Evolutionary Game Analysis of Stakeholder Strategies in China's Basic Education under the "Double Reduction" Policy

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Abstract: The reduction of excessive academic burden in China's basic education system has been established as a central objective of national education reform and has become a subject of intense policy debate. To elucidate the complex strategic interactions that shape the implementation of the "Double Reduction" policy, a multi-agent evolutionary game model was constructed incorporating three principal stakeholder groups: government authorities, schools and teachers, and students and parents. Replicator dynamic equations were employed to examine the evolutionary stability of stakeholder strategies and the conditions under which equilibrium outcomes emerge. Through numerical simulations, the influence of regulatory enforcement intensity on behavioral trajectories and convergence patterns was evaluated. The results reveal that asymptotically stable equilibria exist, with optimal system performance achieved when government bodies maintain active and credible regulatory oversight, educational institutions engage in substantive and sustained burden-reduction efforts, and families adopt cooperative and adaptive responses. By clarifying the mechanisms through which stakeholder interactions determine collective outcomes, this study provides theoretical support for the refinement of policy coordination and the long-term enhancement of education governance capacity. These findings contribute not only to the understanding of the "Double Reduction" policy's systemic impact but also to broader discussions on the role of evolutionary game theory in evaluating multi-agent policy interventions in education systems.

Keywords: Double Reduction policy; Evolutionary game theory; Multi-agent interaction; Replicator dynamics; Education governance

1. Introduction

The practice and theoretical research on reducing the burden of basic education has a long history. Scholars have explored this topic from different perspectives and aspects based on various educational systems. Research has discussed issues related to policy formulation, academic pressure and mental health, extracurricular training institutions and education effectiveness, school-family cooperation, and the implementation of national education policies. The unique contributions of this study lie in three aspects: (a) While existing research has predominantly focused on the game between "the government and after-school tutoring institutions," this study is the first to integrate the government, schools and teachers, as well as students and parents into a unified evolutionary game framework, thereby aligning more closely with the practical implementation logic of the "Double Reduction" policy; (b) Moving beyond theoretical analysis of the game model, it incorporates numerical simulations to reveal the dynamic impact of multidimensional mechanisms—such as incentives and penalties, benefits, and parental feedback—on the evolutionary process; (c) The proposal of tiered and differentiated governance recommendations enhances the operational applicability of the research.

U.S. scholar Epstein (Cooper, 1989) proposed the Theory of Overlapping Spheres of Influence to improve home-school collaboration practices. This theory posits that "the three environments of family, school, and community are both independent and interdependent, exerting overlapping influences on student development. During their growth, students are impacted not only individually by these three environments but also collectively by their interactions. Therefore, Epstein advocates for establishing a new partnership among families, schools, and

communities."

Through investigative research, Cooper (1989) found that an appropriate amount of homework is beneficial to student development, whereas excessive homework increases student burden. The duration of homework has a certain impact on academic performance, but longer homework time does not necessarily lead to better grades. Oyoo et al. (2020) also pointed out that students experience academic pressure when striving to meet standards set by others. Teo & Koh (2020) questioned the educational effectiveness of after-school tutoring institutions. Additionally, many scholars have researched government regulation of after-school tutoring institutions. Yan (2019) emphasized that government management is crucial for addressing issues emerging from reforms.

Applying the Complex Adaptive Systems Theory and Loose-Tight Coupling Theory, Dimmock et al. (2021) found that under the guidance of the Ministry of Education's reform framework, grassroots schools possess a certain degree of autonomy, which can facilitate the effective implementation of national education policies. Furthermore, providing teachers with guidance on interpreting national education policies and offering professional training significantly influences schools' ability to implement these policies effectively. Oguri & Takano (2021) suggested that the central government should appropriately decentralize authority to local governments, and local governments should combine national education policies with their own characteristics to develop distinctive education policies, thereby better implementing national education policies.

In recent years, international academic research on educational governance and evolutionary game theory has continued to expand. Muñet al. (2023) employed evolutionary game theory to explore the dynamic evolution of teaching strategies in the post-pandemic era. These studies not only broaden the international perspective of educational policy research but also provide methodological insights for a comparative understanding of the "Double Reduction" policy.

Jia et al. (2021) argued that students and their families, teachers and schools, and after-school tutoring institutions collectively form a stakeholder matrix in alleviating the burden on primary and secondary school students. They highlighted the existence of multi-directional interactions in the burden reduction process and proposed breakthrough strategies, including exploring the establishment of a dynamic monitoring mechanism for student academic workload, improving the comprehensive enforcement and regulatory mechanisms for burden reduction policies, constructing a collaborative education system involving family, school, and society, and enhancing the supply of diversified high-quality basic education resources. Xing & Yang (2022) developed a game model involving local governments and educational training institutions. By setting specific parameter values, the model showed that governments tend to increase supervision of these institutions, while the institutions choose to operate legally to increase revenue. Xu & Yang (2022) emphasized that the government, families, schools, teachers, and after-school academic tutoring institutions collectively constitute the stakeholder matrix for reducing the educational burden. In addition, all stakeholders were called for to work together to build a healthy educational ecosystem and truly alleviate the burden of compulsory education.

