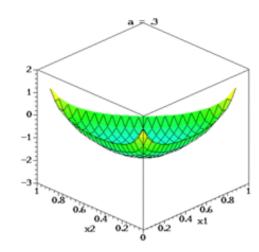
Machine Learning

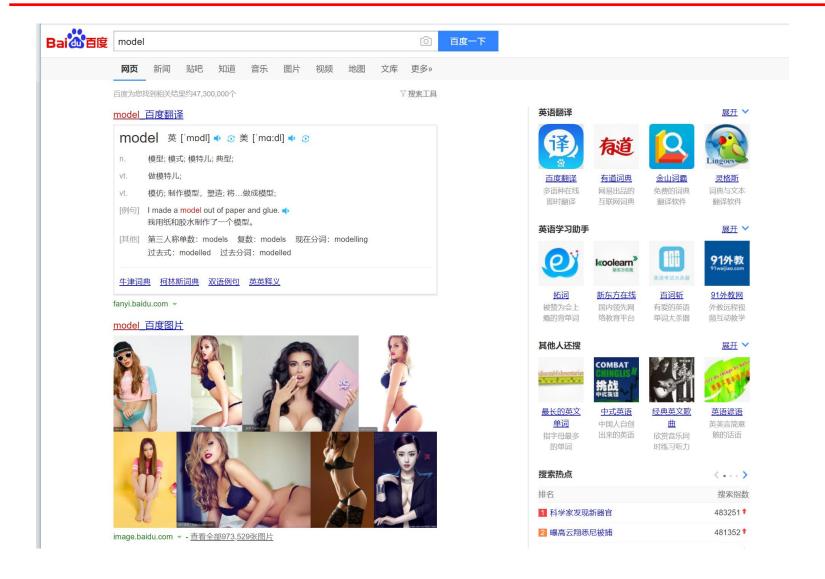
Part 4: Classical Machine Learning Model

Zengchang Qin (Ph.D.)

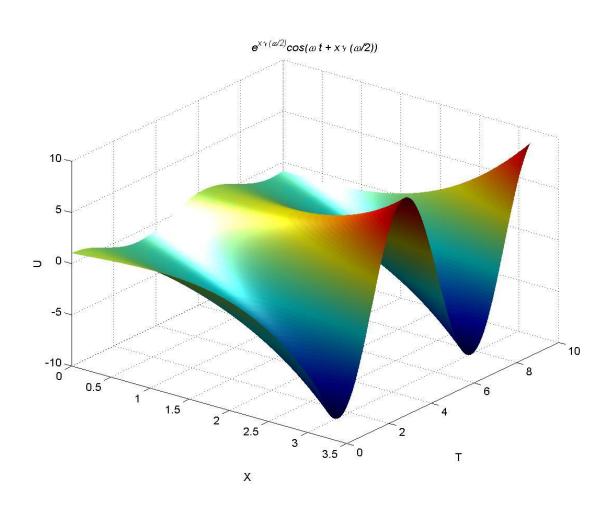


Case Based Reasoning

Model



The Model We are Interested

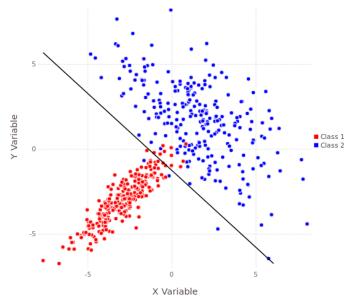


Parametric Model

Parametric models assume some finite set of parameters θ . Given the parameters, future predictions, x, are independent of the observed data, \mathcal{D} :

$$P(x|\theta, \mathcal{D}) = P(x|\theta)$$

therefore θ capture everything there is to know about the data.

