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Enhanced Decision-Making with Advanced Algebraic Techniques in Complex Fermatean Fuzzy Sets under Confidence Levels

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Abstract: This study introduces novel algebraic techniques within the framework of complex Fermatean fuzzy sets (CFFSs) by incorporating confidence levels, presenting a suite of operators tailored for advanced decision-making. Specifically, the confidence complex Fermatean fuzzy weighted geometric (CCFFWG) operator, the confidence complex Fermatean fuzzy ordered weighted geometric (CCFFOWG) operator, and the confidence complex Fermatean fuzzy hybrid geometric (CCFFHG) operator are developed to address multi-attribute group decision-making (MCGDM) challenges. These methodologies are designed to enhance decision-making in scenarios where decision-makers provide asymmetric or imprecise information, often encountered in environmental and industrial contexts. To validate the applicability of the proposed approach, a practical case study involving the selection of an optimal fire extinguisher from several alternatives is conducted. The performance of the newly developed operators is benchmarked against established methods from prior studies, with results demonstrating superior decision outcomes in terms of precision and reliability. By embedding confidence levels into complex Fermatean fuzzy operations, the proposed techniques offer greater robustness in managing uncertainty and variability across multiple attributes. These findings suggest that the advanced algebraic framework contributes significantly to improving decision quality in complex group decision-making environments.

Keywords: Confidence levels; Complex Fermatean fuzzy sets (CFFSs); Decision-making; Multi-attribute group decision-making (MCGDM); Advanced algebraic methods; Complex fuzzy numbers (CFNs)

1 Introduction and Literature Reviewer

The decision-making process (DMP) in fuzzy set theory (FST) [1] contains dealing with imprecision and uncertainty, which are common in real-world problems (RWP). In classical mathematical set theory (CMST), an element either belongs to a set or it doesn't (binary membership), but FST presents partial membership, where elements can belong to a set to varying degrees called membership degree (MD), represented by values between 0 and 1. In DMP under fuzzy conditions, alternatives or criteria are evaluated using fuzzy numbers or MD. These degrees describe the degree of satisfaction related to a particular decision. Common techniques used in fuzzy DMP include fuzzy multi-criteria group decision-making (FMCGDM), fuzzy ranking approaches, and fuzzy logic-based systems, which help in selecting the more suitable alternative when exact data is not available or when imprecision plays a noteworthy and important role. The flexibility of FS allows decision-makers to incorporate subjective judgments, linguistic terms (e.g., "low," "medium," "high"), and inaccurate data, thus providing a more accurate and adaptable method to solving complex decision problems (CDP). Later on, several scholars, such as Jin et al. [2] and Li et al. [3], have added expressively to this domain, offering extensive and related research that has further expanded the applicability and understanding of FS. They show the importance of FS in handling ambiguous and complex information, proving its value in many RWP.

1.1 Drawbacks and Weakness of FST

I. MD: This model discussed only the degree of satisfaction called the degree of membership and has no idea about the degree of dissatisfaction called the MD.

II. Defining the MD in FS is often subjective and can vary between experts or decision-makers. This presents impression and uncertainty in the accuracy of the proposed model.

III. Complexity in Calculation: Handling operations and fuzzy logic, particularly in larger or more complex systems, can lead to increased computational complexity. This makes fuzzy systems slower and harder to implement compared to traditional methods.

IV. Lack of Interpretability: While fuzzy systems can handle uncertainty, the resulting decisions may sometimes be hard to interpret due to the non-binary nature of fuzzy outputs, which can be confusing for users who prefer clear yes/no answers.

V. Dependence on Expert Knowledge: Many fuzzy systems rely heavily on expert knowledge for rule creation and the design of membership functions, which could lead to errors if the experts' inputs are inaccurate or biased.

1.2 Intuitionistic Fuzzy Set Theory

To address the above-mentioned problems, Atanassov [4] presented the idea of an intuitionistic fuzzy set (IFS), categorized by the MD and no-membership degree (NMD), both defined within the closed interval [0, 1]. Importantly, the sum of these degrees also lies within the [0, 1] interval. This approach emphasizes that values should not be considered in isolation from their associated attributes. In IFS every element can be presented as $(\ell, \hbar)$ under limitation: $\ell + \hbar \leq 1$. Building on this foundation, Yager et al. [5, 6] prolonged the IFS framework, leading to the development of the Pythagorean fuzzy set (PFS), which relaxes the limitation of IFS by allowing a broader domain, such as $\ell + \hbar \leq 1$ to $\ell^2 + \hbar^2 \leq 1$. Later on, Rahman et al. [7–10] developed various aggregation methods under a Pythagorean fuzzy environment. Senapati and Yager [11] introduced the concept of Fermatean fuzzy sets (FFSs). FFSs represent a significant enhancement in FS theory, offering a more flexible and accurate framework for managing intricate systems and DMP under uncertainty and vagueness. In FFS, each element can be written as $(\ell, \hbar)$ under limitation: $\ell^3 + \hbar^3 \leq 1$. By accommodating higher levels of uncertainty and providing a robust structure for complex evaluations, FFSs pave the way for more effective and reliable outcomes in various fields, thereby extending the frontier of fuzzy set applications. Figure 1 shows the effectiveness and flexibility of complex Fermatean Fuzzy Set.

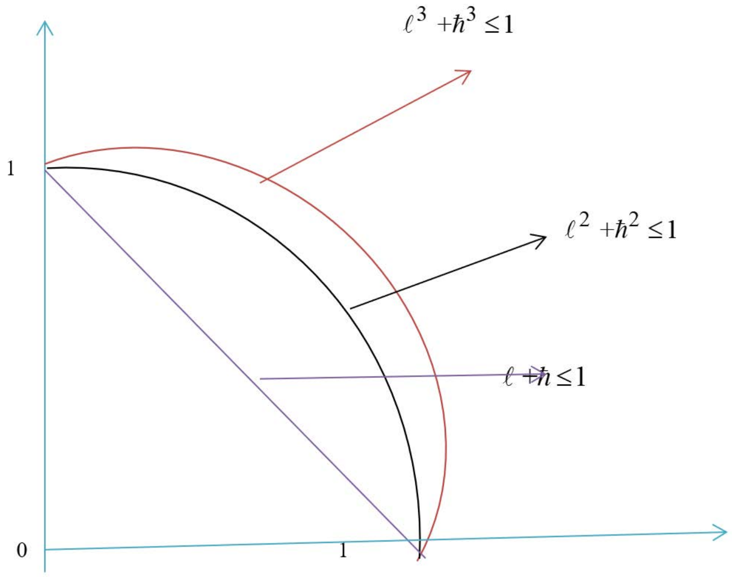

Figure 1. Space comparison of IFS, PyFS and FFS

Subsequently, Buckley [12] introduced the concept of complex fuzzy numbers (CFNs), enriching the field with new mathematical perspectives. Zhang et al. [13] introduced the notion of complex fuzzy set (CFS) and presented some some important operations. Moreover, Nguyen et al. [14] presented the idea of complex-valued physical quantities. These advancements collectively highlight the dynamic and evolving nature of research, underscoring its possible for varied applications.

