AUTOMATIC CONTROL

Computer, Electronic and Communications Engineering

Laboratory practice n. 3

Objectives: Stability of feedback systems, Nyquist stability criterion, frequency response of feedback systems, graphical representation of the frequency response (Bode, polar, Nyquist and Nichols plots), feedback control systems transfer functions.

Problem 1: feedback systems frequency response analysis

Packet information flow in a router working under TCP/IP can be modelled, in a suitable working condition, through the following transfer function[1]:

$$G(s) = \frac{q(s)}{p(s)} = \frac{\frac{c^2}{2N} e^{-sR}}{\left(s + \frac{2N}{R^2 c}\right)\left(s + \frac{1}{R}\right)}$$

where:

- q = queue length (packets)
- p = probability of packet mark/drop
- c = link capacity (packets/s)
- N = load factor (number of TCP sessions)
- R = round trip time (s)

The objective of an active queue management (AQM) algorithm is to choose automatically the packet mark/drop probability p, so that the sender can tune the window size to keep the queue length at a constant level. This system can be represented by the standard feedback structure reported in Figure 1.

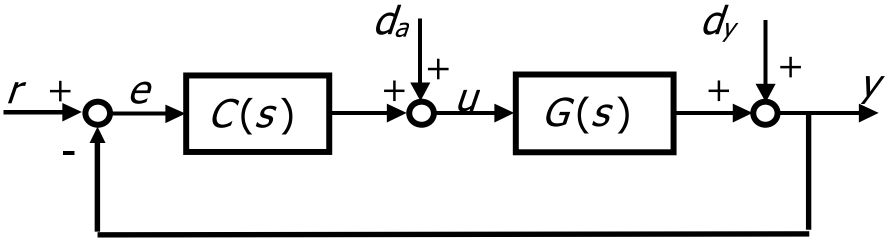


Figure 1

Several AQM algorithms are available, but the one that has received special attention is the random early detection (RED) algorithm. The RED algorithm dynamics can be approximated through the linear controller:

$$C(s) = \frac{\ell}{1 + \frac{s}{\kappa}}$$

———

[1] C. V. Hollot, V. Misra, D. Towsley, W. Gong, "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows", IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 47, NO. 6, pp. 945-959, 2002.

Given the following controller and plant parameter values:

$$\ell = 1.86 \cdot 10^{-4}, \kappa = 0.005, c = 3750, N = 60, R = 0.246$$

- Plot the Bode diagram of the loop function L(s) to evaluate the crossover frequency ω_c and the ω=0 and ω=∞ behaviour. (Answer: ω_c = 0.05 rad/s, |L(j0)| = 20.12 dB arg(L(j0)) = 0°, |L(j∞)| = 0 = – ∞ dB arg(L(j∞)) → – ∞)
- Plot the Bode magnitude plot of T(s) to evaluate its resonant peak T_p and bandwidth ω_B. (Hint: to compute T(s), replace the time delay term e^{-sR} through its 2nd order Padé approximation, see pag. AC-L07 71, then use the statement `bodemag`)
  (Answer: T_p = 0 dB, ω_B = 0.062 rad/s).
- Plot the Bode magnitude plot of the sensitivity function S(s) to evaluate its resonant peak S_p and bandwidth ω_{BS}. (Hint: to compute S(s), replace the time delay term e^{-sR} through its 2nd order Padé approximation, see pag. AC-L07 71, then use the statement `bodemag`)
  (Answer: S_p = 0.74 dB, ω_{BS} = 0.0496 rad/s).
- Use the Nichols plot of the given loop function L(s) to evaluate the crossover frequency ω_c. Moreover, evaluate the resonant peak T_p and the bandwidth ω_B of the complementary sensitivity function T(s), the resonant peak S_p and the bandwidth ω_{BS} of the sensitivity function S(s) through the constant magnitude loci procedure using the `T_grid` and `S_grid` statements. Are all the results in agreement with the ones obtained in the previous points? If not, try to explain why and tell which are the correct ones.
- Repeat the previous points when:

$$C(s) = \frac{K_c\left(1+\frac{s}{z}\right)}{s}, K_c = 9.64 \cdot 10^{-6}, z = 0.53$$

  (Answer: ω_c = 0.51 rad/s, |L(j0)| → ∞ dB arg(L(j0)) = – 90°, |L(j∞)| = 0 → – ∞ dB arg(L(j∞)) → – ∞, T_p = 0 dB, ω_B = 0.722 rad/s, S_p = 1.76 dB, ω_{BS} = 0.425 rad/s)

