# Energy efficient path planning for low speed autonomous electric vehicle

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Abstract—This work presents an energy efficient approach for autonomous electric vehicle path planning. When the vehicle is moving at low speed, the rolling resistance losses can be more than the aerodynamic losses, on a flat ground. In particular, for warehouse low-speed electric vehicles, different road-tire frictions may lead to varying rolling resistance which impacts the energy consumption. The minimization of the energy consumed by a vehicle is important in the context where the number of charging stations is limited. The proposed method aims at planning the most energy efficient path by taking into account the rolling resistance and the path length. Unlike most studies reported in the literature, this energy efficient path planner can achieve a good trade-off between preserving battery energy and not extending to much the path length. The preliminary results obtained through extensive simulation indicates that the optimized path planner is effective and robust.

Index Terms—energy consumption; battery management; path planning; autonomous vehicles; rolling resistance; vehicle dynamics

## I. Introduction

URING the previous years, mobile robots and autonomous vehicles have witnessed a remarkable evolution. These vehicles play an unquestionable role in the military, manufacturing and industrial fields [1]. They are also getting more and more included in other fields of study. A lot of progress has been achieved in the navigational aspect and platform security. However, the autonomy remains energetically unexplored. Nonetheless, the batteries represent an important part of the cost of the equipment and its autonomy remains limited. In fact, the use of the batteries as a source of nourishment in a given electric vehicle raises several challenges such as the recharging period which is relatively long or the limited lifetime of the battery [2]. In order to contest with other conventional inner burning engines, there should be a development in these aspects [3]. An average daily use of these platforms usually produces a full discharge with negatively affect the batteries lifespan [4]. Actually, the lifetime of the battery is directly linked to the way it is used (the number or charge/discharge cycles and the depth-of-discharge) [4], [5]. It is recommended to have the battery in a given interval of Stateof-Charge (SOC) in order to 1) maximize its performance and its range 2) minimize its depth discharge (DOD) to optimize its performance and lifetime.

The number of charging stations is generally weak compared to the number of the functional electric vehicles in a warehouse. The solution that consists in increasing the

batteries capacities would boost the costs: in terms of vehicle costs, recharging energy costs, supplementary energy costs related to the weight of the batteries as well as the maintenance costs. Even if we opt for this solution and we increase the capacity of batteries, the risk of the vehicles failure to meet its objectives will be more probable if we do not take into account the environmental factors.

However, the level of the used energy is straightly linked to the path planning. It is fundamental to take into consideration the energy standard in path planning. Consequently, implementing a planner energetically optimal which will contest with traditional planners generally based on distance standard. Many parameters influence the used battery energy. They can be classified in two categories: (i) intrinsic parameters essentially linked to the structure of the platform: the type of the energy, the powertrain architecture, the mechanical structure and aerodynamic, etc; (ii) extrinsic parameters due to platform-environment interaction on one hand (Slope, friction, number of turn, etc) and on the other hand due to driving style. Since the intrinsic parameters cannot be modulated easily, understanding and quantifying the losses associated with the extrinsic parameters becomes necessary in order to identify key point for energy efficiency.

In this paper, we will be tackling the effects of rolling resistance for the purpose of making an energetically optimal path planning.

- Estimation of the consumed energy for the different subtrajectory in mind the change in the features of the land.
- 2) The use of an algorithm specialized in path planning with the energy standard.

Many researchers worked on the estimation of consumed energy through a platform either mobile robot or generally speaking electric vehicle [2], [6]. However, some people neglected crucial parameters for a good prediction such as the weight, the friction of the floor, etc, which affect the recharging planning and the success of the mission.

On the other hand, many researchers worked on path planning by considering the shortest path criteria for autonomous navigation. Few reported works have included the energetic cost associated the selection of trajectories. In fact, the majority of analysis only consider the shortest path as a performance indicator in order to move point to point [7]–[11].

The problem of the optimal path selection is one of the most typical problems in graph theory, including Floyd algorithm, Johnson algorithms, Dijkstra algorithm, etc. In this paper, Dijkstra's algorithm is used for its ability to explore the entire environment.

The rest of the paper is organized as follows: First, we describe the proposed approach. We explain in this section the representation of the environment, the discretization of space, the graph representation, the approach for solving the optimization problem. Then, in section III, we present the results and the discussion. Finally, in Section IV. we provide a short conclusion.

