Automatic Control AC_L13

Steady state design examples

Example 1

$|e_r^\infty| \leq 0{,}1 \qquad r(t) = 0{,}5t\,\varepsilon(t)$

$\rightarrow$ type 1 system is needed
L(s) with 1 pole at 0
since G(s) has not any pole at 0

$\Rightarrow C_{ss}(s)$ is of the form $\boxed{C_{ss}(s) = \frac{K_c}{s}}$

$$|e_r^\infty| = \left|\frac{\rho}{K_1}\right| \qquad \rho = 0{,}5$$

$$K_1 = \lim_{s\to 0} s\,L(s) \underset{\substack{\uparrow \\ L = C\cdot G}}{=} \lim_{s\to 0} s\,C(s)\,G(s) =$$

$$= \lim_{s\to 0} s\;C_{ss}(s)\,C_r(s)\cdot \underbrace{\frac{s+1}{(s+2)(s+4)}}_{G(s)} =$$

$$= \lim_{s\to 0} \cancel{s}\,\underbrace{\frac{K_c}{\cancel{s}}}_{C_{ss}(s)}\;\underbrace{\cancel{C_r(s)}}_{1}\;\frac{s+1}{(s+2)(s+4)} = \frac{K_c}{8}$$

$$|e_r^\infty| = \left|\frac{\rho}{K_1}\right| = \left|\frac{0{,}5}{K_c/8}\right| \leq 0{,}1$$

$$\rightarrow \left|\frac{4}{K_c}\right| \leq 0{,}1 \rightarrow \boxed{|K_c| \geq 40}$$

- $|y_{d_y}^{\infty}| \leq 0{,}001$ $d_y(t) = \delta_y\, \varepsilon(t)$

$|d_y| \leq 0{,}1$

→ type ⓪ system is needed

On the basis of the type 1 request by $|e_r^{\infty}| \Rightarrow |y_{d_y}^{\infty}| = 0$

Summarizing:

