Automatic Control

Dynamical systems simulation using Simulink

Dynamical systems solution

Problem Given the dynamical system described by the state space representation

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

and

- the time course of u(t)
- the initial state x(0)

compute the system responses x(t) and y(t) through integration of the state equation

$$\dot{x}(t) = f(x(t), u(t))$$

In general, for nonlinear system it is not possible to compute the solution in closed form → perform simulation

Dynamical systems simulation

Simulation solves a dynamical system through numerical integration of the state equation

$$\dot{x}(t) = f(x(t), u(t))$$

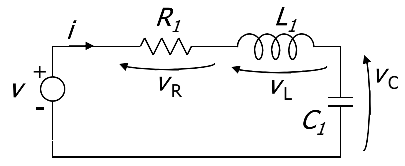
In this case, the system responses $x(t)$ and $y(t)$ can be directly plotted.

Simulation can be easily performed in MatLab environment by means of its extension Simulink.

Basically, Simulink is a tool that allows us to easily build a model of dynamical system using block diagram notation and, at the same time, to compute the solution by means of already implemented numerical solvers.

Simulation of an electric system

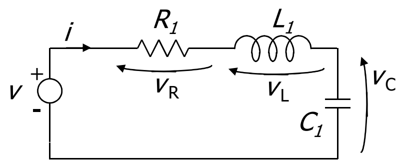
Consider the dynamical system below.



- Voltages v(t) and v_c(t) are the system input and output respectively
- The numerical values of the components are

$$R_1 = 68\ \Omega\ ,\ C_1 = 4\mu F\ ,\ L_1 = 10\ mH$$

Simulation of an electric system



• Derive the state space representation of the system assuming as system states the current i(t) through L_1 and voltage $v_C(t)$ across C_1 respectively

$$u(t)=v\,(t),\, x(t)=\begin{bmatrix} i(t) \\ v_C(t)\end{bmatrix}=\begin{bmatrix} x_1(t) \\ x_2(t)\end{bmatrix}, y(t)=v_C(t)=x_2(t)$$

$$\begin{cases} \dot{x}_1(t)=\dfrac{1}{L_1}\left[-R_1\,x_1(t)-x_2(t)+u(t)\right] \\ \dot{x}_2(t)=\dfrac{1}{C_1}x_1(t) \end{cases} \rightarrow A=\begin{bmatrix} -\dfrac{R_1}{L_1} & -\dfrac{1}{L_1} \\ \dfrac{1}{C_1} & 0 \end{bmatrix}, B=\begin{bmatrix} \dfrac{1}{L_1} \\ 0 \end{bmatrix}$$

$$y(t)=x_2(t) \qquad C=\begin{bmatrix} 0 & 1 \end{bmatrix}, D=\begin{bmatrix} 0 \end{bmatrix}$$

Simulation of an electric system

$$\begin{cases} \dot{x}_1(t)=\dfrac{1}{L_1}\left[-R_1\,x_1(t)-x_2(t)+u(t)\right] \\ \dot{x}_2(t)=\dfrac{1}{C_1}x_1(t) \end{cases}$$

$$y(t)=x_2(t)$$

• Simulate the time behaviour of i(t) and $v_C(t)$ when:
 v(t) = ε(t) V, i(0) = 0 A $v_{C1}(0)$ = 0 V

• Simulate the time behaviour of i(t) and $v_C(t)$ when:
 v(t) = ε(t) V, i(0) = 10 mA $v_{C1}(0)$ = 50 mV

