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Isogeometric Finite Element Analysis with Machine Learning Integration for Piezoelectric Laminated Shells

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Abstract: Innovative lightweight smart structures incorporating piezoelectric material-based active elements, both as sensors and actuators, have been identified to present manifold advantages over traditional passive systems. Such structures have become intrinsically integrated into smart mechatronic systems, necessitating advanced design, testing, and control techniques. Real-time simulation of shell-type deformable objects, especially when employing the finite element method for non-linear analysis and control, has been challenging due to the extensive computational demand. Presented herein is an efficacious implementation leveraging machine learning with the isogeometric finite element formulation. This implementation focuses on shell-like smart mechatronic structures crafted from composite laminates comprising piezoelectric layers, which are characterised by electro-mechanical coupling. The foundation for the shell kinematics is derived from the Mindlin-Reissner assumptions, effectively incorporating transverse shear effects. While the inclusion of machine learning facilitates real-time efficient operations, the isogeometric finite element analysis (FEA) introduces pronounced advantages over conventional finite element method (FEM), also serving as a valuable source of offline data crucial for the training phases of machine learning algorithms. A piezo-laminated semicircular arch has been analysed to exemplify the effectiveness and performance of the presented methodology. Explorations into further machine learning techniques and intelligent control schemes are also contemplated.

Keywords: Machine learning, Isogeometric analysis (IGA), Laminated structure, Smart mechatronic systems, Piezoelectric shells, Intelligent control

1 Introduction

Modern architectural constructs are meticulously designed to serve specific functions. In this endeavour, a preference for lightweight structures is observed, and this inclination has been facilitated by the advent of advanced fibre-reinforced composite materials. Such materials not only enhance the weight-to-stiffness ratio of these structures but also contribute to a reduction in operational costs. The incorporation of intricate multifunctional materials, including piezoelectric substances and shape memory alloys, further augments structural capabilities. By introducing these active elements, structures are rendered adaptive, often earning them the moniker of "smart." These smart structures, rather than passively conforming to deformations, actively adapt to prevailing conditions. These adaptive frameworks comprise components for structural monitoring (sensors), signal processing (controllers), and those influencing structural behaviour (actuators).

Concurrent advancements in mechatronics and intelligent materials have led to the birth of intelligent mechatronic systems. These systems amalgamate the principles of intelligent structures with mechatronic applications. Designing, testing, and controlling these sophisticated systems requires the integration of advanced finite element methods and computational intelligence.

Khan et al. [1] explored the intersection of machine learning and smart structures by leveraging a deep learning framework to detect delamination in smart composite laminates using low-frequency structural vibration responses. A convolutional neural network (CNN) was employed to autonomously extract features from spectrograms, and impressive accuracies were achieved in differentiating between healthy and compromised laminates. In a parallel vein, Badarinath et al. [2] introduced a method that amalgamated machine learning algorithms with FEA, aiming to optimise maintenance schedules in mechanical systems. Machine learning regression models, particularly artificial neural networks (ANN), were deployed and demonstrated precise predictions of time-varying stress distributions and mechanical states, thereby enhancing operational safety and efficiency. Perfetto et al. [3] furthered this fusion by integrating machine learning models with FEA, leading to the creation of a wave-based ANN. This ANN, once trained with Finite Element Method data, displayed remarkable precision in detecting damage within aluminium and composite plates.

Piezoelectric active thin-walled structures have attracted significant attention in recent academic discourses. Central to this discussion is the search for precise numerical tools adept at modelling and simulating these structures. A noteworthy contribution in this field was presented in the study [4], where an isogeometric finite element formulation was tailored for composite laminated shells embedded with piezoelectric layers exhibiting electro-mechanical coupling. Another pivotal study [5] examined IGA, a nuanced subset of FEA, in the context of active composite laminates integrating piezoelectric layers. IGA, by harnessing the direct representation capabilities of Non-Uniform Rational B-Splines (NURBS) from computer-aided design (CAD), obviates the need for geometric approximations and mesh generation. As a result, enhanced analytical precision and efficiency are attained. The formulated isogeometric shell, based on the Reissner-Mindlin kinematic model, facilitates active behaviours through its electro-mechanical coupling capabilities.

