Acadlore Transactions on Applied Mathematics and Statistics
https://www.acadlore.com/journals/ATAMS

Complex Intuitionistic Hesitant Fuzzy Aggregation Information and Their Application in Decision Making Problems

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Received: 12-05-2023 **Revised:** 01-03-2024 **Accepted:** 01-09-2024

Citation: M. Ahmed, S. Ashraf, and D. S. Mashat, "Complex intuitionistic hesitant fuzzy aggregation information and their application in decision making problems," *Acadlore Trans. Appl Math. Stat.*, vol. 2, no. 1, pp. 1–21, 2024. https://doi.org/10.56578/atams020101.

Abstract: In the realm of decision-making, the delineation of uncertainty and ambiguity within data is a pivotal challenge. This study introduces a novel approach through complex intuitive hesitant fuzzy sets (CIHFS), which offers a unique multidimensional perspective for data analysis. The CIHFS framework is predicated on the concept that membership degrees reside within the unit disc of the complex plane, thereby providing a more nuanced representation of data. This method stands apart in its ability to simultaneously process and analyze data in a two-dimensional format, incorporating additional descriptive elements known as phase terms into the membership degrees. The study is bifurcated into two primary phases. Initially, a possibility degree measure is proposed, facilitating the ranking of numerical values within the CIHFS context. Subsequently, the development of innovative operational rules and aggregation operators (AOs) is undertaken. These AOs are instrumental in amalgamating diverse options within a CIHFS framework. The research dissects and deliberates on various AOs, including weighted average (WA), ordered weighted average (OWA), weighted geometric (WG), ordered weighted geometric (OWG), hybrid average (HA), and hybrid geometric (HG). Furthermore, the study extends to the realm of multi-criteria decision making (MCDM), where it proposes a methodology utilizing intricate intuitive and fuzzy information. This methodology emphasizes the objective management of weights, thereby enhancing the decision-making process. The study's findings hold significant implications for the optimization of resources and decision-making strategies, providing a robust framework for the application of CIHFS in practical scenarios.

Keywords: Complex intuitive hesitant fuzzy sets (CIHFS); Multi-criteria decision making (MCDM); Aggregation operators (AOs)

1 Introduction

The field of MCDM is concerned with the formulation and resolution of decision and planning issues in which many criteria are taken into consideration concurrently [1]. This subject of decision making (DM) has piqued the attention of a large number of scholars, who have used a variety of methodologies to study it [2]. The DM procedure has always provided precise results without checking for errors. It is common practice for MCDM issues to call for evaluation information on the criteria and the alternatives to be provided in the form of a fuzzy set (FS) [3], intuitionistic fuzzy set (IFS) [4], hesitant fuzzy sets (HFS) [5], complex intuitionistic fuzzy sets (CIFS) [6], or other extended sets. In order to more precisely describe the uncertainty and ambiguity in the data, a CIHFS has been developed as a subset of the intuitionistic hesitant fuzzy sets (IHFS) [7]. The membership degree and non-membership degree were used to draw each element in these sets, with the goal of ensuring that the total of these factors does not exceed one [8]. These studies show that there are three essential steps in any DM process, which are: (1) Methods for obtaining data interpretation standards; (2) Method(s) for compiling an aggregate preference value by adding up the scores assigned to each attribute; (3) How to order to discover the best option(s).

As a result, this study's objective is to come up with a creative DM strategy for MCDM problems by combining CIHFS with strong data ranking using AOs and a probability density measure (PDM). A lot of researchers have come up with different algorithms to deal with MCDM problems using the FS [9] and IFS [10] theories. These algorithms try to use the imperfect information that is available in the decision-making processes (DMPs). Under it, AO stands for one of the basic rules for combining data using a class of mathematical functions. Therefore, Xu and Yager [11] provided averaging and geometric (AG) AOs for intuitionistic fuzzy numbers within the context of IFS (IFNs). Garg and Kumar [12] added the degree of hesitance between MD and NMD pairings to the study, which led to the granting of AG AOs. In this study, Gou et al. [13]'s IFNs' exponentially expanding operating regulations were devised. According to Ye [14], a hybrid AO is formed when an aspect type parameter, an averaging operator, and a geometric operator are combined. Fan et al. [15] presented the AOs and operational regulations necessary to address the MCDM challenge as it pertains to IFNs. Beyond this, Huang [16] provide other experiments using AOs to address the DMPs in the IFS setting. In addition, a score function, or PDM, is required to determine the values before ranking the provided alternatives. Because of this, researchers have come up with a wide range of ways to measure the process in the IFS environment. Xu and Yager [11], for instance, published the IFN scoring function. The PDM approaches for ranking the IFNs were provided by Wei and Tang [17]. Wan and Dong [18] introduced a PDM-based approach to addressing the MCDM issue. Wei and Tang [17] proposed a method for ranking the IFNs, while Garg and Rani [19] presented an enhanced PDM that addresses the problems with previous methods. Several current PDMs were compared to IFSs and interval-valued PDMs by Dammak et al. [20]. Blanco-Mesa et al. [21] provide a thorough analysis of the AOs, and here the authors provide a concise summary of their findings. Extensive studies and DMPs have shown that their use is limited to dealing with the uncertainty in the data itself but not with its oscillations at any given point in time. Data collected through "medical research, a database for biometric and face identification," is nevertheless subject to constant evolution as time progresses. So, Ramot et al. [22] came up with the idea of the complex fuzzy set (CFS) [23] to deal with these kinds of problems. They did this by making MD's domain bigger from a real subset to the unit disc of the complex plane. This article introduces the fundamental characteristics of CFS by Ramot et al. [24]. The CFS and Pythagorean FS were compared and contrasted by Dick et al. [25].

This article is centered on the CIHFS since it has been shown to be the most effective and helpful technique for dealing with the ambiguity and fuzziness inherent in real-world MCDM challenges. As a result, we created a novel MCDM framework that incorporates CIHFS as the preference data. The management of decision-making has been receiving more and more attention as of late, prompting this essay, in which the challenge of choosing among viable MCDM strategies is examined from a sustainability viewpoint. Even though several MCDM techniques have been developed for CIHFS, no research has been released on how to evaluate and choose the best DM option using the expanded framework with aggregation operators for this purpose. This article proposes a unique paradigm to deal with this problem by addressing the ambiguity and unpredictability of decision experts (DEs)' views. The following are some of the contributions made by this article: (1) Developed a brand-new method known as CIHFS and applied it to the IHF environment; (2) A new formula was made and used to figure out the weights of the DEs in a CIHF setting; (3) A technique known as agreegations, which is based on CIHFS, is used in order to assess the attribute weights; (4) Presented a real-life case study of choosing an alternative for managing healthcare waste in the context of CIHFS to show that the proposed approach is useful for aggregation operators. The case study focused on picking an alternative for managing healthcare waste; (5) Following this, a comparison analysis and a sensitivity analysis are carried out in order to confirm the result that was produced by using the newly proposed method.

The remaining portions of this article are structured as described below. In Section II, the fundamental ideas of CIHFS are laid forth. In Section III, the aggregation operators for CIHFS are discussed, along with an example. Within the context of the CIHFS, Section IV covers the algorithm for the aggregation operator technique. The established technique is put to use in a case study of choosing a suitable healthcare waste management option in the context of Section V, which proves the applicability and power of the given methodology. In addition, Section VI contains the contrastive analysis and sensitivity tests that prove the technique described is consistent and reliable. The article is finished with its seventh and final section.

Since CFS does not provide any knowledge of the object's discords, the resulting approaches are always constrained [26]. To broaden the scope of the CIFS paradigm, the complex-valued MD of the unit disc was added.

As an extension of the IHFS, the CIHFS uses the amplitude term to characterize the scope of an object's properties, just as the time period of the phase characterizes its periodicity. CIHFS is unique from conventional IHFS theories due to the use of these phase terms. With IHFS theory, data is handled by compensating just for the degree of possessions, while the periodicity component is disregarded altogether.

2 Preliminaries

Definition 1. A FS form:

$$\breve{A} = \{ < \Phi_{\breve{A}}(x) > | \; x \in X \}$$

with a condition $0 \leqslant \Phi_{\breve{A}}(x) \leqslant 1$, where $\Phi_{\breve{A}}(x)$ shows the membership of $\breve{A}$ in X. Throughout this article, the collection of all Fuzzy Numbers on X is denoted by $FS(X)$.

Definition 2. A CFS $\acute{A}$ is of the form:

$$\acute{A} = \{< \Phi_{\acute{A}}(x) >| \, x \in X\}$$

where, $\Phi_{\acute{A}}(x) = \Gamma_{\acute{A}}(x).\ddot{e}^{\iota 2\pi(\mu_{\Gamma\acute{A}}(x))}$ shows the complex membership in the form of polar coordinate, where, $\Gamma_{\acute{A}}(x), \mu_{\Gamma\acute{A}}(x) \in [0, 1]$.

Definition 3. A IFS $\bar{A}$ is of the form:

$$\bar{A} = \{< \Phi_{\bar{A}}(x), \lambda_{\bar{A}}(x) >| \, x \in X\}$$

where, $\Phi_{\bar{A}}(x), \lambda_{\bar{A}}(x) : X \to [0, 1]$ shows the degree of membership and degree of non membership of x in $\breve{A}$ and its value lies between zero and one such that $\Phi_{\bar{A}} + \lambda_{\bar{A}} \leq 1$ and the pair $< \Phi_{\bar{A}}, \lambda_{\bar{A}} >$ is called IFN.

Definition 4. A HFS [27] is a set that, when applied to a given set, x, and defined using the function X, produces a subset that is comprised of the values 0 and 1. HFS is expressed as a mathematical symbol for clarity:

$$\ddot{e} = \left\{ \left\langle x, \hat{h}_{\ddot{e}}\left(x\right) \right\rangle \mid x \in X \right\}$$

where, $\hat{h}_{\ddot{e}}(x)$ is a collection of numbers in $[0, 1]$ that represent the potential degrees of membership of the element $x \in X$ to the set $\ddot{e}$. H is the collection of all HFEs, and $h = \hat{h}_{\ddot{e}}(x)$ is a hesitant fuzzy element (HFE). Several operations on the three HFEs $\hat{h}$, $\hat{h}_1$, and $\hat{h}_2$ have been defined as follows:

(1) $\hat{h}_1 \cup \hat{h}_2 = \cup_{\psi_1 \in \hat{h}_1, \psi_2 \in \hat{h}_2} \max\{\psi_1, \psi_2\}$;

(2) $\hat{h}_1 \cap \hat{h}_2 = \cup_{\psi_1 \in \hat{h}_1, \psi_2 \in \hat{h}_2} \min\{\psi_1, \psi_2\}$;

(3) $\hat{h}^c = \cup_{\psi \in \hat{h}} \{1 - \psi\}$.

Definition 5. A CHFS G is of the form:

$$G = \{(x, \xi_{\hat{h}}(x)) | x \in X\}$$

where,

$$\xi_{\hat{h}}(x) = \bigcup_{\ddot{k}=1}^{\#} \left\{ \phi_{\ddot{k}}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\phi_{\ddot{k}}}(x))} \right\} = \left\{ \phi_1 \ddot{e}^{\ddot{\imath}2\pi(\omega_{\phi_1})}, \phi_2 \ddot{e}^{\ddot{\imath}2\pi(\omega_{\phi_2})}, \phi_3 \ddot{e}^{\ddot{\imath}2\pi(\omega_{\phi_3})}, ..., \phi_{\#} \ddot{e}^{\ddot{\imath}2\pi(\omega_{\phi_{\#}})} \right\}$$

represented the complex-valued truth grade which is subset of unit disc in complex plane with a condition $\phi_{\ddot{k}}, \omega_{\phi_{\ddot{k}}} \in [0, 1]$.

