Power Engineering and Engineering Thermophysics
https://www.acadlore.com/journals/PEET

An Innovative Heat Rejection System for High Altitude Unmanned Aerial Vehicles

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Received: 05-03-2023 **Revised:** 06-14-2023 **Accepted:** 06-22-2023

Citation: L. Piancastelli, "An innovative heat rejection system for high altitude Unmanned Aerial Vehicles," *Power Eng. Eng. Thermophys.*, vol. 2, no. 2, pp. 110–119, 2023. https://doi.org/10.56578/peet020205.

Abstract: The development of an effective cooling system is paramount for the optimal design of high altitude Unmanned Aerial Vehicles (UAVs). These vehicles often operate at or near supersonic speeds in thin atmospheric conditions to generate sufficient lift. It is emphasized that the necessity for air-cooling mandates the incorporation of cooling ducts into the initial design, striving for a balance between low-speed, high-density cooling air for efficient heat rejection, minimal drag, or even potential thrust augmentation. The proposition is that dedicated, meticulously optimized cooling air pathways may facilitate superior performance at high altitudes. The abstract further underscores that the longevity and efficiency of solar panels, commonplace in solar-powered UAVs, are substantially temperature-dependent. As such, high-altitude cooling poses a complex challenge. For conventionally fueled jet-powered UAVs, fuel may serve as a viable heat sink, necessitating a design approach that integrates Peltier cells within electronic components. An alternative approach involves the installation of a subsonic Meredith duct within the primary air intake of the main turbo engine. This duct operates by reducing air speed at the face of a high-efficiency air-to-liquid radiator and then expanding the heated air into a nozzle, making the application of radiators feasible, even for supersonic UAVs. The feasibility of deploying the Meredith duct with direct exposure to external air in subsonic UAVs is also explored. This investigation thus sheds light on innovative cooling mechanisms for UAVs operating at high altitudes, potentially leading to improved efficiency and lifespan of critical components. The findings are poised to enhance the understanding of UAV design and operation, contributing to their overall performance and effectiveness.

Keywords: Heat rejection; High altitude; Air-to-liquid heat exchanger; Meredith duct; Aircraft

1 Introduction

UAVs designed to operate at high subsonic or supersonic speeds above 21,336m (70,000ft) necessitate propulsion systems equipped with high-efficiency cooling designs. The intake systems must effectively condense the thin air density into the cooling duct, facilitating the required mass flow while minimizing temperature rise to decrease radiator size. The inclusion of a Meredith cooling duct, a secondary thrusting device that repurposes dissipated heat into jet propulsion, can serve to enhance system performance. This subsonic ramjet engine incorporates a diffuser, a radiator system replacing the combustor, and a nozzle. Notably, the duct can be added to the main engine intake of supersonic vehicles by harnessing already decelerated subsonic airflow. The preliminary design of this heat rejection system warrants discussion, and upon completion, optimization using CFD should be employed.

Efficient and cost-effective solutions are sought after to address the current predicament involving Jet-Powered High-Altitude Unmanned Aerial Vehicles (JHAUV) tasked with intercepting lighter-than-air crafts such as balloons, blimps, and airships. To circumvent undue pressure on manufacturing and financial systems, a necessity emerges for specialized systems to handle the dismantling of a plethora of technologically advanced lighter-than-air craft. This issue is far from novel; past endeavours include the installation of high-powered laser systems on high-altitude bombers and other aircraft operating at 10,972.8m (36,000ft). Development efforts are currently underway for UAVs equipped with classified Laser High Altitude Long Endurance (LHALE) technology. Despite the high associated costs, the utility of these flying laser systems is undeniable.

With advancements in mini and micro-jets, highly reliable autopilot systems, and affordable mini-computers employing Global Positioning System (GPS), the design of JHAUV Interceptors (JHAUVI) for operation at remarkable speeds and altitudes up to 28,956m (95,000ft) has become possible. However, thermal management in electronic, electric, and mechanical systems remains a significant challenge for such specialized JHAUVIs. At altitudes exceeding 15,240m (50,000ft), heat rejection is especially critical due to the extremely thin air. This paper delves into this pressing issue, proposing the Meredith duct for JHAUVIs as a viable solution, albeit not the most efficient. Offering a modest thrust while facilitating heat rejection, this system outperforms traditional heat sinks which warm the fuel to cool the onboard systems.

