Mobile Netw Appl (2009) 14:309–321
DOI 10.1007/s11036-008-0150-5

Minimal Energy Path Planning for Wireless Robots

Chia Ching Ooi · Christian Schindelhauer

Published online: 24 January 2009

Abstract In this paper, the problem of optimizing energy for communication and motion is investigated. We consider a single mobile robot with continuous high bandwidth wireless communication, e.g. caused by a multimedia application like video surveillance. This robot is connected to radio base station(s), and moves with constant speed from a given starting point on the plane to a target point. The task is to find the best path such that the energy consumption for mobility and the communication is optimized. This is motivated by the fact that the energy consumption of radio devices increases polynomially (at least to the power of two) with the transmission distance. We introduce efficient approximation algorithms to find the optimal path given the starting point, the target point and the position of the radio stations. We exemplify the influence of the communication cost by a starting scenario with one radio station. We study the performance of the proposed algorithm in simulation, compare it with the scenario without applying our approach, and present the results.

Keywords networked robots · optimal energy path planning · wireless communications cost · mobility cost

C. C. Ooi (✉) · C. Schindelhauer
Department of Computer Science,
Georges-Köhler-Allee 51,
79110 Freiburg, Germany
e-mail: ooi@informatik.uni-freiburg.de

C. Schindelhauer
e-mail: schindel@informatik.uni-freiburg.de

1 Introduction

Over the past decade, research community in the communications has been active in studying the energy efficiency of wireless communication protocols. On the other hand, researchers in robotics focus on the energy reduction for the motion planning of mobile robots. These two researches are conducted separately, and the study of networked robotics [7, 11, 18] is comparably less.

Similar to other mobile computing systems, the energy resource of most mobile robots is limited. Mobile robots have to accomplish their assigned tasks before deadlines by using the limited energy resources carried by them. The energy are meant for a number of operations: mobility, communications, performing the assigned task, computation and sensing the environment. Among them, the motion and wireless communications are two major consumers of the robot energy, apart from the processing power. The overall lifetime of the robot can be maximized by efficiently distributing its energy resource.

The study of mobile ad hoc networks based on the mobile *Khepera* robots [10] gave us the practical insights that in certain environments, the energy consumption of communication and mobility are the two highest parameters. While the mobility cost grows linearly, the energy consumption of radio communication grows at least quadratically with the distance of two communicating robots. Therefore, at a certain range, it is advantageous to move a robot towards its communication partner. In order to achieve the goals more effectively, there are a number of challenges to judge the tradeoffs of the energy usage between robot movement and communications.

In addition, the standard theoretical model for energy consumption in mobile ad hoc networks considers the distances of communication partners and the amount of data transmission, namely the flow cost model of [17]. The quadratic increase of this model is motivated by the path loss in radio communications, which can be approximated for a fixed scenario by $O(d^{\alpha})$, where d is the distance and α is the path loss exponent. This approximation has been established by extensive tests in several real environments leading to different path loss exponents for different environments. In [19] and [13], an additive constant is added to this term to take into account the signal processing before sending and after receiving messages.

As stated before, considering the combined energy consumption of communication and mobility of robots is new, though there is an extensive line of research for each model. One might think that mobility has only a negative impact on the behavior of wireless networks. But, recent work has shown that this is not the case. Mobility improves the coverage of wireless sensor networks [14], and helps security in ad hoc networks [6]. Furthermore, it can help with network congestion as shown in [9]. This approach overcomes the natural lower bound for throughput of $\Omega(\sqrt{n})$ by instrumenting the random movement of nodes. They design a protocol where mobile nodes relay packets and literally transport them towards the destination node. Adopting the advantages of mobility into robot communications is a new challenge that needs to be studied thoroughly.

One might assume that communication cost is always much lower than motion cost. In [12], the author states that the energy required for robot movements is generally much bigger than for communications. However, it is not true when the amount of data to be transmitted is very high. The communication cost should not be neglected. An example of such situation is when the overall data transmission comprises mainly the multimedia data in such applications as video streaming and surveillance. The volume of the data transmission grows as the video quality required by an application is increased. Wireless multimedia sensor network (WMSN) [4] and the mobile robot video surveillance system [23] are some of the existing applications.

Moreover, a case study [16] has shown the power breakdown of a robot, in which the motion power is not the component consuming the highest percentage of total energy. Instead, the embedded computer that is capable of wireless communications accounts for up to 65.3% of the total energy consumption.

Our work is motivated by the applications in which a team of mobile robots has to exchange a high volume of data over wireless medium among themselves, or to base stations during the exploration. Thus, we concentrate on optimizing the energy consumed by both mobility and communications.

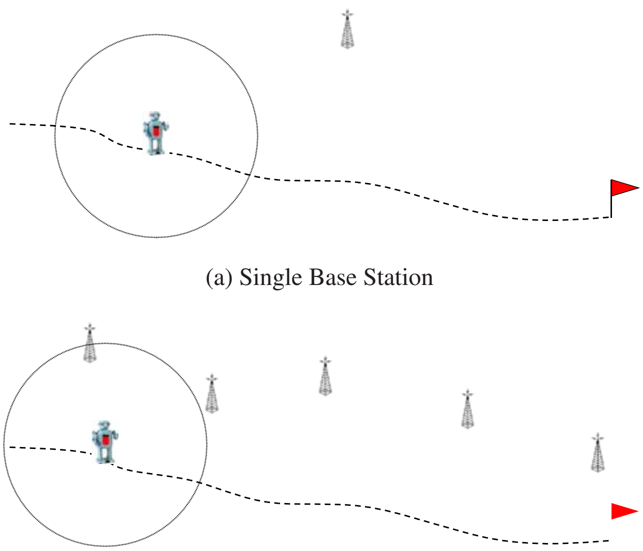
The remainder of the paper is organized as follows. In Section 2, we present the system and the energy models used in our study and define the problem of computing an optimal energy path for a mobile robot. Based on these foundations, we propose approximation algorithms in Section 3. The details of the simulation setup and results are then presented in Section 4, and lastly, we make concluding remarks and discuss future works.

2 Preliminaries

We consider a team of exploration robots and base stations that form a wireless mobile network. Each mobile robot is battery-powered and has limited lifetime. The robots and the base stations can communicate with each other through wireless transmission medium. Every robot is assigned with different task(s): searching, exploring, sensing, foraging, marking, working on target, and so on. Among them, some robots are equipped with video cameras to capture the video or pictures of the environment while they are exploring, and transmit the captured data back to the base station by either single hop or multiple hop communications. Either the base station or the mobile robot can serve as the intermediate node for communications.

Referring to Fig. 1, we consider two general cases, in which a mobile robot communicates with single base station and multiple base stations respectively. At any instant, a mobile robot is assumed to know the Euclidean distance to reach its next target destination and the Euclidean distance to the base station that serves as the destination node of video transmission. In addition, to communicate with multiple base stations, the mobile robot is assumed capable of selecting the base station to interact with during its transition between adjacent cells. For instance, this can be determined based on the channel quality or the signal strength received from base stations [24]. Smooth transitioning between cells are assumed. Therefore, the robot knows the position it has to switch to a new base station to enable seamless video transmission.