In 2021, China introduced its strictest "Double Reduction" policy, which includes two aspects: first, reducing the amount of homework for students; second, alleviating the extracurricular burden on students. This policy aims to address the current problem of excessive student burden in China and improve students' physical and mental health and comprehensive development, and it also requires joint efforts from teachers, parents, and all sectors of society to create a more relaxed and comfortable learning environment for children. According to the theory of stakeholders, the "Double Reduction" policy involves multiple stakeholders, including administrative departments, schools, teachers, students, parents, and education and training institutions. They interact with each other in the policy implementation process, and their conflicts and cooperation affect policy implementation. In addition, for exam-oriented countries like China, teachers, students, and parents are very anxious about this issue. In this context, it is necessary to analyze the strategies and benefits of various parties and explore the deep-rooted issues of the policy to reduce the burden on students using the ideas of evolutionary game theory. For the convenience of research, the main stakeholders affected by the "Double Reduction" policy are grouped into three categories: administrative departments, schools and teachers, and students and parents. In this study, a three-party evolutionary game model was constructed for the government, schools, teachers, students, and parents, and its strategies, benefits, and stability were analyzed.

Evolutionary game theory has been deeply researched by numerous domestic and international scholars such as Smith (1982), Friedman (1991), Tadj & Touzene (2003), and Sun et al. (2003), forming a complete theoretical system. It includes two basic concepts: evolutionarily stable strategy (ESS) and replicator dynamic equation (Hodgson, 1996; Tian & Li, 2025; Wei & Chen, 2013). The model considers the combined benefits of the three parties as a whole, which can better analyze, explain, and predict dynamic game behaviors (Zhang, 2015). Evolutionary game theory has a very wide range of applications in education, especially in the fields of education policy formulation and implementation, teacher behavior research, student behavior research, and educational market competition. It helps educators better understand and solve some complex problems in the education field and provides more accurate policy recommendations. Educational administrators can use the methods of evolutionary game theory to optimize resource allocation strategies to achieve the maximum benefit. In the field of educational market competition, evolutionary game theory can be used to analyze the strategy selection and

market positioning issues between different educational institutions.

2. Establishment of an Evolutionary Game Model

2.1 Basic Assumptions of the Model

In the process of implementing the "Double Reduction" policy, the main participants include administrative departments, schools and teachers, students, and parents. The game participants form a complete system. All parties are assumed to be boundedly rational individuals with learning, imitation, and adjustment capabilities in an environment of incomplete information. Their goal is to maximize their own interests, and they do not consider other factors that may affect the game during the game process. In the game process, parties continuously learn, imitate, and adjust their strategies to adopt equilibrium-stable strategies, forming the optimal strategy for joint governance and achieving the maximum common interest.

Table 1. Main parameters and corresponding meanings

Parameter	Meaning	Parameter	Meaning
C_1	The cost of administrative departments' active supervision	F	The quantitative indicator of negative impacts on schools and teachers caused by students and parents cooperating to discover passive execution by schools and teachers
C_2	The cost of active execution by schools and teachers	G_1	The quantitative indicator of adverse impacts on the government caused by passive execution by schools and teachers
C_3	The cost of cooperation by students and parents	G_2	The quantitative indicator of adverse impacts on students and parents caused by passive execution by schools and teachers, which is a relatively large value
R_1	The benefit obtained by administrative departments' active supervision	G_3	The quantitative indicator of beneficial impacts on the government caused by active execution by schools and teachers
R_2	The economic benefit obtained by active execution by schools and teachers	G_4	The quantitative indicator of beneficial impacts on students and parents caused by active execution by schools and teachers (set to a small value, reflecting their obligation to fulfill social responsibility)
R_3	The quantitative indicator of the positive effect obtained by cooperation from students and parents	R_4	The incentive given by administrative departments to schools and teachers for their active execution
M	The economic benefit brought by the cost saving of administrative departments' inactive supervision	R_5	The incentive given by students and parents to schools and teachers for their active execution
S_1	The quantitative indicator of the negative impact on the government brought by administrative departments' inactive supervision	W_1	The quantitative indicator of the beneficial impact on students and parents brought by administrative departments' active supervision
S_2	The loss caused by passive execution by schools and teachers	W_2	The quantitative indicator of the adverse impact on students and parents brought by administrative departments' inactive supervision
S_3	The quantitative indicator of the adverse impact on students and parents caused by their non-cooperation	P_1	The quantitative indicator of the beneficial impact on administrative departments brought by cooperation by students and parents
D	The punishment given by administrative departments to schools and teachers for their passive execution ($D > C_2$)	P_2	The quantitative indicator of the adverse impact on administrative departments brought by non-cooperation by students and parents

For the convenience of the study, relevant parameters were defined below. The cost-related parameters (C_1, C_2, and C_3) were determined based on the regulatory inputs of education administrative departments and surveys on teachers' workload. The reward and penalty parameters (R_4 and D) were defined with reference to the implementation rules of the local "Double Reduction" policy. The parent feedback parameters (R_5 and F) were derived from parent associations and public opinion. To ensure the robustness of the results, a sensitivity analysis

was incorporated into the numerical simulation to examine the impact of parameter variations on the system evolution outcomes.

The strategy space of administrative departments includes two choices: active and passive supervision. The strategy space of schools and teachers includes two choices: active and passive execution. The strategy space of students and parents includes two choices: cooperation and non-cooperation. The game combinations are as follows: (active supervision, active execution, cooperation), (active supervision, active execution, non-cooperation), (active supervision, passive execution, cooperation), (active supervision, passive execution, non-cooperation), (passive supervision, active execution, cooperation), (passive supervision, active execution, non-cooperation), (passive supervision, passive execution, cooperation), and (passive supervision, passive execution, non-cooperation). For the convenience of research, relevant parameters were set, with all letter symbols being positive (Zhu & Dou, 2007), as shown in Table 1.

2.2 Model of Revenue Collection

Based on the above assumptions and analysis, the revenue combinations of the administrative departments, schools and teachers, students and parents under different game strategy combinations were constructed, as shown in Table 2.

