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Enhanced Performance Evaluation Through Neutrosophic Data Envelopment Analysis Leveraging Pentagonal Neutrosophic Numbers

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Abstract: Neutrosophic sets, expanded from the constructs of fuzzy and intuitionistic fuzzy sets, can accommodate degrees of truth, indeterminacy, and falsity for each element. This attribute equips them with an aptitude for a more refined interpretation of ambiguous or uncertain data. This study presents an innovative application of Neutrosophic Data Envelopment Analysis (Neu-DEA), incorporating pentagonal neutrosophic numbers in both input and output data. This novel methodology involves the transformation of traditional DEA models into a Pentagonal neutrosophic DEA model, subsequently converting it into a Crisp Linear Programming (CrLP) model. A unique ranking function is integral to this process. Performance evaluation of decision-making units (DMUs) is accomplished through the resolution of the CrLP model, with subsequent ranking of the DMUs based on their relative efficiency scores. The utility and effectiveness of this novel technique is validated through a numerical example.

Keywords: Neutrosophic Data Envelopment Analysis; Performance evaluation; Pentagonal neutrosophic number; Ranking function; Efficiency score

1 Introduction

Data Envelopment Analysis (DEA), with its robust analytical framework and inherent flexibility, has emerged as a pivotal tool for performance appraisal. This methodology, proposed initially by Charnes, Cooper, and Rhodes in 1978 [1], leverages mathematical programming to benchmark multiple Decision Making Units (DMUs) that employ various inputs to generate a range of outputs. The underlying aim of DEA is to discern the most resource-efficient DMUs that maximize output through minimal input utilization. The assessment is conducted by establishing a production frontier—a mathematical depiction of an efficient production process—and comparing the position of each DMU relative to this frontier.

DEA's strength lies in its assumption-free approach towards the functional form of the production function, and its ability to manage multiple inputs and outputs. It delivers a quantitative basis for identifying efficient DMUs and setting targets for performance enhancement. In recent decades, DEA's usage has expanded across diverse sectors, such as healthcare, banking, energy, manufacturing, and education, to name a few [2–5]. Despite its benefits, the traditional DEA model is predicated on the certainty and preciseness of the input and output data of DMUs, a condition not always feasible in real-life scenarios. Consequently, a need has arisen for the evolution of traditional DEA models to incorporate fuzzy logic, thereby affording a more nuanced portrayal of DMU performance in the face of ambiguity and uncertainty [6].

One such evolution led to the development of Fuzzy Data Envelopment Analysis (FDEA), a derivative of the traditional DEA model that accommodates imprecise or uncertain data. Since Sengupta's pioneering work on fuzzy DEA, where both inputs and outputs were treated as fuzzy numbers [7], numerous scholars have refined and extended the fuzzy DEA models [8, 9]. These models have been classified into six distinct approaches, including the stochastic fuzzy DEA models developed to handle uncertain data [10] and used wide range of real-world applications [11].

Further, to embody uncertainty more accurately in performance analysis, Intuitionistic fuzzy sets (IFSs) [12] were developed as extensions of fuzzy sets. By encapsulating both the degree of membership and non-membership of an element in a set, IFSs afford a greater depth of understanding for decision-makers. Different methodologies, including weighted entropy approach [13], ranking approach [14], expected value approach [15, 16], parametric approach [17], (α, β)-cut approach [18, 19], and MCDM approach, have been developed to solve Intuitionistic Fuzzy DEA models [20, 21].

In recent decades, the extension of fuzzy sets and intuitionistic fuzzy sets into neutrosophic sets, as first proposed by Smarandache in 1999 [22], has demonstrated exceptional applicability and adaptability in handling situations of high indeterminacy or contradiction [23, 24]. An exemplary demonstration of this adaptability lies in the usage of these sets in performance analysis, contributing significantly to modeling uncertainty with greater precision.

In 2018, a noteworthy advancement was made by Edalatpanah who introduced the Neu-DEA model, establishing a new dimension to handle uncertainty in DEA analysis [25]. This model has since been embraced by numerous researchers to address various uncertainty problems across diverse fields. For instance, it was deployed by Kahraman and colleagues to evaluate the performance of private universities in Turkey by integrating a neutrosophic version of AHP and DEA models [26]. It was also instrumental in Abdelfattah's proposed ranking and parametric approach to tackle the Neu-DEA model, taking into account neutrosophic inputs and outputs [27]. Further development of this model was made by Edalatpanah and Smarandache, who converted an input-oriented Neu-DEA model into a corresponding crisp DEA model using natural logarithms [28]. Edalatpanah continued to build on the model, incorporating Triangular neutrosophic number into the Neu-DEA model [29]. The model was also adapted by Mao and colleagues to accommodate single-valued neutrosophic sets in an undesirable DEA model using a logarithm approach [30]. Yang and team took it a step further by measuring hospital efficiency based on the Neu-DEA model with a single-valued triangular neutrosophic number [31]. Another innovative application of the model was proposed by Tapia, who devised an MCDA technique based on the neutrosophic DEA model to assess risks tied to uncertainties in emerging technologies [32]. More recently, Abdelfattah applied the model to measure the performance of regional hospitals in Tunisia [33]. The performance of All India Institute of Medical Science (AIIMS) are measured by using Neu-DEA model [34].

Advancements have not been limited to the neutrosophic sets, however. Other extensions of fuzzy set, such as Fermatean fuzzy set [35], Plithogenic Set [36], and Spherical fuzzy set [37, 38], have been constructively employed to develop DEA models and establish solution techniques. These extensions have proven invaluable for decision-making under uncertainty, with widespread applications across multiple disciplines.

