

## 9.2 阻抗（导纳）的联接

### 1. 阻抗的串联

串联：瞬时值方程  $\sum u = 0$       相量形式方程  $\sum \dot{U} = 0$

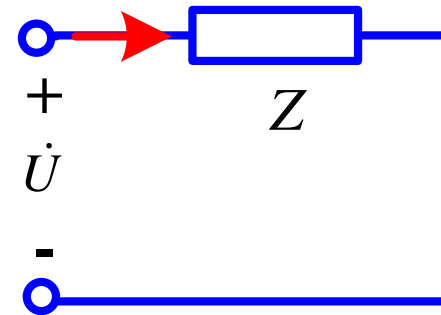
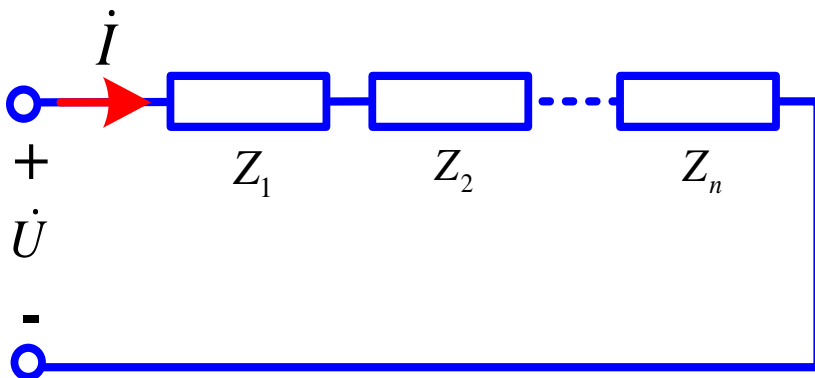
$$\dot{U} = \dot{U}_1 + \dot{U}_2 + \dots + \dot{U}_n = Z_1 \cdot \dot{I} + Z_2 \cdot \dot{I} + \dots + Z_n \cdot \dot{I}$$

$$\dot{U} = Z \dot{I}$$

$$\therefore Z = Z_1 + Z_2 + \dots + Z_n$$



$$\dot{U}_i = \frac{Z_i}{Z} \dot{U}$$



## 9.2 阻抗（导纳）的联接

### 2. 阻抗的并联

并联：瞬时值方程  $\sum i = 0$       相量形式方程  $\sum \dot{I} = 0$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 + \dots + \dot{I}_n = Y_1 \cdot \dot{U} + Y_2 \cdot \dot{U} + \dots + Y_n \cdot \dot{U}$$

