第3章 作业习题

1. 计算行列式

(1) $\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 \\ b^2 & (b+1)^2 & (b+2)^2 \\ c^2 & (c+1)^2 & (c+2)^2 \end{vmatrix}$ (2) $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$

(3) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 0 & 2 \\ 4 & 0 & 0 & 1 \end{vmatrix}$ (4) $\begin{vmatrix} (1+a) & 1 & 1 & 1 \\ 1 & (1-a) & 1 & 1 \\ 1 & 1 & (1+b) & 1 \\ 1 & 1 & 1 & (1-b) \end{vmatrix}$

2.已知x, y, z两两不相等,求证：$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \neq 0.$

3.计算n阶行列式

(1) $\begin{vmatrix} a & b & 0 & \cdots & 0 & 0 \\ 0 & a & b & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a & b \\ b & 0 & 0 & \cdots & 0 & a \end{vmatrix}$ (2) $\begin{vmatrix} x_1+a_1 & a_2 & a_3 & \cdots & a_n \\ a_1 & x_1+a_2 & a_3 & \cdots & a_n \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & x_1+a_n \end{vmatrix}$

(3) $\begin{vmatrix} a & b & 0 & \cdots & 0 & 0 \\ c & a & b & \cdots & 0 & 0 \\ 0 & c & a & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a & b \\ 0 & 0 & 0 & \cdots & c & a \end{vmatrix}$

4.求4阶行列式$D_n = \begin{vmatrix} 3 & 0 & 4 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & -7 & 0 & 0 \\ 5 & 3 & -2 & 2 \end{vmatrix}$中第四行各元素余子式之和.

5.证明n阶行列式：

$$(1)\begin{vmatrix} a_0 & 1 & 1 & \cdots & 1 \\ 1 & a_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & 0 & \cdots & a_n \end{vmatrix} = a_1a_2\cdots a_n(a_0 - \textstyle\sum_{i=1}^n \frac{1}{a_i});$$

$$(2)\begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & \cdots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \cdots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & \alpha+\beta \end{vmatrix} = \tfrac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta};$$

$$(3)\begin{vmatrix} \cos\theta & 1 & 0 & \cdots & 0 & 0 \\ 1 & 2\cos\theta & 1 & \cdots & 0 & 0 \\ 0 & 1 & 2\cos\theta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2\cos\theta & 1 \\ 0 & 0 & 0 & \cdots & 0 & 2\cos\theta \end{vmatrix} = \cos n\theta.$$

6.证明：$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$

7.由$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & 1 \end{vmatrix} = 0$证明奇偶排列各半.

8.证明：$\begin{vmatrix} a_{11} & \cdots & a_{1k} & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ a_{k1} & \cdots & a_{kk} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1k} & b_{11} & \cdots & b_{1r} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ c_{r1} & \cdots & c_{rk} & b_{r1} & \cdots & b_{rr} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & \cdots & \vdots \\ a_{k1} & \cdots & a_{kk} \end{vmatrix}\begin{vmatrix} b_{11} & \cdots & b_{1r} \\ \vdots & \cdots & \vdots \\ b_{r1} & \cdots & b_{rr} \end{vmatrix}$

9.用克莱姆法则解下列线性方程组：

$$(1)\begin{cases} 5x_1+6x_2=1 \\ x_1+5x_2+6x_3=0 \\ x_2+5x_3+6x_4=0 \\ x_3+5x_4+6x_5=0 \\ x_4+5x_5=0 \end{cases} \qquad (2)\begin{cases} 2x_1+x_2-5x_3+x_4=0 \\ x_1-3x_2-6x_4=3 \\ 2x_2-x_3+2x_4=-5 \\ x_1+4x_4-7x_3+6x_4=0 \end{cases}$$

10.设线性空间$a_1, a_2, \cdots, a_n$是数域P中互不相同的数,$b_1, b_2, \cdots, b_n$是数域P中任意一组给定的数,用克莱姆法则证明:存在唯一的数域P上的多项式$f(x)=c_0x^{n-1}+c_1x^{n-2}+\cdots+c_{n-1}$ 使$f(a_i)=b_i, i=1,2,\cdots,n$