The robot can move straight on the Euclidean path to reach its next target destination. We call this movement the *Straight-Line* movement in the paper. However, the Euclidean path might not be the optimal energy path since the robot consumes the energy not only for the movement but also for the wireless



(a) Single Base Station

(b) Multiple Base Stations

Fig. 1 Single mobile robot communicates with base station(s)

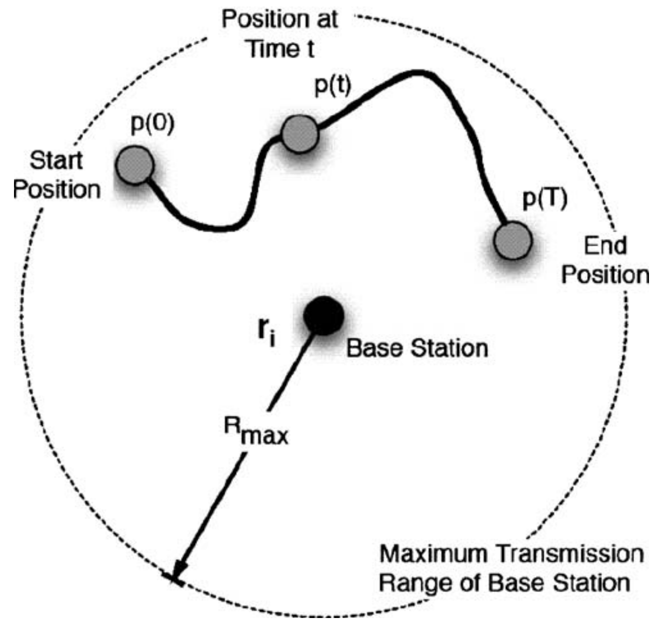


Fig. 2 Exploration area within communication range

communications. Therefore, a minimal energy path has to be computed. We call the resulting robot movement on this path the *Smart* movement.

In the following sub-sections, we introduce the system and the energy models, and define the problem of computing an optimal energy path.

2.1 System model

We consider an exploration area of a mobile robot on a two dimensional Euclidean space as shown in Fig. 2. A base station is located at $r_i = (r_{i,x}, r_{i,y})$ on this exploration area. We indicate the robot position as continuous function of time $p : [0, T] \mapsto \mathbb{R} \times \mathbb{R}$ such that $p(t)$ gives the position of the mobile robot at time t in Cartesian Coordinates $p(t) = (p_x(t), p_y(t))$. At the beginning we have $p(0) = s$ where s is the start position of the node and at the end, we have $p(T) = g$, where g is the target position. When we compute the path, we approximate the path by n path segments of constant speed $P = ((t_0, x_0, y_0), (t_1, x_1, y_1), \ldots, (t_n, x_n, y_n))$ for $t_0 < t_1 < \cdots < t_n$ where the corresponding path function is given by

$$p(t) = \left(x_i + \frac{(t - t_i)(x_{i+1} - x_i)}{t_{i+1} - t_i}, y_i + \frac{(t - t_i)(y_{i+1} - y_i)}{t_{i+1} - t_i} \right),$$

for $t \in [t_i, t_{i+1})$. Let the maximum radio range of both nodes be $R_{\max}$. We limit the movement of the mobile robot to be within $R_{\max}$ so that the two nodes can communicate with each other, i.e. for all $t \in [0, T] : ||p(t) - r_i||_2 \leq R_{\max}$, where $||u - v||_2 = \sqrt{(u_x - v_x)^2 + (u_y - v_y)^2}$ denotes the Euclidean distance between robot and base station. The initial position and the target position of a mobile robot must be within $R_{\max}$. For the single base station communication, if the base station is located at origin $(0, 0)$, then $t \in [0, T] : ||p(t)||_2 \leq R_{\max}$.

On the other hand, in the case of multiple base stations shown in Fig. 1b, a set of base stations are defined as $R = \{r_1, ..., r_m\}$, where m represents the number of base station interacting with mobile robot moving on path P, and $r_i = (r_{i,x}, r_{i,y})$ represents the position of base station. Let the base station in use at t be r_i, the transmission range between robot and base station for all $t \in [0, T] : ||p(t) - r_i||_2 \leq R_{\max}$.

Two models are introduced in this paper. Some applications require the mobile robot to continuously transmit data at a fixed bit rate, e.g. the live video feed of a camera mounted on the mobile robot . This model is called the *constant bit-rate communications model*. Other applications require data transmission only at critical positions, e.g. a mobile surveillance camera in a museum which periodically transmits pictures from certain view points.

We call this model the *position-critical communications model* and illustrate it in Fig. 3. Another example application that fits both models is a robot-assisted wireless (multimedia) sensor network, in which a robot is assigned to collect the data gathered by the sensor nodes in its exploration area. The robot may or may not be required to transmit the data collected to a base station on a real time basis.

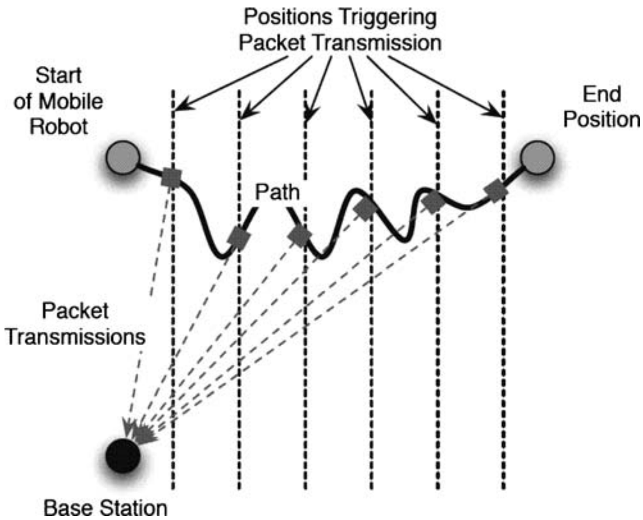


Fig. 3 Position-critical communications model

2.2 Energy models for robots

In our study, the energy model reflects two facets: energy for communication and energy for mobility. For the communications, we base the energy model on that defined in [21] and [5]. The energy required for successful wireless data transmission is affected by the distance between two communication nodes and other factors like interferences, multi-path fading, and other noises, in the transmission medium.

The energy consumed to transmit ℓ bits of data over the distance d_c measured in *meter* is defined as:

$$E_{tx}(\ell, d_c) = \ell \cdot (d_c^{\alpha} \cdot e_{tx} + e_{cct}),$$

where e_{tx} is the energy required by the power amplifier of transceiver to transmit one bit data over the distance of 1 m, and e_{cct} is the energy consumed in the electronic circuits of the transceiver to transmit or receive one bit, measured in the unit of Joule/bit. Depending on the transceiver sensitivity, the value of e_{tx} ranges from some pico- to nano-Joule per bit per meter$^{\alpha}$. α is called the path loss exponent of the transmission medium that ranges from 2 to 6, where $\alpha \in [2, 6]$ in our model.

Table 1 shows the path loss exponent corresponding to different types of environment in which the wireless communication takes place. In [5], it states that α= 2 and α=3,4 is used for short and long distance or multi-path model respectively.

On the other hand, the energy consumption for receiving ℓ bits of data is defined as:

$$E_{rc}(\ell) = \ell \cdot e_{cct}.$$

Table 1 Path loss exponents for various radio environments

Environment	Path loss exponent, α
Free space	2
Urban	2.7 to 3.5
Shadowed urban	3 to 5
In-building	4 to 6

The energy consumption for receiving is independent of the distance between communicating nodes.

For the mobility, we base the energy model used in our study on that defined in [21] and [16]. The mobility energy depends on the mass of the robot, the friction to the surface (air or ground), gravity and acceleration, and the distance travelled. For simplicity, we adopt the mobility energy model that is proportional to distance used in [21]. This model is reasonable for the wheeled robots [8]. It is defined as:

$$E_m = m \cdot d_m,$$

where the movement parameter, m, measured in Joule per meter (J/m), is a constant based on the aforementioned factors, and d_m is the distance traversed by the robot in *meter*.

Constant bit-rate communications model The bit-rate B describes the number of bits that are sent per second. In this model, this rate is constant over the time period $[0, T]$. The total number of bits N transmitted from the mobile robot to the base station is given by

$$N = B \cdot T \,.$$

Based on the piecewise differentiable path function $p(t)$ defined in Section 2.1, using the arc length function and the chain rule, the length of the path $p(t)$ can be described by

$$D = \int_{t=0}^{T} \sqrt{p_x'^2(t) + p_y'^2(t)}\, dt \,,$$

where $p_x'(t) = \frac{dp_x}{dt}$ and $p_y'(t) = \frac{dp_y}{dt}$.

