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# Enhanced Decision-Making Through Induced Confidence-Level Complex Polytopic Fuzzy Aggregation Operators

Khaista Rahman[1*], Jan Muhammad[2]

[1] Department of Mathematics, Shaheed Benazir Bhutto University, 1800 Upper Dir, Pakistan
[2] Department of Mathematics Shanghai University, 200444 Shanghai, China

[*] Correspondence: Khaista Rahman (khaista@sbbu.edu.pk)

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**Abstract:** This study introduces novel aggregation operators aimed at enhancing data analysis and decision-making processes through the induction of confidence levels into complex polytopic fuzzy systems. Specifically, the induced confidence complex polytopic fuzzy ordered weighted averaging aggregation (ICCPoFOWAA) operator and the induced confidence complex polytopic fuzzy hybrid averaging aggregation (ICCPoFHAA) operator are proposed. By integrating confidence levels into the aggregation process, these operators facilitate a more nuanced interpretation of fuzzy data, allowing for the incorporation of expert judgment and uncertainty in decision-making frameworks. A practical demonstration is provided to validate the efficacy and proficiency of these innovative techniques. Through a comprehensive example, the ability of the ICCPoFOWAA and ICCPoFHAA operators to enhance decision-making accuracy and reliability is substantiated, showcasing their potential as powerful tools in the realms of data analysis and complex decision-making scenarios. The incorporation of confidence levels into fuzzy aggregation processes represents a significant advancement in the field, offering a sophisticated approach to handling uncertainty and expert opinions in multi-criteria decision-making problems. This work not only introduces groundbreaking aggregation operators but also sets a new standard for research in fuzzy decision-making, underscoring the importance of confidence levels in the analytical process.

**Keywords:** Confidence level; Complex polytopic fuzzy systems; Decision-making process; Fuzzy aggregation operators

## 1 Introduction

Zadeh's fuzzy set (FS) [1] theory has found diverse applications across various fields due to its ability to handle uncertainty and imprecision. In control systems, particularly in industrial automation and robotics, fuzzy logic controllers provide a flexible framework for managing complex and non-linear processes. In medical diagnosis, Zadeh's theory has been applied to capture the uncertainty inherent in medical data, aiding in more nuanced and personalized patient assessments. FS theory encounters limitations when dealing with data that involves both unsatisfactory and satisfactory information of interest. In such cases, the conventional FS framework may struggle to adequately represent the nuanced and varied degrees of satisfaction or dissatisfaction within the data.

To address the limitations of FS, Atanassov [2] introduced the concept of intuitionistic fuzzy sets (IFSs). The idea behind IFSs is to extend the FS theory by introducing an additional parameter that represents the degree of non-membership or hesitation associated with an element's membership in a set. In IFSs, each element can be presented as $(r, s)$ with $0 \leq r+s \leq 1$. Yager [3] introduced the Pythagorean fuzzy sets (PyFSs), which are extensions of fuzzy sets. In PyFSs, each element can be presented as $(r, s)$ with $0 \leq r^2 + s^2 \leq 1$. Sanapati and Yagar [4] introduced the Fermatean fuzzy sets (FFSs). In FFSs, each element can be presented as $(r, s)$ with $0 \leq r^3 + s^3 \leq 1$. Later on, Yager [5] introduced the more generalized form of the all-mentioned modes, which is called the q-rung orthopair fuzzy sets (q-ROFSs). Each element can be mathematically presented as $(r, s)$ with $0 \leq r^q + s^q \leq 1$, where $q$ is any positive real numbers. They provide a flexible framework for expressing varying degrees of membership and non-membership within specified intervals.

In the context of the mentioned models, researchers have devised various aggregation operators to combine and integrate information derived from FSs, IFSs, PyFSs, FFSs, and q-ROFSs. These aggregation operators play a crucial role in group decision-making processes, where multiple opinions or pieces of information need to be synthesized to reach a collective decision. Scholars have explored and developed these operators to effectively handle uncertainties and complexities inherent in real-world decision scenarios. Several researchers, including Xu [6], Xu and Yager [7], Wang et al. [8, 9], Rahman et al. [10], and Rahman [11], have contributed significantly to the field of aggregation operators by employing intuitionistic fuzzy numbers. They have introduced various aggregation techniques to handle uncertainties and imprecisions in decision-making scenarios. Additionally, Garg [12, 13] and Rahman et al. [14–17] have extended their work by utilizing Pythagorean fuzzy numbers, introducing novel techniques that leverage these numbers for improved decision-making strategies. Liu and Wang [18] has made contributions by introducing the q-ROFWA operator and the q-ROFWG operators, which are novel aggregation operators designed to work with specific mathematical structures. Furthermore, Peng and Liu [19] have presented innovative measures such as entropy, inclusion, and distance measures. These measures are formulated using q-rung orthopair fuzzy numbers, extending the applicability of their techniques to scenarios involving uncertainties described by this mathematical framework.

