$\sigma < 20\%$. $t_s < 2s$ $t_r < 0.7s$. $k_v > 5$. $T = 0.1s$

$$\sigma \Rightarrow e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} < 20\% \Rightarrow \xi = \frac{|\ln\frac{1}{\sigma}|}{\sqrt{\pi^2 + (\ln\frac{1}{\sigma})^2}} \Rightarrow \xi \geq 0.4559.$$

由 $t_r \Rightarrow \dfrac{\pi - \arccos\xi}{Im(s)} < 0.7 \Rightarrow Im(s) \geq 2.9203 \Rightarrow T \cdot Im(s) = 0.292\ rad = 16.73°$.

$t_s \Rightarrow \dfrac{3.5}{Re(s)} < 2 \Rightarrow Re(s) \geq 1.75 \Rightarrow e^{-TRe(s)} = 0.8395$.

$r(t) = t$. $G(s) = \dfrac{1}{(s+1)(s+2)}$ $G(z) = Z\left[\dfrac{1-e^{-Ts}}{s} G(s)\right]$

$$G(z) = \frac{0.0047z + 0.0044}{z^2 - 1.8006z + 0.8187} = 0.0047 \frac{z + 0.936}{[z-(0.9+0.0903i)][z-(0.9-0.0903i)]}$$

$\therefore$ 令 $D(z) = k_c \cdot \dfrac{[z-(0.9+0.0903i)][z-(0.9-0.0903i)]}{z(z-1)}$

$\therefore D(z)G(z) = 0.0047k_c \cdot \dfrac{z+0.936}{z(z-1)} = k \cdot \dfrac{z+0.936}{z(z-1)}$, $k = 0.0047k_c$.

$$k_v = \frac{1}{T}\lim_{z\to1}(z-1)D(z)G(z) = 10 \cdot k \cdot 1.936 > 5$$

$\therefore k > 0.2583$.

取 $k = 0.3646$, $z = 0.3177 \pm 0.4902i$, $k_c = 77.574$.

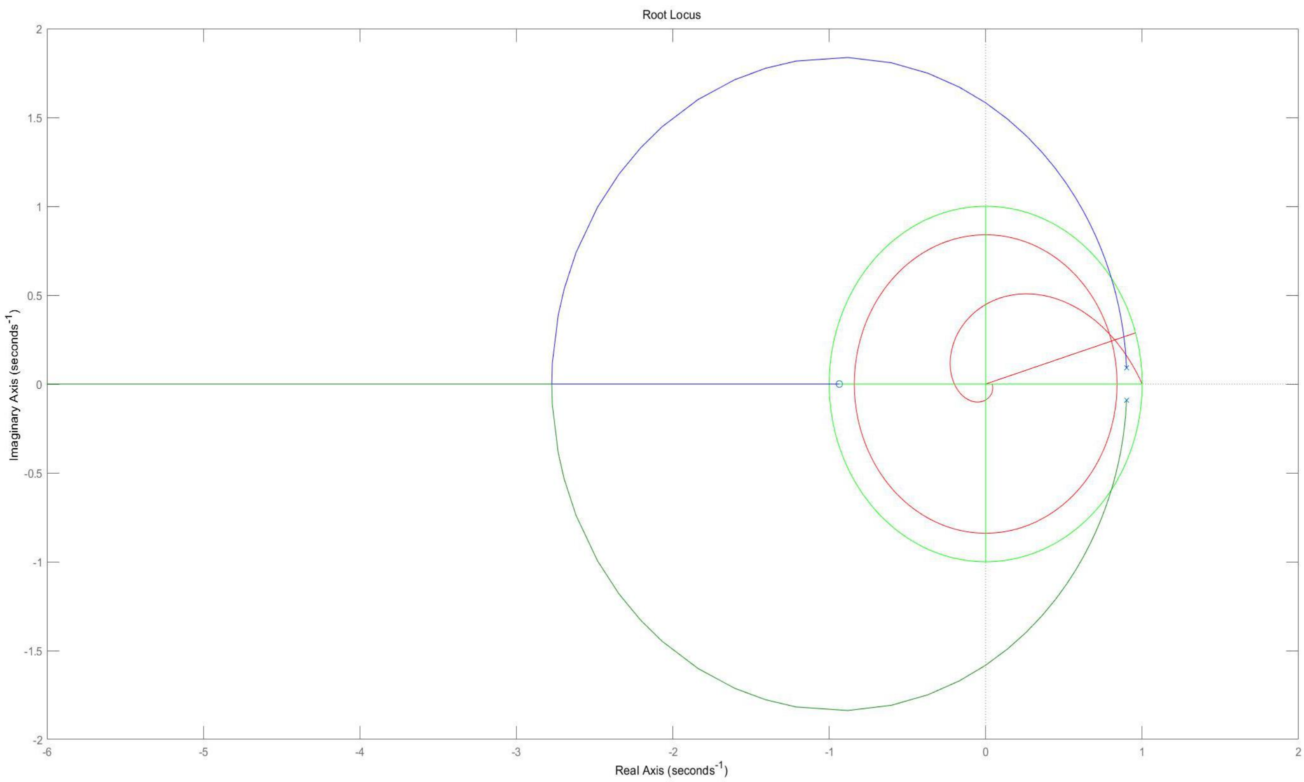
$\therefore$ [illegible]

