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Optimal Motion Planning for Minimizing Energy Consumption of Wheeled Mobile Robots

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***Abstract*—This paper presents a novel approach of finding energy-efficient trajectories for mobile robots. The approach integrates new cost and heuristic functions into the conventional A* algorithm while considering ground conditions and obstacle positions. The resulting planner helps to manage obstacle avoidance and to choose intelligent displacements of the robot. A heuristic function with energy-related criterion is defined in order to generate energy-efficient paths. η^3-Splines continuity property is exploited to generate smoothed energy-paths. The optimal velocity profile for minimum travel time is found by solving Sequential Quadratic Problem. A series of simulations demonstrate the energy saving efficiency of the proposed method.**

***Index Terms*—mobile robots, Minimum energy consumption, motion planning, and A* algorithm.**

I. Introduction

Energy saving techniques for mobile robots abound in the literature [1]–[5]. They empower mobile robots to perform more complex and long-lasting missions. Despite the existence of various energy saving techniques [1]–[4], minimizing the energy consumption of mobile robots remains a real challenge.

In [3], an energy-saving approach for mobile robots by avoiding torque saturation that generally occurs at the wheels DC motors while climbing hills was proposed. A predictive control was implemented to solve the torque saturation problem. Recently, motion planning has emerged as one of the best ways to minimize energy consumption of mobile robots. As a result, velocity planning which could save battery energy by up 5% compared with the widely used trapezoidal velocity profile was proposed in [4]. [6] compared the energy consumption of different routes at different velocities by considering the energy consumed for accelerations and turns. Other contributions based on the optimal motion planning approach and using various energies criteria were also proposed [2], [4].

Among existing contributions in energy savings, based on effective motion planning, some stand out thanks to the choice of the energy criterion. The shortest path length criterion was used by many researchers to minimize energy consumption [7]–[11]. However, the shortest route may not necessarily result in minimum energy consumption. Some factors such as the surface of the navigation or the shape of the planned trajectory might affect significantly the energy consumption. The reduction of the steering actuation was used in [10] as an energy criterion for mobile robots energy minimization. A smoothness criterion depending on the acceleration was used in [11] for energy saving. However, in the aforementioned techniques, a model that can be used to simulate the energy consumption of the mobile robot was not fully investigated. An interesting technique was proposed in [2]. From an energy consumption model of a two-wheeled mobile robot, the authors defined an energy-related criterion. However, the proposed algorithm does not always find the energy-saved path.

To overcome the aforementioned problems, this work integrates new cost and heuristic functions into the conventional A* algorithm. The former helps to manage obstacle avoidance with minimum energy cost, and the last one to better deal with ground conditions. The proposed approach can be summarized in three main steps. Energy consumption model of the mobile robot is first developed. Based on the energy model, a new energy-related criterion is defined and collision-free paths are generated, thereafter. Finally, the waypoints are selected and η^3-splines with optimal shaping parameters are used to smooth the energy-saved paths.

The remainder of the paper is organized as follows. The energy model of a Three-Wheeled Omnidirectional mobile robot (TOMR) is developed in Section II. In Section III, an A* path planner with new cost and heuristic functions is first introduced and η^3-splines for path smoothing is presented, thereafter. Simulation results are provided in Section IV, and concluding remarks are given in Section V.

II. Energy model of Three-wheeled Omnidirectional Mobile Robot

In this section, an energy consumption model for a TOMR is developed. TOMR is composed of three wheels of the same radius r and driven by three identical DC motors Fig. 1. The kinematic equations relating linear and angular velocities of the robot to the linear velocities of the wheels can be expressed

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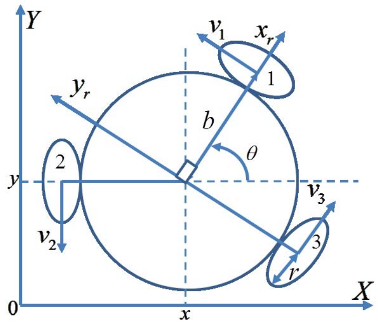

Fig. 1. Kinematic model of a TOMR

as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 & b \\ -\sin\left(\pi/3\right) & b \\ \sin\left(\pi/3\right) & b \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (1)$$

where b is the distance between wheel and robot center.

