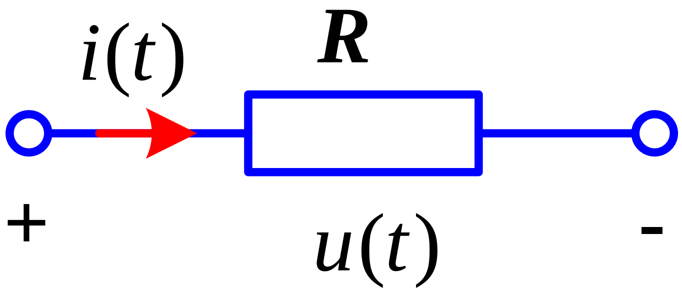
14.4 运算电路

1. 基尔霍夫定律的运算形式

对任一结点 $\sum I(s) = 0$ **KCL**

对任一回路 $\sum U(s) = 0$ **KVL**

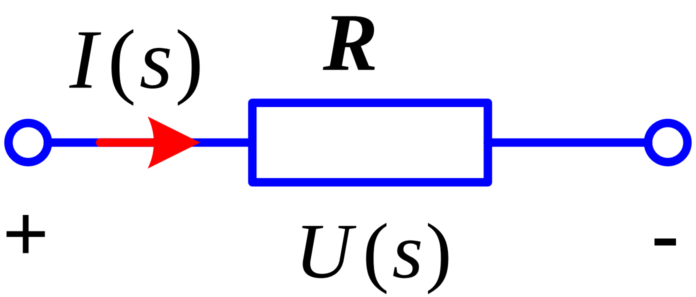
2. 元件电压、电流关系的运算形式VCR



$$u(t) = Ri(t)$$

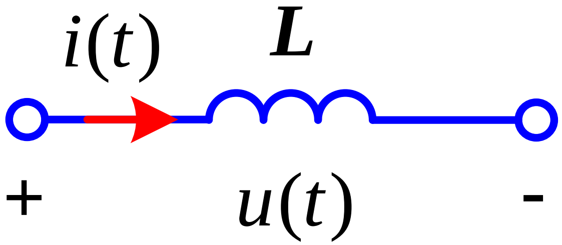
$$\int_{0-}^{\infty} u e^{-st} dt = \int_{0-}^{\infty} R i e^{-st} dt = R\int_{0-}^{\infty} i e^{-st} dt$$

R 运算电路图



$$U(s) = RI(s)$$

R ——运算电阻



$$u(t)=L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

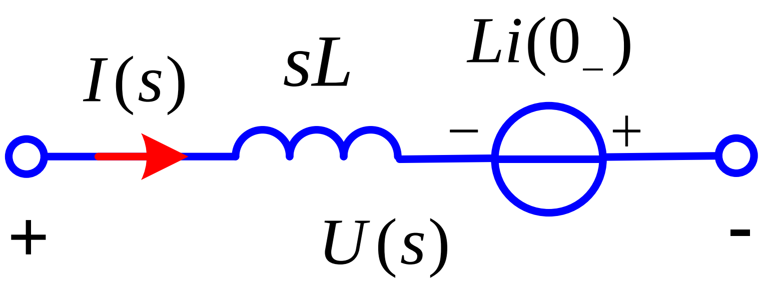
$$\int_{0-}^{\infty}ue^{-st}dt=\int_{0-}^{\infty}L\frac{di}{dt}e^{-st}dt=L\int_{0-}^{\infty}\frac{di}{dt}e^{-st}dt$$

$$U(s)=L\left[sI(s)-i(0\_)\right]$$

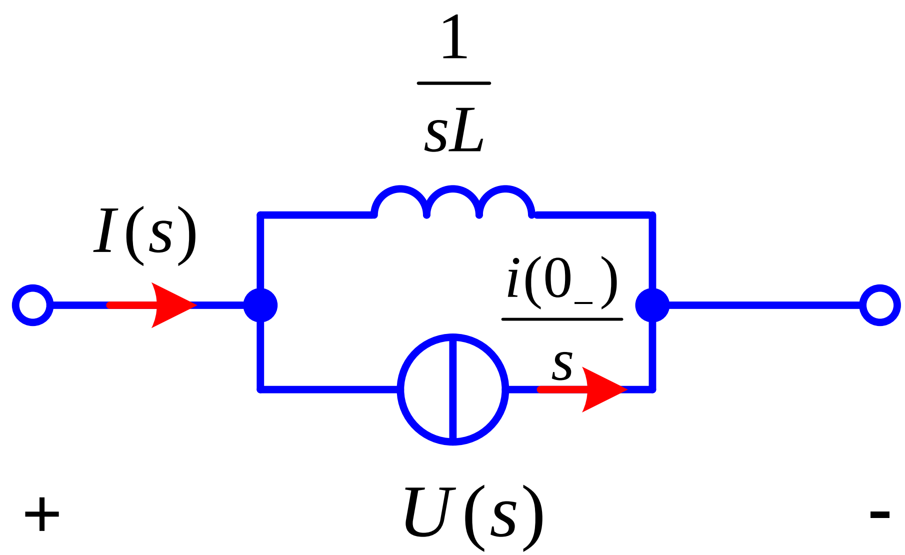
$$U(s)=sLI(s)-Li(0\_)$$

$$I(s)=\frac{U(s)}{sL}+\frac{i(0\_)}{s}$$

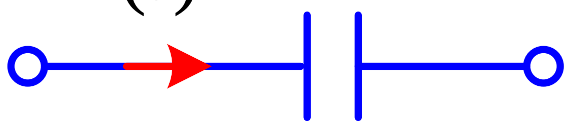
L 运算电路图



运算阻抗



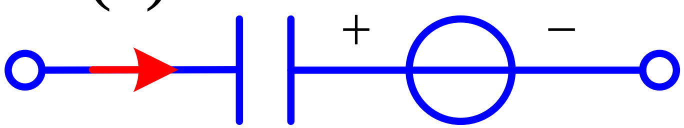
运算导纳



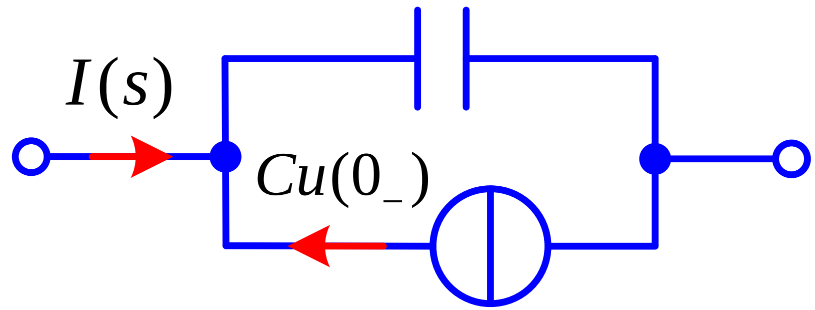
$$u(t) = \frac{1}{C}\int_{0\_}^{t} i(\xi)\,\mathrm{d}\xi + u(0\_)$$

$$U(s) = \frac{1}{sC}I(s) + \frac{u(0\_)}{s} \qquad I(s) = sCU(s) - Cu(0\_)$$

C 运算电路图

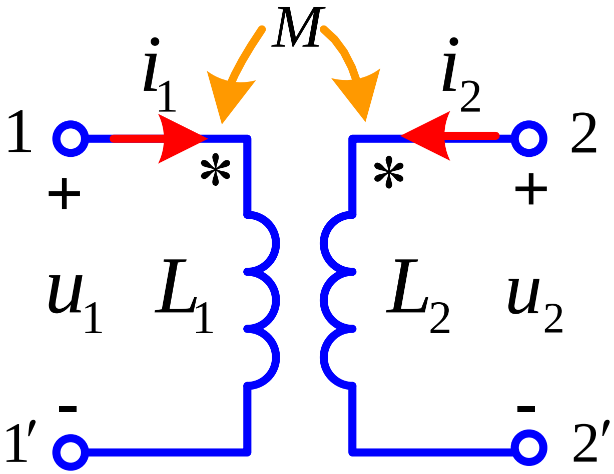


运算阻抗



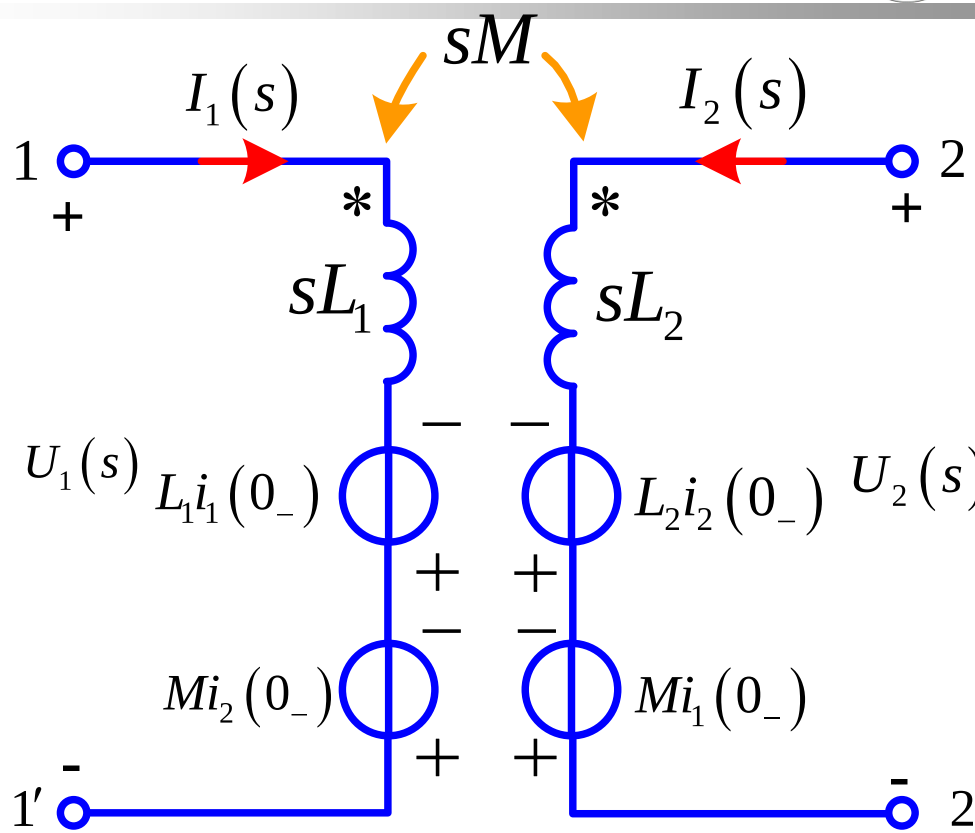
运算导纳

耦合电感 运算电路图



$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = M \frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