## II. ENERGY EFFICIENT PATH PLANNING

In this work, we deal with the off-line optimal path planning needed for guiding an autonomous vehicle to move from its actual position to the destination, to be used later by a controller system [12]. Among different frameworks available to tackle the planning problem, the graph representation has some nice features: synthetic representation between the navigation features and the reachable poses in the workspace. So in the sequel, we present how this representation is used to solve the energy optimal path planning problem.

## A. Graph representation

- 1) Space configuration: To find trajectory in a configuration space, discrete and continuous approaches are used in literature, but both share the same data and they only differ on how to find a solution. We consider the discrete space configuration of the vehicle, introduce in [13] constituted by all the triplets of values  $q_i = (x_i, y_i, \theta_i)$  where  $x_i, y_i$  and  $\theta_i$  represent the vehicle abscissa, ordinate and orientation, respectively. However, for simplification, we consider only a point representation of the vehicle using its pose  $q_i = (x_i, y_i)$ . Therefore the configuration space is the set of all reachable  $q_i$ .
- 2) Map representation: The configuration space is represented with a map and in this work, the metric map is used. An example of such a map is shown in Fig. (1). All cells have the same dimension (uniform cell decomposition). Cells occupied by obstacles have a black color where others have a white color. Therefore, the set of white cells is the workspace or the configuration space.

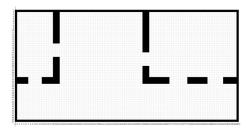


Fig. 1. Occupancy grid (matrix of 50\*100 cells)

3) Building a graph representation: We assume that the vehicle can move to any adjacent cell of the configuration space (a maximum of 8 possible moves at any time) [14]. By connecting each configuration space cell to its neighbors a lattice or a mathematical structure called a graph is constructed.

The unoccupied cells are nodes, and links between adjacent nodes are edges. The vehicle moves between nodes (the white dots on Fig.(2)) which represent the points reachable by the vehicle.

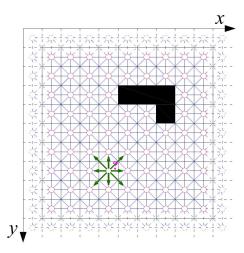


Fig. 2. Example of graph representation in the configuration space (each cell / node has Cartesian coordinates)

## B. Energy optimal path planning problem

Assume that the vehicle can move uniformly (constant speed in both coordinates axis) from one admissible cell to another following cell (node). Given:

- the graph representation of the configuration,
- the initial pose of the vehicle,
- the final or destination pose

how can we compute the path with the shortest traveling length (minimum time) and the minimum energy consumption?

Before presenting the optimization strategy, let introduce the adjacency matrix.

- 1) Adjacency matrix: In our approach, we tried to minimize two different criteria: distance criteria that also reflects traveling time and energy consumption criteria by considering the impact of the rolling resistance. So, we define two adjacency matrices  $A_D$  and  $A_E$  as follows:
  - If there is connexion between cells i (represented by  $q_i$ ) and j (represented by  $q_j$ ) then  $A_D(i,j)$  is the euclidean distance between  $q_i$  and  $q_j$ , otherwise  $A_D(i,j)$  is  $\infty$ .
  - If there is connexion between cells i and j then  $A_E(i, j)$  is the energy used by the vehicle to move from  $q_i$  to  $q_j$ .

Fig. 3 illustrates a distance based adjacency matrix in which a vertical or horizontal motion have a 10m distance, whereas a diagonal motion distance is  $10\sqrt(2) \approx 14$ . The left figure of Fig. 3 represents the computed distance-based graph.

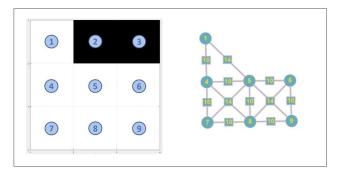


Fig. 3. Representation of distance-based graph: the left figure is the configuration space in which white cells are obstacle-free and black cells are occupied by obstacles. The right figure is the computed distance-based graph.

To build the energy-based adjacency matrix, the vehicle dynamic needs to be used and we consider the most used longitudinal dynamic model for ground vehicles. This model takes into account the tire/road rolling Resistance. Therefore, a rolling resistance coefficient map is required. For each cell in this paper, we assume that this rolling resistance coefficient exist. In [?], [6], methods for identifying the rolling resistance coefficient as the vehicle is moving are proposed.

