CREDIT: The questions on this document were written by Erik Packard, PhD, Associate Professor of Mathematics at Colorado Mesa University.

1. (11 pts) On the website www.listsofjohn.com it is reported that there are 59,817 ranked peaks in Alaska. I took a random sample of 8 peaks and found a sample mean elevation of 4029.1 feet and a sample standard deviation of 2329.8 feet. Does this provide good evidence at the 5% significance level that the population standard deviation of all 59,817 peaks is over 2000 feet?

   H_0 : $\sigma \leq 2000$ ft

   H_a : $\sigma > 2000$ ft

   Critical Value(s): Given that $\alpha = 0.05$, $n = 8$, and $\text{df} = 7$ $\left(\chi^2\right)_{\text{Table}} = 14.07$

   Test Statistic: $\left(\chi^2\right)_{\text{Data}} = \dfrac{\text{df}(s^2)}{\sigma^2} = \dfrac{(7)(2329.8)^2}{(2000)^2} = 9.50$

   Yes/No Answer: Since $\left(\chi^2\right)_{\text{Data}} < \left(\chi^2\right)_{\text{Table}}$, No.

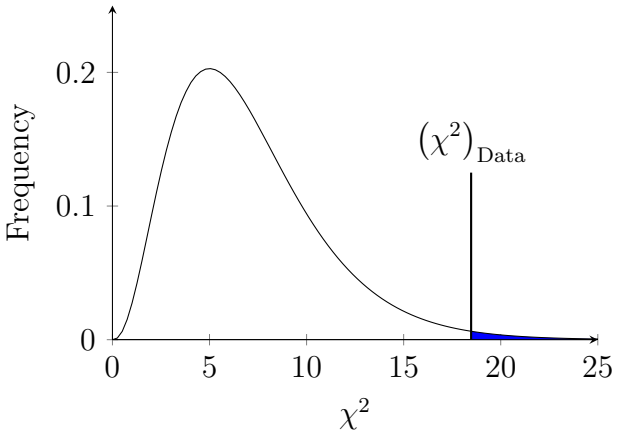
2. (5 pts) Suppose somebody else collects data to answer #1. What is the probability they will conclude the standard deviation is over 2000 when it is not?

   5% or less.

3. (5 pts) Suppose somebody else collects data to answer #1. What is the probability they won't conclude the standard deviation is over 2000 when it is?

   Unknown.

4. (8 pts) Suppose somebody else collects data to answer #1 and their test statistic is $\left(\chi^2\right)_{\text{Data}} = 18.475$. What is their p-value and explain what it means in everday terms.



Since $(\chi^2)_{\text{Data}} = 18.475$ and $\text{df} = 7$, the p-value is equal to 0.01.

This means that if the population standard deviation is less than 2000 ft, then the chance of finding as strong or stronger evidence suggesting that the population standard deviation is over 2000 ft is 0.01.

5. (9 pts) Data was collected from random samples from the website www.listsofjohn.com for how many people had climbed peaks in Mesa County and Garfield County. At the 5% significance level can we conclude that the variance in number of climbers for Mesa County peaks is higher than it is for Garfield County peaks on this website?

	Sample Mean	**Sample Standard Deviation**	**Sample Size**
Garfield County	5.4	3.050	5
Mesa County	8.0	8.188	8

H_0 : $(\sigma^2)_{\text{MC}} \leq (\sigma^2)_{\text{GC}}$ or $\dfrac{(\sigma^2)_{\text{MC}}}{(\sigma^2)_{\text{GC}}} \leq 1$

H_a : $(\sigma^2)_{\text{MC}} > (\sigma^2)_{\text{GC}}$ or $\dfrac{(\sigma^2)_{\text{MC}}}{(\sigma^2)_{\text{GC}}} > 1$