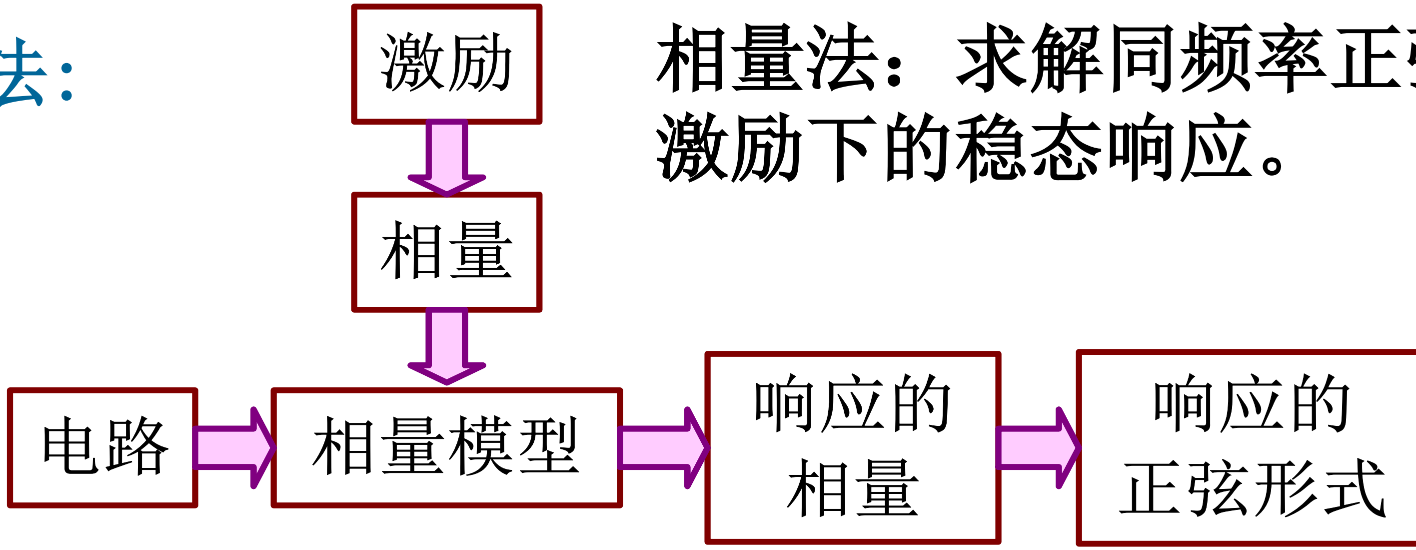


$$\begin{cases} U_1(s) = sL_1I_1(s) - L_1i_1(0\_) + sMI_2(s) - Mi_2(0\_) \\ U_2(s) = sL_2I_2(s) - L_2i_2(0\_) + sMI_1(s) - Mi_1(0\_) \end{cases}$$

运算法与相量法的比较(VCR)：

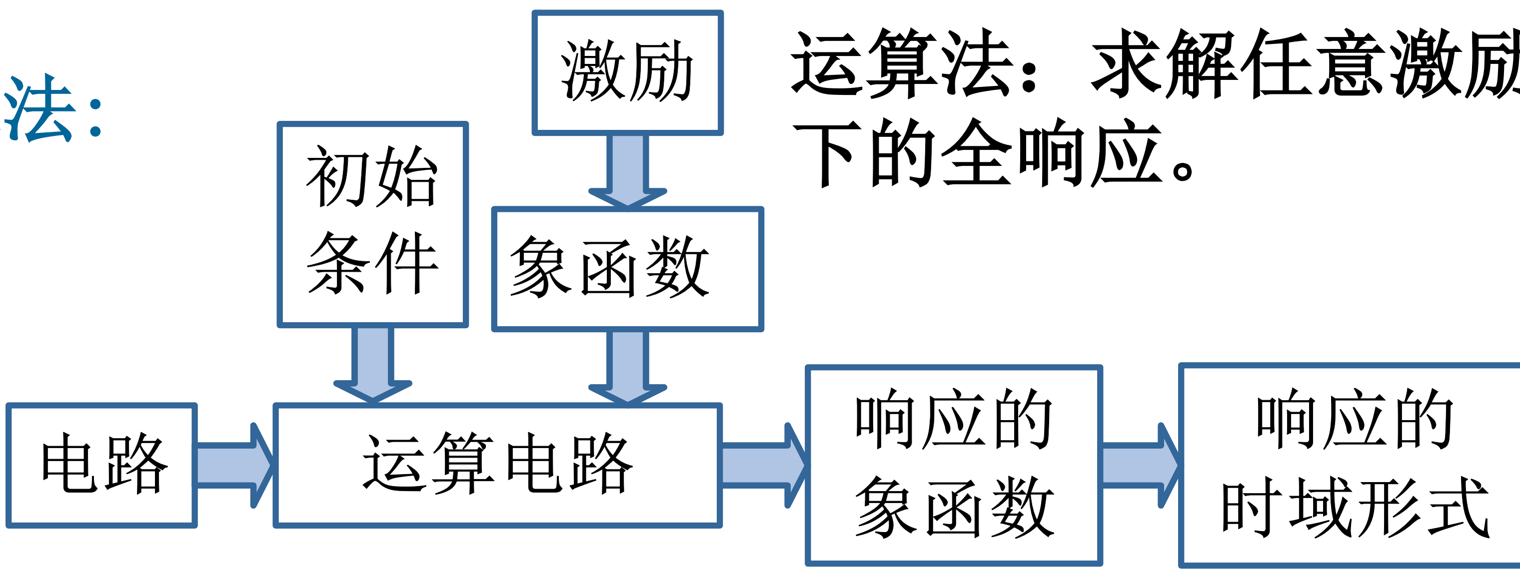
元　件	复阻抗 (相量法)	运算阻抗 (运算法)
R	R	R
L	$\mathrm{j}\omega L$	sL
C	$\dfrac{1}{\mathrm{j}\omega C}$	$\dfrac{1}{sC}$

相量法:



相量法：求解同频率正弦激励下的稳态响应。

运算法:



运算法：求解任意激励下的全响应。

运算法步骤：

（1）由换路前电路求出 $i_L(0_-)$、$u_C(0_-)$

（2）将激励变换成象函数

（3）作运算电路图

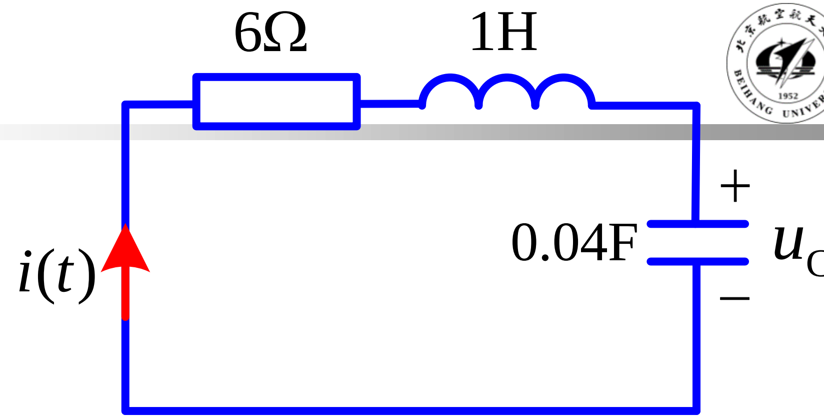
（4）求出响应的象函数

（5）拉氏反变换，求出响应的时域形式

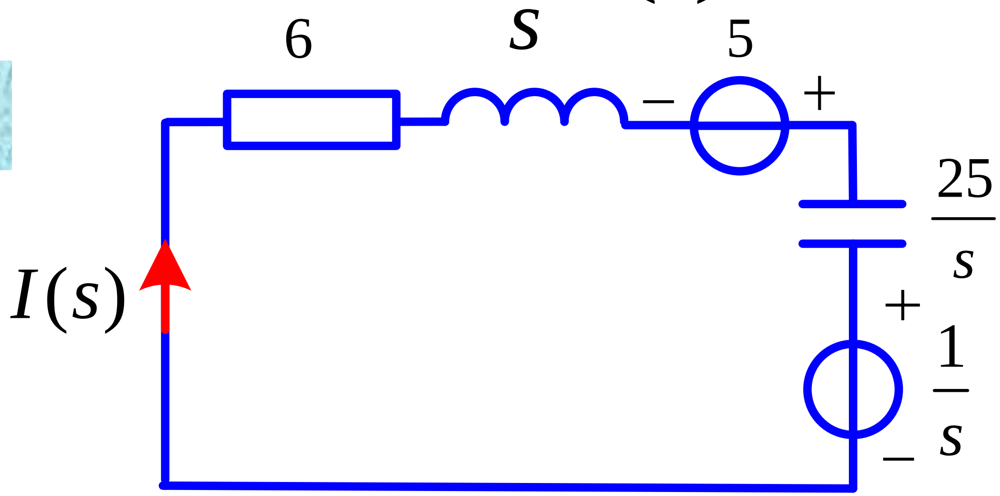
【例】 已知：$i(0_-) = 5\text{A}$

$u_C(0_-) = 1\text{V}$

求：零输入响应$i(t)$



解



$$\left(6 + s + \frac{25}{s}\right) I(s) = 5 - \frac{1}{s}$$

$$I(s)=\frac{5s-1}{s^2+6s+25}$$

$$=\frac{5s-1}{(s+3)^2+4^2}$$

$$=\frac{5(s+3)}{(s+3)^2+4^2}-\frac{4\times 4}{(s+3)^2+4^2}$$

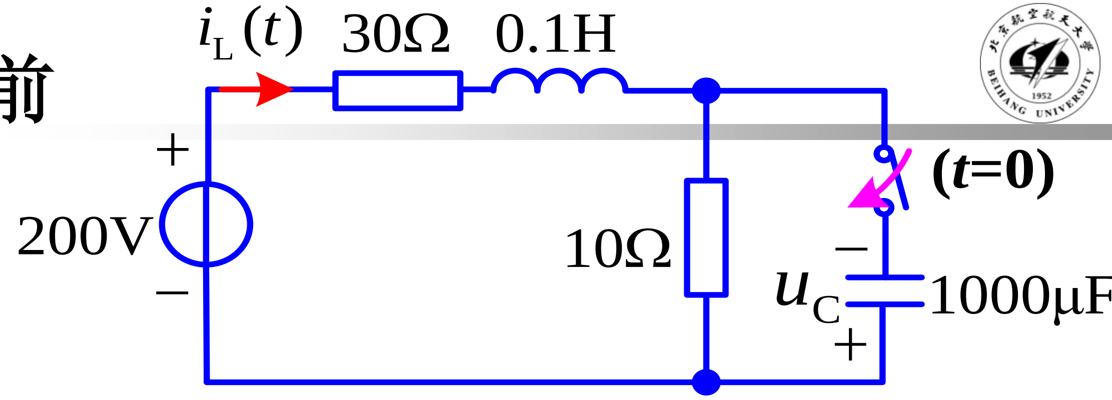
$$L\left[\mathrm{e}^{-\alpha t}\cos\omega t\right]=\frac{s+\alpha}{(s+\alpha)^2+\omega^2} \quad L\left[\mathrm{e}^{-\alpha t}\sin\omega t\right]=\frac{\omega}{(s+\alpha)^2+\omega^2}$$

$$i(t)=L^{-1}\left[I(s)\right]=5\mathrm{e}^{-3t}\cos 4t-4\mathrm{e}^{-3t}\sin 4t\mathrm{A}$$

【例】已知：开关闭合前电路已达稳态

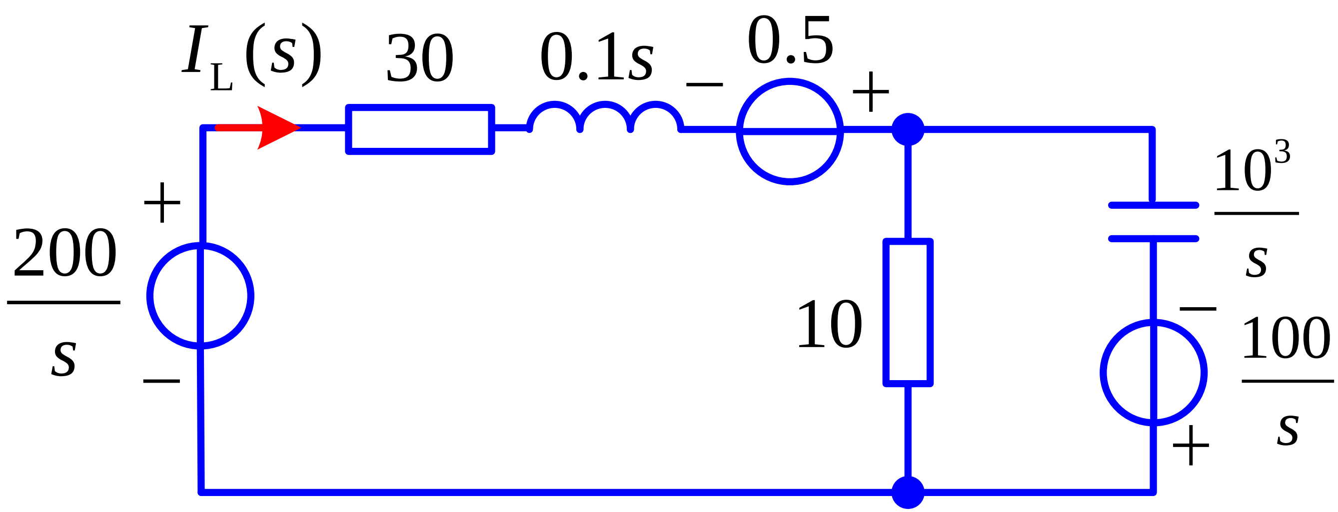
$u_C(0_-) = 100\text{V}$

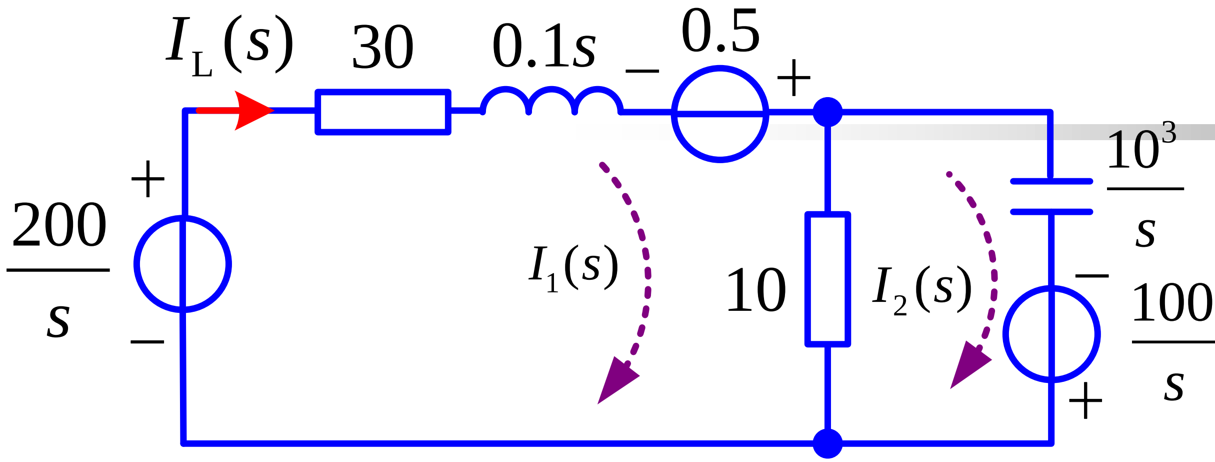
求：$t>0$ 时 $i_L(t)$



解

$$i_L(0\_) = \frac{200}{30+10} = 5\text{A} \qquad u_C(0\_) = 100\text{V}$$



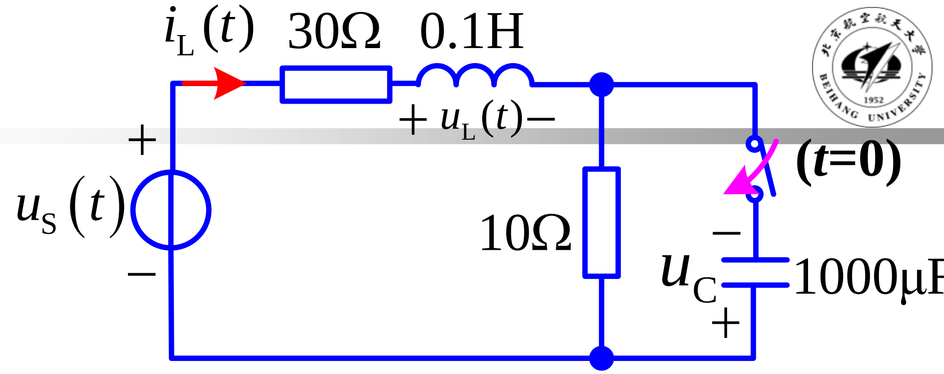


$$\begin{cases} (40+0.1s)I_1(s)-10I_2(s)=\dfrac{200}{s}+0.5 \\ (10+\dfrac{10^3}{s})I_2(s)-10I_1(s)=\dfrac{100}{s} \end{cases}$$

$$I_L(s)=I_1(s)=\frac{5(s^2+700s+40000)}{s(s+200)^2}$$

$$I_L(s)=\frac{5}{s}+\frac{1500}{(s+200)^2} \qquad i_L(t)=5+1500t\,e^{-200t}\ \text{A}$$

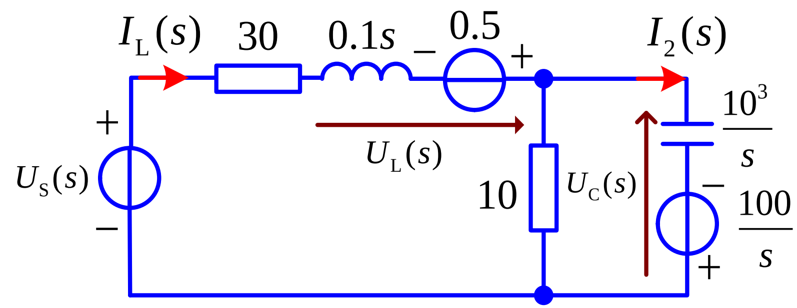
$$u_{\mathrm{L}}(t)=?\,u_{\mathrm{C}}(t)=?$$



