Automatic Control

AC - L17
18

Loopshaping design examples

Ex. Real negative zero as lead network AC_L17

after steady state design $L'(s) = C_{ss}(s)G(s)$
@ $\omega_{c,des} = 1$ rad/s

$$|L'(j\omega_{c,des})| \simeq -4{,}6\ \text{dB}$$

$$\angle L'(j\omega_{c,des}) \simeq -176°$$

$$\downarrow \qquad \rightarrow \Delta\varphi \simeq 60°$$

$$-123° \div -108°$$

→ phase lead is needed

↓ realized through

real negative zero - - - - - - - →

In the presence of $\max_t |u(t)| \leq 60$
$r(t) = \varepsilon(t)$
(see AC_L18)
set $\omega_{norm} = 1{,}5$

$$C_z(z) = \left(1 + \frac{s}{\omega_z}\right)$$

$$\omega_{norm} = 1{,}8$$

$$\omega_z = \frac{\omega_{c,des}}{\omega_{norm}} = 0{,}\bar{5}\ \text{rad/s}$$

after "zero" → $L''(s) = L'(s)\,C_z(s)$

$$|L''(j\omega_{c,des})| \simeq 1{,}62\ \text{dB}$$

$$\angle L''(j\omega_{c,des}) = -115°$$

$\omega_c \simeq 1{,}17$ rad/s

OK

↓

run simulation

$\hat{S} \cong 7,18\,\%$ $\quad t_r = 1,74\,s$ $\quad t_{s,5\%} \cong 2,92\,s$

$|e_r^\infty| = 0,196$ $\quad r(t) = 0,25\,t\,\varepsilon(t)$

Final controller

$$C(s) = \frac{K_c}{s}\left(1 + \frac{s}{\omega_z}\right)$$

$$K_c = 34 \qquad \omega_z = 0,\bar{5}\ \text{rad/s}$$

```
clear all
close all
clc

s=tf('s');

% Plant tf
G=0.045/(s^2+2.6*s+1.2);

% steady state controller
Kc=34
C_SS=Kc/s;
L1=G*C_SS; % loop function update

% transient requirements
T_p=0.42;
S_p=2.68;
wc_des=1;

% nichols diagram for L1
figure(1)
nichols(L1,'b'), hold on
T_grid(T_p)
S_grid(S_p)
%return

% zero design
wnorm=1.8; % in the example of AC_L18 wnorm = 1.5;
wz=wc_des/wnorm
C_Z=(1+s/wz);
L2=C_Z*L1; % loop function update
nichols(L2,'r')
C=C_SS*C_Z; % controller tf update
%return
%simulation
% simulation with step reference signal
r_s = 1; % switch impose step reference
rho = 1;
delta_a = 0;
delta_y = 0;
t_stop = 10;
sim('control_structure_sim')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
```

```matlab
ylabel('y(t)')
legend('r(t)','y(t)')

figure
grid on
hold on
plot(u.time,u.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('u(t)')
%return

% simulation with ramp reference signal
r_s = -1; % switch imposes ramp reference
rho = 0.25;
delta_a = 0;
delta_y = 0;
t_stop = 50;
sim('control_structure_sim')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')

figure
plot(e.time,e.data,'b','linewidth',1.5)
grid on
xlabel('t (s)')
ylabel('e(t)')
```

Ex. sinusoidal disturbance

$|y^{\infty}_{d_y}| \leq 6 \cdot 10^{-3}$ $\quad d_y(t) = \delta_y \sin(\omega_y t)$

$|\delta_y| \leq 6 \cdot 10^{-2}$ $\quad \omega_y \leq 0.08$ rad/s

$$|y^{\infty}_{d_y}| = \delta_y \, |S(j\omega)| \leq 6 \cdot 10^{-3}$$

$\forall \omega \leq 0{,}08$

$\Downarrow$

$$|S(j\omega)| \leq \frac{6 \cdot 10^{-3}}{6 \cdot 10^{-2}} = 10^{-1} = -20\,dB = M_S^{LF}$$

$\omega_{c,des} \gg \omega_y = 0{,}08$ rad/s

$\omega_{c,des} = 10\,\omega_y = 0{,}8$ rad/s

$(t_r, t_{s,2\%} \rightarrow \omega_{c,des} = 1$ rad/s$)$

- <u>After steady state design</u> $\quad L'(s) = C_{ss}(s) G(s)$

$|L'(j\omega_{c,des})| = -17{,}4$ dB $\quad \boxed{\omega_{c,des} = 1 \text{ rad/s}}$

