7-6 Control System Design by Frequency Response Approach

1. Introduction: Basic requirements for closed-loop systems in frequency domain

1) Performance specifications: A desired control system should be stable with sufficiently fast transient response, sufficiently small steady-state error, and strong ability to counteract external disturbance.

In frequency domain, the performance specifications are usually given by

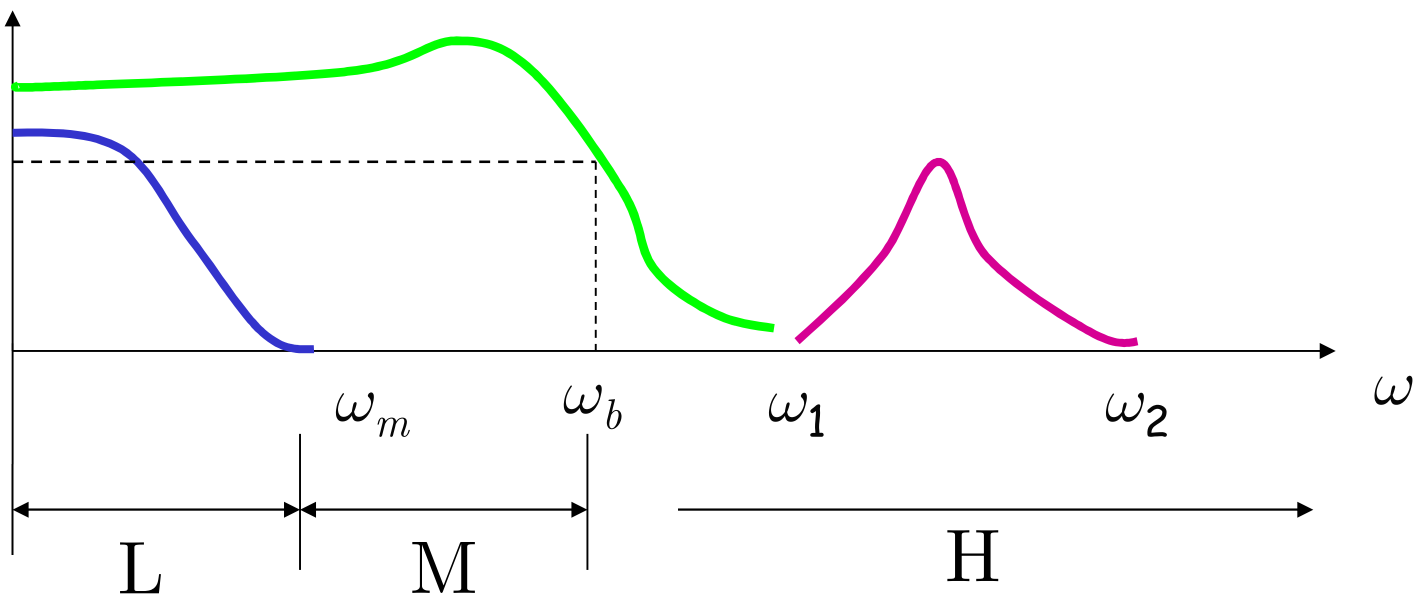
$$\gamma,\ K_g,\ \omega_c,\ K$$

or closed-loop performance indices M_r, ω_b and so on.

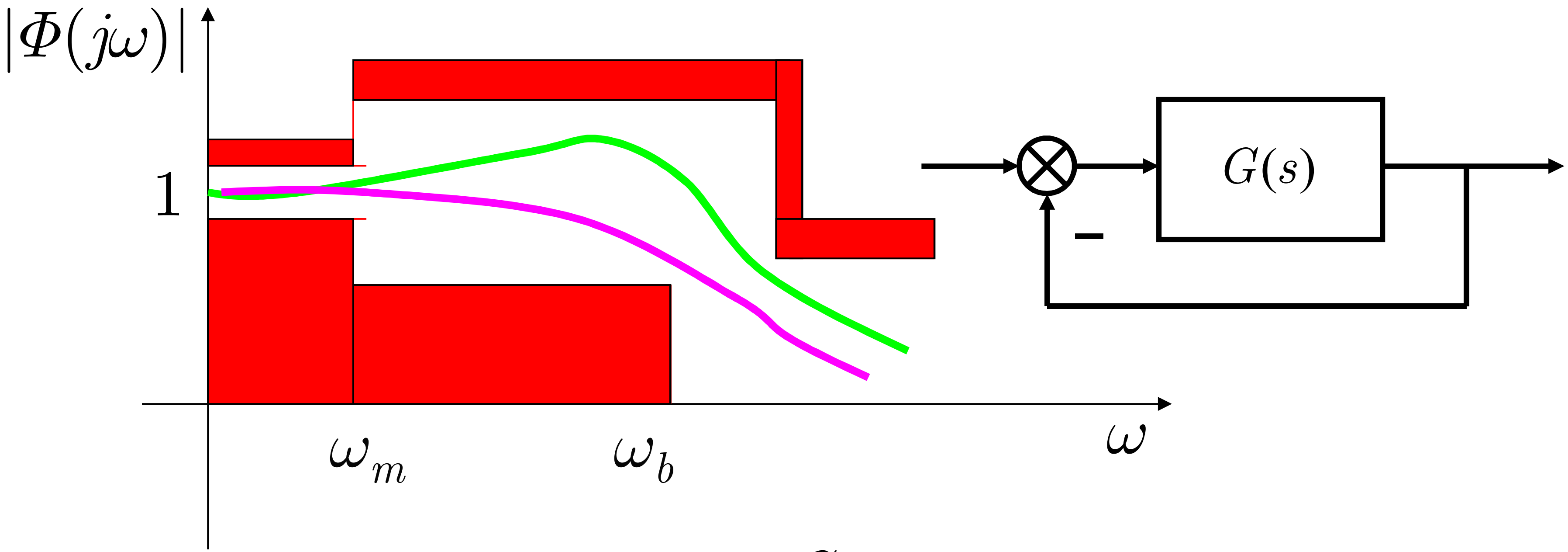
Remark:

$$\begin{cases} \omega_c = \omega_n \sqrt{\sqrt{(4\zeta^4+1)} - 2\zeta^2} \\ \omega_b = \omega_n [(1-2\zeta^2) + \sqrt{(1-2\zeta^2)^2+1}]^{\frac{1}{2}} \end{cases}$$

2)Frequency ranges: In general, a control system operates in low and medium frequency bands or ranges while disturbance operates in high frequency range.

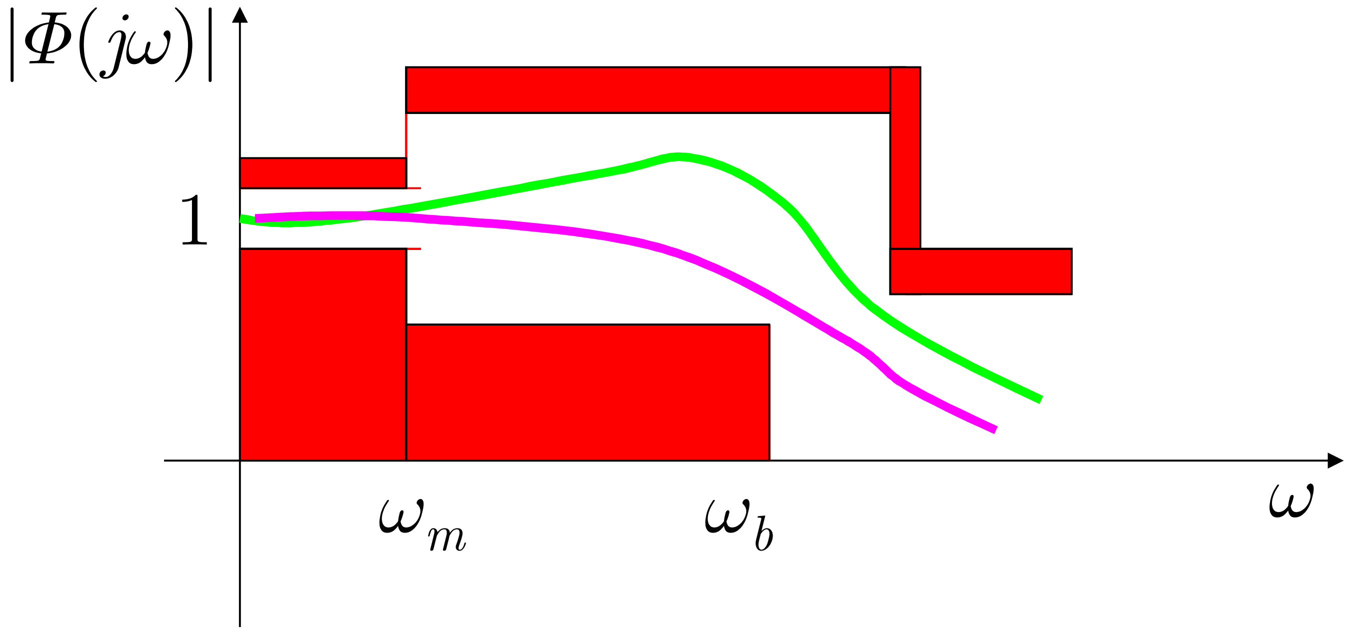


Low frequency range represents the steady-state performance and is expected to be unity, that is, with higher open-loop gain or some integral factors in open-loop transfer function.

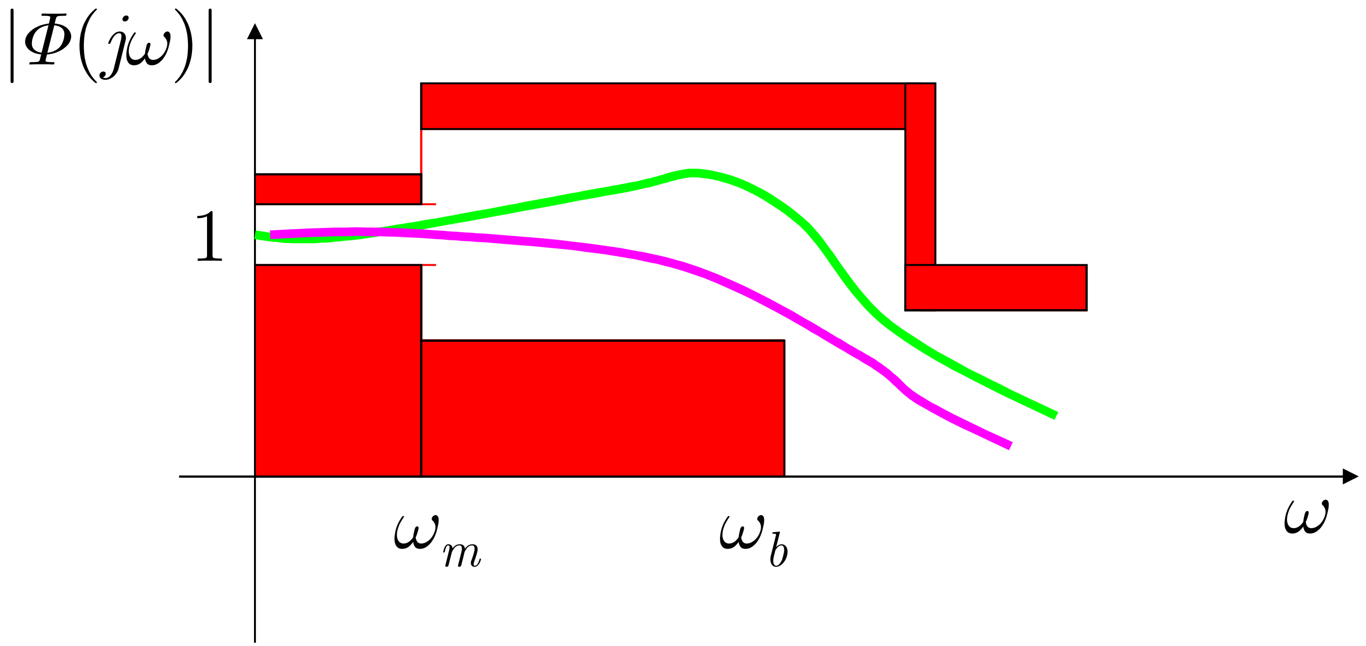


$$\Phi(s) = \frac{G}{1+G}$$

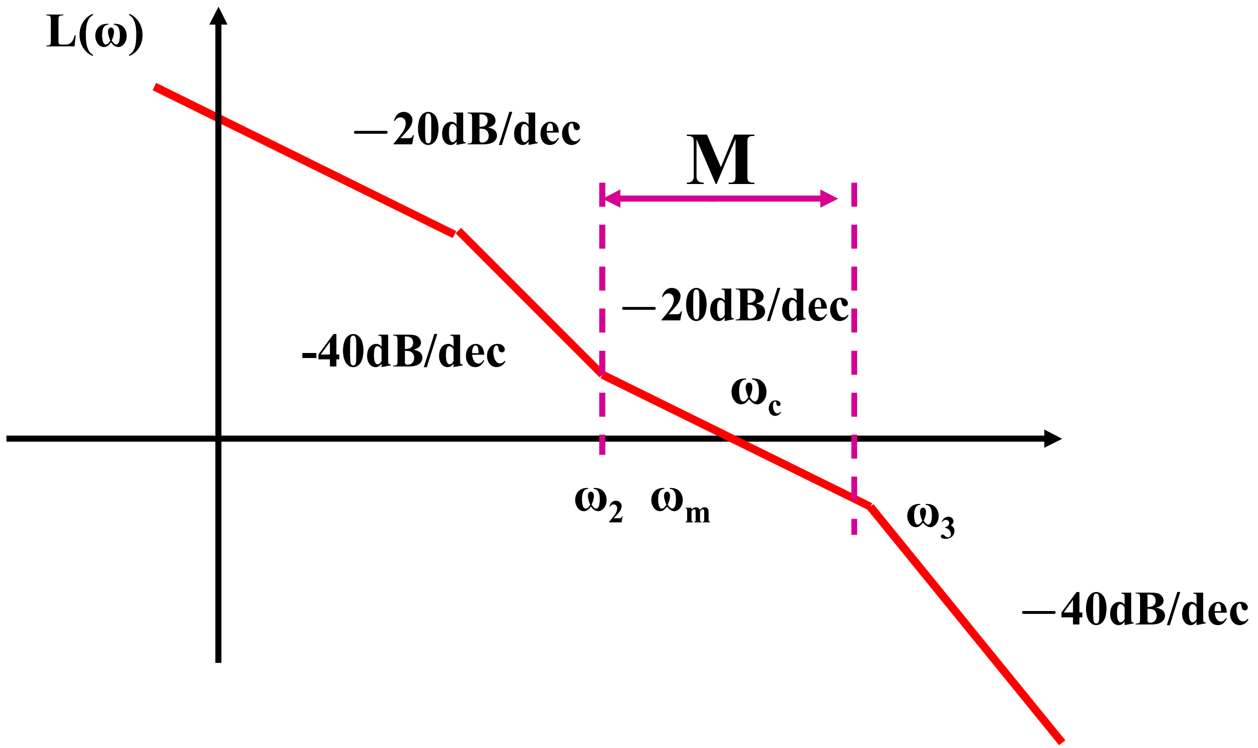
Medium frequency range represents the transient response and is expected to be sufficiently fast and well damped, that is, with higher ω_b and $0 \leq M_r \leq 1.4$.



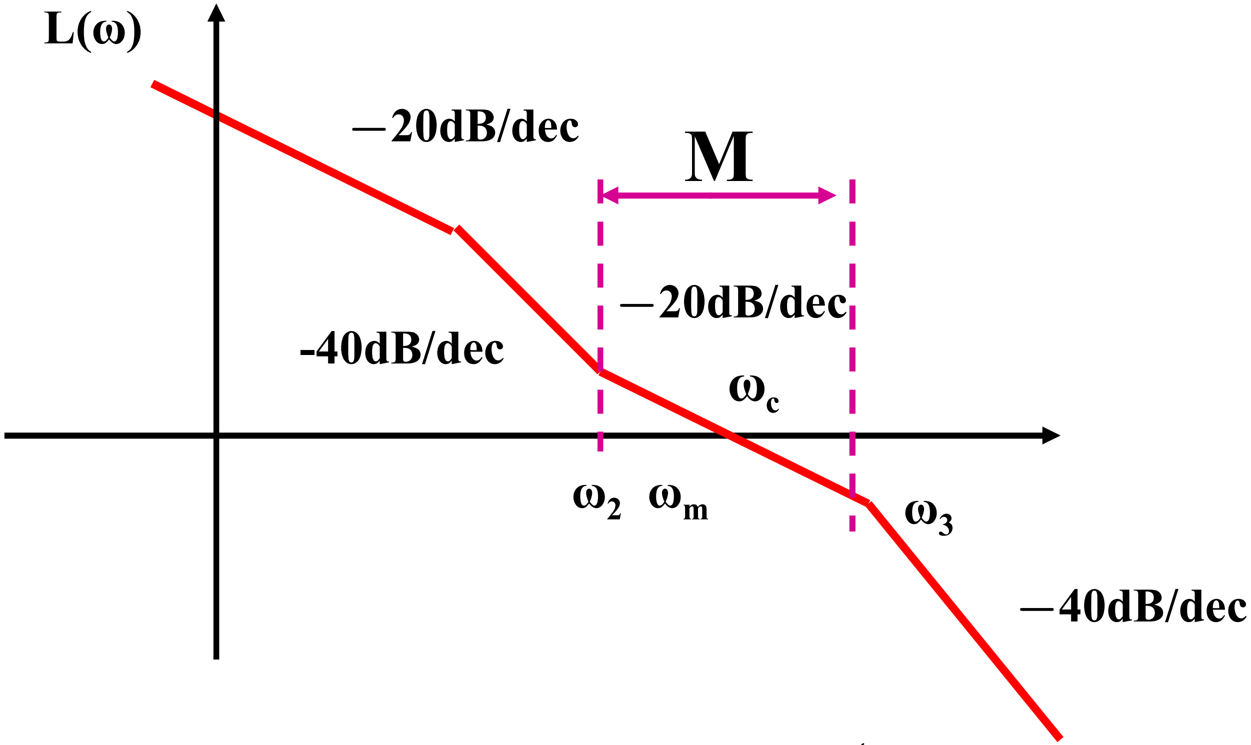
High frequency range represents the disturbance attenuation ability and is expected to be sufficiently attenuated, that is, the frequency response in high frequency range be decreased quickly.



3) Information Obtainable from Open-Loop Frequency Response



The **low frequency region** (the range far below ω_c) of the locus indicates the steady-state behavior of the **closed-loop** system.

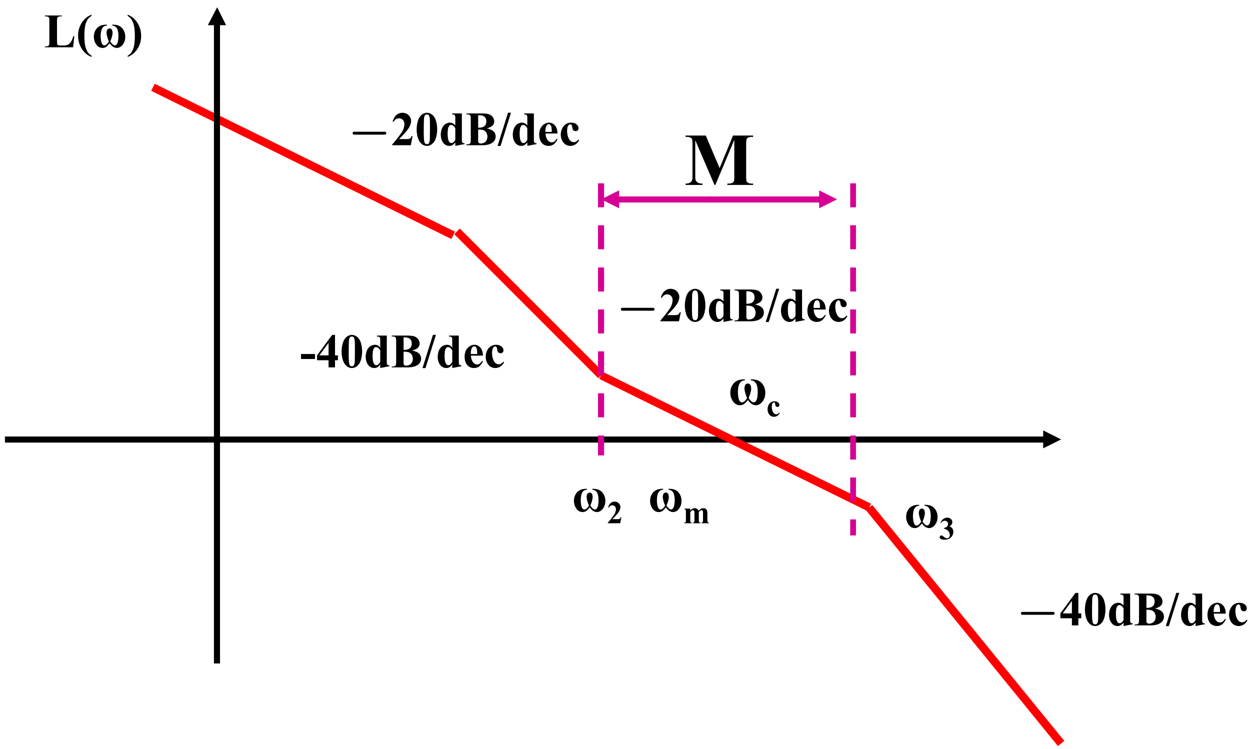


The **medium-frequency region** (the range near ω_c) of the locus indicates transient response and relative stability.