Table 2. Revenue combinations of government, enterprises, and society under eight strategy combinations

Revenue of the Administrative Department	Revenue of Schools and Teachers	Revenue of Students and Parents
$-C_1+R_1+G_3+P_1$	$-C_2+R_2+R_4+R_5$	$-C_3+R_3+G_4+W_1$
$-C_1+R_1+G_3-P_2$	$-C_2+R_2+R_4$	0
$-C_1+R_1+D-G_1+P_1$	$M+-S_2-D-F$	$-C_3+R_3-G_2+W_1$
$-C_1+R_1+D-G_1-P_2$	$M+-S_2-D$	$-S_3-G_2+W_1$
0	$-C_2+R_2+R_5$	$-C_3+R_3+G_4$
0	$-C_2+R_2$	0
$-S_1-G_1$	$M-S_2-F$	$-C_3+R_3-W_2$
$-S_1-G_1-P_2$	$M-S_2$	$-G_2-W_2-S_3$

3. Model Analysis

Assuming the proportion of active supervision chosen by the administrative department is x, then the proportion of passive supervision is $1-x$; the proportion of active performance chosen by schools and teachers is y, then the proportion of passive performance is $1-y$; the proportion of cooperation chosen by students and parents is z, then the proportion of non-cooperation is $1-z$.

3.1 Administrative Department

Given the expected benefits and average group returns of the administrative department's active supervision, U_{1Y} and $\bar{U}_1$ (Jiang et al., 2020), respectively, then:

$$U_{1Y} = yz(-C_1 + R_1 + G_3 + P_1) + y(1 - z)(-C_1 + R_1 + G_3 - P_2) + (1 - y)z(-C_1 + R_1 + D - G_1 + P_1) + (1 - y)(1 - z)(-C_1 + R_1 + D - G_1 - P_2) \tag{1}$$

$$\bar{U}_1 = xyz(-C_1 + R_1 + G_3 + P_1) + xy(1 - z)(-C_1 + R_1 + G_3 - P_2) + x(1 - y)z(-C_1 + R_1 + D\_G_1 + P_1) + x(1 - y)(1 - z)(-C_1 + R_1 + D -G_1 - P_2) + (1 - x)(1 - y)z(-S_1 - G_1) + (1 - x)(1 - y)(1 - z)(-S_1 - G_1 - P_2) \tag{2}$$

According to evolutionary game theory, the replicator dynamic equation for the administrative department's active supervision is:

$$F(x) = \frac{dx}{dt} = x(U_{1Y} - \bar{U}_1) = x(1 - x)[yzP_2 + y(G_3 - P_2 - D - S_1) + zP_1 - C_1 + R_1 + D + S_1] \tag{3}$$

(a) When $z=[y(P_2+D+S_1-G_3)+G_1-R_1-D-S_1]/(yP_2+P_1)$, then $F(x) = 0$, this implies that all levels are stable states.

(b) When $z \neq [y(P_2+D+S_1-G_3)+C_1-R_1-D-S_1]/(yP_2+P_1)$, let $F(x) = 0$, then $x = 0$ and $x = 1$ can be obtained, which are two stable points for x. Taking $F(x)$ derivative gives the following equation (Smith, 1982):

$$\frac{dF(x)}{dx} = (1-2x)[yzP_2 + y(G_3 - P_2 - D - S_1) + zP_1 - C_1 + R_1 + D + S_1]$$
$$= (1-2x)[z(yP_2 + P_1) + y(G_3 - P_2 - D - S_1) - C_1 + R_1 + D + S_1] \tag{4}$$

When $z>[y(P2+D+S1-G3)+C1-R1-D-S1]/(yP2+P1)$, $\left.\frac{dF(x)}{dx}\right|_{x=1} < 0$ and $\left.\frac{dF(x)}{dx}\right|_{x=0} < 0$, then $x = 1$ is the equilibrium point.

When $z<[y(P_2+D+S_1-G_3)+C_1-R_1-D-S_1]/(yP_2+P_1)$, $\left.\frac{dF(x)}{dx}\right|_{x=1} < 0$, and $\left.\frac{dF(x)}{dx}\right|_{x=0} < 0$, then $x = 0$ is the equilibrium point.

Based on the above analysis, the dynamic trend and stability of the administrative department can be concluded, as shown in Figure 1.

(a)

(b)

(c)
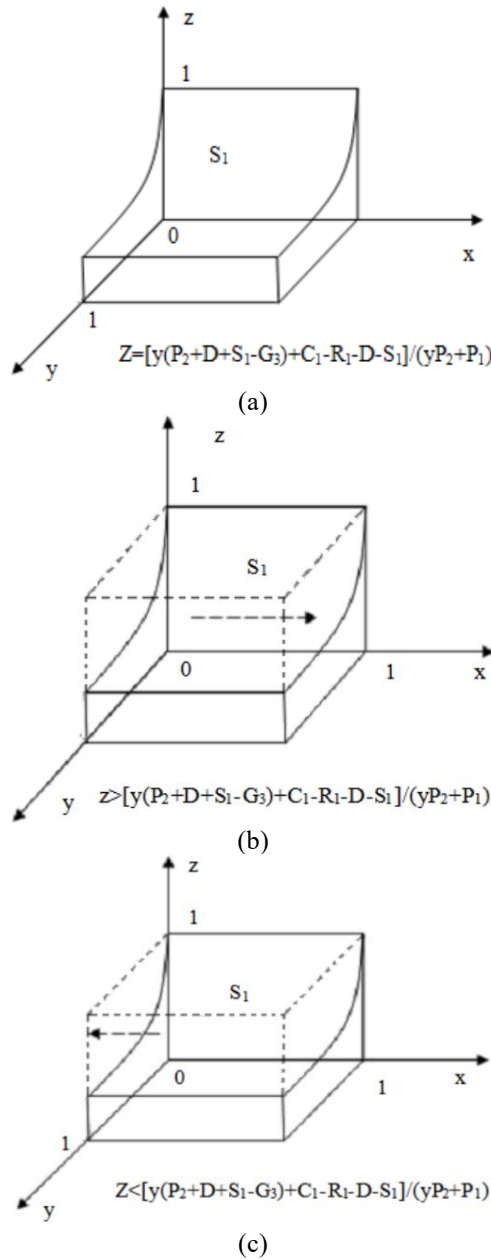
Figure 1. Dynamic trend diagrams of the administrative department: (a) All levels are steady states; (b) $x=1$ is the equilibrium point; and (c) $x=0$ is the equilibrium point

