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Complex Polytopic Fuzzy Model and Their Induced Aggregation Operators

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Abstract: Inducing variables are the parameters or conditions that influence the membership value of an element in a fuzzy set. These variables are often linguistic in nature and represent qualitative aspects of the problem. Thus, the objective of this paper is introduce some aggregation operators based on inducing variable, such as induced complex Polytopic fuzzy ordered weighted averaging aggregation operator (I-CPoFOWAAO) and induced complex Polytopic fuzzy hybrid averaging aggregation operator (I-CPoFHAAO). Induced aggregation operators in decision-making process are indispensable tools for managing uncertainty, integrating multiple criteria, facilitating consensus, and providing a formal and flexible framework for modeling and solving complex decision problems. At the end of the paper, we make an illustrative example to prove the ability and efficiency of the novel proposed aggregation operators.

Keywords: CPoF-Sets; I-CPoFOWAA operator; I-CPoFHAA operator; Decision-making

1 Introduction

Decision-making plays an imperative role in various fields, such as finance, computer science, investment decisions, business, management science, natural science, social science, healthcare, robotics, automotive industries, human resources, medicine industries, engineering science, manufacturing and operations, information technology, environmental management, and public police. In the early stages of decision-making problems it was considered that the data related for making good decisions known as alternatives is consist of clear numbers. Nevertheless, in the actual world, most decisions are made under conditions where objectives can often be imprecise or poorly defined in general. Various ideas have developed with passage of time to address these issues in real-world scenarios. Some theories of them are soft sets (SSs) [1] theory, which as the generalization of classical set theory. It provides a flexible and intuitive way to handle uncertainties and vagueness in decision-making and information processing. Fuzzy sets (FSs) [2] theory is particularly useful in handling ambiguity and vagueness in decision-making. It allows for the manipulation, management and representation of ambiguous or imprecise information, which is often encountered in real-world scenarios. Rough sets (RSs) [3] theory is also a mathematical framework that deals with vagueness and incomplete information in decision-making. It provides a formal methodology for handling imprecise and uncertain data by defining sets based on their discernibility and equivalence relations.

All of the abovementioned valuable theories have their own specific, significant, noteworthy, striking and attractive applications. It is crucial and noteworthy to remember that Zadeh [2], the pioneer of fuzzy set theory, was the one who continued this research by introducing fuzzy sets (FSs), which contains one term considered as membership function that expresses an object's level of satisfaction without exposing its level of dissatisfaction. For instance, if the satisfaction an element is considered to be 0.50, then that's element dissatisfaction can be computed mathematically as $1 - 0.50 = 0.50$.

But fuzzy sets theory fails to handle data that acquire unsatisfactory and satisfactory information of interest. Atanassov and Stoeva [4] introduced intuitionistic fuzzy set (IFS). In IFS each element can be presented as (r, s) with condition: $0 \leq r + s \leq 1$. Later on, Yager [5] introduced Pythagorean fuzzy sets (PyFS), which reduced the limitation of IFS, that is $0 \leq r + s \leq 1$ to $0 \leq r^2 + s^2 \leq 1$. Sanapati and Yagar [6] introduced Fermatean fuzzy set (FFS), which is the generalization of PyFS. It reduced its limitations, such as $0 \leq r^2 + s^2 \leq 1$ to $0 \leq r^3 + s^3 \leq 1$.

Later on, Yager [7] introduced the more generalized form of the all mentioned model, called q-rung orthopair fuzzy set (q-ROFS). Its each element can be mathematically presented as (r, s) under condition, such as $0 \leq r^q + s^q \leq 1$, where q is any positive real numbers.

On the bases of the mentioned models, scholars developed several operators and applied them also on group decision-making. Xu [8], Xu and Yager [9], Wang and Liu [10, 11], Rahman et al. [12] and Rahman [13] introduced many aggregation operators using intuitionistic fuzzy numbers. Garg [14, 15] and Rahman et al. [16–19] using PyFNs and developed several new techniques. Liu and Wang [20] introduced q-ROFWA operator and the q-ROFWG operators. Peng and Liu [21] presented entropy, and distance measures using q-rung orthopair fuzzy numbers.

The above models having many applications in real life problems, but these oerators fail to produce the expected result. For the solution of these types of problems, Cuong [22] introduced picture fuzzy set (PcFS). In PcFS each element mathematically may be written as (r, g, s) under condition $0 < r + g + s \leq 1$. Later on, Ashraf et al. [23] presented spherical fuzzy set (SpFS), which is the generalization of PcFS. In SpFS each element can be written as (r, g, s) with condition $0 < r^2 + g^2 + s^2 \leq 1$. Later on, Beg et al. [24] introduced Polytopic fuzzy set (PoFS), which relaxes the limitation of SpFS to $0 < r^q + g^q + s^q \leq 1$. Garg [25], Beg et al. [24] introduced new techniques using picture fuzzy numbers and Polytopic fuzzy numbers respectively and applied them on in real problems.

