Automatic Control

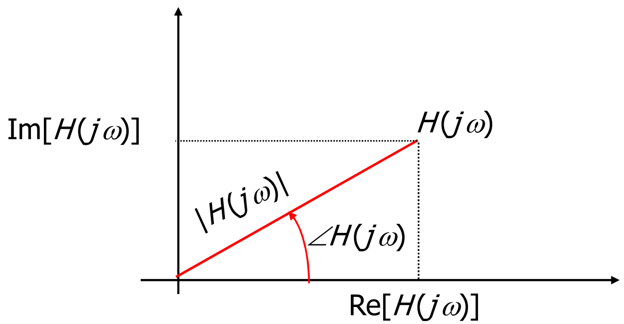
Frequency response tools for analysis and design of feedback control systems

- Part II: Polar diagram, Nyquist diagram and Nichols diagram

Frequency response graphical representations

Frequency response function

The function $H(j\omega) : \mathbb{R}^+ \to \mathbb{C}$ of the variable $\omega \in \mathbb{R}^+$ is called **frequency response function** of the system:



$H(j\omega) = \mathrm{Re}[H(j\omega)] + j\,\mathrm{Im}[H(j\omega)] \rightarrow$ **Cartesian representation**

$H(j\omega) = |H(j\omega)| e^{j\angle H(j\omega)} \rightarrow$ **Polar representation**

Frequency response: graphical representations

The **frequency response function** of a dynamic system can be graphically represented through:

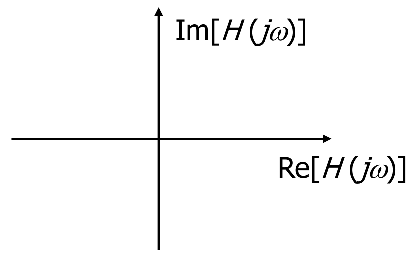
- **Bode diagrams** $\rightarrow$ representation of $|H(j\omega)|$ and $\angle H(j\omega)$ in function of $\omega \in \mathbb{R}^+$
- **Polar diagram** $\rightarrow$ representation of $\mathrm{Im}[H(j\omega)]$) vs. $\mathrm{Re}[H(j\omega)]$ parameterized in $\omega \in \mathbb{R}^+$
- **Nichols diagram** $\rightarrow$ representation of $|H(j\omega)|$ vs. $\angle H(j\omega)$ parameterized in $\omega \in \mathbb{R}^+$

Polar diagram

Graphical representations: polar diagram

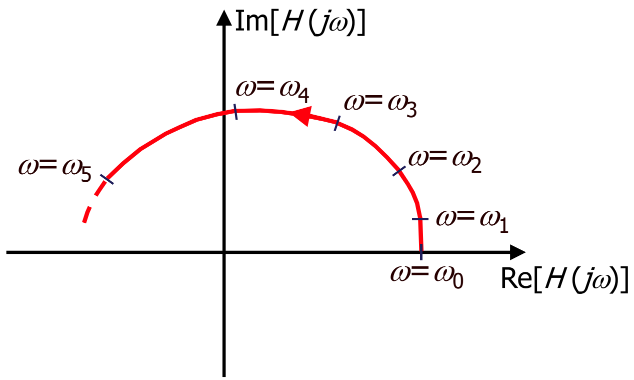
Polar diagram $\rightarrow$ Representation of Im[$H(j\omega)$]) vs. Re[$H(j\omega)$] parametrized in $\omega \in \mathbb{R}^+$

- The polar diagram is obtained by representing Im[$H(j\omega)$] as a function of Re[$H(j\omega)$] in a single plot parameterized and oriented wrt ω
- Each point of the plot corresponds to a value of the frequency $\omega \in \mathbb{R}^+$



Polar diagram

Polar diagram $\rightarrow$ Representation of Im[$H(j\omega)$]) vs. Re[$H(j\omega)$] parametrized and oriented in $\omega \in \mathbb{R}^+$



