①.

$$\dot x_1 = \sin x_2.$$
$$\dot x_2 = u$$
$$\dot x_3 = -\sin x_3 + (1+\cos x_3)u.$$
$$y = x_1$$

$$\dot y = \dot x_1 = \sin x_2.$$
$$\ddot y = (\sin x_2)' = \cos x_2 \cdot \dot x_2 = u\cdot \cos x_2.$$
$$\therefore \rho = 2. \quad \forall x \in D_0 = \{x \in R^3 \mid x_2 \neq \tfrac{k}{2}\pi\}.$$

$$\gamma = \cos x_2. \quad \alpha = -\frac{0}{\gamma(x)} = 0.$$

$$\xi = \begin{bmatrix} h \\ L_f h \end{bmatrix} = \begin{bmatrix} x_1 \\ \sin x_2 \end{bmatrix}.$$

$$T(x) = \begin{bmatrix} \eta \\ \xi \end{bmatrix} = \begin{bmatrix} \phi(x) \\ x_1 \\ \sin x_2 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ 1 \\ 1+\cos x_3 \end{bmatrix}.$$

$$\therefore \frac{\partial \phi(x)}{\partial x_2} + \frac{\partial \phi(x)}{\partial x_3}(1+\cos x_3) = 0, \quad 令\ \phi(x) = -x_2 + \tan\frac{x_3}{2}$$

$$\therefore T(x) = \begin{bmatrix} -x_2 + \tan\frac{x_3}{2} \\ x_1 \\ \sin x_2 \end{bmatrix}. \quad \dot\eta = \dot\phi(x) = -\dot x_2 + \dot x_3 \frac{1}{1+\cos x_3}$$

$$\begin{cases} \eta = -x_2 + \tan\frac{x_3}{2} \\ \xi_1 = x_1 \\ \xi_2 = \sin x_2 \end{cases} \qquad \dot\eta = -u + u - \frac{\sin x_3}{1+\cos x_3}$$
$$\dot\eta = -\frac{\sin x_3}{1+\cos x_3}$$

$$\begin{cases} x_1 = \xi_1 \\ x_2 = \sin^{-1}\xi_2 \\ x_3 = 2\tan^{-1}(\eta + \sin^{-1}\xi_2). \end{cases} \qquad \begin{cases} \dot\eta = -\dfrac{\sin[2\tan^{-1}(\eta+\sin^{-1}\xi_2)]}{1+\cos[2\tan^{-1}(\eta+\sin^{-1}\xi_2)]} \\ \dot\xi_1 = \sin x_2 = \xi_2. \\ \dot\xi_2 = u\cdot\cos(\sin^{-1}\xi_2). \end{cases}$$

$\therefore$ 可设计 $u = \beta(x)(-k\xi)$. $\beta(x) = \gamma^{-1}(x)$.

②.

$$\dot x_1 = u$$
$$\dot x_2 = \sin x_2 + (1+\cos x_2)u.$$

$$f(x) = \begin{bmatrix} 0 \\ \sin x_2 \end{bmatrix} \quad g(x) = \begin{bmatrix} 1 \\ 1+\cos x_2 \end{bmatrix}$$

$$ad_f g = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g = \begin{bmatrix} 0 & 0 \\ 0 & -\sin x_2 \end{bmatrix}\begin{bmatrix} 0 \\ \sin x_2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \cos x_2 \end{bmatrix}\begin{bmatrix} 1 \\ 1+\cos x_2 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ -\sin^2 x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \cos x_2 + \cos^2 x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1-\cos x_2 \end{bmatrix}.$$

$\therefore G(x) = \begin{bmatrix} 1 & 0 \\ 1+\cos x_2 & -1-\cos x_2 \end{bmatrix} \quad \forall\ x_2 \neq (2k+1)\pi.\ R(G(x)) = 2.$

$\mathbb{D} = \left\{ \begin{bmatrix} 1 \\ 1+\cos x_2 \end{bmatrix} \right\}$，显然对合.

$\therefore \exists\, y = h(x).\quad \frac{\partial h}{\partial x} g = 0,\quad \frac{\partial L_f h}{\partial x} g \neq 0.$

$\therefore \frac{\partial h}{\partial x_1} \cdot 1 + \frac{\partial h}{\partial x_2}(1+\cos x_2) = 0$. 令 $h(x) = -x_1 + \tan\frac{x_2}{2}$.

