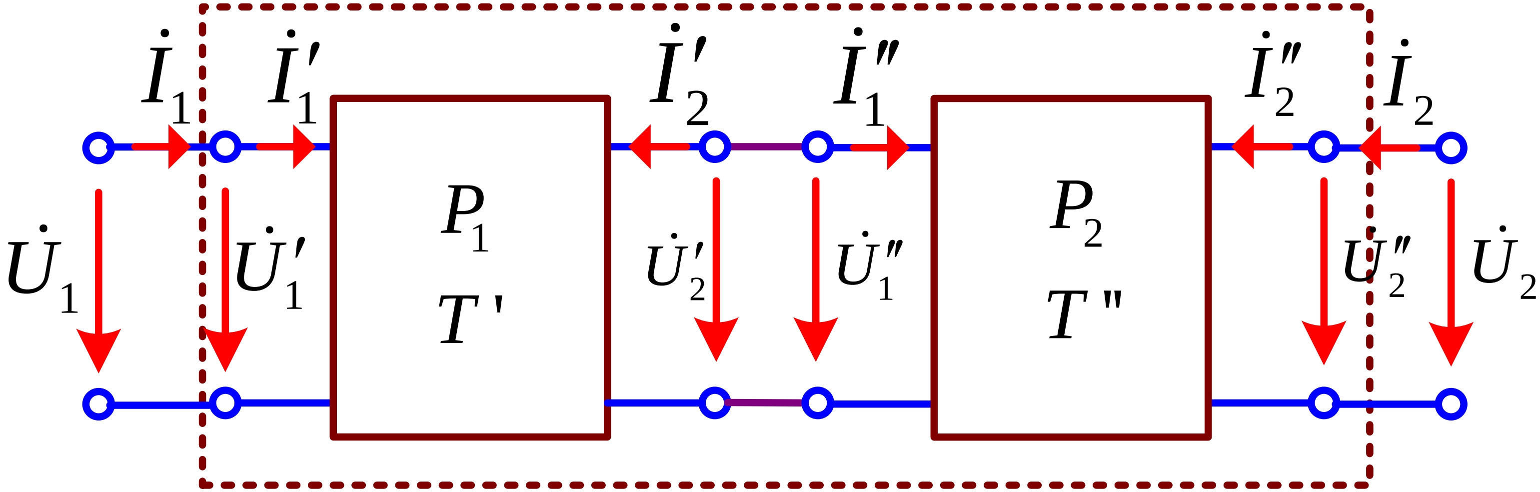
16.5 二端口的连接

一个复杂二端口网络可以看作是由若干简单的二端口按某种方式联接而成，使电路分析得到简化；

1. 级联（链联）



$$\mathbf{T}=\mathbf{T}'\mathbf{T}''$$

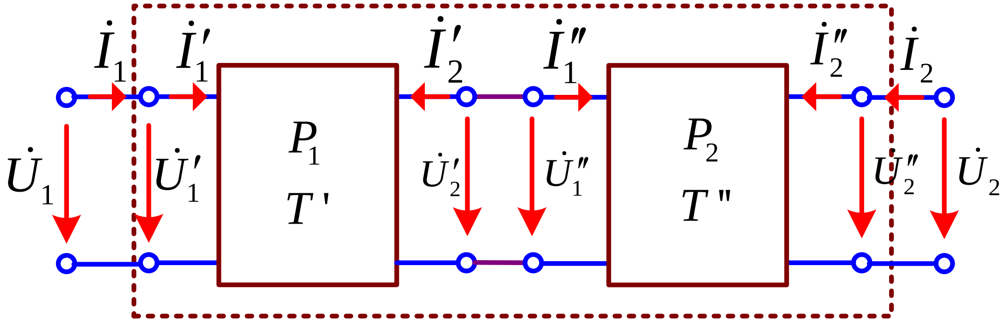
设 $[\mathbf{T}'] = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$ $\quad [\mathbf{T}''] = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$

即 $\begin{bmatrix} \dot{U}_1' \\ \dot{I}_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} \dot{U}_2' \\ -\dot{I}_2' \end{bmatrix}$ $\quad \begin{bmatrix} \dot{U}_1'' \\ \dot{I}_1'' \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} \dot{U}_2'' \\ -\dot{I}_2'' \end{bmatrix}$

级联后 $\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} \dot{U}_1' \\ \dot{I}_1' \end{bmatrix}$ $\quad \begin{bmatrix} \dot{U}_2' \\ -\dot{I}_2' \end{bmatrix} = \begin{bmatrix} \dot{U}_1'' \\ \dot{I}_1'' \end{bmatrix}$ $\quad \begin{bmatrix} \dot{U}_2'' \\ -\dot{I}_2'' \end{bmatrix} = \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$

则 $\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} \dot{U}_1' \\ \dot{I}_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} \dot{U}_2' \\ -\dot{I}_2' \end{bmatrix}$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & B \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$



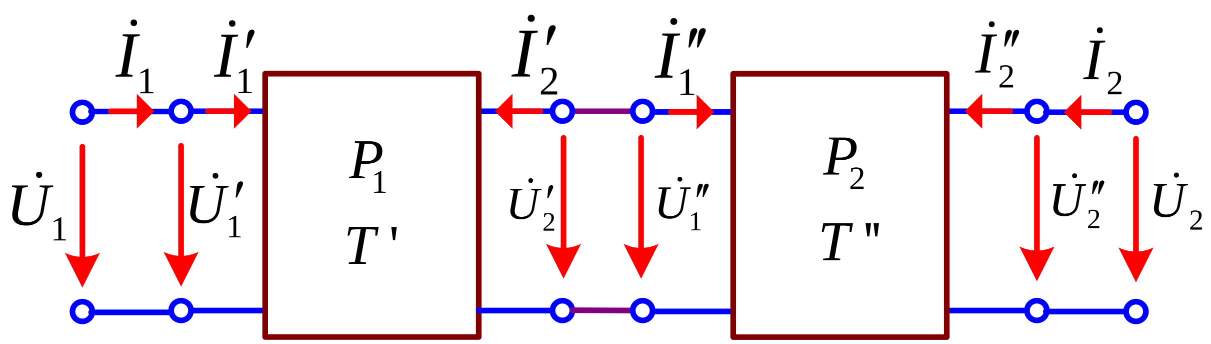
则 $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$

$$= \begin{bmatrix} A'A''+B'C'' & A'B''+B'D'' \\ C'A''+D'C'' & C'B''+D'D'' \end{bmatrix}$$

即：$[\mathbf{T}] = [\mathbf{T}'][\mathbf{T}'']$

结论 级联后所得复合二端口 T 参数矩阵等于级联的二端口 T 参数矩阵相乘。

可推广到 n 个二端口级联的关系。



注意

（1） 级联时T 参数是矩阵相乘的关系，不是对应元素相乘。

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

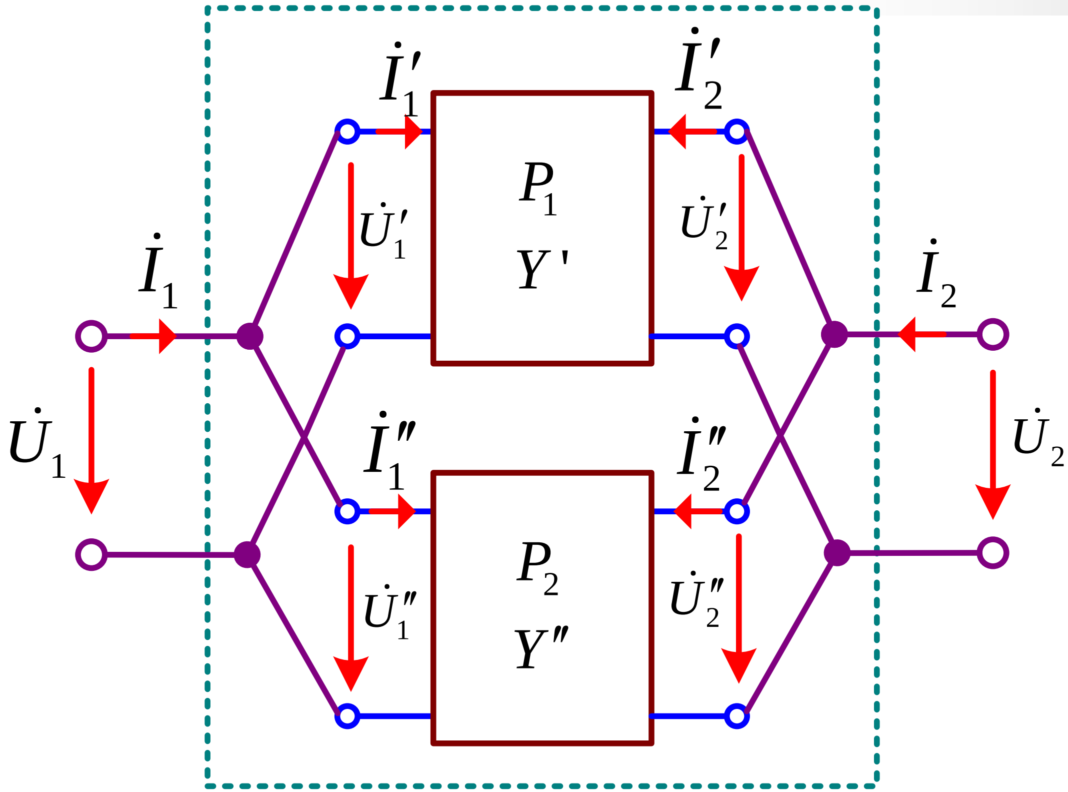
$$= \begin{bmatrix} A'A'' + B'C'' & A'B'' + B'D'' \\ C'A'' + D'C'' & C'B'' + D'D'' \end{bmatrix}$$

