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Enhanced Acoustic Attenuation Performance of a Novel Absorptive Muffler: A Helmholtz Equation-Based Simulation Study

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Abstract: This investigation delves into the noise attenuation capabilities of an innovatively designed muffler, which integrates additional piping and perforation to augment sound reflection. The enhanced muffler's design was rigorously simulated using the Helmholtz equation through the application of COMSOL Multiphysics software, aiming to delineate its acoustic performance relative to conventional models. The analysis underscored the superior efficacy of the optimized model in elevating transmission loss, diminishing acoustic pressure, and concurrently attenuating noise and frequency levels. A comparative evaluation of the transmission loss between the traditional and the novel muffler revealed a significant amelioration in the latter, highlighting its advanced noise reduction capabilities. The study further illuminated that exhaust pressure and back pressure contribute to acoustic wave generation, prompting the optimization of the muffler design to mitigate pressure, thereby circumventing potential damage. Notably, despite the analytical complexity, the construction of the proposed muffler remains straightforward, representing a pivotal advantage. This research contributes to the acoustic engineering field by presenting a muffler design that not only significantly reduces noise pollution but also demonstrates an ease of construction, making it a viable solution for widespread application. The findings advocate for the muffler's potential in enhancing acoustic comfort and environmental compliance in automotive and industrial settings.

Keywords: Exhaust; Muffler; Back-pressure; Helmholtz equation; Acoustic performance; Transmission loss

1 Introduction

A vehicle exhaust system takes on the responsibility of reducing noxious byproducts, high pressure, and hot gases and removing them from the engine to the atmosphere. Due to the emission of gas from the engine, pressure and back pressure are produced in the form of alternative high- and low-pressure pulse waves, which can spark off resonance and, subsequently, vibration and frequency to bring about dire consequences and fatigue failure [1]. Due to malfunctions caused by sound and noise from the exhaust and body of the engine or transmission system, mufflers were invented to minimize noise and sound. The muffler is a part of the exhaust system that is located between the manifold and the catalytic converter, and gas is emitted from the exhaust piped directly into it. Silencers suppress noise through chambers, partitions, and tubes. Furthermore, it can act as a cushion to absorb noise or a chamber to reduce high-pitched frequencies. Finally, gases are released into the atmosphere through the tail pipe. In terms of increasing the efficiency of muffler performance, pipe diameters, fracture characteristics, baffle locations, and absorption material can be changed. The perforation of muffler tubes allows sound inside the muffler to reflect and spread in various directions, resulting in irreparable damage. To minimize the resonance and noise, an absorptive muffler is utilized, which is able to convert the energy of sound into heat [2].