The existing studies on fuzzy sets have proven valuable in addressing uncertainties in various real-world problems. However, one notable limitation is their inability to effectively handle periodic information. In our daily lives, many datasets involve not only ambiguity and vagueness but also variations in periodic patterns. This is especially relevant in complex datasets related to facial recognition, image analysis, health analysis, biometric databases, and audio processing. To navigate through challenging situations, Ramot et al. [20] introduced a concept known as complex fuzzy sets (CFSs). This innovative approach was devised as a solution to address specific circumstances or difficulties. The complex fuzzy set extends the traditional notions of fuzzy sets, providing a more sophisticated framework to handle complex and intricate scenarios. Later, Alkouri and Salleh [21] introduced complex intuitionistic fuzzy sets (CIFs). In CIFS, each element can be mathematically presented as: $\left(re^{i2\pi x}, se^{i2\pi y}\right)$ with $0 \prec r+s \leq 1$ and $0 \prec \frac{x}{2\pi}+\frac{y}{2\pi} \leq 1$. Ma et al. [22], Rani and Garg [23], Kumer and Bajej [24], and Garg and Rani [25] have contributed to this area with their respective studies. Ullah et al. [26] introduced the concept of complex Pythagorean fuzzy sets (CPyFSs), in which each element can be presented as: $\left(re^{i2\pi x}, se^{i2\pi y}\right)$ under $0 \prec r^2+s^2 \leq 1$ and $0 \prec \left(\frac{x}{2\pi}\right)^2+\left(\frac{y}{2\pi}\right)^2 \leq 1$. Rahman et al. [27], Hezam et al. [28], Rahman and Iqbal [29], and Rahman et al. [30] have proposed numerous aggregation operators that rely on CPyFNs. These operators are designed to facilitate the combination of information represented by CPyFNs, offering a more comprehensive and sophisticated approach to aggregating uncertain and imprecise data. Some related works have been found in the studies [31, 32]. Rahman [33] introduced the Complex Polytopic fuzzy set (CPoFS), and its corresponding aggregation operators and applied them to group decision-making. These innovative approaches represent a more generalized form of fuzzy sets.

Thus, keeping the applications of the mentioned models and their corresponding techniques, in this paper, we introduced the induced complex Polytopic fuzzy aggregation operators based on confidence level, namely the I-CCPoFOWAA operator, and the I-CCPoFHAA operator. Finally, consider a practical example to illustrate the efficiency and effectiveness of the proposed techniques.

The subsequent sections of this paper are organized as follows: In Section two, fundamental definitions will be presented. Section three will elaborate on basic operational laws. The fourth section will delve into induced operators. Section five will showcase the application of the operators. Moving on to Section six, an illustrative example will be provided. Finally, in Section seven, a conclusion will be drawn based on the presented content.

## 2 Preliminaries

This section presents foundational definitions pivotal to the research.

**Definition 1:** [20] Let C be a CFS, then mathematically, it is represented over a universal set $Z$ as:

$C = \left\{z, r(z)e^{ix(z)} \,|z \in Z\right\}$, where $r(z) : Z \to [0,1]$, and $r(z)$ is a complex number associated with each element for all $z \in Z$.

**Definition 2:** [21] Let $I$ be a CIFS, then mathematically, it is represented over a universal set $Z$ as:

$I = \left\{\left\langle z, r(z)e^{ix(z)}, s(z)e^{iy(z)}\right\rangle \middle| z \in Z\right\}$, where $r(z) : Z \to [0,1]$, $s(z) : Z \to [0,1]$, $x(z) \in [0,2\pi]$, $y(z) \in [0,2\pi]$ with $0 \prec r(z)+s(z) \leq 1$ and $0 \prec \frac{x(z)}{2\pi}+\frac{y(z)}{2\pi} \leq 1$.

**Definition 3:** [26] Let $P$ be a CPyFS, then mathematically, it is represented over a universal set $Z$ as:

$P = \left\{\left\langle z, r(z)e^{ix(z)}, s(z)e^{iy(z)}\right\rangle \middle| z \in Z\right\}$, where $r(z) : Z \to [0,1]$, $s(z) : Z \to [0,1]$, $x(z) \in [0,2\pi]$, $y(z) \in [0,2\pi]$ with $0 \prec r^2+s^2 \leq 1$ and $0 \prec \left(\frac{x}{2\pi}\right)^2+\left(\frac{y}{2\pi}\right)^2 \leq 1$.

The above models defined the membership and non-membership degrees. But in CPoFS, we discussed the membership function, the neutral function, and the non-membership function.

**Definition 4:** [33] Let $L$ be a CPoFS, then it can be represented over a universal set $Z$ as:

$L = \left\{\left\langle z, r(z)e^{ix(z)}, g(z)e^{iy(z)}, s(z)e^{iz(z)}\right\rangle \middle| z \in Z\right\}$, where $r(z) : Z \to [0,1]$, $g(z) : Z \to [0,1]$, $s(z)$ :

$Z \to [0,\ 1]$, $x(z) \in [0, 2\pi]$, $y(z) \in [0, 2\pi]$, $z(z) \in [0, 2\pi]$ with $0 \prec (r)^q + (g)^q + (s)^q \leq 1$ $(1 \leq q)$, and $0 \prec \left(\frac{x}{2\pi}\right)^q + \left(\frac{y}{2\pi}\right)^q + \left(\frac{z}{2\pi}\right)^q \leq 1$.

