



Enhancement of the Defining Interrelationships Between Ranked Criteria II Method Using Interval Grey Numbers for Application in the Grey-Rough MCDM Model

Duško Tešić^{1*}, Darko Božanić¹, Mohammad Khalilzadeh²

¹ Military Academy, University of Defence, 11042 Belgrade, Serbia

² CENTRUM Católica Graduate Business School, Pontificia Universidad Católica del Perú, 15001 Lima, Peru

* Correspondence: Duško Tešić (dusko.tesic@va.mod.gov.rs)

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Abstract: Multi-Criteria Decision-Making (MCDM) represents a critical area of research, particularly in artificial intelligence, through the modeling of real-world decision-making scenarios. Numerous methods have been developed to address the challenges of integrating non-quantitative, incomplete, and imprecise information under conditions of uncertainty. This paper presents the enhancement of the Defining Interrelationships Between Ranked Criteria II (DIBR II) method by incorporating interval grey numbers, in accordance with the principles of Grey theory, its arithmetic operations, and the DIBR II methodology. The enhancement includes the introduction of a conviction degree to reflect decision-makers' or experts' confidence in their assertions. The application of this enhanced method is demonstrated through an illustrative example, following the procedural steps. Additionally, its efficacy is validated in a real-world scenario involving the selection of Lean organization system management techniques, utilizing the Rough Multi-Attributive Border Approximation Area Comparison (Rough MABAC) method. The results indicate that the enhanced DIBR II method is effective in determining criteria weight coefficients, offering a more nuanced distribution compared to traditional crisp methods. Furthermore, when implemented in a multi-criteria model, it yields a more refined ranking of alternatives, contingent on the degree of confidence in the given claims.

Keywords: Multi-Criteria Decision-Making (MCDM); Defining Interrelationships Between Ranked Criteria II (DIBR II); Grey theory; Criteria weights; Rough Multi-Attributive Border Approximation Area Comparison (Rough MABAC); Lean concept; Selection

1 Introduction

Every day in their lives, people make a series of decisions, both personal and professional. All those decisions are based on certain experiences, knowledge, needs, etc. In order for the decision to be of high quality, it is necessary to look at all the aspects on which the decision depends. MCDM enables us to solve decision-making problems that need to be viewed from different aspects (criteria). For such situations, many MCDM methods have been developed, which have been applied in various fields: logistic [1–5], military [6–9], medicine [10–13], education [14–17], industry [18–22], agriculture [23–26], etc.

Considering the concept of this way of decision-making, to reach a final decision and choose the optimal solution to the problem (alternative), it is first necessary to define the weight coefficients of the criteria, because each of them affects the decision in a certain way and with a certain significance. The stated significance needs to be calculated, which is done with one of the MCDM methods that are either only intended for the calculation of criteria weights or both for the calculation of weights and for the selection of the optimal alternative [27–35]. One of the methods that is intended exclusively for the calculation of criteria weights is the method DIBR II [36]. The DIBR II method was published in 2023, but even though it is a young method, it has so far found its application in various researches. When presenting the method, its application is shown in the example of choosing a car supplier and evaluating the effectiveness of social media [36]. After that, the method was used to determine the weights of the criteria in the problem of choosing a complex combat system [37], for the assault boat selection [38], for the selection of the optimal set of pontoon bridges [39], for the choice of technique in the management of lean organizational systems [40], etc.

To consider information that is imprecise, incomplete and consider uncertainties when determining the weight of criteria, it is necessary to implement theories that treat these areas well, such as fuzzy theory, rough theory, grey system theory and others, into existing methods. The need to do this arises from the fact that classical methods do not take such information into account.

The DIBR II method has so far been improved by means of fuzzy theory and by using triangular [41] and spherical fuzzy numbers [42]. Given that fuzzy theory can be complex due to the need to define fuzzy sets, rules, and membership functions, that is, it can require a lot of time and resources, that it requires a large amount of data and can be less flexible in dynamic systems [43–45], and that there is a real need for simplicity and efficiency, for processing small amounts of incomplete information and more flexible use in situations where there are changes in real time, and for gray theory to treat such data and situations well [46–48], the implementation of gray theory in the DIBR II method was carried out in the paper. So far, the method has not been improved using this theory. The integration was performed in accordance with the algorithm shown in Figure 1.

