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Probabilistic quantification of ship collision risk considering trajectory uncertainties

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Abstract: This paper presents the probabilistic quantification of collision risk for surface ships to safely navigate in ocean environments. A semi-analytical approach is proposed which allows an accurate and fast evaluation of collision probability considering time-varying trajectory uncertainties of both the navigating vehicle and the other ships. For efficient computation, the collision probability is separated into diffusion and drift components based on the concept of probability flow, and these two probability components are combined in order to obtain the instantaneous collision probability (ICP). To demonstrate the validity of the proposed approach, numerical simulations were performed for marine traffic scenarios and the feasibility was assessed by field experiment data obtained in a real-sea environment.

Keywords: Collision probability, probability flow, trajectory prediction, maritime safety

1. INTRODUCTION

Over the last several decades, technological advancements for safe navigation of surface ships have been greatly improved with advances in onboard computer capabilities in many aspects of ship navigation, guidance, and control in ocean environments. A key capability of maritime navigation in riverine and/or coastal environment is to recognize a collision risk and to avoid the potential hazard object in accordance with maritime traffic rules. In particular, it is necessary to inform the ship's operator of all navigation dangers and hazard factors effectively in order to avoid immediate collisions. However, the rules for safe navigation of surface ships do not specify any quantitative criteria regarding the procedures to detect and evaluate the collision risk until now. Consequently, the quantification of collision risk could provide important value for safe navigation of encountering ships, and it has attracted much research interest in the last few decades.

Technically, ship collisions can be predicted if the trajectories of two encountering ships are known or available in advance. However, it is difficult to accurately predict ship trajectory due to various uncertainties such as navigation sensor error, control and path tracking error, and environmental disturbances. The prediction error tends to increase rapidly over time, and the rate of the increase in error is highly dependent on the accuracy of the navigation sensors. This uncertainty characteristic needs to be considered for accurate evaluation of the collision risk.

To evaluate the collision risk between marine surface ships, a number of studies have been performed and various approaches have been suggested using the distance to the closest point of approach (DCPA) and/or the time to the closest point of approach (TCPA). Mou et al. (2010) presented a dynamic method based on safety assessment model for shipping and offshore on the North sea (SAMSON) with TCPA, CPA, and encountering angle. A traffic flow theory, a fuzzy expert system, an artificial neural network, and variation of compass degree have been suggested in Tam et al. (2009); Bukhari et al. (2013); Simsir et al. (2014); Goerlandt et al. (2015). Fujii and Tanaka (1971), Goodwin (1975), and Szlapczynski (2006) proposed the concept of ship domain to indicate the risk of collision. Montewka et al. (2012) also presented a collision criterion obtained by taking into account traffic patterns, maneuvering patterns, and statistical analysis in special water areas. However, most of the existing approaches did not consider the inherent system uncertainty, including sensor noise, uncertainty in the trajectories of objects and environmental disturbances.

In this paper, a semi-analytical approach is proposed for evaluating the collision probability between two encountering ships using the concept of probability flow. The procedure starts with modeling the position uncertainty of each ship as a multivariate Gaussian distribution. The position uncertainties of the two ships are combined through a coordinate transformation for an efficient problem formulation, and a safe separation zone for each ship is defined and then combined. Consequently, the instantaneous probability of collision at a given time is determined by taking into account the rate of change of the probability and the overlap between the expanded safe separation zone and the combined uncertainty.

2. PROBLEM STATEMENT

Consider two ships, labeled $\mathbf{x}_{\mathrm{O}}$ as the own ship and $\mathbf{x}_{\mathrm{T}}$ as the target ship, moving in a horizontal plane. The state vector in state-space form representing the position of each

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ship is defined as $\mathbf{x}_\mathrm{O} = [\ x_\mathrm{O}\ y_\mathrm{O}\ ]^\intercal$ and $\mathbf{x}_\mathrm{T} = [\ x_\mathrm{T}\ y_\mathrm{T}\ ]^\intercal$, where $\mathbf{x}_\mathrm{O}$ represents the own ship's position, $\mathbf{x}_\mathrm{T}$ represents the target ship's position, both defined in the global frame.

