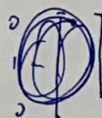


A6-3.

$$x(k+1) = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(k)$$



(1). $x(1) = F \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(0) = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(0)$

令 $u(0) = -3$. $x(1) = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$x(2) = F \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(1)$

令 $u(1) = 0$. $x(2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$

$\therefore u(0) = -3, u(1) = 0$

(2). \therefore 显然, $x(1) = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(0)$ 无法将三个状态全部归 0.

$\therefore N=2$.

(3). $W_R = [F^2 G \quad F G \quad G] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. $\text{rank } W_R = 2 \neq 3 = n$.

\therefore 系统不可达.

$\therefore F = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$, $x_3(i)(k)$ 不受 F 控制

\therefore 无法控制 $x(N) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

A6-4.

$$x(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

$\xi = 0.46 \quad \omega_n = 4.2 \text{ rad/s.} \quad T = 0.15$

$s_{1,2} = -\xi \omega_n \pm j \omega_n \sqrt{1-\xi^2} = -1.932 \pm j 3.73$

$z_{1,2} = e^{-s_{1,2}T} = 0.8243 \cdot e^{-j0.373} = 0.7676 \pm j0.3004$

对应的特征方程: $z^2 - 1.535z + 0.6795 = 0$.

$$K = [1, 0, \dots, 0] W_c^{-1} \alpha_c(F). \quad W_c = [G, FG] \Rightarrow G = \begin{bmatrix} 0.0044 & 0.0912 \\ 0.0912 & 0.805 \end{bmatrix}$$

$$W_c^{-1} = \begin{bmatrix} -95.13 & 15.34 \\ 105.1 & -5.342 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\alpha_c(F) = \begin{bmatrix} 1 & 0.0912 \\ 0 & 0.905 \end{bmatrix}^2 - 1.54 \begin{bmatrix} 1 & 0.0912 \\ 0 & 0.905 \end{bmatrix} + 0.6798 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1446 & 0.0353 \\ 0 & 0.106 \end{bmatrix}$$

$$K = [1, 0] \begin{bmatrix} -95.13 & 15.34 \\ 105.1 & -5.342 \end{bmatrix} \begin{bmatrix} 0.1446 & 0.0353 \\ 0 & 0.106 \end{bmatrix}$$

$$K = [15.17 \quad 3.10]$$

$$0.01446 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.0353 \\ 0.106 \end{bmatrix} = \begin{bmatrix} 0.01446 & 0.0353 \\ 0 & 0.106 \end{bmatrix}$$

$$G = W$$

$$W = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [0 \quad 0 \quad 0] = 0$$

$$W = 0$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = F$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = F$$

$$\begin{bmatrix} 0.0044 & 0.0912 \\ 0.0912 & 0.805 \end{bmatrix} + \begin{bmatrix} 0.0044 & 0.0912 \\ 0.0912 & 0.805 \end{bmatrix} = \begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix}$$

$$2 \begin{bmatrix} 0.0044 & 0.0912 \\ 0.0912 & 0.805 \end{bmatrix} = \begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix}$$

$$\begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix} = \begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix}$$

$$\begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix} = \begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix}$$

$$\begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix} = \begin{bmatrix} 0.0088 & 0.1824 \\ 0.1824 & 1.61 \end{bmatrix}$$