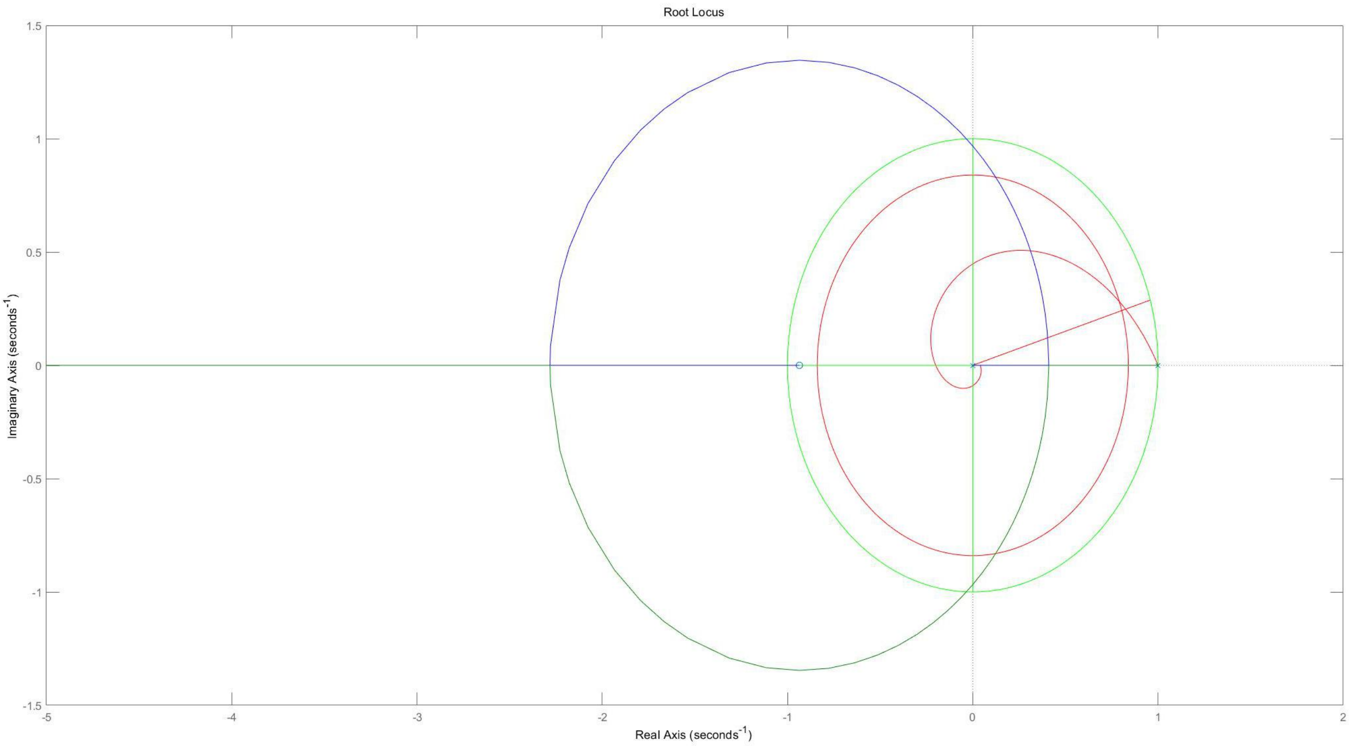
$\therefore D(z) = 77.574 \cdot \dfrac{z^2 - 1.8006z + 0.8187}{z(z-1)}$

仿真, $\sigma \approx 17.7\%$, $t_r \approx 0.35s$, $t_s \approx 0.55s$. 符合.

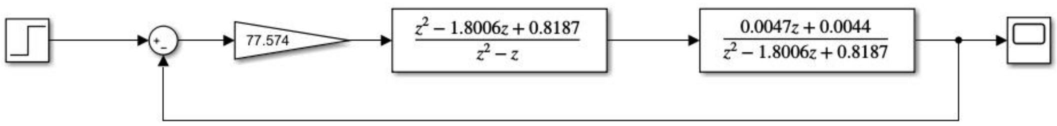
Matlab 代码：

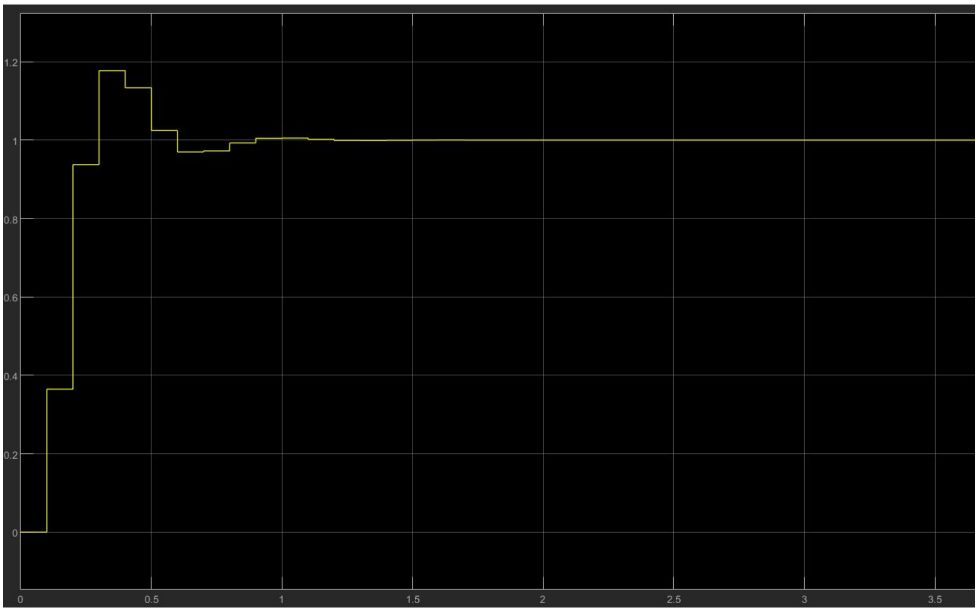
```
clc;
clear all;

% 对数螺旋线
Kexi = 0.4559;
B = acos(Kexi);
TB = -1/tan(B);
WT = 0:0.01:2*pi;
EW = exp(WT*TB);
x = EW.*cos(WT);
y = EW.*sin(WT);
plot(x, y, 'r'),grid;
hold on

% Re(s)同心圆
t = 0:0.01:2*pi;
R = 0.8395;
xR = R*cos(t);
yR = R*sin(t);
x1 = cos(t);
y1 = sin(t);
plot(x1, y1, 'g');
plot(xR, yR, 'r');

% Im(s)射线
thita = 16.73;
temp = thita*pi/180;
x2 = cos(temp);
y2 = sin(temp);
plot([0,x2], [0,y2], 'r');
plot([-1,1], [0,0], 'g');
plot([0,0], [-1,1], 'g');

% 计算脉冲传递函数
[wnun, wdes] = c2dm([1], [1,2,2], 0.1, 'zoh');

% 常值控制器
% rlocus(wnun, wdes)

% 二阶控制器
numGD = [1 0.936];
denGD = [1 -1 0];
rlocus(numGD, denGD)
[K,pole] = rlocfind(numGD, denGD)
```