Next, we defined score and accuracy functions, which are used to quantify the quality of a model.

**Definition 5:** [33] Let $R = \left(re^{ix}, ge^{iy}, se^{iz}\right)$ be a CPoFV, then the score and accuracy functions can be written mathematically as:

$S(R) = \frac{1}{3}\left[(1 + r^q + g^q - s^q) + (1 + x^q + y^q - z^q)\right]$ with $S(R) \in [-2, 2]$ and

$A(R) = \frac{1}{2}\left[(1 + \max(r^q, g^q) - s^q) + (1 + \max(x^q, y^q) - z^q)\right]$ with $A(R) \in [0, 2]$ respectively.

## 3 Basic Operational Laws

In this section, foundational laws are presented, which provide a robust and flexible framework essential for managing uncertainty, representing complex systems, and enhancing decision-making processes across various fields.

**Definition 6:** Let $R_j = \left(r_j e^{ix_j}, g_j e^{iy_j}, s_j e^{iz_j}\right)$ $(1 \leq j \leq 2)$ be a group of CPoFVs and $\hbar \succ 0$, then

i) $R_1 \oplus R_2 = \left( (r_1^q + r_2^q - r_1^q r_2^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{x_1}{2\pi}\right)^q + \left(\frac{x_2}{2\pi}\right)^q - \left(\frac{x_1}{2\pi}\right)^q \left(\frac{x_2}{2\pi}\right)^q\right)^{\frac{1}{q}}}, (g_1 g_2) e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, (s_1 s_2) e^{i2\pi\left(\frac{z_1}{2\pi}\right)\left(\frac{z_2}{2\pi}\right)} \right)$

ii) $R_1 \otimes R_2 = \left( (r_1 r_2) e^{i2\pi\left(\frac{x_1}{2\pi}\right)\left(\frac{x_2}{2\pi}\right)}, (g_1 g_2) e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, (s_1^q + s_2^q - s_1^q s_2^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{z_1}{2\pi}\right)^q + \left(\frac{z_2}{2\pi}\right)^q - \left(\frac{z_1}{2\pi}\right)^q \left(\frac{z_2}{2\pi}\right)^q\right)^{\frac{1}{q}}} \right)$

iii) $\hbar(R) = \left( \left(1 - (1 - r^q)^\hbar\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{x}{2\pi}\right)^q\right)^\hbar\right)^{\frac{1}{q}}}, (g)^\hbar e^{i2\pi\left(\frac{y}{2\pi}\right)^\hbar}, (s)^\hbar e^{i2\pi\left(\frac{z}{2\pi}\right)^\hbar} \right)$

iv) $(R)^\hbar = \left( (r)^\hbar e^{i2\pi\left(\frac{x}{2\pi}\right)^\hbar}, (g)^\hbar e^{i2\pi\left(\frac{y}{2\pi}\right)^\hbar} \left(1 - (1 - s^q)^\hbar\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{z}{2\pi}\right)^q\right)^\hbar\right)^{\frac{1}{q}}} \right)$

**Theorem 1:** Let $R_j = \left(r_j e^{ix_j}, g_j e^{iy_j}, s_j e^{iz_j}\right)$ $(1 \leq j \leq 3)$ be a family of CPoFVs, then:

i) $R_1 \oplus R_2 = R_2 \oplus R_1$

ii) $R_1 \otimes R_2 = R_2 \otimes R_1$

iii) $(R_1 \oplus R_2) \oplus R_3 = R_1 \oplus (R_2 \oplus R_3)$

iv) $(R_1 \otimes R_2) \otimes R_3 = R_1 \otimes (R_2 \otimes R_3)$

v) $R_1 \otimes (R_2 \oplus R_3) = (R_1 \otimes R_2) \oplus (R_1 \otimes R_3)$

vi) $(R_1 \oplus R_2) \otimes R_3 = (R_1 \otimes R_3) \oplus (R_2 \otimes R_3)$

**Proof:** The demonstration is provided for parts (i) and (ii) only, with the understanding that the remaining sections can be validated using a similar methodology.

i) Since $R_j = \left(r_j e^{ix_j}, g_j e^{iy_j}, s_j e^{iz_j}\right)$ $(1 \leq j \leq 2)$ are CPoFVs, then

$$\begin{aligned} R_1 \oplus R_2 &= \left( (r_1^q + r_2^q - r_1^q r_2^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{x_1}{2\pi}\right)^q + \left(\frac{x_2}{2\pi}\right)^q - \left(\frac{x_1}{2\pi}\right)^q \left(\frac{x_2}{2\pi}\right)^q\right)^{\frac{1}{q}}}, (g_1 g_2) e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, (s_1 s_2) e^{i2\pi\left(\frac{z_1}{2\pi}\right)\left(\frac{z_2}{2\pi}\right)} \right) \\ &= \left( (r_2^q + r_1^q - r_2^q r_1^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{x_2}{2\pi}\right)^q + \left(\frac{x_1}{2\pi}\right)^q - \left(\frac{x_2}{2\pi}\right)^q \left(\frac{x_1}{2\pi}\right)^q\right)^{\frac{1}{q}}}, (g_2 g_1) e^{i2\pi\left(\frac{y_2}{2\pi}\right)\left(\frac{y_1}{2\pi}\right)}, (s_2 s_1) e^{i2\pi\left(\frac{z_2}{2\pi}\right)\left(\frac{z_1}{2\pi}\right)} \right) \\ &= R_2 \oplus R_1 \end{aligned}$$