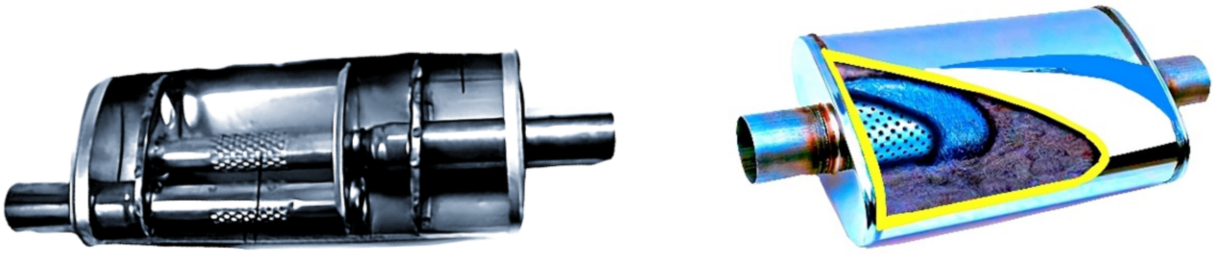
Researchers have shown great interest in sound and noise-reduction components. Belingardi and Leonti [3] implement the finite element method to solve the problem of the complex geometry of exhaust engines. Lee and Seock [4] performed numerical and experimental investigations on the acoustic responses of various vehicles to reduce the noise of vehicle components. Wang et al. [5] presented a new method to address problems caused by noise in fine-pitch gears and optimize their efficiency. Okada and Abe [6] studied the frequency of abnormal noise emitted from exhaust. Recently, mufflers have attracted the attention of a considerable number of researchers. For instance, El-Sharkeway and El-Chazly [7] analyzed plane wave propagation, gas flow conditions, and all possible radial and spinning modes by governing various theories to determine the transmission, reflection, and attenuation of muffler sound. Cheng et al. [8] applied the multi-domain boundary element technique to address the problem of the acoustic segment of the muffler using the Helmholtz equation. Dokumaci [9] analyzed the influences of mean flow and slip velocity on different types of pipe elements by reformulating existing theories. Barbieri and Barbieri [10] by governing the finite element (FE) method optimized the four parameters and Zoulendijk's feasible direction method with the aim of achieving the dimensions. Chiu and Chang [11] derived a four-pole system matrix and designed a multi-cost cross muffler to address the acoustic problems and optimize the performance of the pure-tone and dual-tone of them. Yasuda et al. [12] simulated the acoustic performance of the experimental and CFD model of the muffler' under the wide-open throttle acceleration condition by using one-dimensional computational fluid dynamics. The consequence of the simulation and experimental model, which was carried out in an anechoic chamber according to the Japanese Standard, was noticeably accurate. Similarly, Yasuda et al. [13], by considering the effect of structure parameters according to frequency and time, performed a series of studies on the acoustic performance of low-pass filters and Helmholtz resonances of the interconnecting hole on the tail tube of mufflers, which sparked an improvement in the sound attenuation performance of typical mufflers. Furthermore, the work of Talegaonkar et al. [14] added to this body of research by presenting an improvement in tunable mufflers for vehicles utilized for racing using butterfly valves that could be opened and closed to reduce back pressure. Sagar and Munjal [15], to enhance the transmission loss of the muffler's baffles, improved the performance of the muffler by developing a new three-pass muffler design, which was able to reduce acoustic flow induced in vehicle exhaust. Elsayed et al. [16] were apparently the first to demonstrate the responses of acoustic and back-pressure under the influence of cut ratio shape on baffles by using COMSOL and Multiphasic software. Moreover, Zhang et al. [17] presented a novel principle for reducing the velocity of airflow according to split-stream flow in the muffler to decrease the turbulence noise and back pressure of the muffler. Fan and Ji [18] used inlet and outlet tube analysis of mufflers to investigate the acoustic attenuation performance of mufflers through the transfer matrix method. Interestingly, changing the perforation brought about variation in frequency. Consequently, it left a noticeable influence on the attenuation performance of the muffler. In addition, Zhu et al. [19] presented novel H-Q tubes using semi-active mufflers to tackle the low-frequency noise problem. Lee et al. [20] also investigated the noise problem of mufflers, proposed a method for reducing noise in ducts, and optimized it by comparing some mathematical parameters of mufflers. Similarly, Lee et al. [21], by governing the gradient-based optimization, carried out an investigation on mufflers and their acoustic issues under different conditions, such as noise frequency and unpredictable temperature, and consequently proposed the result of their experimental investigation according to the performance of acoustic attenuation. Xue et al. [22] derived the multi-patch technique to analyze interior acoustic pressure in the framework of the Galerkin formulation of circular, elliptical, and reaction expansion chamber mufflers. In addition, with the aid of the FEM method and non-uniform rational B-splines, they obtained the Helmholtz equation. Shao et al. [23] simulated the acoustic pressure and noise effect of mufflers based on Helmholtz caving structure through COMSOL software to analyze and compare the experimental and numerical results of tunable metamaterial mufflers.

Furthermore, some researchers have recently investigated the effect of muffler structure on its acoustic attenuation. For example, Zhu et al. [24] used several interconnected structures of duck-like components to analyze the acoustic attenuation of mufflers. Also, it is worth mentioning that the step-by-step process used in this study proved to be a valuable and novel method due to the reliable results acquired. Fu et al. [25] investigated the variation in transmission loss of absorptive mufflers in high frequency using finite element automobile-matched layers. Similarly, Fu et al. [26] analyzed the noise pressure of a diesel engine exhaust muffler under high-frequency circumstances in order to calculate transmission loss according to the quadruple method. Adibi et al. [27] investigated the noise reduction performance of a multi-designed muffler based on the Helmholtz equation, considering different parameter effects on transmission loss. Kim et al. [28] fabricated a metamaterial broadband muffler system that consists of a Helmholtz resonator and a membrane cavity. Liu et al. [29] theoretically and numerically examined the vibration response of phononic crystals with Helmholtz mufflers by incorporating a piezoelectric shunt. Zhao et al. [30] investigated the vibration and temperature fields of an absorptive muffler based on the resistant anechoic principle. He et al. [31] developed an acoustic computational FE model to predict the transmission loss performance of hybrid mufflers by using Navier-Stokes equations.