2) Computing energy consumption between two adjacent points: The consumed energy estimation step is elaborated in order to build the energy-based adjacency matrix. From the configuration space, each cell rolling resistance coefficient  $\mu_i$  is retrieved. Assume that the speed is constant, the path from  $q_i$  to  $q_j$  is discretized into N small steps and k is the timestamp. Therefore, the electric energy consumed  $E_(i,j)$  to move from  $q_i$  to  $q_j$  is given by:

$$E_{i}(i,j)(k) = \sum_{n=0}^{N} P_{e}(k)\Delta t$$
 (1)

where  $\Delta t$  is the duration between two consecutive timestamps and where  $P_e$  represents the electric power delivered by the battery pack (see eq. (6)).

The vehicle discrete longitudinal dynamics [15] is:

$$M\dot{v}(k) = F_x(k) - F_a(k) - F_r(k) - F_a(k)$$
 (2)

where k, M and v represent the timestamp, the vehicle total mass and longitudinal speed and where  $F_x$ ,  $F_a$ ,  $F_g$  and  $F_r$  represent the longitudinal traction force provided by the powertrain, the aerodynamic force (see eq. (3)), the gravity force on a road slope (see eq. (4)) and the rolling resistance force (see eq. (5)), respectively.

The aerodynamic force defined in equation (3) is the force that opposes the vehicle's advancement in the air. It depends to the density of air  $\rho$  in  $kg/m^3$ , to the front surface of the vehicle  $A_v$  in  $m^2$ , drag coefficient  $C_d$  and the square of vehicle speed v.

$$F_a(k) = \frac{1}{2} \times \rho \times C_d \times A_v \times v^2(k)$$
 (3)

where  $\rho$ ,  $A_v$ ,  $C_d$  and v represent the density of air, the active front surface of the vehicle, the drag coefficient and the vehicle speed relatively to the air.

$$F_q(k) = M \times g \times \sin(\beta(k)) \tag{4}$$

where  $\beta$  is the road grade (road slope) and where g is the gravity constant.

$$F_r(k) = M \times g \times \mu(k) \times \cos(\beta(k)) \tag{5}$$

where  $\mu$  is the given rolling resistance at the vehicle position q(k).

$$P_e(k) = \eta F_x(k) \times v(k) \tag{6}$$

Using the rolling resistance coefficient map is shown in Fig. (5), we showed in Fig.(4) an example of energy-based adjacency matrix for the configuration space of Fig. (3).

To highlight the impact of the rolling resistance on the consumed energy, we showed in Fig. (5) an example of optimal path selection. It can be observed that the blue path is the shortest length path, however, the red path is the energy efficient one.

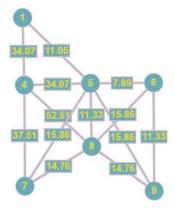


Fig. 4. Representation of energy-based graph using the rolling resistance map

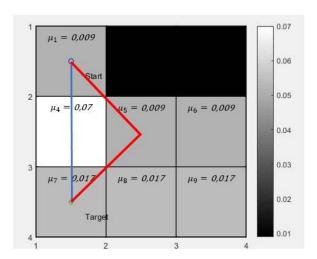


Fig. 5. An explanatory example of the coefficient of friction impact on the decision

# C. Solving the energy optimal path planning problem

Dijkstra's algorithm is the most classic and famous path planning algorithm based on the length criterion. It's an efficient algorithm to find the shortest path from a starting point to a destination point in a graph with non-negative weighted edges. The principle of this algorithm is to examine the sub-trajectories and to exclude the non-favorable ones until finding the most optimal trajectory [16], [17].

The adjacency matrix and other storage structures are used in the algorithm, for storing the weight information between each pair of adjacent nodes. The entries of this matrix often indicate weights between nodes. Initially, all weights (from each vertex at the starting vertex) are initialized to infinity, except for the starting vertex which initialized to zero. The weights here represent the energy consumed to pass from one cell to another  $(E_{i,j})$  by considering the dynamics of the robot and the considerations of the ground, as explained previously.

A subgraph is constructed, gradually, in which are classified the different vertices in ascending order with respect to their minimum weight from the starting vertex. The weight here is the sum of the costs of the selected edges. For each iteration, a minimum distance vertex is chosen outside the subgraph and added to the subgraph. Next, the cost of the vertices adjacent to the selected vertex is updated. This continues until the target vertex is selected.