Definition 6. A CIHFS F is of the form:

$$F = \{\langle x, \xi_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\xi_{\hat{h}_\jmath}}(x))}, \lambda_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\lambda_{\hat{h}_\jmath}}(x))} \rangle \mid x \in X\}$$

where, $\xi_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\xi_{\hat{h}_\jmath}})}$ and $\lambda_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\lambda_{\hat{h}_\jmath}})}$ are the complex hesitant conditional membership and non-membership grades that $0 \leq \{\max\left(\xi_{\hat{h}_\jmath}\right) + \min\left(\lambda_{\hat{h}_\jmath}(x)\right)\} \leq 1$ and $0 \leq \{\max\left(\omega_{\xi_{\hat{h}_\jmath}}(x)\right) + \min\left(\omega_{\lambda_{\hat{h}_\jmath}}(x)\right)\} \leq 1$ such that $\xi_{\hat{h}_\jmath}, \lambda_{\hat{h}_\jmath}, \omega_{\xi_{\hat{h}_\jmath}}, \omega_{\lambda_{\hat{h}_\jmath}}$ are the subset of the unit interval $[0, 1]$.

For simplicity we express $F = \left\{ \xi_{\hat{h}_\jmath} \ddot{e}^{\ddot{\imath}2\pi(\omega_{\xi_{\hat{h}_\jmath}})}, \lambda_{\hat{h}_\jmath} \ddot{e}^{\ddot{\imath}2\pi(\omega_{\lambda_{\hat{h}_\jmath}})} \right\}$ be the complex intuitionistic hesitant fuzzy number (CIHFN).

Let $F_\jmath = \{\xi_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\xi_{\hat{h}_\jmath}})}, \lambda_{\hat{h}_\jmath}(x)\ddot{e}^{\ddot{\imath}2\pi(\omega_{\lambda_{\hat{h}_\jmath}})}\}$ $(\jmath = 1, 2)$ be any two CIHFNs, then

$$1. \quad F_1 \cup F_2 = \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}}}} \left\{ \begin{array}{c} \max\left(\phi_{\ddot{k}_1}, \phi_{\ddot{k}_2}\right) \ddot{e}^{\iota 2\pi\left(\max\left(\omega_{\phi_{\ddot{k}_1}}, \omega_{\phi_{\ddot{k}_2}}\right)\right)}, \\ \min\left(\Gamma_{\ddot{k}_1}, \Gamma_{\ddot{k}_2}\right) \ddot{e}^{\iota 2\pi\left(\min\left(\omega_{\Gamma_{\ddot{k}_1}}, \omega_{\Gamma_{\ddot{k}_2}}\right)\right)} \end{array} \right\}$$

$$2. \quad F_1 \cap F_2 = \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}}}} \left\{ \begin{array}{c} \min\left(\phi_{\ddot{k}_1}, \phi_{\ddot{k}_2}\right) \ddot{e}^{\iota 2\pi\left(\min\left(\omega_{\phi_{\ddot{k}_1}}, \omega_{\phi_{\ddot{k}_2}}\right)\right)}, \\ \max\left(\Gamma_{\ddot{k}_1}, \Gamma_{\ddot{k}_2}\right) \ddot{e}^{\iota 2\pi\left(\max\left(\omega_{\Gamma_{\ddot{k}_1}}, \omega_{\Gamma_{\ddot{k}_2}}\right)\right)} \end{array} \right\}$$

3. $F_1^c = \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}}} \left\{ \Gamma_{\ddot{k}} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\Gamma_{\ddot{k}}})}, \phi_{\ddot{k}} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\phi_{\ddot{k}}})} \right\}$

4. $F_1 \oplus F_2 \{ \xi_{\hat{h}_1}(x) \oplus \xi_{\hat{h}_2}(x), \lambda_{\hat{h}_1}(x) \oplus \lambda_{\hat{h}_2}(x) \}$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}}}} \left\{ \left(\begin{array}{c} \left(\phi_{\ddot{k}_1} + \phi_{\ddot{k}_2} - \phi_{\ddot{k}_1} \phi_{\ddot{k}_2} \right) \ddot{e}^{\iota 2\pi \left(\omega_{\phi_{\ddot{k}_1}} + \omega_{\phi_{\ddot{k}_2}} - \omega_{\phi_{\ddot{k}_1}} \omega_{\phi_{\ddot{k}_2}} \right)}, \\ \Gamma_{\ddot{k}_1} \Gamma_{\ddot{k}_2} \ddot{e}^{\iota 2\pi \left(\omega_{\Gamma_{\ddot{k}_1}} \omega_{\Gamma_{\ddot{k}_2}} \right)} \end{array} \right) \right\}$$

5. $F_1 \otimes F_2 = \{ \xi_{\hat{h}_1}(x) \otimes \xi_{\hat{h}_2}(x), \lambda_{\hat{h}_1}(x) \otimes \lambda_{\hat{h}_2}(x) \}$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}}}} \left\{ \left(\begin{array}{c} \phi_{\ddot{k}_1} \phi_{\ddot{k}_2} \ddot{e}^{\iota 2\pi \left(\omega_{\phi_{\ddot{k}_1}} \omega_{\phi_{\ddot{k}_2}} \right)}, \\ \Gamma_{\ddot{k}_1} + \Gamma_{\ddot{k}_2} - \Gamma_{\ddot{k}_1} \Gamma_{\ddot{k}_2} \ddot{e}^{\iota 2\pi \left(\omega_{\Gamma_{\ddot{k}_1}} + \omega_{\Gamma_{\ddot{k}_2}} - \omega_{\Gamma_{\ddot{k}_1}} \omega_{\Gamma_{\ddot{k}_2}} - \right)} \end{array} \right) \right\}$$

6. $\lambda.F = \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}}} \left\{ \left(1 - (1 - \Gamma_{\ddot{k}}\right)^{\lambda} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \left(1 - \omega_{\Gamma_{\ddot{k}}}\right)\right)^{\lambda}}, \left(\phi_{\ddot{k}} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\phi_{\ddot{k}}})} \right)^{\lambda} \right\}, \lambda > 0$

7. $F^{\lambda} = \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}}} \left\{ \left(\Gamma_{\ddot{k}} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\Gamma_{\ddot{k}}})} \right)^{\lambda}, \left(1 - (1 - \phi_{\ddot{k}}\right)^{\lambda} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \left(1 - \omega_{\phi_{\ddot{k}}}\right)\right)^{\lambda}} \right\}, \lambda > 0$

Definition 7. For an CIHFS:

$$S\left(\hat{h}\right) = \left(\begin{array}{c} \left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\jmath=1}^{n} \xi_{\hat{h}_\jmath}(x) + \frac{1}{n} \sum_{\jmath=1}^{n} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\xi_{\hat{h}_\jmath}}(x))} \right\} \right\} \times \\ \left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\jmath=1}^{n} \lambda_{\hat{h}_\jmath} + \frac{1}{n} \sum_{\jmath=1}^{n} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\lambda_{\hat{h}_\jmath}}(x))} \right\} \right\} \end{array} \right)$$

is called a score function. And

$$A\left(\hat{h}\right) = \left(\begin{array}{c} \left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\jmath=1}^{n} \xi_{\hat{h}_\jmath}(x) - \frac{1}{n} \sum_{\jmath=1}^{n} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\xi_{\hat{h}_\jmath}}(x))} \right\} \right\} \times \\ \left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\jmath=1}^{n} \lambda_{\hat{h}_\jmath} - \frac{1}{n} \sum_{\jmath=1}^{n} \ddot{e}^{\bar{\imath} 2\pi (\omega_{\lambda_{\hat{h}_\jmath}}(x))} \right\} \right\} \end{array} \right)$$

is called a accuracy function.

3 Aggregation Operators for Complex Intuitionistics Hesitant Fuzzy Information

Yager [28] presented ordered weighted averaging aggregation operations, which have gained increasing interest since their introduction. Below is a list of CIHFS aggregation providers:

For a collection of CIHFS $F_\jmath = \{ \xi_{\hat{h}_\jmath}(x) \ddot{e}^{\bar{\imath} 2\pi (b_{\xi_{\hat{h}_\jmath}})}, \lambda_{\hat{h}_\jmath}(x) \ddot{e}^{\bar{\imath} 2\pi (\omega_{\lambda_{\hat{h}_\jmath}})} \}$ $(\jmath = 1, 2)$, then

1. The complex intuitionistic hesitant fuzzy weighted averaging (CIHFWA)operator:

$$CIHFWA\left(F_1, F_2, ..., F_n\right) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath F_\jmath\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(1 - \prod_{\jmath=1}^{n} \left(1 - \Gamma_{F_{\ddot{k}}}\right)^{\varpi_\jmath}\right) \ddot{e}^{\bar{\imath} 2\pi \left(1 - \prod_{\jmath=1}^{n} \left(1 - \omega_{\Gamma_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n} \left(\phi_{F_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\phi_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}} \right) \end{array} \right\}$$

where, $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is the weight vector of $F_\jmath = (\jmath = 1, 2...n)$ with $\varpi_\jmath \in [0, 1]$ and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$.

2. The complex intuitionistics hesitant fuzzy weighted geometeric (CIHFWG) operator:

$$CIHFWG(F_1, F_2, ..., F_n) = \bigotimes_{\jmath=1}^{n} \left(F_\jmath^{\varpi_\jmath}\right) =$$

$$\bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(\prod_{\jmath=1}^{n} \left(\Gamma_{F_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\Gamma_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}} \right), \\ 1 - \prod_{\jmath=1}^{n} \left(1 - \phi_{F_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \prod_{\jmath=1}^{n} (1 - \omega_{\phi_{F_{\ddot{k}}}})^{\varpi_\jmath}\right)} \end{array} \right\}$$

where, $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is the weight vector of $F_\jmath = (\jmath = 1, 2...n)$ with $\varpi_\jmath \in [0, 1]$ and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$.

3. The complex intuitionistic hesitant fuzzy ordered weighted averaging (CIHFOWA) operator:

$$CIHFOWA(F_1, F_2, ..., F_n) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath F_{\beta(\jmath)}\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} 1 - \prod_{\jmath=1}^{n} \left(1 - \Gamma_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \prod_{\jmath=1}^{n} \left(1 - \omega_{\Gamma_{F_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n} \left(\phi_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\phi_{F_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}} \right) \end{array} \right\}$$

where, $F_{\beta_\jmath}$ is the $\jmath$th largest $\beta_\jmath = n\varpi_l F_l \, (l = 1, 2, ..., n)$, $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ with $\varpi_\jmath \in [0, 1]$ and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$. is the vector that is related to aggregation such that $\varpi_\jmath \in [0, 1]$, $\jmath = 1, 2, ..., n$, and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$.

4. The complex intuitionistic hesitant fuzzy ordered weighted geometric (CIHFOWG) operator:

$$CIHFOWG(F_1, F_2, ..., F_n) = \bigotimes_{\jmath=1}^{n} \left(F_{\beta(\jmath)}^{\varpi_\jmath}\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(\prod_{\jmath=1}^{n} \left(\Gamma_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\Gamma_{F_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}} \right), \\ 1 - \prod_{\jmath=1}^{n} \left(1 - \phi_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \prod_{\jmath=1}^{n} (1 - \omega_{\phi_{F_{\beta_{\ddot{k}}}}})^{\varpi_\jmath}\right)} \end{array} \right\}$$

where, $F_{\beta_\jmath}$ is the $\jmath$th largest $\beta_\jmath = n\varpi_l F_l \, (l = 1, 2, ..., n)$, $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ with $\varpi_\jmath \in [0, 1]$ and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$. is the vector that is related to aggregation such that $\varpi_\jmath \in [0, 1]$, $\jmath = 1, 2, ..., n$, and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$.

5. The complex intuitionistic hesitant fuzzy hybrid averaging (CIHFHA) operator:

$$CIHFHA(F_1, F_2, ..., F_n) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath \dot{F}_{\beta(\jmath)}\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} 1 - \prod_{\jmath=1}^{n} \left(1 - \Gamma_{\dot{F}_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \left(1 - \prod_{\jmath=1}^{n} \left(1 - \omega_{\Gamma_{\dot{F}_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n} \left(\phi_{\dot{F}_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath} 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\phi_{\dot{F}_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}} \right) \end{array} \right\}$$

where, $\dot{F}_{\beta_\jmath}$ is the $\jmath$th largest $\dot{F}_l = n\varpi_l F_l \, (l = 1, 2, ..., n)$, $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is the weight vector of $F_\jmath = (\jmath = 1, 2...n)$ with $\varpi_\jmath \in [0, 1]$, $\jmath = 1, 2, ..., n$, $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$ and $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is the vector that is related to aggregation such that $\varpi_\jmath \in [0, 1]$, $\jmath = 1, 2, ..., n$, and $\sum_{\jmath=1}^{n} \varpi_\jmath = 1$.