ii) Again, we have

$$\begin{aligned} R_1 \otimes R_2 &= \left( (r_1 r_2) e^{i2\pi\left(\frac{x_1}{2\pi}\right)\left(\frac{x_2}{2\pi}\right)}, (g_1 g_2) e^{i2\pi\left(\frac{y_1}{2\pi}\right)\left(\frac{y_2}{2\pi}\right)}, (s_1^q + s_2^q - s_1^q s_2^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{z_1}{2\pi}\right)^q + \left(\frac{z_2}{2\pi}\right)^q - \left(\frac{z_1}{2\pi}\right)^q \left(\frac{z_2}{2\pi}\right)^q\right)^{\frac{1}{q}}} \right) \\ &= \left( (r_2 r_1) e^{i2\pi\left(\frac{x_2}{2\pi}\right)\left(\frac{x_1}{2\pi}\right)}, (g_2 g_1) e^{i2\pi\left(\frac{y_2}{2\pi}\right)\left(\frac{y_1}{2\pi}\right)}, (s_2^q + s_1^q - s_2^q s_1^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{z_2}{2\pi}\right)^q + \left(\frac{z_1}{2\pi}\right)^q - \left(\frac{z_2}{2\pi}\right)^q \left(\frac{z_1}{2\pi}\right)^q\right)^{\frac{1}{q}}} \right) \\ &= R_2 \otimes R_1 \end{aligned}$$

## 4 Algebraic Aggregation Operators under Confidence Level

In this section, induced complex operators under the framework of confidence levels are introduced, specifically the I-CCPoFEOWGA operator and the I-CCPoFEHGA operator.

**Definition 7:** Let $(\langle u_j, R_j \rangle, \chi_j)$ $(1 \leq j \leq n)$ be a group of 2-tuple, $m = (m_1, m_2, ..., m_n)^T$ be their weights with $m_j \in [0, 1]$ and $\sum_{j=1}^{n} m_j = 1$, and let $\chi_j \in [0, 1]$ be their corresponding confidence level. Then the I-CCPoFEOWGA operator can be mathematically expressed as:

$$\text{I-CCPoFOWAA}_m\left((\langle u_1, R_1 \rangle, \chi_1), (\langle u_2, R_2 \rangle, \chi_2), ..., (\langle u_n, R_n \rangle, \chi_n)\right)$$
$$= \left( \begin{array}{c} \left(1 - \prod_{j=1}^{n} \left(1 - r_{\propto(j)}^q\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - \left(\frac{x_{\propto(j)}}{2\pi}\right)^q\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n} \left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{n} \left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array} \right)$$

**Theorem 2:** Let $\langle u_j, R_j \rangle$ $(1 \leq j \leq n)$ be a finite group of 2-tuple of CPoFVs, and $\chi_j$ $(1 \leq j \leq n)$ be their corresponding confidence level, then their resulting value by using the I-CCPoFOWAA operator is still a CPoFV, such that:

I-CCPoFOWAA$_m$ $\left(\left(\langle u_1, R_1\rangle, \chi_1\right), \left(\langle u_2, R_2\rangle, \chi_2\right), ..., \left(\langle u_n, R_n\rangle, \chi_n\right)\right)$

$$= \left(\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-r_{\propto(j)}^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{x_{\propto(j)}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}\left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{n}\left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right) \quad (1)$$

**Proof:** Theorem 2 can be proved using the mathematical principles of induction.

**Step 1:** Since, for $n = 2$, we have

$$\chi_1 m_1\left(R_1\right) = \left(\left(1-\left(1-r_1^q\right)^{\chi_1 m_1}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{x_1}{2\pi}\right)^q\right)^{\chi_1 m_1}\right)^{\frac{1}{q}}}, \left(g_j\right)^{\chi_1 m_1} e^{i2\pi\left(\frac{y_1}{2\pi}\right)^{\chi_1 m_1}}, \left(s_j\right)^{\chi_1 m_1} e^{i2\pi\left(\frac{z_1}{2\pi}\right)^{\chi_1 m_1}}\right)$$

$$\chi_2 m_2\left(R_2\right) = \left(\left(1-\left(1-r_2^q\right)^{\chi_2 m_2}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{x_2}{2\pi}\right)^q\right)^{\chi_2 m_2}\right)^{\frac{1}{q}}}, \left(g_2\right)^{\chi_2 m_2} e^{i2\pi\left(\frac{y_2}{2\pi}\right)^{\chi_2 m_2}}, \left(s_2\right)^{\chi_2 m_2} e^{i2\pi\left(\frac{z_2}{2\pi}\right)^{\chi_2 m_2}}\right)$$