We are interested in the energy consumption of the mobile robot $E_{cbr}(p)$ consisting of transmission energy and mobility energy:

$$E_{cbr}(p) := E_{tx}(p) + E_m(p) \,,$$

where $E_m(p) = m \cdot D$. For the transmission energy, we have to take into account that the mobile robot communicates while moving. This is reflected by the following term

$$E_{tx}(p) = \int_{t=0}^{T} B \cdot (||p(t) - r_i||_2)^{\alpha} \cdot e_{tx}\, dt \,.$$

Note that if the robot does not move, this term reduces to $B \cdot T \cdot d_{\mathrm{c}}^{\alpha} \cdot e_{\mathrm{tx}}$.

If the mobile robot is bounded by a maximum velocity $v_{\max}$, the following lemma shows that the best strategy is to move at maximum speed according to this model.

Lemma 1 *If $B > 0$ and $||p(t)||_2 > 0$ for $t \in [0, T]$, considering the mobility and communications costs, we have $||p'(t)||_2 = v_{\max}$ for all $t \in [0, T]$ for every minimal energy path in the constant bit-rate communications model.*

Proof On the contrary, assume that $||p'(t)||_2 < v_{\max}$ for some interval $t \in [t_0, t_1]$ with $t_0 < t_1$. Let δ be the total time saved when the robot moves at $v_{\max}$ instead of $||p'(t)||_2$. We construct a new path $q(f(t)) = p(t)$ using a continuous monotone increasing function $f : [0, T] \to [0, T - \delta]$, where $\delta = (t_1 - t_0) - \frac{1}{v_{\max}} \int_{t=t_0}^{t_1} ||p'(t)||_2 \, \mathrm{d}t$. Define $f(t) := t$ for $t < t_0$, $f(t) := t - \delta$ for $t \in [t_1, T]$, and $f(t) := t_0 + \frac{1}{v_{\max}} \int_{t=t_0}^{t_1} ||p'(t)||_2 \, \mathrm{d}t$ for $t \in [t_0, t_1]$.

Observe that $||p'(t)||_2 \leq v_{\max}$ for all $t \in [0, T - \delta]$, $q(0) = p(0)$, and $q(T - \delta) = p(T)$. Clearly, the mobile energy remains the same on the path q since only the speed has been changed. But, the energy consumption for communication is decreased since

$$\int_{t=0}^{T} (||p(t) - r_i||_2)^{\alpha} \, \mathrm{d}t > \int_{t=0}^{T-\delta} (||q(t) - r_i||_2)^{\alpha} \, \mathrm{d}t \, .$$

So the path p was not optimal which proves the claim. □

Position-critical communications model In the above model, the number of transmitted bits depends on the time the robots need to reach the destination. In some application, the number of transmissions is independent of the duration of the mission, only a number of certain tasks needs to be performed, e.g. taking a picture of a certain area with a camera, measuring environmental data, etc. For these tasks, the robot needs to move to certain areas, and immediately after performing this task, the robot communicates the data from this area to the base station, e.g. see Fig. 3 where the mobile robot communicates a message after crossing each line.

Formally, we define a sequence of tasks (or quests) $Q_1, \ldots, Q_n$ for the mobile robot, where each quest $Q_i = (A_i, N_i)$ consists of a region A_i from which the robot may choose a point $p_i \in A_i$ and a number of messages N_i the robot needs to transmit after performing the task. A path of a robot solves the task $Q = ((A_1, N_1), \ldots, (A_n, N_n))$ at points $(x_1, \ldots, x_n)$ if $\exists t_1 < t_2 < t_3 < \cdots < t_n$ with $p(t_i) = x_i$ and $x_i \in A_i$.

The energy consumption of the *position-critical communications model* for a robot with path p and solution points $x = (x_1, \ldots, x_n)$ is then defined as:

$$E_{\mathrm{cbr}}(Q, p, x) := E_{\mathrm{tx}}(Q, p, x) + E_m(p) \, ,$$

where $E_m(p) = m \cdot D$ and D is the path length of p.

$$E_{\mathrm{tx}}(Q, p, x) = \sum_{i=1}^{n} N_i \cdot (||x_i - r_i||_2)^{\alpha} \cdot e_{\mathrm{tx}} \, .$$

This definition can be simplified using the following lemma.

Lemma 2 *For every sequence of tasks Q and matching solution set x, a path is optimal if and only if the mobile robot moves from x_i to x_{i+1} on a straight line.*

Proof Note that for all such solutions the costs for communication energy is the same, since it depends only on the positions of x_i. Clearly the mobility energy is minimized if the mobile robot uses the straight line. □

An immediate implication of this lemma is, that the speed of the robot has no influence to the energy consumption of the *position-critical model*. The solution set $x = (x_1, \ldots, x_n)$ gives all the necessary information for finding the optimal route and determining the energy consumption. Therefore, we refer to the position-critical energy simply by $E_{\mathrm{cbr}}(Q, x)$.

2.3 Optimal energy path problems

In the optimal energy path problem, the initial and target position of the mobile robot are given. The mobile robot communicates with the base station during its movement. The goal is to find the optimal energy path to reach the given target position.

Definition 1 The path optimization problem for position-critical communications model.

Given the position of base station, r_i and a sequence of tasks $Q = ((s, 0), (A_1, N_1), \ldots, (A_n, N_n), (g, 0))$, the mobile robot has to find a discrete path $(s, x_1, \ldots, x_n, g)$ that solves the task, i.e. $x_i \in A_i$ for all $i \in \{1, \ldots, n\}$, and minimizes the position-critical energy $E_{\mathrm{cbr}}(Q, x)$.

Definition 2 The path optimization problem for constant-bit-rate communications model.

Given the position of base station, r_i, a start position s, a target g, a maximal speed $v_{\max}$ and a bit-rate B, find a time T and a path $p : [0, T] \to \mathbb{R}^2$ such that $p(0) = s$,

$p(T) = g$, $||p'(t)||_2 \leq v_{\max}$ for all $t \in [0, T]$ and the bitrate energy $E_{\mathrm{cbr}}(p)$ is minimized.

In the following section, we describe approximation algorithms for the energy-optimal paths used in both models.

3 Algorithms

For the *position-critical model*, we have to find a solution set $x_1, x_2, \ldots, x_n$ such that $x_i \in A_i$, where A_1 and A_n are single points describing the start node s and the end point g. As a first approach, we choose a finite candidate set $V_{i,\epsilon} = y_{i,1}, y_{i,2}, \ldots \in A_i$ such that for all $u \in A_i$ inside the transmission range of the base station, there exists a candidate $y_{i,j}$ within distance ϵ. This can be done by using a two-dimensional grid positions with distances of at most $\frac{\epsilon}{\sqrt{2}}$. If the task areas are lines, then this candidate can be placed with distance ϵ.

Define the edge set $E_\epsilon = \bigcup_{i\in\{1,\ldots,n-1\}} V_{i,\epsilon} \times V_{i+1,\epsilon}$ and the node set $V_\epsilon = \bigcup_{i\in\{1,\ldots,n\}} V_{i,\epsilon}$ constituting the graph $G_\epsilon = (V_\epsilon, E_\epsilon)$.

