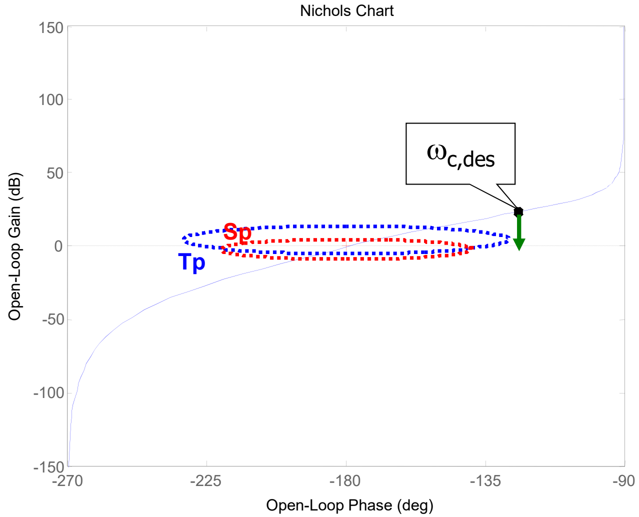
Automatic Control

Loop shaping design of feedback control systems

Part II: Lag network

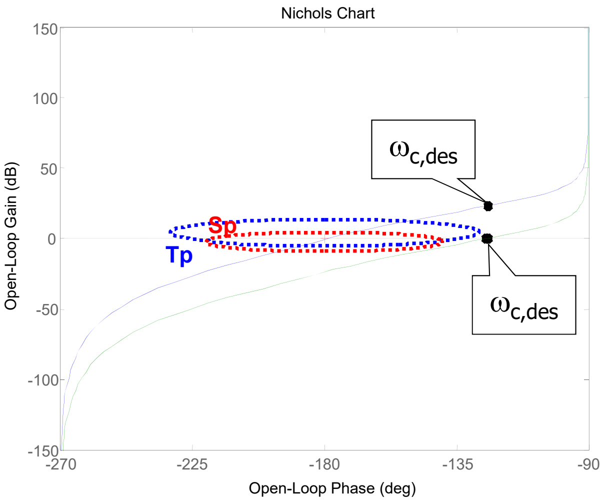
Example



In this situation, $\angle L(j\omega_{c,des})$ is such that the point corresponding to $\omega_{c,des}$ is outside the influence of the constant magnitude loci.

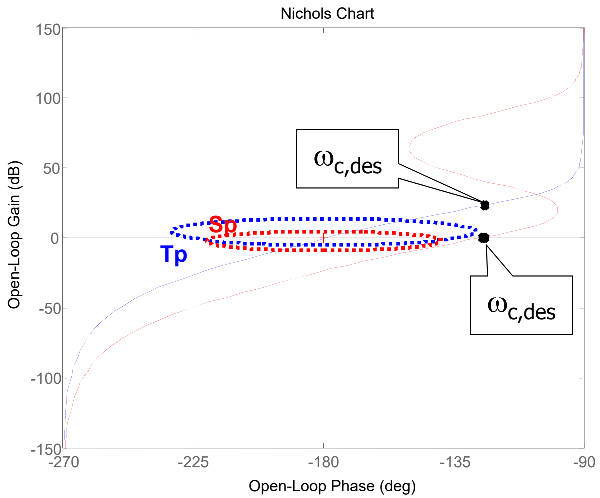
It can be noticed that, in order to make $\omega_{c,des}$ crossover frequency , a magnitude attenuation action is required.

Example



- The simplest way to obtain magnitude attenuation is through a gain attenuation.
- In fact, if L(s) is multiplied by a gain K, such that $|K|<1$, the resulting Nichols diagram is down-shifted by the quantitiy $K|_{dB} < 0$.
- However, since this procedure reduces the dc-gain of L(s), leads <u>in general</u>, to steady state performance degradation.

Example



- In this regard, a viable solution is to introduce magnitude attenuation in a neighbourhood of $\omega_{c,des}$ only, without modifying the dc-gain of L(s).
- This can be achieved thorugh a suitable controller which does not modify the dc-gain (i.e. its dc-gain is 1) and introduces magnitude attenuation from a given frequency.

