]$	$[1, 2, 5, 3, 4, 8, 6, 7]$	$= C_3^2$
$i \circ S_6^5$	$= i \circ [2, 1, 8, 6, 7, 3, 4, 5]$	$[1, 2, 4, 5, 3, 7, 8, 6]$	$= C_3$
$i \circ i$	$= i \circ [2, 1, 7, 8, 6, 5, 3, 4]$	$[1, 2, 3, 4, 5, 6, 7, 8]$	$= E$

Tabela de Multiplicação Resultante:

	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6	S_6^5	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$
E	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6	S_6^5	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$
C_3	C_3	C_3^2	E	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	S_6^5	i	S_6	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$
C_3^2	C_3^2	E	C_3	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$	S_6	S_6^5	i	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$
$C_2^{(a)}$	$C_2^{(a)}$	$C_2^{(c)}$	$C_2^{(b)}$	E	C_3^2	C_3	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	i	S_6^5	S_6
$C_2^{(b)}$	$C_2^{(b)}$	$C_2^{(a)}$	$C_2^{(c)}$	C_3^2	C_3	E	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	S_6^5	S_6	i
$C_2^{(c)}$	$C_2^{(c)}$	$C_2^{(b)}$	$C_2^{(a)}$	C_3	E	C_3^2	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	S_6	i	S_6^5
i	i	S_6^5	S_6	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	E	C_3^2	C_3	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$
S_6	S_6	i	S_6^5	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	$\sigma_d^{(a)}$	C_3	E	C_3^2	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$
S_6^5	S_6^5	S_6	i	$\sigma_d^{(a)}$	$\sigma_d^{(b)}$	$\sigma_d^{(c)}$	C_3^2	C_3	E	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$
$\sigma_d^{(a)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	i	S_6^5	S_6	$C_2^{(c)}$	$C_2^{(a)}$	$C_2^{(b)}$	E	C_3^2	C_3
$\sigma_d^{(b)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	$\sigma_d^{(c)}$	S_6^5	i	S_6	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	C_3	E	C_3^2
$\sigma_d^{(c)}$	$\sigma_d^{(c)}$	$\sigma_d^{(b)}$	$\sigma_d^{(a)}$	S_6	S_6^5	i	$C_2^{(b)}$	$C_2^{(c)}$	$C_2^{(a)}$	C_3^2	C_3	E

Resposta para o Ítem 2.3 - Cíclico

Embora o grupo D_{3d} contenha rotações C_3 e C_3^2 , que geralmente formam grupos cíclicos, as demais operações de simetria, como as rotações de 180° em torno dos três eixos C_2 perpendiculares ao eixo principal, as reflexões diagonais σ_d , e também as rotações impróprias S_6 e S_6^5 , não podem ser obtidas apenas por repetições de qualquer uma dessas rotações isoladamente. Por exemplo, aplicar sucessivas potências de C_3 nunca resultará em uma reflexão σ_d , nem em uma inversão i , nem nas rotações C_2 .

Assim, devido à presença de múltiplos tipos de operações de simetria que não são geradas a partir de um único elemento, o grupo D_{3d} não pode ser classificado como cíclico.

Resposta para o Ítem 2.4 - Abeliano

O grupo D_{3d} não é abeliano, pelo mesmo motivo de que a ordem das operações afeta o resultado final. Em um grupo abeliano, qualquer par de elementos pode ser composto em qualquer ordem sem alterar o produto. No entanto, no D_{3d} , isso não acontece. Por exemplo, $C_3 \circ C_2^{(a)} \neq C_2^{(a)} \circ C_3$. Essa não comutatividade pode ser verificada diretamente na tabela de multiplicação, onde a ausência de simetria em relação à diagonal principal evidencia que as composições não são comutativas. Logo, concluímos que D_{3d} é um grupo não abeliano.

Resposta para o Ítem 2.5 - Classes

De acordo com a definição, dois elementos A e B pertencem à mesma classe de conjugação se existe um elemento $g \in G$ tal que:

$$B = g \circ A \circ g^{-1}$$

Portanto, para determinar as classes de conjugação do grupo D_{3d} , é necessário calcular o conjugado de cada operação por todas as demais:

$$g \circ A \circ g^{-1}, \quad \text{para todo } g \in D_{3d}$$

Após realizar esses cálculos, agrupamos os elementos $A \in D_{3d}$ que produzem o mesmo conjunto de conjugados. Esses agrupamentos formam as classes de conjugação do grupo D_{3d} , refletindo as equivalências sob a ação de conjugação interna ao grupo.

