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Advanced Logarithmic Aggregation Operators for Enhanced Decision-Making in Uncertain Environments

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Abstract: This study introduces logarithmic operations tailored to intuitionistic fuzzy sets (IFSs) aimed at mitigating uncertainty in decision-making processes. Through logarithmic transformations, the membership and non-membership degrees are effectively scaled, thereby enhancing interpretability and facilitating the assessment of uncertainty. Advanced logarithmic aggregation operators have been developed, specifically the Induced Confidence Logarithmic Intuitionistic Fuzzy Einstein Ordered Weighted Geometric Aggregation (ICLIFEOWGA) operator and the Induced Confidence Logarithmic Intuitionistic Fuzzy Einstein Hybrid Geometric Aggregation (ICLIFEHGA) operator. These operators serve as versatile tools, providing robust frameworks for integrating diverse information sources in decision-making and assessment processes. The versatility of the operators is demonstrated through their application across various industries and domains, where they support the integration of multiple criteria in complex decision-making scenarios. An algorithm for the decision-making process is presented, and the effectiveness and efficiency of the proposed techniques are illustrated through a case study on laptop selection.

Keywords: Logarithmic aggregation operator; Intuitionistic Fuzzy Set (IFS); Uncertainty Management; Multi-Attribute Decision-Making (MADM) process

1 Introduction

Decision-making plays a pivotal role across various domains, spanning from medicine to business management, impacting critical sectors like computer science, automotive industries, and robotics. It serves as a logical and comprehensive approach to selecting the most suitable option from a range of possibilities, enhancing the dependability and efficiency of emergency response efforts, and reducing casualties, ecological damage, and financial losses. Traditionally, decision-making was seen as a process where all data related to criteria and their weights were clear numerical values. However, in reality, decisions often occur in environments where goals and constraints are unclear or ambiguous. This ambiguity underscores the importance of decision-making frameworks that can handle uncertainty, vagueness, and imprecision common in real-world scenarios. These frameworks provide valuable resources for decision-makers facing intricate and unclear situations, offering a structured approach to evaluating alternatives and selecting the optimal course of action. By incorporating multiple criteria, decision-making processes become more robust and adaptable, enabling organizations and individuals to navigate complex challenges effectively. In essence, decision-making is not just about choosing between options; it's about strategically managing uncertainty and ambiguity to achieve desired outcomes across diverse fields and industries.

Zadeh's fuzzy set [1], a pivotal concept in fuzzy logic and mathematics, extends the conventional idea of a classical set. This theory provides a robust framework for addressing uncertainty and vagueness in a mathematical manner. Its notable contribution lies in its capacity to handle real-world problems across various domains by offering a more adaptable and nuanced approach to representing and reasoning with imprecise data and concepts. Unlike classical set theory, fuzzy set theory introduces the notion of partial membership, allowing elements to belong to a set to varying degrees. This flexibility proves invaluable when dealing with ambiguous or unclear information. Fuzzy sets have diverse applications in decision-making across fields such as computer science, finance, management science, investment decisions, business, natural science, healthcare, robotics, automotive industries, social science, human resources, medicine industries, engineering science, manufacturing and operations, information technology, public policy, environmental management, and more. The inclusion of membership degrees facilitates a more comprehensive understanding of the degrees of belongingness to a set, thereby enhancing its relevance across a multitude of disciplines. However, challenges may arise in cases involving both unsatisfactory and satisfactory information simultaneously.

Atanassov [2] significantly expanded the concept of fuzzy sets by introducing IFSs, which incorporate a non-membership function alongside the traditional membership function. This innovation, governed by the constraint that their sum should not exceed one, was designed to address uncertainties characterized by vagueness, doubt, and ambiguity. IFSs have proven versatile, finding applications across diverse domains such as expert systems, management, computer science, pattern recognition, decision-making, and risk assessment. Particularly valuable in scenarios demanding decisions amid uncertainty or ambiguity, IFSs offer a nuanced framework for navigating real-world situations marked by incomplete or hesitant information. The inclusion of the non-membership function in IFSs enriches the representation of complex, uncertain, and incomplete data, facilitating more effective information processing across various applications. This approach enables a comprehensive understanding and adept management of situations where traditional models might struggle to capture the intricate nature of uncertain and incomplete data.

Aggregation operators play a crucial role in decision-making problems, especially in the context of the IF environment. Researchers such as Xu and Yager [3], Yager [4], and Xu [5] have introduced various operators using IFNs. These operators are designed to consolidate and combine diverse pieces of information, facilitating effective decision-making in complex scenarios. The applications and advantages of these operators have been explained, showcasing their relevance in addressing real-world problems. Yager and Kacprzyk [6] have dedicated significant efforts to understanding the importance of operators tailored for IFNs, contributing to the development of fundamental roles. Zhao and Wei [7] and Wang and Liu [8, 9] have introduced innovative methodologies by incorporating Einstein's operational principles. Through their research, they have not only developed new aggregation operators, but also thoroughly examined their structural properties and applications. These inherent structural characteristics play a pivotal role in enhancing the efficiency of these operators within computational processes. The versatility of these operators is evident as they find applications across diverse fields, showcasing their adaptability and utility in addressing a wide range of challenges. Researchers such as Garg [10, 11], Xu et al. [12], Dahlman et al. [13], Yu and She [14], Kumar and Garg [15], and Garg [16] have significantly advanced the field by introducing pioneering concepts and innovative approaches in their respective studies. Their work has played a crucial role in broadening our comprehension of IF operators and their practical applications. Rahman et al. [17], Gou et al. [18], Nancy and Garg [19], Jamil et al. [20], Garg and Nancy [21], and Garg et al. [22] have significantly contributed by introducing versatile and broadly applicable approaches that encompass IFSs and IVIFSs. Their work involves the presentation of diverse and generalized methodologies, highlighting the valuable insights and advancements they have brought to the field. Induced aggregation operators are mathematical functions crucial for combining multiple inputs into a unified output based on specific rules. Their purpose is to capture the aggregation process, reflecting relationships and interactions among input elements. These operators play a pivotal role in decision-making, fuzzy logic, and data analysis, serving as essential tools for amalgamating and summarizing information from diverse sources. Their significance lies in their ability to facilitate the integration and synthesis of data, enabling more informed and nuanced outcomes in fields reliant on intricate information processing. Logarithmic aggregation operators, a subset of induced aggregation operators, are particularly important in decision-making processes. These operators use logarithmic functions to analyze and combine information from various sources or criteria. The use of logarithmic functions allows for a comprehensive consideration of a broad range of values while emphasizing the significance of key factors. This approach enhances the decision-making process by providing a balanced and nuanced perspective, taking into account the varying degrees of importance assigned to different elements. Rahman [23], Li and Wei [24], Rahman et al. [25], and Qiyas et al. [26], presented logarithmic techniques, namely LIFWAA, LIFOWAA, LIFEWAA, LIFEOWAA, LIFEHAA, LIFEWGA, LIFEOWGA, LIFEHGA, LIFWGA, LIFOWGA, CLIFEWGA, CLIFEOWGA, CLIFEHGA, CLIFEWAA, CLIFEOWAA, CLIFEHAA, LCWAA, LCOWAA, LCWGA, LCOWGA operators. These methods offer significant value in tackling certain decision-making problems. These techniques prove particularly effective in addressing specific challenges inherent to decision-making processes. Researchers consistently operate under the assumption that decision-makers possess pertinent knowledge concerning the information linked to the objects under evaluation. This foundational understanding underpins the efficacy of logarithmic aggregation approaches in navigating complex decision scenarios. Their utility becomes apparent in situations where precise knowledge plays a crucial role in arriving at informed decisions. Nevertheless, in practical scenarios, this assumption may not be universally applicable. Yu [27, 28], Ma and Zeng [29], as well as pioneered the concept of a confidence level and subsequently developed various methodologies grounded in this notion. They introduced innovative techniques that leverage confidence levels, contributing to the advancement of this approach in their respective works. These methods were carefully crafted to handle challenges that arise when decision-makers lack expertise in a particular domain. The objective is to present a systematic approach that acknowledges and manages uncertainties, promoting more informed and resilient decision-making. The strategies are particularly tailored for situations where decision-makers may not possess expert-level knowledge, providing a structured framework that enhances adaptability and effectiveness when dealing with intricate problems. Wei [30, 31], Su et al. [32], and Xu et al. [33] introduced various induced aggregation operators, such as the I-IFOWA, I-IFOWG, I-IFHA, I-IFHG, I-IFEOWA operators, respectively. These operators are designed to aggregate data in various ways, offering diverse approaches to combining information for analysis or decision-making purposes. Induced aggregation operators are specifically crafted to combine data in diverse ways, allowing for nuanced and comprehensive analysis. Each operator offers a distinct approach to data aggregation, catering to specific needs and objectives in various contexts.