Further insights into the expansive FEM landscape were provided in the study [6], emphasizing real-time simulations of deformable shell structures. Neural networks were utilized for the real-time prediction of thin-walled structural behaviours within FEM models, marrying offline FEM computations with real-time neural network estimations. Consequently, rapid generation of displacements under specific loads was achieved. These neural networks, when trained using data reflecting the mechanical responses of materials, prove invaluable for stress and displacement estimations, thus providing pivotal insights for design and optimisation processes [7–9].

A profound integration of machine learning techniques with IGA emerges as a salient objective of the discourse. Neural networks trained with data sourced from the FEM—representing structural displacement and voltage responses to applied forces—promise swifter, more accessible results extraction. This potential integration offers intriguing prospects for piezostructures, particularly piezoelectric laminated shells, merging active piezoelectric materials with conventional structural components [10, 11].

The core aspiration of this discourse is to forge a system adept at simulating real-time behaviours in piezoelectric shell structures. Central to this vision is the seamless amalgamation of the Isogeometric Finite Element Method with machine learning methodologies, primarily ANN. In the proposed schematic, the neural network expedites the retrieval of essential results for stipulated loads. Training is undertaken using raw numerical outputs from the IGA method, effectively mapping the structure's displacement and sensor voltage trajectories under applied load dynamics.

The subsequent sections of this exposition are organised thusly: Section 2 delineates the modelling of the shell structure via nonlinear FEM analysis. Section 3 proposes a machine learning-augmented isogeometric FEA of smart piezoelectric shells. Insights and avenues for further research are explored in Section 4, with conclusions drawn in Section 5.

2 Methodology

2.1 Isogeometric Shell Formulation and FEA for Shell Structure Modelling

Understanding the behaviour of thin-walled structures is pivotal when aiming to simplify their complex three-dimensional characteristics into a two-dimensional representation. Such simplifications have been demonstrated to enable more efficient representations and analyses of these structures [12]. Thin-walled structures are acknowledged to accommodate various configurations, including those exhibiting irregular curvatures, a facet often referred to as a "general shape". Regardless of the nature of the applied loads, these structures invariably undergo both membrane strains—alterations in shape due to forces parallel to the surface—and flexural strains, the changes resulting from bending forces [12].

When these thin-walled structures comprise composite laminates, the adoption of two-dimensional theories that factor in transverse shear strains and stresses becomes indispensable. Such considerations cover changes in configuration and the internal forces that appear perpendicular to the plane of the structure. To provide a precise depiction of the behaviour of these structures, incorporating these aspects into modelling is deemed necessary. One notable theory frequently applied in this context is the First-order Shear Deformation Theory (FSDT). This theory, rooted in the Mindlin-Reissner kinematical assumptions, posits that transverse shear strains and stresses maintain consistency across the structure's thickness. Such an assumption has been identified to simplify the structure's mathematical modelling, thus aiding in straightforward analyses and predictions of its diverse responses [13].

In the domain of IGA, the model's geometry is described through functions, with NURBS being prominent, a technique often adopted in CAD processes. By utilising the same functions to detail both geometry and solution field, a seamless integration between the CAD model and FEA is ensured.

A NURBS curve is defined by a combination of control points and a knot vector. While the control points delineate the curve's shape, the knot vector ascertains the points and nature of the curve's bends. The subsequent mesh is generated by segmenting the curve into elements, a division determined by the knots. The underlying basis for NURBS functions is the Cox-de Boor recursion formula, an established technique for delineating such curves.

