Cooperative Mapping and Exploration using Counter-rotational Potential Fields

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***Abstract*—This paper presents a novel technique for mapping and exploration using cooperating autonomous underwater vehicles. Rather than using the typical lawnmower sweep pattern to search an entire area, the proposed navigational plan involves guiding the formation directly towards each object of interest in turn, before arriving at a final goal position. This is achieved by the use of traditional artificial potential fields alongside counter-rotational potential fields. These clockwise and counter-clockwise fields are employed simultaneously by vehicles to ensure that the entire object is scanned rather than simply avoided as is the case with traditional collision avoidance techniques. The proposed methodology allows a formation to have fluid-like motion whilst a separation distance between cooperating agents (free of angular constraints) is maintained with a greater degree of flexibility than traditional formation control approaches. Owing to its nature, this technique is suited for applications such as exploration, mapping and underwater inspection to name a few. Simulation results demonstrate the efficacy of the proposed approach.**

***Keywords* - counter-rotational, artificial potential fields, obstacle avoidance, cooperative control, mapping and surveying**

I. Introduction

Over the past number of years there has been a significant increase in the range of uses for autonomous underwater vehicles (AUVs). This has largely been driven by technological improvements alongside a decrease in manufacturing costs with a market value predicted to reach $4.84 Billion by 2019 [1]. Their applications range from military to commercial, right down to the individual hobbyist. The defence industry regularly employs them for surveillance, communications, intelligence-gathering, reconnaissance, oceanography, navigation etc. with low-cost models (designed to be expendable) used to detect and eradicate underwater mines [2]. At present their commercial applications are dominated by the oil and gas industries [3] and include underwater inspection of pipelines and oil rigs - the 2010 BP oil spill disaster highlighted the importance of such a technology [4][5]. The scientific establishment employs them to explore and investigate underwater biodiversity, water clarity, coastline exploration and data-gathering to name a few [6][7]. While the reduction in manufacturing costs has added hobbyists to the list of available users it has also created an avenue for use by terrorists and drug barons [8].

Research has recently been maturing and, coupled with developments in acoustic communications, the trend is now shifting towards using multiple AUVs or MAUVs [9]. MAUVs have a number of advantages compared to single vehicles, for example, in the fields of mapping and exploration a team of cooperating vehicles can ensure that a much greater area is covered in a reduced amount of time. A leader vehicle (which could be a surface vessel) can guide a fleet of less complex agents to perform simple or monotonous tasks which provides a reduction in overall costs. Heterogeneous vehicles can work within a team to perform a variety of tasks in a single visit depending on the objective [10], for example, sending in agents capable of high-resolution image capture when an object has been detected by a sweeping vehicle. In recent times the greatest need for MAUVs has been displayed by the (so far unsuccessful) search for the missing Malaysia Airlines flight MH370 [11].

MAUVs rely on a cooperative control framework within which a number of challenges exist, including localisation, communications and collision avoidance to name a few. Localisation is a critical issue especially when a vehicle's depth increases - the lack of GPS underwater forces vehicles to keep track of their position using dead reckoning techniques and/or periodically resurface for a GPS fix. Underwater communication poses challenges and can be affected by interference, limited-bandwidth, salinity and real-time reception [12]; proper design of control architecture must enable cooperation between necessary platforms for successful missions [13]. Operating within a dynamic and uncertain environment also raises the threat of vehicle failure, alongside power and energy consumption during long-term missions. Group formation combined with path-planning and collision avoidance (including both static and dynamic objects alongside other vehicles) poses further research challenges.

II. Background and Overview

As mentioned previously, when working with fleets of autonomous vehicles, the challenges include maintaining formation and/or communication channels, task allocation, costs, arriving at goal, object avoidance (including other vehicles) etc. Over the past number of years, various techniques have been developed to tackle these problems. These can be broadly classified as centralised (where a vehicle or permanent station

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coordinates the movement of other vehicles) or decentralised (each vehicle is independent but can work cooperatively with others if required). The two methods are not mutually exclusive and commonly draw on aspects of each other for differing missions and working environments. Under this classification the research has been broadly defined under three further categories - behaviour-based, leader-follower and virtual structures. Again, aspects of these three techniques are frequently employed together within individual research projects.