For the edges, we define the following weight function

$$w(y_{i,j}, y_{i+1,k}) = N_i \cdot (||y_{i,j} - r_i||_2)^\alpha \cdot e_{\mathrm{tx}} + m \cdot ||y_{i,j}, y_{i+1,k}||_2$$

for $i < n-1$ and all j, k. Further, we define the following function:

$$\begin{aligned} w(y_{n-1,j}, y_{n,k}) &= e_{\mathrm{tx}} \cdot (N_{n-1} \cdot (||y_{n-1,j} - r_i||_2)^\alpha + N_n \cdot (||y_{n,j}||_2)^\alpha) \\ &\quad + m \cdot ||y_{n-1,j}, y_{n,k}||_2 \end{aligned}$$

Note that every path p in G_ϵ from the start node is a valid solution of the *position-critical communications model*. The weight of $w(p)$ equals the energy consumption of a mobile robot on this path. Let $p_{\min}$ be the minimal energy consuming path in the original problem. By the definition of G_ϵ, there exists a path p in G_ϵ such that $||p_{\min,i} - p_i||_2 \leq \epsilon$. Therefore,

$$|E_m(p_{\min}) - E_m(p)| \leq m \cdot \epsilon \cdot (n-1) \,.$$

Furthermore, one can show that

$$|E_{\mathrm{tx}}(p_{\min}) - E_{\mathrm{tx}}(p)| \leq e_{\mathrm{tx}} \cdot \epsilon \cdot \alpha \cdot (R_{\max} + \epsilon)^{\alpha-1} \cdot \sum_{i=1}^{n} N_i \,,$$

where $R_{\max}$ is the maximum transmission distance of the base station. From this, the theorem below follows:

Theorem 1 *The minimal weighted path in G_ϵ with respect to the weight function w approximates the minimum position-critical energy by an additive term of $m \cdot \epsilon \cdot (n-1) + e_{\mathrm{tx}} \cdot \epsilon \cdot \alpha \cdot (R_{\max} + \epsilon)^{\alpha-1} \cdot \sum_{i=1}^{n} N_i$.*

PCM-Dijkstra-Refinement
- Carefully choose algorithm parameters $c, k > 1$
- $\epsilon' \leftarrow \frac{||s,g||_2}{c}$
- Construct $G_{\epsilon'}$
- Use Dijkstra's algorithm to compute optimal path $p_{\epsilon'}$ in $G_{\epsilon'}$
- **while** $\epsilon' > \epsilon$
  - $\epsilon' \leftarrow \epsilon'/c$
  - **Refine around** $p_{\epsilon'}$:
    - Construct graph $G_{\epsilon'}$
    - Erase all nodes in $V_{\epsilon'}$ which are not within $k \cdot \epsilon'$ distance to a node of p
    - Erase all edges adjacent to erased nodes
  - Use Dijkstra's algorithm to compute optimal path $p_{\epsilon'}$ in resulting graph $G_{\epsilon'}$
- **end of while**

return $p_{\epsilon'}$

Fig. 4 A refinement strategy for the *position-critical communications model*

So an approximation of the minimum energy path can be solved by using Dijkstra's shortest path algorithm. However, if the task areas A_i are regions, i.e. containing some small-sized disk, the number of nodes of G_ϵ grows proportional to $\Theta(\frac{1}{\epsilon^2})$ and the size of the edge set grows by $\Theta(\frac{1}{\epsilon^4})$, which is the decisive term of Dijkstra's algorithm.

With the heuristic refinement strategy of Fig. 4, the running time can be considerately improved. It is an open problem whether this PCM[1]-Dijkstra-Refinement algorithm always finds a path as good as the Dijkstra algorithm on the graph G_ϵ, while all simulation runs show no differences for our test scenario. The following theorem shows that this algorithm is very efficient.

Theorem 2 *The PCM-Dijkstra-Refinement algorithm has an asymptotic running time of $\mathcal{O}(n \cdot \log(n + \frac{1}{\epsilon}))$ for convex task areas aiming at an additive error bound of $\mathcal{O}(\epsilon)$.*

Proof The runtime of Dijkstra's algorithm is linear with respect to the number of edges. From the initial choice of $||s, g||_2$ it follows that the initial graph G_ϵ has at most $\mathcal{O}(c^2 n)$ nodes in V_ϵ and $\mathcal{O}(c^4 n)$ edges. The latter follows from the definition of E_ϵ. After each refinement at most $\mathcal{O}(k^2)$ are chosen around the nodes of the optimized path p' resulting in a total number of

[1] Position-critical Communications Model

at most $\mathcal{O}(k^4 n)$ edges in each round. The number of rounds is bounded by $\log_c \frac{||s,g||_2}{\epsilon''} = \frac{\log \frac{1}{\epsilon''} + \log ||s,g||_2}{\log c}$ where ϵ'' is the final choice for ϵ' in the algorithm such that the additive error term of Theorem 1 fulfills

$$m \cdot \epsilon \cdot (n-1) + e_{\text{tx}} \cdot \epsilon \cdot \alpha \cdot (R_{\max} + \epsilon)^{\alpha-1} \cdot \sum_{i=1}^{n} N_i \le \epsilon'' .$$

We concentrate on the asymptotic behavior of this algorithm and regard the amount of tasks and the original distance $||s, g||_2$ as non-zero constants. The constant $c \ge 2$ can be chosen arbitrary. For the constant k it must be ensured, that the search range is not reduced too fast. It can be shown that a choice of $k = \mathcal{O}(c^\alpha)$ ensures that the optimal path is still covered by the approximating path p. Hence, c and k are constant terms. Then, the runtime can be simplified to

$$\mathcal{O}\left(k^4 n \log \frac{n}{\epsilon} + c^4 n\right) = \mathcal{O}\left(n \log(n + \left(\frac{1}{\epsilon}\right)\right) . \qquad \square$$

Note that this runtime is linear if the additive error bound is chosen to be smaller than $\frac{1}{n}$. Furthermore, this proof is a straight-forward generalization of this proof for non-convex connected task areas leading to the same bounds.

For the *constant bit-rate energy* model, we use a similar approach. For $\epsilon_2 > \epsilon_1 > 0$, define the node set V_{ϵ_1} such that $s, g \in V_{\epsilon_1}$ and for all nodes u in the transmission range of the base station, there exists a node $v \in V_{\epsilon_1}$ with $||u, v||_2 \le \epsilon_1$. The edge set E_{ϵ_2} is defined by

$$E_{\epsilon_2} = \{(u, v) \mid u, v \in V_{\epsilon_1} : \ ||u, v||_2 \le \epsilon_2\}$$

resulting in the graph $G_{\epsilon_1,\epsilon_2} = (V_{\epsilon_1}, E_{\epsilon_2})$. We define the weights for the edges in E_{ϵ_2} as

$$w(u, v) = B \cdot \frac{||u, v||_2}{v_{\max}} \cdot (||u - r_i||_2)^\alpha \cdot e_{\text{tx}} + m \cdot ||u, v||_2 .$$

Theorem 3 *The minimal weighted path in $G_{\epsilon_1,\epsilon_2}$ with respect to the weight function w approximates the minimum bit-rate energy optimal path by a multiplicative factor of at most $1 + \mathcal{O}(\frac{\epsilon_1}{\epsilon_2} + \epsilon_2 \cdot (\frac{1}{||s||_2} + \frac{1}{||g||_2}))$.*

Proof Sketch Consider the energy optimal path $p_{\min}$ according to the bitrate model, there exists a path p in $G_{\epsilon_1,\epsilon_2}$ such that for the length $D_{\min}$ of $p_{\min}$ and the length D of p, it holds: $|D - D_{\min}| \le 2\frac{\epsilon_1}{\epsilon_2} D$.

We have shown in Lemma 1 that the mobile robot moves along $p_{\min}$ with maximum speed. To approximate the communication energy, we model the number of messages to be sent between u and v as $B \cdot \frac{||u,v||_2}{v_{\max}}$. As the sending location, we use the starting node that leads to an error of at most $\mathcal{O}(\epsilon_2 \, (||u - r_i||_2)^{\alpha-1} \cdot e_{\text{tx}})$. If a path segment is fairer than the start point s or end point g from the base station, the relative error per step is smaller than $\Theta\left(\epsilon_2 \cdot (\frac{1}{||s||_2} + \frac{1}{||g||_2})\right)$. Otherwise, the summed error margin is small compared to the errors occurring in the vicinity of s or g. This leads to the asymptotic bound of $\mathcal{O}(\epsilon_2 \cdot (\frac{1}{||s||_2} + \frac{1}{||g||_2}))$. $\square$

Using a good node placement, the graph $G_{\epsilon_1,\epsilon_2}$ has $\Theta\left(\frac{(R_{\max})^2}{(\epsilon_1)^2}\right)$ nodes and $\Theta\left(\frac{(R_{\max})^2(\epsilon_2)^2}{(\epsilon_1)^4}\right)$ edges. So, to achieve an approximation factor of $1 \pm \epsilon$, it is necessary to choose $\epsilon_2 = \epsilon$ and $\epsilon_1 = \epsilon^2$ which leads to a running time of the Dijkstra algorithm of $\mathcal{O}(\frac{1}{\epsilon^6})$. This running time is too high for practical applications. Using the iterative refinement method in the *position-critical model*, it is possible to improve the running time considerably. Such algorithm has a running time of $\mathcal{O}(\frac{1}{\epsilon^3})$ for finding a path within the same error bound. However, the correctness of this method is yet unproven.