Now, by applying Definition 7, we have:

I-CCPoFOWAA$_m$ $\left(\left(\langle u_1, R_1\rangle, \chi_1\right), \left(\langle u_2, R_2\rangle, \chi_2\right)\right)$

$$= \left(\begin{array}{c} \left(1-\prod_{j=1}^{2}\left(1-r_{\propto(j)}^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{2}\left(1-\left(\frac{x_{\propto(j)}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{2}\left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{2}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{2}\left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{2}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right)$$

**Step 2:** Eq. (1) is verified for $n = 2$. Next, assuming that Eq. (1) holds for $n = k$, with then it can be deduced that:

I-CCPoFOWAA$_m$ $\left(\left(\langle u_1, R_1\rangle, \chi_1\right), \left(\langle u_2, R_2\rangle, \chi_2\right), ..., \left(\langle u_k, R_k\rangle, \chi_k\right)\right)$

$$= \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-r_{\propto(j)}^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{x_{\propto(j)}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k}\left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{k}\left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right)$$

**Step 3:** Assuming Eq. (1) holds for $n = k$, then the objective is to demonstrate its applicability for $n = k + 1$:

I-CCPoFOWAA$_m$ $\left(\left(\langle u_1, R_1\rangle, \chi_1\right), ..., \left(\langle u_k, R_k\rangle, \chi_k\right), \left(\langle u_{k+1}, R_{k+1}\rangle, \chi_{k+1}\right)\right)$

$$= \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-r_{\propto(j)}^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{x_{\propto(j)}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k}\left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{k}\left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right) \oplus$$

$$\left(\begin{array}{c} \left(1-\left(1-r_{k+1}^q\right)^{\chi_{k+1} m_{k+1}}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{x_{k+1}}{2\pi}\right)^q\right)^{\chi_{k+1} m_{k+1}}\right)^{\frac{1}{q}}}, \\ \left(g_{k+1}\right)^{\chi_{k+1} m_{k+1}} e^{i2\pi\left(\frac{y_{k+1}}{2\pi}\right)^{\chi_{k+1} m_{k+1}}}, \left(s_{k+1}\right)^{\chi_{k+1} m_{k+1}} e^{i2\pi\left(\frac{z_{k+1}}{2\pi}\right)^{\chi_{k+1} m_{k+1}}} \end{array}\right)$$

$$= \left(\begin{array}{c} \left(1-\prod_{j=1}^{k+1}\left(1-r_{\propto(j)}^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{k+1}\left(1-\left(\frac{x_{\propto(j)}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{k+1}\left(g_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k+1}\left(\frac{y_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{k+1}\left(s_{\propto(j)}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{k+1}\left(\frac{z_{\propto(j)}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right)$$

Hence, it holds for . As a result, Eq. (1) holds for all positive integers n according to the principle of mathematical induction. Thus, the proof is completed.

**Definition 8:** Let $\langle u_j, R_j\rangle$ $(1 \leq j \leq n)$ be a group of 2-tuple of CPoFVs, and $\chi_j$ $(1 \leq j \leq n)$ is their corresponding confidence level. Let $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)$ and $m = (m_1, m_2, ..., m_n)$ respectively present weighted vector and associated vector under conditions, both belong to closed interval and their sum is equal to one. Also, there is $\dot{R}_{\propto(j)} = n\varpi_j R_j$, with $\dot{R}_{\propto(j)}$ $(1 \leq j \leq n)$ being the greatest value, and the positive number n is called the balancing coefficient, which plays a vigorous role in balancing the given equation. Moreover, if the weighted vector $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$, then $(n\varpi_1 R_1, n\varpi_2 R_2, ..., n\varpi_n R_n)$ approaches to $(R_1, R_2, ..., R_n)$. Furthermore, $u_j \in \langle u_j, R_j\rangle$ is the ordered pair of CPoFOWA, having the jth maximum value, referred to as the order inducing variable, represented as $u_j \in \langle u_j, R_j\rangle$, and let $R_j$ $(j = 1, 2, ..., n)$ symbolize the CPoF argument. Given these conditions, the I-CCPoFHAA operator can be presented mathematically as follows:

$$\text{I-CCPoFHAA}_{\varpi,m}\left(\left(\langle u_1,R_1\rangle,\chi_1\right),\left(\langle u_2,R_2\rangle,\chi_2\right),...,\left(\langle u_n,R_n\rangle,\chi_n\right)\right)$$
$$=\left(\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\left(r_{\dot{R}_{\propto(j)}}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{x_{\dot{R}_{\propto(j)}}}{2\pi}\right)^{q}\right)^{\chi_j m_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}\left(g_{\dot{R}_{\propto(j)}}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{y_{\dot{R}_{\propto(j)}}}{2\pi}\right)^{\chi_j m_j}}, \prod_{j=1}^{n}\left(s_{\dot{R}_{\propto(j)}}\right)^{\chi_j m_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{z_{\dot{R}_{\propto(j)}}}{2\pi}\right)^{\chi_j m_j}} \end{array}\right)$$

## 5 An Application of the Novel Operators

**Algorithm:** Let $A = \{A_1, A_2, ..., A_n\}$ be a set of $n$ attributes, $m = (m_1, m_2, ..., m_n)$ be their weights, and $X = \{X_1, X_2, ..., X_m\}$ be a set of $m$ alternatives. Let $D = \{D_1, D_2, ..., D_k\}$ be a group of k-decision makers, whose weights are $\phi = (\phi_1, \phi_2, ..., \phi_k)$.