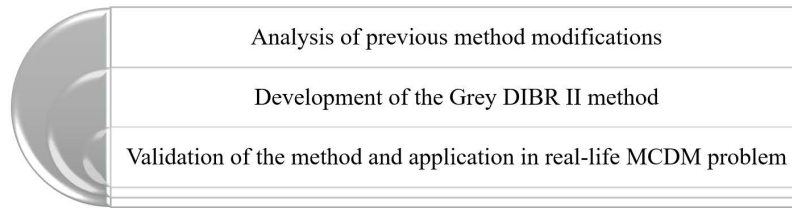


Figure 1. Development algorithm of the Grey DIBR II method

Note: This figure was prepared by the authors

Many MCDM methods have so far been improved using grey theory [49–54]. In accordance with the algorithm (Figure 1), first an analysis of the application of the method in different areas was performed, and then its modifications with theories that treat uncertainty and imprecise and incomplete data well. Because the grey theory method has not been improved so far, the need for the same has arisen. In the continuation of the paper, using interval grey numbers and their arithmetic operations, the grey DIBR II method was developed, and its application was shown on an illustrative example, as well as its application to a real-life problem from existing research, with the aim of validating the method and comparative analysis of results.

2 Methodology

In this part of the paper, the developed grey DIBR II method is presented, with all steps and sub-steps, as well as a numerical illustration of the application of the method. In addition to the above, the degree of confidence in the claims was used to form the interval number, which is described in detail.

2.1 Grey DIBR II Method

The application of grey theory is suitable in situations where there is incomplete information and a small sample [55, 56]. According to Liu et al. [57], the basic concept in grey systems is interval grey numbers. Interval grey number ($\otimes X$) is presented as $\otimes X = [\underline{X}, \bar{X}]$, where \underline{X} and \bar{X} are the lower and upper limits of grey number $\otimes X$, and $\underline{X} < \bar{X}$ [57, 58]. Given that experts are not absolutely sure of their claims when expressing their position on certain problems, for the purposes of this research, the level of confidence (δ) has been used to form the limits of the interval grey number [41, 59]:

$$\otimes X = (\underline{X}, \bar{X}) = \begin{cases} \underline{X} = \delta X, 0 < \delta \leq 1, X \geq 0 \\ \bar{X} = X - ((1 - \delta)\underline{X}) \\ \bar{X} > \underline{X} \end{cases} \quad (1)$$

where, X represents the crisp number, that is, in this case, the value of the comparison of criteria given by the expert or Decision maker (DM). If the DM is confident in his claim 100%, then the value δ is 1.0, if he is 90% sure, $\delta = 0.9$, etc. For example, if the expert compares criterion C_1 with criterion C_2 and states that the comparison value is 1.5 and if he is 90% sure of that statement, the interval number is formed by the following, expression (1): $\underline{X} = \delta X = 0.9 \times 1.5 = 1.35$ and $\bar{X} = X - ((1 - \delta)\underline{X}) = 1.5 - ((1 - 0.9) \times 1.35 = 1.365$, that is,

$\otimes X = (\underline{X}, \bar{X}) = (1.35, 1.365)$. By applying expression (1), an interval number is formed whose crisp value depends on the degree of confidence. Specifically, by reducing the level of confidence of the DM in the given

statement, the crisp value of the comparison decreases, while if the level of confidence is 1.0, the crisp value remains unchanged. All these changes in the crisp value reflect the actual opinion of the DM. The comparison values are reduced to a crisp value of 1, which in this method indicates that the two criteria are equally important. If the criteria are equally important, it means that all criteria have the same weight coefficient, that is, they have an equal influence on the final decision. Or, if the DM is absolutely unsure of his claims, then all criteria have equal weight. Also, during other empirical research by the authors [41, 60], in which the level of conviction was used, it was established that experts who are sure of their claims $\delta \leq 0.5$, have a very low coefficient of competence, and due to the frequent inconsistency of their opinions with other experts and the low level of competence, their opinions were rejected. Figure 2 shows an example of the interval grey number $\otimes 2$ depending on the level of confidence.