6. The complex intuitionistic hesitant fuzzy hybrid geometric (CIHFHG) operator:

$$CIHFHG(F_1, F_2, ..., F_n) \bigotimes_{\jmath=1}^{n} \left(F_{\beta(\jmath)}^{\varpi_\jmath}\right) =$$

$$\bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}\ldots\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}\ldots\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(\prod_{\jmath=1}^{n}\left(\Gamma_{\ddot{F}_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi \prod_{\jmath=1}^{n}\left(\omega_{\Gamma_{\ddot{F}_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}} \right), \\ 1-\prod_{\jmath=1}^{n}\left(1-\phi_{\ddot{F}_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{n}(1-\omega_{\phi_{\ddot{F}_{\beta_{\ddot{k}}}}})^{\varpi_\jmath}\right)} \end{array} \right\}$$

where, $\ddot{F}_{\beta_\jmath}$ is the $\jmath$th largest $\ddot{F}_l = n\varpi_l F_l\,(l=1,2,...,n)$, $\varpi=(\varpi_1,\varpi_2,...,\varpi_n)^T$ is the weight vector of $F_\jmath =$ $(\jmath=1,2...n)$ with $\varpi_\jmath\in[0,1]$, $\jmath=1,2,...,n$, $\sum_{\jmath=1}^{n}\varpi_\jmath=1$ and $\varpi=(\varpi_1,\varpi_2,...,\varpi_n)^T$ is the vector that is related to aggregation such that $\varpi_\jmath\in[0,1]$, $\jmath=1,2,...,n$, and $\sum_{\jmath=1}^{n}\varpi_\jmath=1$.

Example 1. Let

$$F_1=\left\langle\left(0.1\ddot{e}^{\imath 2\pi(0.2)},0.7\ddot{e}^{\imath 2\pi(0.3)},0.4\ddot{e}^{\imath 2\pi(0.16667)}\right),\left(0.3\ddot{e}^{\imath 2\pi(0.1)},0.2\ddot{e}^{\imath 2\pi(0.8)}\right)\right\rangle$$

$$F_2=\left\langle\left(0.3\ddot{e}^{\imath 2\pi(0.2)},0.5\ddot{e}^{\imath 2\pi(0.4)}\right),\left(0.1\ddot{e}^{\imath 2\pi(0.1)},0.2\ddot{e}^{\imath 2\pi(0.8)},0.5\ddot{e}^{\imath 2\pi(0.4)}\right)\right\rangle$$

be two CIHFS and $\varpi=(0.4,0.6)^T$ be the corresponding weight vector of $F_\jmath\,(\jmath=1,2)$.

$$CIHFWA(F_1,F_2)=\bigoplus_{\jmath=1}^{2}(\varpi_\jmath F_\jmath)=\{\xi_{\hat{h}_\jmath}(x),\lambda_{\hat{h}_\jmath}(x)\}$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}\ldots\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}\ldots\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(1-\prod_{\jmath=1}^{n}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath}\right)\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n}\left(\phi_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{n}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}}\right) \end{array} \right\}$$

$$\left[\begin{array}{c} \left\{ \begin{array}{c} 1-(1-0.1)^{0.4}\times(1-0.3)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.2)^{0.4}\times(1-0.2)^{0.6}\right\}}, \\ 1-(1-0.1)^{0.4}\times(1-0.5)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.2)^{0.4}\times(1-0.4)^{0.6}\right\}}, \\ 1-(1-0.7)^{0.4}\times(1-0.3)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.3)^{0.4}\times(1-0.2)^{0.6}\right\}}, \\ 1-(1-0.7)^{0.4}\times(1-0.5)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.3)^{0.4}\times(1-0.4)^{0.6}\right\}}, \\ 1-(1-0.4)^{0.4}\times(1-0.3)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.16667)^{0.4}\times(1-0.2)^{0.6}\right\}}, \\ 1-(1-0.4)^{0.4}\times(1-0.5)^{0.6}\,\ddot{e}^{\imath 2\pi\left\{1-(1-0.16667)^{0.4}\times(1-0.4)^{0.6}\right\}} \end{array} \right\}, \\ \left\{ \begin{array}{c} \left(0.3^{0.4}\times0.1^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.1^{0.4}\times0.1^{0.6}\right)}, \\ \left(0.3^{0.4}\times0.2^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.1^{0.4}\times0.8^{0.6}\right)}, \\ \left(0.3^{0.4}\times0.5^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.1^{0.4}\times0.4^{0.6}\right)}, \\ \left(0.2^{0.4}\times0.1^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.8^{0.4}\times0.1^{0.6}\right)}, \\ \left(0.2^{0.4}\times0.2^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.8^{0.4}\times0.8^{0.6}\right)}, \\ \left(0.2^{0.4}\times0.5^{0.6}\right)\ddot{e}^{\imath 2\pi\left(0.8^{0.4}\times0.4^{0.6}\right)} \end{array} \right\} \end{array} \right]$$

$$= \left(\begin{array}{c} \left\langle \begin{array}{c} 0.2260\ddot{e}^{\imath 2\pi(0.2000)},0.3675\ddot{e}^{\imath 2\pi(0.3268)},0.5012\ddot{e}^{\imath 2\pi(0.2416)} \\ ,0.5924\ddot{e}^{\imath 2\pi(0.3618)},0.3419\ddot{e}^{\imath 2\pi(0.1881)},0.4622\ddot{e}^{\imath 2\pi(0.3168)} \end{array} \right\rangle \\ ,\left\langle \begin{array}{c} 0.1552\ddot{e}^{\imath 2\pi(0.1000)},0.2352\ddot{e}^{\imath 2\pi(0.3482)},0.4076\ddot{e}^{\imath 2\pi(0.2297)} \\ ,0.1320\ddot{e}^{\imath 2\pi(0.2297)},0.2000\ddot{e}^{\imath 2\pi(0.8000)},0.3466\ddot{e}^{\imath 2\pi(0.5278)} \end{array} \right\rangle \end{array} \right)$$

$$S\left(\hat{h}\right)=\left(\ \left\{\frac{1}{2}\left\{\frac{1}{n}\sum_{\imath=1}^{n}\xi_{\hat{h}_\jmath}(x)+\frac{1}{n}\sum_{\imath=1}^{n}\ddot{e}^{\imath 2\pi(\omega_{\xi_{\hat{h}_\jmath}}(x))}\right\}\right\}\times\left\{\frac{1}{2}\left\{\frac{1}{n}\sum_{\imath=1}^{n}\lambda_{\hat{h}_\jmath}+\frac{1}{n}\sum_{\imath=1}^{n}\ddot{e}^{\imath 2\pi(\omega_{\lambda_{\hat{h}_\jmath}}(x))}\right\}\right\}\ \right)$$

$$\left(\begin{array}{c}\left\{\frac{1}{2}\left\{\begin{array}{c}\frac{1}{6}\left(\begin{array}{c}0.2260+0.3675+0.5012+\\0.5924+0.3419+0.4622\end{array}\right)+\\\frac{1}{6}\left(\begin{array}{c}0.2000+0.3268+0.2416+\\0.3618+0.1881+0.3168\end{array}\right)\end{array}\right\}\right\}\times\\\left\{\frac{1}{2}\left\{\begin{array}{c}\frac{1}{6}\left(\begin{array}{c}0.1552+0.2352+0.4076+\\0.1320+0.2000+0.3466\end{array}\right)+\\\frac{1}{6}\left(\begin{array}{c}0.1000+0.3482+0.2297+\\0.2297+0.8000+0.5278\end{array}\right)\end{array}\right\}\right\}\end{array}\right) \text{by using score function} = 0.1064$$

Example 2. Let

$$F_1=\left\langle\left(0.1\ddot{e}^{\bar{\imath}2\pi(0.2)},0.7\ddot{e}^{\bar{\imath}2\pi(0.3)},0.4\ddot{e}^{\bar{\imath}2\pi(0.16667)}\right),\left(0.3\ddot{e}^{\bar{\imath}2\pi(0.1)},0.2\ddot{e}^{\bar{\imath}2\pi(0.8)}\right)\right\rangle$$

$$F_2=\left\langle\left(0.3\ddot{e}^{\bar{\imath}2\pi(0.2)},0.5\ddot{e}^{\bar{\imath}2\pi(0.4)}\right),\left(0.1\ddot{e}^{\bar{\imath}2\pi(0.1)},0.2\ddot{e}^{\bar{\imath}2\pi(0.8)},0.5\ddot{e}^{\bar{\imath}2\pi(0.4)}\right)\right\rangle$$

be two CIHFS and $\varpi=(0.4,0.6)^T$ be the corresponding weight vector of $F_\jmath\,(\jmath=1,2)$.

By Definition 2.7, we calculate the score values of F_1 and F_2:

$$S(F_1)=\left(\begin{array}{c}\left\{\frac{1}{2}\left\{\frac{1}{3}(0.1+0.7+0.4)+\frac{1}{3}\ddot{e}^{\bar{\imath}2\pi(0.2+0.3+0.16667)}\right\}\right\}\times\\\left\{\frac{1}{2}\left\{\frac{1}{2}(0.3+0.2)+\frac{1}{2}\ddot{e}^{\bar{\imath}2\pi(0.1+0.8)}\right\}\right\}\end{array}\right)$$

$$S(F_2)=\left(\begin{array}{c}\left\{\frac{1}{2}\left\{\frac{1}{2}(0.3+0.5)+\frac{1}{2}\ddot{e}^{\bar{\imath}2\pi(0.2+0.4)}\right\}\right\}\times\\\left\{\frac{1}{2}\left\{\frac{1}{3}(0.1+0.2+0.5)+\frac{1}{2}\ddot{e}^{\bar{\imath}2\pi(0.1+0.8+0.4)}\right\}\right\}\end{array}\right)$$

$$S(F_1)=0.1089,S(F_2)=0.1225$$

Since $S(F_2)>S(F_1)$, then

$$F_{\beta_{(1)}}=S(F_2)=\left\langle\left(0.3\ddot{e}^{\bar{\imath}2\pi(0.2)},0.5\ddot{e}^{\bar{\imath}2\pi(0.4)}\right),\left(0.1\ddot{e}^{\bar{\imath}2\pi(0.1)},0.2\ddot{e}^{\bar{\imath}2\pi(0.8)},0.5\ddot{e}^{\bar{\imath}2\pi(0.4)}\right)\right\rangle$$

$$F_{\beta_{(2)}}=S(F_1)=\left\langle\left(0.1\ddot{e}^{\bar{\imath}2\pi(0.2)},0.7\ddot{e}^{\bar{\imath}2\pi(0.3)},0.4\ddot{e}^{\bar{\imath}2\pi(0.16667)}\right),\left(0.3\ddot{e}^{\bar{\imath}2\pi(0.1)},0.2\ddot{e}^{\bar{\imath}2\pi(0.8)}\right)\right\rangle$$

$$CIHFOWA(F_1,F_2,...,F_n)=\bigoplus_{\jmath=1}^{n}\left(\varpi_\jmath F_{\beta_{(\jmath)}}\right)$$

$$=\bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}\dots\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\\omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}\dots\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}}\left\{\begin{array}{c}1-\prod_{\jmath=1}^{n}\left(1-\Gamma_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\Gamma_{F_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}\right)},\\\left(\prod_{\jmath=1}^{n}\left(\phi_{F_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{n}\left(\omega_{\phi_{F_{\beta_{\ddot{k}}}}}\right)^{\varpi_\jmath}}\right)\end{array}\right\}$$