Conjugações do elemento E

$$\begin{aligned} E \circ E \circ E^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ C_3 \circ E \circ C_3^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ C_3^2 \circ E \circ C_3^{2-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ C_2^{(a)} \circ E \circ C_2^{(a)-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ C_2^{(b)} \circ E \circ C_2^{(b)-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ C_2^{(c)} \circ E \circ C_2^{(c)-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ \sigma_{d1} \circ E \circ \sigma_{d1}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ \sigma_{d2} \circ E \circ \sigma_{d2}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ \sigma_{d3} \circ E \circ \sigma_{d3}^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ S_6 \circ E \circ S_6^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ S_6^5 \circ E \circ S_6^{5-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \\ i \circ E \circ i^{-1} &= [1, 2, 3, 4, 5, 6, 7, 8] = E \end{aligned}$$

Conjugações de C_3

$$\begin{aligned} E \circ C_3 \circ E^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\ C_3 \circ C_3 \circ C_3^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\ C_3^2 \circ C_3 \circ C_3^{2-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\ C_2^{(a)} \circ C_3 \circ C_2^{(a)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ C_2^{(b)} \circ C_3 \circ C_2^{(b)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ C_2^{(c)} \circ C_3 \circ C_2^{(c)-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ \sigma_{d1} \circ C_3 \circ \sigma_{d1}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ \sigma_{d2} \circ C_3 \circ \sigma_{d2}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ \sigma_{d3} \circ C_3 \circ \sigma_{d3}^{-1} &= [1, 2, 5, 3, 4, 8, 6, 7] = C_3^2 \\ S_6 \circ C_3 \circ S_6^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\ S_6^5 \circ C_3 \circ S_6^{5-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \\ i \circ C_3 \circ i^{-1} &= [1, 2, 4, 5, 3, 7, 8, 6] = C_3 \end{aligned}$$

Conjugações de $C_2^{(a)}$

$$\begin{aligned} E \circ C_2^{(a)} \circ E^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\ C_3 \circ C_2^{(a)} \circ C_3^{-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\ C_3^2 \circ C_2^{(a)} \circ C_3^{2-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\ C_2^{(a)} \circ C_2^{(a)} \circ C_2^{(a)-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\ C_2^{(b)} \circ C_2^{(a)} \circ C_2^{(b)-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\ C_2^{(c)} \circ C_2^{(a)} \circ C_2^{(c)-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\ \sigma_{d1} \circ C_2^{(a)} \circ \sigma_{d1}^{-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\ \sigma_{d2} \circ C_2^{(a)} \circ \sigma_{d2}^{-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\ \sigma_{d3} \circ C_2^{(a)} \circ \sigma_{d3}^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \\ S_6 \circ C_2^{(a)} \circ S_6^{-1} &= [2, 1, 7, 6, 8, 4, 3, 5] = C_2^{(c)} \\ S_6^5 \circ C_2^{(a)} \circ S_6^{5-1} &= [2, 1, 6, 8, 7, 3, 5, 4] = C_2^{(b)} \\ i \circ C_2^{(a)} \circ i^{-1} &= [2, 1, 8, 7, 6, 5, 4, 3] = C_2^{(a)} \end{aligned}$$

Conjugações de σ_{d1}

$$\begin{aligned} E \circ \sigma_{d1} \circ E^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\ C_3 \circ \sigma_{d1} \circ C_3^{-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \end{aligned}$$

$$\begin{aligned}
C_3^2 \circ \sigma_{d1} \circ C_3^{2-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
C_2^{(a)} \circ \sigma_{d1} \circ C_2^{(a)-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
C_2^{(b)} \circ \sigma_{d1} \circ C_2^{(b)-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
C_2^{(c)} \circ \sigma_{d1} \circ C_2^{(c)-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\
\sigma_{d1} \circ \sigma_{d1} \circ \sigma_{d1}^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1} \\
\sigma_{d2} \circ \sigma_{d1} \circ \sigma_{d2}^{-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
\sigma_{d3} \circ \sigma_{d1} \circ \sigma_{d3}^{-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
S_6 \circ \sigma_{d1} \circ S_6^{-1} &= [1, 2, 5, 4, 3, 7, 6, 8] = \sigma_{d2} \\
S_6^5 \circ \sigma_{d1} \circ S_6^{5-1} &= [1, 2, 4, 3, 5, 6, 8, 7] = \sigma_{d3} \\
i \circ \sigma_{d1} \circ i^{-1} &= [1, 2, 3, 5, 4, 8, 7, 6] = \sigma_{d1}
\end{aligned}$$

