

# NMPC-based Trajectory Tracking and Collision Avoidance of Unmanned Surface Vessels with Rule-based Colregs Confinement

Mohamed Abdelaal , Axel Hahn  
Computer Science department  
University of Oldenburg, Oldenburg, Germany  
{mohamed.abdelaal, axel.hahn}@uni-oldenburg.de

**Abstract**—This paper presents a Nonlinear Model Predictive Control (NMPC) approach for critical maneuvering of unmanned surface vessels (USV) in near-collision situation. The algorithm is formulated as a nonlinear optimization problem that minimize the vessel states' deviation from the time varying reference with collision avoidance as a time varying constraint. The International Regulations for Preventing Collisions at Sea (Colregs) rules are obeyed by employing a rule-based decision support system to decide which vessel is give-way and which is stand-on, and a nonlinear constraint that achieve Colregs confined maneuvers. Moreover, the algorithm can also drive the vessel to its planned trajectory in collision-free situations. Counteraction of the disturbances is achieved by estimating them using Moving Horizon Estimation (MHE). The proposed technique assumes a communication media to get predicted positions of the encounter vessel or assumes straight line motion based on the position and velocity obtained from the radar system or the Automatic Identification Systems (AIS). A real-time C-code is generated using the ACADO toolkit and the qpOASES solver with multiple shooting technique for discretization and Gauss-Newton iteration algorithm. MATLAB simulations are used to assess the validity of the proposed technique for Head-on, overtaking, and crossing from the right situations.

**Keywords**—Nonlinear model predictive control; Autonomous Vessels; Colregs; Tracking; Collision Avoidance

## I. INTRODUCTION

Marine transportation has been the largest carrier of freight over the history. Although the importance of sea travel for passengers has been decreased, marine transportation is still widely used for cargo and military purposes. The tracking control problem of surface vessels is an open research problem that has gained a lot of attention over the last few decades, especially for the future expectations of using autonomous vessels or adding an autopilot feature to commercial ones. In this context, underactuated unmanned surface vessels (USV) should be able to track reference trajectories with the ability to avoid obstacles and counteract disturbances. Underactuated vessels are those vessels with only surge force and yaw moment, and without sway force, they are usually equipped with two independent aft thrusters or with one main aft thruster and a rudder.

There is an increasing number of research publications about different techniques that deal with the tracking problems of surface vessels, mobile robots and air-crafts. Dynamic surface control (DSC) technique is used in [1] for the tracking problem of underactuated vessels by linearizing the dynamics. In [2], an adaptive steering control law is

designed for uncertain ship dynamics with saturation limits. In [3], Lyapunov's direct method and backstepping are used for path following of underactuated vessels employing Serret-Frenet frame. Trajectory tracking problem of an underactuated Unmanned Surface Vessel (USV) is presented in [4] based on a modified backstepping approach. Although the aforementioned techniques give good tracking results, they don't address the collision avoidance problem into the controller design. In [5], a nonlinear adaptive feedback controller is developed by employing a kind of time-varying sliding manifold for the formation problem of a group of spacecrafts with collision avoidance among these spacecrafts; However, collision avoidance of external objects is not addressed.

Model predictive control (MPC) has been used for the tracking problem. In [6], model predictive control is applied for trajectory tracking of underactuated surface vessels by using linear MPC techniques after evaluating the nonlinear functions using optimal states obtained at the previous instant. NMPC is used in [7] for trajectory tracking of underactuated vessels and in [8] for path following of full actuated autonomous surface craft (ASC) considering input constraints, but without addressing collision avoidance. In [9], MPC is used for target approaching of a kinematically redundant space robot with Anti-collision constraints. This is achieved by using feedback linearization to obtain a linear dynamics suitable for applying linear MPC techniques.

In this paper, nonlinear state feedback control law is presented, based on solving a finite horizon optimization problem, at every instant, and using the current state measurement as initial condition for the problem. Moving Horizon Estimation (MHE) is used as a nonlinear disturbance observer for the external forces as they can not be considered as a simple additive disturbance. Collision avoidance constraints are added to the problem with the aid of a rule-based decision support system for obeying Colregs rules for the behaviour of a vessel in a near collision situation. The output of the problem is an optimal control sequence, of length  $N$ , of the vessel's force and moment, where only the first element is applied and then the whole process is repeated at the next instant. The finite horizon optimization problem is discretized and then formulated as a quadratic problem (QP) which is solved by the aid of ACADO toolkit[10] and qpOASES solver[11].