显然 $A = A'A'' + B'C'' \neq A'A''$

（2） 级联时各二端口的端口条件不会被破坏。

2. 并联

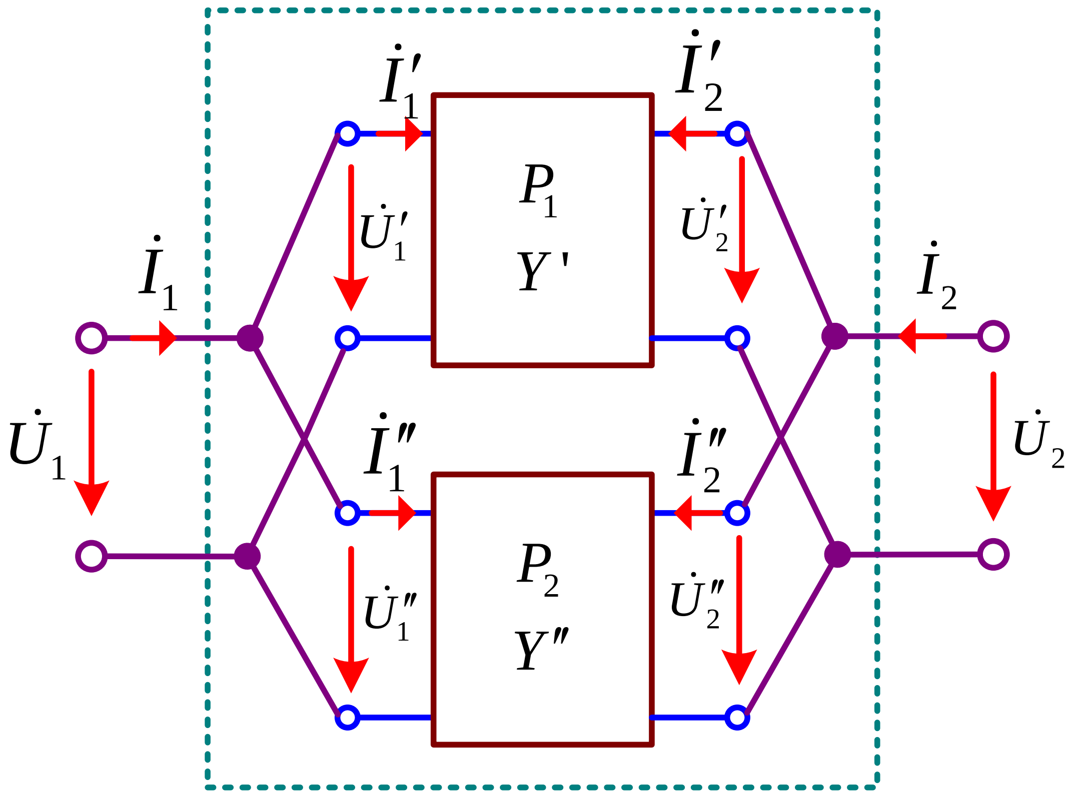
并联采用 Y 参数方便。



若满足端口条件

$$\mathbf{Y}=\mathbf{Y}'+\mathbf{Y}''$$

$$\begin{bmatrix} \dot{I}'_1 \\ \dot{I}'_2 \end{bmatrix} = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \begin{bmatrix} \dot{U}'_1 \\ \dot{U}'_2 \end{bmatrix} \qquad \begin{bmatrix} \dot{I}''_1 \\ \dot{I}''_2 \end{bmatrix} = \begin{bmatrix} Y''_{11} & Y''_{12} \\ Y''_{21} & Y''_{22} \end{bmatrix} \begin{bmatrix} \dot{U}''_1 \\ \dot{U}''_2 \end{bmatrix}$$



并联后

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} \dot{U}'_1 \\ \dot{U}'_2 \end{bmatrix} = \begin{bmatrix} \dot{U}''_1 \\ \dot{U}''_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} \dot{I}'_1 \\ \dot{I}'_2 \end{bmatrix} + \begin{bmatrix} \dot{I}''_1 \\ \dot{I}''_2 \end{bmatrix}$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}+\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}=\begin{bmatrix}Y'_{11} & Y'_{12}\\ Y'_{21} & Y'_{22}\end{bmatrix}\begin{bmatrix}\dot{U}'_1\\ \dot{U}'_2\end{bmatrix}+\begin{bmatrix}Y''_{11} & Y''_{12}\\ Y''_{21} & Y''_{22}\end{bmatrix}\begin{bmatrix}\dot{U}''_1\\ \dot{U}''_2\end{bmatrix}$$

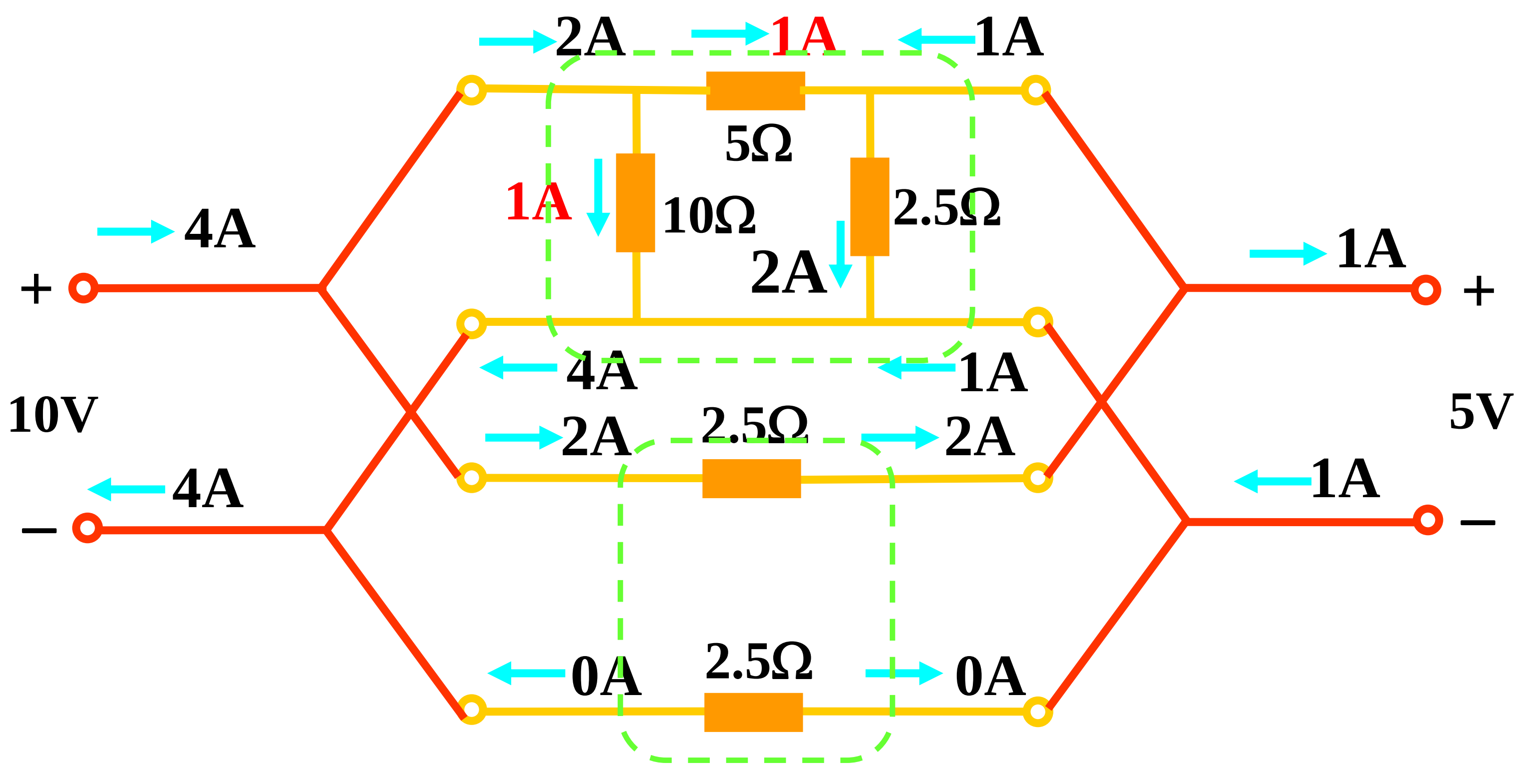
$$=\left\{\begin{bmatrix}Y'_{11} & Y'_{12}\\ Y'_{21} & Y'_{22}\end{bmatrix}+\begin{bmatrix}Y''_{11} & Y''_{12}\\ Y''_{21} & Y''_{22}\end{bmatrix}\right\}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$$

$$=\begin{bmatrix}Y'_{11}+Y''_{11} & Y'_{12}+Y''_{12}\\ Y'_{21}+Y''_{21} & Y'_{22}+Y''_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=[\mathrm{Y}]\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$$