Assume that these two ships move along predefined courses at constant speed: v_O for the own ship and v_T for the target ship. The equation of motion for each ship can be represented using a Wiener process model considering the ship's trajectory uncertainty, which can be written as:

$$\begin{aligned}\dot{\mathbf{x}}_\mathrm{O} &= [\ \dot{x}_\mathrm{O}\ \dot{y}_\mathrm{O}\ ]^\intercal + \mathbf{w}_\mathrm{O} \\ &= [\ v_\mathrm{O}\cos\psi_\mathrm{O}\ \ v_\mathrm{O}\sin\psi_\mathrm{O}\ ]^\intercal + \mathbf{w}_\mathrm{O}\end{aligned} \tag{1}$$

$$\begin{aligned}\dot{\mathbf{x}}_\mathrm{T} &= [\ \dot{x}_\mathrm{T}\ \dot{y}_\mathrm{T}\ ]^\intercal + \mathbf{w}_\mathrm{T} \\ &= [\ v_\mathrm{T}\cos\psi_\mathrm{T}\ \ v_\mathrm{T}\sin\psi_\mathrm{T}\ ]^\intercal + \mathbf{w}_\mathrm{T}\end{aligned} \tag{2}$$

where ψ_O and ψ_T are the heading angles of the own and target ships. $\mathbf{w}_\mathrm{O} \sim \mathbf{N}(0, \mathcal{Q}_\mathrm{O})$ and $\mathbf{w}_\mathrm{T} \sim \mathbf{N}(0, \mathcal{Q}_\mathrm{T})$ denote the zero-mean Gaussian process noise vectors where $\mathcal{Q}_\mathrm{O}$ and $\mathcal{Q}_\mathrm{T}$ are the covariance matrices of the process noise vectors associated with the trajectory uncertainties. Here, the covariance matrices can be determined considering the uncertainties in measurement and modeling errors.

The ship's position uncertainty can be represented using a probability density function (pdf), and its distribution in the horizontal plane can be expressed using probability ellipses under the assumption of multivariate Gaussian distribution. When a ship maintains its heading, the probability ellipse associated with the predicted position of the ship increases in time, as illustrated in Fig. 1.

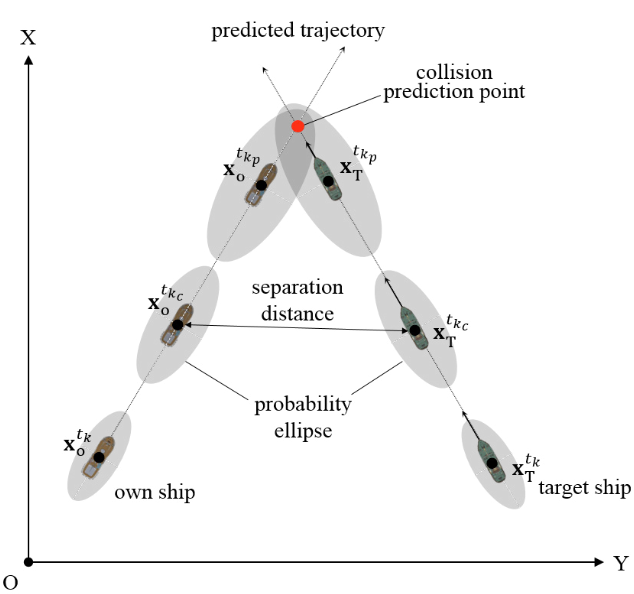

Fig. 1. Two ships are crossing in the horizontal plane. The spread of the probability ellipse representing the position uncertainty of each ship increases in time. t_k, t_{k_c} and t_{k_p} denote time index, current time, and look-ahead time, respectively.