To maintain lift and thrust, JHAUVIs are designed to operate at high speeds and altitudes. A high-speed intake duct is typically incorporated for the jet engine, with the aim of converting incoming air velocity into pressure as efficiently as possible, and slowing the intake air to between 0.3-0.7M. This allows for the installation of the Meredith duct inlet inside the jet engine intake manifold at the most convenient position, optimizing Meredith's intake speed for maximum efficiency. Any velocity reduction at high altitudes can result in an immediate descent due to lack of lift, leading to an increase in airspeed and heat rejection capability of the heat exchanger inside the Meredith duct as a result of thicker air at lower altitudes.

2 High Altitude Aerial Vehicles

High Altitude Long Endurance/High Altitude Platform (HALE/HAP) Unmanned Aerial Vehicles (UAVs) represent a distinct class of high-altitude aviation solutions. Unfortunately, a host of issues beleaguer these configurations. It has been determined that for minimal turbulence and wind speeds under 5kn, HALE/HAP UAVs should operate above 19,812m (65,000ft). Moreover, regulatory altitude limits by the Federal Aviation Administration (FAA) and European Union Aviation Safety Agency (EASA) for Class-A airspace cap at 18,288m (60,000ft), further challenging the design of these UAVs.

Consideration of in-flight refueling is moot due to the lack of tankers able to operate at these altitudes. Disregarding the nuclear option, the sun remains the only feasible energy source. During the day, solar panels power the UAV and charge the energy storage system for night-time use. Potential energy storage solutions include batteries and fuel cells that can store charge or hydrogen. However, limited solar radiation and low efficiency necessitate extensive solar panel areas.

The substantial cooling challenges associated with consistent temperatures around 333.15K lead to a reduction in solar panel efficiency and lifespan. This necessitates the use of high-cost solar panels, typically utilized in satellite applications. Additionally, due to the limited speed achievable in thin air, these panels are installed on the wings, which possess an extensive area.

The design considerations for HALE/HAP UAVs demand an exceptionally lightweight structure. While turbulence and winds are limited at 19,812m (65,000ft), these UAVs still traverse turbulent atmospheres during ascents and descents. Consequently, meteorological conditions severely constrain the launch and recovery windows for such aircraft. The likelihood of flutter and overloads increases in the denser air at lower altitudes, even under optimal conditions.

In comparison to a UAV like the Lockheed U-2, the HALE/HAP UAVs' maneuverability on taxing, takeoff, and landing is markedly reduced. Sensitivity to crosswinds and a propensity to float over the runway make these aircraft difficult to land. This is further complicated by the narrow margin between the minimum and cruising speeds, known as the "coffin corner". Flight safety is compromised due to this small difference, which can lead to loss of altitude from a stall, a scenario undesirable for these aircraft types.

A requirement for large propellers to maintain efficiency in thin air and low speeds necessitates the incorporation of multiple powerplants to avoid large, heavy landing gears. The motors and their controllers, required to be of considerable size, are also impacted by cooling issues due to limited air mass flow. Unfortunately, increasing the number of solar cells, motors, and controllers diminishes overall system reliability. Additionally, the necessity for extended missions to ensure economic viability exacerbates the likelihood of failure.

A pertinent comparison is made with the Lockheed U-2 airplane, extensively redesigned in the 1980s. This aircraft can traverse 10,186km (5500nmi) in 12 hours without refueling, cruising at 190.344m/s (370kn). However, the U-2 has to operate extremely close to the Velocity-Never-Exceed (VNE) to maintain its operational ceiling of 23,164.8m (76,000ft). At this altitude, a mere 5.15m/s (10kn) separates the maximum speed from the stall speed.

In contrast, the Lockheed SR71 Blackbird adopts a radically different approach, built to fly at 3.3M@25,908m (85,000ft). However, the maximum permissible Mach number is contingent on the outside air temperature's effect on the compressor inlet temperature, limited to 700.15K. This permits the pilot to increase the speed up to 3.32M. Despite its range of 5,370.8km (2,900nmi), the SR71 often requires refueling after take-off.

These aircraft, while demonstrating the feasibility and limitations of high-altitude flight, underscore the technical challenges that must be addressed to develop efficient and reliable high-altitude UAVs. The development of HALE/HAP UAVs thus presents a multi-faceted engineering problem demanding innovative solutions for energy harvesting, thermal management, structural integrity, and overall system reliability.

3 Dissipation Mechanisms of Thermal Energy

A variety of methods commonly referred to as "heat sinks" are utilized by modern aircraft for the dissipation of thermal energy. Expendable Heat Sinks (EHS), which encompass materials such as cryogenic gases capable of heat absorption at standard and sub-zero temperatures, and endothermic chemical reactions, are among these methods. EHS are employed when other solutions become impractical during specific flight phases.