**Step 1:** Collect all data connected to each alternative under the different proposed criteria.

**Step 2:** Aggregate all decision matrices into a single collective decision matrix.

**Step 3:** Again, utilize all of the proposed operators to compute the preference values

**Step 4:** Compute and calculate the score function.

**Step 5:** Rank all alternatives with respect to the score function and choose the one with the highest score value.

## 6 Illustrative Example

Case study: Suppose a company is scouting for an ideal location for a new plantation. Firstly, the company assigned this task to a committee of four experts $Đ = \{Đ_1, Đ_2, Đ_3, Đ_4\}$ whose weight vector is $\phi = (0.1, 0.2, 0.3, 0.4)$. The experts considered four locations, represented as $X = \{X_1, X_2, X_3, X_4\}$ for further selection process. When selecting an appropriate location for a plant, it's essential to consider numerous factors. However, let's focus on four of the most common and crucial attributes whose weight vector is $m = (0.3, 0.3, 0.2, 0.2)$.

$A_1$: Cost Efficiency: One of the primary considerations is the overall cost associated with establishing and maintaining the plantation. This encompasses factors such as land acquisition costs, labor expenses, and operational overheads.

$A_2$: Favorable Climatic Conditions: The success of a plantation heavily depends on the climatic conditions of the chosen area. Different plants thrive in specific climates, so selecting a region with suitable temperature ranges, rainfall patterns, and sunlight exposure is imperative.

$A_3$: Safety and Security: Ensuring the safety of the plantation, its workforce, and the surrounding environment is of utmost importance. Choosing a location with a low risk of external threats contributes to the long-term stability and resilience of the plantation.

$A_4$: Environmental Sustainability: With growing concerns about ecological impact, selecting a location that aligns with environmental sustainability is increasingly critical. This involves assessing the impact of the plantation on local ecosystems, water resources, and biodiversity.

**Step 1:** Putt all the data of the experts in the form of matrices based on inducing variables. (Tables 1- 4)

**Step 2:** Combine all individual matrices into a single matrix by using the I-CCPoFOWAA approach, where $\phi = (0.1, 0.2, 0.3, 0.4)$ and $q = 4$, then we have Table 5:

**Step 3:** Next, again, using the I-CCPoFOWAA operator, with $m = (0.3, 0.3, 0.2, 0.2)$, we have:

$r_1 = \left(0.74e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.49)}, 0.73e^{i2\pi(0.58)}\right), r_2 = \left(0.83e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.68)}, 0.70e^{i2\pi(0.69)}\right)$
$r_3 = \left(0.85e^{i2\pi(0.68)}, 0.68e^{i2\pi(0.49)}, 0.80e^{i2\pi(0.79)}\right), r_4 = \left(0.82e^{i2\pi(0.64)}, 0.69e^{i2\pi(0.51)}, 0.75e^{i2\pi(0.67)}\right)$

**Step 4:** Compute the score functions as:

$S(r_1) = 0.83$, $S(r_2) = 0.89$, $S(r_3) = 0.73$,$S(r_4) = 0.79$

**Table 1.** Decision of $1^{st}$ expert based on inducing variable

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $X_1$ | $\left\langle 0.8, \left(0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}\right), 0.3\right\rangle$ | $\left\langle 0.7, \left(0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}\right), 0.5\right\rangle$ | $\left\langle 0.6, \left(0.36e^{i2\pi(0.57)}, 0.49e^{i2\pi(0.54)}, 0.59e^{i2\pi(0.38)}\right), 0.3\right\rangle$ | $\left\langle 0.5, \left(0.48e^{i2\pi(0.55)}, 0.47e^{i2\pi(0.29)}, 0.56e^{i2\pi(0.54)}\right), 0.5\right\rangle$ |
| $X_2$ | $\left\langle 0.5, \left(0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}\right), 0.3\right\rangle$ | $\left\langle 0.5, \left(0.48e^{i2\pi(0.55)}, 0.47e^{i2\pi(0.29)}, 0.56e^{i2\pi(0.54)}\right), 0.5\right\rangle$ | $\left\langle 0.5, \left(0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}\right), 0.3\right\rangle$ | $\left\langle 0.6, \left(0.58e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}\right), 0.4\right\rangle$ |
| $X_3$ | $\left\langle 0.6, \left(0.58e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}\right), 0.4\right\rangle$ | $\left\langle 0.6, \left(0.36e^{i2\pi(0.57)}, 0.49e^{i2\pi(0.54)}, 0.59e^{i2\pi(0.38)}\right), 0.3\right\rangle$ | $\left\langle 0.5, \left(0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}\right), 0.3\right\rangle$ | $\left\langle 0.2, \left(0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.64)}, 0.38e^{i2\pi(0.71)}\right), 0.3\right\rangle$ |
| $X_4$ | $\left\langle 0.7, \left(0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}\right), 0.8\right\rangle$ | $\left\langle 0.5, \left(0.78e^{i2\pi(0.57)}, 0.67e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}\right), 0.5\right\rangle$ | $\left\langle 0.5, \left(0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}\right), 0.3\right\rangle$ | $\left\langle 0.3, \left(0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}\right), 0.8\right\rangle$ |