$$\dot{I} = Y \dot{U}$$

$$\therefore Y = Y_1 + Y_2 + \dots + Y_n$$

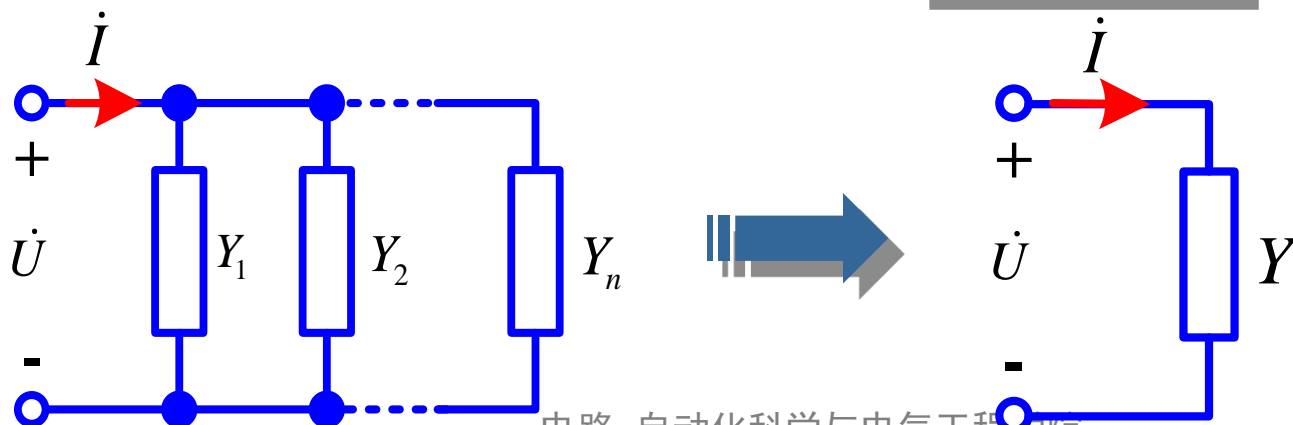
分流公式



$$\dot{I}_i = \frac{Y_i}{Y} \dot{I}$$

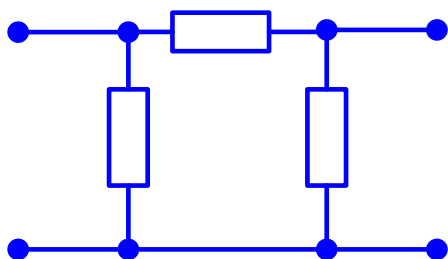
两个阻抗 $Z_1$ 、 $Z_2$ 的并联等效阻抗为：

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

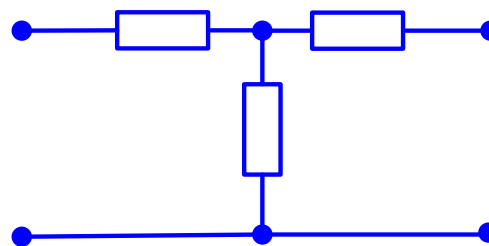


## 9.2 阻抗（导纳）的联接

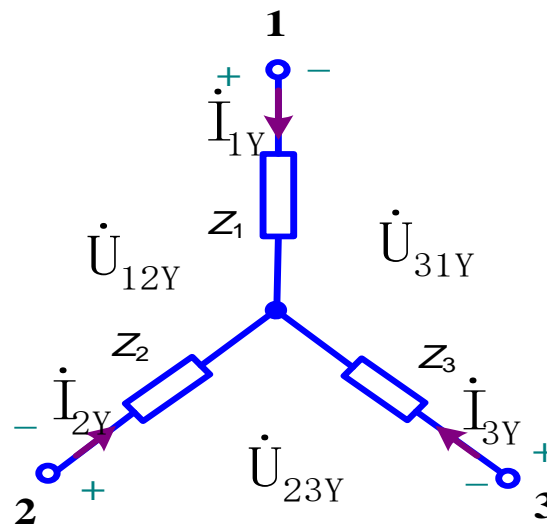
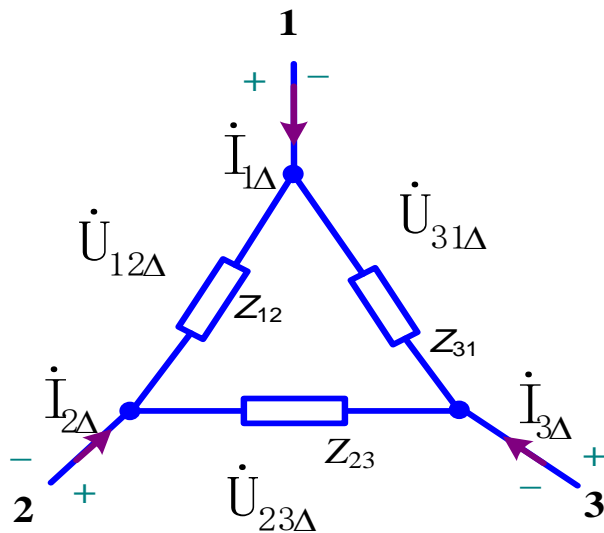
### 3. 阻抗的Y- $\Delta$ 联接