• for degree 0:

$$N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

• for degree p > 0:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,\,p-1}(\xi) \tag{2}$$

In the realm of IGA, NURBS basis functions have been articulated as:

$$C(\xi) = \frac{\sum_{i=0}^{n} N_{i,\,p}(\xi) w_i P_i}{\sum_{i=0}^{n} N_{i,\,p}(\xi) w_i} = \sum_{i=0}^{n} R_{i,\,p}(\xi) P_i, \quad a \leq \xi \leq \mathrm{b} \tag{3}$$

where, w_i represents the weights associated with the control points. This mathematical expression plays a pivotal role in ensuring the robust integration between CAD geometry and FEA. An integral step of this integration, driven by IGA, revolves around the meshing process. The initial number of elements along each axis can be described as:

$$n_e = (n - p)(m - q) \tag{4}$$

Here, m and n denote the count of control points or basic functions across the ξ and η axes respectively, with p and q defining their respective degrees.

To facilitate the creation of a mesh for two or three-dimensional models, NURBS surfaces or volumes, defined by a control polygon with constituent points P_i, are employed. The role of these points is instrumental in deducing and visualising the surface within a physical space as opposed to the parameter space governed by parameters ξ and η, as illustrated in Figure 1.

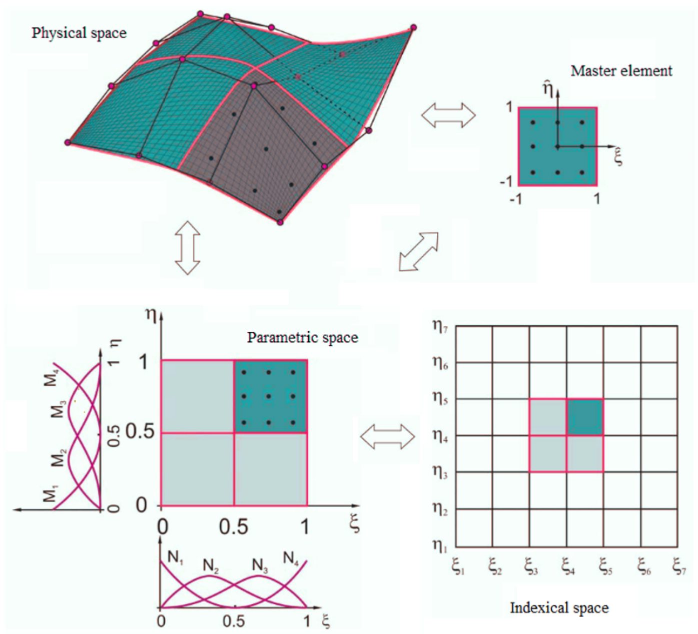

Figure 1. Illustrating the intricate relationship between indexical, parametric, and physical spaces in IGA

The control points are systematically arranged into a grid formation. Knot vectors, in turn, are delineated in each grid direction. The foundational mesh can be modified using a spectrum of techniques. The h-refinement process is identified to add knots to the knot vectors, whereas p-refinement elevates the degree of the basic functions. Conversely, k-refinement merges both methods, initially enhancing the basic functions' degree followed by the addition of a novel knot to the knot vectors. This consolidated approach primarily aims at refining the continuity at element boundaries through concurrent adjustments to the basic functions' degree and mesh density.

For heightened computational efficiency and accuracy, segmentation of the overarching NURBS into smaller constructs, termed as 'elements', is executed. This segmentation task involves the translation of element boundaries from the parameter space to the physical domain, giving rise to segments of the NURBS surface. The demarcated elements fall within defined half-open intervals, exemplified by $\xi \in [\xi_i, \xi_{i+1})$ and $\eta \in [\eta_i, \eta_{j+1})$.

Vectors normal to the reference surface at the control polygon points must be identified for the construction of a finite element model employing the isogeometric approach with shell elements [14]. Contrary to traditional finite element methods where nodes are posited on the reference surface, in IGA, a majority of control polygon points are observed off the reference surface, devoid of a singular projection onto it. Among the methodologies devised to pinpoint the normal vector to the reference surface at a control polygon point, the Closest Point Projection has been reported. This technique determines the minimal distance between the control polygon point and the reference surface, typically harnessing iterative methodologies like the Newton-Raphson process. Upon discovery of the projection point, an orthonormalized coordinate system is subsequently established [4].

