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Observer Based Path Following for Underactuated Marine Vessels in the Presence of Ocean Currents: A Local Approach

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Abstract: In this article a solution to the problem of following a curved path in the presence of a constant unknown ocean current disturbance is presented. The path is parametrised by a path variable that is used to propagate a path-tangential reference frame. The update law for the path variable is chosen such that the motion of the path-tangential frame ensures that the vessel remains on the normal of the path-tangential reference frame. As shown in the seminal work Samson [1992] such a parametrisation is only possible locally. A tube is defined in which the aforementioned parametrisation is valid and the path-following problem is solved within this tube. The size of the tube is proportional to the maximum curvature of the path. It is shown that within this tube, the closed-loop system of the proposed controller, guidance law, and the ocean current observer provides exponential stability of the path-following error dynamics. The sway velocity dynamics are analysed taking into account couplings previously overlooked in the literature, and is shown to remain bounded. Simulation results are presented.

1. INTRODUCTION

In this work we consider path following of an underactuated marine vessel. The problem of path following for underactuated marine vessels has its parallel in the field of mobile robotics. Therefore, a solution for 2D path following for underactuated marine vessels based on the tools developed in Samson [1992] and Micaelli and Samson [1993] was proposed in Encarnaçao et al. [2000], where the path parametrization is used to define the path-following problem and a solution is presented using a nonlinear controller. An observer is used to incorporate the effects of the unknown, but constant ocean current. Part of the state is shown to be stable and the zero dynamics are analysed and shown to be well behaved. However, this is done under the assumption that the total speed is constant. This requires active control of the forward velocity to cancel the effect of the sideways velocity induced by turning. Moreover, the parametrisation from Micaelli and Samson [1993] is only valid locally, making the path-following result valid locally. Another local result based on the same parametrisation is obtained in Do and Pan [2004]. In this work a practical stability result is shown for the path-following states of an underactuated surface vessel in the presence of an environmental disturbance. However, in Do and Pan [2004] there is a problem with the bound of the practical stability result and the error cannot be made arbitrary small. Moreover, a simplified model with diagonal system matrices is used and the interconnection between the total velocity and the sideways velocity is not taken into account in the analysis of the zero dynamics. In Lapierre and Soetanto [2007] the path parametrisation is used to solve the path-following problem globally. This is done using another result, first described for the control of mobile robots in Soetanto et al. [2003]. In particular, it is achieved by adapting the parametrisation of the path in order to avoid singularities in the parametrisation of the path. The work in Lapierre and Soetanto [2007] does not consider environmental disturbances, however. It focuses on stabilisation of the path-following states but does not analyse the zero dynamics. A similar approach is taken in Børhaug and Pettersen [2006] in which the frame is propagated differently. In Børhaug and Pettersen [2006] a look-ahead based steering law is used to guide the vehicle to the path. Stability of the path-following errors is shown using cascaded systems theory, and the zero dynamics are analysed and shown to be well behaved. To take into account ocean currents, the work in Børhaug and Pettersen [2006] is extended in Børhaug et al. [2008] by adding integral action to the steering laws. However, the results in Børhaug et al. [2008] are only valid for straight-line path following. The work in Børhaug et al. [2008] was revisited in Caharija et al. [2012] for surface vessels using a relative velocity model. Experimental results were added in Caharija et al. [2016]. To address curved paths,

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the work of Børhaug and Pettersen [2006] is extended with an ocean current observer in Moe et al. [2014]. However, in Moe et al. [2014] the zero dynamics are not analysed and the suggested input signals contain the unknown ocean current. Another line-of-sight (LOS) guidance approach for path following is presented in Fossen et al. [2003], which is used to follow a path made of straight-line sections connecting way points. These concepts are further developed to circles in Breivik and Fossen [2004] where the vessel is regulated to the tangent of its projection on the circle. These works do not consider environmental disturbances.

This paper considers path-following of underactuated marine vessels in the presence of constant ocean currents, for general paths. A line-of-sight guidance law, an ocean current observer, and a local parametrisation of the path are used. Compared to Do and Pan [2004], in this work the parametrisation is adapted to include the effect of the unknown ocean currents and a complete analysis of the sway velocity dynamics are given, taking into account the coupling between the total velocity and the sway velocity. Moreover, the mass and damping matrix are allowed to be non-diagonal and we avoid the problems with the practical stability result. Due to the locality of the parametrisation it can only be used in a certain tube around the path whose size depends on the maximum curvature of the path. When in this tube, it is shown that the closed-loop system of the controllers and the ocean current observer provides exponential stability of the path-following error dynamics.

The article is organized as follows: in Section 2 the vessel model and the problem definition are presented. The path parametrisation is introduced in Section 3. Section 4 presents the ocean current observer, the guidance law, and controllers. The closed-loop system is then formulated and analysed in Section 5. A simulation case study is presented in Section 6 and conclusions are given in Section 7.

2. VESSEL MODEL AND PROBLEM DEFINITION

We consider the following model, given in Fossen [2011], for an ASV or an AUV moving in a plane:

$$\dot{x} = u_r\cos(\psi) - v_r\sin(\psi) + V_x, \tag{1a}$$
$$\dot{y} = u_r\sin(\psi) + v_r\cos(\psi) + V_y, \tag{1b}$$
$$\dot{\psi} = r, \tag{1c}$$
$$\dot{u}_r = F_{u_r}(v_r, r) - \frac{d_{11}}{m_{11}}u_r + \tau_u, \tag{1d}$$
$$\dot{v}_r = X(u_r)r + Y(u_r)v_r, \tag{1e}$$
$$\dot{r} = F_r(u_r, v_r, r) + \tau_r, \tag{1f}$$

The functions $X(u_r)$, $Y(u_r)$, F_u, and F_r are given by

$$F_{u_r}(v_r, r) \triangleq \frac{1}{m_{11}}(m_{22}v_r + m_{23}r)r, \tag{2a}$$
$$X(u_r) \triangleq \frac{m_{23}^2 - m_{11}m_{33}}{m_{22}m_{33} - m_{23}^2}u_r + \frac{d_{33}m_{23} - d_{23}m_{33}}{m_{22}m_{33} - m_{23}^2}, \tag{2b}$$
$$Y(u_r) \triangleq \frac{(m_{22} - m_{11})m_{23}}{m_{22}m_{33} - m_{23}^2}u_r - \frac{d_{22}m_{33} - d_{32}m_{23}}{m_{22}m_{33} - m_{23}^2}, \tag{2c}$$
$$\begin{aligned} F_r(u_r, v_r, r) \triangleq {} & \frac{m_{23}d_{22} - m_{22}(d_{32} + (m_{22} - m_{11})u_r)}{m_{22}m_{33} - m_{23}^2}v_r \\ & + \frac{m_{23}(d_{23} + m_{11}u_r) - m_{22}(d_{33} + m_{23}u_r)}{m_{22}m_{33} - m_{23}^2}r. \end{aligned} \tag{2d}$$

Note that the functions $X(u_r)$ and $Y(u_r)$ are linear functions of the velocity. The kinematic variables are illustrated in Figure 1. Moreover, we assume that the following assumptions hold.