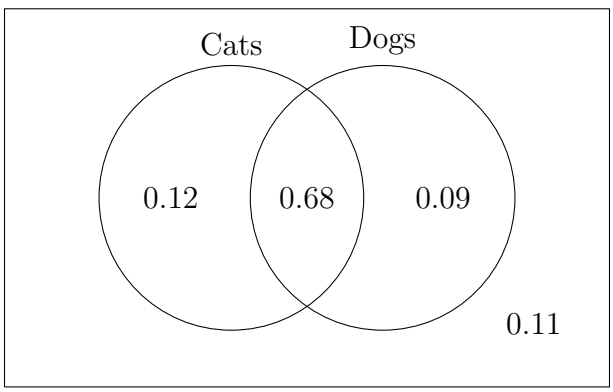
Critical Value(s): Since $\alpha = 0.05$, $\text{df}_{\text{Top}} = 7$, and $\text{df}_{\text{Bottom}} = 4$ $\text{F}_{\text{Table}} = 6.09$

Test Statistic: $\text{F}_{\text{Data}} = \dfrac{(s^2)_{\text{MC}}}{(s^2)_{\text{GC}}} = \dfrac{(8.188)^2}{(3.050)^2} = 7.21$

Yes/No Answer: Since $\text{F}_{\text{Data}} > \text{F}_{\text{Table}}$, Yes.

6. Suppose in a STAT 200 class that 80% of students like cats and 77% like dogs and 68% like both.

Venn Diagram:



A) (4 pts) What percent like only cats? 12%

B) (4 pts) What percent like neither? 11%

C) (6 pts) What is the probability that a person that likes cats also likes dogs?

$$P(\text{Dogs} \mid \text{Cats}) = \frac{P(\text{Dogs \& Cats})}{P(\text{Cats})} = \frac{0.68}{0.80} = 0.85$$

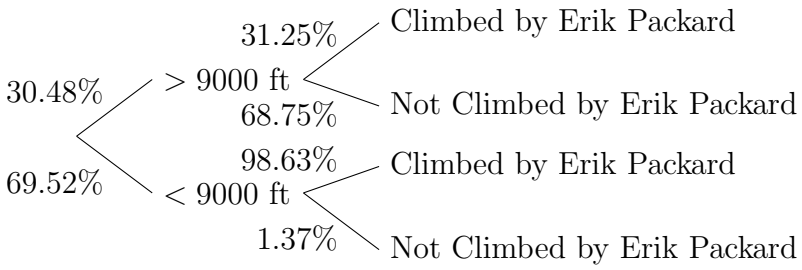
7. (7 pts) Suppose that 8% of people in Colorado are New England Patriot fans. If you ask 9 people at random in Colorado, what is the probability that at least one will not be a New England fan?

$p = 0.08 \quad q = 0.92 \quad n = 9$

$P(0) = {}_9\mathsf{C}_0\ (0.08)^0(0.92)^9 = 0.47 \quad \therefore \quad P(x \geq 1) = 1 - P(x < 1) = 1 - 0.47 = 0.53$

8. According to the website www.listsofjohn.com , 30.48% of the ranked peaks in Rio Blanco County are over 9000 feet, and 69.52% are under 9000 feet. Erik Packard has climbed 31.25% of the peaks over 9000 feet and 98.63% of the peaks are under 9000 feet.

Tree Diagram:



A) (8 pts) What percent of the ranked peaks in Rio Blanco County has Erik Packard climbed?

$(0.3125 \cdot 30.48\%) + (0.9863 \cdot 69.52\%) = 78.10\%$

B) (10 pts) If a ranked peak in Rio Blanco County has been climbed by Erik Packard, what is the probability it is over 9000 feet?

$$P(> 9000 \text{ ft} \mid \text{Climbed by Erik Packard}) = \frac{P(> 9000 \text{ ft \& Climbed by Erik Packard})}{P(\text{Climbed by Erik Packard})}$$

$P(> 9000 \text{ ft} \mid \text{Climbed by Erik Packard}) = 0.12$

9. A club has 20 members.

A) (6 pts) How many different possibilities are there for choosing a president, vice president and secretary?

$${}_{20}\mathsf{P}_3 = \frac{20!}{(20-3)!} = (20)(19)(18) = 6840$$

B) (6 pts) How many different possible committees of 4 people are there?

$${}_{20}\mathsf{C}_4 = \frac{20!}{(20-4)!4!} = \boxed{4845}$$

10. Suppose Chico makes 76% of his free-throws.

A) (5 pts) If he shoots 9 free-throws, what is the probability he will make exactly 7?

