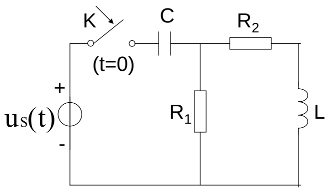
三系学习生活部

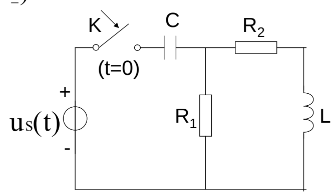
例6-1



已知：开关K闭合前电容和电感无贮能，t=0时K闭合。
求：　t=0_+时，各元件上电压、电流。

1

解：1. 求　$u_C(0_-)$　$i_L(0_-)$



$u_C(0_-)=0V$

$i_L(0_-)=0A$

$u_C(0_+)=u_C(0_-)=0V$

$i_L(0_+)=i_L(0_-)=0A$

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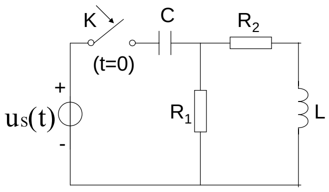
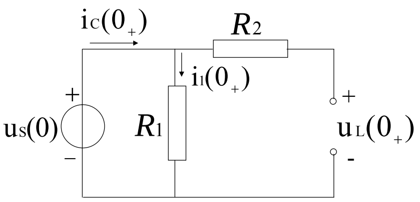
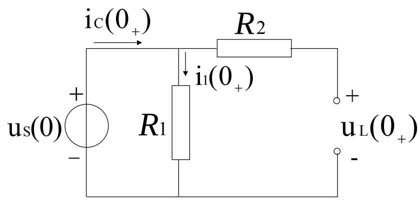
2. 作0_+等效电路





3

3. 求初值



$i_C(0_+)=i_1(0_+)=\dfrac{u_s(0)}{R_1}$

$u_L(0_+)=u_s(0)$

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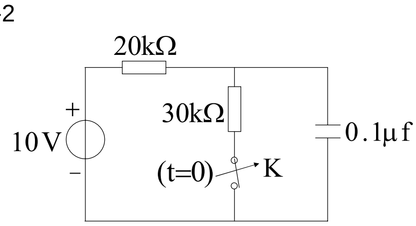
例6-2



已知：开关K打开前电路已达稳定，t=0时开关打开。

求：　t=0_+时刻，各电压、电流值。

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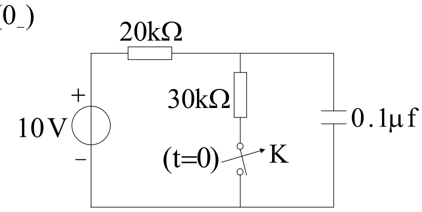
解：1. 求　$u_C(0_-)$



$u_C(0_-)=\dfrac{10}{20+30}\times 30=6V$

$u_C(0_+)=u_C(0_-)=6V$

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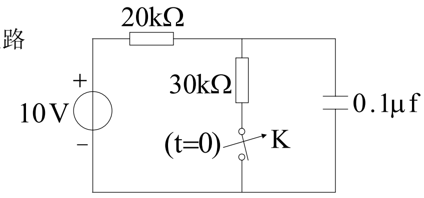
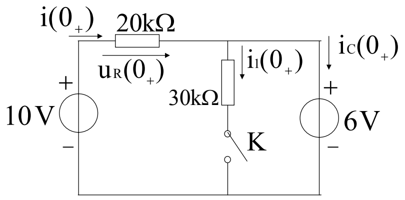
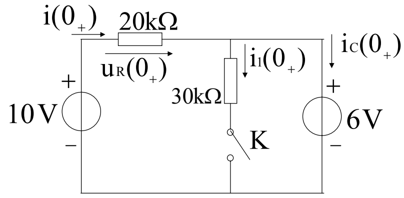
2. 作0_+等效电路





