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Brief paper

Global κ-exponential way-point maneuvering of ships: Theory and experiments☆

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Abstract

The paper considers way-point maneuvering of ships with only two control inputs available (surge force and yaw moment). A line-of-sight (LOS) motivated control law that globally κ-exponentially stabilizes an underactuated ship in three degrees of freedom is derived using cascaded control theory. In addition to incorporating the nice features of the LOS algorithm, the control law is easy to tune using linear control techniques. Furthermore, experimental results with a model ship, scale 1:70, are presented to verify and illustrate the theory.

Keywords: Maneuvering; Nonlinear control; Cascaded theory; Experimental results

1. Introduction

The route of a ship is usually specified in terms of way-points (Fossen, 2002). The way-points are selected taking into account a number of features like weather conditions, obstacle avoidance and mission planning. Each way-point is defined in Cartesian coordinates (x_i, y_i) for $i = 1 \dots p$. The reference trajectory is then made up by straight lines connecting the p way-points. When the ship reaches a prescribed circle, the circle of acceptance, about way-point i, the reference trajectory is switched to the straight line connecting way-points i and $i+1$. Alternatively, circle arcs connecting the straight lines can be used to define the desired turn at each way-point. In this paper, we consider the problem of designing a controller that makes the ship follow the straight lines between way-points, and at the same time controls the forward speed to a possibly time-varying desired speed u_d, i.e. we consider a way-point maneuvering control problem.

The way-point maneuvering control problem can be defined by two control objectives. The first objective is to reach and follow the desired path given in Earth-fixed coordinates. The second control objective, *the speed assignment*, is to keep a prescribed speed along the body-fixed x-axis of the ship (Skjetne, Fossen, & Kokotovic, 2002). Note that in literature the terms path following, tracking and maneuvering are used interchangeably. *The path following problem* consists of stabilizing the system about a prescribed path, without any time specifications, i.e. without specifying when the ship is to be at a given point at this path. Following further the notion of Fossen (2002) and Skjetne et al. (2002), *the tracking problem* consists of forcing the system output to track a desired time-varying output. The output reference is then given as a time-varying function $y_\mathrm{d}(t)$, and steering the ship to this reference trajectory will typically include changes in the magnitude of the forward speed (acceleration and retardation will often be necessary in order to be at the right place at the right time). Large changes in the forward speed is however not preferred for most (in particular larger) ships. Ship tracking control is thus mainly used when it is necessary to coordinate the behavior of a number of ships, like during underway replenishment and formation control. *The maneuvering control problem* consists of solving two tasks. The first, *geometric task*, is to force the output to converge to a

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prescribed, parameterized path, $y_d(\theta(t))$, where θ is the path parameter. The second, *dynamic task*, is to force the speed to converge to a desired speed $v_s(\theta(t))$. In other words, the maneuvering control problem consists of making the ship converge to a given path, and at the same time solve a dynamic task which typically can be to keep a constant forward speed. Ship maneuvering is much used in commercial applications where it is more important to keep a constant speed than to specify when the ship is to be at a particular point along the route.

Conventional ships are usually equipped with one or two main propellers for forward speed control, and rudders for course control. Even if equipped with side thrusters, these will not give a significant force at transit speeds. This means that we only have independent controls in two degrees of freedom (DOF), surge and yaw, while we seek to control all three degrees of freedom (surge, sway and yaw) of the ship. The path following, tracking and maneuvering problems are hence underactuated control problems. In conventional way-point guidance systems, the output space is reduced such that the number of outputs equals the number of control inputs, and in this way a fully actuated control problem is obtained (Fossen, Breivik, & Skjetne, 2003). It is most often the yaw angle and surge velocity that are controlled. The desired yaw angle ψ_d is then usually chosen using a line-of-sight (LOS) algorithm, while the desired forward speed is constant. Recent research, however, aims at *controlling all 3DOF* of the ship using only the two available control inputs. Furthermore, while conventional way-point guidance systems usually are based on linear approximations of the ship models, recent research has focused on using models that also take into account the nonlinear effects. In addition to aiming at improving the control performance by *using more accurate models of the ship*, another important aim is to achieve global results, something which is highly preferable for practical purposes.

In Godhavn (1996), a 3DOF nonlinear model with the simplifying assumption that the sway and yaw dynamics are decoupled, i.e. having diagonal inertia and damping matrices, is considered. The output space is here reduced to 2DOF such that the tracking control problem is fully actuated. Using feedback linearization and backstepping, a controller is developed that guarantees global exponential stability of straight line and circle trajectories (under the mild assumption that the forward velocity is positive). Toussaint, Basar, and Bullo (2000b) generalizes the work of Godhavn (1996) to ships with two generalized forces. Also, here the output space is reduced to 2DOF, and systematically using vectorial backstepping a feedback control law is developed that stabilizes the system and provides global exponential convergence of the two outputs. A trajectory planning approach based on the perspective of Liouvillian systems is presented for the underactuated ship in Sira-Ramirez (1999). Full state, i.e. 3DOF, tracking control based on the first mentioned ship model is considered in Pettersen and Nijmeijer (1999, 2000, 2001), Lefeber, Pettersen, and Nijmeijer (2003) and Jiang (2002), where controllers are derived that stabilize both the position variables and the heading of the ship to the desired trajectory. However, the reference trajectories are here restricted to having a non-zero curvature, and hence these controllers cannot be used for straight-line tracking.

Various results are reported that use simplified 2DOF ship models. In Zhang, Chen, Zengqi, Fuchun, and Hanzhen (1998), the nonlinear ship model with diagonal matrices is further simplified by assuming that the forward velocity is kept constant by the main propellers, such that a 2DOF model describing the sway and yaw dynamics and kinematics can be used. Having only one control input, i.e. the rudder control, both 2DOF are controlled. Using coordinate transformations, input–output linearization and sliding mode control, a path following controller is developed that guarantees local asymptotic stability of the straight-line reference path. In Encarnação, Pascoal, and Arcak (2000), the path following of both straight lines and circles are addressed. Also, here a 2DOF nonlinear model with rudder control is considered. Furthermore, the impact of ocean currents on the ship are included in the model. The ship model is expressed in the Serret-Frenet frame, and using feedback linearization and backstepping a controller and a current estimator are developed that guarantee local convergence to the trajectory in the presence of constant, unknown currents. Indiveri, Aicardi, and Casalino (2000) consider the straight-line path following control problem. Also here, the forward speed is assumed to be kept constant by the main propellers, and a 2DOF nonlinear model with one (yaw) control input is considered. The inertia and damping matrices are assumed to be diagonal. A globally asymptotically stabilizing controller is developed by first defining a yaw velocity that will steer the ship parallel to a given velocity vector field, designed such that an ideal point moving at this velocity will converge to the desired straight line, and then designing a yaw moment control law by integrator backstepping of the yaw velocity. It is furthermore shown how sliding mode techniques can be used to deal with modelling parameter uncertainties. Also in Pettersen and Lefeber (2001), the path following for the 2DOF nonlinear model with diagonal matrices and one control input is considered. The approach is motivated by LOS methods, which are much used in way-point ship control practice, but which are ad hoc methods for which, to the authors' best knowledge, stability and convergence have not been proved. Defining the desired ship course angle as a LOS motivated nonlinear function of the cross-track error (the cross-track error is the shortest distance to the path, i.e. normal to the path), and furthermore using coordinate transformations and cascaded systems theory, a controller is developed that guarantees global asymptotic stability of the straight-line path. In Do, Jiang, and Pan (2003), straight-line path following for the 2DOF model is considered, with the relaxed assumption that the forward speed is allowed to be time varying as long as its derivative is bounded. Furthermore, environmental disturbances are taken into account in the control development. Based on the backstepping technique and some technical lemmas introduced for nonlinear systems with non-vanishing disturbances, a controller is developed that guarantees global asymptotic stability of the straight-line trajectory in the case without disturbances, and global ultimate boundedness of the tracking errors in the presence of environmental disturbances. This result is furthermore extended to output feedback control.

