# **Automatic Control**

# **Dynamical systems simulation using Simulink**

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# Dynamical systems simulation

Simulation solves a dynamical system through numerical integration of the state equation

$$\dot{x}(t) = f(x(t), u(t))$$

In this case, the system responses x(t) and y(t) can be directly plotted.

Simulation can be easily performed in MatLab environment by means of its extension Simulink.

Basically, Simulink is a tool that allows us to easily build a model of dynamical system using block diagram notation and, at the same time, to compute the solution by means of already implemented numerical solvers.

# Dynamical systems solution

**<u>Problem</u>** Given the dynamical system described by the state space representation

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = g(x(t), u(t))$$

and

- the time course of u(t)
- the initial state x(0)

compute the system responses x(t) and y(t) through integration of the state equation

$$\dot{x}(t) = f(x(t), u(t))$$

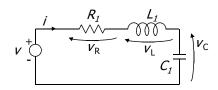
In general, for nonlinear system it is not possible to compute the solution in closed form  $\rightarrow$  perform simulation

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# Simulation of an electric system

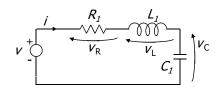
Consider the dynamical system below.



- ullet Voltages v(t) and  $v_c(t)$  are the system input and output respectively
- The numerical values of the components are

$$R_1 = 68 \ \Omega$$
 ,  $C_1 = 4 \mu F$  ,  $L_1 = 10 \ mH$ 

# Simulation of an electric system



• Derive the state space representation of the system assuming as system states the current i(t) through L<sub>1</sub> and voltage v<sub>C</sub>(t) across C<sub>1</sub> respectively

$$u(t) = v(t), x(t) = \begin{bmatrix} i(t) \\ v_{C}(t) \end{bmatrix} = \begin{bmatrix} x_{1}(t) \\ x_{2}(t) \end{bmatrix}, y(t) = v_{C}(t) = x_{2}(t)$$

$$\begin{cases} \dot{x}_{1}(t) = \frac{1}{L_{1}} [-R_{1} x_{1}(t) - x_{2}(t) + u(t)] \\ \dot{x}_{2}(t) = \frac{1}{C_{1}} x_{1}(t) \end{cases} \rightarrow A = \begin{bmatrix} -\frac{R_{1}}{L_{1}} & -\frac{1}{L_{1}} \\ \frac{1}{C_{1}} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L_{1}} \\ 0 \end{bmatrix}$$

$$y(t) = x_{2}(t) \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$$

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# Simulation of an electric system

In order to effectively handle the simulation procedure a suitable MatLab script file is developed.

```
clc
clear all
close all
% define parameter values
R1=68;
L1=10e-3;
C1=4e-6;
A=[-R1/L1 -1/L1;1/C1 0];
B=[1/L1;0];
C=[0 1];
D=0:
% define initial condition
x0=[0;0];
```

# Simulation of an electric system

Simulation of an electric system

$$\begin{cases} \dot{X}_{1}(t) = \frac{1}{L_{1}} \left[ -R_{1} X_{1}(t) - X_{2}(t) + u(t) \right] \\ \dot{X}_{2}(t) = \frac{1}{C_{1}} X_{1}(t) \end{cases}$$

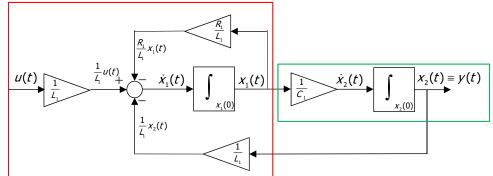
$$y(t) = X_{2}(t)$$

- Simulate the time behaviour of i(t) and v<sub>c</sub>(t) when:  $v(t) = \varepsilon(t) V$ ,  $i(0) = 0 A V_{C1}(0) = 0 V$
- Simulate the time behaviour of i(t) and v<sub>c</sub>(t) when:  $v(t) = \varepsilon(t) V$ ,  $i(0) = 10 \text{ mA } V_{C1}(0) = 50 \text{ mV}$