The probability ellipse increases with time in the along-track and cross-track directions according to the assumption of the Wiener process. As the two ships get close, the overlap between two probability ellipses (i.e., joint probability) increases, which directly reflects the probability of collision. The integrated joint probability over the region $\mathcal{D} \subset \mathrm{R}^2$ at a specific time is defined as the instantaneous collision probability (ICP).

$$\mathbf{P}_c^i \triangleq \iint_{\mathcal{D}} p_\mathrm{O}(\mathbf{x}_\mathrm{O};t)\ p_\mathrm{T}(\mathbf{x}_\mathrm{T};t)\mathrm{d}x\mathrm{d}y \tag{3}$$

Here, $p_\mathrm{O}(\mathbf{x}_\mathrm{O};t)$ and $p_\mathrm{T}(\mathbf{x}_\mathrm{T};t)$ denote the pdfs for the own ship and the target ship, each of which can be expressed using a two-dimensional Gaussian distribution at time t.

The exact collision probability can be dependent on the geometric shapes of the ships involved in a collision. However, for an analytically tractable formulation, a circular boundary for safety is considered. That is, the minimum allowable distance between two ships is defined as the safe separation distance. Any violation of this safe separation is defined as a conflict, and the probability of conflict at a specific time is regarded to be ICP.

In general, there exists no analytical solution to ICP in (3). Therefore, numerical methods and approximations have commonly been used to evaluate the ICP, according to Paielli and Erzberger (1997); Lambert et al. (2008); Hwang and Seah (2008). In this study, we adopt a systematic semi-analytical procedure using the concept of probability flow which allows for efficient evaluation of the collision probability.

3. COLLISION PROBABILITY

The position uncertainty of each ship is represented using a Gaussian pdf with a probability ellipse (i.e., cofidence ellipse), and the geometric configuration of the confidence ellipse is determined by the associated error covariance matrix. The covariance increases in time with the progressively growing uncertainty in trajectory prediction. To simplify the problem formulation, a safe separation zone with a circular boundary for each ship is defined, and the position uncertainties of two ships are combined by merging two Gaussian pdfs into a single pdf through coordinate transformation (Paielli and Erzberger (1997)). The combined pdf determines a new probability ellipse associated with the combined uncertainty. Here, no cross-correlation between the motion of two ships is assumed. The combined probability ellipse is applied to one of the ships (target ship), and the safe separation zone is expanded around the other ship (own ship) by adding the safe separations of the two ships, as illustrated in Fig. 2.

The combined pdf represents the spatial separation between the two ships with uncertainty. The chance of violating the safe separation at a specific time (i.e., ICP) can be estimated by integrating the combined pdf over the overlap between the combined probability ellipse and the combined safe separation zone. In general, there exists no analytical solution to this integral, and the direct integration of the probability is computationally too expensive to be evaluated at every time step for real-time applications such as online path planning and vessel traffic management.

For efficient computation of ICP, the concept of probability flow is introduced in this study. Instead of the volume

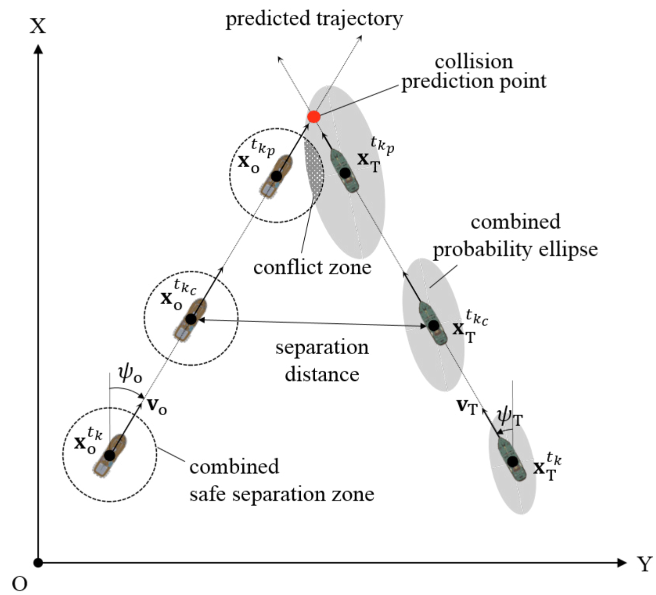

Fig. 2. Illustration of the combined probability ellipse and the combined safe separation zone: the combined probability ellipse is obtained through coordinate transformation. The combined safe separation zone is denoted by a circle whose radius is the sum of the safe separation distances of the own and target ships.