3.2 Schools and Teachers

Let the expected value and average group benefit of schools and teachers actively implementing the "Double Reduction" policy be U_{2Y} and $\bar{U}_2$, respectively, then:

$$U_{2Y} = xz(-C_2 + R_2 + R_4 + R_5) + x(1-z)(-C_2 + R_2 + R_4) + (1-x)z(-C_2 + R_2 + R_5) + (1-x)(1-z)(-C_2 + R_2) \tag{5}$$

$$\bar{U}_2 = xyz(-C_2 + R_2 + R_4 + R_5) + xy(1-z)(-C_2 + R_2 + R_4) + x(1-y)z(-S_2 - D - F) + x(1-y)(1-z)(-S_2 - D) + (1-x)yz(-C_2 + R_2 + R_5) + (1-x)y(1-z)(-C_2 + R_2) + (1-x)(1-y)z(-S_2 - F) + (1-x)(1-y)(1-z)(-S_2) \tag{6}$$

According to evolutionary game theory, the replication dynamic equation of schools and teachers actively implementing the "Double Reduction" policy is:

$$F(y) = \frac{dy}{dt} = y(U_{2Y} - \bar{U}_2) = y(1-y)[x(R_4 + D) + z(R_5 + F) - C_2 + R_2 + S_2 - M] \tag{7}$$

(a) When $z = [C_2 - R_2 - S_2 + M - x(R_4 + D)]/(R_5 + F)$, then $F(y) = 0$, meaning that all levels are stable states.

(b) When $z \neq [C_2 - R_2 - S_2 + M - x(R_4 + D)]/(R_5 + F)$, let $F(y) = 0$, then $y = 0$ and $y = 1$ can be obtained, which are the two stable points of y. Taking $F(y)$ derivative obtains the following equation:

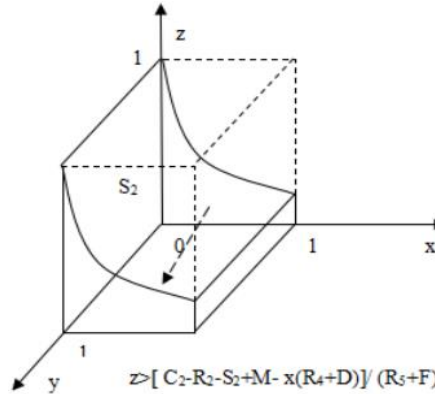$$\frac{dF(y)}{dy} = (1 - 2y)[x(R_4 + D) + z(R_5 + F) - C_2 + R_2 + S_2 - M] \tag{8}$$

When $z > [C_2 - R_2 - S_2 + M - x(R_4 + D)]/(R_5 + F)$, and $\left.\frac{dF(y)}{dy}\right|_{y=1} < 0$, then $y = 1$ is the equilibrium point.

When $z < [C_2 - R_2 - S_2 + M - x(R_4 + D)]/(R_5 + F)$, and $\left.\frac{dF(y)}{dy}\right|_{y=0} < 0$, then $y = 0$ is the equilibrium point.

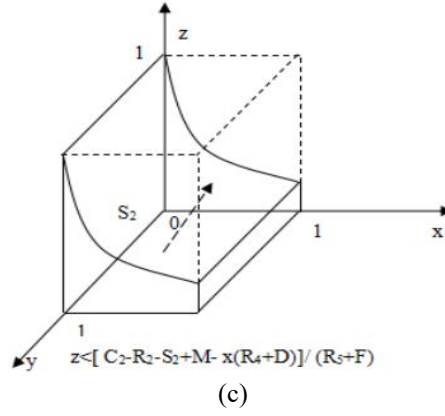Based on the above analysis, the dynamic trends and stability of schools and teachers can be concluded, as shown in Figure 2.

(a)

(b)

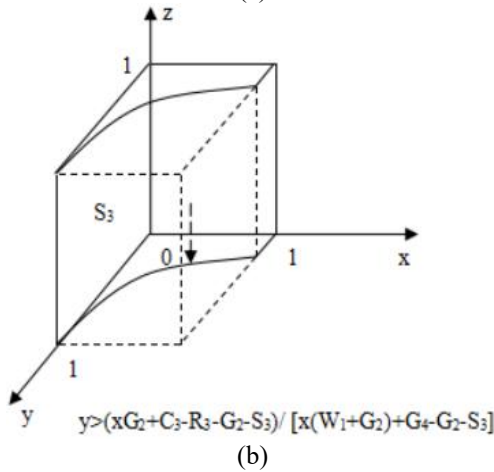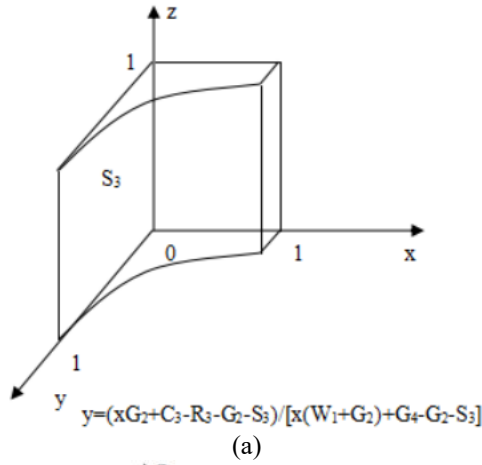$$z < [\; C_2\text{-}R_2\text{-}S_2\text{+}M\text{-}\; x(R_4\text{+}D)]/\;(R_5\text{+}F)$$

