Automatic Control

Loop shaping design of feedback control systems

Part III: real negative zero lead network and PID controllers

Real negative zero as lead network

Real negative zero as lead network

- A phase lead controller can also be designed through a suitable use of a **real negative zero** of the form

$$C_z(s) = 1 + \frac{s}{\omega_Z}, \omega_Z > 0, \lim_{s\to 0} C_z(s) = 1$$

- Note that $C_z(s)$ is a special case of the lead network when $m_D \to \infty$

$$C_D(s) = \frac{1 + \dfrac{s}{\omega_D}}{1 + \dfrac{s}{m_D\omega_D}} \underset{m_D\to\infty}{=} 1 + \frac{s}{\omega_D}$$

- The maximum phase lead introduced by $C_z(s)$ is 90° → when more than 90° are required, multiple real negative zeros can be employed.

$$C_z(s) = \left(1 + \frac{s}{\omega_{Z_1}}\right)\left(1 + \frac{s}{\omega_{Z_2}}\right)\cdots$$

Real negative zero as lead network: some issues

$$C_z(s) = 1 + \frac{s}{\omega_Z}$$

- Since the transfer function of a real negative zero is not proper, it is not guaranteed, in general, that the final overall controller C(s) is proper.
- To discuss such issue, let us consider the general form of the steady state controller

$$C_{SS}(s) = \frac{K_c}{s^{g_c}}, g_c \geq 0$$

and of a "network" $C_z(s)$ made up by a series of n_z real negative zeros

$$C_z(s) = \left(1 + \frac{s}{\omega_{Z_1}}\right)\left(1 + \frac{s}{\omega_{Z_2}}\right)\cdots\left(1 + \frac{s}{\omega_{Z_{n_z}}}\right)$$

Real negative zero as lead network: some issues

$$C_{SS}(s) = \frac{K_C}{s^{g_c}},\ g_c \geq 0 \qquad C_z(s) = \left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)\cdots\left(1+\frac{s}{\omega_{z_{n_z}}}\right)$$

- The overall controller

$$C(s) = C_{SS}(s)C_z(s) = \frac{K_C \overbrace{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)\cdots\left(1+\frac{s}{\omega_{z_{n_z}}}\right)}^{\text{degree } n_z}}{\underbrace{s^{g_c}}_{\text{degree } g_c}}$$

is proper if $g_c \geq n_z$

Examples

$$C(s) = \frac{K_C}{s}\left(1+\frac{s}{\omega_z}\right), C(s) = \frac{K_C}{s^2}\left(1+\frac{s}{\omega_z}\right), C(s) = \frac{K_C}{s^2}\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)$$

Real negative zero as lead network: some issues

$$C(s) = C_{SS}(s)C_z(s) = \frac{K_C \overbrace{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)\cdots\left(1+\frac{s}{\omega_{z_{n_z}}}\right)}^{\text{degree } n_z}}{\underbrace{s^{g_c}}_{\text{degree } g_c}}$$

- When $g_c < n_z$ the controller is not proper.
- Examples

$$C(s) = K_C\left(1+\frac{s}{\omega_z}\right), C(s) = \frac{K_C}{s}\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)$$

Real negative zero as lead network: some issues

- To make proper the controller, a common procedure consists in adding $n_p = n_z - g_c$ **closure poles** to the controller tf
- Such closure poles have to be placed at higher frequencies with respect to the highest frequency of the controller zeros

$$C(s) = K_C\left(1+\frac{s}{\omega_z}\right) \rightarrow C(s) = K_C\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)},\ \omega_p \gg \omega_z$$

