Automatic Control

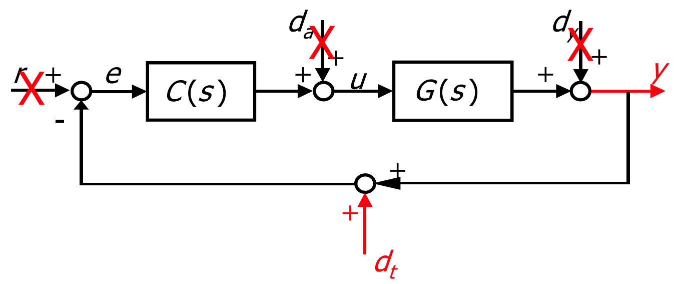
Sinusoidal disturbances attenuation requirements

Control input requirements

Sinusoidal disturbance attenuation

Sensor noise

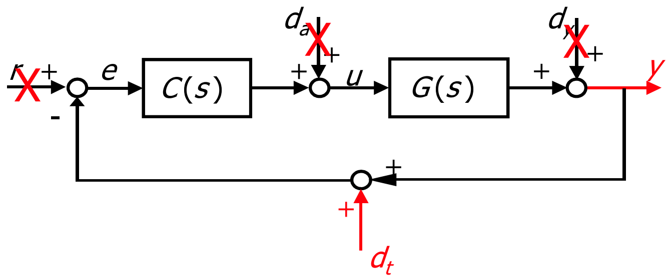
Steady-state response to sinusoidal disturbances



- Here the problem of attenuating the effect of the sensor noise d_t on the controlled output y at steady state is considered.
- The focus is restricted to the class of sinusoidal signals, i.e.:

$$d_t(t) = \delta_t \sin(\omega t) \quad \omega \geq \omega_t \quad \text{given } \delta_t \text{ and } \omega_t$$

Steady-state response to sinusoidal disturbances



- At steady state, in the presence of d_t, we have:

$$y_{ss}(t) = \delta_t \left|T(j\omega)\right| \sin(\omega t + \angle(-T(j\omega)))$$

- The steady-state output error $\left|y_{d_t}^{\infty}\right|$ is defined as:

$$\left|y_{d_t}^{\infty}\right| = \max_t \left|y_{ss}(t)\right| = \delta_t \left|T(j\omega)\right|$$

Steady-state response to sinusoidal disturbances

- The steady-state output error is required to be bounded by a given constant:

$$\left|y_{d_t}^{\infty}\right| \leq \rho_t \qquad \text{given} \quad \rho_t > 0$$

- A design constraint on $|T(j\omega_t)|$ is obtained as

$$\left|y_{d_t}^{\infty}\right| = \delta_t \left|T(j\omega)\right| \leq \rho_t \Rightarrow$$

$$\Rightarrow \left|T(j\omega)\right| \leq \frac{\rho_t}{\delta_t} = M_T^{HF} \qquad \forall \omega \geq \omega_t$$

Design constraints on $T(j\omega)$ and $L(j\omega)$ due to d_t

$$\left|T(j\omega)\right| \leq \frac{\rho_t}{\delta_t} = M_T^{HF} \qquad \forall \omega \geq \omega_t$$

- Note that, disturbance attenuation is obtained if $|T(j\omega)| << 1$
- Since $|T(j\omega)| << 1$ holds for $\omega >> \omega_c$, the requirement on $|y^{\infty}_{dt}|$ introduces a constraint on the value of ω_c such that:

$$\omega_c \ll \omega_t$$

- Rule of thumb:

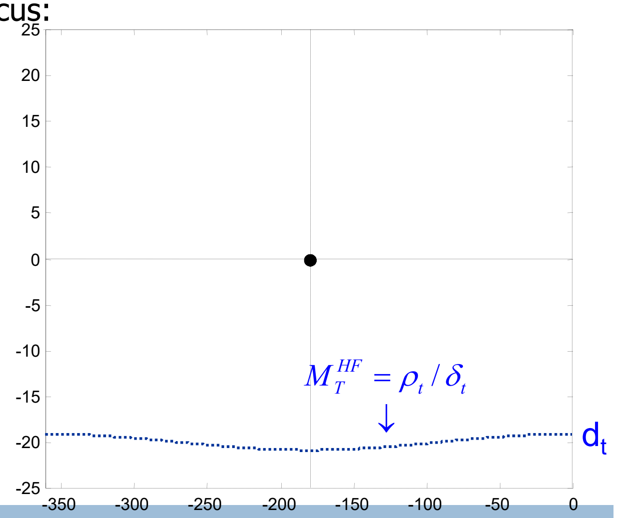
$$\omega_{c,des} \leq 0.1\omega_t$$

Design constraints on $T(j\omega)$ and $L(j\omega)$ due to d_t

On the Nichols plane, the constraint on $T(j\omega)$ can be represented as a constant magnitude locus:

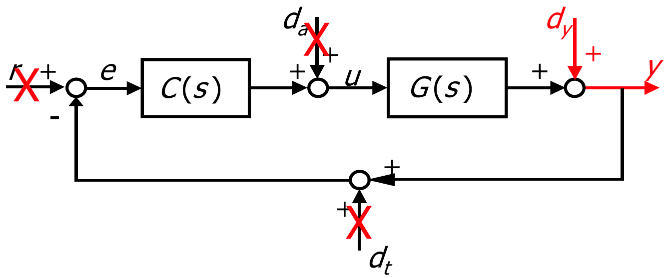
The points of the loop function $L(j\omega)$ corresponding to frequencies greater than ω_t, must lie below the constant magnitude locus defined by

$$\frac{\rho_t}{\delta_t} = M_T^{HF}$$



Output disturbance

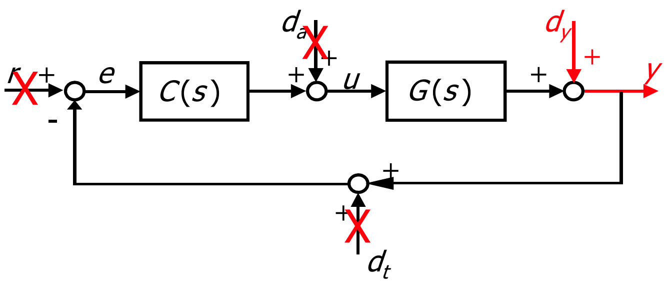
Steady-state response to sinusoidal disturbances



- Now, the problem of attenuating the effect of the output disturbance d_y on the controlled output y at steady state is considered
- The focus is restricted to the class of sinusoidal signals, i.e.:

$$d_y(t) = \delta_y \sin(\omega t) \quad \omega \le \omega_y \quad \text{given } \delta_y \text{ and } \omega_y$$

Steady-state response to sinusoidal disturbances



- At steady state, in the presence of d_y, we have:

$$y_{ss}(t) = \delta_y \left|S(j\omega)\right| \sin(\omega t + \angle S(j\omega))$$

- The steady-state output error $\left|y_{d_y}^{\infty}\right|$ is defined as:

$$\left|y_{d_y}^{\infty}\right| = \max_t \left|y_{ss}(t)\right| = \delta_y \left|S(j\omega)\right|$$

Steady-state response to sinusoidal disturbances

- The steady-state output error is required to be bounded by a given constant:

$$\left|y_{d_y}^{\infty}\right| \le \rho_y \quad \text{given } \rho_y > 0$$

- A design constraint on $|S(j\omega)|$ is obtained as

$$\left|y_{d_y}^{\infty}\right| = \delta_y \left|S(j\omega)\right| \le \rho_y \Rightarrow$$

$$\Rightarrow \left|S(j\omega)\right| \le \frac{\rho_y}{\delta_y} = M_S^{LF} \quad \forall \omega \le \omega_y$$

Design constraints on $S(j\omega)$ and $L(j\omega)$ due to d_y

$$|S(j\omega)| \le \frac{\rho_y}{\delta_y} = M_S^{LF} \quad \forall \omega \le \omega_y$$

- Note that, disturbance attenuation is obtained if $|S(j\omega)| << 1$
- Since $|S(j\omega)| << 1$ holds for $\omega << \omega_c$, the requirement on $|y^{\infty}_{dy}|$ introduces a constraint on the value of ω_c such that:

$$\omega_c \gg \omega_y$$

- Rule of thumb:

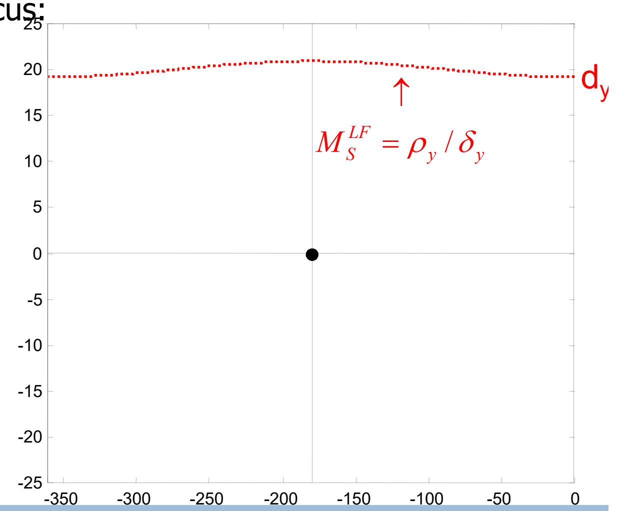
$$\omega_{c,des} \ge 10\omega_y$$

Design constraints on $S(j\omega)$ and $L(j\omega)$ due to d_y

On the Nichols plane, the constraint on $S(j\omega)$ can be represented as a constant magnitude locus:

The points of the loop function $L(j\omega)$ corresponding to frequencies smaller than ω_y, must lie above the constant magnitude locus defined by

$$\frac{\rho_y}{\delta_y} = M_S^{LF}$$



Design example

- Given the plant tf $G(s) = \dfrac{-0.3}{s^2 + 1.75s + 0.37}$

design a cascade controller C(s) in order to satisfy the following requirements.

$$|y^{\infty}_{d_a}| \le 0.02,\ d_a(t) = \delta_a t\varepsilon(t), |\delta_a| \le 0.03 \rightarrow C_{SS}(s) = \frac{-1.5}{s}$$

$$|y^{\infty}_{d_y}| \le 6 \cdot 10^{-3},\ d_y(t) = \delta_y \sin(\omega_y t), |\delta_y| \le 6 \cdot 10^{-2},$$

$$\omega_y \le 0.08 \text{ rad/s} \rightarrow \omega_{c,des} \gg 0.08 \text{ rad/s} \rightarrow$$

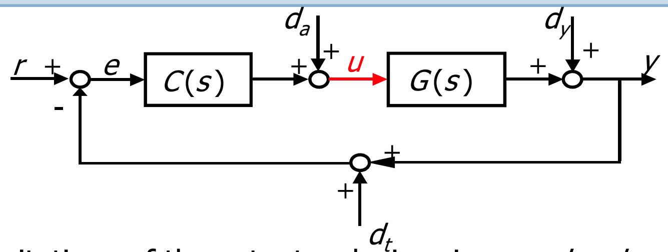
$$\omega_{c,des} \ge 0.8 \text{ rad/s}, M_S^{LF} = -20 \text{ dB}$$

$$\hat{s} \le 10\% \rightarrow T_p = 0.42 \text{ dB}, S_p = 2.68 \text{ dB}$$

$$t_r \le 2.5\ s, t_{s,2\%} \le 8\ s \rightarrow \omega_{c,des} = 1 \text{ rad/s}$$

Actuator saturation and control input requirements

Introduction



Physical limitations of the actuator devices impose *hard constraints* on the control input u(t). Examples:

- amplifiers input voltage is bounded by the input swing (about $\pm$ 22 mV for OpAmps);
- aircraft control moving surfaces deflections (i.e. ailerons, elevators, rudders) have a limited working range (about $\pm$ 5°);
- actuators employed to enhance vehicle lateral dynamics, such as active differentials, can provide limited values of the yaw moment (typically about $\pm$ 2500 Nm);

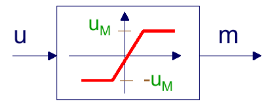
Input saturation constraint

The above described actuator limitations, impose to the control input u(t) a **saturation constraint** of the form

$$-u_M \le u(t) \le u_M, \forall t \ge 0$$
$$\rightarrow |u(t)| \le u_M, \forall t \ge 0$$

The saturation constraints can be described as a nonlinear static function of the control input as

$$m(t) = \begin{cases} u(t), \text{ if } -u_M \le u(t) \le u_M \\ -u_M, \text{ if } u(t) < -u_M \\ u_M, \text{ if } u(t) > u_M \end{cases}$$