Several results are reported that achieve full state control based on 3DOF ship models with diagonal matrices. Trajectory tracking control of the 3DOF model with diagonal inertia and damping matrices are addressed in Do, Jiang, and Pan (2002a,b). In Do et al. (2002a) Lyapunov techniques are used to develop a control law that makes the ship globally asymptotically track a reference trajectory generated by a virtual ship. The reference trajectories that can be generated include straight lines and curves. When the reference trajectory is not a straight line, and the yaw angle satisfies a persistently exciting condition, then global κ-exponential stability is guaranteed. In Do et al. (2002b) this result is extended to a universal controller that solves both the problem of ship stabilization and tracking simultaneously. This controller is time varying since the ship model with zero surge and yaw reference velocities does not satisfy Brockett's necessary condition, and hence cannot be stabilized by time-invariant state feedback (Brockett, 1983; Pettersen, 1996; Zabczyk, 1989). Toussaint, Basar, and Bullo (2000a) consider trajectory tracking for the 3DOF ship model with diagonal matrices, with the addition that the effects of disturbances are added. The output space is three dimensional, and the reference trajectory is generated by a reference model copying the system dynamics without disturbances. Linearizing the equations of motion about the reference trajectory and applying H^{∞}-optimal design tools, a controller is developed that guarantees local exponential stability. In Behal, Dawson, Dixon, and Fang (2002), the maneuvering problem is considered. The desired forward speed is constant and the reference curve is constructed by a reference trajectory generator that can generate both straight lines and circles amongst others. The 3DOF nonlinear model is also here simplified by assuming that the inertia matrix is diagonal. Using coordinate transformations and Lyapunov theory, a continuous, time-varying controller is developed that guarantees global exponential practical stability. These results are extended to the adaptive case in Behal, Dawson, Xian, and Setlur (2001), where a continuous time-varying controller and a parameter estimation law are developed that ensure that the error variables are ultimately confined to a ball that can be made arbitrarily small, in the presence of uncertainty in the hydrodynamic damping coefficients.

Recent research has aimed at developing results that are valid for the full ship model without the simplifying assumptions of having diagonal damping and inertia matrices. For these matrices to be diagonal, the ship should have both port/starboard and fore/aft symmetry. Most ships have port/starboard symmetry, but they do not have symmetry fore/aft, and their models will hence include off-diagonal matrix elements. Control design in the general case where also the non-diagonal matrix elements are taken into account is trivial in the fully actuated case, while it is an active topic of research in the underactuated case. In Do and Pan (2003b), the 2DOF model without the simplifying assumption of diagonal matrices is considered. Using coordinate transformations and Lyapunov theory, a controller is developed that under given conditions on the inertia and damping matrices, gives global κ-exponential stability of the straight-line reference trajectory. The control scheme is also extended to output feedback control. In Do and Pan (2003a), trajectory tracking control of the 3DOF ship model with non-diagonal inertia and damping matrices is considered. Using coordinate transformations and backstepping techniques, a controller is developed that gives global κ-exponential stability of a transformed tracking error, something which guarantees that the actual position and orientation κ-exponentially converge to a neighborhood about the reference trajectory. Also in Fossen et al. (2003), the full 3DOF ship model with non-diagonal matrices is considered, while the output space is reduced from 3 to 2DOF. The way-point maneuvering problem is considered, with the objectives of forcing the yaw angle to converge to a commanded LOS heading and the forward speed to a desired time-varying speed reference. The backstepping technique is used together with a new idea of assigning dynamics to the stabilizing function of the unactuated DOF. The resulting controller becomes a dynamic feedback controller that guarantees stability and global asymptotic convergence in 2DOF.

The purpose of this paper is to develop a LOS motivated way-point maneuvering controller based on a general 3DOF nonlinear ship model, giving global results and exponential convergence properties in all 3DOF. Specifically, using the same LOS motivated nonlinear course angle reference as in Pettersen and Lefeber (2001), we develop a controller that incorporates the nice features of the LOS algorithms that are much used in ship control practice. As opposed to Pettersen and Lefeber (2001), however, we consider the general 3DOF ship model of Fossen (2002). In particular, we do not make the assumption of fore/aft symmetry (that would imply that the sway and yaw dynamics were decoupled), and the system matrices are hence non-diagonal. In addition, the model takes into account that a deflection of the rudder gives a sway force in addition to the desired yaw moment used for course control, something which complicates the control design. This gives the problem of stabilizing an underactuated system, where one of the controls also affects the unactuated dynamics. We use coordinate transformations to decouple the unactuated dynamics from the controls, and thus facilitate a cascaded systems structure. Cascaded systems theory is then used to develop surge propeller force and rudder control laws that give global κ-exponential stability of the closed-loop system. We furthermore present experimental results to verify the theoretical results and to illustrate the advantages and the shortcomings of the theory.

The paper is organized as follows: in Section 2 the nonlinear ship model is presented along with the coordinate transformations used to facilitate the cascaded control design. In Section 3, a full state feedback control law is derived, and the closed-loop system is proved to be globally κ-exponentially stable. Experimental results are presented in Section 4 and conclusions are given in Section 5.

2. The ship model and control objective

We consider the ship model (Fossen, 2002)

$$\mathbf{M}\dot{\boldsymbol{v}} + \mathbf{C}(\boldsymbol{v})\boldsymbol{v} + \mathbf{D}\boldsymbol{v} = \boldsymbol{\tau}, \tag{1}$$

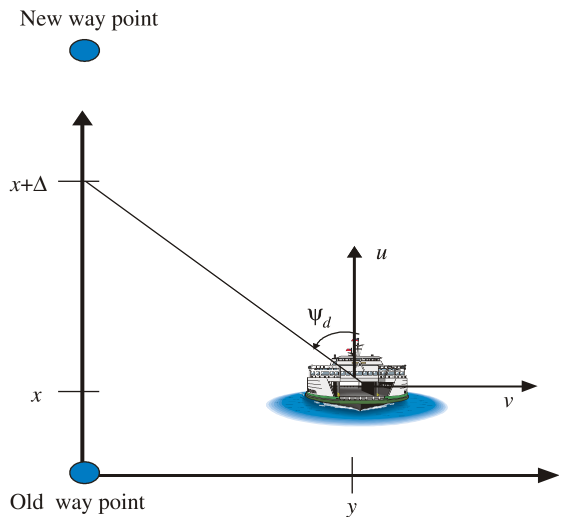

Fig. 1. An interpretation of ψ_{d} and Δ.

where $\boldsymbol{v}=[u, v, r]^{\mathrm{T}}$ denotes the surge, sway and yaw velocities. The control vector $\boldsymbol{\tau}=[\tau_u, Y_\delta\delta, N_\delta\delta]^{\mathrm{T}}$, where τ_u is the control force in surge, while $Y_\delta\delta$ and $N_\delta\delta$ are rudder force and moment affecting sway and yaw, respectively. The system matrices are given as

$$\mathbf{M}=\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}, \quad \mathbf{D}=\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \tag{2}$$

$$\mathbf{C}(\boldsymbol{v})=\begin{bmatrix} 0 & 0 & -m_{22}v-m_{23}r \\ 0 & 0 & m_{11}u \\ m_{22}v+m_{23}r & -m_{11}u & 0 \end{bmatrix}, \tag{3}$$

where the matrices $\mathbf{M}$, $\mathbf{C}(\boldsymbol{v})$ and $\mathbf{D}$ are the inertia, Coriolis/centripetal and damping matrices, respectively.