This paper is organized as follows. Section II describes the vessel nonlinear dynamics, the generation of the reference trajectory, and clarifies the control objective. In section III, a

brief introduction of NMPC is given followed by MHE design as a disturbance observer. After that, the rule-based decision support system is presented followed by an algorithm flow of the whole controller. Simulation results done on MATLAB are given in Section IV. Section V concludes this paper.

## II. PROBLEM FORMULATION

The autonomous vessel model has 6-DOF: surge, sway, yaw, heave, roll, and pitch, which can be simplified to 3-DOF motion under the following assumptions[1]:

- 1) The heave, roll, and pitch modes induced by wind and currents are negligible,
- 2) The inertia, added mass, and hydrodynamic damping matrices are diagonal,
- 3) The available control variables are surge force and yaw moment.

These assumptions are valid for vessels with two aft thruster and without a rudder. Based on that, the 3-DOF model will be[12]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}\mathbf{u} + \mathbf{g}_w(\mathbf{x})\mathbf{w} \quad (1)$$

Here,  $\mathbf{x} = [x \ y \ \psi \ u \ v \ r]^T \in \mathbb{R}^6$  is the state vector,  $\mathbf{u} = [\tau_u \ \tau_r]^T \in \mathbb{R}^2$  is the input vector,  $\mathbf{w} = [w_u \ w_v \ w_r]^T \in \mathbb{R}^3$  is the unknown disturbance vector,  $x_p$  and  $y_p$  are the positions,  $\psi$  is the heading angle of the ship with respect to the earth-fixed frame,  $u$  and  $v$  are longitudinal and transverse linear velocities with respect to the body fixed frame,  $r$  is the angular velocity in yaw around body-fixed  $z$  axis (see Fig. 1).  $\mathbf{f}(\cdot)$ ,  $\mathbf{g}(\cdot)$ , and  $\mathbf{g}_w(\cdot)$  are continuous functions in  $\mathbf{x}$ ,  $\mathbf{u}$  and  $\mathbf{w}$ .  $\mathbf{f}(\cdot)$  is locally Lipschitz in  $\mathbf{x}$  that satisfies  $\mathbf{f}(0) = 0$ ,

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} u \cos(\psi) - v \sin(\psi) \\ u \sin(\psi) + v \cos(\psi) \\ r \\ \frac{m_2}{m_1}vr - \frac{d_1}{m_1}u \\ \frac{m_1}{m_2}ur - \frac{d_2}{m_2}v \\ \frac{(m_1 - m_2)}{m_3}uv - \frac{d_3}{m_3}r \end{bmatrix} \in \mathbb{R}^6,$$

$$\mathbf{g} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_3} \end{bmatrix}^T \in \mathbb{R}^6 \times \mathbb{R}^2, \text{ and}$$

$$\mathbf{g}_w(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & \frac{\cos(\psi)}{m_1} & -\frac{\sin(\psi)}{m_2} & 0 \\ 0 & 0 & 0 & \frac{\sin(\psi)}{m_1} & \frac{\cos(\psi)}{m_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_3} \end{bmatrix}^T \in \mathbb{R}^6 \times \mathbb{R}^3.$$

The parameters  $m_1, m_2, m_3$  are the ship inertia including added mass effect, and  $d_1, d_2, d_3$  are the hydrodynamic damping coefficients.

The controller objective is to steer the vessel states  $\mathbf{x}$  to follow a reference states  $\mathbf{x}_r$ , while avoiding collision of encountered vessels or objects according to Colregs rules.

The following assumptions are utilized throughout this paper:

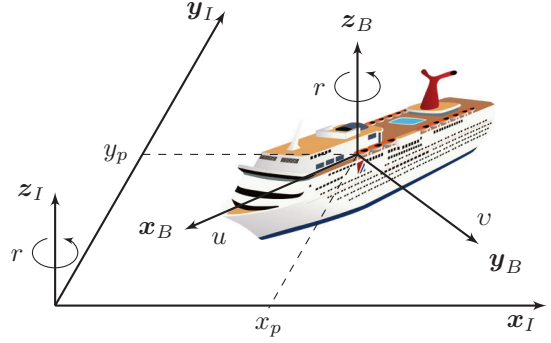


Fig. 1. Earth-fixed  $(x_I, y_I, z_I)$  and body-fixed  $(x_B, y_B, z_B)$  frames.

*Assumption 1:* All ship state variables (position, orientation, and velocities) are measurable or can be accurately estimated.

*Assumption 2:* The reference velocities and positions are smooth over time.