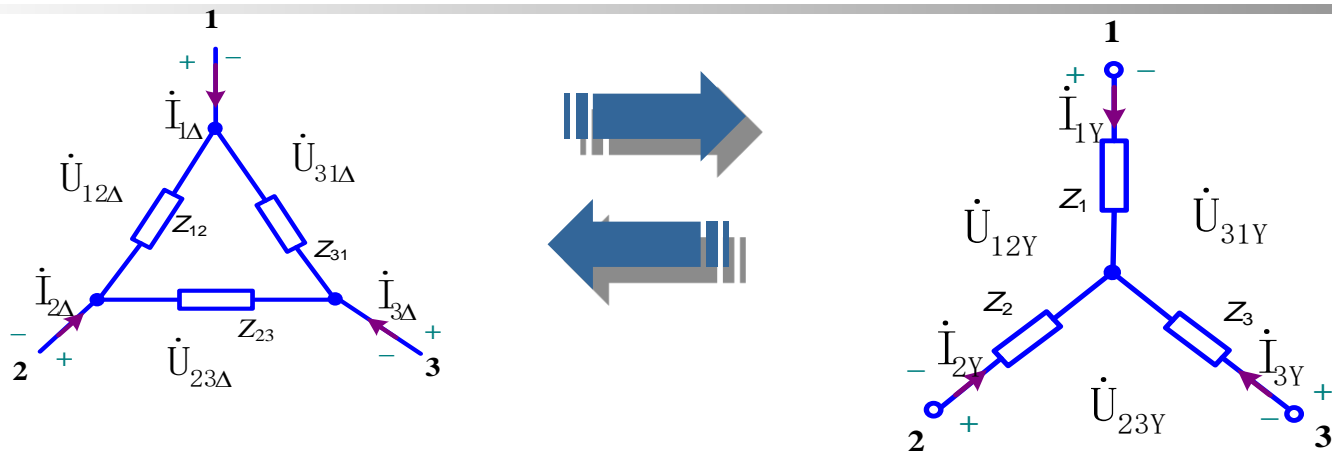
$\pi$  型电路 ( $\Delta$  型)



T 型电路 (Y、星型)



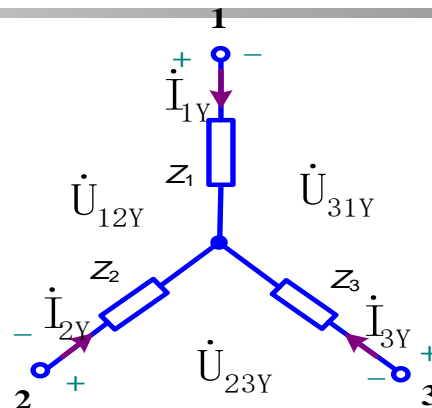
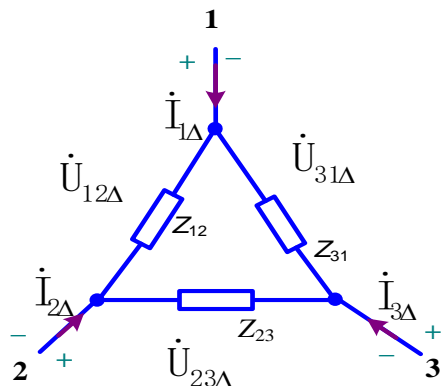
## 9.2 阻抗（导纳）的联接



$$\left. \begin{aligned} i_{1Y} &= \frac{\dot{U}_{12Y}Z_3 - \dot{U}_{31Y}Z_2}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \\ i_{2Y} &= \frac{\dot{U}_{23Y}Z_1 - \dot{U}_{12Y}Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \\ i_{3Y} &= \frac{\dot{U}_{31Y}Z_2 - \dot{U}_{23Y}Z_1}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \end{aligned} \right\} \quad \left\{ \begin{aligned} i_{1\Delta} &= \frac{\dot{U}_{12\Delta}}{Z_{12}} - \frac{\dot{U}_{31\Delta}}{Z_{31}} \\ i_{2\Delta} &= \frac{\dot{U}_{23\Delta}}{Z_{23}} - \frac{\dot{U}_{12\Delta}}{Z_{12}} \\ i_{3\Delta} &= \frac{\dot{U}_{31\Delta}}{Z_{31}} - \frac{\dot{U}_{23\Delta}}{Z_{23}} \end{aligned} \right.$$

根据端电压相等则电流相等的等效条件，得Y型→Δ型的变换条件

## 9.2 阻抗（导纳）的联接



得Y型→Δ型的变换条件:

$$Y_{12} = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}$$

$$Y_{23} = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}$$

$$Y_{31} = \frac{Y_3 Y_1}{Y_1 + Y_2 + Y_3}$$

Δ型→ Y型的变换条件:

$$Z_1 = \frac{Z_{12} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{23} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$

