北京航空航天大学

2013－2014 学年 第一学期期末

Modern Control Engineering

考 试 A 卷

班 级＿＿＿＿＿＿＿＿＿＿学 号 ＿＿＿＿＿＿

姓 名＿＿＿＿＿＿＿＿＿＿成 绩 ＿＿＿＿＿＿

2014 年 1 月 15 日

班号__________ 学号_________ 姓名________ 成绩________

Examination Questions

1. **(15 points)** A network is shown in Figure 1:

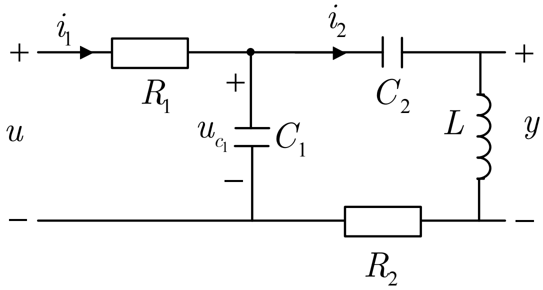


Figure 1

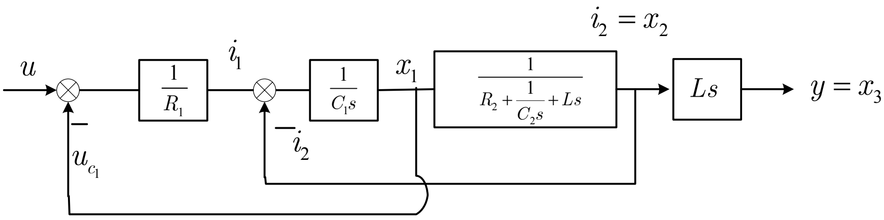
where u is the input signal, y, the voltage across the inductor L, is the output signal, and i_1 and i_2 are the currents passing through the resistor R_1 and the capacitor C_2, respectively.

(1). Draw the block diagram according to Figure 1;

(2). If $R_1 = R_2 = 1\Omega$, $C_1 = C_2 = 1F$ and $L = 1H$, find the transfer function $Y(s)/U(s)$.

Solution:

(1) By applying the concept of complex impedance, it is easy to obtain the diagram of the network:



(2) Letting $R_1 = R_2 = 1\Omega$, $C_1 = C_2 = 1F$ and $L = 1H$, the transfer function from the above diagram is

$$G(s)=\frac{s^2}{s^3+2s^2+3s+1}$$

2. **(15 points, 5 points each)**

(1). Let the closed loop transfer function of a system be

$$\frac{C(s)}{R(s)}=\frac{1}{(s+1)(s+2)}$$

Determine its unit step response $c(t)$.

(2). Let the transfer function of a system be

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\text{ , }(\zeta\ge 0,\omega_n>0)$$

where $0<\zeta<1$. Write the response $c(t)$ when the input signal is a unit-step function.

(3). The block diagram of a system is shown in Figure 2. Design a feedforward compensator $G_c(s)$, such that the steady-state error e_{ss} of the system is zero when the input signal is $r(t)=t, t\ge 0$（The error is defined as $e=r-c$）.

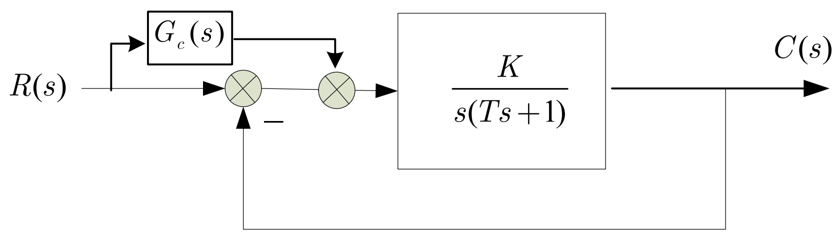


Figure 2

4. (15 points, 5 points each).

(1). The system transfer function is

$$\frac{Y(s)}{X(s)}=G(s)=\frac{4(s+1)}{(s+2)(s+3)}$$

If the input signal $x(t)=A\sin\omega t$, determine the system output $y(t)$ when the system reaches the steady-state.

(2). The *Nyquist* curves of two open-loop transfer functions are shown in Figure 4, where P denotes the number of unstable open-loop poles. Determine the stability of the two systems by using *Nyquist* stability criterion.