1.1 Motivation

This paper draws inspiration from three separate ideas introduced in previous studies: the logarithmic concept [23–26], the confidence concept [29], and the induced concept [30–33]. Despite their relevance and potential synergies, prior research has, surprisingly, not attempted to integrate these three concepts. Our research focuses on merging three key concepts: the logarithmic concept, the confidence concept, and the induced concept. By integrating logarithmic aggregation operators within an inducing variable under a confidence level framework, we seek to capitalize on the strengths of all approaches, potentially offering fresh insights and better outcomes across various applications. The concept of an inducing variable is typically associated with decision-making scenarios involving uncertainty or when an additional influential factor needs consideration in the aggregation process. Our study stands out for its unique contribution to advancing existing methodologies by refining and expanding upon established models. Through this, we anticipate achieving enhanced performance and broader applicability in practical settings.

1.2 Contribution

i) *Induced Logarithmic Aggregation Operators under Confidence Level*: The paper introduces the I-CLIFEOWGA and I-CLIFEHGA operators, crafted with unique structural features for enhanced flexibility and effectiveness. These operators showcase innovation in design, aimed at improving overall performance. Their distinct qualities underscore their potential impact across diverse applications, highlighting their tailored characteristics for specific needs.

ii) *Development of an Algorithm for Decision-Making Process*: This paper introduces a tailored algorithm for the innovative IFS-based model, streamlining its application in real-world decision-making scenarios. Its structured framework enhances the usability of newly proposed operators, facilitating their practical implementation.

iii) *Illustrative Example to Prove the Reliability of the Proposed Techniques*: The paper substantiates the visibility and trustworthiness of newly introduced operators by presenting their practical utility and consistency. Utilizing real-world scenarios extensively, it not only affirms the efficacy of these operators but also offers valuable practical applications. This methodology not only validates the proposed operators but also emphasizes their significance in decision-making contexts.

1.3 Organization

The forthcoming paper is structured meticulously, comprising several sections that delve deeply into the intricacies of logarithmic operations and aggregation operators.

In Section 2, we lay down fundamental definitions that will underpin our subsequent investigations. This section serves as the groundwork upon which the rest of our research builds.

Moving on to Section 3, our focus shifts to elucidating the laws governing logarithmic operations. Here, we aim to provide a clear and concise presentation of these laws for a better comprehension of their application. Section 4 introduces two operators, namely I-CLIFEOWGA and I-CLIFEHGA, wherein we conduct a meticulous analysis of their essential properties like idempotency, boundedness, and monotonicity. This thorough examination helps in understanding the behavior and characteristics of these operators. In Section 5, we present a novel emergency decision-making model, offering a fresh perspective on this critical subject matter. Our intent here is to shed light on a unique aspect of decision-making, particularly in situations of utmost importance. Section 6 is dedicated to demonstrating the practical utility of our proposed methods. Through an illustrative example, we employ various techniques to showcase how our approaches can be applied effectively in real-world scenarios. Following this, in Section 7, we conduct a comparative and sensitive analysis. This allows us to evaluate the efficacy and efficiency of our methods in comparison to existing approaches, thereby providing a deeper understanding of their strengths and limitations. In Section 8, the proposed techniques were subjected to experimental validation, where their effectiveness was rigorously assessed. Following this, in Section 9, a conclusive summary was provided, encapsulating the key findings and contributions of the research. This section serves to consolidate the outcomes of the study, offering a comprehensive understanding of the advancements made in the field.

2 Preliminaries

In our research, we devised induced logarithmic operators that incorporate a confidence level. An inducing variable is a factor that directly impacts or triggers a particular outcome or phenomenon, eliciting a specific response within a system or process. The confidence level denotes the extent of certainty or trust in the accuracy of a result or decision, serving as a metric of reliability. Typically expressed as a percentage, it signifies the likelihood that the actual value falls within a specified range or interval, thereby indicating the level of confidence associated with the outcome.

Definition 1 [24]: Let $I = \{\langle q, \gamma_I(q), \chi_I(q)\rangle \mid q \in \Re\}$ is an IFS, $\Re$ is a universal set, and then $\log_r I = \{\langle q, 1-\log_r \gamma_I(q), \log_r(1-\chi_I(q))\rangle \mid q \in \Re\}$ with $r \neq 1$ and $0 < \mathrm{r} \leq \gamma \leq 1$. Membership and non-membership are denoted by $1-\log_r \gamma_I(q) : \Re \to [0,1], \forall q \in \Re \to 1-\log_r \gamma_I(q) \in [0,1]$ and $1-\log_r \chi_I(q) : \Re \to [0,1], \forall q \in \Re \to 1-\log_r \chi_I(q) \in [0,1]$ respectively, under conditions: $0\mathrm{p}1 - \log_r \chi_I(q) + \log_r(1-\chi_I(q)) \leq 1$.

When comparing two or more methods in research or data analysis, using score functions and accuracy functions can be valuable metrics. A score function, often referred to as a performance metric or evaluation measure, is a quantitative measure used to assess the quality or effectiveness of a model, algorithm, or method in various tasks, such as machine learning, optimization, or data analysis. The choice of a specific score function depends on the nature of the problem and the goals of the analysis. Accuracy is a specific type of score function that measures the correctness of predictions made by a model. In classification tasks, accuracy is often used and represents the ratio of correct predictions to the total number of predictions. It is a straightforward measure of overall model performance.

Definition 2 [3]: Let $\mathrm{h} = (\gamma, \chi)$, then its score and accuracy can be computed mathematically as: $s(\mathrm{h}) = \gamma - \chi$ and $h(\mathrm{h}) = \gamma + \chi$ respectively.

Definition 3 [3]: Let h_1 and h_2 be IFNs, then

i) If $s(\mathrm{h}_1) < s(\mathrm{h}_2)$, this show that $\mathrm{h}_1 < \mathrm{h}_2$

ii) If $s(\mathrm{h}_2) < s(\mathrm{h}_1)$, this show that $\mathrm{h}_2 < \mathrm{h}_1$

iii) If $s(\mathrm{h}_1) = s(\mathrm{h}_2)$, this show that $\mathrm{h}_1 = \mathrm{h}_2$,

then we have some more conditions:

a) If $h(\mathrm{h}_1) < h(\mathrm{h}_2)$, this show that $\mathrm{h}_1 < \mathrm{h}_2$

b) If $h(\mathrm{h}_2) < h(\mathrm{h}_1)$, this show that $\mathrm{h}_2 < \mathrm{h}_1$

c) If $h(\mathrm{h}_1) = h(\mathrm{h}_2)$, this show that $\mathrm{h}_1 = \mathrm{h}_2$

3 Logarithmic Operational Laws

Operational laws, particularly in the realm of mathematical operations and algebraic structures, are principles that dictate how specific operations behave. This paper focuses on logarithmic operational laws, which are mathematical rules governing the manipulation and simplification of logarithmic expressions. Logarithms, in this context, are mathematical functions depicting the exponent to which a certain base must be raised to produce a specified number. The developed laws in this study aim to elucidate the systematic procedures for handling and streamlining expressions involving logarithms.