Conjugações de S_6

$$\begin{aligned}
E \circ S_6 \circ E^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
C_3 \circ S_6 \circ C_3^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
C_3^2 \circ S_6 \circ C_3^{2-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
C_2^{(a)} \circ S_6 \circ C_2^{(a)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
C_2^{(b)} \circ S_6 \circ C_2^{(b)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
C_2^{(c)} \circ S_6 \circ C_2^{(c)-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d1} \circ S_6 \circ \sigma_{d1}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d2} \circ S_6 \circ \sigma_{d2}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
\sigma_{d3} \circ S_6 \circ \sigma_{d3}^{-1} &= [2, 1, 8, 6, 7, 3, 4, 5] = S_6^5 \\
S_6 \circ S_6 \circ S_6^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
S_6^5 \circ S_6 \circ S_6^{5-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6 \\
i \circ S_6 \circ i^{-1} &= [2, 1, 6, 7, 8, 4, 5, 3] = S_6
\end{aligned}$$

Conjugações de i

$$\begin{aligned}
E \circ i \circ E^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_3 \circ i \circ C_3^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_3^2 \circ i \circ C_3^{2-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(a)} \circ i \circ C_2^{(a)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(b)} \circ i \circ C_2^{(b)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
C_2^{(c)} \circ i \circ C_2^{(c)-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d1} \circ i \circ \sigma_{d1}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d2} \circ i \circ \sigma_{d2}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
\sigma_{d3} \circ i \circ \sigma_{d3}^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
S_6 \circ i \circ S_6^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
S_6^5 \circ i \circ S_6^{5-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i \\
i \circ i \circ i^{-1} &= [2, 1, 7, 8, 6, 5, 3, 4] = i
\end{aligned}$$

Portanto o grupo de simetria D_{3d} possui as seguintes classes de conjugação:

$$[\{E\}, \{C_3, C_3^2\}, \{C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}, \{i\}, \{\sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}, \{S_6, S_6^5\}]$$

Resposta para o Ítem 2.6 - Subgrupos

Apenas lembrando, um subconjunto $H \subseteq G$ é um **subgrupo** de G se ele for um grupo sob a mesma operação que define o grupo G .

Isso equivale a exigir para H todos os axiomas que definem um grupo:

- $e \in H$ — a identidade de G pertence a H ;
- $\forall a, b \in H, a \circ b \in H$ — H é fechado sob a mesma operação que define G ;
- $\forall a \in H, a^{-1} \in H$ — cada elemento de H possui um elemento inverso, que também pertence a G ;

- $\forall a, b, c \in H, (a \circ b) \circ c = a \circ (b \circ c)$ — a operação herdada de G que define H é associativa em H .

Ou seja, um subgrupo é um subconjunto que também satisfaz os axiomas de grupo, sob a mesma operação do grupo de origem. Portanto, qualquer subconjunto de G que contenha a identidade, seja fechado sob a mesma operação que define G , que seja associativa e contenha os inversos dos seus elementos é um subgrupo G .

Para identificar os subgrupos de D_{3d} podemos utilizar:

1. **Teorema de Lagrange:** os subgrupos devem ter ordem divisora de 12, ou seja: 1, 2, 3, 4, 6, 12, embora como verificado não existe nenhum subconjunto de 4 elementos do grupo D_{3d} que seja fechado sob composição e contenha a identidade.
2. **Elementos geradores similares:** concentração em rotações, reflexões, inversão ou improvisas.
3. **Classes de conjugação:** se um elemento está num subconjunto e um elemento conjugado pertence ao grupo, então todo seu conjunto conjugado também precisa estar.
4. **Fechamento:** verificar tabela de multiplicação para confirmar fechamento, presença da identidade e inversos.

Elemento	E	C_3	C_3^2	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6	S_6^5	σ_{d1}	σ_{d2}	σ_{d3}
Inverso	E	C_3^2	C_3	$C_2^{(a)}$	$C_2^{(b)}$	$C_2^{(c)}$	i	S_6^5	S_6	σ_{d1}	σ_{d2}	σ_{d3}

Essas técnicas permitiram encontrar todos os subgrupos próprios do grupo D_{3d} , excetuando-se os subgrupos triviais $\{E\}$ e o grupo total D_{3d} , que são considerados subgrupos impróprios e são deduzidos trivialmente.