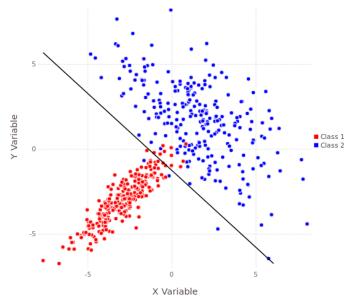


Parametric Model

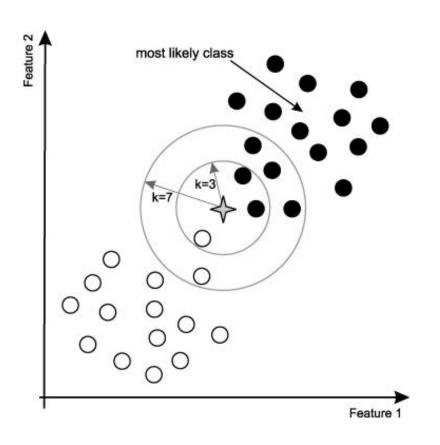
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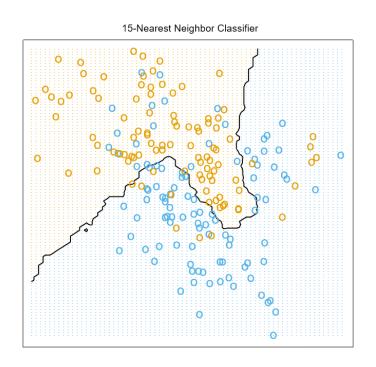
K-Nearest Neighbor (k-NN)

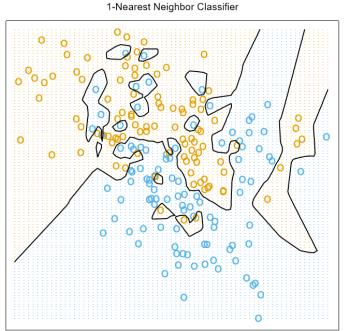


k-nearest neighbors algorithm (k-NN) is a non-parametric method used for classification and regression. The input consists of the k closest training examples in the feature space.

Q: *k* has to be an odd number?

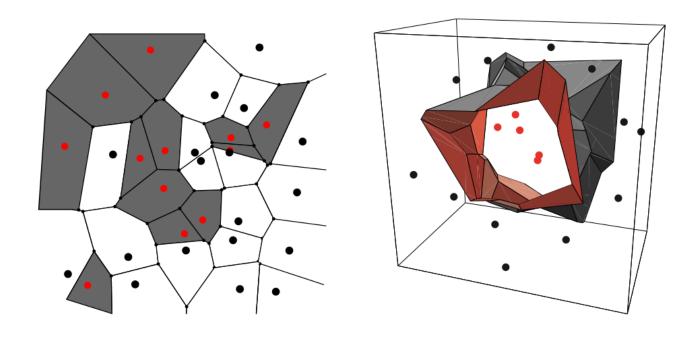
K-Nearest Neighbor (KNN)





Q: What if K becomes very large?

Parametric Model



In two dimensions, the nearest-neighbor algorithm leads to a partitioning of the input space into Voronoi cells, each label led by the category of the training point it contains. In three dimensions, the cells are three-dimensional, and the decision boundary resembles the surface of a crystal.

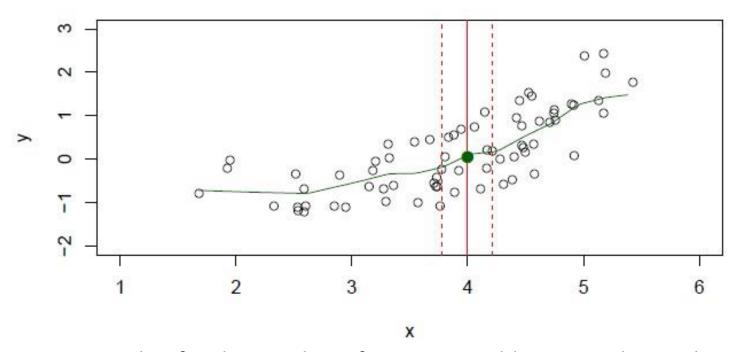
Can k-NN Work for Prediction?



K-Nearest Neighbors for Machine Learning, Photo by Valentin Ottone

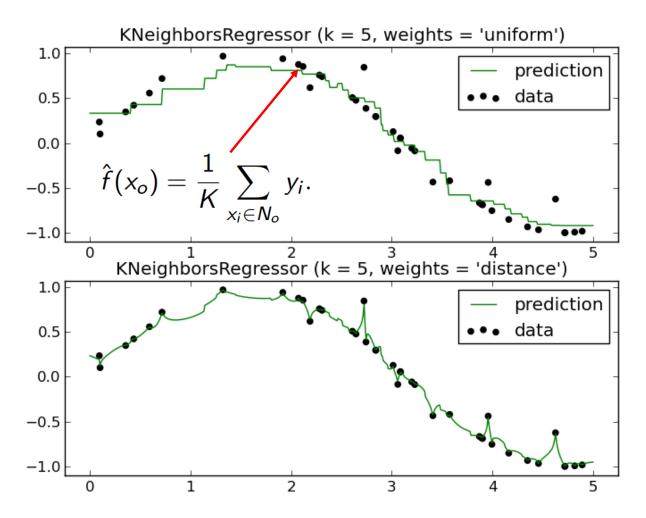
Regression Problem

KNN for Regression (Prediction)



- 1. Assume a value for the number of nearest neighbors K and a prediction point xo.
- 2. KNN identifies the training observations No closest to the prediction point xo.
- 3. KNN estimates f (xo) using the average of all the responses in No

k-NN for Prediction



Q: Any better non-parametric model, do we need to adjust the weights?