We place the origin of the Earth-fixed coordinate system in the previous way-point, with its x-axis in the direction of the current way-point, see Fig. 1. The cross-track error then equals the sway position y of the ship, and the ship is parallel to the straight-line trajectory when $\psi=0$. The kinematics is then given by

$$\dot{y}=\sin(\psi)u+\cos(\psi)v, \tag{4}$$

$$\dot{\psi}=r. \tag{5}$$

The control objective is to solve the way-point maneuvering problem consisting of a geometric and a dynamic task. We want the ship to track the straight line $y=0$ between two way-points with its heading parallel to the straight line trajectory, i.e. $\psi=0$. At the same time, we want the ship to keep a desired constant forward speed $u_{\mathrm{d}}>0$. This desired surge speed is both upper and lower bounded. Note that $u_{\mathrm{d}}=0$ would give a stabilization control problem instead of a tracking control problem, and this is not solvable using static state feedback control (Pettersen & Egeland, 1996). Also note that we are not interested in controlling the Earth-fixed position coordinate x as we would if trajectory tracking was the control objective. Under maneuvering control, it is considered more important that the ship keeps a constant desired forward speed than controlling where along this path (the x-coordinate) the ship should be at any given time ($x_{\mathrm{d}}(t)$). The control problem is therefore

The control problem. Find feedback control laws

$$\tau_u=\tau_u(y, \psi, u, v, r),$$

$$\delta=\delta(y, \psi, u, v, r)$$

such that $(y, \psi, u, v, r)=(0, 0, u_{\mathrm{d}}, 0, 0)$ is an asymptotically stable equilibrium point of (1), (4)–(5).

To facilitate the cascaded control design, we define some coordinate transformations. First, we shift the desired equilibrium point to the origin. In particular, we transform the surge velocity to the surge deviation, which can be written as

$$\bar{u}=u-u_{\mathrm{d}}. \tag{6}$$

More importantly for the cascaded control design, we note that (1), (4)–(5) is an underactuated system, where one of the controls also affects the unactuated dynamics. In particular, the system is underactuated as there is only two independent controls, τ_u and δ, to control the motion in surge, sway and yaw. Specifically, the ship is not equipped with a side thruster, such that there is no independent sway control force. The sway dynamics is hence the unactuated dynamics. The rudder, which mainly affects the course (yaw) dynamics, also influences the unactuated sway dynamics. This complicates the control design in general, and in particular it destroys the cascaded structure that we seek in the system equations. To decouple the transformed sway dynamics from the rudder control, we use the following coordinate transformation, inspired by Do and Pan (2003a), which removes the effect from δ in the $\bar{v}$-dynamics:

$$\bar{x}=x+\varepsilon\cos\psi, \tag{7}$$

$$\bar{y}=y+\varepsilon\sin\psi, \tag{8}$$

$$\bar{v}=v+\varepsilon r, \tag{9}$$

where

$$\varepsilon=-\left(\frac{m_{33}Y_\delta-m_{23}N_\delta}{m_{22}N_\delta-m_{23}Y_\delta}\right). \tag{10}$$

This corresponds to moving the origin along the x-axis of the body-fixed coordinate system to that point where the rudder gives only a rotational moment and no sway force. The transformed system equations are

$$\dot{\bar{y}}=\sin(\psi)(\bar{u}+u_{\mathrm{d}})+\cos(\psi)\bar{v}, \tag{11}$$

$$\dot{\psi}=r, \tag{12}$$

$$\begin{aligned}\dot{\bar{v}}&=\dot{v}+\varepsilon\dot{r}\\&=(\Upsilon\bar{u}+\Theta u_{\mathrm{d}}+M)r+(\Lambda\bar{u}+\Xi u_{\mathrm{d}}+N)\bar{v},\end{aligned} \tag{13}$$

$$\dot{r}=\frac{\delta}{\Gamma}(m_{22}N_\delta-m_{23}Y_\delta)+\Omega r+\mathsf{F}\bar{v}, \tag{14}$$

$$\dot{\bar{u}}=\frac{1}{m_{11}}(\tau_u+(m_{22}v+m_{23}r)r-d_{11}u), \tag{15}$$

where

$$\Gamma = m_{22}m_{33} - m_{23}^2 > 0, \tag{16}$$

$$\Upsilon = \frac{1}{\Gamma}(-2m_{23}m_{22}\varepsilon + m_{22}^2\varepsilon^2 + m_{23}^2 - m_{22}m_{11}\varepsilon^2 - m_{33}m_{11} + 2m_{23}m_{11}\varepsilon), \tag{17}$$

$$\Theta = \frac{1}{\Gamma}(-2m_{23}m_{22}\varepsilon + m_{22}^2\varepsilon^2 + m_{23}^2 - m_{22}m_{11}\varepsilon^2 - m_{33}m_{11} + 2m_{23}m_{11}\varepsilon), \tag{18}$$

$$M = \frac{1}{\Gamma}(m_{23}d_{33} - \varepsilon m_{22}d_{33} - m_{33}d_{23} - m_{23}d_{32}\varepsilon + \varepsilon m_{23}d_{23} + m_{22}d_{32}\varepsilon^2 + m_{33}d_{22}\varepsilon - m_{23}d_{22}\varepsilon^2), \tag{19}$$

$$\Lambda = \frac{1}{\Gamma}(m_{22}m_{11}\varepsilon - m_{22}^2\varepsilon - m_{23}m_{11} + m_{22}m_{23}), \tag{20}$$

$$\Xi = \frac{1}{\Gamma}(m_{22}m_{11}\varepsilon - m_{22}^2\varepsilon - m_{23}m_{11} + m_{22}m_{23}), \tag{21}$$

$$N = \frac{1}{\Gamma}(m_{23}d_{22}\varepsilon - m_{22}d_{32}\varepsilon - m_{33}d_{22} + m_{23}d_{32}), \tag{22}$$

$$\Omega = \frac{1}{\Gamma}(m_{23}m_{11}(\bar{u} + u_{\mathrm{d}}) + m_{22}^2(\bar{u} + u_{\mathrm{d}})\varepsilon - m_{23}d_{22}\varepsilon + m_{23}d_{23} - m_{22}m_{11}(\bar{u} + u_{\mathrm{d}})\varepsilon - m_{22}(\bar{u} + u_{\mathrm{d}})m_{23} - m_{22}d_{33} + m_{22}d_{32}\varepsilon), \tag{23}$$

$$\mathsf{F} = \frac{1}{\Gamma}(m_{23}d_{22} - m_{22}^2(\bar{u} + u_{\mathrm{d}}) - m_{22}d_{32} + m_{22}m_{11}(\bar{u} + u_{\mathrm{d}})). \tag{24}$$

We have now obtained a system where a rudder deflection δ used for course control will affect one equation only (14). Forward thrust τ_u is available for surge control, and the next section treats the control design for δ and τ_u.

3. Problem statement and control law

Finding control laws for δ and τ_u that stabilize the origin of (11)–(15) solves the control problem stated in Section 2. In this section, we propose control laws which produce a closed-loop system consisting of two exponentially stable subsystems. Cascaded theory is then used to show that the origin of the resulting closed-loop system is globally κ-exponentially stable. It is furthermore a goal of this paper to derive a controller that mimics the actions of a helmsman, who typically will pick a point at the path a certain distance Δ ahead of the ship (along the path), see Fig. 1, that he will aim at, and hence choose the desired heading ψ_{d} as a function of both the cross-track error and Δ. Moreover, we want the ship heading to stay within certain bounds, typically $[-\pi/2, \pi/2]$, and we therefore choose (Pettersen & Lefeber, 2001)

$$\psi_{\mathrm{d}} = -\arctan\left(\frac{\bar{y}}{\Delta}\right). \tag{25}$$

We conjecture that a control law δ designed to make ψ converge to the LOS motivated $\psi_{\mathrm{d}} = -\arctan(\bar{y}/\Delta)$ will stabilize both the sway and yaw dynamics, and thus together with a stabilizing surge control law solve the control problem posed in Section 2.