One such extension, the pentagonal neutrosophic numbers, demonstrates a marked increase in nuance and flexibility in portraying uncertain or imprecise information compared to other neutrosophic numbers. The use of five parameters enables complex degrees of truth, falsity, and indeterminacy to be expressed more comprehensively [39–42]. As a result, decision-makers are better equipped to model uncertainty or vagueness with greater accuracy, potentially leading to more informed decision-making and improved problem-solving outcomes [43–46].

The study delineated here introduces three key contributions: Firstly, the development of a new ranking function for pentagonal neutrosophic numbers, designed to assist decision-makers in prioritizing PNNs. Secondly, the proposal of a Pentagonal neutrosophic DEA model, which integrates PNN inputs and outputs. Lastly, the innovative use of the proposed ranking function to convert the Pentagonal neutrosophic DEA model into a crisp LP model, enabling the determination of relative efficiency of DMUs and their subsequent ranking based on their efficiency score.

The organization of the study follows a systematic approach: Section 2 defines the concept of neutrosophic number and pentagonal neutrosophic numbers, and introduces an accuracy function, demonstrated through theorems, to establish its linearity. Section 3 discusses the development of Neutrosophic Data Envelopment Analysis from traditional Data Envelopment Analysis, accounting for the presence of pentagonal neutrosophic numbers. The same section also includes the conversion of the Pentagonal Neutrosophic DEA model into a corresponding crisp LP Model using the Ranking function. Section 4 offers a pertinent numerical example to illustrate the methodology. Finally, Section 5 presents the conclusion and provides directions for future research.

2 Preliminaries

This section commences with an exploration of fundamental aspects of neutrosophic sets and pentagonal neutrosophic number, as well as the associated arithmetic operations. The first subsection offers an original ranking function that is predicated on the pentagonal neutrosophic number.

Definition 1 [22]: A single valued neutrosophic set X^N in a universe of discourse Ω is given by

$$X^N = \{\langle x; T(x), I(x), F(x)\rangle : x \in \Omega\}$$

where $T : \Omega \to [0, 1], I : \Omega \to [0, 1]$ and $F : \Omega \to [0, 1]$, are the truth, indeterminacy, and falsity membership degrees with satisfy the condition $0 \leq T_A + I_A + F_A \leq 3, \forall x \in X$.

Definition 2: [Pentagonal Neutrosophic Number] A pentagonal neutrosophic number of a neutrosophic set X^N is defined as $X^{PN} = \langle x^{p_1}, x^{p_2}, x^{p_3}, x^{p_4}, x^{p_5}; \mu_x, \vartheta_x, \pi_x \rangle$ and whose truth, indeterminacy and falsity membership function are defined as

$$T(x) = \begin{cases} 0, x \leq x^{p_1} \\ \mu_x \left(\frac{x - x^{p_1}}{x^{p_2} - x^{p_1}} \right), \quad x^{p_1} \leq x \leq x^{p_2} \\ \mu_x + (1 - \mu_x) \left(\frac{x - x^{p_2}}{x^{p_3} - x^{p_2}} \right), x^{p_2} \leq x \leq x^{p_3} \\ 1, x = x^{p_3} \\ \mu_x + (1 - \mu_x) \left(\frac{x^{p_4} - x}{x^{p_4} - x^{p_3}} \right), x^{p_3} \leq x \leq x^{p_4} \\ \mu_x \left(\frac{x^{p_5} - x}{x^{p_5} - x^{p_4}} \right), \quad x^{p_4} \leq x \leq x^{p_5} \\ 0, x \geq x^{p_5} \end{cases}$$

$$I(x) = \begin{cases} 1, x \leq x^{p_1} \\ \vartheta_x + (1 - \vartheta_x) \left(\frac{x^{p_2} - x}{x^{p_2} - x^{p_1}} \right), x^{p_1} \leq x \leq x^{p} \\ \vartheta_x \left(\frac{x^{p_3} - x}{x^{p_3} - x^{p_2}} \right), \quad x^{p_2} \leq x \leq x^{p_3} \\ 0, x = x^{p_3} \\ \vartheta_x \left(\frac{x - x^{p_3}}{x^{p_4} - x^{p_3}} \right), x^{p_3} \leq x \leq x^{p_4} \\ \vartheta_x + (1 - \vartheta_x) \left(\frac{x - x^{p_4}}{x^{p_5} - x^{p_4}} \right), x^{p_4} \leq x \leq x^{p_5} \\ 1, x \geq x^{p_5} \end{cases}$$

$$F(x) = \begin{cases} 1, x \leq x^{p_1} \\ \pi_x + (1 - \pi_x) \left(\frac{x^{p_2} - x}{x^{p_2} - x^{p_1}} \right), x^{p_1} \leq x \leq x^{p_2} \\ \pi_x \left(\frac{x^{p_3} - x}{x^{p_3} - x^{p_2}} \right), \quad x^{p_2} \leq x \leq x^{p_3} \\ 0, x = x^{p_3} \\ \pi_x \left(\frac{x - x^{p_3}}{x^{p_4} - x^{p_3}} \right), \quad x^{p_3} \leq x \leq x^{p_4} \\ \pi_x + (1 - \pi_x) \left(\frac{x - x^{p_4}}{x^{p_5} - x^{p_4}} \right), x^{p_4} \leq x \leq x^{p_5} \\ 1, x \geq x^{p_5} \end{cases}$$

Satisfy $0 \leq T(x) + I(x) + F(x) \leq 3$. Figure 1 show the graphically representation of $T(x)$, $I(x)$, and $F(x)$.