Furthermore, the Exact Basis Systems Calculation method is often leveraged. This approach capitalises on the fact that the reference surface's normal vectors can be ascertained at integration points through two avenues: either via the position vector's derivative with respect to the variable ξ or η, or through interpolation over the control polygon points' normal vector. Such a procedure culminates in a system of $(p + 1)(q + 1)$ equations for every component of all element control points. However, it has been observed that as the degree of the basic functions escalates, the complexity of the normal vector calculation also witnesses an augmentation.

2.2 Nonlinear Analysis of Piezo-Laminated Semicircular Arch

An evaluation was conducted on a piezoelectric laminated semi-cylindrical shell, as depicted in Figure 2, to assess the efficacy of the proposed machine learning approach and to draw comparisons with results from direct FEM of varied formulations [15]. The structure under consideration is distinguished by its central metallic layer, which acts as a core. Sandwiching this metallic core are two piezoelectric (PZT) layers, affixed to its external and internal surfaces.

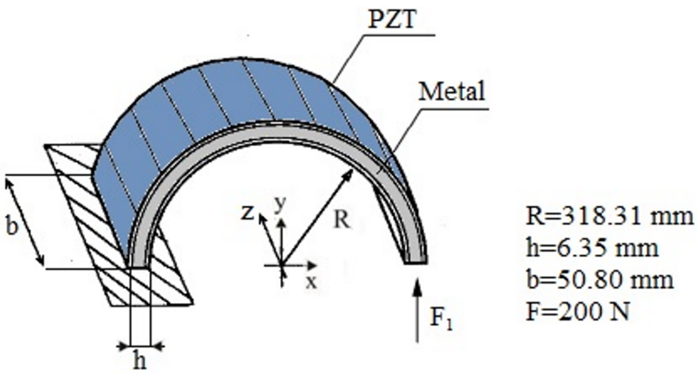

Figure 2. Schematic representation of the piezoelectric laminated semi-cylindrical shell

Dimensional assessments revealed that the core metallic layer possesses a thickness of 5.842 mm. The core's material properties were characterised by a Young's modulus of 68.95×10^3 and a Poisson's ratio of 0.3. Adjacent to this central layer, the piezoelectric layers were identified, each demonstrating a thickness of 0.254 mm. The characterisation of these layers revealed a Young's modulus of 63×10^3, a Poisson's ratio of 0.3, a piezoelectric constant e31 (identical to e32) of 16.11×10^{-6}, and a dielectric constant of 1.65×10^{-11}.

From an examination of boundary conditions, it was noted that one straight edge of the semi-cylindrical shell is restrained, while its opposite edge remains unrestrained. A force, with magnitudes described in various studies to lie between 100N and 200N, is exerted vertically on this unrestrained edge, instigating deformation. Close observation was conducted on the displacements, particularly at the unrestrained edge, in both radial and circumferential orientations. Furthermore, the intrinsic piezoelectric nature of the external layers meant that an induced electrical potential was detected upon the application of the force, with the values being documented for subsequent comparative evaluations.

In efforts to model the behaviour of this laminated structure, varied computational methods have been employed across studies. For instance, Zhang [16] leveraged a discretization method with 1×10 elements in the relevant directions. Another model [17] consisted of 160 triangular elements, whereas in the study [4], a NURBS model, utilising quadratic base functions, was employed with a defined count of elements in both width and hoop orientations.

It was discerned that the forces, which emerge from piezoelectric coupling and are influenced by it, are contingent upon the structural configuration. The new increment of actuating bending moments was deduced from the piezoelectric coupling stiffness matrix, integrated over the extant structural configuration. Both the intensity and directionality of the induced loads depend on the current configuration and are termed as follower forces. Given the follower nature of these forces or moments, minuscule increments in the electric voltage became requisite. Some approaches viewed the scenario as solely mechanical, disregarding follower type loads.

