习题1.

$$\begin{cases} \dot{x} = f(x) + g(x)(v + \delta(x, v)). \\ \|\delta(x, v)\|_2 \le \rho(x) + k_0\|v\|_2 \quad , 0 \le k_0 < 1. \\ \frac{\partial V}{\partial x} f(x) \le -\alpha_3(x). \quad \alpha_3(x) \ge \phi^2(x). \quad \rho(x) \le \rho_1 \phi(x). \quad w = (\frac{\partial V}{\partial x} g)^T. \end{cases}$$

当 $v = -kw$ 时.

$\because \frac{\partial V}{\partial x} f(x) \le -\alpha_3(x)$. 即 $\dot{V}$ 对标称模型 $\dot{x} = f(x) + g(x)v$ 为Lyapunov函数

$\therefore \dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + Gv) + \frac{\partial V}{\partial x} G(v + \delta)$ -

$\dot{V} \le -\alpha_3(x) + \frac{\partial V}{\partial x} G(v + \delta)$. 代入 $w = (\frac{\partial V}{\partial x} g)^T$.

$\dot{V} \le -\alpha_3(x) + w^T v + w^T \delta$.

$\because \|\delta(x, v)\|_2 \le \rho(x) + k_0\|v\|_2$. $0 \le k_0 < 1$.

$\therefore w^T v + w^T \delta \le \|w\|_2 \|\delta\|_2 + w^T v \le w^T v + \|w\|_2 [\rho + k_0\|v\|_2]$.

此时. $v = -kw$

$\therefore w^T v + w^T \delta \le -k\|w\|_2 + \rho\|w\|_2 + k_0 k\|w\|_2$

$= -k(1-k_0)\|w\|_2 + \rho\|w\|_2$.

$\therefore$ 当 $k \ge \frac{\rho}{1-k_0}$ 时. $w^T v + w^T \delta \le -\rho\|w\|_2 + \rho\|w\|_2 = 0$

$\therefore$ 此时 $\dot{V} \le -\alpha_3(x) + w^T v + w^T \delta = -\alpha_3(x)$

$\therefore$ 闭环系统原点在 $v = -kw$ 时渐近稳定.

习题2.

$$\begin{cases} \dot{x} = f(x) + g(x)(u + \delta(t, x, u)). \\ \dot{x} = f(x) + g(x)\psi(x) \text{ 渐近稳定} \\ \exists V, \ s.t \ \frac{\partial V}{\partial x}(f + g\psi) \le -\alpha_3(x). \quad \alpha_3(x) > 0 \end{cases}$$

求证：if $\|\psi(x) - \delta(t, x, u)\|_2 \le \rho(x) + k_0 \|u\|_2$. $0 \le k_0 < 1$.

采用 $u = v = -\frac{\eta w}{\|w\|_2}$，$\eta \ge \frac{\rho(x)}{1-k_0}$ 也可S.

原先证明策略中，构造 $u = \psi \Rightarrow u = \psi + v$，为的是将

$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + G\psi) \le \alpha \Rightarrow \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + G\psi) + \frac{\partial V}{\partial x} G(v + \delta - \psi) \le \alpha$. 新

此时以 $u = v = -\eta\frac{w}{\|w\|_2}$ 替代 $u = \psi + v$. 则说明 $\psi = 0$ 时仍然可证，

$\because \|-\delta(t, x, u)\|_2 \le \rho(x) + k_0\|u\|_2$. $0 \le k_0 < 1$.

$\therefore$ 定有 $u = \frac{-\eta w}{\|w\|_2}$，使 $w^T u + w^T \delta \le -\eta(1 - k_0)\|w\|_2 + \rho\|w\|_2$

$\eta \ge \frac{\rho}{1-k_0}$，$w^T u + w^T \delta \le -\rho\|w\|_2 + \rho\|w\|_2 = 0$

$\dot{x} = (f + G\psi) + G(u + \delta - \psi) \Rightarrow \dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{\partial V}{\partial x}[G\psi] + \frac{\partial V}{\partial x}[G(u + \delta - \psi)] \le 0$

$\therefore$ 原点依然稳定

习题3. $\dot{x} = u + \delta(t, x)$ $\quad \delta(x) = a_0 + a_1 x + a_2 x^2$.

$\psi(x, t) = -x$. $\quad V = \frac{1}{2}x^2$

$\therefore x = 0$ 对于 $\dot{x} = -x$（标称）为 G.A.S.

$w = \frac{\partial V}{\partial x} \cdot g = x$.

$\therefore v = -kw\|T\|_2^2 = -kx$

$\dot{x} = -x - kx + \delta$. $\quad \dot{V} = x\dot{x} = -x^2 - kx^2 + \frac{\partial \delta}{\partial x}$.

$\therefore$ 当 $\delta(t)$ 为有界函数且 $|\delta(t)| \le k_0$ 时，总能设计 v 使系统毕竟一致有界.