*Assumption 3:* The vessel is equipped with a Radar, Lidar, or Cameras that provide the position of the near by vessels, or there is a communication media among different vessels and position information is exchanged.

## III. CONTROLLER SYNTHESIS

### A. Nonlinear Model Predictive Control

Consider a continuous time-invariant nonlinear state space model of the vessel in the form of (1) that needed to be controlled while satisfying some states and control constraints:

$$\mathbf{x}(t) \in \mathcal{X}, \quad (2a)$$

$$\mathbf{u}(t) \in \mathcal{U}, \quad t \geq 0, \quad (2b)$$

where  $\mathcal{X} \subset \mathbb{R}^6$  and  $\mathcal{U} \subset \mathbb{R}^2$  are compact sets.

In general, the scheme of MPC is a form of control law in which the control value is obtained by solving an online finite optimal control problem utilizing a nominal model, as in (1), for the system and the last available measurement or accurate estimation as an initial state for the optimization solver. The output is a sequence of  $N$  control actions over a finite horizon  $T_p$ , that steer the system states to follow a time-varying reference. Then, the first element of the control sequence is applied and the whole process is repeated at the next sample to account for nominal model uncertainty.

Hence, NMPC is considered as a nonlinear state feedback  $\mathbf{u}(t) = \mathcal{K}(\mathbf{x}(t))$  obtained online from an optimal control problem that minimizes a least squares (LS) objective function by penalizing the deviation of the system inputs and states from the reference trajectories. It takes the form:

$$\min_{\mathbf{x}(t), \mathbf{u}(t)} J(\mathbf{x}, \mathbf{u}) = \int_{t=T_0}^{T_p+T_0} \ell(\mathbf{x}(t), \mathbf{u}(t)) dt \quad (3)$$

s.t. (1) and (2).

where  $\ell(\mathbf{x}(t), \mathbf{u}(t))$  is the stage cost function and must satisfy the following conditions[13]:

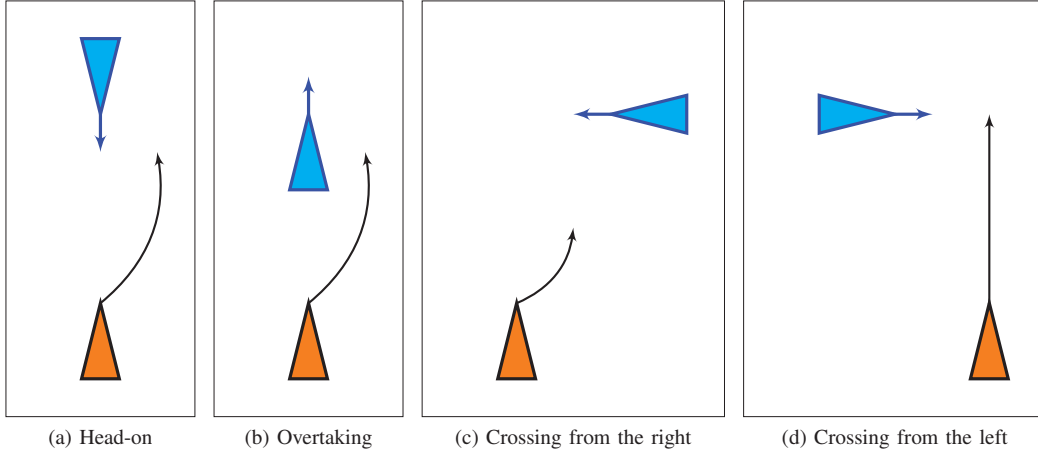


Fig. 2. Colregs maneuvers for different situations

- $\ell(\mathbf{x}_r, \mathbf{u}_r) = 0$
- $\ell(\mathbf{x}(t), \mathbf{u}(t)) > 0, \forall \mathbf{x}(t) \in \mathcal{X}, \mathbf{u}(t) \in \mathcal{U}, \mathbf{x}(t) \neq \mathbf{x}_r(t)$ .

We define the stage cost function as:

$$\ell(\mathbf{x}(t), \mathbf{u}(t)) = \|\mathbf{x}(t) - \mathbf{x}_r(t)\|_Q + \|\mathbf{u}(t) - \mathbf{u}_r(t)\|_R \quad (4)$$

where  $Q, R$  are positive definite weighing matrices. Control law is penalized because that may make optimization problem easier and avoid control values of high energy[14]. In practice the control law constraints are written in the form of linear inequalities to represent the actuator limitations, and hence (2b) is reformulated as

$$\mathbf{u}_{min} \leq \mathbf{u}(t) \leq \mathbf{u}_{max} \quad (5)$$

### B. Moving Horizon Estimation

Unlike NMPC formulation, that solves an online optimization problem that minimize the states deviation of the predicted states from the reference, Moving Horizon Estimation (MHE) solves a similar online optimization problem that minimize the past deviation of the trajectory from the measured output values[13] with respect to unmeasured states.