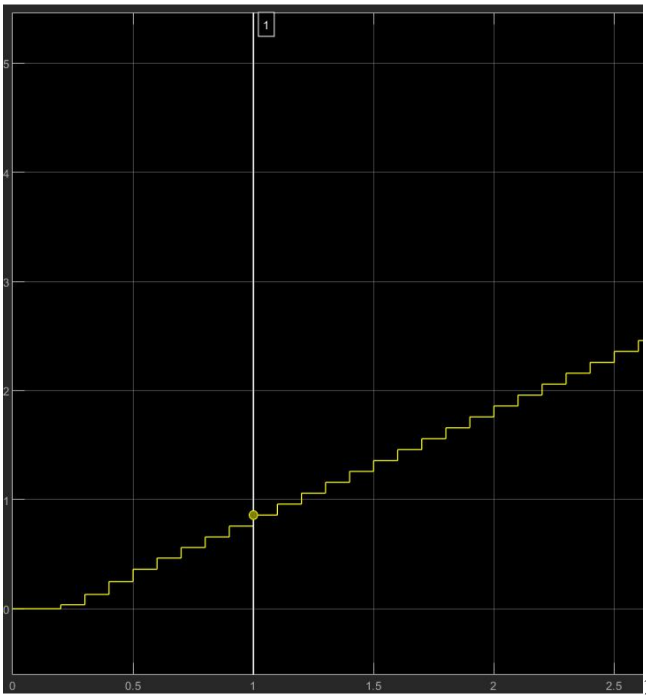



Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
1.5
1
0.5
0
-0.5
-1
-1.5
-5
-4
-3
-2
-1
0
1
2

Simulink:


77.574
$\frac{z^2 - 1.8006z + 0.8187}{z^2 - z}$
$\frac{0.0047z + 0.0044}{z^2 - 1.8006z + 0.8187}$



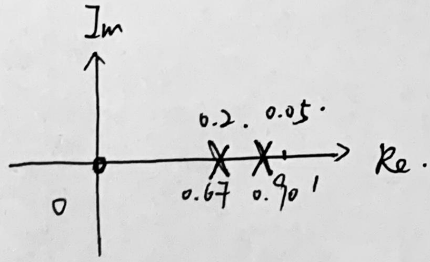


斜坡响应 ess 约为 0.143， Kv=1/ess>5，符合要求。

$D(s) = \frac{2}{s+2}$, $T = 0.05s / 0.2s$.

(1)(2). ①. Z变换： $D(z) = \frac{2z}{z-0.9048}$, $T=0.05s$.

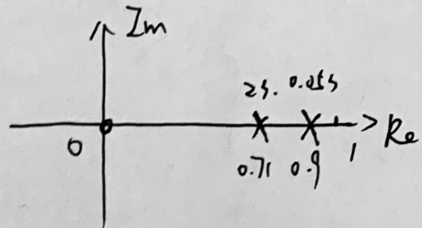
$D(z) = \frac{2z}{z-0.6703}$, $T=0.2s$.



---

②. 向后差分： $D(z) = D(s)\big|_{s=\frac{z-1}{Tz}} = \frac{0.1z}{1.1z-1}$, $T=0.05$.

$D(z) = \frac{0.4z}{1.4z-1}$, $T=0.2s$



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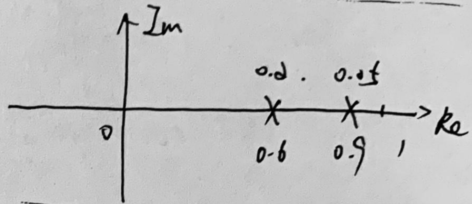
③. 向前差分： $D(z) = D(s)\big|_{s=\frac{z-1}{T}} = \frac{0.1}{z-0.9}$, $T=0.05$.

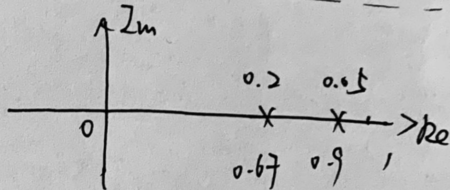
$D(z) = \frac{0.4}{z-0.6}$, $T=0.2s$.



