1.2.3.4.5.6

§生存分布与生命表.

① 死亡年龄：X

② $F(x) = Pr(X \le x)$. $x \ge 0$. 新生儿在x岁前死亡的概率.

③ $s(x) = 1 - F(x) = Pr(X > x)$. $x \ge 0$. ⇒ 生存函数
新生儿活到x岁的概率.

④ x岁人的剩余寿命：$T(x)$.

⑤ ${}_tq_x = Pr[T(x) \le t]$ (x)在t年内死去的概率.
${}_tp_x = Pr[T(x) > t] = 1 - {}_tq_x$. (x)活到x+t的概率.

· t=1时, 可省略. ${}_xq_0 = F(x)$. ${}_xp_0 = s(x)$.

⑥ ${}_{t|u}q_x = Pr(t < T(x) \le t+u)$. 生存t年后在其后u年内死亡.
· u=1时, 可省略为 ${}_{t|}q_x$.

⑦. x岁人的整值剩余寿命：$K(x)$. $k \le T(x) < k+1$ 时. $K(x)=k$.

⑧. 死亡效力 / 瞬时死亡率：μ_x.

${}_tp_x = e^{-\int_0^t \mu_{x+s} ds}$ $S(x) = e^{-\int_0^x \mu_s ds}$. $f(x) = {}_xp_0 \cdot \mu_x$

由 l_0 个新生命组成的个体.

l_x：l_0个新生命能存活到x年龄的期望个数.

${}_nd_x = E[{}_nD_x] = l_x - l_{x+n}$. : l_0个……中在x和x+n之间死亡的个数

L_x：l_0个……在x到x+1生存总年数.

T_x：……在x以后生存总年数.

期望剩余寿命 $\mathring{e}_x = E[T(x)]$.

$\mathring{e}_x = \int_0^\infty {}_tp_x\,dt$.　　$E[T^2(x)] = 2\int_0^\infty t\cdot{}_tp_x\,dt$.

$Var[T(x)] = 2\int_0^\infty t\cdot{}_tp_x\,dt - \mathring{e}_x^2$

期望整值剩余寿命. $e_x = E[K(x)]$

$e_x = \sum_{k=0}^\infty {}_{k+1}p_x$.　　$E[K^2(x)] = \sum_{k=0}^\infty (2k+1)\,{}_{k+1}p_x$.

$Var[K(x)] = \sum_{k=0}^\infty (2k+1)\,{}_{k+1}p_x - e_x^2$.

关于分数年龄的假设：①. 死亡是均匀分布的. ☆.
②. 死亡力是常数.
③. Balducci 假设.

${}_tq_x = t\cdot q_x$.
${}_tp_x = 1 - t\cdot q_x$.

> 例：${}_{2.7}q_{2.5} = 1 - {}_{2.7}p_{2.5} = 1 - {}_{0.5}p_{2.5}\cdot p_3\cdot p_4\cdot{}_{0.2}p_5$
$= 1 - (1-0.5\cdot q_{2.5})\cdot(1-q_3)(1-q_4)(1-0.2\cdot q_5)$.

$[x]$：投保年龄.

${}_rq_{[x]+t}$ ：x 岁投保，现 $x+t$ 岁，在 $x+t+r$ 岁之前死亡的概率.

一般情况下，$q_{[x]} \le q_{[x-1]+1} \le q_{[x-2]+2} \le \cdots$

1.2.3.4.5.6

§人寿保险.

t：保单从签发到被保人死亡的时间长度.
v_t：从赔付时刻回溯到保单签发时的贴现因子.
b_t：受益赔付额.
z_t：受益赔付额在保单发行时现时值. $z_t = b_t\cdot v_t$.
x：投保年龄；$T = T(x)$.
$Z = Z_T = b_T\cdot v_T$，现时值随机变量.

• n年期人寿保险：
(x)死亡即刻应付1个单位金额的n年期保险的净趸缴保费：

$E[Z] = \bar{A}^1_{x:\overline{n}|} = \int_0^\infty z_t\,g(t)\,dt = \int_0^n v^t\,{}_tp_x\,\mu_{x+t}\,dt$.

$(v^t = e^{-\delta t}$　　${}^2\bar{A}^1_{x:\overline{n}|} = \int_0^n e^{-2\delta t}\,{}_tp_x\,\mu_{x+t}\,dt$

• 终身人寿保险：
$\bar{A}_x = E[Z] = \int_0^\infty v^t\,{}_xp_t\,\mu_{x+t}\,dt$.