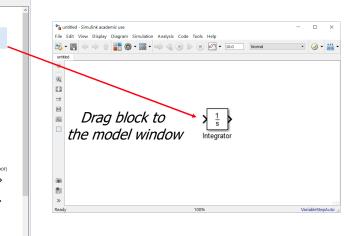
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$$\begin{cases} \dot{x}_{1}(t) = \frac{1}{L_{1}} \left[ -R_{1} x_{1}(t) - x_{2}(t) + u(t) \right] \\ \dot{x}_{2}(t) = \frac{1}{C_{1}} x_{1}(t) \end{cases}$$

$$y(t) = x_{2}(t)$$



# Simulation of an electric system Simulink Library Browse A - 🛂 - 🛅 💣 = 3 mulank/Continuous Commonly Used Blocks Continuous Continuous Discontinuities Math Operations Motol Verifications Signal Routing Signal Routing Signal Routing Signal Routing Signal Routing Communications System Tobooks Disports Signal Routing Signal R



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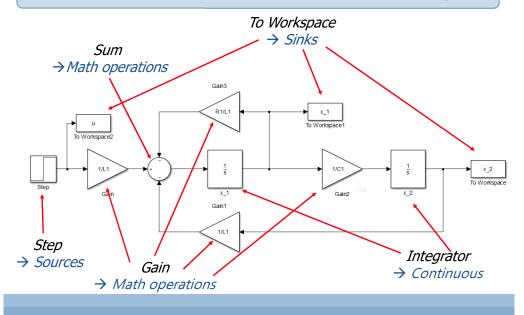
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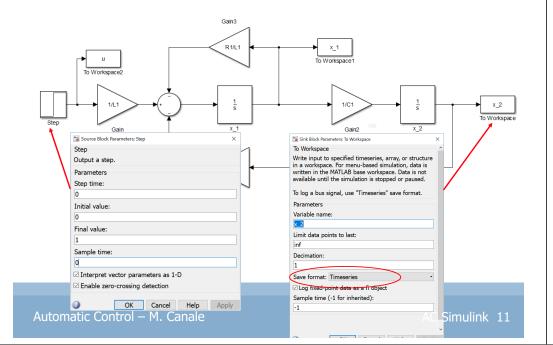
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# Simulation of an electric system

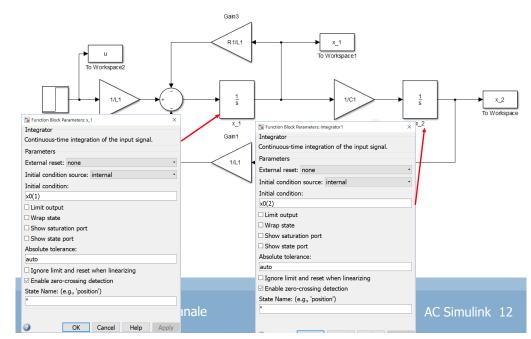


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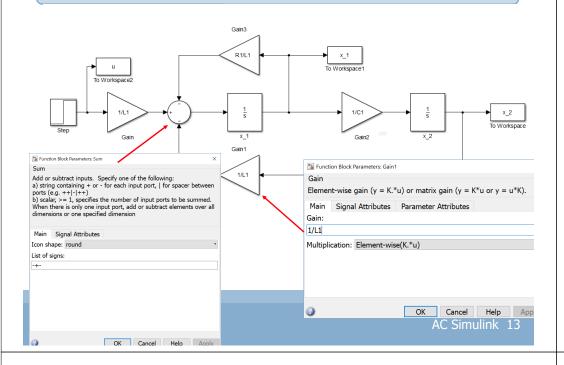
# Simulation of an electric system