Behaviour-based strategies can also be referred to as reactive strategies due to their animal-like behaviour where a vehicle will sense an object and then react to it [13]. Rules for vehicle behaviour can vary greatly and will depend upon the working environment and mission objectives. These rules can be prioritised by assigning cost functions to evaluate their level of priority at any instant of time [9]. Fish schooling behaviour has been employed as a technique for underwater vehicles [14][15].

Leader-follower, as the name suggests, is a technique whereby a leader (or leaders) will guide a number of vehicles alongside its trajectory towards a goal position. Follower vehicles can move at an angle specific to the leader (or directly behind it) meaning a reduction in the complexity of their hardware/software in comparison to the leader vehicle, thus reducing overall operation costs. A follower (or followers) can also be designed as a back-up leader should any issues arise with the leader vehicle [16].

The virtual structure technique involves a number of vehicles which move in a specific formation or shape. One method of achieving this is to use a virtual vehicle at the centre of the formation and place the remainder of the fleet at pre-specified distances and angles from it [17]. The virtual vehicle can be guided along a trajectory towards the goal and bring the formation along with it, or the trajectory can be assigned to the entire formation [18].

A number of further research areas focus on techniques which are used within the aforementioned categories including communications [19], task allocation [13], stability [9], mapping [20], path-planning [21] and collision avoidance [17].

A. Artificial Potential Fields

A pioneering technique originally developed by Khatib in the 1980s likens the vehicles' working environment to an electrical potential field. Using both attractive and repulsive potentials, a vehicle is guided away from areas of high potential (repulsion - objects) towards a low potential (attraction - goal position) [22].

Equation 1 below describes a typical attractive potential field function at a goal position:

$$Ux_d(x) = \frac{1}{2}k(x - x_d)^2 \tag{1}$$

Where $Ux_d(x)$ is the attractive potential field from the goal, x is the current position of the vehicle, x_d is desired goal and k is a scaling factor.

Equation 2 below is a typical example of a repulsive force function used around objects to repel vehicles away from their boundaries during a mission.

$$U_o(x) = \begin{cases} \frac{1}{2}\eta(\frac{1}{\rho} - \frac{1}{\rho_o})^2 & \text{if } \rho \leq \rho_o \\ 0 & \text{if } \rho \geq \rho_o \end{cases} \tag{2}$$

Where $U_o(x)$ is the repulsive force around each object, ρ_o is the limit distance of the potential field and ρ the shortest distance to object O, η is a scaling factor.

Artificial potential fields (APFs) are computationally inexpensive and the technique has also been extended by a number of researchers to include multiple vehicles [14][23][24][25]. One major problem with this method is the ability to get trapped in a position where the cumulative sum of the attractive/repulsive vectors around it produce a null force. This is known as a local minimum which prevents the vehicle from moving forward towards the goal. A number of solutions have been proposed in order to solve it such as simulated annealing [26], creating a nearby virtual object [27] or rotational potential fields created around objects [28][29][30].

Work to the present has been concerned with reaching a target and includes object-avoidance as one of the necessary challenges to be overcome, usually focusing on the shortest path to goal. This paper presents a novel approach to guide a pair of autonomous vehicles across a workspace to survey the boundaries of every object of interest within an environment using counter-rotational potential fields. In [28]and [30] a rigid formation of vehicles was shown to be guided around objects towards a goal position. The entire formation travels towards the goal without splitting or deforming its shape. In both these papers, rotational potential fields were used to guide vehicle formations around objects in either a clockwise or anticlockwise direction. A virtual vehicle is used in [29] to guide a rigid formation to an intended target, this structure splits when meeting an object head on and recombines when it is passed.

