CREDIT: The questions on this document were written by Erik Packard, PhD, Associate Professor of Mathematics at Colorado Mesa University.

1. (6 pts) According to the website www.listsofjohn.com there are 154 ranked peaks in Garfield County, CO and Erik Packard has climbed 149 of them. If a ranked Garfield County, CO peak is picked at random, what is the probability it has NOT been climbed by Erik Packard?

$$P(\text{Packard}) = \frac{149}{154} \quad \therefore \quad P(\overline{\text{Packard}}) = 1 - P(\text{Packard}) = 1 - \frac{149}{154} = \boxed{\frac{5}{154}}$$

2. A bag of candies has 4 colors as given in the chart:

Color:	Blue	Red	Yellow	Green
Percentage:	50%	20%	5%	?

A) (6 pts) What percent are Green?

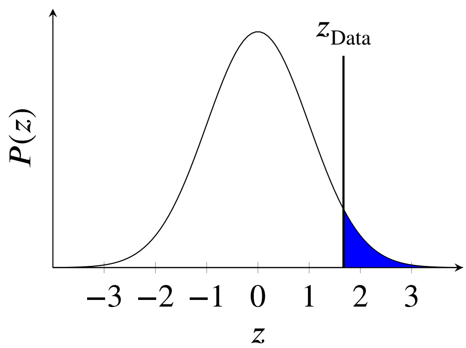
$$\sum P(x) = 1 \quad \therefore \quad 50\% + 20\% + 5\% + x = 100\% \quad \therefore \quad x = 100\% - (50\% + 20\% + 5\%) = \boxed{25\%}$$

B) (6 pts) If a candy is picked at random, what is the probability it is either Red or Yellow?

$$P(\text{Red or Yellow}) = \frac{20}{100} + \frac{5}{100} = \frac{25}{100} = \boxed{\frac{1}{4}}$$

3. Suppose male tigers have a mean weight of 400 lbs and a standard deviation of 30 lbs and the weights are normally distributed.

A) (8 pts) What is the probability of one male tiger weighing over 450 lbs?



$$z = \frac{x - \mu}{\sigma} = \frac{(450 - 400)\text{ lbs}}{30\text{ lbs}} = 1.67$$

Area $= P(x > 450\text{ lbs}) = 0.50 - 0.4525 = \boxed{0.0475}$

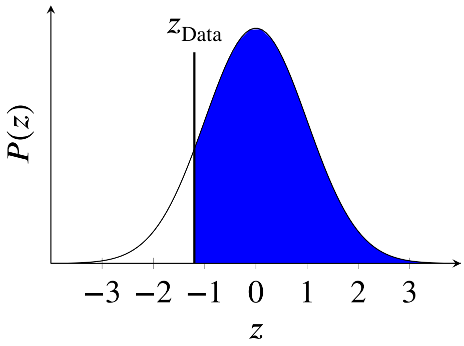
B) (12 pts) How heavy are the 5% heaviest male tigers?

Area $= 0.05 \quad \therefore \quad z = 1.645 \quad \therefore \quad x = z\sigma + \mu = (1.645)(30\text{ lbs}) + 400\text{ lbs} = 449.35\text{ lbs}$

Therefore, the 5% heaviest male tigers will weigh 449.35 lbs or greater.

C) (8 pts) What is the probability that the average weight of 9 male tigers will be over 388 lbs?

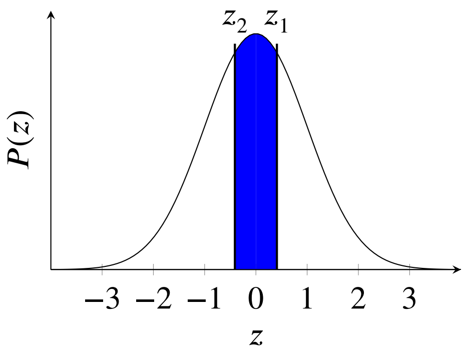
$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{(388 - 400)\text{ lbs}}{\frac{30\text{ lbs}}{\sqrt{9}}} = -1.20$$



Area $= P(x > 388\text{ lbs}) = 0.50 + 0.3849 = \boxed{0.8849}$

D) (8 pts) You pick 6 male tigers at random, what is the probability the average weight will be between 395 lbs and 405 lbs?

$$z_2 = \frac{(395 - 400)\text{ lbs}}{\frac{30\text{ lbs}}{\sqrt{6}}} = -0.41 \qquad z_1 = \frac{(405 - 400)\text{ lbs}}{\frac{30\text{ lbs}}{\sqrt{6}}} = 0.41$$



Area $= P(395\text{ lbs} < x < 405\text{ lbs}) = 2(0.1591) = \boxed{0.3182}$

4. Consider the sample data set of 11 numbers: {-2, 1, 7, 7, 7, 8, 13, 13, 14, 19, -12}. Note almost ranked smallest to biggest, but not quite ... easy to fix this for you.