Here, MHE is used as a disturbance observer by considering the external forces  $\mathbf{w}$  as a constant extra states over the optimization horizon  $T_e$ . The problem is formulated as follows:

$$\begin{aligned} \min_{\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)} J(\mathbf{x}, \mathbf{u}) &= \int_{t=T_0-T_e}^{T_0} \ell_e(\mathbf{x}(t), \mathbf{u}(t)) dt \\ \text{s.t. (1),} \\ \mathbf{w}_{min} &\leq \mathbf{w} \leq \mathbf{w}_{max} \end{aligned} \quad (6)$$

where  $\mathbf{w}_{min}$  and  $\mathbf{w}_{max}$  are the known minimum and maximum values of the disturbances respectively, and the cost function is defined by:

$$\ell_e(\mathbf{x}(t), \mathbf{u}(t)) = \|\mathbf{x}(t) - \hat{\mathbf{x}}(t)\|_{Q_e} + \|\mathbf{u}(t) - \hat{\mathbf{u}}(t)\|_{R_e} \quad (7)$$

The solution of the this optimization problem is the external forces estimation  $\hat{\mathbf{w}}$ .

### C. Collision Avoidance

Conventionally, collision avoidance is treated as controller-independent planning problem [9] that might not be achievable by the controller. To overcome that and due to NMPC ability to handle the constraints of the control inputs and states systematically into its optimization problem, collision avoidance is translated into an inequality constraint. By defining the vessel ship domain as the area around the ship that should be free from other ships and obstacles [15] and by choosing a circular type, the collision avoidance constraint can take the form:

$$\sqrt{(x(t) - x_o(t))^2 + (y(t) - y_o(t))^2} \geq R, \forall t \in (0, T_p] \quad (8)$$

where  $x$  and  $y$  are the position of the vessel,  $x_o^i$  and  $y_o^i$  are the position of the  $i^{th}$  obstacle, and  $R$  is the safety distance that is measured from the center of our vessel and any of the encountered vessel and is selected based on the geometry of the vessels.

### D. Colregs Confinement

The maneuvering occurred due to the control action generated by the NMPC problem (3) with collision avoidance constraint (8) is collision-free, but is not necessarily confined to the International Regulations for Preventing Collisions at Sea(Colregs)[16] that set rules for the actions a vessel should follow when encountering other vessels at sea. We will focus on three main situations; namely Head-on, Overtaking, and Crossing (see Fig.2). In a head-on situation between two vessels, the vessel must turn to starboard (right) so that they pass on the port side (left) of each other. In an overtaking situation, the overtaking vessel must turn to starboard so that they pass on the port side of the overtaken one. When two vessels are crossing each other, the vessel which has the other on the starboard side must give way and avoid crossing ahead of her.

To integrate Colregs confinement into our NMPC approach, we impose another nonlinear state constraint into the problem (3) that force the encounter vessel to be on the left side of

the straight line generated by our vessel's position  $(x, y)$  and heading angle  $\psi$ . It takes the form:

$$\sin(\psi)(x(t) - x_o^i) - \cos(\psi)(y(t) - y_o^i) \geq 0 \quad (9)$$

The collision avoidance (8) and the Colregs confinement (9) constraints can only be imposed at the risk of collision for the vessel that must do the maneuvering according to the situation. Hence, the rule-based decision support system presented in [17] is adapted to identify the collision risk and detect which vessel will stand on its course and which one must do the maneuvering (give-away). The rule-based system evaluated over the whole prediction horizon  $T_p$  and is as follow:

- The USV is in overtaking situation if:
  - $\sqrt{(x(t) - x_o(t))^2 + (y(t) - y_o^i(t))^2} \leq R$
  - $|\psi(t) - \psi_o^i(t)| \leq \frac{\pi}{4}$
- The USV is in head-on situation if:
  - $\sqrt{(x(t) - x_o(t))^2 + (y(t) - y_o^i(t))^2} \leq R$
  - $|\psi(t) - \psi_o^i(t) + \pi| \leq \frac{\pi}{4}$
- The USV is in crossing from the right situation if:
  - $\sqrt{(x(t) - x_o(t))^2 + (y(t) - y_o^i(t))^2} \leq R$
  - $\frac{\pi}{4} \leq \psi_o^i(t) - \psi(t) \leq \frac{3\pi}{4}$
- The USV is in crossing from the left situation if:
  - $\sqrt{(x(t) - x_o(t))^2 + (y(t) - y_o^i(t))^2} \leq R$
  - $\frac{5\pi}{4} \leq \psi_o^i(t) - \psi(t) \leq \frac{7\pi}{4}$