Simulation of an electric system

In order to effectively handle the simulation procedure a suitable MatLab script file is developed.

```
clc
clear all
close all

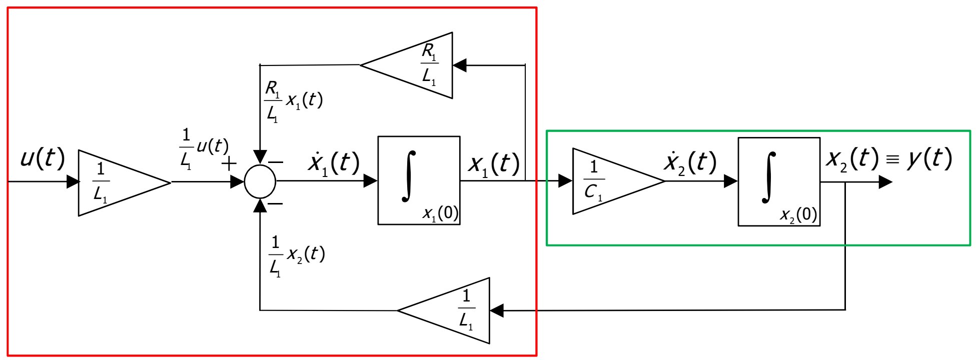
% define parameter values
R1=68;
L1=10e-3;
C1=4e-6;

A=[-R1/L1 -1/L1;1/C1 0];
B=[1/L1;0];
C=[0 1];
D=0;
% define initial condition
x0=[0;0];
```

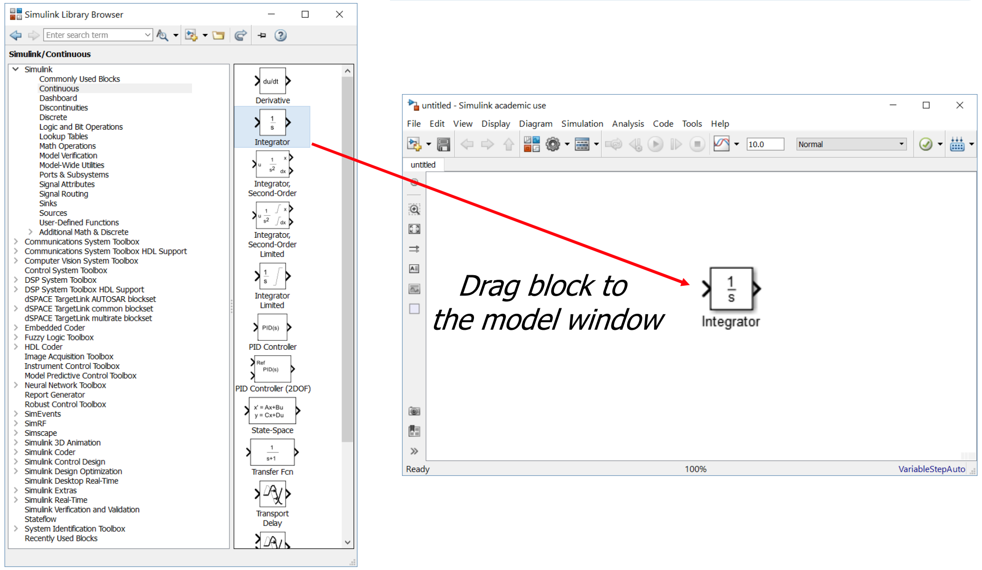

Simulation of an electric system

$$\begin{cases} \dot{x}_1(t)=\dfrac{1}{L_1}\left[-R_1\,x_1(t)-x_2(t)+u(t)\right] \\ \dot{x}_2(t)=\dfrac{1}{C_1}x_1(t) \end{cases}$$