$$=\left[\begin{array}{c}\left\{\begin{array}{c}1-(1-0.3)^{0.4}\times(1-0.1)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.2)^{0.4}\times(1-0.2)^{0.6}\right\}},\\1-(1-0.3)^{0.4}\times(1-0.7)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.2)^{0.4}\times(1-0.3)^{0.6}\right\}},\\1-(1-0.3)^{0.4}\times(1-0.4)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.2)^{0.4}\times(1-0.16667)^{0.6}\right\}},\\1-(1-0.5)^{0.4}\times(1-0.1)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.4)^{0.4}\times(1-0.2)^{0.6}\right\}},\\1-(1-0.5)^{0.4}\times(1-0.7)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.4)^{0.4}\times(1-0.3)^{0.6}\right\}},\\1-(1-0.5)^{0.4}\times(1-0.4)^{0.6}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.4)^{0.4}\times(1-0.16667)^{0.6}\right\}}\end{array}\right\},\\\left\{\begin{array}{c}\left(0.1^{0.4}\times0.3^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.1^{0.4}\times0.1^{0.6}\right)},\\\left(0.1^{0.4}\times0.2^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.1^{0.4}\times0.8^{0.6}\right)},\\\left(0.2^{0.4}\times0.3^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.8^{0.4}\times0.1^{0.6}\right)},\\\left(0.2^{0.4}\times0.2^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.8^{0.4}\times0.8^{0.6}\right)},\\\left(0.5^{0.4}\times0.3^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.4^{0.4}\times0.1^{0.6}\right)},\\\left(0.5^{0.4}\times0.2^{0.6}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.4^{0.4}\times0.8^{0.6}\right)}\end{array}\right\}\end{array}\right]$$

$$= \left(\begin{array}{c} \left\langle \begin{array}{c} 0.1861\ddot{e}^{\imath 2\pi(0.2000)}, 0.5790\ddot{e}^{\imath 2\pi(0.2616)}, 0.3618\ddot{e}^{\imath 2\pi(0.1802)} \\ , 0.1000\ddot{e}^{\imath 2\pi(0.2870)}, 0.5344\ddot{e}^{\imath 2\pi(0.3419)}, 0.2944\ddot{e}^{\imath 2\pi(0.2693)} \end{array} \right\rangle \\ , \left\langle \begin{array}{c} 0.1933\ddot{e}^{\imath 2\pi(0.1000)}, 0.2551\ddot{e}^{\imath 2\pi(0.2297)}, 0.3680\ddot{e}^{\imath 2\pi(0.1741)} \\ , 0.1516\ddot{e}^{\imath 2\pi(0.3482)}, 0.2000\ddot{e}^{\imath 2\pi(0.8000)}, 0.2885\ddot{e}^{\imath 2\pi(0.6063)} \end{array} \right\rangle \end{array} \right)$$

$$S\left(\hat{h}\right) = \left(\left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\imath=1}^{n} \xi_{\hat{h}_\jmath}(x) + \frac{1}{n} \sum_{\imath=1}^{n} \ddot{e}^{\imath 2\pi(\omega_{\xi_{\hat{h}_\jmath}}(x))} \right\} \right\} \times \left\{ \frac{1}{2} \left\{ \frac{1}{n} \sum_{\imath=1}^{n} \lambda_{\hat{h}_\jmath} + \frac{1}{n} \sum_{\imath=1}^{n} \ddot{e}^{\imath 2\pi(\omega_{\lambda_{\hat{h}_\jmath}}(x))} \right\} \right\} \right)$$

$$\left(\begin{array}{c} \left\{ \frac{1}{2} \left\{ \begin{array}{c} \frac{1}{6} \left(\begin{array}{c} 0.1861 + 0.5790 + 0.3618 + \\ 0.1000 + 0.5344 + 0.2944 \end{array} \right) + \\ \frac{1}{6} \left(\begin{array}{c} 0.2000 + 0.2616 + 0.1802 + \\ 0.2870 + 0.3419 + 0.2693 \end{array} \right) \end{array} \right\} \right\} \times \\ \left\{ \frac{1}{2} \left\{ \begin{array}{c} \frac{1}{6} \left(\begin{array}{c} 0.1933 + 0.2551 + 0.3680 + \\ 0.1516 + 0.2000 + 0.2885 \end{array} \right) + \\ \frac{1}{6} \left(\begin{array}{c} 0.1000 + 0.2297 + 0.1741 + \\ 0.3482 + 0.8000 + 0.6063 \end{array} \right) \end{array} \right\} \right\} \end{array} \right) \text{by using score function} = 0.0928$$

Example 3. Let

$$F_1 = \left\langle \left(0.1\ddot{e}^{\imath 2\pi(0.2)}, 0.7\ddot{e}^{\imath 2\pi(0.3)}, 0.4\ddot{e}^{\imath 2\pi(0.16667)} \right), \left(0.3\ddot{e}^{\imath 2\pi(0.1)}, 0.2\ddot{e}^{\imath 2\pi(0.8)} \right) \right\rangle$$

$$F_2 = \left\langle \left(0.3\ddot{e}^{\imath 2\pi(0.2)}, 0.5\ddot{e}^{\imath 2\pi(0.4)} \right), \left(0.1\ddot{e}^{\imath 2\pi(0.1)}, 0.2\ddot{e}^{\imath 2\pi(0.8)}, 0.5\ddot{e}^{\imath 2\pi(0.4)} \right) \right\rangle$$

be two CIHFS and $\varpi = (0.4, 0.6)^T$ be the corresponding weight vector of $F_\jmath\,(\jmath = 1, 2)$ and the aggregation-associated vector is $\varpi = (0.6, 0.4)^T$ the we can obtained

$$\dot{F}_1 = \left(\begin{array}{c} \left\langle \begin{array}{c} 1 - (1 - 0.1)^{2*0.4}\, \ddot{e}^{\imath 2\pi(1-(1-0.2)^{2*0.4})}, \\ 1 - (1 - 0.7)^{2*0.4}\, \ddot{e}^{\imath 2\pi(1-(1-0.0.3)^{2*0.4})}, \\ 1 - (1 - 0.4)^{2*0.4}\, \ddot{e}^{\imath 2\pi(1-(1-0.16667)^{2*0.4})} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} 1 - (1 - 0.3)^{2*0.6}\, \ddot{e}^{\imath 2\pi(1-(1-0.1)^{2*0.6})}, \\ 1 - (1 - 0.2)^{2*0.6}\, \ddot{e}^{\imath 2\pi(1-(1-0.8)^{2*0.6})} \end{array} \right\rangle \end{array} \right)$$

$$\dot{F}_2 = \left(\begin{array}{c} \left\langle \begin{array}{c} 1 - (1 - 0.3)^{2*0.4}\, \ddot{e}^{\imath 2\pi(1-(1-0.2)^{2*0.4})}, \\ 1 - (1 - 0.5)^{2*0.4}\, \ddot{e}^{\imath 2\pi(1-(1-0.4)^{2*0.4})} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} 1 - (1 - 0.1)^{2*0.6}\, \ddot{e}^{\imath 2\pi(1-(1-0.1)^{2*0.6})}, \\ 1 - (1 - 0.2)^{2*0.6}\, \ddot{e}^{\imath 2\pi(1-(1-0.0.8)^{2*0.6})}, \\ 1 - (1 - 0.5)^{2*0.6}\, \ddot{e}^{\imath 2\pi(1-(1-0.4)^{2*0.6})} \end{array} \right\rangle \end{array} \right)$$

$$\dot{F}_1 = \left(\left\langle \begin{array}{c} 0.0808\ddot{e}^{\imath 2\pi(0.1635)}, \\ 0.6183\ddot{e}^{\imath 2\pi(0.2482)}, \\ 0.3355\ddot{e}^{\imath 2\pi(0.1357)} \end{array} \right\rangle, \left\langle \begin{array}{c} 0.3482\ddot{e}^{\imath 2\pi(0.2349)}, \\ 0.5647\ddot{e}^{\imath 2\pi(0.4583)} \end{array} \right\rangle \right)$$

$$\dot{F}_2 = \left(\left\langle \begin{array}{c} 0.2482\ddot{e}^{\imath 2\pi(0.0808)}, \\ 0.1635\ddot{e}^{\imath 2\pi(0.7241)} \end{array} \right\rangle, \left\langle \begin{array}{c} 0.1188\ddot{e}^{\imath 2\pi(0.1188)}, \\ 0.2349\ddot{e}^{\imath 2\pi(0.8550)}, \\ 0.5647\ddot{e}^{\imath 2\pi(0.4583)} \end{array} \right\rangle \right)$$

By Definition 7, we calculate the score values of $\dot{F}_1$ and $\dot{F}_2$:

$$CIHFHA\,(F_1, F_2, ..., F_n) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath \dot{F}_{\beta(\jmath)} \right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} \ldots \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} \ldots \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} 1 - \prod_{\jmath=1}^{n} \left(1 - \Gamma_{\dot{F}_{\beta_{\ddot{k}}}} \right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi \left(1 - \prod_{\jmath=1}^{n} \left(1 - \omega_{\Gamma_{\dot{F}_{\beta_{\ddot{k}}}}} \right)^{\varpi_\jmath} \right)}, \\ \left(\prod_{\jmath=1}^{n} \left(\phi_{\dot{F}_{\beta_{\ddot{k}}}} \right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi \prod_{\jmath=1}^{n} \left(\omega_{\phi_{\dot{F}_{\beta_{\ddot{k}}}}} \right)^{\varpi_\jmath}} \right) \end{array} \right\}$$

$$= \left[\begin{array}{c} \left\{ \begin{array}{c} 1-(1-0.2482)^{0.6}\times(1-0.0808)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.0808)^{0.6}\times(1-0.1635)^{0.4}\right\}}, \\ 1-(1-0.2482)^{0.6}\times(1-0.6183)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.0808)^{0.6}\times(1-0.2482)^{0.4}\right\}}, \\ 1-(1-0.2482)^{0.6}\times(1-0.3355)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.0808)^{0.6}\times(1-0.1357)^{0.4}\right\}}, \\ 1-(1-0.1635)^{0.6}\times(1-0.0808)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.7241)^{0.6}\times(1-0.1635)^{0.4}\right\}}, \\ 1-(1-0.1635)^{0.6}\times(1-0.6183)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.7241)^{0.6}\times(1-0.2482)^{0.4}\right\}}, \\ 1-(1-0.1635)^{0.6}\times(1-0.3355)^{0.4}\,\ddot{e}^{\bar{\imath}2\pi\left\{1-(1-0.7241)^{0.6}\times(1-0.1357)^{0.4}\right\}} \end{array} \right\}, \\ \left\{ \begin{array}{c} \left(0.1188^{0.6}\times 0.3482^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.1188^{0.6}\times 0.2349^{0.4}\right)}, \\ \left(0.1188^{0.6}\times 0.5647^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.1188^{0.6}\times 0.4583^{0.4}\right)}, \\ \left(0.2349^{0.6}\times 0.3482^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.8550^{0.6}\times 0.2349^{0.4}\right)}, \\ \left(0.2349^{0.6}\times 0.5647^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.8550^{0.6}\times 0.4583^{0.4}\right)}, \\ \left(0.5647^{0.6}\times 0.3482^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.4583^{0.6}\times 0.2349^{0.4}\right)}, \\ \left(0.5647^{0.6}\times 0.5647^{0.4}\right)\ddot{e}^{\bar{\imath}2\pi\left(0.4583^{0.6}\times 0.4583^{0.4}\right)} \end{array} \right\} \end{array} \right]$$