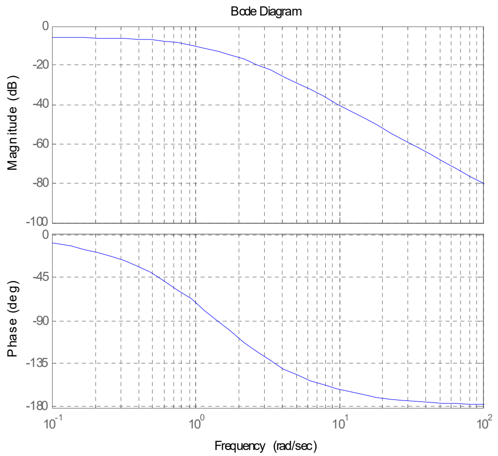
Polar diagram: approximate drawing

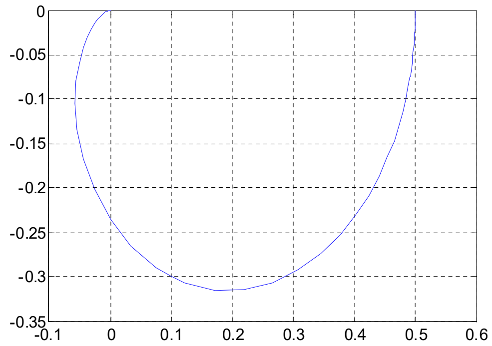
An approximate polar diagram can be obtained from the Bode diagram according to the following procedure:

- **Real and imaginary part for $\omega = 0^+$:** take from the Bode diagram the values of $|H(j0^+)|$ and $\angle H(j0^+)$, and mark the corresponding point on the plane (Re[$H(j\omega)$], Im[$H(j\omega)$])
- **Real and imaginary part for $\omega \rightarrow \infty$** : take from the Bode diagram the values of $|H(j\infty)|$ and $\angle H(j\infty)$, and mark the corresponding point on the plane (Re[$H(j\omega)$], Im[$H(j\omega)$])
- **Real and imaginary part for $0 < \omega < \infty$:** consider, on the $\angle H(j\omega)$ diagram, the points corresponding to: $\angle H(j\omega) = \pm k 90°$, $k = 0,1,2,...$ $\rightarrow$ these points identify the intersections of the polar diagram with the axes of the (Re[$H(j\omega)$], Im[$H(j\omega)$]) plane