1.3 CFS Theory

Ramot et al. [15] have associated this gap by offering CFSs, an extension of traditional fuzzy sets. CFSs offer a refined framework for managing ambiguity and uncertainty, enabling a more nuanced representation of intricate relations within dynamic datasets. The primary difference between CFS and FS lies in the range and application of their MD. While FSs are confined to a range between [0, 1], CFSs extend this range onto a unit disc, providing a more intricate illustration. This enhanced scope of CFS has garnered significant attention within the FS theory due to its potential for capturing more detailed information. Yazdanbakhsh and Dick [16] explored this potential by employing CF logic for time series forecasting, demonstrating the advanced capabilities and applications of CFS in predictive modeling. Bi et al. [17] and Chen et al. [18] introduced a range of innovative approaches centered on CFNs, showcasing significant advancements in this area. Their contributions have laid a strong foundation for further research, offering novel solutions and techniques that enhance the application and efficiency of CFNs in complex systems. Alkouri and Salleh [19] introduced the concept of CIFSs, characterized by MD and NMD. Expanding on this, Ma et al. [20] proposed CFS to address challenges posed by multiple periodic factors. Dick et al. [21] conducted an extensive study on various CFS models, contributing to a deeper understanding of their applications. Hu et al. [22] focused on evaluating the consistency of CFS operations and introduced novel procedures to enhance their functionality. Greenfield et al. [23] made a significant contribution by redefining Complex Interval-Valued Fuzzy Sets (CIVFS), thereby advancing the concept of CFSs and extending the interval-valued fuzzy sets (IVFS) framework. Ullah et al. [24] introduced complex Pythagorean fuzzy sets (CPyFSs), which extend the concept of CIFS. Rahman et al. [25] introduced some novel approaches based on complex Pythagorean fuzzy information. Chen et al. [26] introduced CFFSs as a novel approach to enhance the handling of uncertainty in DMP. This innovative method aims to improve the precision and reliability of decisions in situations where uncertainty is a significant factor.

The theory of confidence level within the CFF-Model introduces a robust framework for handling uncertainty in MCGDM. The integration of various averaging and geometric operators allows for the effective fusion of diverse data types, providing a more nuanced and accurate representation of RWP. An illustrative example within the study demonstrates the practicality and effectiveness of the operators based on confidence level in MCGDM, showcasing the model's potential for application in various fields in real life where MD under uncertainty is crucial.

Building on earlier studies of CPyFSs [24, 25] and their use in decision-making, this research presents CFFSs, which offer greater elasticity by loosening the strict constraints of CPyFSs. New operators have been developed to enable more precise modifications, enhancing their application. As a result, CFFSs provide a more robust and effective framework for addressing decision-making challenges.

This research is planned as follows: Section 2 presents the main definitions for all the ideas and methods explored in later sections. These foundational terms provide the basis for understanding the material that follows. Section 3 introduces the CCFFWGA operator, CCFFOWGA operator, and CCFFHGA operator. These operators are crafted to adeptly aggregate complex fuzzy information, which is crucial for processing and analyzing data in environments characterized by uncertainty and complexity. Section 4 presents a novel emergency decision-making model designed to expand decision efficacy under pressure. This approach directly addresses a key challenge in emergency management, enhancing rapid and effective response. To demonstrate the practical applicability of these theoretical constructs, Section 5 offers a detailed example that illustrates how the proposed methods can be implemented in real-world situations. Section 6 presents the main contributions and findings of the proposed study, highlighting the importance of its results and their potential to advance the field. These results offer valuable insights that could shape future research and practical applications.

2 Preliminaries

In this section, we present the main concepts of presented models such as IFS, PFS, and FFS under complex fuzzy information. The following Figure 2 shows the structure of the paper.

I) Complex Intuitionistic Fuzzy Set:

Alkouri and Salleh [19] presented the idea of the CIFS. This model allows for a richer demonstration of uncertainty by expanding the range of both degrees, including MD and NMD, within the closed interval [0, 1], to the entire unit circle in the complex plane.

Definition 1: The CIFS $\boldsymbol{C}$ on a universal set $\boldsymbol{X}$ can be presented mathematically as [19]:

$$C = \left\{ \left\langle x, \ell(x)e^{i2\pi d(x)}, \hbar(x)e^{i2\pi f(x)} \right\rangle \mid x \in X \right\}$$

where, ℓ and $\hbar$ are called complex valued MD and NMD, respectively, with conditions: $\mu, \hbar \in [0,1]$ and $\ell + \hbar \leq 1$. Moreover, d and f are called the phase terms with conditions: $d, f \in [0, 2\pi]$ and $\frac{d}{2\pi} + \frac{f}{2\pi} \leq 1, \forall x \in X$.

II) Complex Pythagorean Fuzzy Set:

Ullah et al. [24] presented the CPyFS as an extension of the PyFS, allowing for values within a complex subset controlled by a unit disc.

Definition 2: The CPyFS P on a universal $\boldsymbol{X}$ can be mathematically presented as [24]:

$$P=\left\{\left\langle x,\ell(x)e^{i2\pi d(x)},\hbar(x)e^{i2\pi f(x)}\right\rangle \mid x\in X\right\} \text{ with } \ell^2+\hbar^2\leq 1 \text{ and } \left(\frac{d}{2\pi}\right)^2+\left(\frac{f}{2\pi}\right)^2\leq 1, \forall x\in X$$

III) CFFSs:

Chen et al. [26] offered complex CFFSs as a groundbreaking tool for addressing uncertainty in decision-making. This model is designed to enhance the dependability and the accuracy of decisions, especially in situations where uncertainty plays a crucial role.

Definition 3: The CFFS $\boldsymbol{F}$ on a universal set X can be defined mathematically as [26]:

$$F=\left\{\left\langle x,\ell(x)e^{i2\pi d(x)},\hbar(x)e^{i2\pi f(x)}\right\rangle \mid x\in X\right\}$$

where, ℓ and $\hbar$ are called complex valued DoS and DoD respectively with conditions: $\ell^3+\hbar^3\leq 1$ and $\left(\frac{d}{2\pi}\right)^3+\left(\frac{f}{2\pi}\right)^3\leq 1, \forall x\in X$.