In the domain of nonlinear analysis regarding the piezoelectric laminated semi-cylindrical shell element, when juxtaposed with the innovative isogeometric method, discrepancies between computations with ACShell9 and NURBS finite elements were found to be minimal, with variations of approximately 0.1% [4].

3 Machine Learning Integrated Isogeometric FEA of Intelligent Piezoelectric Shells

To address the challenges presented by the robust analysis capabilities of the FEM and to optimise the computation time required, an integration of the nonlinear isogeometric FEM with a standard feed-forward, back-propagation multilayer perceptron (MLP) has been proposed. This integration facilitates a machine learning-based isogeometric FEA.

For the analysed case of a piezoelectric laminated semi-cylindrical shell, as illustrated in Figure 2, an ANN mirroring the isogeometric FEA was designed. Its purpose was to predict three primary outputs: displacement in the radial direction, displacement in the hoop direction, and sensor voltage. Predictions were made based on two input variables: the magnitude of the applied force and its point of application along the arch of the shell (with the force directed vertically upwards).

The architecture of this network was strategically chosen to capture the complex, non-linear relationship between the force variables and the consequent structural behaviours. Initialisation of the neural network involved the specification of three hidden layers, each containing ten neurons, as depicted in Figure 3.

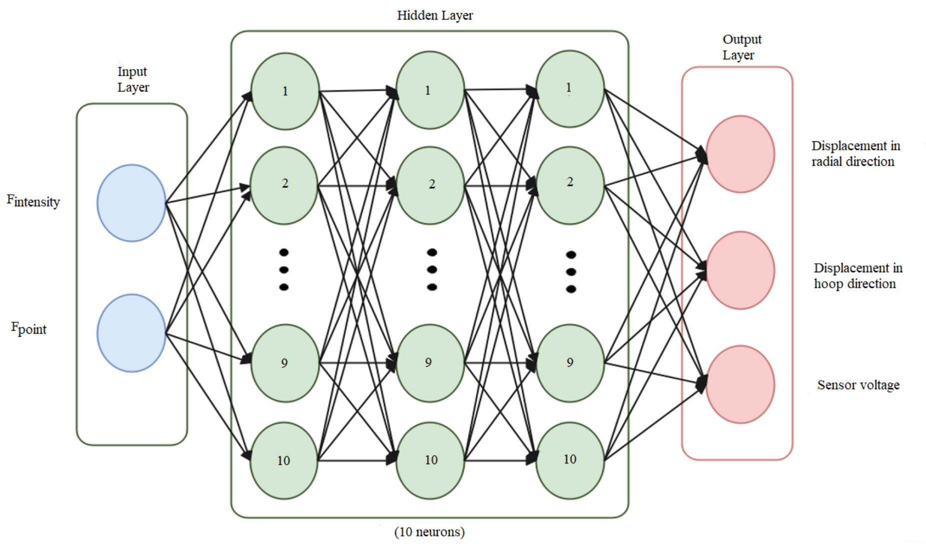

Figure 3. Feed-forward MLP with 2 input variables, 3 hidden layers with 10 neurons each, and 3 output variables

This chosen architecture was determined to strike a balance between the complexity of the model and the risk of overfitting, taking the input parameters into account [18]. The Scaled Conjugate Gradient (SCG) back-propagation algorithm, a variant of the Conjugate Gradient (CG) method, was employed for network training. The SCG algorithm enhanced the CG method by infusing a scaling factor into the update rule, aiding in step size modification contingent on the curvature of the error surface. The mathematical representation for the update rule of the SCG algorithm is as follows:

$$p(k+1) = -g(k+1) + \beta(k) * p(k) \tag{5}$$

where, $p(k)$ signifies the search direction at iteration k, $g(k)$ is the gradient at iteration k, and $\beta(k)$ is a scaling factor determined by the curvature of the error surface [19].