The present study adds to the body of research by performing precise analysis by COMSOL software based on the Helmholtz equation on new-design mufflers with several added holes and tubes to reduce failure caused by noise and sound. The main aim of this study was to analyze and compare the amount of variation in some parameters of absorptive mufflers, such as transmission loss and acoustic pressure, and identify the attenuation of resonance before and after improvement. Finally, evaluating the noise attenuation performance of a muffler design showed that optimization of the muffler led to a considerable decrease in transmission loss and acoustic pressure. The validity of the optimal muffler achieved by the simulation of the formulated problem was experimentally supported.

2 Methodology

From an acoustic point of view, to reduce the amount of noise emitted by the exhaust of an internal combustion engine, the exhaust pipe is made of two pipe structures with different lengths separated by an intersection. Since the difference between two pipes is equal to half of the wave-length sound generated by cars, utilizing two columns of sound brings about superposition interference, which reduces the intensity of sound. Figure 1 shows both reactive and absorptive mufflers, which destroy acoustic pressure by utilizing the phenomenon of destructive interference. Moreover, the absorptive muffler takes responsibility for reducing sound wave energy by transforming it into heat energy. In this investigation, to improve the function of eradicating noise, a new muffler was designed as shown in Figure 2. As can be seen, the deceptively simple set of tubes with holes is offered to reflect the sound waves produced by the engine. Sound waves lose energy as they travel through a porous medium. The absorptive material causes the fluctuating gas particles to convert acoustic energy into heat. An absorptive silencer produces a more consistent transmission loss. The expansion chamber transmission loss curve is typically dosed to shape these humps but also dramatically increases transmission loss, especially at higher frequencies.



(a) (b)

Figure 1. A prevalent muffler for automobile a) reactive muffler b) muffler with absorptive layer

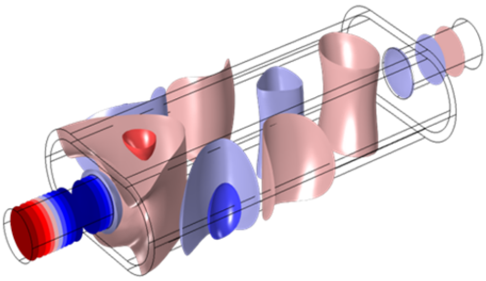


Figure 2. Schematic of optimized absorptive muffler

3 Theory and Formulation

The Helmholtz equation is defined as the primary equation of continuum mechanics. Explaining the conservation of mass, the conservation of momentum, which is known as the Nervier-Stokes equation, the energy conservation in medium, and the state equation, which explains the thermodynamic variables [32]. The equations used to measure acoustic vibration and efficiency are derived from the Helmholtz equation as follows:

$$\nabla . \left(-\frac{1}{\rho_c} \left(\nabla \rho_t - q_d \right) \right) - \frac{K_{eq}^2 \rho_t}{\rho_c} = Q_m \tag{1}$$

where:

$$\begin{aligned} p_t &= p_a + p_b \\ K_{eq}^2 &= \left(\frac{\omega}{C_c} \right)^2 \end{aligned} \tag{2}$$

where, p_a and p_b are pressure from each tube separately, q and Q are possible acoustic sources, K_{eq} is acoustic wave-number, ω is a circular value, C_c and ρ_c are complex spread of sound and complex density of fluid, respectively;

definitions of these parameters are given as follows:

$$
\begin{aligned}
C_c &= \frac{c}{\left(1 + C_1\left(\rho_f \frac{f}{R_f}\right)^{-C_2} - iC_3\left(\rho_f \frac{f}{R_f}\right)^{-C_4}\right)} \\
\rho_c &= \frac{\rho_f c}{C_c}\left(1 + C_5\left(\rho_f \frac{f}{R_f}\right)^{-C_6} - iC_7\left(\rho_f \frac{f}{R_f}\right)^{-C_8}\right)
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
R_f &= \frac{3.18 * 10^{-19} * \rho_{ap}^{1.53}}{d_{av}^2} \\
10^3 &< R_f < 50 * 10^3 \text{ pa.s }/\mathrm{m}^2 \\
0.01 &< \frac{\rho_f . f}{R_f} < 1
\end{aligned}
\tag{4}
$$

where, R_f is the flow resistivity, ρ_{ap} is the apparent density of material, d_{av} is the mean fiber diameter, C is the sound of speed, f is the resonance frequency, and ρ_f is the density of material.

In this investigation, vibrations were evaluated by FEM approaches. This design was simulated in a 3D model environment of COMSOL software to achieve the utilized parameters, which are tabulated in Table 1.