# Simulation of an electric system

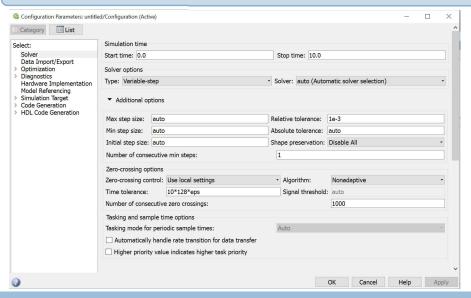


# Simulation of an electric system

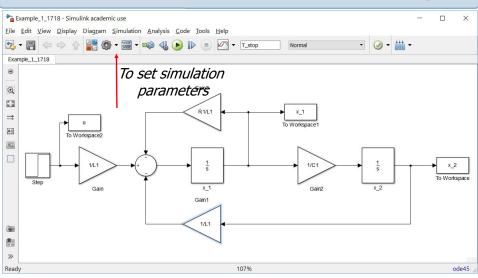


# Simulation of an electric system

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Simulation of an electric system



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# Hints on main simulation parameters

In order to perform a "good" simulation for a continuous time LTI system suitable numerical integration algorithms\* should be chosen:

- Variable step algorithms such as ode45 and ode23 should be chosen since they are quite efficient and provide more accurate results.
- In the presence of "stiff" systems, i.e. system with eigenvalues characterized by large stiffness ratio

$$\frac{\max_{i}(\operatorname{Re}(\lambda_{i}))}{\min_{i}(\operatorname{Re}(\lambda_{i}))}, i=1,\ldots,n$$

try different solvers such as ode15s, ode23s, ... to determine which one performs best.

\* See textbooks 1. (Chapter 13) and 3. (Chapter 12) for more details.

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# Hints on main simulation parameters

Moreover configuration parameters such as Stop Time, Max Step size and Min Step size, should be chosen\* according to

- the time constants of the system  $\tau_i = \frac{1}{|\operatorname{Re}(\lambda_i)|}, i = 1, ..., n$
- the period T of the involved periodic signals (e.g. sinusoidal inputs,...)

Some hints are

• Stop Time 
$$\rightarrow \begin{cases} (5 \div 10) \cdot \tau_{\text{max}} \\ (5 \div 10) \cdot \mathcal{T}_{\text{max}} \end{cases}$$

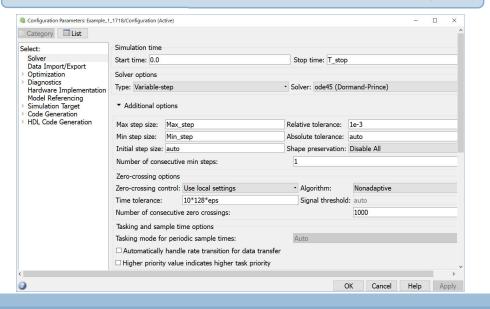
- Max Step size  $\Rightarrow \begin{cases} (0.01 \div 0.1) \cdot \tau_{\text{max}} \\ (0.05 \div 0.1) \cdot \mathcal{T}_{\text{max}} \end{cases}$
- Min Step size  $\rightarrow$  (0.001 ÷ 0.01) ·  $\tau_{\min}$

\* See textbook 3. (Chapter 12) for more details.

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# Simulation of an electric system