   A) (8 pts) Give a histogram for this data set. Use the intervals $-15 < x \leq -10$, $-10 < x \leq -5$, etc.

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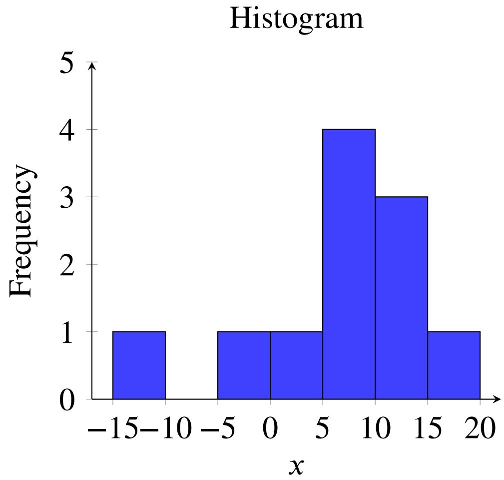

B) (10 pts) Find the sample mean.

$$\overline{x} = \frac{\sum x_i}{n} = \frac{(-12 - 2 + 1 + 7 + 7 + 7 + 8 + 13 + 13 + 14 + 19)}{11} == \frac{75}{11}$$

C) (13 pts) Find the sample standard deviation.

$$s = \sqrt{\frac{\sum\left(\left(x_i\right)^2\right) - \frac{\left(\sum x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{1255 - \frac{(75)^2}{11}}{10}} = 8.6234$$

D) (8 pts) Give the 5# summary (Min, Q1, Median, Q3, Max).

Rank	1	2	3	4	5	6	7	8	9	10	11
Number from Set	-12	-2	1	7	7	7	8	13	13	14	19

Min $= -12$

Q1 $= 2.50$ $\quad r = \frac{np}{100} + \frac{1}{2} = \frac{(11)(25)}{100} + \frac{1}{2} = 3.25 \quad \therefore \quad \text{Q1} = 1 + 0.25(7-1) = 2.50$

Med $= 7$ $\quad r = \frac{(11)(50)}{100} + \frac{1}{2} = 6 \quad \therefore \quad \text{Med} = 7$

Q3 $= 13$ $\quad r = \frac{(11)(75)}{100} + \frac{1}{2} = 8.75 \quad \therefore \quad \text{Q3} = 13 + 0.75(13-13) = 13$

Max $= 19$

5. On the website www.listsofjohn.com it is reported that there are 59,817 ranked peaks in Alaska. It is also reported that the standard deviation of the elevation of all those peaks is 2165 feet. I took a random sample of 8 peaks and found a sample mean of 3900.5 feet. Assume the elevations are normally distributed.

A) (16 pts) Give a 95% confidence interval for the mean elevation of all the peaks in Alaska.

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$$95\% \text{ CI} = 3900.5 \text{ ft} \pm (1.960)\frac{2165 \text{ ft}}{\sqrt{8}} \longrightarrow 2400.23 \text{ ft} < \mu < 5400.77 \text{ ft}$$

B) (8 pts) How large of a sample would be needed to estimate the mean elevation to within +/− 500 feet with 95% confidence?

$$n = \left[\frac{(z_{\alpha/2})\sigma}{E}\right]^2 = \left[\frac{(1.960)(2165 \text{ ft})}{500 \text{ ft}}\right]^2 \approx 72.0 \text{ Peaks}$$

The rest are worth 1 point each.

6. Which are likely to be closer? The percentages in two samples of size 5 from the same population, or the percentages in two samples of size 500 from the same population?

   The percentages in two samples of size 500 from the same population are likely to be closer.

7. In a discrete probability model all the probabilities of all outcomes add up to what number?

   One.

8. Three ways of determining probability are guess, theory, and experiment.

9. Chance behavior has what property in the short run?

   Chance is unpredictable in the short run.

10. What is the notation for the population mean?

    μ

11. What is the formula for the z curve?

    $$z = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

12. On the z curve how much of the data is within 2 standard deviations of the mean?

    About 95%

13. For any probability distribution how much of the data is within 1 standard deviation of the mean?

    About 68%

14. What is the notation for the sample mean?

    $\overline{x}$

15. If you flip a fair coin 1000 times and get close to 50% it will be mostly due to what?

    Having a large sample size.

16. If you were to get all samples with replacement of size n from a population with standard deviation σ, the standard deviation of all these sample means would be what?

    $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$

17. If the original data is normal, what about the shape of all sample means from samples of the same size?

    The shape of all sample means will follow that of a normal distribution curve.

18. If the original data is not normal, what happens to the shape of all sample means from samples of size n as n goes up?

    As n increases, the shape of all sample means gets closer to that of a normal distribution curve.

19. Which two of the three should be close? σ, $\sigma_{\overline{x}}$, and s

    σ and s

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