3. 求初值



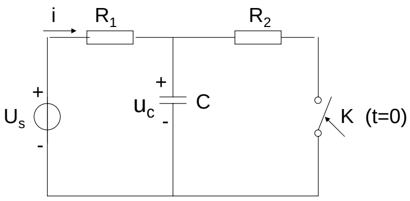
0_+等效电路

$$i(0_+)=i_C(0_+)=\frac{10-6}{20}=0.2\text{mA}$$

$$i_1(0_+)=0$$

$$u_R(0_+)=20\times0.2=4\text{V}$$

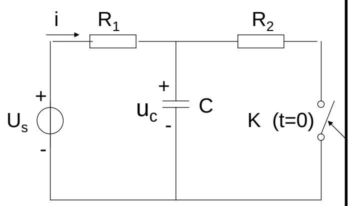
例6-3



求 t>0 时 $u_c(t)$ u_{c0i} u_{c0s} u_c' u_c''

$i(t)$ i_{0i} i_{0s} i' i''

解： 求 $u_c(t)$ u_c' u_c''



$$u_c(0_+)=u_c(0_-)=U_s$$

$$u_c(\infty)=\frac{R_2}{R_1+R_2}U_S$$

$$\tau=R_{eq}C=\frac{R_1R_2}{R_1+R_2}C$$

全响应

$$u_C(t)=u_C(\infty)+\left[u_C(0_+)-u_C(\infty)\right]e^{-\frac{t}{\tau}}$$

$$=\frac{R_2U_S}{R_1+R_2}+\left[U_S-\frac{R_2U_S}{R_1+R_2}\right]e^{-\frac{R_1+R_2}{R_1R_2C}t}$$

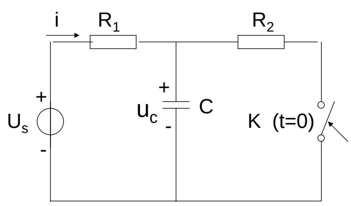
稳态响应

$$u_c'=\frac{R_2}{R_1+R_2}U_S$$

暂态响应

$$u_c''=\left(U_S-\frac{R_2U_S}{R_1+R_2}\right)e^{-\frac{R_1+R_2}{R_1R_2C}t}$$

求 $i(t)$ i' i''



$$i(\infty)=\frac{U_S}{R_1+R_2}$$

$$i(0_+)=\frac{U_S-u_c(0_+)}{R_1}=\frac{U_S-U_S}{R_1}=0$$

全响应
$$i(t)=\frac{U_S}{R_1+R_2}+\left(0-\frac{U_S}{R_1+R_2}\right)e^{-\frac{t}{\tau}}$$
$$=\frac{U_S}{R_1+R_2}\left(1-e^{-\frac{t}{\tau}}\right)$$

稳态响应
$$i'=\frac{U_S}{R_1+R_2}$$

暂态响应
$$i''=-\frac{U_S}{R_1+R_2}e^{-\frac{t}{\tau}}$$

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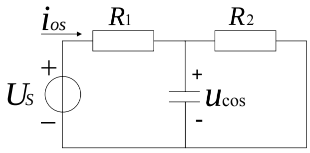
求 u_{cos} i_{os}



$$u_{cos}(0_+)=0$$
$$u_{cos}(\infty)=\frac{R_2}{R_1+R_2}U_S$$
$$u_{cos}(0_+)=0$$
$$u_{cos}(t)=\frac{R_2U_S}{R_1+R_2}+\left(0-\frac{R_2U_S}{R_1+R_2}\right)e^{-\frac{t}{\tau}}$$
$$=\frac{R_2U_S}{R_1+R_2}\left(1-e^{-\frac{t}{\tau}}\right)$$

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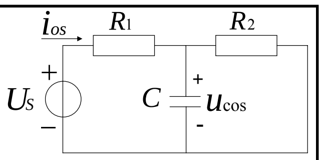

$$i_{os}(\infty)=\frac{U_S}{R_1+R_2}$$
$$u_{cos}(0_+)=0$$
$$i_{os}(0_+)=\frac{U_S-U_{cos}(0_+)}{R_1}=\frac{U_S-0}{R_1}=\frac{U_S}{R_1}$$
$$i_{os}(t)=\frac{U_S}{R_1+R_2}+\left(\frac{U_S}{R_1}-\frac{U_S}{R_1+R_2}\right)e^{-\frac{t}{\tau}}$$