Ordem 2 (cíclicos e abelianos):

- $\langle C_2^{(a)} \rangle = \{E, C_2^{(a)}\}$
- $\langle C_2^{(b)} \rangle = \{E, C_2^{(b)}\}$
- $\langle C_2^{(c)} \rangle = \{E, C_2^{(c)}\}$
- $\langle i \rangle = \{E, i\}$
- $\langle \sigma_{d1} \rangle = \{E, \sigma_{d1}\}$
- $\langle \sigma_{d2} \rangle = \{E, \sigma_{d2}\}$
- $\langle \sigma_{d3} \rangle = \{E, \sigma_{d3}\}$

Ordem 3 (cíclicos e abelianos):

- $\langle C_3 \rangle = \{E, C_3, C_3^2\}$
- $\langle S_6 \rangle = \{E, S_6, S_6^5\}$

Ordem 6 (não abelianos):

- $D_3 = \{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}$
- $C_{3v} = \{E, C_3, C_3^2, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$

Ordem	Subgrupos
1	$\{E\}$
2	$\{E, C_2^{(a)}\}, \{E, C_2^{(b)}\}, \{E, C_2^{(c)}\}, \{E, i\}, \{E, \sigma_{d1}\}, \{E, \sigma_{d2}\}, \{E, \sigma_{d3}\}$
3	$\{E, C_3, C_3^2\}, \{E, S_6, S_6^5\}$
6	$\{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}\}, \{E, C_3, C_3^2, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$
12	$\{E, C_3, C_3^2, C_2^{(a)}, C_2^{(b)}, C_2^{(c)}, i, S_6, S_6^5, \sigma_{d1}, \sigma_{d2}, \sigma_{d3}\}$

Todas as ordens aparecem como divisores de 12, mostrando a conformidade ao teorema de Lagrange. A presença de rotações, reflexões e inversão agrupadas em cada subgrupo respeita fechamento e classes de conjugação.

Questão 3. Considere as simetrias da molécula do hexafluoreto de enxofre SF_6 , pertencente ao grupo O_h . Liste os 48 elementos de simetria desta molécula.

Resposta para o Ítem 3 - Lista de simetrias

Considerando cada elemento de fluor alinhado aos eixos cartesianos x, y e z, podemos numerar os fluors que compõe a molécula da seguinte forma: 1: z, 2: x, 3: y, 4: -z, 5: -x, 6: -y. As operações de simetria são as que seguem:

- A simetria identidade E :

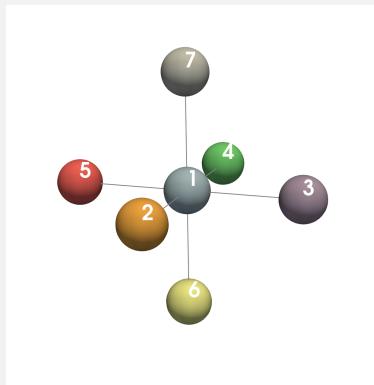


Figure 29: Simetria E

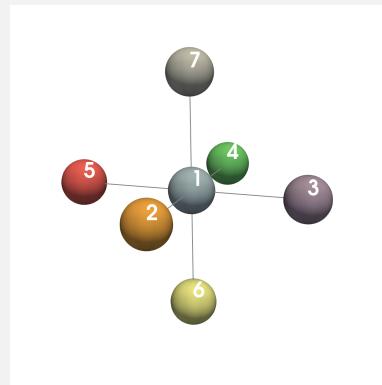


Figure 30: Operação E aplicada

$$\text{Identidade: } E(1, 2, 3, 4, 5, 6, 7) = (1, 2, 3, 4, 5, 6, 7)$$

- Duas rotações próprias C_4 não triviais em torno de cada eixos x, y e z:

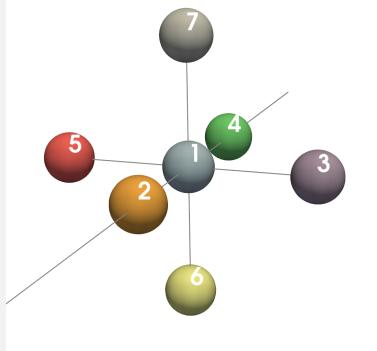


Figure 31: Eixo de simetria C_{4x}

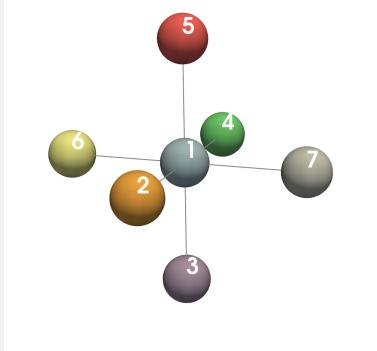


Figure 32: Operação C_{4x}^+ aplicada

$$\text{Rotações } C_4: \quad C_{4x}^+(1, 2, 3, 4, 5, 6, 7) = (1, 2, 7, 4, 6, 3, 5)$$