Definition 4 [24]: Let $\mathrm{h}_j = (\gamma_j, \chi_j)\ (1 \leq j \leq 2)$ and a real number $\wp > 0$, then

i) $\mathrm{h}_1 + \mathrm{h}_2 = \left(\frac{\gamma_1+\gamma_2}{1+\gamma_1\gamma_2}, \frac{\chi_1\chi_2}{1+(1-\chi_1)(1-\chi_2)}\right)$

ii) $\mathrm{h}_1 \times \mathrm{h}_2 = \left(\frac{\gamma_1\gamma_2}{1+(1-\gamma_1)(1-\gamma_2)}, \frac{\chi_1+\chi_2}{1+\chi_1\chi_2}\right)$

iii) $\wp(\mathrm{h}) = \left(\frac{(1+\gamma)^{\wp}-(1-\gamma)^{\wp}}{(1+\gamma)^{\wp}+(1-\gamma)^{\wp}}, \frac{2\chi^{\wp}}{(2-\chi)^{\wp}+\chi^{\wp}}\right)$

iv) $(\mathrm{h})^{\wp} = \left(\frac{2\gamma^{\wp}}{(2-\gamma)^{\wp}+\gamma^{\wp}}, \frac{(1+\chi)^{\wp}-(1-\chi)^{\wp}}{(1+\chi)^{\S}+(1-\chi)^{\wp}}\right)$

v) $\mathrm{h}_1 \cup \mathrm{h}_2 = (\max\{\gamma_1, \gamma_2\}, \min\{\chi_1, \chi_2\})$

vi) $\mathrm{h}_1 \cap \mathrm{h}_2 = (\min\{\gamma_1, \gamma_2\}, \max\{\chi_1, \chi_2\})$

vii) $(\mathrm{h})^c = (\chi, \gamma)$

Theorem 1: Let $\mathrm{h}_j = (\gamma_j, \chi_j)\ (1 \leq j \leq 3)$ with $r_j \neq 1$ and $0\mathrm{p}r_j \leq \min\{\gamma_j, (1-\chi_j)\} \leq 1$, then

i) $\log_r \mathrm{h}_1 + \log_r \mathrm{h}_2 = \log_r \mathrm{h}_2 + \log_r \mathrm{h}_1$

ii) $\log_r \mathrm{h}_1 \times \log_r \mathrm{h}_2 = \log_r \mathrm{h}_2 \times \log_r \mathrm{h}_1$

iii) $(\log_r \mathrm{h}_1 + \log_r \mathrm{h}_2) + \log_r \mathrm{h}_3 = \log_r \mathrm{h}_1 + (\log_r \mathrm{h}_2 + \log_r \mathrm{h}_3)$

iv) $(\log_r \mathrm{h}_1 \times \log_r \mathrm{h}_2) \times \log_r \mathrm{h}_3 = \log_r \mathrm{h}_1 \times (\log_r \mathrm{h}_2 \times \log_r \mathrm{h}_3)$

Proof: This theorem comprises four components, but our focus will be on demonstrating the validity of parts (i, ii). The remaining portions of the theorem can be straightforwardly proven employing a similar approach.

i) Since, by using Definition 4, we have

$$
\begin{aligned}
&\log_r \mathrm{h}_1 + \log_r \mathrm{h}_2 \\
&= ((1-\log_r \gamma_1, \log_r (1-\chi_1)) + (1-\log_r \gamma_2, \log_r (1-\chi_2))) \\
&= \left(\frac{(1-\log_r \gamma_1)+(1-\log_r \gamma_2)}{1+(1-\log_r \gamma_1)(1-\log_r \gamma_2)}, \frac{\log_r (1-\chi_1)\log_r (1-\chi_2)}{1+(1-\log_r (1-\chi_1))(1-\log_r (1-\chi_2))}\right) \\
&= \left(\frac{1-\log_r \gamma_2+1-\log_r \gamma_1}{1+(1-\log_r \gamma_2)(1-\log_r \gamma_1)}, \frac{\log_r (1-\chi_2)\log_r (1-\chi_1)}{1+(1-\log_r (1-\chi_2))(1-\log_r (1-\chi_1))}\right) \\
&= \log_r \mathrm{h}_2 + \log_r \mathrm{h}_1
\end{aligned}
$$

ii) Again, by using Definition 4, we have

$$
\begin{aligned}
&\log_r \mathrm{h}_1 \times \log_r \mathrm{h}_2 \\
&= ((1-\log_r \gamma_1, \log_r (1-\chi_1)) \otimes (1-\log_r \gamma_2, \log_r (1-\chi_2))) \\
&= \left(\frac{\log_r (1-\gamma_1)\log_r (1-\gamma_2)}{1+(1-\log_r (1-\gamma_1))(1-\log_r (1-\gamma_2))}, \frac{(1-\log_r \chi_1)+(1-\log_r \chi_2)}{1+(1-\log_r \chi_1)(1-\log_r \chi_2)}\right) \\
&= \left(\frac{\log_r (1-\gamma_2)\log_r (1-\gamma_1)}{1+(1-\log_r (1-\gamma_2))(1-\log_r (1-\gamma_1))}, \frac{1-\log_r \chi_2+1-\log_r \chi_1}{1+(1-\log_r \chi_2)(1-\log_r \chi_1)}\right) \\
&= \log_r \mathrm{h}_2 \times \log_r \mathrm{h}_1
\end{aligned}
$$

Thus, the proof is completed.

Theorem 2: Let $\mathrm{h}_j = (\gamma_j, \chi_j)\,(1 \le j \le 3)$ with $r_j \ne 1$ and $0 < r_j \le \min\{\gamma_j, (1-\chi_j)\} \le 1$, then

i) $\log_r (\mathrm{h}_1 \cup \mathrm{h}_2) \cap \log_r \mathrm{h}_2 = \log_r \mathrm{h}_2$

ii) $\log_r (\mathrm{h}_1 \cap \mathrm{h}_2) \cup \log_r \mathrm{h}_2 = \log_r \mathrm{h}_2$

iii) $\log_r (\mathrm{h}_1 \cup \mathrm{h}_2) \cap \log_r \mathrm{h}_3 = \log_r (\mathrm{h}_1 \cap \mathrm{h}_3) \cup \log_r (\mathrm{h}_2 \cap \mathrm{h}_3)$

iv) $\log_r (\mathrm{h}_1 \cap \mathrm{h}_2) \cup \log_r \mathrm{h}_3 = \log_r (\mathrm{h}_1 \cup \mathrm{h}_3) \cap \log_r (\mathrm{h}_2 \cup \mathrm{h}_3)$

Proof: This theorem consists of ten parts, but our attention will be directed towards proving the accuracy of parts (i, ii). The other aspects of the theorem can be easily demonstrated using a similar method. Therefore, applying Definition 4, we can express the following:

i) Since, by using Definition 4, we have

$$
\begin{aligned}
&\log_r (\mathrm{h}_1 \cup \mathrm{h}_2) \cap \log_r \mathrm{h}_2 \\
&= \log_r (\max\{\gamma_1, \gamma_2\}, \min\{(1-\chi_1), (1-\chi_2)\}) \cap \log_r (\gamma_2, (1-\chi_2)) \\
&= \log_r (\min\{\max\{\gamma_1, \gamma_2\}, \gamma_2\}, \max\{\min\{(1-\chi_1), (1-\chi_2)\}, (1-\chi_2)\}) \\
&= \log_r (\gamma_2, (1-\chi_2)) \\
&= \log_r \mathrm{h}_2
\end{aligned}
$$

ii) By using Definition 4, we have

$$
\begin{aligned}
&\log_r (\mathrm{h}_1 \cap \mathrm{h}_2) \cup \log_r \mathrm{h}_2 \\
&= \log_r (\min\{\gamma_1, \gamma_2\}, \max\{(1-\gamma_1), (1-\gamma_2)\}) \cup \log_r (\gamma_2, (1-\gamma_2)) \\
&= \log_r (\max\{\min\{\gamma_1, \gamma_2\}, \gamma_2\}, \min\{\max\{(1-\gamma_1), (1-\gamma_2)\}, (1-\gamma_2)\}) \\
&= \log_r (\eta_2, (1-\gamma_2)) \\
&= \log_r \mathrm{h}_2
\end{aligned}
$$

iii) Again, by using Definition 4, we have

$$
\begin{aligned}
&\log_r (\mathrm{h}_1 \cup \mathrm{h}_2) \cap \log_r \mathrm{h}_3 \\
&= \log_r (\max\{\gamma_1, \gamma_2\}, \min\{(1-\chi_1), (1-\chi_2)\}) \cap \log_r (\gamma_3, (1-\chi_3)) \\
&= \log_r (\min\{\max\{\gamma_1, \gamma_2\}, \gamma_3\}, \max\{\min\{(1-\chi_1), (1-\chi_2)\}, (1-\chi_3)\}) \\
&= \log_r (\mathrm{h}_1 \cap \mathrm{h}_3) \cup \log_r (\mathrm{h}_2 \cap \mathrm{h}_3)
\end{aligned}
$$

Thus, the proof is completed.