• n年期生存保险：
$A_{x:\overline{n}|}^{\;\;1} = E[Z] = v^n\cdot{}_np_x$.
${}^2A_{x:\overline{n}|}^{\;\;1} = v^{2n}\cdot{}_np_x$.

• n年期两全保险：
$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\;\;1}$

• m年递延保险（延期保险）：
${}_{m|}\bar{A}^1_{x:\overline{n}|} = \int_m^{m+n} v^t\,{}_tp_x\,\mu_{x+t}\,dt$　…
${}_{m|}\bar{A}_x = \int_m^\infty v^t\,{}_tp_x\,\mu_{x+t}\,dt$.

• 递增终身寿险：$(I\bar{A})_x = \int_0^\infty [t+1]\,v^t\,{}_tp_x\,\mu_{x+t}\,dt$.
• 年递增m次终身寿险：$(I^{(m)}\bar{A})_x = \int_0^\infty \frac{v^t}{m}[mt+1]\,{}_tp_x\,\mu_{x+t}\,dt$
$m\to\infty \Rightarrow (\bar{I}\bar{A})_x = \int_0^\infty t\cdot v^t\,{}_tp_x\,\mu_{x+t}\,dt = \int_0^\infty {}_{s|}\bar{A}_x\,ds$.

• 递减n年期：$(D\bar{A})^1_{x:\overline{n}|} = \int_0^n v^t\,(n-[t])\,{}_tp_x\,\mu_{x+t}\,dt$.

死亡即刻赔付：

1. n年定期寿险：$A^1_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1} {}_kp_x\, q_{x+k}$.

2. 终身寿险：$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x\, q_{x+k}$.

3. 递延m年的n年期寿险：${}_{m|n}A_x = A^1_{x:\overline{m+n|}} - A^1_{x:\overline{m|}}$

4. 递延m年的终身寿险：${}_{m|}A_x = A_x - A^1_{x:\overline{m|}}$

5. n年期两全险：$A_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1} {}_kp_x\, q_{x+k} + v^n {}_np_x$

$= A^1_{x:\overline{n|}} + A_{x:\overline{n|}}^{\ \ 1}$

递推公式：$\bar{A}_x = \frac{i}{\delta} A_x$.　$(I\bar{A})^1_{x:\overline{n|}} = \frac{i}{\delta}(IA)^1_{x:\overline{n|}}$

1. $A_x = vq_x + vA_{x+1}p_x$.

2. $A_x = vq_x + vA_{x+1}(1-q_x)$.

3. $l_x(1+i)A_x = l_xA_{x+1} + d_x(1-A_{x+1})$.

4. $A_{x+1} - A_x = iA_x - q_x(1-A_{x+1})$.

5. $A_y = \sum_{x=y}^{\infty} v^{x-y+1} q_x (1-A_{x+1})$.

计算基数：$D_x = v^x l_x$　　$C_x = v^{x+1} d_x = D_x v q_x$.

$M_x = \sum_{k=0}^{\infty} C_{k+x} = \sum_{j=x}^{\infty} C_j$

$R_x = \sum_{j=0}^{\infty} M_j = \sum_{k=0}^{\infty} (k+1) C_{x+k}$.

例：某人25岁投保定期35年的寿险，死亡年末赔付，i=6%.

(1). 保险金100000元，求净趸缴保费.

(2). ～ 1500元，求保险金.

I. 35年寿险.　(1). $A^1_{25:\overline{35|}} \times 100000 = \frac{M_{25}-M_{60}}{D_{25}} \times 100000$

$= \frac{1819.6709 - 916.2413}{22286.38} \times 100000 = 4053.73$.

(2). 37002.967

II. 终身寿险.

(1). $100000 \cdot A_{25} = 100000 \cdot \frac{M_{25}}{D_{25}} = 8164.95$

(2) 183835.82.

III. 两全险：

(1). $100000 \cdot A_{25:\overline{35|}} = 100000 \cdot \frac{M_{25}-M_{60}+D_{60}}{D_{25}}$

$= 6844.54$

(2). 21924.888.

对于年龄x的人在y～z间死亡年末赔付的寿险：

$\frac{M_y - M_z}{D_x}$

n年期生存险：$\frac{D_{x+n}}{D_x}$

n年期两全险：$\frac{M_x - M_{x+n} + D_{x+n}}{D_x}$

n年期递增：$\frac{R_x - R_{x+n} - n \cdot M_{x+n}}{D_x}$

2.4.5.6.7.8.

