Automatic Control

Loop shaping design of feedback control systems

Part I: Lead network

Time domain requirements translation: resume

The properties of the 2nd order prototype model

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

allowed the translation of the time domain requirements $\hat{s}, t_r, t_{s,\alpha\%}$ into the relevant indices of the frequency response of the functions T(s), S(s) and L(s).

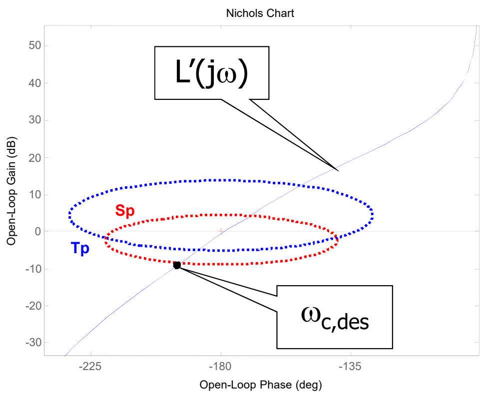
$$\hat{s} \rightarrow \begin{cases} T_p \rightarrow \text{resonant peak of } |T(j\omega)| \\ S_p \rightarrow \text{resonant peak of } |S(j\omega)| \end{cases}$$

$$\left.\begin{matrix} t_r \\ t_{s,\alpha\%} \end{matrix}\right\} \rightarrow \omega_{c,des} \rightarrow \text{crossover frequency of } |L(j\omega)|$$

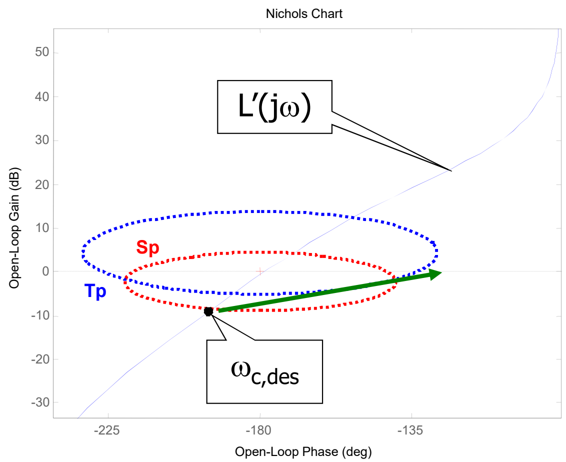
$C_T(s)$ design procedure: preliminary considerations

After the first step of the design procedure, the steady state controller is obtained → $C_{ss}(s)$. Before going on with the design of the transient controller $C_T(s)$, the following preliminary steps need to be performed.

1. consider the loop function obtained after the steady state design $L'(s) = C_{ss}(s)G(s)$.
2. plot the frequency response of $L'(j\omega)$ on the Nichols plane and mark the point corresponding to $\omega_{c,des}$.
3. plot the constant magnitude loci T_p and S_p obtained by transient requirements analysis.



Example

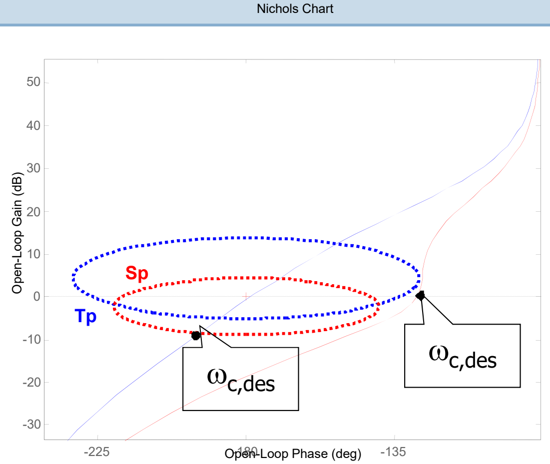


In this case, in order to

- make the course of $L'(j\omega)$, tangent to the loci T_p and S_p,
- make $\omega_{c,des}$ cross-over frequency

phase lead and magnitude increase actions are required in the middle frequency range (i.e. in a suitable neighborhood of $\omega_{c,des}$)

Example



In order to achieve the desired performance, the frequency response of the loop function L'(s) needs to be suitably "shaped" → **loop shaping design procedure**

The lead network

The just introduced example motivates the use of the

$$\textbf{lead network} \rightarrow \; C_D(s) = \frac{1+\dfrac{s}{\omega_D}}{1+\dfrac{s}{m_D\omega_D}}, \; \omega_D > 0, m_D > 1$$