$$= \left(\begin{array}{c} \left\langle \begin{array}{c} 0.1852\ddot{e}^{\bar{\imath}2\pi(0.1148)}, 0.4267\ddot{e}^{\bar{\imath}2\pi(0.1518)}, 0.2844\ddot{e}^{\bar{\imath}2\pi(0.1032)} \\ , 0.1314\ddot{e}^{\bar{\imath}2\pi(0.5700)}, 0.3888\ddot{e}^{\bar{\imath}2\pi(0.5880)}, 0.2371\ddot{e}^{\bar{\imath}2\pi(0.5644)} \end{array} \right\rangle \\ ,\left\langle \begin{array}{c} 0.1827\ddot{e}^{\bar{\imath}2\pi(0.1560)}, 0.2216\ddot{e}^{\bar{\imath}2\pi(0.2039)}, 0.2750\ddot{e}^{\bar{\imath}2\pi(0.5100)} \\ , 0.3336\ddot{e}^{\bar{\imath}2\pi(0.6663)}, 0.4654\ddot{e}^{\bar{\imath}2\pi(0.3508)}, 0.5647\ddot{e}^{\bar{\imath}2\pi(0.4583)} \end{array} \right\rangle \end{array} \right)$$

$$S\left(\hat{h}\right) = \left(\left\{\tfrac{1}{2}\left\{\tfrac{1}{n}\sum_{\bar{\imath}=1}^{n}\xi_{\hat{h}_\jmath}(x) + \tfrac{1}{n}\sum_{\bar{\imath}=1}^{n}\ddot{e}^{\bar{\imath}2\pi(\omega_{\xi_{\hat{h}_\jmath}}(x))}\right\}\right\} \times \left\{\tfrac{1}{2}\left\{\tfrac{1}{n}\sum_{\bar{\imath}=1}^{n}\lambda_{\hat{h}_\jmath} + \tfrac{1}{n}\sum_{\bar{\imath}=1}^{n}\ddot{e}^{\bar{\imath}2\pi(\omega_{\lambda_{\hat{h}_\jmath}}(x))}\right\}\right\} \right)$$

$$\left(\begin{array}{c} \left\{ \frac{1}{2}\left\{ \begin{array}{c} \frac{1}{6}\left(\begin{array}{c} 0.1852+0.4267+0.2844+ \\ 0.1314+0.3888+0.2371 \end{array} \right) + \\ \frac{1}{6}\left(\begin{array}{c} 0.1148+0.1518+0.1032+ \\ 0.5700+0.5880+0.5644 \end{array} \right) \end{array} \right\} \right\} \times \\ \left\{ \frac{1}{2}\left\{ \begin{array}{c} \frac{1}{6}\left(\begin{array}{c} 0.1827+0.2216+0.2750+ \\ 0.3336+0.4654+0.5647 \end{array} \right) + \\ \frac{1}{6}\left(\begin{array}{c} 0.1560+0.2039+0.5100+ \\ 0.6663+0.3508+0.4583 \end{array} \right) \end{array} \right\} \right\} \end{array} \right) \text{ by using score function } = 0.1141$$

Theorem 1. Let $p_\jmath\,(\jmath = 1, 2, ..., n)$ be a collection of CIHFS then their aggregated valued calculated using the CIHFWA operator using the CIHFWA operator is an CIHFWA $(F_1, F_2, ..., F_n)$,

$$CIHFWA\,(F_1, F_2, ..., F_n) = \bigoplus_{\jmath=1}^{n} (\varpi_\jmath F_\jmath)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} 1-\prod_{\jmath=1}^{n}\left(1-\Gamma_{F_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\Gamma_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n}\left(\phi_{F_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{n}\left(\omega_{\phi_{F\ddot{k}}}\right)^{\varpi_\jmath}}\right) \end{array} \right\}$$

Proof.

$$\varpi_1 F_1 = \left\{ 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)}, \left(\left(\phi_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}}\right) \right\}$$

$$\varpi_2 F_2 = \left\{ 1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)}, \left(\left(\phi_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}}\right) \right\}$$

we have

$$\varpi_1 F_1 \bigoplus \varpi_2 F_2 =$$

$$\left\{\begin{matrix} 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)}, \\ \left(\left(\phi_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}}\right) \end{matrix}\right\}\bigoplus\left\{\begin{matrix} 1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)}, \\ \left(\left(\phi_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}}\right) \end{matrix}\right\}$$

$$\left\{\left(\left[\begin{matrix} 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \\ +1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)} \\ -\left(1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)}\right) \\ \left(1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)}\right) \end{matrix}\right],\left[\begin{matrix} \left(\phi_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}} \\ ,\left(\phi_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}} \end{matrix}\right]\right)\right\}$$

$$\left\{\left(\left[\begin{matrix} 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \\ \left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)} \end{matrix}\right],\left[\begin{matrix} \left(\phi_{\ddot{k}_1}\right)^{\varpi_1}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}} \\ ,\left(\phi_{\ddot{k}_2}\right)^{\varpi_2}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}} \end{matrix}\right]\right)\right\}$$

for $n=l$, other words CIHFWA

$$\left\{\left[\begin{matrix} 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \\ ,\left(\prod_{\jmath=1}^{l}\left(\phi_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}}\right) \end{matrix}\right]\right\}$$

then, when $n=l+1$,

$$CIHFWA\left(F_1,F_2,...,F_l,F_{l+1}\right)=\left(\bigoplus_{\jmath=1}^{n}\left(\varpi_\jmath F_\jmath\right)\right)\bigoplus\left(\varpi_{l+1}F_{l+1}\right)=\{\xi_{\hat{h}_\jmath}(x),\lambda_{\hat{h}_\jmath}(x)\}$$

$$=\bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}}\left\{\begin{matrix}\left(\left[\begin{matrix} 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{l}\left(\phi_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}}\right) \end{matrix}\right]\right) \\ \mid\Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l\end{matrix}\right\}$$

$$\bigoplus\left\{\begin{matrix}\left(\left[\begin{matrix} 1-\left(1-\Gamma_{l+1}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)}, \\ \left(\left(\phi_{l+1}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{l+1}}\right)^{\varpi_\jmath}}\right) \end{matrix}\right]\right) \\ \mid\Gamma_l\in F_l\end{matrix}\right\}$$

$$\left\{\begin{matrix}\left(\begin{matrix}\left[\begin{matrix} 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \\ +1-\left(1-\Gamma_{l+1}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)} \\ -\left(1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)}\right) \\ \left(1-\left(1-\Gamma_{l+1}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)}\right) \end{matrix}\right], \\ \left[\begin{matrix}\left(\prod_{\jmath=1}^{l}\left(\phi_{\ddot{k}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}}\right) \\ \left(\left(\phi_{l+1}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{l+1}}\right)^{\varpi_\jmath}}\right)\end{matrix}\right]\end{matrix}\right) \\ \mid\Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l,,\Gamma_{l+1}\in F_{l+1}\end{matrix}\right\}$$

$$\left\{ \left(\begin{array}{c} \left[1-\prod_{\jmath=1}^{l+1}\left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l+1}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)}\right], \\ \left[\left(\prod_{\jmath=1}^{l+1}\left(\phi_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l+1}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}}\right)\right] \end{array} \right) \\ \mid \Gamma_1 \in F_1, \Gamma_2 \in F_2......, \Gamma_l \in F_l, , \Gamma_{l+1} \in F_{l+1} \right\}.$$

So we obtain the required result.

Theorem 2. Let $p_\jmath \, (\jmath = 1, 2, ..., n)$ be a collection of CIHFS, then their aggregated valued calculated using the CIHFWG operator using the CIHFWG operator is an CIHFWG $(F_1, F_2, ..., F_n)$,

$$CIHFWG\,(F_1, F_2, ..., F_n) = \bigotimes_{\jmath=1}^{n}\left(F_\jmath^{\varpi_\jmath}\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(\prod_{\jmath=1}^{n}\left(\Gamma_{F_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{n}\left(\omega_{\Gamma_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right), \\ 1-\prod_{\jmath=1}^{n}\left(1-\phi_{F_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\phi_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)} \end{array} \right\}$$

Proof.

$$\varpi_1 F_1 = \left\{ \left(\left(\phi_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}} \right), 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \right\}$$

$$\varpi_2 F_2 = \left\{ \left(\left(\phi_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}} \right), 1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)} \right\}$$

we have

$$F_1^{\varpi_1} \bigotimes F_2^{\varpi_2} = \left\{ \left(\left(\phi_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}} \right), 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \right\}$$

$$\bigotimes \left\{ \left(\left(\phi_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}} \right), 1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)} \right\}$$

$$\left\{ \left(\begin{array}{c} \left[\left(\phi_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}}, \left(\phi_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}}\right], \\ 1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \\ +1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)} \\ -\left(1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)}\right) \\ \left(1-\left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)}\right) \end{array} \right) \right\}$$

$$\left\{ \left(\begin{array}{c} \left[\left(\phi_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_1}}\right)^{\varpi_1}}, \left(\phi_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\ddot{k}_2}}\right)^{\varpi_2}}\right], \\ \left[1-\left(1-\Gamma_{\ddot{k}_1}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_1}}\right)^{\varpi_1}\right)} \left(1-\Gamma_{\ddot{k}_2}\right)^{\varpi_2} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\ddot{k}_2}}\right)^{\varpi_2}\right)}\right] \end{array} \right) \right\}$$

for $n = l$, other words CIHFWG

$$\left\{ \left[\left(\prod_{\jmath=1}^{l} \left(\phi_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi \prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}} \right), 1 - \prod_{\jmath=1}^{l} \left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \right] \right\}$$

then, when $n = l+1$,

$$CIHFWG\left(F_1, F_2, ..., F_l, F_{l+1}\right) = \left(\bigotimes_{\jmath=1}^{n} \left(F_\jmath^{\varpi_\jmath}\right) \right) \bigotimes \left(\varpi_{l+1} F_{l+1}\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1}, \phi_{\ddot{k}_2}\in\xi_{\hat{h}_2} ... \phi_{\ddot{k}_n}\in\xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} \left(\left[\begin{array}{c} \left(\prod_{\jmath=1}^{l} \left(\phi_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi \prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}} \right), \\ 1 - \prod_{\jmath=1}^{l} \left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \end{array} \right] \right) \\ \mid \Gamma_1 \in F_1, \Gamma_2 \in F_2 ......, \Gamma_l \in F_l \end{array} \right\}$$

$$\bigotimes \left\{ \begin{array}{c} \left(\left[\begin{array}{c} \left(\left(\phi_{l+1}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{l+1}}\right)^{\varpi_\jmath}} \right), \\ 1 - \left(1-\Gamma_{l+1}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)} \end{array} \right] \right) \\ \mid \Gamma_l \in F_l \end{array} \right\}$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \left(\prod_{\jmath=1}^{l} \left(\phi_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi \prod_{\jmath=1}^{l}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}} \right) \\ \left(\left(\phi_{l+1}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{l+1}}\right)^{\varpi_\jmath}} \right) \end{array} \right], \\ \left[\begin{array}{c} 1 - \prod_{\jmath=1}^{l} \left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \\ +1 - \left(1-\Gamma_{l+1}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)} \\ - \left(1 - \prod_{\jmath=1}^{l} \left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \right) \\ \left(1 - \left(1-\Gamma_{l+1}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{l+1}}\right)^{\varpi_\jmath}\right)} \right) \end{array} \right] \end{array} \right) \\ \mid \Gamma_1 \in F_1, \Gamma_2 \in F_2 ......, \Gamma_l \in F_l, , \Gamma_{l+1} \in F_{l+1} \end{array} \right\}$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\left(\prod_{\jmath=1}^{l+1} \left(\phi_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi \prod_{\jmath=1}^{l+1}\left(\omega_{\phi_{\ddot{k}}}\right)^{\varpi_\jmath}} \right) \right], \\ \left[1 - \prod_{\jmath=1}^{l+1} \left(1-\Gamma_{\ddot{k}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l+1}\left(1-\omega_{\Gamma_{\ddot{k}}}\right)^{\varpi_\jmath}\right)} \right], \end{array} \right) \\ \mid \Gamma_1 \in F_1, \Gamma_2 \in F_2 ......, \Gamma_l \in F_l, , \Gamma_{l+1} \in F_{l+1} \end{array} \right\}.$$