(c)
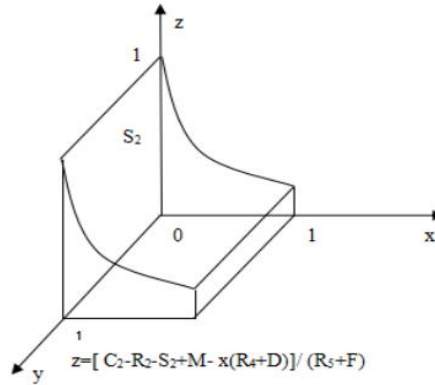
Figure 2. Dynamic trend diagrams of schools and teachers: (a) All levels are steady states; (b) *y*=1 is the equilibrium point; and (c) *y*=0 is the equilibrium point
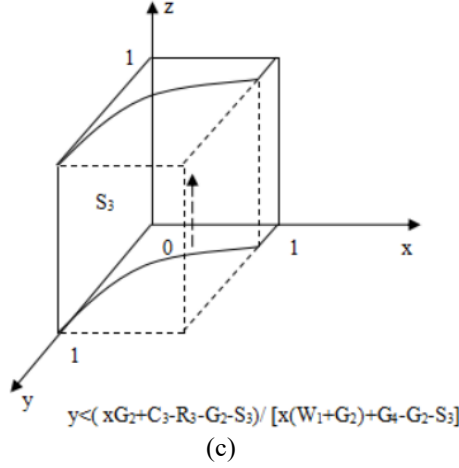
3.3 Students and Parents

Assuming the expected value and population average payoff of students and parents adopting a cooperative strategy are U_{3Y} and $\bar{U}_3$, respectively, then:

$$U_{3Y} = xy(-C_3 + R_3 + G_4 + W_1) + x(1 - y)(-C_3 + R_3 - G_2 + W_1) + (1 - x)y(-C_3 + R_3 + G_4) \\ + (1 - x)(1 - y)(-C_3 + R_3 - W_2)$$

(9)

$$\bar{U}_3 = xyz(-C_3 + R_3 + G_4 + W_1) + x(1 - y)z(-C_3 + R_3 - G_2 + W_1) + x(1 - y)(1 \\ - z)(-S_3 - G_2 + W_1) + (1 - x)yz(-C_3 + R_3 + G_4) + (1 - x)(1 \\ - y)z(-C_3 + R_3 - W_2) + (1 - x)(1 - y)(1 - z)(-G_2 - W_2 - S_3)$$

(10)

$$y = (xG_2\text{+}C_3\text{-}R_3\text{-}G_2\text{-}S_3)/[x(W_1\text{+}G_2)\text{+}G_4\text{-}G_2\text{-}S_3]$$

(a)

$$y > (xG_2\text{+}C_3\text{-}R_3\text{-}G_2\text{-}S_3)/\;[x(W_1\text{+}G_2)\text{+}G_4\text{-}G_2\text{-}S_3]$$

(b)

$$y < (xG_2+C_3-R_3-G_2-S_3) / [x(W_1+G_2)+G_4-G_2-S_3]$$

(c)

Figure 3. Dynamic trend diagrams of students and parents: (a) All levels are steady states; (b) $z=1$ is the equilibrium point; and (c) $z=0$ is the equilibrium point

According to evolutionary game theory, the replication dynamic equation of students and parents adopting a cooperative strategy is:

$$F(z) = \frac{dz}{dt} = z(U_{3Y} - \bar{U}_3) = z(1-z)\{y[x(W_1 + G_2) + G_4 - G_2 - S_3] - xG_2 - C_3 + R_3 + G_2 + S_3\} \quad (11)$$

(a) When $y = (xG_2+C_3-R_3-G_2-S_3)/[x(W_1+G_2)+G_4-G_2-S_3]$, then $F(z) = 0$, indicating that all levels represent stable states.

(b) When $y \neq (xG_2+C_3-R_3-G_2-S_3)/[x(W_1+G_2)+G_4-G_2-S_3]$, let $F(z) = 0$, then $z = 0$ and $z = 1$ can be obtained, which are two stable points of z. Taking $F(z)$ derivative obtains the following equation:

$$\frac{dF(z)}{dz} = (1 - 2z)\{y[x(W_1 + G_2) + G_4 - G_2 - S_3] - xG_2 - C_3 + R_3 + G_2 + S_3\} \quad (12)$$

According to the assumptions, $x(W_1+G_2)+G_4-G_2-S_3 < 0$, so:

When $y > (xG_2+C_3-R_3-G_2-S_3)/[x(W_1+G_2)+G_4-G_2-S_3]$, and $\left.\frac{dF(z)}{dz}\right|_{z=0} < 0$, then $z = 0$ is the equilibrium point.

When $y < [C_2-R_2-S_2+M- x(R_4+D)]/(R_5+F)$, and $\left.\frac{dF(z)}{dz}\right|_{z=1} < 0$, then $z=1$ is the equilibrium point.

