AUTOMATIC CONTROL

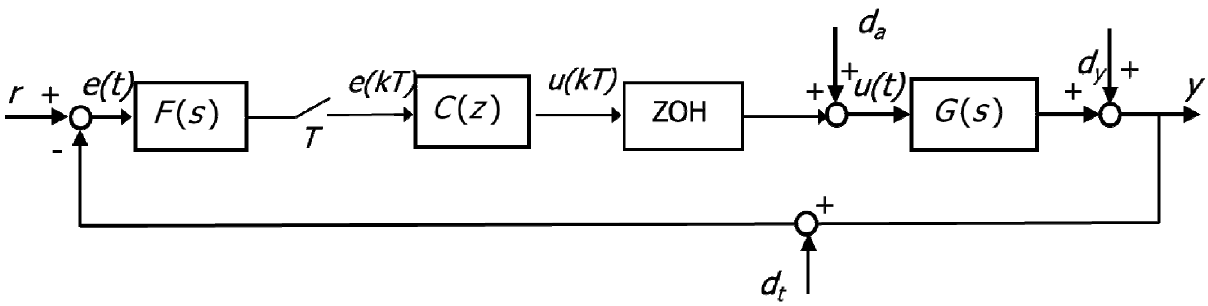
Computer and Electronic Engineering

Laboratory practice n. 6

Objectives: Digital control design, DT LTI systems solution

Problem 1 digital control design through emulation

Given the control structure



where

$$G(s) = \frac{40}{s^2 + 4s - 9.81}$$

Assume a sampling time $T_s = 0.02$ s. Design a digital controller C(z) in order to meet the following requirements:

- $\left|e_r^{\infty}\right| \le 0.25,\ r(t) = 2t\varepsilon(t)$
- $\left|y_{d_t}^{\infty}\right| \le 0.01,\ d_t(t) = \delta_t \sin(\omega_t t),\ \left|\delta_t\right| \le 0.1,\ \omega_t \ge 90$ rad/s
- $\hat{S} \le 20\%$
- $t_r \le 0.3$s
- $\max_t \left|u(t)\right| \le 5$ when $r(t) = 0.2\varepsilon(t)$

At the end of the design evaluate, through time domain simulation, the maximum amplitude of the input signal *u*(*t*) in the presence of the disturbance d_t, only

Steady state requirements analysis and design (4 points, quit the exercise evaluation in the presence of either a "destabilizing" steady state controller or the wrong type of the control system)

Report here all the steps needed to analyse the steady state requirements including

- How you handle steady state errors in the presence of both polynomial and sinusoidal signals
- The steady state design procedure (i.e. system type, choice of K_c)
- The K_c sign discussion (with detailed motivations obtained using the nyquist diagram)

Report the expression of the steady state controller in the form $C_{SS}(s) = \frac{K_c}{s^h}, K_c = ..., h = ...$

$C_{SS}(s) =$

Transient and other requirements analysis (2 points)

Describe here how you analyzed the transient requirements in order to get the data useful for the design ($\omega_{c,des}$, T_p, S_p, …), including considerations on the input saturation constraints (if present)

Design procedure description (5 points)

Please resume and deeply motivate all the design steps performed to obtain the final controller

Describe here the design procedure including all the choices you did in order to get the final controller (in order to better describe the procedure, include also qualitative but conceptually correct sketches of the obtained Nichols plots). Please **avoid the inclusion of any Matlab code**. All the design steps must be adequately motivated.
The evaluation of this part includes:

- Completeness of the reported discussion and documentation in support of the made design choices, e.g. why controller parameters have been chosen in a given way
- Choice of the controller form, e.g. too complicated as the case of not necessary networks
- Details on the design of the anti-aliasing filter (if needed and designed)

Report the expression of the final controller in the **dc-gain form**

(e.g. $C_0(s) = K_c \dfrac{1+s/\omega_d}{1+s/(m_d\omega_d)}, K_c = \ldots, \omega_d = \ldots, m_d = \ldots$ **this only an example!!**)

(**If the expression of C0(s) is missing quit the exercise evaluation -1 point if provided in the wrong form**)

$C_0(s) =$

Report the expression of the final digital controller in the polynomial form

$C_d(z) =$ discretization method

Details on the Butterworth anti-aliasing filter (if designed and not needed: –2 points)

$\omega_h =$ $\gamma =$ $\omega_f =$ n =

Performance evaluation (5 points)

Use simulation in order to evaluate the achieved performance (5 points)
(0,5 each correct evaluation, 0 if the evaluation is wrong or missing
0,5 if the requirement has been satisfied (within 5%),
0 for each unsatisfied requirement with an error > 5%
-0,5 for each unsatisfied requirement with an error > 15%
-1 for each unsatisfied requirement with an error > 30%)

Report below the exact (i.e. not approximate) values obtained in simulation.

- $|e_r^\infty| =$
- $|y_{d_t}^\infty| =$
- $\hat{S} =$
- $t_r =$
- $\max_t |u(t)| =$ in the presence of $r(t) = 0.2\varepsilon(t)$

Final evaluation after design

(1 if the evaluation and the employed procedure are correct (within 10%), 0 if it is wrong or missing)

$u_{MAX} =$

Report here details on the procedure performed to compute/evaluate the requested quantity.

Save results >> `save Results_AC_s123456 G C0 Ts Cd F` **(-3 if not done)**

Problem 2 (DT LTI systems solution)

Given the LTI system:

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ 0.1 & -0.3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 7 & 7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Compute the state x(k) and the output y(k) responses when $x(0)=[0,0]^T$, and u(k) = ε(k).

(Answer:
$$x(k) = \begin{bmatrix} 0.8\overline{3} + 0.9524(-0.5)^k - 1.7857(0.2)^k \\ 0.8\overline{3} - 0.4761(-0.5)^k - 0.3571(0.2)^k \end{bmatrix} \varepsilon(k), y(k) = (11.\overline{6} + 3.3341(-0.5)^k - 15(0.2)^k)\varepsilon(k))$$
)

Problem 3 (DT LTI systems solution, modal analysis and stability)

Given the discrete time LTI system (sampling time is supposed T = 1 s):

$$x(k+1) = \begin{bmatrix} 0.5 & -1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

- Compute the output y(k) response when $x(0)=[2,1]^T$, and u(k) = $0.9^k\varepsilon$(k).
- Compute the system natural modes and perform the modal analysis.
- Study the internal and BIBO stability properties of the given system.

(Answer: $y(k) = (-2 \cdot 0.5^k + 10 \cdot 0.9^k)\varepsilon(k)$, the natural modes are 0.5^k (geometrically convergent) and 1^k (bounded), the system is internally stable and BIBO stable))

Problem 4 (DT LTI systems internal stability)

Study the internal stability properties of a discrete time LTI system characterized by the following dynamic matrix *A* (sampling time is supposed T = 1 s).

$$A = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Answer: the system is stable)