The above studies having much application, in daily life problems, but none of them are capable of handling periodic information. However, ambiguity, vagueness, and variations in the data's periodicity all coexist in complex datasets. These databases also contain an immense amount of information that is insufficient and incomplete. In order to circumvent these circumstances, Ramot et al. [26] developed complex fuzzy set (CFS). Later Alkouri and Salleh [27] introduced complex intuitionistic fuzzy set (CIFS). In CIFS each element mathematically can be presented as: $\left(re^{i2\pi p}, se^{i2\pi q}\right)$, where $r \in [0,1]$, $s \in [0,1]$, $p \in [0, 2\pi]$ and $p \in [0, 2\pi]$ with $0 \leq r + s \leq 1$ and $0 < \frac{p}{2\pi} + \frac{q}{2\pi} \leq 1$. Ma et al. [28], Rani and Garg [29], Kumer and Bajej [30], Garg and Reni [31], presented related work using complex intuitionistic fuzzy numbers. Ullah et al. [32] presented complex Pythagorean fuzzy set (CPyFS). It is the generalization of CIFS, and reduced its conditions: $0 < r + s \leq 1$ to $0 < r^2 + s^2 \leq 1$ and $0 < \frac{p}{2\pi} + \frac{q}{2\pi} \leq 1$ to $0 < (\frac{p}{2\pi})^2 + (\frac{q}{2\pi})^2 \leq 1$. Rahman et al. [33] and Hezam et al. [34] introduced novel operational laws, and techniques based on CPyFNs. Some new research about CPyFS found in the studies [35, 36]. Later, Liu et al. [37] developed complex q-rung orthopair fuzzy set (Cq-ROFS). Each element of Cq-ROFS can be written as: $\left(re^{i2\pi p}, se^{i2\pi q}\right)$ with $0 < r^q + s^q \leq 1$ and $0 < (\frac{p}{2\pi})^2 + (\frac{q}{2\pi})^2 \leq 1$. Later on, Rahman [38] introduced complex Polytopic fuzzy set (CPoFS), complex Polytopic fuzzy numbers (CPoFNs) and some aggregation operators. Each element of CPoFS can be written mathematically as: $\left(re^{i2\pi p}, ge^{i2\pi y}, se^{i2\pi q}\right)$ with $0 < r^q + g^q + s^q \leq 1, (1 \leq q)$ and $0 < (\frac{p}{2\pi})^q + (\frac{y}{2\pi})^q + (\frac{q}{2\pi})^q \leq 1$.

Thus, keeping the applications of the mentioned models and their corresponding techniques, in this paper, we introduced induced aggregation operators based on complex Polytopic fuzzy numbers, namely I-CPoFOWAA operator, I-CPoFHAA operator. Finally, a practical example is considered showing the efficiency and effectiveness of the proposed model and their corresponding induced aggregation operators.

The rest of this paper is planned as: Section 2 contains basic definitions, Section 3 contains basic operational laws based on CPoFNs, Section 4 contains induced aggregation operators, Section 5 contains an application of the proposed operators, Section 6 contains an example, Section 7 contains sensitivity analysis, and Section 8 contains conclusion.

2 Preliminaries

In this part, we develop complex fuzzy set and complex fuzzy numbers. Complex fuzzy set theory extends the fuzzy set theory by introducing complex numbers to represent uncertainty and imprecision in a more nuanced manner.

Definition 1: [26] The complex fuzzy set C, can be mathematically defined on a universal set M as: $C = \left\{m, r_C(m)e^{ip_C(m)} | m \in M\right\}$, where $r_C(m) : M \to [0,1]$ and $r_C(M)$ is said to be the membership degree.

Definition 2: [27] The complex intuitionistic fuzzy set I, can be mathematically defined on a universal set M as: $I = \left\{\langle m, r_I(m)e^{ip_I(x)}, s_I(m)e^{iq_I(m)}\rangle | m \in M\right\}$, where $r_I(m) : M \to [0,1]$, $s_I(m) : M \to [0,1]$, $p_I(m) \in [0, 2\pi]$ and $q_I(m) \in [0, 2\pi]$ under the conditions: $0 < r_I(m) + s_I(m) \leq 1$ and $0 < \frac{p_I(m)}{2\pi} + \frac{q_I(m)}{2\pi} \leq 1$.

Definition 3: [32] The complex Pythagorean fuzzy set P, can be mathematically defined on a universal set M as: $P = \left\{\langle m, r_P(m)e^{ip_P(x)}, s_P(m)e^{in_P(m)}\rangle | m \in M\right\}$, where $r_P(m) : M \to [0,1]$, $r_(m) : M \to [0,1]$, $p_P(m) \in [0, 2\pi]$ and $q_P(m) \in [0, 2\pi]$ under the conditions: $0 < (r_P(m))^2 + (s_P(m))^2 \leq 1$ and $0 < \left(\frac{p_P(m)}{2\pi}\right)^2 + \left(\frac{q_P(m)}{2\pi}\right)^2 \leq 1$.

Definition 4: [38] The complex Polytopic fuzzy set L, can be defined on a universal set M as: $L = \left\{\langle m, r_L(m)e^{ip_L(xm)}, g_L(m)e^{iy_L(m)}, s_L(m)e^{iq_L(m)}\rangle | m \in M\right\}$ where $r_L(m) : M \to [0,1]$, $g_L(m) : M \to [0,1]$, $s_L(m) : M \to [0,1]$, $p_L(m) \in [0, 2\pi]$, $y_L(m) \in [0, 2\pi]$ and $q_L(m) \in [0, 2\pi]$ under the conditions: $0 <$

$(r_L(m))^q + (g_L(m))^q + (s_L(m))^q \leq 1$ and $0 < \left(\frac{p_L(m)}{2\pi}\right)^q + \left(\frac{y_L(m)}{2\pi}\right)^q + \left(\frac{q_L(m)}{2\pi}\right)^q \leq 1$.

Definition 5: [38] Let $H = \left(re^{ip}, ge^{iy}, se^{iq}\right)$ be a CPoFN, then the score $S(H)$ CPoFNs and the accuracy $A(H)$ can be presented mathematically as: $S(H) = \frac{1}{3}\left[(1 + r^q + g^q - s^q) + (1 + p^q + y^q - q^q)\right]$ with condition: $S(H) \in [2, 2]$ and $A(H) = \frac{1}{2}\left[(1 + max(r^q, g^q) - s^q) + (1 + max(p^q, y^q) - q^q)\right]$, with condition: $A(H) \in [0, 2]$ respectively.