Based on the above analysis, the dynamic trends and stability of students and parents can be concluded, as shown in Figure 3.

4. Stability Analysis of the Game Model

The equilibrium points and stability of the game system were analyzed based on the Jacobian matrix and Lyapunov stability theory. According to the replicator dynamic equation system, there are eight special equilibrium points, namely $E_1(1, 1, 1)$, $E_2(1, 1, 0)$, $E_3(1, 0, 1)$, $E_4(1, 0, 0)$, $E_5(0, 1, 1)$, $E_6(0, 1, 0)$, $E_7(0, 0, 1)$, and $E_8(0, 0, 0)$, which are (active supervision, active execution, cooperation), (active supervision, active execution, non-cooperation), (active supervision, passive execution, cooperation), (active supervision, passive execution, non-cooperation), (passive supervision, active execution, cooperation), (passive supervision, active execution, non-cooperation), (passive supervision, passive execution, cooperation), and (passive supervision, passive execution, non-cooperation), respectively.

If all the eigenvalues of the Jacobian matrix have positive real parts, the equilibrium point is an unstable point. If at least one eigenvalue of the Jacobian matrix has a positive real part, the equilibrium point is a saddle point. If all the eigenvalues of the Jacobian matrix have negative real parts, the equilibrium point is a stable point. According to the characteristic of the equilibrium state being robust against small disturbances, the system has only one stable equilibrium point, which is $x=1$, $y=1$, and $z=1$, namely $E_1(1, 1, 1)$, and it is the ESS of this game. No matter what its initial state is, it tends to eventually converge to this ESS due to the lack of robustness against small disturbances of a certain strategy (Ma et al., 2023).

From Eq. (3), it can be seen that as long as the ratio of student and parent cooperation exceeds a certain value, the administrative department tends to actively perform supervision responsibilities. If students and parents choose not to cooperate, the administrative department tends to choose passive supervision strategies. The administrative

department, students, and parents are positively correlated, and all parties must coordinate to play their roles effectively; they cannot rely solely on one party. Otherwise, the administrative department, students, and parents tend to fall into a state of mutual buck-passing, leading to the administrative department choosing passive supervision and students and parents choosing not to cooperate.

From Eq. (7), it can be seen that as long as the ratio of student and parent cooperation is greater than a certain value, the school and teacher group tends to ultimately choose to actively perform the "Double Reduction" strategy. This means that if schools and teachers choose active strategies, students and parents must fulfill certain cooperation functions. From Eq. (11), it can be seen that as the ratio of the active performance strategy of the school and teacher group approaches 1, the ratio of student and parent cooperation approaches 0; as the ratio of the active performance strategy approaches 0, the ratio of student and parent cooperation approaches 1. This indicates that although an internal driving force exists for schools and teachers to fulfill their obligations, the model shows that they are unlikely to act voluntarily; without external intervention from the government department, students, and parents, they tend not to consciously assume these responsibilities.

5. Numerical Simulation

To intuitively analyze the evolutionary game strategies of all interested parties in the implementation process of the "Double Reduction" policy, the model was numerically analyzed using MATLAB. For easy analysis, the initial values of the parameters are assumed in this study as follows: $C = (3, 2, 3)$, $R = (4, 6, 2, 2)$, $M = 1$, $S = (6, 6, 6)$, $D = 8$, $F = 6$, $G = (2, 8, 2, 0.5)$, $W = (3, 3)$, and $P = (3, 3)$. The simulation results are shown in Figure 4.
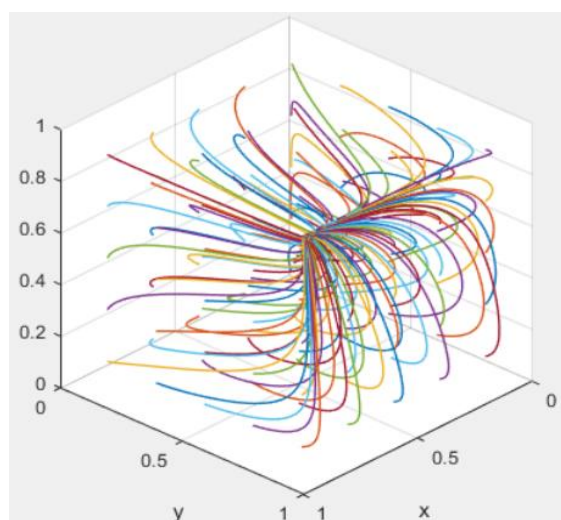

Figure 4. Results of the dataset evolving 200 times

As shown in Figure 4, it can be seen that when the condition x(W1+G2)+G4-G2-S3<0 is satisfied, the system has a stable point $E_1(1, 1, 1)$, where the administrative department actively supervises, schools and teachers actively perform, and student and parent cooperation is the stable state of the game. This indicates that while the final stable point is unique, the convergence speed and trajectory are significantly influenced by parameters. The effectiveness of policy implementation depends not only on institutional design but also on the intensity of different incentive mechanisms and the level of parental cooperation.

5.1 Influence of the Awards and Penalties of the Administrative Department on the Tripartite Management Game

The impact of the awards and penalties of the administrative department on the fluctuations of the tripartite evolutionary game model was analyzed in this study. The rewards for schools and teachers were assigned values of $R_4 = 1, 20, 50$, and the penalties were assigned values of $D = 0, 1, 3, 8, 50, 100, 200$. The simulation results are shown in Figure 5.