The yaw error variables are then

$$z_1 = \psi - \psi_{\mathrm{d}} = \psi + \arctan\left(\frac{\bar{y}}{\Delta}\right), \tag{26}$$

$$z_2 = \dot{z}_1 \tag{27}$$

and the corresponding kinematic and dynamic equations are

$$\dot{z}_1 = \dot{\psi} + \frac{\dot{\bar{y}}\Delta}{\Delta^2 + \bar{y}^2} = r + \frac{\dot{\bar{y}}\Delta}{\Delta^2 + \bar{y}^2}, \tag{28}$$

$$\dot{z}_2 = \frac{\delta}{\Gamma}(m_{22}N_\delta - m_{23}Y_\delta) + \Omega r + \mathsf{F}\bar{v} + \left(\frac{\Delta\ddot{\bar{y}}}{\Delta^2 + \bar{y}^2} - \frac{2\Delta\bar{y}(\dot{\bar{y}})^2}{(\Delta^2 + \bar{y}^2)^2}\right). \tag{29}$$

The following control laws τ_u and δ are chosen with the objective of obtaining a cascaded systems structure and exponentially stable subsystems:

$$\tau_u = -(m_{22}v + m_{23}r)r + d_{11}u - m_{11}k_u\bar{u}, \tag{30}$$

$$\delta = \frac{\Gamma}{(m_{22}N_\delta - m_{23}Y_\delta)}\left(-\Omega r - F\bar{v} + \frac{2\Delta\bar{y}(\dot{\bar{y}})^2}{(\Delta^2 + \bar{y}^2)^2} - \frac{\Delta\ddot{\bar{y}}}{\Delta^2 + \bar{y}^2} - k_1 z_2 - k_0 z_1\right). \tag{31}$$

Remark 1. Note that the constant $(m_{22}N_\delta - m_{23}Y_\delta)/\Gamma$ is nonzero. In particular, it can be seen from (1) that this is the coefficient of δ in the yaw dynamics. Therefore, if this was zero this would mean that the rudder had no effect on the ship course. This is of course unfeasible for any practical ship. For any rudder-controlled ship, the control law (31) thus has no singularities.

With this choice of control laws the resulting closed-loop system can be written as

$$\dot{\bar{y}} = \frac{\sin(z_1)(\Delta(\bar{u} + u_{\mathrm{d}}) + \bar{y}\bar{v})}{\sqrt{\Delta^2 + \bar{y}^2}} + \frac{\cos(z_1)(\Delta\bar{v} - (\bar{u} + u_{\mathrm{d}})\bar{y})}{\sqrt{\Delta^2 + \bar{y}^2}}, \tag{32}$$

$$\dot{\bar{v}} = (\Lambda\bar{u} + \Xi u_{\mathrm{d}} + N)\bar{v} + (\Upsilon\bar{u} + \Theta u_{\mathrm{d}} + M)\left(z_2 - \frac{\dot{\bar{y}}\Delta}{\Delta^2 + \bar{y}^2}\right), \tag{33}$$

$$\dot{z}_1 = z_2, \tag{34}$$

$$\dot{z}_2 = -k_1 z_2 - k_0 z_1, \tag{35}$$

$$\dot{\bar{u}} = -k_u\bar{u}. \tag{36}$$

We have then obtained a cascaded structure, which is easily seen when we rewrite the system as

$$
\begin{bmatrix} \dot{\bar{y}} \\ \dot{\bar{v}} \end{bmatrix} = \begin{bmatrix} -\dfrac{u_{\mathrm{d}}}{\sqrt{\Delta^2+\bar{y}^2}} & \dfrac{\Delta}{\sqrt{\Delta^2+\bar{y}^2}} \\ \left(X\dfrac{\Delta u_{\mathrm{d}}}{\left(\sqrt{\Delta^2+\bar{y}^2}\right)^3} \right) & \left(Y - X\dfrac{\Delta^2}{\left(\sqrt{\Delta^2+\bar{y}^2}\right)^3} \right) \end{bmatrix} \times \begin{bmatrix} \bar{y} \\ \bar{v} \end{bmatrix} + \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \bar{u} \end{bmatrix}, \tag{37}
$$

$$
\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{\bar{u}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_0 & -k_1 & 0 \\ 0 & 0 & -k_u \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \bar{u} \end{bmatrix}, \tag{38}
$$

where $X = (\Theta u_{\mathrm{d}} + M)$, $Y = (\Xi u_{\mathrm{d}} + N)$ and

$$
h_{11} = \frac{\sin(z_1)}{z_1}\frac{\Delta(\bar{u}+u_{\mathrm{d}})+\bar{y}\bar{v}}{\sqrt{\Delta^2+\bar{y}^2}} + \frac{1-\cos(z_1)}{z_1}\frac{u_{\mathrm{d}}\bar{y}-\Delta\bar{v}}{\sqrt{\Delta^2+\bar{y}^2}}, \tag{39}
$$

$$
h_{12} = 0, \tag{40}
$$

$$
h_{13} = -\cos(z_1)\frac{\bar{y}}{\sqrt{\Delta^2+\bar{y}^2}}, \tag{41}
$$

$$
\begin{aligned}
h_{21} &= -\left(\frac{\Delta}{\Delta^2+\bar{y}^2}(\Theta u_{\mathrm{d}} + M)h_{11} + \Upsilon\frac{\Delta}{\Delta^2+\bar{y}^2}\bar{u}h_{11} \right) \\
&= \frac{-\Delta}{\Delta^2+\bar{y}^2}((\Theta u_{\mathrm{d}} + M)h_{11} + \Upsilon\bar{u}h_{11}),
\end{aligned} \tag{42}
$$

$$
h_{22} = (\Upsilon\bar{u} + \Theta u_{\mathrm{d}} + M), \tag{43}
$$

$$
\begin{aligned}
h_{23} = &-\left(\frac{\Delta}{\Delta^2+\bar{y}^2}(\Theta u_{\mathrm{d}} + M)h_{13} - \Lambda\bar{v} \right. \\
&-\Upsilon\frac{\Delta}{\left(\sqrt{(\Delta^2+y^2)}\right)^3}u_{\mathrm{d}}\bar{y} + \Upsilon\frac{\Delta^2}{\left(\sqrt{(\Delta^2+y^2)}\right)^3}\bar{v} \\
&\left. -\Upsilon\frac{\Delta}{\Delta^2+\bar{y}^2}h_{13}\bar{u} \right) \\
= &-\frac{\Delta}{\Delta^2+\bar{y}^2}\left((\Theta u_{\mathrm{d}} + M)h_{13} - \Upsilon\frac{u_{\mathrm{d}}\bar{y}}{\sqrt{(\Delta^2+y^2)}} \right. \\
&\left. +\Upsilon\frac{\Delta\bar{v}}{\sqrt{(\Delta^2+y^2)}} + \Upsilon h_{13}\bar{u} \right) + \Lambda\bar{v}.
\end{aligned} \tag{44}
$$

Remark 2. It can be seen that the control laws (30) and (31) are easy to tune using linear control techniques applied to (38). Furthermore, note that this control approach is straightforward to transform to a control law for ships equipped with thrusters (instead of rudders), where the couplings in sway and yaw will give similar effects in the sway dynamics in addition to the desired yaw moment used for course control.

Remark 3. In the following we will assume that X and Y are negative. This gives the following restrictions on the desired surge speed:

$$
X = (\Theta u_{\mathrm{d}} + M) < 0 \tag{45}
$$

$$
Y = (\Xi u_{\mathrm{d}} + N) < 0. \tag{46}
$$

How hard these constraints on u_{d} are will depend on the specific ship. For most ships $\Theta, \Xi \leqslant 0$, such that (45)–(46) give lower bounds on u_{d}.

The cascaded system (37)–(38) can be written in a more compound form

$$
\dot{\mathbf{x}} = f_1(t, \mathbf{x}) + g(t, \mathbf{x}, \xi)\xi, \tag{47}
$$

$$
\dot{\xi} = f_2(t, \xi), \tag{48}
$$

where $\mathbf{x} = [\bar{y}, \bar{v}]^{\mathrm{T}}$ and $\xi = [z_1, z_2, \bar{u}]^{\mathrm{T}}$.

