AUTOMATIC CONTROL

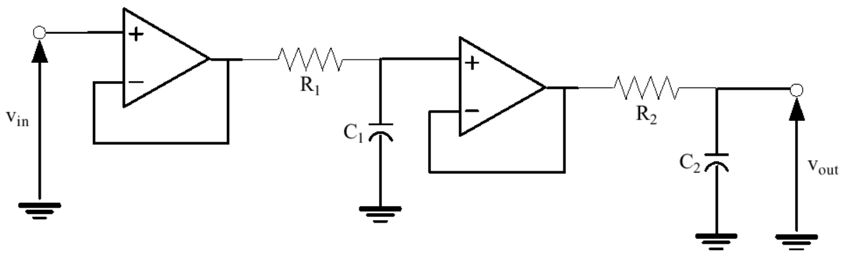
Computer, Electronic and Telecommunications Engineering

Laboratory practice n. 1

Objectives: simulation of LTI dynamical systems through the MatLab/Simulink environment, computation of the time response of LTI dynamical systems through the solution of the system state equations (in Laplace domain), state space and transfer function representation of electric circuits.

Problem 1

Consider the dynamic system below:



Voltages $v_{in}(t)$ and $v_{out}(t)$ are the system input and output respectively. Suppose that the operational amplifiers are ideal and that the numerical values of resistances and capacitances are: $R_1 = R_2 = 1k\Omega$, $C_1 = C_2 = 1\mu F$.

1. Derive the system state space representation assuming as system states the voltages $v_{C1}(t)$ and $v_{C2}(t)$ across capacitors C_1 e C_2 respectively.
2. Build a suitable Simulink model for the simulation of the given system.
3. Simulate the time behaviour of $v_{C1}(t)$ and $v_{C2}(t)$ when: $v_{in}(t) = 2\varepsilon(t)$ V, $v_{C1}(0) = v_{C2}(0) = 0$ V. Plot the obtained time courses of $v_{C1}(t)$ e $v_{C2}(t)$.
4. Repeat the simulation using the initial conditions $v_{C1}(0) = 100$ mV, $v_{C2}(0) = 50$ mV.
5. Simulate the time behaviour of $v_{C1}(t)$ and $v_{C2}(t)$ when: $v_{in}(t) = 2 \sin(100t)$ V, $v_{C1}(0) = v_{C2}(0) = 0$ V. Plot the obtained time courses of $v_{C1}(t)$ e $v_{C2}(t)$ together with the input $v_{in}(t)$.
6. Simulate the time behaviour of $v_{C1}(t)$ and $v_{C2}(t)$ when: $v_{in}(t) = 2 \sin(1000t)$ V, $v_{C1}(0) = v_{C2}(0) = 0$ V. Plot the obtained time courses of $v_{C1}(t)$ e $v_{C2}(t)$. Plot the obtained time courses of $v_{C1}(t)$ e $v_{C2}(t)$ together with the input $v_{in}(t)$.
7. Compute the state responses $v_{C1}(t)$ and $v_{C2}(t)$ when: $v_{in}(t) = 2\varepsilon(t)$ V, $v_{C1}(0) = 100$ mV, $v_{C2}(0) = 50$ mV. Use MatLab to plot the time behaviour of $v_{C1}(t)$ and $v_{C2}(t)$.
8. Plot the obtained time behavior of $v_{C1}(t)$ e $v_{C2}(t)$. (compare with the plots you obtained at point 4.)

(Answer:

$$x(t)=\begin{bmatrix} v_{C_1}(t) \\ v_{C_2}(t) \end{bmatrix}=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad u(t)=v_{in}(t) \quad y(t)=v_{out}(t)=x_2(t)$$

$$A=\begin{bmatrix} -1/R_1C_1 & 0 \\ 1/R_2C_2 & -1/R_2C_2 \end{bmatrix}=\begin{bmatrix} -1000 & 0 \\ 1000 & -1000 \end{bmatrix} \quad B=\begin{bmatrix} 1/R_1C_1 \\ 0 \end{bmatrix}=\begin{bmatrix} 1000 \\ 0 \end{bmatrix} \quad C=\begin{bmatrix} 0 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 0 \end{bmatrix}$$

$$v_{C1}(t)=(-1.9\,e^{-1000t}+2)\varepsilon(t) \quad v_{C2}(t)=(-1.95\,e^{-1000t}-1900t\,e^{-1000t}+2)\varepsilon(t)$$)

Problem 2

Consider the following LTI system:

$$\dot{x}(t)=\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}x(t)+\begin{bmatrix} 5 \\ 8 \end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix} -1 & 3 \end{bmatrix}x(t)+8u(t)$$

Compute the state x(t) and the ouput y(t) responses when *x(0)=[2,2]^T* and the input u(t) is a step signal with amplitude 2.