Problem 2: Practice on the graphical representation of the frequency response

Consider the following transfer functions

1. $L(s) = \dfrac{s+1}{(s-1)^2}$
2. $L(s) = \dfrac{5}{s^3}$
3. $L(s) = \dfrac{2(s^2+80s+40000)}{s^2(s^2+160s+80000)}$
4. $L(s) = \dfrac{2.4 \cdot 10^5 s^2}{(s^2+4.2s+49)(s^2+35s+4900)}$

for each:

- write the transfer function in the "dc-gain" form (see AC_L03 12)
- plot (using MatLab) the Bode diagrams. Discuss about the correctness of each plot based on the zeros and poles properties
- draw (by hand) the polar and the Nyquist plots (verify the results using MatLab); plot (using MatLab) the Nichols diagram.

Problem 3: Computation of transfer functions of a feedback control system. Review of time response computation and steady state response.

Consider the feedback control system reported in Figure 1

where $G(s) = \dfrac{1}{(s+1)^2}$ and $C(s) = \dfrac{(1+s)^2}{s\left(1+s/4\right)}$

- Compute the time response of the control input u(t), when r(t) is a unit step and the other inputs (i.e. d_a and d_y) are set to zero.
- Compute, if possible, the steady state response $e_{ss}(t)$ of the tracking error e(t) when $d_y(t)=0.5\sin(t)\varepsilon(t)$ and the other inputs (i.e. r (t) and $d_a(t)$) are set to zero.
- Compute, if possible, the steady state response $y_{ss}(t)$ of the controlled output y(t) when $r(t) = 3\varepsilon(t)$, $d_y(t)= 2\varepsilon(t)$ and $d_a(t)= 0$.

(Answer:

$u(t) = \left(3e^{-2t} - 2te^{-2t} + 1\right)\varepsilon(t)$, $e_{ss}(t) = 0.4123\sin(t\text{-}2.2531)$, $y_{ss}(t) = 3\varepsilon(t)$)

Conceptual problems

Problem 4: stability of feedback systems

Consider the feedback control system of Figure 1 and the transfer matrix M(s):

$$\begin{bmatrix} e(s) \\ u(s) \\ y(s) \end{bmatrix} = M(s) \begin{bmatrix} r(s) \\ d_a(s) \\ d_y(s) \end{bmatrix}, \quad M(s) = \begin{bmatrix} S(s) & -Q(s) & -S(s) \\ R(s) & S(s) & -R(s) \\ T(s) & Q(s) & S(s) \end{bmatrix}$$

$$T(s) = \frac{L(s)}{1+L(s)} \quad S(s) = \frac{1}{1+L(s)} \quad R(s) = \frac{C(s)}{1+L(s)} \quad Q(s) = \frac{G(s)}{1+L(s)}$$

$$L(s) = C(s)G(s)$$

1. Compute all the entries of M(s)

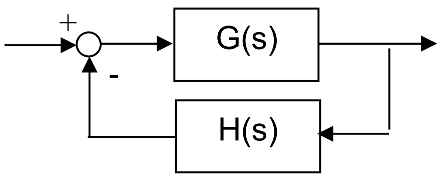
2. The definition of stability is that all the nine transfer functions should be stable. Show that in the considered unitary-feedback case, it actually suffices to check only two of the nine. Which two?

Problem 5: frequency response of feedback systems

Suggest a procedure to evaluate the resonant peak of the closed loop transfer function

$$Q(s) = \frac{G(s)}{1+L(s)} \quad L(s) = G(s)H(s)$$

of the feedback system below



supposing that the following information only is available:
- Nichols plot of L(s) (and constant magnitude loci M_T and M_S)
- H(s) = h = const.

Problem 6: Nyquist criterion

Show that, if the polar plot of the loop function L(s) crosses the critical point (-1, j0) of the Nyquist plane for a given frequency ω_0, then the feedback system reported in Figure 1 (supposed well-posed) is not stable since the complementary sensitivity function T(s) has a couple of imaginary poles at $s = \pm j\omega_0$.

Problem 7: frequency response of feedback systems

Show that

if $|L(j0)| \to \infty$ (due to the presence of poles at the origin in $L(s)$)

$$\text{then} \begin{cases} |S(j0)| \to |L(j0)|^{-1} \to 0 \\ |T(j0)| = 1 \end{cases}$$

if $|L(j\infty)| \to 0$ (due to the fact that $L(s)$ is strictly proper)

$$\text{then} \begin{cases} |S(j\infty)| \to 1 \\ |T(j\infty)| \to |L(j\infty)| = 0 \end{cases}$$