$C_{SS}(s) = \dfrac{K_c}{s}$ $|K_c| \geq 40$

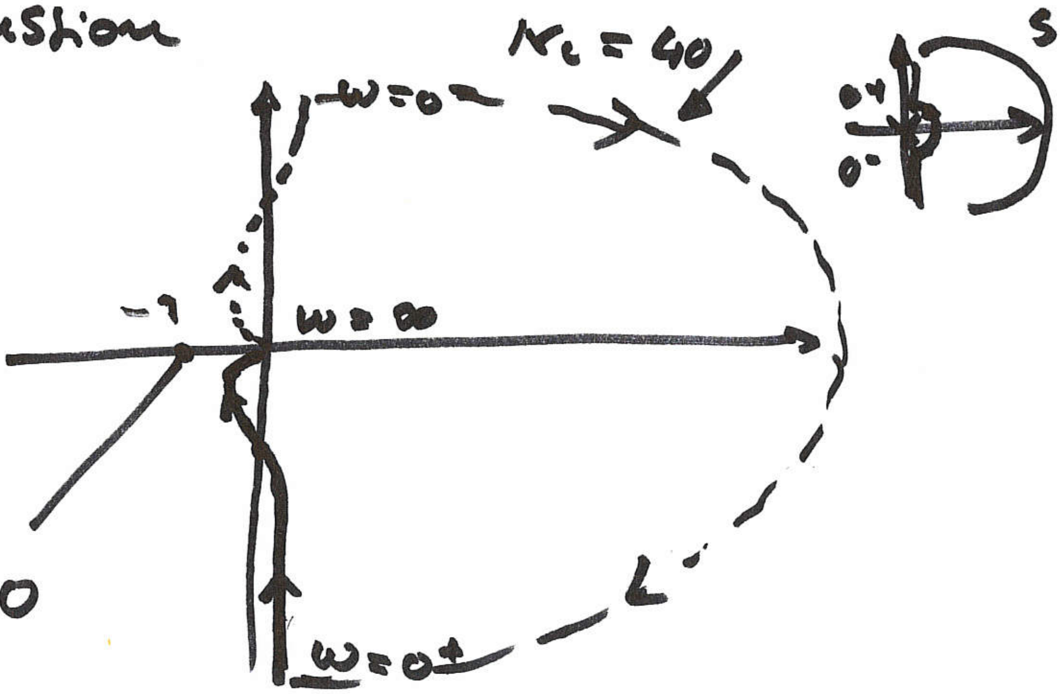
$\Downarrow$

$|K_c| = 40$

K_c Sign discussion

$L'(s) = C_{SS}(s)\,G(s)$

$P = 0$



$N = 0$

closed loop system is stable $\Rightarrow C_{SS}(s) = \dfrac{40}{s}$

Example 2

- $|e_r^\infty| \leq 0{,}1 \qquad r(t) = t^2 \varepsilon(t)$

$\rightarrow$ type 2 system is needed

$L(s)$ with 2 poles at 0

$G(s)$ has 1 pole at 0

$$\Rightarrow \boxed{C_{ss}(s) = \frac{K_c}{s}}$$

$$|e_r^\infty| = \left|\frac{\rho}{K_2}\right| \qquad \rho = 2$$

$$K_2 = \lim_{s\to 0} s^2 L(s) = \lim_{s\to 0} \cancel{s^2}\, \underbrace{\frac{K_c}{\cancel{s}}\, \cancel{C_1(s)}_{1}}_{C(s)}\, \frac{(s+3)(s+8)}{\cancel{s}(s+1)^2} =$$

$$= 24 K_c$$

$$|e_r^\infty| = \left|\frac{\rho}{K_2}\right| = \left|\frac{2}{24 K_c}\right| \leq 0{,}1$$

$$\ldots \quad \boxed{|K_c| \geq 0{,}8\bar{3}}$$

- $|y_{d_y}^\infty| = 0 \qquad d_y(t) = \delta_y\, t\, \varepsilon(t) \qquad |\delta_y| \leq 0{,}1$

$\rightarrow$ type 2 system is needed

$$\ldots \quad \rightarrow \boxed{C_{ss}(s) = \frac{K_c}{s}}$$

$|e_r^{\infty}| \rightarrow \quad C_{ss}(s) = \dfrac{K_c}{s} \qquad |K_c| \geq 0{,}8\dot{3}$

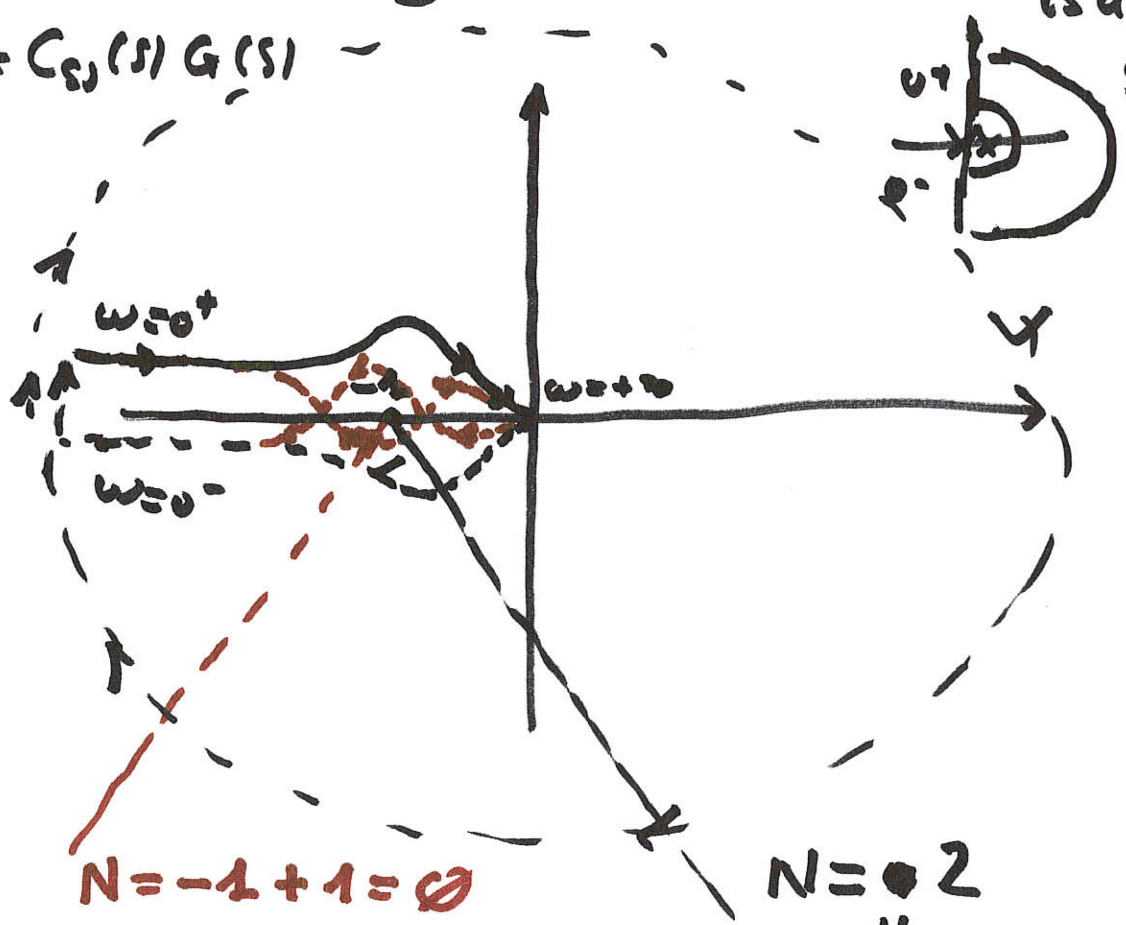
$|y_{dr}^{\infty}| \rightarrow \quad C_{ss}(s) = \dfrac{K_c}{s} \qquad \forall K_c$ (stabilizing)