4 Performance evaluation

In this section, we present the simulation results for both models described in Section 2.1. First, the simulation setup is described. The simulation results and the analysis of the total energy consumed by the mobile robot with and without applying our proposed algorithm are presented next.

4.1 Simulation setup

We break the simulation works into two parts: communications with single base station and multiple base stations. In the first part, a mobile robot and a base station are placed in the simulation area that is bounded by the maximum communication range between these two nodes. The data sheet of *Lucent Orinoco PC card* [15] for 802.11b technology shows that the communication range ranges from 25 to 550 m using varying bit rate from 1 to 11 Mbps, for both indoor and outdoor environment. In our simulation study, the maximum communication range is configured to approximately 100 to 115 m, which is sufficient to show the total energy saved using our proposed algorithm. A higher maximum communication range further increases the total energy saved.

Figure 5 shows the location of the mobile robot and the base station in the simulation. To vary the distance the robot has to travel and the transmission range, for

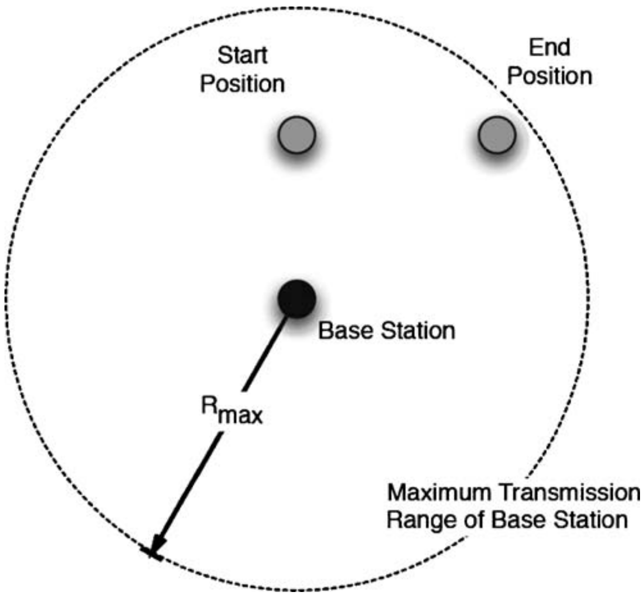


Fig. 5 Location of mobile robot and base station in simulation

simplification, we change the initial location of mobile robot to different points along the y-axis of the base station, while the target location is varied along the x-axis of its initial position, up to the point that yields the maximum radio range.

Based on the communication and the mobility energy model described in Section 2.2, the simulation runs are performed using the parameters specified in Table 2. We configure e_{cct}, e_{tx} and e_{rc} according to [17], and the moving parameter, m to 1–2 J m^{-1}, which is reasonable and realistic, as stated in [8]. The parameter m is influenced by several factors including robot weight or type. The value used in the simulation is applicable to wheeled robot moving on flat concrete terrain at constant friction. According to [22], a wheeled vehicle with rubber tires at 1 kg moving on concrete has to overcome 0.1 N force of dynamic friction. In other words, it has to expend 0.1 J m^{-1}. Thus, our parameter $m = 1$ J m^{-1} is applicable for robot up to the weight of 10 kg. Some example wheeled robots include *Khepera II* (80–250 g), and *Khepera III* (690 g–2 kg) [3], *s-bot* (660 g) [1], and *e-puck* (150 g) [2].

Table 2 Simulation parameters

Parameter	Value(s)
Energy consumed by transceiver circuitry to transmit or receive a bit, e_{cct} (J)	10^{-7}
Energy consumed by transceiver amplifier to transmit one bit data over 1 m, e_{tx} (J)	10^{-12}
Energy to receive a bit, e_{rc} (J)	10^{-7}
Energy to move robot over 1 m, m (J)	1, 2
Path loss exponent, α	3, 4
Data size, N (MB)	1, 2, 3
Video bit rate, B (Mbps)	1, 2, 3
Size of navigation region allowed, A (m)	1, 5, 10

Since path loss exponent hardly achieves 2 in reality, we choose the path loss exponent of 3 and 4 in our simulation. The path loss exponent may change dynamically based on the environment being explored by the robot. A robot can adopt a path loss prediction method, e.g. the ray-tracing algorithm [20] to compute the path loss exponent value. Lastly, the data size, N and video bit rate, B in Table 2 ranges from 1 to 3 MB and 1 to 3 Mbps respectively. They determine the volume of data transmission in PCM and CBR models respectively. In PCM model, the robot is simulated to transmit N data at every 1 m of the Euclidean path.

In the second part, the simulation setup remains the same as the first part except the followings. We increase the number of base station up to 6, with each located along the x-axis of the first base station. A base station is placed next to its neighbouring station such that the robot has to switch to the next base station whenever the maximum communication range is reached. For example, the adjacent base stations are separated by about 70.71 m for a maximum radio range of 100 m. The goal is to analyze to what extent the parameters affect the simulation output over increasing number of base station, and thus, increasing traveling distance of robot.

We vary the values of the parameters in Table 2 to form a number of combination sets and evaluate their impacts on computing the optimal energy path. In the following sections, we present the simulation results of computing optimal energy path for single and multiple base station communications.

4.2 Simulation results: single base station communications

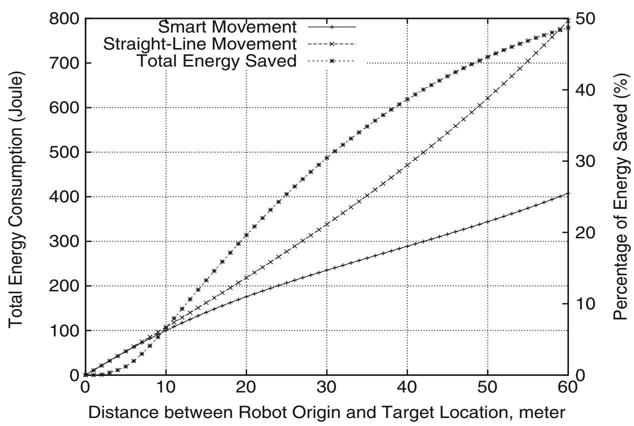
First, we simulate our algorithm in both *Position Critical Communications Model* and *Constant Bit-Rate Communications Model* for single base station scenario. The aim is to determine if the minimal energy path exists in both models. If so, we study the percentage of energy that can be saved by applying our algorithm in each model. We use the terms *Smart* movement and *Straight-Line* movement to indicate radio energy-aware path and radio energy unaware path respectively. The *Smart* movement is computed using PCM-refinement algorithm. Once the target location of the mobile robot is determined, the algorithm is applied to compute the optimal energy path.