$$U_{\mathrm{L}}(s)=0.1sI_{\mathrm{L}}(s)-0.5$$

$$U_{\mathrm{C}}(s)=-U_{\mathrm{S}}(s)+30I_{\mathrm{L}}(s)+U_{\mathrm{L}}(s)$$

或$U_{\mathrm{C}}(s)=-\dfrac{10^3}{s}I_2(s)+\dfrac{100}{s}$

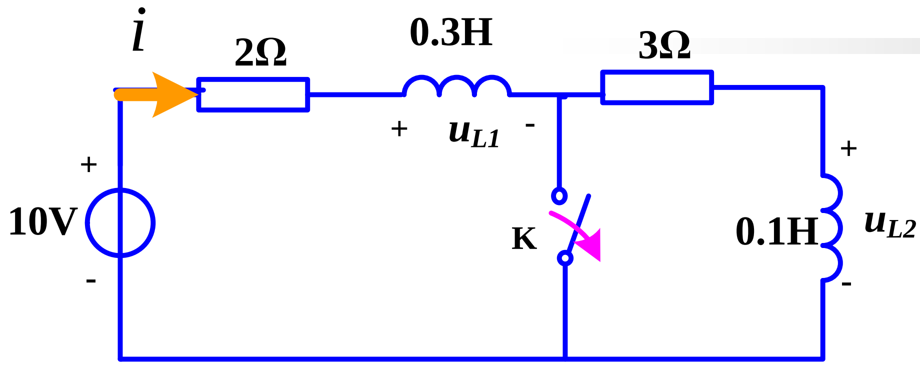


若$u_{\mathrm{S}}(t)=\mathrm{e}^{-2t}\varepsilon(t)\,\mathrm{V}$

$$U_{\mathrm{S}}(s)=\frac{1}{s+2}$$

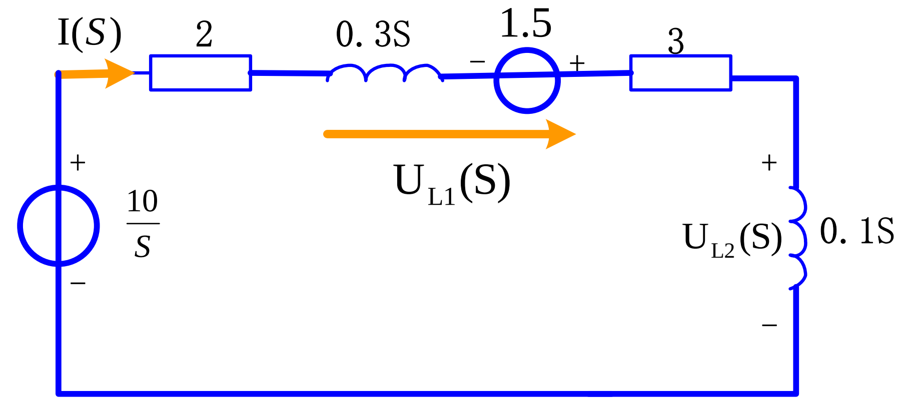
若 $\boxed{u_{\mathrm{S}}(t)=\varepsilon(t-t_0)\,\mathrm{V}}$ $\boxed{U_{\mathrm{S}}(s)=e^{-t_0 s}\dfrac{1}{s}}$

【例】 求：t>0时的 $u_{L1}(t)$、$u_{L2}(t)$。



解

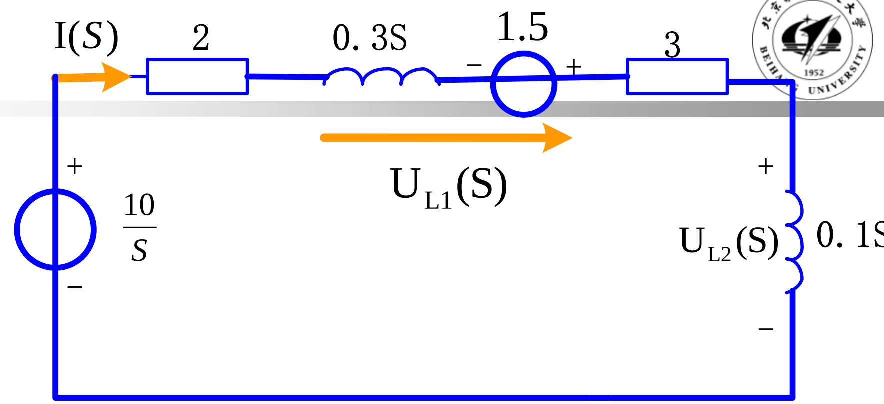
$$i(0_-)=\frac{10}{2}=5\text{A}$$



$$I(S)=\frac{\frac{10}{S}+1.5}{5+0.4S}=\frac{2}{S}+\frac{1.75}{S+12.5}$$

$$i(t)=2+1.75e^{-12.5t}(A)$$

在t=0时刻有跳变



$$I(S)=\frac{\frac{10}{S}+1.5}{5+0.4S}=\frac{2}{S}+\frac{1.75}{S+12.5}$$

$$U_{L1}(S)=0.3S\times I(S)-1.5=-0.375+\frac{-6.56}{S+12.5}$$

$$u_{L1}(t)=-0.375\delta(t)-6.56e^{-12.5t}(V)$$

在t=0时刻有冲激项

$$U_{L2}(S)=0.1S\times I(S)=0.375+\frac{-2.19}{S+12.5}$$

$$u_{L2}(t)=0.375\delta(t)-2.19e^{-12.5t}(V)$$

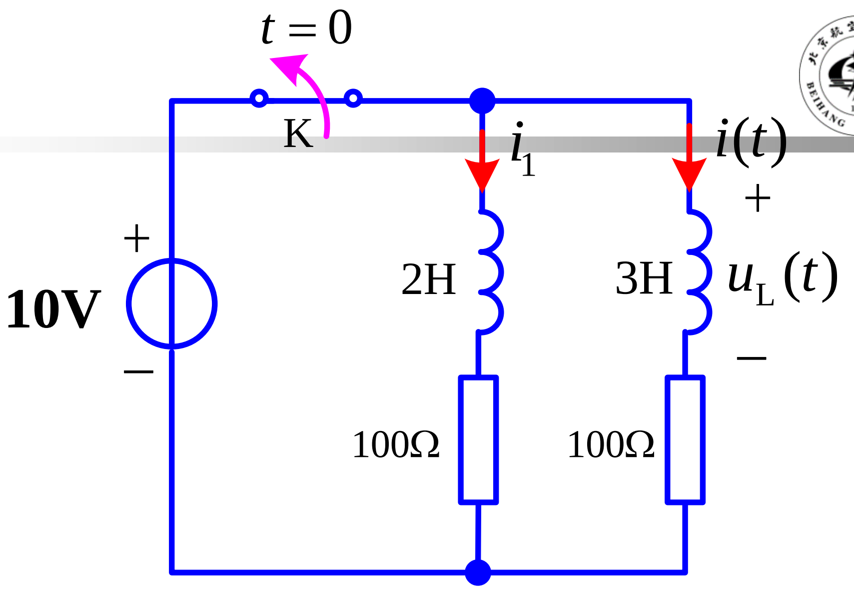
在t=0时刻有冲激项

$$u_{L1}+u_{L2}$$

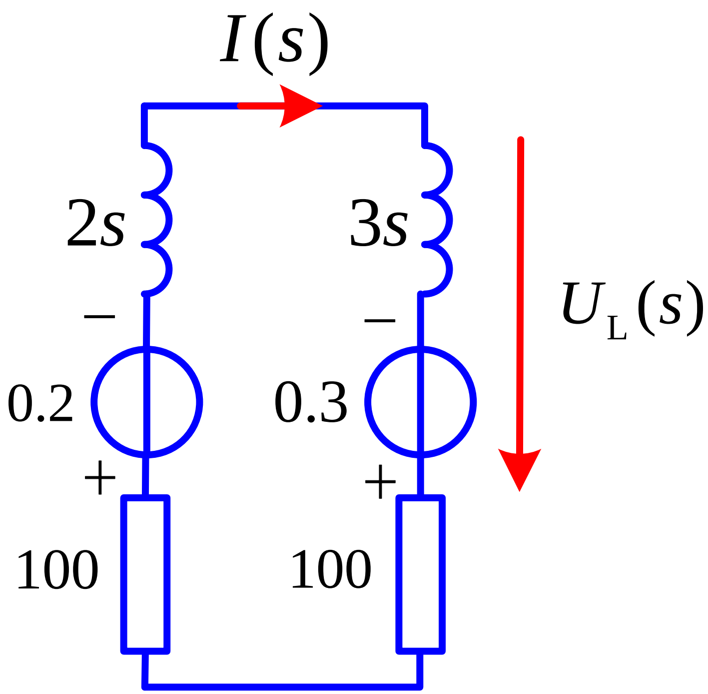
在t=0时刻无冲激项

【例】已知：开关打开前电路已达稳态

求：$t>0$时 $i(t)$ $u_L(t)$



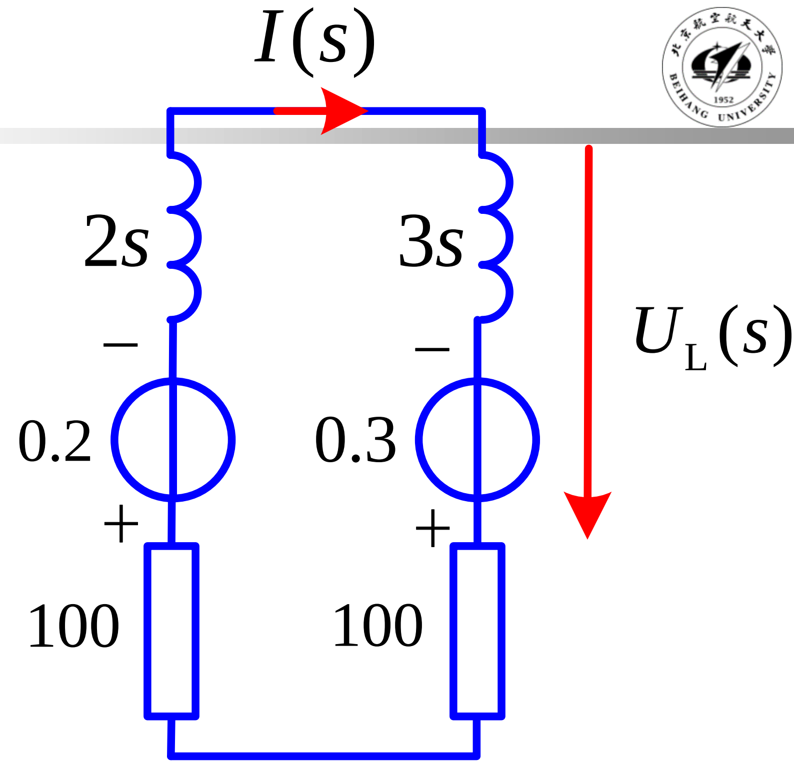
解



$$i(0_-)=\frac{10}{100}=0.1\text{A}$$

$$i_1(0_-)=\frac{10}{100}=0.1\text{A}$$

$$I(s)=\frac{0.3-0.2}{5s+200}=\frac{0.02}{s+40}$$



$$U_L(s)=3sI(s)-0.3$$

$$=\frac{0.06s}{s+40}-0.3$$

$$=-0.24-\frac{2.4}{s+40}$$

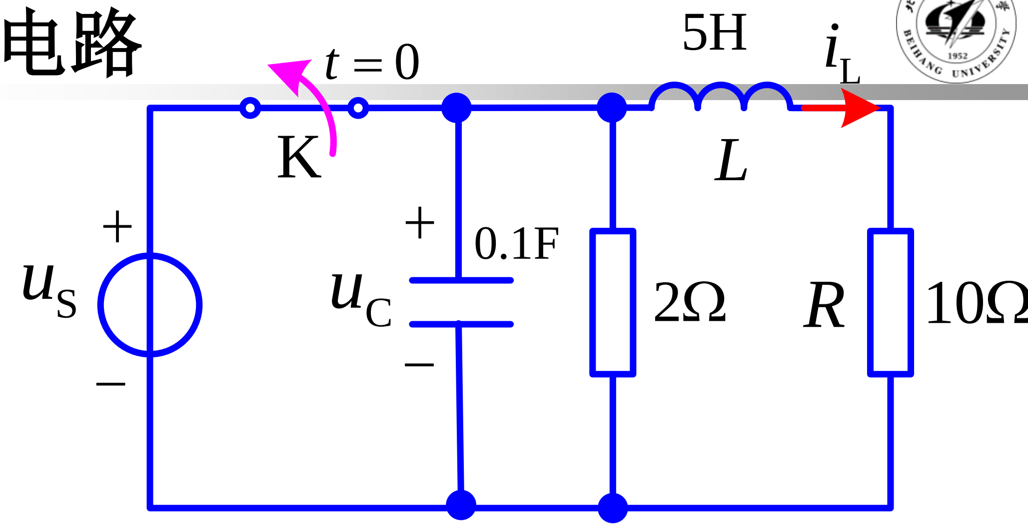
$$i(t)=L^{-1}[I(s)]=0.02\mathrm{e}^{-40t}\ \mathrm{A}$$