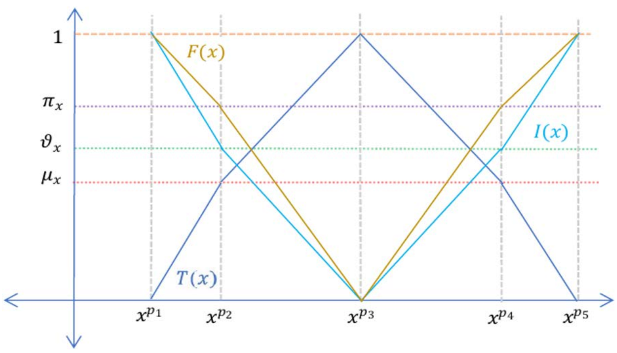

Figure 1. Representation of truth, indeterminacy and falsity membership grades of a PNN

Definition 3: [Arithmetic Operation]
Let $X_1^{PN} = \langle x_1^{p_1}, x_1^{p_2}, x_1^{p_3}, x_1^{p_4}, x_1^{p_5}; \mu_{x_1}, \vartheta_{x_1}, \pi_{x_1} \rangle$ and $X_2^{PN} = \langle x_2^{p_1}, x_2^{p_2}, x_2^{p_3}, x_2^{p_4}, x_2^{p_5}; \mu_{x_2}, \vartheta_{x_2}, \pi_{x_2} \rangle$ are the pentagonal neutrosophic numbers. The following arithmetic operation satisfies

1. Addition Rule:

$$X_1^{PN} \oplus X_2^{PN} = \langle x_1^{p_1} + x_2^{p_1}, x_1^{p_2} + x_2^{p_2}, x_1^{p_3} + x_2^{p_3}, x_1^{p_4} + x_2^{p_4}, x_1^{p_5} + x_2^{p_5}; \mu_{x_1} \wedge \mu_{x_2}, \vartheta_{x_1} \vee \vartheta_{x_2}, \pi_{x_1} \vee \pi_{x_2} \rangle$$

2. Difference Rule:

$$X_1^{PN} \ominus X_2^{PN} = \langle x_1^{p_1} - x_2^{p_5}, x_1^{p_2} - x_2^{p_4}, x_1^{p_3} - x_2^{p_3}, x_1^{p_4} - x_2^{p_2}, x_1^{p_5} - x_2^{p_1}; \mu_{x_1} \wedge \mu_{x_2}, \vartheta_{x_1} \vee \vartheta_{x_2}, \pi_{x_1} \vee \pi_{x_2} \rangle$$

3. Scalar Multiplication Rule:

$$\lambda X_1^{PN} = \begin{cases} \langle \lambda x_1^{p_1}, \lambda x_1^{p_2}, \lambda x_1^{p_3}, \lambda x_1^{p_4}, \lambda x_1^{p_5}; \mu_{x_1}, \vartheta_{x_1}, \pi_{x_1} \rangle \ \lambda \geq 0 \\ \langle \lambda x_1^{p_5}, \lambda x_1^{p_4}, \lambda x_1^{p_3}, \lambda x_1^{p_2}, \lambda x_1^{p_1}; \mu_{x_1}, \vartheta_{x_1}, \pi_{x_1} \rangle \ \lambda \leq 0 \end{cases}$$

4. Product Rule:

$$X_1^{PN} \otimes X_2^{PN} = \langle x_1^{p_1} x_2^{p_1}, x_1^{p_2} x_2^{p_2}, x_1^{p_3} x_2^{p_3}, x_1^{p_4} x_2^{p_4}, x_1^{p_5} x_2^{p_5}; \mu_{x_1} \vee \mu_{x_2}, \vartheta_{x_1} \wedge \vartheta_{x_2}, \pi_{x_1} \wedge \pi_{x_2} \rangle$$

5. Quotient Rule:

$$\frac{X_1^{PN}}{X_2^{PN}} = \left\langle \frac{x_1^{p1}}{x_2^{p5}}, \frac{x_1^{p2}}{x_2^{p4}}, \frac{x_1^{p3}}{x_2^{p3}}, \frac{x_1^{p4}}{x_2^{p2}}, \frac{x_1^{p5}}{x_2^{p1}}; \mu_{x_1} \vee \mu_{x_2}, \vartheta_{x_1} \wedge \vartheta_{x_2}, \pi_{x_1} \wedge \pi_{x_2} \right\rangle$$

2.1 Proposed Ranking Function of Pentagonal Neutrosophic Number

Definition 4: [Ranking Function] Let $X^{PN} = \langle x^{p_1}, x^{p_2}, x^{p_3}, x^{p_4}, x^{p_5}; \mu_x, \vartheta_x, \pi_x \rangle$ be a PNN. The ranking function for X^{PN} is denoted by $R\left(X^{PN}\right)$ and defined as

$$R\left(X^{PN}\right) = \frac{x^{p_1} + 3x^{p_2} + 5x^{p_3} + 3x^{p_4} + x^{p_5}}{39} \times [\mu_x + (1 - \vartheta_x) + (1 - \pi_x)]$$

If $X^{PN} = x$ be any real crisp number, which can be represent in PNN form i.e., $X^{PN} = \langle x, x, x, x, x, ; 1, 0, 0 \rangle$ then $R\left(X^{PN}\right) = x$.

Definition 5: [Ordering of PNNs]

Let $X_1^{PN} = \langle x_1^{p_1}, x_1^{p_2}, x_1^{p_3}, x_1^{p_4}, x_1^{p_5}; \mu_{x_1}, \vartheta_{x_1}, \pi_{x_1} \rangle$ and $X_2^{PN} = \langle x_2^{p_1}, x_2^{p_2}, x_2^{p_3}, x_2^{p_4}, x_2^{p_5}; \mu_{x_2}, \vartheta_{x_2}, \pi_{x_2} \rangle$ are the PNNs. Then

1. $R\left(X_1^{PN}\right) \leq R\left(X_2^{PN}\right)$ that implies $X_1^{PN} \leq X_2^{PN}$.
2. $R\left(X_1^{PN}\right) \geq R\left(X_2^{PN}\right)$ that implies $X_1^{PN} \geq X_2^{PN}$.
3. $R\left(X_1^{PN}\right) = R\left(X_2^{PN}\right)$ that implies $X_1^{PN} \approx X_2^{PN}$.