integration of the probability over the entire overlap region which is computationally expensive, the rate of change of ICP is evaluated by integrating probability flow along the boundary of the combined safe separation circle. The ICP at time t is obtained by integrating the rate of change of the probability over the time interval $[t_0, t]$, which can be expressed as:

$$\mathbb{P}_c^i(t) = \mathbb{P}_c^i(t_0) + \int_{t_0}^{t} \dot{\mathbb{P}}_c^i(\tau)\mathrm{d}\tau \tag{4}$$

where $\dot{\mathbb{P}}_c^i(t)$ is the rate of change of probability through the boundary of the combined safe separation zone.

In evaluating (4), the rate of change of ICP has to be equal to the net rate of the probability flow into the safety boundary. In addition, the combined probability ellipse is neither created nor disappeared in the combined safe separation zone. This can be expressed using a differential form of the continuity equation as follows:

$$\frac{\partial}{\partial t} f(\mathbf{x}, t) = -\nabla \cdot \mathcal{J} \tag{5}$$

where $f(\mathbf{x}, t)$ is the pdf associated with the combined probability ellipse and $\mathcal{J}$ denotes the probability flux representing the rate of probability flow per unit element of the safety boundary. The combined probability undergoes translation and expansion in time due to the change of motion and uncertainty increase. Considering this, the probability flux can be decomposed into drift and diffusion flux components as:

$$\mathcal{J} = \mathcal{J}_{\text{drift}} + \mathcal{J}_{\text{diff}} \tag{6}$$

where $\mathcal{J}_{\text{drift}}$ and $\mathcal{J}_{\text{diff}}$ denote the drift flux and the diffusion flux, respectively.

The drift flux, $\mathcal{J}_{\text{drift}}$, is due to the translation of the pdf that can be represented by

$$\mathcal{J}_{\text{drift}} \triangleq f(\mathbf{x}, t)\mathbf{v} \tag{7}$$

where $\mathbf{v}$ is the relative velocity between the two ships.

The diffusion flux, $\mathcal{J}_{\text{diff}}$, is contributed by the amount of the probability flow that spreads out through the safety boundary, as follows:

$$\mathcal{J}_{\text{diff}} \triangleq -\mathbf{D}(t)(\nabla f(\mathbf{x}, t)) \tag{8}$$

Here, ∇ denotes the vector differential operator with respect to the position of the target ship, and $\mathbf{D}(t)$ is the diffusion coefficient, such that $\mathbf{D}(t) \triangleq \mathcal{Q}_c\mathcal{Q}_c^\intercal/2 > 0$ where $\mathcal{Q}_c$ is the 2×2 standard deviation matrix representing the combined probability in two-dimensional coordinates.

The pdf in (5) is expressed using a multivariate normal distribution as:

$$\begin{aligned} f(\mathbf{x}, t) =& p(\mathbf{x}; \mu, \Sigma) \\ =& \frac{1}{(2\pi) \mid \det \Sigma \mid^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^\intercal \Sigma^{-1}(\mathbf{x}-\mu)\right) \end{aligned} \tag{9}$$

where μ denotes the predicted position of the target ship and $\Sigma \triangleq \mathcal{Q}_c\mathcal{Q}_c^\intercal$.

Therefore, the time rate of the probability flow can be obtained by integrating the probability flux along the boundary of the combine safe separation as follows:

$$\begin{aligned} \dot{\mathbb{P}}_c^i &= -\iint_{\mathcal{S}} (-\nabla \cdot \mathcal{J}) d\mathcal{S} \\ &= -\iint_{\mathcal{S}} (-\nabla \cdot (-\mathbf{D}(\nabla p(\mathbf{x}; \mu, \Sigma)) + \mathbf{v}p(\mathbf{x}; \mu, \Sigma))) d\mathcal{S} \\ &= -\int_{\mathcal{L}} ((-\mathbf{D}(\nabla p(\mathbf{x}; \mu, \Sigma)) + \mathbf{v}p(\mathbf{x}; \mu, \Sigma)) \cdot \mathbf{n}) d\mathcal{L} \end{aligned} \tag{10}$$

where $\mathcal{S}$ and $d\mathcal{S}$ denote the area of conflict zone and its infinitesimal area element, respectively. $\mathcal{L}$ represents the boundary of the combined safe separation zone, $d\mathcal{L}$ denotes its infinitesimal arc element of the boundary, and $\mathbf{n}$ is the outward-pointing normal vector on the safety boundary. The negative sign of the dot product with $\mathbf{n}$ denotes an inflow of the probability flux, while the positive sign represents an outflow.