结论 $[\mathbf{Y}]=[\mathbf{Y}']+[\mathbf{Y}'']$

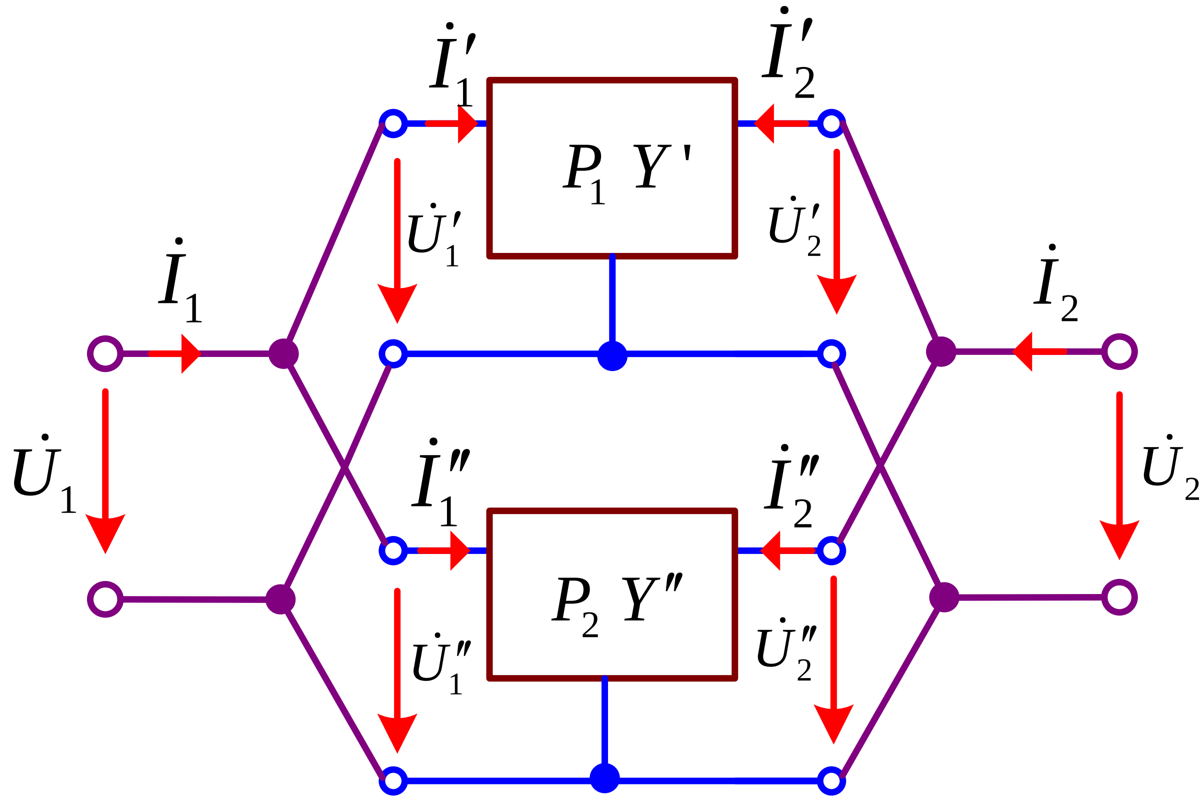
二端口并联所得复合二端口的Y 参数矩阵等于两个二端口Y参数矩阵相加。

注意：（1）两个二端口并联时，其端口条件可能被破坏此时上述关系式就不成立。

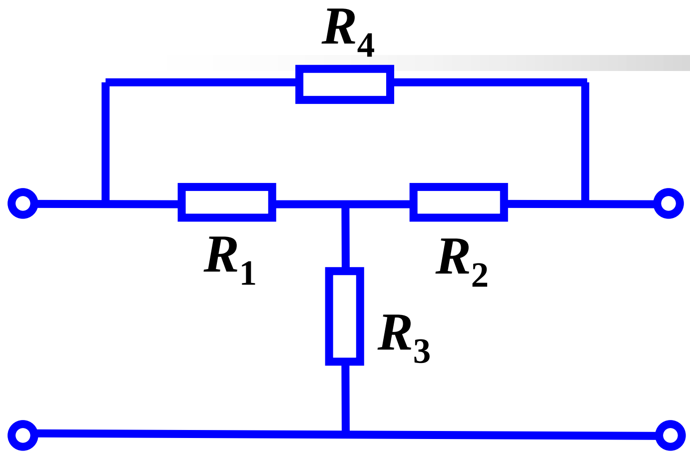


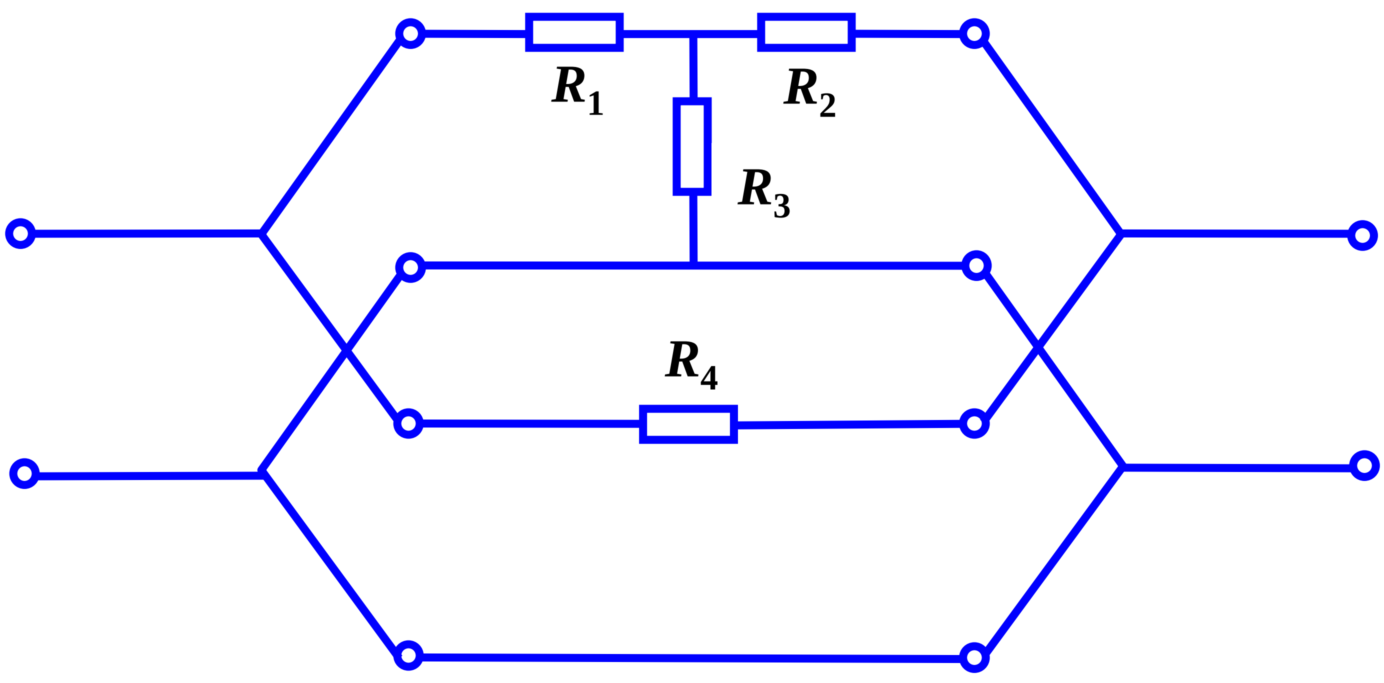
并联后端口条件破坏。

(2)　具有公共端的二端口(三端网络形成的二端口)，将公共端并在一起将不会破坏端口条件。

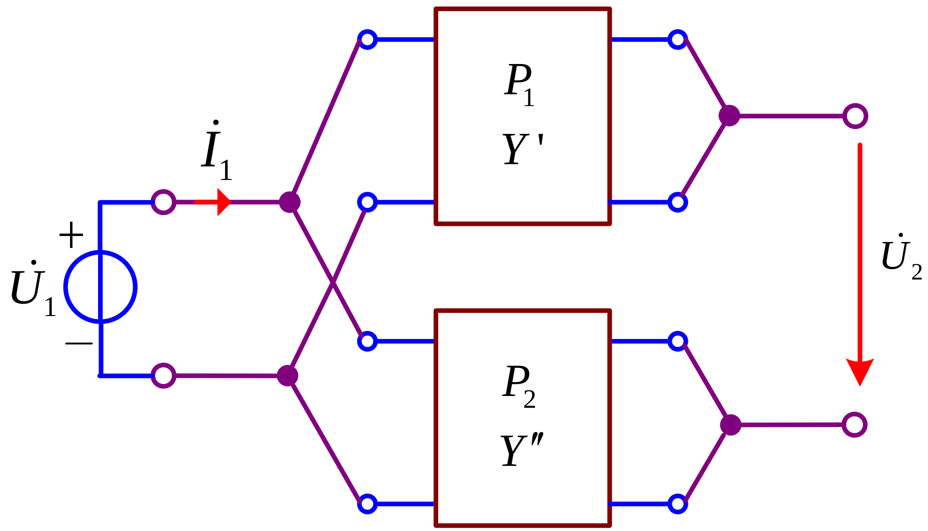


【例】



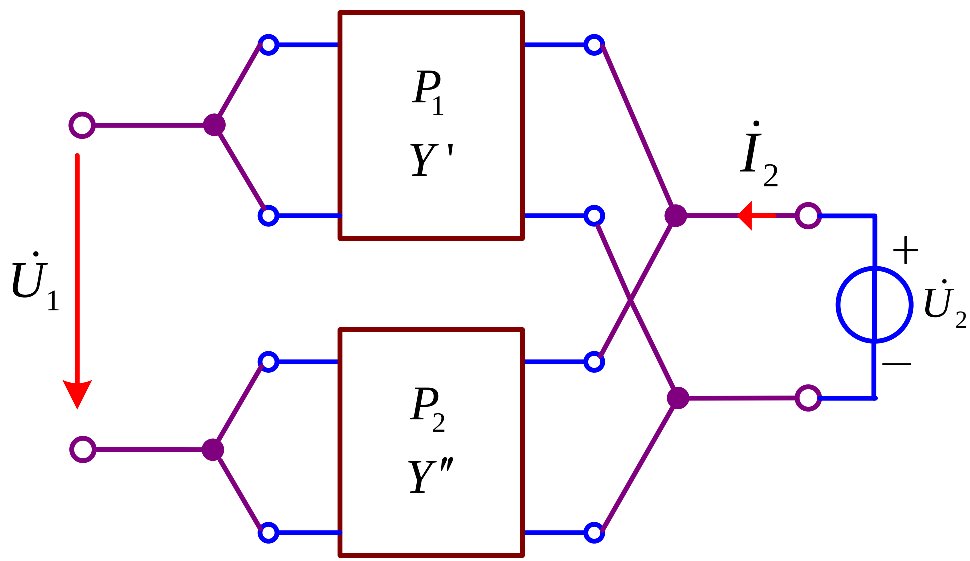


(3) 检查是否满足并联端口条件的方法：



$$\dot{U}_2 ?= 0$$

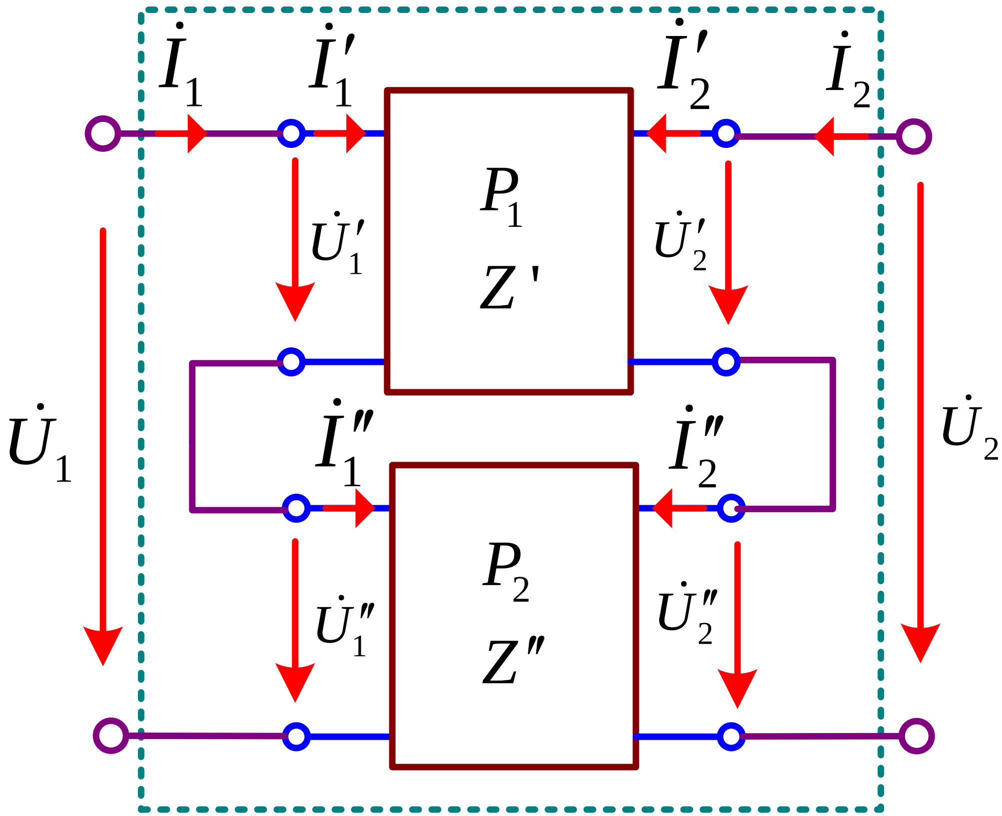
输入端是否满足端口条件。



$$\dot{U}_1 ?= 0$$

输出端是否满足端口条件。

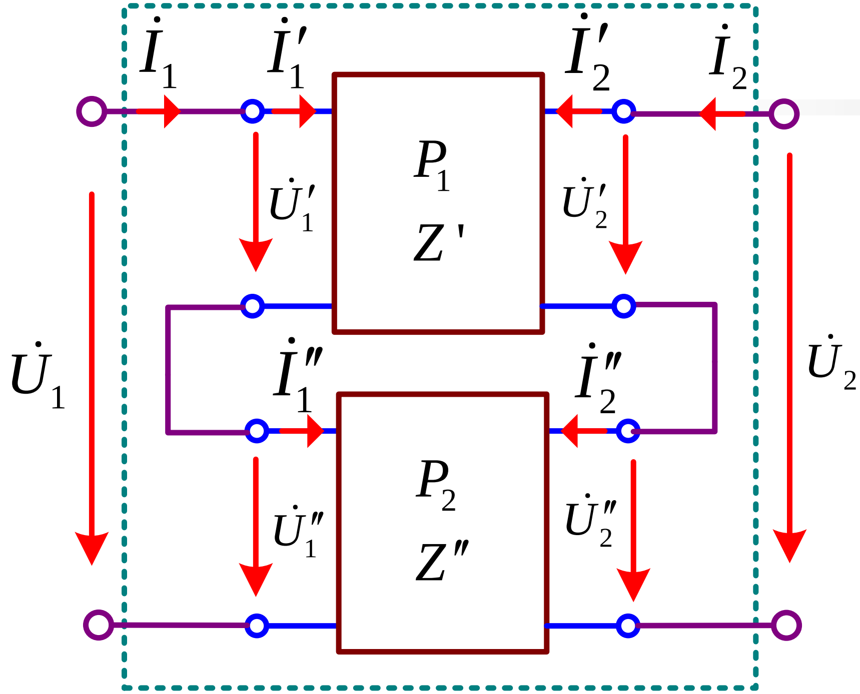
3. 串联　采用Z 参数方便。



若满足端口条件

$$\mathbf{Z}=\mathbf{Z}'+\mathbf{Z}''$$

$$\begin{bmatrix}\dot U_1'\\ \dot U_2'\end{bmatrix}=\begin{bmatrix}Z_{11}' & Z_{12}'\\ Z_{21}' & Z_{22}'\end{bmatrix}\begin{bmatrix}\dot I_1'\\ \dot I_2'\end{bmatrix} \quad \begin{bmatrix}\dot U_1''\\ \dot U_2''\end{bmatrix}=\begin{bmatrix}Z_{11}'' & Z_{12}''\\ Z_{21}'' & Z_{22}''\end{bmatrix}\begin{bmatrix}\dot I_1''\\ \dot I_2''\end{bmatrix}$$



$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}=\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1\\ \dot{U}'_2\end{bmatrix}+\begin{bmatrix}\dot{U}''_1\\ \dot{U}''_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1\\ \dot{U}'_2\end{bmatrix}+\begin{bmatrix}\dot{U}''_1\\ \dot{U}''_2\end{bmatrix}=[Z']\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}+[Z'']\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}$$

$$=\{[Z']+[Z'']\}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=[\mathbf{Z}]\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}$$