The energy consumption model of differential drive mobile robots has been developed in [2], [12]. Based on [2], [12], an energy consumption model for TOMR can be easily derived. By considering the losses due to transform back the kinetic energy to the equivalent electric energy, the losses due to friction, and the losses in the electronic components, the energy consumption model is expressed as follows:

$$\begin{aligned} E_T &= E_k + E_F + E_e \\ &= \tfrac{1}{2}\left(mv\left(t\right)^2 + Iw\left(t\right)^2\right) \\ &+ \mu mg \int \left(\left|b\omega\left(t\right)\right| + 2max\left(\left|b\omega\left(t\right)\right|, \left|\tfrac{\sqrt{3}}{2}v\left(t\right)\right|\right)\right) dt \\ &+ \int P_e dt \end{aligned}, \quad (2)$$

where m is the mass of the robot, I is the moment of inertia of the robot, $v\left(t\right)$ is the linear velocity, $\omega\left(t\right)$ is the angular velocity, μ is the rolling friction coefficient that depends on the surface type of the ground, g is the gravitational acceleration, and P_e is the power consumption by on-board electronic components.

III. Efficiency motion planning for energy minimization

In this section, the modified A* algorithm with new cost and heuristic functions is first introduced and η^3-splines with optimal shaping parameters for path smoothing is presented, thereafter.

A. A* algorithm

A* algorithm has been widely used in path planning [2], [10], [11]. In robotics, path planning consists of finding successive states (cells) on a grid map which allows the robot to move from an initial state (start) to a final state (goal) by avoiding obstacles. A* algorithm plans the path on a grid map. Each grid constitutes a node that can be free or occupied by an obstacle. The path is planned from a start node (s_{start}) to a goal node (s_{goal}), with s_{start}, $s_{goal} \in S$, and S being the possible set of robot locations [13]. Let s be the actual node, an evaluation function $f\left(s'\right)$ is used to determine which node should be expanded next. The next node denoted s' is one of the successors of the node s. The actual node has a total of eight successors. The values $f\left(s'\right)$ of each successor of the node s are calculated and the successor with the smallest value of $f\left(s'\right)$, denoted s', is considered. The evaluation function $f\left(s'\right)$ is the sum of two functions, a heuristic function $h\left(s', s_{goal}\right)$ that represents the estimated cost of an optimal path from the node s' to the node s_{goal}, and a function $g\left(s'\right)$ representing the actual cost of the path from the node s_{start} to the node s' passing through s node [14].

$$f\left(s'\right) = g\left(s'\right) + h\left(s', s_{goal}\right) \quad (3)$$

The values of $g\left(s'\right)$ are derived from the value of $g\left(s\right)$ as follows:

$$g\left(s'\right) = g\left(s\right) + c\left(s, s'\right), \quad (4)$$

where $c\left(s, s'\right)$ represents the cost to move from node s to node s'. This cost can be designed to suit the need [13]. The travel distance was widely used as a cost function to minimize energy consumption [7]–[11]. To plan energy-saved paths, a valuable contribution was proposed in [2] where, a part of total energy representing loss due to friction zones has been used in path finding. The cost function (5) and the heuristic function (6) were defined, and a penalty factor $\rho\left(s'\right) \in [0,1]$ was integrated into the cost function to maintain a safety distance from obstacles.

$$\begin{aligned} c\left(s, s'\right) &= \sqrt{3}\mu_{s,s'} mg \left(\int_{t_s}^{t_{s'}} \left|v\left(t\right)\right| dt\right) \times \frac{1}{\rho(s')} \\ &= \sqrt{3}\mu_{s,s'} mg d_{s,s'} \frac{1}{\rho(s')} \end{aligned} \quad (5)$$

$$h\left(s', s_{goal}\right) = \sqrt{3}\mu_{s,s'} mg d_{s',s_{goal}}, \quad (6)$$

Where $\mu_{s,s'}$ and $d_{s,s'}$ are the friction coefficient and distance between the two nodes s and s', respectively.