Polar diagram: example 1

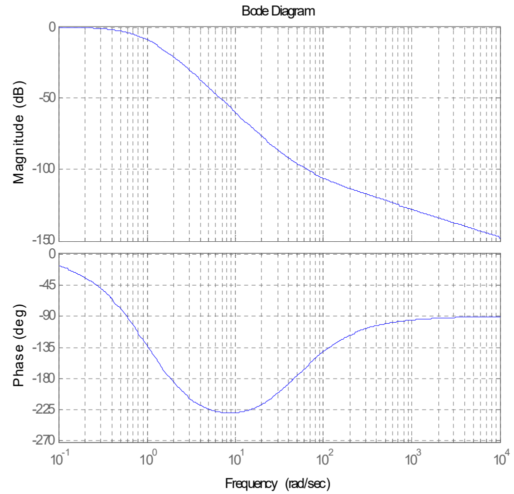
$$H(s) = \frac{1}{s^2 + 3s + 2}$$

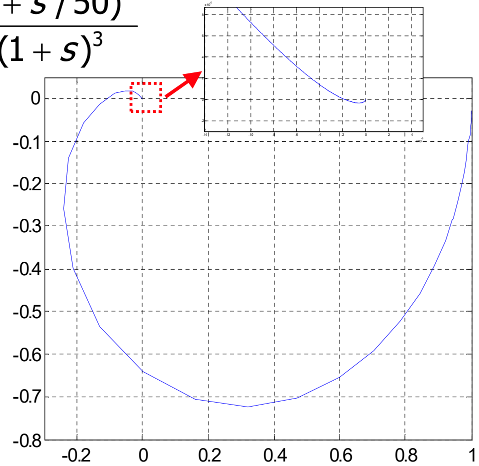




Polar diagram: example 2

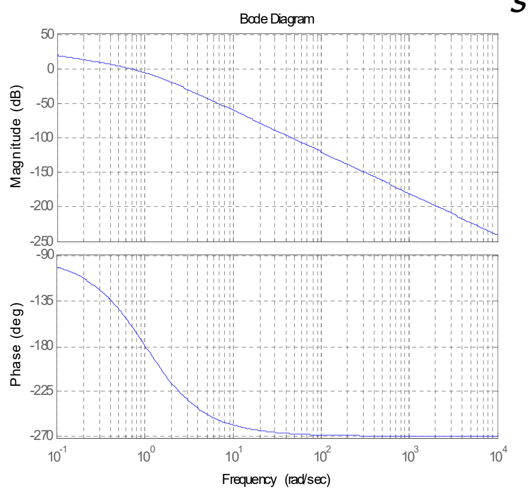
$$H(s) = \frac{(1 + s/50)^2}{(1+s)^3}$$

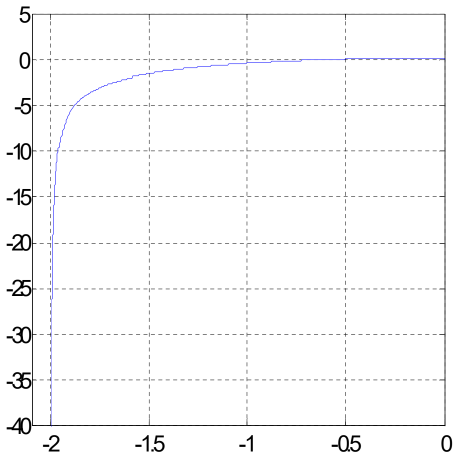




Polar diagram: example 3

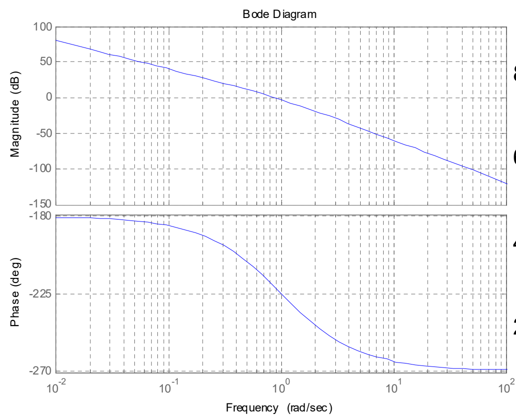
$$H(s) = \frac{1}{s(1+s)^2}$$

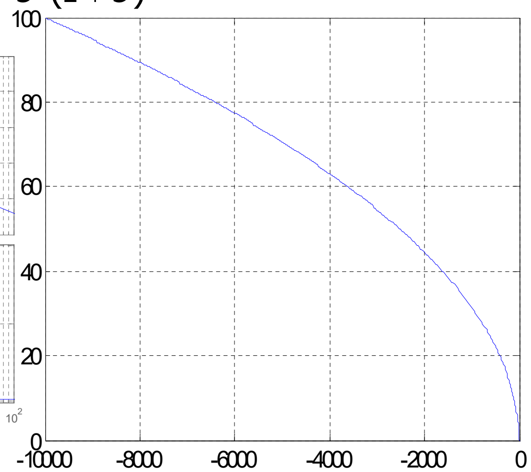




Polar diagram: example 4

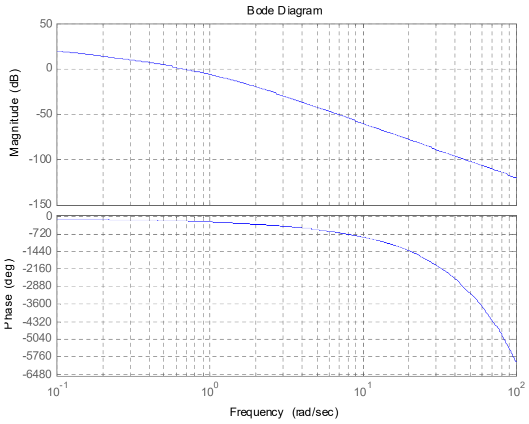
$$H(s) = \frac{1}{s^2(1+s)}$$

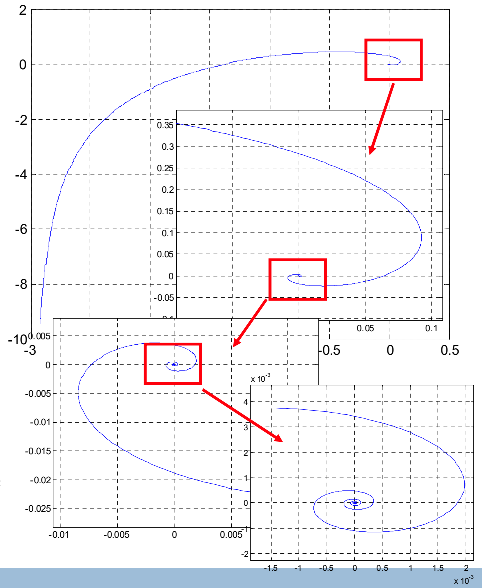




Polar diagram: example 5

$$H(s) = \frac{e^{-s}}{s(1+s)^2}$$

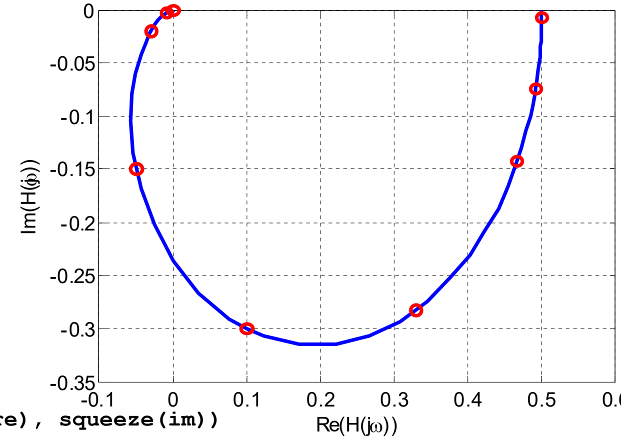




Polar diagram

- Polar diagram with MatLab
  - Statement **nyquist**