#### Simulation of an electric system

$$\begin{cases} \dot{x_1}(t) = \frac{1}{L_1} \left[ -R_1 \, x_1(t) - x_2(t) + u(t) \right] & R_1 = 68 \, \Omega \, , \, C_1 = 4 \mu F \, , \, L_1 = 10 \, mH \\ \dot{x_2}(t) = \frac{1}{C_1} \, x_1(t) & \begin{cases} \dot{x_1}(t) = -6800 \, x_1(t) - 100 x_2(t) + 100 u(t) \\ \dot{x_2}(t) = 250000 x_1(t) \end{cases} \\ \dot{x_2}(t) = 250000 \, x_1(t) & \lambda_{1,2} = \left( -3.4 \pm j \, 3.6 \right) \cdot 10^3 \rightarrow \tau_{1,2} = -\frac{1}{\left| \mathbb{R}e(\lambda_{1,2}) \right|} = 0.29 \, \text{ms} \end{cases}$$
Stop Time  $\rightarrow 0.003 \, \text{s}$ 
Max Step size  $\rightarrow 0.00003 \, \text{s}$ 
Min Step size  $\rightarrow 0.000003 \, \text{s}$ 
% define simulation parameters
% constant
lambda=eig(A);
tau=1./abs(real(lambda))
$$\begin{cases} \mathbf{x_1}(t) = -6800 \, x_1(t) - 100 x_2(t) + 100 u(t) \\ \dot{x_2}(t) = 250000 x_1(t) \\ \hline{\mathbf{x_2}(t)} = 2500000 x_1(t) \\ \hline{\mathbf{x_2}(t)} = 250000 x_1(t) \\ \hline{\mathbf{x_2}(t)} = 2500000 x_1(t) \\ \hline{\mathbf{x_2}(t)} = 2500000 x_1(t$$

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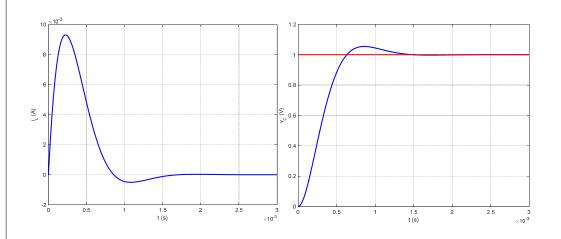
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# Simulation of an electric system

```
% run simulation
sim('Example_1_1617')
% plot results
figure
plot(x_1.time,x_1.data,'b','linewidth',2)
grid on
xlabel('t (s)'), ylabel('i_L (A)')

figure
plot(x_2.time,x_2.data,'b','linewidth',2)
grid on
hold on
plot(u.time,u.data,'r','linewidth',2)
xlabel('t (s)'), ylabel('v_C (V)')
```

# Simulation of an electric system

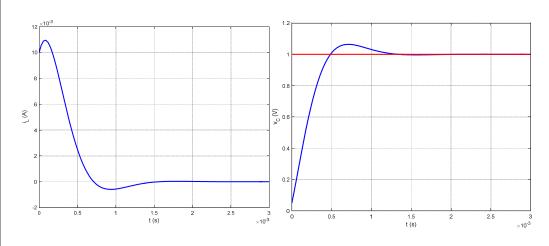


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# Simulation of an electric system

x0=[10e-3; 50e-3];



# Simulation of an electric system

```
% define simulation parameters
clc
                                 T stop = 0.003; % about 10*tau
clear all
                                 Max step = 0.00003; % about 0.1*tau
close all
                                 Min step = 0.000003; % about 0.01*tau
% define parameter values
R1=68:
                                 % run simulation
L1=10e-3;
                                 sim('Example 1 1617')
C1=4e-6;
                                 % plot results
A=[-R1/L1 -1/L1;1/C1 0];
                                 figure
B=[1/L1;0];
                                plot(x 1.time,x 1.data,'b','linewidth',2)
C=[0 1];
                                 grid on
D=0:
                                 xlabel('t (s)'), ylabel('i L (A)')
% define initial condition
x0=[0;0];
                                 plot(x 2.time,x 2.data,'b','linewidth',2)
% compute eig and time
                                 grid on
% constant
                                hold on
lambda=eig(A);
                                plot(u.time, u.data, 'r', 'linewidth', 2)
tau=1./abs(real(lambda))
                                 xlabel('t (s)'), ylabel('v C (V)')
```

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#### **Textbooks about Simulink**

- 1. James B. Dabney, Thomas L. Harman, *Mastering Simulink*, Prentice Hall, 2004
- 2. A. Cavallo, R. Setola, F. Vasca, *Using Matlab, Simulink and Control System ToolBox: A Practical Approach*, Pearson, 1996
- 3. A. Cavallo, R. Setola, F. Vasca, *La nuova guida a Matlab Simulink e Control Toolbox*, Liguori, 2002

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