3 Basic Operational Laws under CPoFNs

Fuzzy sets allow for the representation of uncertainty and vagueness in a more flexible way compared to classical set theory. Basic operational laws provide a formal framework for manipulating and combining fuzzy sets, allowing for the modeling of imprecise or uncertain information.

Definition 6: Let $H_j = \left(r_j e^{ip_j}, g_j e^{iy_j}, s_j e^{iq_j}\right)$ $(j = 1, 2)$ be a family of CPoFNs and $t > 0$, then

i) $H_1 \oplus H_2 = \begin{pmatrix} \left(r_1^q + r_2^q - r_1^q r_2^q\right)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{p_1}{2\pi}\right)^q + \left(\frac{p_2}{2\pi}\right)^q - \left(\frac{p_1}{2\pi}\right)^q \left(\frac{p_2}{2\pi}\right)^q\right)^{\frac{1}{q}}}, \\ (g_1 g_2)\, e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, (s_1 s_2)\, e^{i2\pi\left(\frac{q_1}{2\pi}\right)\left(\frac{q_2}{2\pi}\right)} \end{pmatrix}$

ii) $H_1 \otimes H_2 = \begin{pmatrix} (r_1 r_2)\, e^{i2\pi\left(\frac{p_1}{2\pi}\right)\left(\frac{p_2}{2\pi}\right)}, (g_1 g_2)\, e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, \\ \left(s_1^q + s_2^q - s_1^q s_2^q\right)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{q_1}{2\pi}\right)^q + \left(\frac{q_2}{2\pi}\right)^q - \left(\frac{q_1}{2\pi}\right)^q \left(\frac{q_2}{2\pi}\right)^q\right)^{\frac{1}{q}}} \end{pmatrix}$

iii) $t(H) = \left(\left(1 - (1 - r^q)^t\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{p}{2\pi}\right)^q\right)^t\right)^{\frac{1}{q}}}, g^t e^{i2\pi\left(\frac{y}{2\pi}\right)^t}, s^t e^{i2\pi\left(\frac{q}{2\pi}\right)^t}\right)$

iv) $(H)^t = \left(r^t e^{i2\pi\left(\frac{p}{2\pi}\right)^t}, g^t e^{i2\pi\left(\frac{y}{2\pi}\right)^t} \left(1 - (1 - s^q)^t\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{q}{2\pi}\right)^q\right)^t\right)^{\frac{1}{q}}}\right)$.

4 Complex Polytopic Fuzzy Aggregation Operators under Inducing Variables

In this section, we present some novel aggregation operators under inducing variables, namely I-CPoFEOWGA operator and I-CPoFEHGA operator. These operators allow for a more nuanced representation of preferences and relationships among input elements, contributing to more sophisticated decision-making processes.

Definition 7: Let $\langle u_j, H_j \rangle$ $(j = 1, 2, \ldots, n)$ be a finite group of 2-tuple of CPoFNs, where $m = (m_1, m_2, ..., m_n)^T$ be their weighted vector with conditions: $m_j \in [0, 1]$ and $\sum_{j=1}^{n} m_j = 1$ Then the I-CPoFOWAA operator can be mathematically written as follows:

$$\text{I-CPoFOWAA}_{\,m}\left(\langle u_1, H_1 \rangle, \langle u_2, H_2 \rangle, \ldots, \langle u_n, H_n \rangle\right)$$
$$= \begin{pmatrix} \left(1 - \prod_{j=1}^{n}\left(1 - r_{\propto(j)}^q\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - \left(\frac{p_{\propto(j)}}{2\pi}\right)^q\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{n}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix} \quad (1)$$

where, $u_j \in \langle u_j, H_j \rangle$ be the ordered pair of CPoFOWA having the jth greatest value and called the order inducing variable, such as $u_j \in \langle u_j, H_j \rangle$ and $H_j (j = 1, 2, ..., n)$ as the complex Polytopic fuzzy argument.

Theorem 1: Let $\langle u_j, H_j \rangle$ $(j = 1, 2, \ldots, n)$ be a finite group of 2-tuple of CPoFNs, then their resulting value by using CPoFWAA operator still CPoFN, such that:

$$\text{I-CPoFOWAA}_{\,m}\left(\langle u_1, H_1 \rangle, \langle u_2, H_2 \rangle, \ldots, \langle u_n, H_n \rangle\right)$$
$$= \begin{pmatrix} \left(1 - \prod_{j=1}^{n}\left(1 - r_{\propto(j)}^q\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - \left(\frac{p_{\propto(j)}}{2\pi}\right)^q\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{n}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix} \quad (2)$$

Proof. Theorem 1 can be proved using mathematical principles of induction.

Step 1: Since $\langle u_j, H_j \rangle$ $(j = 1, 2, \ldots, n)$, then for $n = 2$, we have

$$m_1 H_1 = \begin{pmatrix} \left(1 - (1 - r_1^q)^{m_1}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{p_1}{2\pi}\right)^q\right)^{m_1}\right)^{\frac{1}{q}}}, \\ (g_1)^{m_1} e^{i2\pi\left(\frac{y_1}{2\pi}\right)^{m_1}}, (s_1)^{m_1} e^{i2\pi\left(\frac{q_1}{2\pi}\right)^{m_1}} \end{pmatrix}$$

$$m_2 H_2 = \begin{pmatrix} \left(1 - (1 - r_2^q)^{m_2}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{p_2}{2\pi}\right)^q\right)^{m_2}\right)^{\frac{1}{q}}}, \\ (g_2)^{m_2} e^{i2\pi\left(\frac{y_2}{2\pi}\right)^{m_2}}, (s_2)^{m_2} e^{i2\pi\left(\frac{q_2}{2\pi}\right)^{m_2}} \end{pmatrix}$$