As shown in Figure 5, adjusting the amount of rewards and penalties imposed by administrative authorities on schools and teachers does not alter the final stable point, but significantly affects the speed and trajectory at which the system reaches stability. If rewards and penalties are insufficient, schools and teachers may maintain passive compliance over an extended period, manifesting as "inertia" in policy implementation. Conversely, when the reward-penalty mechanism is strengthened, schools and teachers converge more rapidly toward an active compliance strategy, indicating that the initial phase of the "Double Reduction" policy must rely on robust

administrative incentives.

(a)   (b)
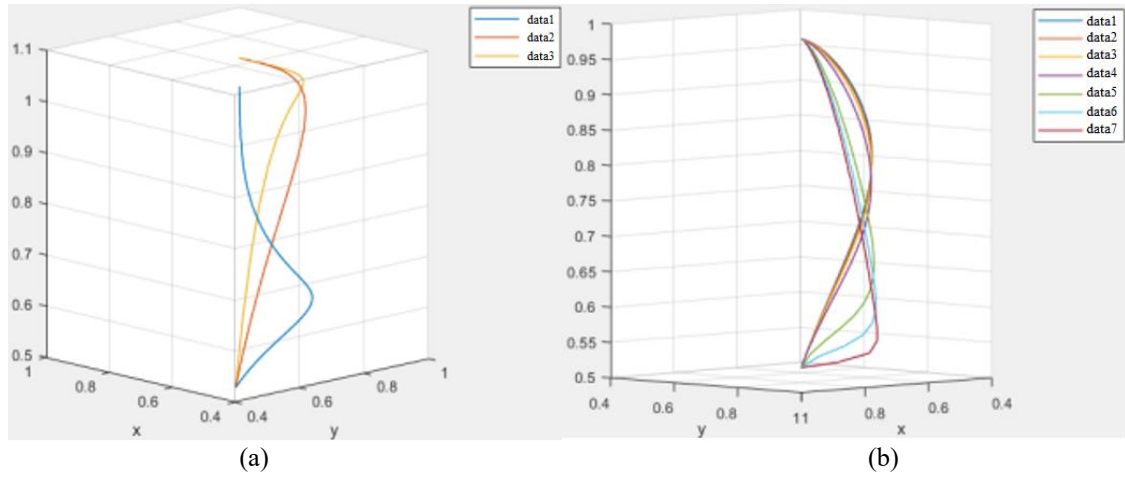
Figure 5. Influence of the awards and penalties of the administrative department on the tripartite management behavior game: (a) Rewards and (b) Penalties

5.2 Impact of School and Teacher Behavior on the Tripartite Management Game

The impact of the behavior of schools and teachers on the fluctuations of the tripartite evolutionary game model was analyzed in this study. The rewards for schools and teachers were assigned values of $R_2 = 1, 20, 50$, and the penalties were assigned values of $S_2 = 1, 50, 200$. The simulation results are shown in Figure 6.


(a)

(b)
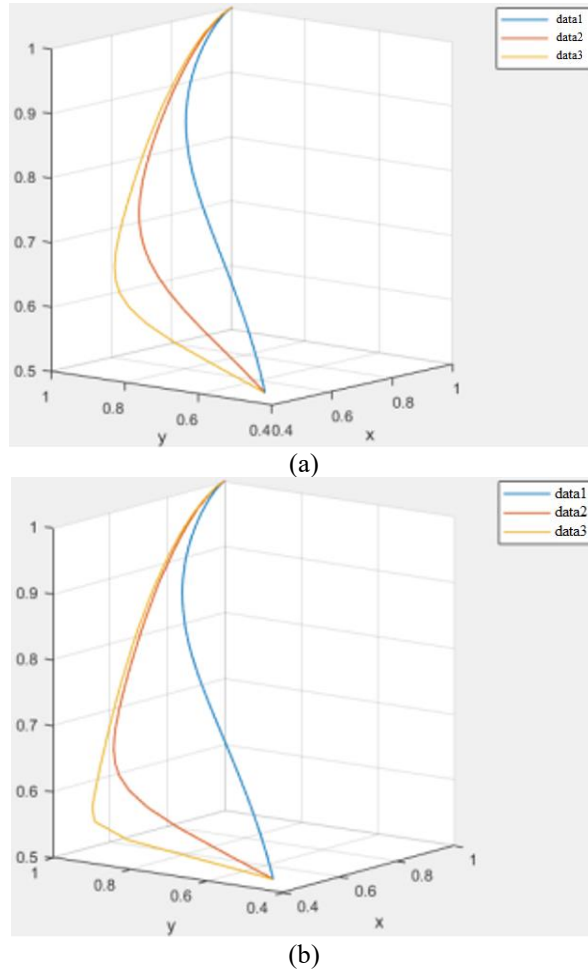
Figure 6. Impact of school and teacher behavior on the tripartite management game: (a) Rewards and (b) Penalties

As shown in Figure 6, the magnitude of gains or losses for schools and teachers significantly influences their strategic choices. In the initial period, as the values of gains and losses increase, the proportion of schools and teachers opting to actively comply with the "Double Reduction" policy rises markedly, eventually converging to a stable state. This suggests that the "Double Reduction" policy cannot rely solely on restricting tutoring to alleviate burdens—it must also simultaneously enhance teachers' professional returns (such as compensation, promotion channels, and career development) to ensure endogenous motivation for policy implementation at the behavioral level.