### E. Multiple Shooting Method

In order to solve the NMPC optimization problem (3), a direct multiple shooting technique[18], derived from the solution of boundary value problems of differential equations, is used. The concept of this technique is based on uniform discretization of the horizon into  $N$  smaller intervals such that  $t_0 = 0 < t_1 < \dots < T_N = T_p$ , solves an initial value problem in each of the smaller intervals  $[t_j, t_{j+1}]$ , and imposes additional matching conditions to form a solution on the whole horizon. The problem constraints are evaluated on the grid nodes  $t_j$  and the control vector has been parametrized as piecewise constant. The result of this technique is a least-squares nonlinear programming (NLP) that can be solved employing Gauss-Newton method. The same is applied for MHE problem with  $N_e$  smaller intervals.

ACADO toolkit[19] is used for solving the underlying NMPC and MHE problem by generating a highly efficient C-code, used for real time implementation, and MATLAB executable (mex) files, used for simulation with MATLAB.

### F. Algorithm Flow

In this subsection, the whole algorithm used that combine NMPC with collision avoidance and Colregs confinement, is briefed as follow:

### Algorithm 1 NMPC with Collision Avoidance and Colregs Confinement

- 1: Initialize the external forces estimation  $\hat{w} = \mathbf{0}$ .
- 2: Get the states measurement  $\mathbf{x}$  of our USV, and predict the position over the prediction horizon  $T_p$ .
- 3: Get the position, velocity, and heading of the nearby vessels using a communication media or via radar and predict the position over the prediction horizon  $T_p$  using constant velocity (CV) model [20].
- 4: Execute the rule-based decision support system.
- 5: **if** the USV is in overtaking, head-on, or crossing from the right **then**
- 6:   Execute the following NMPC problem:

$$\min_{\mathbf{x}(t), \mathbf{u}(t)} J(\mathbf{x}, \mathbf{u}) = \int_{t=T_0}^{T_p+T_0} \ell(\mathbf{x}(t), \mathbf{u}(t)) dt \quad (10)$$

s.t. (1), (5), (8) and (9).

7: **else**

- 8:   Execute the following NMPC problem:

$$\min_{\mathbf{x}(t), \mathbf{u}(t)} J(\mathbf{x}, \mathbf{u}) = \int_{t=T_0}^{T_p+T_0} \ell(\mathbf{x}(t), \mathbf{u}(t)) dt \quad (11)$$

s.t. (1) and (5).

9: **end if**

- 10: Apply only the first element of the resulting control sequence.
- 11: Exute the MHE problem (6) to get estimation of the external forces  $\hat{w}(\cdot)$
- 12: wait for the next sampling instance and then go to step 2.

It is worth to mention that all the constraints (5), (8) and (9) are included in the programming and can be deactivated by relaxing the upper and lower limits of the inequalities during the execution.

### G. Stability of NMPC

To guarantee asymptotic stability by using the control law  $u(k) = \mathcal{K}(x(k))$ , it is desirable to use infinite prediction and control horizons, i.e., set  $N = \infty$  in (3), but it is not applicable to get the solution of the infinite horizon nonlinear optimization problem[21]. On the other hand, stability can be guaranteed for finite horizon problems by suitably choosing a terminal cost  $F$  and terminal attractive region  $\Omega$ . This result has been studied in [22], [21], [23], [24] and conditions required for that are summarized in[6]. Although, there are clear conditions for the asymptotic stability, but designing the terminal cost  $F$  and the attractive region  $\omega$  is still an open problem and may make the online optimizations more difficult and time consuming to solve[25]. In [14], it was shown that asymptomatic stability can be guaranteed just by tuning  $N$ ,  $Q$ , and  $R$ . Closed loop stability is achieved for relatively long horizons without the need to use terminal cost or terminal constraint [25]. Based on that, cost function (3) with  $\ell$  selected as in (4) is used in this paper.



