

Table 1

Equations for all dTMM solvers of electron structure

Solver type	Equation	p/q	q_{pq}
Parabolic	$-\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z)} \frac{d\psi_n(z)}{dz} + U_{\text{eff}}(z)\psi_n(z) = E_n\psi_n(z)$	$\sqrt{\frac{2m_{i/i+1}(U_{i/i+1}-E)}{\hbar^2}}$	$\frac{m_{i+1}}{m_i} \frac{p}{q}$
Two band	$-\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z)(1+\alpha(E-U_{\text{eff}}(z)))} \frac{d\psi_n(z)}{dz} + U_{\text{eff}}(z)\psi_n(z) = E_n\psi_n(z)$	$\sqrt{\frac{2m_{i/i+1}(1+\alpha_{i/i+1}(E-U_{i/i+1}))(U_{i/i+1}-E)}{\hbar^2}}$	$\frac{m_{i+1}(1+\alpha_{i+1}(E-U_{i+1}))}{m_i(1+\alpha_i(E-U_i))} \frac{p}{q}$
Taylor	$-\frac{\hbar^2}{2} \frac{d}{dz} \frac{(1-\alpha(E-U_{\text{eff}}(z)))}{m(z)} \frac{d\psi_n(z)}{dz} + U_{\text{eff}}(z)\psi_n(z) = E_n\psi_n(z)$	$\sqrt{\frac{2m_{i/i+1}(U_{i/i+1}-E)}{(1-\alpha_{i/i+1}(E-U_{i/i+1}))\hbar^2}}$	$\frac{m_{i+1}(1-\alpha_i(E-U_i))}{m_i(1-\alpha_{i+1}(E-U_{i+1}))} \frac{p}{q}$
14 k·p	$\frac{d^2}{dz^2} \alpha^E(z) \frac{d^2\psi_n(z)}{dz^2} - \frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z)} \frac{d\psi_n(z)}{dz} + U_{\text{eff}}(z)\psi_n(z) = E_n\psi_n(z)$	$\sqrt{\frac{\hbar^2 \left[1 - \sqrt{1 - 16 \alpha_{i/i+1}^E (E-U_{i/i+1}) \left(\frac{m_{i/i+1}}{\hbar^2} \right)^2} \right]}{4 \alpha_{i/i+1}^E m_{i/i+1}}}$	$\frac{2\alpha_i^E p^2 - \frac{\hbar^2}{2m_i}}{2\alpha_{i+1}^E q^2 - \frac{\hbar^2}{2m_{i+1}}} \frac{p}{q}$

Table 2

Derivatives for all dTMM solvers of electron structure

Solver type	$\frac{dp}{dE}$	$\frac{dq}{dE}$	$\frac{dq_{pq}}{dE}$
Parabolic	$-\frac{m_i}{\hbar^2 p}$	$-\frac{m_{i+1}}{\hbar^2 q}$	$\frac{m_{i+1}}{m_i} \frac{\frac{dp}{dE} q - \frac{dq}{dE} p}{q^2}$
Two band	$-\frac{m_i(1+2\alpha_i(E-U_i))}{\hbar^2 p}$	$-\frac{m_{i+1}(1+2\alpha_{i+1}(E-U_{i+1}))}{\hbar^2 q}$	$\frac{m_{i+1}(1+\alpha_{i+1}(E-U_{i+1}))}{m_i(1+\alpha_i(E-U_i))} \frac{\frac{dp}{dE} q - \frac{dq}{dE} p}{q^2} + \frac{p}{q} \frac{m_{i+1}(\alpha_{i+1}-\alpha_i+\alpha_i\alpha_{i+1}(U_{i+1}-U_i))}{m_i(1+\alpha_i(E-U_i))^2}$
Taylor	$-\frac{m_i}{(1-\alpha_i(E-U_i))^2 \hbar^2 p}$	$-\frac{m_{i+1}}{(1-\alpha_{i+1}(E-U_{i+1}))^2 \hbar^2 q}$	$\frac{m_{i+1}(1-\alpha_i(E-U_i))}{m_i(1-\alpha_{i+1}(E-U_{i+1}))} \frac{\frac{dp}{dE} q - \frac{dq}{dE} p}{q^2} + \frac{p}{q} \left(\frac{\alpha_{i+1}m_{i+1}(1-\alpha_i(E-U_i))}{m_i(1-\alpha_{i+1}(E-U_{i+1}))^2} - \frac{m_{i+1}\alpha_i}{m_i(1-\alpha_{i+1}(E-U_{i+1}))} \right)$
14 k·p	$\frac{m_i}{\hbar^2} \frac{1}{p + \frac{4 \alpha_i^E m_i}{\hbar^2} p^3}$	$\frac{m_{i+1}}{\hbar^2} \frac{1}{q + \frac{4 \alpha_{i+1}^E m_{i+1}}{\hbar^2} q^3}$	$\frac{6\alpha_i^E p^2 - \frac{\hbar^2}{2m_i}}{2\alpha_{i+1}^E q^3 - \frac{\hbar^2}{2m_{i+1}} q} \frac{dp}{dE} - \frac{\left(2\alpha_i^E p^3 - \frac{\hbar^2}{2m_i} p \right) \left(6\alpha_{i+1}^E q^2 - \frac{\hbar^2}{2m_{i+1}} \right)}{\left(2\alpha_{i+1}^E q^3 - \frac{\hbar^2}{2m_{i+1}} q \right)^2} \frac{dq}{dE}$