$\Downarrow$

Summarizing: $C_{ss}(s) = \dfrac{K_c}{s} \qquad |K_c| \geq 0{,}8\dot{3}$

K_c sign discussion → $K_c = 0{,}9$ ($K_c = 1$ is also ok)

$L'(s) = C_{ss}(s)\, G(s)$



$P = 0$

$N = -1 + 1 = \emptyset$

$\Downarrow$

closed loop stable

$\Downarrow$

$C_{ss}(s) = \dfrac{0{,}9}{s}$ ($C_{ss}(s) = \dfrac{1}{s}$ is also ok)

$N = 2$

$\Downarrow$

closed loop not stable

Example 3

$|e_r^\infty| = 0 \qquad r(t) = \varepsilon(t)$

→ type 1 system is needed

$L(s)$ with 1 pole at $\emptyset$
$(G(s)$ has not poles at $\emptyset)$ } → $\boxed{C_{ss}(s) = \frac{K_c}{s}}$

the requirement does not provide indications for the numerical value of K_c

$$\Rightarrow \quad C_{ss}(s) = \frac{K_c}{s} \quad \forall K_c \text{ stabilizing}$$

for simplicity, $|K_c| = 1$ is chosen

K_c sign discussion $\quad L'(s) = \dfrac{-500}{s(s^2-900)}$

$$G(s) = \frac{500}{s^2-900} = \frac{500}{(s+30)(s-30)} =$$

$$= \frac{500}{30\left(1+\frac{s}{30}\right)\cdot(-30)\left(1-\frac{s}{30}\right)} =$$

$$= -\frac{5}{9}\,\frac{1}{\left(1+\frac{s}{30}\right)\left(1-\frac{s}{30}\right)}$$