The lag network

The just introduced example motivates the use of the

lag network → $C_I(s) = \dfrac{1+\dfrac{s}{m_I\omega_I}}{1+\dfrac{s}{\omega_I}}, \; \omega_I > 0, m_I > 1$

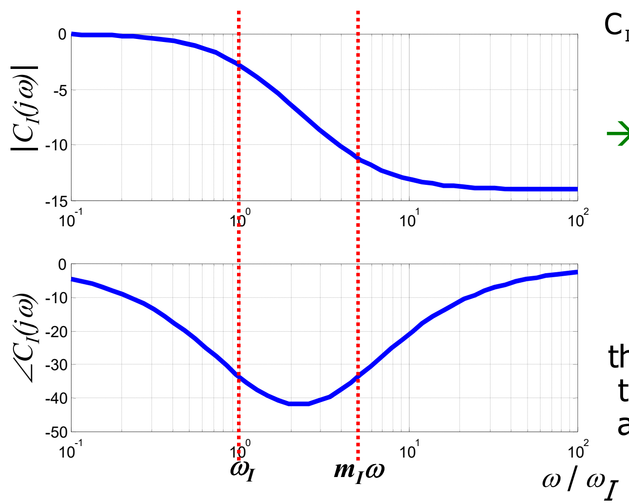
A lag network is described by a proper tf with:

- a real negative pole at $-\omega_I$
- a real negative zero at $-m_I\,\omega_I$

Note also that:

$$\lim_{s\to 0} C_I(s) = \lim_{s\to 0} \frac{1+\dfrac{s}{m_I\omega_I}}{1+\dfrac{s}{\omega_I}} = 1$$

The lag network: frequency response



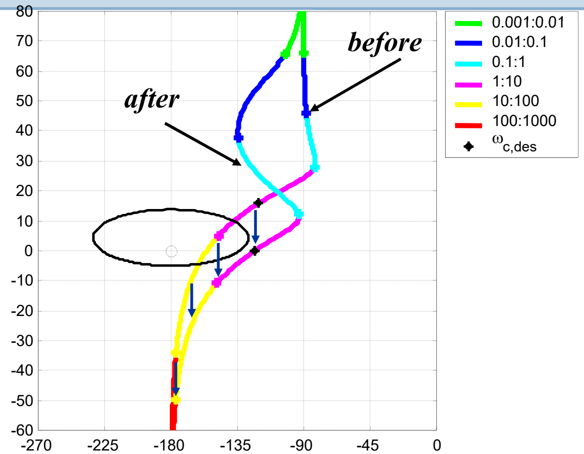
$C_I(s) = \dfrac{1+\dfrac{s}{m_I\omega_I}}{1+\dfrac{s}{\omega_I}}, \; \omega_I > 0, m_I > 1$

→ Magnitude attenuation ←

Phase lag

the greater is m_I, the larger is the amount of the phase lag and the magnitude decrease

The lag network: effects



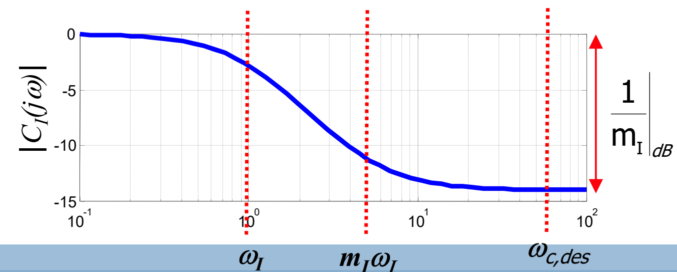
In the Nichols plane, the magnitude attenuation effect produces a negative vertical shift of the frequency interval of interest

The lag network: design

- Parameter m_I is designed on the basis of the magnitude attenuation needed at $\omega_{c,des}$.
- In this regard, the key idea is to place the lag network so that the "flat" zone of its magnitude behavior is located in a suitable neighbourhood of $\omega_{c,des}$.
- In this way, the lag network produces a vertical negative shift of the Nichols diagram in a neighbourhood of $\omega_{c,des}$ as shown in the previous page.