The penalty factor related to the distance to the obstacle is given by:

$$\rho(s') = \begin{cases} 1, & \lambda_{s'} > \lambda_{safe} \\ \frac{\lambda_{s'} - b}{\lambda_{safe} - b}, & b < \lambda_{s'} \le \lambda_{safe} \\ 0, & \lambda_{s'} \le b \end{cases} \quad (7)$$

where $\lambda_{s'}$ is the distance of the cell s' to the nearest obstacle, λ_{safe} is the safety distance defined for safe motion of the robot [2], b is the half size of the robot. However, the fact that in the heuristic function (6), the friction ($\mu_{s,s'}$) is multiplied by the distance from s' node to goal node s_{goal} suggests that $\mu_{s,s'}$ extends up to goal node; which is not always the case. In this paper, a new heuristic function (10) is defined to better deal with friction zones, and a turning angle is introduced as a penalty factor (8) through a Gaussian function to reduce the number of turns. Fig. 2 shows the effect of this function in the grid map. The latter passes from a flat surface (node plane) to a hollow surface allowing the robot to move in the same direction. The penalty function affects high costs to nodes that

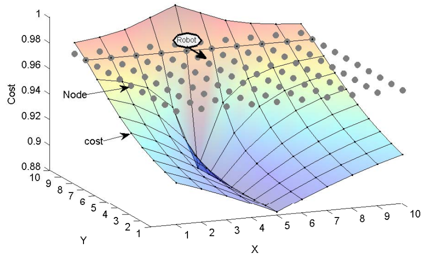

Fig. 2. Turning angle penalty factor function

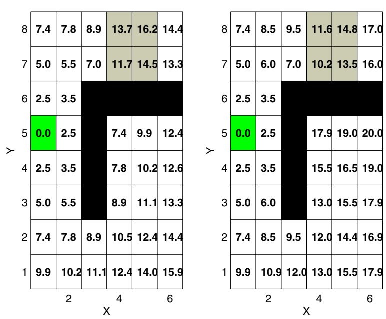

Fig. 3. Heuristic value: (left) Liu and Sun (5) and (right) Proposed (10)

are not in the robot direction and lower costs to those are oriented towards the goal node.

$$\phi(\alpha) = 1 - e^{(-\frac{\alpha^2}{8})} \tag{8}$$

The cost function used in this paper is defined as follows:

$$c(s, s') = \sqrt{3}\mu_{s,s'} mgd_{s,s'} \left(\phi(\alpha) + \frac{1}{\rho(s')}\right) \tag{9}$$

where $\alpha \in [-1, 1]$ is the normalized angle formed by the segment $s's$ and the previous direction of the robot, $\rho(s')$ is a penalty factor related to the distance to the obstacles (7).

The heuristic function is modified as follows: the node goal s_{goal} introduced in Liu and Sun's distance $d_{s',s_{goal}}$ is replaced by a neighbor of the node s' with minimum heuristic value denotes s'', and its heuristic $h(s'', s_{goal})$ is added to heuristic function (10).

$$h(s', s_{goal}) = \sqrt{3}\mu_{s',s''} mgd_{s',s''} + h(s'', s_{goal}), \tag{10}$$

where $h(s'', s_{goal})$ is the minimum heuristic function value of the successors of the node s'. Heuristic function values of all free nodes are calculated as follows: Start at goal node with value 0 and extend to it neighbors using (10) until map are covered. Fig. 3 represents on a 6x8 grid map, the heuristic values of the proposed heuristic function (right-hand side) and Liu and Sun's heuristic function (left-hand side). In both diagrams, grey spaces represent friction zones, the black boxes are the obstacles, and the green circle is the goal node. We notice that the values obtained from Liu and Sun's heuristic