When aircraft operate below an altitude of 12,192m (40,000ft), Ram Air (RA) is directed towards equipment racks or separate cooling Heat Exchangers (HX) through protruding ram scoops or flush inlets. This methodology, however, proves ineffective for high-altitude aircraft due to the insufficiency of air density for use as RA.

An alternative approach involves the placement of an external duct at the main fan's exit point, thus deriving Engine Fan Air (EFA) for cooling purposes. The deployment of this method becomes impractical with high-altitude aircraft that employ low-bypass turbofans or pure jets for propulsion.

Skin Heat Exchangers (SHX) serve as another common method, expelling thermal loads directly through the aircraft's fuselage to the external environment. Implementations of SHX typically fall into one of three categories: air loop, liquid loop, or heat pipe systems. Among these, the liquid loop system is the most frequently employed due to its superior heat transfer capacity in comparison to the air loop, and more controllable properties relative to the heat pipe system. Nonetheless, the thin atmosphere at supersonic speeds renders the SHX method impractical for Jet High Altitude Unmanned Vehicle Inspection (JHAUVI) due to reduced overall heat exchange efficiency and increased skin heating of the aircraft.

The use of fuel as a heat sink has a long-standing history in aviation. Many aircraft dissipate heat via a fuel-oil heat exchanger. If the fuel flow required for cooling surpasses the fuel flow demand of the engine, recirculation systems are installed. This is particularly pertinent in low-thrust flight situations such as descent and taxiing. Moreover, extensive research has been conducted into Thermal Management Systems (TMS) and the enhancement of jet fuel properties to increase its heat capacity.

During the Concorde's supersonic flight, for instance, the fuel serves as a heat sink, first heating up within the tanks and then further in the fuel system, thereby cooling auxiliary components and engine oil. Before the fuel is further heated in the engine oil cooler, it is warmed up to approximately 323.15K during a supersonic cruise. Indeed, the thermal instability of the fuel may result in deposits that decrease the effectiveness of heat transfer through various exchangers within the aircraft's fuel system and lead to the clogging of filters or tiny passages in various fuel system control units.

Another modern solution to heat sinks is the Peltier module, which relies on the Peltier effect to generate a heat flux at the junction of two different materials. Here, a Peltier cooler—a solid-state active heat pump—transfers heat from one side of the device to the other, driven by electricity. In this setup, the cold junction can be embedded in the equipment racks or a cold plate can be installed on the device that needs cooling. The hot junction is installed within the fuel heat exchanger or submerged directly in the fuel tank. This solution can be employed for small JHAUVIs, albeit with limitations related to energy rejection and an appropriately managed TMS.

Further research is necessary to develop more efficient methods of thermal management for high-altitude aircraft, taking into consideration the unique operational and environmental constraints they face. It is therefore important to continue to investigate heat sinks as well as alternative methods of thermal energy dissipation in order to improve the longevity and operational safety of these vehicles.

4 Air-to-Liquid Heat Exchangers

In the realm of heat dissipation, air-to-liquid radiators remain a widely adopted mechanism. A prevalent approximation postulates that the heat transferred from the radiator core to the ambient air is dictated by three factors: the temperature differential between the air and the liquid, the air mass flow, and the mean air velocity traversing the fins. This power required for cooling is directly proportional to the square of the airspeed across the fins while being inversely proportional to the temperature difference between the air and the core or cylinder fins.

However, challenges arise as the gas flow's Reynolds number affects the boundary layer thickness and subsequently the radiator's overall airside performance. Additionally, the large drag coefficient of the radiator poses difficulties when directly exposed to the air flow, leading to issues such as air bypassing the radiator. Ducted radiators face their own challenges, with non-uniform airflow and the formation of vortexes reducing the active radiator area. Thus, careful design of the ducts for and aft the radiator becomes crucial.

Airflow manipulation through the radiator involves drawing in air through a small intake and then slowing it down to a sufficient area core. This allows for a slower airstream speed than the aircraft, reducing internal drag. To maximize heat transfer, the coolant should be as hot as possible. For instance, in Formula 1, oil temperatures were noted to reach 423.15K and liquid temperatures reached 418.15K.

Contemporary radiator cores consist of finned tubes, with coolant circulating through these tubes. Fins increase the area available for air-side exchange, and end tanks supply the coolant for these tubes. A low Reynolds number in the air passages is necessary for substantial heat transfer, hence necessitating a small cross-section through the fin. While surface roughness in these passages can increase the pressure drop, it doesn't significantly enhance heat transfer.

Materials used in radiator production have evolved over time. Historically, brass and copper were commonly used in aviation and racing. Today, aluminium alloy is the preferred choice. This shift is mainly due to aluminium and copper exhibiting similar thermal conductivities, but copper fins being soldered to a brass tube with lead, which exhibits poor conductivity.