$$u_L(t)=L^{-1}[U_L(s)]=-0.24\delta(t)-2.4\mathrm{e}^{-40t}\ \mathrm{V}$$

【例】 已知：开关打开前电路已达稳态

$$u_S = 20\sin(2t+\frac{\pi}{2})\text{V}$$

求：t>0时 $u_C(t)$



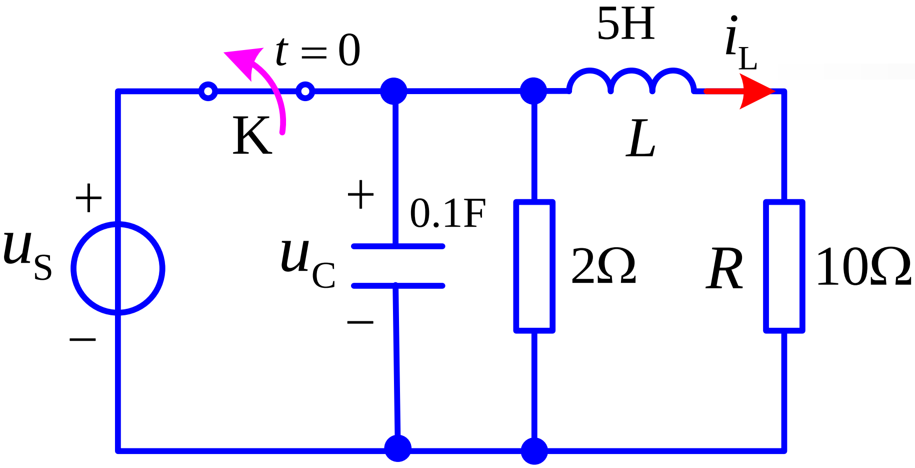
解

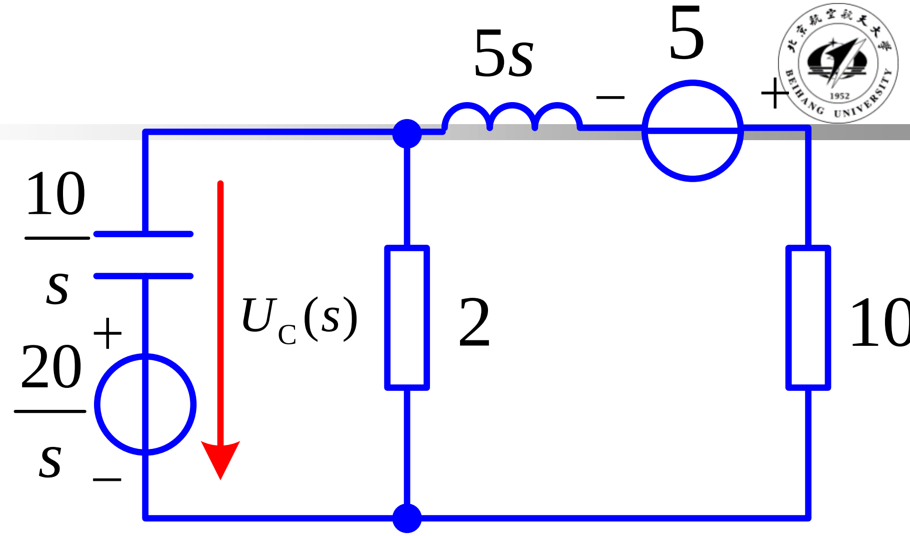
先求0_时刻初值

用相量法

$$\dot{U}_S = \frac{20}{\sqrt{2}}\angle\frac{\pi}{2}\text{V}$$

$$\dot{I}_L = \frac{\dot{U}_S}{R+\mathrm{j}\omega L} = \frac{\frac{20}{\sqrt{2}}\angle\frac{\pi}{2}}{10+\mathrm{j}2\times5} = 1\angle\frac{\pi}{4}\text{A}$$



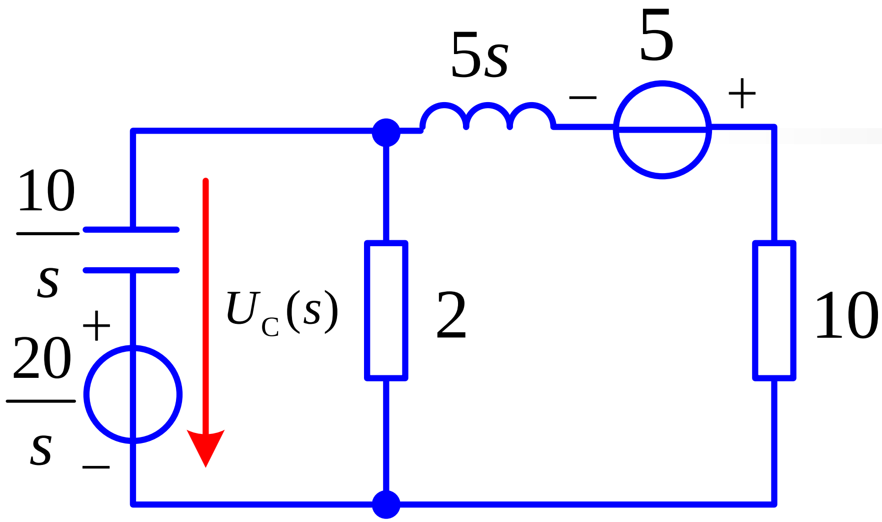


$$\dot{I}_{\mathrm{L}} = 1\angle\frac{\pi}{4}\mathrm{A} \quad u_{\mathrm{S}} = 20\sin(2t + \frac{\pi}{2})\mathrm{V}$$

$$i_{\mathrm{L}}(t) = \sqrt{2}\sin(2t + 45°)\mathrm{A}$$

$$i_{\mathrm{L}}(0\_) = \sqrt{2}\sin 45° = 1\mathrm{A}$$

$$u_{\mathrm{C}}(0\_) = u_{\mathrm{S}}(0) = 20\sin\frac{\pi}{2} = 20\mathrm{V}$$



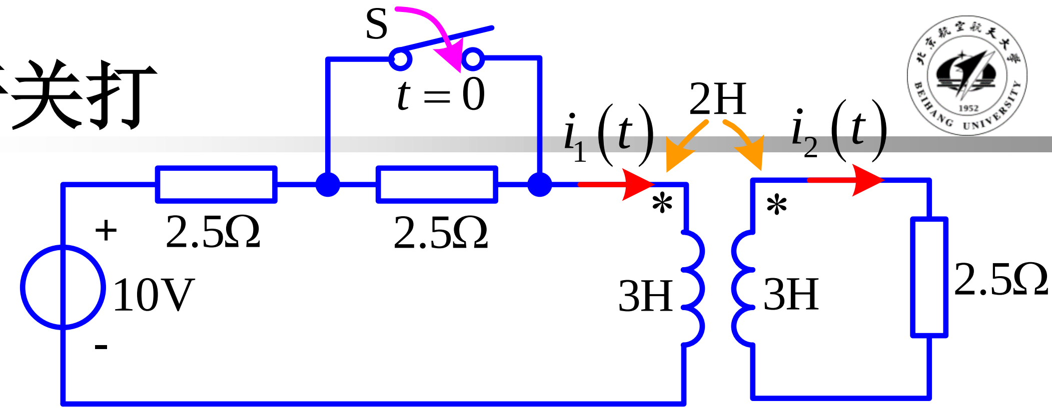
$$\left(\frac{1}{2}+\frac{s}{10}+\frac{1}{5s+10}\right)U_C(s)=\frac{\frac{20}{s}}{\frac{10}{s}}-\frac{5}{5s+10}$$

$$U_C(s)=\frac{10(2s+3)}{s^2+7s+12}=\frac{-30}{s+3}+\frac{50}{s+4}$$

$$u_C(t)=L^{-1}\left[U_C(s)\right]=-30\mathrm{e}^{-3t}+50\mathrm{e}^{-4t}\ \mathrm{V}$$