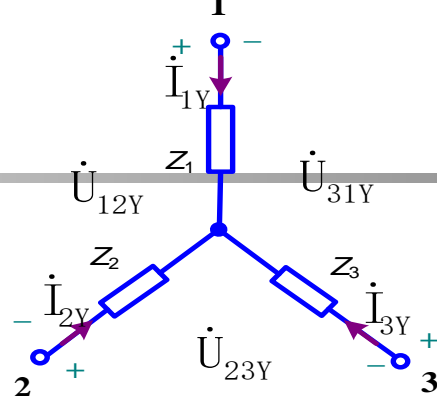
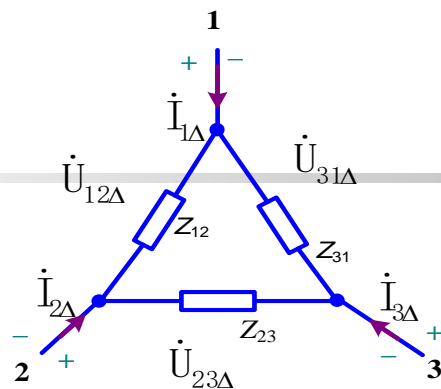
$$Z_3 = \frac{Z_{31} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Y_{\Delta} = \frac{Y \text{ 相邻导纳乘积}}{\sum Y_Y}$$

动化

$$Z_Y = \frac{\Delta \text{ 相邻阻抗乘积}}{\sum Z_{\Delta}}$$

# 单选题



三个相等的阻抗 $Z$ ，由三角形连接关系变成星形连接后，下列说法正确的是：

- ☐ A 阻抗模变大了；
- ☐ B 阻抗角变大了；
- ☐ C 阻抗角变小了；
- ☒ D 阻抗模变小了。

提交

## 9.2 阻抗（导纳）的联接

【例】 已知： $\omega = 1000 \text{ rad/s}$ ,  $U = 100 \text{ V}$ ,  $R = 10 \Omega$ ,  $R_1 = 50 \Omega$ ,

$$L = 20 \text{ mH}, C = 10 \mu\text{F}$$

求：各支路电流。

解

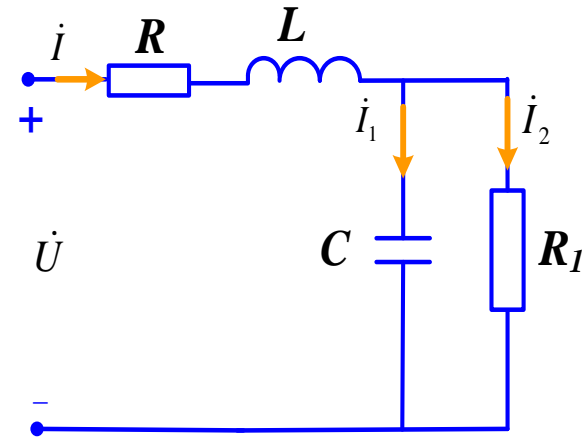
$$\text{设 } \dot{U} = 100 \angle 0^\circ \text{ V}$$

$$Z = R + j\omega L + \frac{R_1(-j\frac{1}{\omega C})}{R_1 - j\frac{1}{\omega C}} = 50 \angle 0^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{100 \angle 0^\circ}{50 \angle 0^\circ} = 2 \angle 0^\circ \text{ A}$$

$$\dot{I}_2 = \frac{-j\frac{1}{\omega C}}{R_1 - j\frac{1}{\omega C}} \dot{I} = 1.79 \angle -26.6^\circ \text{ A}$$

$$\dot{I}_1 = \frac{R_1}{R_1 - j\frac{1}{\omega C}} \dot{I} = 0.894 \angle 63.4^\circ \text{ A}$$