生存年金

存活后n年末支付1单位保金期望现值:

$$_nE_x = A_{x:\overline{n}|}^{\ \ 1} = v^n\,{}_np_x$$

精算积累值:

$$S = \frac{1}{_nE_x} = \frac{1}{v^n\,{}_np_x} = (1+i)^n\frac{l_x}{l_{x+n}}$$

(x)活着时每年1单位连续支付的终身年金.

精算现值 $\bar{a}_x = \int_0^\infty v^t\,{}_tp_x\,dt$

$$1 = \delta\bar{a}_x + \bar{A}_x$$

活着时每年年初支付一单位的终身年金，精算现值为 $\ddot{a}_x$

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k\,{}_kp_x = \frac{1}{l_x}\sum_{k=0}^{\infty} v^k l_{x+k} = 1+\sum_{k=0}^{\infty} v^{k+1}\,{}_{k+1}p_x$$

$$1 = d\ddot{a}_x + A_x$$

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k\,{}_kp_x \qquad 1 = d\ddot{a}_{x:\overline{n}|} + A_{x:\overline{n}|}$$

$_{n|}\ddot{a}_x = {}_nE_x\cdot\ddot{a}_{x+n}$ (x)从x+n岁起每年年初支付1.

$$\ddot{s}_{x:\overline{n}|} = \frac{1}{_nE_x}\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1}\frac{1}{_{n-k}E_{x+k}}$$

$$a_x = \ddot{a}_x - 1 = \sum_{k=1}^{\infty} v^k\,{}_kp_x \qquad 1 = ia_x + (1+i)A_x$$

$$a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x = \sum_{k=1}^{n} v^k\,{}_kp_x$$

$\ddot{a}_x^{(m)}$ 每年m次支付生存年金(期初). exp: 每月, m=12.

$$\ddot{a}_x^{(m)} = \alpha(m)\ddot{a}_x - \beta(m) \approx \ddot{a}_x - \frac{m-1}{2m}$$

死亡均匀分布假设

Woolhouse公式

$$\alpha(m) = s_{\overline{1}|}^{(m)}\cdot\ddot{a}_{\overline{1}|}^{(m)} = \frac{id}{i^{(m)}d^{(m)}}$$

$$\beta(m) = \frac{s_{\overline{1}|}^{(m)}-1}{d^{(m)}} = \frac{i-i^{(m)}}{i^{(m)}d^{(m)}}$$

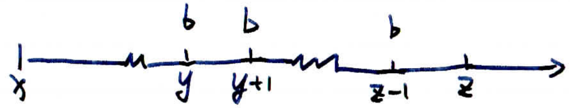
$$\ddot{a}_{x:\overline{n}|}^{(m)} = \ddot{a}_{\overline{1}|}^{(m)}\cdot\ddot{a}_{x:\overline{n}|} - \beta(m)A_{x:\overline{n}|}^{1}$$

$$_{n|}\ddot{a}_x^{(m)} = \ddot{a}_{\overline{1}|}^{(m)}\,{}_{n|}\ddot{a}_x - \beta(m)\,{}_{n|}A_x$$

$$a_x^{(m)} = \ddot{a}_x^{(m)} - \frac{1}{m} \qquad a_{x:\overline{n}|}^{(m)} = \ddot{a}_{x:\overline{n}|}^{(m)} - \frac{1}{m}(1-{}_nE_x)$$

$$1 = d^{(m)}\ddot{a}_x^{(m)} + A_x^{(m)} = i^{(m)}a_x^{(m)} + \left(1+\frac{i^{(m)}}{m}\right)A_x^{(m)}$$

$$N_x = \sum_{u=x}^{\infty} D_u$$



① y~z-1岁每年支付b.

精算现值为: $\frac{b}{D_x}(N_y - N_z)$.

$$N_x^{(m)} = \alpha(m)N_x - \beta(m)D_x \approx N_x - \frac{m-1}{2m}D_x$$

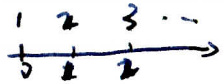
② 一年m期, 现值为 $\frac{b}{D_x}(N_y^{(m)} - N_z^{(m)})$

$$\ddot{a}_x^{(m)} = \frac{N_x^{(m)}}{D_x}$$

b: 定额年收入, 每期 $\frac{b}{m}$.

$$S_x^{(m)} = \sum_{y=x}^{\infty} N_y^{(m)} \qquad N_y^{(m)} = S_y^{(m)} - S_{y+1}^{(m)}$$

$$= \alpha(m)S_x - \beta(m)N_x.$$



第一年收入1, 第2年收入2, …… : $\frac{1}{D_x}\cdot S_x^{(m)}$

3.4.6.

