(A2-11) 3.

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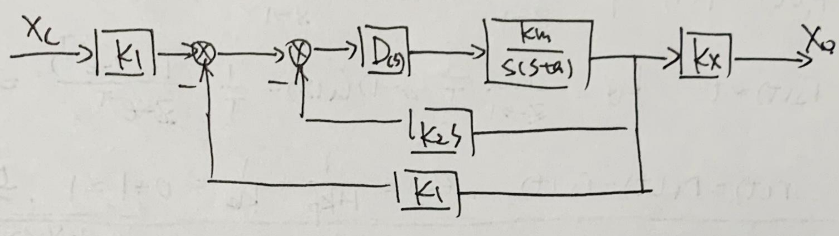
$$c(k+3)+a_1c(k+2)+a_3c(k)=b_0r(k+3)+b_2r(k+1)+b_3r(k).$$

∵初始条件均为零，则

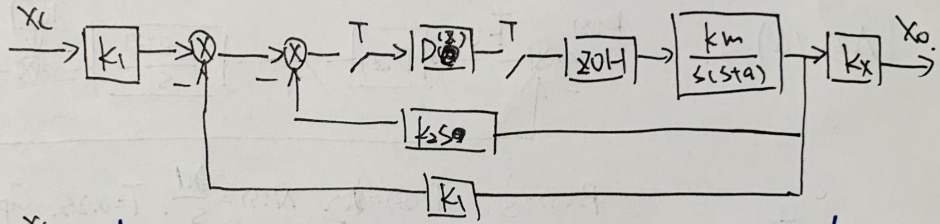
$$G(z)=\frac{C(z)}{R(z)},\quad z^3C(z)+a_1z^2C(z)+a_3C(z)=b_0z^3R(z)+b_2z^1R(z)+b_3R(z)$$

$$\therefore G(z)=\frac{(b_0z^3+b_2z^1+b_3)}{(z^3+a_1z^2+a_3)}$$

(B2-13) 5. 原图:



增加T.ZOH:



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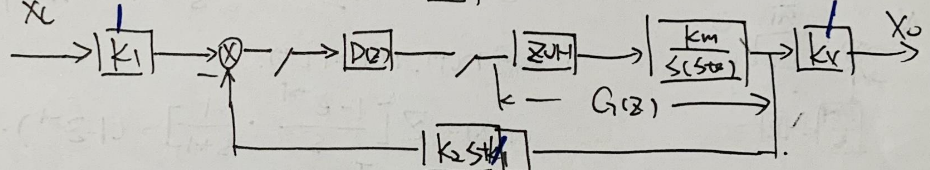
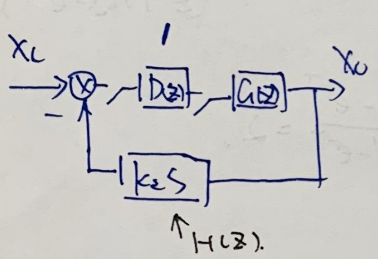




$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{k_m}{s(s+a)}\right]=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{30}{s(s+2)}\right]$$

$$G(z)=(1-z^{-1})\cdot Z\left[\frac{30}{s^2(s+2)}\right]=(1-z^{-1})\cdot Z\left[\frac{7.5}{s+2}-\frac{7.5}{s}+\frac{15}{s^2}\right]$$

$$G(z)=(1-z^{-1})\left(\frac{7.5z}{z-e^{-2T}}-\frac{7.5z}{z-1}+\frac{15Tz}{(z-1)^2}\right)$$

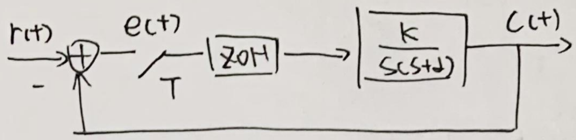
$$G(z)=\frac{7.5(z-1)^2-7.5(z-1)(z-e^{-0.2})+1.5(z-e^{-0.2})}{(z-e^{-0.2})(z-1)}$$

$$GH(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{k_m}{s(s+a)}\cdot k_2s\right]=(1-z^{-1})\cdot Z\left[\frac{1.5\times 2}{s(s+2)}\right]$$

$$GH(z)=(1-z^{-1})\cdot 1.5\cdot\frac{(1-e^{-2T})z}{(z-1)(z-e^{-2T})}=1.5\,\frac{1-e^{-0.2}}{z-e^{-0.2}}$$

$$\Phi(z)=\frac{D(z)G(z)}{1+D(z)GH(z)}=\frac{\boxed{G(z)}}{\frac{1.5(1-e^{-0.2})+(z-e^{-0.2})}{(z-e^{-0.2})}}=\frac{[1.5(1-e^{-0.2})+(z-e^{-0.2})][7.5(z-1)(e^{-0.2}-1)+1.5(z-e^{-0.2})]}{z-1}$$

(A3-6) 1.