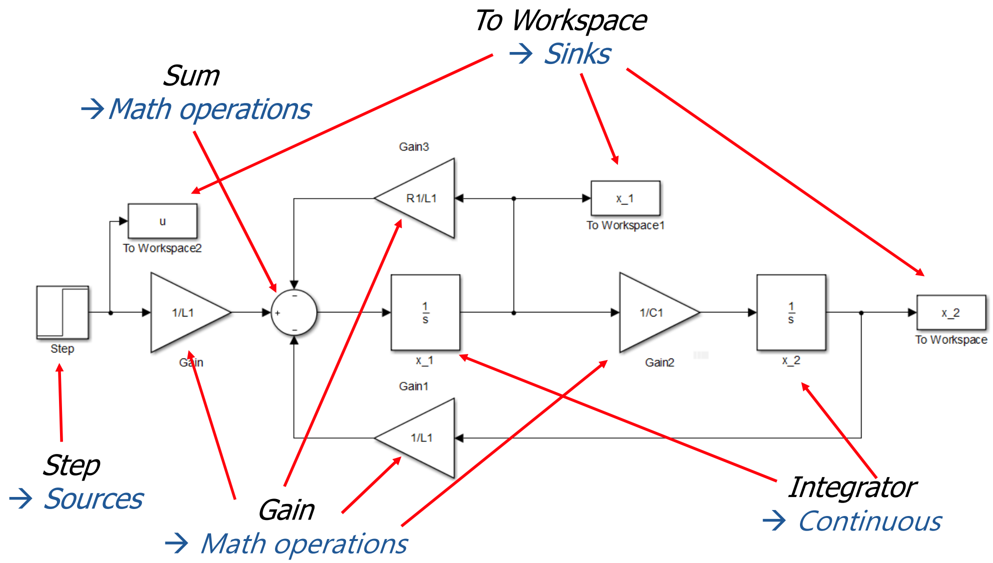
$$y(t)=x_2(t)$$



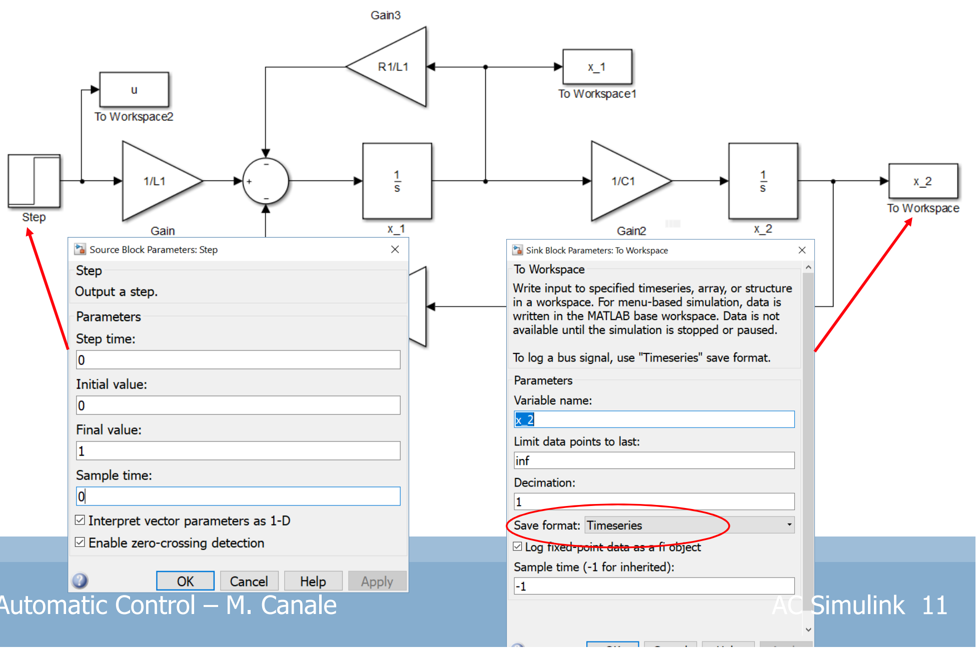
Simulation of an electric system



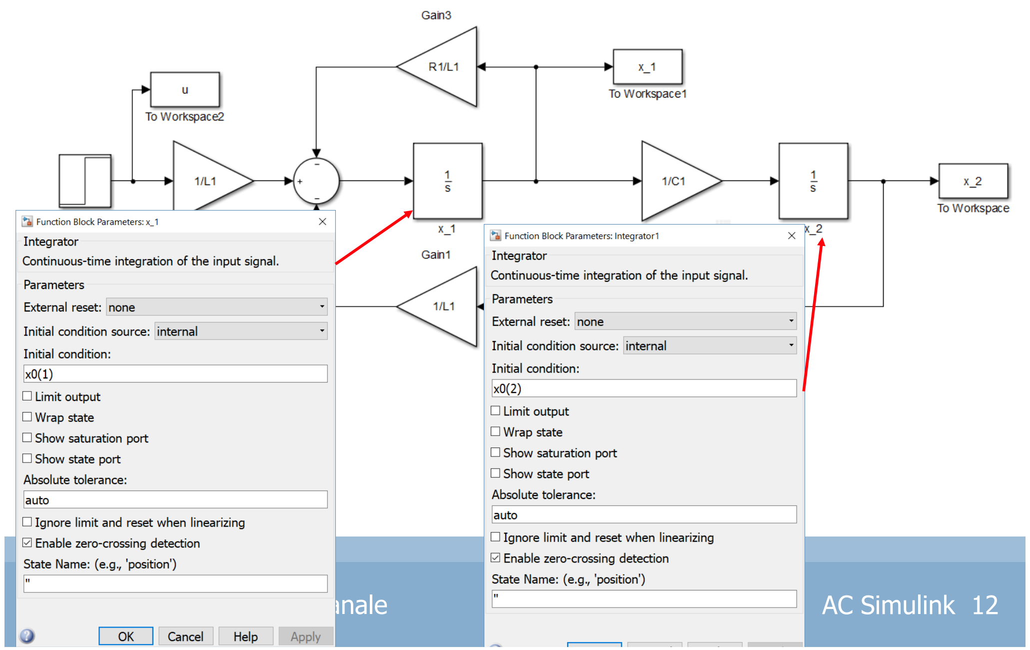
Simulation of an electric system



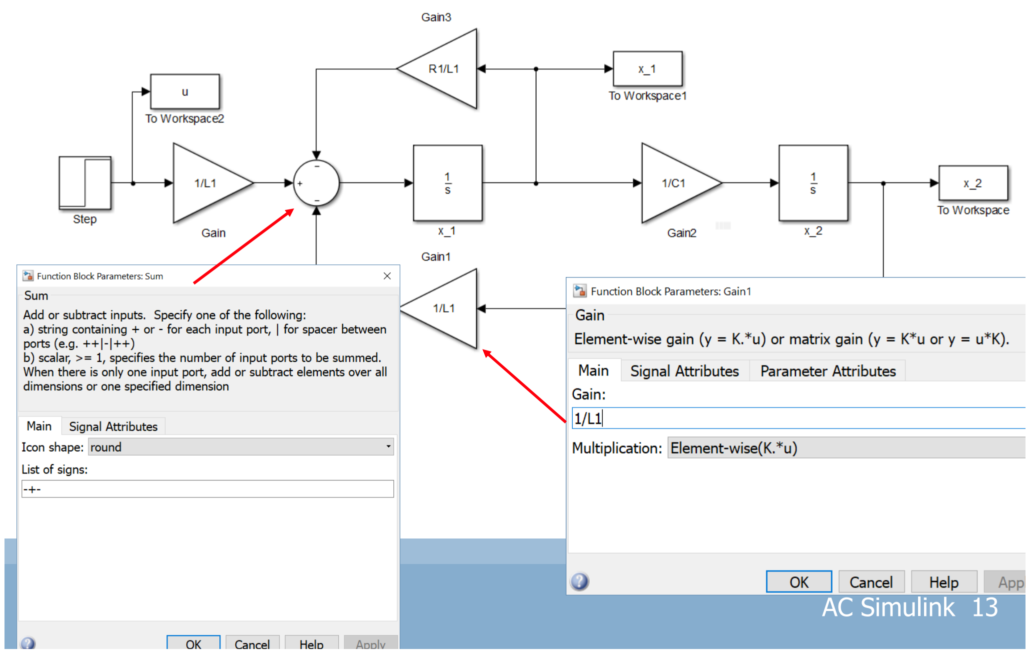
Simulation of an electric system



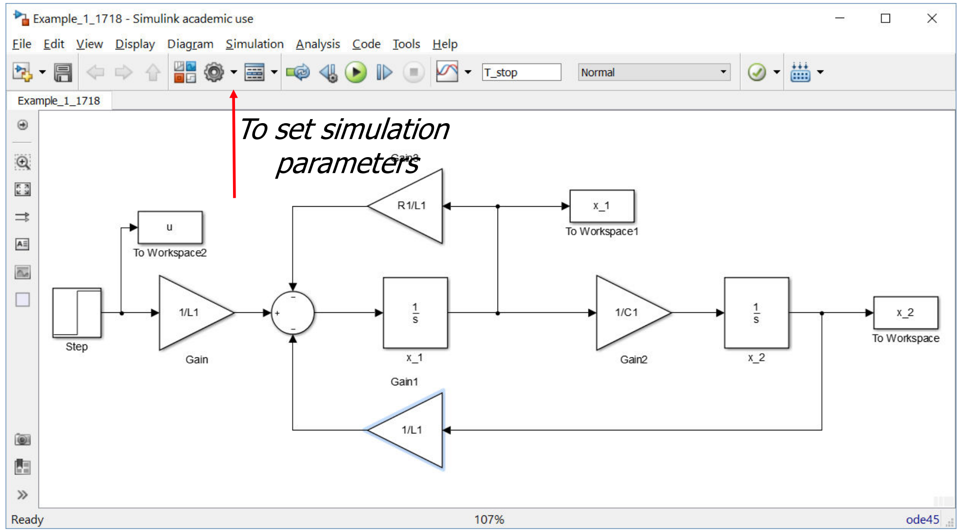
Simulation of an electric system



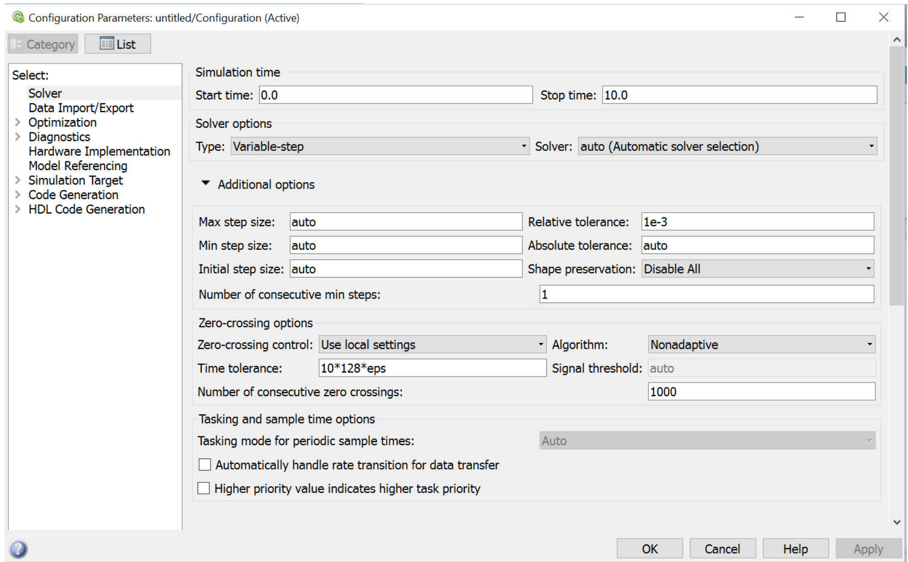
Simulation of an electric system



Simulation of an electric system



Simulation of an electric system



Hints on main simulation parameters

In order to perform a "good" simulation for a continuous time LTI system suitable numerical integration algorithms* should be chosen:

- Variable step algorithms such as `ode45` and `ode23` should be chosen since they are quite efficient and provide more accurate results.
- In the presence of "stiff" systems, i.e. system with eigenvalues characterized by large stiffness ratio

$$\frac{\max_i(\mathrm{Re}(\lambda_i))}{\min_i(\mathrm{Re}(\lambda_i))}, i = 1,\dots,n$$

  try different solvers such as `ode15s` , `ode23s` , … to determine which one performs best.

* See textbooks 1. (Chapter 13) and 3. (Chapter 12) for more details.

Hints on main simulation parameters

Moreover configuration parameters such as `Stop Time`, `Max Step size` and `Min Step size`, should be chosen* according to

- the time constants of the system $\tau_i = \dfrac{1}{|\mathrm{Re}(\lambda_i)|}, i = 1,\dots,n$
- the period T of the involved periodic signals (e.g. sinusoidal inputs,…)

Some hints are

- `Stop Time` $\rightarrow \begin{cases}(5 \div 10)\cdot\tau_{\max} \\ (5 \div 10)\cdot T_{\max}\end{cases}$
- `Max Step size` $\rightarrow \begin{cases}(0.01 \div 0.1)\cdot\tau_{\max} \\ (0.05 \div 0.1)\cdot T_{\max}\end{cases}$
- `Min Step size` $\rightarrow (0.001 \div 0.01)\cdot\tau_{\min}$

* See textbook 3. (Chapter 12) for more details.