The frontal surface area of a radiator has been observed to have a significant impact on heat rejection, with a doubling of the frontal surface area increasing heat rejection. Conversely, doubling the thickness of a core does not yield a similar effect. Key variables in determining core heat rejection and airside drag are fin pitch (Fins Per Inch-FPI) and tube spacing. As FPI increases and tube space decreases, the surface area and heat rejection increase but so does the airside drag. Currently, the optimal FPI for the best applications is 25.

Historically, single-pass radiators had the inlet and outlet on different tanks and lacked internal baffles. Designs varied, with cross-flow and down-flow models. However, the advent of double-pass radiators, with a central baffle requiring the water to pass through twice, has largely rendered single-pass radiators obsolete. Triple-pass radiators are seldom used due to their significant waterside pressure drop.

The design and manufacturing process of radiators includes a plethora of shapes for tubes and fins. The optimization of cooling tank shapes is another area that poses common challenges. Polymeric tanks allow for significant shape changes without increasing costs or fundamentally altering the design. The most common tubes are oval-shaped, but others including flat, ovoidal, rounded rectangular, B-shapes, and airfoils also exist. The fins, to maximize airflow, are often multi-louvered and perfectly aligned.

The complexities of air-to-liquid radiator design necessitate ongoing research and optimization, to accommodate the evolving needs of aviation applications. It is therefore critical to explore various design possibilities and continually seek improvement in the thermal management of these systems.

5 Exploring the Meredith Effect in Duct Design

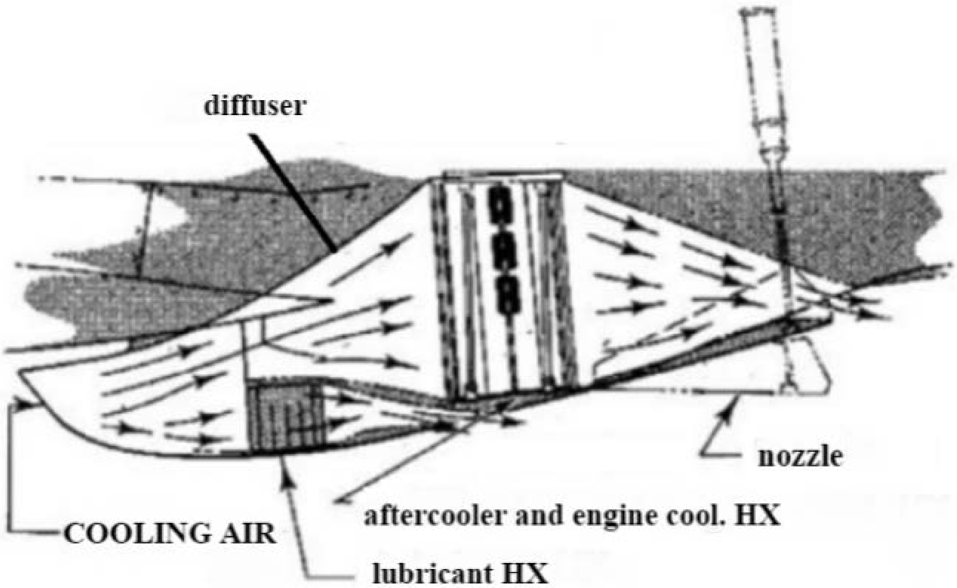

Figure 1. P51D cooling duct (rearranged from the flight manual)

The Meredith effect allows for the integration of a radiator into a ramjet to generate thrust. This theory posits that the aerodynamic drag of a cooling radiator can be offset by the thrust procured from the ramjet nozzle. However, meticulous design of the cooling duct becomes crucial due to the low temperatures of the hot air in the duct that expands in the nozzle to produce thrust. This phenomenon was first recognized during World War I, with F.W. Meredith elucidating and documenting the effect in 1936.

In subsequent years, and notably in the aircraft deployed during World War II, the speed of piston-engine aircraft saw an increase, making the Meredith effect a vital component in propulsion design. In the current era, this phenomenon is employed in the majority of aircraft powerplants requiring a cooling radiator and in Formula 1 racing.

The Meredith effect is triggered when air, heated by a Heat Exchanger (HX) containing a hot working fluid such as lubricant, aftercooler-air, or engine coolant, traverses through a duct. For the effect to be manifested, the duct must be installed on a vehicle that is operating at the design speed. The diffuser within the system slows down and compresses the incoming air as it encounters drag resistance from the HX. The external air is then heated in the HX, causing a rise in temperature and an expansion in volume. This pressurized, hot air subsequently expands, accelerates, and exits the vehicle through a nozzle. If the speed of this accelerated air outstrips that of the vehicle, a variation in momentum generates thrust.