Next, by Definition 7, we have

$$\text{I-CPoFOWAA}_{m}\left(\langle u_1, H_1\rangle, \langle u_2, H_2\rangle\right)$$
$$= \begin{pmatrix} \left(1-\prod_{j=1}^{2}\left(1-r_{\propto(j)}^{q}\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{2}\left(1-\left(\frac{p_{\propto(j)}}{2\pi}\right)^{q}\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{2}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{2}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{2}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{2}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix}$$

Step 2: Eq. (2) holds for $n = 2$. Next, we suppose that Eq. (2) holds for $n = k$, with $k > 0$ and then it follows that

$$\text{I-CPoFOWAA}_{m}\left(\langle u_1, H_1\rangle, \langle u_2, H_2\rangle, \ldots, \langle u_k, H_k\rangle\right)$$
$$= \begin{pmatrix} \left(1-\prod_{j=1}^{k}\left(1-r_{\propto(j)}^{q}\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{p_{\propto(j)}}{2\pi}\right)^{q}\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{k}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix}$$

Step 3: Suppose Eq. (2) holds for $n = k$, next, we show for $n = k + 1$,

$$\text{I-CPoFOWAA}_{m}\left(\langle u_1, H_1\rangle, \langle u_2, H_2\rangle, \ldots, \langle u_k, H_k\rangle, \langle u_{k+1}, H_{k+1}\rangle\right)$$
$$= \begin{pmatrix} \left(1-\prod_{j=1}^{k}\left(1-r_{\propto(j)}^{q}\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{p_{\propto(j)}}{2\pi}\right)^{q}\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{k}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix}$$
$$\oplus \begin{pmatrix} \left(1-\left(1-r_{k+1}^{q}\right)^{k+1}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{p_{k+1}}{2\pi}\right)^{q}\right)^{k+1}\right)^{\frac{1}{q}}}, \\ \left(g_{k+1}\right)^{k+1} e^{i2\pi\left(\frac{y_{k+1}}{2\pi}\right)^{k+1}}, \left(s_{k+1}\right)^{k+1} e^{i2\pi\left(\frac{q_{k+1}}{2\pi}\right)^{k+1}} \end{pmatrix}$$
$$= \begin{pmatrix} \left(1-\prod_{j=1}^{k+1}\left(1-r_{\propto(j)}^{q}\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k+1}\left(1-\left(\frac{p_{\propto(j)}}{2\pi}\right)^{q}\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k+1}\left(g_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k+1}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{k+1}\left(s_{\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{k+1}\left(\frac{q_{\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix}$$

Hence, it holds for $n = k + 1$. As a result, Eq. (2) holds for all positive integers according to the principle of mathematical induction. Thus, the proof is completed.

Definition 8: Let $\langle u_j, H_j\rangle\,(j = 1, 2, ..., n)$ be a finite group of 2-tuple, and $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)$, $m = (m_1, m_2, ..., m_n)$ respectively presented weighted vector and associated vector under conditions, such as both are belong to closed interval and their sum is equal to one. And also $H_{\propto(j)} = n\varpi_j H_j$ with $H_{\propto(j)}$ be the greatest value, and n is called the balancing coefficient. Moreover, if the weighted vector $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$, then $(n\varpi_1 H1, n\varpi_2 H_2, ..., n\varpi_n H_n)$ approaches to $(H_1, H_2, ..., H_n)$. Furthermore, $u_j \in \langle u_j, H_j\rangle$ be the ordered pair of CPoFOWA having the jth maximum value is called as the order inducing variable, such as $u_j \in \langle u_j, H_j\rangle, H_j (j = 1, 2, ..., n)$ as the CPoF argument. Then the I-CPoFHAA operator can be presented mathematically as follows:

$$\text{I-CPoFHAA}_{\varpi,m}\left(\langle u_1, H_1\rangle, \langle u_2, H_2\rangle, \ldots, \langle u_n, H_n\rangle\right)$$
$$= \begin{pmatrix} \left(1-\prod_{j=1}^{n}\left(1-r_{H\cdot\propto(j)}^{q}\right)^{m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{p_{H\cdot\propto(j)}}{2\pi}\right)^{q}\right)^{m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}\left(g_{H\cdot\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{y_{H\cdot\propto(j)}}{2\pi}\right)^{m_j}}, \prod_{j=1}^{n}\left(s_{H\cdot\propto(j)}\right)^{m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{q_{H\cdot\propto(j)}}{2\pi}\right)^{m_j}} \end{pmatrix} \tag{3}$$

5 An Application of the Proposed Approaches

In this paper, we consider a decision making using complex Polytopic fuzzy information, and their corresponding techniques, namely I-CPoFOWAA operator, and I-CPoFHAA operator. In this process, we consider a group n criteria, $H = \{H_1, H_2, ..., H_n\}$ and $N = \{N_1, N_2, ..., N_m\}$ be a group of $\boldsymbol{m}$ options, whose weights is $m = m_1, m_2, ..., m_n$. Moreover, let $D = \{D_1, D_2, ..., D_k\}$ be a finite fixed group of decision-makers, whose weights are $\omega = (\omega_1, \omega_2, ..., \omega_k)$.

Step 1: In this step, we present expert information in the form of a matrix.

Step 2: Creating a single decision matrix from multiple decision matrices involves combining the information from each matrix.

Step 3: To find preference values, such as $p_i(i = 1, 2, ..., m)$.

Step 4: To find the score functions of all preference values.

Step 5: Ranking of all alternative according to the score functions, and choose that alternative having the highest score value.

6 Illustrative Example

Case study: Suppose Shaheed Benazir Bhutto University, wants to buy a car for Pro-Vice Chancellor. To choose the more suitable car, the university makes a commute of three experts, whose weighted vector is $w = (0.3, 0.3, 0.4)$ and $q = 4$. In the first selection, experts select only four different cars of different companies.