4 Induced Logarithmic Geometric Operators

Logarithmic aggregation operators offer several advantages in decision-making processes. Firstly, they accommodate a wide range of input values, allowing for the integration of diverse data sets with varying scales. Secondly, they effectively handle extreme values, preventing them from disproportionately influencing the final outcome. Additionally, logarithmic aggregation operators maintain the multiplicative property, ensuring consistency in their application across different contexts. Moreover, they facilitate intuitive interpretation, as the logarithmic scale aligns with human perceptions of relative differences. Lastly, these operators promote robustness by reducing the impact of uncertainties or inaccuracies in the data, leading to more reliable decision outcomes. We present two novel techniques: the I-CLIFEOWGA operator and the I-CLIFEHGA operator. Notably, the I-CLIFEHGA operator represents a more comprehensive iteration of the I-CLIFEOWGA operator, incorporating additional elements, methods, or rules beyond simple averaging. Our analysis will delve into each operator, providing detailed insights along with illustrative examples to elucidate their structural nuances. We will specifically explore fundamental characteristics like monotonicity, idempotency, and boundedness inherent in these operators. These traits are pivotal in the decision-making process, offering a foundational understanding of how these aggregation methods contribute to the overall decision-making framework.

Definition 5: Let $(\ell_j, \langle \mathrm{h}_j, \phi_j \rangle)\,(j \leq n)$ be a finite family of IFNs, with their corresponding weights $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ with certain limitations, such as their sum is less than one or equal to one; let $\mathrm{h}_j \in \langle \mathrm{h}_j, \phi_j \rangle$ be the ordered pair of IFEOWG where the j$^{\text{th}}$ largest value is known as the order inducing variable, with $\mathrm{h}_j \in \langle \mathrm{h}_j, \phi_j \rangle$ and $\mathrm{h}_j (1 \leq j \leq n)$ as the IF arguments. Then the I-CLIFEOWGA operator can be expressed mathematically as:

I-CLIFEOWGA$_\xi$ $(\langle \ell_1, (\mathrm{h}_1, \phi_1) \rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n) \rangle)$

$$
= \begin{cases}
\left(\dfrac{2\prod_{j=1}^{n}\left(1-\log_r \gamma_{\tau(j)}\right)^{\phi_j \xi_j}}{\prod_{j=1}^{n}\left(1+\log_r \gamma_{\tau(j)}\right)^{\phi_j \xi_j}+\prod_{j=1}^{n}\left(1-\log_r \gamma_{\tau(j)}\right)^{\phi_j \xi_j}}, \dfrac{\prod_{j=1}^{n}\left(1+\log_r (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}-\prod_{j=1}^{n}\left(1-\log_r (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}}{\prod_{j=1}^{n}\left(1+\log_r (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}+\prod_{j=1}^{n}\left(1-\log_r (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}} \right) \\
\text{where } r \neq 1 \text{ and } 0 < \frac{1}{r} \leq \min\left\{\gamma_{\tau(j)}, (1-\chi_{\tau(j)})\right\} \leq 1, \\
\left(\dfrac{2\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} \gamma_{\tau(j)}\right)^{\phi_j \xi_j}}{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}} \gamma_{\tau(j)}\right)^{\phi_j \xi_j}+\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} \gamma_{\tau(j)}\right)^{\phi_j \xi_j}}, \dfrac{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}} (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}-\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}}{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}} (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}+\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} (1-\chi_{\tau(j)})\right)^{\phi_j \xi_j}} \right) \\
\text{where } r \neq 1 \text{ and } 0 < \frac{1}{r} \leq \min\left\{\gamma_{\tau(j)}, (1-\chi_{\tau(j)})\right\} \leq 1
\end{cases}
$$

Theorem 3: Let $(\ell_j, \langle \mathrm{h}_j, \phi_j \rangle)\,(1 \leq j \leq n)$ be IFNs and confidence level $\phi_j (1 \leq j \leq n)$, then the following four properties hold:

1) Commutatively: Assuming a collection of two tuples, such as $\left(\ell_j^*, \left\langle \mathrm{h}_j^*, \phi_j^* \right\rangle\right)(1 \leq j \leq n)$, then

$$
\begin{aligned}
&\text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1, (\mathrm{h}_1, \phi_1) \rangle, \langle \ell_2, (\mathrm{h}_2, \phi_2) \rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n) \rangle\right) \\
&= \text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1^*, (\mathrm{h}_1^*, \phi_1^*) \rangle, \langle \ell_2^*, (\mathrm{h}_2^*, \phi_2^*) \rangle, \ldots, (\ell_n^*, \langle \mathrm{h}_n^*, \phi_n^* \rangle)\right)
\end{aligned} \tag{1}
$$

where, $\left(\ell_j^*, \left\langle \mathrm{h}_j^*, \phi_j^* \right\rangle\right)(1 \leq j \leq n)$ is the rearrangement and reordering of $(\ell_j, \langle \mathrm{h}_j, \phi_j \rangle)\,(1 \leq j \leq n)$

2) Idempotency: Let $(\ell_j, \langle \mathrm{h}_j, \phi_j \rangle)\,(1 \leq j \leq n) = \mathrm{h}$ with $\phi_1 = \phi_2 = \ldots = \phi_n = \phi$, then

$$
\text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1, (\mathrm{h}_1, \phi_1) \rangle, \langle \ell_2, (\mathrm{h}_2, \phi_2) \rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n) \rangle\right) = \log_{\Psi} \langle \ell, (\mathrm{h}, \phi) \rangle) \tag{2}
$$

3) Boundedness: Assuming $(\ell_j, \langle \mathrm{h}_j, \phi_j \rangle)\,(1 \leq j \leq n)$ with $\mathrm{h}_{\max} = (\max_j \{\phi_j \gamma_j\}, \min_j \{\phi_j \chi_j\})$ and $\mathrm{h}_{\min} = (\min_j \{\phi_j \gamma_j\}, \max_j \{\phi_j \chi_j\})$, then the following holds.

$$
\log_r(\mathrm{h}_{\min}) \leq \text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1, (\mathrm{h}_1, \phi_1) \rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n) \rangle\right) \leq \log_r(\mathrm{h}_{\max}) \tag{3}
$$

4) Monotonicity: If $\left\langle \ell_j^*, \left(\mathrm{h}_j^*, \phi_j^*\right) \right\rangle$ is another collection IFNs, with $\gamma_j \leq \gamma_j^*$ and $\chi_j \geq \chi_j^*$, then

$$
\begin{aligned}
&\text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1, (\mathrm{h}_1, \phi_1) \rangle, \langle \ell_2, (\mathrm{h}_2, \phi_2) \rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n) \rangle\right) \\
&\leq \text{I-CLIFEOWGA}_{\xi}\left(\langle \ell_1^*, (\mathrm{h}_1^*, \phi_1^*) \rangle, \langle \ell_2^*, (\mathrm{h}_2^*, \phi_2^*) \rangle, \ldots, \langle \ell_n^*, (\mathrm{h}_n^*, \phi_n^*) \rangle\right)
\end{aligned} \tag{4}
$$