## 9.3 电路的相量图

### ■ 相量图

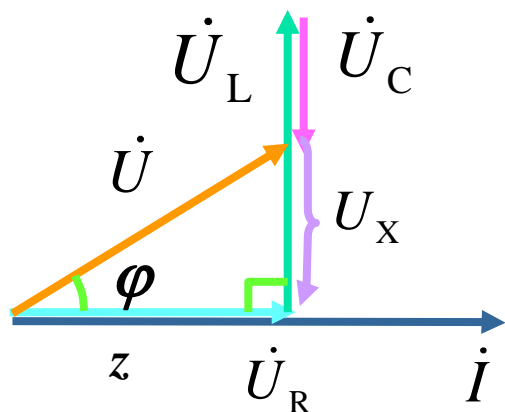
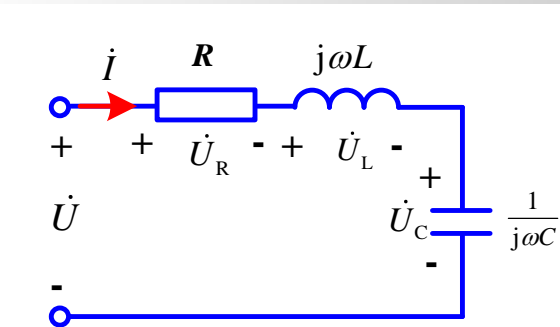
- 描述电路中电流、电压相量关系的图叫相量图
- 利用相量图分析求解

### ■ 相量图画法

- 任取一相量作为参考相量（相位为零），不需画出复平面的实轴、虚轴；
- 具有一般性，参考相量的初相可以不为零，其它相量与参考相量之间的相位关系为相对关系；
- 选择距离电源最远端的一个复杂环节，串联则以电流作为参考相量；并联则以电压作为参考相量；
- 相量间首尾相连以反映KCL与KVL，不要画成放射状。

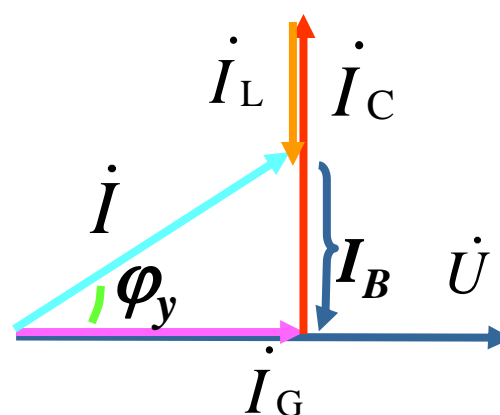
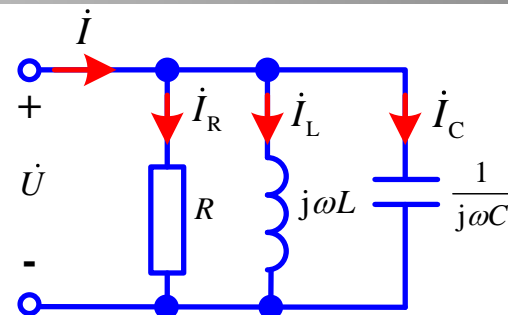


## 9.3 电路的相量图



$$U = \sqrt{U_R^2 + U_X^2}$$

$$= \sqrt{U_R^2 + (U_L - U_C)^2}$$



$$I = \sqrt{I_G^2 + I_B^2} = \sqrt{I_G^2 + (I_L - I_C)^2}$$

电流参考 → VCR → KVL

电压参考 → VCR → KCL

## 【例】

$R = 800\Omega, U = 220V, f = 50Hz, U_R = 110V,$   
求：元件L参数值。

**解** （一）解析法

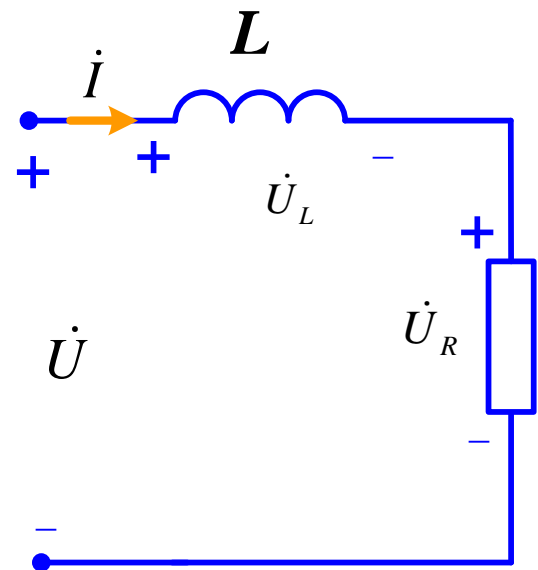
$$\text{设 } \dot{I} = I \angle 0^\circ \quad I = \frac{U_R}{R} = 0.1375(A)$$