```
>> s=tf('s')
 Transfer function:
s
 >> H=1/(s^2+3*s+2)
 Transfer function:
      1
-------------
s^2 + 3 s + 2
>> [re,im]=nyquist(H);
>> figure, plot(squeeze(re), squeeze(im))
```

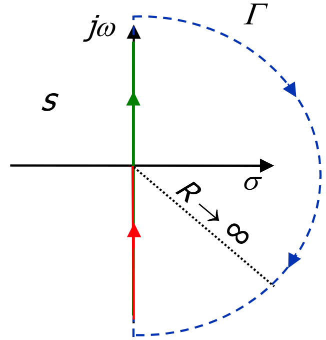


Nyquist diagram

Nyquist contour

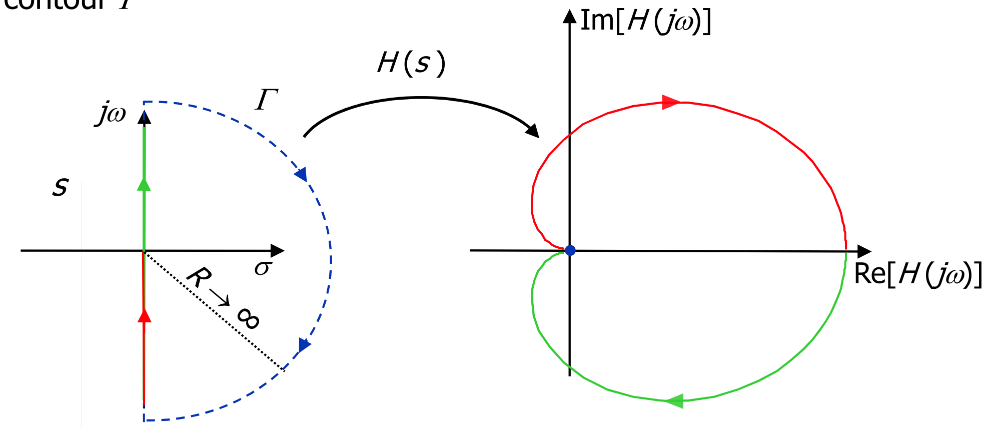
The **Nyquist contour** is defined as the closed curve Γ on the complex plane s given by the union of the following set of points:

- the negative imaginary axis
  → $s = \sigma + j\omega : \sigma = 0,\ \omega \in (-\infty, 0)$
- the positive imaginary axis
  → $s = \sigma + j\omega : \sigma = 0,\ \omega \in [0, +\infty)$
- a semicircle of radius $R \to \infty$, centered at the origin, connecting clockwise the points $(0 + j\infty)$ and $(0 - j\infty)$