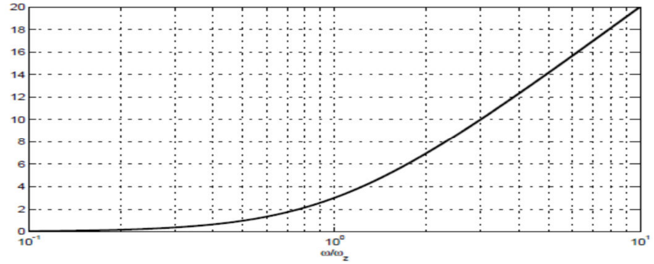
$$C(s) = \frac{K_C}{s}\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right) \rightarrow C(s) = \frac{K_C\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{s\left(1+\frac{s}{\omega_p}\right)},\ \omega_p \gg \max(\omega_{z_1},\omega_{z_2})$$

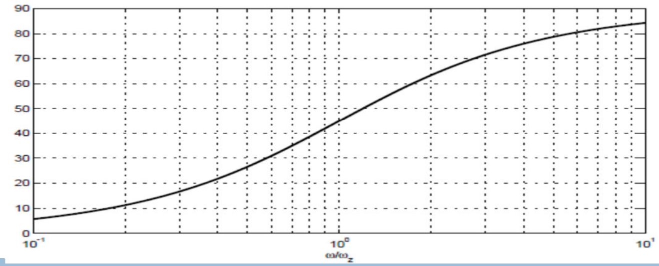
Real negative zero as lead network: remarks

- <u>Remark 1</u>: the addition of a closure pole can be considered also in the presence of already proper controllers when a magnitude attenuation of L(jω) at high frequency is needed.
- <u>Remark 2</u>: the disjoint design of a real negative zero and a closure pole makes up an indirect procedure to design a "canonical" lead network.
- <u>Remark 3</u>: real negative zeros can be designed in combination with a lag network, if needed.

Real negative zero design

- The zero(s) ω_z are tuned in order to get the needed phase lead.
- To this aim, a similar procedure as the one for the lead controller design can be suitably employed through the use of an "universal diagram" of the zero.





Real negative zero design

The frequency normalized (w.r.t. ω_z) Bode plots of a real negative zero is used to tune ω_z

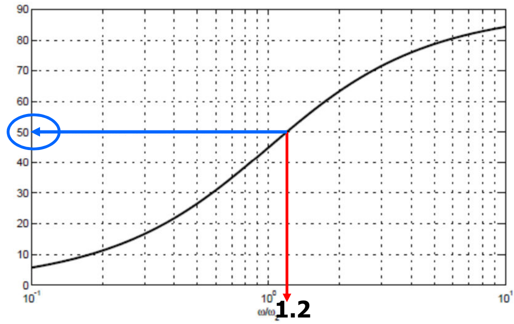
The procedure is the same as the design of a lead network.

In particular, it sufficies to select the normalized frequency at which the needed phase lead $\Delta\varphi$ occurs.

The value of ω_z is chosen through "frequency denormalization" (wrt $\omega_{c,des}$).

Example :

$\Delta\varphi = 50° @ \omega_{c,des} = 2\,\text{rad/s}$



$\omega_{norm} = \omega/\omega_z = 1.2$

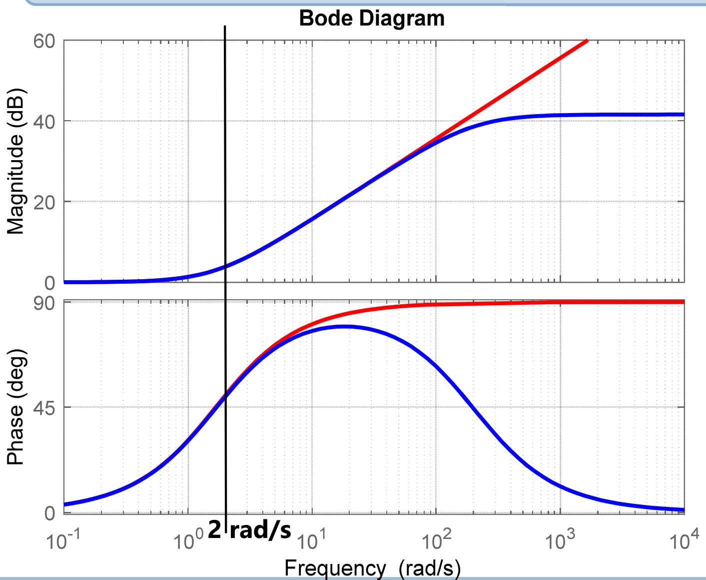
$(\omega/\omega_z)|_{\omega=\omega_{c,des}} = 1.2$