Lemma 1: Let $X^{PN} = \langle x^{p_1}, x^{p_2}, x^{p_3}, x^{p_4}, x^{p_5}; \mu_x, \vartheta_x, \pi_x \rangle$ be a PNN and $\lambda \in \mathbb{R}$. Then

$$R\left(\lambda X^{PN}\right) = \lambda R\left(X^{PN}\right)$$

Proof. Since $\lambda \in \mathbb{R}$, then there are two cases arises.

Case 1: (when $\lambda \geq 0$ **)**

From Definition 3,

$$\lambda X^{PN} = \langle \lambda x^{p_1}, \lambda x^{p_2}, \lambda x^{p_3}, \lambda x^{p_4}, \lambda x^{p_5}; \mu_x, \vartheta_x, \pi_x \rangle$$

From Definition 4,

$$\begin{aligned} R\left(\lambda X^{PN}\right) &= \frac{\lambda x^{p_1} + 3\lambda x^{p_2} + 5\lambda x^{p_3} + 3\lambda x^{p_4} + \lambda x^{p_5}}{39} \times [\mu_x + (1 - \vartheta_x) + (1 - \pi_x)] \\ &= \lambda \left(\frac{x^{p_1} + 3x^{p_2} + 5x^{p_3} + 3x^{p_4} + x^{p_5}}{39} \times [\mu_x + (1 - \vartheta_x) + (1 - \pi_x)] \right) = \lambda R\left(X^{PN}\right) \end{aligned}$$

Case 2: (when $\lambda \leq 0$**)**

From Definition 3,

$$\lambda X^{PN} = \langle \lambda x^{p_5}, \lambda x^{p_4}, \lambda x^{p_3}, \lambda x^{p_2}, \lambda x^{p_1}; \mu_x, \vartheta_x, \pi_x \rangle$$

From Definition 4,

$$\begin{aligned} R\left(\lambda X^{PN}\right) &= \frac{\lambda x^{p_5} + 3\lambda x^{p_4} + 5\lambda x^{p_3} + 3\lambda x^{p_2} + \lambda x^{p_1}}{39} \times [\mu_x + (1 - \vartheta_x) + (1 - \pi_x)] \\ &= \lambda \left(\frac{x^{p_1} + 3x^{p_2} + 5x^{p_3} + 3x^{p_4} + x^{p_5}}{39} \times [\mu_x + (1 - \vartheta_x) + (1 - \pi_x)] \right) = \lambda R\left(X^{PN}\right) \end{aligned}$$

Note: The ranking function(R) is not linear i.e., $R\left(X_1^{PN} \oplus X_2^{PN}\right) \neq R\left(X_1^{PN}\right) + R\left(X_2^{PN}\right)$ for $X_1^{PN} \neq X_2^{PN}$.

Example 1: Let $X_1^{PN} = \langle 11, 16, 23, 26, 30; 0.8, 0.3, 0.5 \rangle$ and $X_2^{PN} = \langle 10, 18, 22, 25, 32; 0.7, 0.4, 0.3 \rangle$ are two PNNs. Then

a. $R\left(X_1^{PN}\right) = \frac{11+3\times16+5\times23+3\times26+30}{39} \times (0.8 + 1 - 0.3 + 1 - 0.5) = 14.4615$

$R\left(X_2^{PN}\right) = \frac{10+3\times18+5\times22+3\times25+32}{39} \times (0.7 + 1 - 0.4 + 1 - 0.3) = 14.4103$

Hence $R\left(X_1^{PN}\right) > R\left(X_1^{PN}\right)$ that implies $X_1^{PN} > X_2^{PN}$.

b. Let $3X_1^{PN} = \langle 33, 48, 69, 78, 90; 0.8, 0.3, 0.5 \rangle$, then $R\left(3X_1^{PN}\right) = 43.3846 = 3R\left(X_1^{PN}\right)$.

c. $X_1^{PN} + X_2^{PN} = \langle 21, 34, 45, 51, 62; 0.7, 0.4, 0.5 \rangle$, then

$R\left(X_1^{PN} + X_2^{PN}\right) = 27.4282 \neq R\left(X_1^{PN}\right) + R\left(X_2^{PN}\right) = 28.8718$

Definition 6: [Aggregation ranking function] Let $X_i^{PN} = \langle x_i^{p_1}, x_i^{p_2}, x_i^{p_3}, x_i^{p_4}, x_i^{p_5}; \mu_{x_i}, \vartheta_{x_i}, \pi_{x_i} \rangle, i = 1, 2, \cdots, n$ be n PNNs. The aggregation ranking function $\Re$ is defined as

$$\Re\left(\sum_{i=1}^{n} X_i^{PN}\right) = [\bigwedge_{i=1}^{n}\mu_{x_i} + (1-\bigvee_{i=1}^{n}\vartheta_{x_i}) + (1-\bigvee_{i=1}^{n}\pi_{x_i})] \times \sum_{i=1}^{n} \frac{R\left(X_i^{PN}\right)}{\mu_{x_i}+\left(1-\vartheta_{x_i}\right)+\left(1-\pi_{x_i}\right)}$$

Example 2: Let $X_1^{PN} = \langle 11, 16, 23, 26, 30; 0.8, 0.3, 0.5\rangle$ and $X_2^{PN} = \langle 10, 18, 22, 25, 32; 0.7, 0.4, 0.3\rangle$ are two PNNs. Then $R\left(X_1^{PN} + X_2^{PN}\right) = 27.4282$ and $\Re\left(X_1^{PN} + X_2^{PN}\right) = 27.4282$.