Theorem 3. Let $p_\jmath\,(\jmath = 1, 2, ..., n)$ be a collection of CIHFS then their aggregated valued calculated using the CIHFOWA operator using the CIHFOWA operator is an CIHFOWA $(F_1, F_2, ..., F_n)$,

$$CIHFOWA\left(F_1, F_2, ..., F_n\right) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath F_{\beta(\jmath)}\right) =$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1}, \phi_{\ddot{k}_2}\in\xi_{\hat{h}_2} ... \phi_{\ddot{k}_n}\in\xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{ \begin{array}{c} 1 - \prod_{\jmath=1}^{n} \left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{n} \left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\bar{\imath}2\pi \prod_{\jmath=1}^{n}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}} \right) \end{array} \right\}$$

Proof.

$$\varpi_1 F_1 = \left\{ 1 - \left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)}, \left(\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}} \right) \right\}$$

$$\varpi_2 F_2 = \left\{ 1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}, \left(\left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\right) \right\},$$

we have

$$\left\{\left(\begin{array}{c} \left[\begin{array}{c} 1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)} \\ +1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)} \\ -\left(1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)}\right) \\ \left(1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\right) \end{array}\right], \\ \left[\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}}, \left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\right] \end{array}\right)\right\}$$

$$\left\{\left(\begin{array}{c} \left[1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)} \left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\right], \\ \left[\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}}, \left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\right] \end{array}\right)\right\}$$

for $n = l$, other words CIHFOWA

$$\left\{\left[1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}, \left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi \prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right)\right]\right\}$$

then, when $n = l + 1$,

$$CIHFOWA\left(F_1, F_2, ..., F_l, F_{l+1}\right) = \left(\bigoplus_{\jmath=1}^{n}\left(\varpi_\jmath F_\jmath\right)\right) \bigoplus \left(\varpi_{l+1} F_{l+1}\right) = \{\xi_{\hat{h}_\jmath}(x), \lambda_{\hat{h}_\jmath}(x)\}$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1} \in \xi_{\hat{h}_1}, \phi_{\ddot{k}_2} \in \xi_{\hat{h}_2} ... \phi_{\ddot{k}_n} \in \xi_{\hat{h}_n} \\ \omega_{\phi_{\ddot{k}_1}} \in \omega_{\xi_{\hat{h}_1}}, \omega_{\phi_{\ddot{k}_2}} \in \omega_{\xi_{\hat{h}_2}} ... \omega_{\phi_{\ddot{k}_n}} \in \omega_{\xi_{\hat{h}_n}}}} \left\{\begin{array}{c} \left(\left[\begin{array}{c} 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi \prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right) \end{array}\right]\right) \\ \mid \Gamma_1 \in F_1, \Gamma_2 \in F_2 ......, \Gamma_l \in F_l \end{array}\right\}$$

$$\bigoplus \left\{\begin{array}{c} \left(\left[\begin{array}{c} 1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)}, \\ \left(\left(\phi_{\beta_{l+1}}\right)^{\varpi_\jmath} \ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{l+1}}}\right)^{\varpi_\jmath}}\right) \end{array}\right]\right) \\ \mid \Gamma_l \in F_l \end{array}\right\}$$

$$\left\{\left(\begin{bmatrix} 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)} \\ +1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)} \\ -\left(1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\right) \\ \left(1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)}\right) \end{bmatrix}, \begin{bmatrix} \left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right) \\ \left(\left(\phi_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{l+1}}}\right)^{\varpi_\jmath}}\right) \end{bmatrix}\right) \mid \Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l,,\Gamma_{l+1}\in_{l+1}\right\}$$

$$\left\{\left(\begin{matrix}\left[1-\prod_{\jmath=1}^{l+1}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l+1}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\right], \\ \left[\left(\prod_{\jmath=1}^{l+1}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{l+1}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right)\right]\end{matrix}\right) \mid \Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l,,\Gamma_{l+1}\in F_{l+1}\right\}.$$

This is the the complete the proof.

Theorem 4. Let $p_\jmath\,(\jmath=1,2,...,n)$ be a collection of CIHFS then their aggregated valued calculated using the CIHFOWG operator using the CIHFOWG operator is an CIHFOWG $(F_1,F_2,...,F_n)$,

$$CIHFOWG\,(F_1,F_2,...,F_n)=\bigotimes_{\jmath=1}^{n}\left(F_{\beta(\jmath)}^{\varpi_\jmath}\right)$$

$$=\bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}}\left\{\begin{matrix}\left(\prod_{\jmath=1}^{n}\left(\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{n}\left(\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right), \\ 1-\prod_{\jmath=1}^{n}\left(1-\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\end{matrix}\right\}$$

Proof.

$$F_1^{\varpi_1}=\left\{\left(\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}}\right),1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)}\right\}$$

$$F_2^{\varpi_2}=\left\{\left(\left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\right),1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\right\}$$

we have

$$\left\{\left(\begin{array}{c}\left[\begin{array}{c}\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}}\\ ,\left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\end{array}\right],\\ \left[\begin{array}{c}1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)}\\ +1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\\ -\left(1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta\ddot{k}_1}}\right)^{\varpi_1}\right)}\right)\\ \left(1-\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\right)\end{array}\right]\end{array}\right)\right\}$$

$$\left\{\left(\begin{array}{c}\left[\left(\phi_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}},\left(\phi_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}}\right],\\ \left[1-\left(1-\Gamma_{\beta_{\ddot{k}_1}}\right)^{\varpi_1}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_1}}}\right)^{\varpi_1}\right)}\left(1-\Gamma_{\beta_{\ddot{k}_2}}\right)^{\varpi_2}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{\ddot{k}_2}}}\right)^{\varpi_2}\right)}\right]\end{array}\right)\right\}$$

for $n=l$, other words CIHFOWG

$$\left\{\left[\left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right),1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\right]\right\}$$

then, when $n=l+1$,

$$CIHFOWG\left(F_1,F_2,...,F_l,F_{l+1}\right)=\left(\bigotimes_{\jmath=1}^{n}\left(\varpi_\jmath F_\jmath\right)\right)\bigotimes\left(\varpi_{l+1}F_{l+1}\right)=\{\xi_{\hat{h}_\jmath}(x),\lambda_{\hat{h}_\jmath}(x)\}$$

$$=\bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}}\left\{\begin{array}{c}\left(\left[\begin{array}{c}\left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right),\\ 1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\end{array}\right]\right)\\ \mid\Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l\end{array}\right\}$$

$$\bigotimes\left\{\begin{array}{c}\left(\left[\begin{array}{c}\left(\left(\phi_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(\omega_{\phi_{\beta_{l+1}}}\right)^{\varpi_\jmath}}\right),\\ 1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\imath 2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)}\end{array}\right]\right)\\ \mid\Gamma_l\in F_l\end{array}\right\}$$

$$\left\{\left(\begin{array}{c}\left[\begin{array}{c}\left(\prod_{\jmath=1}^{l}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right)\\ \left(\left(\phi_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(\omega_{\phi_{\beta_{l+1}}}\right)^{\varpi_\jmath}}\right)\end{array}\right],\\ \left[\begin{array}{c}1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\\ +1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)}\\ -\left(1-\prod_{\jmath=1}^{l}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\right)\\ \left(1-\left(1-\Gamma_{\beta_{l+1}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\left(1-\omega_{\Gamma_{\beta_{l+1}}}\right)^{\varpi_\jmath}\right)}\right)\end{array}\right]\end{array}\right)\\ \mid\Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l,,\Gamma_{l+1}\in_{l+1}\right\}$$

$$\left\{\left(\begin{array}{c}\left[\left(\prod_{\jmath=1}^{l+1}\left(\phi_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{l+1}\left(\omega_{\phi_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right)\right],\\ \left[1-\prod_{\jmath=1}^{l+1}\left(1-\Gamma_{\beta_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{l+1}\left(1-\omega_{\Gamma_{\beta_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)}\right]\end{array}\right)\\ \mid\Gamma_1\in F_1,\Gamma_2\in F_2......,\Gamma_l\in F_l,,\Gamma_{l+1}\in F_{l+1}\right\}$$

4 Algorithm

Consider an MCDM with complex intuitionistic hesitant fuzzy information and assume that p is different alternatives $\Delta = (\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, ..., \Delta_\jmath)$; $F = (1, 2, 3..., \jmath)$ and q criteria $K = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4, ..., \kappa_d\}$; $q = (1, 2, 3..., d)$ and the weight vectors for the criteria is $\varpi = \{\varpi_1, \varpi_2, \varpi_3, ..., \varpi_d\}$, where $\varpi_q \geq 0$ and $\sum_{q=1}^{d} \varpi_q = 1$.

The procedure of the presented MCDM technique by using CIHFS is provided below of help in the selection of best alternative.

Step 1. We will gather the information in complex Intuitionistic hesitant fuzzy form by the experts as follows:

$$\hat{S} = \left[\langle \Xi_{\hat{S}}(\kappa), \lambda_{\hat{S}}(\kappa)\rangle_{pq}\right]_{\jmath d} ; \kappa \in \chi$$

Step 2. Create the CIHF-decision matrix that is combined. In order to compute the CIHF-DM, it is necessary for the DEs to make use of views in order to aggregate all of the individual matrices into one group decision matrix. Operators using CIHF-weighted averaging may achieve this

$$CIHFWA\left(F_1, F_2, ..., F_n\right) = \bigoplus_{\jmath=1}^{n} \left(\varpi_\jmath F_\jmath\right)$$

$$= \bigcup_{\substack{\phi_{\ddot{k}_1}\in\xi_{\hat{h}_1},\phi_{\ddot{k}_2}\in\xi_{\hat{h}_2}...\phi_{\ddot{k}_n}\in\xi_{\hat{h}_n}\\ \omega_{\phi_{\ddot{k}_1}}\in\omega_{\xi_{\hat{h}_1}},\omega_{\phi_{\ddot{k}_2}}\in\omega_{\xi_{\hat{h}_2}}...\omega_{\phi_{\ddot{k}_n}}\in\omega_{\xi_{\hat{h}_n}}}} \left\{\begin{array}{c}\left(1-\prod_{\jmath=1}^{n}\left(1-\Gamma_{F_{\ddot{k}}}\right)^{\varpi_\jmath}\right)\ddot{e}^{\bar{\imath}2\pi\left(1-\prod_{\jmath=1}^{n}\left(1-\omega_{\Gamma_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}\right)},\\ \left(\prod_{\jmath=1}^{n}\left(\phi_{F_{\ddot{k}}}\right)^{\varpi_\jmath}\ddot{e}^{\bar{\imath}2\pi\prod_{\jmath=1}^{n}\left(\omega_{\phi_{F_{\ddot{k}}}}\right)^{\varpi_\jmath}}\right)\end{array}\right\}$$

are similar $CIHFWG, CIHFOWA, CIHFOWG, CIHFHA, CIHFHG$.

Step 3. Utilize the CIHF technique to calculate the criterion weights. how to use the CIHF model to evaluate attribute weights.

Step 4. First, the score values $S\left(\hat{h}_\jmath\right)$ of CIHFs are calculated using Definition 2.7.

Step 5. We will rank the alternative in the descending

Note that the alternative ranking order varies when the modified the values of the parameters. The flowchart of the proposed algorithm is shown in Figure 1.