4.2.1 Position-critical communications model

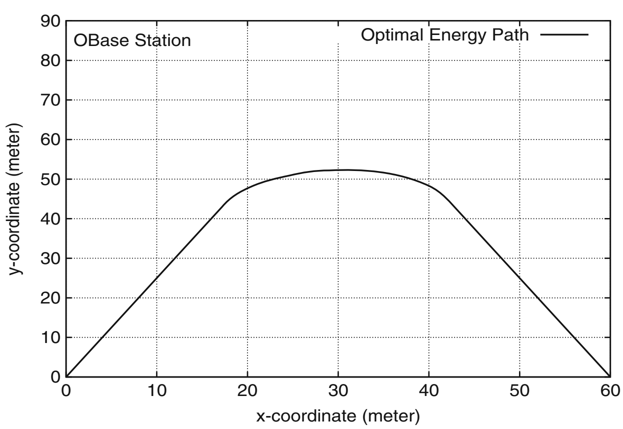
In Fig. 6, we show a sample result of simulation runs that are configured with $\alpha = 4$, $m = 1$, $N = 3$. The base station is located 85 m away from the initial position of the robot, while the target point is varied up to $R_{\max}$. Total energy consumption of both *Smart* and *Straight-Line* movement, and the amount of energy saved are illustrated in Fig. 6a. It shows that using our algorithm, total energy saved grows over the robot's traveling distance. It achieves 48.71% when the target point is 60 m from the origin, which is the maximum value allowed by the constraint of transmission range. This optimal path is depicted in Fig. 6b.

We introduce the parameter *Closeness*, c, which defines the size of region A a robot can navigate and thus, the maximum gap between the optimal and the Euclidean paths. The value of c is determined based on specific application, and the environment. For instance, the presence of an obstacle prevents a robot from selecting certain values of c. A path closer to the Euclidean line is indicated by a lower c. We vary the value of c to analyze its impact on total energy savings.

To study the impact of c, we vary c based on a precision set of 0.1 m, where $c = 1$ allows the mobile robot to move 0.1 m further away from Euclidean path at every step ϵ, 0.2 m for $c = 2$ and so on. We show two example results in Fig. 7 using five different values of c, ranging from 1 to 5. The simulation uses $\alpha = 4$, $m = 1$, $N = 3$. Figure 7a and b illustrate the amount of energy saved when the robot origin is 85 and 52 m away from base station respectively. Both results indicate that the further a robot is allowed to move towards the base station, the more energy it can save. Moreover, it shows that the amount of energy saved grows with the distance between the two communicating nodes.

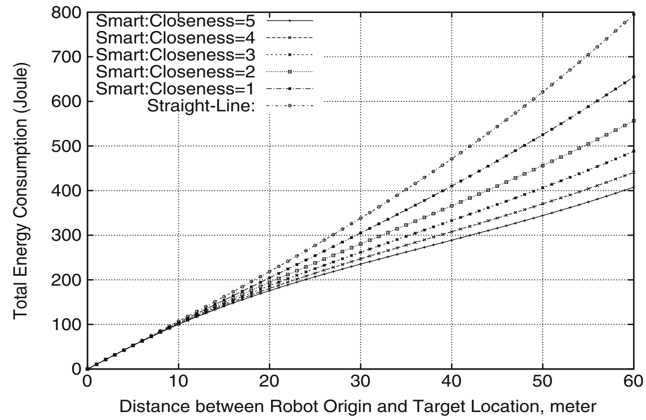


(a) Total energy consumed and percentage saved

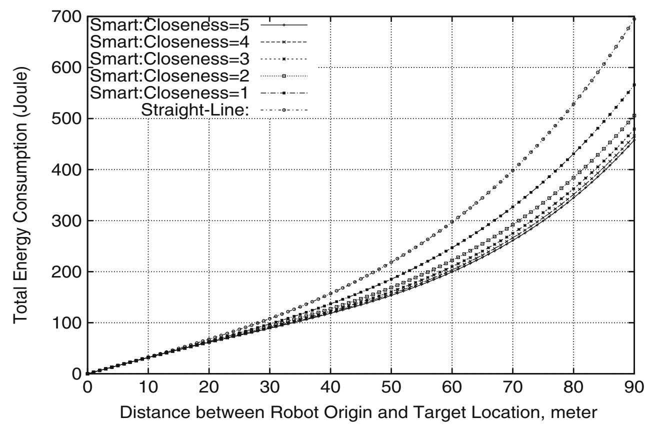


(b) Computed optimal energy path

Fig. 6 PCM: distance between robot origin and base station = 85 m



(a) Distance between robot origin and base station = 85 meters

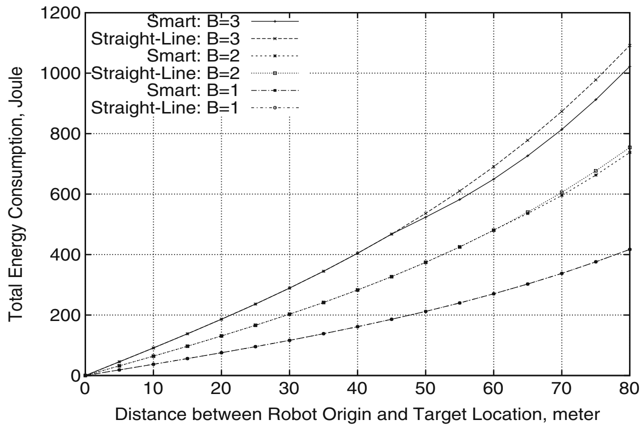


(b) Distance between robot origin and base station = 52 meters

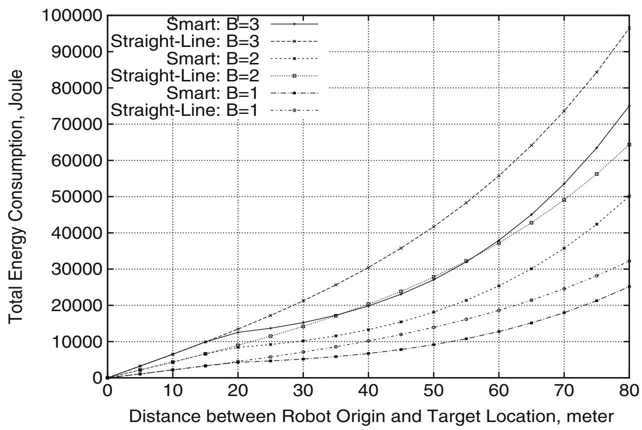
Fig. 7 PCM: total energy consumption for varying *closeness*

4.2.2 Constant bit-rate communications model

In Fig. 8, we show the sample results of the simulation runs that are configured with $\alpha = 3, 4$, $m = 1$, $B = 1$,



(a) Path loss exponent of 3 and varying video bitrate, B (Mbps)



(b) Path loss exponent of 4 and varying video bitrate, B (Mbps)

Fig. 8 CBR: total energy consumption for distance between robot origin and base station = 80 m

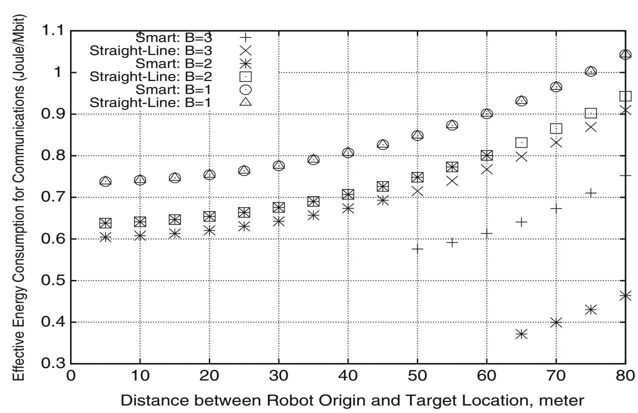
2, 3, and the distance between robot origin and base station = 80 m. No limitations on the size of region a robot can travel are imposed. In its presence, the Euclidean path is found to be the optimal path in most cases. This can be observed in Fig. 8a and b. In Fig. 8a, all optimal paths fall on the Euclidean paths when $B = 1$ and $\alpha = 3$. When the traveling distance reaches a certain point ($>$ 60 m when $B = 2$ and $>$ 45 m when $B = 3$ approximately), which yields a higher communication distance, the following optimal paths generated using our algorithm save an amount of energy: a maximum of 2.21% when $B = 2$ and 6.36% when $B = 3$. On the other hand, Fig. 8b shows that only a maximum energy savings of 22.18% is achieved when $\alpha = 4$. This amount is comparably lower than that in PCM model (48.71%).