---

④. 零极点： $D(z) = \frac{0.09516}{z-0.9048}$, $T=0.05$

$D(z) = \frac{0.3297}{z-0.6703}$, $T=0.2$

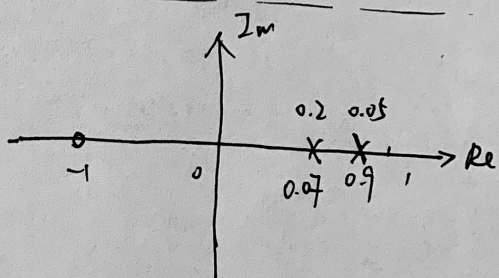


---

⑤. Tustin 变换： $D(z) = D(s)\big|_{s=\frac{2}{T}\cdot\frac{z-1}{z+1}}$ 40/10

$= \frac{2(z+1)}{42z-38}$, $T=0.05s$

$D(z) = \frac{2(z+1)}{12z-8}$, $T=0.2s$



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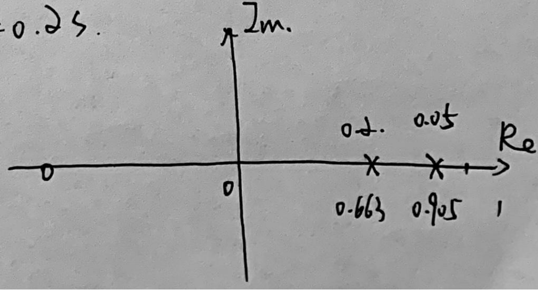
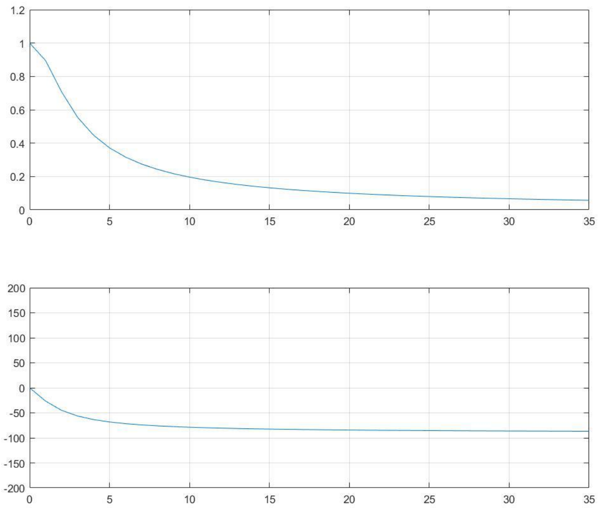
⑥. 预修正Tustin变换： $D(z) = D(s)\big|_{s=\left(\frac{\omega_1}{\tan\frac{\omega_1 T}{2}}\right)\frac{z-1}{z+1}}$, $\omega_1=2$ $\frac{\omega_1}{\tan\frac{\omega_1 T}{2}} = 39.97 / 9.866$.

$= \frac{2(z+1)}{41.97z-37.97}$, $T=0.05s$.

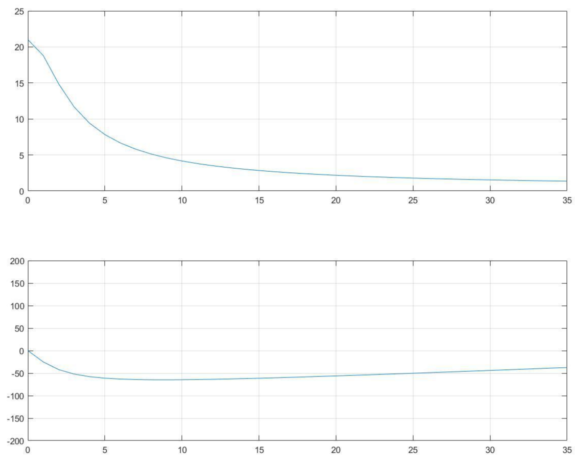
$D(z) = \frac{2(z+1)}{11.866z-7.866}$, $T=0.2s$.

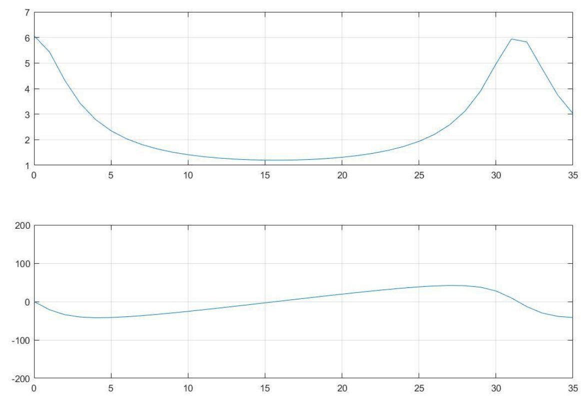


（3） D(jω)