Note that:

$$\lim_{\omega\to\infty} \left| \frac{1+\dfrac{j\omega}{m_I\omega_I}}{1+\dfrac{j\omega}{\omega_I}} \right| = \frac{1}{m_I}$$



The lag network: design

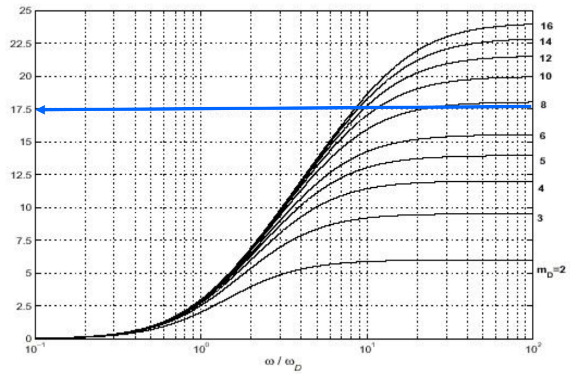
- In order to fix parameter m_I, we note that, after the introduction of the lag network, the loop function becomes $L'(s)=C_I(s)L(s)$

- Then, since at $\omega_{c,des}$, the magnitude of $|C_I(j\omega)|$ must be $1/m_I$ (flat zone), we have

$$\left.\begin{aligned} |L'(j\omega_{c,des})| &= |C_I(j\omega_{c,des})||L(j\omega_{c,des})| = 1 \\ |L'(j\omega_{c,des})| &\underset{\substack{\uparrow \\ |C_I(j\omega_{c,des})|=1/m_I}}{=} \frac{1}{m_I}|L(j\omega_{c,des})| = 1 \end{aligned}\right\} \Rightarrow m_I = |L(j\omega_{c,des})|$$

The lag network: design

- It is worth noting that a rough evaluation of m_I can be obtained using the universal magnitude diagram of a *lead* network*.
- Suppose that a magnitude decrease of about 17.5 dB is needed at $\omega_{c,des}$



In this case, a value of $m_I < 8$ can been chosen:
→ roughly $m_I = 7.7, 7.8$)

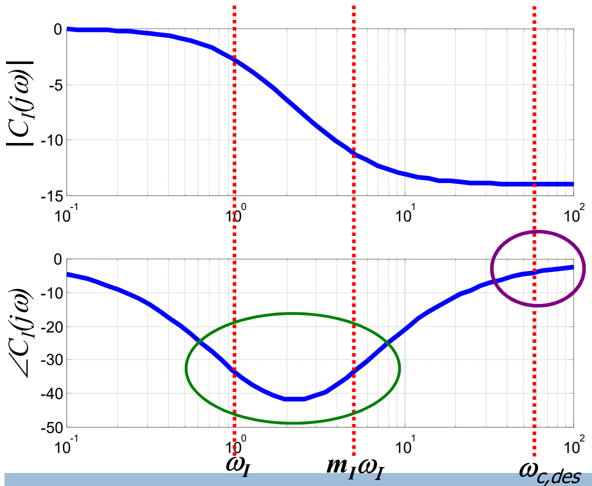
The exact procedure leads to:

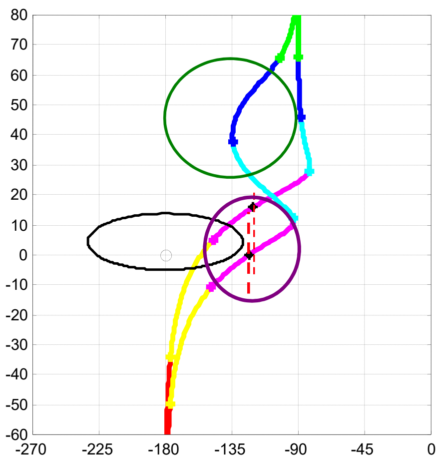
$$m_I = |L(j\omega_{c,des})| = \\ = 17{,}5\,dB = 7.5$$

* Note that the magnitude diagram of a lag network has the same behavior of the lead network one except for the sign.

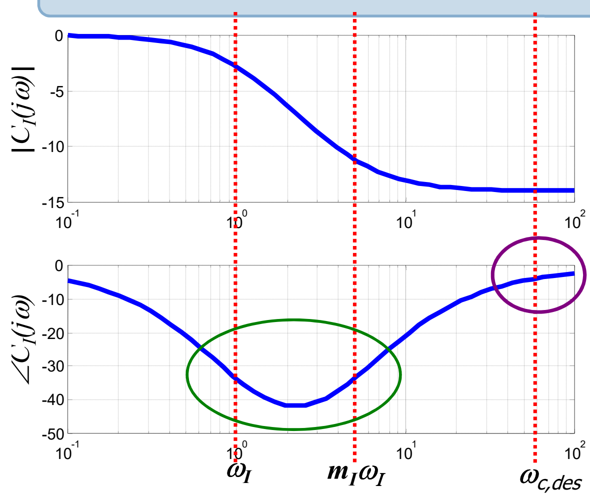
The lag network: design

- Parameter ω_I is designed in order to place the flat zone at $\omega_{c,des}$ and to limit the phase lag that occurs near $\omega_{c,des}$