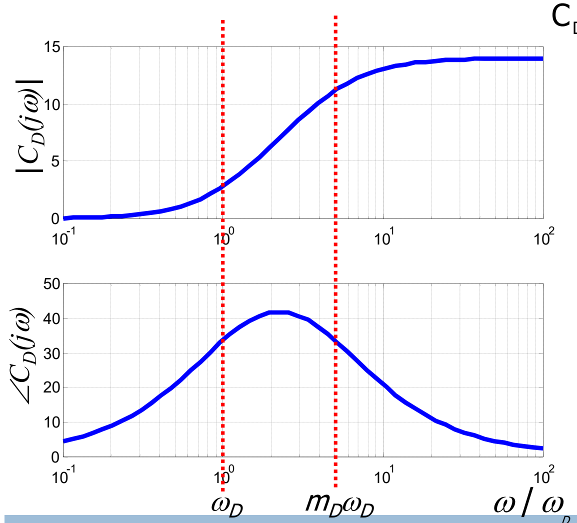
A lead network is described by a proper tf with

- a real negative zero at $-\omega_D$
- a real negative pole at $-m_D\,\omega_D$

Note also that

$$\lim_{s\to 0} C_D(s) = \lim_{s\to 0} \frac{1+\dfrac{s}{\omega_D}}{1+\dfrac{s}{m_D\omega_D}} = 1$$

The lead network: frequency response



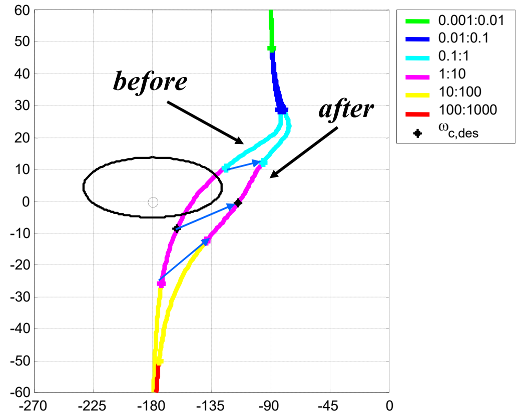
$$C_D(s) = \frac{1+\dfrac{s}{\omega_D}}{1+\dfrac{s}{m_D\omega_D}}, \; \omega_D > 0, \; m_D > 1$$

Magnitude increase

→ Phase lead ←

the greater is m_D, the larger is the amount of the phase lead and the magnitude increase

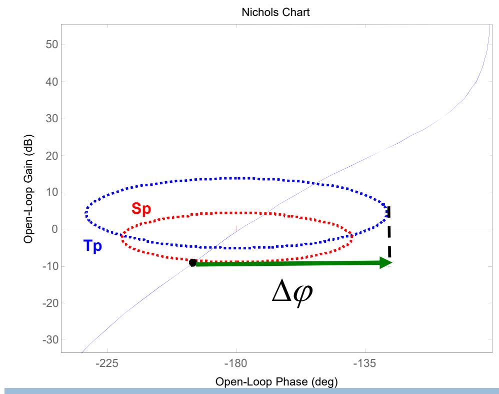
The lead network: effects



In the Nichols plane, phase lead and magnitude increase introduced by a lead network produce an oblique shift of the loop function frequency response in the frequency range of interest.

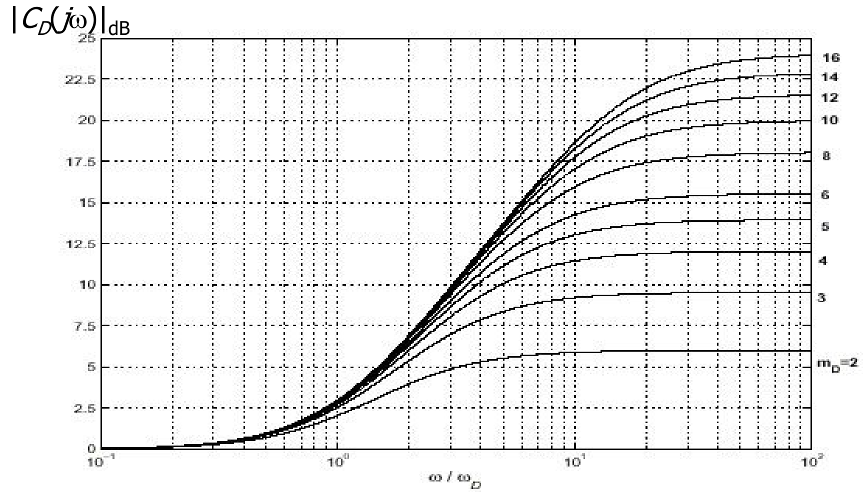
The lead network: basic guidelines for design