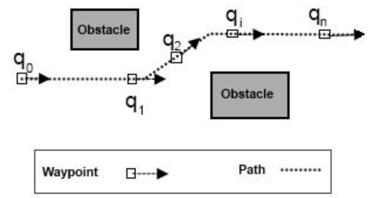

Fig. 4. Waypoints selection

function are very large for friction zones, which discriminate passages through the latter, even if passages through them yield a minimum energy consumption. However, we observe reasonable values in the proposed heuristic function giving a good estimation of friction zones.

B. η^3-Splines smooth path with minimum energy

This subsection focuses on the smoothing of the generated paths while minimizing energy consumption. The logical step following the path generation is the tracking of the latter with a minimum time. Thus, in order to obtain continuous paths, η^3-Splines, thanks to its third order geometric continuity with continuous tangent vector, curvature, and curvature derivative along the arc length, was used for path smoothing. The smoothing can be summarized in three steps: the selection of the Waypoints along the path generated, the selection of the optimal shaping parameters via an optimization problem, and the tracking of the smoothed path with minimum time through another optimization problem.

The Waypoints are selected on the generated path as follows: The start and goal positions as well as closed neighbors of knee points along the path are selected. Two neighboring waypoints are combined if they are close enough, and new waypoints may be inserted if they are extremely distant. The orientation at each waypoint is set along the next segment of the generated path Fig. 4; concerning the endpoint, the robot orientation is set to the previous orientation.

η^3-Splines are seventh order polynomial curves which smoothly connect two arbitrary points $q_{i-1} = \begin{bmatrix} X_{i-1} & Y_{i-1} & \theta_{i-1} \end{bmatrix}^T$ and $q_i = \begin{bmatrix} X_i & Y_i & \theta_i \end{bmatrix}^T$ where $[X, Y]$ and θ are the robot position and orientation, respectively. η^3-Splines is parameterized by: $q(u) = \begin{bmatrix} x(u) & y(u) \end{bmatrix}^T, u \in [0, 1]$ with $x(u)$ and $y(u)$ defined as:

$$x(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 + a_6 u^6 + a_7 u^7 \tag{11}$$

$$y(u) = b_0 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4 + b_5 u^5 + b_6 u^6 + b_7 u^7, \tag{12}$$

where the polynomial coefficients $a_i, b_i, i = 0, ..., 7$ depend on six shaping parameters $\eta_i, i = 1, ..., 6$ [15]. These parameters influence only the path shape; the endpoints interpolating conditions i.e. the position, the unit tangent vector, the curvature, and the curvature derivative, remain unchanged.

To maintain a safe distance to the obstacle, the curvatures of η^3-splines curves are constrained to the maximal bound k_{max} while the path length is minimized [16]. This constitutes the first optimization problem, defined as follows:

$$\min_{\eta \in \Re} s_q \tag{13}$$

$$|k(u)| \leq k_{\max}, u \in [0, 1], \tag{14}$$

where s_q is the arc length of a spline curve between two endpoints. The curvature $k(u)$ with respect to u is described in (15)

$$k(u) = \frac{(\dot{x}\ddot{y} - \ddot{x}\dot{y})}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \tag{15}$$

The above problem is a sequential quadratic problem, its resolution leads to the selection of optimal shaping parameters.