$\angle L'(j\omega_{c,des}) = -200°$

$\downarrow$

$-123° \div -108°$

$77° \quad 92°$

phase lead needed

$\downarrow$

real negative zero $\quad 1 + \frac{s}{\omega_z}$

$\omega_{norm} = 5{,}5$ (6 is ok)

$C_z(s) = 1 + \frac{s}{\omega_z}$

$\omega_z = \frac{\omega_{c,des}}{\omega_{norm}} = 0{,}1818$ rad/s

after zero $L''(s) = C_z(s)\, L'(s)$

$|L''(j\omega_{c,des})| = 2{,}64$ dB

$\angle L''(j\omega_{c,des}) = -120°$

(there are intersections with S_p and T_p)

$|S(j\omega_y)| = -23$ dB

$\downarrow$

OK!

run simulation anyway

$\downarrow$

$\hat{s} \cong 13{,}5\ \%$

$\downarrow$

→ Magnitude attenuation through Lag Network

$$C_I(s) = \frac{1 + \frac{s}{m_I \omega_I}}{1 + \frac{s}{\omega_I}}$$

$$m_I = 10^{\frac{2{,}64}{20}} = 1.349$$

$$\omega_I = \frac{\omega_{c,des}}{\alpha\, m_I} \underset{\alpha = 10}{=} 0{,}0741 \text{ rad/s}$$

after Lag $\quad L'''(s) = C_I(s)\, L''(s)$

$$|L'''(j\omega_{c,des})| \cong 0\,dB$$

$$\angle L'''(j\omega_{c,des}) = -121^\circ$$

$|L'''(j\omega_y)| = 22{,}8\,dB$

$\downarrow$

$|S(j\omega_y)| \cong -22{,}8\,dB$

$\downarrow$

OK

Run simulation

- $\hat{s} = 9{,}57\,\%$ $\boxed{OK}$ $\quad t_r = 1{,}99\,s$ $\boxed{OK}$ $\quad t_{s,2\%} = 4{,}092$ $\boxed{OK}$

- $|Y_{d_y}^{\infty}| = 4{,}401 \cdot 10^{-3}$ $\boxed{OK}$

($\omega_y = 0{,}08$ rad/s $\rightarrow T_y = \frac{2\pi}{\omega_y} \cong 78{,}5\,s$ $\rightarrow$ run simulation with t_stop = 400 s)

sinusoid period

- $|Y_{d_a}^{\infty}| = 0{,}02$ $\boxed{OK}$ $\quad d_a(t) = 0{,}03\,t\,\varepsilon(t)$

All the requirements have been satisfied

- Final controller

$$C(s) = \frac{K_c}{s}\left(1 + \frac{s}{\omega_z}\right)\frac{1 + \frac{s}{m_I\,\omega_I}}{1 + \frac{s}{\omega_I}}$$

$K_c = -1{,}5$

$\omega_z = 0{,}1818$

$\omega_I = 0{,}0741$

$m_I = 1{,}349$

```
clear all
close all
clc

s=tf('s');

% Plant tf
G=-0.3/(s^2+1.75*s+0.37);

% steady state controller
Kc=-1.5
C_SS=Kc/s;
L1=G*C_SS;% loop function update
M_S_LF=-20;

% transient requirements
T_p=0.42;
S_p=2.68;
wc_des=1;

% nichols diagram for L1
figure(1)
nichols(L1,'b'), hold on
T_grid(T_p)
S_grid(S_p)
S_grid(M_S_LF)

%return


% lead network design
mD=10
wnorm=1.5;
wD=wc_des/wnorm
C_D=(1+s/wD)^2/((1+s/(mD*wD)))^2;
L2=C_D*L1; % loop function update
C=C_D*C_SS; % controller tf update
figure(1), hold on
nichols(L2,'r')
%return




% gain adjustment
K=10^(4/20)
L3=K*L2 % loop function update
```