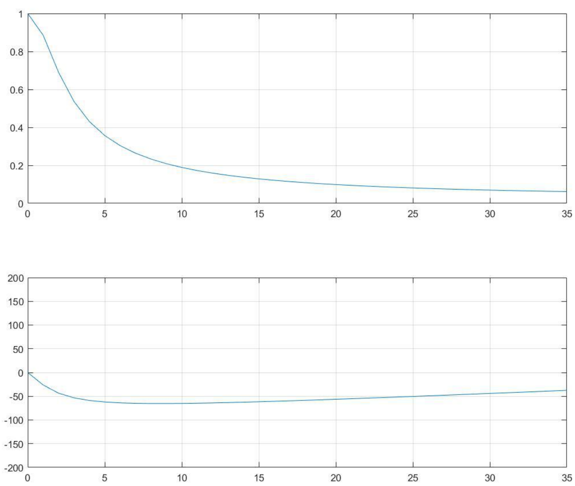


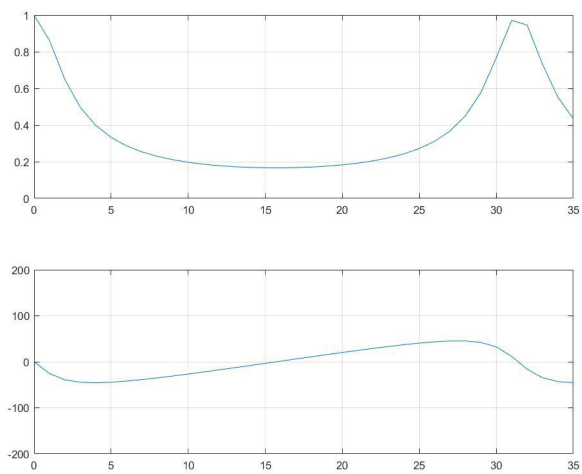
Z 变换（左图为 T=0.05s，右图为 T=0.2s，下同）



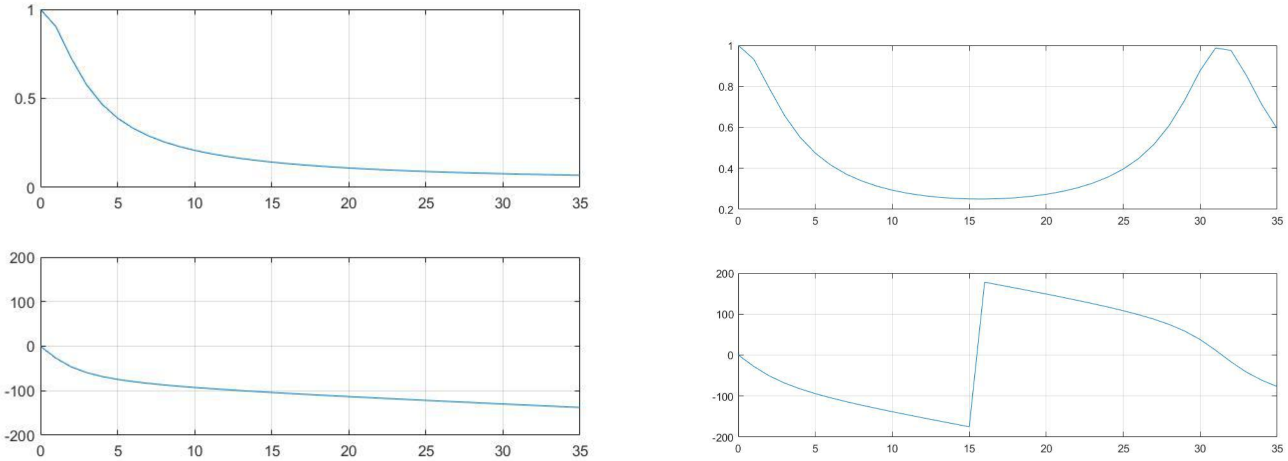


向后差分

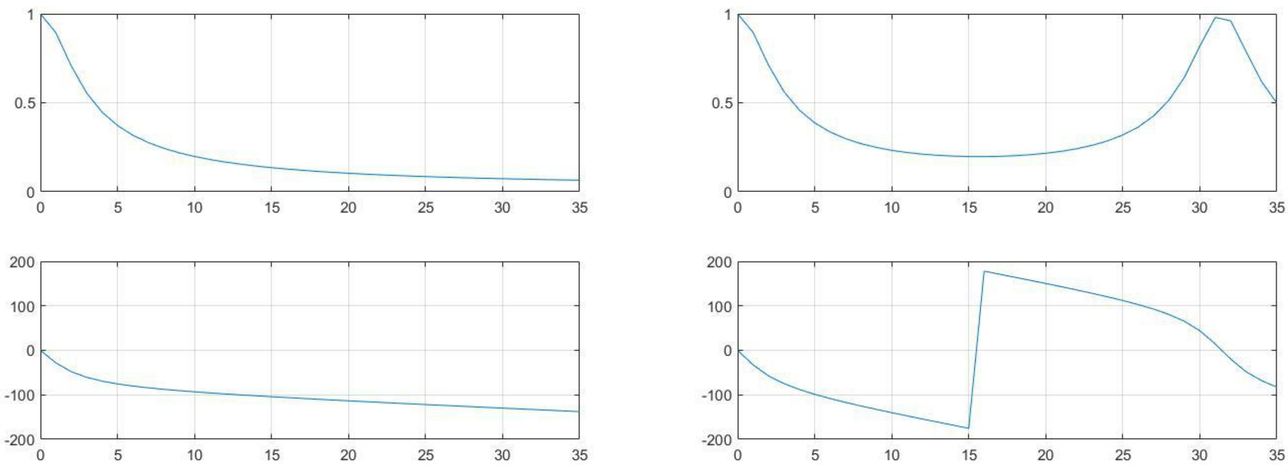




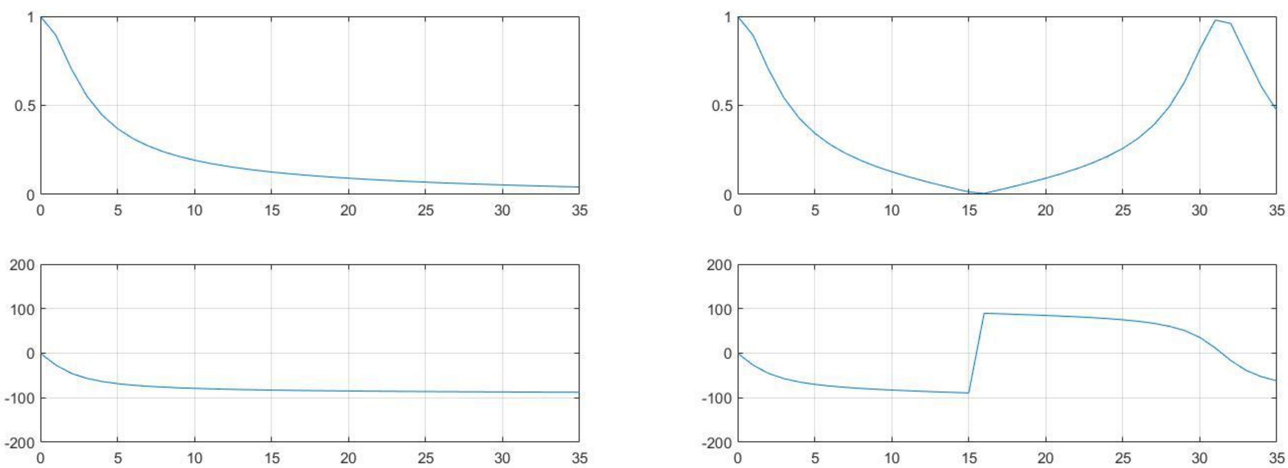
向前差分