$K=1.\quad r(t)=1(t)+t.\quad T=0.1s$

判断是否稳定:

$1+D(z)G(z)=0$

$\Downarrow$

$(z-e^{-T})T+(z-1)^2=0$

$z^2-1.9z+0.91=0$

$|a_2|=0.91<1$

$\Delta(1)=0.01>0$

$\Delta(-1)=1+1.9+0.91>0$

$\therefore$ 稳定

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{K}{s(s+2)}\right]=(1-z^{-1})\,Z\left[\frac{1}{s^2(s+1)}\right].$$

$$G(z)=(1-z^{-1})\cdot\left[\frac{z}{z-e^{-T}}-\frac{z}{z-1}+\frac{zT}{(z-1)^2}\right]=\left(\frac{z-1}{z-e^{-T}}+\frac{T}{z-1}-1\right)$$

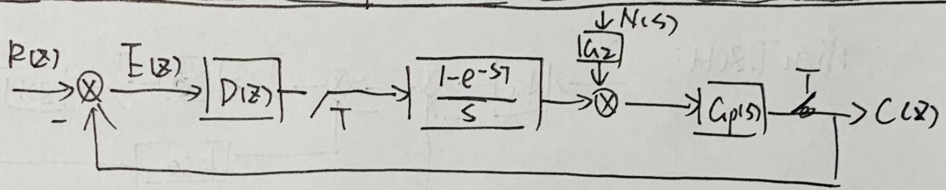
$$G(z)=\frac{(e^{-T}-1)(z-1)+T(z-e^{-T})}{(z-e^{-T})(z-1)}$$

$r_1(t)=1(t)\quad k_p=\lim\limits_{z\to1}D(z)G(z)=\lim\limits_{z\to1}G(z)=\infty$

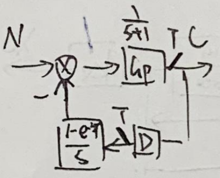
$r_2(t)=t\quad k_v=\lim\limits_{z\to1}\frac{1}{T}(z-1)G(z)=\frac{1}{T}\cdot\frac{T\cdot(z-e^{-T})}{z-e^{-T}}=1$

$\therefore r(t)=r_1(t)+r_2(t).\quad e_{ss}=\frac{1}{1+k_p}+\frac{1}{k_v}=0+1=1$, 与T无关.

(A3-10) 2.



$R(s)=\frac{1}{s}.\quad D(z)=2.\quad N(s)=\frac{1}{s}.\quad T=0.2s.\quad G_p(s)=\frac{1}{s+1}\quad G_2(s)=1$



① 对于 r(t), r(t)=1(t).

$$G(z)=Z\left[\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right]=(1-z^{-1})\cdot Z\left[\frac{1}{s(s+1)}\right]=\frac{(1-z^{-1})(1-e^{-T})z}{(z-1)(z-e^{-T})}$$

$$G(z)=\frac{1-e^{-T}}{z-e^{-T}}$$

判断是否稳定:

$1+D(z)G(z)=0$

$[2\cdot(1-e^{-T})]+(z-e^{-T})=0$

代入 T=0.2.

$(z-0.819)+0.363=0$

$z=0.819-0.363<1$

$\therefore$ 稳定.

$e_{ssr}=\frac{1}{1+k_p}.\quad k_p=\lim\limits_{z\to1}D(z)G(z)=2\cdot\frac{1-e^{-T}}{1-e^{-T}}=2.$

$\therefore e_{ssr}=\frac{1}{1+2}=\frac{1}{3}=0.333$

② 对于 n(t). n(t)=1(t).

$$C_N(z)=\frac{NG_p(z)}{1+\underbrace{G_p(z)D(z)Z\frac{1-e^{sT}}{s}}_{G(z)}}=\frac{Z\left[\frac{1}{s(s+1)}\right]}{1+2\cdot\frac{1-e^{-T}}{z-e^{-T}}}=\frac{\frac{(1-e^{-T})z}{(z-1)(z-e^{-T})}}{\frac{z-e^{-T}+2-2e^{-T}}{(z-e^{-T})}}$$

$C_N(z)=\frac{(1-e^{-T})z}{(z+2-3e^{-T})(z-1)}$, $c_{ssn}=\lim\limits_{z\to1}(z-1)C_N(z)$. 代入 T=0.2s

$\therefore c_{ssn}=\frac{0.181}{1-0.456}=0.333$

$\therefore e_{ss}=e_{ssr}+c_{ssn}=0.666.$