Here the need for a virtual vehicle is removed, as is a rigid structure. These will be replaced by a novel approach using a more fluid-like formation whereby vehicles will maintain an approximate separation distance while dispensing with angular constraints. Both attractive and repulsive forces are assigned to vehicles which can be switched in order to maintain the flexible separation distance as they head towards a final goal position. Counter-rotational potential fields will be used to ensure that every object within a defined working area is deliberately scanned on opposing sides of its boundary. A range of possibilities exist where this technique could be employed e.g. seabed mapping or exploration, an initial scouting mission to check an area before other work ensues, inspection of underwater structures such as piers or oil rigs etc.

The rest of this paper is laid out as follows. Vehicle guidance and the method of assigning rotational potential fields to objects is covered in Section III. Section IV describes how the fluid-like structure and the vehicle inter-separation distances are maintained. Sections V and VI deal with the simulation results and conclusions.

III. Vehicle Guidance

In this paper, APFs will be used to guide an ensemble of vehicles to a number of waypoints within a defined, planar environment. Attractive forces are generated at places of interest (such as a target position or waypoint) while repellent forces are used around objects to be avoided (obstacles or approaching vehicles). The cumulative sum of these forces then determines an overall resultant field which guides the vehicles. The potential field changes as the formation navigates in the environment due to the proximity of the vehicles to each other. The formation then make its way from high potential towards the point of lowest potential.
Both attractive and repulsive equations used in this paper are shown below in Equations 3 and 4 respectively-

$$V_a = \frac{1}{2} K_a((X - xp(t))^2 + (Y - yp(t))^2) \tag{3}$$

Where V_a is the attractive force around a goal or vehicle, K_a is a scaling factor, X and Y are matrices of all coordinates within the defined workspace and $xp(t)$ and $yp(t)$ are the coordinates within the defined workspace of the goal or vehicle at time *t*.

$$V_r = \frac{K_r}{\sqrt{((X - xp(t))^2 + (Y - yp(t))^2)}} \tag{4}$$

Where V_r is the repulsive force around an object or vehicle and K_r is a scaling factor.

Equations 3 and 4 above are used simultaneously to create both attractive and repulsive potentials for target locations and objects. These potentials are also used both to maintain formation and/or avoid collisions with other vehicles and depend on inter-vehicle separation distance (see Section IV). The combined equations are calculated at every iteration to create a cumulative potential sum at each position within the entire workspace. Each vehicle will have its immediate surrounding area checked for the point of lowest potential and a force will be used to drive it to the appropriate point at each iteration.

$$D = \sqrt{(X - x_p(t))^2 + (Y - y_p(t))^2} \tag{5}$$

$$x_c, y_c = min\{D\} \tag{6}$$

$$f(X_c(x_c, y_c)) = -fx_r - fx_{r2} - fx_{va} - fx_a - fx_o - fy_o \tag{7}$$

$$f(Y_c(x_c, y_c)) = -fy_r - fy_{r2} - fy_{va} - fy_a - fy_o + fx_o \tag{8}$$

Where D is the distance from all coordinates within the defined workspace to the coordinates of each vehicle at time *t*, x_c, y_c are the coordinates of the minimum D value, $f(X_c(x_c, y_c))$ is the horizontal component of the combined clockwise potential at (x_c, y_c), fx_r and fx_{r2} are initial repulsive forces at the starting points of vehicles 1 and 2 respectively, fx_{va} is an attractive force around vehicle 2, fx_a is the attractive force at the goal and fx_o and fy_o are the horizontal and vertical repulsive components around the object's perimeter.

Equation 8 is the vertical component of the clockwise field and is used in conjunction with the horizontal component (Equation 7) to create the final overall field. A simultaneous anticlockwise field is created in a similar manner. Both techniques also incorporate either attractive or repulsive forces on vehicles depending on the formation at any instant of time *t*.