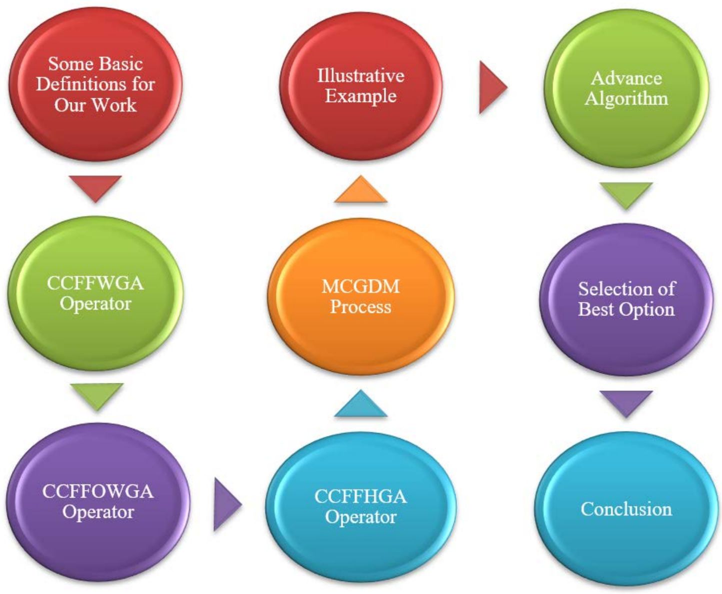

Figure 2. Structure procedure of the paper

3 Novel Approaches Using Complex Numbers

In the following section, we present three methods based on confidence level, such as the CCFFWGA operator, the CCFFOWGA operator, and the CCFFHGA operator, which provide a more reliable and flexible assessment.

Figure 3 presents the novel methods.

Definition 4: Let $(\phi_j,\eta_j)\,(1\leq j\leq n)$ be a group of CFFNs, and η be their confidence level with $\eta_j\in[0,1]$. Moreover $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$, be their weighted vector with condition: $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$. Then mathematically, the CCFFWGA can be defined as follows:

$$\begin{aligned}&\mathrm{CCFFWGA}_{\,\omega}\left(\left(\phi_1,\eta_1\right),\left(\phi_2,\eta_2\right),\ldots,\left(\phi_n,\eta_n\right)\right)\\&=\left(\begin{array}{c}\prod_{j=1}^{n}\left(\ell_j\right)^{\eta_j\omega_j}\exp\left(i2\pi\prod_{j=1}^{n}\left(\frac{d_j}{2\pi}\right)^{\eta_j\omega_j}\right),\\ \sqrt[3]{1-\prod_{j=1}^{n}\left(1-\hbar_j^3\right)^{\eta_j\omega_j}}\exp\left(i2\pi\left(\sqrt[3]{1-\prod_{j=1}^{n}\left(1-\left(\frac{f_j}{2\pi}\right)^3\right)^{\eta_j\omega_j}}\right)\right)\end{array}\right)\end{aligned}$$

Theorem 1: Let $(\phi_j,\eta_j)\,(1\leq j\leq 2)$ be a collection of two CFFNs, then:

I) $\phi_1\cup\phi_2=\phi_2\cup\phi_1$

II) $\phi_1\cap\phi_2=\phi_2\cap\phi_1$

III) $(\phi_1 \cup \phi_2) \cap \phi_1 = \phi_1$
IV) $(\phi_1 \cap \phi_2) \cup \phi_1 = \phi_1$

Proof: To prove (I) and (II), follow the outlined approach step by step. The remainder of the proof can be completed similarly.

I) As (ϕ_j, η_j) $(1 \leq j \leq 2)$ are two CFFNs, we have:

$$\begin{aligned}\phi_1 \cup \phi_2 &= \left(\max\{\ell_1, \ell_2\} \mathrm{e}^{i2\pi[\max\{d_1,d_2\}]}, \min\{\hbar_1, \hbar_2\} \mathrm{e}^{i2\pi[\min\{f_1,f_2\}]}\right) \\ &= \left(\max\{\ell_2, \ell_1\} \mathrm{e}^{i2\pi[\max\{d_2,d_1\}]}, \min\{\hbar_2, \hbar_1\} \mathrm{e}^{i2\pi[\min\{f_2,f_1\}]}\right) \\ &= \phi_2 \cup \phi_1\end{aligned}$$

II) Again, we have:

$$\begin{aligned}\phi_1 \cap \phi_2 &= \left(\min\{\ell_1, \ell_2\} \mathrm{e}^{i2\pi[\min\{d_1,d_2\}]}, \max\{\hbar_1, \hbar_2\} \mathrm{e}^{i2\pi[\max\{f_1,f_2\}]}\right) \\ &= \left(\min\{\ell_2, \ell_1\} \mathrm{e}^{i2\pi[\min\{d_2,d_1\}]}, \max\{\hbar_2, \hbar_1\} \mathrm{e}^{i2\pi[\max\{f_2,f_1\}]}\right) \\ &= \phi_2 \cap \phi_1\end{aligned}$$

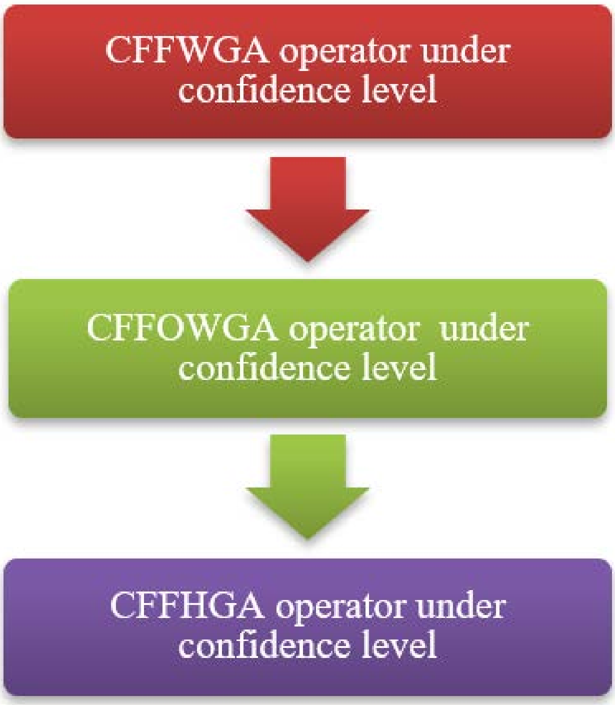

Figure 3. Order of the novel methods

Theorem 2: Let (ϕ_j, η_j) $(1 \leq j \leq n)$ be a family of CFFNs, and η be their confidence level with $\eta_j \in [0, 1]$. Let (ϕ^*, η^*) be another CFFN, η^* be their confidence level with $\eta_j^* \in [0, 1]$ under condition $\phi_j = \phi^*$, then the following condition holds:

$$\begin{aligned}\mathrm{CCFFWGA}_\omega\left((\phi_1, \eta_1), (\phi_2, \eta_2), \ldots, (\phi_n, \eta_n)\right) &= (\phi_1)^{\eta_1\omega_1} \otimes (\phi_2)^{\eta_2\omega_2} \otimes \ldots \otimes (\phi_n)^{\eta_n\omega_n} \\ &= (\phi^*, \eta^*)\end{aligned}$$