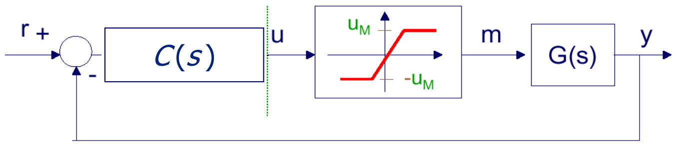


$m(t) \rightarrow$ saturated input

Note that, when $u_M \le u(t) \le u_M, m(t) = u(t)$

Input saturation constraint

In the presence of an **input saturation constraint**, the considered feedback control systems becomes



- When the input saturation is active (i.e. when either u(t) > u_M or u(t) < - u_M), the feedback control system becomes non linear.
- Exceeding the input prescribed bounds causes unexpected behaviour of the systems such as large overshoots, low performance or, in the worst case, unstability.
- Note that, in this case, stability analysis of the feedback system can not be performed through the Nyquist criterion.

Input saturation constraint

- Thus, in order to avoid performance degradation in the presence of a control input saturation, a constraint of the form

  $$|u(t)| \le u_M, \forall t \ge 0$$

  has to be taken into account during the design procedure.

- Typically, the input saturation requirement is provided in the presence of a specific working situation.

  Example: $|u(t)| \le 10, \forall t \ge 0, r(t) = 2\varepsilon(t)$

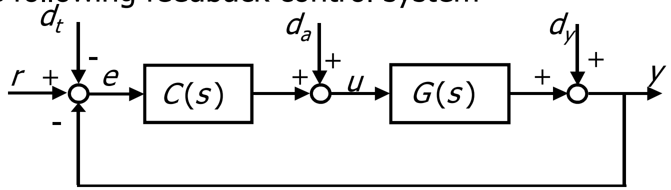
- In this context, a step reference is usually considered since it introduces critical solicitations to the control input during the transient phase.

Input saturation constraint

- Unfortunately, the input saturation constraint can not be handled directly by the loop-shaping design procedure but it has to be checked "a posteriori" .
- If it has not been satisfied, a common procedure is to reduce the value of the actual crossover frequency ω_c.
- However, reduction of ω_c may cause unsatisfaction of the transient requirements (i.e. rise time and settling time) → conflicting requirements.

Input saturation requirement: a special case

Consider the following feedback control system



and suppose that

- the reference is a step signal of the form r(t) = $\rho\varepsilon$(t);
- the plant transfer function G(s) is strictly proper;
- the controller transfer function C(s) is proper and does not contain any poles at the origin, i.e. it is (e.g.) of the form

$$C(s) = K_C \frac{1+\frac{s}{\omega_D}}{1+\frac{s}{m_D\omega_D}} \frac{1+\frac{s}{m_I\omega_I}}{1+\frac{s}{\omega_I}}$$

Input saturation requirement: a special case

… then

Property: the amplitude of the control input at time t = 0 is given by:

$$u_0 = u(0) = \rho K_C \frac{m_D}{m_I}, \text{ Proof} \rightarrow \text{apply Initial Value Theorem}$$

Remarks:

- The Property still holds in the presence of multiple lead and lag networks.
- In the presence of a pole at the origin in the controller, we have that u(0) = 0 (still apply IVT).
- In general, the greater are the values of K_C and m_D the greater is the maximum control input u_{MAX} while the greater is m_I the smaller is u_{MAX} → aspects to be taken into account during the design.

Design example

- A plant to be controlled is described by

$$G(s) = \frac{0.045}{s^2 + 2.6s + 1.2}$$

design a cascade controller C(s) in order to satisfy the following requirements:

$$|e_r^\infty| \le 0.2,\ r(t) = 0.25t\varepsilon(t) \rightarrow C_{SS}(s) = 34/s$$

$$\hat{s} \le 10\% \rightarrow T_p = 0.4\text{ dB}, S_p = 2.66\text{ dB}$$

$$t_r \le 2\ s, t_{s,5\%} \le 4\ s \rightarrow \omega_{c,des} = 1\text{ rad/s}$$

$$\max_t |u(t)| \le 60, \text{when } r(t) = \varepsilon(t)$$