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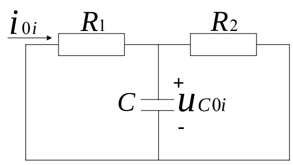
求 u_{coi} i_{oi}



$$u_{coi}(0_+)=U_S$$
$$u_{coi}(\infty)=0$$
$$u_{coi}(0_+)=U_S$$
$$u_{coi}(t)=0+(U_S-0)e^{-\frac{t}{\tau}}=U_Se^{-\frac{t}{\tau}}$$

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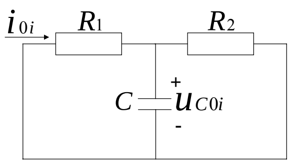

$$i_{oi}(0_+)=-\frac{U_S}{R_1}$$
$$i_{oi}(\infty)=0$$
$$u_{coi}(0_+)=U_S$$
$$i_{oi}(t)=0+\left(-\frac{U_S}{R_1}-0\right)e^{-\frac{t}{\tau}}=-\frac{U_S}{R_1}e^{-\frac{t}{\tau}}$$

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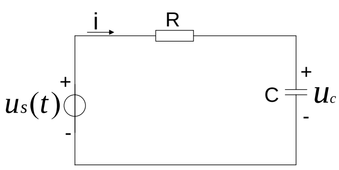
例6-4



1. 求 u_c i 的阶跃响应。

2. 当 $u_c(0_-)=0$, $u_s(t)=\varepsilon(t-t_1)$时, 求 u_c i 的响应。

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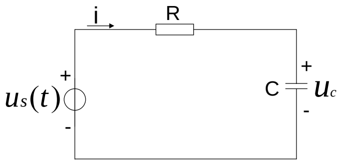
解：



令 $u_S(t)=\varepsilon(t)$

$u_C(0)=0V$

u_c的阶跃响应 $S_u(t)=\left(1-e^{-\frac{1}{RC}t}\right)\varepsilon(t)$

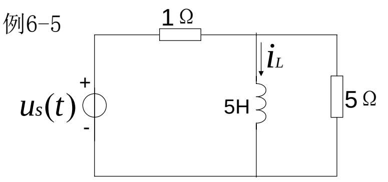
i的阶跃响应 $S_i(t)=C\frac{du_c}{dt}=\frac{1}{R}e^{-\frac{1}{RC}t}\varepsilon(t)$

若 $u_s(t)=\varepsilon(t-t_1)$

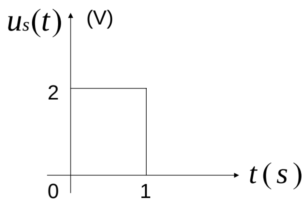
$$u_C(t)=\left(1-e^{-\frac{t-t_1}{RC}}\right)\varepsilon(t-t_1)$$

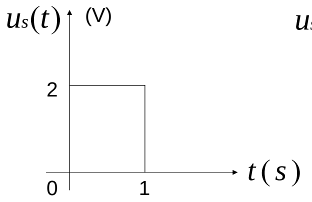
$$i(t)=\frac{1}{R}e^{-\frac{t-t_1}{RC}}\varepsilon(t-t_1)$$

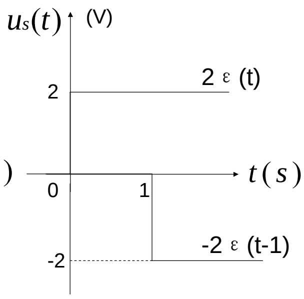
例6-5



求 t>0 时 $i_L(t)$

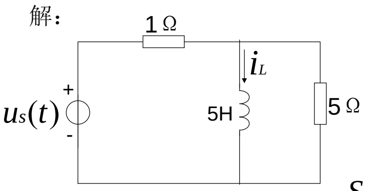






$$u_s(t)=2\varepsilon(t)-2\varepsilon(t-1)$$

解：



令 $u_s(t)=\varepsilon(t)$

求 $i_L(t)=S(t)$

$S(0_+)=S(0_-)=0A$

$S(\infty)=\frac{1}{1}=1A$

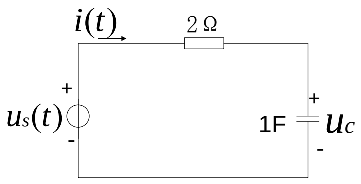
$\tau=\frac{5}{\frac{5\times1}{5+1}}=6\ s$

$$S(t)=\left(1-e^{-\frac{t}{6}}\right)\varepsilon(t)$$

当 $u_s(t)=2\varepsilon(t)-2\varepsilon(t-1)$

$$i_L(t)=2\left(1-e^{-\frac{t}{6}}\right)\varepsilon(t)-2\left(1-e^{-\frac{t-1}{6}}\right)\varepsilon(t-1)A$$