Proof: For proof using Definition 4, we have:

$$
\begin{aligned}
&\text{CCFFWGA}_{\omega}\left(\left(\phi_1,\eta_1\right),\left(\phi_2,\eta_2\right),\ldots,\left(\phi_n,\eta_n\right)\right)\\
&=\left(\prod_{j=1}^{n}\left(\ell_j\right)^{\eta_j\omega_j}\exp\left(i2\pi\prod_{j=1}^{n}\left(\frac{d_j}{2\pi}\right)^{\eta_j\omega_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\hbar_j^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}\exp i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{f_j}{2\pi}\right)^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}\right)\\
&=\left(\left(\ell^*\right)^{\eta^*}\sum_{j=1}^{n}\omega_j\exp i2\pi\left(\frac{d^*}{2\pi}\right)^{\eta^*\sum_{j=1}^{n}\omega_j},\left(1-\left(1-\left(\hbar^*\right)^3\right)^{\eta^*\sum_{j=1}^{n}\omega_j}\right)^{\frac{1}{3}}\exp i2\pi\left(1-\left(1-\left(\frac{f^*}{2\pi}\right)^3\right)^{\eta^*\sum_{j=1}^{n}\omega_j}\right)^{\frac{1}{3}}\right)\\
&=\left(\ell^*e^{i2\pi d^*},\hbar^*e^{i2\pi f^*}\right)=\left(\phi^*,\eta^*\right)
\end{aligned}
$$

Theorem 3: Let $(\phi_j,\eta_j)\,(1\le j\le n)$ be a family of CFFNs, where $\phi_{\max}=\left(\ell_{\max}e^{i2\pi d_{\max}},\hbar_{\max}e^{i2\pi f_{\max}}\right)$ and $\phi_{\min}=\left(\ell_{\min}e^{i2\pi d_{\min}},\hbar_{\min}e^{i2\pi f_{\min}}\right)$, then:

$$
\phi_{\min}\le\text{CCFFWGA}_{\omega}\left(\left(\phi_1,\eta_1\right),\left(\phi_2,\eta_2\right),\left(\phi_3,\eta_3\right),\ldots,\left(\phi_n,\eta_n\right)\right)\le\phi_{\max}
$$

Proof: Since $(\phi_j,\eta_j)\,(1\le j\le n)$, we have:

$$
\Leftrightarrow\left(\left(\min_j\{\ell_{\min}\}\right)^3\right)^{\frac{1}{3}}\le\left(\ell_j^3\right)^{\frac{1}{3}}\le\left(\left(\max_j\{\ell_{\max}\}\right)^3\right)^{\frac{1}{3}}
$$

$$
\Leftrightarrow\left(1-\left(\max_j\{\ell_{\max}\}\right)^3\right)^{\frac{1}{3}}\le\left(1-\ell_j^3\right)^{\frac{1}{3}}\le\left(1-\left(\min_j\{\ell_{\min}\}\right)^3\right)^{\frac{1}{3}}
$$

$$
\Leftrightarrow\left(\prod_{j=1}^{n}\left(1-\left(\max_j\{\ell_{\max}\}\right)^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}\le\left(\prod_{j=1}^{n}\left(1-\ell_j^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}\le\left(\prod_{j=1}^{n}\left(1-\left(\min_j\{\ell_{\min}\}\right)^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}
$$

$$
\min_j\{\ell_{\min}\}^{\eta_j}\le\left(1-\prod_{j=1}^{n}\left(1-\ell_j^3\right)^{\eta_j\omega_j}\right)^{\frac{1}{3}}\le\max_j\{\ell_{\max}\}^{\eta_j}
$$

Similarly, we have to prove $\min_j\{i2\pi d_j\}\le i2\pi d_j\le\max_j\{i2\pi d_j\}$.

Next, we have to prove the non-membership function:

$$
\Leftrightarrow\min_j\{\hbar_j\}\le\hbar_j\le\max_j\{\hbar_j\}
$$

$$
\Leftrightarrow\prod_{j=1}^{n}\left(\min_j\{\hbar_j\}\right)^{\eta_j\omega_j}\le\prod_{j=1}^{n}\left(\hbar_j\right)^{\eta_j\omega_j}\le\prod_{j=1}^{n}\left(\max_j\{\hbar_j\}\right)^{\eta_j\omega_j}
$$

$$
\Leftrightarrow\left(\min_j\{\hbar_j\}\right)^{\eta_j\sum_{j=1}^{n}\omega_j}\le\amalg_{j=1}^{n}\left(\hbar_j\right)^{\eta_j\omega_j}\le\left(\max_j\{\hbar_j\}\right)^{\eta_j\sum_{j=1}^{n}\omega_j}
$$

$$
\Leftrightarrow\min_j\{\hbar_j\}^{\eta_j}\le\amalg_{j=1}^{n}\left(T_j\right)^{\eta_j\epsilon_j}\le\max_j\{\hbar_j\}^{\eta_j}
$$

Based on the similar approach, we have: $\min_j\{i2\pi f_j\}\le x_j\le\max_j\{i2\pi f_j\}$. Thus, from the above inequalities, we have: $\phi_{\min}\le\text{CCFFWGA}_{\omega}\left(\left(\phi_1,\eta_1\right),\left(\phi_2,\eta_2\right),\left(\phi_3,\eta_3\right),\ldots,\left(\phi_n,\eta_n\right)\right)\le\phi_{\max}$.

Thus, the proof is completed.

Definition 5: Let $(\phi_j,\eta_j)\,(1\le j\le n)$ be a family of CFFNs, where $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$ is their weighted vector, with condition: $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$. Moreover, let η be their confidence level with $\eta_j\in[0,1]$ and $(æ(1),æ(2),\ldots,æ(n))$ be any permutation of $(1,2,\ldots,n)$ under condition: $\phi_{æ(j-1)}\ge\phi_{æ(j)}$ for all j. Then mathematically, the CCFFOWGA can be defined as follows:

$$
\begin{aligned}
&\text{CCFFOWGA}_{\omega}\left(\left(\phi_1,\eta_1\right),\left(\phi_2,\eta_2\right),\ldots,\left(\phi_n,\eta_n\right)\right)\\
&=\left(\begin{array}{c}\prod_{j=1}^{n}\left(\ell_{æ(j)}\right)^{\eta_j\omega_j}\exp\left(i2\pi\prod_{j=1}^{n}\left(\frac{d_{æ(j)}}{2\pi}\right)^{\eta_j\omega_j}\right),\\ \sqrt[3]{1-\prod_{j=1}^{n}\left(1-\hbar_{æ(j)}^3\right)^{\eta_j\omega_j}}\ \text{expi}2\pi\left(\sqrt[3]{1-\prod_{j=1}^{n}\left(1-\left(\frac{f_{æ(j)}}{2\pi}\right)^3\right)^{\eta_j\omega_j}}\right)\end{array}\right)
\end{aligned}
$$

The complex Fermatean fuzzy hybrid geometric aggregation (CCFFHGA) operator is a unified framework that blends the principles of weighted geometric and ordered weighted geometric approaches within the context of complex Fermatean fuzzy operations. By integrating these methods, the CCFFHGA offers enhanced flexibility and adaptability, enabling it to handle diverse scenarios and preferences in aggregating complex Fermatean fuzzy information. This hybrid operator captures the strengths of both approaches, allowing for a more comprehensive and robust aggregation process that leverages geometric principles alongside weighted and ordered considerations.