Nyquist diagram

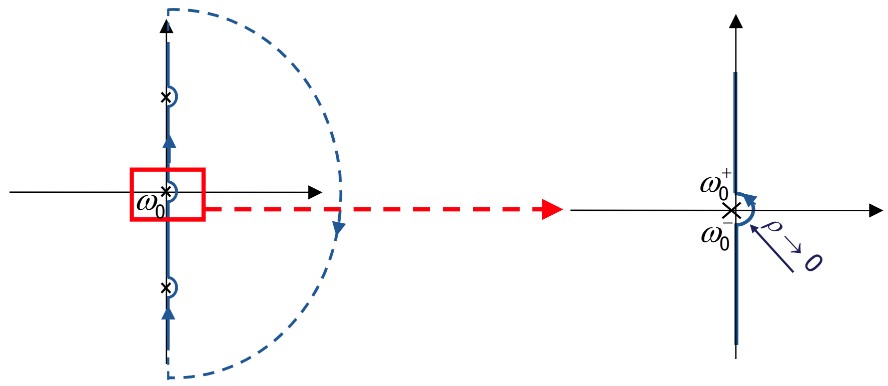
The **Nyquist diagram** is defined as the image on the complex plane (Re[$H(j\omega)$],Im[$H(j\omega)$]) of the function $H(s)$ computed on the Nyquist contour Γ



Nyquist contour: a critical case

If the Nyquist contour has some poles on the imaginary axis (e.g. in the origin), the function $H(s)$ cannot be computed

In this case, the Nyquist contour has to be modified:



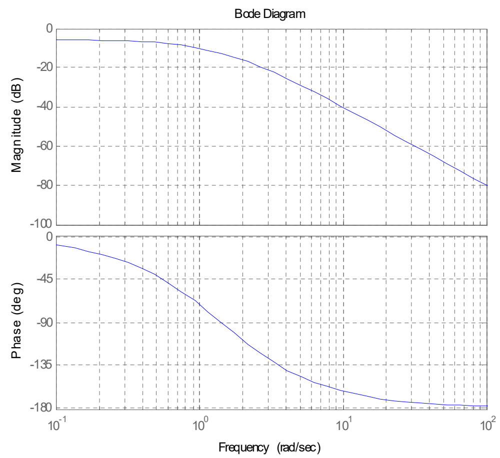
Nyquist diagrams: approximate drawing

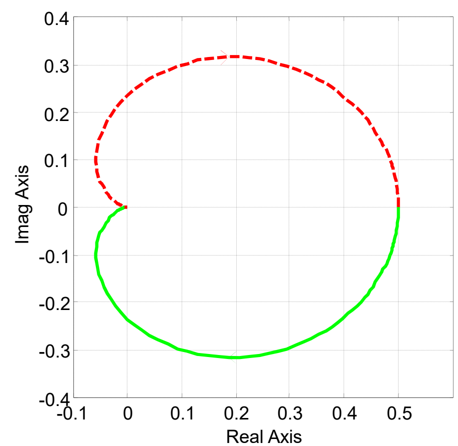
An approximate Nyquist diagram can be obtained from the polar diagram

- **Axis $j\omega > 0$**: the image is the polar diagram
- **Axis $j\omega < 0$**: since $H(j\omega) = H^*(-j\omega)$, the image is the symmetric reflection of the polar diagram w.r.t. the real axis Re[$H(j\omega)$]
- **Semicircle R → ∞**: the image is given by $H(j\infty)$
- **Semicircle $\rho \to 0$** → related to the presence of a pole in $s = j\omega_0$ with mulitplicity μ : the image is given by μ semicircles which clockwise connect the image of ω_0^-, $(H(j\omega_0^-))$ with the image of ω_0^+, $(H(j\omega_0^+))$ on the (Re[$H(j\omega)$], Im[$H(j\omega)$]) plane