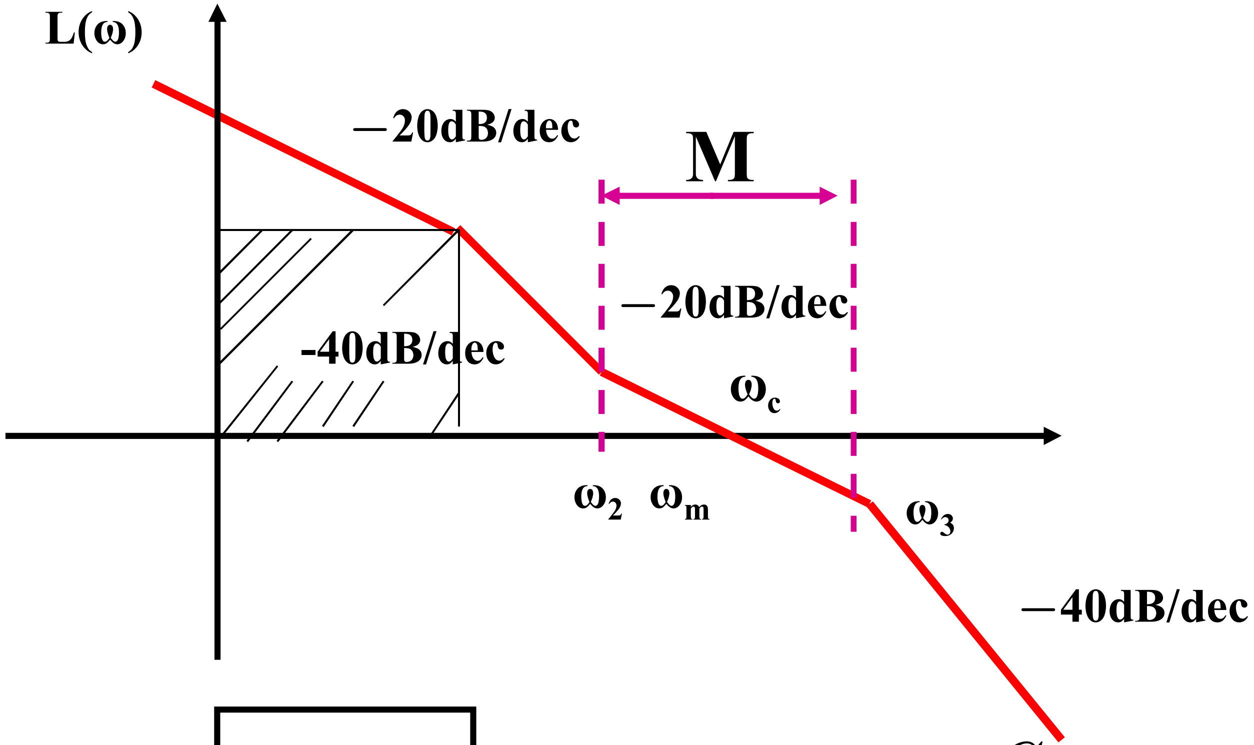
The **high frequency region** (the range far above ω_c) indicates disturbance attenuation.

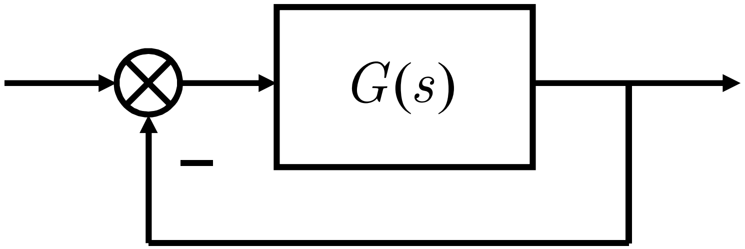
Requirements on Open-Loop Frequency Response

To have a high value of the velocity error constant and yet satisfactory relative stability, it is necessary to reshape the open-loop frequency-response curve through a compensator.



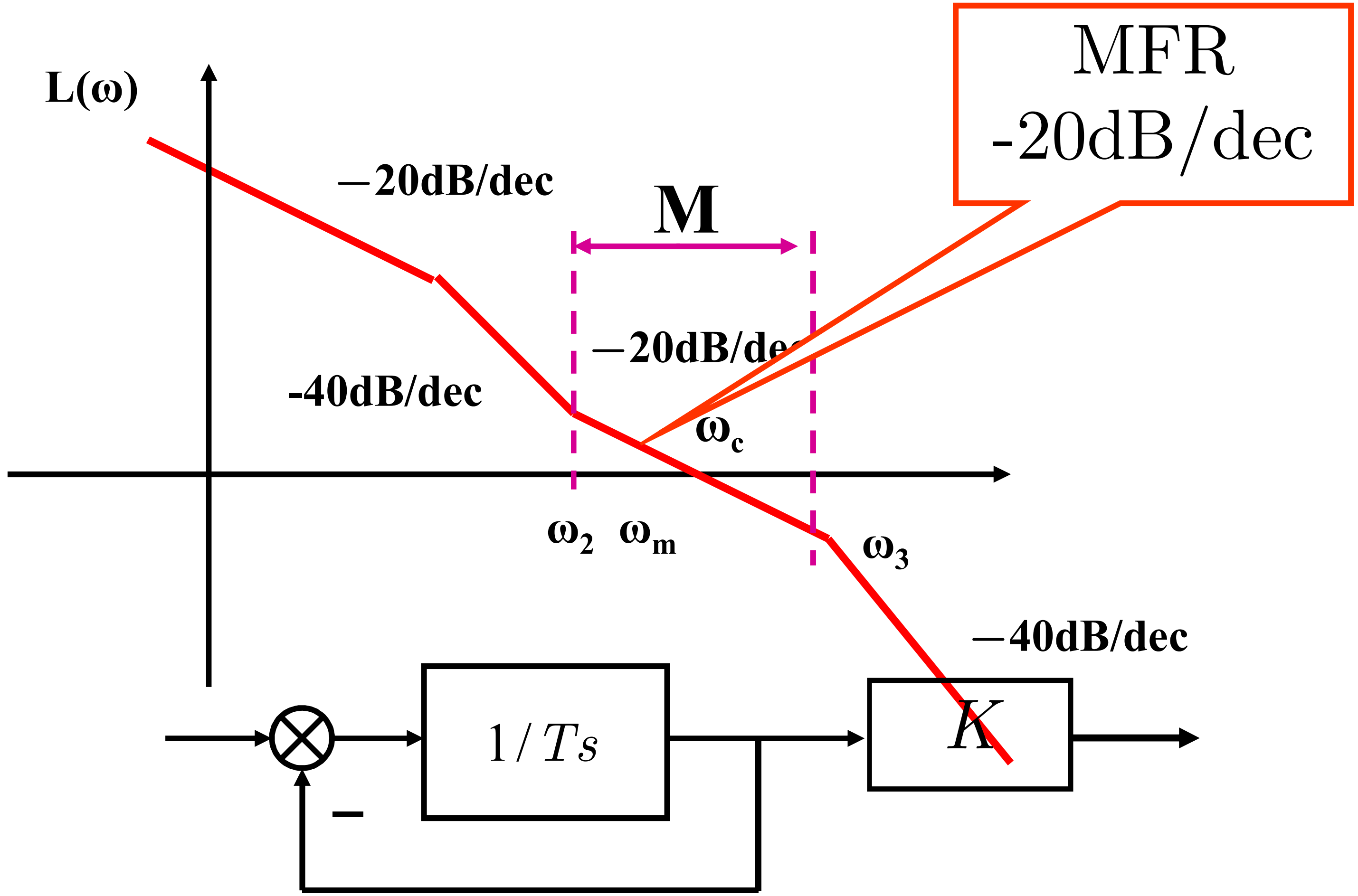
The requirement on open-loop frequency response for low frequency region: The gain in low-frequency region should be large enough and should have an integral factor.



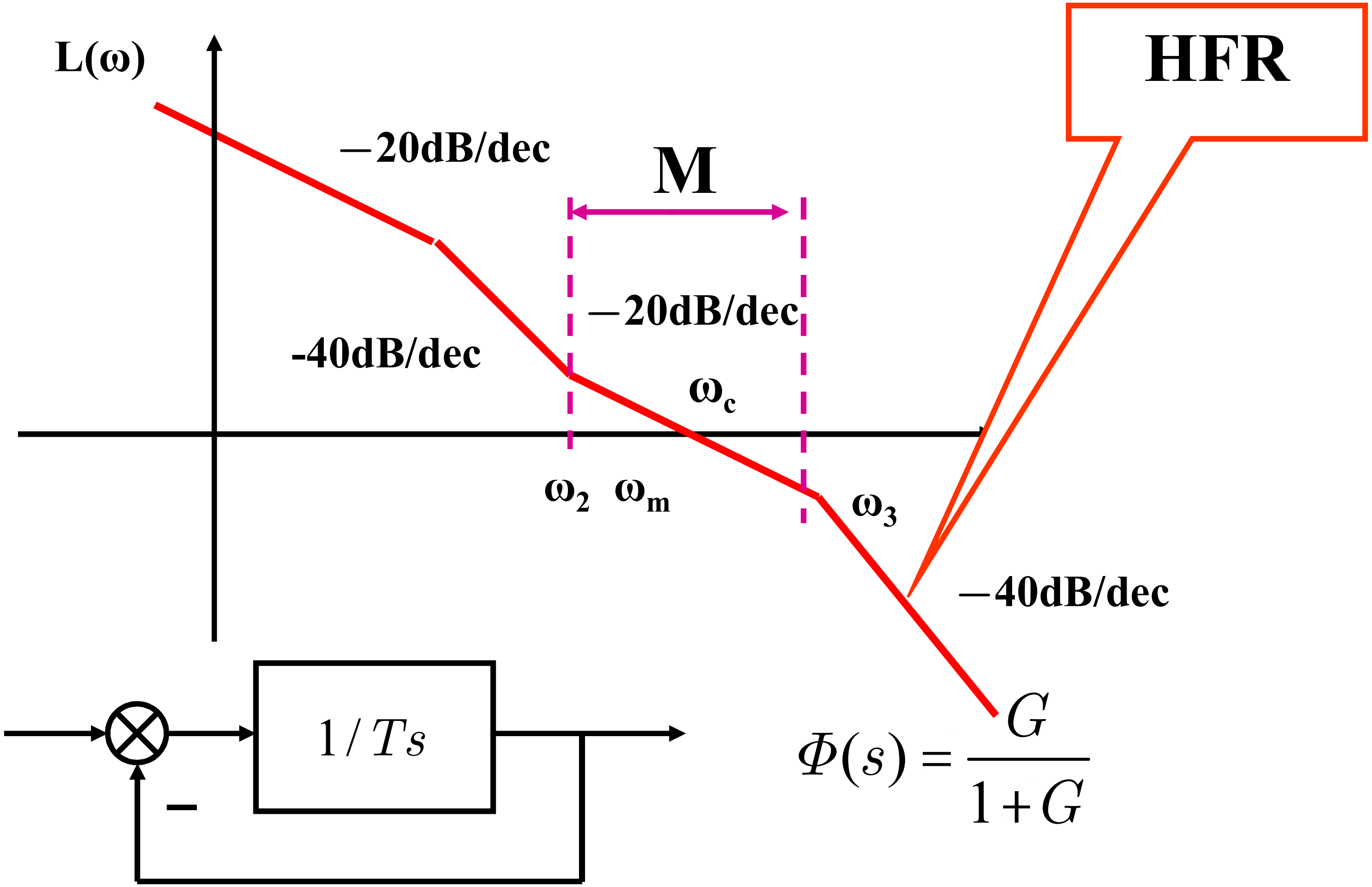


$$\Phi(s) = \frac{G}{1+G}$$

The requirement on open-loop frequency response for medium frequency region: Near the gain crossover frequency, the slope of the magnitude curve in the Bode diagram should be -20 dB/decade.



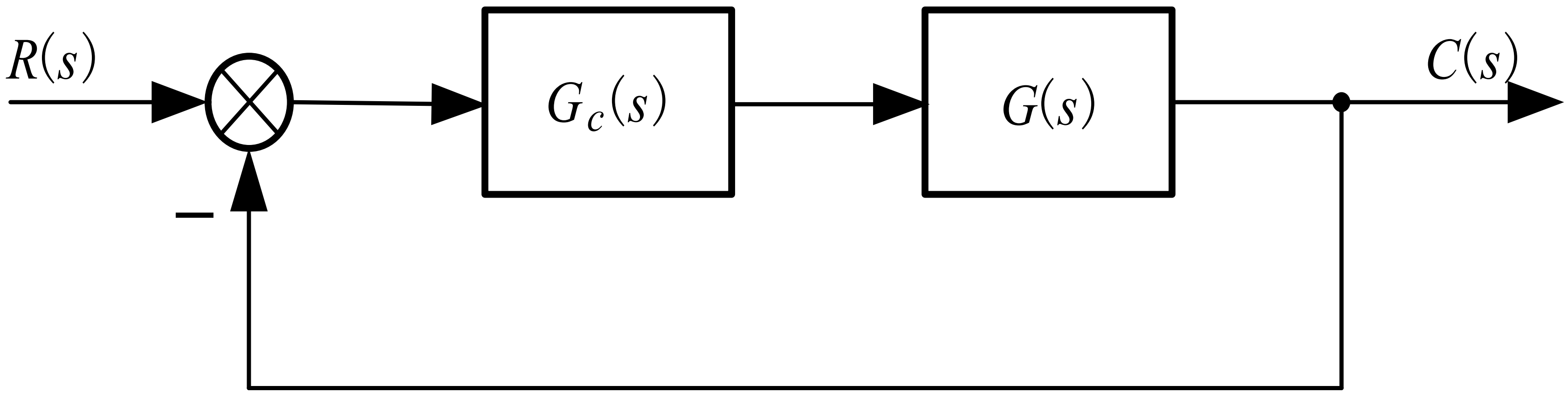
The requirement on open-loop frequency response for high frequency region: For the high-frequency region, the gain should be attenuated as rapidly as possible to minimize the effects of noise.



$$\Phi(s) = \frac{G}{1+G}$$

2. Lead Compensation

- Lead compensation essentially yields an appreciable improvement in transient response and a small change in steady-state accuracy.
- It may accentuate high-frequency noise effects.



Series compensation

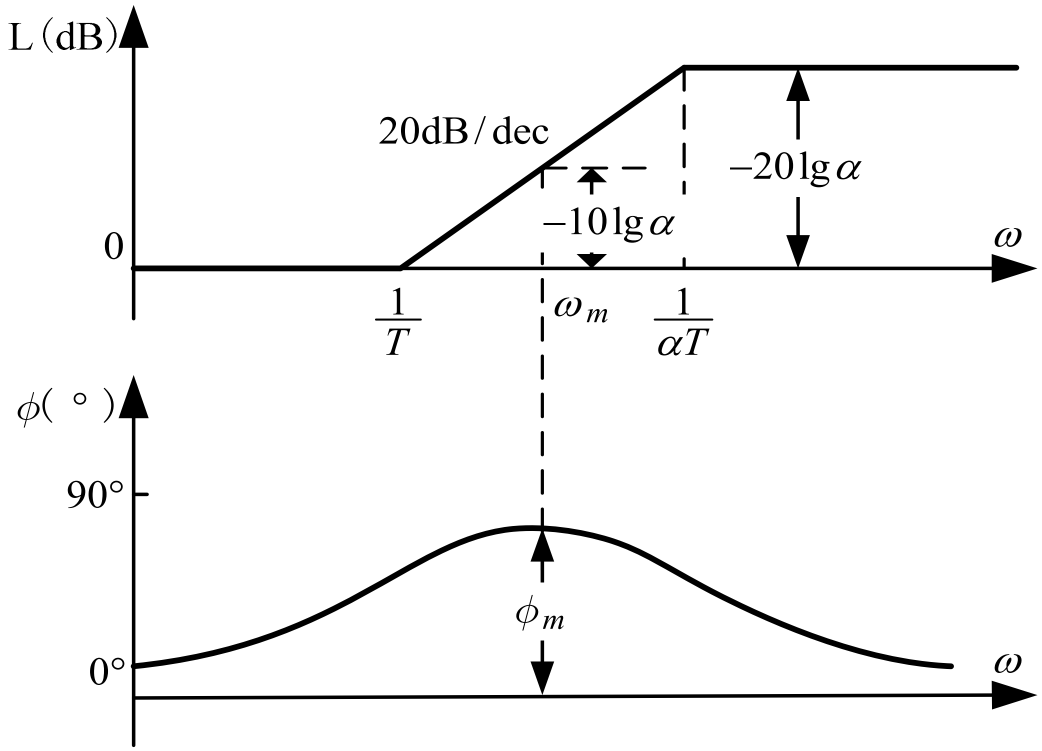
1). Mathematical model:

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\dfrac{1}{T}}{s+\dfrac{1}{\alpha T}}, \quad 0<\alpha<1$$

$$\phi(\omega) = \angle G_c(j\omega) = \tan^{-1}T\omega - \tan^{-1}\alpha T\omega$$

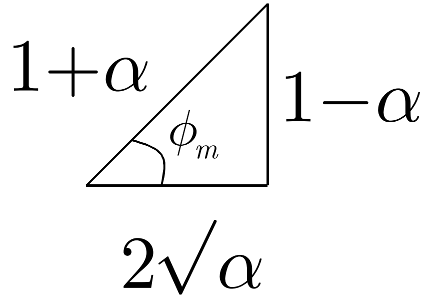
Bode diagram for

$$\frac{G_c(s)}{K_c\alpha} = \frac{Ts+1}{\alpha Ts+1}$$



where it can be calculated by letting $d\phi/d\omega=0$ that (see **Appendix**)

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$



Therefore, we have

$$20\lg\frac{|G_c(j\omega_m)|}{K_c\alpha} = -10\lg\alpha$$

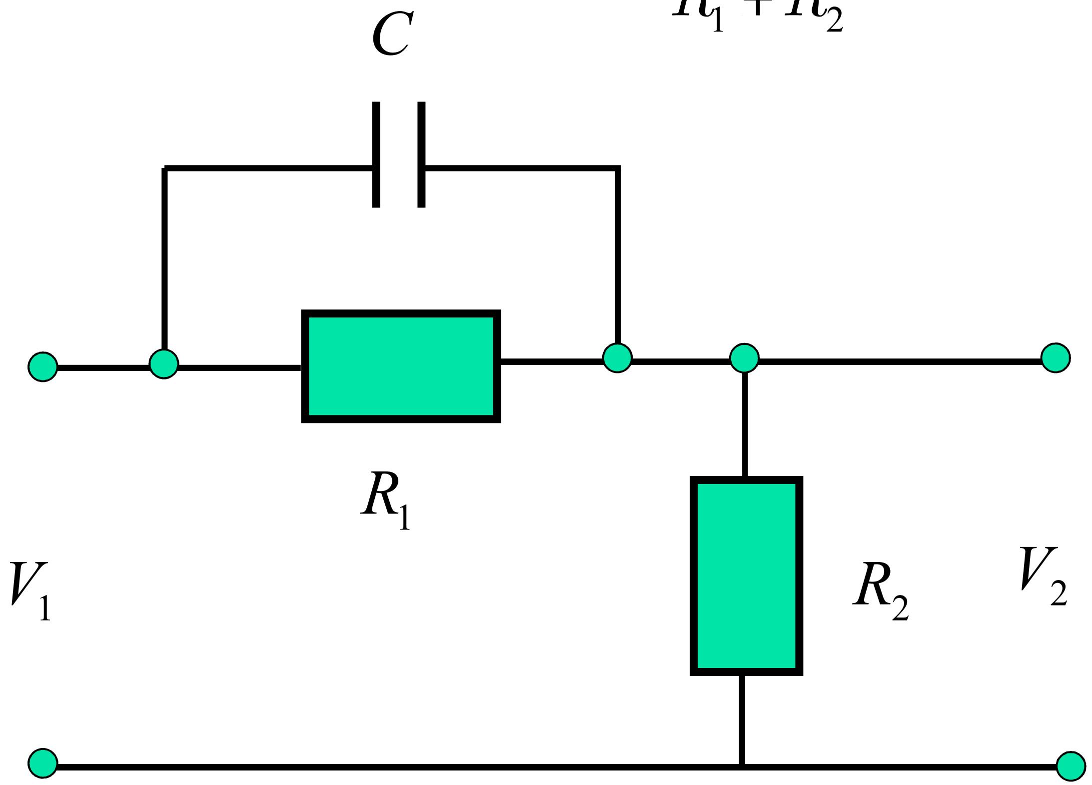
$$\tan\phi(\omega_m) = \left.\frac{T\omega(1-\alpha)}{1+\alpha T^2\omega^2}\right|_{\omega_m=\frac{1}{\sqrt{a}T}} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

which relates the maximum phase-lead angle and the value of α.