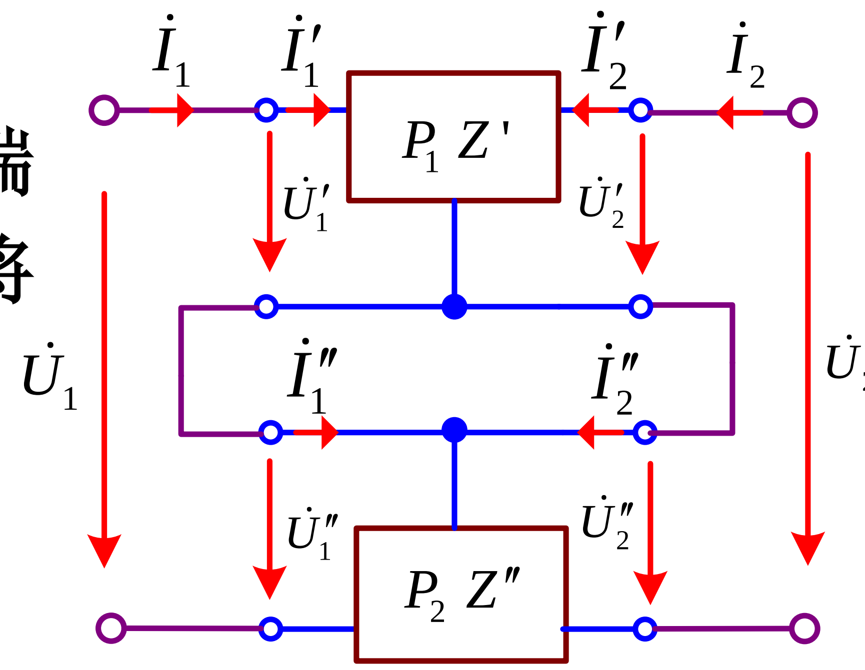
$$[Z]=[Z']+[Z'']$$

结论: 串联后复合二端口Z 参数矩阵等于原二端口Z 参数矩阵相加。

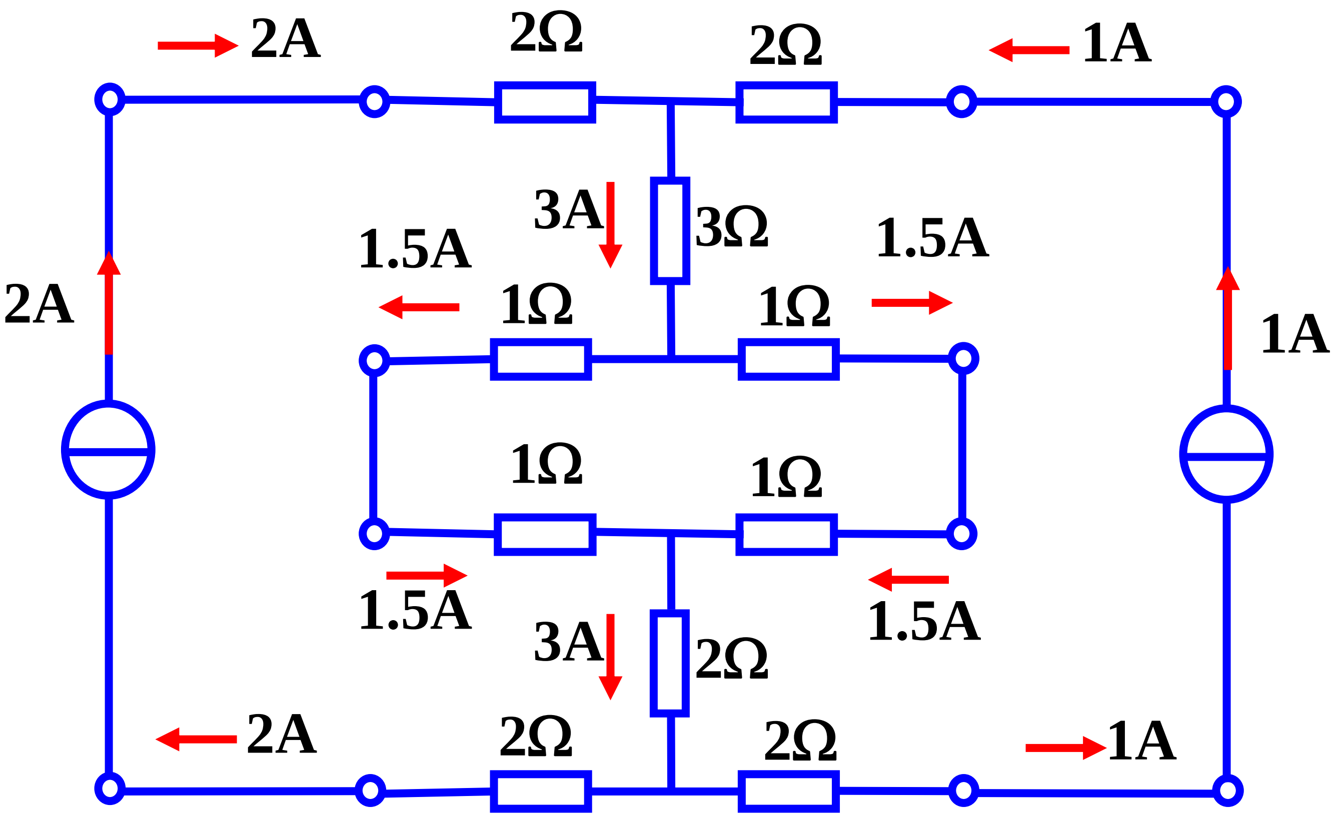
可推广到n端口串联。

注意:（1） 串联后端口条件可能被破坏。需检查端口条件。

（2） 具有公共端的二端口，将公共端串联时将不会破坏端口条件。

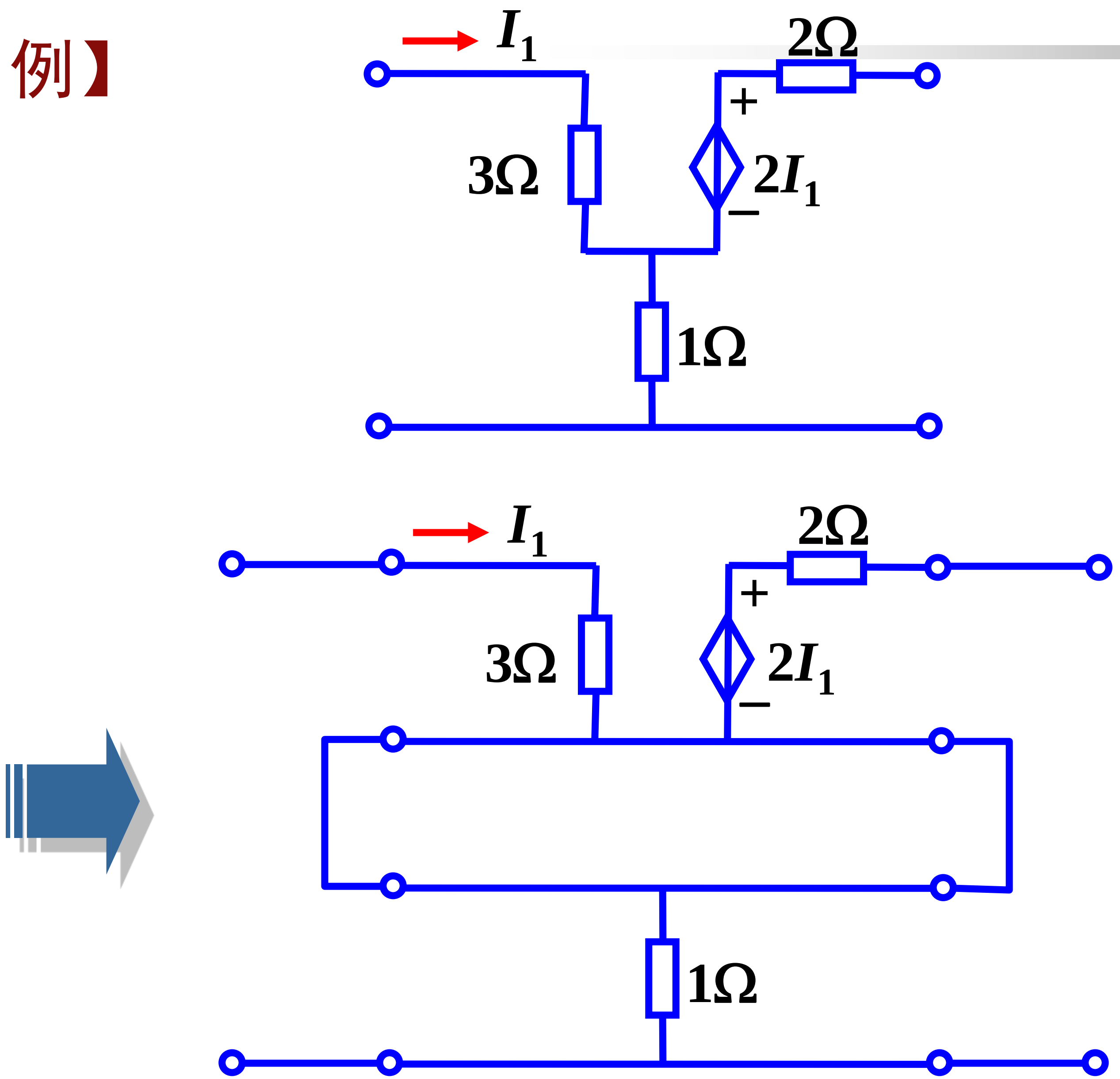


端口条件不会破坏

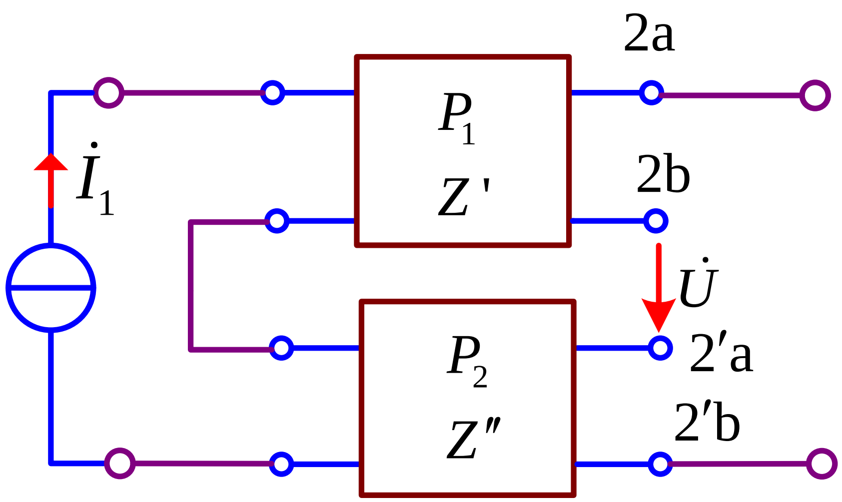


端口条件破坏 ！

【例】

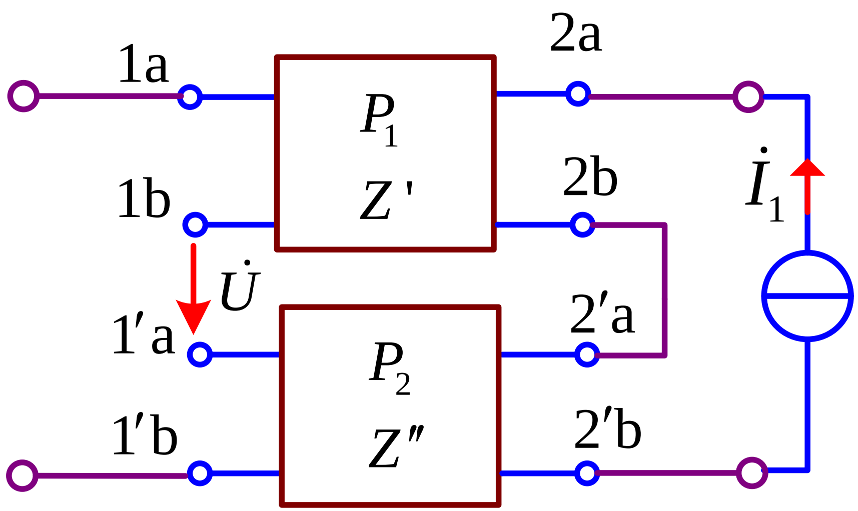


(3) 检查是否满足串联端口条件的方法:



$\dot{U} ? = 0$

输入端是否满足端口条件。

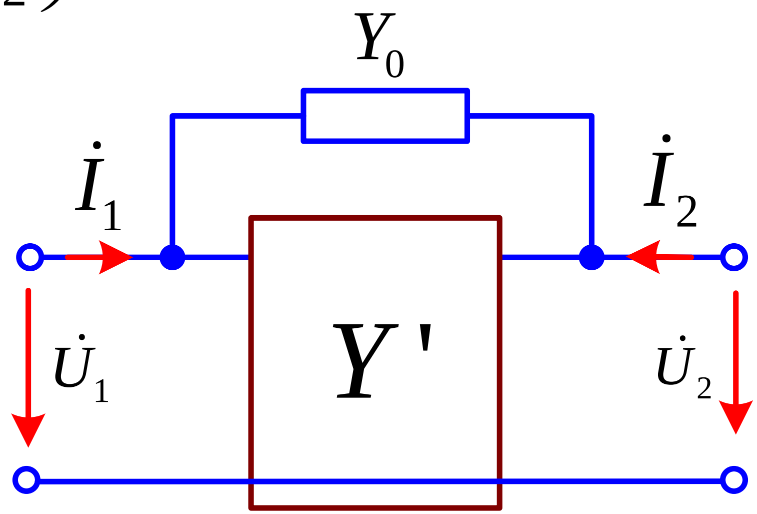


$\dot{U} ? = 0$

输出端是否满足端口条件。

【例】 已知 $\mathbf{Y}' = \begin{pmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{pmatrix}$ 求Y矩阵。

解 法1：

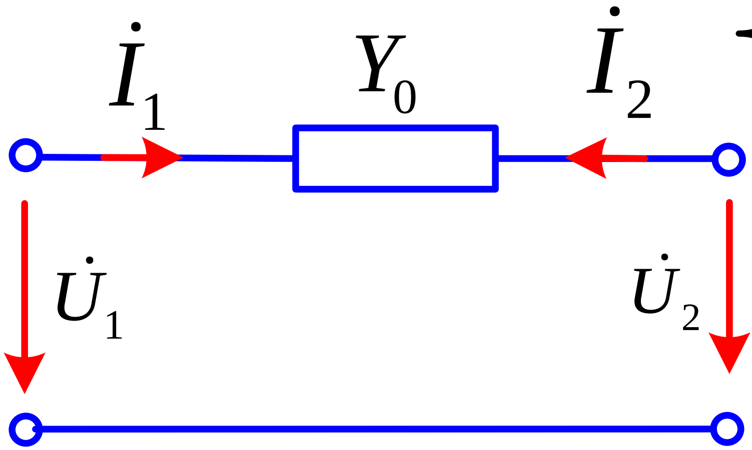


二端口并联

具有公共端，公共端并在一起，不会破坏端口条件。

$$[\mathbf{Y}] = [\mathbf{Y}'] + [\mathbf{Y}'']$$

解 法1：

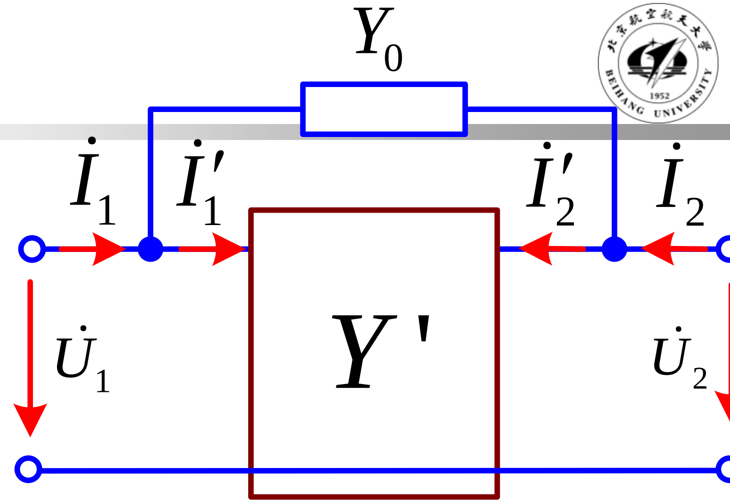