The second optimization problem concerns the tracking of the smoothed paths with minimum time. Let $d(t)$, the arc length measured along the smoothed path at time t, d_f the total length of the smoothed path, and t_f the time required to travel the smoothed path. To minimize energy along this path, $\bar{t}_f$ need to be minimal while satisfying the velocity and acceleration constraints. This problem has been addressed in detail in [17]. The smoothed path is discredited in $N-1$ elementary equal parts of length $l = \frac{d_f}{N-1}$. Velocity along the $j-th$ elementary part is considered constant and set as the average $\frac{v_j + v_{j+1}}{2}$ of the velocities of the two endpoints. The total time t_f is the sum $\frac{2l}{v_j + v_{j+1}}$ of the time taken to cover the elementary parts.

$$t_f = 2l \sum_{j=1}^{N-1} \frac{1}{v_j + v_{j+1}} \tag{16}$$

This optimization problem is defined as follows:

Find the velocity sequence $\mathbf{v} := (v_1, ..., v_N) \in \Re^N$ which $\min_{v \in \Re^N} 2l \sum_{j=1}^{N-1} \frac{1}{v_j + v_{j+1}}$ subject to constraints (17-20).

$$v_1 = v_{start}, v_N = v_{goal} \tag{17}$$

$$0 \leq v_j \leq v_{\max}, j = 1, ..., N \tag{18}$$

$$a^h_{\min} \leq a^h_j \leq a^h_{\max}, j = 1, ..., N-1 \tag{19}$$

$$v_j^2 |k_j| \leq a^n_{\max}, j = 1, ..., N. \tag{20}$$

Where a^h and a^n are the longitudinal and normal accelerations, and k_j the path curvature of the j^{th} elementary part.

Finally, after the selection of shaping parameters $\begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 & \eta_6 \end{bmatrix}^T$ and the velocities $(v_1, ..., v_N)$ along the smoothed trajectory, the energy model (2) is reformulated as follows:

$$E_T = \sum_{j=1}^{N-1} E_j, \tag{21}$$

TABLE I
ROBOTINO PARAMETERS

Parameters	Value	Parameters	Value
Wheel radius	$40mm$	Battery voltage	$24V$
Robot radius	$175mm$	Robot mass	$11kg$
M. of inertia	$0.16245kg.m^2$	Max velocity	$1.325m.s^{-1}$

with E_j the energy consumed on $j-th$ part of the elementary path, and defined as follows:

$$\begin{aligned} E_j = & \tfrac{1}{2}\left(mV_j^2 + I(k_jV_j)^2\right) \\ & + \mu mg \left(|bk_jV_j| + 2\max\left(|bk_jV_j|, \left|\tfrac{\sqrt{3}}{2}V_j\right|\right)\right) \times t_j \\ & + P_e t_j, \end{aligned} \tag{22}$$

where $V_j = \frac{v_j + v_{j+1}}{2}$, k_j, and t_j are the average velocity, the path curvature, and the travel time of the j^{th} elementary part, respectively.

IV. MODEL VALIDATION AND SIMULATION RESULTS

In this section, simulations are conducted to demonstrate the effectiveness of the proposed algorithm. The section starts by the identification and validation of the energy model, and ends by simulation results and a discussion.

A. Identification and validation of the energy model of the Robotino mobile robot

The energy model (2) developed in section II can be used for any TOMR platform; however, the platform parameters must be first identified before the implementation of the energy model. Robotino mobile robot is and TOMR platform supplied by two $12V$ batteries which permit a running time up to two hours. The robot's dimensions are $350mm$ in diameter and $210mm$ in height with an overall weight of approximately $11kg$. The experimental tests were performed on a platform to identify the modeling parameters of the Robotino. The electric power $P_e = 1.46W$, the gravitational acceleration $g = 10m.s^{-2}$, the friction coefficient μ on the flat zone 0.013 has been identified. Table I gives some Robotino's parameters.