If the origin of (37)–(38) is asymptotically stable, then the control problem stated in Section 2 is solved. In the following we will show that both subsystems $\dot{\mathbf{x}} = f_1(t, \mathbf{x})$ and $\dot{\xi} = f_2(t, \xi)$ are exponentially stable, and that the origin of the total system (37)–(38) is globally κ-exponentially stable.

Remark 4. κ-exponential stability is defined in Sørdalen and Egeland (1995). As noted in Lefeber (2000) this is equivalent to LUES + GUAS.

Proposition 5. *The origin* $(\bar{y}, \bar{v}, z_1, z_2, \bar{u}) = (0, 0, 0, 0, 0)$ *of the system* (37)–(38) *with control laws* (30) *and* (31), *control parameters* $k_0, k_1, k_u > 0$ *and* Δ *satisfying* $\Delta > (\sqrt{2}+1/2)(X/Y)$ *and* $\Delta > (X + u_{\mathrm{d}})/Y$ *with* $X, Y < 0$ *is globally* κ*-exponentially stable*.

Proof. To prove the above proposition, we use Panteley and Loria (1998, Theorem 2) and Panteley, Lefeber, Loria, and Nijmeijer (1998, Lemma 8). We will first show that the interconnection term $g(t, \mathbf{x}, \xi)\xi$ has linear growth in $\mathbf{x}$. This can be seen from

$$
\begin{aligned}
\|h_{11}z_1 + h_{12}z_2 + h_{13}\bar{u}\| \leqslant{}& (2\|u_{\mathrm{d}}\| + 1)\|\xi\| + \|\xi\|^2 \\
&+ 2\|\xi\|\|\mathbf{x}\|,
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
&\|h_{21}z_1 + h_{22}z_2 + h_{23}\bar{u}\| \\
&\quad\leqslant |\Upsilon|\|\xi\|^3 + (|\Theta|\|u_{\mathrm{d}}\| + |M| + 2|\Upsilon|\|u_{\mathrm{d}}\| + 2|\Upsilon|)\|\xi\|^2 \\
&\qquad + (2|\Theta|\|u_{\mathrm{d}}^2\| + 2|M|\|u_{\mathrm{d}}\| + 2|\Theta|\|u_{\mathrm{d}}\| \\
&\qquad + 2|M| + |\Upsilon|\|u_{\mathrm{d}}\|)\|\xi\| + (2|\Upsilon|\|\xi\|^2 + (2|\Theta|\|u_{\mathrm{d}}\| \\
&\qquad + 2|M| + (|\Upsilon| + |\Lambda|))\|\xi\|)\|\mathbf{x}\|.
\end{aligned} \tag{50}
$$

The linearity assumption A2 of Panteley and Loria (1998, Theorem 2) is thus satisfied.

The subsystem (48) is globally uniformly exponentially stable. In particular, this is a linear time-invariant system. Since $k_0, k_1, k_u > 0$, the system matrix is Hurwitz, something which guarantees global (uniform) exponential stability of subsystem

(48). Assumption A3 of Panteley and Loria (1998, Theorem 2) is thus satisfied. We will furthermore show that the system (47) with $\boldsymbol{\xi}=\mathbf{0}$, i.e. the system $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is globally κ-exponentially stable, under given conditions on Δ.

We first show global uniform asymptotic stability of $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$, by using the Lyapunov function candidate (LFC)

$$W=\frac{1}{2}\bar{y}^2+\frac{\gamma}{2}\bar{v}^2, \tag{51}$$

where $\gamma=-2X(1-\alpha)/\alpha^3Y^2u_d>0,\quad 0<\alpha<1$. (Note that $X<0$.) The time derivative along the trajectories of $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is

$$\begin{aligned}\dot{W}&=\bar{y}\dot{\bar{y}}+\gamma\bar{v}\dot{\bar{v}}\\ &=\bar{y}\left(\frac{-u_d}{\sqrt{\Delta^2+\bar{y}^2}}\bar{y}+\frac{\Delta}{\sqrt{\Delta^2+\bar{y}^2}}\bar{v}\right)\\ &\quad+\gamma\bar{v}\left(X\frac{\Delta u_d}{\left(\sqrt{(\Delta^2+\bar{y}^2)}\right)^3}\bar{y}\right.\\ &\quad\left.+\left(Y-X\frac{\Delta^2}{\left(\sqrt{(\Delta^2+\bar{y}^2)}\right)^3}\right)\bar{v}\right)\\ &=\frac{-\alpha Y\Delta u_d}{X\sqrt{(\bar{y}^2+\Delta^2)}}\left(\bar{y}-\frac{1}{2}\left(\frac{X}{\alpha Yu_d}+\frac{\gamma\alpha Y\Delta^2}{(\bar{y}^2+\Delta^2)}\right)\bar{v}\right)^2\\ &\quad-\left(-\gamma Y+\frac{1}{2}\frac{\gamma\alpha Y\Delta^3}{(\bar{y}^2+\Delta^2)^{3/2}}-\frac{1}{4\alpha Y}\frac{\Delta}{u_d}\frac{X}{\sqrt{(\bar{y}^2+\Delta^2)}}\right.\\ &\quad\left.-\frac{1}{4}\alpha^3Y^3\Delta^5\frac{u_d}{X\left(\sqrt{(\bar{y}^2+\Delta^2)}\right)^5}\gamma^2\right)\bar{v}^2,\end{aligned} \tag{52}$$

where $\alpha=X/\Delta Y$.

Note that $0\leqslant\Delta/\sqrt{(\bar{y}^2+\Delta^2)}\leqslant 1\ \forall\bar{y}$. With negative X and Y, the derivative of W thus satisfies

$$\begin{aligned}\dot{W}&\leqslant\frac{-\alpha Y\Delta u_d}{X\sqrt{(\bar{y}^2+\Delta^2)}}\left(\bar{y}-\frac{1}{2}\left(\frac{X}{\alpha Yu_d}+\frac{\gamma\alpha Y\Delta^2}{(\bar{y}^2+\Delta^2)}\right)\bar{v}\right)^2\\ &\quad-\left(-\gamma Y+\frac{1}{2}\gamma\alpha Y-\frac{X}{4\alpha Yu_d}-\frac{1}{4}\alpha^3Y^3\frac{u_d}{X}\gamma^2\right)\bar{v}^2.\end{aligned} \tag{53}$$

We can conclude that $\dot{W}$ is negative definite if

$$-\gamma Y+\frac{1}{2}\gamma\alpha Y-\frac{X}{4\alpha Yu_d}-\frac{1}{4}\alpha^3Y^3\frac{u_d}{X}\gamma^2>0. \tag{54}$$

Recalling that $0<\alpha<1$ we see that this is equivalent to

$$\begin{aligned}&1-\alpha-\frac{1}{4}\alpha^2>0,\\ &\Downarrow\\ &0<\alpha<-2+2\sqrt{2},\\ &\Downarrow\\ &\Delta>\frac{\sqrt{2}+1}{2}\frac{X}{Y}.\end{aligned} \tag{55}$$

Hence, when (55) is satisfied $\dot{W}$ is upper bounded by a continuous, negative definite function, and we have thus shown that the origin of $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is GUAS. We will furthermore show that the origin of $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is locally uniformly exponentially stable under given conditions on Δ. We investigate its local stability properties by considering its linearization about the origin:

$$\dot{\mathbf{x}}=\begin{bmatrix}-\frac{u_d}{\Delta} & 1\\ X\frac{u_d}{\Delta^2} & Y-\frac{X}{\Delta}\end{bmatrix}\mathbf{x}. \tag{56}$$