This process can be encapsulated into the three steps of an open Brayton cycle: compression, heat exchange at nearly constant pressure, and expansion. The thrust produced by the cycle is dependent on the working fluid temperature and the pressure ratio between the interior and exterior of the duct. The net thrust is thus given by the cycle thrust minus the duct drag. In the P51H Mustang duct, for instance, three radiators were positioned in series to achieve the maximum possible air temperature.

When dealing with a supersonic vehicle speed, a carefully constructed air intake is required to efficiently convert velocity into pressure. This becomes particularly crucial in high-altitude airplanes, where speed plays a vital role in increasing air density at the jet engine face. Since jet engine thrust hinges on mass flow and temperature at the engine intake, it is feasible to install the cooling Meredith duct ahead of the main jet air intake, following an arrangement analogous to that of the P51D Mustang (Figure 1).

This approach proves especially advantageous in supersonic airplanes due to the intricate design of the air intake. Designing an efficient fixed air intake is relatively straightforward up to approximately Mach 1.6. Beyond this speed, the air intake necessitates variable geometry, adding complexity to the supersonic air intake part. By installing an auxiliary air scoop for the Meredith cooling duct within the subsonic portion of the main air intake, the overall design of the system can be simplified.

The Meredith effect offers an intriguing and efficient way to manage the cooling and thrust generation in high-speed vehicles. The complexity of these processes underlines the importance of continued research and development in this field to further refine and enhance these methods. This will help better accommodate the evolving needs of aviation applications and enhance the performance of these vehicles.

6 Meredith Duct for High Altitude-High Speed Application

This section employs a practical instance to assess the radiator's cooling capability. Figure 1 illustrates the oil-cooling duct positioned within the primary Meredith duct of a P51D Mustang. When flying at altitudes reaching 28,956m (95,000ft), air density emerges as a crucial factor. Indeed, both thrust and cooling are reliant on the mass flow. Therefore, high-altitude aircraft must maintain high speeds to transmute velocity into pressure and air density. Supersonic flight is feasible only with appropriate aerodynamics and highly efficient air intakes.

Contemporary turbofans necessitate subsonic speeds for proper functioning, generally within the 0.3M-0.7M range. Consequently, the air intake of the main engine assumes a critical role. It is advantageous to incorporate a cooling Meredith duct within the primary engine duct. This cooling ram-jet will utilize the already decelerated air and employ a subsonic Meredith duct coupled with a radiator. The dimensions of this duct are determined by equations 1 through 6. In these equations, please employ the International System of Units and verify the consistency of formulas prior to computation. Figure 1 is applicable for speeds between 0.3M and 0.7M at the engine face, and γ denotes the heat capacity ratio for air. The airplane turbofan with less air intake diversion presents a pressure recovery curve as illustrated in Figure 2. In Figure 2, the x-axis represents the ratio of the true total pressure at the engine face to the ideal one. Eq. (1) provides the isentropic total pressure for compressible flow. p_{ideal} is the total pressure at the engine face, 'M' represents the airspeed at the engine intake, and 'p' stands for the true total pressure at the engine face. The ratio between p and p_{ideal} is referred to in this article as "intake efficiency" η. p signifies the static pressure.

$$p_{\text{ideal}} = p_{\text{static}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{1}$$

The static pressure $p_{\text{static\_Meredith\_mouth}}$ in face of the Meredith's duct intake depend on the speed at the duct mouth/face $M_{\text{Meredith\_mouth}}$ and can be calculated using Eq. (2).

$$p_{\text{ideal}}\,\eta = p_{\text{static\_Meredith\_mouth}} \left(1 + \frac{\gamma - 1}{2} M^2_{\text{Meredith\_mouth}}\right)^{\frac{\gamma}{\gamma - 1}} \tag{2}$$

The compression ratio r of the air intake is in Eq. (3).