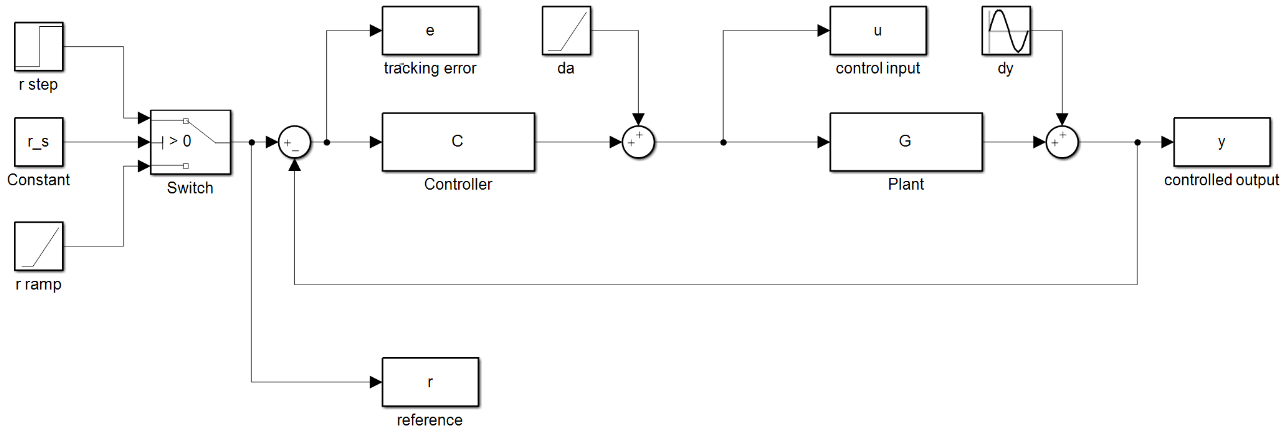
```
C=K*C;  % controller tf update
figure(1), hold on
nichols(L3,'k')

%return
% simulation with step reference signal
r_s = 1; % switch impose step reference
rho = 1;
delta_a = 0;
delta_y = 0;
wy = 0.08;
t_stop = 20;
sim('control_structure_sim_2')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')

%return
% simulation with disturbance dy
r_s = 1; % switch impose step reference
rho = 0;
delta_a = 0;
delta_y = 6e-2;
wy = 0.08;
t_stop = 400;
sim('control_structure_sim_2') % modify control_structure_sim.slx
                               % since disturbance d_a is a ramp
figure
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')

% simulation with disturbance da
r_s = 1; % switch impose step reference
rho = 0;
delta_a = 0.03;
delta_y = 0;
t_stop = 100;
sim('control_structure_sim_2') % modify control_structure_sim.slx
                               % since disturbance d_a is a ramp
figure
```

```
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
```



Solution 2.

- realize the phase lead throug a double real negative zero (to avoid the use of a lag network)

$$C_z(s) = \left(1 + \frac{s}{\omega_z}\right)^2$$

suppose $\Delta\varphi = 90^\circ$

($\rightarrow$ 45° each zero)

$\rightarrow$ $\omega_{norm} = 1$

$$\omega_z = \frac{\omega_{c,des}}{\omega_{norm}} = 1 \text{ rad/s}$$

After double zero $\quad L''(s) = C_z(s)\, L'(s)$

$$|L''(j\omega_{c,des})| = -6{,}29 \text{ dB}$$

$$\angle L''(j\omega_{c,des}) = -110^\circ$$

A magnitude increase is needed but it is not sufficient to move the Nichols plot outside the T_p locus