The phase-lead compensator

$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_1 + R_2} \cdot \frac{R_1 C s + 1}{\frac{R_2}{R_1 + R_2} R_1 C s + 1} = \alpha \cdot \frac{Ts + 1}{\alpha T s + 1}$$



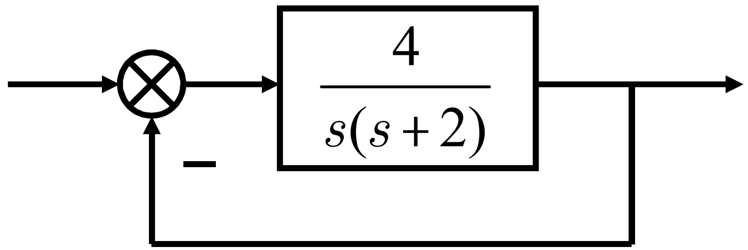
$$\alpha = \frac{R_2}{R_1 + R_2}$$

$$T = R_1 C$$

2). Lead Compensation Design Procedure

The design procedure is introduced via the following example.

Example. Consider the following system with open-loop transfer function $G(s)$:



It is desired to design a compensator for the system so that the static velocity error constant K_v is 20 sec^{-1}, the phase margin is at least 50^0, and the gain margin is at least 10 dB.

Step 1: Determine the gain K to meet the steady-state performance specification.
In this example, it is required that K_v=20 sec^{-1}. Hence from

$$K\frac{4}{s(s+2)} = \frac{2K}{s(0.5s+1)}$$

we obtain that K=10.

Step 2: Let

$$G_1(s) = KG(s)$$

Using the gain K thus determined, draw a Bode diagram of $G_1(j\omega)$, the gain adjusted but uncompensated system. Evaluate the phase margin.

In this example,

$$G_1(s) = KG(s) = \frac{20}{s(0.5s+1)}$$

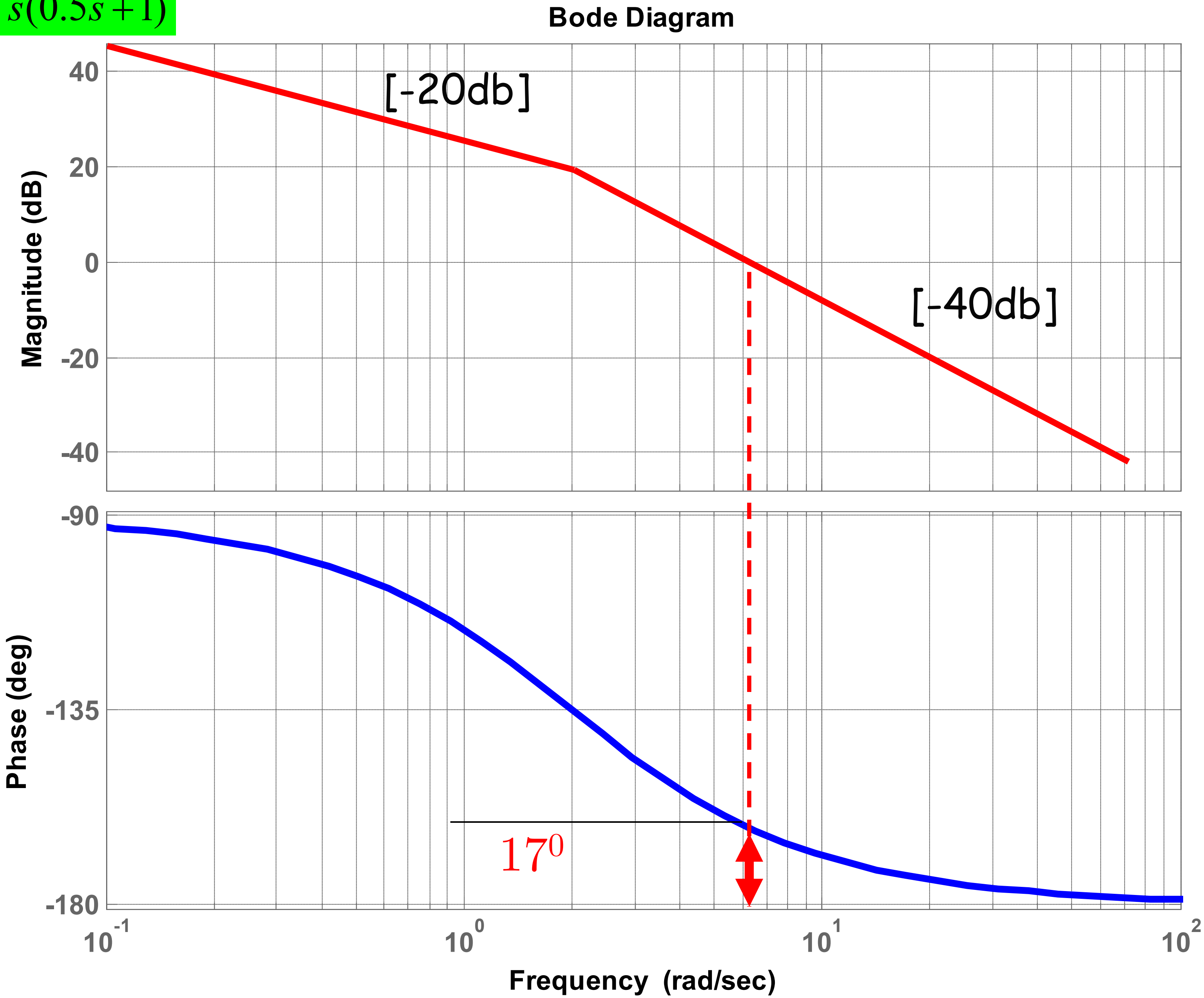
Draw the Bode diagram of $G_1(j\omega)$, from which it can be evaluated that

$$L(\omega_c) = 20\lg\left|\frac{20}{0.5\omega_c^2}\right| = 0 \;\Rightarrow\; \omega_c = 6.32$$

$$\gamma = 180^\circ + \phi(\omega_c) = 180^\circ - 90^\circ - \tan^{-1}(0.5\omega_c) = 17^\circ$$

A phase margin of 17^0 implies that the system is quite oscillatory. Thus, satisfying the specification on the steady state yields a poor transient-response performance, and therefore, **a phase lead compensator** should be designed.

$$G(s) = \frac{20}{s(0.5s+1)}$$



Step 3: Determine the necessary phase-lead angle to be added to the system. Add an additional 5^0 to 12^0 to the phase-lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin.

In this example:

$$\phi_m = 50^\circ - 17^\circ + 5^\circ = 38^\circ$$

Let the phase lead compensator be of the form

$$G_c(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1}$$

Then the open loop transfer function becomes

$$G_c(s)G(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1} G(s)$$

Let

$$K = K_c\alpha$$

$$G_1(s) = KG(s)$$

where K has been obtained as 10. Then,

$$G_c(s)G(s) = \frac{Ts+1}{\alpha Ts+1} G_1(s)$$

Step 4: 1)Determine the attenuation factor α by using the formula $(1-\alpha)/(1+\alpha)=\sin\phi_m$ (the main idea is to use ϕ_m to compensate for the lack of the phase margin).

In this example, let

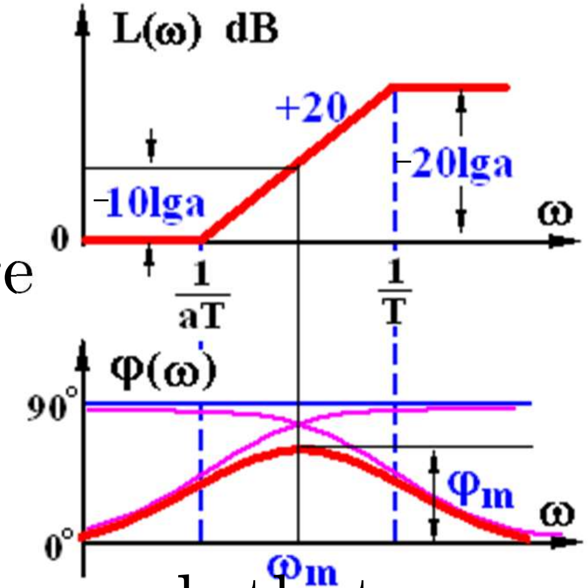
$$\phi_m = \arcsin\frac{1-\alpha}{1+\alpha} = 38^\circ \quad \Rightarrow \quad \frac{1-\alpha}{1+\alpha} = \sin 38^\circ = 0.62 \;\Rightarrow\; \alpha=0.24$$

Hence,

$$-10\lg\alpha = 6.2\,\text{dB}$$

which should correspond to ω_m, where

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

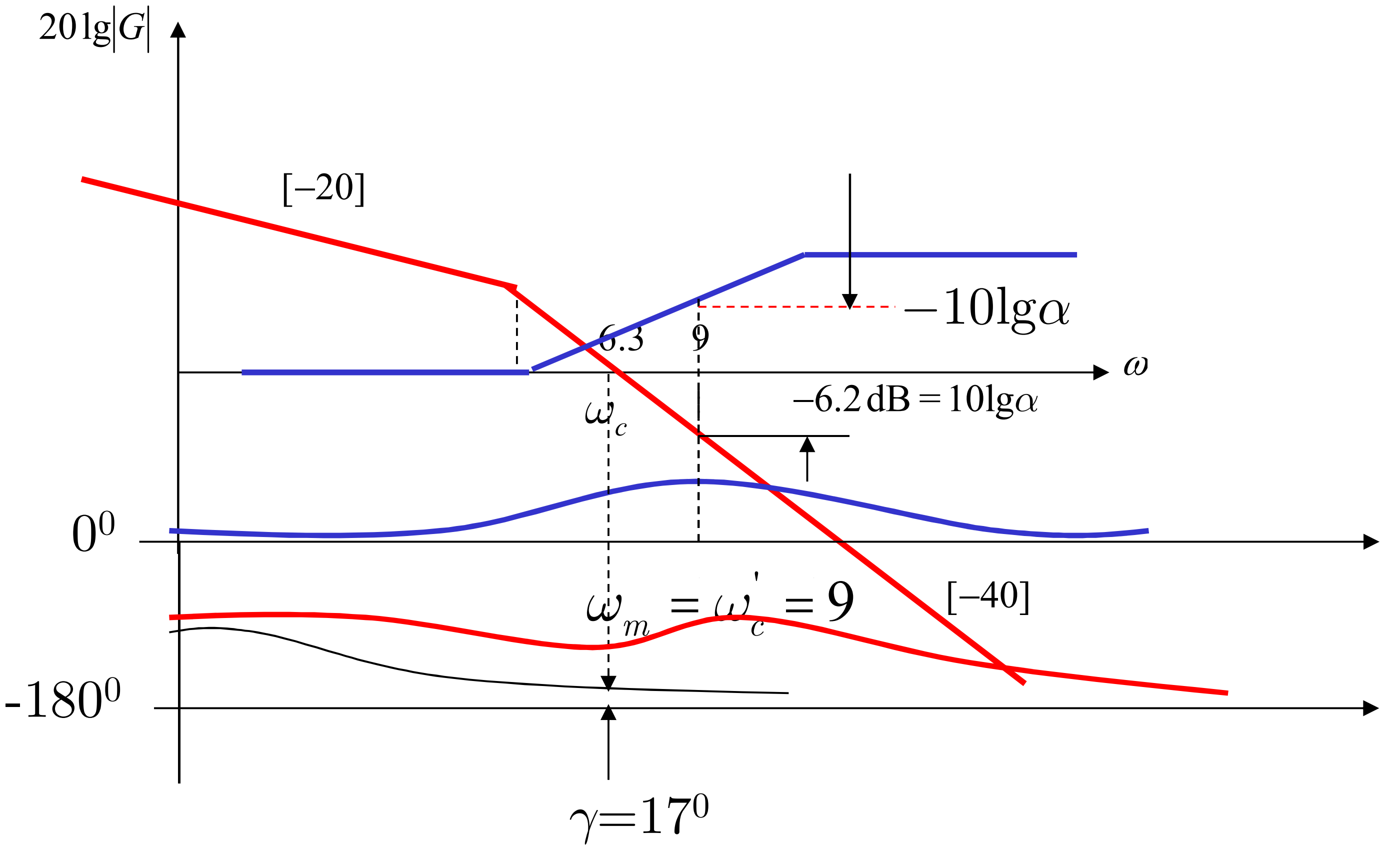


2) Select new gain crossover frequency such that

$$\omega_c' = \omega_m$$

which can be found from the point ω at which $20\lg|G_1(j\omega)|=10\lg\alpha=-6.2$ dB, that is, $\omega_c'=9$ rad/sec:

$$\omega_m = \frac{1}{T\sqrt{\alpha}} = \omega_c' = 9$$



Step 5. Determine the corner frequencies of the lead compensator.

In this example, with α=0.24, we obtain

$$\frac{1}{T} = 4.41 \qquad \frac{1}{\alpha T} = 18.4$$

Step 6. Using the value of K determined in step 1 and that of α determined in step 4, calculate constant K_c from

$$K_c = K / \alpha$$

In this example,

$$K = K_c \alpha = 10 \Rightarrow K_c = 10 / 0.24 = 41.7$$

Therefore, the compensator is

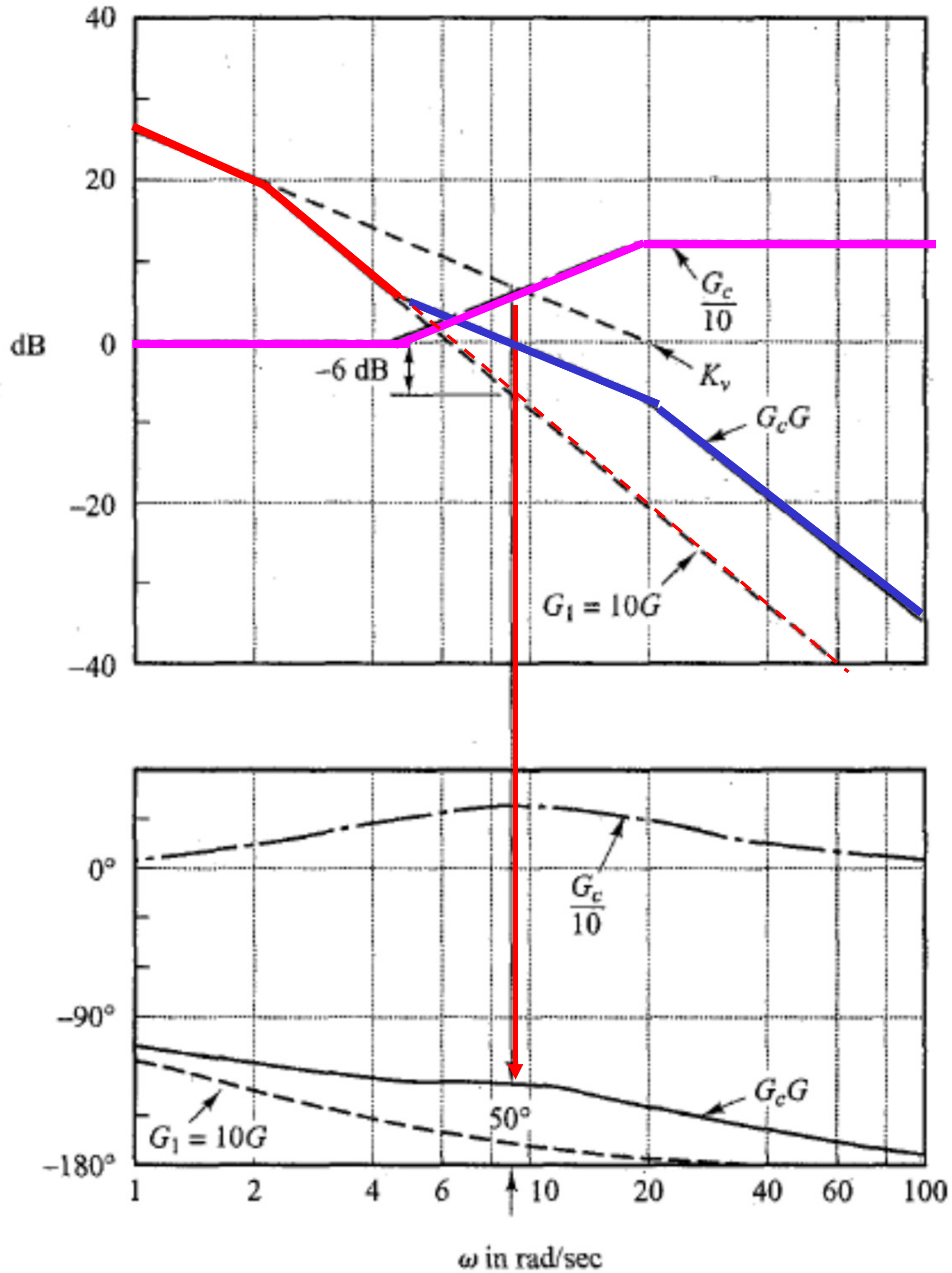
$$G_c(s) = 41.7\frac{s+4.41}{s+18.4}$$

$$= 10\frac{0.227s+1}{0.054s+1}$$

where

$$K = K_c\alpha = 10$$

$$\frac{G_c(s)}{10} = \frac{0.227s+1}{0.054s+1}$$



Step 7. Check the gain margin to be sure it is satisfactory. If not, repeat the design process by modifying the pole-zero locations of the compensator until a satisfactory result is obtained.

In this example, since after the compensation, the phase angle never crosses -180^0 line, the gain margin always satisfies the requirement. This completes the design.

Summary of the design procedure

1. Determine ϕ_m, the necessary phase-lead angle to be added to the system:

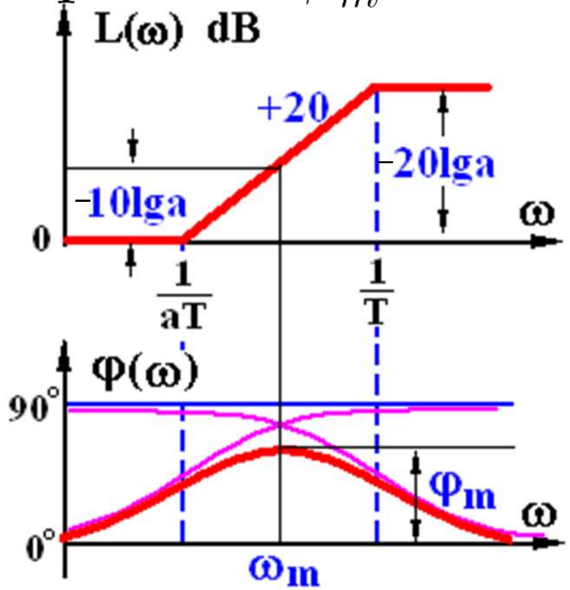
$$\phi_m = \gamma' - \gamma + (5^0 \sim 12^0)$$

where γ' is the required phase margin.