$\therefore \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} h \\ L_f h \end{bmatrix} = \begin{bmatrix} -x_1 + \tan\frac{x_2}{2} \\ \frac{\sin x_2}{1+\cos x_2} \end{bmatrix} = T(x).$

$$\frac{\partial L_f h}{\partial x} = \frac{\cos x_2(1+\cos x_2) - \sin x_2(-\sin x_2)}{(1+\cos x_2)^2} \cdot (\sin x_2 + (1+\cos x_2)u)$$

$$= \frac{2\sin x_2 + (2+2\cos x_2)u}{(1+\cos x_2)^2}$$

$\therefore \cancel{\alpha(x)} = \gamma(x) = \frac{2(1+\cos x_2)}{(1+\cos x_2)^2} = \frac{2}{1+\cos x_2}$

$\alpha(x) = -\frac{\frac{2\sin x_2}{(1+\cos x_2)^2}}{\frac{2}{1+\cos x_2}} = \frac{-2\sin x_2}{2(1+\cos x_2)} = \frac{-\sin x_2}{1+\cos x_2}$

$\therefore$ 设计 $u = \alpha(x) + \beta(x)(-k\xi)$，$\beta(x) = \gamma^{-1}(x)$.

18.1. $\dot{x}_1 = x_2$

$\dot{x}_2 = a\sin x_1 + b + u$, $f(x) = a\sin x_1 + b$. $g(x) = 1$. $\dot{x}_2 = f(x) + g(x)\cdot u$.

设计滑动面：$S = x_1 + x_2 = 0$.

滑动面上动态为 $\dot{x}_1 = x_2 = -x_1$，系统 A.S.

$\dot{S} = \dot{x}_1 + \dot{x}_2 = x_2 + f + gu = g\left[u + \frac{f + x_2}{g}\right]$. 代入 f.g.

$\frac{f + x_2}{g} = \frac{a\sin x_1 + b + x_2}{1} = a\sin x_1 + b + x_2 \le x_2 + a_2 + b_2 = F(x)$.

$\therefore$ 设计 $u = -\mathrm{sgn}(g(x))\,[F(x) + \varepsilon]\,\mathrm{sgn}(s)$

$u = -(x_2 + a_2 + b_2 + \varepsilon)\,\mathrm{sgn}(s)$, $\varepsilon > 0$

18.2. $\dot{x}_1 = u_1 + d_1(x)$

$\dot{x}_2 = u_2 + d_2(x)$. , $|d_1| \le D_1$, $|d_2| \le D_2$.

$\dot{x}_3 = x_1 x_2^2$.

设计滑动面： $S_1 = x_1 + x_3 = 0$ $S_2 = x_2 - x_3 = 0$.

滑动面上动态：$\dot{x}_3 = x_1 x_2^2 = -x_3 \cdot x_3^2 = -x_3^3$，系统 A.S.

$\dot{S}_1 = \dot{x}_1 + \dot{x}_3 = u_1 + d_1(x) + x_1 x_2^2$.

$\dot{S}_2 = \dot{x}_2 - \dot{x}_3 = u_2 + d_2(x) - x_1 x_2^2$.

$\therefore$ 设计：$u_1 = -x_1 x_2^2 - (D_1(x) + \varepsilon_1)\,\mathrm{sgn}(S_1)$.

$u_2 = x_1 x_2^2 - (D_2(x) + \varepsilon_2)\,\mathrm{sgn}(S_2)$.

其中 $\varepsilon_1 > 0$，$\varepsilon_2 > 0$.

18.3. $\dot{x}_1 = u_1 + d_1(x)$

$\dot{x}_2 = u_2 + d_2(x)$.

$\dot{x}_3 = x_1 x_2 + x_3^2$.

设计滑动面：$S_1 = x_1 - x_3^2 = 0$. $S_2 = x_2 + x_3 + 1 = 0$.

滑动面上系统动态：$\dot{x}_3 = x_1 x_2 + x_3^2 = x_3^2(-x_3 - 1) + x_3^2 = -x_3^3$，A.S.

$\dot{S}_1 = \dot{x}_1 - 2x_3 \cdot \dot{x}_3 = u_1 + d_1(x) - 2x_3(x_1 x_2 + x_3^2)$

$= u_1 + d_1(x) - 2x_1 x_2 x_3 - 2x_3^3$

$\dot{S}_2 = \dot{x}_2 + \dot{x}_3 = u_2 + d_2(x) + x_1 x_2 + x_3^2$.

∴设计 $u_1 = 2x_1 x_2 x_3 + 2x_3^3 - (D_1(x) + \varepsilon_1)\,\mathrm{sgn}(S_1)$.

$u_2 = -x_1 x_2 - x_3^2 - (D_2(x) + \varepsilon_2)\,\mathrm{sgn}(S_2)$.

其中 $\varepsilon_1 > 0$. $\varepsilon_2 > 0$.