Table 1. Parameter description of absorptive muffler

Description	Name	Expression	Value
Amplitude of incoming pressure wave	P_{in}	1 Pa	1 Pa
Apparent density of glass wool	ρ_{ap}	12 kg/m^3	12 kg/m^3
Mean fiber diameter	d_{av}	10 μm	$1E-5m$
Ambient Temperature	T_0	20 C	293.15 K
Flow resistivity	R_f	$3.18e-9\left[N.s/m^2\right]^* \left(\rho_{ap}/1\left[Kg/m^3\right]\right)^{1.53}/d_{av}^2$	1424.2 kg/m^{3}s
Ambient pressure	P_0	1 atm	1.0133E5 Pa
Muffler length	L	600 mm	0.6 m
Muffler height	H	150 mm	0.15 m
Muffler width	W	300 mm	0.3 m
Inlet and outlet length	L_{io}	150 mm	0.15 m
Inlet and outlet radius	R_{io}	40 mm	0.04 m
Linear thickness	D	15 mm	0.015 m

Based on the obtained results, the effect of tubes and perforation on the noise-canceling property of the muffler was identified. The inlet port as the first boundary of the simulation where the air enters the exhaust is illustrated in Figure 3. Similarly, Figure 4 illustrates the outlet port where the air leaves the exhaust.

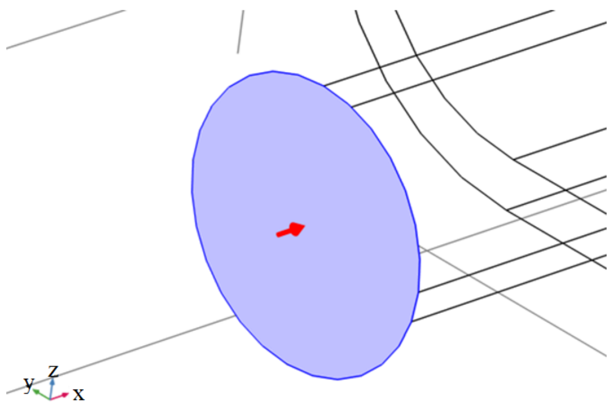


Figure 3. Inlet port: $p_t = \sum_{i \in bnd} A^{in} e^{i\phi} \left(S_{ij} + \delta_{ij}\right) p_i, j =$ incident wave port number ($\delta =$ Term representing attemuation, $S =$ Microphone Spacing)

Figure 5 and Figure 6 illustrate the interior and exterior final shapes of the developed muffler, respectively.

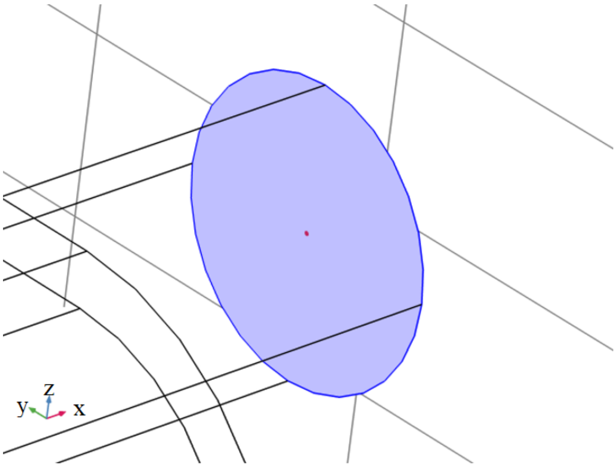


Figure 4. Outlet port: $p_t = \sum_{i \in bnd} A^{in} e^{i\phi} \left(S_{ij} + \delta_{ij}\right) p_i, j =$ incidentwave port number ($\delta =$ Term representing attenuation, $S =$ Microphone Spacing)