Hence, $\Re\left(X_1^{PN} + X_2^{PN}\right) = R\left(X_1^{PN} + X_2^{PN}\right)$.

Lemma 2: Let $X_i^{PN} = \langle x_i^{p_1}, x_i^{p_2}, x_i^{p_3}, x_i^{p_4}, x_i^{p_5}; \mu_{x_i}, \vartheta_{x_i}, \pi_{x_i}\rangle, i = 1, 2, \cdots, n$ be the n PNNs and $\lambda_i \in \mathbb{R}$. Then

$$\Re\left(\sum_{i=1}^{n} \lambda_i X_i^{PN}\right) = [\bigwedge_{i=1}^{n}\mu_{x_i} + (1-\bigvee_{i=1}^{n}\vartheta_{x_i}) + (1-\bigvee_{i=1}^{n}\pi_{x_i})] \times \sum_{i=1}^{n} \frac{R\left(X_i^{PN}\right)}{\mu_{x_i}+\left(1-\vartheta_{x_i}\right)+\left(1-\pi_{x_i}\right)}\lambda_i$$

Proof.

From Definition 3, we have

$$\sum_{i=1}^{n} \lambda_i X_i^{pN} =$$

$$\langle \sum_{i=1}^{n} \lambda_i x_i^{p_1}, \sum_{i=1}^{n} \lambda_i x_i^{p_2}, \sum_{i=1}^{n} \lambda_i x_i^{p_3}, \sum_{i=1}^{n} \lambda_i x_i^{p_4}, \sum_{i=1}^{n} \lambda_i x_i^{p_5}; \bigwedge_{i=1}^{n}\mu_{x_i}, \bigvee_{i=1}^{n}\vartheta_{x_i}, \bigvee_{i=1}^{n}\pi_{x_i}\rangle$$

From Definition 6 and Lemma 1, we have

$$\begin{aligned}\Re\left(\sum_{i=1}^{n} \lambda_i X_i^{PN}\right) &= [\bigwedge_{i=1}^{n}\mu_{x_i} + (1-\bigvee_{i=1}^{n}\vartheta_{x_i}) + (1-\bigvee_{i=1}^{n}\pi_{x_i})] \times \sum_{i=1}^{n} \frac{R\left(\lambda_i X_i^{PN}\right)}{\mu_{x_i}+\left(1-\vartheta_{x_i}\right)+\left(1-\pi_{x_i}\right)} \\ &= [\bigwedge_{i=1}^{n}\mu_{x_i} + (1-\bigvee_{i=1}^{n}\vartheta_{x_i}) + (1-\bigvee_{i=1}^{n}\pi_{x_i})] \times \sum_{i=1}^{n} \frac{R\left(X_i^{PN}\right)}{\mu_{x_i}+\left(1-\vartheta_{x_i}\right)+\left(1-\pi_{x_i}\right)}\lambda_i\end{aligned}$$

3 Data Envelopment Analysis

Suppose that there are n decision making units (DMUs) each having m inputs and r outputs as represented by the vectors $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^r$, respectively. We define the input matrix X as $X = [x_1, \cdots, x_m] \in \mathbb{R}^{m\times n}$, and the output matrix Y as $Y = [y_l, \cdots, y_r] \in \mathbb{R}^{r\times n}, x_i \in R^m, \forall i = 1, 2, \cdots, m, y_k \in \mathbb{R}^r, \forall k = 1, 2, 3, \cdots, r$ and assume that $X > 0$ and $Y > 0$. Charnes et al. [1] developed this model for measuring the efficiency of $DMU_o, (o = 1, 2, 3, \cdots, n)$, that is,

$$\left.\begin{array}{rl} \max_{u_k,v_i} & \theta = \frac{\sum_{k=1}^{r} u_k y_{ko}}{\sum_{i=1}^{m} v_i x_{io}}, \\ \text{subject to} & \frac{\sum_{k=1}^{r} u_k y_{kj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, 2, \cdots n, \\ \text{and} & u_k \ge 0, k = 1, 2, \cdots, r, \\ & v_i \ge 0, i = 1, 2, \cdots, m, \end{array}\right\} \text{(Model 1)}$$

which is the non-linear programming problem. The corresponding linear programming (LP_o) problem of the given non-linear programming problem is

$$\begin{aligned} \max_{u_k,v_i}\theta &= \sum_{k=1}^{r} u_k y_{ko}, \\ \text{subject to } &\sum_{i=1}^{m} v_i x_{io} = 1, \\ &\sum_{k=1}^{r} u_k y_{kj} \le \sum_{i=1}^{m} v_i x_{ij}, j = 1, 2, \cdots, n, \\ \text{and } u_k &\ge 0, \quad k = 1, 2, \cdots, r, \\ v_i &\ge 0, \quad i = 1, 2, \cdots, m, \end{aligned}$$

which is called CCR model. This mathematical model can be solved easily using LPP solving techniques. The optimal solution (θ_o^*) of the above models gives the efficiency score of the $DMU_o, \quad o = 1, 2, \cdots, n$.

Definition 7: A DMU is said to be efficient if the efficiency score (θ^*) is 1; Otherwise, the DMU is inefficient.