The system matrix A is Hurwitz if

$$\mathrm{Tr}(A)=-\frac{u_d}{\Delta}+Y-\frac{X}{\Delta}<0, \tag{57}$$

$$\mathrm{Det}(A)=-\frac{u_d}{\Delta}\left(Y-\frac{X}{\Delta}\right)-X\frac{u_d}{\Delta^2}>0, \tag{58}$$

$$\Updownarrow\ \begin{aligned}&\Delta>0,\\ &X,Y<0,\end{aligned}$$

$$\Delta>\frac{X+u_d}{Y} \tag{59}$$

$$-u_dY\Delta>0. \tag{60}$$

The latter inequality is always satisfied as $u_d,\Delta>0$, $Y<0$. We thus have that if (59) is satisfied, the origin of $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is locally uniformly exponentially stable. Thus, in addition to satisfying the assumptions in Panteley and Loria (1998, Theorem 2) $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$ is globally κ-exponentially stable and $\dot{\boldsymbol{\xi}}=f_2(t,\boldsymbol{\xi})$ is globally exponentially stable, and by Panteley et al. (1998, Lemma 8), the origin of the cascaded system (37)–(38) is globally κ-exponentially stable. □

4. Experimental results

To verify and illustrate the theoretical results, the proposed control law was implemented at the Marine Cybernetics Laboratory (MCLab), Norwegian University of Science and Technology. The model ship used in the experiments was CybershipII, a 1:70 scale model of an offshore supply vessel with mass 15 kg and length (LOA) = 1.255 m. The thruster allocation algorithm presented in Lindegaard and Fossen (2003) was used. The commanded yaw moment was thus mainly thruster generated, and we hence transformed the control scheme to fit to a thruster controlled vessel. This was easily done by setting the hydrodynamic parameters to $Y_\delta=0$ and $N_\delta=1$, something which led to a

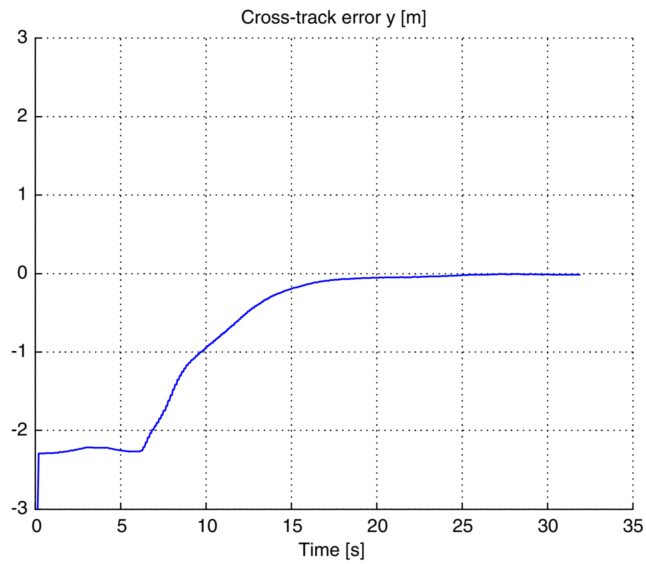

Fig. 2. Cross-track error y (m).

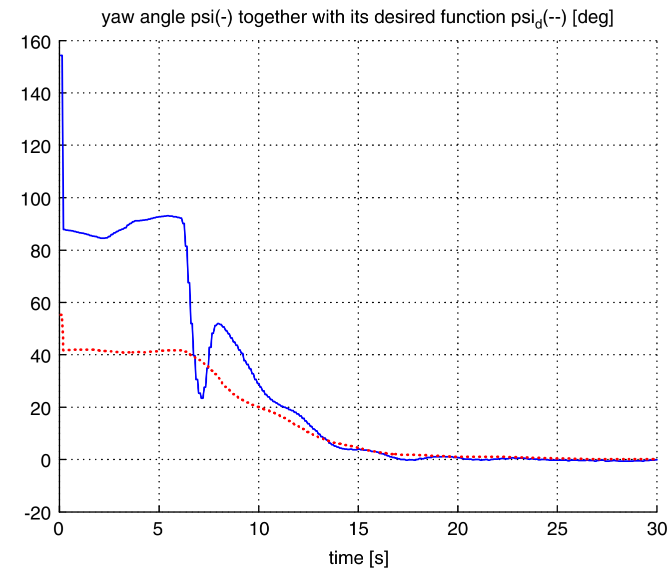

Fig. 3. The yaw angle ψ (solid) together with its desired function ψ_{d} (dotted) (deg).

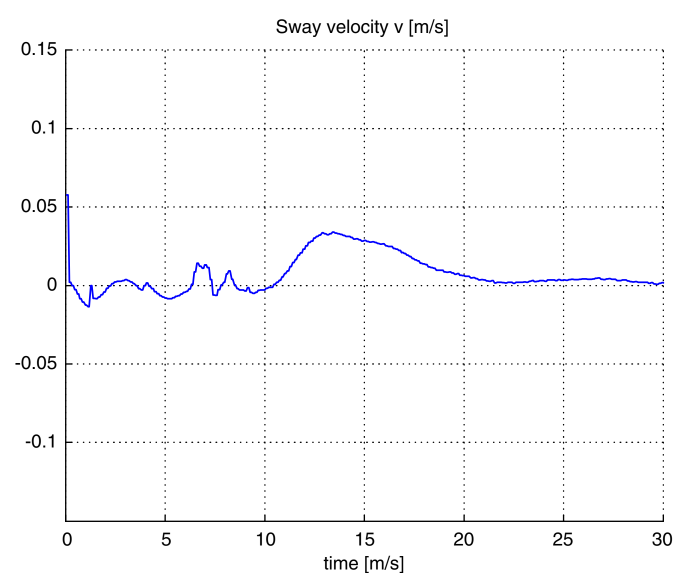

Fig. 4. The sway velocity v (m/s).

new control vector $\boldsymbol{\tau}$ and, consequently, a new coordinate transformation (7)–(9). Specifically, the dynamics for CybershipII can be described by (1), where $\boldsymbol{\tau}=[\tau_u, 0, \delta]^{\mathrm{T}}=[\tau_u, 0, \tau_r]^{\mathrm{T}}$ and

$$\mathbf{M}=\begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix},$$
$$\mathbf{D}=\begin{bmatrix} 0.9257 & 0 & 0 \\ 0 & 2.8909 & -0.2601 \\ 0 & -0.2601 & 0.5 \end{bmatrix}. \tag{61}$$

The maximum magnitude of the actuated forces and moments are 2 N in surge and 1.5 Nm in yaw. For CybershipII, the minimum reference surge speed to guarantee GUES for the $\dot{\mathbf{x}}=f_1(t,\mathbf{x})$-system with the LFC in Eq. (51) was found to be 0.014 m/s (in order to obtain negative values for X and Y), which evidently is no hard restriction. In the experiments, the desired surge speed was set to $u_{\mathrm{d}}=0.2$ m/s, corresponding to a velocity of 3.5 knots for the full scale ship. This gives $X=-0.1424$ and $Y=-0.0855$. We chose $\Delta=2*\mathrm{LOA}$, which is a common choice for LOS algorithms and which satisfies (55) with the given X and Y. The control parameters were chosen to be $k_0=4.2$, $k_1=3.5$, $k_u=10$. These parameters correspond to poles $\lambda_{1,2}=-1.75\pm 1.07i$ and a phase margin of 71.9° at the crossover frequency 3.68 rad/s.

