Phys 345 Homework 8 (Optional)

1. Quantum computers can simulate the dynamics of quantum systems. Consider a chain of N qubits with Hamiltonian

$$H = \sum_{j=1}^{N-1} Z_j Z_{j+1} \; + \; B \sum_{j=1}^{N} X_j,$$

where B is a scalar parameter and Z_j, X_j are Pauli operators.

Starting from an initial state $|\psi_0\rangle$, the state after evolving for a time t is

$$|\psi(t)\rangle = e^{-itH}|\psi_0\rangle.$$

We can approximate this evolution using *Trotterization*, dividing the total time into M small steps and writing

$$|\psi(t)\rangle \approx \prod_{m=1}^{M} \left[\left(\prod_{j=1}^{N-1} e^{-i(t/M)\, Z_j Z_{j+1}} \right) \left(\prod_{j=1}^{N} e^{-i(t/M)\, B X_j} \right) \right] |\psi_0\rangle.$$

The single-qubit term $e^{-i(t/M)BX_j}$ corresponds to an R_x gate, and the two-qubit term $e^{-i(t/M)Z_jZ_{j+1}}$ is implemented in Qiskit as the `RZZ` gate.

Write Qiskit code that takes B, N, M, and t as inputs and constructs a circuit that prepares the final state starting from the initial state $|\psi_0\rangle = |00\ldots0\rangle$. Print your circuit for $N = 6$, $M = 5$, $B = 0.4$, and $t = 2$.