## III. RESULTS AND DISCUSSIONS

We have considered a static metric navigation map with the obstacles represented in black. We have also considered the rolling resistance coefficient map (see the grayscale on each map on Fig.(6), Fig.(7), Fig.(8) and Fig.(9). Different type grounds are used: mud, grass, concrete, asphalt etc. The corresponding values of rolling resistance are presented in Table (I). We used the parameters of a small mobile robotic platform powered by a battery (see Table (II)).

In Fig.(6), Fig.(7), Fig.(8) and Fig.(9), the paths drawn with transparent circles presents the shortest path, whilst the path

 $\label{thm:table I} \textbf{TABLE I}$  Interval value of friction's coefficients of different areas

Interval value of friction's	0.07
coefficients of Mud zone	0.077
Interval value of friction's	0.044
coefficients of Grass zone	0.051
Interval value of friction's	0.017
coefficients of concrete zone	0.023
Interval value of friction's	0.014
coefficients of Asphalt zone	0.019

drawn in blue represents the most energetic path. These results show that if a path contains different types of grounds, the shortest path in not the energy efficient one.

TABLE II
PARAMETERS OF THE VEHICLE (MOBILE ROBOT)

Mass (M)	17 kg
Global car efficacity $(\eta)$	0.8
Wheels radius $(R_w)$	0.1 m
Air density $(\rho)$	$1.292 \ kg/m^3$
Rolling friction $(\mu)$ ,	{0.014;; 0.077}
Front area surface $(A_v)$	$0.077955 \ m^2$
Gravity (g)	9.81 $m/s^2$
Air speed $(V_{air})$	$0 \ m/s^2$
Drag Coefficient $(C_d)$	1.05

These results indicate that our planning method succeeds in going around the most difficult surfaces in order to reduce energy consumption. Whilst the traditional Dijkstra's algorithm succeeds to select the shortest path but fails to find the most energy efficient.

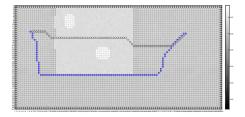


Fig. 6. Comparison of Dijkstra's approach and energy-efficient approach (scenario 1)



Fig. 7. Comparison of Dijkstra's approach and energy-efficient approach (scenario 2)

TABLE III SIMULATION RESULTS OF DIJKSTRA'S APPROACH AND ENERGY EFFICIENT APPROACH

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		Path 1	Path 2	Difference	ľ
Scenario 1	Energy consumption (J)	2341.75	1615.91	31 %	۱ [
	Distance (m)	803.85	1054.56	-31 %	] t
Scenario 2	Energy consumption (J)	1446.24	1252.49	13 %	1
	Distance (m)	683.55	851.42	-25 %	1.
Scenario 3	Energy consumption (J)	2090.36	1128.51	46 %	1
	Distance (m)	621.42	761.42	-23 %	
Scenario 4	Energy consumption (J)	747.79	633.62	15 %	t
	Distance (m)	143.14	154.85	-8 %	٦.

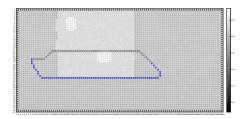


Fig. 8. Comparison of Dijkstra's approach and energy-efficient approach (scenario 3)

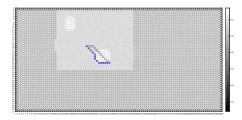


Fig. 9. Comparison of Dijkstra's approach and energy-efficient approach (scenario 4)

The comparative analysis between using Dijkstra method and energy-efficient approach is presented in Table (III). Scenario 2 (Fig.(7)) shows a limitation of our approach. Indeed, we notice a low energy gain compared to the high percentage of travel distance. On the other hand, scenario 3 shows a highenergy gain of 46% is observed. In Scenario 4, we assumed that there is a wall along the street (barred street) to test the ability of our tool to bypass the sludge. Here again, our method provides satisfactory results.

The proposed approach deals with global (offline) planning. Therefore, only static obstacles included in maps are considered. If a moving or unexpected obstacle appears or if the soil conditions change (e.g. winter context), a local re-planning must be carried out. In addition, other factors can influence energy consumption, such as slope and number of gyration. We will address these aspects in a future work.

## IV. CONCLUSION

In this work, we proved that the shortest path is not always the most energy efficient one. Since we know that at low speed, the energy due to rolling resistance is much more important that aerodynamic energy, it is, therefore, valuable to select the path accordingly. In addition, for an autonomous vehicle, an energy efficient path planner should take into account the obstacles. Using the optimal planning framework, we select the Dijkstra method as the basis of finding an energy-efficient path. Therefore, the proposed smart planning algorithm used:

- 1) The environment map which includes the workspace.
- 2) The environment rolling resistance map.
- 3) The vehicle dynamics.