2. Determine α and $-10\lg\alpha$. Let the maximum phase angle of the compensator be equal to ϕ_m:

$$\phi_m = \arcsin\frac{1-\alpha}{1+\alpha} \Rightarrow \alpha$$

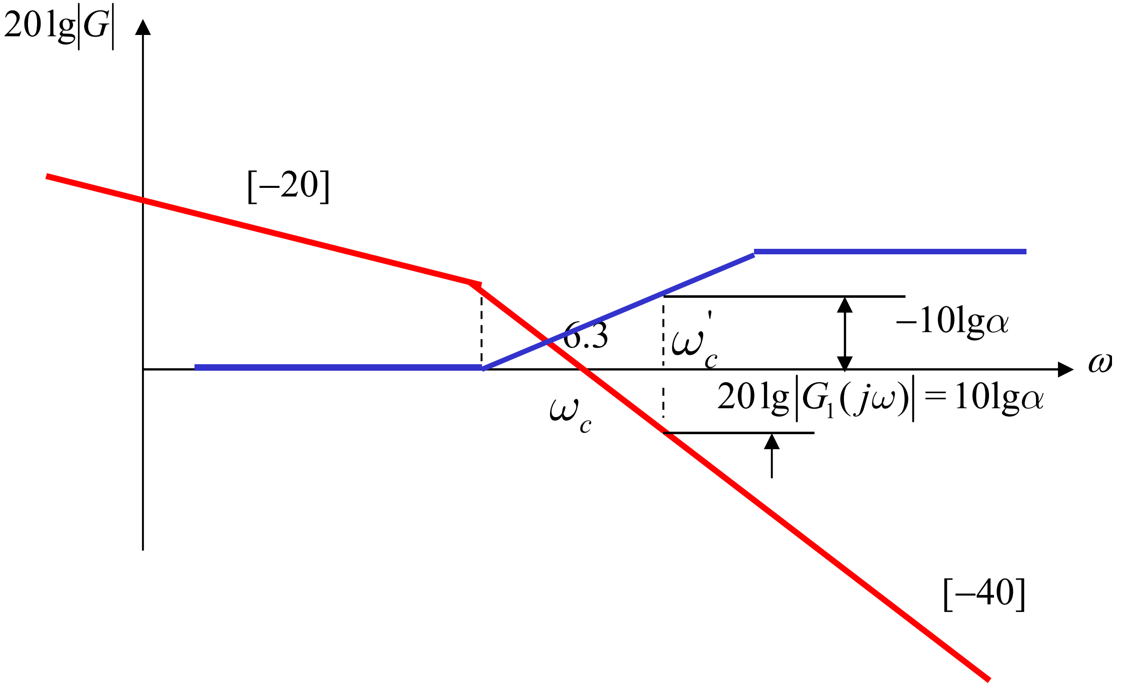
$$\Rightarrow -10\lg\alpha$$



3. Determine ω_c' such that

$$20\lg\left|G_1(j\omega_c)\right| = 10\lg\alpha$$

which can be obtained directly from Bode plot:

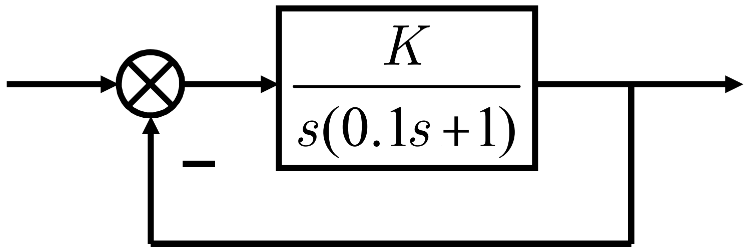


4. Determine the corner frequencies by using

$$\omega_c' = \omega_m = \frac{1}{T\sqrt{\alpha}}$$

Remark: If the required ω'_c is given, the design procedure can be simplified as follows.

Example. Consider the following system with open-loop transfer function $G(s)$:



It is desired to design a compensator for the system so that the static velocity error $e_{ss} \le 0.01$, $\gamma \ge 45^0$, and $\omega_c \ge 40$.

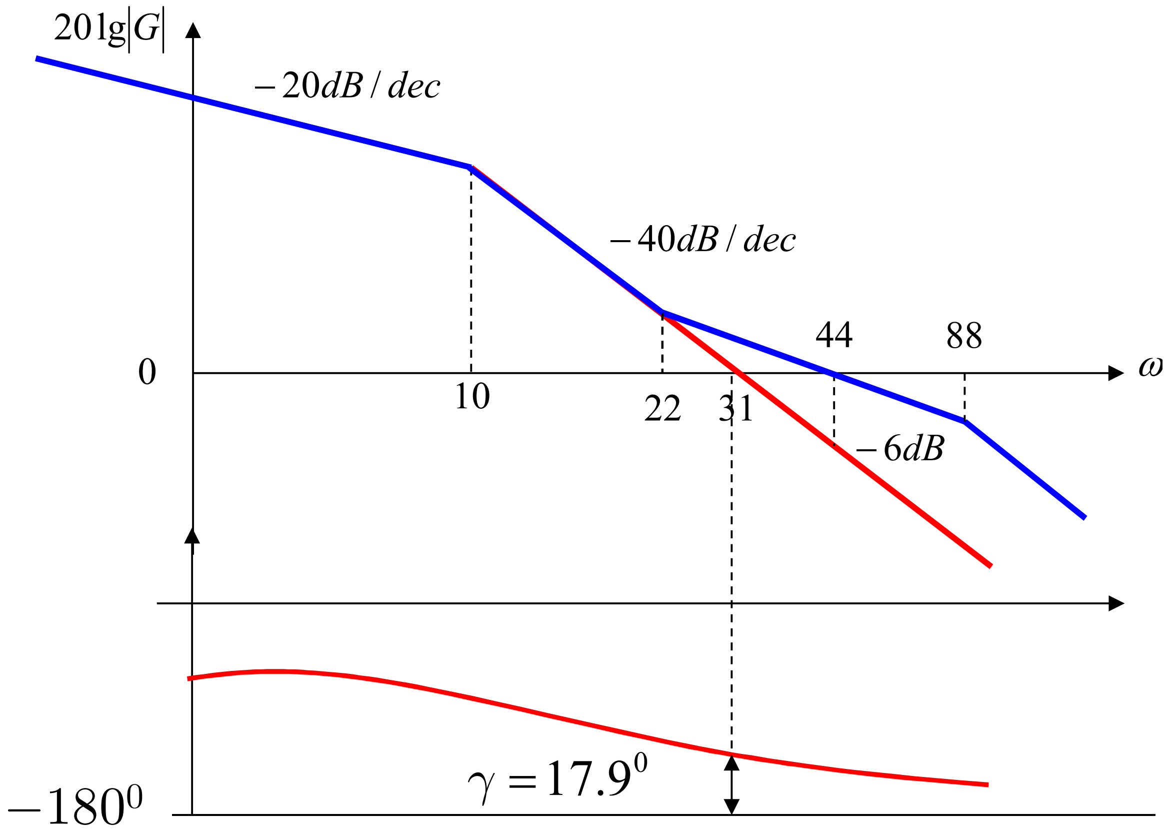
Step 1: Determine the gain K to meet the steady-state performance specification.
In this example, it is required that $e_{ss} \leq 0.01$. Hence $K \geq 100$.

Step 2: Using the gain K thus determined, draw a Bode diagram of $G\ (j\omega)$, the gain adjusted but uncompensated system. Evaluate the phase margin.

In this example, with $K=100$,

$$G(s) = \frac{100}{s(0.1s+1)}$$

From its Bode diagram (or by calculation),



$$\omega_c = 31 \mathrm{rad} / s$$

$$\gamma = 17.9^0$$

Obviously, a lead compensator is necessary.

Step 3: Determine the new crossover frequency. Since it is required that $\omega_c' = \omega_m \geq 40$, we choose

$$\omega_c' = 44 \text{ rad/s}$$

Step 4: Evaluate

$$20\lg|G(j\omega_c')|$$

of the uncompensated system. In this example,

$$20\lg|G(j\omega_c')| = -6 \text{ dB}$$

Step 5: Determine the corner frequencies of the compensator. From

$$-10\lg\alpha = 6 \text{ dB}$$

we obtain

$$\alpha = 1/4$$

Let

$$\omega_m = \omega_c' = \frac{1}{\sqrt{\alpha}T} = 44$$

we obtain

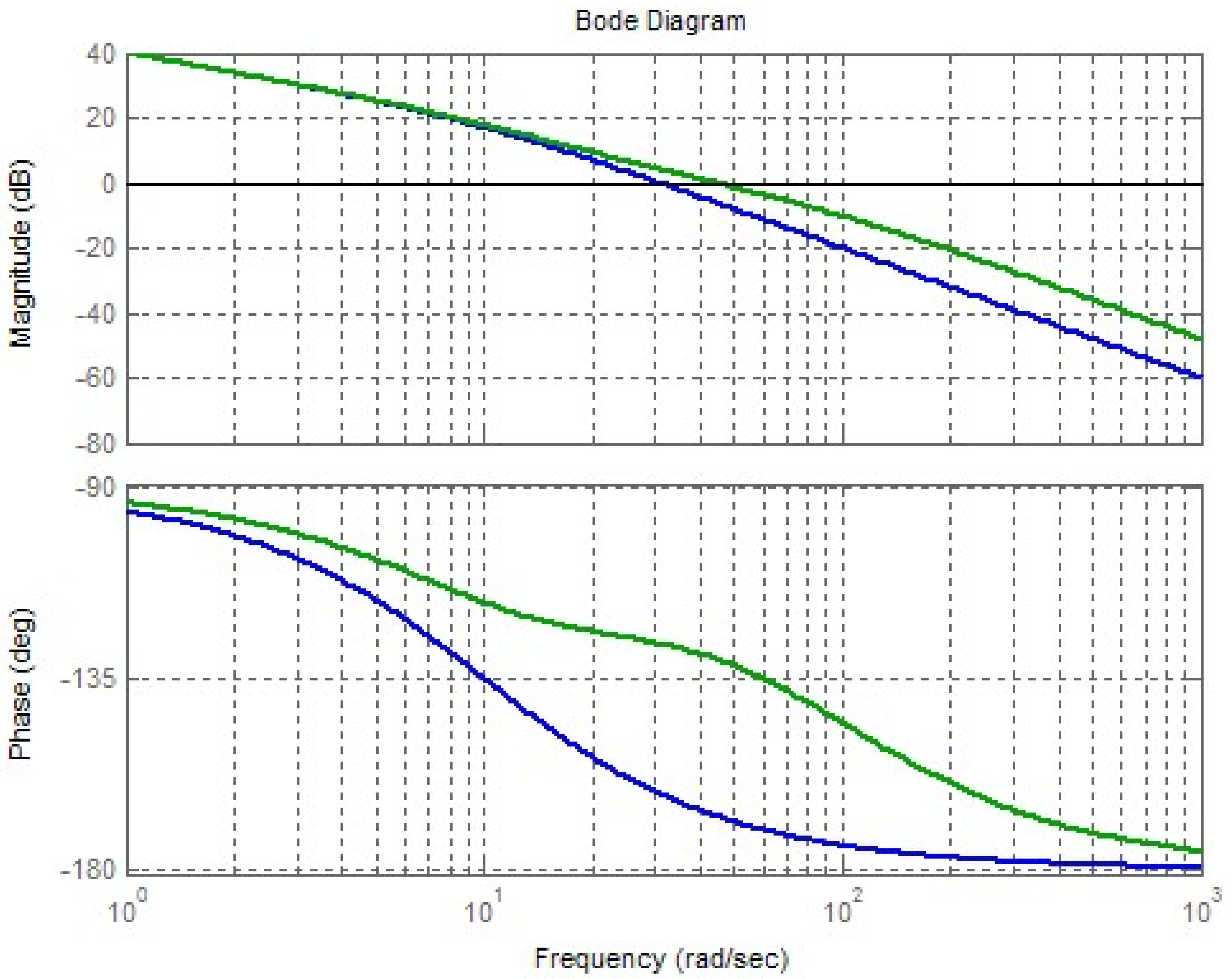
$$T = 0.04544$$

Hence,

$$\frac{G_c(s)}{K_c\alpha} = \frac{Ts+1}{\alpha Ts+1} = \frac{0.04544s+1}{0.01136s+1}$$

This ends the design.

Bode diagrams for the system before and after the lead compensation by using Matlab.



3. Lag Compensation

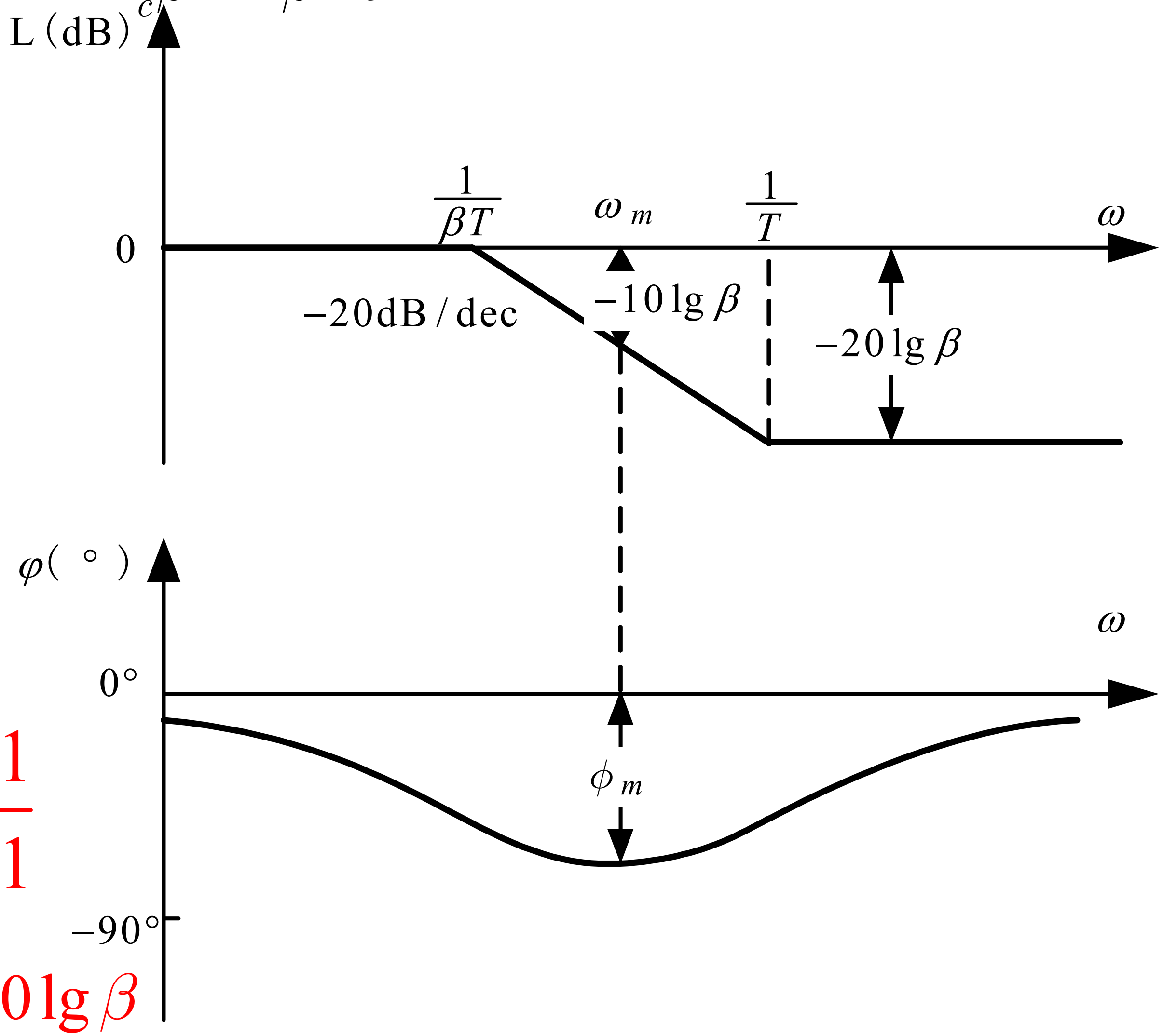
The primary function of a lag compensator is to provide attenuation in high frequency range to give a system sufficient phase margin.

1). Mathematical model:

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}, \quad \beta > 1$$

$$\phi(\omega) = \angle G_c(j\omega) = \tan^{-1} T\omega - \tan^{-1} \beta T\omega$$

Bode diagram for $\dfrac{G_c(s)}{K_c\beta} = \dfrac{Ts+1}{\beta Ts+1}, \beta > 1$



From which we can obtain that

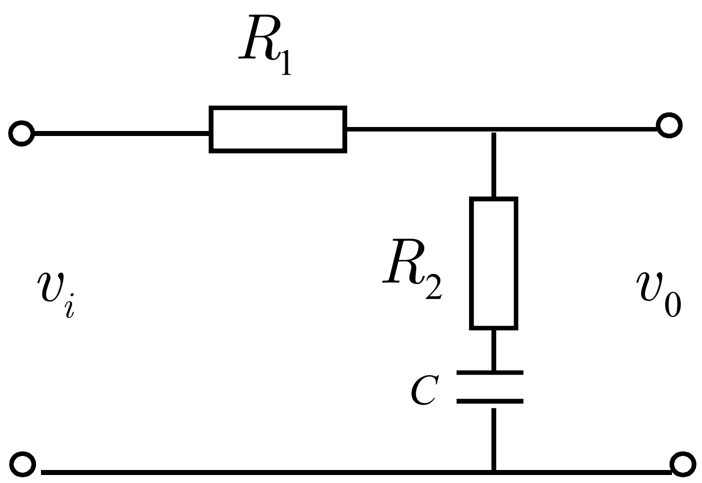
$$\omega_m = \frac{1}{T\sqrt{\beta}}$$

$$\phi(\omega_m) = \arcsin\frac{\beta-1}{\beta+1}$$

$$20\lg\left|\frac{G_c(j\omega)}{K_c\beta}\right|_{\omega=\frac{1}{T}} = -20\lg\beta$$

The phase-lag compensator

The phase-lag compensation transfer function can be obtained by using the network shown in the following Figure:



$$G_c(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

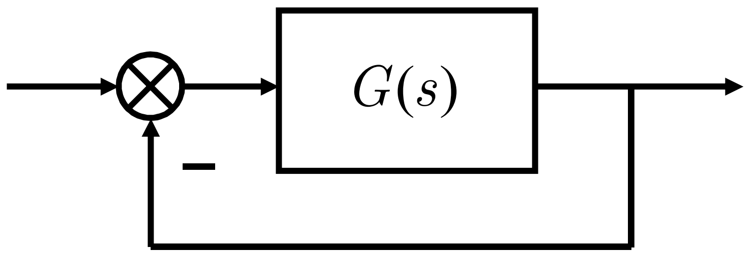
$$\beta = \frac{R_1 + R_2}{R_2} \qquad T = (R_1 + R_2)C$$

$$G_c(s) = \frac{Ts + 1}{\beta Ts + 1}$$

2). Lag Compensation Design Procedure

The design procedure is introduced via the following example.

Example. Consider the following system with open-loop transfer function $G(s)$:



$$G(s) = \frac{1}{s(s+1)(0.5s+1)}$$

It is desired to compensate the system so that the static velocity error constant K_v is 5 sec^{-1}, the phase margin is at least 40° and the gain margin is at least 10 dB.

Step 1: Determine gain K to satisfy the requirement on the given static velocity error constant.

