Weighted Component Analysis

The objective here is to find the optimal combination of sensors to find the hidden state.

We begin with PCA & define our it as a projection P from sensor space Y to hidden state X, s.t. $P_{pca}(Y,X) \simeq Y \xrightarrow{P_{pca}} X$

$$P_{pca} = \min Cor(X)$$

$$P_{ICA} = \min Cor(X) + \min Cor(x)$$

$$\boxed{P_{WCA} = {}^{\min} Cor(X) + {}^{\min} CostSensor(Y)}$$

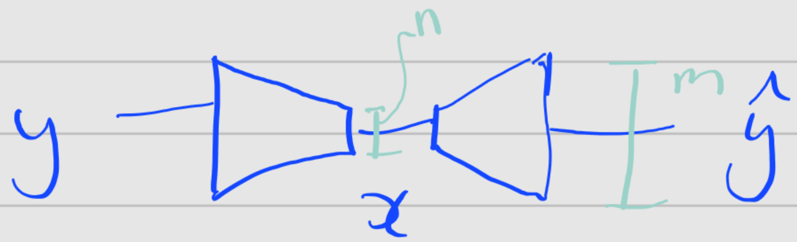
Now Φ is a map

$\|\Phi(Y,X)\|$ is our objective function that balances our concerns, namely

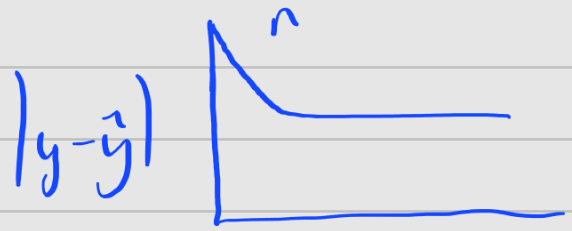
- Cost of DAG
- hidden state estimation

$$\boxed{P_{WCA} = \min Cor(X) + \min CostSensor(Y)}$$

From the AE, we know $\min Cor(X) = n, \; x \in \mathbb{R}^n$



Therefore n is a binary search



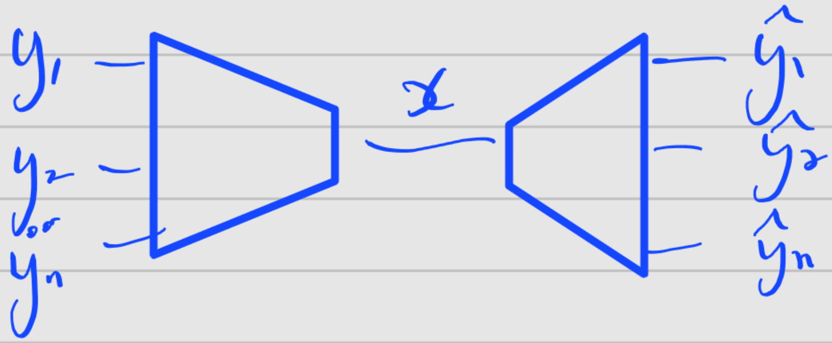
Now let us select the sensor combination

→ Combinatorial search, not good

$$CostSensor(Y) = Cost(y_1) + \dots + Cost(y_n) + \|y - \hat{y}\|$$

Can we find a continuous gradient based search for this problem?

Gradient based sensor selection



The reconstruction loss may hold the answer.

We want to be able to reconstruct $|y-\hat{y}|$ with the fewest sensor cost. So we will introduce the cost of sensor deployment α_i.

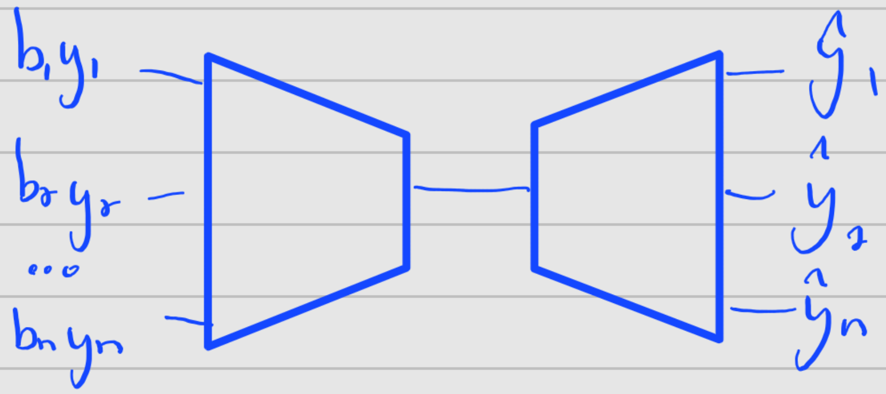
Now $|y-\hat{y}| \propto \delta$

We want to $[\min \|y-\hat{y}\|]$ & $[\min \sum \alpha_i]$

Introduce binary switches.

if sensor i is included $b_i = 1$, else $b_i = 0$

$b \in \{1, 0\}$



$$L = \underbrace{\sum \alpha_i b_i}_{\text{cost of sensor inclusion}} + \underbrace{\|y-\hat{y}\|}_{\text{Reconstruction loss}}$$

So A is designed by the cost of including sensers.

β is the binary vector representing the sensers.

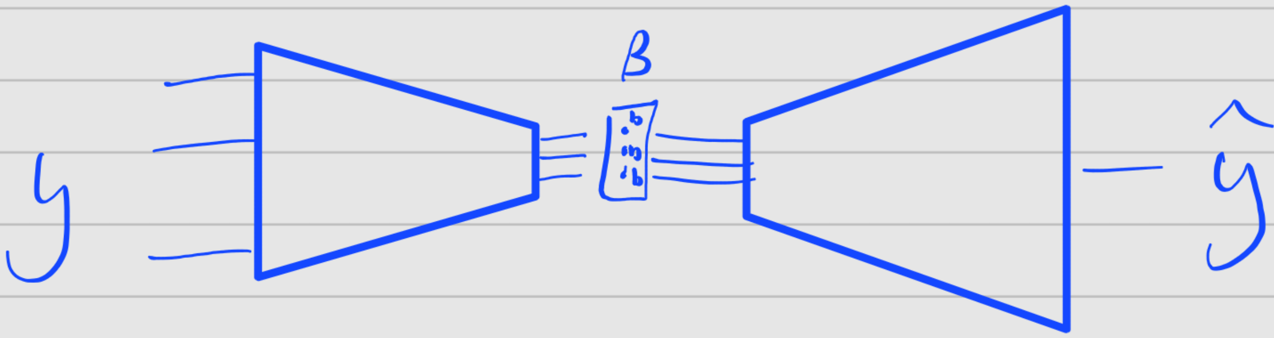
$$\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \text{ include sensers } \begin{bmatrix} \text{Yes} \\ \text{No} \\ \text{No} \\ \text{Yes} \end{bmatrix}$$

A different question is whether we can use this for the latent space selection?

Let us start by specifying $f(x_t) = x_{t+1}$

We can't, because that would imply we know n.

Given a static or steady-state problem (QSS), can we search for n.



Let β be the model of B s.t.

for $\beta = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}$

$$\begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} - \overset{\beta}{\begin{bmatrix}1\\0\\0\\1\end{bmatrix}} - \begin{bmatrix}x_1\\0\\0\\x_4\end{bmatrix}$$

Then the cost should be

$$L = \underbrace{\sum \beta}_{\text{minimize latent space size}} + \underbrace{\|y - \hat{y}\|}_{\text{min reconstruction error.}}$$