Assumption 1. The ocean current is assumed to be constant and irrotational with respect to the inertial frame, i.e. $\boldsymbol{V}_c \triangleq [V_x, V_y, 0]^T$. Furthermore, it is bounded by $V_{\max} > 0$ such that $\|\boldsymbol{V}_c\| = \sqrt{V_x^2 + V_y^2} \leq V_{\max}$.

Moreover, for the considered range of values of the desired surge velocity u_{rd}, the following assumption holds.

Assumption 2. It is assumed that $Y(u_r)$ satisfies $Y(u_r) \leq -Y_{\min} < 0$, $\forall u_r \in [-V_{\max}, u_{rd}]$, i.e. $Y(u_r)$ is negative for the range of desired velocities considered.

Assumption 2 is satisfied for commercial vessels by design, since the converse would imply an undamped or nominally unstable vessel in sway.

Assumption 3. It is assumed that $2V_{\max} < u_{rd}(t)$ $\forall t$, i.e. the desired relative velocity of the vessel is sufficiently larger than the maximum value of the ocean current.

Assumption 3 assures that the vessel can overcome the ocean current affecting it. The factor two in Assumption 3 adds some extra conservativeness to bound the solutions of the ocean current observer. This is discussed further in Section 4.

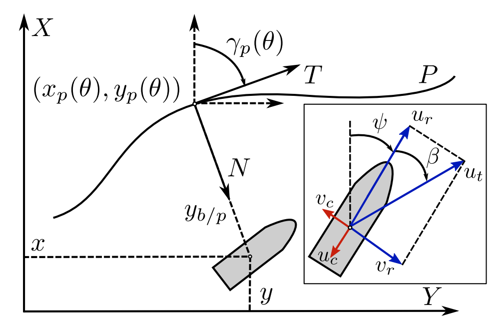

Fig. 1. Definition of the path with its parameterisation and the ship's kinematic variables given inside the box.

2.1 Problem definition

The goal is to follow a smooth path P, parametrised by a path variable θ, by controlling the ship's surge velocity and yaw rate. To follow a path, the direction of the motion needs to be tangential to the path. Therefore, for an underactuated vessel, path following can be achieved by positioning the vessel on the path with the total velocity $u_t \triangleq \sqrt{u_r^2 + v_r^2}$ (see Figure 1) tangential to the path. To express the path-following error, we propagate a path-tangential frame along P such that the vessel will be on the normal of the frame at all time. This is illustrated in Figure 1. The preceding implies that the progression of the path-tangential frame should be controlled such that the path-following error takes the form:

$$\begin{bmatrix} x_{b/p} \\ y_{b/p} \end{bmatrix} = \begin{bmatrix} \cos(\gamma_p) & \sin(\gamma_p) \\ -\sin(\gamma_p) & \cos(\gamma_p) \end{bmatrix} \begin{bmatrix} x - x_p \\ y - y_p \end{bmatrix} = \begin{bmatrix} 0 \\ y_{b/p} \end{bmatrix} \tag{3}$$

where $\gamma_p(\theta)$ is the angle of the path with respect to the X-axis, $x_{b/p}$ is the deviation from the normal in tangential direction, and $y_{b/p}$ is the deviation from the tangent in normal direction. Consequently, the goal is to regulate $x_{b/p}$ and $y_{b/p}$ to zero. Note that the time derivative of the angle $\gamma_p(\theta)$ is given by $\dot{\gamma}_p(\theta) = \kappa(\theta)\dot{\theta}$ where $\kappa(\theta)$ is the curvature of P at θ.

3. LOCALLY VALID PARAMETRISATION

In this section we will find an update law for the path variable θ that keeps the error in the tangential direction $x_{b/p}$ at zero. It is well known that such a parametrisation will only be unique locally (Samson [1992]). The error dynamics of the vessel with respect to the path frame is given by:

$$\dot{x}_{b/p} = -\dot{\theta}(1-\kappa(\theta)y_{b/p}) + u_t\cos(\chi-\gamma_p(\theta)) + V_T, \quad (4a)$$

$$\dot{y}_{b/p} = u_t\sin(\chi-\gamma_p(\theta)) + V_N - \kappa(\theta)\dot{\theta}x_{b/p}, \quad (4b)$$

where $\chi \triangleq \psi+\beta$ is the course angle (see Figure 1) and $V_T \triangleq V_x\cos(\gamma_p(\theta)) + V_y\sin(\gamma_p(\theta))$ and $V_N \triangleq V_y\cos(\gamma_p(\theta)) - V_x\sin(\gamma_p(\theta))$ are the ocean current component in the tangential direction and normal direction of the path-tangential reference frame, respectively. Consequently, if the path variable θ is updated according to

$$\dot{\theta} = \frac{u_t\cos(\chi-\gamma_p(\theta)) + V_T}{1-\kappa(\theta)y_{b/p}}, \quad (5)$$

the vessel stays on the normal when it starts on the normal. To make sure that the update law (5) is well defined the following condition should be satisfied

Condition 1. It should hold that $1-\kappa(\theta)y_{b/p} \neq 0$ such that the update law for the path variable θ is well defined for all time.

Note that Condition 1 implies that the update law is well defined within the tube of radius $y_{b/p} < 1/\kappa_{\max}$, which results in the parametrisation being only locally valid.

The update law (5) depends on the current component V_T. However, since the current is assumed unknown we have to replace V_T by its estimate $\hat{V}_T \triangleq \hat{V}_x\cos(\gamma(\theta)) + \hat{V}_y\sin(\gamma(\theta))$. Consequently, the last equality of (3) does not hold until the current is estimated correctly. Therefore, we augment the parametrisation of Samson [1992], i.e (5), to be

$$\dot{\theta} = \frac{u_t\cos(\chi-\gamma_p(\theta)) + \hat{V}_T + k_\delta x_{b/p}}{1-\kappa(\theta)y_{b/p}}, \quad (6)$$

such that the path-tangential reference frame propagates based on an estimate of the ocean current and has a restoring term to drive $x_{b/p}$ to zero. This extension allows us to use the parametrisation in the presence of an unknown disturbance. Hence, substituting (6) in (4a) gives

$$\dot{x}_{b/p} = -k_\delta x_{b/p} + \tilde{V}_T, \quad (7)$$

which shows that if the estimate of the current has converged, the restoring term $k_\delta x_{b/p}$ remains to drive $x_{b/p}$ to zero. With this choice of parametrisation and update law, the error dynamics of the vessel is given by (7)-(4b), and the control objective is to stabilize the origin $(x_{b/p}, y_{b/p}) = (0,0)$. In the next section, we will develop a guidance law that stabilizes the origin, and thus achieves the goal of path following.