Example 1: Assuming five values, such as: $\mathrm{h}_1 = \langle 0.20, (0.40, 0.50), 0.50 \rangle, \mathrm{h}_2 = \langle 0.40, (0.40, 0.50), 0.40 \rangle$, $\mathrm{h}_3 = \langle 0.50, (0.40, 0.40), 0.70 \rangle, \mathrm{h}_4 = \langle 0.60, (0.50, 0.30), 0.60 \rangle, \mathrm{h}_5 = \langle 0.80, (0.60, 0.20), 0.80 \rangle$ whose weights are $\xi = (0.1, 0.2, 0.2, 0.2, 0.3)$ and $r = 0.2$. Next, we need to arrange these values according to the inducing variable:

$\mathrm{h}_5 = \langle 0.80, (0.60, 0.20), 0.80 \rangle$,
$\mathrm{h}_4 = \langle 0.60, (0.50, 0.30), 0.60 \rangle$,
$\mathrm{h}_3 = \langle 0.50, (0.40, 0.40), 0.70 \rangle$,
$\mathrm{h}_2 = \langle 0.40, (0.40, 0.50), 0.40 \rangle$,
$\mathrm{h}_1 = \langle 0.20, (0.40, 0.50), 0.50 \rangle$.

Then, we have the new arrangement as:

$\mathrm{h}_{\tau(1)} = \langle 0.80, (0.60, 0.20), 0.80\rangle,$
$\mathrm{h}_{\tau(2)} = \langle 0.60, (0.50, 0.30), 0.60\rangle,$
$\mathrm{h}_{\tau(3)} = \langle 0.50, (0.40, 0.40), 0.70\rangle,$
$\mathrm{h}_{\tau(4)} = \langle 0.40, (0.40, 0.50), 0.40\rangle,$
$\mathrm{h}_{\tau(5)} = \langle 0.20, (0.40, 0.50), 0.50\rangle.$

The required values are computed as:

$$\prod_{j=1}^{5}\left(1-\log_r \gamma_{\tau(j)}\right)^{\phi_j\xi_j} = (1-\log_{0.2}(0.60))^{0.80\times 0.1}(1-\log_{0.2}(0.50))^{0.60\times 0.2}$$
$$(1-\log_{0.2}(0.40))^{0.70\times 0.2}(1-\log_{0.2}(0.40))^{0.40\times 0.2}(1-\log_{0.2}(0.40))^{0.50\times 0.3}$$
$$= 0.663$$
$$\prod_{j=1}^{5}\left(1+\log_r \gamma_{\tau(j)}\right)^{\phi_j\xi_j} = (1+\log_{0.2}(0.60))^{0.80\times 0.1}(1+\log_{0.2}(0.50))^{0.60\times 0.2}$$
$$(1+\log_{0.2}(0.40))^{0.70\times 0.2}(1+\log_{0.2}(0.40))^{0.40\times 0.2}(1+\log_{0.2}(0.40))^{0.50\times 0.3}$$
$$= 1.260$$
$$\prod_{j=1}^{5}(1-\log_r(1-\chi_\tau(1)))^{\phi_j\xi_j} = (1-\log_{0.2}(1-0.20))^{0.80\times 0.1}(1-\log_{0.2}(1-0.30))^{0.60\times 0.2}$$
$$(1-\log_{0.2}(1-0.40))^{0.70\times 0.2}(1-\log_{0.2}(1-0.50))^{0.40\times 0.2}(1-\log_{0.2}(1-0.50))^{0.50\times 0.3}$$
$$= 0.798$$
$$\prod_{j=1}^{5}(1+\log_r(1-\chi_\tau(j)))^{\phi_j\xi_j} = (1+\log_{0.2}(1-0.20))^{0.80\times 0.1}(1+\log_{0.2}(1-0.30))^{0.60\times 0.2}$$
$$(1+\log_{0.2}(1-0.40))^{0.70\times 0.2}(1+\log_{0.2}(1-0.50))^{0.40\times 0.2}(1+\log_{0.2}(1-0.50))^{0.50\times 0.3}$$
$$= 1.168$$

Next, using the I-LIFEOWGA operator, we have

$$\text{I-CLIFEOWGA}_\xi\left(\langle \ell_1, (\mathrm{h}_1, \phi_1)\rangle, \langle \ell_2, (\mathrm{h}_2, \phi_2)\rangle, \ldots, \langle \ell_5, (\mathrm{h}_5, \phi_5)\rangle\right)$$
$$\left(\frac{2\prod_{j=1}^{n}(1-\log_r \gamma_\tau(j))^{\phi_j\xi_j}}{\prod_{j=1}^{n}\left(1+\log_r \gamma_{\tau(j)}\right)^{\phi_j\xi_j}+\prod_{j=1}^{n}\left(1-\log_r \gamma_{\tau(j)}\right)^{\phi_j\xi_j}}, \frac{\prod_{j=1}^{n}(1+\log_r(1-\chi_\tau(j)))^{\phi_j\xi_j}-\prod_{j=1}^{n}(1-\log_r(1-\chi_\tau(j)))^{\phi_j\xi_j}}{\prod_{j=1}^{n}(1+\log_r(1-\chi_\tau(j)))^{\phi_j\xi_j}+\prod_{j=1}^{n}(1-\log_r(1-\chi_\tau(j)))^{\phi_j\xi_j}}\right)$$
$$= \left(\frac{2(0.663)}{1.260+0.663}, \frac{1.168-0.798}{1.168+0.798}\right)$$
$$= (0.689, 0.188)$$

Definition 6: Let $(\ell_j, \langle \mathrm{h}_j, \phi_j\rangle)\,(j \le n)$ be IFNs, then the I-CLIFEHAGA operator can be mathematically expressed as:

$$\text{I-CLIFEHGA}_{\zeta,\xi}\left(\langle \ell_1, (\mathrm{h}_1, \phi_1)\rangle, \langle \ell_2, (\mathrm{h}_2, \phi_2)\rangle, \ldots, \langle \ell_n, (\mathrm{h}_n, \phi_n)\rangle\right)$$

$$= \begin{cases} \left(\dfrac{2\prod_{j=1}^{n}\left(1-\log_r \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}}{\prod_{j=1}^{n}\left(1+\log_r \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}+\prod_{j=1}^{n}\left(1-\log_r \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}}, \dfrac{\prod_{j=1}^{n}\left(1+\log_r\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}-\prod_{j=1}^{n}\left(1-\log_r\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}}{\prod_{j=1}^{n}\left(1+\log_r\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}+\prod_{j=1}^{n}\left(1-\log_r\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}}\right) \\ \text{where } r \neq 1, \text{ and } 0 < r \le \min\left\{\dot{\gamma}_{\tau(j)}, \left(1-\dot{\chi}_{\tau(j)}\right)\right\} \le 1, \\ \left(\dfrac{2\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}}{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}} \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}+\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}} \dot{\gamma}_{\tau(j)}\right)^{\phi_j\xi_j}}, \dfrac{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}}\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}-\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}}\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}}{\prod_{j=1}^{n}\left(1+\log_{\frac{1}{r}}\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}+\prod_{j=1}^{n}\left(1-\log_{\frac{1}{r}}\left(1-\dot{\chi}_{\tau(j)}\right)\right)^{\phi_j\xi_j}}\right) \\ \text{where} r \neq 1, \text{ and } 0 < \frac{1}{r} \le \min\left\{\dot{\gamma}_{\tau(j)}, \left(1-\dot{\chi}_{\tau(j)}\right)\right\} \le 1 \end{cases}$$

In the context of vector representation, assuming two vectors $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$. These vectors are associated with specific elements or entities, each having a corresponding vector representation. Additionally, there are weighted vectors, which can be represented as combinations of these associated vectors. The conditions for these vectors are as follows: both must belong to the closed interval $[0, 1]$, meaning that each component of the vectors must fall within the range of 0 to 1 , inclusive. Furthermore, the sum of he components in both vectors must equal one. In simpler terms, the maximum value $\dot{h}_{\tau(j)}$ in the equation $\dot{h}_{\tau(j)} = n\lambda_j \mathrm{h}_j$ is influenced by a parameter called n. This parameter, referred to as the balancing coefficient, is a positive number. Essentially, the value of $\dot{h}_{\tau(j)} = n\lambda_j \mathrm{h}_j$ is determined and constrained by the value assigned to the parameter n. In essence, the balancing coefficient plays a crucial role in shaping the outcome of the equation, ensuring its effectiveness.