The lag network: design



In order to limit the phase lag at $\omega_{c,des}$, the zero of the lag network, $m_I\omega_I$ should be sufficiently far from $\omega_{c,des}$

$$\omega_{c,des} \approx \alpha\, m_I \omega_I, \alpha \gg 1$$
$$\Rightarrow \omega_I = \omega_{c,des} / (\alpha m_I)$$

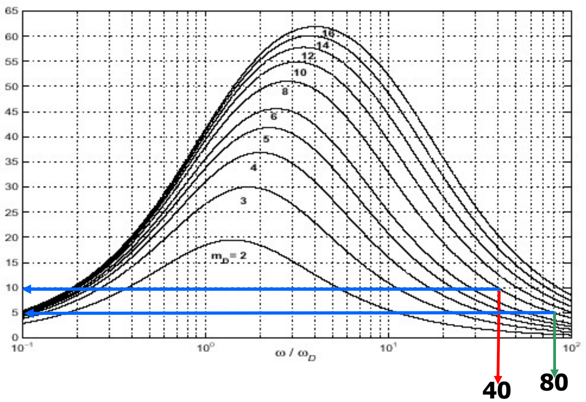
rule of thumb: start with $\alpha = 10$

The greater is α, the smaller is the phase lag introduced at $\omega_{c,des}$

Remark: the greater is α the lower is the frequency of the zero $m_I\omega_I$ → the longer is the transient extinction → avoid the use of large α

The lag network: design

- A more efficient procedure can be employed to fix ω_I in order to quantify the amount of the phase lag that occurs at $\omega_{c,des}$
- In this context, the *lead* universal phase diagram* can be employed



Suppose that for $\omega_{c,des}$ = 3 *rad/s*, a value of m_I = 8 has been chosen

$\omega_{norm} = \omega/\omega_I = 40$
$\angle C_I(j\omega_{norm}) = -10°$
$(\omega/\omega_I)|_{\omega=\omega_{c,des}} = 40$
$\omega_I = \omega_{c,des}/40 = 0.075$ rad/s

$\omega_{norm} = \omega/\omega_I = 80$
$\angle C_I(j\omega_{norm}) = -5°$
$(\omega/\omega_I)|_{\omega=\omega_{c,des}} = 80$
$\omega_I = \omega_{c,des}/80 = 0.037$ rad/s

* Note that the phase diagram of a lag network has the same behavior of the lead network one except for the sign.

The lag network: design

- We have established two different procedure to tune the value of ω_I of a lag network
  1. (more empirical) : $\omega_I = \omega_{c,des}/(\alpha m_I)$
  2. (more precise) : $\omega_I = \omega_{c,des}/\omega_{norm}$

- Procedure 1.→ α defines the "distance" of the network zero $m_I\omega_I$ wrt $\omega_{c,des}$.
- Procedure 2. → ω_{norm} quantifies the phase lag introduced at $\omega_{c,des}$.
- Comparing the two procedures, we can also compute α in order to get a given amount of the phase lag at $\omega_{c,des}$

$$\alpha = \frac{\omega_{norm}}{m_I}$$

Lag network: design example 1

- A plant to be controlled is described by the transfer function

$$G(s) = \frac{2}{(1+0.2s)(1+0.1s)}$$

design a cascade controller C(s) in order to satisfy the requirements below.

- $\left|y_{d_a}^{\infty}\right| \le 0.1,\ d_a(t) = \delta_a t\varepsilon(t), |\delta_a| \le 1 \rightarrow C_{SS}(s) = \frac{10}{s}$
- $\hat{s} \le 20\% \rightarrow T_p = 1.72\text{dB}, S_p = 3.63\text{dB}$
- $t_r \le 1\ s \rightarrow \omega_{c,des} = 1.9$ rad/s

Lag network: design example 2

- A plant to be controlled is described by the transfer function

$$G(s) = \frac{s+0.2}{s(s+0.4)(s+1)}$$

design a cascade controller C(s) in order to satisfy the requirements below.

- $\left|e_r^{\infty}\right| \le 0.2,\ r(t) = 2t\varepsilon(t) \rightarrow C_{SS}(s) = 20$
- $\hat{s} \le 10\% \rightarrow T_p = 0.42\text{ dB}, S_p = 2.68\text{ dB}$
- $t_r \le 0.5\ s, \rightarrow \omega_{c,des} = 4$ rad/s