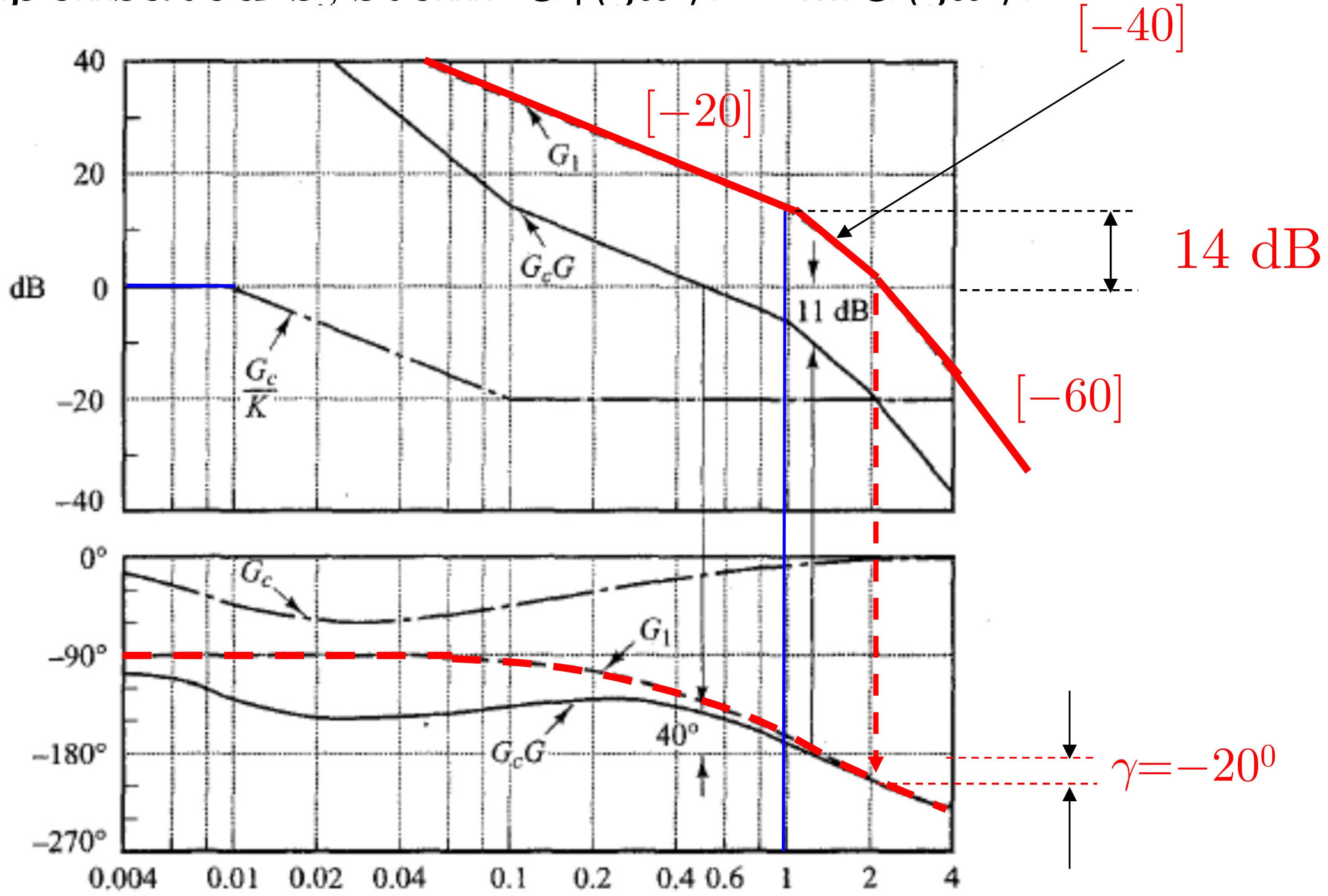
In this example, from

$$G_1(s) = K\frac{1}{s(s+1)(0.5s+1)}$$

we obtain that

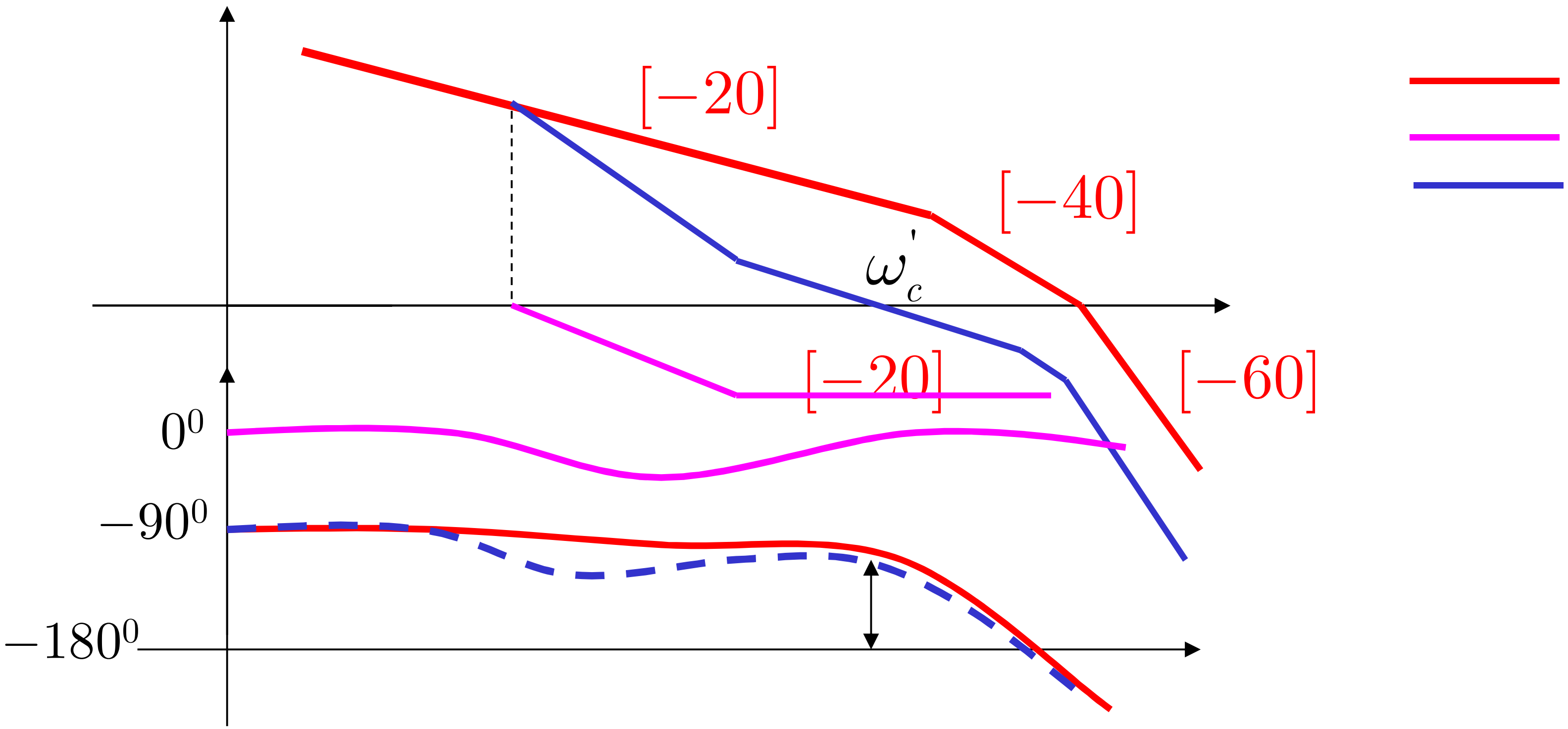
$$K_v = K = 5$$

Draw the Bode diagram of the gain-adjusted but uncompensated system $G_1(j\omega) := KG(j\omega)$.



$\gamma = -20^0$ implies that the gain-adjusted but uncompensated system is unstable.

Since in this example, there is no requirement about ω_c, it is possible to use a **lag compensator** to satisfy the required performance specifications.

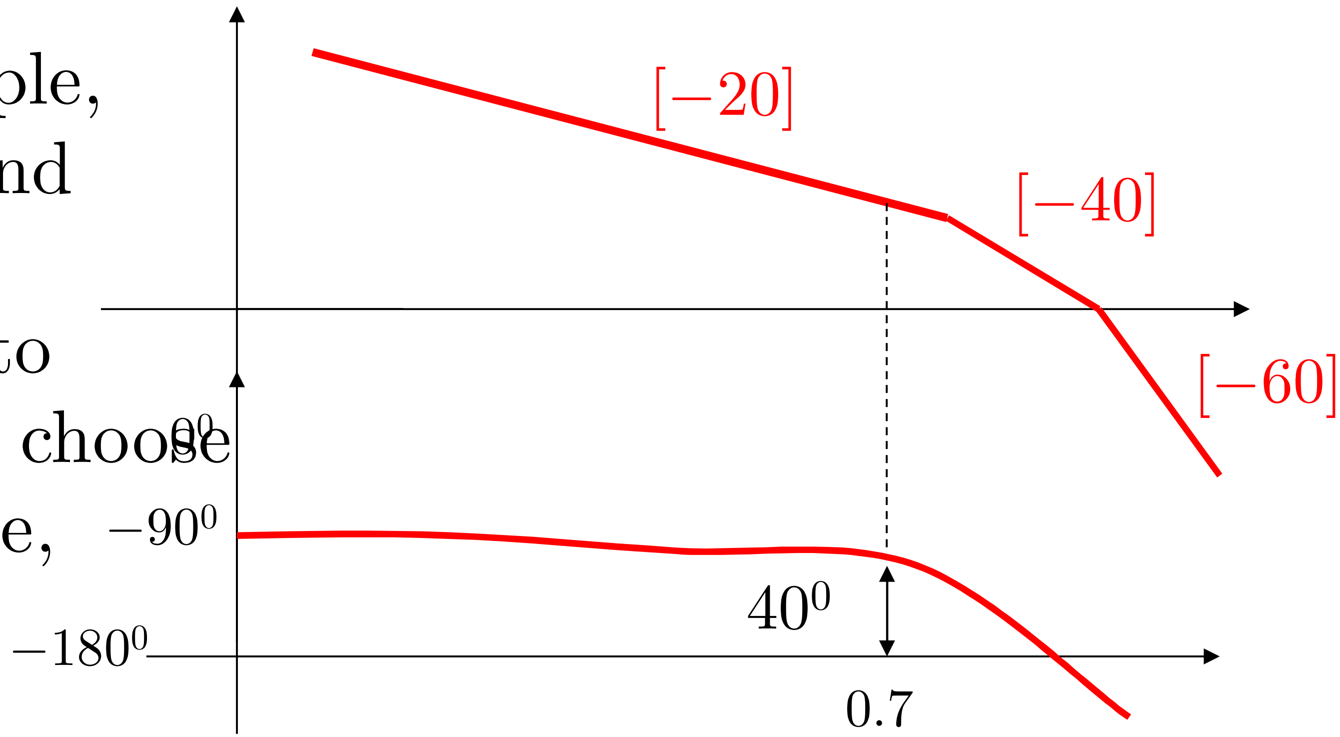


Step 2: Find the new crossover frequency point ω_c' such that

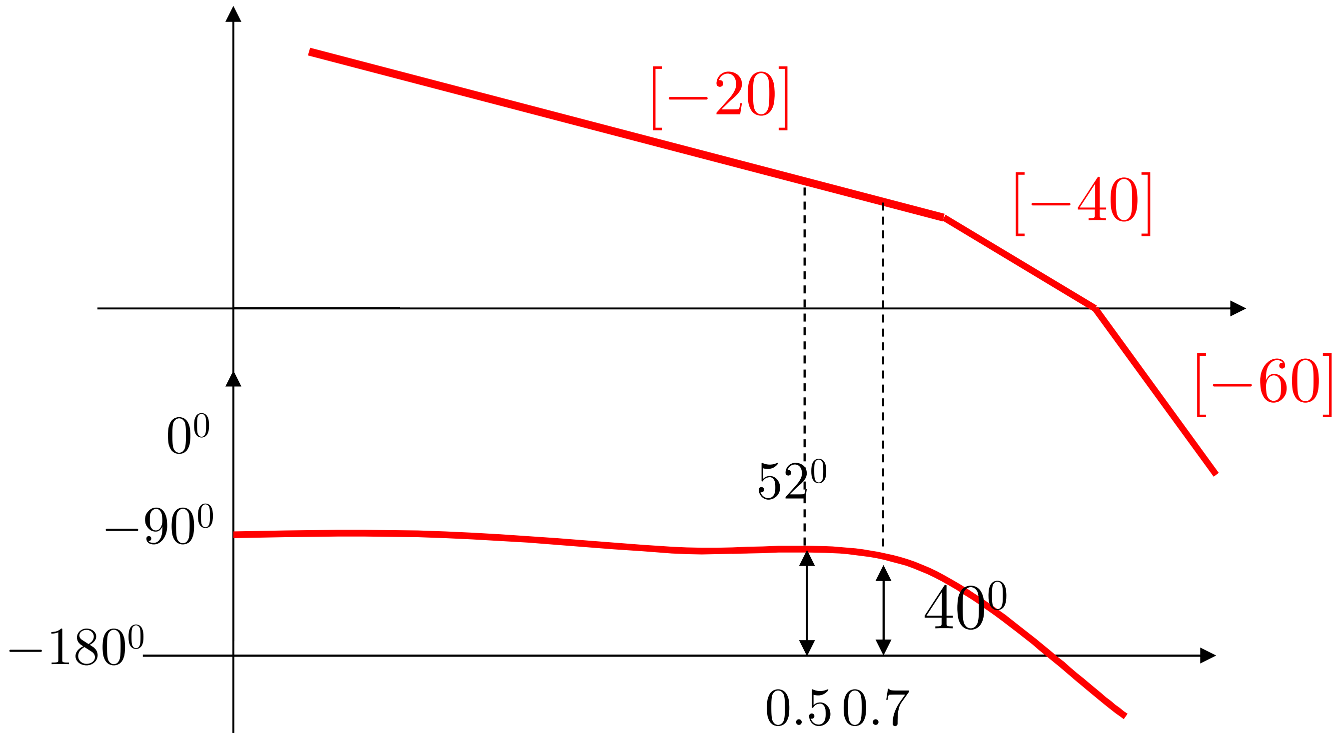
$$\gamma' = \gamma^* + \delta = \gamma^* + (5^0 \sim 12^0)$$

where γ^* is the required phase margin and the additional 5^0 to 12^0 is used to compensate for the phase lag of the lag compensator.

In this example, it can be found that $\gamma^*=40^0$ corresponds to 0.7rad/s. We choose $\delta=12^0$. Hence, $\gamma'=52^0$.



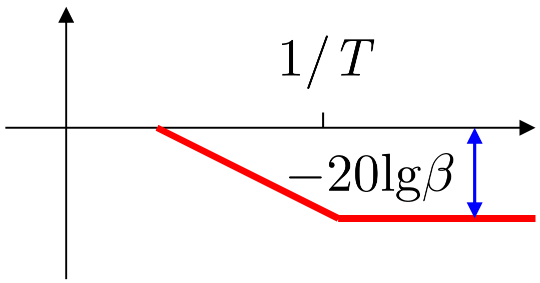
With γ'=52^0, it can be found from the Bode diagram that γ' corresponds to 0.5 rad/s. Therefore, we choose ω_c'=0.5 rad/s.



Step 3: Choose the corner frequency $\omega = 1/T$ (corresponding to the zero of the lag compensator) 1 octave to 1 decade **below** the new gain crossover frequency.

In this example, we choose

$$\omega = \frac{1}{T} = 0.1$$



Step 4: Determine β such that

$$-20\lg\beta + 20\lg\left|G_1(j\omega_c^{'})\right| = 0$$

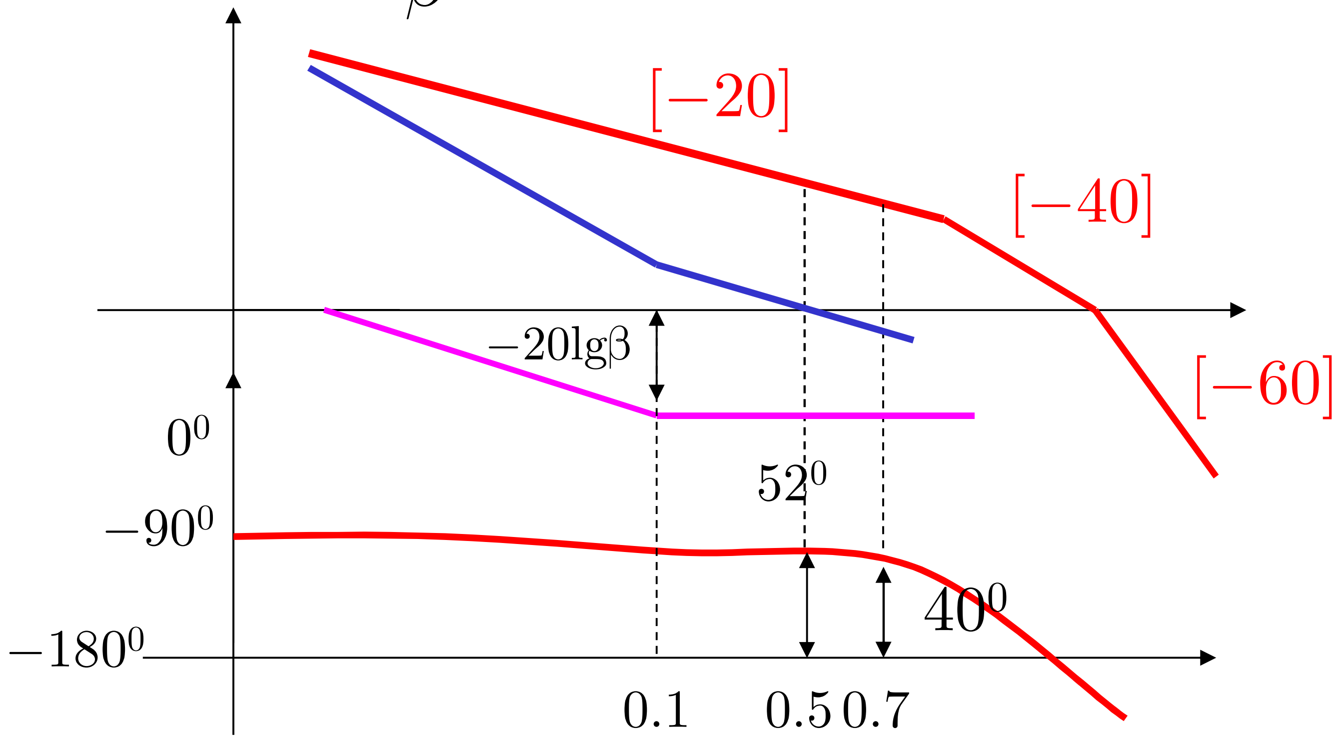
which will bring the magnitude curve down to 0 dB at this new gain crossover frequency. Then determine the corner frequency $\omega = 1/\beta T$.

In this example, it can be measured that

$$20\lg\left|G_1(j\omega_c^{'})\right| = 20\text{ dB}$$

Therefore,

$$20\lg\frac{1}{\beta} + 20\lg 10 = 0 \Rightarrow \beta = 10$$



Since we have obtained that β=10 and

$$\omega = \frac{1}{T} = 0.1$$

the corner frequency $\omega = \dfrac{1}{\beta T} = 0.01$

Step 5: Finally, let

$$K = K_c\beta \Rightarrow K_c = \frac{K}{\beta}$$

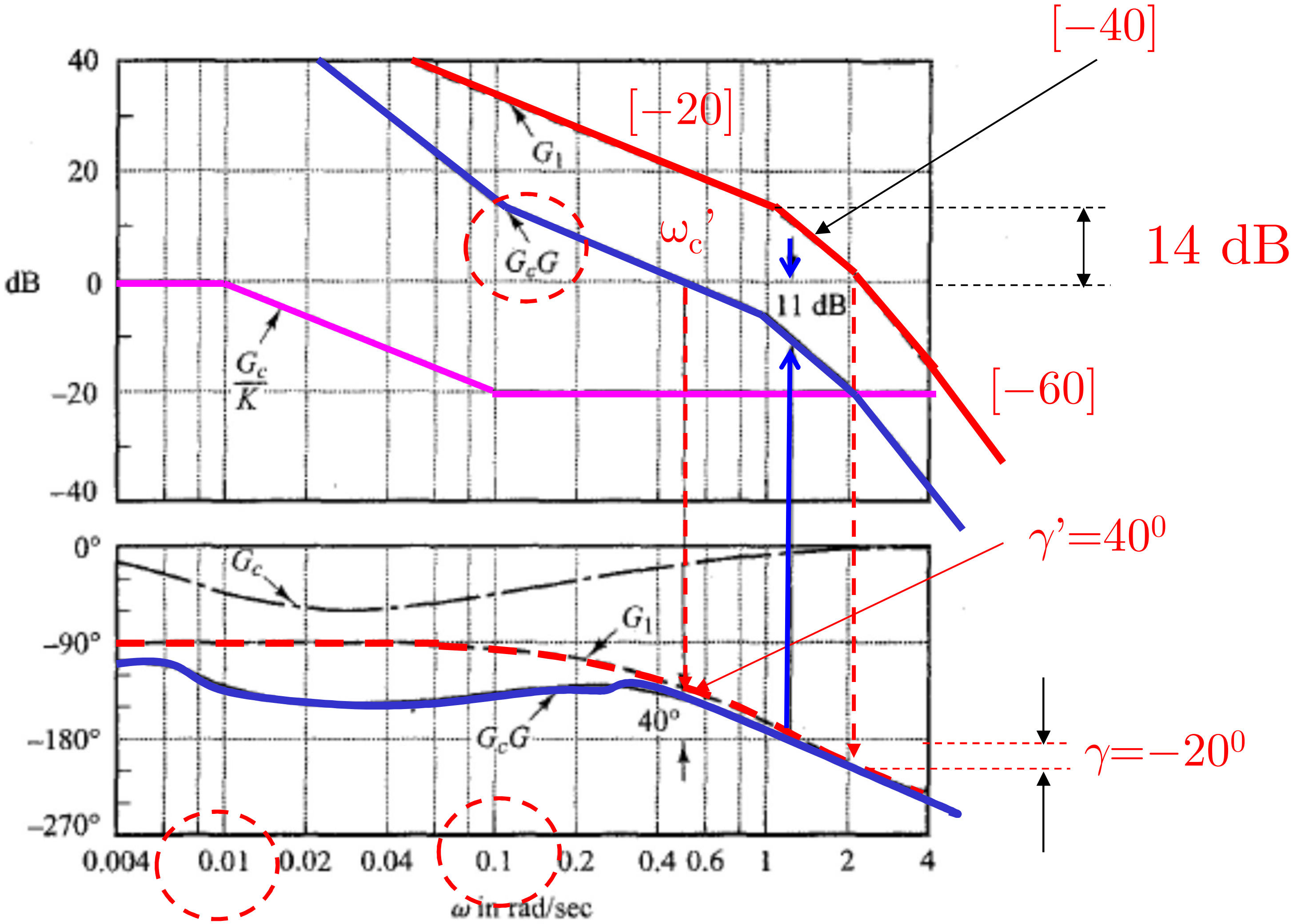
In this example,

$$K = K_c\beta = 5 \Rightarrow K_c = \frac{5}{10} = 0.5$$

Therefore, the compensator is

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} = 5\frac{10s+1}{100s+1}$$

$$G_c(s)G(s) = \frac{5(10s+1)}{s(s+1)(0.5s+1)(100s+1)}$$



Phase margin=40^0, gain margin=11dB.

Example. Consider a unity-feedback system with its open-loop transfer function as

$$G(s) = \frac{1}{s(0.5s+1)}$$

It is required that the steady-state error for unit ramp input be 0.05 and the phase margin of the system be at least 45^0.