Note that since the path parametrisation is only local, we can only utilise it within a tube of radius $1/\kappa_{\max}$ around the path. To achieve global results this tube would need to be made attractive and invariant. The disadvantage of this is that a two-step approach is needed to solve the path-following problem, which complicates the analysis. There is, however, also a big advantage to this approach, since extra design freedom is available when making the tube attractive. In particular, for a global one-step approach the behaviour will be a compromise between the desired behaviour far away from the path and the desired behaviour close to the path, while for a two-step approach the two controllers can be tuned independently. This allows faster convergence to the path and can be particularly advantageous in cluttered environments.

4. CONTROLLER, OBSERVER, AND GUIDANCE

In this section we design the two control laws for τ_u and τ_r, and the ocean current estimator, that are used to achieve path-following. In the first subsection we present the velocity control law τ_u. The second subsection presents the ocean current estimator. The third subsection presents the guidance law we propose to use within the tube.

4.1 Surge velocity control

The velocity control law is a feedback-linearising P-controller that is used to drive the relative surge velocity to a desired, possibly time-varying, reference u_{rd} and is given by

$$\tau_u = -F_{u_r}(v_r, r) + \dot{u}_{rd} + \frac{d_{11}}{m_{11}}u_{rd} - k_u(u_r - u_{rd}), \quad (8)$$

where $k_u > 0$ is a constant controller gain. It is straightforward to verify that (8) ensures global exponential tracking of the desired velocity. In particular, when (8) is substituted in (1d) we obtain

$$\dot{\tilde{u}}_r = -k_u(u_r - u_{rd}) = -k_u\tilde{u}_r, \quad (9)$$

where $\tilde{u}_r \triangleq u_r - u_{rd}$. Consequently, the velocity error dynamics are described by a stable linear systems, which assures exponential tracking of the desired velocity u_{rd}.

4.2 Ocean current estimator

This subsection presents the ocean current estimator introduced in Aguiar and Pascoal [2007]. This observer provides the estimate of the ocean current needed to implement (6) and the guidance law developed in the next subsection. Rather than estimating the time-varying current components in the path frame V_T and V_N, the observer is used to estimate the constant ocean current components in the inertial frame V_x and V_y. The observer from Aguiar and Pascoal [2007] is based on the kinematic equations of the vehicle, i.e. (1a) and (1b), and requires measurements of the vehicle's x and y coordinates in the inertial frame. The observer is formulated as

$$\dot{\hat{x}} = u_r\cos(\psi) - v_r\sin(\psi) + \hat{V}_x + k_{x_1}\tilde{x} \quad (10a)$$

$$\dot{\hat{y}} = u_r\sin(\psi) + v_r\cos(\psi) + \hat{V}_y + k_{y_1}\tilde{y} \quad (10b)$$

$$\dot{\hat{V}}_x = k_{x_2}\tilde{x} \quad (10c)$$

$$\dot{\hat{V}}_y = k_{y_2}\tilde{y} \quad (10d)$$

where $\tilde{x} \triangleq x - \hat{x}$ and $\tilde{y} = y - \hat{y}$ are the positional errors and k_{x_1}, k_{x_2}, k_{y_1}, and k_{y_2} are constant positive gains. Consequently, the estimation error dynamics are given by

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \\ \dot{\tilde{V}}_x \\ \dot{\tilde{V}}_y \end{bmatrix} = \begin{bmatrix} -k_{x_1} & 0 & 1 & 0 \\ 0 & -k_{y_1} & 0 & 1 \\ -k_{x_2} & 0 & 0 & 0 \\ 0 & -k_{y_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{V}_x \\ \tilde{V}_y \end{bmatrix}. \quad (11)$$

which is a linear system with negative eigenvalues. Hence, the observer error dynamics are globally exponentially stable at the origin. Note that this implies that also $\hat{V}_T$ and $\hat{V}_N$ converge to V_T and V_N, respectively, with global uniform exponential convergence.

For implementation of the controllers, it is desired that $\|\hat{V}_N(t)\| < u_{rd}(t)\ \forall t$. To achieve this, we first choose the initial conditions of the estimator as

$$[\hat{x}(t_0), \hat{y}(t_0), \hat{V}_x(t_0), \hat{V}_y(t_0)]^T = [x(t_0), y(t_0), 0, 0]^T. \quad (12)$$

Consequently, the initial estimation error is given by

$$[\tilde{x}(t_0), \tilde{y}(t_0), \tilde{V}_x(t_0), \tilde{V}_y(t_0)]^T = [0, 0, V_x, V_y]^T, \quad (13)$$

which has a norm smaller than or equal to $V_{\max}$ according to Assumption 1. Now consider the function

$$W(t) = \tilde{x}^2 + \tilde{y}^2 + \tfrac{1}{k_{x_2}}\tilde{V}_x^2 + \tfrac{1}{k_{y_2}}\tilde{V}_y^2, \quad (14)$$

which has the following time derivative

$$\dot{W}(t) = -2k_{x_1}\tilde{x}^2 - 2k_{y_1}\tilde{y}^2 \leq 0. \quad (15)$$

This implies that $W(t) \leq \|W(t_0)\|$ and it can be verified that $\|\tilde{\boldsymbol{V}}_c(t)\|^2/\max(k_{x_2}, k_{y_2}) \leq W(t)$. The choice of initial conditions implies that $\|W(t_0)\| \leq V_{\max}^2/\min(k_{x_2}, k_{y_2})$. Combining the observations given above we obtain

$$\tfrac{1}{\max(k_{x_2}, k_{y_2})}\|\tilde{\boldsymbol{V}}_c(t)\|^2 \leq \tfrac{1}{\min(k_{x_2}, k_{y_2})}V_{\max}^2. \quad (16)$$

Consequently, we obtain

$$\|\tilde{\boldsymbol{V}}_c(t)\| \leq \sqrt{\tfrac{\max(k_{x_2}, k_{y_2})}{\min(k_{x_2}, k_{y_2})}}V_{\max} < \sqrt{\tfrac{\max(k_{x_2}, k_{y_2})}{\min(k_{x_2}, k_{y_2})}}u_{rd}(t),$$

for all time, which implies that if the gains are chosen as $k_{x_2} = k_{y_2}$, we have $\|\hat{V}_N\| \leq 2V_{\max} \leq u_{rd}(t),\ \forall t$. Hence, $\|\hat{V}_N\| < u_{rd}(t),\ \forall t$ if $2V_{\max} < u_{rd}(t),\ \forall t$.