$$\begin{cases} \dot{I}_1 = (\dot{U}_1 - \dot{U}_2)Y_0 = Y_0\dot{U}_1 - Y_0\dot{U}_2 \\ \dot{I}_2 = (\dot{U}_2 - \dot{U}_1)Y_0 = -Y_0\dot{U}_1 + Y_0\dot{U}_2 \end{cases}$$

$$\mathbf{Y}'' = \begin{pmatrix} Y_0 & -Y_0 \\ -Y_0 & Y_0 \end{pmatrix}$$

$$\mathbf{Y} = \mathbf{Y}' + \mathbf{Y}'' = \begin{pmatrix} Y_{11}' + Y_0 & Y_{12}' - Y_0 \\ Y_{21}' - Y_0 & Y_{22}' + Y_0 \end{pmatrix}$$

法2：$\begin{cases} \dot{I}_1' = Y_{11}'\dot{U}_1 + Y_{12}'\dot{U}_2 \\ \dot{I}_2' = Y_{21}'\dot{U}_1 + Y_{22}'\dot{U}_2 \end{cases}$



$$\begin{cases} \dot{I}_1 = \dot{I}_1' + (\dot{U}_1 - \dot{U}_2)Y_0 \\ \dot{I}_2 = \dot{I}_2' + (\dot{U}_2 - \dot{U}_1)Y_0 \end{cases}$$

$$\begin{cases} \dot{I}_1 = Y_{11}'\dot{U}_1 + Y_{12}'\dot{U}_2 + Y_0\dot{U}_1 - Y_0\dot{U}_2 \\ \dot{I}_2 = Y_{21}'\dot{U}_1 + Y_{22}'\dot{U}_2 - Y_0\dot{U}_1 + Y_0\dot{U}_2 \end{cases}$$