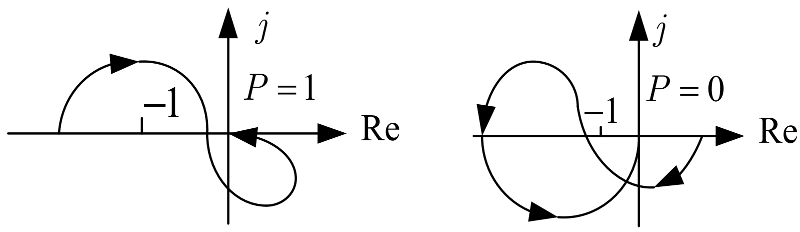


Figure 4

3. **(15 points)** The block diagram of a system is shown in Figure 5:

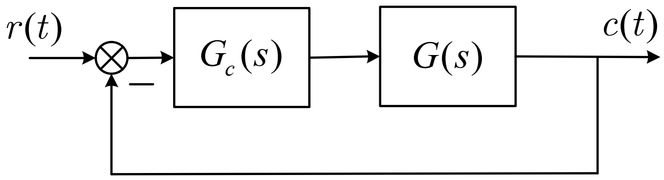


Figure 5

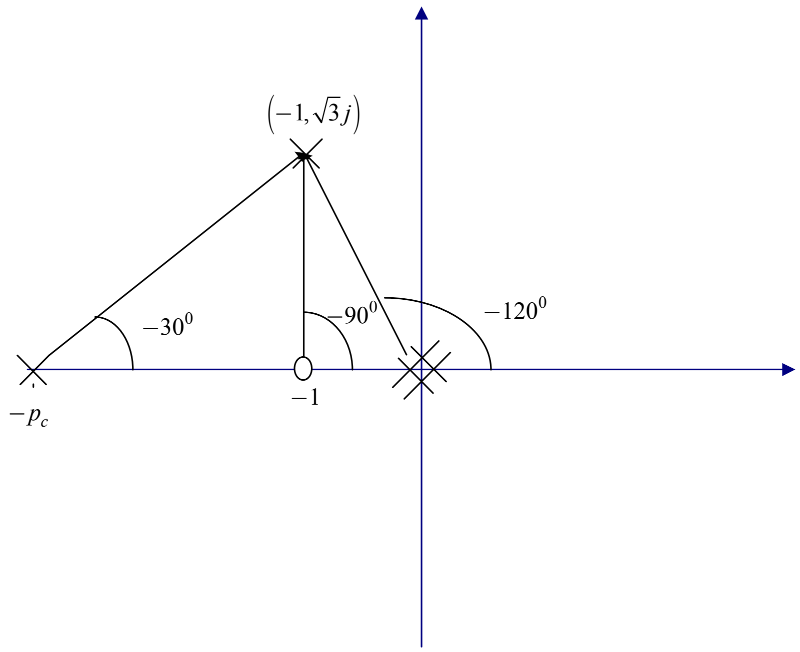
where $G(s) = \dfrac{1}{s^2}$ and the transfer function of the compensator is

$$G_c(s) = \frac{K_c^*(s+1)}{(s+p_c)}$$

(1). By using root locus method to determine the values of K_c^* and p_c, so that a pair of closed-loop poles are located at $-1 \pm \sqrt{3}j$ after the compensation (hint: using angle and magnitude conditions);

(2). With the determined p_c, draw the root loci as $K_c^* : 0 \rightarrow +\infty$.

Solution: (1) From the phase angle condition, we obtain that $p_c = 4$.



由此图易得$p_c = 4$。

根据模值方程可得$K_c^* = 8$。

（2）开环传递函数极点 0,0,-4，零点-1；

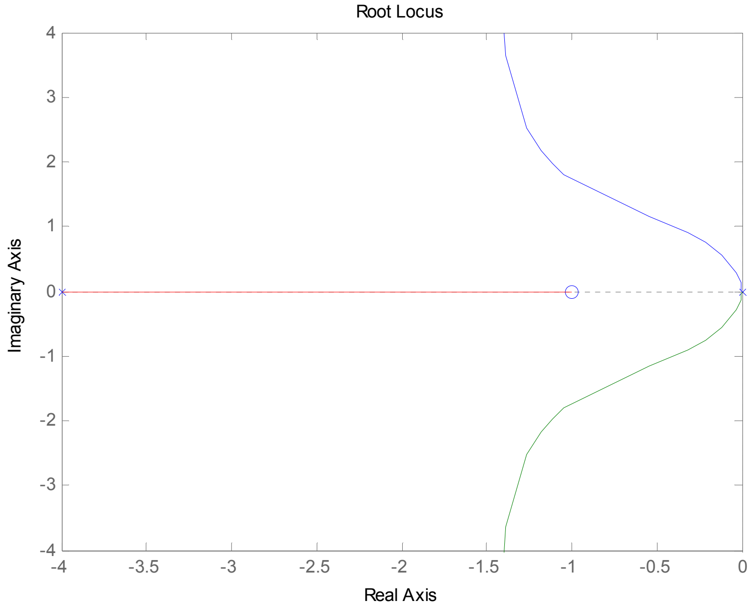
实轴上根轨迹$\left(-4,-1\right)$；

渐近线$\varphi_a = \frac{\pi}{2},-\frac{\pi}{2}$，$\sigma_a = -1.5$；

起始角$\frac{\pi}{2},-\frac{\pi}{2},0$；

分离点坐标$\frac{1}{d+1} = \frac{2}{d} + \frac{1}{d+4}$，舍去

由以上法则可绘制出如下根轨迹



5. (10 points） The open-loop transfer function of a unity-feedback system is

$$G(s) = \frac{3}{s(s-1)}$$

Draw its *Nyquist* curve and determine

closed-loop stability by using *Nyquist* stability criterion.