Remark 1. The bound $2V_{\max} < u_{rd},\ \forall t$, is only required when deriving the bound on the solutions of the observer. In particular, it is required to guarantee that $\|\hat{V}_N\| < u_{rd}(t),\ \forall t$. For the rest of the analysis it suffices that $V_{\max} < u_{rd},\ \forall t$. Therefore, if the more conservative bound $2V_{\max} < u_{rd},\ \forall t$, is not satisfied the observer can be changed to an observer that allows explicit bounds on the estimate $\hat{V}_N$, e.g. the observer developed in Narendra and Annaswamy [1987], rather than an observer that only provides a bound on the error $\tilde{\boldsymbol{V}}_c$ as is the case here. For practical purposes the estimate can also be saturated such that $\|\hat{V}_N\| < u_{rd},\ \forall t$, which is the approach taken in Moe et al. [2014]. However, in the theoretical analysis of the yaw controller we use derivatives of $\hat{V}_N$ which will be discontinuous when saturation is applied.

4.3 Guidance

This subsection presents the guidance law that is used in combination with the local parametrisation. Since, the chosen parametrisation is only valid in a tube around the path, the proposed guidance is designed for operation in the tube. Inside the tube we propose the following guidance law

$$\psi_d = \gamma(\theta) - \operatorname{atan}\left(\frac{v_r}{u_{rd}}\right) - \operatorname{atan}\left(\frac{y_{b/p} + g}{\Delta}\right). \quad (17)$$

The guidance law consists of three terms. The first term is a feedforward of the angle of the path with respect to the inertial frame. The second part is the desired side-slip angle, i.e. the angle between the surge velocity and the total speed when $u_r \equiv u_{rd}$. This side-slip angle is used to make the vehicle's total speed tangential to the path when the sway velocity is non-zero. The third term is a line-of-sight (LOS) term that is intended to steer the vessel to the path, where, as in Moe et al. [2014], g is term dependent on the ocean current. The choice of g provides extra design freedom to compensate for the component of the ocean current along the normal axis V_N. To analyse the effect of this guidance law and to design g, we consider the error dynamics along the normal (4b). To this end we substitute (17) in (4b) and obtain

$$\dot{y}_{b/p} = -u_{td}\frac{y_{b/p} + g}{\sqrt{(y_{b/p} + g)^2 + \Delta^2}} + V_N + G_1(\cdot) \quad (18a)$$

where $G_1(\tilde{\psi}, \tilde{u}_r, x_{b/p}, \psi_d, y_{b/p}, u_{td}, \dot{\gamma}_p(\theta))$ is a perturbing term given by

$$G_1(\cdot) = u_{td}\left[1 - \cos(\tilde{\psi})\right]\sin\left(\operatorname{atan}\left(\tfrac{y_{b/p}+g}{\Delta}\right)\right) + \tilde{u}_r\sin(\psi - \gamma_p(\theta)) + u_{td}\cos\left(\operatorname{atan}\left(\tfrac{y_{b/p}+g}{\Delta}\right)\right)\sin(\tilde{\psi}) - x_{b/p}\dot{\gamma}_p(\theta) \quad (19)$$

and $u_{td} \triangleq \sqrt{u_{rd}^2 + v_r^2}$ is the desired total velocity. Note that $G_1(\cdot)$ satisfies

$$G_1(0, 0, 0, \psi_d, y_{b/p}, u_{td}, \dot{\gamma}_p(\theta)) = 0 \quad (20a)$$

$$\|G_1(\cdot))\| \leq \zeta(\dot{\gamma}_p(\theta), u_{td})\|(\tilde{\psi}, \tilde{u}, x_{b/p})\|, \quad (20b)$$

where $\zeta(\dot{\gamma}_p(\theta), u_{td}) > 0$, which shows that $G_1(\cdot)$ is zero when the perturbing variables are zero and that it has maximal linear growth in the perturbing variables.

To compensate for the ocean current component V_N the variable g is now chosen to satisfy the equality

$$u_{td}\tfrac{g}{\sqrt{\Delta^2 + (y_{b/p}+g)^2}} = \hat{V}_N. \quad (21)$$

which is a choice inspired by Moe et al. [2014]. In order for g to satisfy the equality above, g should be the solution of the following second order equality

$$\underbrace{(u_{td}^2 - \hat{V}_N^2)}_{-a}\left(\tfrac{g}{\hat{V}_N}\right)^2 = \underbrace{\Delta^2 + y_{b/p}^2}_{c} + 2\underbrace{y_{b/p}\hat{V}_N}_{b}\left(\tfrac{g}{\hat{V}_N}\right), \quad (22)$$

hence we choose g to be

$$g = \hat{V}_N\tfrac{b + \sqrt{b^2 - ac}}{-a}, \quad (23)$$

which has the same sign as $\hat{V}_N$ and is real and well defined for $(u_{rd}^2 - \hat{V}_N^2) > 0$.

Consequently if we substitute this choice for g in (18) we obtain

$$\dot{y}_{b/p} = -u_{td}\tfrac{y_{b/p}}{\sqrt{(y_{b/p}+g)^2 + \Delta^2}} + \tilde{V}_N + G_1(\cdot). \quad (24)$$

The desired yaw rate can be found by taking the time derivative of (17) resulting in

$$\dot{\psi}_d = \kappa(\theta)\dot{\theta} + \tfrac{\dot{v}_r u_{rd} - \dot{u}_{rd}v_r}{u_{rd}^2 + v_r^2} + \tfrac{\Delta(\dot{y}_{b/p} + \dot{g})}{\Delta^2 + (y_{b/p}+g)^2}, \quad (25)$$

where $\dot{v}_r$ is given in (1e), $\dot{y}_{b/p}$ in (24), and $\dot{g}$ is given by

$$\dot{g} = \dot{\hat{V}}_N\tfrac{b + \sqrt{b^2 - ac}}{-a} + \tfrac{\partial g}{\partial a}\dot{a} + \tfrac{\partial g}{\partial b}\dot{b} + \tfrac{\partial g}{\partial c}\dot{c}, \quad (26)$$

Note that $\dot{y}_{b/p}$ appears many times in the expression for $\dot{\psi}_d$ and that $\dot{y}_{b/p}$ depends on $\tilde{V}_N$. Consequently, $\dot{\psi}_d$ depends on an unknown variable and cannot be used to control the yaw rate. This was not considered in Moe et al. [2014] where the proposed controller contained both $\dot{\psi}_d$ and $\ddot{\psi}_d$.