The utilised dataset comprised 150 data entries, each containing two input variables (force intensity and application point) and three target outcomes: radial displacement, hoop displacement, and sensor voltage. These outcomes were meticulously obtained through NURBS FEA. The activation functions were selected, with ReLU being deployed in the hidden layers due to its efficiency and ability to introduce non-linearities. Simultaneously, a linear activation function was adopted in the output layer, considering the continuous nature of the target variable.

Hyperparameters were judiciously configured, setting a learning rate of 0.001 to foster consistent convergence and extending training over 200 epochs, ensuring a balance between learning capability and overfitting risk. Given the dataset's size, a batch size of 5 was deemed optimal to balance stability and convergence rate. The Adam optimizer was employed, renowned for its adaptability, especially with smaller datasets. With the regression task in mind, the Mean Squared Error (MSE) was chosen as the loss function. The addition of L2 regularization, with a coefficient of 0.01, coupled with He initialization for the initial weights, further enhanced training robustness.

Data segmentation occurred randomly into training, validation, and testing sets. The training set, encompassing 70% of the data, was presented to the neural network during training phases, adjusting the network based on estimation errors. The validation set, 15% of the data, evaluated network generalization throughout training, halting the process when generalization ceased to improve. In contrast, the test set, also 15% of the data, remained untouched during training, providing an independent performance measure of the neural network post-training.

Lastly, performance metrics such as the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) were employed on the validation set, yielding critical insights into the model's capacity for generalization. With meticulous implementation, this strategy offers a tailored MLP fitting the dataset and problem nuances.

4 Comparative Analysis and Prospective Research Directions

Upon juxtaposition of results derived from the ANN simulations with findings from prior studies depicted in Figure 4, a significant alignment was observed with the data presented by Zhang [16], as well as with results previously documented in studies [4, 17]. Remarkably, discrepancies of merely 0.01% were identified when contrasting the trained model with the NURBS finite analysis method. Such minimal divergence underscores the impressive concordance between the two methods. Further, the inherent parallel processing capability of the neural network ensures expedited execution during the application phase. This swift functionality renders the system amenable for real-time operations and potential integrations within isogeometric FEM loop simulations, thereby serving as a crucial component within the control loop. The proposed machine learning integration of isogeometric FEM for intelligent structures is posited as a pivotal avenue for future investigations.