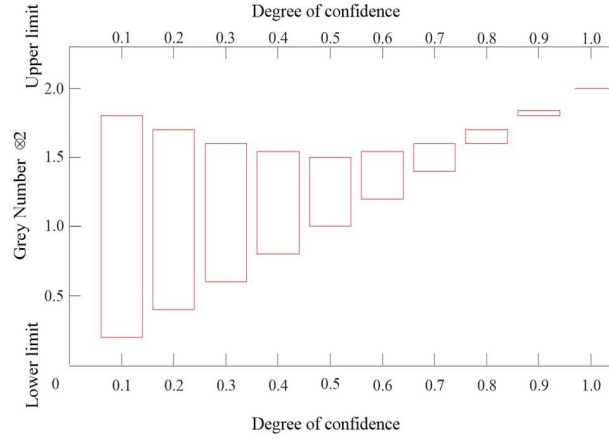


Figure 2. Example of interval grey number with degree of confidence

Note: This figure was prepared by the authors

Based on the steps of the DIBR II method [36] and basic regularities and arithmetic operations of interval grey numbers, the method was improved, according to the following:

Step 1: Defining a set of m criteria $C = \{C_1, C_2, \dots, C_m\}$ that condition the choice and importance of each of the criteria $C_1 > C_2 > \dots > C_m$.

Step 2: Determining the relationship between adjacent criteria ($\Omega_{m-1,m}$).

$$\otimes w_1 : \otimes w_2 = \otimes \Omega_{1,2} : 1 \mapsto \frac{\otimes w_1}{\otimes w_2} = \otimes \Omega_{1,2} \quad (2)$$

$$\otimes w_2 : \otimes w_3 = \otimes \Omega_{2,3} : 1 \mapsto \frac{\otimes w_2}{\otimes w_3} = \otimes \Omega_{2,3} \quad (3)$$

...

$$\otimes w_{m-1} : \otimes w_m = \otimes \Omega_{m-1,m} : 1 \mapsto \frac{\otimes w_{m-1}}{\otimes w_m} = \otimes \Omega_{m-1,m} \quad (4)$$

$$\otimes w_1 : \otimes w_m = \otimes \Omega_{1,m} : 1 \mapsto \frac{\otimes w_1}{\otimes w_m} = \otimes \Omega_{1,m} \quad (5)$$

where, $\otimes w$ represents the grey value of the weight coefficients of the criteria. Relationships between adjacent criteria are defined based on the level of DM confidence in the given claims, that is, using Eq. (1), with a condition if $\Omega, \bar{\Omega} < 1$ then $\Omega, \bar{\Omega} = 1$, and due to the specificity of the concrete method.

Step 3: Based on certain relationships, the relationships between the most significant criteria and the others are \mp defined, Eqs. (6) to (8).

$$\otimes w_2 = \frac{\otimes w_1}{\otimes \Omega_{1,2}} \quad (6)$$

$$\otimes w_3 = \frac{\otimes w_1}{\otimes \Omega_{1,2} \bullet \otimes \Omega_{2,3}} \quad (7)$$

...

$$\otimes w_n = \frac{\otimes w_1}{\otimes \Omega_{1,2} \bullet \otimes \Omega_{2,3} \bullet \dots \bullet \otimes \Omega_{m-1,m}} \quad (8)$$

Step 4: According to Eqs. (7) to (9), the grey value of the weight coefficient of the most significant criterion Eq. (9) is defined.

$$\otimes w_1 = \frac{1}{1 + \frac{1}{\otimes \Omega_{1,2}} + \frac{1}{\otimes \Omega_{1,2} \bullet \otimes \Omega_{2,3}} + \dots + \frac{1}{\otimes \Omega_{1,2} \bullet \otimes \Omega_{2,3} \bullet \dots \bullet \otimes \Omega_{m-1, m}}} \quad (9)$$

Step 5: After obtaining the grey value of the weight of the most important criterion, the grey values of the weights of the other criteria is determined, based on Eqs. (6) to (8).

Step 6: After obtaining the grey values of the weights for each of the criteria, the values are converted into crisp numbers, using the Eq. (10) [58, 61].

$$w_\lambda = (1 - \lambda) \cdot \underline{w} + \lambda \cdot \bar{w} \quad (10)$$

where, λ represents the whitening coefficient $\lambda \in [0, 1]$ [58, 61].

Step 7: In order to check the quality of the relationship between the criteria, it is necessary to calculate the control value w_m^K and the deviation W_m , where the above must satisfy the condition that taking the values of the weights of the criteria and the values of the comparison when $\delta = 1$.