Moreover, since $\dot{\psi}_d$ contains $\dot{v}_r$, which depends on $r = \dot{\psi}$, the yaw rate error $\dot{\tilde{\psi}} \triangleq \dot{\psi} - \dot{\psi}_d$ grows with $\dot{\psi}$, which leads to a necessary condition for a well defined yaw rate error. The yaw rate error dynamics are given by

$$\dot{\tilde{\psi}} = r\left[1 + \frac{X(u_r)u_{rd}}{u_{rd}^2+v_r^2} - \frac{\Delta}{\Delta^2+\left(y_{b/p}+g\right)^2}\frac{\partial g}{\partial a}\left(2v_r X(u_r)\right)\right] + \Gamma, \quad (27)$$

$$\begin{aligned}\Gamma \triangleq & -\kappa(\theta)\dot{\theta} + \frac{Y(u_r)v_r u_{rd} - \dot{u}_{rd}v_r}{u_{rd}^2+v_r^2} \\ & + \frac{\Delta}{\Delta^2+\left(y_{b/p}+g\right)^2}\left[\dot{\hat{V}}_N\left(\frac{b+\sqrt{b^2-ac}}{-a} + 2\frac{\partial g}{\partial b}y_{b/p}\right)\right. \\ & + \frac{\partial g}{\partial a}\left(2\hat{V}_N\dot{\hat{V}}_N - 2u_{rd}\dot{u}_{rd} - 2v_r Y(u_r)v_r\right) \\ & \left. + \left(1 + \frac{\partial g}{\partial c}2y_{b/p} + 2\frac{\partial g}{\partial b}\hat{V}_N\right)\dot{y}_{b/p}\right].\end{aligned} \quad (28)$$

which leads to the following necessary condition for a well defined yaw rate, i.e. existence of the yaw controller,

Condition 2. In the tube, it should hold that

$$C_r \triangleq 1 + \frac{X(u_r)u_{rd}}{u_{rd}^2+v_r^2} - \frac{\partial g}{\partial a}\frac{2v_r X(u_r)\Delta}{\Delta^2+\left(y_{b/p}+g\right)^2} \neq 0. \quad (29)$$

such that the yaw rate controller is well defined.

Remark 2. The condition above can be verified for any positive velocity, for the vehicles considered in the case study of this work. Note that for most vessels this condition is verifiable, since standard ship design practices will result in similar properties of the function $X(u_r)$. Besides, having a lower bound greater then zero, C_r is also upper-bounded since the term between brackets can be verified to be bounded in its arguments.

Since $\dot{\psi}_d$ depends on the unknown signal $\tilde{V}_N$ we cannot take $\dot{\psi}_d = r_d$. To define an expression for r_d without requiring the knowledge of $\tilde{V}_N$ we use (27) to define $r_d \triangleq \Gamma/C_r$, which results in the following yaw angle error dynamics

$$\dot{\tilde{\psi}} = C_r\tilde{r} + \left[1 + \frac{\partial g}{\partial c}2y_{b/p} + \frac{\partial g}{\partial b}\left(2\hat{V}_N\right)\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2} \quad (30)$$

where $\tilde{r} \triangleq r - r_d$ is the yaw rate error. From (30) it can be seen that choosing r_d as above, results in yaw angle error dynamics that have a term dependent on the yaw rate error $\tilde{r}$ and a perturbing term that vanishes when the estimation error $\tilde{V}_N$ goes to zero.

To add acceleration feedforward to the yaw rate controller, the derivative of r_d needs to be calculated. However, when we analyse the dependencies of r_d we obtain $r_d = r_d(h, y_{b/p}, x_{b/p}, \tilde{\psi}, \tilde{x}, \tilde{y})$, where $h = [\theta, v_r, u_r, u_{rd}, \dot{u}_{rd}, \hat{V}_T, \hat{V}_N]^T$ is introduced for the sake of brevity and represents all the variables whose derivatives do not contain $\tilde{V}_N$ or $\tilde{V}_T$. Consequently, the acceleration feedforward cannot be taken as $\dot{r}_d$, since using (27), (7), and (4b) it is straightforward to verify that this signal contains the unknowns $\tilde{V}_T$ and $\tilde{V}_N$. Therefore we define the yaw rate controller in terms of only known signals as:

$$\begin{aligned}\tau_r = & -F(u_r, v_r, r) + \frac{\partial r_d}{\partial h^T}\dot{h} - \frac{\partial r_d}{\partial \tilde{x}}k_x\tilde{x} - \frac{\partial r_d}{\partial \tilde{y}}k_y\tilde{y} \\ & + \frac{\partial r_d}{\partial y_{b/p}}\left(-u_{td}\frac{y_{b/p}}{\sqrt{\Delta^2+(y_{b/p}+g)^2}} + G_1(\cdot)\right) \\ & - k_{\delta_x}\frac{\partial r_d}{\partial x_{b/p}}x_{b/p} + \frac{\partial r_d}{\partial \tilde{\psi}}C_r\tilde{r} - k_1\tilde{r} - k_2\tilde{\psi}\end{aligned} \quad (31)$$

Using (31) in (1f) we then obtain the following yaw rate error dynamics

$$\begin{aligned}\dot{\tilde{r}} = & -k_1\tilde{r} - k_2 C_r\tilde{\psi} - \frac{\partial r_d}{\partial \boldsymbol{p}_{b/p}}[\tilde{V}_T, \tilde{V}_N]^T + \frac{\partial r_d}{\partial \tilde{x}}\tilde{V}_x + \frac{\partial r_d}{\partial \tilde{y}}\tilde{V}_y \\ & - \frac{\partial r_d}{\partial \tilde{\psi}}\left[1 + \frac{\partial g}{\partial c}2y_{b/p} + \frac{\partial g}{\partial b}\left(2\hat{V}_N\right)\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2}\end{aligned} \quad (32)$$

where $\boldsymbol{p}_{b/p} \triangleq [x_{b/p}, y_{b/p}]^T$ and that has a term depending on the yaw angle error, a term depending on the yaw rate error, and perturbing terms depending on the unknown ocean current estimation error.

Remark 3. It is straightforward to verify that all the terms in (25) are smooth fractionals that are bounded with respect to $(y_{b/p},\ x_{b/p},\ \tilde{x},\ \tilde{y},\ \tilde{\psi})$ or are periodic functions with linear arguments, and consequently the partial derivatives in (31) and (32) are all bounded. This is something that is used when showing closed-loop stability in the next section.

5. CLOSED-LOOP ANALYSIS

In this section we analyse the closed-loop system consisting of the model (1) with controllers (8) and (31) and observer (10) when the frame propagates by the update law (6) along the path P. To show that path following is achieved we have to show that the following error dynamics converge to zero

$$\dot{x}_{b/p} = -k_\delta x_{b/p} + \tilde{V}_T \quad (33a)$$

$$\dot{y}_{b/p} = -u_{td}\frac{y_{b/p}}{\sqrt{\Delta^2+(y_{b/p}+g)^2}} + G_1(\cdot) + \tilde{V}_N \quad (33b)$$

$$\dot{\tilde{\psi}} = C_r\tilde{r} + \left[1 + 2\frac{\partial g}{\partial c}y_{b/p} + 2\frac{\partial g}{\partial b}\hat{V}_N\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2} \quad (33c)$$

$$\begin{aligned}\dot{\tilde{r}} = & -k_1\tilde{r} - k_2 C_r\tilde{\psi} - \frac{\partial r_d}{\partial \boldsymbol{p}_{b/p}}[\tilde{V}_T, \tilde{V}_N]^T + \frac{\partial r_d}{\partial \tilde{x}}\tilde{V}_x + \frac{\partial r_d}{\partial \tilde{y}}\tilde{V}_y \\ & - \frac{\partial r_d}{\partial \tilde{\psi}}\left[1 + \frac{\partial g}{\partial c}2y_{b/p} + 2\frac{\partial g}{\partial b}\hat{V}_N\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2}\end{aligned} \quad (33d)$$

$$\dot{\tilde{u}} = -\left(k_u + \frac{d_{11}}{m_{11}}\right)\tilde{u} \quad (33e)$$