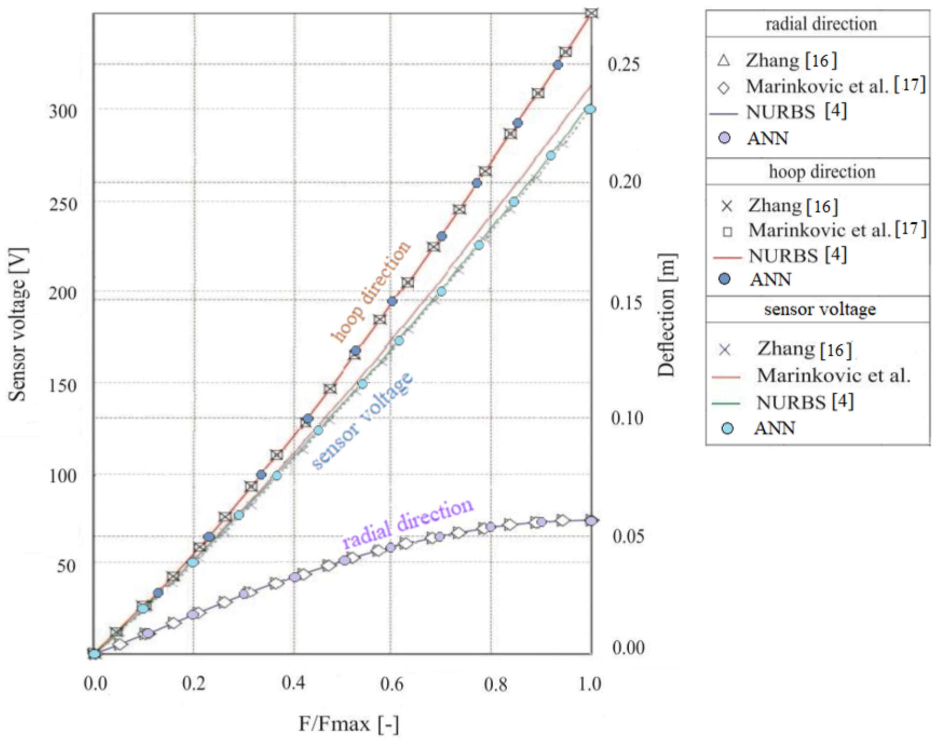

Figure 4. Comparing the analysis of the sensor voltage of the inner piezolayer and the displacement of the free arc edge tip in the radial and hoop directions

For an exhaustive appraisal, consideration of supplementary analytical techniques is advocated. The incorporation of cross-validation might be pivotal in discerning the model's robustness across varied data subsets, ensuring that its efficacy isn't disproportionately contingent on the demarcation of training and validation data. Furthermore, residual analysis could be instrumental in pinpointing systematic deviations in model predictions, potentially unearthing biases or latent issues within the model structure. An amalgamation of these evaluative tools, inclusive of correlation coefficient assessment, could proffer a multifaceted comprehension of the model's aptness for the designated assignment.

Whilst the current machine learning paradigm adeptly navigates intricate non-linear associations, its opaqueness might label it as a "black box" model. Prospective investigations could delve into methodologies aimed at enhancing model transparency, such as scrutinising the weightage of individual predictors in influencing outcomes. Gaining a deeper understanding of influential variables might unearth invaluable insights into intrinsic physical mechanisms, subsequently guiding strategic design or control implementations.

In light of the data's complexity, there is an impetus to extrapolate the findings of this study towards more advanced frameworks such as deep learning architectures. Such an extension could potentially bolster predictive accuracy while affording a more granular insight into the nuanced interrelationships embedded within the dataset [20].

5 Conclusion and Implications

In the investigation at hand, the feasibility of amalgamating machine learning techniques, notably MLP ANN, with isogeometric FEA for the meticulous and expedient examination of smart piezoelectric shell structures was elucidated. The advanced approach, as demonstrated, capitalises on the merits of both paradigms: it harnesses the computational agility and real-time functionality of machine learning and melds it with the intricate and precise modelling prowess inherent to isogeometric FEA.

The ensuing results, centred around a simply supported plate endowed with piezoelectric layers, have compellingly shown the proficiency of the ANN in discerning the multifaceted, non-linear interplay among active force, shell deflection, and output sensor voltage. Evaluation of the network's efficacy was undertaken via a correlation coefficient (R) metric, shedding light on a discernable alignment between the prognosticated and verifiable values. Such findings suggest that the MLP possesses the capability to anticipate voltage fluctuations and deflections—both radial and hoop in nature—requisite for reinstating the arc to its primordial configuration, contingent on the magnitude and locus of the imposed force.

Yet, while the inherent power of the ANN in real-time predication of the behaviour of smart piezoelectric shell structures cannot be understated, its modus operandi echoes the opaqueness of a "black box" model. Subsequent studies may be inclined towards augmenting the model's transparency, potentially by probing into the salience of individual predictors in rendering outcomes. Such explorations could unearth pivotal insights into intrinsic physical mechanisms, thus refining design and modulatory blueprints for intelligent mechatronic infrastructures. It is essential that the resilience of this novel method be ascertained through diversified case analyses and myriad shell structure archetypes. Both cross-validation and residual scrutiny might be pivotal for a rounded critique of the model's aptness.

Given sustained evolution and rigorous validation, the emergent machine learning-centric methodology for examining active piezoelectric shells might indeed revolutionise the ideation, experimentation, and modus operandi of advanced mechatronic systems, catalysing the inception of supremely efficient and state-of-the-art smart constructs.

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Data Availability

The data used to support the findings of this study is available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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