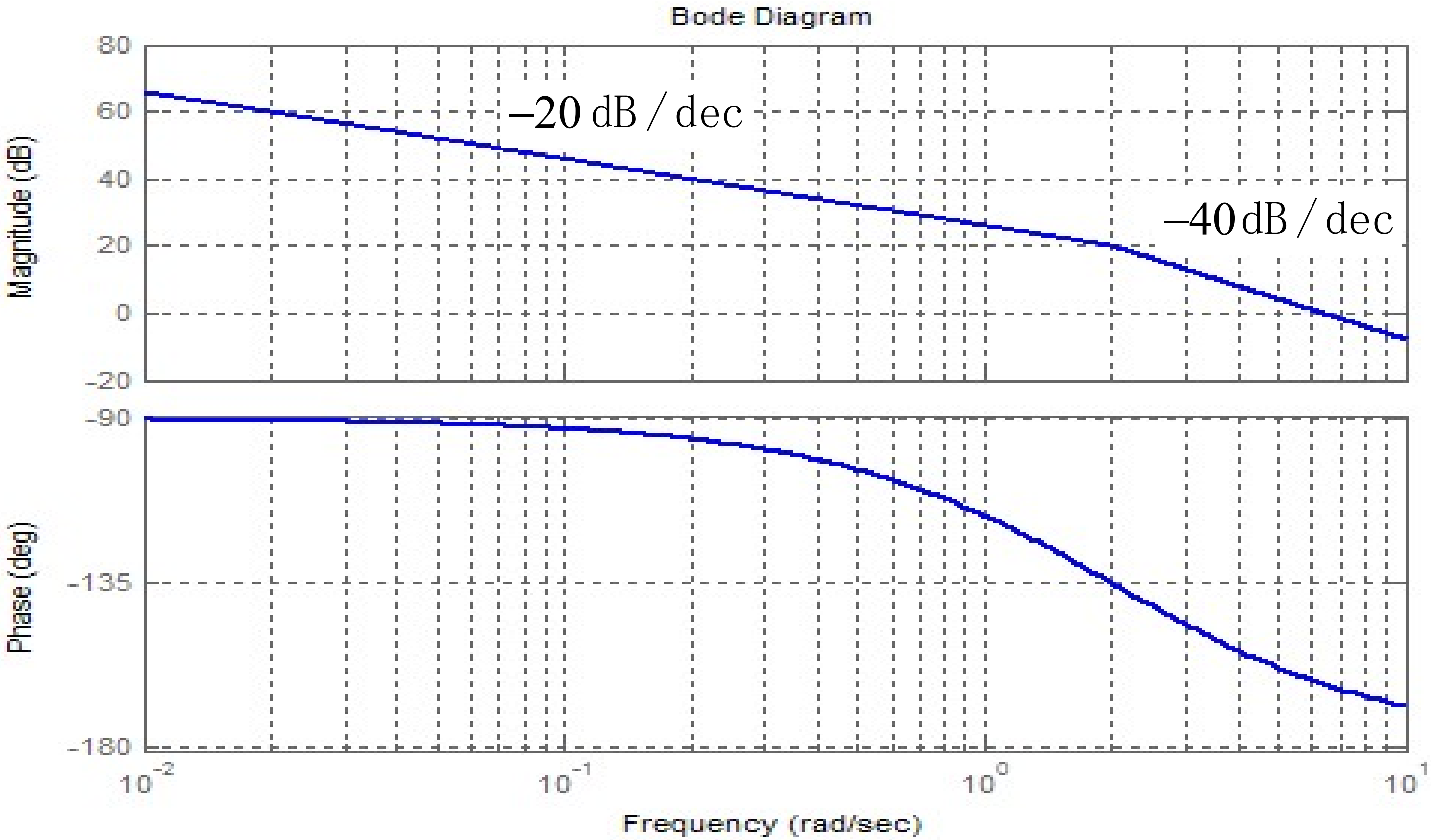
Step 1: According to the requirement of the steady-state error, we need that

$$G_1(s) = K\frac{1}{s(0.5s+1)}$$

with

$$K = K_v = 20$$

Then, plotting the Bode diagram of the gain-adjusted but uncompensated transfer function yields



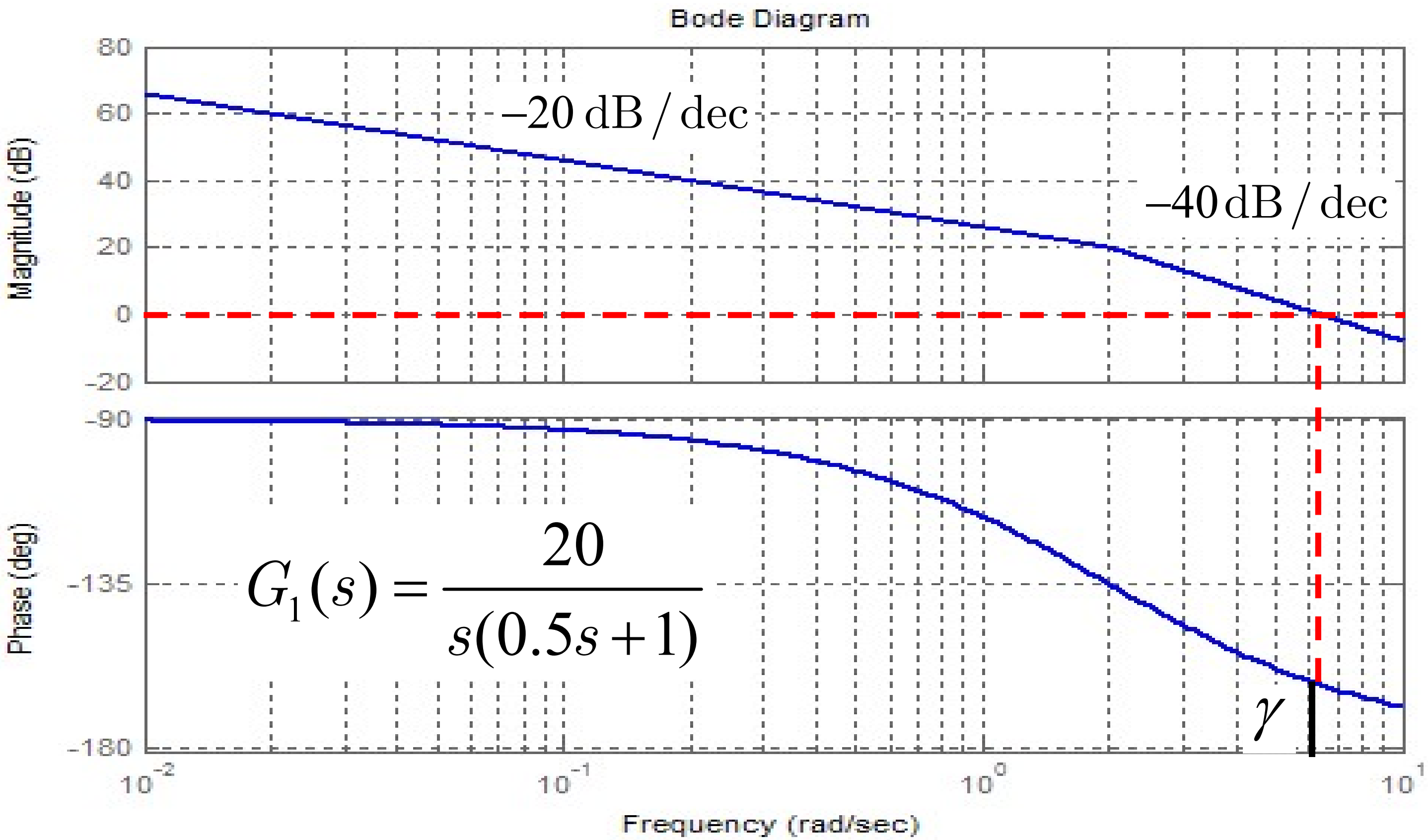
$$G_1(s) = \frac{20}{s(0.5s+1)}$$

from which it can be obtained that

$$\omega_c = 6.3\,\mathrm{rad/s}$$

$$\phi(\omega_c) = -90^0 - \tan^{-1}(0.5\omega_c) = -162^0$$

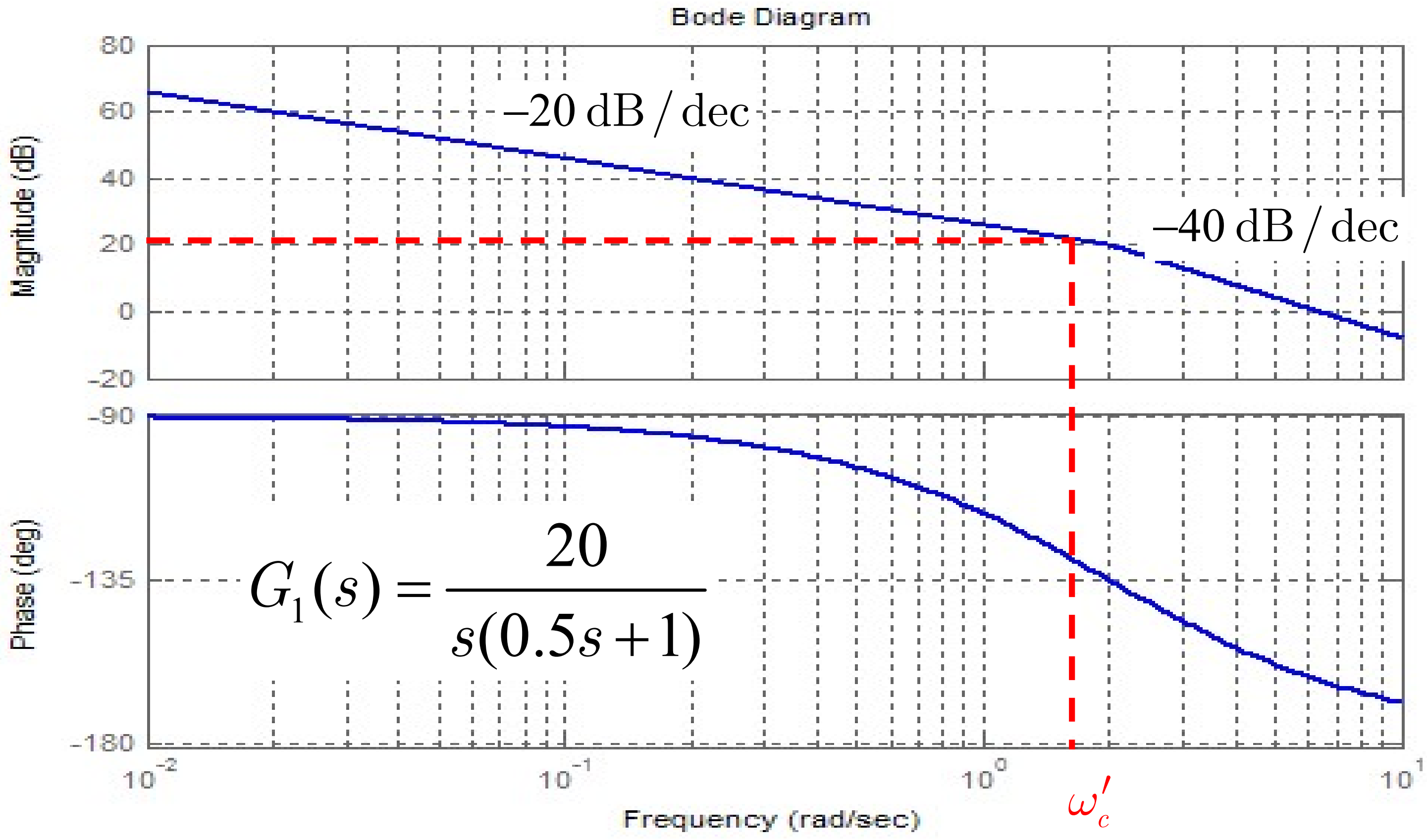
Therefore, the phase margin is $\gamma = 18^0$.



Step 2: With γ'=50^0 (where an additional 5^0 is given to compensate for the phase-lag), we obtain

$$\omega_c' = 1.5\text{rad}/\text{s} \qquad \phi(\omega_c') = -130^0$$

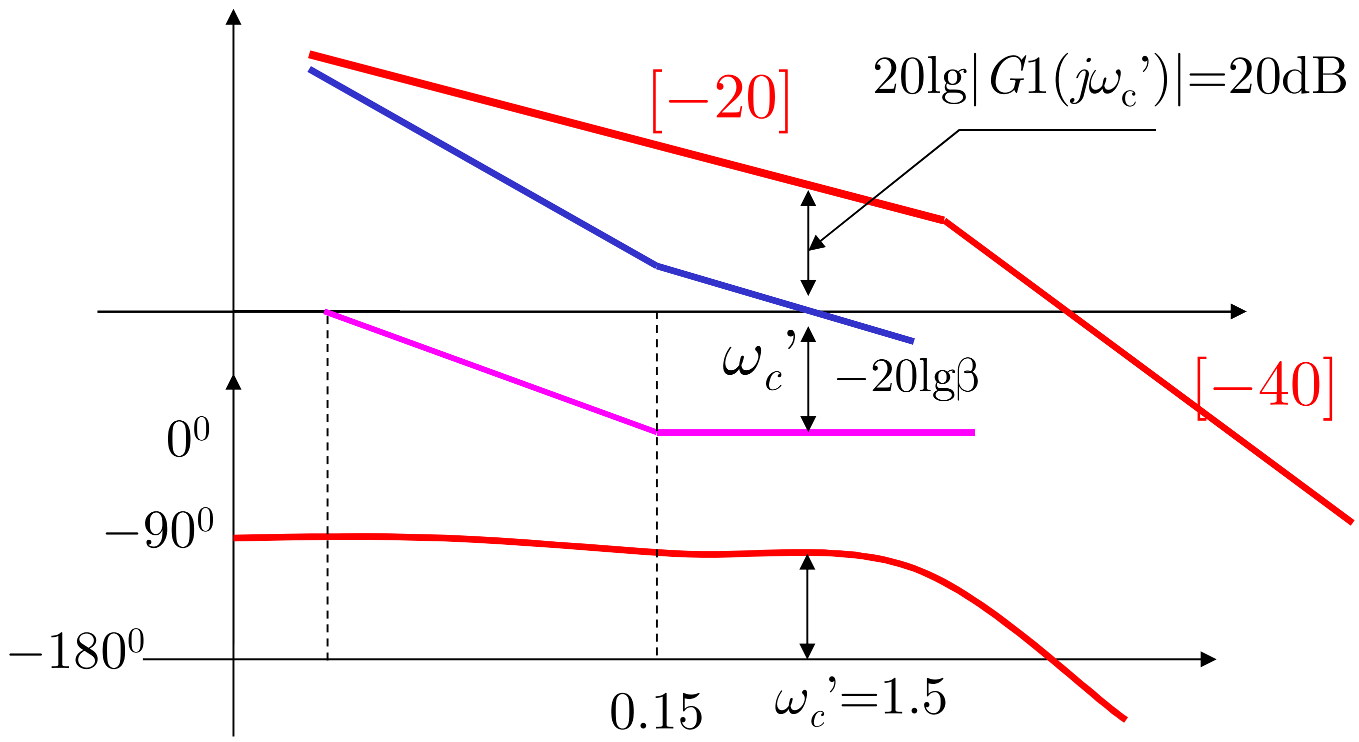
at which it can be found that 20lg$|G_1(j\omega_\text{c}')|$=20dB.



To make ω_c' be the new crossover frequency, the following equation should be satisfied

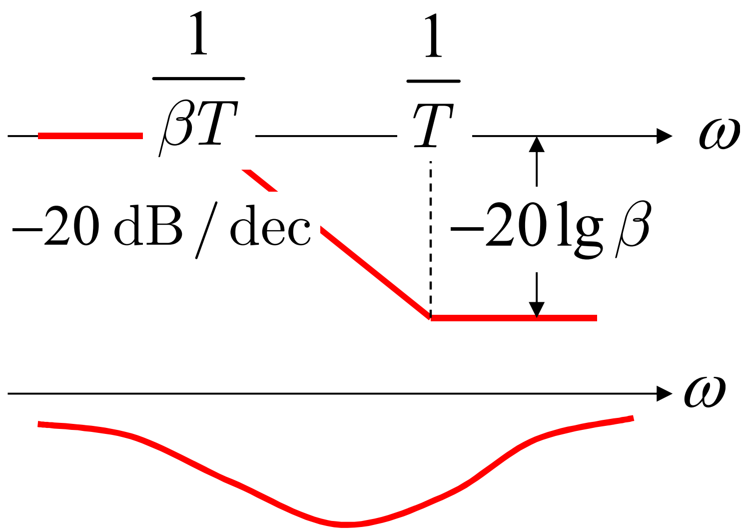
$$-20\lg\beta = -20\lg\left|G_1(j\omega_c^{'})\right| = -20\text{ dB}$$

$$\Rightarrow \beta = 10$$



Step 3: Determine corner frequencies of the compensator:

$$\frac{G_c(s)}{K_c\beta} = \frac{Ts+1}{\beta Ts+1} \qquad \beta = 10$$



Choose $\frac{1}{T} = 0.1\cdot\omega_c' = 0.15$

Then $\frac{1}{\beta T} = 0.015$

Therefore, the compensator is

$$G_c(s) = 20\frac{6.67s+1}{66.7s+1} = K_c\beta\frac{6.67s+1}{66.7s+1}$$

Step 4: Finally, let

$$K = K_c\beta \Rightarrow K_c = \frac{K}{\beta}$$

In this example,

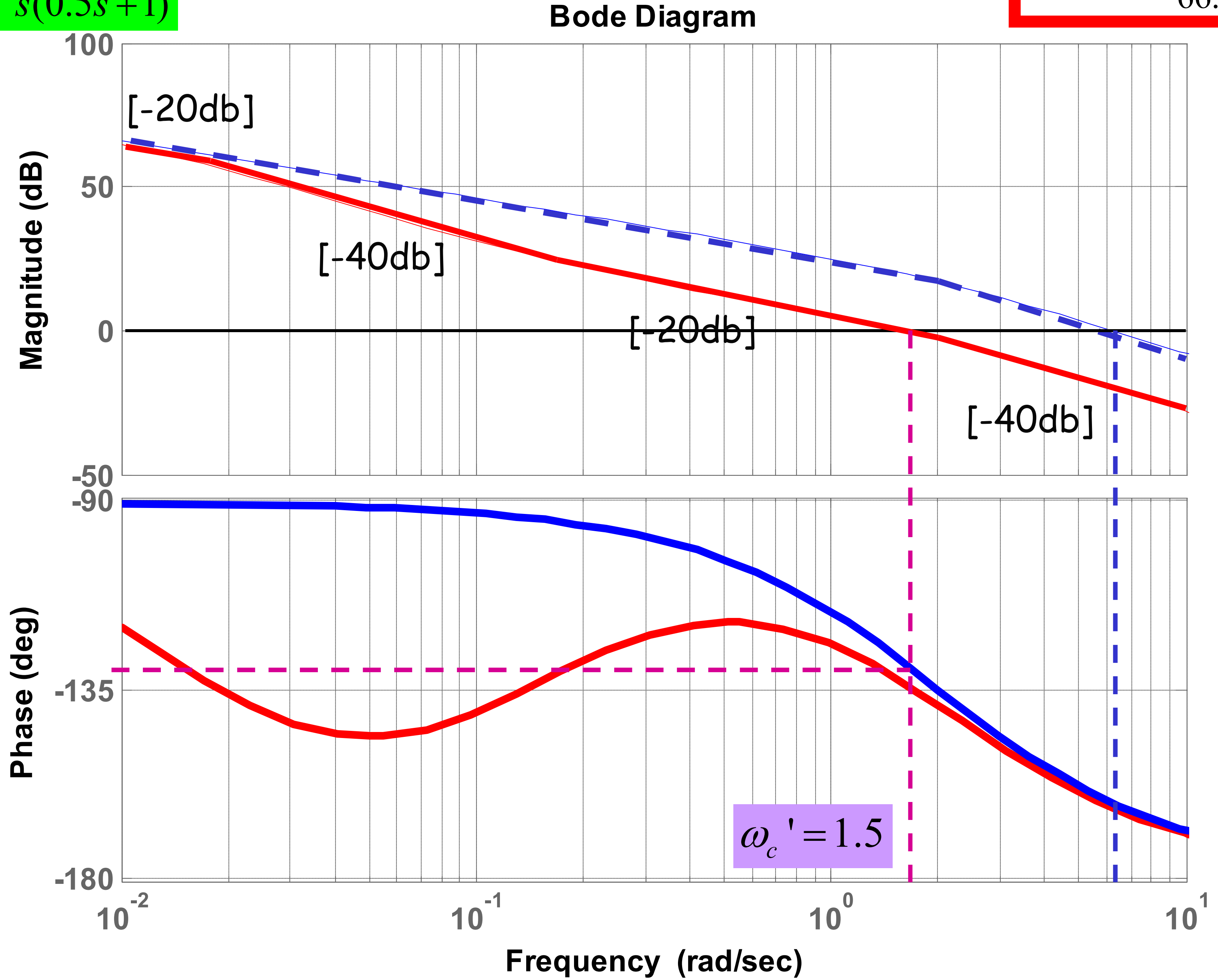
$$K = K_c\beta = 20 \Rightarrow K_c = \frac{20}{10} = 2$$