Definition 6: Let $(\phi_j, \eta_j)\,(1 \leq j \leq n)$ be a finite group of CFFNs, where associated and weighted vectors are presented by $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$, respectively, and both vectors satisfy the conditions of belonging to the closed interval $[0,1]$ and having a sum equal to one. Furthermore, $\dot{\phi}_{æ}(j) = (\phi_j)^{n\varphi_j}$, if $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$, then $\left((\phi_1)^{n\varphi_1}, (\phi_2)^{n\varphi_2}, \ldots, (\phi_n)^{n\varphi_n}\right)$ approaches to $(\phi_1, \phi_2, \ldots, \phi_n)$. Moreover, let η be their confidence level with $\eta_j \in [0,1]$ and $(æ(1), æ(2), \ldots, æ(n))$ be any reordering of $(1, 2, \ldots, n)$ under condition: $\phi_{æ(j-1)} \geq \phi_{æ(j)}$ for all j. Then the CCFFHGAO can be presented as follows:

$$\begin{aligned}
&\mathrm{CCFFOWGA}_{\,\omega}\left((\phi_1, \eta_1), (\phi_2, \eta_2), \ldots, (\phi_n, \eta_n)\right)\\
&= \left(\begin{array}{c} \prod_{j=1}^{n} \left(\ell_{æ(j)}\right)^{\eta_j \omega_j} \exp\left(i2\pi \prod_{j=1}^{n} \left(\frac{d_{æ(j)}}{2\pi}\right)^{\eta_j \omega_j}\right), \\ \sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - \hbar^3_{æ(j)}\right)^{\eta_j \omega_j}} \exp i2\pi \left(\sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - \left(\frac{f_{æ(j)}}{2\pi}\right)^3\right)^{\eta_j \omega_j}}\right) \end{array} \right)
\end{aligned}$$

4 DMP Using CFF-Aggregation Operators

The DMP involves identifying a problem or opportunity, gathering and analyzing relevant information, evaluating possible options, and selecting the most appropriate course of action. It requires careful consideration of potential outcomes, risks, and benefits to ensure that the chosen decision aligns with goals and values.

In a decision-making scenario involving a finite group of k decision-makers, denoted by $D = \{D_1, D_2, \ldots, D_k\}$, each member of the group holds a certain influence or importance, represented by a weighted vector $f = (f_1, f_2, \ldots, f_k)$. The experts also rate their familiarity with the options being evaluated and assign confidence levels $\eta_{ij} = 0 \leq \eta_{ij} \leq 1$ to reflect how certain they are about their assessments. This approach ensures that their confidence in their judgments is taken into account. This group is tasked with evaluating a set of alternatives $\Im = \{\Im_1, \Im_2, \ldots, \Im_m\}$, each is assessed under a specific set of criteria $\Re = \{\Re_1, \Re_2, \ldots, \Re_n\}$; where each criterion is assigned a weight $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. The collective decision process requires synthesizing the preferences of each expert, taking into account the weighted implication of both the experts and the alternatives. This structure ensures that the group's DMP is systematic and balanced, integrating diverse perspectives and prioritizing the most critical factors in evaluating the alternatives.

Step 1: In this first step, we included all data of decision-makers with Fermatean fuzzy matrices:

The decision matrix $D^k = \left(D^k_{ij}, \eta^k_{ij}\right)_{m\times n}$ is constructed where η^k_{ij} represents the complex Fermatean fuzzy rating for alternative $\Im_i (i = 1, 2, \ldots, m)$ under criterion $\Re_j (j = 1, 2, \ldots, n)$ provided by expert κ is the associated level of confidence. This process is repeated for each expert, resulting in all such matrices.

$$D = \begin{array}{c} \\ \Im_1 \\ \Im_2 \\ \vdots \\ \Im_m \end{array} \begin{array}{c} \begin{array}{ccccc} \Re_1 & \Re_2 & \Re_3 & \cdots & \Re_n \end{array} \\ \begin{bmatrix} (\phi^k_{11}, \eta^k_{11}) & (\phi^k_{12}, \eta^k_{12}) & (\phi^k_{13}, \eta^k_{13}) & \cdots & (\phi^k_{1n}, \eta^k_{1n}) \\ (\phi^k_{21}, \eta^k_{21}) & (\phi^k_{22}, \eta^k_{22}) & (\phi^k_{23}, \eta^k_{23}) & \cdots & (\phi^k_{2n}, \eta^k_{2n}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (\phi^k_{m1}, \eta^k_{m1}) & (\phi^k_{m2}, \eta^k_{m2}) & (\phi^k_{m3}, \eta^k_{m3}) & \cdots & (\phi^k_{mn}, \eta^k_{mn}) \end{bmatrix} \end{array}$$

Step 2: In this step, convert all cost type criteria into normalized decision matrices:

When dealing with decision matrices that involve both benefit and cost criteria, it's often useful to convert cost criteria into benefit criteria to facilitate a more straightforward comparison. This is typically achieved by normalizing the decision matrices. To convert a decision matrix $D^k = \left(\ell^k_{ij}, \hbar^k_{ij}\right)_{m\times n}$ into a normalized decision matrix $R^k = \left(\hbar^k_{ij}, \ell^k_{ij}\right)_{m\times n}$, follow these steps:

• Identify cost and benefit criteria: Determine which criteria are categorized as benefits and which are categorized as costs.

• Normalization of cost criteria: For each cost criterion, invert the values such that lower costs are converted into higher benefits.

Step 3: Combining individual matrices into a single matrix:

To combine all individual matrices into a single matrix as per Definition 4: This process involves aligning the matrices in a manner that preserves their individual structures while forming a cohesive, larger matrix.

$$D = \begin{array}{c} \\ \Im_1 \\ \Im_2 \\ \vdots \\ \Im_m \end{array} \begin{array}{c} \begin{array}{ccccc} \Re_1 & \Re_2 & \Re_3 & \cdots & \Re_n \end{array} \\ \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \phi_{23} & \cdots & \phi_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{m1} & \phi_{m2} & \phi_{m3} & \cdots & \phi_{mn} \end{bmatrix} \end{array}$$

Step 4: Calculating all preference values.
Step 5: Calculating the score function:
To calculate the score function using the preference values $\aleph_i (i = 1, 2, \ldots, m)$.
Step 6: Ranking alternatives based on score function:
By ranking the scores and selecting the highest value, the most optimal choice is determined. This method ensures that the best option, based on the given criteria, is accurately identified.