$$C_{4x}^-(1, 2, 3, 4, 5, 6, 7) = (1, 2, 6, 4, 7, 5, 3)$$

$$C_{4y}^+(1, 2, 3, 4, 5, 6, 7) = (1, 6, 3, 7, 5, 4, 2)$$

$$C_{4y}^-(1, 2, 3, 4, 5, 6, 7) = (1, 7, 3, 6, 5, 2, 4)$$

$$C_{4z}^+(1, 2, 3, 4, 5, 6, 7) = (1, 3, 4, 5, 2, 6, 7)$$

$$C_{4z}^-(1, 2, 3, 4, 5, 6, 7) = (1, 5, 2, 3, 4, 6, 7)$$

- Duas rotações próprias C_3 não triviais em torno de cada um dos 4 eixos diagonais que ligam vértices opostos do cubo em que a molécula se inscreve em configuração de face centrada:

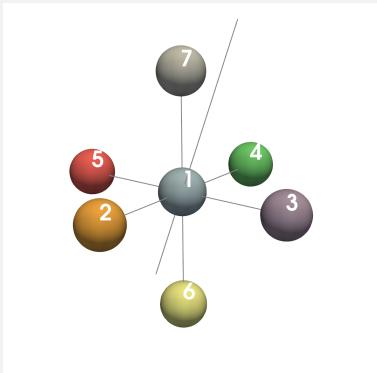


Figure 33: Eixo de simetria $C_{3<111>}$

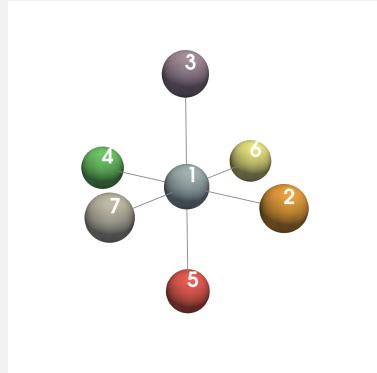


Figure 34: Operação $C_{3<(111)>}^+$ aplicada

$$\text{Rotações } C_3 : \quad C_{3(111)}^+(1, 2, 3, 4, 5, 6, 7) = (1, 7, 2, 6, 4, 5, 3)$$

$$C_{3(111)}^-(1, 2, 3, 4, 5, 6, 7) = (1, 3, 7, 5, 6, 4, 2)$$

$$C_{3(1\bar{1}\bar{1})}^+(1, 2, 3, 4, 5, 6, 7) = (1, 6, 4, 7, 2, 5, 3)$$

$$C_{3(1\bar{1}\bar{1})}^-(1, 2, 3, 4, 5, 6, 7) = (1, 5, 7, 3, 6, 2, 4)$$

$$C_{3(\bar{1}1\bar{1})}^+(1, 2, 3, 4, 5, 6, 7) = (1, 7, 4, 6, 2, 3, 5)$$

$$C_{3(\bar{1}1\bar{1})}^-(1, 2, 3, 4, 5, 6, 7) = (1, 5, 6, 3, 7, 4, 2)$$

$$C_{3(\bar{1}\bar{1}1)}^+(1, 2, 3, 4, 5, 6, 7) = (1, 6, 2, 7, 4, 3, 5)$$

$$C_{3(\bar{1}\bar{1}1)}^-(1, 2, 3, 4, 5, 6, 7) = (1, 3, 6, 5, 7, 2, 4)$$

- Uma rotação própria (C_2) não trivial por eixo x, y e z:

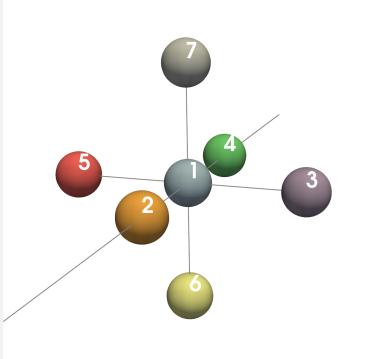


Figure 35: Eixo de simetria C_{2z}

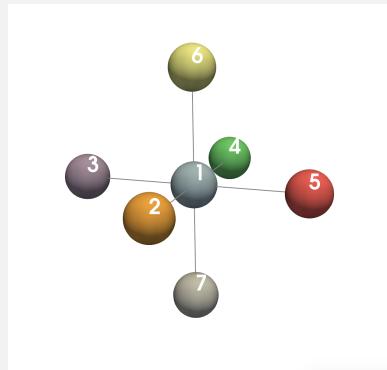


Figure 36: Operação C_{2z} aplicada

Rotações C_2 : $C_{2z}^\pm(1, 2, 3, 4, 5, 6, 7) = (4, 5, 6, 1, 2, 3)$

$$C_{2x}^\pm(1, 2, 3, 4, 5, 6, 7) = (4, 5, 6, 1, 2, 3)$$

$$C_{2y}^\pm(1, 2, 3, 4, 5, 6, 7) = (4, 2, 6, 1, 5, 3)$$

- Uma rotação própria C_2 não trivial por cada eixo formado pelo ponto médio de arestas opostas no cubo inscrito pela molécula na configuração de face centrada:

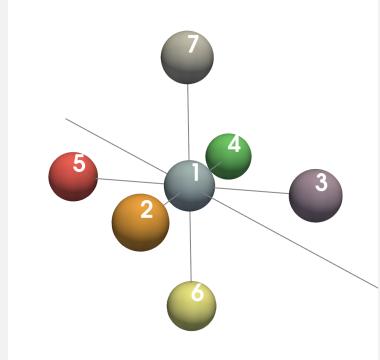


Figure 37: Eixo de simetria $C_{2<110>}$

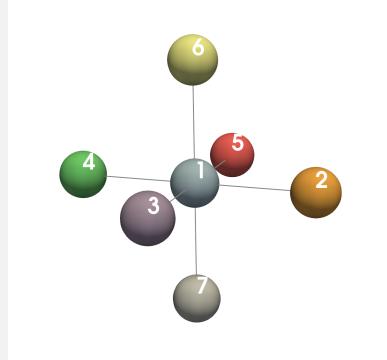


Figure 38: Operação $C_{2<110>}$ aplicada

Rotações C_2 : $C_{2<110>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 3, 2, 5, 4, 7, 6)$

$$C_{2<1\bar{1}0>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 5, 4, 3, 2, 7, 6)$$

$$C_{2<101>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 4, 7, 2, 6, 5, 3)$$

$$C_{2<10\bar{1}>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 4, 6, 2, 7, 3, 5)$$

$$C_{2<011>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 7, 5, 6, 3, 4, 2)$$

$$C_{2<01\bar{1}>}^\pm(1, 2, 3, 4, 5, 6, 7) = (1, 6, 5, 7, 3, 2, 4)$$

- Uma reflexão σ_h horizontal em cada plano xy, zy e xz:

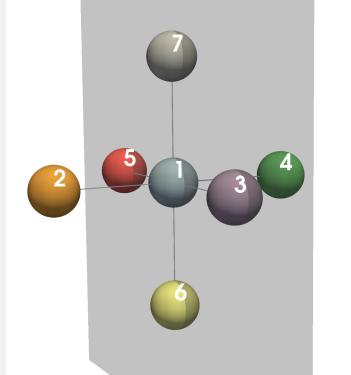


Figure 39: Plano de reflexão σ_{yz}

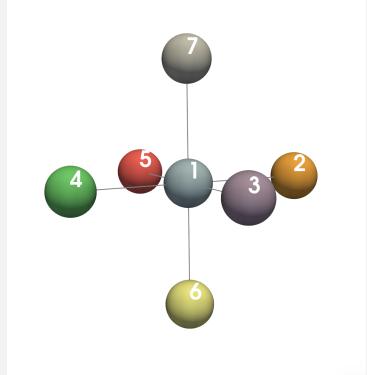


Figure 40: Operação σ_{yz} aplicada

Reflexão σ_h : $\sigma_{yz}(1, 2, 3, 4, 5, 6, 7) = (1, 4, 3, 2, 5, 6, 7)$

$$\sigma_{xy}(1, 2, 3, 4, 5, 6, 7) = (1, 2, 3, 4, 5, 7, 6)$$

$$\sigma_{xz}(1, 2, 3, 4, 5, 6, 7) = (1, 2, 5, 4, 3, 6, 7)$$

- Uma reflexão para cada um dos planos de simetria diagonais σ_d (diagonais do cubo inscrito pela molécula na configuração de face centrada)