B. Simulations results and discussion

The effectiveness of the proposed motion planning was demonstrated by conducting a series of simulations. Based on the energy model (22), simulations were performed in an environment containing 53×54 grids with wall-like obstacles Fig. 5-(a). The grid size is set based on the robot and obstacles dimensions. It is considered as a square of $350mm$ of side. The performance of the proposed energy saving method was assessed for different values of friction areas Fig. 5-(a), (b) and (c). The proposed energy saving approach is compared with Liu & Sun's method which showed satisfactory performance over existing approaches [2], [10], [11]. In order to preserve the main property of the

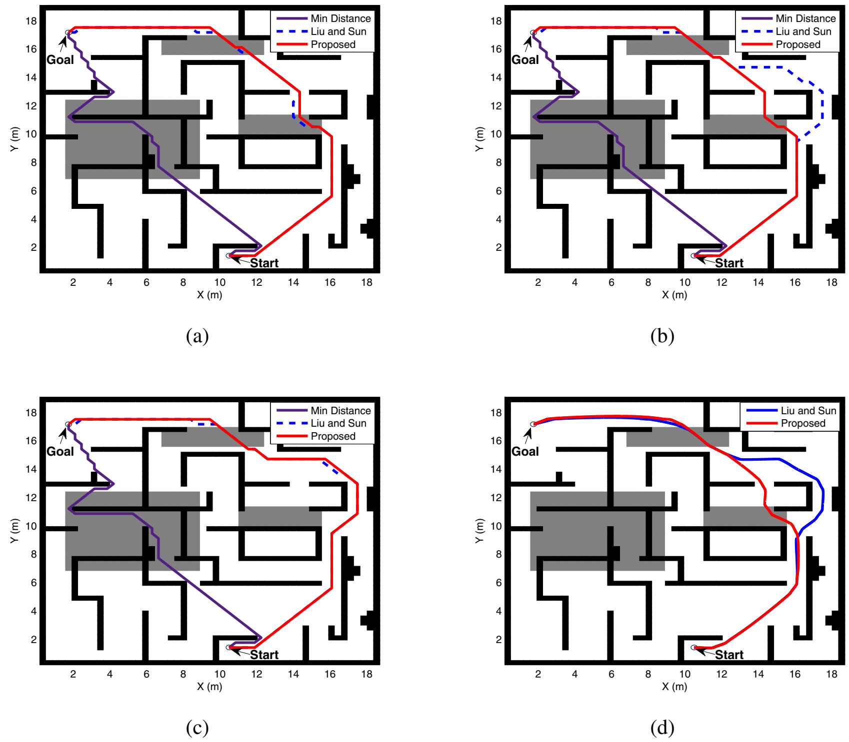

Fig. 5. Optimal path for the three methods: (a) $\mu = 0.015$ for friction zone, (b) $\mu = 0.01628$ for friction zone, (c) $\mu = 0.03005$ for friction zone, (d) smoothed trajectory of Liu & Sun and the proposed methods.

A* algorithm, the proposed approach is also compared with a motion planning method based on the Newton algorithm [7]. Computational costs were measured in MATLAB under a personal computer with a $1.5GHz$ Intel Pentium M CPU processor and 1.00Go Random Access Memory. The simulation times, the number of expanded nodes to get the optimal path, and the path lengths were computed and compared, Table II. Regarding the simulation results, μ value of friction zone has been gradually increased on the same grid map and three scenarios were retained. The ability to generate energy-saving paths was assessed in the first and third scenarios while a good visibility of friction zones was evaluated in the second. The generated paths obtained with Liu & Sun's and proposed methods are smoothed, the energy and travel times are computed and compared, Table III.