【例】已知：当$t<0$时，开关打开电路已稳定；当$t=0$时，开关闭合。

求：$t>0$时的全响应$i_2(t)$

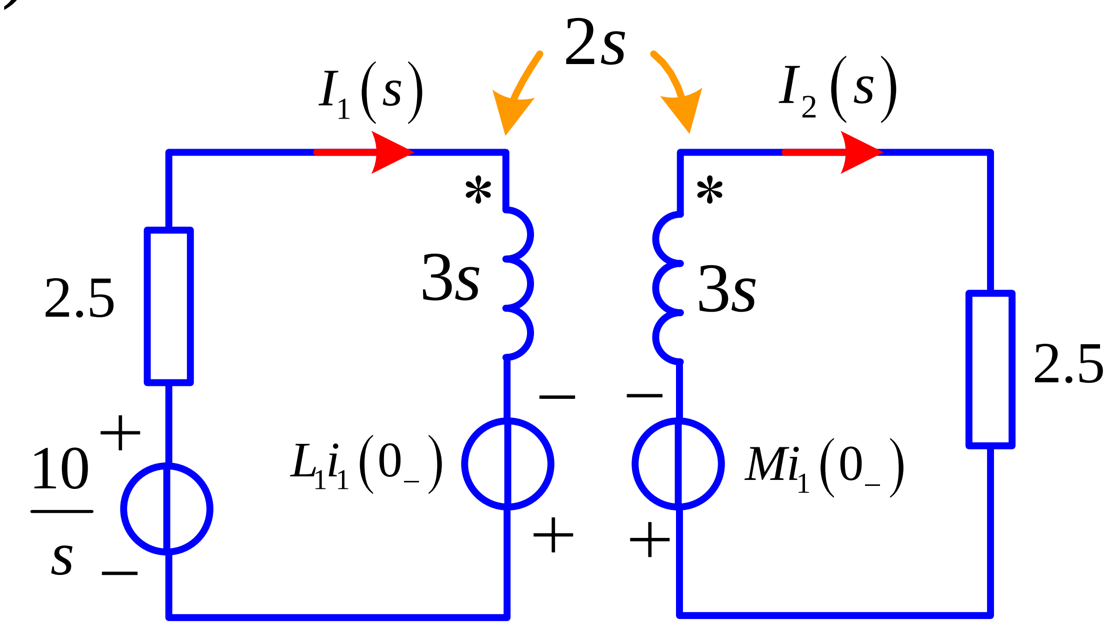


解 方法一

$$i_1(0_-)=\frac{10}{2.5+2.5}$$

$$=2\text{A}$$

$$i_2(0_-)=0\text{A}$$

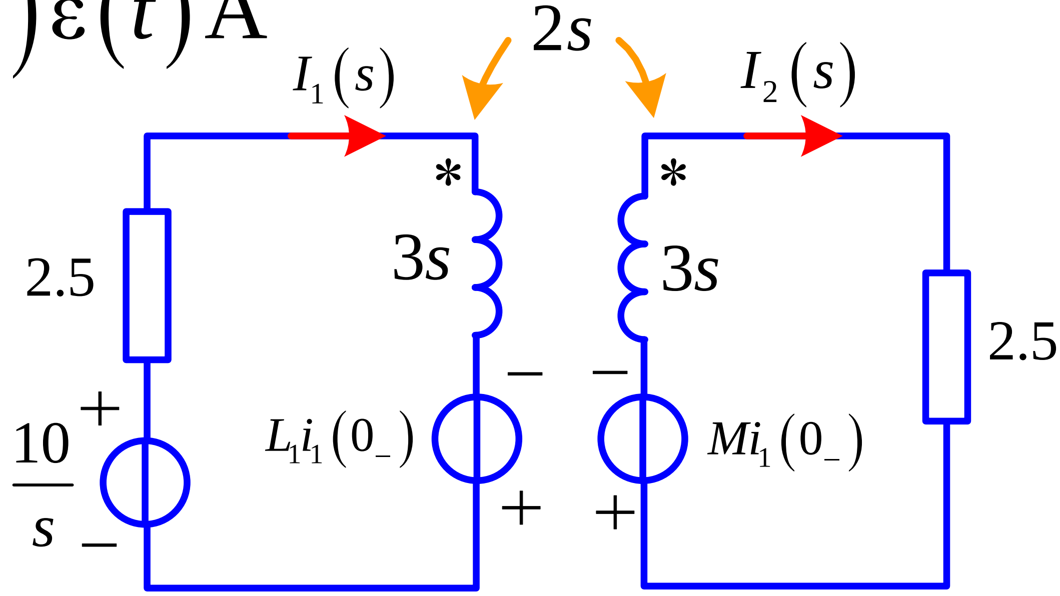


$$\left(2.5+3s\right)I_1\left(s\right)-2sI_2\left(s\right)=\frac{10}{s}+6$$

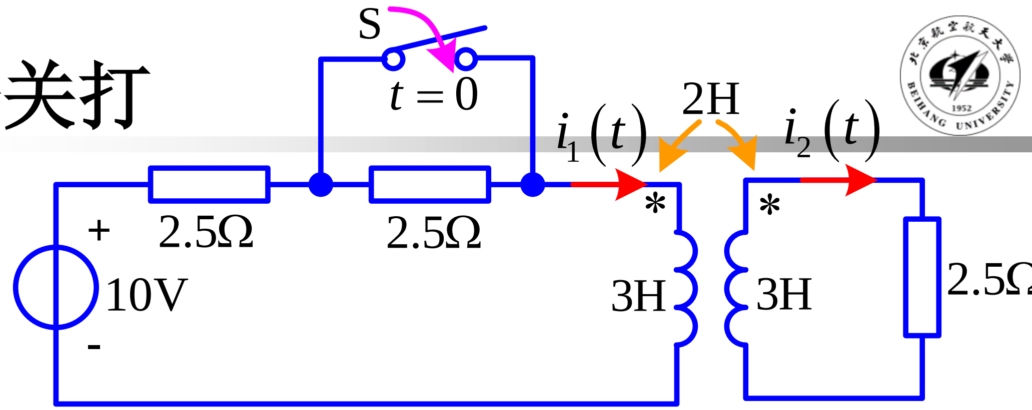
$$-2sI_1\left(s\right)+\left(2.5+3s\right)I_2\left(s\right)=-4$$

$$I_2\left(s\right)=\frac{2}{s^2+3s+1.25}=\frac{-1}{s+2.5}+\frac{1}{s+0.5}$$

$$i_2\left(t\right)=\left(-\mathrm{e}^{-2.5t}+\mathrm{e}^{-0.5t}\right)\varepsilon\left(t\right)\mathrm{A}$$