If $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$, then $(n\lambda_1\mathrm{h}_1, n\lambda_2\mathrm{h}_2, \ldots, n\lambda_n\mathrm{h}_n)$ approaches to $(\mathrm{h}_1, \mathrm{h}_2, \ldots, \mathrm{h}_n)$. In the IFEOWG system, the order-inducing variable is the element in the ordered pair $\mathrm{h}_j \in \langle \mathrm{h}_j, \phi_j\rangle$ that has the jth largest value. This specific variable plays a crucial role in shaping the overall sequence or ranking within the system. It significantly affects the prioritization process, contributing to the assessment of importance or weight in decision-making.

5 Application of the Proposed Techniques

Decision-making is a systematic and methodical process that starts with recognizing a problem or seizing an opportunity. It involves gathering relevant information, analyzing available alternatives, and ultimately selecting the most suitable course of action. This structured approach is essential for individuals or organizations to make informed and rational decisions. This research focuses on decision-making processes dealing with complex polytopic fuzzy information, introducing techniques specifically designed for handling intricate data sets. These techniques are represented by operators, each serving a specific purpose. The paper emphasizes two significant operators: the I-CLIFEOWGA operator and the I-CLIFEHGA operator. These operators are tailor-made to address the complexities associated with intuitionistic fuzzy information, presenting unique strategies for decision-making in this specialized context. Let $\mathcal{B} = \{\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_m\}$ be a finite set of m alternatives. These could be different options, choices, or solutions that you are considering. $\partial = \{\partial_1, \partial_2, \dots, \partial_n\}$ be a set of $\boldsymbol{n}$ criteria. These criteria represent the different factors or attributes that you will use to evaluate and compare the alternatives. Criteria could include things like cost, performance, reliability, etc., with a weighted vector $\partial = (\partial_1, \partial_2, \dots, \partial_m)$. Let $E = \{E_1, E_2, \dots, E_k\}$ be a set of decision-makers whose weighted vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)$.

Step 1: In the first stage, the goal is to organize the information given by decision-makers in an intuitionistic, fuzzy environment. This involves using fuzzy numbers to capture the uncertainties and imprecisions inherent in the decision-makers' inputs. The objective is to develop a thorough and adaptable representation that considers the nuanced nature of the information provided by decision-makers.

Step 2: In the second phase, the goal is to form a unified matrix by merging distinct matrices through designated operations such as addition, subtraction, multiplication, or other pertinent actions, depending on the context. The value of each element in the resulting matrix is determined by executing the specified operation on the corresponding elements from the initial matrices.

Step 3: In the third step, the focus is once again on applying the suggested techniques or aggregation operators to calculate preference values. This step involves utilizing the proposed methods to evaluate and determine the preference levels for each option. The goal is to obtain a comprehensive set of preference values that will aid in making the best selection from the available choices. This process ensures that all relevant criteria and considerations are considered, providing a solid foundation for the final decision-making process.

Step 4: In the fourth step of the process, the objective is to give numerical scores to preferences, considering their significance in a specific context. This requires assessing and assigning values to preferences, allowing for a meaningful comparison and ranking. The aim is to create a quantifiable representation of how each preference influences the overall decision-making process within the given scenario.

Step 5: During the fifth step of the process, the focus is on assessing and sorting items according to the scores they've been assigned. The objective is to identify the item with the highest score, ultimately selecting or prioritizing it at the top of the ranking. This stage involves a detailed review of the given scores to make informed decisions about the order in which items are addressed or given priority.

Step 6: End of the algorithm.

6 Illustrative Example

An illustrative example serves as a concise portrayal, offering a tangible scenario to enhance comprehension of a concept. Through a brief demonstration or instance, it brings clarity and vividness, making the underlying idea more accessible. By presenting a practical case, it becomes a valuable tool for elucidation, facilitating a better understanding of the concept at hand. In essence, it acts as a clear and tangible representation to drive home a point effectively.

Case study: In their joint pursuit of aiding the customer's quest for the ideal laptop, the five consultants $(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \mathrm{E}_4, \mathrm{E}_5)$ have united their expertise to meticulously evaluate five distinct laptops from various manufacturers. Each consultant is committed to unraveling the intricacies of these laptops, providing the customer with a holistic understanding of their unique features and qualities. Through this collaborative effort, the team aims to empower the customer with comprehensive insights, enabling them to make a well-informed and satisfying purchase decision. By synergizing their individual perspectives, the consultants aspire to offer a nuanced and detailed assessment that goes beyond mere specifications, ensuring the customer receives personalized guidance tailored to their specific needs

$\mathcal{B}_1$ - Gaming Laptops: Gaming laptops are specialized devices tailored for gaming enthusiasts, featuring robust processors and dedicated graphics cards to deliver high-performance gaming experiences. With advanced cooling systems to handle intense graphics processing, these laptops prioritize immersive gameplay. Their design often incorporates vibrant displays, customizable RGB lighting, and ergonomic keyboards for an optimal gaming setup on the go.

$\mathcal{B}_2$ - Business laptops: Business laptops are tailored to meet the professional needs of users, emphasizing features such as security, durability, and productivity. These laptops often incorporate advanced security measures, robust build quality, and business-focused software and applications. With a focus on reliability and performance, business laptops are designed to enhance efficiency and facilitate seamless collaboration in a corporate environment.

$\mathcal{B}_3$ - Workstation laptops: Workstation laptops are specialized devices tailored for professionals in fields requiring high computing power, such as design, engineering, or content creation. These laptops feature robust hardware configurations, including powerful processors and dedicated graphics, to seamlessly handle resource-intensive tasks. With high-resolution displays and ample memory, workstation laptops provide an optimal platform for demanding applications, making them indispensable for users engaged in complex and creative endeavors.

$\mathcal{B}_4$ - Budget laptops: Budget laptops cater to users seeking cost-effective yet functional computing solutions. Designed for affordability, these laptops offer a balance between performance and price, making them ideal for students, casual users, and professionals on a budget. Despite their economical nature, many budget laptops now incorporate features like decent processing power, ample storage, and reliable connectivity options. With advancements in technology, budget-friendly options continue to provide viable alternatives for individuals who prioritize value without compromising essential computing capabilities.

$\mathcal{B}_5$ - Rugged laptops: Rugged laptops are specifically designed to endure challenging environments and conditions, featuring robust construction to withstand factors like dust, moisture, and physical impacts. Ideal for outdoor, industrial, or military use, these durable devices offer reliability and resilience in situations where standard laptops might falter. With reinforced materials and specialized engineering, rugged laptops ensure consistent performance in demanding settings.

To pinpoint the ideal timepiece, a meticulous examination of four crucial factors $(\partial_1, \partial_2, \partial_3, \partial_4)$ is indispensable. The comprehensive assessment and comparison of these elements are pivotal in the selection process, ensuring the chosen watch perfectly resonates with the customer's distinct preferences and specific requirements. By delving into the nuances of these key attributes, one can navigate through the myriad options available, ultimately zeroing in on the watch that seamlessly blends functionality, style, and personalized appeal. This methodical approach empowers individuals to make an informed decision, elevating the significance of each factor in the overall watch-selection journey.

∂_1 - Performance: When evaluating a laptop's performance, it is crucial to focus on key components such as the processor speed and type, ensuring efficient task execution. Additionally, the amount and type of RAM play a vital role in enabling seamless multitasking. To enhance overall responsiveness, it's essential to consider the storage capacity and type, opting for fast and reliable options like SSDs for improved performance.