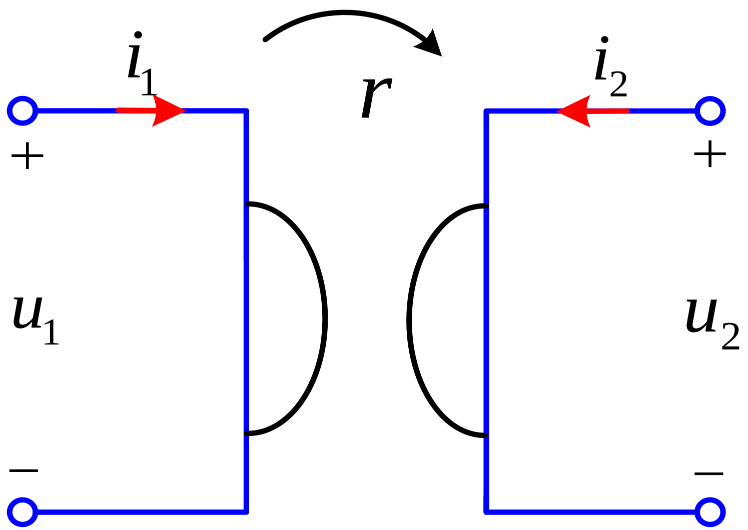
$$\begin{cases} \dot{I}_1 = (Y_{11}' + Y_0)\dot{U}_1 + (Y_{12}' - Y_0)\dot{U}_2 \\ \dot{I}_2 = (Y_{21}' - Y_0)\dot{U}_1 + (Y_{22}' + Y_0)\dot{U}_2 \end{cases}$$

$$\mathbf{Y} = \begin{pmatrix} Y_{11}' + Y_0 & Y_{12}' - Y_0 \\ Y_{21}' - Y_0 & Y_{22}' + Y_0 \end{pmatrix}$$

1. 回转器

电路符号、参数、VCR、功能关系

电路符号：



方程：

$$u_1 = -ri_2 \quad \text{或} \quad i_1 = gu_2$$

$$u_2 = ri_1 \quad\quad i_2 = -gu_1$$

r ——回转电阻

g ——回转电导

把一个端口的电流回转为另一个端口的电压或逆过程

特点1：*不满足互易定理

$$u_1 = -ri_2 \qquad i_1 = gu_2$$

$$u_2 = ri_1 \qquad i_2 = -gu_1$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & o \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

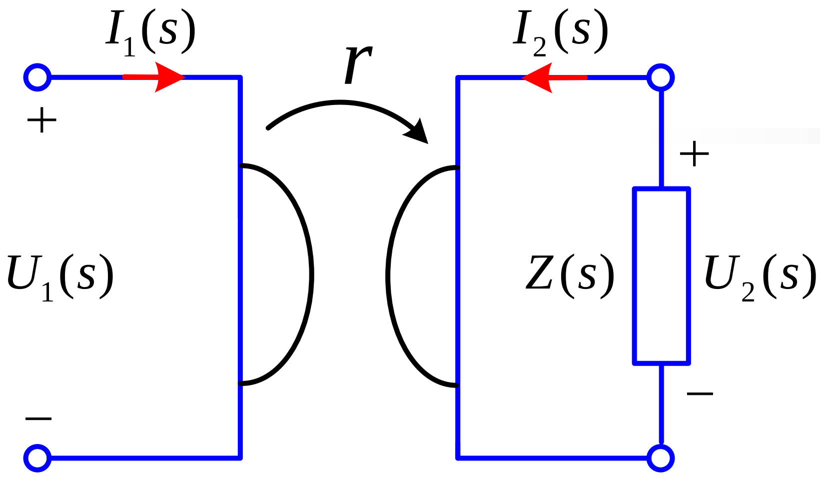
$$\mathbf{Z} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$u_1 i_1 + u_2 i_2 = -r i_2 i_1 + r i_1 i_2 = 0$$

$$u_1 i_1 + u_2 i_2 = g u_2 u_1 - g u_2 u_1 = 0$$

特点2： 回转器是线性无源元件，不发出也不消耗功率。

特点3：倒换阻抗



$$Z_i(s)=r^2\frac{1}{Z(s)}$$

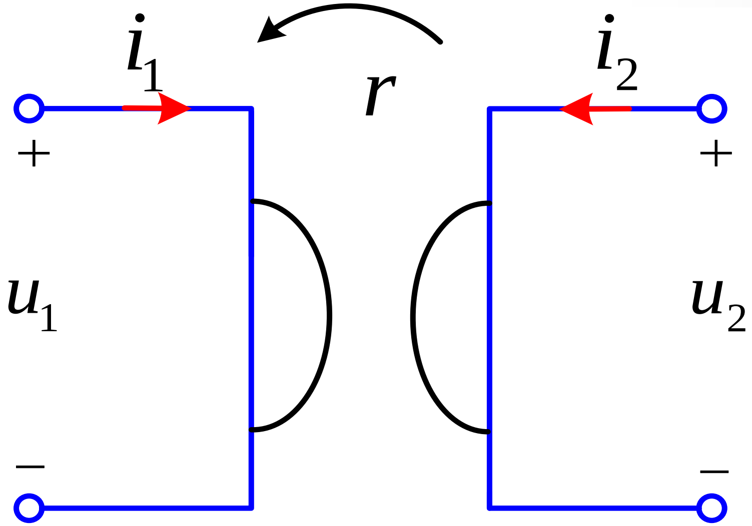
$$Z_i(s)=\frac{U_1(s)}{I_1(s)}=\frac{-rI_2(s)}{I_1(s)}=\frac{r\dfrac{U_2(s)}{Z(s)}}{I_1(s)}=\frac{\dfrac{r^2}{Z(s)}I_1(s)}{I_1(s)}=r^2\frac{1}{Z(s)}$$

若 $Z(s)=\dfrac{1}{sC}$ $\quad Z_i(s)=r^2sC \quad L_e=r^2C$

$Z(s)=sL \quad Z_i(s)=r^2\dfrac{1}{sL} \quad C_e=\dfrac{L}{r^2}$

$Z(s)=0$(短路) $\quad Z_i(s)=\infty$(开路)

$Z(s)=\infty$(开路) $\quad Z_i(s)=0$(短路)



方程：

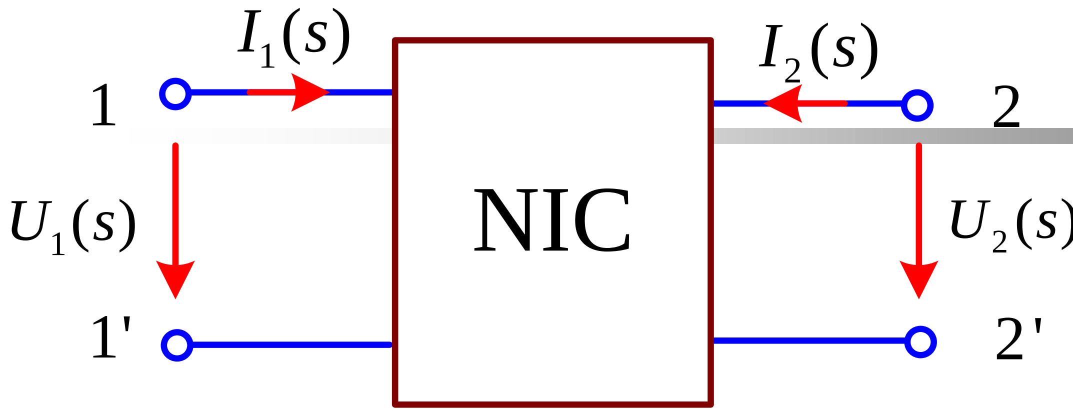
$$u_1 = r i_2 \quad 或 \quad i_1 = -g u_2$$

$$u_2 = -r i_1 \quad\quad i_2 = g u_1$$

$$Z_i(s) = r^2 \frac{1}{Z(s)}$$

2. 负阻抗变换器

电路符号：



Negative Impedance Converter

电流反向型

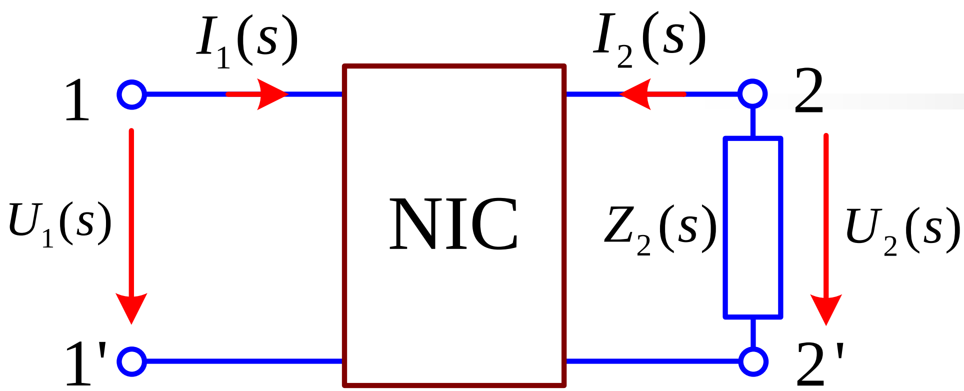
$$U_1(s) = U_2(s)$$
$$I_1(s) = kI_2(s)$$

$$\begin{bmatrix} U_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -k \end{bmatrix} \begin{bmatrix} U_2(s) \\ -I_2(s) \end{bmatrix}$$

电压反向型

$$U_1(s) = -kU_2(s)$$
$$I_1(s) = -I_2(s)$$

$$\begin{bmatrix} U_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} -k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_2(s) \\ -I_2(s) \end{bmatrix}$$