Simulation of an electric system

$$\begin{cases}\dot{x}_1(t) = \dfrac{1}{L_1}\left[-R_1\, x_1(t) - x_2(t) + u(t)\right] \\ \dot{x}_2(t) = \dfrac{1}{C_1} x_1(t)\end{cases} \qquad R_1 = 68\ \Omega,\ C_1 = 4\mu F,\ L_1 = 10\ mH$$

$$y(t) = x_2(t)$$

$$\begin{cases}\dot{x}_1(t) = -6800\, x_1(t) - 100\, x_2(t) + 100\, u(t) \\ \dot{x}_2(t) = 250000\, x_1(t)\end{cases}$$

$$\lambda_{1,2} = (-3.4 \pm j3.\bar{6})\cdot 10^3 \rightarrow \tau_{1,2} = -\frac{1}{\left|\mathbb{R}\mathrm{e}(\lambda_{1,2})\right|} = 0.29\ \mathrm{ms}$$

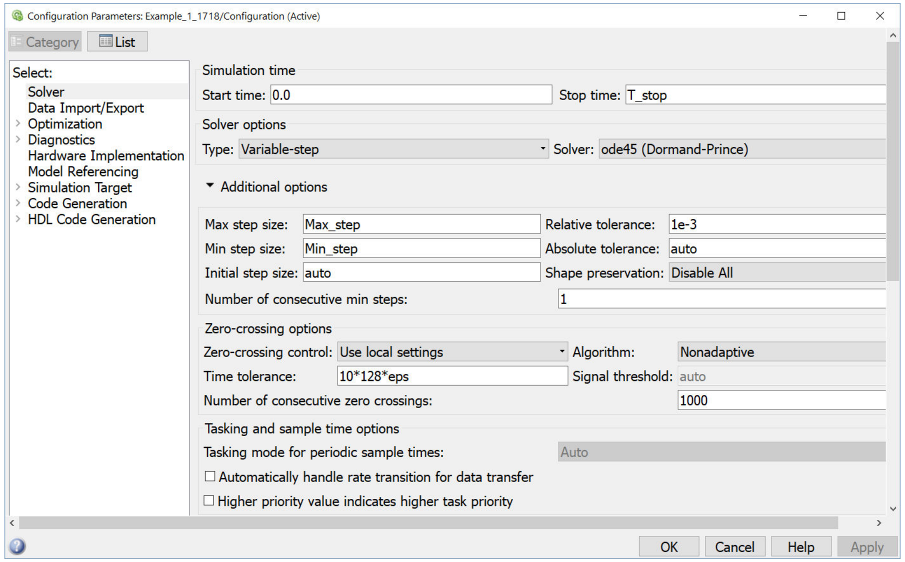
`Stop Time` → `0.003 s`
`Max Step size` → `0.00003 s`   `Min Step size` → `0.000003 s`

```
% compute eig and time
% constant
lambda=eig(A);
tau=1./abs(real(lambda))
```

```
% define simulation parameters

T_stop = 0.003; % about 10*tau
Max_step = 0.00003; % about 0.1*tau
Min_step = 0.000003; % about 0.01*tau
```