Distance Measure





```
d(A,B) = d(B,A) Symmetry

d(A,A) = 0 Constancy of Self-Similarity

d(A,B) = 0 iff A=B Positivity Separation

d(A,B) \le d(A,C) + d(B,C) Triangular Inequality
```

Distance Measure (Look Familiar?)

Minkowski Distance

$$dist(\mathbf{x}, \mathbf{y}) = \sqrt[r]{\sum_{i=1}^{d} |x_i - y_i|^r}$$

Euclidean distance (r=2)

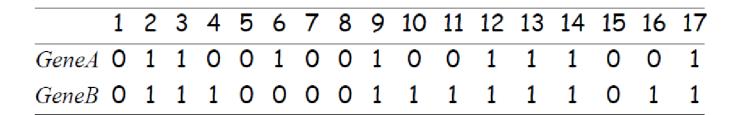
$$dist(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{k=1}^{d} (x_k - y_k)^2} = \|\mathbf{x} - \mathbf{y}\|_2$$

Manhattan distance (r=1)

$$dist(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^{d} |x_k - y_k| = ||\mathbf{x} - \mathbf{y}||_1$$

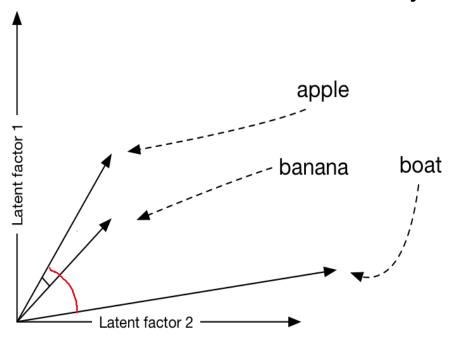


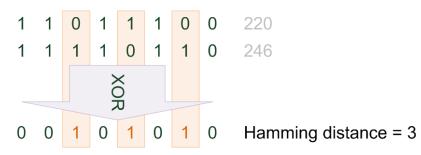
Distance Measure

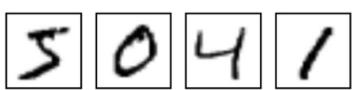


Hamming distance

when all features are binary





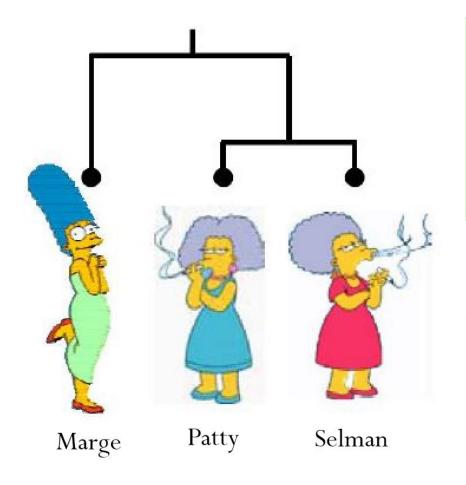


Cosine Similarity

$$\cos(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x}^{\top} \mathbf{y}}{\|\mathbf{x}\|_2 \|\mathbf{y}\|_2}$$

Edited Measure

To measure the similarity between two objects, transform one into the other, and measure how much effort it took. The measure of effort becomes the distance measure.



The distance between Marge and Selma

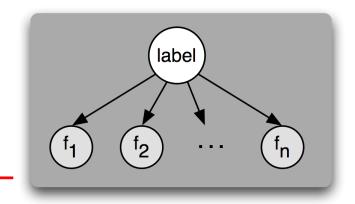
Change dress color, 1 point Add earrings, 1 point Decrease height, 1 point Take up smoking, 1 point Loss weight, 1 point

D(Marge, Selma) = 5

The distance between Patty and Selma.