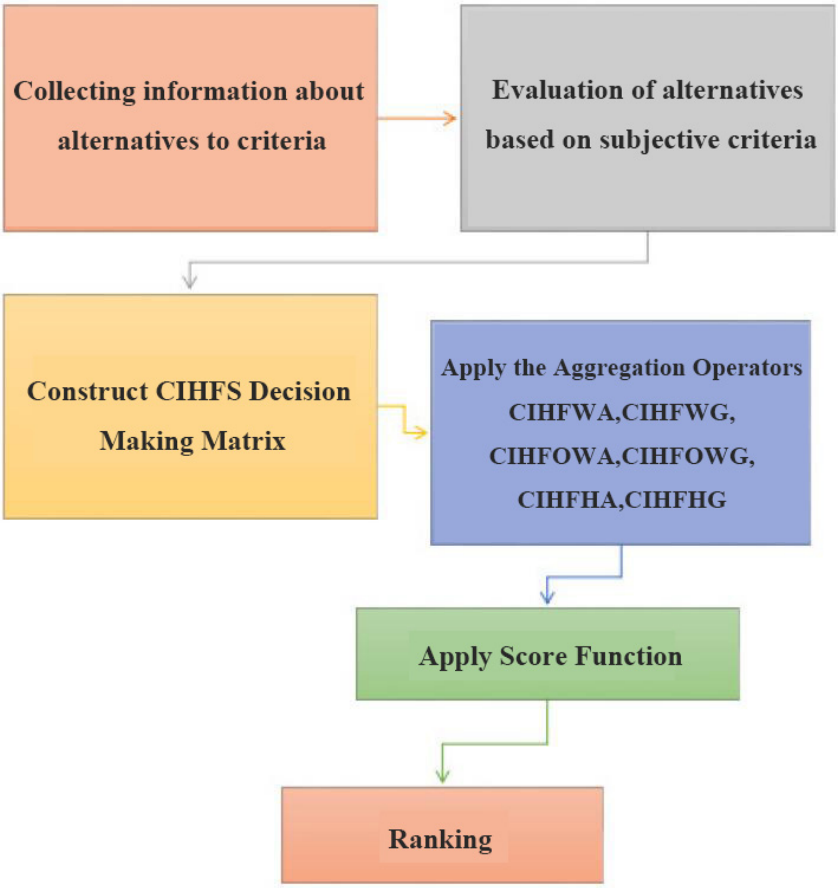

Figure 1. Flowchart of the proposed algorithm

5 Case Study

The vast amounts of healthcare waste (HCW) produced by hospitals and other healthcare institutions pose a threat to the health of patients, healthcare workers, and the general public, as well as to the quality of the surrounding environment. Over the last few months, there has been alarming news about the rise of HCW [29]. As a result, HCW management (HCWM) has become a worldwide environmental and public health concern, especially in developing countries where HCW use is on the rise [30]. incorporates MSW as a common kind of differentiation. In its definition of healthcare waste, the World Health Organization (WHO) notes that it "includes a wide variety of items, from used needles and syringes to soiled dressings, body parts, diagnostic samples, blood, chemicals, medications, medical equipment, and radioactive materials" [31]. According to the WHO, only around 15% of HCW pose a risk because of their potentially infectious, poisonous, or radioactive nature. If not handled or disposed of properly, approximately 15% of HCW poses a number of environmental and health risks. Therefore, it is crucial that biomedical waste be separated out at the point of production, stored appropriately, and disposed of in a methodical manner. The environment and public health are negatively impacted when HCW are treated improperly [32]. This is why HCWM has garnered so much interest from scientists, policymakers, and activists in the environmental, academic, and professional communities [33]. Healthcare facility waste management (HCWM) systems assess several transportation modes and routes for delivering trash to treatment facilities, as well as providing assistance in deciding which treatment method and final disposal location to choose. The potential negative effects of HCW on the economy and the environment have made it a priority throughout the world to evaluate the best possible HCW treatment (HCWT) [34]. Decision experts must balance a number of conflicting material and immaterial factors in order to settle on the optimal HCWT solution. Performance ratings for each HCWT option are calculated individually using a variety of metrics. However, there is no one HCWT solution that outperforms the others on every criterion [35]. Due to the presence of several alternatives and evaluation criteria, evaluating HCWT possibilities is seen as a difficult MCDM challenge. Since this is an issue that needs to be solved, an effective and precise method for evaluating HCWTs is essential [36].

In this study, Pakistan's options for getting rid of medical waste are looked at using the CIHFS framework. Pakistan's healthcare system, particularly its super-speciality hospitals, has expanded rapidly during the last decade.

As more people have access to health care, hospitals and clinics produce more biological waste. Because there are more people in hospitals and clinics, the amount of biological waste they make has gone up. However, the current healthcare waste treatment alternatives are inadequate to deal with such massive quantities of medical waste, and as a result, a lot of it ends up in landfills. Further, due to ineffective treatment alternatives and a dearth of appropriate authorizations for disposing of biological waste, a significant amount of healthcare waste gets disposed of as general trash [37].

Therefore, it is crucial to develop and implement alternative methods of treating HCW disposal. Since different treatment methods may significantly affect the economy, the environment, and the general public, identifying the optimal approach is crucial for handling medical waste. We did this by investigating and reviewing the various methods already in use for treating medical waste. We also talked to ecologists, government officials, and university professors who study waste management [38] to find out what they thought about how HCW is managed in Pakistan right now. Five potential methods for dealing with HCW were evaluated after an initial assessment was made. Treatment options for medical waste include: Δ_1 = autoclave sterilization; Δ_2 = thermal decomposition of plasma; Δ_3 = microwave; Δ_4 = eliminating germs using chemicals; and Δ_1 = sanitary landfill. Assume we need to evaluate four distinct parameters: $\kappa_{\tilde{\imath}} = (\tilde{\imath} = 1, 2, 3, 4)$. Using the four specified qualities from the balanced scorecard technique (it should be emphasized that they are all of the maximizing type), it is required to compare these healthcare waste treatment measures in order to identify the most significant of them and to arrange them based on their significance. κ_1 = waste residuals; κ_2 = energy consumption; κ_3 = treatment effectiveness; and κ_4 = public acceptance. These professionals come from a variety of backgrounds and industries and include a waste-treatment company expert, an HCWM specialist, an industrial engineer, and an environmental engineer. The steps required to implement the CIHFS strategy in this context are outlined below. Consider the attribute weight vector to be $\varpi = (0.15, 0.3, 0.2, 0.35)^T$.

Here, we put the devised strategy into practice in order to produce a HCWT with the best possible outcome.

Step 1. The decision matrix $F = \left(p_{\tilde{\imath}\jmath}\right)_{n\times n}$ is shown in Table 1, where $p_{\tilde{\imath}\jmath}$ $(\tilde{\imath}, \jmath = 1, 2, 3, 4)$ are in the form of CIHFSs to prevent decision makers from being swayed by one another's preferences.

Table 1. Basic information Healthcare waste treatment in CIHFSs

Alternative	κ_1	κ_2
Δ_1	$\left\{ \left(0.4\ddot{e}^{\tilde{\imath}2\pi(0.24)}, 0.5\ddot{e}^{\tilde{\imath}2\pi(0.25)}\right), \left(0.4\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right) \right\}$	$\left\{ \left(0.7\ddot{e}^{\tilde{\imath}2\pi(0.1)}\right), \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.7)}, 0.1\ddot{e}^{\tilde{\imath}2\pi(0.3)}\right) \right\}$
Δ_2	$\left\{ \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.3)}, 0.3\ddot{e}^{\tilde{\imath}2\pi(0.2)}\right), \left(0.4\ddot{e}^{\tilde{\imath}2\pi(0.6)}\right) \right\}$	$\left\{ \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right), \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.2)}, 0.6\ddot{e}^{\tilde{\imath}2\pi(0.4)}\right) \right\}$
Δ_3	$\left\{ \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.3)}\right), \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.5)}, 0.3\ddot{e}^{\tilde{\imath}2\pi(0.3)}\right) \right\}$	$\left\{ \left(0.2\ddot{e}^{\tilde{\imath}2\pi(0.22)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.13)}\right) \right\}$
Δ_4	$\left\{ \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.3)}\right), \left(0.2\ddot{e}^{\tilde{\imath}2\pi(0.25)}\right) \right\}$	$\left\{ \left(0.6\ddot{e}^{\tilde{\imath}2\pi(0.50)}, 0.5\ddot{e}^{\tilde{\imath}2\pi(0.25)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.30)}\right) \right\}$
Δ_5	$\left\{ \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right), \left(0.2\ddot{e}^{\tilde{\imath}2\pi(0.3)}, 0.5\ddot{e}^{\tilde{\imath}2\pi(0.7)}\right) \right\}$	$\left\{ \left(0.6\ddot{e}^{\tilde{\imath}2\pi(0.4)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.4)}, 0.6\ddot{e}^{\tilde{\imath}2\pi(0.2)}\right) \right\}$
Alternative	κ_3	κ_4
Δ_1	$\left\{ \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.4)}\right), \left(0.1\ddot{e}^{\tilde{\imath}2\pi(0.1)}\right) \right\}$	$\left\{ \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.5)}, 0.5\ddot{e}^{\tilde{\imath}2\pi(0.1)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.33)}\right) \right\}$
Δ_2	$\left\{ \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.2)}\right), \left(0.1\ddot{e}^{\tilde{\imath}2\pi(0.2)}, 0.8\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right) \right\}$	$\left\{ \left(0.4\ddot{e}^{\tilde{\imath}2\pi(0.5)}, 0.2\ddot{e}^{\tilde{\imath}2\pi(0.3)}\right), \left(0.1\ddot{e}^{\tilde{\imath}2\pi(0.33)}\right) \right\}$
Δ_3	$\left\{ \left(0.5\ddot{e}^{\tilde{\imath}2\pi(0.25)}\right), \left(0.7\ddot{e}^{\tilde{\imath}2\pi(0.1)}, 0.3\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right) \right\}$	$\left\{ \left(0.4\ddot{e}^{\tilde{\imath}2\pi(0.2)}, 0.6\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.2)}, 0.4\ddot{e}^{\tilde{\imath}2\pi(0.4)}\right) \right\}$
Δ_4	$\left\{ \left(0.1\ddot{e}^{\tilde{\imath}2\pi(0.3)}, 0.3\ddot{e}^{\tilde{\imath}2\pi(0.35)}\right), \left(0.2\ddot{e}^{\tilde{\imath}2\pi(0.2)}\right) \right\}$	$\left\{ \left(0.2\ddot{e}^{\tilde{\imath}2\pi(0.25)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.1)}, 0.5\ddot{e}^{\tilde{\imath}2\pi(0.45)}\right) \right\}$
Δ_5	$\left\{ \left(0.8\ddot{e}^{\tilde{\imath}2\pi(0.4)}, 0.7\ddot{e}^{\tilde{\imath}2\pi(0.15557)}\right), \left(0.6\ddot{e}^{\tilde{\imath}2\pi(0.33)}, 0.1\ddot{e}^{\tilde{\imath}2\pi(0.4)}\right) \right\}$	$\left\{ \left(0.1\ddot{e}^{\tilde{\imath}2\pi(0.5)}\right), \left(0.3\ddot{e}^{\tilde{\imath}2\pi(0.35)}\right) \right\}$

Step 2. For tasks $\kappa_{\tilde{\imath}}$ $(\tilde{\imath}= 1, 2, 3, 4)$, the CIHFWA, CIHFWG, CIHFOWA, CIHFOWG, CIHFHA,CIHFHG operator is used to derive the CIHFS $p_{\tilde{\imath}}(\tilde{\imath}= 1, 2, 3, 4)$.

Step 3. Compute the score values $S\left(\hat{h}\right) = (\tilde{\imath}= 1, 2, 3, 4)$ of $\hat{h}_{\tilde{\imath}}(\tilde{\imath}= 1, 2, 3, 4)$ by 2.7. Table 2 presents the score values for the options.

Step 4. Based on the rankings of $S(\hat{h}_{\tilde{\imath}})(\tilde{\imath}= 1, 2, 3, 4)$. Table 2 shows the relative importance of the options $\kappa_{\tilde{\imath}}$ $(\tilde{\imath}= 1, 2, 3, 4)$ as the changed operator.

Step 5. Using $(\hat{h}_{\imath}) = (\imath = 1, 2, 3)$, arrange the possible solutions $p_{\imath\jmath}\,(\imath, \jmath = 1, 2, 3, 4)$ from most preferred to least preferred.

Step 6. Sort the available choices $p_{\imath\jmath}\,(\imath, \jmath = 1, 2, 3, 4)$ by $S(\hat{h}_{\imath})(\imath = 1, 2, 3, 4)$ in descending order

$$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$$

Thus the best alternative is Δ_5.