The ICP along the own ship's trajectory can be estimated using (4). The maximum ICP (MICP) in (11), which is the peak value of $\mathbb{P}_c^i(t)$ within any fixed time interval $[t_0, t_f]$, is also used as a practically useful measure of collision probability.

$$\mathbb{P}_c \triangleq \max\{\mathbb{P}_c^i(t) | t \in [t_0, t_f]\} \tag{11}$$

4. TRAFFIC SIMULATION WITH COLREGS RULES

Numerical simulations have been performed to verify the utility and effectiveness of the proposed approach, and the computational efficiency in evaluating the ICP is compared with numerical methods applied in marine traffic simulations.

Three traffic simulation scenarios were considered: 1) overtaking, 2) head-on, and 3) crossing. Primary marine traffic rules defined in the COLREGs rules (see Cockcroft and Lameijer (2003)) relevant to these encountering situations are as follows:

- **Overtaking (Rule 13)**: Any vessel overtaking any other (give-way vessel) shall keep out of the way of the vessel being overtaken (stand-on vessel).
- **Head-on (Rule 14)**: When two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other. (Both are give-way vessels.)
- **Crossing (Rule 15)**: When two power-driven vessels are crossing so as to involve risk of collision, the vessel (give-way vessel) which has the other (stand-on vessel) on her own starboard side shall keep out of the way and shall, if the circumstances of the case admit, avoid crossing ahead of the other vessel.

In such encountering situations, the give-way vessel is required to alter its course to avoid the stand-on vessel, and the stand-on vessel should maintain its course and speed in accordance with the COLREGs rules. To clearly show the utility of the proposed approach, it is assumed that the collision occurs at the point (see Fig. 3) with DCPA = 0 and both ships take no evasive maneuver.

Table 1 shows the simulation settings including the initial conditions for the traffic simulation scenarios. Two traffic ships were assumed to be small boats or unmanned surface vehicles (USVs). Considering the uncertainty in trajectory prediction, it is also assumed that the along-track error grows linearly at 3 m/sec, and that the cross-track error grows at 1 m/sec, for each ship. The ICP is computed over a 60 sec time horizon at 1 sec intervals.

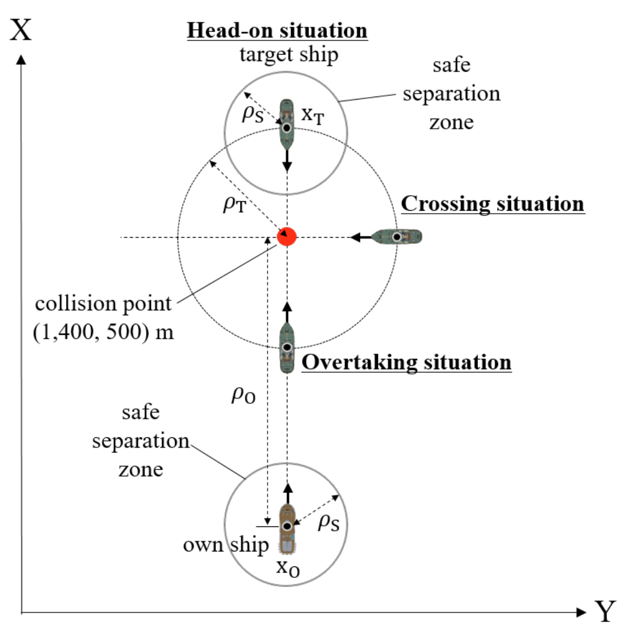

Fig. 3. Traffic simulation settings: The heading and position of the ships were set on collision courses.