N_1: Honda Accord, N_2: Mercedes Benz S Class, N_3: BMW 7 Series, N_4: Lexus LS. The experts take decision about the above four short listed cars according to the following four criteria, such as H_1: Price, H_2: Safety Features, H_3: Performance, H_4: Style and Design, whose weighted vector is $m - (0.3, 0.3, 0.2, 0.2)$ and $q = 4$.

Step 1: Putting all data of the experts in the form of matrices form based on inducing variable (Tables 1–4).

Step 2: Convert cost type's criteria to benefit type's criteria; means make normalized decision making matrices (Tables 5–8).

Step 3: By using the I-CPoFOWAA operator, to make a single matrix, where the weighted vector is $w = (0.2, 0.3, 0.3, 0.2)$ and $q = 4$, we have Table 9.

Table 1. Decision of D_1

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.8, \left(0.65e^{i2\pi(0.45)}, 0.87e^{i2\pi(0.64)}, 0.78e^{i2\pi(0.73)} \right) \right\rangle$	$\left\langle 0.5, \left(0.89e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.90)}, 0.67e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.6, \left(0.87e^{i2\pi(0.70)}, 0.67e^{i2\pi(0.40)}, 0.58e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.4, \left(0.89e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.90)}, 0.67e^{i2\pi(0.40)} \right) \right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.5, \left(0.68e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.40)}, 0.86e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.4, \left(0.58e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)}, 0.67e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.3, \left(0.57e^{i2\pi(0.80)}, 0.48e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.2, \left(0.78e^{i2\pi(0.40)}, 0.56e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.60)} \right) \right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.6, \left(0.88e^{i2\pi(0.60)}, 0.67e^{i2\pi(0.30)}, 0.78e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.5, \left(0.89e^{i2\pi(0.50)}, 0.67e^{i2\pi(0.60)}, 0.57e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.4, \left(0.58e^{i2\pi(0.72)}, 0.47e^{i2\pi(0.63)}, 0.79e^{i2\pi(0.44)} \right) \right\rangle$	$\left\langle 0.3, \left(0.58e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.90)} \right) \right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.6, \left(0.78e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.46)}, 0.88e^{i2\pi(0.71)} \right) \right\rangle$	$\left\langle 0.5, \left(0.78e^{i2\pi(0.70)}, 0.67e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.3, \left(0.57e^{i2\pi(0.80)}, 0.48e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.8, \left(0.58e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.70)}, 0.67e^{i2\pi(0.50)} \right) \right\rangle$

Table 2. Decision of D_2

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.5, \left(0.58e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.50)}, 0.79e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.4, \left(0.48e^{i2\pi(0.70)}, 0.89e^{i2\pi(0.60)}, 0.78e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.3, \left(0.78e^{i2\pi(0.40)}, 0.67e^{i2\pi(0.70)}, 0.56e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.2, \left(0.78e^{i2\pi(0.60)}, 0.86e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.6, \left(0.59e^{i2\pi(0.80)}, 0.81e^{i2\pi(0.70)}, 0.79e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.5, \left(0.47e^{i2\pi(0.90)}, 0.86e^{i2\pi(0.50)}, 0.69e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.4, \left(0.78e^{i2\pi(0.60)}, 0.86e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.3, \left(0.62e^{i2\pi(0.80)}, 0.69e^{i2\pi(0.50)}, 0.72e^{i2\pi(0.60)} \right) \right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.7, \left(0.59e^{i2\pi(0.30)}, 0.87e^{i2\pi(0.60)}, 0.78e^{i2\pi(0.70)} \right) \right\rangle$	$\left\langle 0.6, \left(0.68e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.80)}, 0.48e^{i2\pi(0.80)} \right) \right.$	$\left\langle 0.5, \left(0.78e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.50)}, 0.56e^{i2\pi(0.90)} \right) \right\rangle$	$\left\langle 0.4, \left(0.39e^{i2\pi(0.80)}, 0.58e^{i2\pi(0.90)}, 0.79e^{i2\pi(0.40)} \right) \right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.8, \left(0.78e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.53)}, 0.92e^{i2\pi(0.81)} \right) \right\rangle$	$\left\langle 0.7, \left(0.69e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.40)}, 0.78e^{i2\pi(0.90)} \right) \right\rangle$	$\left\langle 0.8, \left(0.78e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.53)}, 0.92e^{i2\pi(0.81)} \right) \right\rangle$	$\left\langle 0.5, \left(0.68e^{i2\pi(0.65)}, 0.32e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.62)} \right) \right\rangle$

Table 3. Decision of D_3

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.9, \left(0.68e^{i2\pi(0.80)}, 0.69e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.7, \left(0.57e^{i2\pi(0.80)}, 0.39e^{i2\pi(0.70)}, 0.52e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.6, \left(0.48e^{i2\pi(0.70)}, 0.87e^{i2\pi(0.40)}, 0.79e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.2, \left(0.87e^{i2\pi(0.60)}, 0.82e^{i2\pi(0.70)}, 0.89e^{i2\pi(0.50)}\right)\right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.3, \left(0.68e^{i2\pi(0.80)}, 0.47e^{i2\pi(0.70)}, 0.69e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.2, \left(0.62e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.90)}, 0.72e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.4, \left(0.67e^{i2\pi(0.80)}, 0.51e^{i2\pi(0.60)}, 0.89e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.1, \left(0.76e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.70)}, 0.57e^{i2\pi(0.70)}\right)\right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.5, \left(0.79e^{i2\pi(0.50)}, 0.38e^{i2\pi(0.90)}, 0.78e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.3, \left(0.59e^{i2\pi(0.80)}, 0.73e^{i2\pi(0.60)}, 0.49e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.2, \left(0.78e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.70)}, 0.58e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.1, \left(0.89e^{i2\pi(0.70)}, 0.61e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)}\right)\right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.7, \left(0.47e^{i2\pi(0.90)}, 0.68e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.6, \left(0.58e^{i2\pi(0.70)}, 0.39e^{i2\pi(0.50)}, 0.92e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.4, \left(0.89e^{i2\pi(0.50)}, 0.79e^{i2\pi(0.90)}, 0.38e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.2, \left(0.79e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.50)}, 0.67e^{i2\pi(0.40)}\right)\right\rangle$