5 Illustrative Example

Suppose in a hospital, the manager of the hospital wants to select a new information system for the purpose of the best productivity. To achieve this, the manager has made a committee of three experts $\{D_1, D_2, D_3\}$ with weights of $f = (0.4, 0.3, 0.3)$. This committee is tasked with evaluating and selecting the most suitable system for the hospital from the available systems. In the first selection, the committee considered only four systems as an alternative $\{\Im_1, \Im_2, \Im_3, \Im_4\}$ based on four specific criteria, $\{\Re_1, \Re_2, \Re_3, \Re_4\}$, with $\omega = (0.1, 0.4, 0.3, 0.2)$.

Optimizing costs of software and hardware investments ($\Re_1$): Optimizing costs of software and hardware investments involves carefully assessing business needs, leveraging scalable cloud solutions, and prioritizing open-source or subscription-based software to reduce upfront expenses. Additionally, implementing energy-efficient hardware and regularly auditing resource utilization can minimize ongoing operational costs.

Table 1. Decision of the 1st expert

	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$
$\Im_1$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.58e^{i2\pi(0.50)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.67)} \\ 0.61e^{i2\pi(0.50)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.69e^{i2\pi(0.63)} \\ 0.51e^{i2\pi(0.49)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.6 \right\rangle$
$\Im_2$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.6 \right\rangle$	$\left\langle \begin{pmatrix} 0.68e^{i2\pi(0.53)} \\ 0.58e^{i2\pi(0.46)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.67)} \\ 0.61e^{i2\pi(0.50)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.60e^{i2\pi(0.53)} \\ 0.72e^{i2\pi(0.46)} \end{pmatrix}, 0.7 \right\rangle$
$\Im_3$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.53e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.63)} \\ 0.57e^{i2\pi(0.49)} \end{pmatrix}, 0.4 \right\rangle$
$\Im_4$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.63)} \\ 0.48e^{i2\pi(0.49)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.64e^{i2\pi(0.53)} \\ 0.53e^{i2\pi(0.46)} \end{pmatrix}, 0.5 \right\rangle$

Table 2. Decision of the 2nd expert

	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$
$\Im_1$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.67)} \\ 0.61e^{i2\pi(0.50)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.63)} \\ 0.48e^{i2\pi(0.49)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.6 \right\rangle$
$\Im_2$	$\left\langle \begin{pmatrix} 0.60e^{i2\pi(0.53)} \\ 0.72e^{i2\pi(0.46)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.58e^{i2\pi(0.50)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.72e^{i2\pi(0.53)} \\ 0.60e^{i2\pi(0.46)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle \begin{pmatrix} 0.53e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.6 \right\rangle$
$\Im_3$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.62e^{i2\pi(0.67)} \\ 0.61e^{i2\pi(0.50)} \end{pmatrix}, 0.61 \right\rangle$	$\left\langle \begin{pmatrix} 0.68e^{i2\pi(0.53)} \\ 0.58e^{i2\pi(0.46)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.9 \right\rangle$
$\Im_4$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.1 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.6 \right\rangle$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.7 \right\rangle$

Table 3. Decision of the 3rd expert

	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$
$\Im_1$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.63)} \\ 0.57e^{i2\pi(0.49)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.64e^{i2\pi(0.53)} \\ 0.53e^{i2\pi(0.46)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.7 \right\rangle$
$\Im_2$	$\left\langle \begin{pmatrix} 0.64e^{i2\pi(0.53)} \\ 0.53e^{i2\pi(0.46)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.66e^{i2\pi(0.67)} \\ 0.61e^{i2\pi(0.50)} \end{pmatrix}, 0.6 \right\rangle$	$\left\langle \begin{pmatrix} 0.67e^{i2\pi(0.63)} \\ 0.56e^{i2\pi(0.49)} \end{pmatrix}, 0.7 \right\rangle$
$\Im_3$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.67)} \\ 0.58e^{i2\pi(0.50)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.6 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.63)} \\ 0.57e^{i2\pi(0.49)} \end{pmatrix}, 0.8 \right\rangle$
$\Im_4$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.45)} \\ 0.63e^{i2\pi(0.51)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.60e^{i2\pi(0.53)} \\ 0.72e^{i2\pi(0.46)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle \begin{pmatrix} 0.72e^{i2\pi(0.53)} \\ 0.60e^{i2\pi(0.46)} \end{pmatrix}, 0.7 \right\rangle$

Hospital support services ($\Re_2$): Hospital support services are essential non-clinical functions that ensure the smooth operation of healthcare facilities. These services include housekeeping, food service, maintenance, security, and patient transportation, all of which contribute to a safe, clean, and comfortable environment for patients, staff, and visitors.

Transforming current systems for a better future ($\mathfrak{R}_3$): Transforming current systems for a better future requires innovative thinking and collaborative effort. By embracing sustainability, inclusivity, and technological advancements, ensuring reliability in outsourced software development ($\mathfrak{R}_4$): Ensuring reliability in outsourced software development involves clear communication, thorough vetting of partners, and well-defined contracts that outline expectations, deliverables, and timelines.

When dealing with DM models where attributes are categorized as either cost or benefit criteria, it's important to standardize them for accurate comparisons. Cost-type criteria ($\mathfrak{R}_1$ and $\mathfrak{R}_3$) are usually undesirable, so they should be converted into benefit-type criteria (like $\mathfrak{R}_2$ and $\mathfrak{R}_4$) to ensure consistency. This conversion can be done by inverting the cost criteria, such as by taking the reciprocal or subtracting them from a constant, thus allowing all criteria to be maximized, facilitating a uniform assessment.

Step 1: To provide a concise summary, please share the details or content of Tables 1, 2, and 3. This will help me understand the decision-makers' decisions and write an accurate response.