Through extensive simulation, the preliminary results suggested that the proposed approach can select the energetically optimal path without increasing too much the whole motion duration. Furthermore, the flexibility of the path planner can be applied to all type autonomous vehicles for unmarked navigation environments either inside of a building (museum, hotel, store, etc.) or outside a building (desert, port, etc.). The future works will focus on integrating other criteria (battery degradation for an electric vehicle, path slope, etc.) to the smart path planner.

#### V. ACKNOWLEDGEMENT

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#### REFERENCES

- [1] M. József, "Concept of an UGV with Arduino device," 2014.
- [2] K. Maalej, S. Kelouwani, K. Agbossou, Y. Dubé, and N. Henao, "Long-Trip Optimal Energy Planning With Online Mass Estimation for Battery Electric Vehicles," IEEE Transactions on Vehicular Technology, vol. 64, no. 11, pp. 4929-4941, 2015.
- S. G. Wirasingha and A. Emadi, "Classification and Review of Control Strategies for Plug-In Hybrid Electric Vehicles," IEEE Transactions on Vehicular Technology, vol. 60, no. 1, pp. 111–122, 2011.
- [4] F. Martel, Y. Dubé, L. Boulon, and K. Agbossou, "Hybrid electric vehicle power management strategy including battery lifecycle and degradation model," in 2011 IEEE Vehicle Power and Propulsion Conference, 2011, pp. 1–8.
  [5] Usbattery, "No Title." [Online]. Available: http://usbattery.com/
- [6] A. Sadrpour, J. Jin, and A. G. Ulsoy, "Experimental validation of mission energy prediction model for unmanned ground vehicles," in American Control Conference (ACC), 2013. IEEE, 2013, pp. 5960-5965.
- [7] K. Al-Muteb, M. Faisal, M. Emaduddin, M. Arafah, M. Alsulaiman, M. Mekhtiche, R. Hedjar, H. Mathkoor, M. Algabri, and M. A. Bencherif, "An autonomous stereovision-based navigation system (ASNS) for mobile robots," Intelligent Service Robotics, vol. 9, no. 3, pp. 187-205, 2016.
- [8] H. Che, Z. Wu, R. Kang, and C. Yun, "Global path planning for explosion-proof robot based on improved ant colony optimization,' 2016 Asia-Pacific Conference on Intelligent Robot Systems (ACIRS), 2016, pp. 36-40.
- E. G. Tsardoulias, A. Iliakopoulou, A. Kargakos, and L. Petrou, "A Review of Global Path Planning Methods for Occupancy Grid Maps Regardless of Obstacle Density," Journal of Intelligent & Robotic Systems, vol. 84, no. 1, pp. 829-858, 2016.

- [10] Y. Zhang, G. Guan, and X. Pu, "The Robot Path Planning Based on Improved Artificial Fish Swarm Algorithm," *Mathematical Problems in Engineering*, vol. 2016, 2016.
- [11] W. Huijuan, Y. Yuan, and Y. Quanbo, "Application of Dijkstra algorithm in robot path-planning," in 2011 Second International Conference on Mechanic Automation and Control Engineering, 2011, pp. 1067–1069.
- [12] S. Kelouwani, C. Ouellette, and P. Cohen, "Stable and Adaptive Control for Wheeled Mobile," vol. 2013, no. November, pp. 391–405, 2013.
- [13] T. Perez-Lozano, "Spatial planning: A configuration space approach," IEEE Trans. Computers, vol. 32, 1983.
- [14] K.-Y. Chang, G. E. Jan, and I. Parberry, "A method for searching optimal routes with collision avoidance on raster charts," *The Journal* of Navigation, vol. 56, no. 03, pp. 371–384, 2003.
- [15] R. Rajamani, "Longitudinal Vehicle Dynamics," in Vehicle Dynamics and Control. Boston, MA: Springer US, 2006, pp. 95–122.
- [16] E. W. Dijkstra, "A note on two problems in connexion with graphs:(numerische mathematik, \_1 (1959), p 269-271)," 1959.
- [17] R. Bellman, "On the Theory of Dynamic Programming," Proceedings of the National Academy of Sciences of the United States of America, vol. 38, no. 8, pp. 716–719, 1952.