3.1 Proposed Pentagonal Neu-DEA Model

This subsection provides a brief overview of the Pentagonal Neutrosophic Data Envelopment Analysis (PNNeu-DEA) model, which is an extension of the traditional Data Envelopment Analysis (DEA) method. The PNNeu-DEA model utilizes pentagonal neutrosophic numbers to represent imprecise and uncertain input and output data. Pentagonal neutrosophic numbers are made up of five components and three membership degree that are represented as real numbers between 0 and 1. Let us consider the input and output for the $DMU_j, j = 1, 2, \cdots n$ are $x_{ij}^{PN}, i = 1, 2, \cdots, m$ and $y_{kj}^{PN}, k = 1, 2, \cdots, r$. The input and output weights are considered as $v_i \in \mathbb{R}, i = 1, 2, \cdots, m$ and $u_k \in \mathbb{R}, k = 1, 2, \cdots, r$ respectively. The Mathematical model in the presence of PNN inputs and outputs is defined as below.

$$\left.\begin{array}{rl} \max_{u_k, v_i} & \theta = \sum_{k=1}^{r} u_k y_{ko}^{PN}, \\ \text{subject to} & \sum_{i=1}^{m} v_i x_{io}^{PN} = 1^{PN}, \\ & \sum_{k=1}^{r} u_k y_{kj}^{PN} \leq \sum_{i=1}^{m} v_i x_{ij}^{PN}, j = 1, 2, \cdots n, \\ \text{and} & u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ & v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{array}\right\} \text{(Model 2)}$$

where the input and outputs are PNNs i.e., $y_{kj}^{PN} = \left\langle y_{kj}^{p_1}, y_{kj}^{p_2}, y_{kj}^{p_3}, y_{kj}^{p_4}, y_{kj}^{p_5}; \mu_{y_{kj}}, \vartheta_{y_{kj}}, \pi_{y_{kj}} \right\rangle$ and $x_{ij}^{PN} = \left\langle x_{ij}^{p_1}, x_{ij}^{p_2}, x_{ij}^{p_3}, x_{ij}^{p_4}, x_{ij}^{p_5}; \mu_{x_{ij}}, \vartheta_{x_{ij}}, \pi_{x_{ij}} \right\rangle$, and $1^{PN} = \langle 1, 1, 1, 1, 1; 1, 0, 0 \rangle$. The DEA model base on pentagonal neutrosophic numbers is defined as

$$\begin{aligned} \max_{u_k, v_i} \theta &= \sum_{k=1}^{r} u_k \left\langle y_{ko}^{p_1}, y_{ko}^{p_2}, y_{ko}^{p_2}, y_{ko}^{p_4}, y_{ko}^{p_5}; \mu_{y_{ko}}, \vartheta_{y_{ko}}, \pi_{y_{ko}} \right\rangle, \\ \text{subject to} & \sum_{i=1}^{m} v_i \left\langle x_{ij}^{p_1}, x_{ij}^{p_2}, x_{ij}^{p_3}, x_{ij}^{p_4}, x_{ij}^{p_5}; \mu_{x_{ij},}, \vartheta_{x_{ij},}, \pi_{x_{ij}} \right\rangle = \langle 1, 1, 1, 1, 1; 1, 0, 0 \rangle \\ & \sum_{k=1}^{r} u_k \left\langle y_{kj}^{p_1}, y_{kj}^{p_2}, y_{kj}^{p_3}, y_{kj}^{p_4}, y_{kj}^{p_5}; \mu_{y_{kj}}, \vartheta_{y_{kj}}, \pi_{y_{kj}} \right\rangle \\ & \leq \left\langle x_{ij}^{p_1}, x_{ij}^{p_2}, x_{ij}^{p_3}, x_{ij}^{p_4}, x_{ij}^{p_5}; \mu_{x_{ij},}, \vartheta_{x_{ij}}, \pi_{x_{ij}} \right\rangle, j = 1, 2, \cdots n, \\ \text{and} \quad & u_k \geq 0, k = 1, 2, \cdots, r, \\ & v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{aligned}$$

which is the Pentagonal Neutrosophic DEA (PN-Neu-DEA) model. This model can't be solved directly using the existing LP techniques. The aggregation ranking function is used in the constraints and objective function of the PN-Neu-DEA model to convert its corresponding crisp form.

$$\max_{u_k, v_i} \theta = \Re\left(\sum_{k=1}^{r} u_k \left\langle y_{ko}^{p_1}, y_{ko}^{p_2}, y_{ko}^{p_3}, y_{ko}^{p_4}, y_{ko}^{p_5}; \mu_{y_{ko}}, \vartheta_{y_{ko'}}, \pi_{y_{ko}} \right\rangle\right),$$

$$\text{subject to } \Re\left(\sum_{i=1}^{m} v_i \left\langle x_{io}^{p_1}, x_{io}^{p_2}, x_{io}^{p_3}, x_{io}^{p_4}, x_{io}^{p_5}; \mu_{x_{io}}, \vartheta_{x_{io}}, \pi_{x_{io}} \right\rangle\right) = R(\langle 1, 1, 1, 1, 1; 1, 0, 0 \rangle)$$

$$\Re\left(\sum_{k=1}^{r} u_k \left\langle y_{kj}^{p_1}, y_{kj}^{p_2}, y_{kj}^{p_3}, y_{kj}^{p_4}, y_{kj}^{p_5}; \mu_{y_{kj},}, \vartheta_{y_{kj}}, \pi_{y_{kj}} \right\rangle\right)$$

$$\leq \Re\left(\left\langle x_{ij}^{p_1}, x_{ij}^{p_2}, x_{ij}^{p_3}, x_{ij}^{p_4}, x_{ij}^{p_5}; \mu_{x_{ij}}, \vartheta_{x_{ij},}, \pi_{x_{ij}} \right\rangle\right), j = 1, 2, \cdots, n,$$