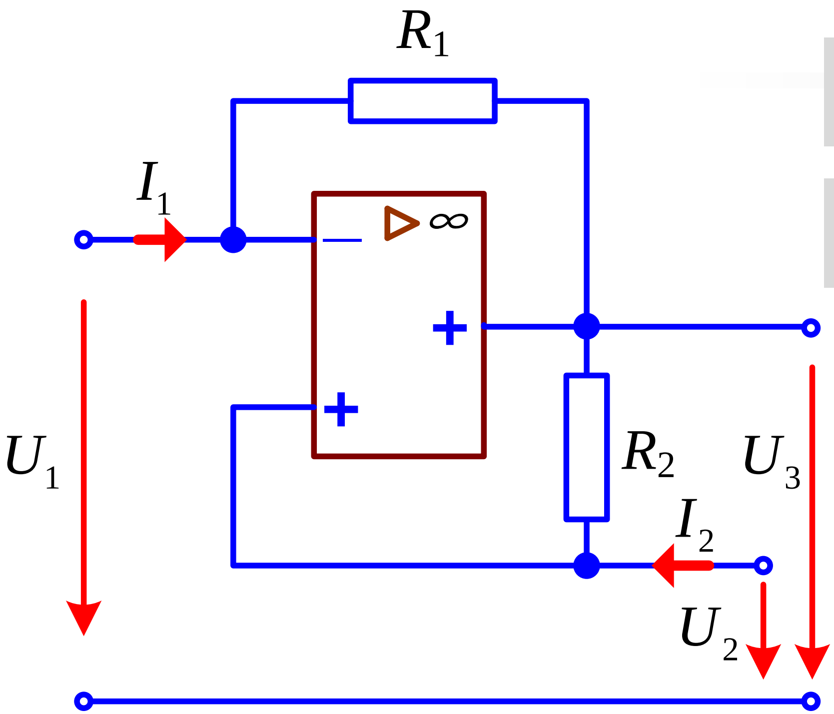
$$Z_i(s) = -\frac{Z_2(s)}{k}$$

$$Z_i(s) = \frac{U_1(s)}{I_1(s)} = \frac{U_2(s)}{kI_2(s)} = -\frac{Z_2(s)}{k}$$

端口$2-2'$接R　　端口$1-1'$为$-\frac{1}{k}R$

端口$2-2'$接L　　端口$1-1'$为$-\frac{1}{k}L$

端口$2-2'$接C　　端口$1-1'$为$-kC$



由虚短： $U_1 = U_2$

由虚断： $= \dfrac{U_1 - U_3}{R_1}$

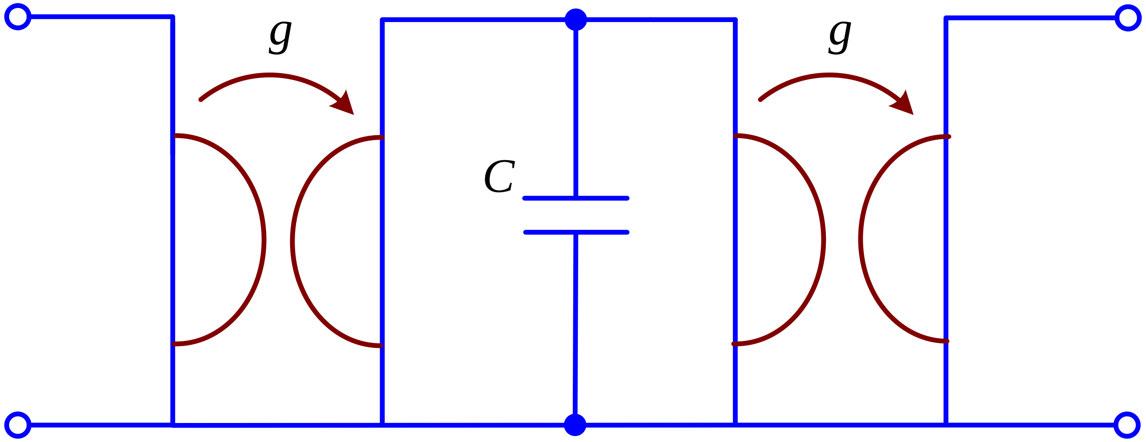
$$I_2 = \frac{U_2 - U_3}{R_2}$$

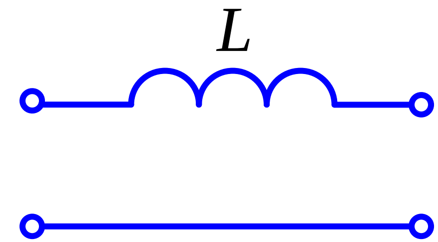
$$I_1 = \frac{R_2}{R_1} I_2$$

$$k = \frac{R_2}{R_1}$$

电流反向型负阻抗变换器

【例】 求两电路等效的条件。



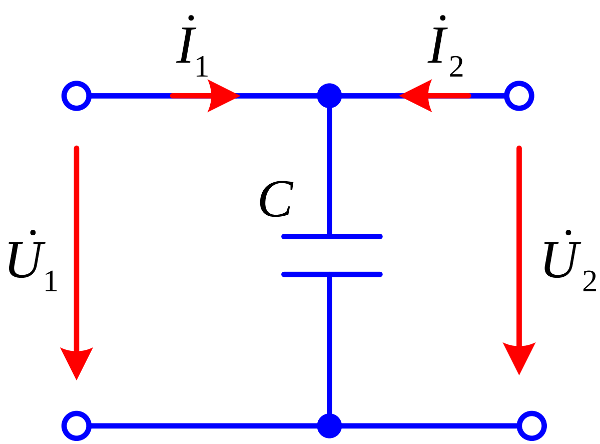


解

$$u_1 = -ri_2$$

$$u_2 = ri_1$$

$$\mathbf{T}_1 = \begin{pmatrix} 0 & \dfrac{1}{g} \\ g & 0 \end{pmatrix}$$

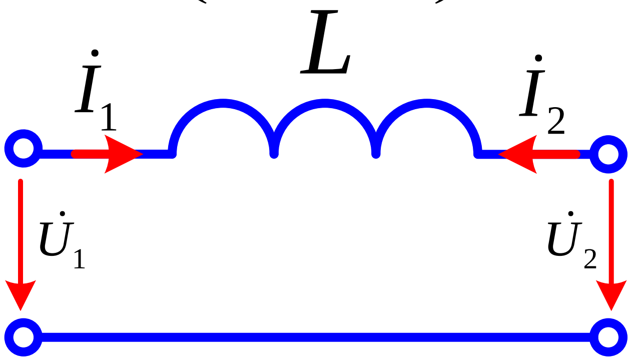


$$\dot{U}_1 = \dot{U}_2$$

$$\dot{I}_1 = \mathrm{j}\omega C\dot{U}_2 - \dot{I}_2$$

$$\mathbf{T}_2 = \begin{pmatrix} 1 & 0 \\ \mathrm{j}\omega C & 1 \end{pmatrix}$$

$$\mathbf{T}=\begin{pmatrix}0 & \frac{1}{g}\\ g & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ \mathrm{j}\omega C & 1\end{pmatrix}\begin{pmatrix}0 & \frac{1}{g}\\ g & 0\end{pmatrix}=\begin{pmatrix}1 & \mathrm{j}\frac{\omega C}{g^2}\\ 0 & 1\end{pmatrix}$$

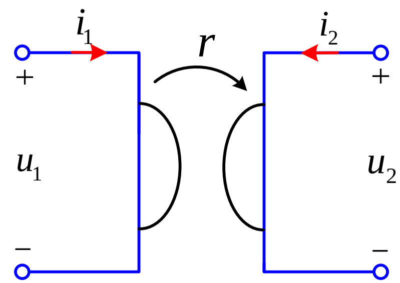


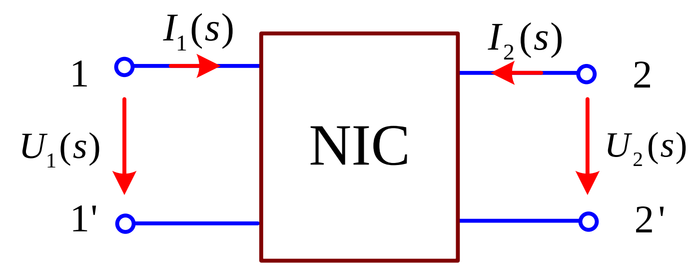
$$\mathbf{T}'=\begin{pmatrix}1 & \mathrm{j}\omega L\\ 0 & 1\end{pmatrix}$$

$$\dot{U}_1=\dot{U}_2-\mathrm{j}\omega L\dot{I}_2$$

$$\dot{I}_1=-\dot{I}_2$$

$$\Rightarrow \quad L=\frac{C}{g^2}$$





相同点 四端元件

有源元件组成

不同点

$$u_1 = -ri_2 \qquad i_1 = gu_2$$

$$u_2 = ri_1 \qquad i_2 = -gu_1$$

$$U_1(s) = U_2(s)$$

$$I_1(s) = kI_2(s)$$

$$U_1(s) = -kU_2(s)$$

$$I_1(s) = -I_2(s)$$

阻抗的倒数

$$Z_i(s) = r^2 \frac{1}{Z(s)}$$

无源元件

阻抗为负数

$$Z_i(s) = -\frac{Z_2(s)}{k}$$

不是无源元件

作业

- 16-11　　【T型并联】
- 16-12　　【T1，T2】
- 16-14　　【回转器】