TABLE I  
SURFACE VESSEL PARAMETERS

Para.	Value	Unit	Para.	Value	Unit
$m_1$	$120.0 \times 10^3$	kg	$d_1$	$215.0 \times 10^2$	$\text{kg} \cdot \text{s}^{-1}$
$m_2$	$172.9 \times 10^3$	kg	$d_2$	$97.0 \times 10^3$	$\text{kg} \cdot \text{s}^{-1}$
$m_3$	$636.0 \times 10^5$	kg	$d_3$	$802.0 \times 10^4$	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$

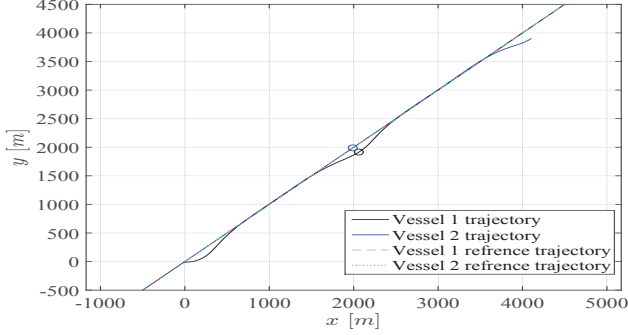


Fig. 3. Trajectory of both vessels at head-on situation

#### IV. SIMULATION RESULTS

In this section, simulation results are presented to demonstrate the validity and assess the performance of the proposed NMPC for tracking of the autonomous underactuated vessel (1). Simulation is done with the mex files exported using ACADO on the MATLAB. These results have been obtained on 3.3 GHz core i5 CPU with 8 GB RAM. The vessel parameters are chosen as in Table I [26], the maximum surge force is  $125kN$ , and the maximum yaw moment is  $10.0 \times 10^5 kN \cdot m$ . The prediction horizon length is selected to be  $T_p = 300s$  and the sampling interval is selected to be  $T_s = 5$ , which both lead to a discrete horizon of  $N = 60$  samples. The reason of choosing such a long prediction horizon is to detect the collision risk early enough for the algorithm to take the necessary maneuver. The matrices  $Q$  and  $R$  are chosen to be diagonal to weigh each state and control law independently with the values given in Table II. The safety radius  $R$  is selected to be  $50m$  which is suitable for our vessel of length  $L = 32m$ . MHE estimation horizon is selected to be  $T_e = 25s$ , that lead to a discrete horizon of  $N_e = 10$  samples. The matrices  $Q_e$  and  $R_e$  are chosen to be diagonal as in Table III. The encountered vessels are assumed to have the same dynamics and parameters, but without any collision avoidance scheme.

Crossing, head-on, and overtaking situations are evaluated under a constant disturbance of  $w = [2000 \ 800 \ 1200]^T$ , for the case where MHE is used as a disturbance observer.

Fig.3 shows the trajectory when our vessel is in a head-on situation with another vessel. The initial condition of our vessel is  $x(0) = [0 \ 0 \ 0 \ 5 \ 0 \ 0]^T$  and the reference course is  $\frac{\pi}{4}$ . It shows the ability of the NMPC to achieve offset free trajectory tracking by employing the estimated external forces using MHE. Moreover, our vessel perform the necessary Colregs-confined maneuver to avoid collision with the encountered

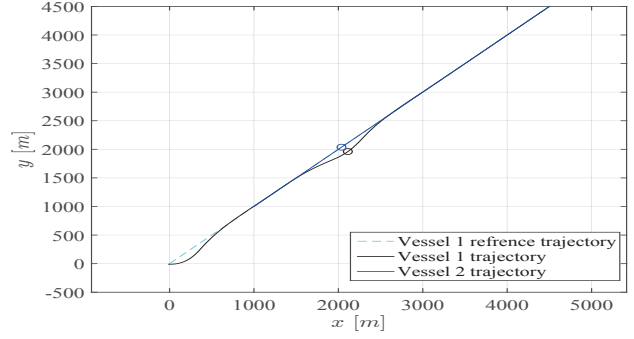


Fig. 4. Trajectory of both vessels at overtaking situation

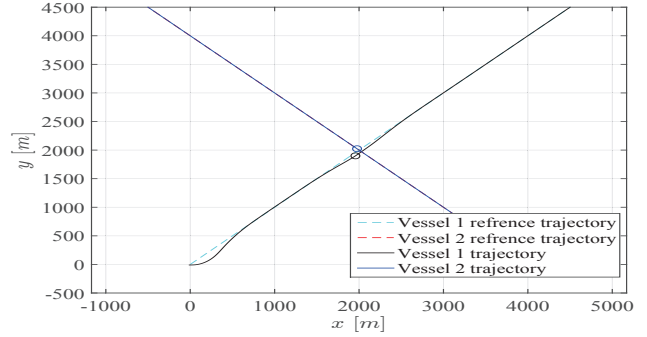


Fig. 5. Trajectory of both vessels at crossing situation

vessel with the aid of the rule-based decision support system. The two vessels are drawn as circles at the closest point of approach (CPA).