To create counter rotational fields around each object, a vector of equidistant points along its circumference is used to create a repulsive force at each point in both the vertical and horizontal directions. Both clockwise and anticlockwise potential fields are simultaneously formed by the addition or subtraction of these orthogonal forces. In order to assign a field to a particular vehicle in the formation, a number of conditions including vehicle's direction of approach to the object and orientation towards other vehicles are used.

IV. Structure Formation with Object Detection and Avoidance

A. Flexible Formation

The proposed method removes the need for a virtual vehicle whilst also dispensing with a rigid structure. Without having to adhere to a set distance and angle from their nearest neighbours, vehicles have a greater degree of flexibility to manoeuvre in the planar environment in a fluid-like formation. This paper uses an example of a 2-vehicle formation where Equations 3 and 4 are used to assign either an attractive or repulsive field to vehicle 2 based upon its distance from vehicle 1. This allows both vehicles to be guided in a flexible formation, towards each waypoint, with their separation distance being the only inter-dependent constraint.

Both repulsive and attractive fields assigned to vehicle 2 are given a much lower scaling factor than those used for the counter-rotational or waypoint fields (see Fig 5 for an example of high potential fields around objects). This ensures that when an object is encountered, the vehicle formation splits naturally, before recombining when the object has been successfully passed.

B. Object Surveying

Using this technique, the formation is guided towards each object of interest in turn. When an object is encountered, the vehicles manoeuvre around both sides of its boundary, thus allowing surveying/scanning etc. to be performed. This is achieved by assigning temporary subgoal positions (or waypoints) at opposing sides of each object. The vehicles will then be guided to each of these in turn. To create waypoints for the first object of interest for example, the starting positions of vehicles 1 and 2 are used to determine a midpoint between themselves. This midpoint is then used to draw an imaginary line passing through it and the object's centre. Two waypoints are then placed on this line on either side of the object, based on a set distance from its centre, using the equations below:

$$x_n = \frac{a_2 x_1}{a_1 + a_2} + \frac{a_1 x_2}{a_1 + a_2} \tag{9}$$

$$y_n = \frac{a_2 y_1}{a_1 + a_2} + \frac{a_1 y_2}{a_1 + a_2} \tag{10}$$

$$x_f = \frac{x_2(a_1 + 2a_2) - a_2 x_1}{a_1 + a_2} \tag{11}$$

$$y_f = \frac{y_2(a_1 + 2a_2) - a_2 y_1}{a_1 + a_2} \tag{12}$$

Where a_1 is the distance from the midpoint to the nearside waypoint, a_2 is the distance from the nearside waypoint to the object's centre, (x_1,y_1) are the coordinates of the midpoint, (x_2,y_2) are the coordinates of the object centre, (x_n,y_n) and (x_f,y_f) are the coordinates of the near and farside waypoints respectively.

C. *Object Avoidance*

In order to determine which rotational field is assigned to each vehicle, a number of conditions are tested. Initially the slope of the line between the object's centre and subsequent waypoint is measured; if it is non-horizontal for example, the approach direction of both vehicles will be used alongside their *x*-coordinates in order to determine which rotational field is assigned to each vehicle. Counter-rotational directions minimise the potential for inter-vehicle collisions, whilst ensuring that both sides of the object are covered.

1) Pseudo Code: Pseudo code for the proposed algorithm is provided below using the 2-vehicle example. Note that the locations of objects to be scanned are known *a priori* and stored in the mission planner.