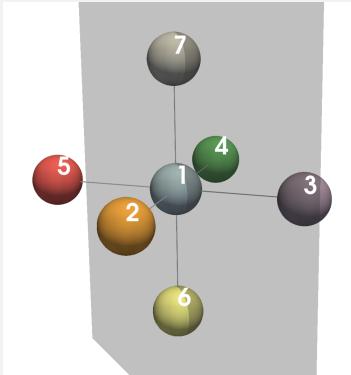


Figure 41: Plano de reflexão σ_d^{xz}

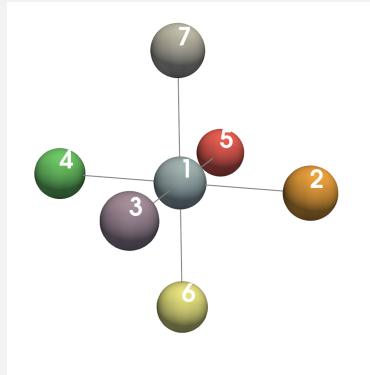


Figure 42: Operação σ_d^{xz} aplicada

Reflexões σ_d : $\sigma_d^{xz}(1, 2, 3, 4, 5, 6, 7) = (1, 3, 2, 5, 4, 6, 7)$

$$\sigma_d^{xy}(1, 2, 3, 4, 5, 6, 7) = (1, 5, 4, 3, 2, 6, 7)$$

$$\sigma_d^{yz}(1, 2, 3, 4, 5, 6, 7) = (1, 2, 6, 4, 7, 3, 5)$$

$$\sigma_d^{x\bar{y}}(1, 2, 3, 4, 5, 6, 7) = (1, 2, 7, 4, 6, 5, 3)$$

$$\sigma_d^{x\bar{z}}(1, 2, 3, 4, 5, 6, 7) = (1, 6, 3, 7, 5, 2, 4)$$

$$\sigma_d^{y\bar{z}}(1, 2, 3, 4, 5, 6, 7) = (1, 7, 3, 6, 5, 4, 2)$$

- Um ponto de inversão i :

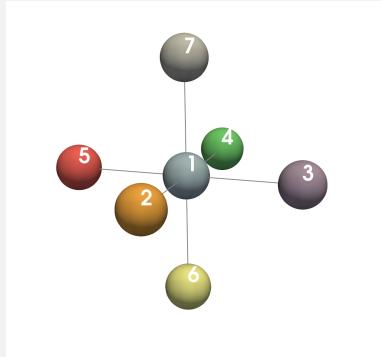


Figure 43: Ponto de inversão i na origem

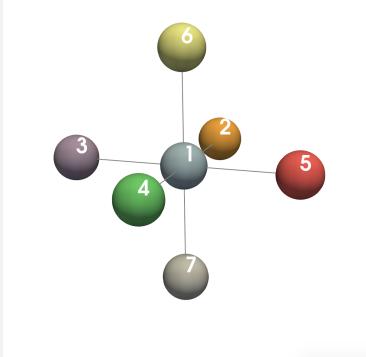


Figure 44: Operação i aplicada

Inversão: $i(1, 2, 3, 4, 5, 6, 7) = (1, 4, 5, 2, 3, 7, 6)$

- Duas rotações impróprias S_4 (rotação 90° e 270° [-90°] seguida de reflexão no plano perpendicular a este eixo) por cada eixo C_4

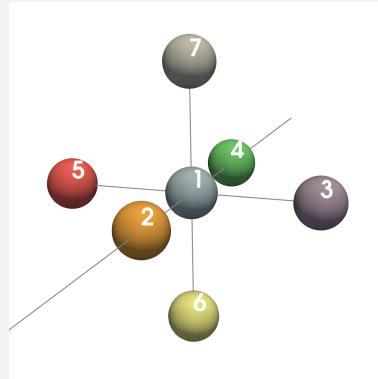


Figure 45: Eixo de rotação $C4$

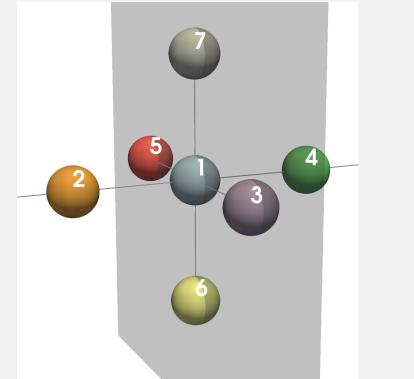


Figure 46: Plano de reflexão σ_{yz}

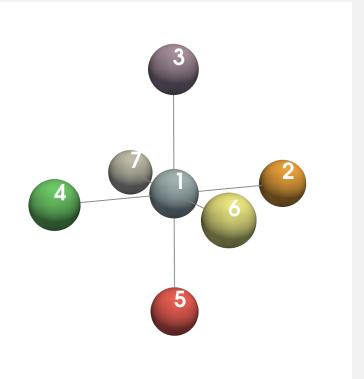


Figure 47: Operação S_4^x aplicada

Rotações impróprias S_4 : $S_4^{x+}(1, 2, 3, 4, 5, 6, 7) = (1, 4, 6, 2, 7, 5, 3)$