$\rightarrow$ More phase lead is needed

$\rightarrow$ $\omega_{norm} = 1{,}4$

$$\omega_z = \frac{\omega_{c,des}}{\omega_{norm}} = 0{,}7143 \text{ rad/s}$$

$\rightarrow$ $|L''(j\omega_{c,des})| = -2{,}91 \text{ dB}$

$$\angle L''(j\omega_{c,des}) = -90{,}9^\circ$$

Magnitude increase of ~ 3 dB needed

$$K = 10^{\frac{3}{20}} \qquad L'''(s) = K\, L''(s)$$

$$|L'''(j\omega_{c,des})| \cong 0{,}09 \text{ dB}$$

$$\angle L'''(j\omega_{c,des}) = -90{,}9^\circ \qquad ✓$$

At this point $C(s) = \dfrac{K_c}{s}\left(1+\dfrac{s}{\omega_z}\right)^2$

$\downarrow$

not proper

→ Add a closure pole $\omega_p \gg \omega_c$

Let's try with $\omega_p = 10$ rad/s

$$C_p(s) = \frac{1}{1+\frac{s}{\omega_p}} \qquad L^{IV}(s) = C_p(s)\, L'''(s)$$

$$|L^{IV}(j\omega_{c,des})| = 0{,}049 \text{ dB}$$

$$\angle L^{IV}(j\omega_{c,des}) = -96{,}6^\circ \qquad ✓$$

$$|L^{IV}(j\omega_y)| \simeq 26{,}3 \text{ dB} \rightarrow |S(j\omega_y)| \simeq -26{,}3 \text{ dB} < 20 \text{ dB} \qquad ✓$$

Simulation

$\hat{s} \cong 6{,}02\,\%$ ✓ $\qquad t_r = 2{,}69\,s$ ✗ $\qquad t_{s,2\%} \cong 8.13\,s$ ✗

→ Increase K → $K = 10^{\frac{4}{20}} = 1{,}5849$

....

$\hat{s} \cong 5{,}68\,\%$ ✓ $\qquad t_r \cong 2{,}49\,s$ ✓ $\qquad t_{s,2\%} \cong 7{,}67\,s$ ✓

$|Y_{d_y}^{\infty}| \cong 2{,}6 \cdot 10^{-3}$ ✓

$|Y_{d_a}^{\infty}| \cong 0{,}012$ ✓

Final controller

$$C(s) = \frac{K_c}{s}\left(1+\frac{s}{\omega_z}\right)^2 \cdot K \cdot \frac{1}{1+\frac{s}{\omega_p}}$$

$K_c = -1{,}5$

$\omega_z = 0{,}7143\ \text{rad/s}$

$\omega_p = 10\ \text{rad/s}$

$K = 1{,}5849$

```
clear all
close all
clc

s=tf('s');

% Plant tf
G=-0.3/(s^2+1.75*s+0.37);

% steady state controller
Kc=-1.5
C_SS=Kc/s;
L1=G*C_SS;% loop function update
M_S_LF=-20;

% transient requirements
T_p=0.42;
S_p=2.68;
wc_des=1;

% nichols diagram for L1
figure(1)
nichols(L1,'b'), hold on
T_grid(T_p)
S_grid(S_p)
S_grid(M_S_LF)
%return

% double zero design
wnorm=1.4;
wz=wc_des/wnorm
C_Z=(1+s/wz)^2;
L2=C_Z*L1; % loop function update
C=C_Z*C_SS; % controller tf update
figure(1), hold on
nichols(L2,'r')
%return

% gain adjustment
K=10^(4/20)
L3=K*L2 % loop function update
C=K*C;  % controller tf update
figure(1), hold on
nichols(L3,'k')
%return

% closure pole design
wp=10
```

```
C_P=1/(1+s/(wp));
L4=L3*C_P; % loop function update
C=C*C_P % controller tf update
figure(1)
nichols(L4,'m')
%return

% simulation with step reference signal
r_s = 1; % switch impose step reference
rho = 1;
delta_a = 0;
delta_y = 0;
wy = 0.08;
t_stop = 20;
sim('control_structure_sim_2')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')


return
% simulation with disturbance dy
r_s = 1; % switch impose step reference
rho = 0;
delta_a = 0;
delta_y = 6e-2;
wy = 0.08;
t_stop = 400; % about 5 periods of the sinusoid
sim('control_structure_sim_2') % modify control_structure_sim.slx
                               % since disturbance d_y is a sinusoid
figure
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')

% simulation with disturbance da
r_s = 1; % switch impose step reference
rho = 0;
delta_a = 0.03;
delta_y = 0;
t_stop = 100;
```

```
sim('control_structure_sim_2') % modify control_structure_sim.slx
                                % since disturbance d_a is a ramp
figure
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
```

Solution 3 (Homework)

- phase lead through double lead network

$$\rightarrow C(s) = \frac{K_c}{s}\left(\frac{1+\frac{s}{\omega_D}}{1+\frac{s}{m_D\omega_D}}\right)^2 \cdot k$$

$\rightarrow$ possible solution:

$K_c = -1,5$

$\omega_D = 0,\bar{6}$ rad/s) ($\omega_{norm} = 1,5$)

$m_D = 10$

$k = 1,5849$