例6-6



已知 $u_C(0)=10V$

$u_s(t)=5\varepsilon(t-2)$

求 t>0时，$i(t)$

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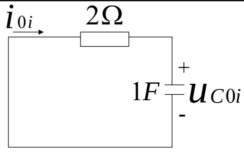
解：求零输入响应

$u_{C0i}(0)=10V$



$$i_{oi}(0_+)=\frac{-u_{coi}(0_+)}{2}=\frac{-10}{2}=-5A$$

$$i_{oi}(\infty)=0$$

$$\tau=RC=2\times1=2s$$

$$i_{oi}(t)=-5e^{-\frac{t}{2}}\varepsilon(t)$$

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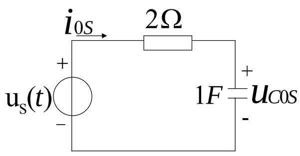
求零状态响应

$u_s(t)=5\varepsilon(t-2)$

$u_{C0S}(0)=0$



令 $u_s(t)=\varepsilon(t)$ 求 $S(t)$

$$s(0_+)=\frac{1}{2}=0.5$$

$$s(\infty)=0$$

$$s(t)=0.5e^{-\frac{t}{2}}\varepsilon(t)$$

$$i_{0s}(t)=5\times0.5e^{-\frac{t-2}{2}}\varepsilon(t-2)$$

$$=2.5e^{-\frac{t-2}{2}}\varepsilon(t-2)$$

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求全响应

$$i(t)=i_{oi}(t)+i_{os}(t)$$

$$=-5e^{-0.5t}\varepsilon(t)+2.5e^{-0.5(t-2)}\varepsilon(t-2)A$$

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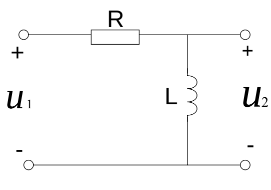
例6-7 求 u_2的冲激响应

解：



令 $u_1=\varepsilon(t)$ $i_L(0)=0$

$$s(0+)=1$$

$$s(\infty)=0$$

$$\tau=\frac{L}{R}$$

$$s(t)=e^{-\frac{R}{L}t}\varepsilon(t)$$

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$$s(t)=e^{-\frac{R}{L}t}\varepsilon(t)$$

$$h(t)=s'(t)=e^{-\frac{R}{L}t}\delta(t)-\frac{R}{L}e^{-\frac{R}{L}t}\varepsilon(t)$$

$$=\delta(t)-\frac{R}{L}e^{-\frac{R}{L}t}\varepsilon(t)$$

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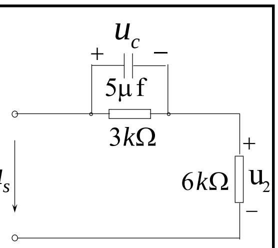
例6-8　求u_2 u_C的冲激响应

解：法1

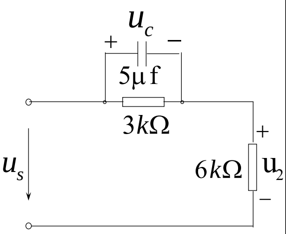


令 $u_s=\varepsilon(t)$ $u_C(0)=0$ 求s(t)

$$u_c(\infty)=\frac{3}{3+6}\times 1=\frac{1}{3}\text{V}$$

$$\tau=(3//6)\times10^{3}\times5\times10^{-6}=10^{-2}(\text{s})$$

$$s_{uc}(t)=\frac{1}{3}(1-e^{-100t})\varepsilon(t)$$



$$u_2(0_+)=1$$

$$u_2(\infty)=\frac{6}{3+6}\times1=\frac{2}{3}$$

$$S_{u2}(t)=\frac{2}{3}+(1-\frac{2}{3})e^{-100t}=(\frac{2}{3}+\frac{1}{3}e^{-100t})\varepsilon(t)$$