Step 2: The criteria are two types, such as cost type and benefit type criteria. So, convert decision matrices $D^k = \left(\ell_{ij}^k, \hbar_{ij}^k\right)_{m\times n}$ into normalized decision matrices $R^k = \left(\hbar_{ij}^k, \ell_{ij}^k\right)_{m\times n}$. Here, $\mathfrak{R}_1$ and $\mathfrak{R}_3$ are cost types criteria, shown in Tables 4, 5 and 6:

Table 4. Normalized decision-matrix of the 1st expert

	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\mathfrak{I}_1$	$\left\langle \binom{0.58e^{i2\pi(0.50)}}{0.63e^{i2\pi(0.67)}}, 0.4 \right\rangle$	$\left\langle \binom{0.62e^{i2\pi(0.67)}}{0.61e^{i2\pi(0.50)}}, 0.3 \right\rangle$	$\left\{ \binom{0.51e^{i2\pi(0.49)}}{0.69e^{i2\pi(0.63)}}, 0.7 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.6 \right\rangle$
$\mathfrak{I}_2$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.6 \right\rangle$	$\left\langle \binom{0.68e^{i2\pi(0.53)}}{0.58e^{i2\pi(0.46)}}, 0.5 \right\rangle$	$\left\langle \binom{0.61e^{i2\pi(0.50)}}{0.62e^{i2\pi(0.67)}}, 0.9 \right\rangle$	$\left\langle \binom{0.60e^{i2\pi(0.53)}}{0.72e^{i2\pi(0.46)}}, 0.7 \right\rangle$
$\mathfrak{I}_3$	$\left\langle \binom{0.56e^{i2\pi(0.49)}}{0.67e^{i2\pi(0.63)}}, 0.4 \right\rangle$	$\left\langle \binom{0.53e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.5 \right\rangle$	$\left\langle \binom{0.56e^{i2\pi(0.49)}}{0.67e^{i2\pi(0.63)}}, 0.7 \right\rangle$	$\left\langle \binom{0.58e^{i2\pi(0.63)}}{0.57e^{i2\pi(0.49)}}, 0.4 \right\rangle$
$\mathfrak{I}_4$	$\left\langle \binom{0.48e^{i2\pi(0.49)}}{0.62e^{i2\pi(0.63)}}, 0.8 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.67)}}{0.56e^{i2\pi(0.49)}}, 0.3 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.8 \right\rangle$	$\left\langle \binom{0.64e^{i2\pi(0.53)}}{0.53e^{i2\pi(0.46)}}, 0.5 \right\rangle$

Table 5. Normalized decision-matrix of the 2nd expert

	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\mathfrak{I}_1$	$\left\langle \binom{0.61e^{i2\pi(0.50)}}{0.62e^{i2\pi(0.67)}}, 0.2 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.67)}}{0.56e^{i2\pi(0.49)}}, 0.8 \right\rangle$	$\left\{ \binom{0.48e^{i2\pi(0.49)}}{0.62e^{i2\pi(0.63)}}, 0.4 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.6 \right\rangle$
$\mathfrak{I}_2$	$\left\langle \binom{0.72e^{i2\pi(0.46)}}{0.60e^{i2\pi(0.53)}}, 0.7 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.67)}}{0.58e^{i2\pi(0.50)}}, 0.8 \right\rangle$	$\left\langle \binom{0.60e^{i2\pi(0.46)}}{0.72e^{i2\pi(0.53)}}, 0.7 \right\rangle$	$\left\langle \binom{0.53e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.6 \right\rangle$
$\mathfrak{I}_3$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.3 \right\rangle$	$\left\langle \binom{0.62e^{i2\pi(0.67)}}{0.61e^{i2\pi(0.50)}}, 0.61 \right\rangle$	$\left\langle \binom{0.58e^{i2\pi(0.46)}}{0.68e^{i2\pi(0.53)}}, 0.5 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.9 \right\rangle$
$\mathfrak{I}_4$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.1 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.67)}}{0.56e^{i2\pi(0.49)}}, 0.5 \right\rangle$	$\left\langle \binom{0.56e^{i2\pi(0.49)}}{0.67e^{i2\pi(0.63)}}, 0.6 \right\rangle$	$\left\langle \binom{0.67e^{i2\pi(0.63)}}{0.56e^{i2\pi(0.49)}}, 0.7 \right\rangle$

Table 6. Normalized decision-matrix of the 3rd expert

	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\mathfrak{I}_1$	$\left\langle \binom{0.57e^{i2\pi(0.49)}}{0.58e^{i2\pi(0.63)}}, 0.8 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.67)}}{0.56e^{i2\pi(0.49)}}, 0.3 \right\rangle$	$\left\langle \binom{0.53e^{i2\pi(0.46)}}{0.64e^{i2\pi(0.53)}}, 0.5 \right\rangle$	$\left\langle \binom{0.67e^{i2\pi(0.63)}}{0.56e^{i2\pi(0.49)}}, 0.7 \right\rangle$
$\mathfrak{I}_2$	$\left\langle \binom{0.53e^{i2\pi(0.46)}}{0.64e^{i2\pi(0.53)}}, 0.3 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.9 \right\rangle$	$\left\langle \binom{0.61e^{i2\pi(0.50)}}{0.66e^{i2\pi(0.67)}}, 0.6 \right\rangle$	$\left\langle \binom{0.67e^{i2\pi(0.63)}}{0.56e^{i2\pi(0.49)}}, 0.7 \right\rangle$
$\mathfrak{I}_3$	$\left\langle \binom{0.58e^{i2\pi(0.50)}}{0.63e^{i2\pi(0.67)}}, 0.3 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.6 \right\rangle$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.4 \right\rangle$	$\left\langle \binom{0.58e^{i2\pi(0.63)}}{0.57e^{i2\pi(0.49)}}, 0.8 \right\rangle$
$\mathfrak{I}_4$	$\left\langle \binom{0.63e^{i2\pi(0.51)}}{0.49e^{i2\pi(0.45)}}, 0.9 \right\rangle$	$\left\langle \binom{0.49e^{i2\pi(0.45)}}{0.63e^{i2\pi(0.51)}}, 0.3 \right\rangle$	$\left\langle \binom{0.72e^{i2\pi(0.46)}}{0.60e^{i2\pi(0.53)}}, 0.2 \right\rangle$	$\left\langle \binom{0.72e^{i2\pi(0.53)}}{0.60e^{i2\pi(0.46)}}, 0.7 \right\rangle$

Step 3: To combine all individual matrices into a single matrix by using the following formula:

$$\begin{aligned}
&\mathrm{CCFFWGA}_{\omega}\left(\left(\phi_1, \eta_1\right), \left(\phi_2, \eta_2\right), \ldots, \left(\phi_n, \eta_n\right)\right) \\
&= \left(\prod_{j=1}^{n} \left(\ell_j\right)^{\eta_j \omega_j} e^{i2\pi \prod_{j=1}^{n} \left(\frac{d_j}{2\pi}\right)^{\eta_j \omega_j}}, \sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - \hbar_j^3\right)^{\eta_j \omega_j}} e^{i2\pi \left(\sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - \left(\frac{f_j}{2\pi}\right)^3\right)^{\eta_j \omega_j}} \right)} \right)
\end{aligned}$$

where, $f = (0.4, 0.3, 0.3)$ represent their weights, and Table 7 can be attained. This process involved applying the specified weights to various elements.