Input: Vehicle positions at time *t*
Output: Vehicle positions at time *t*+1

Initialisation : Set vehicles' starting positions plus object size & position, use midpoint between vehicle start positions and object's centre position to assign near & farside waypoints across object. Set overall potential including counter-rotational fields.
LOOP Process

while Goal not reached **do**
  Calculate distance between the two vehicles, *dv*
  if ($dv \leq$ desired dv) **then**
    Set repulsive force to vehicle two and add this to overall potential field
  else if ($dv >$ desired dv) **then**
    Set attractive force to vehicle two and add this to overall potential field
  end if
  If first waypoint has been reached by both vehicles, determine direction of approach to object
  if (horizontal approach) **then**
    Use vehicles' approach direction plus vertical positions to determine assignment of clockwise & anti-clockwise fields
  else if (non-horizontal approach) **then**
    Use vehicles' approach direction plus horizontal positions to determine assignment of clockwise & anticlockwise fields
  end if
  If second waypoint successfully reached
end while
return

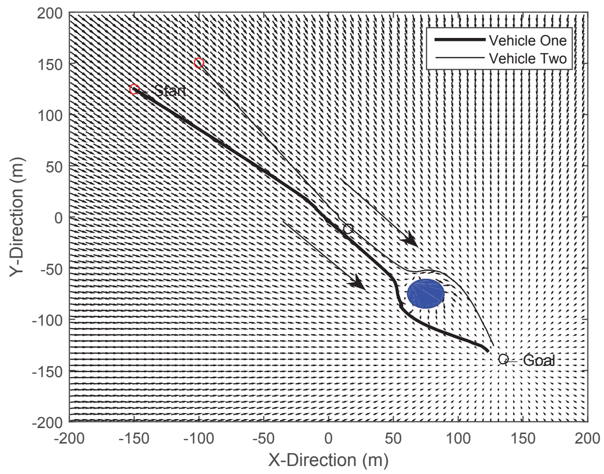

Fig. 1. Complete trajectory of the formation with one object showing overall potential field

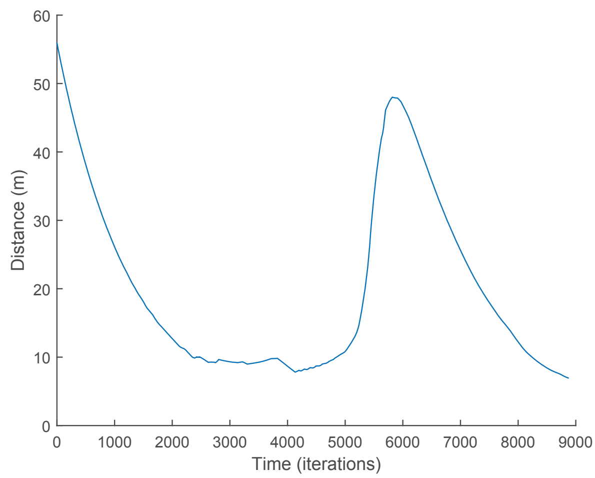

Fig. 2. Inter-vehicle separation distance for one object

Fig. 1 depicts a snapshot of the resultant trajectory taken by both vehicles along with the total potential field for a single object of interest. The figure displays the clockwise vector field around the object (although both fields exist simultaneously). This clockwise field is used to guide vehicle 2 around the object in this particular scenario, with vehicle 1 employing the anticlockwise direction. Both vehicles, starting on the left hand side and guided by APF, make their way towards an initial waypoint on the nearside of the object. The approach conditions are then used to assign opposing rotational fields to each vehicle to guide them to a farside waypoint as determined by Equations 9 - 12.

As can be seen in Fig 2 the inter-vehicle separation distance decreases as both vehicles approach the first waypoint. At around 2200 iterations the repulsion/attraction formation technique starts to impose the predefined 10 m separation distance between both vehicles. They are then each assigned counter-rotational fields (at around 5000 iterations) to successfully avoid the object. The following section includes results using multiple objects within the workspace.