The control algorithm was tested with different initial conditions, and waves (JONSWAP distribution) with significant wave height $H_{\mathrm{s}}=2$ cm and mean period $T_{\mathrm{s}}=0.75$ s (corresponding to sea state code 4 (moderate) with $H_{\mathrm{s}}=1.4$ m and $T_{\mathrm{s}}=0.75\sqrt{70}\approx 6.3$ s (Lindegaard, 2003)) were also applied. In all the experiments, the ship converged to the desired straight line and the forward speed to its reference value. Figs. 2–6 show the time evolution of the states y, ψ, v, r and $\bar{u}$ in an experiment where the ship was placed with initial heading perpendicular ($\psi(0)=90°$) towards the desired straight line, with a cross-track error $y(0)=-2.2$ m, and all the initial velocities were zero. We see in Figs. 2–6 that the state variables converge to their desired values, and in Fig. 7 we see that the natural logarithm of the norm $\sqrt{y^2+\psi^2+v^2+r^2+\bar{u}^2}$ is upper bounded by a straight line, something which complies with the theoretical result of κ-exponential stability for the closed-loop system. The unphysical behavior of the cross-track error shown initially in Fig. 2 does not reflect the physical ship behavior in the experiment, but is rather a drift in the y-position estimate. It is probably caused by camera errors due to the person standing near the boat to give it zero initial velocities when the experiment started. The same behavior can also be seen in Fig. 3, where the yaw angle seems to suddenly decrease from 150° to the correct value of 90°. The reference surge velocity u_{d} was generated by a second-order reference model (Fossen, 2002),

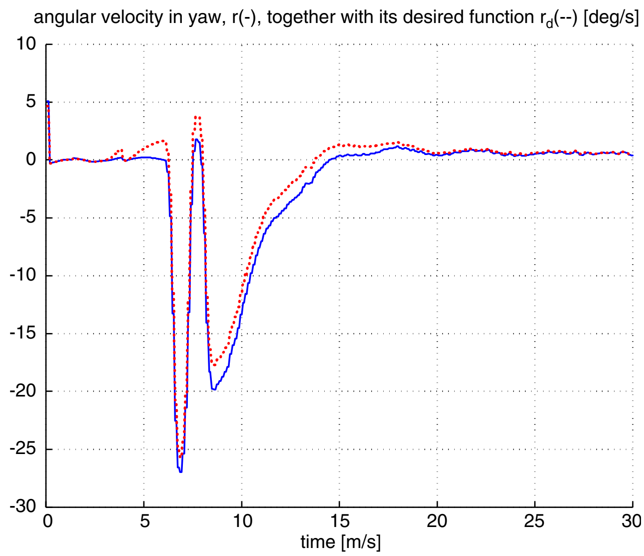

Fig. 5. The yaw velocity r (solid) together with its desired function $\dot{\psi}_d$ (dotted) (deg/s).

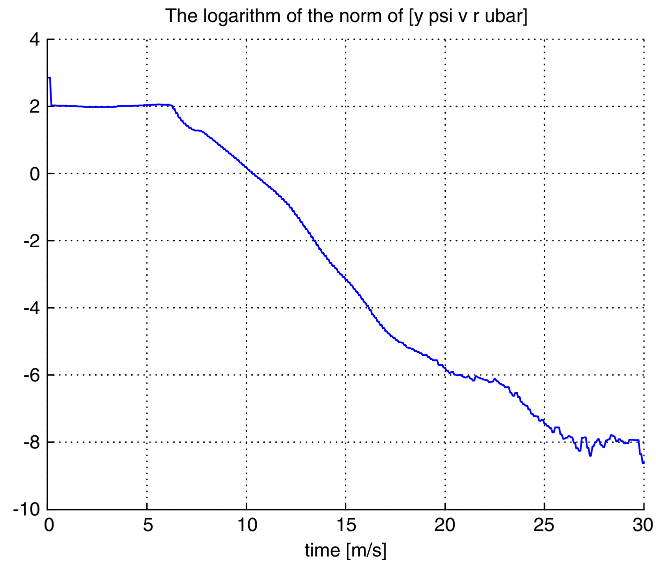

Fig. 7. The natural logarithm of the norm of $[y, \psi, v, r, \bar{u}]$.

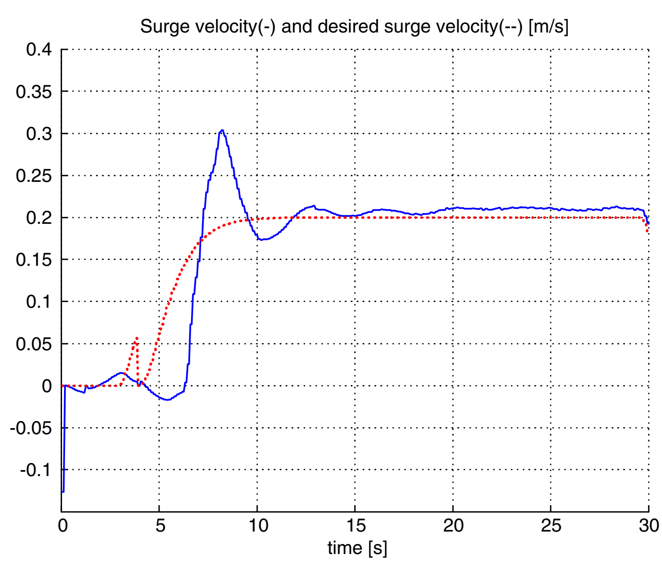

Fig. 6. Surge speed u (solid) together with its desired u_d (dotted) (m/s).

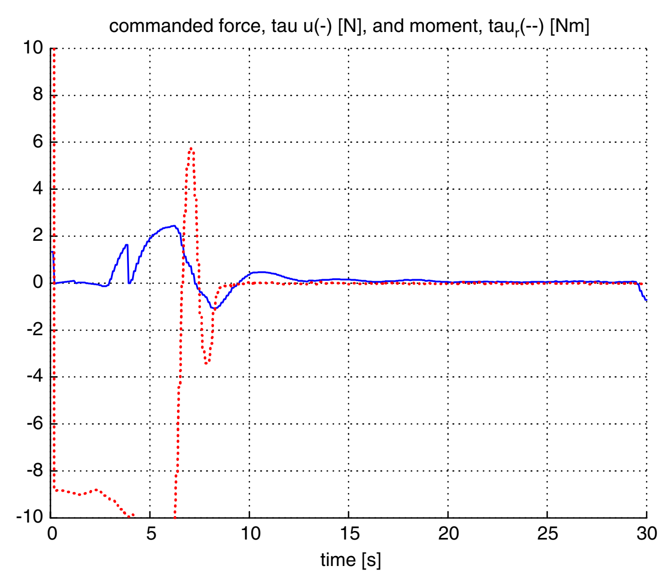

Fig. 8. Commanded control force τ_u (solid) (N) and moment τ_r (dotted) (Nm).

to give a smooth transition from 0 to 0.2 m/s. When the ship moved towards the end of the basin, the desired surge speed was decreased (at time $t = 29$ s), something which causes the drop in surge speed before the ship is out of range for the camera system. As stated above, the minimum reference surge speed to guarantee GUES was 0.014 m/s, something that may explain the behavior in the first 4–5 s of the experiments. Specifically, we see in Figs. 2–6 that the state variables principally started converging after about 7 s. The slow initial convergence may also be due to actuator saturation, as will be discussed later. The surge control law is a proportional controller, and does not include an integral term to compensate for disturbances, and this may be the cause of the steady-state error observed in Fig. 6. In Fig. 8, the commanded control force and moment are plotted. We see that the commanded controls, and in particular the commanded yaw moment, exceeded the saturation limits for CybershipII, and this may be another reason for the slow initial convergence. The experiments show that the ship converges in the presence of waves, something which suggests that the system has some robustness properties. This complies with the fact that exponential stability guarantees some robustness to perturbations (Khalil, 2002).

5. Conclusions and future work

In this paper, a way-point maneuvering controller was developed based on a general nonlinear ship model, and was proven to give global results and exponential convergence properties.

Specifically, the ship model included all 3DOF and no simplifying assumptions of decoupling of the motion in sway and yaw were made. Furthermore, the model took into account that a deflection of the rudder gives a sway force in addition to the desired yaw moment used for course control, something which complicates the control design. This posed the problem of stabilizing an underactuated system where one of the controls also affected the unactuated dynamics. To be able to apply the efficient results of cascaded systems theory, we proposed to use a coordinate transformation to decouple the unactuated dynamics from the control and thus facilitated a cascaded systems structure. The control law was then designed to obtain a cascaded systems structure consisting of exponentially stable subsystems. Using cascaded systems theory, we were then able to show global κ-exponential stability of the closed-loop system. Moreover, the control design was motivated by a desire to mimic the actions of a helmsman, and incorporate the nice features of line-of-sight (LOS) algorithms. A common rule of thumb used in LOS path control schemes is thus embodied in the proposed control law. The control law was seen to be easy to tune and straightforward to transform to a control law for ships equipped with thrusters instead of rudders for course control. Experimental results were presented, illustrating how the ship converged to the desired straight line between two way-points, also when subjected to waves.