$$\text{and} \quad u_k \geq 0, k = 1, 2, \cdots, r,$$

$$v_i \geq 0, i = 1, 2, \cdots, m,$$

By applying Lemma 2, we have

$$\max_{u_k, v_i} \theta = [\wedge_{k=1}^{r} \mu_{y_{ko}} + (1 - \bigvee_{k=1}^{r} \vartheta_{y_{ko}}) + (1 - \bigvee_{k=1}^{r} \pi_{y_{ko}})] \times \sum_{k=1}^{r} \frac{y_{ko}^{p_1} + 3y_{ko}^{p_2} + 5y_{ko}^{p_3} + 3y_{ko}^{p_4} + y_{ko}^{p_5}}{39} u_k \text{ (Model 3)}$$

$$\text{subject to } [\bigwedge_{i=1}^{m} \mu_{x_{io}} + (1 - \bigvee_{i=1}^{m} \vartheta_{x_{io}}) + (1 - \bigvee_{i=1}^{m} \pi_{x_{io}})] \sum_{i=1}^{m} (x_{io}^{p_1} + 3x_{io}^{p_2} + 5x_{io}^{p_3} + 3x_{io}^{p_i} + x_{io}^{p_5}) v_i = 39,$$

$$[\bigwedge_{k=1}^{r} \mu_{y_{kj}} + (1 - \bigvee_{k=1}^{r} \vartheta_{y_{kj}}) + (1 - \bigvee_{k=1}^{r} \pi_{y_{kj}})] \sum_{k=1}^{r} \left(y_{kj}^{p_1} + 3y_{kj}^{p_2} + 5y_{kj}^{p_3} + 3y_{kj}^{p_4} + y_{kj}^{p_5}\right) u_k$$

$$\leq [\bigwedge_{i=1}^{m} \mu_{x_{ij}} + (1 - \bigvee_{i=1}^{m} \vartheta_{x_{ij}}) + (1 - \bigvee_{i=1}^{m} \pi_{x_{ij}})] \sum_{i=1}^{m} (x_{ij}^{p_1} + 3x_{ij}^{p_2} + 5x_{ij}^{p_s} + 3x_{ij}^{p_4} + x_{ij}^{p_5}) v_i,$$

$$j = 1, 2, \cdots, n,$$

$$\text{and} \quad u_k \geq 0, \quad k = 1, 2, \cdots, r$$
$$v_i \geq 0, \quad i = 1, 2, \cdots, m$$

which is the corresponding crisp LP model of the Pentagonal Neutrosophic DEA model.

Theorem 1: The Optimal solution of the Pentagonal Neu-DEA model defined in Model 2 and the corresponding crisp LP model defined in Model 3 are equivalent.

Definition 8: The optimal solution of the crisp LP model is the efficiency score of the DMUs. If efficiency score of the DMU is 1, then it is said to be efficient otherwise it is inefficient.

The provided flow chart in Figure 2 illustrates the methodology employed to solve the Pentagonal Neu-DEA (PN-Neu-DEA) model.

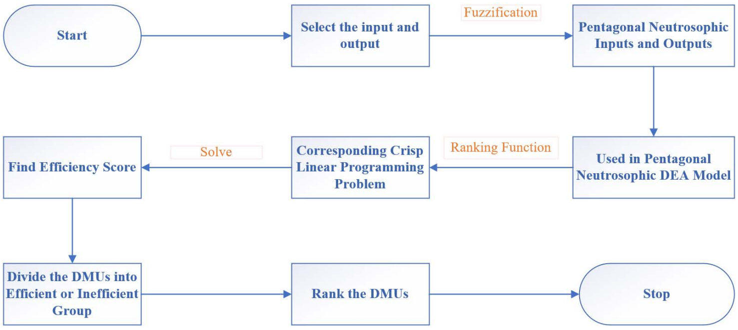

Figure 2. The method of solution for the PN-Neu-DEA model

4 Numerical Example

This section presents a practical example to elucidate the applicability and efficacy of the Pentagonal Neu-DEA model. The illustration incorporates seven Decision-Making Units (DMUs), each characterized by two input and two output variables represented as Pentagonal Neutrosophic Numbers (PNNs). The input-output matrix for the respective DMUs is provided in Table 1.

Table 1. The pentagonal neutrosophic input and output data

DMUs	Input 1	Input 2	Output 1	Output 2
DMU 1	⟨21,25,30,33,36; 0.8,0.4,0.6⟩	⟨108,112,120,125,129; 0.5,0.2,0.6⟩	⟨54,59,63,68,72; 0.5,0.3,0.7⟩	⟨162,168,172,176,180; 0.7,0.3,0.6⟩
DMU 2	⟨8,12,17,23,27; 0.9,0.3,0.4⟩	⟨87,92,95,99,106; 0.7,0.5,0.3⟩	⟨75,79,83,88,92; 0.8,0.5,0.2⟩	⟨144,148,153,156,161; 0.8,0.5,0.3⟩
DMU 3	⟨15,19,25,27,34; 0.4,0.8,0.6⟩	⟨118,122,127,132,136; 0.9,0.5,0.1⟩	⟨64,68,71,76,80; 0.7,0.1,0.6⟩	⟨153,156,161,167,173; 0.7,0.2,0.4⟩
DMU 4	⟨11,15,17,21,26; 0.8,0.4,0.2;⟩	⟨98,106,109,115,121; 0.6,0.2,0.8⟩	⟨71,76,81,85,89; 0.6,0.5,0.3⟩	⟨168,173,178,183,187; 0.5,0.6,0.2⟩
DMU 5	⟨25,29,33,37,40; 0.7,0.5,0.1⟩	⟨105,110,118,125,130; 0.8,0.6,0.4⟩	⟨50,53,58,62,65; 0.4,0.9,0.5⟩	⟨151,156,160,165,169; 0.8,0.3,0.1⟩
DMU 6	⟨18,23,27,31,35; 0.5,0.7,0.4⟩	⟨100,105,109,114,119; 0.7,0.3,0.5⟩	⟨57,63,67,72,75; 0.9,0.6,0.4⟩	⟨173,176,180,183,187; 0.6,0.5,0.4⟩
DMU 7	⟨6,10,18,26,30; 0.8,0.2,0.5;⟩	⟨115,120,126,131,137; 0.9,0.4,0.7⟩	⟨73,78,81,86,88; 0.5,0.4,0.6⟩	⟨162,165,169,174,179; 0.9,0.4,0.3⟩