The computational performance of the proposed approach for evaluating the ICP is compared with two methods: 1) the volume integration method, 2) the Monte Carlo simulation (MCS) method (see Lambert et al. (2008)).

The error statistics for accuracy comparison and computational costs are summarized in Table 2. The result of the volume integration computed through high-precision numerical integration can be regarded as a reference solution. The MCS method requires large computational effort for sufficient accuracy, due to the nature of Monte Carlo simulations. The proposed method provides an accurate result at an acceptable computation cost. The proposed method evaluates the ICP based on cumulative evaluation of the probability flow by recursion, unlike the other methods that compute a snapshot value at a specific time. Because the computation can be distributed in time in the framework of recursive filtering or prediction, the proposed method can be an attractive option for online estimation of collision probability.

Table 1. Parameter settings for ship traffic simulations

Designation	Value
Length (L) [m]	7.5
Breadth (B) [m]	2.5
Speed of own ship (USV) (v_O) [m/sec]	8.0
Speed of target ship (v_T) [m/sec]	5.0
Radius of safe separation zone (ρ_s) [m]	3.0L
Initial along-track error (σ_{ia}) [m]	15
Initial cross-track error (σ_{ic}) [m]	10
Collision point (CP) [m]	(1,400,500)
Range to the CP of own ship (ρ_O) [m]	400.0
Range to the CP of target ship (ρ_T) [m]	250.0

5. FIELD EXPERIMENT

To check the practical feasibility and utility of our approach, the MICP was evaluated for marine traffic scenarios by using field experiment data obtained in a real-sea environment.

5.1 USV system and experimental setup

In the experiment, the ARAGON USV developed by Korea Research Institute of Ships and Ocean Engineering (KRISO) was used as the own ship, and the 8 m boat was used as the target ship (see Fig. 4). The USV was equipped with a suite of communication equipment and navigation sensors including the Real-Time Kinematic Global Positioning System (RTK-GPS) for accurate position sensing. An automatic identification system (AIS) was installed in the target ship. They were controlled and monitored from the ground control through wireless communication.

Fig. 5 shows the snapshots from the experiment for head-on and crossing situations. In the head-on situation, two ships were operated to approach each other head-on, and the target ship was approached from the USV's starboard to port side in the crossing situation. The USV's nominal speed was set to be 7 knots and the speed of target ship was approximately 9 knots.

5.2 Experimental results

The algorithm parameters such as the safe separation zone and path track error uncertainties were tuned through ship traffic simulations (see Table 1). The ICP value was computed over the 30 second time horizon into the future, and MICP was evaluated within the time interval. Figs. 6

Table 2. Comparison of the collision probability and computational efficiency between the proposed method and the other two methods: $\mathbb{P}_c$ is the MICP. $\bar{\epsilon}$ is the MICP error relative to the volume integration method. t_c is the computation time for a single evaluation using MATLAB on a desktop computer with an Intel(R) Core(TM) i7-2600 3.4GHz processor and 4GB RAM.

Traffic situation	**Method**	$\mathbb{P}_c$	$\bar{\epsilon}$	t_c(s)
Overtaking	Volume integration	0.73089	-	0.31310
	MCS method[a]	0.73830	0.00741	0.56081
	Proposed method[b]	0.72954	0.00135	0.01907
Head-on	Volume integration	0.72890	-	0.30385
	MCS method	0.73110	0.00220	0.59596
	Proposed method	0.72314	0.00575	0.01809
Crossing	Volume integration	0.70658	-	0.32662
	MCS method	0.70830	0.00172	0.56002
	Proposed method	0.69962	0.00697	0.01779

[a] Monte Carlo simulations were performed with 10^4 samples.
[b] Time integration for cumulative evaluation was performed at 1 Hz between the 60 second time interval.

(a) (b)

Fig. 4. The own ship and the target ship used for field experiments: (a) ARAGON USV by KRISO (7.5 m in length); (b) 8 m traffic target ship.