↓

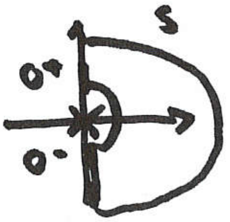
$K_G < 0$

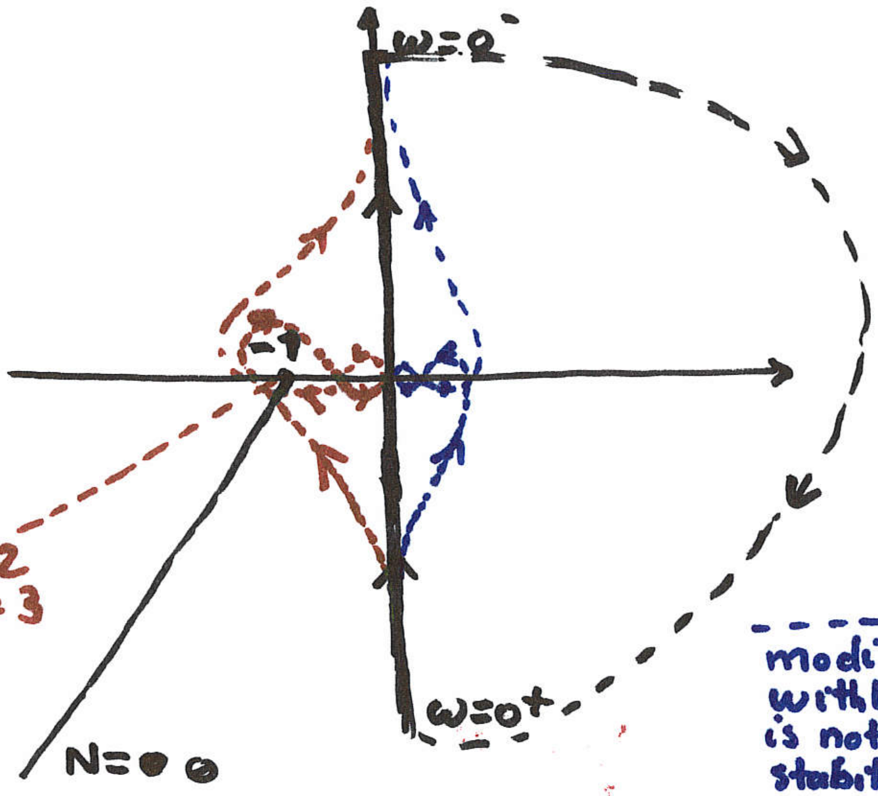
$K_c = -1$
(to have $K_g > 0$)
$C_{ss}(s) = -\frac{1}{s}$

→

$K_c = -1$

$P = 1$





--- modification with lag action is not able to stabilize the system

$N = 2$
$Z = 3$

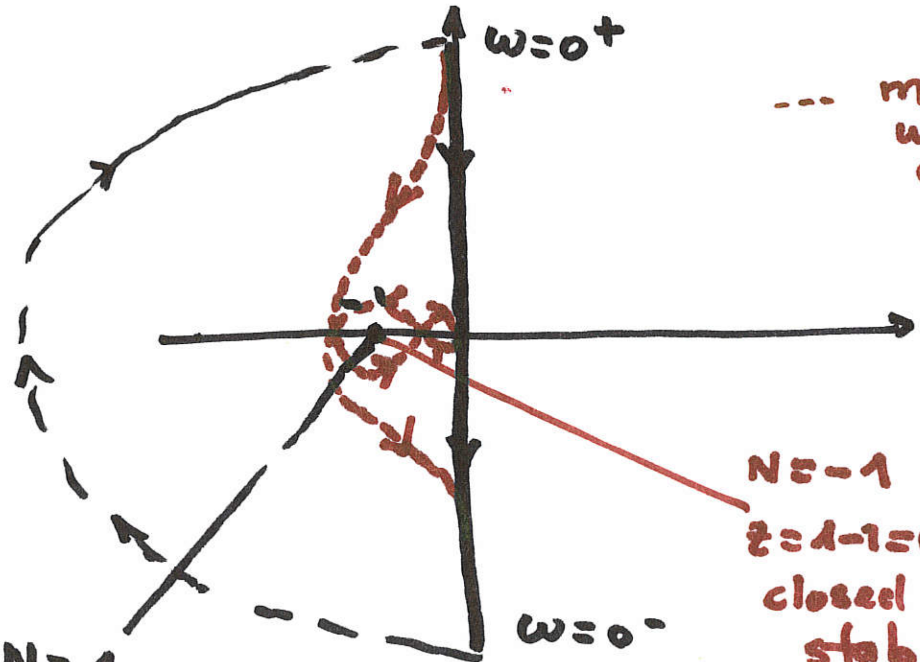
$N = 0$

$Z = 1$
closed loop not stable

--- modification with lead action is not able to stabilize the system ($N=0$)

$K_c < 0$ is the wrong sign

$K_c = 1$
$P = 1$



--- modification with lead action

$N = -1$
$Z = 1-1 = 0$
closed loop stable

⇓

$\boxed{C_{ss}(s) = \frac{1}{s}}$

$N = 1$
$Z = 2$
closed loop not stable