The open-loop transfer function of the compensated system is then obtained as

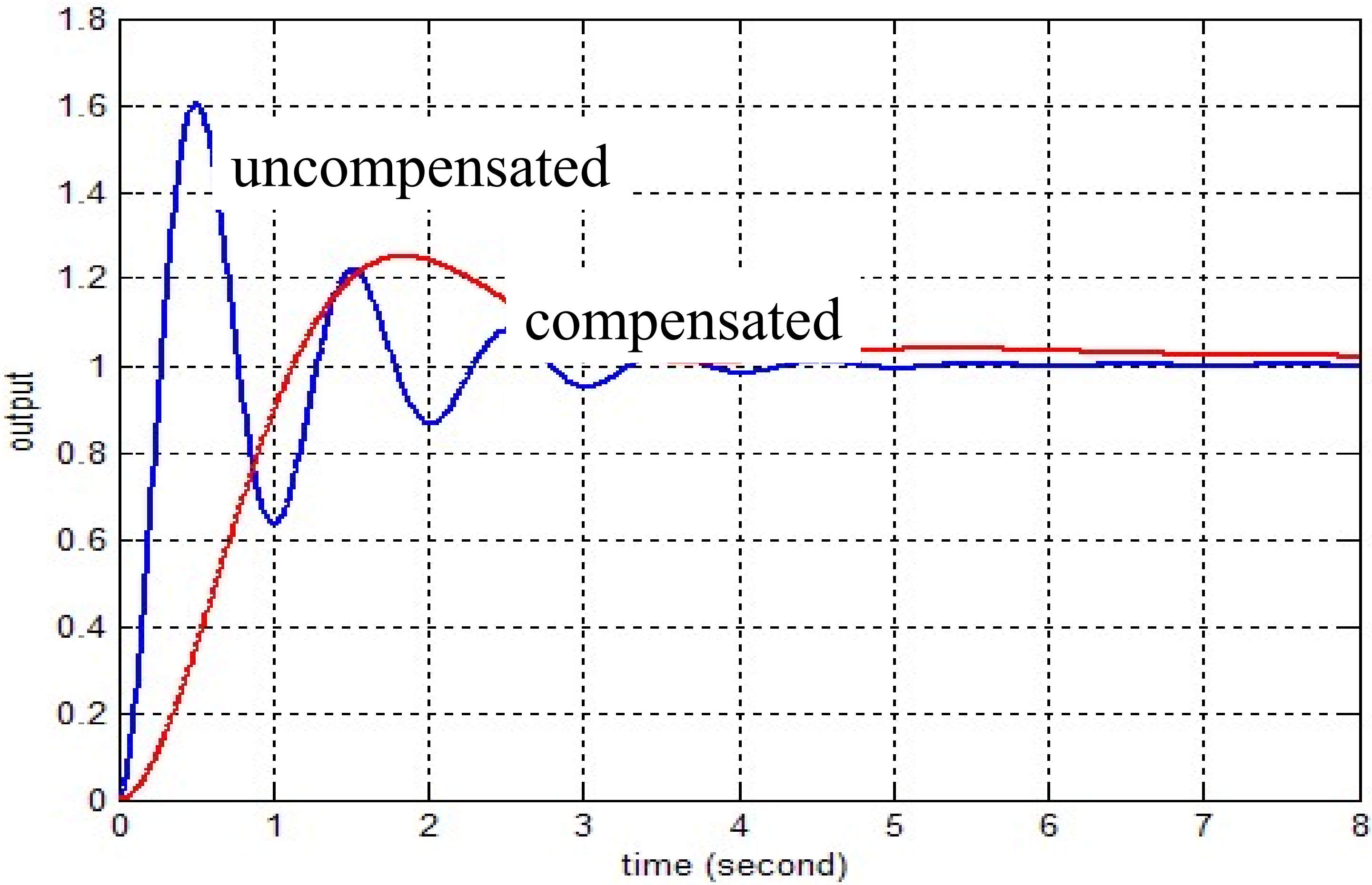
$$G(s)G_c(s) = \frac{20}{s(0.5s+1)} \cdot \frac{6.67s+1}{66.7s+1}$$

$$G(s) = \frac{1}{s(0.5s+1)}$$

$$G_c(s) = 20\frac{6.67s+1}{66.7s+1}$$



The time responses of the uncompensated and compensated systems are show in the following figure:



A Few Comments on Lag Compensation

1. Lag compensators are essentially **low-pass filters**. Therefore, lag compensation permits a **high gain** at low frequencies.
2. The closed-loop pole located near the origin gives a very slowly decaying transient response, although its magnitude will become very small because the zero of the lag compensator will almost cancel the effect of this pole. However, the transient response (decay) due to this pole is so slow that the settling time will be adversely affected.
3. $\omega_c' < \omega_c \rightarrow t_s \uparrow$.

Summary of the design procedure

1. Determine, according to the desired γ^*,

$$\gamma' = \gamma^* + (5^0 \sim 12^0)$$

Then, by measuring the Bode diagram, ω_c' can be determined.

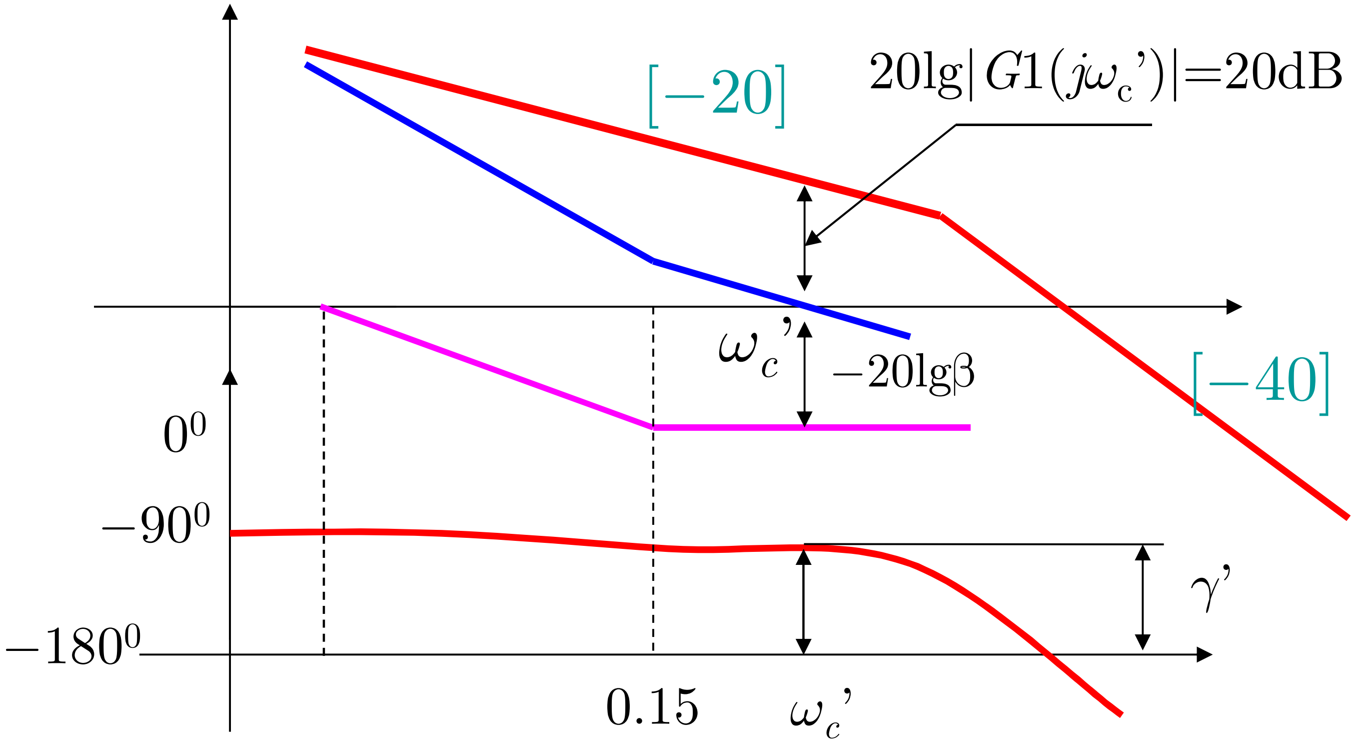
2. Determine β:

$$20\lg\left|G_1(j\omega_c')\right| \Rightarrow -20\lg\beta = -20\lg\left|G_1(j\omega_c')\right| \Rightarrow \beta = 10$$

3. Determine $1/T$:

$$\frac{1}{T} = (0.1 \sim 0.5)\omega_c'$$

$$\gamma' = \gamma^* + (5^0 \sim 12^0)$$



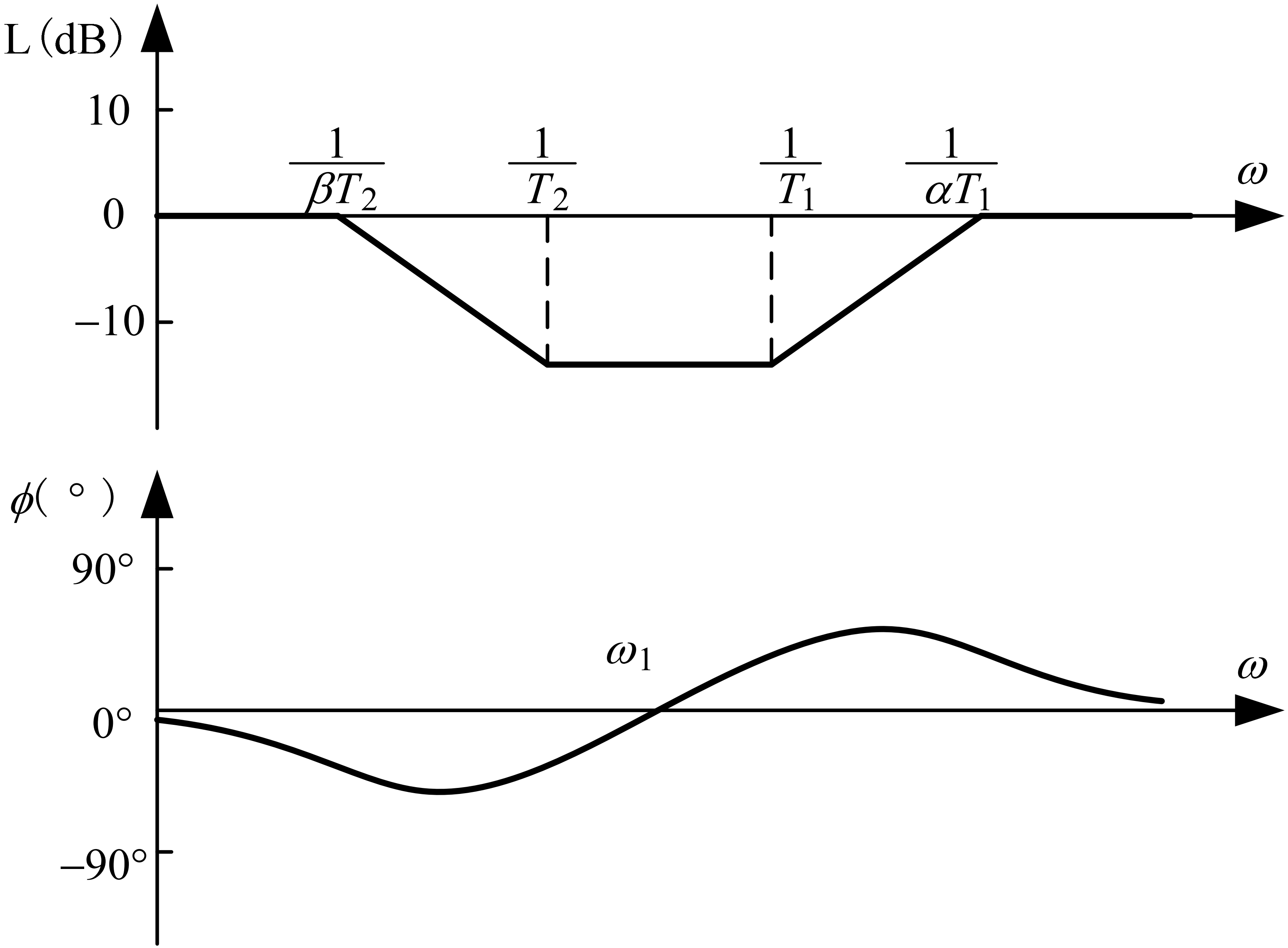
3. The phase lag-lead compensator

- The lead compensator improves settling time and phase margin, but increases the bandwidth.
- However, phase-lag compensator when applied properly improves phase margin but usually results in a longer settling time.
- It is natural, therefore, to consider using a combination of the lead-lag compensator, so that the advantages of both schemes can be utilized.

The transfer function of a lag-lead compensator can be written as

$$G_c(s) = K_c \underbrace{\left(\frac{1 + T_1 s}{1 + \alpha T_1 s} \right)}_{\text{lead}} \underbrace{\left(\frac{1 + T_2 s}{1 + \beta T_2 s} \right)}_{\text{lag}} \qquad \left(\beta > 1, \alpha = \frac{1}{\beta} \right)$$

It is usually assumed that the two corner frequencies of the lag portion are lower than the two corner frequencies of the lead portion.


L（dB）
10
0
−10
$\frac{1}{\beta T_2}$
$\frac{1}{T_2}$
$\frac{1}{T_1}$
$\frac{1}{\alpha T_1}$
ω
ϕ(°)
90°
0°
−90°
ω_1
ω

Example. Consider a unity-feedback system with its open-loop transfer function as

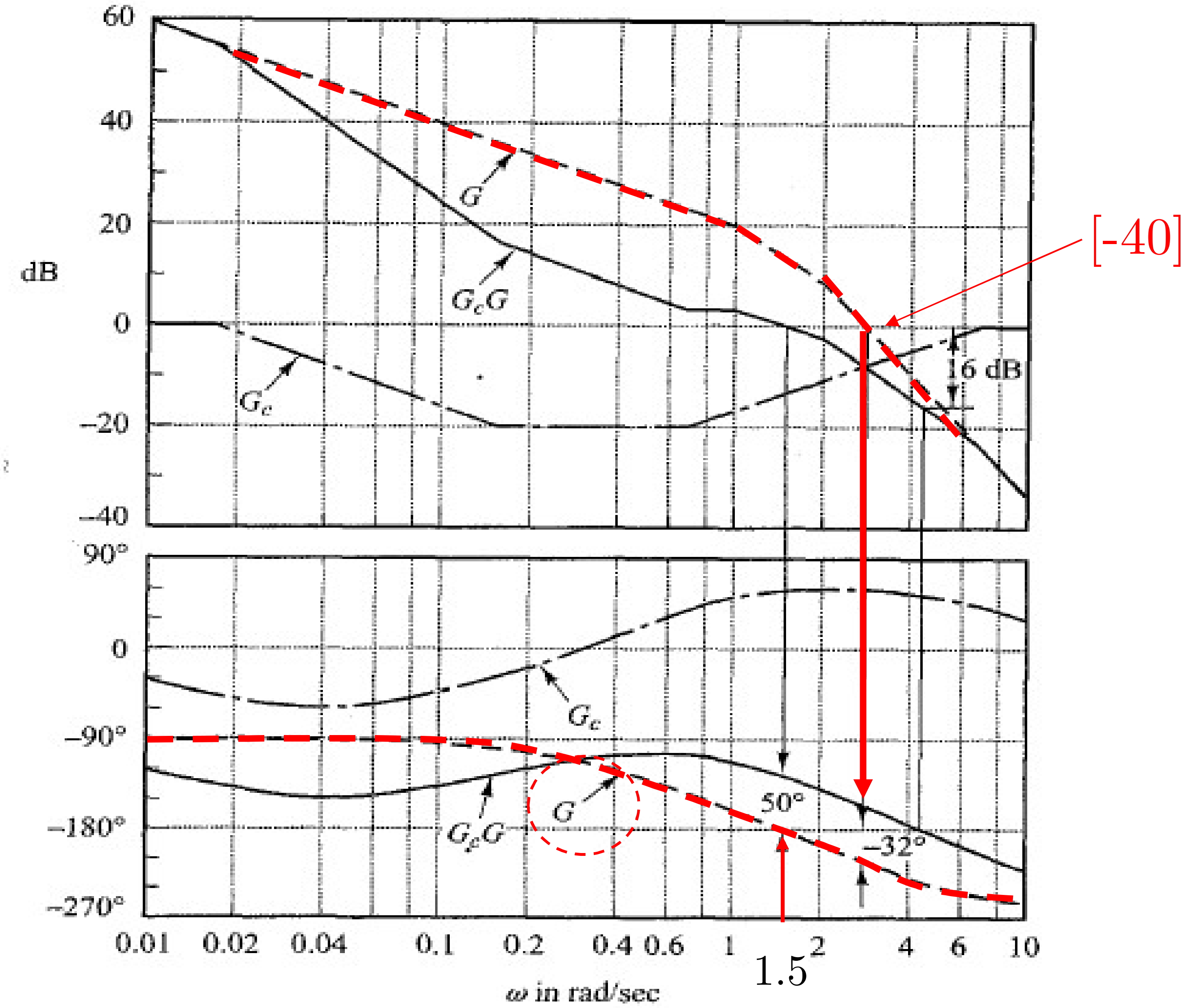
$$G(s) = \frac{K}{s(0.5s+1)(s+1)}$$

It is desired that the static velocity error constant be 10/s, the phase margin be 50^0, and the gain margin be 10 dB or more.