$\omega_z = \omega_{c,des} / 1.2 = 2/1.2$

$= 1.67$ rad/s

$$C_z(s) = 1 + \frac{s}{1.67}$$

Real negative zero design



$$C_z(s) = 1 + \frac{s}{1.67}$$

$$C'_z(s) = \frac{1 + \frac{s}{1.67}}{1 + \frac{s}{200}}$$

Design example

- A plant to be controlled is described by the transfer function

$$G(s) = \frac{0.045}{s^2 + 2.6s + 1.2}$$

design a cascade controller C(s) in order to satisfy the requirements below.

- $|e_r^\infty| \le 0.2,\ r(t) = 0.25t\varepsilon(t) \rightarrow C_{SS}(s) = 34/s$
- $\hat{s} \le 10\% \rightarrow T_p = 0.42\text{ dB}, S_p = 2.68\text{ dB}$
- $t_r \le 2\ s, t_{s,5\%} \le 4\ s \rightarrow \omega_{c,des} = 1\text{ rad/s}$

PID controllers

PID controllers through a formal exercise

$$C(s)=\frac{K_c\left(1+s/\omega_z\right)}{s} \;\Longrightarrow\; C(s)=K_P\left(1+\frac{1}{T_i s}\right)$$

$$C(s)=\frac{K_c\left(1+s/\omega_{z1}\right)\left(1+s/\omega_{z2}\right)}{s} \;\Longrightarrow\; C(s)=K_P\left(1+\frac{1}{T_i s}+T_d s\right)$$

$$C(s)=K_c\left(1+s/\omega_z\right) \;\Longrightarrow\; C(s)=K_P\left(1+T_d s\right)$$

Standard PID controllers: definition

- We get the general form of the controller

$$C(s)=K_P\left(1+\frac{1}{T_i s}+T_d s\right)$$

- In the time domain we have

$$e(t) \longrightarrow \boxed{C(s)} \longrightarrow u(t) \qquad u(t)=K_P\left(e(t)+\frac{1}{T_i}\int_0^t e(\tau)d\tau+T_d\frac{d}{dt}e(t)\right)$$

- The control is thus realized through the sum of the following three elementary actions
  - **P**roportional
  - **I**ntegral
  - **D**erivative

PID Controller

Standard PID controllers: terminology

$$C(s)=K_P\left(1+\frac{1}{T_i s}+T_d s\right)$$

- K_p: gain of the Proportional action;
- T_i: time constant of the Integral action;
- T_d: time constant of the Derivative action;

- In order to guarantee physical reliability of the derivative term, the following substitution is usually introduced ($N = 5 \div 20$)

$$T_d s=\frac{T_d s}{1+(T_d/N)s}$$

Standard PID controllers: different forms

In general, the three actions (P, I and D) can be used independently to obtain different forms for the controller.

P $C(s) = K_P$

PI $C(s) = K_P\left(1 + \frac{1}{T_i s}\right)$

PD $C(s) = K_P(1 + T_d s)$

PID $C(s) = K_P\left(1 + \frac{1}{T_i s} + T_d s\right)$

PID controller loop shaping design

- Using the expressions

$$C(s) = \frac{K_c(1 + s/\omega_z)}{s} \quad C(s) = \frac{K_c(1 + s/\omega_{z1})(1 + s/\omega_{z2})}{s}$$

$$C(s) = K_c(1 + s/\omega_z)$$

- PID controllers can be designed considering the factor K_c/s, (K_c) as the result of a steady state design step.
- The zero(s) ω_z are tuned in order to obtain the phase lead as seen before.