Note that all the perturbing terms disappear as the current estimates converge to zero. In particular, we cannot apply our desired control action whilst the current estimates have not converged yet, since the current cannot be compensated for until it is estimated correctly. The full closed-loop system of the model (1) with controllers (8) and (31) and observer (10) is given by

$$\begin{aligned}\dot{\tilde{X}}_1 \triangleq \begin{bmatrix}\dot{y}_{b/p} \\ \dot{\tilde{\psi}} \\ \dot{\tilde{r}}\end{bmatrix} = & \begin{bmatrix}-u_{td}\frac{y_{b/p}}{\sqrt{\Delta^2+(y_{b/p}+g)^2}} + G_1(\cdot) \\ C_r\tilde{r} \\ -k_1\tilde{r} - k_2 C_r\tilde{\psi}\end{bmatrix} + \\ & \begin{bmatrix}\tilde{V}_N \\ \left[1+\frac{\partial g}{\partial c}2y_{b/p}+\frac{\partial g}{\partial b}(2\hat{V}_N)\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2} \\ -\frac{\partial r_d}{\partial \boldsymbol{p}_{b/p}}\begin{bmatrix}\tilde{V}_T \\ \tilde{V}_N\end{bmatrix} - \frac{\partial r_d}{\partial \tilde{\psi}}\left[1+\frac{\partial g}{\partial c}2y_{b/p}+\frac{\partial g}{\partial b}2\hat{V}_N\right]\frac{\Delta\tilde{V}_N}{\Delta^2+\left(y_{b/p}+g\right)^2} - \frac{\partial r_d}{\partial \tilde{\boldsymbol{p}}}\tilde{\boldsymbol{V}}_c\end{bmatrix}\end{aligned} \quad (34a)$$

$$\dot{\tilde{X}}_2 \triangleq \begin{bmatrix}\dot{x}_{b/p} \\ \dot{\tilde{x}} \\ \dot{\tilde{y}} \\ \dot{\tilde{V}}_x \\ \dot{\tilde{V}}_y \\ \dot{\tilde{u}}\end{bmatrix} = \begin{bmatrix}-k_\delta x_{b/p} + \tilde{V}_T \\ -k_x\tilde{x} - \tilde{V}_x \\ -k_y\tilde{y} - \tilde{V}_y \\ -k_{x1}\tilde{x} \\ -k_{y1}\tilde{y} \\ -k_u\tilde{u}\end{bmatrix} \quad (34b)$$

$$\begin{aligned}\dot{v}_r = & X(u_{rd}+\tilde{u})r_d(h, y_{b/p}, x_{b/p}, \tilde{\psi}, \tilde{x}, \tilde{y}) + \\ & + X(u_{rd}+\tilde{u})\tilde{r} + Y(u_{rd}+\tilde{u})v_r\end{aligned} \quad (34c)$$

Before starting with the stability analysis of (34), we first establish GES of (34b) by using the following lemma.

Lemma 1. The system (34b) is GES.

The proof is omitted due to space constraints, but can be found in Maghenem et al. [2017]. Note that due to the parametrisation the dynamics of $x_{b/p}$ are only defined in the tube and the stability result is only valid in the tube.

The first step in the stability analysis of (34) is to assure that the closed-loop system is forward complete and that the sway velocity v_r remains bounded. Therefore, under the assumption that Condition 1-2 are satisfied, i.e. $1 - \kappa(\theta)y_{b/p} \neq 0$ and $C_r \neq 0$, we take the following steps:

(1) First, we prove that the trajectories of the closed-loop system are forward complete.
(2) Then, we derive a necessary condition such that v_r is locally bounded with respect to $(\tilde{X}_1, \tilde{X}_2)$.
(3) Finally, we establish that for a sufficiently big value of Δ, v_r is locally bounded only with respect to $\tilde{X}_2$.

The above three steps are taken by formulating and proving three lemmas. Due to space constraints the proofs of the lemmas are replaced by a sketch of each proof. The full proofs can be found in Maghenem et al. [2017].

Lemma 2. (Forward completeness). The trajectories of the global closed-loop system (34) are forward complete.

The general idea is as follows. Forward completeness for (34b) is evident since this part of the closed-loop system consists of GES error dynamics. Using the forward completeness and in fact boundedness of (34b), we can show forward completeness of (34c), $\dot{\tilde{\psi}}$, and $\dot{\tilde{r}}$. Hence, forward completeness of (34) depends on forward completeness of $\dot{y}_{b/p}$. To show forward completeness of $\dot{y}_{b/p}$, we consider the $y_{b/p}$ dynamics with $\tilde{X}_2$, $\tilde{\psi}$, $\tilde{r}$, and v_r as input, which allows us to claim forward completeness of $\dot{y}_{b/p}$. Consequently, all the states of the closed-loop system are forward complete and hence the closed-loop system (34) is forward complete.

Lemma 3. (Boundedness near $(\tilde{X}_1, \tilde{X}_2) = 0$). The system (34c) is bounded near $(\tilde{X}_1, \tilde{X}_2) = 0$ if and only if the curvature of P satisfies the following condition:

$$\kappa_{\max} \triangleq \max_{\theta \in P} |\kappa(\theta)| < \frac{Y_{\min}}{X_{\max}}. \tag{35}$$

A sketch of the proof is as follows. The sway velocity dynamics (34c) are analysed using a quadratic Lyapunov function $V = 1/2v_r^2$. It can be shown that the derivative of this Lyapunov function satisfies the conditions for boundedness when the solutions are on or close to the manifold where $(\tilde{X}_1, \tilde{X}_2) = 0$. Consequently, (34c) satisfies the conditions of boundedness near $(\tilde{X}_1, \tilde{X}_2) = 0$ as long as (35) is satisfied.

In Lemma 3 we show boundedness of v_r for small values of $(\tilde{X}_1, \tilde{X}_2)$ and derive a bound on the curvature. However, locality with respect to $\tilde{X}_1$, i.e. the path-following errors and yaw angle and yaw rate errors, is not desired. In the next lemma, boundedness independently of $\tilde{X}_1$ is shown under an extra condition on the look-ahead distance Δ.