Nyquist diagram: example 1

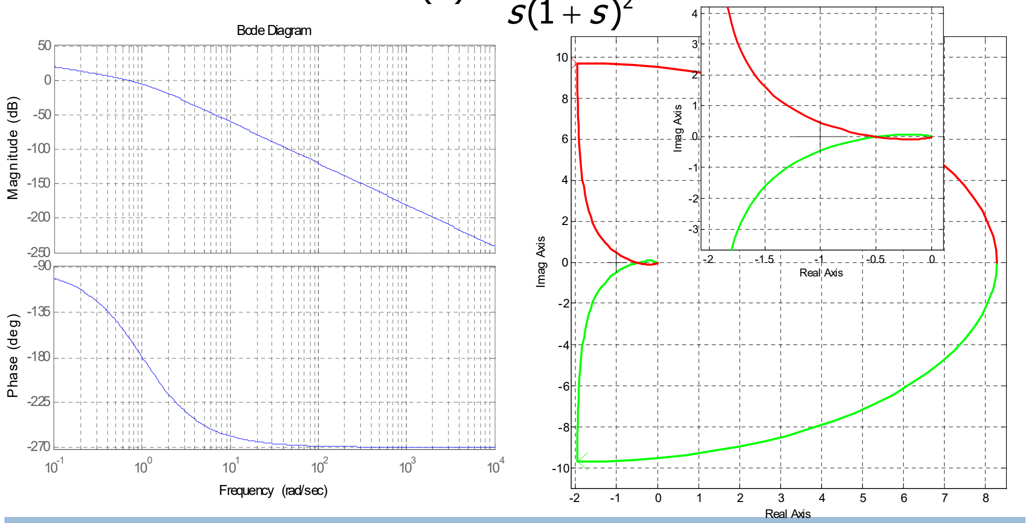
$$H(s) = \frac{1}{s^2 + 3s + 2}$$





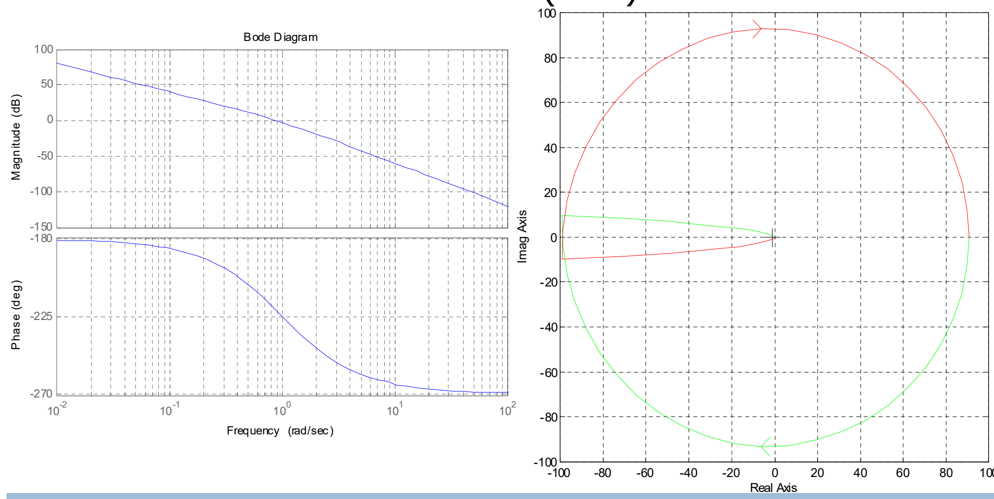
Nyquist diagram: example 2

$$H(s) = \frac{1}{s(1+s)^2}$$



Nyquist diagram: example 3

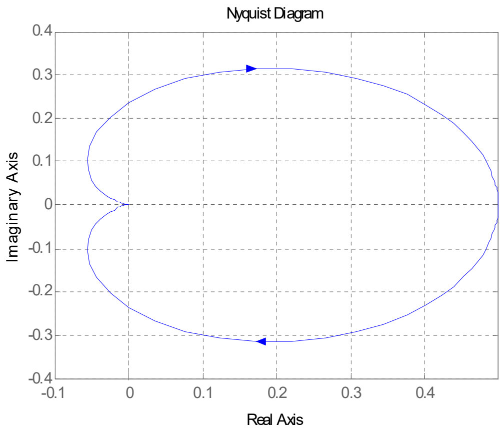
$$H(s) = \frac{1}{s^2(1+s)}$$



Nyquist diagram

- Nyquist diagram with MatLab
  - Command **nyquist**

```
>> s=tf('s')
 Transfer function:
s
 >> H=1/(s^2+3*s+2)
 Transfer function:
      1
-------------
s^2 + 3 s + 2
>> figure, nyquist(H)
```