For the overtaking situation, our vessel has the same initial condition as head-on and the encountered vessel is assumed to have the same course but with a slower speed. The trajectory of both vessel, depicted in Fig.4, validates the ability of the proposed scheme to avoid the collision by maneuvering to the starboard side and bring the vessel back to its planned trajectory once there is no collision risk.

The crossing situation trajectories are presented in Fig. 5. A vessel crosses the trajectory of our USV from the right at a relative course of  $\frac{\pi}{2}$ . The proposed scheme detects the collision according to the Colregs rules and our USV maneuvers to the starboard side while keeping the safety distance and counteracting the external disturbance .

All the aforementioned situations use the MHE for estimating the disturbance and hence including it in the nominal model (1) used for the controller design. Fig. 6 shows the trajectory of both vessels in the overtaking situation while ignoring the external forces ( $w = 0$ ), which leads to an offset in the trajectory. Nevertheless, collision avoidance is achieved while obeying Colregs rules.

#### V. CONCLUSIONS

An NMPC scheme is presented for tracking of underactuated autonomous surface vessel with anti-collision scheme that obey Colregs rules, where a 3-DOF model is used with

TABLE II  
STATES AND CONTROL VARIABLES WEIGHTS FOR NMPC

Variable	$x$	$y$	$\chi$	$u$	$v$	$r$	$\tau_u$	$\tau_r$
Weight	5	5	5	1	1	1	0.001	0.001

TABLE III  
STATES AND CONTROL VARIABLES WEIGHTS FOR MHE

Variable	$x$	$y$	$\chi$	$u$	$v$	$r$	$\tau_u$	$\tau_r$
Weight	0.01	0.01	0.01	0.01	0.01	0.01	1	1

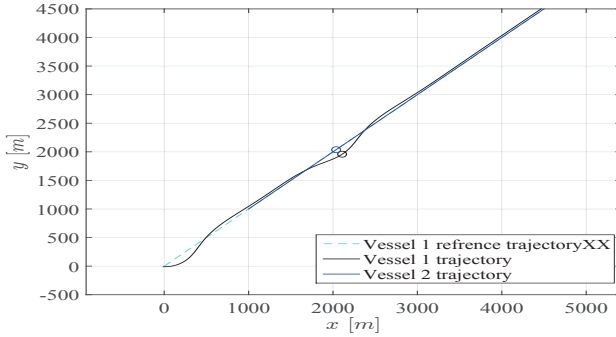


Fig. 6. Trajectory of both vessels at overtaking situation while ignoring the disturbances

only two control variables; surge force and yaw moment. A rule-based decision support system is used to detect the collision and hence activate the collision avoidance constraints. Employing a quadratic cost function, real-time efficient C code is generated using ACADO toolkit and qpOASES solver, which is called by MATLAB at each sampling interval, with collision avoidance as an inequality constraint for the optimization problem. Simulation results show the ability of the proposed scheme to track the reference trajectory while counteracting the disturbance, and achieve Colregs-confined collision avoidance in case of near collision situations. The maximum execution time of the algorithm is less than 20 ms, which amounts to appr. 0.4% of the sampling interval.

Due to the efficient implementation of the optimization routines, future research will aim at implementing that on the University of Oldenburg vessel named *Zuse* while taking into consideration uncertainty of measured data from the radar and the GPS.

#### ACKNOWLEDGMENT

This research is supported supported by the State of Lower Saxony as part of the project Critical Systems Engineering for Socio-Technical Systems (CSE). More data are available: <https://www.uni-oldenburg.de/en/cse/>.

#### REFERENCES

[1] D. Chwa, "Global tracking control of underactuated ships with input and velocity constraints using dynamic surface control method," *IEEE Transactions on Control Systems Technology*, vol. 19, no. 6, pp. 1357–1370, 2011.

[2] N. E. Kahveci and P. a. Ioannou, "Adaptive steering control for uncertain ship dynamics and stability analysis," *Automatica*, vol. 49, no. 3, pp. 685–697, 2013. [Online]. Available: <http://dx.doi.org/10.1016/j.automatica.2012.11.026>

[3] K. D. Do and J. Pan, "State- and output-feedback robust path-following controllers for underactuated ships using Serret-Frenet frame," *Ocean Engineering*, vol. 31, no. 5-6, pp. 587–613, 2004.