Step 8: Due to the specificity of the method and conditions from Step 2 (if $\underline{\Omega}, \bar{\Omega} < 1$ then $\underline{\Omega}, \bar{\Omega} = 1$) because the values of the comparison of adjacent criteria are in the range $\Omega_{m-1, m} \in (1, \infty)$ and that the lower and upper limit values do not decrease by the same percentage, the quality control of the relationship between the criteria must be performed on crisp values, i.e. the weight coefficients of the criteria are calculated when $\delta = 1$, using Eqs. (2) to (10). After obtaining the crisp values of the weights of the criteria, the control value and the deviation value are calculated in the next sub-step.

Step 9: Checking the quality of the relationship between the criteria.

$$W_m = \left| 1 - \frac{w_m}{w_m^K} \right| \quad (11)$$

$$w_m^K = \frac{w_1}{\Omega_{1, m}} \quad (12)$$

If the relations are well defined ($0 \leq W_m \leq 0.1$), it can be stated that the values of the criterion weights from Step 7 are final, if they are not, the relations must be redefined, as described in [36].

2.2 Numerical Example

Let a set of five criteria be defined $C = \{C_1, C_2, \dots, C_5\}$, and let the order of the criteria is in order of importance $C_1 > C_2 > \dots > C_m$. Let the DM be confident in claims $\delta = 0.95$ and present the relationships between the criteria ($\Omega_{m-1, m}$) as shown in Table 1:

Table 1. Relationships between adjacent criteria based on DM's opinion

	$r \otimes \Omega_{m-1, m}$
C ₁ -C ₂	2.00
C ₂ -C ₃	1.80
C ₃ -C ₄	1.80
C ₄ -C ₅	1.20
C ₁ -C ₅	8.00

By applying Eqs. (1) and (6) and the level of confidence, the statements (Table 1) are converted into grey numbers, that is, grey relationships ($\otimes \Omega_{m-1, m}$), as shown in Table 2.

Table 2. Grey relationships between adjacent criteria based on DM's opinion

	$\otimes \Omega_{m-1, m}$	
C ₁ -C ₂	1.900	1.905
C ₂ -C ₃	1.710	1.715
C ₃ -C ₄	1.710	1.715
C ₄ -C ₅	1.140	1.143
C ₁ -C ₅	6.600	6.170

Applying the expressions from (7) to (10) leads to the following grey values of the weight coefficients of the criteria (Table 3).

Table 3. Grey values of weight coefficients of the criteria for DM

	$\otimes w$	
C ₁	0.46	0.462
C ₂	0.242	0.242
C ₃	0.142	0.141
C ₄	0.083	0.082
C ₅	0.073	0.072

To obtain the final values of the criterion weights, the obtained grey values are turned into a crisp using expression (11), where $\lambda = 0.5$, as shown in Table 4.

Table 4. Final values of weight coefficients of the criteria

	W
C ₁	0.461
C ₂	0.242
C ₃	0.142
C ₄	0.083
C ₅	0.072

Given that the relationships between the criteria are well defined (for $\delta = 1$, $w_m^K = 0.061$ and $W_m = 0.003$), criteria weight values from Table 4 are final.

To validate the proposed method and demonstrate its use on a real-life problem, in the next part of the paper, a comparison of the results obtained using the classic DIBR II and Grey DIBR II methods will be performed, as well as the application of the problem of choosing methods and techniques of lean organization systems management [40].

3 Application of the Grey DIBR II Method in the MCDM Model and Comparative Analysis

The presentation of the application of the grey DIBR II method was carried out on an existing study. Božanić et al. [40] chose the method and technique of the lean organization of the management system in the process of technical maintenance using the classic DIBR II and rough MABAC methods, considering the five criteria $C = \{C_1, C_2, \dots, C_5\}$ that determine the choice (“Reduction of a SPTS maintenance cycle time; optimization level of equipment and working position arrangement within the maintenance system structure of technical systems for special purposes; SPTS effectiveness increase percentage; system efficiency increase level”). Considering that the crisp DIBR II method was used in this study and that the experts’ conviction in the given claims was not taken into account, the research represents a good comparative example. The aim of this comparison is to assess the influence of the experts’ uncertainty in their claims on the values of the weight of the criteria and on the final decision.