V. Simulation Results

The simulation study in this paper is carried out using the MATLAB software package. A working environment is used for vehicles to test the validity of the proposed techniques. An initial workspace with an area of 400 x 400 m is considered; object positions and sizes are then determined along with vehicle starting positions (which must lie outside the range $\pm$ 150 m from the centre as both vehicles move across the environment from left to right or visa versa). An overall potential field is then created by assigning an attractive potential to goals (Equation 3) and a repulsion to objects (Equation 4). These equations are also assigned to vehicle 2 in order to maintain an inter-vehicle separation distance (set at 10 m for this simulation) when vehicles are not under the direct influence of other forces. Each object is surrounded by two counter-rotational potential fields, formed using either the addition or subtraction of their horizontal and vertical components. When the two vehicles approach an object they are each assigned an opposing field to ensure that the object is avoided on both sides of its boundary. Each subgoal has a circle of acceptance (determined by the user), which is used as a condition to test for the successful arrival of both vehicles to its position.

Five objects were distributed throughout the workspace to test whether the vehicles could successfully traverse each object in turn. This could represent the pillars of a pier or underwater infrastructure for example. Vehicles 1 and 2 were given starting positions at the bottom left-hand corner of the environment to be inspected. Fig 3 displays both vehicles' trajectories whereas their separation distance is shown in Fig 4.

As can be seen in Fig 3 an initial waypoint has been assigned on the nearside of the first object to be encountered. Both vehicles are guided by APF to this waypoint which has a circle of acceptance of 11 m. Following their arrival, the farside waypoint replaces the current one as the subsequent goal position. The program also tests the vehicles' orientation

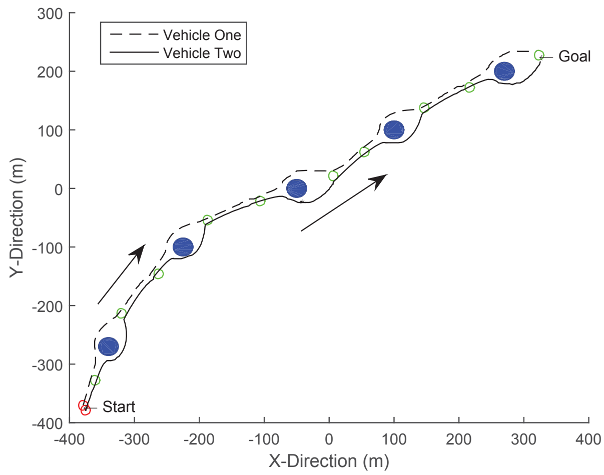

Fig. 3. Complete environment coverage using both vehicles and counter-rotating fields

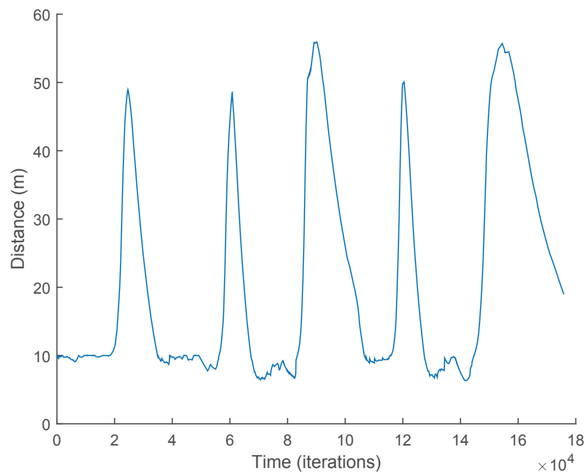

Fig. 4. Inter-vehicle separation distance for multiple objects

and their approach towards the object, which is used to assign each an opposing counter-rotational field. When the first object has been successfully avoided both vehicles then move on to the next one.

Fig 4 displays the inter-vehicle separation distance. It can be seen that when vehicles are not directly passing objects they adhere to the set distance of 10 m apart. This distance is not rigid and allows for a degree of flexibility which can be observed.

During testing it was found that a minimum distance is required, between the object centre and corresponding waypoint positions, for a successful trajectory. When waypoints are placed too close to object boundaries, repulsive/attractive fields can cause local minima. Further, due to the fact that the flexible formation method employs an inter-vehicle separation distance, a minimum circle of acceptance at each waypoint is also necessary for a successful arrival.