$$S_4^{x-}(1, 2, 3, 4, 5, 6, 7) = (1, 4, 7, 2, 6, 3, 5)$$

$$S_4^{y+}(1, 2, 3, 4, 5, 6, 7) = (1, 7, 5, 6, 3, 2, 4)$$

$$S_4^{y-}(1, 2, 3, 4, 5, 6, 7) = (1, 6, 5, 7, 3, 4, 2)$$

$$S_4^{z+}(1, 2, 3, 4, 5, 6, 7) = (1, 5, 2, 3, 4, 7, 6)$$

$$S_4^{z-}(1, 2, 3, 4, 5, 6, 7) = (1, 3, 4, 5, 2, 7, 6)$$

- Duas rotações impróprias S_6 (rotação 60° e 300° [-60°] seguida de reflexão no plano perpendicular a este eixo) para cada eixo $C_{6<110>}$ e equivalentes simétricos:

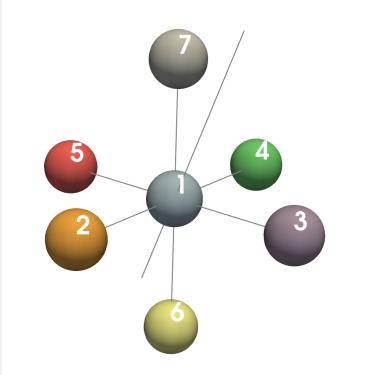


Figure 48: Eixo de rotação $C_{6<111>}$

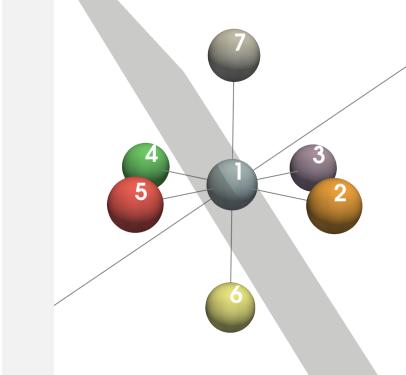


Figure 49: Plano de inversão σ_d

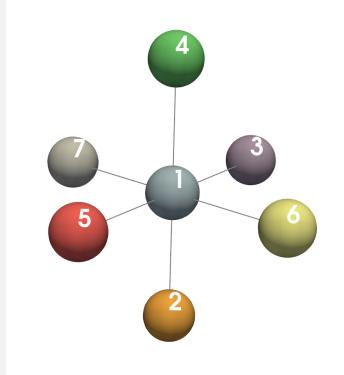


Figure 50: Operação $S_6^{(111)}$ aplicada

Rotações Impróprias S_6 : $S_6^{(111)}+(1, 2, 3, 4, 5, 6, 7) = (1, 6, 4, 7, 2, 3, 5)$

$$S_6^{(111)}-(1, 2, 3, 4, 5, 6, 7) = (1, 5, 6, 3, 7, 2, 4)$$

$$S_6^{(1\bar{1}1)}+(1, 2, 3, 4, 5, 6, 7) = (1, 3, 6, 5, 7, 4, 2)$$

$$S_6^{(1\bar{1}1)}-(1, 2, 3, 4, 5, 6, 7) = (1, 7, 2, 6, 4, 3, 5)$$

$$S_6^{(\bar{1}11)}+(1, 2, 3, 4, 5, 6, 7) = (1, 3, 7, 5, 6, 2, 4)$$

$$S_6^{(\bar{1}11)}-(1, 2, 3, 4, 5, 6, 7) = (1, 6, 2, 7, 4, 5, 3)$$

$$S_6^{(\bar{1}\bar{1}1)}+(1, 2, 3, 4, 5, 6, 7) = (1, 5, 7, 3, 6, 4, 2)$$

$$S_6^{(\bar{1}\bar{1}1)}-(1, 2, 3, 4, 5, 6, 7) = (1, 7, 4, 6, 2, 5, 3)$$

[1] Todas as simetrias e imagens apresentadas neste trabalho foram geradas utilizando o programa desenvolvido no contexto desta disciplina. O código-fonte está disponível publicamente no repositório GitHub: <https://github.com/naavilam/simetria-molecular>. Além disso, uma versão online da ferramenta pode ser acessada em: <https://naavilam.github.io/simetria-molecular/> ou pelo qrcode, permitindo a reprodução de todos os cálculos apresentados.