TABLE II
SIMULATION RESULTS: SIMULATION TIME (AVERAGE OF 5 RUN), EXTENDED NODE, AND PATH LENGTH

Friction	Methods	time (s)	Node	Length (m)
0.015	Minimum distance	0.41	787	25.72
	Liu & Sun	0.68	792	30.74
	Proposed method	0.42(0.18)	359	30.25
0.01628	Minimum distance	0.39	787	25.72
	Liu & Sun	0.76	908	33.05
	Proposed method	0.40(0.18)	344	30.25
0.03005	Minimum distance	0.39	787	25.72
	Liu & Sun	0.72	890	33.05
	Proposed method	0.56(0.29)	396	33.05

In the first scenario, flat zone has a rolling coefficient of 0.013 and friction zones were set at 0.015 Fig. 5-(a). The basic A* algorithm for minimum travel distance found the optimal path in approximately $0.4s$ with 787 expended nodes. The robot moves through narrow spaces and passes too close to the surrounding obstacles. The friction zones are not considered and the displacements on these zones cause more energy consumption. The path generated based on Liu & Sun's method are found in approximately $0.68s$ with more expanded nodes and larger distance. The robot maintains a safe distance to surrounding obstacles, and can move with high velocity to consume less energy. The friction zones are considered to save energy. The optimal path generated by the proposed cost function (9) and heuristic function (10) is found in approximately $0.42s$ where the time required to compute heuristic function values indicated in parentheses is included Table II. The number of expanded nodes is less than the two other methods, which reduces the computation time. The robot also maintains a safe distance to surrounding obstacles. Fig. 5-(a) depicts the results obtained, the proposed method and Liu & Sun's method are almost identical; however, the proposed

TABLE III
SMOOTH PATH RESULTS

Friction	Method	Length (m)	Travel time(s)	Energy (J)
0.015	Liu & Sun	29.84	38.80	183.83
	Proposed	29.72	38.07	179.28
0.01628	Liu & Sun	32.25	42.52	195.24
	Proposed	29.73	38.96	180.60
0.03005	Liu & Sun	32.25	42.52	201.49
	Proposed	32.47	41.14	197.74

method gains approximately $0.50m$ of the path distance and 38% of the simulation time over Liu & Sun's method Table II.

In the second scenario, the rolling coefficient of friction zones was set at 0.01628 Fig. 5-(b). The optimal paths generated based on the shortest travel distance and the proposed approach remain unchanged. The optimal paths generated by Liu & Sun's method pass through the flat zone when they encounter the first friction zone. This is inherent to Liu & Sun's heuristic function which does not give good visibility of friction zones.

In the third scenario, the rolling coefficient of friction zones was set at 0.03005. The paths generated by Liu & Sun's method and the proposed approach are identical, Fig. 5-(c). However, the proposed method gains approximately 22% of the simulation time over Liu & Sun's method.

To evaluate the energy saving aspect of the proposed method, the paths obtained are smoothed using η^3-splines while considering the energy model (21) and (22). The maximum curvature has been fixed as the inverse of the half size of the robot dimension, and the segment joining the two waypoints is divided into 1000 pieces for a smoothed path, Fig. 5-(d) shows smoothed paths of second scenario. The smoothed path was divided into 100 elementary parts and the following parameters were used to find the optimal travel time: $-0.5m.s^{-2}$ and $0.5m.s^{-2}$ for minimal and maximal longitudinal accelerations, respectively, and $0.3m.s^{-2}$ for maximal normal acceleration. Table III shows the travel time and the energy consumption of both paths. The proposed method saves up to 8.3% of travel time and 7.5% of the energy over Liu & Sun's method in the second scenario.

CONCLUSION

An optimal motion planning aiming to achieve minimum energy consumption was proposed in this paper. A Three-wheeled Omnidirectional Mobile Robot (TOMR) named Robotino was used as a study case. The A* path planner and the heuristic function integrating energy saving criterion were used to generate energy-saved paths. The algorithm integrates previous robot orientations and as well a new heuristic function for path generation with minimum energy consumption. By using the waypoints of the generated path, the trajectory was smoothed through optimal η^3-Spline parameters. The velocity profile along the generated path is optimized by solving sequential quadratic problems. The effectiveness of the proposed motion planning was demonstrated by performing a series of simulations. The proposed energy saving approach showed a better performance compared with the existing methods. Future work will include an extension of the proposed approach to dynamic environments.

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