An application of the proposed Pentagonal Neu-DEA (PN-Neu-DEA) model is then made in a neutrosophic environment to evaluate the relative efficiency of the DMUs. Presented in Table 2, the findings are resultant of the PN-Neu-DEA model's application. Efficiency scores are quantified ranging from 0 to 1, where a score of 1 signifies complete efficiency and a score less than 1 indicates inefficiency. From the analysis, it is observed that only DMU 3 managed to obtain a perfect efficiency score of 1, suggesting an optimal resource utilization. The remaining six DMUs demonstrated inefficiencies, with efficiency scores in the range of 0.5972 to 0.9433. Consequently, these inefficient DMUs are identified as potential targets for performance improvement interventions.

Among the DMUs, DMU 5 demonstrated the lowest efficiency, with a score of 0.3314, thus ranking at the bottom. This reflects DMU 5's vast scope for enhancement. DMUs were sequenced in the order of DMU 3 $>$ DMU 2 $>$ DMU 4 $>$ DMU 6 $>$ DMU 7 $>$ DMU 1 $>$ DMU 5 based on their efficiency scores. The sequencing served as an inverse indication of their efficiency levels, with a lower rank signifying higher efficiency. DMU 3 achieved the topmost rank, thereby deemed as the most efficient DMU, while DMU 5, with the lowest rank, was identified as the least efficient. These rankings provide valuable information for decision-makers to allocate resources and identify best practices judiciously.

Table 2. Efficiency score of the DMUs in PN-Neu-DEA model

DMU	Efficiency score	Type	Ranking
DMU 1	0.5972	Inefficient	6
DMU 2	0.9433	Inefficient	2
DMU 3	1	Efficient	1
DMU 4	0.9185	Inefficient	3
DMU 5	0.3314	Inefficient	7
DMU 6	0.8376	Inefficient	4
DMU 7	0.7037	Inefficient	5

The insights derived from the PN-Neu-DEA model findings can contribute to informed decision-making by quantifying efficiency levels and pinpointing areas of improvement. Applying this model for DMU evaluation can enhance resource allocation, mitigate costs, and escalate productivity.

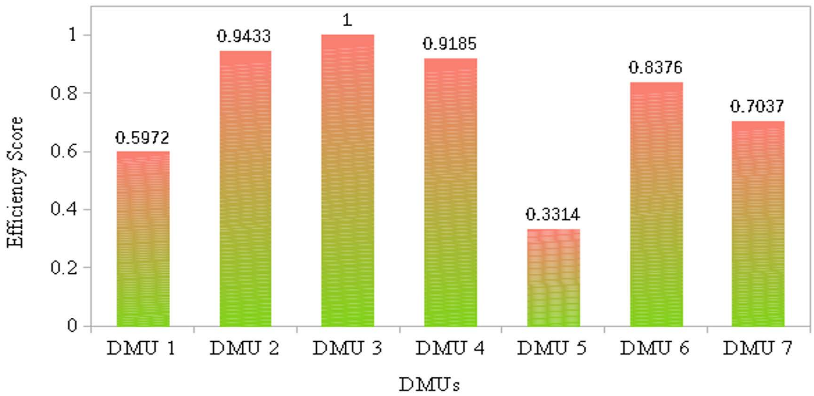

Figure 3. Comparison of the efficiency score of the DMUs in PN-Neu-DEA model

Figure 3 exhibits the efficiency scores of various DMUs as calculated through the PN-Neu-DEA model. The Y-axis depicts the efficiency scores, while the X-axis represents the DMUs. This graphical representation delineates the variation in efficiency among the DMUs, spotlighting those achieving optimal levels of efficiency (i.e., 1), and those falling short of it. This visualization serves as a beneficial tool for decision-makers, promoting an understanding of the overall performance landscape.

5 Conclusions

In this study, a novel ranking function for Pentagonal Neutrosophic Numbers (PNN) was proposed, which takes into consideration the degrees of truth, indeterminacy, and falsity. This ranking function serves as a vital tool in decision-making processes, enabling the prioritization of neutrosophic elements in scenarios characterized by uncertain, vague, or inconsistent information. A traditional Data Envelopment Analysis (DEA) model has been adapted to incorporate pentagonal neutrosophic inputs and outputs, resulting in the development of the Pentagonal Neutrosophic DEA (PN-Neu-DEA) model.

The proposed ranking function was employed to transform the PN-Neu-DEA model into a corresponding crisp Linear Programming (LP) model. By solving this crisp LP model, the efficiency of Decision-Making Units (DMUs) can be assessed, with the DMUs ranked according to their efficiency scores. An efficiency score of 1 indicates an efficient DMU, while scores below 1 signify inefficiency. A numerical example was provided to demonstrate the applicability and validity of the proposed methodology.

A key advantage of the PN-Neu-DEA model is its ability to handle uncertain and incomplete information with greater flexibility compared to other DEA models. Future research directions include applying the PN-Neu-DEA model to real-life performance evaluation cases and extending its reach to address more complex decision-making problems. These may encompass multiple inputs and outputs in uncertain environments, as well as dynamic systems.

Furthermore, the integration of the PN-Neu-DEA model with other decision-making tools could be explored to enhance its effectiveness and utility in practical situations.

Data Availability

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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