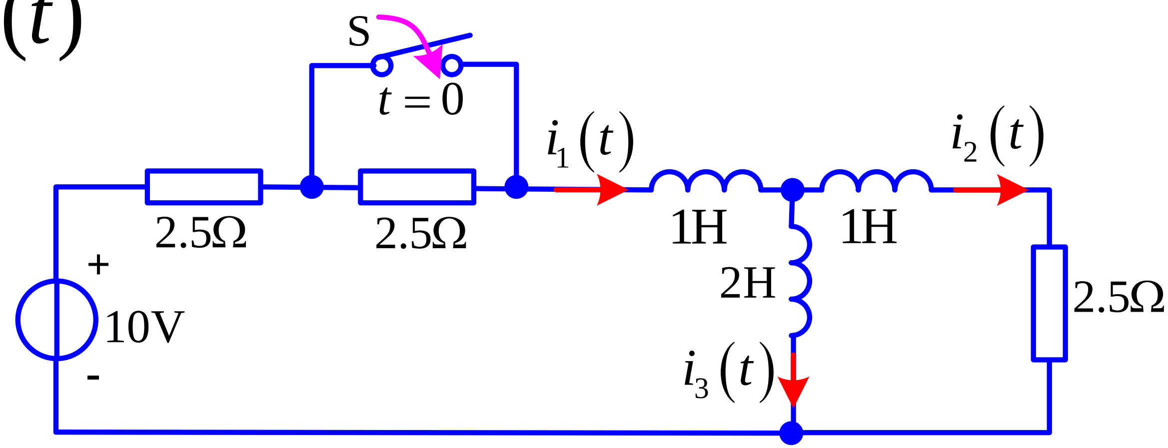


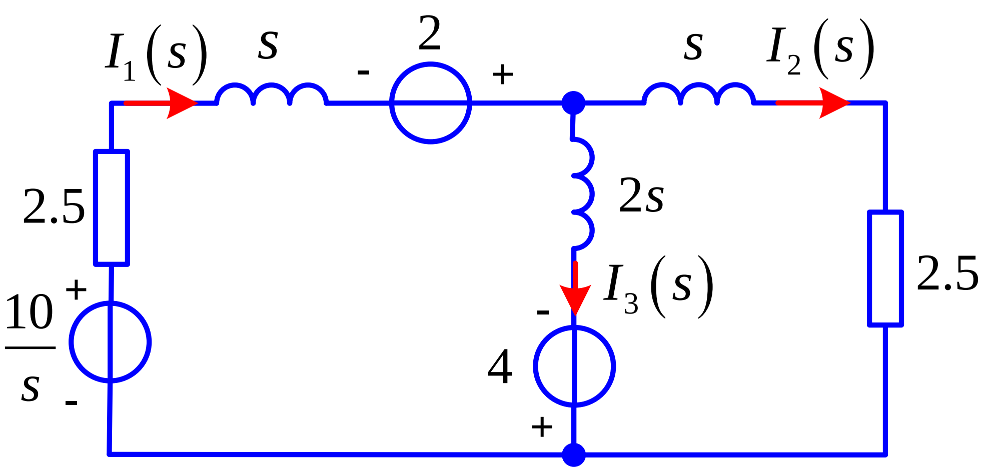
【例】已知：当$t<0$时，开关打开前电路已稳定；当$t=0$时，开关闭合。



求：$t>0$时的全响应$i_2(t)$

解 方法二

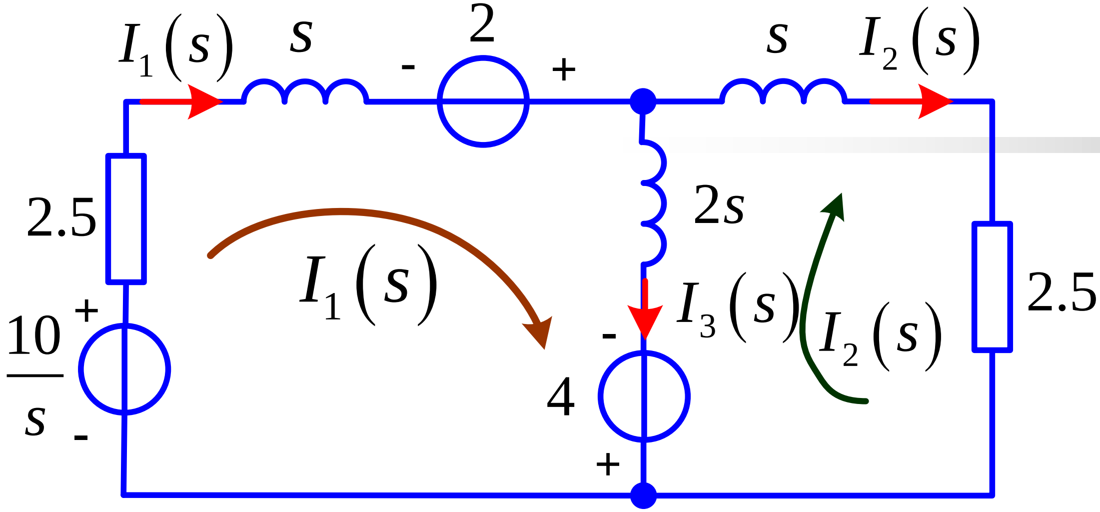




$$i_1(0_-)=2\text{A}$$

$$i_2(0_-)=0\text{A}$$

$$i_3(0_-)=2\text{A}$$

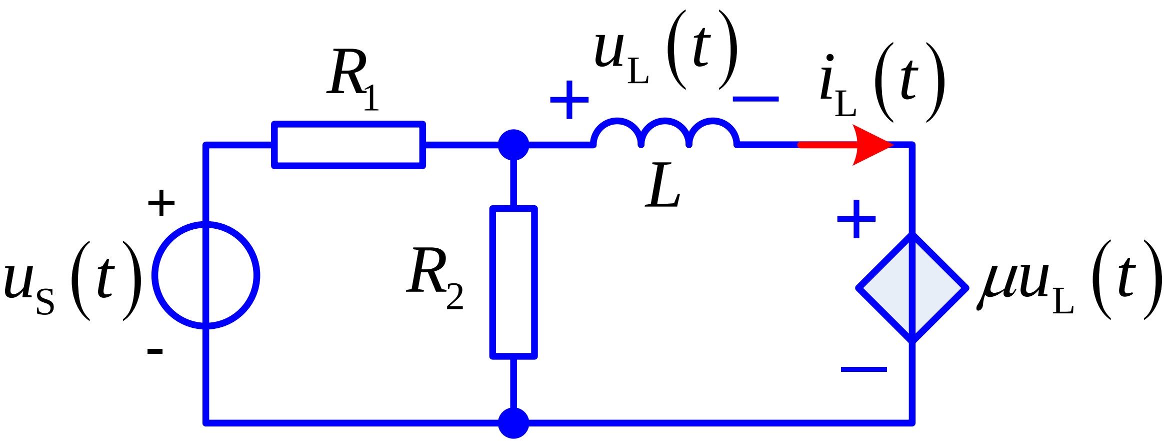


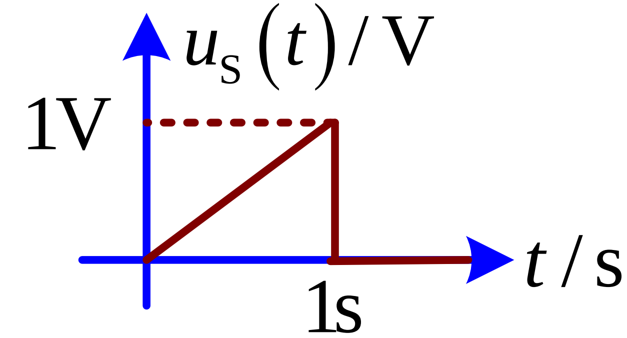
$$\begin{cases} (2.5+s+2s)I_1(s)-2sI_2(s)=\dfrac{10}{s}+2+4 \\ -2sI_1(s)+(2s+s+2.5)I_2(s)=-4 \end{cases}$$

$$I_2(s)=\frac{2}{s^2+3s+1.25}=\frac{-1}{s+2.5}+\frac{1}{s+0.5}$$

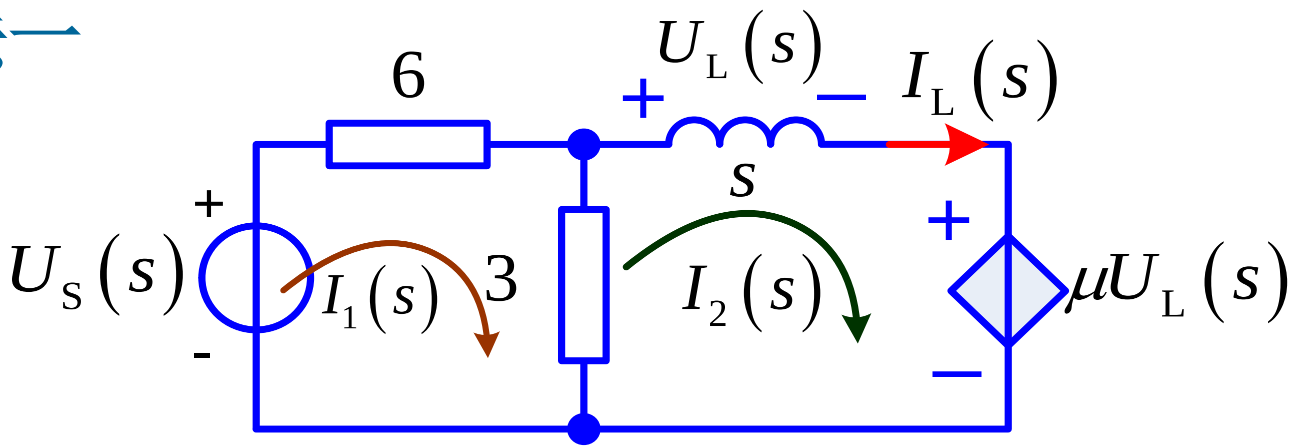
$$i_2(t)=\left(-\mathrm{e}^{-2.5t}+\mathrm{e}^{-0.5t}\right)\varepsilon(t)\,\mathrm{A}$$

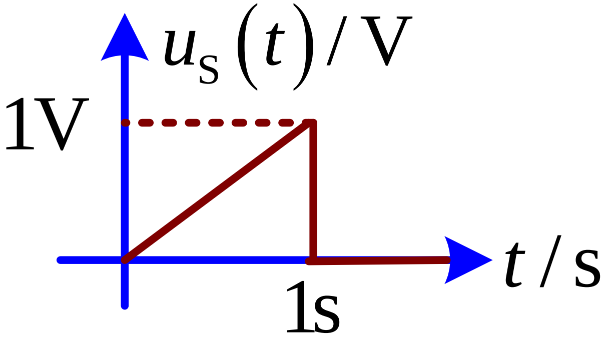
【例】 求当 $u_S(t)$ 波形如图所示时，电路的零状态响应 $i_L(t)$ 。 $R_1=6\Omega, R_2=3\Omega, L=1\text{H}, \mu=1$

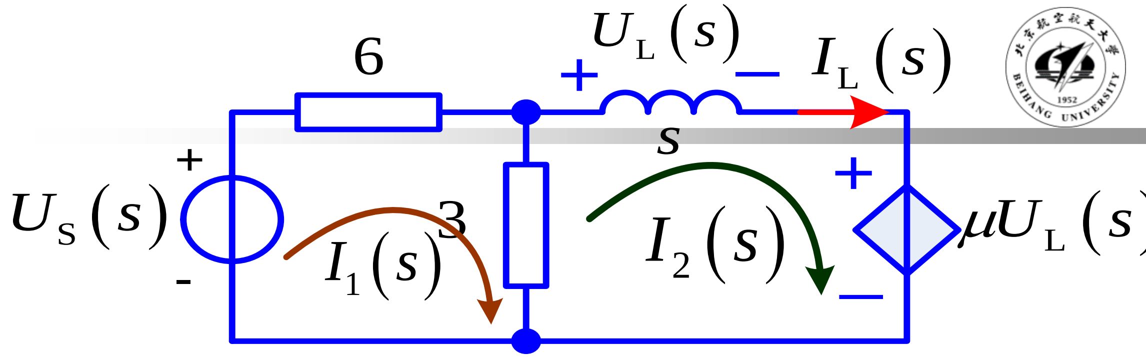




解 方法一





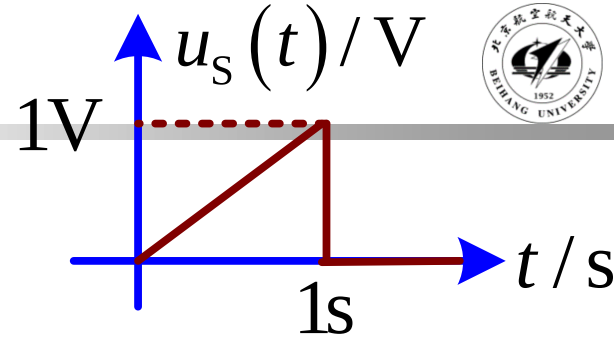


$$\begin{cases} 9I_1(s) - 3I_2(s) = U_S(s) \\ -3I_1(s) + (3+s)I_2(s) = -\mu U_L(s) \end{cases}$$

$$U_L(s) = I_L(s)s = I_2(s)s$$

$$I_L(s) = I_2(s) = \frac{1}{6}\frac{U_S(s)}{s+1}$$

$$I_L(s) = I_2(s) = \frac{1}{6}\frac{U_S(s)}{s+1}$$



$$u_S(t) = t\left[\varepsilon(t) - \varepsilon(t-1)\right]$$

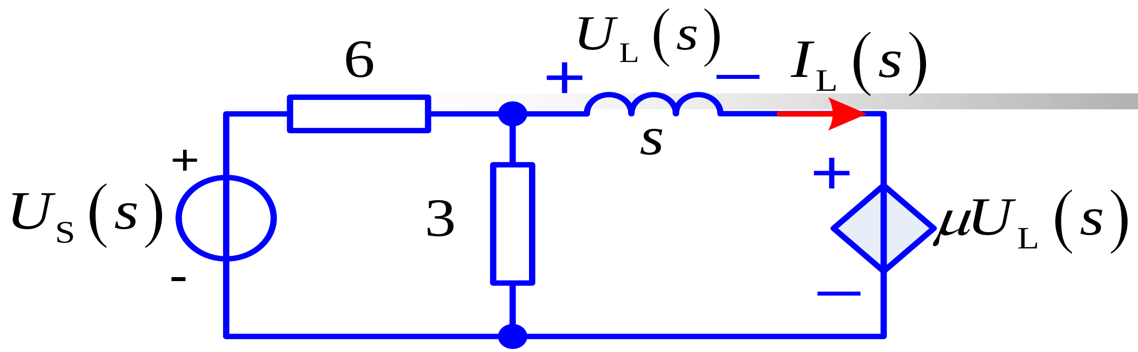
$$= t\varepsilon(t) - (t-1)\varepsilon(t-1) - \varepsilon(t-1)$$