**Table 2.** Decision of $2^{nd}$ expert based on inducing variable

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $X_1$ | $\langle 0.7, (0.58e^{i2\pi(0.49)}, 0.68e^{i2\pi(0.65)}, 0.79e^{i2\pi(0.46)}), 0.4\rangle$ | $\langle 0.9, (0.64e^{i2\pi(0.52)}, 0.72e^{i2\pi(0.62)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ | $\langle 0.3, (0.78e^{i2\pi(0.54)}, 0.67e^{i2\pi(0.37)}, 0.56e^{i2\pi(0.48)}), 0.4\rangle$ | $\langle 0.6, (0.56e^{i2\pi(0.7)}, 0.47e^{i2\pi(0.6)}, 0.38e^{i2\pi(0.8)}), 0.5\rangle$ |
| $X_2$ | $\langle 0.7, (0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}), 0.8\rangle$ | $\langle 0.4, (0.78e^{i2\pi(0.57)}, 0.67e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}), 0.5\rangle$ | $\langle 0.5, (0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.6, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.8\rangle$ |
| $X_3$ | $\langle 0.6, (0.58e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}), 0.4\rangle$ | $\langle 0.2, (0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}), 0.5\rangle$ | $\langle 0.5, (0.78e^{i2\pi(0.4)}, 0.87e^{i2\pi(0.5)}, 0.56e^{i2\pi(0.9)}), 0.8\rangle$ | $\langle 0.5, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ |
| $X_4$ | $\langle 0.8, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ | $\langle 0.5, (0.48e^{i2\pi(0.55)}, 0.47e^{i2\pi(0.29)}, 0.56e^{i2\pi(0.54)}), 0.5\rangle$ | $\langle 0.6, (0.56e^{i2\pi(0.7)}, 0.47e^{i2\pi(0.6)}, 0.38e^{i2\pi(0.8)}), 0.5\rangle$ | $\langle 0.7, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.6\rangle$ |

**Table 3.** Decision of $3^{rd}$ expert based on inducing variable

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $X_1$ | $\langle 0.4, (0.72e^{i2\pi(0.45)}, 0.57e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.5, (0.48e^{i2\pi(0.55)}, 0.47e^{i2\pi(0.29)}, 0.56e^{i2\pi(0.54)}), 0.5\rangle$ | $\langle 0.5, (0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.3, (0.68e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}), 0.4\rangle$ |
| $X_2$ | $\langle 0.9, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ | $\langle 0.8, (0.68e^{i2\pi(0.57)}, 0.57e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}), 0.5\rangle$ | $\langle 0.3, (0.68e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}), 0.4\rangle$ | $\langle 0.7, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.6\rangle$ |
| $X_3$ | $\langle 0.6, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.8\rangle$ | $\langle 0.4, (0.72e^{i2\pi(0.45)}, 0.57e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.7, (0.78e^{i2\pi(0.64)}, 0.89e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.48)}), 0.5\rangle$ | $\langle 0.1, (0.68e^{i2\pi(0.57)}, 0.57e^{i2\pi(0.45)}, 0.69e^{i2\pi(0.64)}), 0.5\rangle$ |
| $X_4$ | $\langle 0.7, (0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}), 0.8\rangle$ | $\langle 0.8, (0.68e^{i2\pi(0.57)}, 0.57e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}), 0.5\rangle$ | $\langle 0.5, (0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.6, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.8\rangle$ |

**Table 4.** Decision of $4^{th}$ expert based on inducing variable

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $X_1$ | $\langle 0.7, (0.48e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}), 0.8\rangle$ | $\langle 0.7, (0.78e^{i2\pi(0.57)}, 0.67e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}), 0.5\rangle$ | $\langle 0.5, (0.62e^{i2\pi(0.45)}, 0.67e^{i2\pi(0.44)}, 0.48e^{i2\pi(0.64)}), 0.3\rangle$ | $\langle 0.4, (0.38e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.8\rangle$ |
| $X_2$ | $\langle 0.6, (0.58e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}), 0.4\rangle$ | $\langle 0.6, (0.68e^{i2\pi(0.63)}, 0.45e^{i2\pi(0.42)}, 0.71e^{i2\pi(0.52)}), 0.5\rangle$ | $\langle 0.5, (0.78e^{i2\pi(0.4)}, 0.87e^{i2\pi(0.5)}, 0.56e^{i2\pi(0.9)}), 0.8\rangle$ | $\langle 0.5, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ |
| $X_3$ | $\langle 0.4, (0.38e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.8\rangle$ | $\langle 0.7, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.6\rangle$ | $\langle 0.9, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ | $\langle 0.9, (0.58e^{i2\pi(0.57)}, 0.67e^{i2\pi(0.45)}, 0.79e^{i2\pi(0.64)}), 0.5\rangle$ |
| $X_4$ | $\langle 0.9, (0.64e^{i2\pi(0.51)}, 0.72e^{i2\pi(0.61)}, 0.38e^{i2\pi(0.71)}), 0.3\rangle$ | $\langle 0.8, (0.68e^{i2\pi(0.57)}, 0.57e^{i2\pi(0.45)}, 0.89e^{i2\pi(0.64)}), 0.5\rangle$ | $\langle 0.3, (0.68e^{i2\pi(0.56)}, 0.67e^{i2\pi(0.53)}, 0.78e^{i2\pi(0.46)}), 0.4\rangle$ | $\langle 0.7, (0.58e^{i2\pi(0.46)}, 0.87e^{i2\pi(0.71)}, 0.67e^{i2\pi(0.57)}), 0.6\rangle$ |