5.3 Impact of Student and Parent Feedback on the Tripartite Management Game

This study analyzed how students and parents provide incentives and negative feedback to schools and teachers depending on whether the latter actively implement the "Double Reduction" policy. To analyze this impact, values of $R_5 = 1, 20, 100$, and $F = 1, 20, 60$ were assigned. The simulation results are shown in Figure 7.
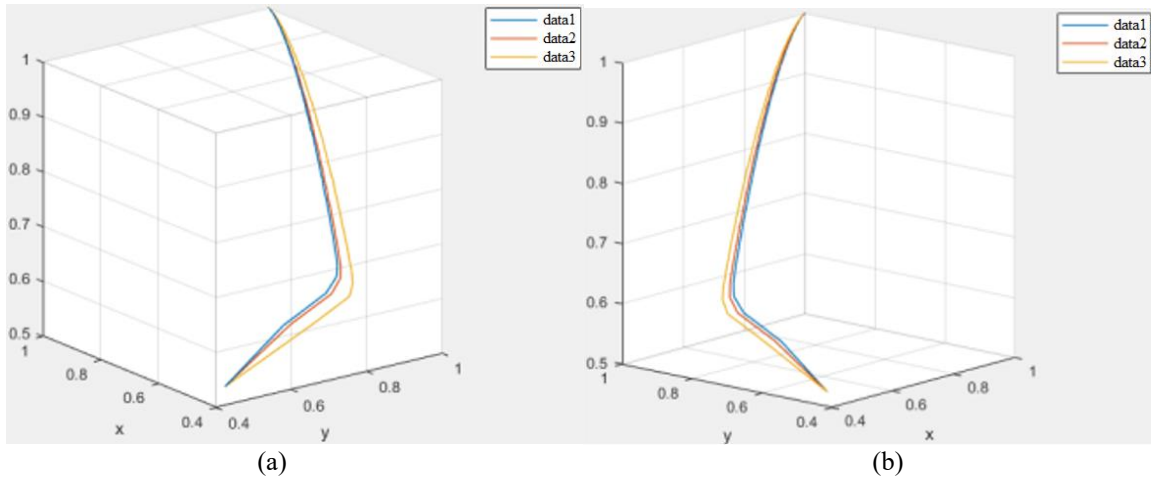
Figure 7. Impact of student and parent feedback on the tripartite management game: (a) Incentives and (b) Negative feedback

As shown in Figure 7, the magnitude of incentives provided by students and parents to schools and teachers for active compliance with the "Double Reduction" policy, as well as the quantified negative impact of passive compliance, has limited influence on teachers' strategic choices. This indicates that individual parental incentives and penalties are insufficient to alter collective teacher behavior. This mechanism must be integrated with government supervision to exert substantive effects. Therefore, social participation in the "Double Reduction" policy needs to be embedded through institutionalized arrangements (e.g., home-school collaboration platforms and community supervision organizations) rather than relying on sporadic feedback from individual households.

To verify the robustness of parameter settings, key parameters (such as C_2, R_4, and D) were selected for simulations across different intervals. The results show that although convergence speeds vary, the system ultimately tends toward the same evolutionary stable point (E_1: active government supervision, active school compliance, and parental cooperation). This suggests that policymakers can flexibly adjust the magnitude of rewards and penalties according to practical circumstances without altering the overall governance direction.

6. Insights and Recommendations

Based on the evolutionary game model and numerical simulation results, the insights and policy recommendations below can be derived.

6.1 Institutionalized and Phased Administrative Incentive Mechanisms

Simulation results indicate that the magnitude of rewards and penalties does not alter the final stable point of the system but significantly affects the convergence speed. Insufficient rewards and penalties may lead schools and teachers to maintain passive compliance over the long term, manifesting as "inertia" in policy implementation. Therefore, during the initial phase of the "Double Reduction" policy, supervision and enforcement should be intensified to ensure rapid policy adoption. As the policy stabilizes, a transition to normalized and refined supervision should be gradually implemented.

6.2 Teacher Career Incentives as a Key to "Double Reduction" Effectiveness

Teachers are highly sensitive to gains and losses. Without positive incentives, merely "banning tutoring" is insufficient to ensure their active participation. Thus, policymakers must simultaneously improve teachers' compensation (e.g., performance subsidies and housing benefits) and expand career advancement opportunities and professional development spaces. This can provide endogenous motivation for teachers to proactively implement the "Double Reduction" policy.

6.3 Institutionalized Channels for Parental and Societal Supervision

Simulation results show that incentives and penalties from individual households are inadequate to alter collective teacher behavior. Therefore, parental feedback should be integrated into the educational governance system through institutionalized channels such as home-school collaboration platforms, parent committees, and community supervision organizations. This approach can alleviate parental anxiety about education while preventing "rat race" comparisons.

6.4 Tiered and Differentiated Policy Pathways

- Major cities: Digital supervision could be promoted, leveraging big data to track hidden operations of off-campus tutoring institutions. In addition, third-party evaluation mechanisms could be introduced to avoid "selective enforcement" by local governments.
- Small and medium-sized cities/counties: Performance subsidies or housing benefits could be increased for teachers. In addition, regional educational resource-sharing platforms could be developed to avoid "resource disparities."
- School level: "Double Reduction" implementation could be incorporated into performance evaluations for principals and teachers. In addition, interest-based courses and extracurricular activities could be expanded to fill students' after-school "gaps."
- Household level: Parental education guidance systems could be established to foster scientific educational perspectives. In addition, parent mutual-aid organizations within communities could be created to promote rational dialogue.

7. Conclusion

Based on evolutionary game theory, a tripartite game model involving the government, schools and teachers, and students and parents was constructed in this study, analyzing their strategic choices and evolutionary patterns in the implementation of the "Double Reduction" policy. Numerical simulation results demonstrate that the system exhibited a unique asymptotically stable point: active government supervision, proactive compliance by schools and teachers, and cooperative engagement by students and parents. Sensitivity analysis reveals that variations in key parameters did not alter the final stable point but influenced the convergence path and speed, confirming the robustness of the findings.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The author declare no conflict of interest.

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