$$s_{uc}(t)=\frac{1}{3}(1-e^{-100t})\varepsilon(t)$$

$$h_{uc}(t)=s'_{uc}(t)=\frac{1}{3}(1-e^{-100t})\delta(t)+\frac{100}{3}e^{-100t}\varepsilon(t)$$

$$=\frac{100}{3}e^{-100t}\varepsilon(t)$$

$$S_{u2}(t)=(\frac{2}{3}+\frac{1}{3}e^{-100t})\varepsilon(t)$$

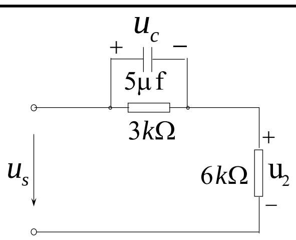
$$h_{u2}(t)=S'_{u2}(t)=(\frac{2}{3}+\frac{1}{3}e^{-100t})\delta(t)-\frac{100}{3}e^{-100t}\varepsilon(t)$$

$$=\delta(t)-\frac{100}{3}e^{-100t}\varepsilon(t)$$

或 $t>0$ 或 $t\geq0_+$

$$h_{u2}(t)=-\frac{100}{3}e^{-100t}\varepsilon(t)$$

法2



$$u_C+6\times10^{3}(5\times10^{-6}\frac{du_C}{dt}+\frac{u_C}{3\times10^{3}})=\delta(t)$$

$$3\times10^{-2}\frac{du_C}{dt}+3u_C=\delta(t)$$

$$3\times10^{-2}\frac{du_C}{dt}+3u_C=\delta(t)$$

$$\int_{0-}^{0+}3\times10^{-2}\frac{du_C}{dt}dt+\int_{0-}^{0+}3u_C dt=\int_{0-}^{0+}\delta(t)dt$$

$\because$ $u_c(0)\neq\infty$

如果 u_C 为$\delta(t)$，$\frac{du_C}{dt}$ 为$\delta'(t)$，KVL 不能满足

$$\therefore 3\times10^{-2}[u_C(0_+)-u_C(0_-)]=1$$

$$\therefore 3\times10^{-2}[u_C(0_+)-u_C(0_-)]=1$$

$$u_C(0_-)=0$$

$$u_C(0_+)=\frac{1}{3\times10^{-2}}=\frac{100}{3}$$

$t>0$时零输入响应 $u_C(\infty)=0$

$$u_C(t)=h(t)=\frac{100}{3}e^{-100t}\quad t\geq0_+$$

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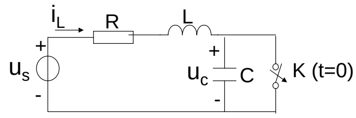
例7-1



已知 u_s=100V　R=10Ω　L=0.5mH　C=2μf
K打开前，电路已达稳态

求 t>0时，$u_c(t)$

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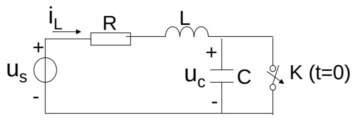
解：



$u_C(0_-)=0V$

$$i_L(0_-)=\frac{100}{10}=10A$$

$$\begin{cases} LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=u_S\ (t\geq0) \\ u_C(0_+)=0 \\ \frac{du_C}{dt}\Big|_{t=0_+}=\frac{i}{c}\Big|_{t=0_+}=\frac{i(0_+)}{c}=\frac{10}{2\times10^{-6}}=5\times10^6 \end{cases}$$

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$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=u_S\ (t\geq0)$$

特征方程 $LCP^2+RCP+1=0$

$$10^{-9}P^2+2\times10^{-5}P+1=0$$

$$P_{1,2}=-10^4\pm j3\times10^4$$

$$u_C(t)=u_C'+u_C''$$

$$=100+Ae^{-10^4t}\sin(3\times10^4t+\beta)$$

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$$u_C(t)=100+Ae^{-10^4t}\sin(3\times10^4t+\beta)$$

由初始条件求出　A=167

β=-36.9°

$$u_C(t)=100+167e^{-10^4t}\sin(3\times10^4t-36.9^\circ)V$$

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