Simulation of an electric system



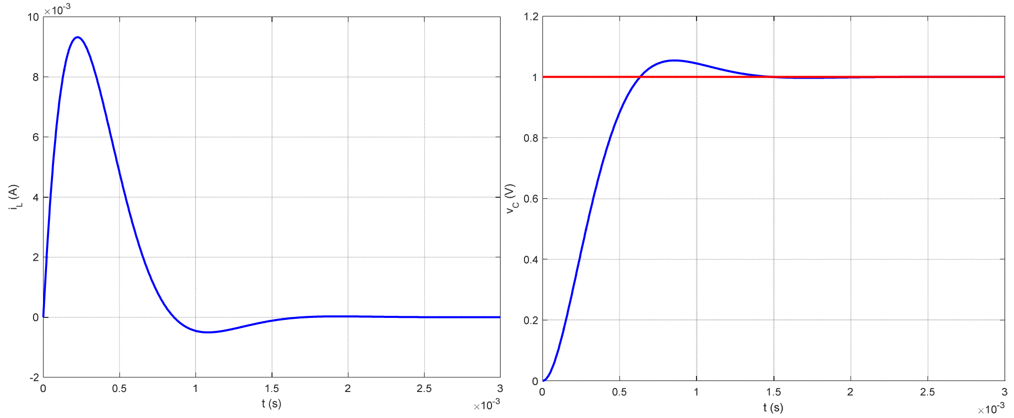
Simulation of an electric system

```
% run simulation
sim('Example_1_1617')

% plot results
figure
plot(x_1.time,x_1.data,'b','linewidth',2)
grid on
xlabel('t (s)'), ylabel('i_L (A)')

figure
plot(x_2.time,x_2.data,'b','linewidth',2)
grid on
hold on
plot(u.time,u.data,'r','linewidth',2)
xlabel('t (s)'), ylabel('v_C (V)')
```

Simulation of an electric system



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B=[1/L1;0];
C=[0 1];
D=0;
% define initial condition
x0=[0;0];

% compute eig and time
% constant
lambda=eig(A);
tau=1./abs(real(lambda))
```

```
% define simulation parameters
T_stop = 0.003; % about 10*tau
Max_step = 0.00003; % about 0.1*tau
Min_step = 0.000003; % about 0.01*tau

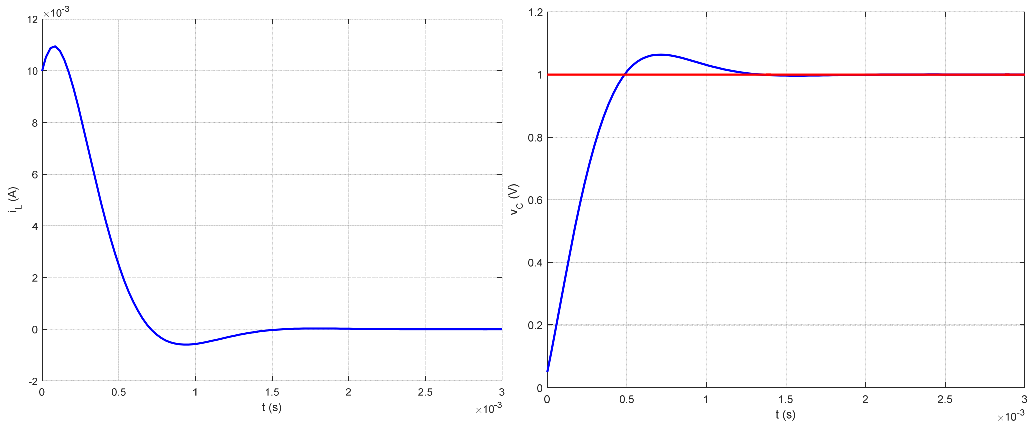
% run simulation
sim('Example_1_1617')

% plot results
figure
plot(x_1.time,x_1.data,'b','linewidth',2)
grid on
xlabel('t (s)'), ylabel('i_L (A)')

figure
plot(x_2.time,x_2.data,'b','linewidth',2)
grid on
hold on
plot(u.time,u.data,'r','linewidth',2)
xlabel('t (s)'), ylabel('v_C (V)')
```


Simulation of an electric system

```
x0=[10e-3; 50e-3];
```



Textbooks about Simulink

1. James B. Dabney, Thomas L. Harman, *Mastering Simulink,* Prentice Hall, 2004

2. A. Cavallo, R. Setola, F. Vasca, *Using Matlab, Simulink and Control System ToolBox: A Practical Approach*, Pearson, 1996

3. A. Cavallo, R. Setola, F. Vasca, *La nuova guida a Matlab Simulink e Control Toolbox ,* Liguori, 2002