6. **(15 points)** The block diagram of a system is shown in Figure 6 with $r(t) = t \ (t \geq 0)$ $(e(t) = r(t) - c(t))$.

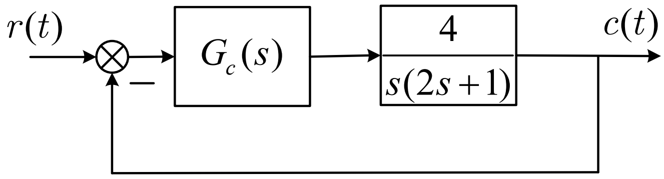


Figure 6

Design the compensator $G_c(s)$ such that there is no steady-state error.

7. (15 points) The open-loop transfer function of a unity-feedback system is

$$G_p(s)=\frac{2000}{s(s+2)(s+20)}$$

The asymptotic Bode magnitude curve of $20\log\left|G_c(j\omega)G_p(j\omega)\right|$ is shown in Figure 7, where $G_c(s)$ is a series compensator.

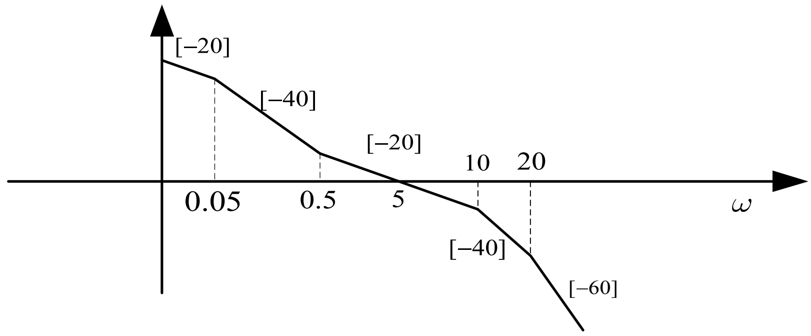


Figure 7: The asymptotic Bode magnitude curve of $20\log\left|G_c(j\omega)G_p(j\omega)\right|$

(1). Determine the transfer function of $G_c(s)$. What kind of a compensator is it?

(2). Draw the asymptotic Bode magnitude curve of $G_c(s)$ for which the corner frequencies and slopes are required;

(3). Calculate the phase margin of the compensated system.

解: $G_p(s)=\dfrac{50}{s(\frac{1}{2}s+1)(\frac{1}{20}s+1)}$

(1) $G_e(s)=\dfrac{K\left(\frac{1}{0.5}s+1\right)}{s\left(\frac{1}{0.05}s+1\right)\left(\frac{1}{10}s+1\right)\left(\frac{1}{20}s+1\right)}$

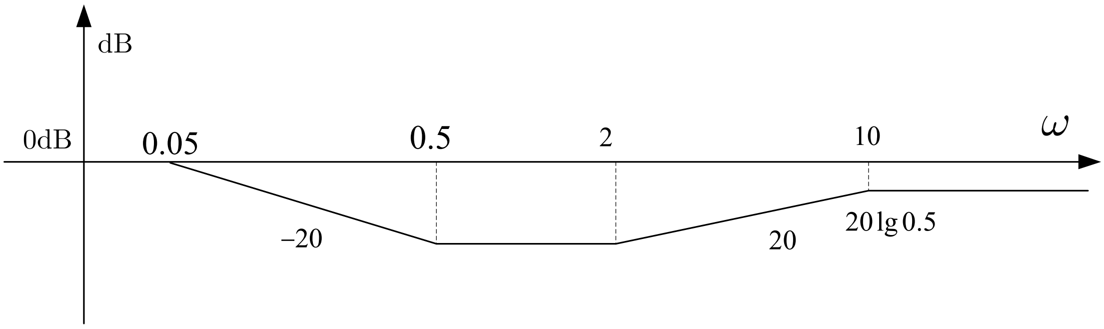
由 $\dfrac{K\frac{1}{0.5}}{5\times\frac{1}{0.05}}=1\Rightarrow K=50$

故 $G_c(s)=\dfrac{G_e(s)}{G_p(s)}=\dfrac{\left(\frac{1}{0.5}s+1\right)\left(\frac{1}{2}s+1\right)}{\left(\frac{1}{0.05}s+1\right)\left(\frac{1}{10}s+1\right)}$

滞后超前校正。

(1) 幅频特性如下图所示。

当$\omega > 10$时，由$|G_c| = \dfrac{\dfrac{1}{0.5}\times\dfrac{1}{2}}{\dfrac{1}{0.05}\times\dfrac{1}{10}} = 0.5$



(3) $\gamma = 180^\circ - 90^\circ + \arctan\dfrac{5}{0.5} - \arctan\dfrac{5}{0.05} - \arctan\dfrac{5}{10} - \arctan\dfrac{5}{20} = 44^\circ$