

A6-5. $C = [1 \ 0]$. 令阶次观测器. ~~现在值观测器~~. $\omega_n = 4.2$. $\zeta = 0.46$.

① 预测观测器:

观测.

$T = 0.1s$.

由上题作业可知, $x(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$

$C = [1 \ 0]$. 原系统特征方程: $z^2 - 1.55z + 0.6795 = 0$

$$L = \alpha_o(T) \omega_n^{-1} [I \ 0 - 0]^\top = a_c \begin{bmatrix} C \\ CT \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

选择观测器极点: 特征根为有复实根, 4倍衰减.

$$\therefore \sigma = \omega_n \zeta = 4.2 \times 0.46 = 1.932. \quad T = 0.1s$$

$$z = e^{-1.932 \times 0.1} = 0.46, \quad (z - 0.46)^2 \text{ 为特征方程.}$$

$$\therefore a_c = z^2 - 0.92z + 0.212 = 0$$

$$a_c(T) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix}^2 - 0.92 \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} + 0.212 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore a_c(T) = \begin{bmatrix} 0.293 & 0.094 \\ 0 & 0.1994 \end{bmatrix}, \quad \begin{bmatrix} C \\ CT \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -10.51 & 10.51 \end{bmatrix}$$

$$\therefore L = a_c(T) \begin{bmatrix} C \\ CT \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.293 & 0.094 \\ 0 & 0.1994 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -10.51 & 10.51 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.988 \\ 2.076 \end{bmatrix}$$

$$\therefore \tilde{x}(k+1) = F \tilde{x}(k) + G u(k) + L [y(k) - C \tilde{x}(k)]$$

$$\therefore \tilde{x}(k+1) = \begin{bmatrix} 0.013 & 0.0952 \\ -2.069 & 0.905 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k) + \begin{bmatrix} 0.987 \\ 2.069 \end{bmatrix} y(k)$$

②. 现在值观测器. $\tilde{x}(k+1) = F \tilde{x}(k) + G u(k) + L \{y(k+1) - C[F \tilde{x}(k) + G u(k)]\}$

$$\tilde{x}(k+1) = [F - LC] \tilde{x}(k) + [G - LCG] u(k) + L y(k+1)$$

$$\text{令 } L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad F - LC = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} C \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix}$$

$$= \begin{bmatrix} 1-L_1 & 0.0952(1-L_1) \\ L_2 & 0.905-0.0952L_2 \end{bmatrix}$$

相应特征方程为 $\begin{cases} z^2 + (L_1 - 1.905 + 0.095L_2)z + 0.905 - 0.905L_1 = 0. \\ a_c. \end{cases}$

$\therefore \begin{cases} L_1 - 1.905 + 0.095L_2 = -0.905 \\ 0.905 - 0.905L_1 = 0.013 \end{cases} \Rightarrow \begin{cases} L_1 = 0.705 \\ L_2 = 2.08 \end{cases}, L = \begin{bmatrix} 0.705 \\ 2.08 \end{bmatrix}$

$\therefore \tilde{x}(k+1) = \begin{bmatrix} 0.235 & 0.0223 \\ -2.31 & 0.685 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0.00114 \\ 0.084 \end{bmatrix} u(k) + \begin{bmatrix} 0.705 \\ 2.08 \end{bmatrix} y(k+1)$

(3). 降维观测器: 极点位于原点.

$\tilde{x}_2(k+1) = [F_{22} - LF_{12}] \tilde{x}_2(k) + [F_{21} - LF_{11}] y(k) + [G_2 - LG_1] u(k) + Ly(k+1)$

\therefore 极点位于 $z=0$.

\therefore 特征方程为 $z=0$. (期望).

$\det[zI - F_{22} + LF_{12}] = z - 0.905 + 0.095L = 0$

$\therefore 0.095L = 0.905, L = 9.506$

$\therefore \tilde{x}_2(k+1) = 9.506 y(k+1) - 9.506 y(k) + 0.0492 u(k)$

根据上同法求 $k = [-15.17 \quad -3.12]$, $u(k) = -15.17 y(k) - 3.12 \tilde{x}_2(k)$

$\therefore \tilde{x}_2(k+1) = 9.506 [y(k+1) - y(k)] + 0.0492 [-15.17 y(k) - 3.12 \tilde{x}_2(k)]$

$\therefore \tilde{x}_2(z) = 9.506 [z y(z) - y(z)] + 0.0492 [-15.17 y(z) - 3.12 \tilde{x}_2(z)]$

$u(z) = -15.17 y(z) - 3.12 \frac{9.506(z-1.078)}{z+0.10135} y(z)$

$\therefore D(z) = \frac{U(z)}{Y(z)} = -\frac{44.86(z-0.6614)}{z+0.153}$

④. \tilde{x}_1 设计.

B6-2.

$$x(k+1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$\textcircled{1}. F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$W_R = [F^2 G \quad F G \quad G] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.25 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\text{rank } W_R = 2 \neq 3$$

$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0$: 系统不可达.

$$\textcircled{2} \quad u(k) = -[k_1 \ k_2 \ k_3] x(k)$$

$$\therefore x(k+1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} x(k) - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} [k_1 \ k_2 \ k_3] x(k)$$

$$x(k+1) = \begin{bmatrix} -k_1 & -k_2 & -k_3 \\ 0 & 0.5 & 0 \\ -k_1 & -k_2 & 2-k_3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} -k_1 x_1 - k_2 x_2 - k_3 x_3 \\ 0.5 x_2 \\ -k_1 x_1 - k_2 x_2 - (2-k_3) x_3 \end{bmatrix}$$

\therefore 不稳定.