Assignment 1

MAT216

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S-17

1.

$$5x_3 + 15x_5 = 5$$
$$2x_2 + 4x_3 + 7x_4 + x_5 = 3$$
$$x_2 + 2x_3 + 3x_4 = 1$$
$$x_2 + 2x_3 + 4x_4 + x_5 = 2$$

Augmented Matrix:

$$\left[\begin{array}{cccc|c} 0 & 5 & 0 & 15 & 5 \\ 2 & 4 & 7 & 1 & 3 \\ 1 & 2 & 3 & 0 & 1 \\ 1 & 2 & 4 & 1 & 2 \end{array}\right] \quad R_1 \Leftrightarrow R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 2 & 4 & 7 & 1 & 3 \\ 0 & 5 & 0 & 15 & 5 \\ 1 & 2 & 4 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 - 2R_1 \\ R_4' = R_4 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 5 & 0 & 15 & 5 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \quad R_2 \Leftrightarrow R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & 5 & 0 & 15 & 5 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \quad R_2' = \frac{R_2}{5}$$

$$R_1' = R_1 - 2R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$R_1' = R_1 - 3R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 3 & -6 & -1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \quad R_4' = R_4 - R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & -9 & -4 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

so, we get , let $x_5 = t$, $t \in \mathbb{R}$, we get,

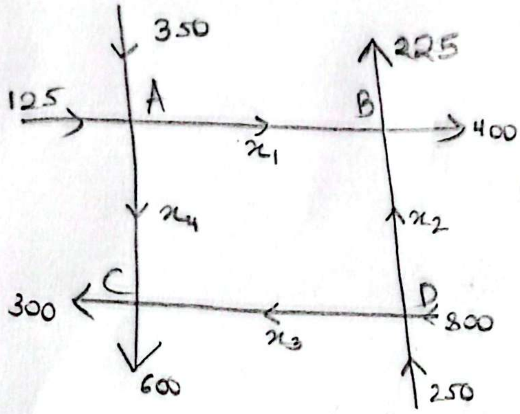
$x_2 - 9x_5 = -4$ $\qquad$ $x_2 = 9t - 4$

$x_3 + 3x_5 = 1$ $\qquad$ $x_3 = 1 - 3t$

$x_4 + x_5 = 1$ $\qquad$ $x_4 = 1 - t$

$$\begin{pmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 9t-4 \\ 1-3t \\ 1-t \\ t \end{pmatrix} \text{ (Ans)}$$

2. 1)



A

$350+125 = x_1+x_4$

B

$225+400 = x_1+x_2$

C

$x_4+x_3 = 300+600$

D

$800+250 = x_2+x_3$

So, we have

$x_1+x_4 = 475$ —— (I)

$x_1+x_2 = 625$ —— (II)

$x_3+x_4 = 900$ —— (III)

$x_2+x_3 = 1050$ —— (IV)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 475 \\ 1 & 1 & 0 & 0 & 625 \\ 0 & 0 & 1 & 1 & 900 \\ 0 & 1 & 1 & 0 & 1050 \end{array}\right] \quad R_2' = R_2 - R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 475 \\ 0 & 1 & 0 & -1 & 150 \\ 0 & 0 & 1 & 1 & 900 \\ 0 & 1 & 1 & 0 & 1050 \end{array}\right] \quad R_4' = R_4 - R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 475 \\ 0 & 1 & 0 & -1 & 150 \\ 0 & 0 & 1 & 1 & 900 \\ 0 & 0 & 1 & 1 & 900 \end{array}\right] \quad R_4' = R_4 - R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 475 \\ 0 & 1 & 0 & -1 & 150 \\ 0 & 0 & 1 & 1 & 900 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

← we get degenerate equation so, x_1, x_2, x_3, x_4 have infinite solutions.

one such combination can be,

$x_1 = 175$, $x_4 = 300$

$x_2 = 450$, $x_3 = 600$

} these values would balance the traffic flow

2ii) if $x_4 = 0$,

$x_1 = 475$
$x_2 = 150$
$x_3 = 900$

(Solved from equations in (i))
The effect is car flow increases on the road from A to B, and C to D, while it decreases from C to B.

iii) if $x_4 = 100$,

$x_1 = 375$
$x_2 = 250$
$x_3 = 800$

3. $V = \{(x,y) \in \mathbb{R}^2 ; y = 2x\}$, $f(x) = 2x$, $y = 2x$

$\Rightarrow \{(x, 2x) : x \in \mathbb{R}\}$ $\qquad V = \{(x, 2x) : x \in \mathbb{R}\}$

let $u = (r, 2r)$, $v = (s, 2s)$, $w = (t, 2t)$, $\alpha, \beta \in \mathbb{R}$, $u, v, w \in V$

i) Closure under addition
$u+v = (r, 2r) + (s, 2s) = (r+s, 2r+2s) \in V$

ii) Commutative of addition
$u+v = (r+s, 2r+2s) = (s+r, 2s+2r) = v+u$

iii) Associativity of addition
$(u+v)+w = ((r+s)+t, 2(r+s)+2t) = (r+(s+t), 2r+2(s+t))$
$= u+(v+w)$

iv) Existence of additive identity,
let, $q = 0 = (0,0)$
$u+q = u+0 = (r, 2r) + (0,0) = (r+0, 2r+0) = (r, 2r) = u$

v) Existence of additive inverse
$u = (r, 2r)$, $-u = (-r, -2r)$
$u+(-u) = (r-r, 2r-2r) = (0,0)$

vi) Closure under scalar multiplication
$\alpha u = (\alpha r, 2\alpha r)$
$= (\alpha r, 2(\alpha r)) \in V$

vii) Distributivity of scalar over vector addition
$\alpha(u+v) = \alpha(r+s, 2r+2s)$
$= (\alpha r + \alpha s, 2\alpha r + 2\alpha s)$
$= \alpha u + \alpha v$

3. viii) Distributivity of scalar addition over vector

$(\alpha+\beta)u = ((\alpha+\beta)r, (\alpha+\beta)2r)$

$= (\alpha r+\beta r, 2\alpha r+2\beta r)$

$= \alpha u+\beta u$

ix) Associative property of scalar multiplication

$(\alpha\beta)u = (\alpha\beta r, 2\alpha\beta r) = (\alpha(\beta r), \alpha(2\beta r))$

$= \alpha(\beta u)$

x) Existence of scalar identity

$1.u = (1.r, 2.r) = (r, 2r) = u$

Since all 10 properties satisfied, V is a vector space

4. $V = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \mathbb{R}\right\}$ and $W = \{A \in V : A^2 = A\}$

i) Zero Vector condition

lets say, $v \Rightarrow 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ [A is taken 0]

in w, $0 = 0^2$

ii) Closure under addition

let $A, B \in W$

so, $A^2 = A$, $B^2 = B$

if $A+B \in W$ [this condition must satisfy]

$(A+B)^2$ should be $A+B$

$(A+B)^2 = A^2+BA+AB+B^2 = A+B+AB+BA$

from this we get

$AB+BA$ must be always 0 to satisfy condition, but

thats not always true, so W is not closed under addition

iii) Closure under Scalar Multiplication

$A \in W$ and $k \in \mathbb{R}$

so, $kA \in W$ [this condition must satisfy]

W: $(kA)^2 = k^2A^2 = k^2A$

so, $k^2A = kA \Rightarrow k^2A - kA = 0$

$A(k^2-k) = 0$

this only works for $k = 0, 1$

so it fails closure under scalar multiplication.

since all 3 of these tests didnt pass, W is not a subspace of V