2.23.

李海超
16711094
160324

$$X(z) = \frac{3}{1-0.5z^{-1}} + \frac{2}{1-2z^{-1}}$$

(1). 极点为 $z_1 = 0.5$，$z_2 = 2$

∴ 收敛域 $|z| < 0.5$ 或 $0.5 < |z| < 2$ 或 $|z| > 2$.

(2). ① $|z| < 0.5$：为左边序列.

$$F(z) = X(z) z^{n-1} = \frac{5-7z^{-1} \cdot z^{n-1}}{(1-0.5z^{-1})(1-2z^{-1})} = \frac{(5z-7) \cdot z^n}{(z-2)(z-0.5)}$$

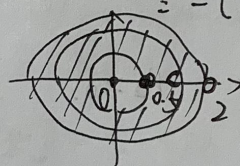考虑 z^n 情况：① 若 $n \geq 0$，C内无极点，$x(n) = 0$.

② 若 $n \leq -1$，$\frac{1}{z^n}$ 为 n 阶极点.

∴ $x(n) = \sum Res[F(z), z_{2k}] = -\frac{(5z-7)z^n}{(z-2)}\bigg|_{0.5} - \frac{(5z-7)z^n}{(z-0.5)}\bigg|_2$

$$= -\left(3 \cdot \left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n\right) \cdot u(-1-n).$$

② $0.5 < |z| < 2$：为双边序列.

① $n \geq 0$ 时，C内有 $z = 0.5$.

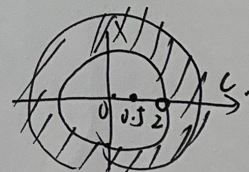$$x(n) = Res[F(z), 0.5] = 3 \cdot \left(\tfrac{1}{2}\right)^n \cdot u(n).$$

② $n \leq -1$ 时，C内有一阶 $z = 0.5$ 与 n 阶 $z = 0$，与①类似.

$$x(n) = -Res[F(z), z_k = 2] = -2 \cdot 2^n \cdot u(-1-n).$$

∴ $x(n) = 3 \cdot \left(\tfrac{1}{2}\right)^n \cdot u(n) - 2 \cdot 2^n \cdot u(-1-n).$

③ $|z| > 2$.

① $n \geq 0$ 时，有 0.5，2 两个一阶极点.

$$x(n) = \sum Res[F(z), z_{1k}] = \left[3 \cdot \left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n\right] \cdot u(n).$$

② $n \leq -1$ 时，0 为 n 阶，但 C 外无留数可求

∴ $x(n) = 0$

∴ $x(n) = \left[3 \cdot \left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n\right] \cdot u(n).$

2.24.

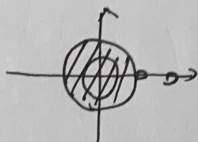$$X(z) = \frac{1}{1 - 1.5 \cdot z^{-1} + 0.5 z^{-2}} = \frac{z^2}{z^2 - \frac{3}{2}z + \frac{1}{2}}$$

$$-\frac{1}{2} \times \frac{1}{1}$$
$$-1 \qquad 1$$

$$= \frac{z^2}{(z - \frac{1}{2})(z - 1)}$$ ∥∥∥∥∥∥∥∥   极点在 1/0.5

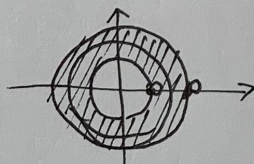$$F(z) = \frac{z^{n+1}}{(z - 0.5)(z - 1)}$$

① $|z| < 0.5$.

n≥-1 时. C内无极点. $x(n) = 0$.

n < -1 时, 存在 0 的高阶极点.

$$x(n) = -\sum Res[F(z), z_{2k}] = -\frac{z^{n+1}}{z-1}\Big|_{0.5} - \frac{z^{n+1}}{z-0.5}\Big|_{1}$$

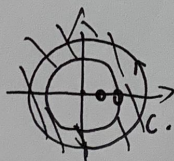$$= \left[2 \cdot \left(\frac{1}{2}\right)^{n+1} - 2\right] u(-n-2).$$

② $0.5 < |z| < 1$.

n≥-1 时.

$$x(n) = Res[F(z), 0.5] = -2 \cdot \left(\frac{1}{2}\right)^{n+1} = -\left(\frac{1}{2}\right)^{n}.$$

n < -1 时.

$$x(n) = -Res[F(z), 1] = -2 \cdot 2^{n+1} = -2^{n+2}.$$

∴ $x(n) = -\left(\frac{1}{2}\right)^{n} u(n+1) - 2^{n+2} u(-n-2).$

③ $|z| > 1$.

n≥-1 时.

$$x(n) = \sum Res[F(z), z_{k}] = -\left(\frac{1}{2}\right)^{n} + 2$$

n < -1 时.

C外无极点. 则 $x(n) = 0$

∴ $x(n) = \left[-\left(\frac{1}{2}\right)^{n} + 2\right] u(n+1)$