Based on the results obtained, we can deduce that using the same parameters, the optimal energy path constructed in CBR model always achieves lower percentage of energy saving than in PCM model. While selecting the right model for an application, one should consider the benefits of each model. For instance, if the total energy saved is too low to compensate other overheads incurred to run the algorithm, the Euclidean path should be used.

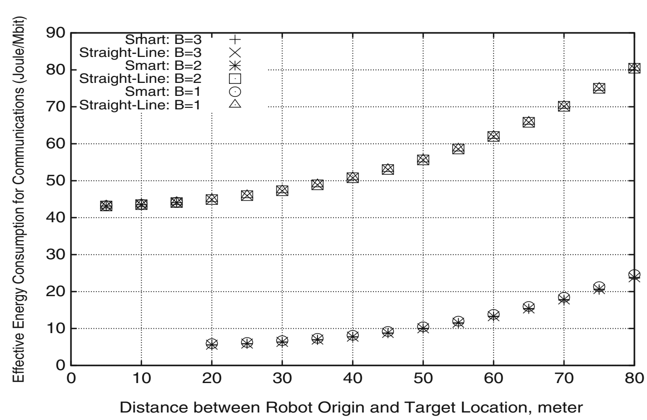
In CBR model, the data transmission rate is constant along all paths. Therefore, if a path computed by our algorithm is longer than the Euclidean path, and the robot moves at constant speed, the amount of video transmitted is higher on this path. By considering both the amount of data transmission and the path length, we introduce the effective energy consumption, E_{eff} to further analyze the energy efficiency of the paths traveled by robot. Given the same target point, we measure E_{eff} for the path computed using our algorithm and the Euclidean path. E_{eff} is computed as below:

$$E_{\text{eff}} = E_{\text{total}}/N \,,$$

where E_{total} is the total energy consumed by a robot for both moving and communications, while N indicates the amount of data transmitted in bits.



(a) Path loss exponent of 3 and varying video bitrate, B (Mbps)



(b) Path loss exponent of 4 and varying video bitrate, B (Mbps)

Fig. 9 CBR: effective energy consumption in data transmission for distance between robot origin and base station = 80 m

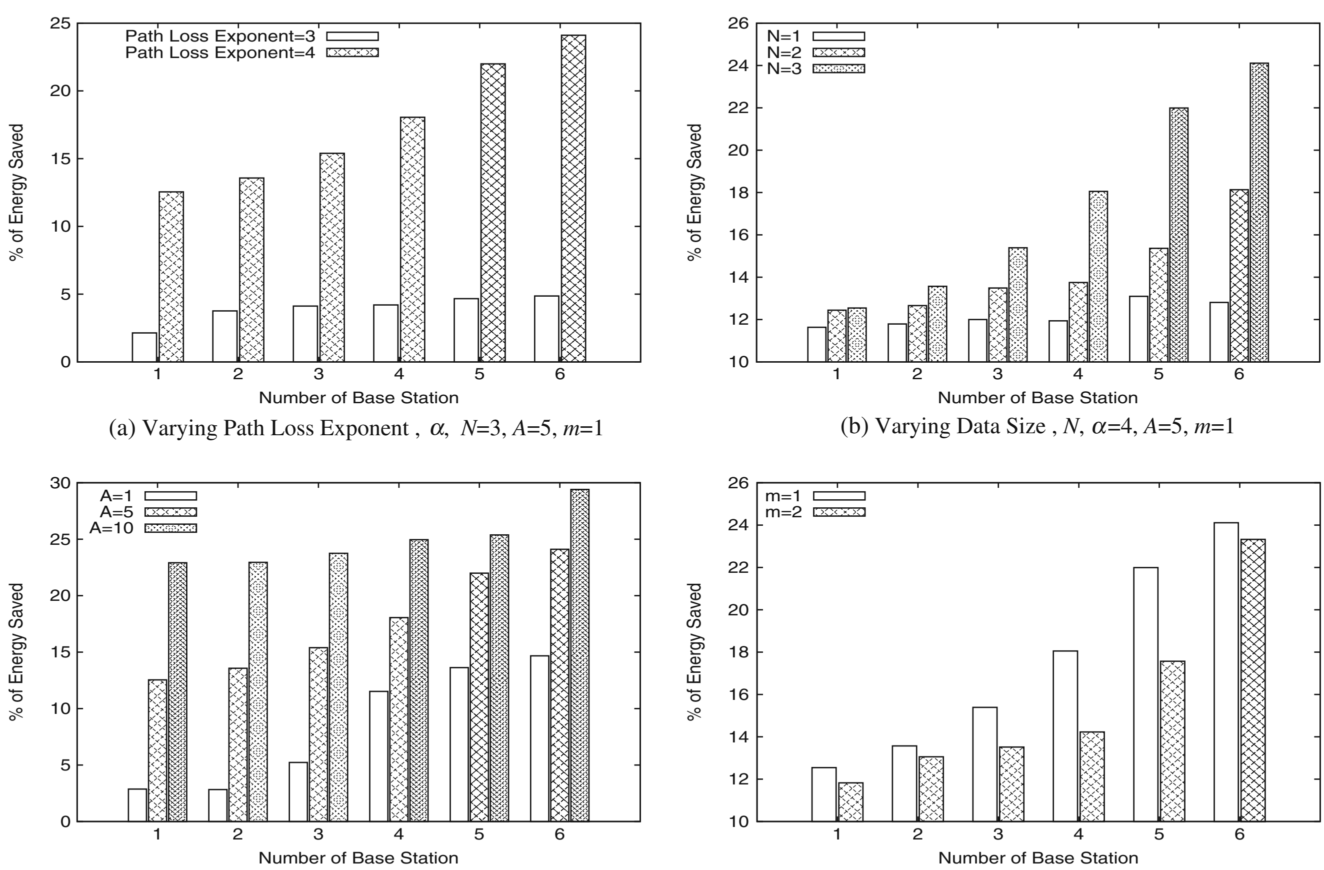
Figure 9 shows the simulation results of measuring energy efficiency of paths when $\alpha = 3$, 4, $m = 1$, the distance between the robot origin and the base station = 80 m and varying B. The corresponding results on energy savings using the same parameter values can be read in Fig. 8. Every optimal energy path generated has lower E_{eff} than that of the Euclidean path, except the Euclidean path itself being the optimal energy path. The improvement in energy efficiency achieves up to 70.61% in Fig. 9b when $\alpha = 4$ and $B = 3$ Mbps. The findings shown in Fig. 9a are significantly relatively lower. Path loss exponent, α, has critical impact on E_{eff}.

4.3 Simulation results: multiple base stations communications

In multiple base station scenario, only one model is simulated because our objective is to analyze the effect of increasing the travelling distance of robot on the energy savings using different parameters. We conduct this experiment in *Position-Critical Communications Model*.

Four parameters used in this simulation are the path loss exponent, α, the amount of data transmission, N, the size of navigation region allowed, A, and the movement parameter, m. The parameter A defines a rectangular region bounded by the Euclidean path, start and end points, and the path parallel to and A away from the Euclidean path. The values of these parameters are varied according to Table 2.

To form an Euclidean path for robot, we randomly pick a start position and a target point, which are located within the radio range of the first and the last base station respectively. For each parameter and number of base station, we execute 100 simulation runs and compute the average percentage of energy saved in each case. The results are illustrated in Fig. 10. In Fig. 10a, as seen in the single base station scenario, α has significant impact on the amount of energy saved in all number of base station. It can be observed through the difference of energy saving between the scenario with $\alpha = 3$ and $\alpha = 4$. On the other hand, higher percentage of energy is saved when the traveling distance increases in all cases. However, lower α only has slight impact over growing distance.