Change dress color, 1 point Change earring shape, 1 point Change hair part, 1 point

D(Patty,Selma) = 3



Naïve Bayes



Bayes Rule

$$P(Y|X_1,\ldots,X_n) = \frac{P(X_1,\ldots,X_n|Y)P(Y)}{P(X_1,\ldots,X_n)}$$
 Normalization Constant

The Naïve Bayes Assumption: Assume that all features are independent given the class label Y:

Equationally speaking:

$$P(X_1, ..., X_n | Y) = \prod_{i=1}^n P(X_i | Y)$$

Independence assumption – Naïve but effective!

Play-Tennis Problem

PlayTennis: training examples

	0 1 1	·		TAT 1	DI T
Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

$$P(\text{Play=}Yes) = 9/14$$

Outlook	Play =Yes	Play =No
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

Learning

$$P(\text{Play}=Yes) = 9/14$$

P(P)	lay=No)	= 5/14
------	---------	--------

Outlook	Play=Yes	Play=No
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

Temperature	Play=Yes	Play=No
Hot	2/9	2/5
Mild	4/9	2/5
Cool	3/9	1/5

Humidity	Play=Yes	Play=No
High	3/9	4/5
Normal	6/9	1/5

Wind	Play=Yes	Play=No
Strong	3/9	3/5
Weak	6/9	2/5

For a new data:

x'=(Outlook=*Sunny*, Temperature=*Cool*, Humidity=*High*, Wind=*Strong*)

Prediction

- Given a new instance, predict its label

```
x'=(Outlook=Sunny, Temperature=Cool, Humidity=High, Wind=Strong)
```

Look up tables achieved in the learning phrase

$$P(Outlook=Sunny | Play=Yes) = 2/9 \qquad P(Outlook=Sunny | Play=No) = 3/5$$

$$P(Temperature=Cool | Play=Yes) = 3/9 \qquad P(Temperature=Cool | Play=No) = 1/5$$

$$P(Huminity=High | Play=Yes) = 3/9 \qquad P(Huminity=High | Play=No) = 4/5$$

$$P(Wind=Strong | Play=Yes) = 3/9 \qquad P(Wind=Strong | Play=No) = 3/5$$

$$P(Play=Yes) = 9/14 \qquad P(Play=No) = 5/14$$

Decision making with the MAP rule

```
P(Yes \mid \mathbf{X}') \approx [P(Sunny \mid Yes)P(Cool \mid Yes)P(High \mid Yes)P(Strong \mid Yes)]P(Play=Yes) = 0.0053

P(No \mid \mathbf{X}') \approx [P(Sunny \mid No) P(Cool \mid No)P(High \mid No)P(Strong \mid No)]P(Play=No) = 0.0206
```

Given the fact $P(Yes | \mathbf{x}') < P(No | \mathbf{x}')$, we label \mathbf{x}' to be "No".

NB for Spam Filtering

But if we have, say, $y \in \{0,1\}$ for being spam or non-spam email, a vocabulary of 50000 words, then $x \in \{0,1\}^{50000}$:

$$p(x_1, \dots, x_{50000}|y)$$

$$= p(x_1|y)p(x_2|y, x_1)p(x_3|y, x_1, x_2) \cdots p(x_{50000}|y, x_1, \dots, x_{49999})$$

$$= p(x_1|y)p(x_2|y)p(x_3|y) \cdots p(x_{50000}|y)$$

$$= \prod_{i=1}^{n} p(x_i|y)$$

• Disadvantages:

- Assumes independence of feature

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \text{aardwolf} \\ \vdots \\ 1 \\ \text{buy} \\ \vdots \\ 0 \\ \text{zygmurgy} \end{bmatrix}$$



NB for Spam Filtering

$$\phi_{j|y=1} = \frac{\sum_{i=1}^{m} 1\{x_j^{(i)} = 1 \land y^{(i)} = 1\}}{\sum_{i=1}^{m} 1\{y^{(i)} = 1\}}$$

$$\phi_{j|y=0} = \frac{\sum_{i=1}^{m} 1\{x_j^{(i)} = 1 \land y^{(i)} = 0\}}{\sum_{i=1}^{m} 1\{y^{(i)} = 0\}}$$

$$\phi_y = \frac{\sum_{i=1}^{m} 1\{y^{(i)} = 1\}}{m}$$

• Advantages:

- Fast to train (single scan).
- Fast to classify
- Not sensitive to irrelevant features
- Handles real and discrete data
- Handles streaming data well

• Smoothing:

- It always happens that a particular word has not appeared in the given text.
- A smoothing factor is needed, e.g. P = 1/|V|, where |V| = 50000 in this example.