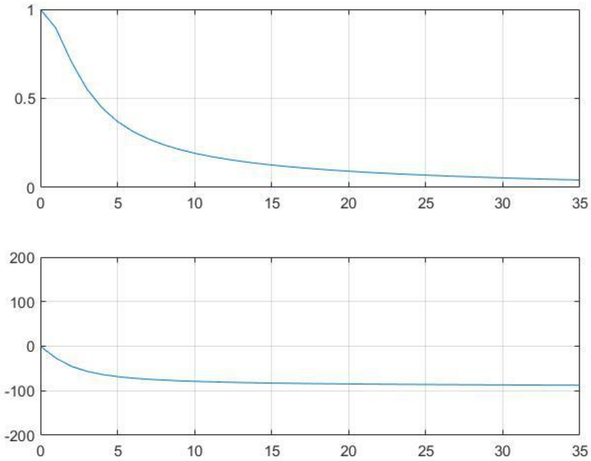
零极点

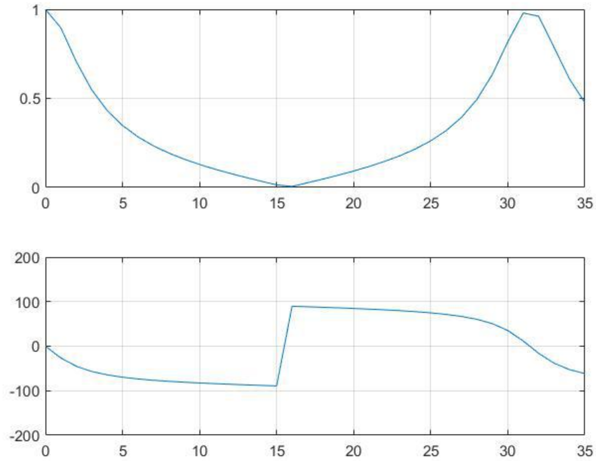


TUSTIN 变换

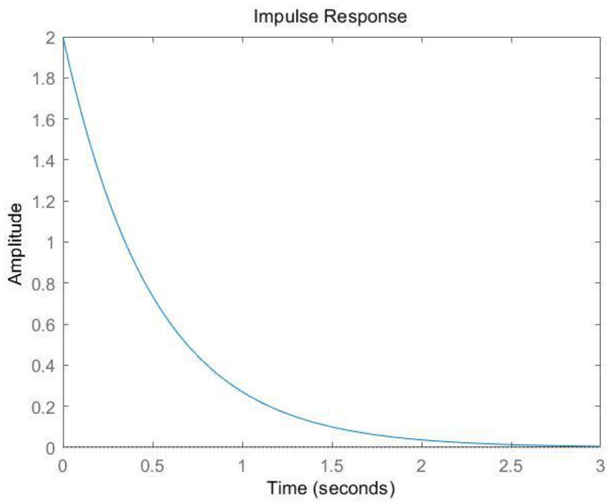


预修正 TUSTIN 变换

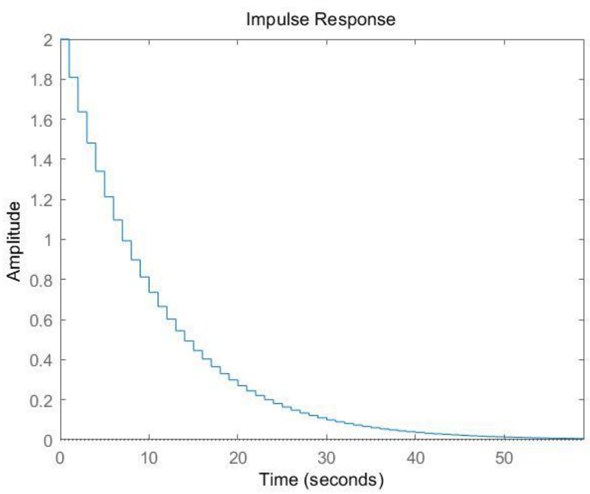


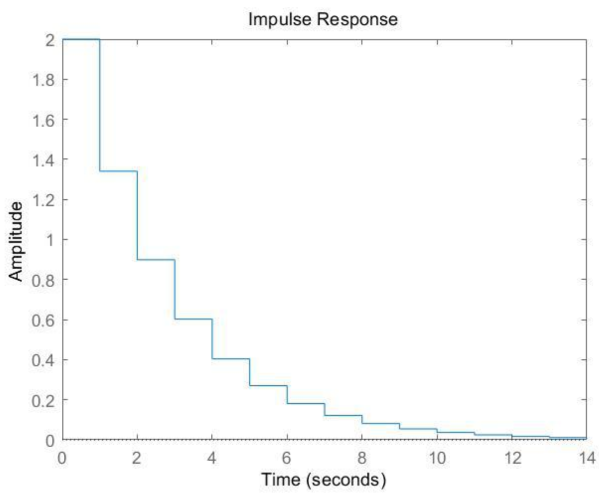


(4) 单位脉冲响应：D(s)