The paper has thus presented an approach for cascaded control of underactuated systems when the controls also affect the unactuated dynamics. Moreover, the paper has presented a method to incorporate the nice features of LOS algorithms in the control scheme when the control problem is that of maneuvering. We conjecture that this method can be used for maneuvering control of a number of other underactuated mechanical systems, like underwater vehicles, spacecraft, mobile carts, etc.

The ship will be subject to environmental forces like wind, currents and waves. Due to the exponential stability results, we know that the controller proposed in this paper has certain robustness properties to disturbances (Khalil, 2002). However, it will be interesting to explore whether improved robustness to environmental disturbances can be achieved, for instance using disturbance adaptation. It is still an open question whether it is possible to fully adapt and counteract disturbances in 3DOF having only two independent control inputs (In Encarnação et al. (2000), Pettersen and Nijmeijer (2000), Behal et al. (2001) and Do et al. (2003) some preliminary results on this topic are given). Furthermore, if only 2DOF of the disturbances are to be accounted for, careful consideration should be made in the choice of which 2DOF to choose.

References

Behal, A., Dawson, D. M., Dixon, W. E., & Fang, Y. (2002). Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics. *IEEE Transactions on Automatic Control*, *47*, 495–500.

Behal, A., Dawson, D. M., Xian, B., & Setlur, P. (2001). Adaptive tracking control of underactuated surface vessels. *Proceedings of the 2001 IEEE international conference on control applications* (pp. 645–650).

Brockett, R. W. (1983). Asymptotic stability and feedback stabilization. In: R.W. Brockett, R.S. Millman, & H.J. Sussmann (Eds.), *Differential geometric control theory* (pp. 181–191). Basel: Birkhäuser.

Do, K. D., Jiang, Z. P., & Pan, J. (2002a). Underactuated ship global tracking under relaxed conditions. *IEEE Transactions on Automatic Control*, *47*, 1529–1536.

Do, K. D., Jiang, Z. P., & Pan, J. (2002b). Universal controllers for stabilization and tracking of underactuated ships. *Systems and Control Letters*, *47*, 299–317.

Do, K. D., Jiang, Z. P., & Pan, J. (2003). Robust global stabilization of underactuated ships on a linear course: State and output feedback. *International Journal of Control*, *76*, 1–17.

Do, K. D., & Pan, J. (2003a). Global tracking control of underactuated ships with off-diagonal terms. *Proceedings of the 42nd IEEE conference on decision and control* (pp. 1250–1255).

Do, K. D., & Pan, J. (2003b). Global way-point tracking control of underactuated ships under relaxed assumptions. *Proceedings of the 42nd conference on decision and control* (pp. 1244–1249).

Encarnação, P., Pascoal, A., & Arcak, M. (2000). Path following for autonomous marine craft. In *Proceedings of the fifth IFAC conference on manoeuvring and control of marine craft* (pp. 117–122). Aalborg, Denmark.

Fossen, T. I. (2002). *Marine control systems: Guidance, navigation and control of ships, rigs and underwater vehicles*. Trondheim, Norway: Marine Cybernetics.

Fossen, T. I., Breivik, M., & Skjetne, R. (2003). Line-of-sight path following of underactuated marine craft. *Proceedings of the sixth IFAC conference maneuvering and control of marine crafts* (pp. 244–249).

Godhavn, J. -M. (1996). Nonlinear tracking of underactuated surface vessels. In *Proceedings of the 35th IEEE conference on decision and control* (pp. 987–991). Kobe, Japan.

Indiveri, G., Aicardi, M., & Casalino, G. (2000). Robust global stabilization of an underactuated marine vehicle on a linear course by smooth time-invariant feedback. In *Proceedings of the 39th IEEE conference on decision and control* (pp. 2156–2161). Sydney, Australia.

Jiang, Z.-P. (2002). Global tracking control of underactuated ships by Lyapunov's direct method. *Automatica*, *38*, 301–309.

Khalil, H. K. (2002). *Nonlinear systems*. (3rd ed.), Englewood Cliffs, NJ: Prentice-Hall.

Lefeber, E. (2000). *Tracking control of nonlinear mechanical systems*. Ph.D. thesis, University of Twente, The Netherlands.

Lefeber, E., Pettersen, K. Y., & Nijmeijer, H. (2003). Tracking control of an underactuated ship. *IEEE Transactions on Control Systems Technology*, *11*, 52–61.

Lindegaard, K. P. (2003). *Acceleration feedback in dynamic positioning*. Ph.D. thesis, Department of Engineering Cybernetics, Norwegian University of Science and Technology.

Lindegaard, K. P., & Fossen, T. I. (2003). Fuel efficient rudder and propeller control allocation for marine craft: Experiments with a model ship. *IEEE Transactions on Control Systems Technology*, *11*(6),

Panteley, E., Lefeber, E., Loria, A., & Nijmeijer, H. (1998). Exponential tracking control, of a mobile car using a cascaded approach. *Proceedings of the IFAC workshop on motion control* (pp. 221–226). ⟨http://seweb.se.wtb.tue.nl/∼lefeber/pub/pdffiles/PLLN98.pdf⟩.

Panteley, E., & Loria, A. (1998). On global uniform asymptotic stability of nonlinear time-varying systems in cascade. *Systems and Control Letters*, *33*, 131–138.

Pettersen, K. Y. (1996). *Exponential stabilization of underactuated vehicles*. Ph.D. thesis, Norwegian University of Science and Technology.

Pettersen, K. Y., & Egeland, O. (1996). Exponential stabilization of an underactuated surface vessel. In *Proceedings of the 35th IEEE conference on decision and control* (pp. 967–971), Kobe, Japan.

Pettersen, K. Y., & Lefeber, E. (2001). Way-point tracking control of ships. *Proceedings of the 40th IEEE conference on decision and control* (pp. 940–945).

Pettersen, K. Y., & Nijmeijer, H. (1999). Global practical stabilization and tracking for an underactuated ship—a combined averaging and backstepping approach. *Modeling, Identification and Control*, *20*(4), 189–199.

Pettersen, K. Y., & Nijmeijer, H. (2000). Semi-global practical stabilization and disturbance adaptation for an underactuated ship. *Proceedings of the 39th IEEE conference on decision and control*, *3*, 2144–2149.

Pettersen, K. Y., & Nijmeijer, H. (2001). Underactuated ship tracking control: Theory and experiments. *International Journal of Control*, *74*(14), 1435–1446.

Sira-Ramirez, H. (1999). On the control of the underactuated ship: A trajectory planning approach. *Proceedings of the 38th IEEE conference on decision and control* (pp. 2192–2197).

Skjetne, R., Fossen, T. I., & Kokotovic, P. (2002). Output maneuvring for a class of nonlinear systems. *Proceedings of the IFAC world congress*. Barcelona.

Sørdalen, O. J., & Egeland, O. (1995). Exponential stabilization of nonholonomic chained systems. *IEEE Transactions on Automatic Control*, *40*(1), 35–49.

Toussaint, G. J., Basar, T., & Bullo, F. (2000a). H^{∞}-optimal tracking control techniques for nonlinear underactuated systems. *IEEE conference on decision and control* (pp. 2078–2083).

Toussaint, G. J., Basar, T., & Bullo, F. (2000b). Tracking for nonlinear surface vessels with generalized forces. *Proceedings of the 2000 IEEE international conference on control applications* (pp. 355–360).

Zabczyk, J. (1989). Some comments on stabilizability. *Applied Mathematics and Optimization*, *19*, 1–9.

Zhang, R., Chen, Y., Zengqi, S., Fuchun, S., & Hanzhen, X. (1998). Path control of a surface ship in restricted waters using sliding mode. In *Proceedings of the 37th IEEE conference on decision and control* (pp. 3195–3200).

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