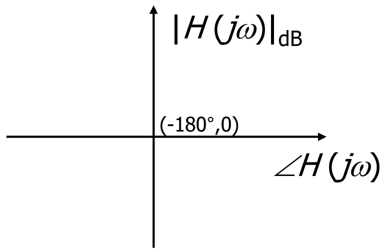
Remark: MatLab **does not plot** the images of the semicircles $\rho \to 0$

Nichols diagram

Graphical representations: Nichols diagram

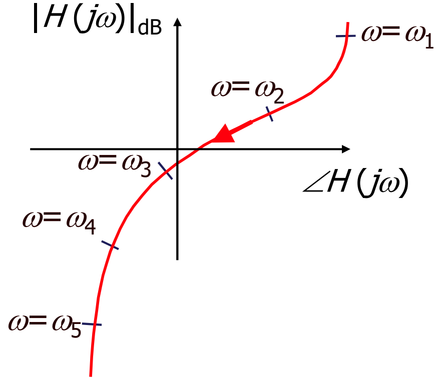
Nichols diagram $\rightarrow$ representation of $|H(j\omega)|$ vs. $\angle H(j\omega)$ parametrized in $\omega \in \mathbb{R}^+$

- The Nichols diagram is obtained by representing $|H(j\omega)|$ in function of $\angle H(j\omega)$ in a single plot parameterized and oriented in ω
- Each point of the plot corresponds to a value of the frequency $\omega \in \mathbb{R}^+$
- The origin of the diagram is conventionally fixed at the point (-180°,0 dB)



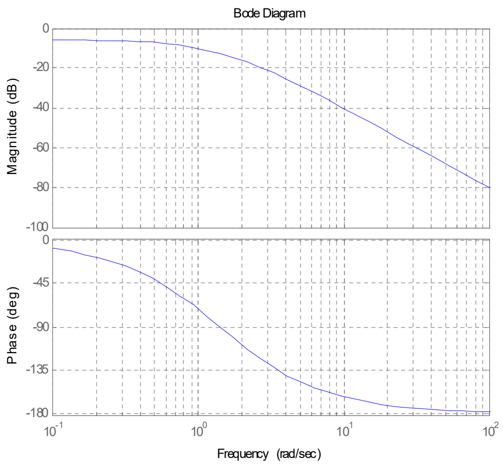
Nichols diagram

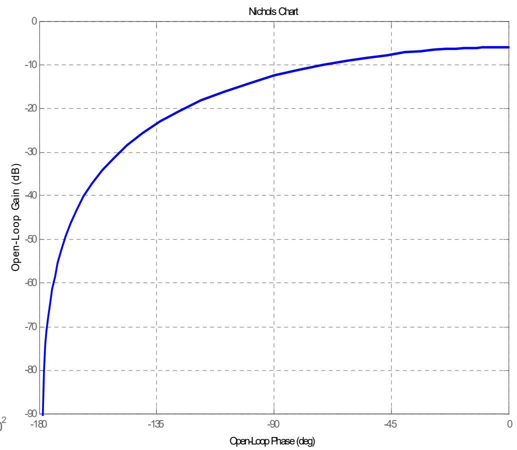
Nichols diagram $\rightarrow$ polar representation of $|H(j\omega)|_{dB}$ vs. $\angle H(j\omega)$ in degrees as parametrized and oriented in $\omega \in \mathbb{R}^+$