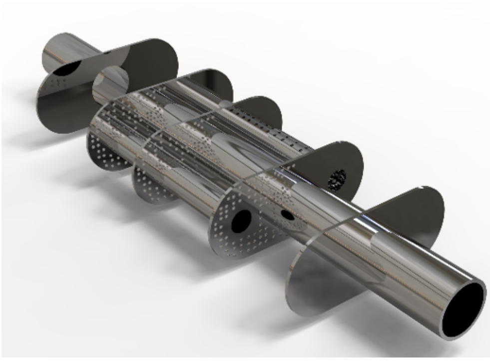


Figure 5. Interior final shape

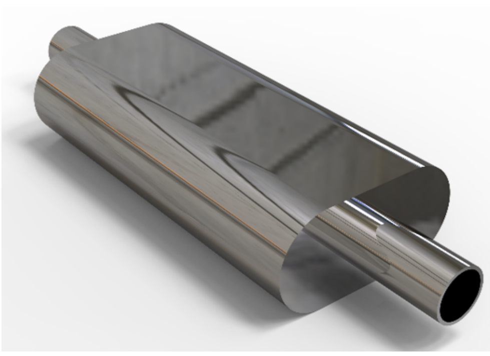


Figure 6. Exterior final shape

This research attempted to achieve absolute sound and pressure through simulation using COMSOL software. In the present simulation, the muffler was optimized by adding partitions and two pipes to reduce noise and vibration. A probabilistic area of the muffler, including pipes and holes, was identified. Similarly, geometric and mechanical data such as custom element size, curvature element size, ambient temperature, and pressure were input into the COMSOL software. As shown in Figure 7 and Figure 8, the positions of both developed and undeveloped mufflers were modified.

Meshing technology was applied for undeveloped and developed objects in COMSOL through 344 edge elements and 40 vertex elements, and other mesh statistics are tabulated in Table 2 and Table 3.

Furthermore, geometric data such as curve factor and max growth rate of both undeveloped and developed are listed in Table 4 and Table 5, respectively. As can be seen, the minimum element size was decreased from 16.2 to 1 mm. Consequently, the holes in partitions called for complex geometry in comparison with undeveloped mufflers.

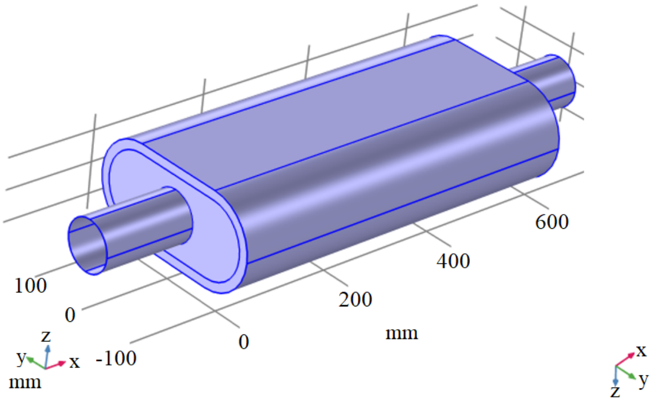


Figure 7. Sound hard boundary of typical muffler

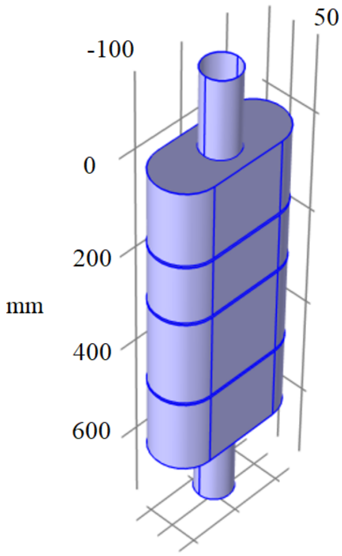


Figure 8. Sound hard boundary of optimized muffler

Table 2. Mesh statistics of typical muffler

Description	Value
Minimum element 'quality	0.2733
Average element quality	0.5818
Prism	2416
Triangle	364
Quad	768
Edge'element	344
Vertex element	40

Table 3. Mesh statistics of optimal muffler

Description	Value
Minimum element quality	0.2054
Average element 'quality	0.6315
Tetrahedron	423995
Triangle	100058
Edge element	11220
Vertex element	1252

Table 4. Geometric data of typical muffler

Description	Value
Maximum element-size	34[m/s]/1500[Hz]/5
Minimum element-size	16.2
Curvature element size	0.6
Resolution of narrow regions	0.5
Maximum element growth rate	1.5
Custom element size	Custom

Table 5. Geometric data of optimal muffler

Description	Value
Maximum element size	34[m/s]/1500[Hz]/5
Minimum element size	1
Curvature element size	0.6
Resolution of narrow regions	0.5
Maximum element growth rate	1.5
Custom element size	Custom

Figure 9 presents meshing on undeveloped and developed mufflers. Moreover, thanks to the complex and rounded shape of the geometry, wall boundaries were meshed by utilizing a free triangular shape after importing air from the pipe. What is more, Figure 10 and Figure 11 show the free triangular and free tetrahedral shape meshing's on the muffler before and after optimization, respectively.