Lemma 4. (Boundedness near $\tilde{X}_2 = 0$). If the following additional assumption is satisfied:

$$\exists\, \sigma > 0\ : 1 - \kappa(\theta)y_{b/p} \geq \sigma > 0 \wedge \left[Y_{\min} - X_{\max}\kappa_{\max}\tfrac{1}{\sigma}\right] > 0$$

the system (34c) is bounded only near $\tilde{X}_2 = 0$ if we have

$$\Delta > \frac{4X_{\max}}{\left[Y_{\min} - X_{\max}\kappa_{\max}\frac{1}{\sigma}\right]}, \qquad \kappa_{\max} < \sigma\frac{Y_{\min}}{X_{\max}} \tag{36}$$

Remark 4. The size of σ depends on the choice of Δ. A large Δ implies a small σ which in turn implies a larger tube defined by $y_{b/p}^{\text{tube}} = (1-\sigma)/\kappa_{\max}$. Note that the tube defined by $y_{b/p}^{\text{tube}}$ is the suitable set of initial conditions for the system. Furthermore, the assumptions reflect that ocean current has to be such that it does not push the vehicle outside the tube during the transient.

The proof follows along the same lines of that of Lemma 3, but solutions are considered close to the manifold $\tilde{X}_2 = 0$ rather than $(\tilde{X}_1, \tilde{X}_2) = 0$. It is shown that boundedness can still be shown if (36) is satisfied additionally to the conditions of Lemma 3.

Theorem 1. Consider a θ-parametrised path denoted by $P(\theta) \triangleq (x_p(\theta), y_p(\theta))$. Then under Conditions 1-2 and the conditions of Lemma 2-4, the system (1) with control laws (8) and (31) and observer (10) follows the path P, while maintaining v_r, τ_r and τ_u bounded. In particular, the origin of the system (34a)-(34b) is exponentially stable in the tube $y_{b/p}^{\text{tube}} = (1-\sigma)/\kappa_{\max}$.

Proof. From the fact that the origin of (34b) is GES, the fact that the closed-loop system (34) is forward complete according to Lemma 2, and the fact that solutions of (34c) are locally bounded near $\tilde{X}_2 = 0$ according to Lemma 4, we can conclude that there is a finite time $T > t$ after which solutions of (34b) will be sufficiently close to $\tilde{X}_2 = 0$ to guarantee boundedness of v_r. Having established that v_r is bounded we first analyse the cascade

$$\begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{r}} \end{bmatrix} = \begin{bmatrix} C_r\tilde{r} \\ -k_1\tilde{r} - k_2C_r\tilde{\psi} \end{bmatrix} + \begin{bmatrix} G_2(\cdot) \\ -\frac{\partial r_d}{\partial \tilde{\psi}}G_2(\cdot) - \frac{\partial r_d}{\partial \boldsymbol{p}_{b/p}}[\tilde{V}_T, \tilde{V}_N]^T + \frac{\partial r_d}{\partial[\tilde{x},\tilde{y}]^T}\tilde{\boldsymbol{V}}_c \end{bmatrix} \tag{37a}$$

$$\begin{bmatrix} \dot{x}_{b/p} \\ \dot{\tilde{x}} \\ \dot{\tilde{y}} \\ \dot{\tilde{V}}_x \\ \dot{\tilde{V}}_y \\ \dot{\tilde{u}} \end{bmatrix} = \begin{bmatrix} -k_\delta x_{b/p} + \tilde{V}_T \\ -k_x\tilde{x} - \tilde{V}_x \\ -k_y\tilde{y} - \tilde{V}_y \\ -k_{x1}\tilde{x} \\ -k_{y1}\tilde{y} \\ -k_u\tilde{u} \end{bmatrix} \tag{37b}$$

The perturbing system (37b) is GES as shown in Lemma 1. The interconnection term, i.e. the second and third term in (37a), satisfies the linear growth criteria from [Panteley et al., 1998, Theorem 2]. More specifically, it has an upperbound that does not grow with $\tilde{\psi}$ and $\tilde{r}$ since all the partial derivatives of r_d and g can be bounded by constants. The nominal dynamics, i.e. the first matrix in (37a), can be analysed with the following quadratic Lyapunov function $V_{(\tilde{r},\tilde{\psi})} = \frac{1}{2}\tilde{r}^2 + \frac{1}{2}k_2\tilde{\psi}^2$,whose derivative along the solutions of the nominal system is given by

$$\dot{V}_{(\tilde{r},\tilde{\psi})} = -k_1\tilde{r}^2 - k_2C_r\tilde{\psi}\tilde{r} + k_2C_r\tilde{r}\tilde{\psi} = -k_2\tilde{r}^2 \leq 0 \tag{38}$$

which implies that $\tilde{r}$ and $\tilde{\psi}$ are bounded. The derivative of (38) is given by

$$\ddot{V}_{(\tilde{r},\tilde{\psi})} = -2k_1^2\tilde{r}^2 - 2k_1k_2C_r\tilde{\psi}\tilde{r} \tag{39}$$

which is bounded since $\tilde{r}$ and $\tilde{\psi}$ are bounded. This implies that (38) is a uniformly continuous function. Consequently, from Barbalat's lemma (Khalil [2002, Lemma 8.2]) we have that

$$\lim_{t\to\infty}\dot{V}_{(\tilde{r},\tilde{\psi})} = \lim_{t\to\infty} -k_1\tilde{r}^2 = 0 \ \Rightarrow\ \lim_{t\to\infty}\tilde{r} = 0. \tag{40}$$

Since C_r is persistently exciting, which follows from the fact that C_r is upper bounded and lower bounded by positive constants, it follows from the expression of the nominal dynamics that

$$\lim_{t\to\infty} \tilde{r} = 0 \;\Rightarrow\; \lim_{t\to\infty} \tilde{\psi} = 0. \tag{41}$$

This implies that the system is globally asymptotically stable. Consequently, from the above it follows that the cascade (37) is GES [Panteley et al., 1998, Theorem 2].

We now consider the following dynamics

$$\dot{y}_{b/p} = -u_{td} \frac{y_{b/p}}{\sqrt{\Delta^2 + (y_{b/p} + g)^2}} + \tilde{V}_N + G_1(\cdot). \tag{42}$$

Note that we can view the systems (37) and (42) as a cascaded system where the nominal dynamics are formed by the first term of (42), the interconnection term is given by the second and third terms of (42), and the perturbing dynamics are given by (37). As we have just shown, the perturbing dynamics are GES. Using the bound on $G_1(\cdot)$ from (20) it is straightforward to verify that the interconnection term satisfies the conditions of [Panteley et al., 1998, Theorem 2]. We now consider the following Lyapunov function for the nominal system $V_{y_{b/p}} = 1/2 y_{b/p}^2$, whose derivative along the solutions of the nominal system is given by

$$\dot{V}_{y_{b/p}} = -u_{td} \frac{y_{b/p}^2}{\sqrt{\Delta^2 + (y_{b/p} + g)^2}} \leq 0, \tag{43}$$

which implies that the nominal system is GAS. Moreover, since it is straightforward to verify that $\dot{V}_{y_{b/p}} \leq \alpha V_{y_{b/p}}$ for some constant α dependent on the initial conditions, it follows from the comparison lemma (Khalil [2002, Lemma 3.4]) that the nominal dynamics are also LES. Consequently, the cascaded system satisfies the conditions of Panteley and Loria [1998, Theorem 2] and Panteley et al. [1998, Lemma 8], and therefore the cascaded system is GAS and LES. This implies that the origin of the error dynamics, i.e. $(\tilde{X}_1, \tilde{X}_2) = (0, 0)$, is globally asymptotically stable and locally exponentially stable. However, since the parametrisation is only valid locally we can only claim exponential stability in the tube.