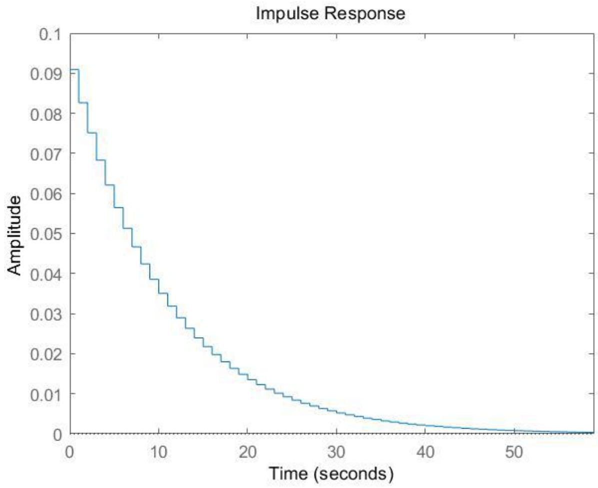


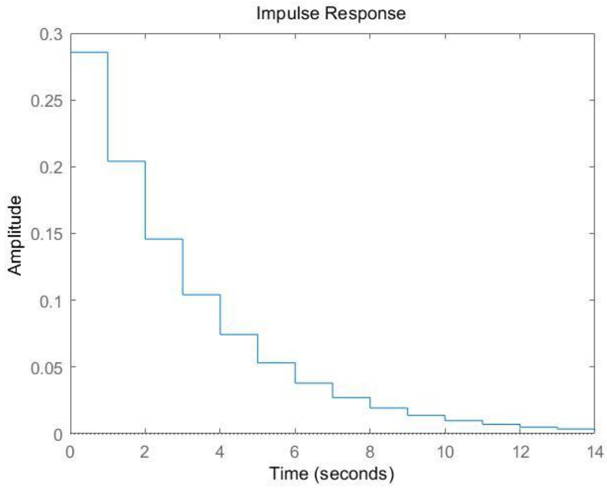
Z 变换（左图为 T=0.05s，右图为 T=0.2s，下同）



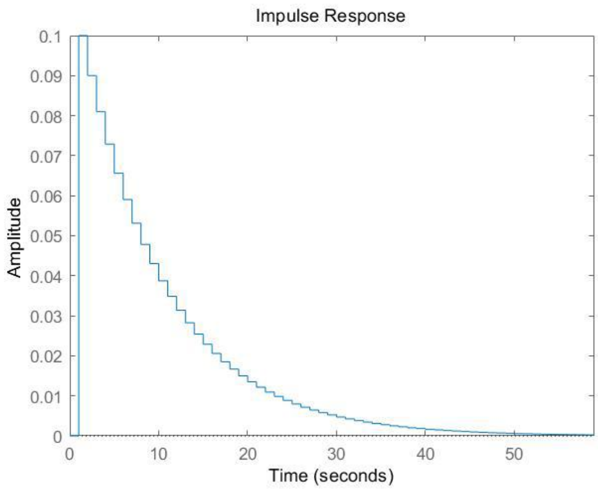


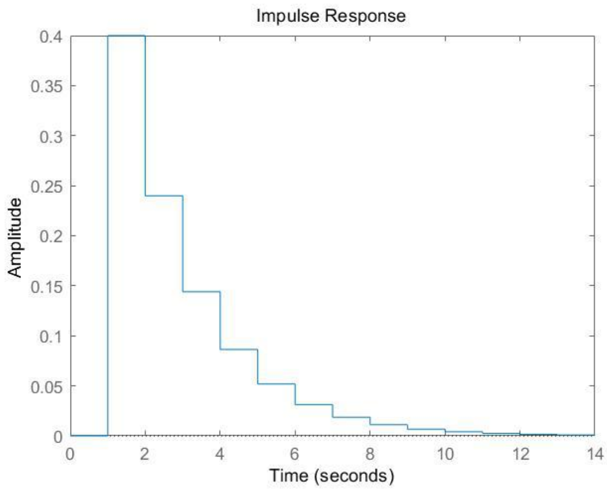
向后差分



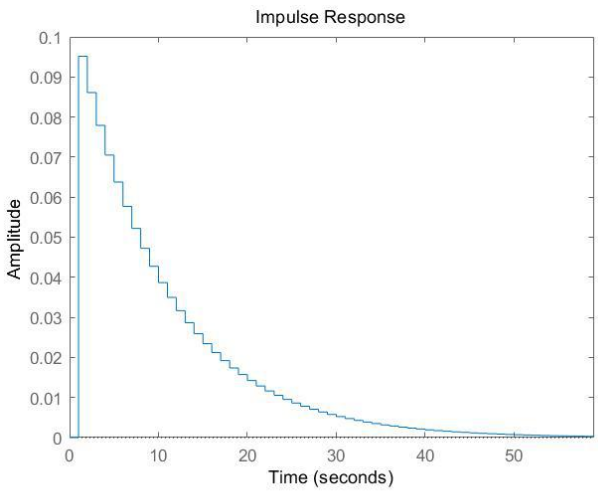


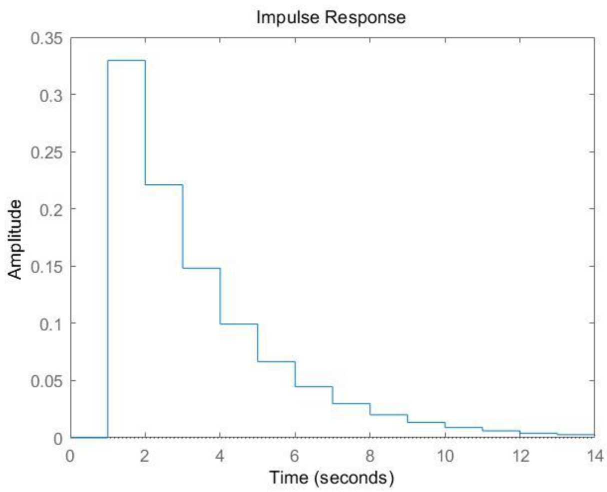
向前差分



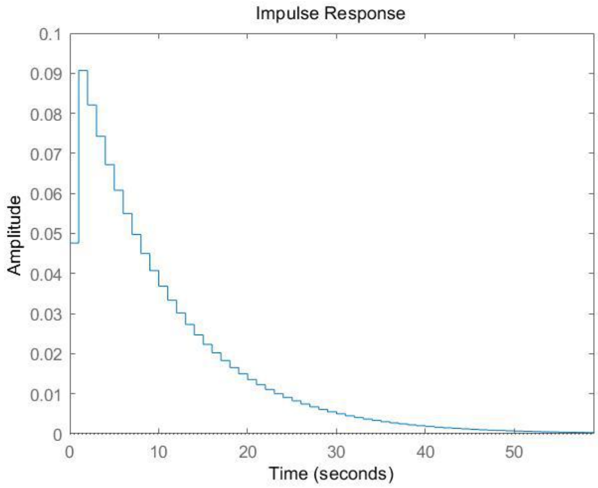