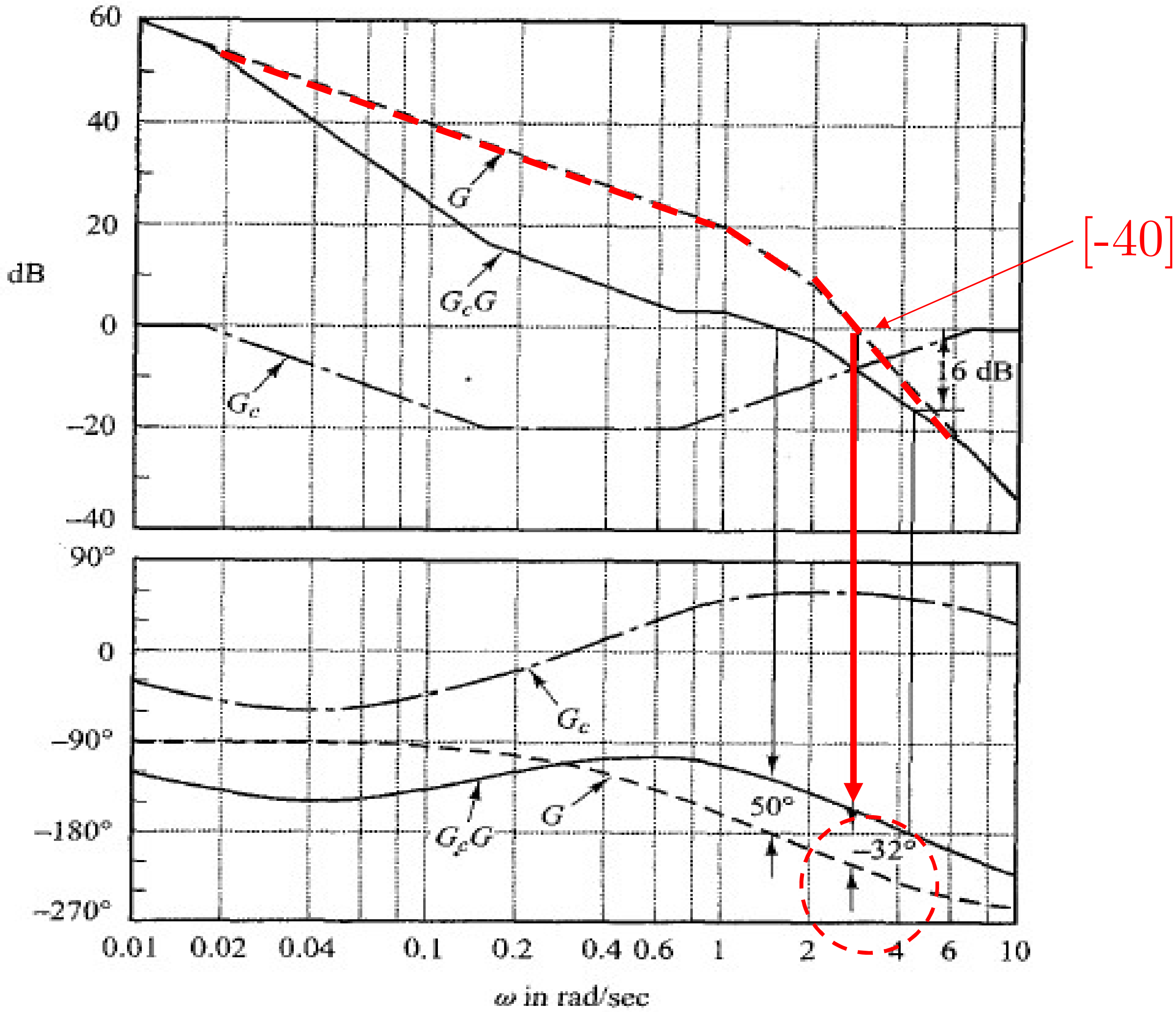
According to the requirement of the steady-state error, we need that

$$K = K_v = 10$$

Then, plotting the Bode diagram of the gain-adjusted but uncompensated transfer function yields

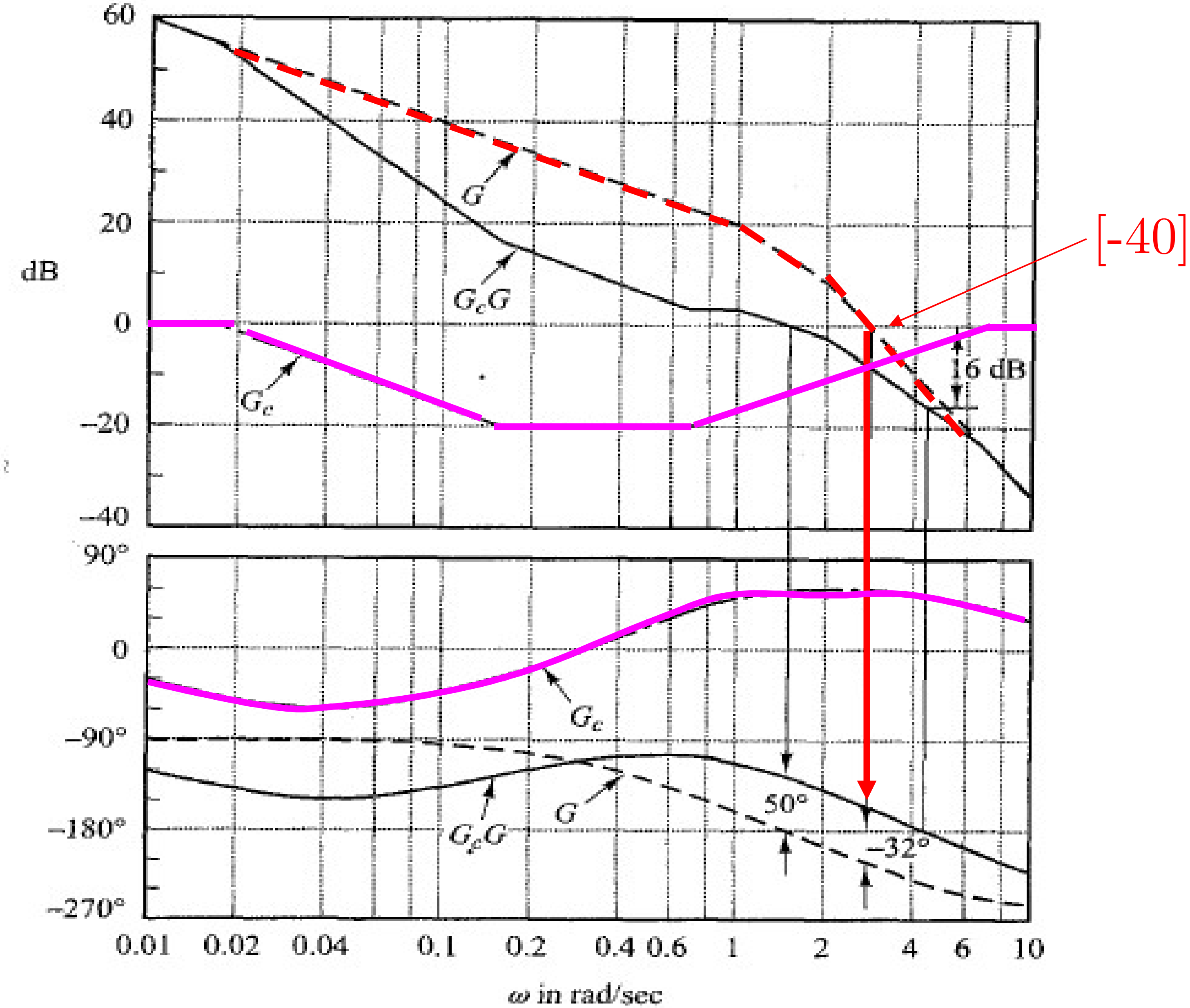


60
40
20
0
-20
-40
dB
G
G_cG
G_c
16 dB
[-40]
90°
0
-90°
-180°
-270°
G_c
G
G_cG
50°
-32°
0.01
0.02
0.04
0.1
0.2
0.4
0.6
1
2
4
6
10
1.5
ω in rad/sec

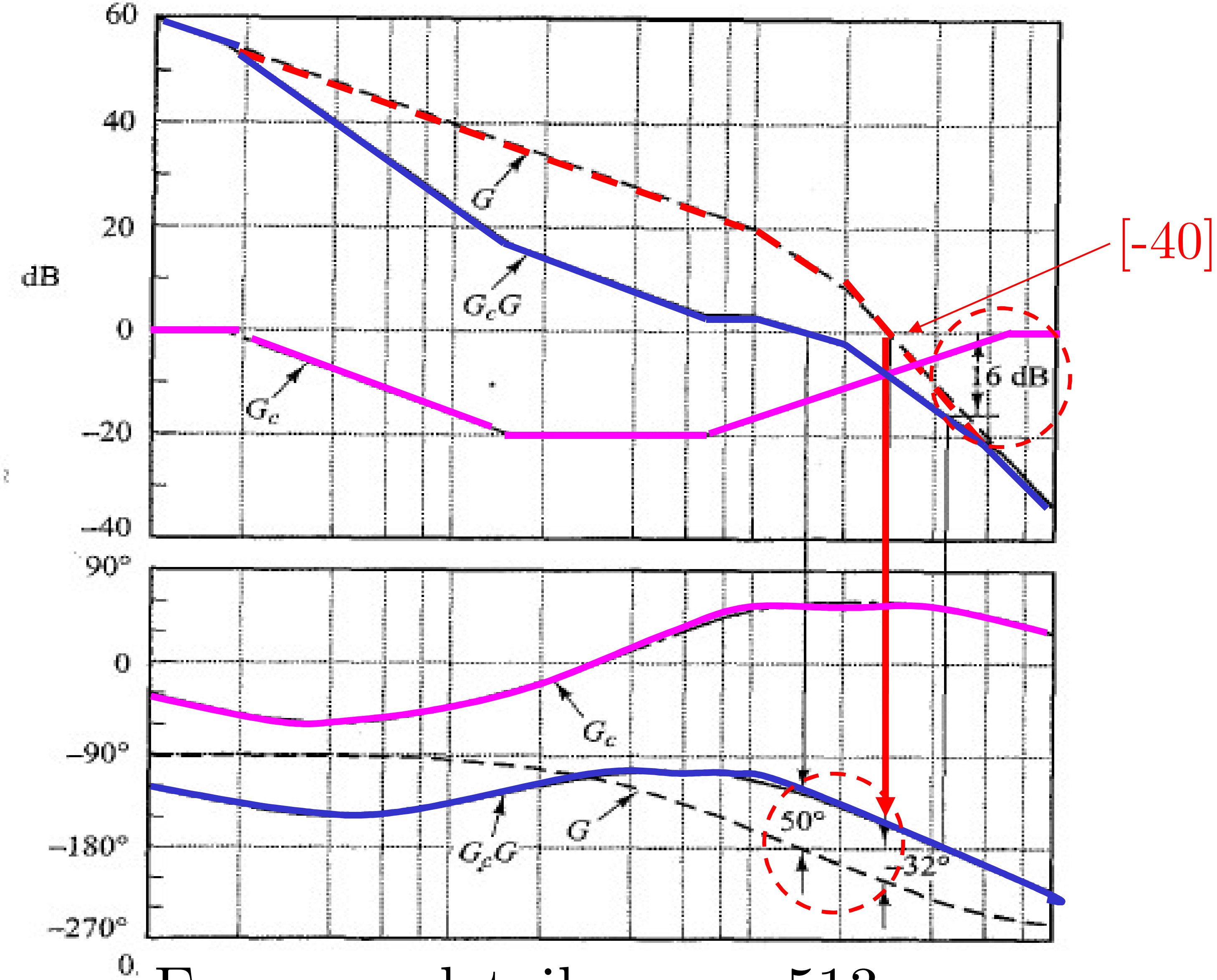


$\gamma = -32^0$ shows that the system is unstable.

A lag-lead compensator is utilized:



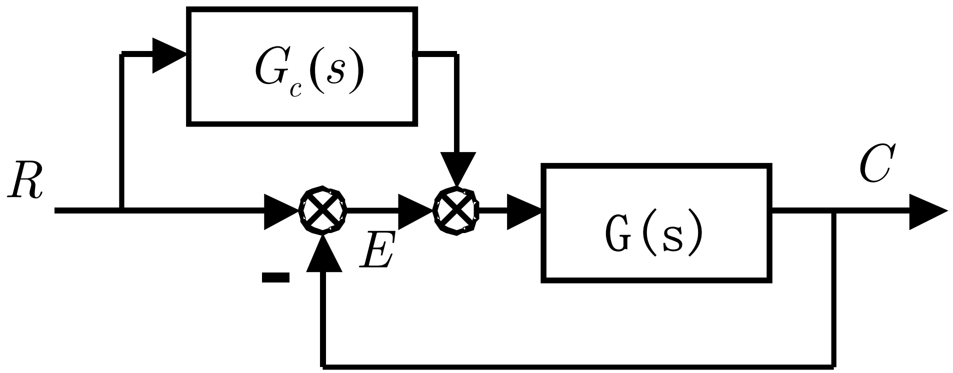
After the compensation:



For more detail, see p. 513.

4. Feedforward Compensation

We consider the following system



where $G_c(s)$ is called feedforward compensator and can be used to improve the steady-state error while keeping the system stability. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \left(1 + G_c(s)\right)\frac{G(s)}{1 + G(s)}$$

It is required that

$$E(s) = R(s) - C(s) = 0$$

Since

$$E(s) = R(s) - C(s) = \frac{1 - G(s)G_c(s)}{1 + G(s)} R(s)$$

a natural choice seems

$$G_c(s) = G^{-1}(s)$$

In that case

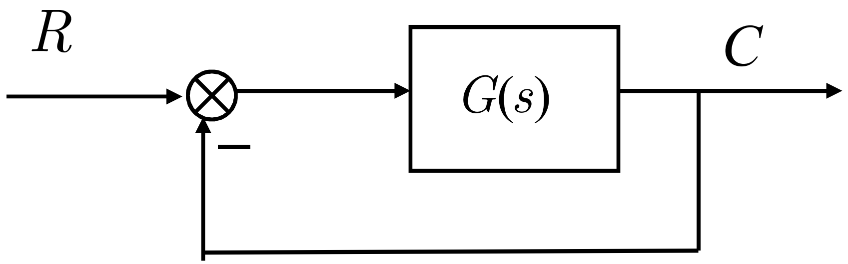
$$E(s) = R(s) - C(s) = 0$$

However,

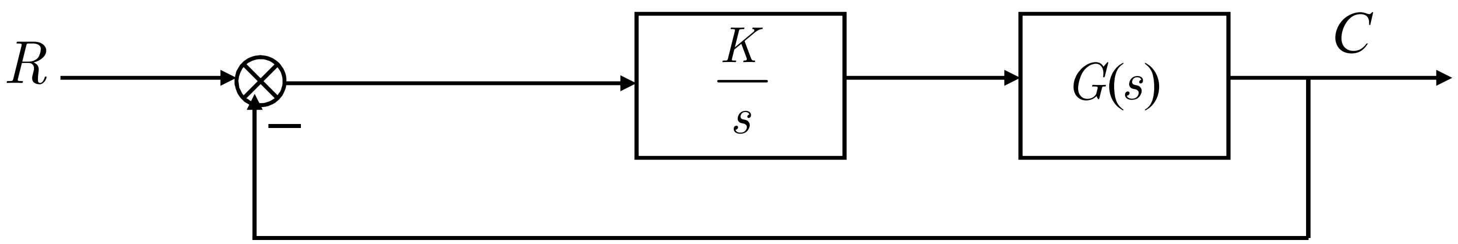
$$G_c(s) = \frac{1}{G(s)}$$

sometimes is difficult to be physically realized, especially when $G(s)$ has zeros in right half s plane. Therefore, approximation compensation is necessary.

Example. The original system is

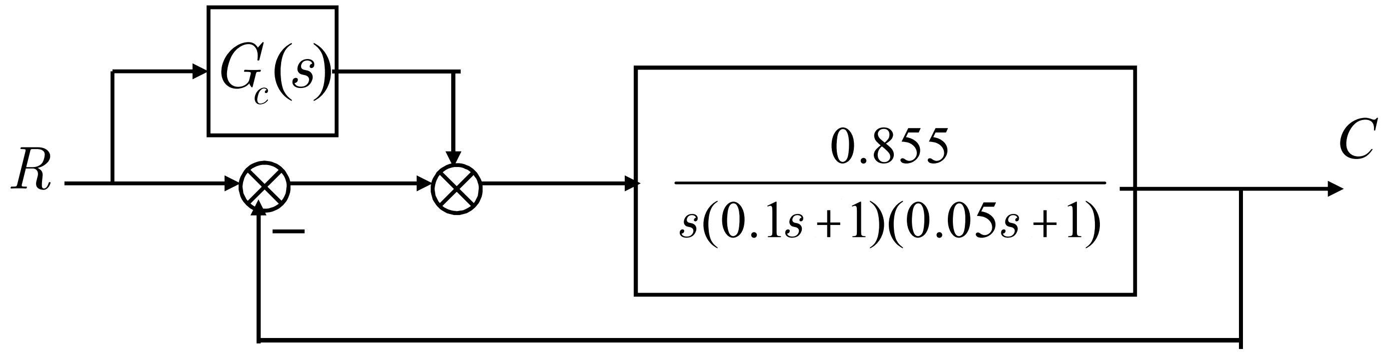


To improve the steady-state performance, an integrator can be used. However, such a compensator might cause instability:



To solve such a conflicting requirement, feedforward compensator can be utilized.

Example. The block diagram of system is



Design a $G_c(s)$ as simply as possible such that there is no steady-state error when $r(t)=t\cdot 1(t)$.

Solution: The original system is Type 1 system. Therefore, without compensation, the static velocity error is nonzero. Since

$$E(s)=R(s)-C(s)=\frac{1-G(s)G_c(s)}{1+G(s)}R(s)$$

We can choose

$$G_c(s) = G^{-1}(s)$$

which, however, is not a simpler one. An alternative is to write

$$E(s) = C(s) - R(s) = \frac{1 - G(s)G_c(s)}{1 + G(s)} R(s)$$

$$= \frac{s(0.1s+1)(0.05s+1) - G_c(s)0.855}{s(0.1s+1)(0.05s+1) + 0.855} \times \frac{1}{s^2}$$

Using Final value theorem,

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{s(0.1s+1)(0.05s+1) - G_c(s)0.855}{s(0.1s+1)(0.05s+1) + 0.855} \times \frac{1}{s}$$

$$e_{ss}=\lim_{s\to 0}\frac{0.005s^3+0.01s^2+s-G_c(s)0.855}{s(0.1s+1)(0.05s+1)+0.855}\times\frac{1}{s}$$

Let

$$G_c(s)0.855=s\Rightarrow G_c(s)=\frac{s}{0.855}$$

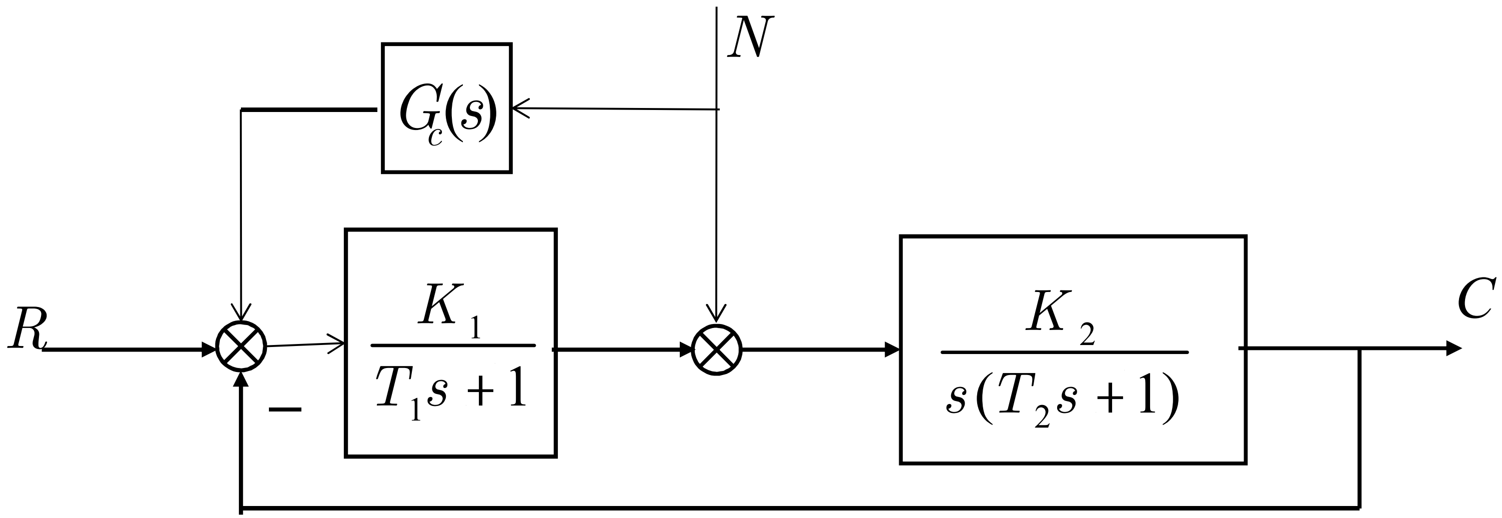
Substituting $G_c(s)$ into $E(s)$ yields

$$e_{ss}=\lim_{s\to 0}\frac{0.005s^3+0.01s^2}{s(0.1s+1)(0.05s+1)+0.855}\times\frac{1}{s}$$

$$=\lim_{s\to 0}\frac{0.005s^2+0.01s}{s(0.1s+1)(0.05s+1)+0.855}=0$$

$G_c(s)=s/0.885$ is obviously simpler than $G_c(s)=1/G(s)$. This completes the design.

Example. Feedforward compensation of external disturbance.



Design a $G_c(s)$ as simply as possible such that there is no steady-state error when $N(t)=1(t)$.

Solution: One can choose

$$N(s)+G_c(s)G_1(s)N(s)=0 \Rightarrow G_c(s)=-\frac{1}{G_1(s)}=-\frac{1}{K_1}(T_1 s+1)$$

If, however, take $N(t)=1(t)$ into consideration,

$$e_{ss} = \lim_{s\to 0} sE(s) = -\lim_{s\to 0} sC(s)$$

$$= -\lim_{s\to 0} s\frac{[1+G_1(s)G_c(s)]G_2(s)}{1+G_1(s)G_2(s)}N(s)$$

$$= -\lim_{s\to 0} s\frac{K_2[T_1 s+1+K_1G_c(s)]}{s(T_1 s+1)(T_2 s+1)+K_1K_2}\frac{1}{s}$$

We can simply let

$$G_c(s) = -\frac{1}{K_1}$$

Then,

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= -\lim_{s\to 0} \frac{K_2 T_1 s}{s(T_1 s+1)(T_2 s+1)+K_1K_2} = 0$$

Appendix

$$\phi(\omega) = \angle G_c(j\omega) = \tan^{-1} T\omega - \tan^{-1} \alpha T\omega = \tan^{-1} \frac{T\omega(1-\alpha)}{1+\alpha T^2\omega^2}$$

$$\frac{d\phi(\omega)}{d\omega} = 0 \quad \Rightarrow \quad \frac{d}{d\omega}\left[\tan\phi(\omega)\right] = 0$$

$$\frac{d}{d\omega}\left[\frac{T\omega(1-\alpha)}{1+\alpha T^2\omega^2}\right] = \frac{T(1-\alpha)[1-\alpha T^2\omega^2]}{(1+\alpha T^2\omega^2)^2} = 0 \Rightarrow \omega_m = \frac{1}{T\sqrt{\alpha}}$$

Therefore,

$$\tan\phi(\omega_m) = \left.\frac{T\omega(1-\alpha)}{1+\alpha T^2\omega^2}\right|_{\omega_m = \frac{1}{\sqrt{\alpha}T}} = \frac{1-\alpha}{2\sqrt{\alpha}}$$