Using the criteria comparison values by experts $E = \{E_1, E_2, E_3, E_4\}$ from existing research [40], let the experts be convinced of their claims by the following $\delta_1^E = 0.9$; $\delta_2^E = 0.8$; $\delta_3^E = 1.0$; $\delta_4^E = 0.8$, and by applying Eqs. (1) and (6), the following grey values of criteria comparison by experts are obtained (see Table 5):

Table 5. Grey relationships between adjacent criteria and their significance based on expert opinion

E1			E2			E3			E4		
C ₁ -C ₂	1.800	1.820	C ₁ -C ₃	2.000	2.100	C ₁ -C ₂	1.800	1.800	C ₁ -C ₂	1.360	1.428
C ₂ -C ₃	1.000	1.000	C ₃ -C ₂	1.000	1.000	C ₂ -C ₃	1.000	1.000	C ₂ -C ₃	1.040	1.092
C ₃ -C ₄	1.440	1.456	C ₂ -C ₄	1.280	1.344	C ₃ -C ₄	1.500	1.500	C ₃ -C ₅	1.200	1.260
C ₄ -C ₅	1.170	1.183	C ₄ -C ₅	1.120	1.176	C ₄ -C ₅	1.100	1.100	C ₅ -C ₄	1.000	1.000
C ₁ -C ₅	4.500	4.550	C ₁ -C ₅	5.600	5.880	C ₁ -C ₅	3.000	3.000	C ₁ -C ₄	2.800	2.940

By applying the steps of the grey DIBR II method and Eqs. (2) to (9), and with a minimal correction of the ratio between the most significant and the least significant criterion, the following values of the weight coefficients of the criteria are reached (see Table 6):

Table 6. Values of weight coefficients of the criteria for experts

	E1		E2		E3		E4	
	$\otimes w$	w	$\otimes w$	w	$\otimes w$	w	$\otimes w$	w
C ₁	0.354	0.358	0.356	0.365	0.383	0.374	0.355	0.355
C ₂	0.197	0.197	0.197	0.183	0.183	0.183	0.197	0.197
C ₃	0.197	0.197	0.197	0.183	0.183	0.183	0.197	0.197
C ₄	0.136	0.135	0.136	0.143	0.136	0.139	0.131	0.131
C ₅	0.117	0.114	0.115	0.127	0.116	0.121	0.119	0.119

By applying Step 8 (calculation of the control value and deviation on the values obtained using the crisp method, that is, when $\delta = 1$), the following deviation values for each of the experts are calculated $W_m^{E_1} = 0.093$; $W_m^{E_2} = 0.042$; $W_m^{E_3} = 0.010$; $W_m^{E_4} = 0.040$, which means that the relations are qualitatively defined, and the obtained values of the weight of the criteria by the experts can be aggregated.

To obtain the final values of the weights of the criteria, the values of the weights of each of the experts are aggregated with the Bonferroni aggregator [40, 62]. The final values of the weights of the criteria are presented in Table 7 ($q = p = 1$).

Table 7. Final values of weight coefficients of the criteria obtained using the grey DIBR II method

	W
C ₁	0.343
C ₂	0.195
C ₃	0.193
C ₄	0.141
C ₅	0.128

Considering that the experts were differently confident in their claims, the distribution of the values of the weights of the criteria is noticeably more nuanced than it is in the case when the degree of confidence is $\delta = 1$, that is, when the weights are determined using the classic DIBR II method, where the values match the weights of the criteria obtained in the existing research [40] (see Table 8).

Table 8. Values of weight coefficients of the criteria ($\delta = 1$ and [40])

	w($\delta = 1$ and [40])
C ₁	0.343
C ₂	0.195
C ₃	0.193
C ₄	0.141
C ₅	0.128

Given that the final values of the weight coefficients of the criteria were obtained (see Table 7), they represent the input data for the application of the MABAC method [63] improved using rough theory [64], based on existing research [40]. The first step of applying the rough MABAC method is the formation of the initial decision matrix (see Table 9), based on previously defined criteria, where the optimal alternative is chosen from six possible alternatives [40].