Fig 5 below shows one example of a 3D plot which displays areas of high potential around objects to be scanned or surveyed.

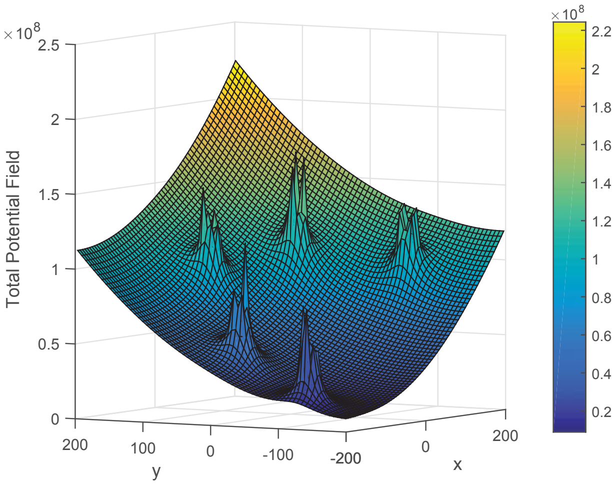

Fig. 5. Overall potential field with multiple objects

A. Scalability

The focus of this paper is to map objects of interest within a defined working environment. While the main thrust of the paper is towards manoeuvring vehicles around objects, in such a way as the objects can be scanned or surveyed, the two vehicles also work cooperatively to maintain a fluid-like formation when in close proximity with each other. Although the scheme is flexible enough to allow guiding a lone vehicle independently (following a failure of either one) to the goal, the overall intended surveying mission could not be completed with a single agent as only half of an object's boundary would be covered. Further vehicles could be added to permit the mission to be accomplished. Additional heterogeneous vehicles could also be added to the initial two e.g. as followers to perform high-resolution image capture of the object's surface. A preliminary simulation has been carried out using 4 vehicles as depicted in Fig. 6. A larger area with hundreds or thousands of objects could contain a fleet of operators, with each assigned a section to explore, before meeting at a specified rendezvous.

VI. Conclusion

A novel technique was presented in this paper to guide an ensemble of autonomous vehicles for surveying tasks. A fluid-like formation was used instead of a strictly rigid structure and without the need of a virtual vehicle as is commonly employed in the literature. While similar papers focus on reaching a goal (using object avoidance on a shortest-path), this methodology ensures that every object is surveyed completely using counter-rotational potential fields. This is achieved by assigning waypoints on opposing sides of each object in turn based on *a priori* information about the objects to be surveyed. Moreover, the counter-rotating potential fields are assigned to each vehicle based on their positions, along

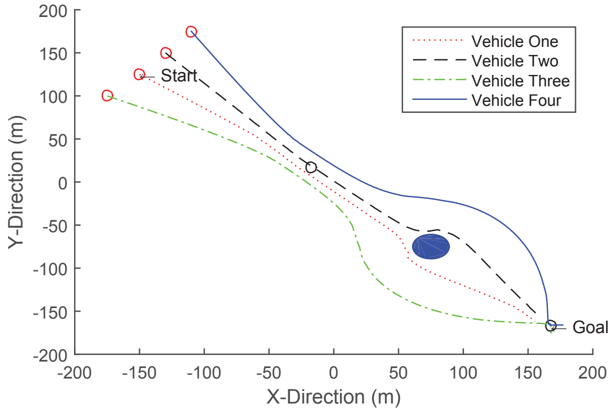

Fig. 6. Complete trajectory for 4 vehicles and a single object of interest

with the object's orientation with respect to the waypoints. The scheme also allows to extend for additional vehicles in the formation but is shown here for 2 vehicles as an example. Simulations carried out within MATLAB have shown the proposed method to be effective in a workspace populated with both single and multiple objects.

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