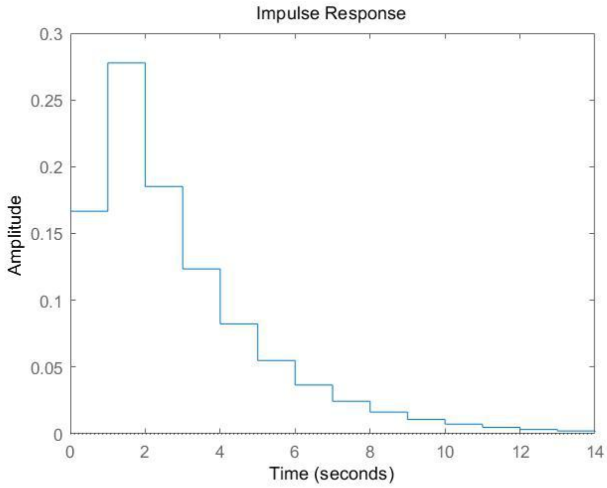
零极点



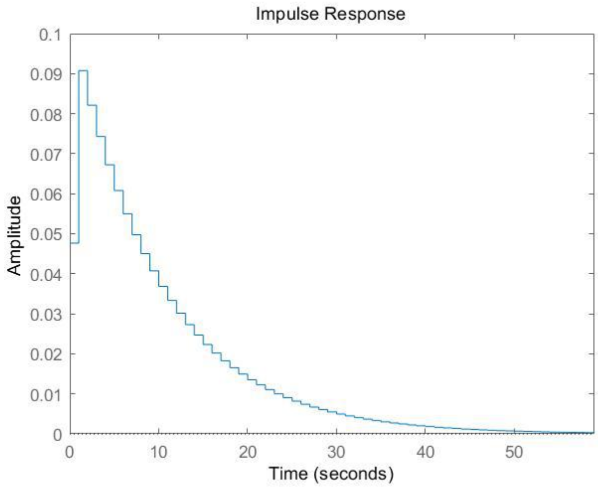


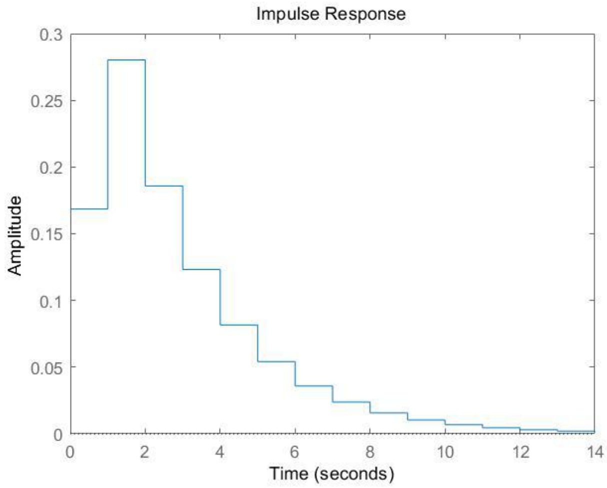
TUSTIN 变换





预修正 TUSTIN 变换





(5) 在本实验中，能够清晰地看出，不存在能够完全还原连续特性的离散化方法。对于同一种方法，显然 T 小，即采样频率大的离散化过程能够更好地还原原始特性。一般而言，零极点、TUSTIN 变换、预修正 TUSTIN 变换会得到更好的结果，但就本实验传递函数而言，向前、向后差分变换得到的结果畸变也都完全可以接受。