Table 7. Resulting matrix of all experts

	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$
$\Im_1$	$\begin{pmatrix}0.66e^{i2\pi(0.70)}\\0.72e^{i2\pi(0.75)}\end{pmatrix}$	$\begin{pmatrix}0.70e^{i2\pi(0.77)}\\0.71e^{i2\pi(0.70)}\end{pmatrix}$	$\begin{pmatrix}0.75e^{i2\pi(0.57)}\\0.56e^{i2\pi(0.76)}\end{pmatrix}$	$\begin{pmatrix}0.67e^{i2\pi(0.77)}\\0.73e^{i2\pi(0.70)}\end{pmatrix}$
$\Im_2$	$\begin{pmatrix}0.72e^{i2\pi(0.59)}\\0.77e^{i2\pi(0.86)}\end{pmatrix}$	$\begin{pmatrix}0.76e^{i2\pi(0.74)}\\0.71e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.82e^{i2\pi(0.66)}\\0.66e^{i2\pi(0.73)}\end{pmatrix}$	$\begin{pmatrix}0.62e^{i2\pi(0.69)}\\0.80e^{i2\pi(0.68)}\end{pmatrix}$
$\Im_3$	$\begin{pmatrix}0.71e^{i2\pi(0.80)}\\0.60e^{i2\pi(0.70)}\end{pmatrix}$	$\begin{pmatrix}0.80e^{i2\pi(0.60)}\\0.70e^{i2\pi(0.86)}\end{pmatrix}$	$\begin{pmatrix}0.83e^{i2\pi(0.55)}\\0.61e^{i2\pi(0.81)}\end{pmatrix}$	$\begin{pmatrix}0.75e^{i2\pi(0.67)}\\0.77e^{i2\pi(0.88)}\end{pmatrix}$
$\Im_4$	$\begin{pmatrix}0.67e^{i2\pi(0.71)}\\0.64e^{i2\pi(0.81)}\end{pmatrix}$	$\begin{pmatrix}0.83e^{i2\pi(0.80)}\\0.51e^{i2\pi(0.77)}\end{pmatrix}$	$\begin{pmatrix}0.75e^{i2\pi(0.42)}\\0.70e^{i2\pi(0.85)}\end{pmatrix}$	$\begin{pmatrix}0.74e^{i2\pi(0.59)}\\0.76e^{i2\pi(0.70)}\end{pmatrix}$

Step 4: Again, by using the CCFFWGA with weights $w = (0.1, 0.4, 0.3, 0.2)$, we need to aggregate the preferences based on their cumulative fuzzy weights.

$$\partial_1 = \left(0.67e^{i2\pi(0.66)}, 0.58e^{i2\pi(0.61)}\right), \; \partial_2 = \left(0.77e^{i2\pi(0.67)}, 0.63e^{i2\pi(0.55)}\right)$$
$$\partial_3 = \left(0.59e^{i2\pi(0.72)}, 0.48e^{i2\pi(0.65)}\right), \; \partial_4 = \left(0.64e^{i2\pi(0.68)}, 0.53e^{i2\pi(0.60)}\right)$$

Step 5: Calculate the score of all values using the formula: $scor(\phi) = \left(\ell^3 - \hbar^3\right) + \frac{1}{8\pi^3}\left(d^3 - f^3\right)$. $scor\left(\partial_1\right) = 0.16$, $scor\left(\partial_2\right) = 0.34$, $scor\left(\partial_3\right) = 0.19$, $scor\left(\partial_4\right) = 0.21$.

Step 6: Thus, the more suitable information system for hospital is $\Im_2$.

The following Table 8 and Table 9 shows the score function of all novel methods and the comparison of the proposed model with some existing fuzzy models, respectively. Figure 4 shows the ranking of all methods.

Table 8. Score functions of all novel methods

	CCFFWGA	**CCFFOWGA**	**CCFFHGA**
$\Im_1$	0.16	0.22	0.21
$\Im_2$	0.34	0.37	0.41
$\Im_3$	0.19	0.26	0.32
$\Im_4$	0.21	0.33	0.35

Table 9. Score functions of all novel methods

	Model	**Vagueness**	**Falsity**	**Indeterminacy**	**Periodicity**	**2-D Information**	**Power in Cube**
	F-set	Yes	No	No	No	No	No
	IF-set	Yes	Yes	Yes	No	No	No
	PyF-set	Yes	Yes	Yes	No	No	No
:	FF-set	Yes	Yes	Yes	No	No	No
	CF-set	Yes	No	No	Yes	Yes	No
	CIF-set	Yes	Yes	Yes	Yes	Yes	No
	CPyF-set	Yes	Yes	Yes	Yes	Yes	No
	CFF-set	Yes	Yes	Yes	Yes	Yes	Yes

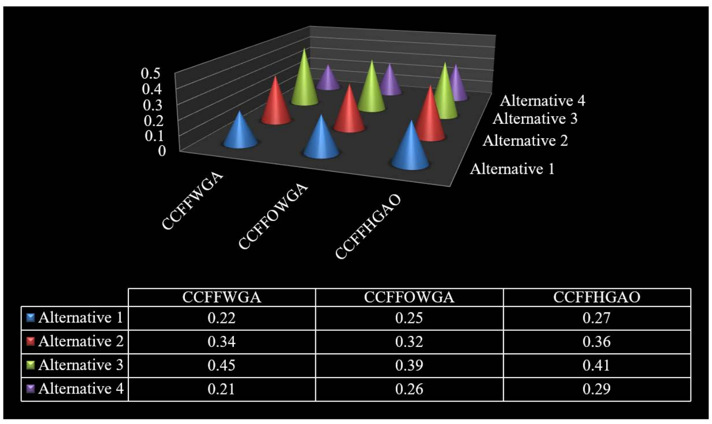

Figure 4. Ranking of all methods

6 Conclusion

In this paper, we introduced the concept of the CFFS, an advanced extension of CFS, CIFS, and CPyFS. Unlike its predecessors, the CFFS offers broader applicability. It can handle not only CFF-data but also can be adapted to IF-data, PyF-data, and FF-data by setting the phase terms to zero. This versatility makes the proposed model particularly beneficial for modeling and controlling dynamic systems with variable or uncertain parameters, offering more elasticity and flexibility than previous models. Additionally, we developed several operators, including the CCFFWGA, the CCFFOWGA, and the CCFFHGA. An illustrative example concerning the selection of the most suitable information system for a hospital demonstrates the model's effectiveness and efficiency. The proposed approach has broad potential applications across various domains, including signal processing, control systems, quantum computing, decision support systems, image processing, and fields such as artificial intelligence, machine learning, and pattern recognition, where handling uncertainty and expressiveness are critical.

In the future, we aim to expand the use of CFF models by applying them to RWP like medical diagnosis, pattern recognition, machine learning, and detecting brain hemorrhages. Through advancements in fuzzy logic, these models can be customized for different environments, improving their adaptability and performance. This will lead to more accurate and reliable decision-making in various fields. Ultimately, our goal is to enhance the effectiveness of complex Fermatean fuzzy models, enabling them to address complex challenges across a broad range of applications.

Data Availability

The data used to support the findings of this study are included in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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