[4] Z. Dong, L. Wan, Y. Li, T. Liu, and G. Zhang, "Trajectory tracking control of underactuated USV based on modified backstepping approach," *international journal of naval architecture and ocean engineering*, vol. 7, no. 5, pp. 817–832, 2015.

[5] Q. Hu, H. Dong, Y. Zhang, and G. Ma, "Tracking control of spacecraft formation flying with collision avoidance," *Aerospace Science and Technology*, vol. 42, pp. 353–364, 2015. [Online]. Available: <http://linkinghub.elsevier.com/retrieve/pii/S1270963815000334>

[6] Z. Yan and J. Wang, "Model Predictive Control for Tracking of Underactuated Vessels Based on Recurrent Neural Networks," *IEEE Journal of Oceanic Engineering*, vol. 37, no. 4, pp. 717–726, 2012.

[7] M. Abdelaal, M. Fränzle, and A. Hahn, "Nonlinear Model Predictive Control for Tracking of Underactuated Vessels under Input Constraints," in *2015 IEEE European Modelling Symposium on Mathematical Modelling and Computer Simulation*, 2015, pp. 313–318.

[8] B. J. Guerreiro, C. Silvestre, R. Cunha, and A. Pascoal, "Trajectory Tracking Nonlinear Model Predictive Control for Autonomous Surface Craft," *IEEE Transaction on Control Systems Technology*, vol. 22, no. 6, pp. 3006–3011, 2014.

[9] M. Wang, J. Luo, and U. Walter, "A non-linear model predictive controller with obstacle avoidance for a space robot," *Advances in Space Research*, 2015.

[10] "ACADO Toolkit." [Online]. Available: <http://www.acadotoolkit.org>

[11] "qpOASES Homepage." [Online]. Available: <http://www.qpoases.org>

[12] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*, 1st ed. John Wiley & Sons Ltd., 2011, no. 1.

[13] J. Rawlings and D. Mayne, "Model Predictive Control : Theory and Design," pp. 1–11, 2012.

[14] L. Grüne and J. Pannek, *Nonlinear Model Predictive Control Theory and Algorithms*. Springer, 2011.

[15] R. Szlapczynski and J. Szlapczynska, "A Simulative Comparison of Ship Domains and Their Polygonal Approximations," *TransNav*, vol. 9, pp. 135–141, 2015.

[16] "COLREGS - International Regulations for Preventing Collisions at Sea," *Convention on the International Regulations for Preventing Collisions at Sea*, 1972, pp. 1–74, 1972.

[17] Y. Kuwata, M. T. Wolf, D. Zarzhitsky, and T. L. Huntsberger, "Safe Maritime Autonomous Navigation With COLREGS, Using Velocity Obstacles," *IEEE Journal of Oceanic Engineering*, vol. 39, no. 1, pp. 110–119, 2014.

[18] H. Bock and K. Plitt, "A multiple shooting algorithm for direct solution of optimal control problems," in *Proceedings of the 9th IFAC World Congress*, 1984.

[19] B. Houska and H. Joachim, "ACADO Toolkit User's Manual," 2014. [Online]. Available: <http://acado.github.io/documentation.html>

[20] X. Rong Li and V. P. Jilkov, "Survey of maneuvering target tracking. Part I. Dynamic models," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 39, no. 4, pp. 1333–1364, 2003. [Online]. Available: <http://dx.doi.org/10.1109/TAES.2003.1261132>

[21] L. Magni, G. D. Nicolao, L. Magnani, and R. Scattolini, "A stabilizing model-based predictive control algorithm for nonlinear systems," *Automatica*, vol. 37, no. 9, pp. 1351–1362, 2001.

[22] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, pp. 789–814, 2000.

[23] H. Chen and F. Allgöwer, "A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability," *Automatica*, vol. 34, no. 10, pp. 1205–1217, 1998.

[24] C. Ong, D. Sui, and E. Gilbert, "Enlarging the terminal region of nonlinear model predictive control using the support vector machine method," *Automatica*, vol. 42, no. 6, pp. 1011–1016, 2006.

[25] A. Jadbabaie and J. Hauser, "On the Stability of Receding Horizon Control With a General Terminal Cost," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 674–678, 2005.

[26] K. D. Do, Z. P. Jiang, and J. Pan, "Universal controllers for stabilization and tracking of underactuated ships," *Systems and Control Letters*, vol. 47, no. 4, pp. 299–317, 2002.