13.1. $u=-kx \ (k>1)$. $\dot{x}=\frac{u}{1+x^2}+x$

将该控制应用，$\dot{x}=\frac{-kx}{1+x^2}+x$.

对原点处线性化. $\frac{\partial f}{\partial x}=A=\frac{-k(1+x^2)-(-kx)(2x)}{(1+x^2)^2}+1 \Big|_{x=0}$

$\therefore \dot{x}=(-k+1)x$, $k>1$.

$\therefore$ $Re(\lambda)<0$，则原点渐近稳定.

$\therefore$ 反馈控制使系统实现了局部稳定. 与12.1同理，系统是区域稳定而非全局稳定的. 但若有 x 在吸引区外 ($x>\sqrt{k-1}$)，则系统不能实现全局稳定.

令 $u=-(k+1)x^3-(k+1)x$. 可消除非线性. 得到 $\dot{x}=-kx$.

13.2.

$\dot{x}_1=x_2$

$\dot{x}_2=u$

$y=x_1$

$u=-k_1x_1-k_2z$.

$\dot{z}=-k_3z-k_4x_1$.

整理，

$\dot{x}_1=x_2$

$\dot{x}_2=-k_1x_1-k_2z$.

$\dot{z}=-k_4x_1-k_3z$.

$y=x_1$.

$$\therefore \sqrt[3]{\left(\frac{k_2k_4}{2}-\frac{k_1k_3}{3}+\left[\left(\frac{k_3^3}{27}+\frac{k_1k_3}{3}-\frac{k_2k_4}{2}\right)^2+\left(\frac{k_1}{3}-\frac{k_3^2}{9}\right)^3\right]^{\frac{1}{2}}-\frac{k_3^3}{27}\right)}>0.$$

14.1. $\ddot{x} = -au_1\sin\theta$. (u_1,u_2)为控制输入. $(a>0,\ g>0$ 为常数.

$\ddot{y} = au_1\cos\theta - g$. $(x, y, \theta, \dot{x}, \dot{y})$可测. $\to (x_d, y_d, 0, 0, 0)$.

$\dot{\theta} = u_2$.

①. $x_1 = y$, $x_2 = \dot{y}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = au_1\cos\theta - g$, 在原点线性化, $\dot{x}_2 = au_1 - g = a(u_1 - \frac{g}{a})$.

保证 $y = y_d$ 时期望平衡点及相应稳态控制为:

$x_{1s} = y_d$, $x_{2s} = 0$. $u_s = \frac{g}{a}$.

令 $e_i = x_i - x_{is}$ $(i=1,2)$, $u = u_s - k_1e_1 - k_2e_2$. 相应闭环系统为:

$\dot{e}_1 = e_2$. $\dot{e}_2 = a(u_s - k_1e_1 - k_2e_2) - g$. 闭环系统在原点线性化:

$\dot{e}_1 = e_2$. $\dot{e}_2 = -k_1ae_1 - k_2ae_2$. 选择 $k_1>0$, $k_2>0$, 控制 y.

②. $x_1 = x$ $x_2 = \dot{x}$ $x_3 = \theta$.

$\dot{x}_1 = x_2$

$\dot{x}_2 = ~~au_1\cos~~ -au_1\sin\theta\, x_3$

$\dot{x}_3 = u_2$.

$x_{1s} = x_d$. $x_{2s} = 0$ $u_2 = 0$.

14.3. 附加积分器得到扩展系统:

$~~\dot{x}_0 = x_1 - y_d~~$ $\dot{x}_0 = x_1 - y_d$, $\dot{x}_1 = x_2$ $\dot{x}_2 = au_1 - g$.

$u_1 = -k_0x_0 - k_1(x_1 - y_d) - k_2x_2$.

$\therefore$ 闭环系统: $\dot{x}_0 = x_1 - y_d$

$\dot{x}_1 = x_2$

$\dot{x}_2 = a(-k_0x_0 - k_1(x_1 - y_d) - k_2x_2) - g$.

选择 $k\neq 0$, 则存在平衡点: $x_{1s} = y_d$ $x_{2s} = 0$ $x_{0s} = -\frac{g}{k_0a}$.

令 $e_i = x_i - x_{is}$ $(i=0.1.2)$，得到線性化闭环系统方程：

$$\dot{e}_0 = e_1$$

$$\dot{e}_1 = e_2.$$

$$\dot{e}_2 = -a(k_0 e_0 + k_1 e_1 + k_2 e_2)$$ ，选择 (k_0, k_1, k_2) 使得其特征多项

式为 Hurwitz 多项式，反馈控制率为 $u_1 = -k_0 x_0 - k_1(x_1 - y_d) - k_2 x_2$

$$x_0 = x_1 - y_d.$$

14.4. 设 $\{M, N\} = \left\{ \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix} \right\}$

$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ 行满秩即 A.B.C 满秩.

$$M = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad N = \begin{bmatrix} B \\ 0 \end{bmatrix}.$$

$$\text{rank}\begin{bmatrix} s \cdot I_{n+p} - M & N \end{bmatrix} = \text{rank}\begin{bmatrix} s \cdot I_n - A & 0 & B \\ -C & sI_p & 0 \end{bmatrix}$$

对于任何 $s \neq 0$，$\text{rank}\begin{bmatrix} sI_n - A & 0 & B \\ -C & sI_p & 0 \end{bmatrix} = p + \text{rank}\begin{bmatrix} sI_n - A & B \end{bmatrix}$.

对于 $s=0$，$\text{rank}\begin{bmatrix} sI_n - A & 0 & B \\ -C & sI_p & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$.

∴ 可知 $\{A, B\}$ 能控为 $\{M, N\}$ 能控的充要条件.