Table 4. Decision of D_4

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.8, \left(0.89e^{i2\pi(0.50)}, 0.62e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.7, \left(0.78e^{i2\pi(0.60)}, 0.69e^{i2\pi(0.40)}, 0.59e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.5, \left(0.68e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)}, 0.92e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.3, \left(0.78e^{i2\pi(0.40)}, 0.69e^{i2\pi(0.70)}, 0.77e^{i2\pi(0.90)}\right)\right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.5, \left(0.78e^{i2\pi(0.60)}, 0.69e^{i2\pi(0.50)}, 0.58e^{i2\pi(0.70)}\right)\right\rangle$	$\left\langle 0.4, \left(0.47e^{i2\pi(0.60)}, 0.89e^{i2\pi(0.70)}, 0.48e^{i2\pi(0.70)}\right)\right\rangle$	$\left\langle 0.3, \left(0.89e^{i2\pi(0.70)}, 0.57e^{i2\pi(0.50)}, 0.49e^{i2\pi(0.90)}\right)\right\rangle$	$\left\langle 0.2, \left(0.76e^{i2\pi(0.60)}, 0.51e^{i2\pi(0.70)}, 0.59e^{i2\pi(0.70)}\right)\right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.6, \left(0.67e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.40)}, 0.48e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.3, \left(0.79e^{i2\pi(0.60)}, 0.61e^{i2\pi(0.70)}, 0.78e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.2, \left(0.68e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.50)}, 0.86e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.1, \left(0.78e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.60)}, 0.37e^{i2\pi(0.80)}\right)\right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.8, \left(0.87e^{i2\pi(0.50)}, 0.56e^{i2\pi(0.60)}, 0.79e^{i2\pi(0.90)}\right)\right\rangle$	$\left\langle 0.6, \left(0.67e^{i2\pi(0.60)}, 0.49e^{i2\pi(0.40)}, 0.78e^{i2\pi(0.90)}\right)\right\rangle$	$\left\langle 0.4, \left(0.68e^{i2\pi(0.60)}, 0.57e^{i2\pi(0.50)}, 0.79e^{i2\pi(0.40)}\right)\right\rangle$	$\left\langle 0.2, \left(0.89e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.50)}\right)\right\rangle$

Step 4: Next, again applying the I-CPoFOWAA operator, with $m = (0.3, 0.3, 0.2, 0.2)$ and get preference values as follows:

$$p_1 = \left(0.73e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.68)}, 0.70e^{i2\pi(0.69)}\right)$$

$$p_2 = \left(0.74e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.49)}, 0.73e^{i2\pi(0.58)}\right)$$

$$p_3 = \left(0.82e^{i2\pi(0.64)}, 0.69e^{i2\pi(0.51)}, 0.75e^{i2\pi(0.67)}\right)$$

$$p_4 = \left(0.85e^{i2\pi(0.68)}, 0.68e^{i2\pi(0.49)}, 0.80e^{i2\pi(0.79)}\right)$$

Table 5. Normalized decision of D_1

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.8, \left(0.78e^{i2\pi(0.73)}, 0.87e^{i2\pi(0.64)}, 0.65e^{i2\pi(0.45)}\right)\right\rangle$	$\left\langle 0.5, \left(0.89e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.90)}, 0.67e^{i2\pi(0.40)}\right)\right\rangle$	$\left\langle 0.6, \left(0.58e^{i2\pi(0.80)}, 0.67e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.70)}\right)\right\rangle$	$\left\langle 0.4, \left(0.89e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.90)}, 0.67e^{i2\pi(0.40)}\right)\right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.5, \left(0.86e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.40)}, 0.68e^{i2\pi(0.50)}\right)\right\rangle$	$\left\langle 0.4, \left(0.58e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)}, 0.67e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.3, \left(0.87e^{i2\pi(0.60)}, 0.48e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.2, \left(0.78e^{i2\pi(0.40)}, 0.56e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.60)}\right)\right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.6, \left(0.78e^{i2\pi(0.60)}, 0.67e^{i2\pi(0.30)}, 0.88e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.5, \left(0.89e^{i2\pi(0.50)}, 0.67e^{i2\pi(0.60)}, 0.57e^{i2\pi(0.40)}\right)\right\rangle$	$\left\langle 0.4, \left(0.79e^{i2\pi(0.44)}, 0.47e^{i2\pi(0.63)}, 0.58e^{i2\pi(0.72)}\right)\right\rangle$	$\left\langle 0.3, \left(0.58e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.90)}\right)\right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.6, \left(0.88e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.46)}, 0.78e^{i2\pi(0.56)}\right)\right\rangle$	$\left\langle 0.5, \left(0.78e^{i2\pi(0.70)}, 0.67e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.60)}\right)\right\rangle$	$\left\langle 0.3, \left(0.87e^{i2\pi(0.60)}, 0.48e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.80)}\right)\right\rangle$	$\left\langle 0.8, \left(0.58e^{i2\pi(0.40)}, 0.87e^{i2\pi(0.70)}, 0.67e^{i2\pi(0.50)}\right)\right\rangle$