$$C_D(s) = \frac{1 + \frac{s}{\omega_D}}{1 + \frac{s}{m_D \omega_D}}, \omega_D > 0, m_D > 1$$

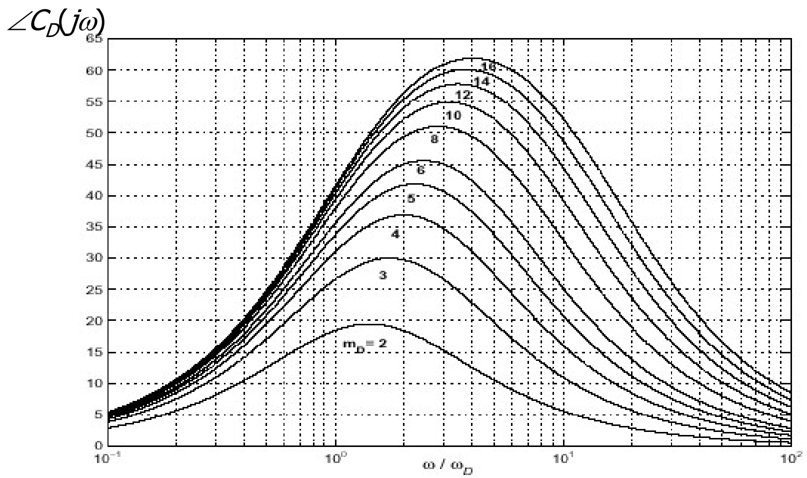


- Quantify the amount of the phase lead $\Delta\varphi$ needed at $\omega_{c,des}$ in order to shift the value of $\angle L'(j\omega_{c,des})$ outside the "influence" of the constant magnitude loci T_p and S_p.
- m_D is choosen on the basis of the required value of $\Delta\varphi$.
- ω_D is fixed to obtain that the phase lead $\Delta\varphi$ occurs exactly at $\omega_{c,des}$.
- magnitude adjustments (if needed) are obtained in a successive step.
- a systematic procedure for the choice of m_D and ω_D can be established using the **universal lead network diagrams** →

The lead network: magnitude design diagram

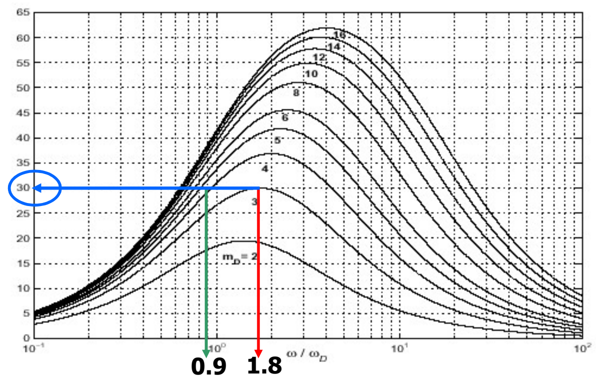


The lead network: phase design diagram



The lead network: design

Example: suppose that a phase lead of $\Delta\varphi$ = 30° is required at $\omega_{c,des}$ = 3 *rad/s*



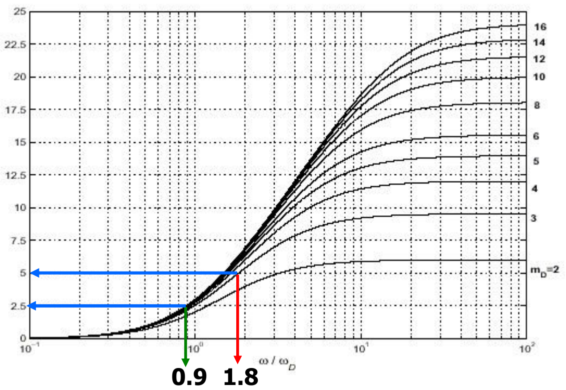
different choices can be made

m_D= 3; ω_{norm} = ω/ω_D = 1.8
$(\omega/\omega_D)|_{\omega=\omega_{c,des}}$ = 1.8
ω_D= $\omega_{c,des}$ /1.8=1.67 rad/s

m_D= 4; ω_{norm} = ω/ω_D = 0.9
$(\omega/\omega_D)|_{\omega=\omega_{c,des}}$ = 0.9
ω_D= $\omega_{c,des}$ /0.9=3.33 rad/s