$$r = \frac{p_{\text{static\_Merdith\_mouth}}}{p_{\text{static}}} \tag{3}$$

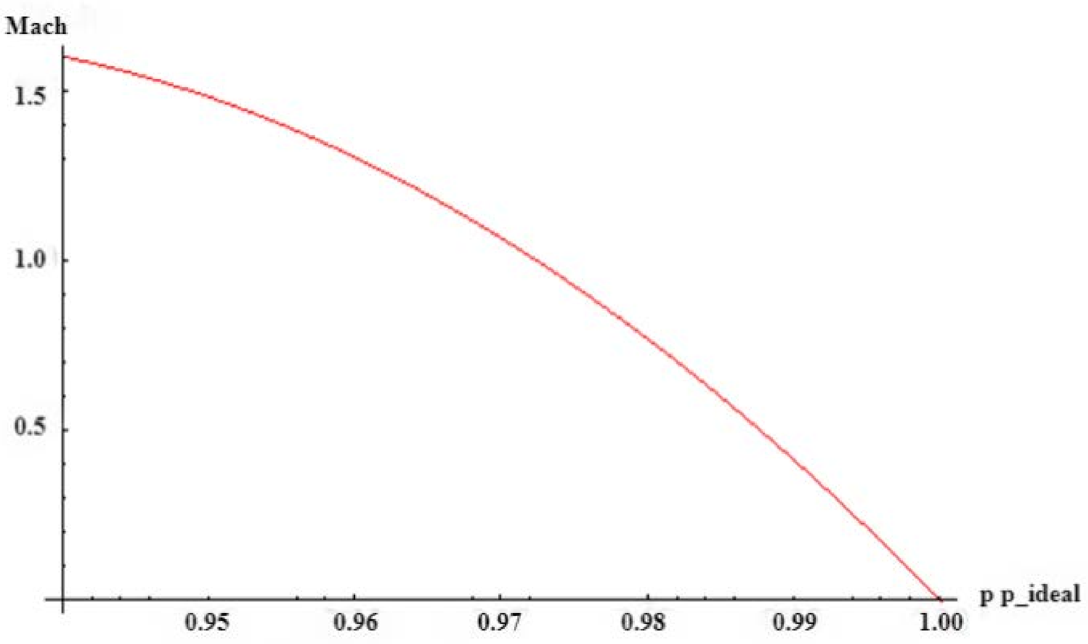

Figure 2. Pressure recovery of a last-generation, fixed-geometry divertless air intake

Therefore, the density ratio is shown in Eq. (4). ρ and $\rho_{\text{Meredith\_mouth}}$ are the air density in front of the air intake and at Meredith's duct mouth.

$$\frac{\rho_{\text{Meredith\_mouth}}}{\rho} = r^{\frac{1}{\gamma}} \tag{4}$$

From the ideal gas law, the temperature $T_{\text{Meredith\_mouth}}$ of the air at the Meredith's duct mouth is therefore in Eq. (5). where, R is the ideal gas constant (Eq. (5)).

$$T_{\text{Meredith\_mouth}} = \frac{p_{\text{static\_Meredith\_mouth}}}{\rho_{\text{Meredith\_mouth}} R} \tag{5}$$

Post intake, the air in the Meredith duct expands in the subsonic diffuser with an efficiency η exceeding 0.995. The precise and optimized shape of the Meredith duct is of utmost importance due to numerous reasons. The overall efficiency of the Brayton cycle won't be significantly high due to the exceedingly low maximum temperature of the air circulating outside the radiator. Therefore, it is imperative to design a Meredith duct with maximal efficiency and to embed it within the aircraft structure to minimize friction and drag from external air. Another crucial reason is the need to prevent the formation of parasitic vortices in front of the radiator, a common cause for the reduction of effective radiator area and heat rejection. Furthermore, the air should ideally pass inside the duct. If the internal air resistance becomes overly high, the incoming air will opt for the path of least energy outside the cooling duct. The most efficient radiator is a 25 Fins-Per-Inch (FPI) radiator designed for Formula 1 racing cars. The optimal speed of the air passing through the radiator is $V_{in\_radiator} = 5\ \mathrm{m/s}$ at 298.15K outside-temperature sea level. Eq. (1) to Eq. (4) are also applicable for the diffuser. The sound speed at duct intake $V_{sound\_Meredith\_mouth}$ can be calculated using Eq. (6).

$$V_{sound\_Meredith\_mouth} = \sqrt{K \times R \times T_{Meredith\_mouth}} \tag{6}$$

To calculate $P_{\text{static\_in\_radiator}}$, it is then possible to use Eq. (2) by replacing P_{ideal} with $p_{\text{total\_Meredith\_mouth}}$ (Eq. (7)) and $M_{\text{Meredith\_mouth}}$ with $M_{\text{in\_radiator}}$ (Eq. (8)). In this way, $P_{\text{static\_in\_radiator}}$ will replace $P_{\text{static\_Meredith\_mouth}}$ in Eq. (2).

$$p_{total\_Meredit\_mouth} = p_{static\_Meredith\_mouth} \left(1 + \frac{\gamma - 1}{2} M^2_{Meredith\_mouth}\right)^{\frac{\gamma}{\gamma-1}} \tag{7}$$

$$M_{in\_radiator} = \frac{V_{in\_radiator}}{V_{sound\_Meredith\_mouth}} \tag{8}$$