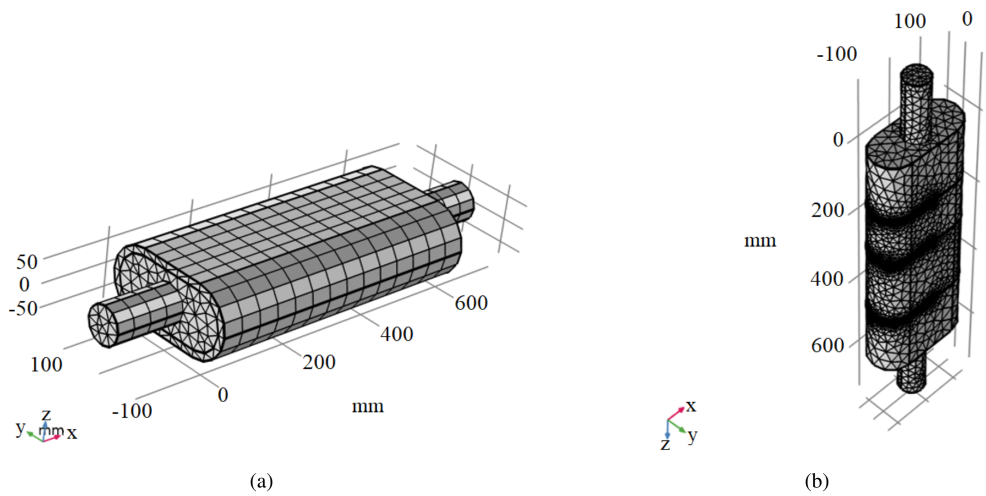


Figure 9. a) Typical muffler mesh, b) Optimal muffler mesh

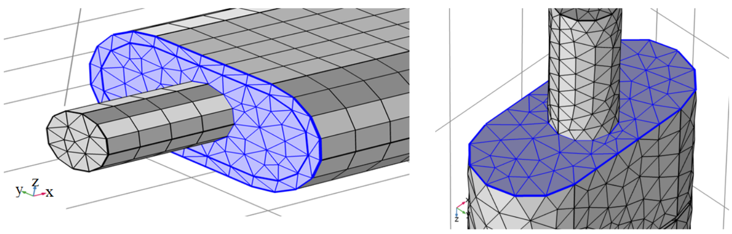
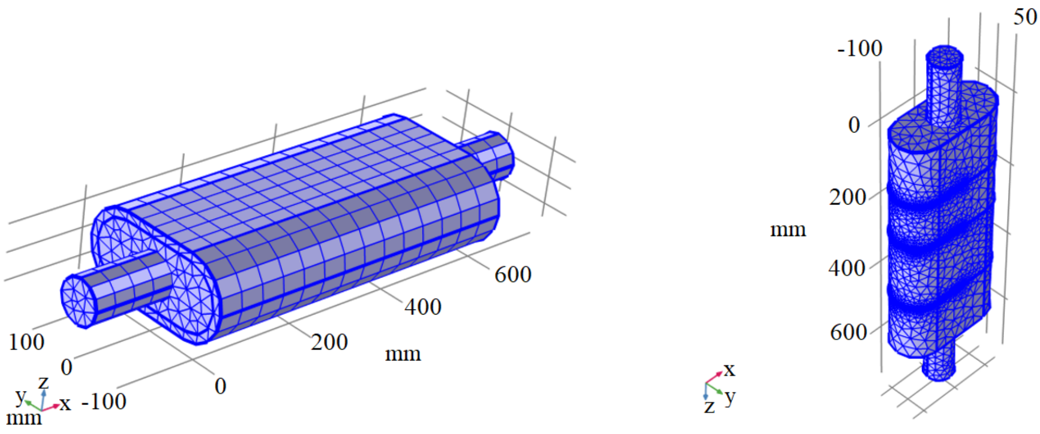


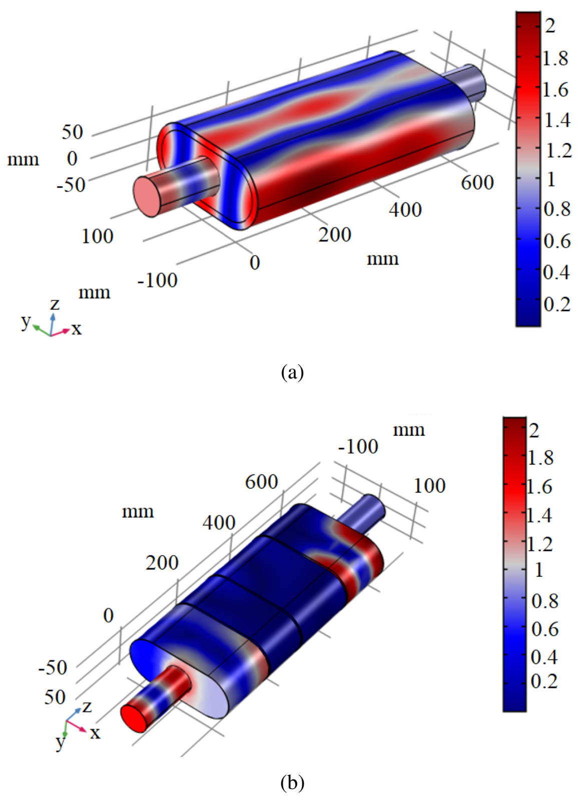
Figure 10. Typical and optimal muffler triangular meshes