Nichols diagram: example 1

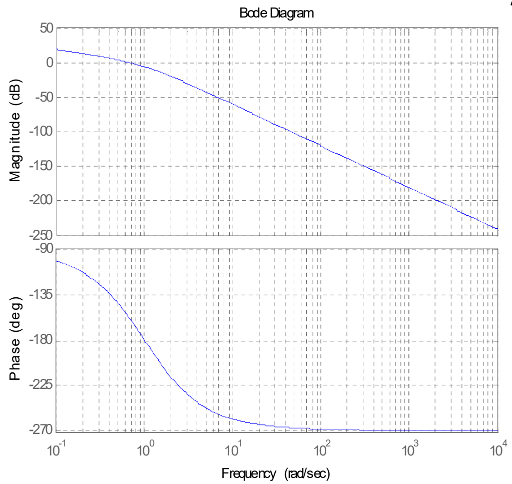
$$H(s) = \frac{1}{s^2 + 3s + 2}$$

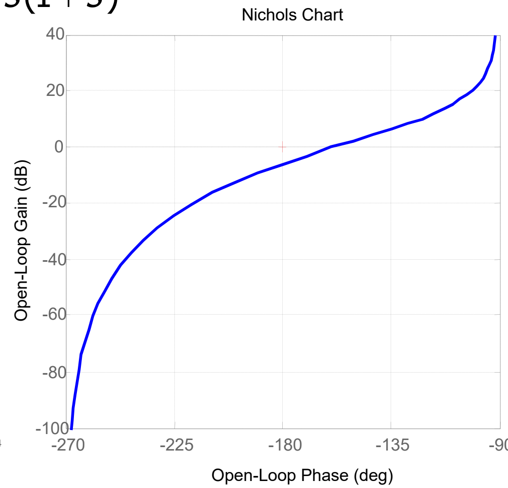




Nichols diagram: example 2

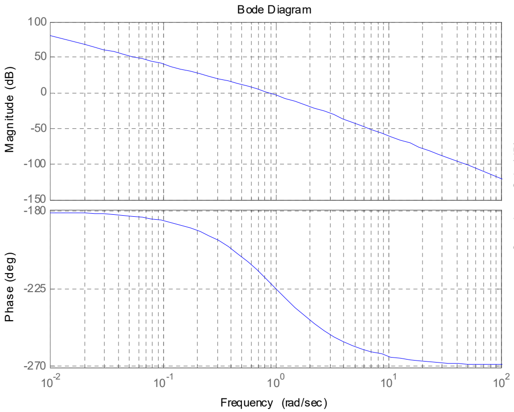
$$H(s) = \frac{1}{s(1+s)^2}$$

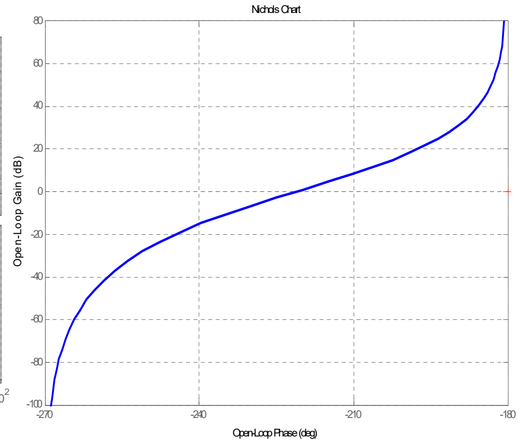




Nichols diagram: example 3

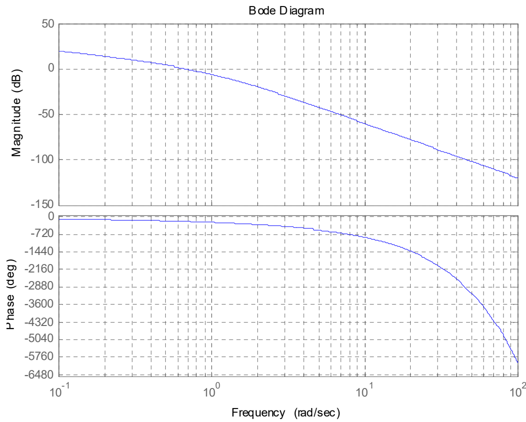
$$H(s)=\frac{1}{s^2(1+s)}$$

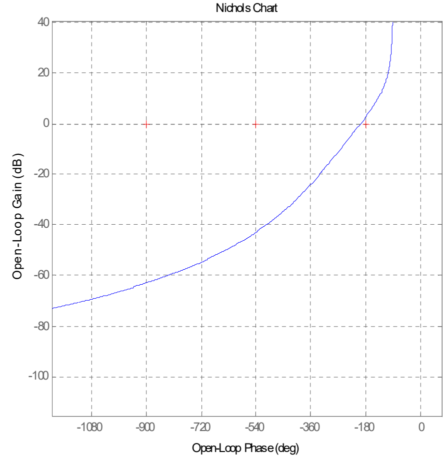




Nichols diagram: example 4

$$H(s)=\frac{e^{-s}}{s(1+s)^2}$$





Nichols diagram

- Nichols diagram with MatLab
  - Statement **nichols**

```
>> s=tf('s')
 Transfer function:
s
 >> H=1/(s*(s+1)^2)
 Transfer function:
        1
---------------
s^3 + 2 s^2 + s
>> figure, nichols(H)
```