Table 9. Initial decision matrix [40]

w	0.343	0.343	0.195	0.195	0.193	0.193	0.141	0.141	0.128	0.128
	Cost		Benefit		Benefit		Benefit		Cost	
	C ₁		C ₂		C ₃		C ₄		C ₅	
A ₁	6.200	6.500	3.000	4.000	13.000	15.000	5.000	6.000	1.000	2.000
A ₂	4.500	5.000	5.000	7.000	10.000	12.000	3.000	4.000	3.000	4.000
A ₃	6.800	7.200	1.000	4.000	17.000	19.000	5.000	7.000	2.000	3.000
A ₄	7.300	7.500	5.000	6.000	18.000	22.000	1.000	4.000	1.000	2.000
A ₅	3.000	3.700	4.000	5.000	12.000	16.000	4.000	5.000	2.000	3.000
A ₆	3.200	4.000	6.000	7.000	15.000	20.000	6.000	7.000	4.000	5.000

By applying the steps of the rough MABAC method, the following ranking of alternatives is reached (see Table 10), while the results from the existing research are shown next to them [40].

Table 10. Rank of alternatives obtained by this and existing research [40]

	Grey DIBR II-Rough MABAC			DIBR II-Rough MABAC [40]		
	$RN(K_i)$	K_i^{crisp}	Rank	$RN(K_i)$	K_i^{crisp}	Rank
A ₁	[0.142,0.486]	0.128	4	[0.15,0.478]	0.106	3
A ₂	[0.046,0.438]	0.053	5	[0.028,0.406]	0.000	5
A ₃	[0.219,0.659]	0.247	2	[0.226,0.647]	0.228	2
A ₄	[0.277,0.692]	0.293	1	[0.317,0.705]	0.307	1
A ₅	[-0.077,0.329]	-0.063	6	[-0.115,0.281]	-0.143	6
A ₆	[0.163,0.592]	0.185	3	[0.103,0.522]	0.096	4

As can be seen from Table 10, the first-ranked and the last-ranked alternatives kept their place, while the alternatives A6 and A1 changed their rank, more precisely, in this research, the alternative A6 moved from the fourth place to the third, while the alternative A1 moved from the third to the fourth place in the ranking, due to the more nuanced distribution of the value of the weight of the criteria and the degree of the expert's confidence in the claims. This is very important in situations where more than one alternative is chosen or when there is no absolute agreement of DMs, when it comes to group decision making.

4 Conclusions

The Grey theory represents a significant contribution to the processing of incomplete and imprecise information. This paper focuses on the application of interval grey numbers in the DIBR II method. In addition, the degree of conviction of decision-makers or experts in the accuracy of certain claims was used to form the grey number, which further enriches the decision-making process.

The paper shows the application of the grey DIBR II method on an illustrative example, which enables better understanding and practical application. In addition, the paper deals with the application of this method to the real problem of choosing methods and techniques for managing lean organizational systems from existing research. This application uses the model with the rough MABAC method, with the aim of validating the method.

It was concluded that the grey DIBR II method can be very useful in decision-making problems when it is necessary to determine the weight coefficients of criteria. Compared to classical methods, this method enables a more precise distribution of criteria weights, thus achieving a more detailed analysis. When this method is applied in a multi-criteria model, there is a finer ranking of alternatives, which depends on the degree of belief in certain statements. The research results show that the grey DIBR II method not only provides more robust and detailed results, but also enables better adaptation to real conditions and uncertainties present in the decision-making process. This confirms its usability and effectiveness in various fields, for example, economic analysis, engineering, military, biology, and medicine, i.e. where the availability of data is often limited. It is particularly important that the method enables decision-making in conditions when information is incomplete and imprecise, which is often the case in the real world. Also, considering that grey's theory uses intervals and series to model uncertainty, the mentioned method can be simpler to interpret and implement. The application of the method to the problem of the choice of methods and techniques for lean organizational systems shows its practical value and potential for further development, through the implementation of other theories that treat the area of imprecision and uncertainty well.

This method provides a robust framework for analysis and decision-making, allowing decision-makers to better understand the interdependencies between different criteria and to determine their weights more precisely. In this way, the grey DIBR II method contributes to the improvement of the decision-making process and enables the achievement of quality decisions. The main limitation of the method is reflected in the need for the quality between the ratios to be performed on crisp values, and due to the limitation of the traditional method that the smallest value of the comparison between the criteria is 1 (then the criteria are of equal importance), which further complicates the procedure itself, but does not threaten the validity of the obtained values.

This paper highlights the importance of Multi-Criteria Decision-Making and the grey theory in contemporary research and practice. The grey DIBR II method, with its innovative approach and practical application, represents a significant step forward in this field. Its potential for further development and application in various fields makes it an important tool for researchers and decision-makers.

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Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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