Comparing the computation time of the muffler before and after improvement showed that the undeveloped computational process took 45 seconds, while this time was around 45 minutes and 45 seconds for the developed muffler. Consequently, it is enough to prove the complexity of the improved muffler. Considering the frequency range, interestingly, both studied mufflers experienced the same amount, between 50 and 150 HZ, with step counts of 25. Also, Figure 12 illustrates the simulated absolute pressure of flowing air through mufflers, which is exported through the second port. It is worth mentioning that the red region is the region with the highest pressure. Considering the developed muffler, as can be seen, the area of the red region decreased and that of the blue region increased in comparison with the undeveloped muffler. In addition, as can be seen from Figure 13 illustrates the sound pressure level around the muffler boundaries before and after optimization. Figure 14, which illustrates the acoustic pressure of mufflers, due to internal partitions, sound level was divided and decreased, and the acoustic pressure of the undeveloped muffler was noticeably high in comparison with the muffler after improvement. Comparing improved and unimproved mufflers revealed that while the local pressure deviation from ambient caused by sound waves was considerably high in a typical muffler, interestingly, comparing the red segment in the figure showed that the amount of sound level was decreased inside the muffler after improvement.



(a) (b)

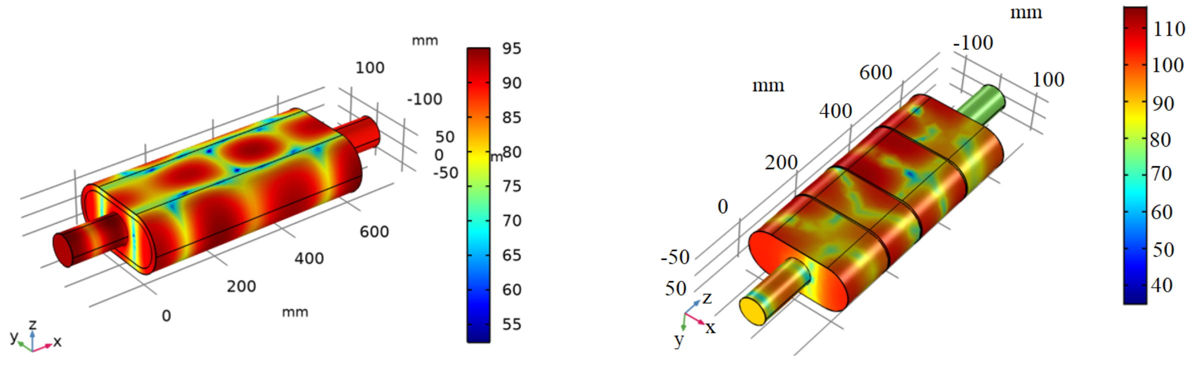
Figure 11. a) Typical muffler mesh, b) Optimal muffler mesh



(a)

(b)

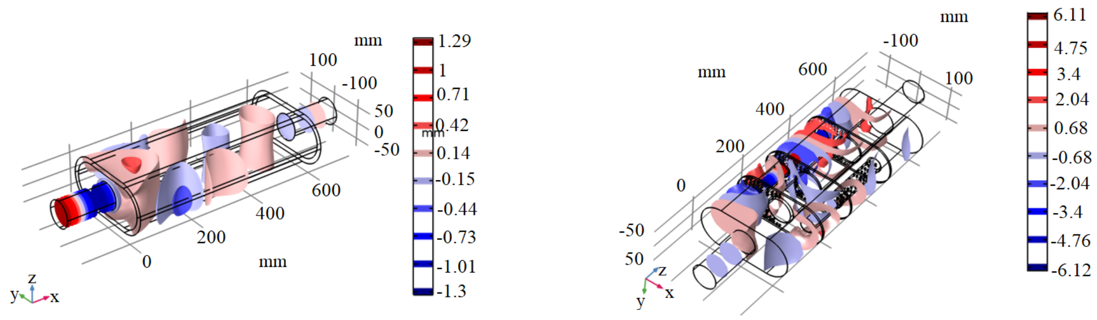
Figure 12. Absolute pressure of flow air of a) typical and b) optimal mufflers