It is then possible to calculate the temperature $T_{in\_radiator}$ and the air-density $\rho_{in\_radiator}$ at the intake of the radiator following the procedure outlined in Eqs. (3)-(5). The Reynolds number at the radiator intake is calculated with Eq. (9), where D_{fins} is the distance between 2 fins (Eq. (10)) and $\mu_{in\_radiator}$ is air viscosity.

$$\text{Re} = \frac{\rho_{in\_radiator} V_{in\_radiator} D_{fins}}{\mu_{in\_radiator}} \tag{9}$$

$$D_{fins} = \frac{Inch/1000}{FPI} = \frac{25.4/1000}{25} \tag{10}$$

It is then possible to calculate the pressure drop in the radiator P_{drop} (Eq. (11)) and the maximum heat rejection H_{rmax} (Eq. (12)) [1]. $T_{out\_radiator}$ depends on the temperature of the liquid to be cooled and $S_{radiator}$ is the radiator frontal area.

$$P_{\text{drop}} = 0.1097 \times \text{Re}^{1.6637} \tag{11}$$

$$H_{rmax} = \alpha \left(T_{in\_radiator} - T_{out\_radiator}\right) S_{radiator} \tag{12}$$

where, the global heat transfer coefficient α can be calculated with Eq. (13). Pr is the Prandtl number.

$$\alpha = \frac{-1.619 \times 10^{-5}\,\text{Re}^2 + 0.018 \times \text{Re} + 0.877}{\text{Pr}^{2/3}} \tag{13}$$

The air flowing from the radiator then expands in the nozzle to produce the thrust F_{nozzle} (Eq. (14)). V_{nozzle_exit} can be calculated with De Laval's nozzle Eq. (15).

$$F_{\text{Meredith}} = \eta_{\text{nozzle}}\, MF_{\text{air}} \left(V_{\text{nozzle-exit}} - V_{\text{airplane}}\right) \tag{14}$$

$$v_{exit\_nozzle} = \sqrt{T_{out\_radiator} R \frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{P_{static\_out\_radiator}}{P_{static\_outside}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{15}$$

where, $P_{static\_out\_radiator}$ is given by Eq. (16). The air flowing from the radiator then expands in the nozzle to produce the thrust F_{nozzle} (Eq. (14)). V_{nozzle_exit} can be calculated with Eq. (15). $P_{static\_out\_radiator}$ is calculated with Eq. (16) and the $P_{static\_outside}$ is the outside air pressure.

$$P_{static\_out\_radiator} = P_{static\_in\_radiator} - P_{drop} \tag{16}$$

The nozzle efficiency η_{nozzle} depends on the design of the nozzle. In any case, it is high, varying from 0.8 to 0.99. Table 1 summarizes the results of a few Meredith ducts installed in an air intake of an airplane flying at 0.9M and 1.6M (Figure 2).

Table 1 demonstrates that the position/speed of/at the Meredith duct mouth is not critical. It is preferable to remain below 0.7M to circumvent compressibility effects [2] that are already incorporated in the main intake design. If the intake nozzle is efficient, the airplane speed is irrelevant for heat rejection and thrust. Conversely, the effect of pressure altitude is crucial. This is due to the fact that the radiator pressure drop becomes significant at extremely low atmospheric pressure. The higher pressure achieved at the radiator face at 1.6M is counterbalanced by the higher temperature [3]. This is further clarified by Table 1, which reveals no advantage in increasing the speed over 0.9M. At 12,192m (40,000ft), the design is less critical, resulting in improved overall performance of the Meredith duct. Table 1 was calculated with unitary nozzle efficiency [4].

Table 1. Calculated data for a few Meredith ducts installed on the jet intake of a vehicle flying in ISA atmosphere-1 m^2 radiator-max oil temperature 421.15K

Altitude	**Vehicle speed** [**M**]	**Mouth speed** [**M**]	**Airspeed at the radiator [m/s]**	**Heat rejection** [**kW**]	**Thrust** [**N**]
$21,336$ m $70,000$ft	0.9	0.7	15	246	71
$21,336$ m $70,000$ft	0.9	0.3	15	246	71
$28,956$ m $95,000$ft	0.9	0.3	15	70	23
$12,192$ m $40,000$ft	1.6	0.3	15	760	359
$21,336$ m $70,000$ft	1.6	0.3	15	256	105
$28,956$ m $95,000$ft	1.6	0.3	15	67	26