§ 净保费

对(x)的一单位终身寿险，其年均衡年缴保费记作P_x，赔付在死亡年末。

保险人亏损为 $L = v^{k+1} - P_x \cdot \ddot{a}_{\overline{k+1|}}$.

平衡原理：$E[L]=0$， $\Rightarrow$ $P_x = \dfrac{A_x}{\ddot{a}_x}$

每年m次分期：每次付 $\dfrac{P_x^{(m)}}{m}$.

$$P_{x:\overline{n|}}^{(m)} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}^{(m)}}.$$

计算基数：

$${}_hP_{x:\overline{n|}} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+h}}.$$

$${}_hP_x = \frac{M_x}{N_x - N_{x+h}}$$

$$P^1_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}.$$

$${}_hP_{x:\overline{n|}}^{(m)} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x^{(m)} - N_{x+h}^{(m)}}$$

4.5.7.8.

§净保费责任准备金.

净保费为 P_x 的 1 单位终身人寿, k 年末的责任准备金:

$$_kV_x = A_{x+k} - P_x \cdot \ddot{a}_{x+k}$$

$$^h_kV^{(m)}_{x:\overline{n}|} = A_{x+k:\overline{n-k}|} - {}_hP^{(m)}_{x:\overline{n}|}\,\ddot{a}^{(m)}_{x+k:\overline{h-k}|}$$

$$V_t = A_{(t)} - P\ddot{a}_{(t)}$$

$v^t = e^{-\delta t}$ | $\int_m^n e^{-\delta t}\,dt = \dfrac{e^{-m\delta} - e^{-n\delta}}{\delta}$

$A^{\ \ 1}_{35:\overline{30}|} = A_{35} - v^{30}\dfrac{l_{65}}{l_{35}}A_{65}$

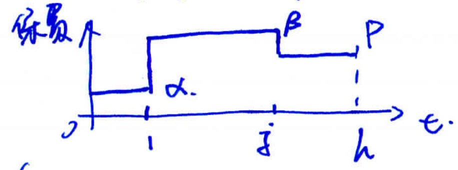
$A^{\ \ 1}_{x:\overline{n}|} = A_x - v^n\,{}_nP_x \cdot A_{x+n}$

$\delta = \dfrac{1}{\ln(1+i)}$ | $Var(1000Z) = 1000^2\,Var(Z)$

4.6.7.

§包括费用的保险模型.

修正责任准备金方法:



$$^h_kV^{Mod}_{x:\overline{n}|} = {}^h_kV_{x:\overline{n}|} - (\beta - {}_hP_{x:\overline{n}|}) \cdot \ddot{a}_{x+k:\overline{j-k}|}.$$

$\alpha^{FPT} = A^{\ \ 1}_{x:\overline{1}|}$ $\beta^{FPT} = P + \dfrac{P - A^{\ \ 1}_{x:\overline{1}|}}{\ddot{a}_{x+1:\overline{n-1}|}}$

例1: 终身 满险 调整

<0.04 ≥ 0.04 $(A + 0.036 + 0.25P^a_x)/\ddot{a}$

$h=10$ 的 $n=15$ 两全险.

$${}_{10}P^a_{20:\overline{15}|} = \frac{A_{20:\overline{15}|} + 0.036 + 0.25P^a_{20}}{\ddot{a}_{20:\overline{10}|}}$$

(35), 前20年150000, 后10年100000, 其时还需交100000 (两全险. 毛保费前10年2500, 后1250. $i=0.06$.

$AAI = 150000.$

$P = 50000(3M_{35} - M_{55} - 2M_{65} + 2D_{65})$

${}_0V_{x:\overline{3}|} = 0$, ${}_1V_{x:\overline{3}|} = 1000 \cdot A_{x+1:\overline{2}|}$

$+2 \cdot \ddot{a}_{x+1:\overline{2}|} + 0.06G\,\ddot{a}_{x+1:\overline{2}|} - G \cdot \ddot{a}_{x+1:\overline{2}|}$

$= 218.41$, ${}_2V_{x:\overline{3}|} = A_{x+2:\overline{1}|}$, $\ddot{a}_{x+2:\overline{1}|}$

${}_3V_{x:\overline{3}|} = 1000.$ $= 559.16$

(6.1). $k=0$. 581.16 0.1 $E=0$

$k=1$ 216.94 0.1