$$\dot{U}_R = \dot{I}R = 110 \angle 0^\circ (V)$$

$$\dot{U}_L = j\omega L \times \dot{I} = 0.1375\omega L \angle 90^\circ (V)$$

$$\dot{U} = \dot{U}_R + \dot{U}_L = \sqrt{110^2 + (0.1375\omega L)^2} \angle \arctan\left(\frac{0.1375\omega L}{110}\right) (V)$$

$$U = 220 = \sqrt{110^2 + (0.1375\omega L)^2} \quad \Rightarrow \quad L = 4.4H$$



## (二) 相量图法

利用相量模（有效值）之间的关系

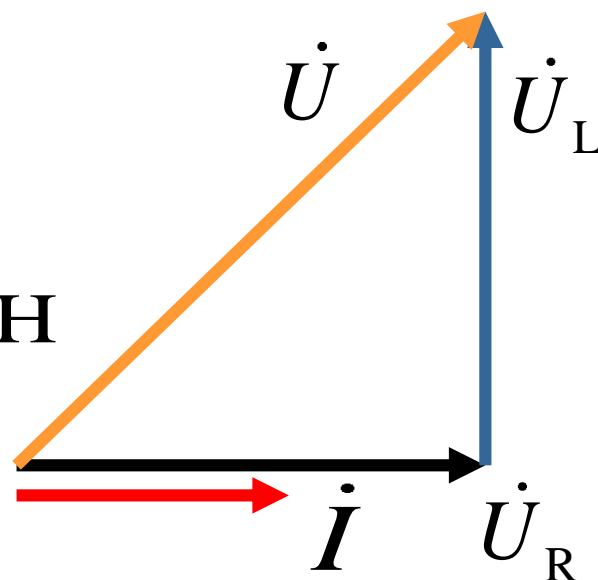
$$U_L = \sqrt{U^2 - U_R^2} = \sqrt{220^2 - 110^2} = 190.5V$$

$$I = \frac{U_R}{R} = \frac{110}{800} = 0.1375A$$

$$X_L = \omega L = \frac{U_L}{I} = \frac{190.5}{0.1375} = 1385.45\Omega$$

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{1385.45}{2\pi \times 50} = 4.4H$$

相量图是很好的工具！



相量图法

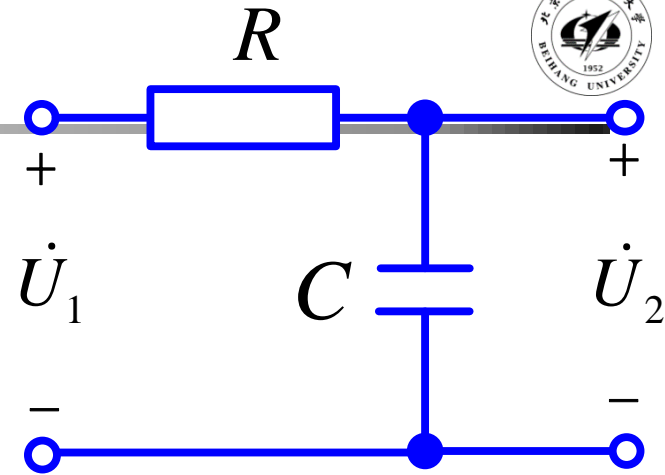
【例】已知： $R=1\text{k}\Omega$   $f=5\text{kHz}$

若使  $\dot{U}_2$  滞后于  $\dot{U}_1$   $30^\circ$  应取  $C$  为何值。

解 法1：解析法

$$\dot{U}_2 = \frac{-j\frac{1}{\omega C}}{R - j\frac{1}{\omega C}} \dot{U}_1$$

$$= \frac{-j\frac{1}{\omega C} (R + j\frac{1}{\omega C})}{R^2 + (\frac{1}{\omega C})^2} \dot{U}_1 = \frac{\frac{1}{\omega C}}{R^2 + (\frac{1}{\omega C})^2} (\frac{1}{\omega C} - jR) \dot{U}_1$$