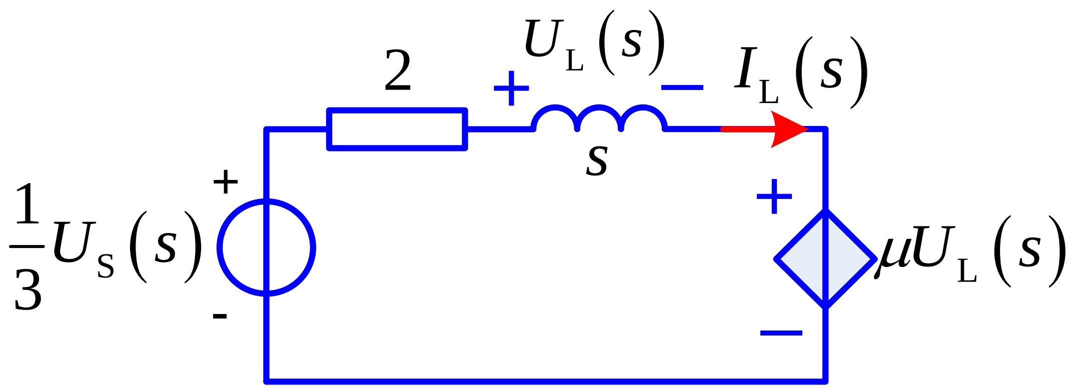
$$U_S(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

$$I_L(s) = \frac{1}{6}\left[\frac{1}{s+1} + \frac{1-e^{-s}}{s^2} + \frac{-1}{s}\right]$$

$$i_L(t) = \frac{1}{6}\left[\left(e^{-t} + t - 1\right)\varepsilon(t) - (t-1)\varepsilon(t-1)\right]$$

解 方法二

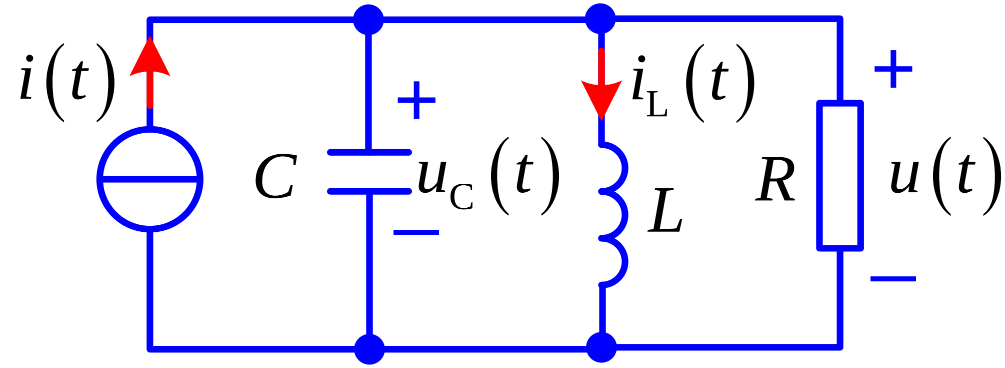




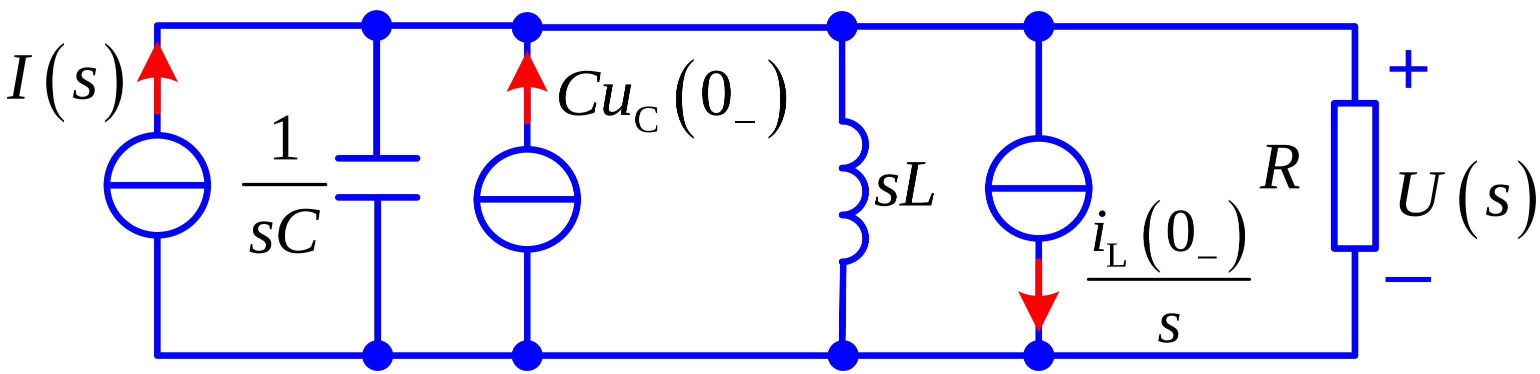
【例】求响应 $u(t)$，并指出稳态响应、暂态响应、零输入响应、零状态响应。$R=\frac{2}{7}\Omega, C=0.5\text{F}, L=0.2\text{H},$

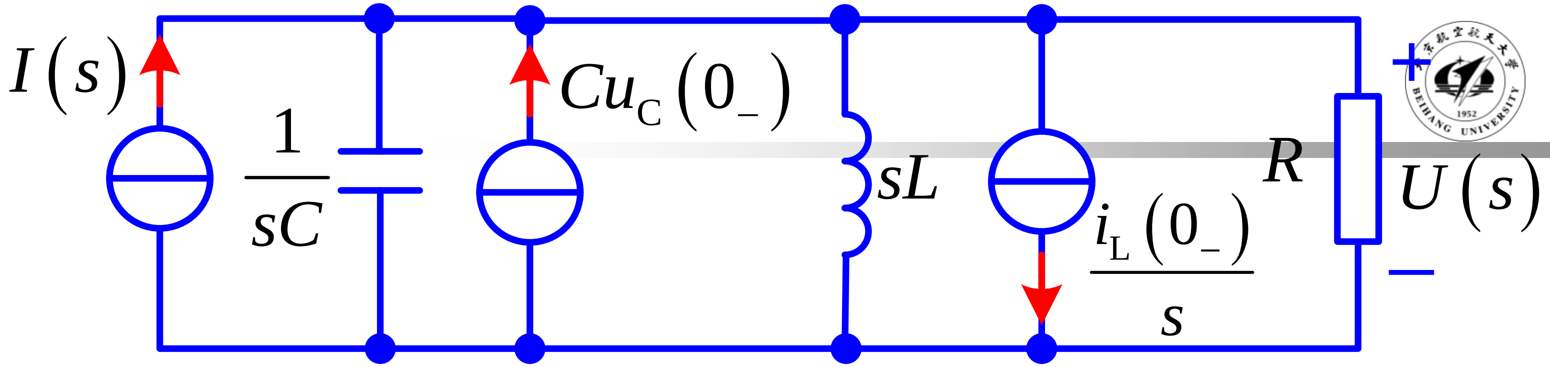
$u_\text{C}(0_-)=2\text{V}, i_\text{L}(0_-)=3\text{A}, i(t)=10\sin 5t\varepsilon(t)\,\text{A}$

解



$$I(s)=\frac{50}{s^2+5^2}$$





$$U(s)\left(sC+\frac{1}{Ls}+\frac{1}{R}\right)=I(s)+Cu_C(0_-)-\frac{i_L(0_-)}{s}$$

$$U(s)\left(0.5s+\frac{1}{0.2s}+3.5\right)=\frac{50}{s^2+5^2}+1-\frac{3}{s}$$

$$U(s)=\frac{50\times 2s}{(s^2+5^2)(s^2+7s+10)}+\frac{2(s-3)}{s^2+7s+10}$$

$$U(s)=\frac{50\times 2s}{\left(s^2+5^2\right)\left(s^2+7s+10\right)}+\frac{2(s-3)}{s^2+7s+10}$$

$$U(s)=\frac{100s}{(s-\mathrm{j}5)(s+\mathrm{j}5)(s+2)(s+5)}+\frac{2s-6}{(s+2)(s+5)}$$

$$U(s)=\left(\frac{-1.31\mathrm{e}^{\mathrm{j}66.8^\circ}}{s-\mathrm{j}5}-\frac{1.31\mathrm{e}^{-\mathrm{j}66.8^\circ}}{s+\mathrm{j}5}-\frac{2.3}{s+2}+\frac{3.33}{s+5}\right)$$

$$+\left(\frac{-3.33}{s+2}+\frac{5.33}{s+5}\right)$$

$$U(s)=\left(\frac{-1.31\mathrm{e}^{\mathrm{j}66.8^\circ}}{s-\mathrm{j}5}-\frac{1.31\mathrm{e}^{-\mathrm{j}66.8^\circ}}{s+\mathrm{j}5}-\frac{2.3}{s+2}+\frac{3.33}{s+5}\right)$$

$$+\left(\frac{-3.33}{s+2}+\frac{5.33}{s+5}\right)$$

零状态响应

$$u(t)=\left[\underbrace{-2\times1.31\cos\left(5t+66.8^\circ\right)-2.3\mathrm{e}^{-2t}+3.33\mathrm{e}^{-5t}}_{\text{零状态响应}}\right]$$

$$+\underbrace{\left(-3.33\mathrm{e}^{-2t}+5.33\mathrm{e}^{-5t}\right)}_{\text{零输入响应}}$$

零输入响应

$$u(t)=\left[\underbrace{2.62\sin\left(5t-23.2^\circ\right)}_{\text{稳态响应}}\underbrace{-5.63\mathrm{e}^{-2t}+8.66\mathrm{e}^{-5t}}_{\text{暂态响应}}\right]$$

稳态响应　暂态响应

作业

- 14-4　　　【运算电路图】
- 14-18　　【 求响应】