**Table 5.** Combine decision of all experts

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $X_1$ | $(0.65e^{i2\pi(0.74)}, 0.75e^{i2\pi(0.53)}, 0.86e^{i2\pi(0.92)})$ | $(0.69e^{i2\pi(0.70)}, 0.68e^{i2\pi(0.40)}, 0.68e^{i2\pi(0.55)})$ | $(0.75e^{i2\pi(0.63)}, 0.68e^{i2\pi(0.56)}, 0.78e^{i2\pi(0.69)})$ | $(0.78e^{i2\pi(0.74)}, 0.88e^{i2\pi(0.65)}, 0.73e^{i2\pi(0.45)})$ |
| $X_2$ | $(0.72e^{i2\pi(0.64)}, 0.57e^{i2\pi(0.56)}, 0.79e^{i2\pi(0.61)})$ | $(0.56e^{i2\pi(0.55)}, 0.69e^{i2\pi(0.61)}, 0.79e^{i2\pi(0.71)})$ | $(0.68e^{i2\pi(0.69)}, 0.56e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.60)})$ | $(0.84e^{i2\pi(0.44)}, 0.66e^{i2\pi(0.57)}, 0.80e^{i2\pi(0.63)})$ |
| $X_3$ | $(0.83e^{i2\pi(0.82)}, 0.75e^{i2\pi(0.35)}, 0.57e^{i2\pi(0.49)})$ | $(0.56e^{i2\pi(0.55)}, 0.69e^{i2\pi(0.61)}, 0.79e^{i2\pi(0.71)})$ | $(0.68e^{i2\pi(0.69)}, 0.56e^{i2\pi(0.50)}, 0.89e^{i2\pi(0.60)})$ | $(0.84e^{i2\pi(0.44)}, 0.66e^{i2\pi(0.57)}, 0.80e^{i2\pi(0.63)})$ |
| $X_3$ | $(0.83e^{i2\pi(0.82)}, 0.75e^{i2\pi(0.35)}, 0.57e^{i2\pi(0.49)})$ | $(0.95e^{i2\pi(0.57)}, 0.71e^{i2\pi(0.66)}, 0.69e^{i2\pi(0.46)})$ | $(0.68e^{i2\pi(0.75)}, 0.59e^{i2\pi(0.64)}, 0.86e^{i2\pi(0.48)})$ | $(0.71e^{i2\pi(0.57)}, 0.72e^{i2\pi(0.60)}, 0.85e^{i2\pi(0.68)})$ |
| $X_4$ | $(0.80e^{i2\pi(0.52)}, 0.66e^{i2\pi(0.74)}, 0.86e^{i2\pi(0.70)})$ | $(0.89e^{i2\pi(0.77)}, 0.75e^{i2\pi(0.55)}, 0.96e^{i2\pi(0.68)})$ | $(0.89e^{i2\pi(0.54)}, 0.75e^{i2\pi(0.68)}, 0.96e^{i2\pi(0.45)})$ | $(0.68e^{i2\pi(0.43)}, 0.95e^{i2\pi(0.72)}, 0.73e^{i2\pi(0.56)})$ |

**Step 5:** Thus, the most suitable location is $X_2$.
Table 6 shows ranking of all methods.

**Table 6.** Ranking of all methods

| Locations | I-CCPoFOWAA | I-CCPoFHAA |
|---|---|---|
| $X_1$ | 0.83 | 0.76 |
| $X_2$ | 0.89 | 0.88 |
| $X_3$ | 0.73 | 0.70 |
| $X_4$ | 0.79 | 0.73 |

## 7 Conclusions

In this study, we have developed a concept known as the complex polytopic fuzzy set within the framework of fuzzy set theory. Additionally, we have introduced two distinct operators, namely the I-CCPoFOWAA operator and the I-CCPoFHAA operator. These operators serve as mathematical functions essential in the realm of complex fuzzy set theory and decision-making. Moreover, this research can be extended to complex Fermatean fuzzy sets, complex Einstein operators, complex Logarithmic operators, complex Dombi operators, complex Power operators, complex interval-valued Power operators, etc.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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