(Answer: $x(t)=\begin{bmatrix} 3.0\overline{6}\,e^{5t}-0.\overline{6}e^{-t}-0.4 \\ 6.1\overline{3}\,e^{5t}+0.\overline{6}e^{-t}-4.8 \end{bmatrix}\varepsilon(t) \quad y(t)=(15.\overline{3}\,e^{5t}+2.\overline{6}e^{-t}+2)\varepsilon(t)$)

Problem 3

Given the LTI system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\begin{bmatrix} 4 & 0 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Compute the output y(t) response when *x(0)=[0,0]^T*, and $u_1(t) = 0$, $u_2(t) = 2\,\delta(t)$.

(Answer: $y(t)=(2.3094e^{-0.5t}\cos(0.866t-1.5708))\varepsilon(t)$)

Problem 4

Given the LTI system:

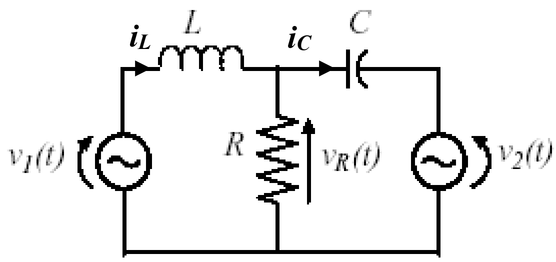
$$\dot{x}(t)=\begin{bmatrix} 0 & 6 \\ -1 & -5 \end{bmatrix}x(t)+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

compute the exponential matrix e^{At} of the system; (hint: recall that $e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$)

(Answer: $e^{At}=\begin{bmatrix} 3e^{-2t}-2e^{-3t} & 6e^{-2t}-6e^{-3t} \\ -e^{-2t}+e^{-3t} & -2e^{-2t}+3e^{-3t} \end{bmatrix}$)

Problem 5

In the electric dynamic system below, generator voltages $v_1(t)$ e $v_2(t)$ are the inputs while the voltage $v_R(t)$ is the output:



Using basic circuit equations derive matrices A, B, C and D of the state space representation. (Hint. Use $i_L(t)$ e $v_C(t)$ as state variables)

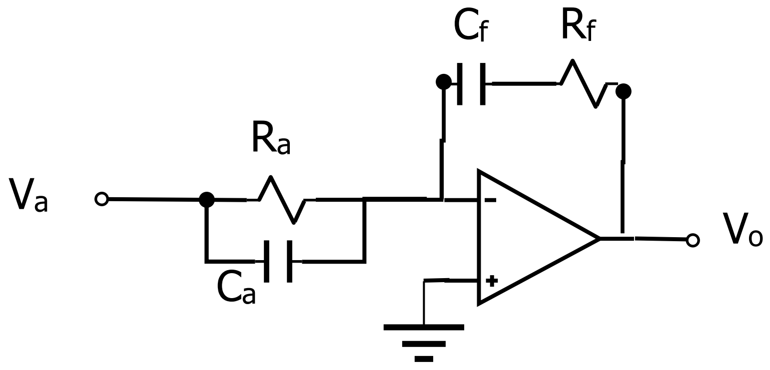
$$u(t)=\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}=\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \quad x(t)=\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad y(t)=v_R(t)=x_2(t)+u_2(t)$$

Answer:

$$A=\begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad B=\begin{bmatrix} 1/L & -1/L \\ 0 & -1/RC \end{bmatrix} \quad C=\begin{bmatrix} 0 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 0 & 1 \end{bmatrix}$$

Problem 6

Show that the transfer function $H(s) = V_o(s)/V_a(s)$ of the circuit below is not proper.



(Answer:

$$H(s)=-\frac{(1+R_fC_fs)(1+R_aC_as)}{R_aC_fs}$$)