The lead network: design

The corresponding magnitude increases are



Lead network: design example 1

A plant to be controlled is described by the following transfer function

$$G(s) = \frac{2}{(1+0.2s)(1+0.1s)}$$

design a cascade controller C(s) in order to satisfy the following requirements.

- $\left|e_r^\infty\right| \le 0.1,\ r(t) = t\varepsilon(t) \rightarrow C_{SS}(s) = \frac{5}{s}$
- $\hat{s} \le 30\% \rightarrow T_p = 3.67\text{dB}, S_p = 5.1\text{dB}$
- $t_r \le 0.3\ s \rightarrow \omega_{c,des} = 7\ \text{rad/s}$

Lead network: design example 2

A plant to be controlled is described by the following transfer function

$$G(s) = \frac{s+1}{s^2(s-1)}$$

design a cascade controller C(s) in order to satisfy the following requirements.

- $\left|e_r^\infty\right| = 0,\ r(t) = 2\varepsilon(t), \left|y_{d_a}^\infty\right| \le 0.1,\ d_a(t) = \varepsilon(t) \rightarrow C_{SS}(s) = 10$
- $\hat{s} \le 25\% \rightarrow T_p = 2.67\text{dB}, S_p = 4.35\text{dB}$
- $t_r \le 0.1\ s, t_{s,1\%} \le 0.7\ s \rightarrow \omega_{c,des} = 18\ \text{rad/s}$

Multiple lead network

- In principle, the maximum phase lead that can be introduced by a lead network is 90° corresponding to the ideal case $m_D \rightarrow \infty$; in this case, the lead network tf degenerates in the non-proper form:

$$C_D(s) = \frac{1+\frac{s}{\omega_D}}{1+\frac{s}{m_D\omega_D}} \xrightarrow[m_D\to\infty]{} 1+\frac{s}{\omega_D}$$

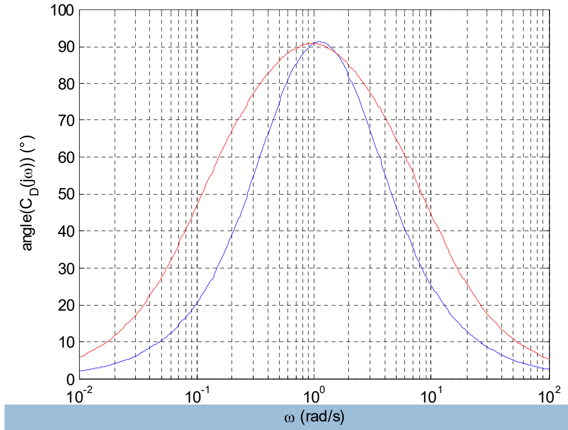
- In practice, it is suggested to use a multiple lead network (e.g. double) when the required phase lead is greater than 60°:

$$C_D(s) = \frac{1+\frac{s}{\omega_{D1}}}{1+\frac{s}{m_{D1}\omega_{D1}}}\,\frac{1+\frac{s}{\omega_{D2}}}{1+\frac{s}{m_{D2}\omega_{D2}}}$$

Multiple lead network

$$C_D(s) = \frac{1+\dfrac{s}{\omega_{D1}}}{1+\dfrac{s}{m_{D1}\omega_{D1}}} \cdot \frac{1+\dfrac{s}{\omega_{D2}}}{1+\dfrac{s}{m_{D2}\omega_{D2}}}$$

Parameters m_{D1} and m_{D2} of a double lead network are chosen in order to introduce the required phase increase at the desired cross-over frequency



Example :

$\Delta\varphi = 90° @ 1\,\text{rad/s}$

$m_{D1} = m_{D2} = 6$

$\omega_{norm,1} = \omega_{norm,2} = 2.2$

$m_{D1} = m_{D2} = 12$

$\omega_{norm,1} = 10, \omega_{norm,2} = 1.3$