Table 6. Normalized decision of D_2

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.5, \left(0.79e^{i2\pi(0.60)}, 0.68e^{i2\pi(0.50)}, 0.58e^{i2\pi(0.70)} \right) \right\rangle$	$\left\langle 0.4, \left(0.48e^{i2\pi(0.70)}, 0.89e^{i2\pi(0.60)}, 0.78e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.3, \left(0.56e^{i2\pi(0.80)}, 0.67e^{i2\pi(0.70)}, 0.78e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.2, \left(0.78e^{i2\pi(0.60)}, 0.86e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.6, \left(0.79e^{i2\pi(0.60)}, 0.81e^{i2\pi(0.70)}, 0.59e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.5, \left(0.47e^{i2\pi(0.90)}, 0.86e^{i2\pi(0.50)}, 0.69e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.4, \left(0.89e^{i2\pi(0.50)}, 0.86e^{i2\pi(0.40)}, 0.78e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.3, \left(0.62e^{i2\pi(0.80)}, 0.69e^{i2\pi(0.50)}, 0.72e^{i2\pi(0.60)} \right) \right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.7, \left(0.78e^{i2\pi(0.70)}, 0.87e^{i2\pi(0.60)}, 0.59e^{i2\pi(0.30)} \right) \right\rangle$	$\left\langle 0.6, \left(0.68e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.80)}, 0.48e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.5, \left(0.56e^{i2\pi(0.90)}, 0.87e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.4, \left(0.39e^{i2\pi(0.80)}, 0.58e^{i2\pi(0.90)}, 0.79e^{i2\pi(0.40)} \right) \right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.2, \left(0.92e^{i2\pi(0.81)}, 0.79e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.74)} \right) \right\rangle$	$\left\langle 0.7, \left(0.69e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.40)}, 0.78e^{i2\pi(0.90)} \right) \right\rangle$	$\left\langle 0.8, \left(0.92e^{i2\pi(0.81)}, 0.79e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.74)} \right) \right\rangle$	$\left\langle 0.5, \left(0.68e^{i2\pi(0.65)}, 0.32e^{i2\pi(0.73)}, 0.47e^{i2\pi(0.62)} \right) \right\rangle$

Table 7. Normalized decision of D_3

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.9, \left(0.87e^{i2\pi(0.50)}, 0.69e^{i2\pi(0.60)}, 0.68e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.7, \left(0.57e^{i2\pi(0.80)}, 0.39e^{i2\pi(0.70)}, 0.52e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.6, \left(0.79e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.40)}, 0.48e^{i2\pi(0.70)} \right) \right\rangle$	$\left\langle 0.2, \left(0.87e^{i2\pi(0.60)}, 0.82e^{i2\pi(0.70)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.3, \left(0.69e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.2, \left(0.62e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.90)}, 0.72e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.4, \left(0.89e^{i2\pi(0.60)}, 0.51e^{i2\pi(0.60)}, 0.67e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.1, \left(0.76e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.70)}, 0.57e^{i2\pi(0.70)} \right) \right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.5, \left(0.78e^{i2\pi(0.50)}, 0.38e^{i2\pi(0.90)}, 0.79e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.3, \left(0.59e^{i2\pi(0.80)}, 0.73e^{i2\pi(0.60)}, 0.49e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.2, \left(0.58e^{i2\pi(0.80)}, 0.89e^{i2\pi(0.70)}, 0.78e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.1, \left(0.89e^{i2\pi(0.70)}, 0.61e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)} \right) \right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.7, \left(0.87e^{i2\pi(0.80)}, 0.68e^{i2\pi(0.60)}, 0.47e^{i2\pi(0.90)} \right) \right\rangle$	$\left\langle 0.6, \left(0.58e^{i2\pi(0.70)}, 0.39e^{i2\pi(0.50)}, 0.92e^{i2\pi(0.80)} \right) \right\rangle$	$\left\langle 0.4, \left(0.38e^{i2\pi(0.60)}, 0.79e^{i2\pi(0.90)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.2, \left(0.79e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.50)}, 0.67e^{i2\pi(0.40)} \right) \right\rangle$

Table 8. Normalized decision of D_4

	$\boldsymbol{H_1}$	$\boldsymbol{H_2}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4}$
$\boldsymbol{N_1}$	$\left\langle 0.8, \left(0.68e^{i2\pi(0.80)}, 0.62e^{i2\pi(0.70)}, 0.89e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.7, \left(0.78e^{i2\pi(0.60)}, 0.69e^{i2\pi(0.40)}, 0.59e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.5, \left(0.92e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)}, 0.68e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.3, \left(0.78e^{i2\pi(0.40)}, 0.69e^{i2\pi(0.70)}, 0.77e^{i2\pi(0.90)} \right) \right\rangle$
$\boldsymbol{N_2}$	$\left\langle 0.5, \left(0.58e^{i2\pi(0.70)}, 0.69e^{i2\pi(0.50)}, 0.78e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.4, \left(0.47e^{i2\pi(0.60)}, 0.89e^{i2\pi(0.70)}, 0.48e^{i2\pi(0.70)} \right) \right\rangle$	$\left\langle 0.3, \left(0.49e^{i2\pi(0.90)}, 0.57e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.70)} \right) \right\rangle$	$\left\langle 0.2, \left(0.76e^{i2\pi(0.60)}, 0.51e^{i2\pi(0.70)}, 0.59e^{i2\pi(0.70)} \right) \right\rangle$
$\boldsymbol{N_3}$	$\left\langle 0.6, \left(0.48e^{i2\pi(0.80)}, 0.89e^{i2\pi(0.40)}, 0.67e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.3, \left(0.79e^{i2\pi(0.60)}, 0.61e^{i2\pi(0.70)}, 0.78e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.2, \left(0.86e^{i2\pi(0.60)}, 0.57e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.40)} \right) \right\rangle$	$\left\langle 0.1, \left(0.78e^{i2\pi(0.40)}, 0.89e^{i2\pi(0.60)}, 0.37e^{i2\pi(0.80)} \right) \right\rangle$
$\boldsymbol{N_4}$	$\left\langle 0.8, \left(0.79e^{i2\pi(0.90)}, 0.56e^{i2\pi(0.60)}, 0.87e^{i2\pi(0.50)} \right) \right\rangle$	$\left\langle 0.6, \left(0.67e^{i2\pi(0.60)}, 0.49e^{i2\pi(0.40)}, 0.78e^{i2\pi(0.90)} \right) \right\rangle$	$\left\langle 0.4, \left(0.79e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.50)}, 0.68e^{i2\pi(0.60)} \right) \right\rangle$	$\left\langle 0.2, \left(0.89e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.40)}, 0.57e^{i2\pi(0.50)} \right) \right\rangle$