(a) (b)

Fig. 5. Snapshots captured by the camera mounted on the USV from the field experiments at sea: (a) and (b) are from the head-on and crossing situation, respectively.

and 7 show the trajectories of the two ships and the change of MICP varying with the relative motion between them. As the time increases (two ships approach closely), the MICP increases. In these scenarios, the USV changed its course to starboard according to the COLREGs rule. Consequently, by changing the heading of the USV to starboard side, the MCIP decreased, as shown in Fig. 7. As the along-track and cross-track errors grow quickly in the trajectory prediction, the ICP gradually increases even when there is a long distance between two ships, i.e., the potential collision risk can be perceived in an early phase of approach.

6. CONCLUSION

In this paper, we introduced and discussed a semi-analytical approach for estimating the collision probability between two surface ships over a prespecified time horizon in the future, considering the trajectory uncertainties and the safe separation zone. For quantitative and efficient evaluation of the potential collision, a probability flow model was proposed and the collision probability was expressed as the sum of drift and diffusion flux components in a manner similar to fluid dynamics. Numerical simulations and a field experiment at sea were performed for various representative maritime traffic situations. The results demonstrated the practical feasibility of the proposed approach.

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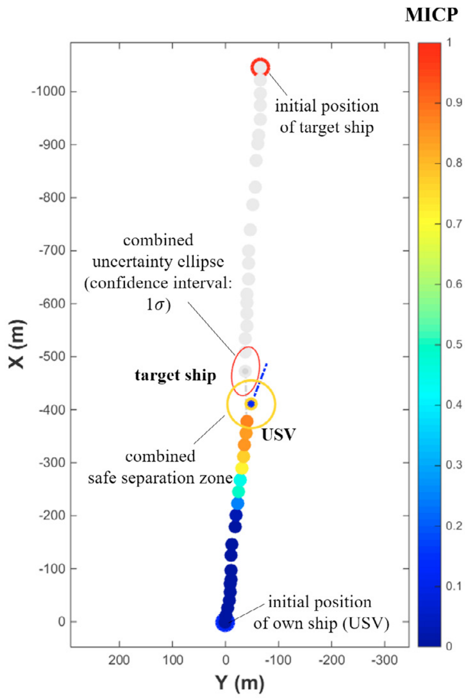
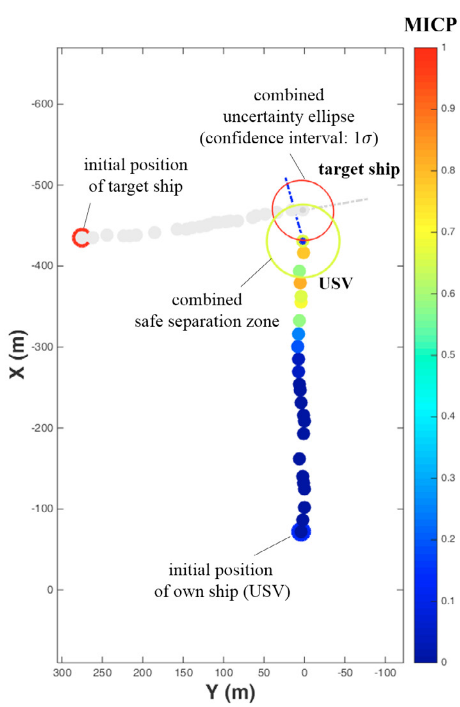


(a) Head-on situation



(b) Crossing situation

Fig. 6. Field experimental results of two ship encountering scenarios. The trajectories of the USV and target ship marked with enlarged dots whose colors represent the MICP.

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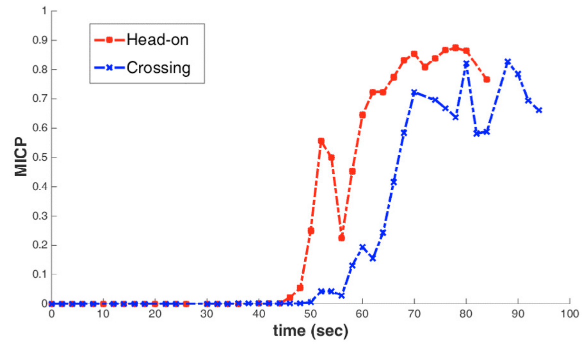

Fig. 7. Experiment results of the MICP from the two encountering scenarios.

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