∂_2 - Operating System: The operating system (OS) serves as the foundational software that manages hardware resources and facilitates user interaction with a computer. It acts as an intermediary between applications and hardware components, providing a platform for software execution and ensuring seamless user experiences. Whether it's Windows, macOS, or Linux, the choice of an operating system significantly influences a user's computing environment and functionality.

∂_3 - When establishing a budget for a laptop purchase, it's crucial to strike a harmonious balance between performance and cost, considering individual financial constraints. Careful consideration of key factors such as processor speed, RAM capacity, and storage type will guide the budgeting process. A realistic approach ensures that the chosen laptop meets personal requirements without unnecessary overspending. Ultimately, finding the sweet spot between affordability and performance is the key to a satisfying laptop purchase.

∂_4 - Brand and Support: When considering brand and support in the selection of a laptop, it is prudent to opt for established and reputable brands that have a track record of reliability and customer satisfaction. Choosing a wellknown brand not only ensures a higher likelihood of product quality but also typically comes with better customer support and warranty services. Prioritizing a brand with a strong reputation contributes to a more positive overall user experience and long-term satisfaction with the chosen laptop.

Step 1: In the initial step (Tables 1-5), matrices are constructed in accordance with the expert's decision. This involves organizing relevant data or variables into a structured format, often represented as matrices. The expert's input guides the arrangement of information, ensuring that key elements are accurately captured and organized for further analysis or computation.

Step 2: The I-CLIFEOWGA operator, employing a weighted vector $\wp = (0.1, 0.2, 0.2, 0.2, 0.3)$ and a designated base $r = 0.2$, serves as a powerful tool in creating Table 6. This operator seamlessly blends intuitionistic fuzzy logic with the weighted geometric average, offering a sophisticated approach to factor evaluation. Through the incorporation of specified weights and the chosen base, Table 6 emerges as a comprehensive depiction. It intricately captures the nuanced relationships and prioritizations assigned to each element within the context, providing a detailed overview essential for informed decision-making in diverse applications.

Table 1. Assessment of expert E_1

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$\langle 0.70,(0.50,0.20),0.70\rangle$	$\langle 0.60,(0.30,0.70),0.60\rangle$	$\langle 0.70,(0.40,0.50),0.30\rangle$	$\langle 0.80,(0.50,0.40),0.60\rangle$
$\mathcal{B}_2$	$\langle 0.50,(0.40,0.60),0.40\rangle$	$\langle 0.70,(0.70,0.20),0.60\rangle$	$\langle 0.50,(0.40,0.40),0.30\rangle$	$\langle 0.50,(0.50,0.30),0.20\rangle$
$\mathcal{B}_3$	$\langle 0.40,(0.40,0.50),0.30\rangle$	$\langle 0.60,(0.60,0.30),0.80\rangle$	$\langle 0.40,(0.40,0.50),0.10\rangle$	$\langle 0.40,(0.60,0.30),0.50\rangle$
$\mathcal{B}_4$	$\langle 0.30,(0.50,0.40),0.30\rangle$	$\langle 0.60,(0.60,0.30),0.20\rangle$	$\langle 0.30,(0.50,0.40),0.40\rangle$	$\langle 0.30,(0.50,0.40),0.50\rangle$
$\mathcal{B}_5$	$\langle 0.40,(0.50,0.40),0.40\rangle$	$\langle 0.40,(0.50,0.40),0.50\rangle$	$\langle 0.40,(0.50,0.40),0.30\rangle$	$\langle 0.60,(0.60,0.30),0.20\rangle$

Table 2. Assessment of expert E_2

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$\langle 0.70,(0.30,0.60),0.30\rangle$	$\langle 0.80,(0.50,0.30),0.60\rangle$	$\langle 0.80,(0.30,0.50),0.60\rangle$	$\langle 0.90,(0.50,0.40),0.70\rangle$
$\mathcal{B}_2$	$\langle 0.60,(0.40,0.50),0.60\rangle$	$\langle 0.60,(0.70,0.20),0.60\rangle$	$\langle 0.60,(0.40,0.60),0.40\rangle$	$\langle 0.80,(0.50,0.30),0.20\rangle$
$\mathcal{B}_3$	$\langle 0.50,(0.40,0.50),0.10\rangle$	$\langle 0.50,(0.60,0.30),0.50\rangle$	$\langle 0.50,(0.40,0.50),0.30\rangle$	$\langle 0.70,(0.60,0.30),0.80\rangle$
$\mathcal{B}_4$	$\langle 0.40,(0.50,0.40),0.40\rangle$	$\langle 0.40,(0.50,0.40),0.50\rangle$	$\langle 0.40,(0.50,0.40),0.30\rangle$	$\langle 0.60,(0.60,0.30),0.20\rangle$
$\mathcal{B}_5$	$\langle 0.30,(0.40,0.50),0.60\rangle$	$\langle 0.30,(0.70,0.20),0.60\rangle$	$\langle 0.30,(0.30,0.50),0.60\rangle$	$\langle 0.10,(0.50,0.30),0.20\rangle$

Table 3. Assessment of expert E_3

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$\langle 0.80,(0.30,0.40),0.70\rangle$	$\langle 0.90,(0.50,0.30),0.60\rangle$	$\langle 0.90,(0.30,0.60),0.80\rangle$	$\langle 0.80,(0.50,0.30),0.60\rangle$
$\mathcal{B}_2$	$\langle 0.70,(0.40,0.50),0.20\rangle$	$\langle 0.70,(0.50,0.40),0.30\rangle$	$\langle 0.80,(0.40,0.50),0.60\rangle$	$\langle 0.70,(0.40,0.50),0.60\rangle$
$\mathcal{B}_3$	$\langle 0.60,(0.30,0.60),0.20\rangle$	$\langle 0.60,(0.60,0.30),0.80\rangle$	$\langle 0.60,(0.50,0.40),0.50\rangle$	$\langle 0.60,(0.60,0.30),0.50\rangle$
$\mathcal{B}_4$	$\langle 0.50,(0.50,0.40),0.30\rangle$	$\langle 0.50,(0.60,0.30),0.20\rangle$	$\langle 0.40,(0.40,0.60),0.04\rangle$	$\langle 0.50,(0.50,0.40),0.40\rangle$
$\mathcal{B}_5$	$\langle 0.80,(0.40,0.50),0.60\rangle$	$\langle 0.80,(0.50,0.30),0.60\rangle$	$\langle 0.90,(0.50,0.30),0.60\rangle$	$\langle 0.60,(0.40,0.50),0.60\rangle$

Table 4. Assessment of expert E_4

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$\langle 0.40,(0.40,0.50),0.10\rangle$	$\langle 0.40,(0.50,0.40),0.50\rangle$	$\langle 0.40,(0.40,0.60),0.40\rangle$	$\langle 0.20,(0.60,0.30),0.20\rangle$
$\mathcal{B}_2$	$\langle 0.30,(0.40,0.50),0.60\rangle$	$\langle 0.30,(0.70,0.20),0.60\rangle$	$\langle 0.30,(0.30,0.50),0.60\rangle$	$\langle 0.10,(0.50,0.30),0.20\rangle$
$\mathcal{B}_3$	$\langle 0.50,(0.40,0.50),0.10\rangle$	$\langle 0.50,(0.60,0.30),0.50\rangle$	$\langle 0.50,(0.40,0.50),0.30\rangle$	$\langle 0.70,(0.60,0.30),0.80\rangle$
$\mathcal{B}_4$	$\langle 0.20,(0.30,0.60),0.30\rangle$	$\langle 0.20,(0.60,0.30),0.50\rangle$	$\langle 0.20,(0.40,0.50),0.30\rangle$	$\langle 0.10,(0.60,0.30),0.80\rangle$
$\mathcal{B}_5$	$\langle 0.10,(0.50,0.40),0.40\rangle$	$\langle 0.10,(0.50,0.30),0.60\rangle$	$\langle 0.10,(0.50,0.40),0.30\rangle$	$\langle 0.20,(0.50,0.40),0.70\rangle$