$p = 0.76 \quad q = 0.24 \quad n = 9 \quad \therefore \quad P(7) = {}_9\mathsf{C}_7\ (0.76)^7(0.24)^2 = \boxed{0.30}$

B) (6 pts) If he shoots 9 free-throws, what is the probability he will make less than 7?

$P(7) = {}_9\mathsf{C}_7\ (0.76)^7(0.24)^2 = 0.30$

$P(8) = {}_9\mathsf{C}_8\ (0.76)^8(0.24)^1 = 0.24$

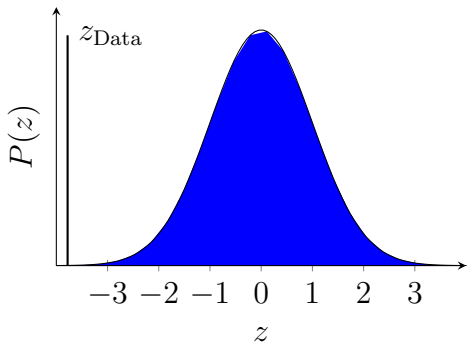
$P(9) = {}_9\mathsf{C}_9\ (0.76)^9(0.24)^0 = 0.08 \quad \therefore$

$P(x < 7) = 1 - \big[P(7) + P(8) + P(9)\big] = 1 - (0.30 + 0.24 + 0.08) = \boxed{0.38}$

C) (6 pts) Suppose he shoots 300 free-throws, what is the probability he will make at least 200?

$\sigma = \sqrt{npq} = \sqrt{(300)(0.76)(0.24)} = 7.40 \quad \text{and} \quad \mu = np = (300)(0.76) = 228 \quad \therefore$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{200 - 228}{7.40} = -3.78 \quad \therefore \quad \text{Area} = P(x \geq 200) \approx \boxed{1}$



11. Suppose that population X has a mean of 25 and a variance of 16, and that population Y has a mean of 19 and a variance of 9.

A) (6 pts) If each piece of data from X has 7 added to it, what will be the mean and variance?

MEAN = $\boxed{32}$ VARIANCE = $\boxed{16}$

B) (4 pts) If each piece of data from X is divided by 2, what will be the mean and the variance?

MEAN = $\boxed{12.5}$ VARIANCE = $\boxed{8}$

C) (6 pts) If we take a piece of data from X at random and then subtract a piece of data from Y at random, what will be the mean and the variance of this process?

MEAN = 6 VARIANCE = 25

The rest are worth 1 point each.

12. What is the area under an F curve? One.

13. Suppose X and Y are independent normal populations. What is the shape of $\frac{(s^2)_X}{(s^2)_Y}$ where the sample sizes are of size 10 for X and size 8 for Y?

F with 9 degrees of freedom on the top and 7 degrees of freedom on the bottom.

14. Variance is similar to $\frac{\sum |x-\mu|}{N}$ which tells what in everyday terms?

On average how far the data is from the mean.

15. We came up with the formula for $P(A|B)$ by taking a sports team and making a fraction for $P(W|H)$ and the top of the fraction represented what?

How many games were both wins and at home.

16. We came up with the formula for $P(A|B)$ by taking a sports team and making a fraction for $P(W|H)$ and the bottom of the fraction represented what?

How many games were at home.

17. Explain what $P(A|B) = P(A)$ means in everyday terms.

B has no effect on the chances of A.

18. If X and Y are independent then $P(X|Y) =$ What?

$P(x)$

19. To figure out how many ways a multi-step process can be done you do what?

Multiply how many choices at each step.

20. When finding out how many ways to pick 6 numbers from 42 numbers in a lottery in which order does not matter we first (incorrectly) came up with what? We realized that each outcome was being counted 6! times? So we divided by this number and came up with the correct answer of (42)(41)(40)(39)(38)(37)/6!

    (42)(41)(40)(39)(38)(37)

21. When finding out how many ways to pick 6 numbers from 42 numbers in a lottery in which order does not matter we first (incorrectly) came up with (42)(41)(40)(39)(38)(37). We realized that each outcome was being counted how many times? So we divided by this number and came up with the correct answer of (42)(41)(40)(39)(38)(37)/6!

    6!