Table 2. Calculated data for a few Meredith ducts installed on the jet intake of a vehicle flying in ISA atmosphere - 1 m^2 radiator - max oil temperature 421.15K - with different air speeds at the radiator

Altitude	**Vehicle speed** [**M**]	**Mouth speed** [**M**]	**Airspeed at the radiator [m/s]**	**Heat rejection** [**kW**]	**Thrust** [**N**]
$21,336$ m $70,000$ft	0.9	0.3	25	410	84
$21,336$ m $70,000$ft	0.9	0.3	25	351	33
$21,336$ m $70,000$ft	0.9	0.3	50	817	0
$28,956$ m $95,000$ft	0.9	0.3	50	408	22

7 Discussion

Table 2 delineates how the position of radiator heat rejection at various altitudes is contingent upon the air speed at the radiator, a key design variable. It was observed that when the speed at the radiator was increased to 25m/s, the overall performance of the duct improved significantly, ranging from an altitude of 21,336m (70,000ft) to 28,956m (95,000ft).

When the speed was further escalated to 50m/s, the duct demonstrated partial efficiency for heat rejection at an altitude of 28,956m (95,000ft) [5]. However, it was found to be non-functional at an altitude of 21,336m (70,000ft), indicating zero or negative thrust, which defies physical principles [6].

Though implementing a variable geometry diffuser could potentially enhance performance, the complexity it adds to the system was deemed unnecessary [7]. In contrast, the inclusion of a variable geometry nozzle, though it introduces some complexity, was regarded as potentially beneficial and warranted, akin to the design of the Mustang P51 [8].

Data presented in Table 3 underlines the importance of optimizing the airspeed at the radiator. In situations where the duct was installed on an airplane operating at 0.3M, it was observed that the speed had to be reduced to 5m/s at the radiator face [9, 10]. If not, the duct remained functional only at 28,956m (95,000ft), where the air is so thin that the pressure drop on the radiator becomes minimal [11].

The intricacies of this study emphasize the importance of striking a balance between multiple design factors, which should be done carefully to ensure optimal system performance. It underlines the complex interactions of factors like radiator position, airspeed, and duct design. Further research is needed to continue to explore these dynamics and how they can be manipulated to improve the efficiency and reliability of the system [12]. For example, future studies could explore the specific mechanisms by which variable geometry nozzles improve performance, with an eye towards the potential development of new, more effective designs.

Table 3. Computational data for select Meredith ducts installed on a vehicle in flight within an ISA atmosphere -1 m^2 radiator-max oil temperature 421.15K-with different air speeds at radiator

Altitude	Vehicle speed [M]	Mouth speed [M]	Airspeed at the radiator [m/s]	Heat rejection [kW]	Thrust [N]
$21,336$ m $70,000$ft	0.7	0.7	15	234	63
$28,956$ m $95,000$ft	0.7	0.7	15	67	21
$21,336$ m $70,000$ft	0.3	0.3	5	71	8
$28,956$ m $95,000$ft	0.3	0.3	5	20	2.5
$28,956$ m $95,000$ft	0.3	0.3	15	61	2.5

8 Conclusions

Cooling challenges at high altitudes are demonstrably manageable by integrating a cooling radiator within a Meredith duct. A design of a Meredith duct has been shown to effectively cool a high-speed airplane, even at altitudes reaching up to 28,956m (95,000ft). The duct proves effective at a lower speed limit of 0.3M. The most optimal arrangement is obtained by positioning the inlet of the cooling duct where the airspeed is approximately 0.3M. This configuration can be achieved by installing the duct within the primary air intake, even in supersonic aircraft. Air velocities at the intake mouth of up to 0.7M are manageable and require relatively straightforward design considerations.

Beyond this speed, however, meticulous design is mandatory to mitigate compressibility effects in the diffuser. The airspeed at the radiator's face has been identified as a significant concern for system optimization. The Meredith duct should ideally be optimized at the cruise altitude, acknowledging that the design may not be optimal at higher or lower altitudes.

The introduction of a variable geometry diffuser could potentially rectify the problem, but the added complexity outweighs the potential performance improvement. Contrarily, the employment of a variable geometry nozzle is seen as beneficial.

The conclusions drawn in this study underscore the importance of a balanced approach to the design of cooling systems for high-speed, high-altitude aircraft. These findings should be of interest to both academic researchers and practitioners in the field of aerospace engineering. Further research could focus on the specifics of the variable geometry nozzle and its potential benefits, as well as on further refining the design of the Meredith duct to improve performance at non-optimal altitudes. Future investigations could also examine the potential applications of these design principles in other high-speed vehicles.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares no conflict of interest.

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