6. CASE STUDY

This section presents a case study to verify the theoretical results. The goal of the case study is to follow a circular path using the model of an underactuated surface vessel from Fredriksen and Pettersen [2004]. The ocean current components are given by $V_x = -1$ [m/s] and $V_y = 1.2$ [m/s], and consequently $V_{\max} \approx 1.562$ [m/s]. The desired relative surge velocity is chosen to be constant and set to $u_{rd} = 5$ [m/s] such that Assumption 3 is verified. It is straightforward to see that the curvature bound from Lemma 3 is given by $\kappa_{\max} < (Y_{\min})/(X_{\max}) \approx 0.1333$ using the parameters of the model in Fredriksen and Pettersen [2004]. The observer is initialised as suggested in Subsection 4.2 and the observer gains are selected as $k_{x_1} = k_{y_1} = 1$ and $k_{x_1} = k_{y_1} = 0.1$. The controller gains are selected as $k_{u_r} = 0.1$ for the surge velocity controller and $k_1 = 1000$ and $k_2 = 400$ for the yaw rate controller.

In this case study the vessel is required to follow a circle with a radius of 400 [m] that is centred around the origin. Consequently, the curvature of the path is given by $\kappa_p = 1/400 = 0.0025$. Choosing $\Delta = 40$ [m] results in $\sigma \approx 0.0268$ which in turn implies $y_{b/p}^{\text{tube}} \approx 369.983$ [m]. Note that this is only slightly smaller then the size of the tube where the parametrisation is valid, i.e. 400 [m]. To stay within this tube we choose the initial conditions as $[u_r, v_r, r, x, y, \psi]^T = [0, 0, 0, 700, 10, \pi/2]^T$.The resulting trajectory for the vessel can be seen in Figure 2. From the plot in Figure 2, the vessel converges to the circle (in red) and starts to follow the path. Moreover, it can be seen from the yellow vessels that the orientation of the ship is not tangential to the circle, but instead automatically adopts a crab angle which is necessary to compensate for the ocean current. The path-following errors can be seen in the top

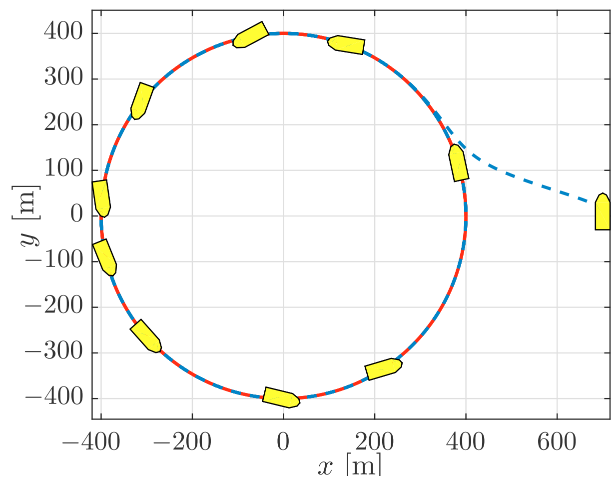

Fig. 2. Path of the vessel in the $x-y$-plane. The dashed blue line is the trajectory of the path and the red line is the reference. The yellow ships denote the orientation of the vessel at certain times.

plot of Figure 3, which confirms that the path-following errors converge to zero. A detail of the steady-state is given to show the reduction of the error. Moreover, note that because of the choice of parametrisation, the error in tangential direction $x_{b/p}$ is zero throughout the motion except during a very small transient at the beginning caused by the transient of the observer. The estimates obtained from the ocean current observer can be seen in the second plot from the top in Figure 3. From this plot it can be seen that the estimates converge exponentially with no overshoot. This underlines the conservativeness of the bound from Assumption 3 that is required for the error bound for the observer as explained in Subsection 4.2. The third plot in Figure 3 depicts the yaw rate and the sway velocity induced by the motion. It can be seen that these do not converge to zero but converge to a periodic motion. Note that for circular motion in the absence of current, the yaw rate would converge to zero. However, when current is present the vessel needs to change its turning rate depending on whether it goes with or against the current. This plot also contains the magnitude of C_r. From this plot it can clearly be seen that Condition 2 is verified in steady-state and during the transient of the velocity controller.

7. CONCLUSIONS

This article considered curved-path following for underactuated marine vessels in the presence of constant ocean currents. In the proposed control approach the path is parametrised by a path variable with an update law that

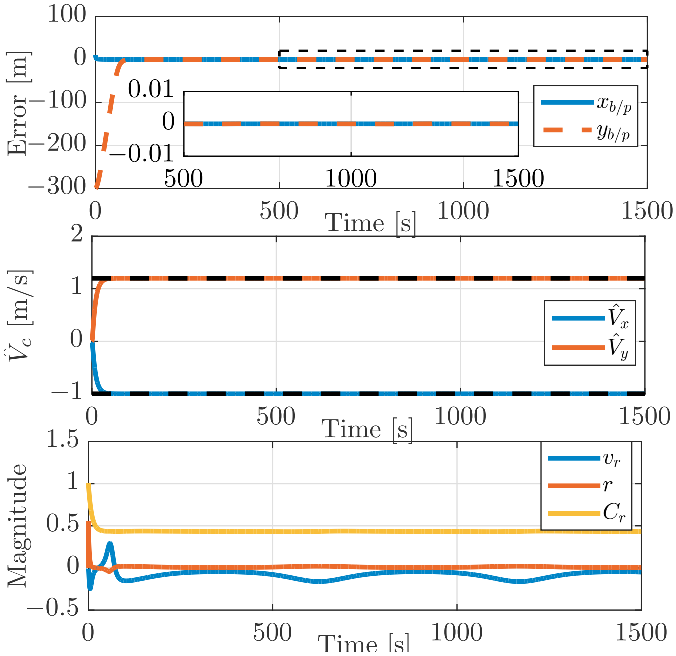

Fig. 3. Path-following errors plotted agains time (top), current estimates against time (second), sway velocity [m/s], yaw rate [rad/s], and C_r against time (bottom).

is designed to keep the vessel on the normal of a path-tangential reference frame. This parametrisation is well defined in a tube around the path, whose size is proportional to the curvature of the path. The vessel is steered using a line-of-sight guidance law, which to compensate for the unknown ocean currents is aided by an ocean current observer. The closed-loop system was analysed and it was shown that the path-following errors are exponentially stable in the tube. Moreover, a complete analysis of the underactuated sway velocity dynamics was given that takes into account couplings previously overlooked in the literature. The analysis showed that boundedness of the sway velocity is guaranteed if certain conditions are verified.

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