(a)

(b)

Figure 13. Sound pressure level around a) typical and b) optimal mufflers boundary



(a)

(b)

Figure 14. Total acoustic pressure field (Pa) of a) typical and b) optimal mufflers

Figure 15 and Figure 16 show transmission loss in terms of the frequency of nonlinear and absorptive linear mufflers in typical and optimal mufflers, respectively. It is noticeable from the graph's transmission loss that the absorptive linear of developed mufflers decreased in comparison with undeveloped mufflers. As can be seen, both the linear and absorptive linear of typical and optimal mufflers saw a fluctuation trend. By comparing the diagrams, approximately between the frequencies of 200 and 1300 Hz, the power of the wave witnessed a steady trend of fluctuation. Apparently, although power at the frequency of 1300 Hz picked up the hit in both graphs, transmission loss in both experienced a considerable decrease. Consequently, it shows that adding the tube and holes inside the typical muffler can have a significant and efficient influence on the attenuation process.

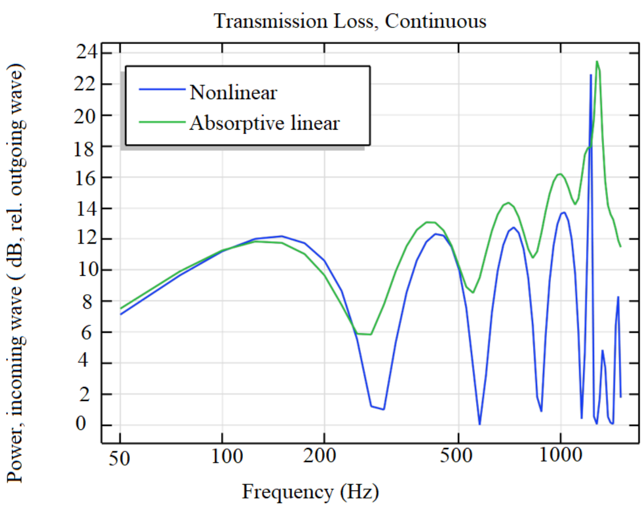


Figure 15. Transmission loss of typical muffler

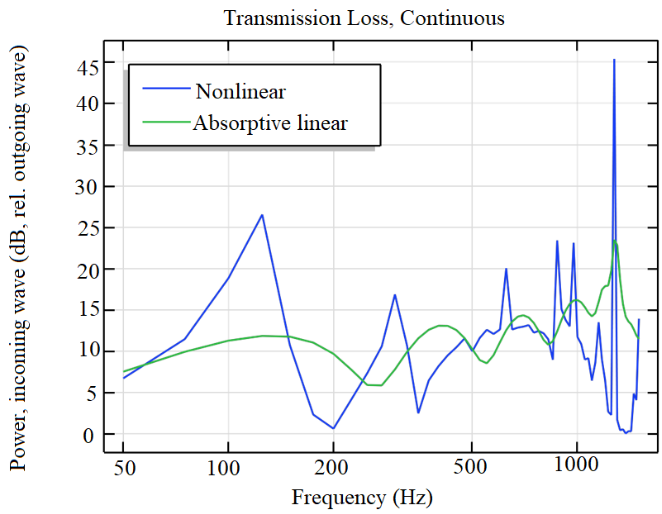


Figure 16. Transmission loss of optimal muffler

4 Conclusions

Since generally the gas discarding from the exhaust is pinned down to change direction, back pressure is generated, which can spark off failure in the exhaust. Normally, mufflers are utilized to increase sound attenuation and reduce the irreparable consequences of back pressure. The current study presents an innovative design for the muffler of an exhaust to reduce the amount of vibration and noise from the exhaust. In addition, this study attempted to compare the obtained results with muffler before improvement. Analysis by COMSOL software was carried out for developed and undeveloped mufflers. Consequently, the data and output of this study were sufficient to prove the superiority of this optimized muffler model in comparison with the typical type of muffler. Due to additional holes and pipes in the muffler's partition, the amount of toxic emitted was divided, which provides a considerable difference in acoustic pressure, transmission loss, vibration, and frequency of mufflers.

Data Availability

Not Available.

Conflicts of Interest

The authors declare no conflict of interest.

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