Table 2. Ranking on the alternatives by score value based on the proposed operator

Proposed Opertors	$\mathbf{\Delta_1}$	$\mathbf{\Delta_2}$	$\mathbf{\Delta_3}$	$\mathbf{\Delta_4}$	$\mathbf{\Delta_5}$	Ranking:
CIHFWA	0.1096	0.1120	0.1010	0.0874	0.1582	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$
CIHFWG	0.1194	0.1338	0.1034	0.0841	0.1384	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$
CIHFOWA	0.0956	0.1112	0.0620	0.0732	0.1581	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_4 > \Delta_3$
CIHFOWG	0.1161	0.1321	0.0956	0.0729	0.1384	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$
CIHFHA	0.0869	0.1103	0.0881	0.0662	0.1432	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$
CIHFHG	0.1061	0.1120	0.0861	0.0724	0.1315	$\Delta_5 > \Delta_2 > \Delta_1 > \Delta_3 > \Delta_4$

The graphical representation of ranking is shown in Figure 2.

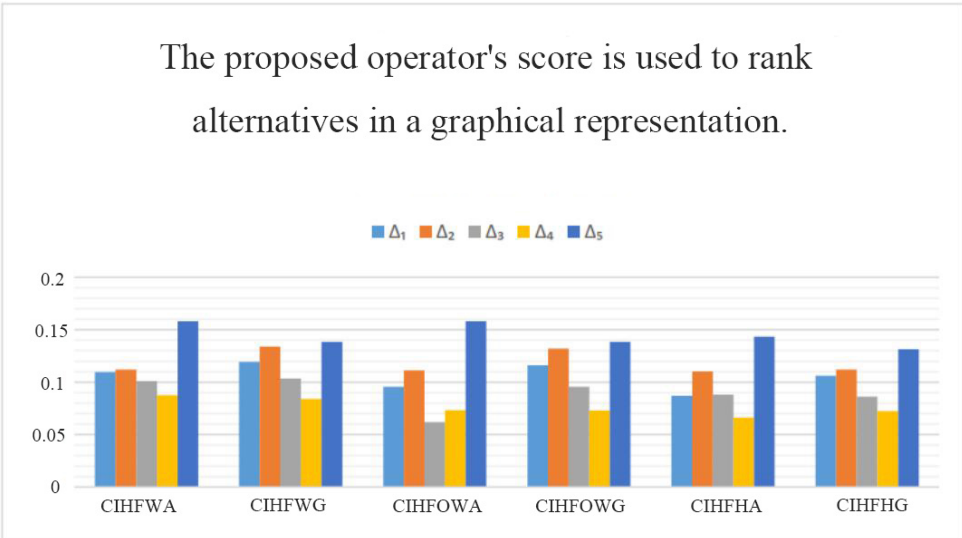

Figure 2. Graphical representation of proposed operators

6 Conclusions

This research has conducted a thorough investigation on the utilization of complex intuitionistic, hesitant fuzzy information aggregation approaches in decision-making. Complex operational rules for intuitionistic hesitant fuzzy sets have been developed based on their link with hesitant fuzzy sets. A collection of operators has been developed in various situations, and their interconnections have been examined, all with the aim of combining intricate intuitionistic, hesitant, and fuzzy information. Furthermore, we have utilized the aggregate operators that we have constructed to discover unidentified answers to predicaments in the process of making decisions. We have demonstrated the overall patterns in the development of the output of aggregation operators through the use of a representative example.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1179–1187, 2007. https://doi.org/10.1109/tfuzz.2006.890678

[2] H. Garg and K. Kumar, "An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making," *Soft Comput.*, vol. 22, no. 15, pp. 4959–4970, 2018. https://doi.org/10.1007/s00500-018-3202-1

[3] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965. https://doi.org/10.1016/s0019-9958(65)90241-x

[4] D. Yu and S. Shi, "Researching the development of Atanassov intuitionistic fuzzy set: Using a citation network analysis," *Appl. Soft Comput.*, vol. 32, pp. 189–198, 2015. https://doi.org/10.1016/j.asoc.2015.03.027

[5] V. Torra, "Hesitant fuzzy sets," *Int. J. Intell. Syst.*, vol. 25, no. 5, pp. 529–539, 2010. https://doi.org/10.1002/int.20418

[6] D. Rani and H. Garg, "Distance measures between the complex intuitionistic fuzzy sets and their applications to the decision-making process," *Int. J. Uncertainty Quantification*, vol. 7, no. 5, pp. 423–439, 2017. https://doi.org/10.1615/int.j.uncertaintyquantification.2017020356

[7] I. Beg and T. Rashid, "Group decision making using intuitionistic hesitant fuzzy sets," *Int. J. Fuzzy Logic Intell. Syst.*, vol. 14, no. 4, pp. 181–187, 2014.

[8] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, and T. Mahmood, "Spherical fuzzy sets and their applications in multi-attribute decision making problems," *J. Int. Fuzzy Syst.*, vol. 36, no. 3, pp. 2829–2844, 2019. https://doi.org/10.3233/jifs-172009

[9] S. Ashraf, S. Abdullah, and Muneeza, "Some novel aggregation operators for cubic picture fuzzy information: Application in multi-attribute decision support problem," *Granul. Comput.*, vol. 6, no. 3, pp. 603–618, 2020. https://doi.org/10.1007/s41066-020-00219-1

[10] Y. Jin, S. Ashraf, and S. Abdullah, "Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems," *Entropy*, vol. 21, no. 7, p. 628, 2019. https://doi.org/10.3390/e21070628

[11] Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *Int. J. Gen. Syst.*, vol. 35, no. 4, pp. 417–433, 2006. https://doi.org/10.1080/03081070600574353

[12] H. Garg and K. Kumar, "Improved possibility degree method for ranking intuitionistic fuzzy numbers and their application in multiattribute decision-making," *Granular Computing*, vol. 4, no. 2, pp. 237–247, 2018. https://doi.org/10.1007/s41066-018-0092-7

[13] X. Gou, Z. Xu, and Q. Lei, "New operational laws and aggregation method of intuitionistic fuzzy information," *J. Intell. Fuzzy Syst.*, vol. 30, no. 1, pp. 129–141, 2015. https://doi.org/10.3233/ifs-151739

[14] J. Ye, "Intuitionistic fuzzy hybrid arithmetic and geometric aggregation operators for the decision-making of mechanical design schemes," *Appl. Intell.*, vol. 47, no. 3, pp. 743–751, 2017. https://doi.org/10.1007/s10489-017-0930-3

[15] C. L. Fan, Y. Song, Q. Fu, L. Lei, and X. Wang, "New operators for aggregating intuitionistic fuzzy information with their application in decision making," *IEEE Access*, vol. 6, pp. 27 214–27 238, 2018. https://doi.org/10.1109/access.2018.2832206

[16] J. Y. Huang, "Intuitionistic fuzzy Hamacher aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 27, no. 1, pp. 505–513, 2014. https://doi.org/10.3233/ifs-131019

[17] C. P. Wei and X. Tang, "Possibility degree method for ranking intuitionistic fuzzy numbers," in *2010 IEEE/WIC/ACM International Conference on Web Intelligence and Intelligent Agent Technology, Toronto, ON, Canada*, 2010, pp. 142–145. https://doi.org/10.1109/wi-iat.2010.239

[18] S. Wan and J. Dong, *A possibility degree method for interval-valued intuitionistic fuzzy multi-attribute group decision making*. Springer, Singapore, 2020, pp. 1–35. https://doi.org/10.1007/978-981-15-1521-7_1

[19] H. Garg and D. Rani, "A robust correlation coefficient measure of complex intuitionistic fuzzy sets and their applications in decision-making," *Appl. Intell.*, vol. 49, no. 2, pp. 496–512, 2018. https://doi.org/10.1007/s10489-018-1290-3

[20] F. Dammak, L. Baccour, and M. Adel Alimi, "An exhaustive study of possibility measures of intervalvalued intuitionistic fuzzy sets and application to multicriteria decision making," *Adv. Fuzzy Syst.*, vol. 2016, pp. 1–10, 2016. https://doi.org/10.1155/2016/9185706

[21] F. Blanco-Mesa, J. M. Merigó, and A. M. Gil-Lafuente, "Fuzzy decision making: A bibliometric-based review," *J. Intell. Fuzzy Syst.*, vol. 32, no. 3, pp. 2033–2050, 2017. https://doi.org/10.3233/jifs-161640

[22] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 171–186, 2002. https://doi.org/10.1109/91.995119

[23] D. E. Tamir, N. D. Rishe, and A. Kandel, *Complex fuzzy sets and complex fuzzy logic an overview of theory and applications*. Springer, Cham, 2015, pp. 661–681. https://doi.org/10.1007/978-3-319-19683-1_31

[24] D. Ramot, M. Friedman, G. Langholz, and A. Kandel, "Complex fuzzy logic," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 450–461, 2003. https://doi.org/10.1109/tfuzz.2003.814832

[25] S. Dick, R. R. Yager, and O. Yazdanbakhsh, "On the properties of pythagorean and complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 5, pp. 1009–1021, 2016. https://doi.org/10.1109/tfuzz.2015.2500273

[26] A. M. D. J. S. Alkouri and A. R. Salleh, "Complex intuitionistic fuzzy sets," *AIP Conf. Proc.*, vol. 1482, pp. 464–470, 2012. https://doi.org/10.1063/1.4757515

[27] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *2009 IEEE International Conference on Fuzzy Systems, Jeju, Korea*, 2009, pp. 1378–1382. https://doi.org/10.1109/fuzzy.2009.5276884

[28] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decisionmaking," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, no. 1, pp. 183–190, 1988. https://doi.org/10.1109/21.87068

[29] E. S. Windfeld and M. S. L. Brooks, "Medical waste management – A review," *J. Environ. Manage.*, vol. 163, pp. 98–108, 2015. https://doi.org/10.1016/j.jenvman.2015.08.013

[30] M. Karamouz, B. Zahraie, R. Kerachian, N. Jaafarzadeh, and N. Mahjouri, "Developing a master plan for hospital solid waste management: A case study," *Waste Manage.*, vol. 27, no. 5, pp. 626–638, 2007. https://doi.org/10.1016/j.wasman.2006.03.018

[31] A. Hinduja and M. Pandey, "Assessment of healthcare waste treatment alternatives using an integrated decision support framework," *Int. J. Comput. Intell. Syst.*, vol. 12, no. 2, pp. 318–333, 2018. https://doi.org/10.2991/ijcis.2019.0022

[32] E. Twinch, *Medical waste management*. International Committee of the Red Cross (ICRC), Geneva, Switzerland, 2011.

[33] S. Lee, M. Vaccari, and T. Tudor, "Considerations for choosing appropriate healthcare waste management treatment technologies: A case study from an East Midlands NHS Trust, in England," *J. Cleaner Prod.*, vol. 135, pp. 139–147, 2016. https://doi.org/10.1016/j.jclepro.2016.05.166

[34] E. A. Voudrias, "Technology selection for infectious medical waste treatment using the analytic hierarchy process," *J. Air Waste Manage. Assoc.*, vol. 66, no. 7, pp. 663–672, 2016. https://doi.org/10.1080/10962247.2016.1162226

[35] H. Shi, H. C. Liu, P. Li, and X. G. Xu, "An integrated decision making approach for assessing healthcare waste treatment technologies from a multiple stakeholder," *Waste Manage.*, vol. 59, pp. 508–517, 2017. https://doi.org/10.1016/j.wasman.2016.11.016

[36] O. Awodele, A. A. Adewoye, and A. C. Oparah, "Assessment of medical waste management in seven hospitals in Lagos, Nigeria," *BMC Public Health*, vol. 16, no. 1, pp. 1–11, 2016. https://doi.org/10.1186/s12889-016-2916-1

[37] P. Rani, A. R. Mishra, R. Krishankumar, K. S. Ravichandran, and A. H. Gandomi, "A new Pythagorean fuzzy based decision framework for assessing healthcare waste treatment," *IEEE Trans. Eng. Manage.*, vol. 69, no. 6, pp. 2915–2929, 2022. https://doi.org/10.1109/tem.2020.3023707

[38] A. R. Mishra, A. Mardani, P. Rani, and E. K. Zavadskas, "A novel EDAS approach on intuitionistic fuzzy set for assessment of health-care waste disposal technology using new parametric divergence measures," *J. Cleaner Prod.*, vol. 272, p. 122807, 2020. https://doi.org/10.1016/j.jclepro.2020.122807