Table 5. Assessment of expert E_5

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$\langle 0.50,(0.40,0.50),0.10\rangle$	$\langle 0.60,(0.40,0.40),0.30\rangle$	$\langle 0.50,(0.30,0.60),0.80\rangle$	$\langle 0.30,(0.50,0.30),0.60\rangle$
$\mathcal{B}_2$	$\langle 0.40,(0.50,0.40),0.40\rangle$	$\langle 0.50,(0.50,0.30),0.60\rangle$	$\langle 0.40,(0.40,0.50),0.60\rangle$	$\langle 0.40,(0.60,0.30),0.50\rangle$
$\mathcal{B}_3$	$\langle 0.30,(0.30,0.60),0.20\rangle$	$\langle 0.30,(0.60,0.30),0.80\rangle$	$\langle 0.30,(0.50,0.40),0.50\rangle$	$\langle 0.90,(0.60,0.30),0.50\rangle$
$\mathcal{B}_4$	$\langle 0.20,(0.50,0.40),0.30\rangle$	$\langle 0.20,(0.60,0.30),0.20\rangle$	$\langle 0.20,(0.40,0.60),0.40\rangle$	$\langle 0.40,(0.50,0.40),0.40\rangle$
$\mathcal{B}_5$	$\langle 0.30,(0.50,0.40),0.30\rangle$	$\langle 0.60,(0.60,0.30),0.20\rangle$	$\langle 0.30,(0.50,0.40),0.40\rangle$	$\langle 0.30,(0.50,0.40),0.50\rangle$

Table 6. Collective decision of all experts

	∂_1	∂_2	∂_3	∂_4
$\mathcal{B}_1$	$(0.630,0.256)$	$(0.547,0.316)$	$(0.663,0.304)$	$(0.521,0.357)$
$\mathcal{B}_2$	$(0.624,0.314)$	$(0.482,0.296)$	$(0.558,0.229)$	$(0.546,0.410)$
$\mathcal{B}_3$	$(0.665,0.321)$	$(0.591,0.357)$	$(0.628,0.246)$	$(0.547,0.316)$
$\mathcal{B}_4$	$(0.568,0.234)$	$(0.536,0.460)$	$(0.619,0.324)$	$(0.596,0.382)$
$\mathcal{B}_5$	$(0.538,0.264)$	$(0.601,0.354)$	$(0.618,0.236)$	$(0.527,0.346)$

Step 3: Utilizing the innovative I-CLIFEOWGA operator, preference values are efficiently computed by considering a comprehensive array of factors. This operator encompasses a holistic approach, integrating individual preferences, context, lifestyle, emotions, objectives, and global awareness, ensuring a nuanced and dynamic evaluation process. Through its multidimensional analysis, the I-CLIFEOWGA operator offers a sophisticated means of determining preferences, reflecting the intricate interplay of diverse elements within decision-making scenarios.

Using the weights $\xi = (0.1, 0.2, 0.3, 0.4)$.

$$r_1 = (0.495, 0.286), r_2 = (0.491, 0.229) r_3 = (0.498, 0.287), r_4 = (0.488, 0.239), r_5 = (0.475, 0.296)$$

Step 4: In this step, compute the score function for all preference values to facilitate subsequent selection. This involves evaluating and assigning numerical scores to each preference, providing a quantitative basis for the subsequent decision-making process.

$$s(r_1) = 0.495 - 0.286 = 0.209, s(r_2) = 0.491 - 0.229 = 0.262, s(r_3) = 0.498 - 0.287 = 0.211$$
$$s(r_4) = 0.488 - 0.239 = 0.249, s(r_5) = 0.475 - 0.296 = 0.179$$

Step 5: Ranking all alternatives as: $\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$. The following Table 7, shows the score function of the proposed methods:

Table 7. The score function of the proposed methods

Operators	Score Functions	Ranking
I-CLIFEOWGA	$0.209, 0.262, 0.211, 0.249, 0.180$	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$
I-CLIFEHGA	$0.230, 0.292, 0.275, 0.276, 0.190$	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$

Step 6: Choosing a business laptop is the more fitting option for your needs. These devices are specifically designed to meet the demands of professional settings, offering the performance and features necessary for efficient work in a business environment.

7 Comparative and Sensitive Analysis

IFSs revolutionize fuzzy set theory by incorporating membership, non-membership, and hesitation degrees, providing a nuanced representation of uncertainty. The hesitation degree quantifies doubt or indecision in assigning elements to sets, enhancing realism in handling imprecise information. With a constraint ensuring coherence in interpreting these components, IFSs offer a flexible framework for decision-making and pattern recognition in scenarios of limited knowledge or conflicting evidence. This comprehensive approach to uncertainty makes IFSs indispensable for real-world applications requiring robust handling of incomplete or uncertain information. The Comparison is given in Table 8.

Table 8. The score function of the proposed methods

Averaging Operators	Ranking	Geometric Operators	Ranking
IFEWA[9]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$	IFEWG[7]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$
IFEOWA[9]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$	IFEOWG[7]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$
IFEHA[8]	$\mathcal{B}_2 > \mathcal{B}_3 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_5$	IFEHG[8]	$\mathcal{B}_2 > \mathcal{B}_3 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_5$
LIFWA[8]	$\mathcal{B}_2 > \mathcal{B}_3 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_5$	LIFWG[23]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
LIFOWA[25]	$\mathcal{B}_2 > \mathcal{B}_3 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_5$	LIFOWG[23]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
CLIFEWA[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$	CLIFEWG[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
CLIFEOWA[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$	CLIFEOWG[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
CLIFEHA[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$	CLIFEHG[25]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
I-IFOWAA[29]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$	I-IFOWGA[30]	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_5$
I-CLIEIEOWGA	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$	I-CLIFEHGA	$\mathcal{B}_2 > \mathcal{B}_4 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_5$

8 Experimental Validation and Discussion

Logarithmic aggregation operators are mathematical tools utilized in aggregating functions to combine values using logarithmic operations. They find applications across diverse fields like computer science, data analysis, and signal processing. These operators prove especially handy when dealing with multiplicative relationships, as they convert products into sums, thereby simplifying calculations. Their significance lies in decision-making processes where non-linear relationships are involved. Logarithmic aggregation operators help in normalizing scale differences, handling sensitivity towards extreme values, aligning with human perception, and offering flexibility in modeling various decision scenarios. They play a crucial role in complex decisionmaking environments where multiple criteria must be considered simultaneously. However, the new model also has limitations and weaknesses, such as $\log_1(\mathrm{h})$ and $\log_r(0)$ cannot be defined for any real number. Also, it's not possible to calculate $\log_r(\mathrm{h})$ for any real numbers r and IFNs h. Therefore, throughout the paper, we consider $\mathrm{h} \neq 0$ and $r \neq 1$.

9 Conclusion

This paper introduces novel mathematical constructs called Einstein sum and Einstein product, which serve as effective alternatives to conventional algebraic sum and product operations. These new constructs are particularly tailored for handling IFSs, characterized by a real base number r and confidence level considerations. Moreover, the paper proposes a set of algorithmic operational laws (LOLs) designed specifically for IFSs, enhancing computational efficiency and accuracy. Additionally, the study introduces induced Einstein aggregation operators, such as the I-CLIFEOWGA and I-CLIFEHGA operators, within a confidence-based framework. To evaluate the effectiveness of these new constructs and operators, the paper conducts a comparative analysis against recent studies. This comparative study highlights the superiority and validity of the proposed approaches, emphasizing their potential for practical applications in various domains. The practical utility of these operators is exemplified through a decision-making problem focused on selecting the most optimal and suitable laptop, underscoring their effectiveness in real-world applications. This exploration shows great potential for branching into various advanced methodologies, such as geometric strategies and linguistic frameworks. By delving deeper into Dombi approaches, symmetric operators, and power operators, the investigation can uncover nuanced insights. Moreover, the inclusion of Hamacher operators provides an additional dimension to the study. The research could further extend into interval-valued methodologies, incorporating Hamacher interval approaches and Einstein interval approaches, among others. This comprehensive expansion of perspectives facilitates a thorough examination of intricate aspects within the chosen domain, promising a rich and multifaceted exploration.

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