(a) Varying Path Loss Exponent , α, N=3, A=5, m=1

(b) Varying Data Size , N, α=4, A=5, m=1

(c) Varying Region Size, A, α=4, N=3, m=1

(d) Varying Movement Parameter , m, α=4, N=3, A=5

Fig. 10 Multiple base stations: percentage of energy saved with varying α, N, A, and m over traveling distance

Figure 10b shows that increasing the value of N yields higher percentage of energy saved in all number of base station. It increases more significantly with longer distance. The percentage of energy saved grows over the distance in all simulated scenarios, except when N is low. It can be observed when $N = 1$, which saves approximately 11–13% energy.

Next, we analyze the impact of the size of travel region, A, and show the results in Fig. 10c. The values of A are set to 1, 5 and 10 m. Overall, the percentage of energy saved grows over A and traveling distance in all simulated scenarios. The robot is allowed to move closer to the base station when A is lower, thereby reducing the communication cost. We also observe that the percentage of energy saved tends to grow slower at higher A.

Lastly, Fig. 10d illustrates higher percentage of energy saved using lower m. It also grows over the number of base stations. However, one can observe that m has less impact compared to that with varying α. This parameter is important when choosing the type of robot to perform in an application that requires energy-efficient radio communications.

Generally, the percentage of energy saving increases with higher α, A and N and lower m for each number of station. Moreover, in all scenarios, it grows over distance except when the data transmission is small, for instance, when $N = 1$. Among these four parameters, α has the most significant impact on the energy savings. Therefore, the navigation environment will affect the total energy consumption of the robot based on its path loss exponent value (Table 1).

Different combination of parameter values determine if the optimal energy path falls on the Euclidean path, the percentage and total energy saved and the energy efficiency of robot using *Smart* movement, considering both the mobility and communication costs. From our simulation, we note that the value of path loss exponent has significant impact on computing the minimal energy path. Based on the simulation conducted in Section 4.2, if the robot navigates in free space, in which path loss exponent, α=2, the robot will always move on the Euclidean path in all simulated scenarios. Otherwise, its impact is influenced by other simulation parameters.

Overall, the total energy saved increases with higher path loss exponent, the size of navigation region allowed, video rate or the amount of data transmission, the distance between robot origin and its target position, as well as the distance between two communicating nodes. In contrast, it decreases over the moving parameter, which depends on the type of robot used in practice.

5 Conclusions and future works

In this paper, we study the problem of optimizing the energy consumption of robot that moves and communicates based on two models: the *position-critical communications model* and *constant bit-rate communications model*. An approximation algorithm based on Dijkstra's algorithm is proposed to compute the minimal energy path. We demonstrate the simulation results by applying our solution to a number of scenarios considering both single and multiple base stations. Then, we compare the results with that of the Euclidean path.

In the *position-critical communications model*, the total energy savings achieve up to nearly 50%. In the *constant bit-rate communications model*, the optimal energy path of the *Smart* movement exists in some scenarios, depending on the combination of parameter values. We show that the total energy consumption can be saved up to 22.18% computed using the proposed algorithm. It also depicts that the generated path is the energy-efficient path for continuous data transmission.

Our solution enables mobile robot to take full advantage of the optimal-energy path to prolong its lifetime while completing its mission. By analyzing the impact of different parameters on performance gain, we show the simulation results, which are promising in improving the energy efficiency of future networked robot system. For further exploration, the proposed strategy can be incorporated into other aspects, such as improving the performance of wireless communications in terms of its Quality of Service (QoS), by utilizing the benefits of controllable node mobility.

Acknowledgements This research is partially supported by *DFG-Sonderforschungsbereich SPP 1183: Organic Computing. Smart Teams: Local, Distributed Strategies for Self-Organizing Robotic Exploration Teams.*

References

1. Swarm-bots Project (2005) The European Commission (IST-2000-31010). http://www.swarm-bots.org. Accessed 16 Jan 2009
2. e-puck Education Robot Project (2009) Ecole Polytechnique Federale de Lausanne (EPFL). http://www.e-puck.org. Accessed 16 Jan 2009
3. Khepera Robots (2008) K-Team Corporation. http://www.k-team.com. Accessed 16 Jan 2009
4. Akyildiz IF, Melodia T, Chowdhury KR (2007) A survey on wireless multimedia sensor networks. Comput Networks 51(4):921–960. doi:10.1016/j.comnet.2006.10.002
5. Anh Tuan Hoang MM (2005) Exploiting wireless broadcast in spatially correlated sensor networks. In: IEEE ICC, pp 2807– 2811

6. Capkun S, Hubaux JP, Buttyán L (2003) Mobility helps security in ad hoc networks. In: Proceedings of the 4th ACM MobiHoc, pp 46–56
7. Das SM, Hu YC, Lee CG, Lu YH (2005) An efficient group communication protocol for mobile robots. In: IEEE ICRA
8. Goldenberg DK, Lin J, Morse AS, Rosen BE, Yang YR (2004) Towards mobility as a network control primitive. In: Proceedings of the 5th ACM MobiHoc. ACM Press, New York, NY, USA, pp 163–174. doi:10.1145/989459.989481
9. Grossglauser M, Tse DNC (2002) Mobility increases the capacity of ad hoc wireless networks. IEEE/ACM Trans Netw 10(4):477–486. doi:10.1109/TNET.2002.801403
10. Gruenewald M, Rueckert U, Schindelhauer C, Volbert K (2003) Directed power-variable infrared communication for the mini robot khepera. In: Proceedings of the 2nd International conference on autonomous minirobots for research and edutainment, pp 113–122
11. Koutsonikolas D, Das SM, Hu YC, Lu YH, Lee CSG (2006) Cocoa: coordinated cooperative localization for mobile multi-robot ad hoc networks. In: Proceedings of the 26th IEEE ICDCSW. IEEE computer society, Washington, DC, USA, p 9. doi:10.1109/ICDCSW.2006.30
12. Labella TH, Fuchs G, Dressler F (2006) A simulation model for self-organised management of sensor/actuator networks. In: GI/ITG KuVS Fachgespr"ach Selbstorganisierende, Adaptive, Kontextsensitive verteilte Systeme (SAKS)
13. Li XY, Wan PJ (2001) Constructing minimum energy mobile wireless networks. ACM Mobile Comp Commun Rev 5(4):55–67
14. Liu B, Brass P, Dousse O, Nain P, Towsley D (2005) Mobility improves coverage of sensor networks. In: Proceedings of the 6th ACM MobiHoc, pp 300–308
15. Lucent Technologies (2000) Orinoco PC card
16. Mei Y, Lu YH, Hu YC, Lee C (2005) A case study of mobile robot's energy consumption and conservation techniques. In: 12th IEEE ICAR, pp 492–497
17. Meyer auf der Heide F, Schindelhauer C, Volbert K, Grünewald M (2004) Congestion, dilation, and energy in radio networks. TOCS 37(3):343–370
18. Powers M, Balch T, Lab B (2004) Value-based communication preservation for mobile robots. In: IEEE ICRA
19. Rodoplu V, Meng T (1998) Minimum energy mobile wireless networks. In: Proceedings of IEEE ICC, vol 3, pp 1633–1639
20. Schaubach K, Davis NJ, Rappaport T (1992) A ray tracing method for predicting path loss and delay spread in microcellular environment. In: Proceedings of IEEE 42nd vehicular technology conference. IEEE, pp 932–935
21. Tang C, McKinley PK (2006) Energy optimization under informed mobility. IEEE Trans Parallel Distrib Syst 17(9):947–962. doi:10.1109/TPDS.2006.122
22. Tipler PA (1991) Physics for scientists and engineers. Worth, New York
23. Tseng YC, Wang YC, Cheng KY, Hsieh YY (2007) Imouse: an integrated mobile surveillance and wireless sensor system. Computer 40:60–66
24. Zeng QA, Agrawal DP (2002) Handoff in wireless mobile networks. In: Handbook of wireless networks and mobile computing, pp 1–25