Step 5: Next, calculate the score function of the above preference values as:

$$Sc(p_1) = 0.81,$$

$$Sc(p_2) = 0.83,$$

$$Sc(p_3) = 0.79,$$

$$Sc(p_4) = 0.73.$$

Step 6: Thus, the more suitable car is Mercedes Benz S Class. Score function of all methods is as shown in Table 10.

Table 9. Combine decision of all experts

	H_1	H_2	H_3	H_4
N_1	$\begin{pmatrix} 0.71e^{i2\pi(0.57)}, \\ 0.72e^{i2\pi(0.60)}, \\ 0.85e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.69e^{i2\pi(0.70)}, \\ 0.68e^{i2\pi(0.40)}, \\ 0.68e^{i2\pi(0.55)} \end{pmatrix}$	$\begin{pmatrix} 0.66e^{i2\pi(0.68)}, \\ 0.80e^{i2\pi(0.46)}, \\ 0.78e^{i2\pi(0.69)} \end{pmatrix}$	$\begin{pmatrix} 0.80e^{i2\pi(0.52)}, \\ 0.66e^{i2\pi(0.74)}, \\ 0.86e^{i2\pi(0.70)} \end{pmatrix}$
N_2	$\begin{pmatrix} 0.72e^{i2\pi(0.64)}, \\ 0.57e^{i2\pi(0.56)}, \\ 0.79e^{i2\pi(0.61)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.55)}, \\ 0.69e^{i2\pi(0.61)}, \\ 0.79e^{i2\pi(0.71)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.69)}, \\ 0.56e^{i2\pi(0.50)}, \\ 0.89e^{i2\pi(0.60)} \end{pmatrix}$	$\begin{pmatrix} 0.84e^{i2\pi(0.44)}, \\ 0.66e^{i2\pi(0.57)}, \\ 0.80e^{i2\pi(0.63)} \end{pmatrix}$
N_3	$\begin{pmatrix} 0.93e^{i2\pi(0.62)}, \\ 0.71e^{i2\pi(0.35)}, \\ 0.87e^{i2\pi(0.59)} \end{pmatrix}$	$\begin{pmatrix} 0.95e^{i2\pi(0.57)}, \\ 0.71e^{i2\pi(0.66)}, \\ 0.69e^{i2\pi(0.46)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.75)}, \\ 0.59e^{i2\pi(0.64)}, \\ 0.86e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.74)}, \\ 0.75e^{i2\pi(0.53)}, \\ 0.86e^{i2\pi(0.92)} \end{pmatrix}$
N_4	$\begin{pmatrix} 0.88e^{i2\pi(0.73)}, \\ 0.78e^{i2\pi(0.45)}, \\ 0.93e^{i2\pi(0.65)} \end{pmatrix}$	$\begin{pmatrix} 0.89e^{i2\pi(0.77)}, \\ 0.75e^{i2\pi(0.55)}, \\ 0.96e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.89e^{i2\pi(0.54)}, \\ 0.75e^{i2\pi(0.68)}, \\ 0.96e^{i2\pi(0.45)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.43)}, \\ 0.95e^{i2\pi(0.72)}, \\ 0.73e^{i2\pi(0.56)} \end{pmatrix}$

Table 10. Scores of the novel methods

	I-CPoFOWAA	I-CPoFHAA
Honda Accord	0.81	0.75
Mercedes Benz S Class	0.83	0.87
BMW 7 Series	0.79	0.72
Lexus LS	0.73	0.69

7 Comparative Sensitivity Analysis

In this part, we compared our proposed model with some existing models. The proposed model may be used for Polytopic fuzzy data by taking the phase terms 0. Moreover, it can be used for q-rung orthopair fuzzy data by setting their neutral and phase terms zero. Thus, our model is more elastic as compared to their existing models, see Table 11.

Table 11. Sensitivity analysis

Sets	Uncertainty	Falsity	Indeterminacy	Periodicity	2-D Information	qth-Power
FS	Yes	No	No	No	No	No
IFS	Yes	Yes	Yes	No	No	No
PyFS	Yes	Yes	Yes	No	No	No
FFS	Yes	Yes	Yes	No	No	No
CFS	Yes	No	No	Yes	Yes	No
CIFS	Yes	Yes	Yes	Yes	Yes	No
CPyFS	Yes	Yes	Yes	Yes	Yes	No
CPoFS	Yes	Yes	Yes	Yes	Yes	Yes

8 Conclusions

In this research, we have developed complex Polytopic fuzzy set and their aggregation operators under inducing variable. A complex Polytopic fuzzy set is a concept used in fuzzy set theory, a mathematical framework for dealing with uncertainty and vagueness. We also introduced I-CPoFOWAA operator and I-CPoFHAA operator. These operators play a crucial role in aggregating fuzzy values or fuzzy relations to derive a comprehensive result that reflects the collective information from different sources or criteria. Finally, we have constructed an example to show the effectiveness and efficiency of the proposed techniques.

Moreover, this research can be extendable to complex Fermatean fuzzy set, complex Einstein operators, complex Logarithmic operators, complex Dombi operators, complex Power operators, complex interval-valued Power operators etc.

Data Availability

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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