常用公式结论

一、三角函数

和差化积：$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x-\sin y=2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$

$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

积化和差：$\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y=\frac{1}{2}[\cos(x-y)+\cos(x+y)]$

$\sin x\cos y=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$

三倍角公式：$\sin 3x=3\sin x-4\sin^3 x$

$\cos 3x=4\cos^3 x-3\cos x$

$\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

二、常用不等关系

1、Bernoulli 不等式：对$x>-1$时

$n\geq 1$时 $(1+x)^n\geq 1+nx$

$0\leq x\leq 1$时 $(1+x)^n\leq 1+nx$

更一般形式：$(1+x_1+x_2+\cdots+x_n)\leq(1+x_1)(1+x_2)\cdots(1+x_n)$

2、Cauchy 不等式：$(\sum_{i=1}^{n} a_i b_i)^2 \le (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)$

3、均值不等式：$\frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}} \le \sqrt[n]{\prod_{i=1}^{n} x_i} \le \frac{\sum_{i=1}^{n} x_i}{n} \le \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$

4、绝对值三角不等式：$\big||a|-|b|\big| \le |a \pm b| \le |a|+|b|$

5、$\sin x \le x \le \tan x (0 \le x \le \frac{\pi}{2})$

三、常用极限

1、$\lim_{n\to\infty}(1+\frac{1}{n})^n = \sum_{i=0}^{\infty}\frac{1}{i!} = e$

2、$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\cos x}{1-\frac{1}{2}x^2} = \lim_{x\to 0}\frac{\ln(x+1)}{x}$

四、常用泰勒级数

$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}+\cdots\cdots$

$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\cdots$

$\cos x = 1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots\cdots$

$\tan x = x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7+\cdots\cdots$

$\cot x = \frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2x^5}{945}-\cdots\cdots$

$\arcsin x = x+\frac{1}{6}x^3+\frac{3}{40}x^5+\cdots\cdots$

$$\arccos x = \frac{\pi}{2} - \arcsin x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\cdots$$

$$= \frac{\pi}{2} - arc\cot x$$

$$= \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots\cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

$$\ln(x+1) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots\cdots$$

$$\ln(\frac{1+x}{1-x}) = 2(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \cdots\cdots)$$

$$\ln(\frac{x+1}{x-1}) = 2(\frac{1}{x} + \frac{1}{3}(\frac{1}{x})^3 + \frac{1}{5}(\frac{1}{x})^5 + \frac{1}{7}(\frac{1}{x})^7 + \cdots\cdots)$$