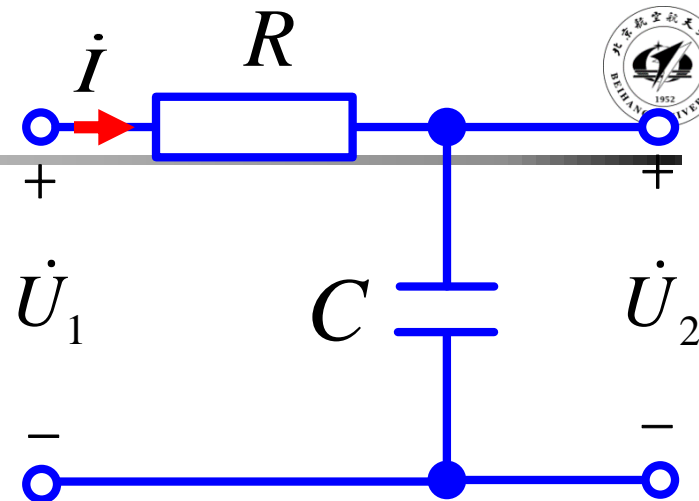
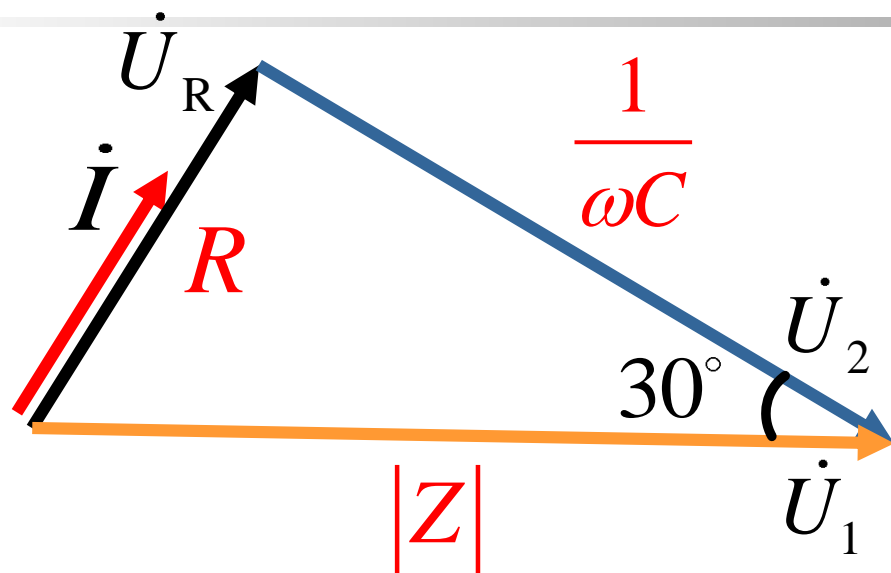
$$\begin{aligned}\dot{U}_2 &= \frac{\frac{1}{\omega C}}{R^2 + (\frac{1}{\omega C})^2} (\frac{1}{\omega C} - jR) \dot{U}_1 \\ &= \dot{U}_1 \frac{\frac{1}{\omega C} \sqrt{R^2 + (\frac{1}{\omega C})^2}}{R^2 + (\frac{1}{\omega C})^2} \angle \arctg \frac{-R}{\frac{1}{\omega C}}\end{aligned}$$

$$\arctg(-\omega CR) = -30^\circ$$

$$\operatorname{tg}(-30^\circ) = -\omega CR = -\frac{\sqrt{3}}{3}$$

$$C = \frac{\frac{\sqrt{3}}{3}}{2\pi f R} = 0.0184 \mu\text{F}$$

# 法2相量图法



利用相量相位之间的关系

$$\tan 30^\circ = \frac{R}{\frac{1}{\omega C}} = \omega CR = 2\pi fCR$$

$$C = \frac{\tan 30^\circ}{2\pi fR} = \frac{\frac{\sqrt{3}}{3}}{2\pi \times 5 \times 10^3 \times 10^3} = 0.0184 \mu\text{F}$$



## ■ 9-5 【串并联，相量图】

原题中  $I_2 = 10\text{A}$  改为  $\dot{I}_2 = 10\angle 90^\circ\text{A}$

## ■ 9-6 【并联电路分析】

## ■ 9-9 【输入阻抗】