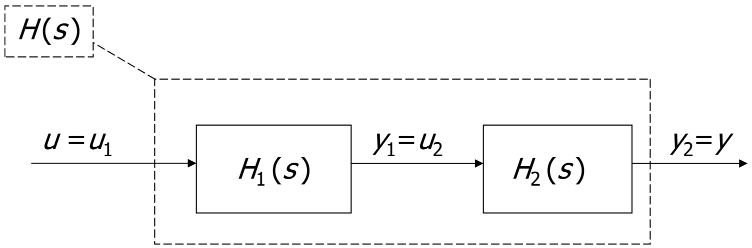
Automatic Control

The structure of feedback control systems

Relevant tf of feedback control systems

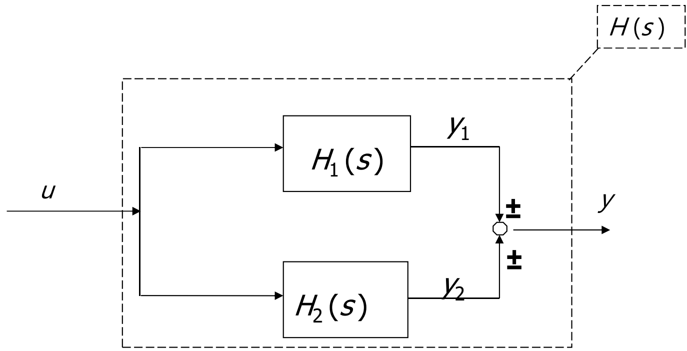
Transfer function connection and block algebra

Series connection of transfer functions



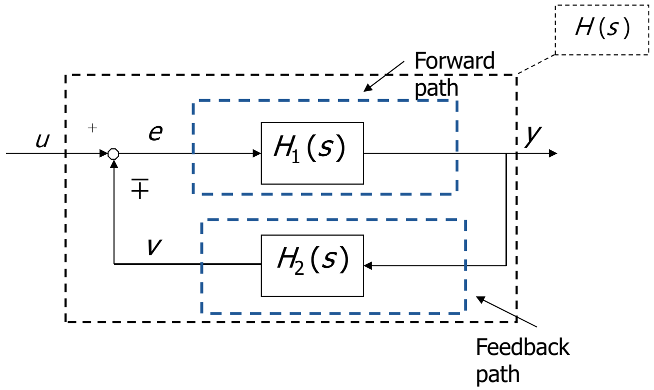
$$Y(s) = Y_2(s) = H_2(s)U_2(s) = H_2(s)Y_1(s) =$$
$$= H_2(s)H_1(s)U_1(s) = H_2(s)H_1(s)U(s) = H(s)U(s)$$
$$\Downarrow$$
$$H(s) = H_2(s)H_1(s)$$

Parallel connection of transfer functions



$$Y(s) = \pm Y_1(s) \pm Y_2(s) = \pm H_1(s)U(s) \pm H_2(s)U(s) =$$
$$= \left[\pm H_1(s) \pm H_2(s)\right]U(s) = H(s)U(s)$$
$$\Downarrow$$
$$H(s) = \pm H_1(s) \pm H_2(s)$$

Feedback connection of transfer functions

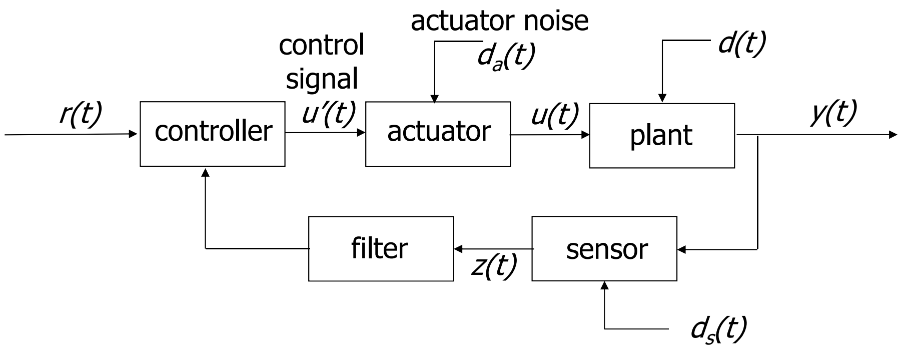


$$H(s) = \frac{H_1(s)}{1 \pm H_1(s)H_2(s)}$$

The structure of a feedback control system

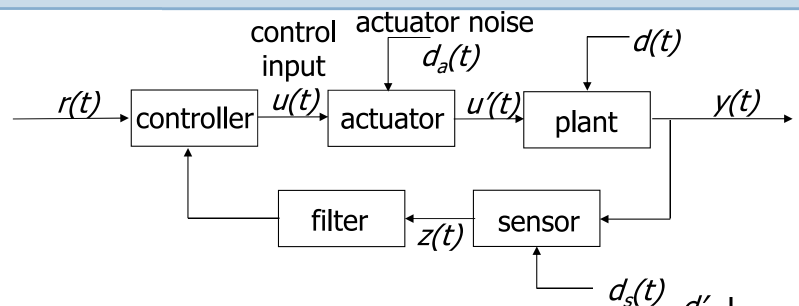
The general structure of a control system

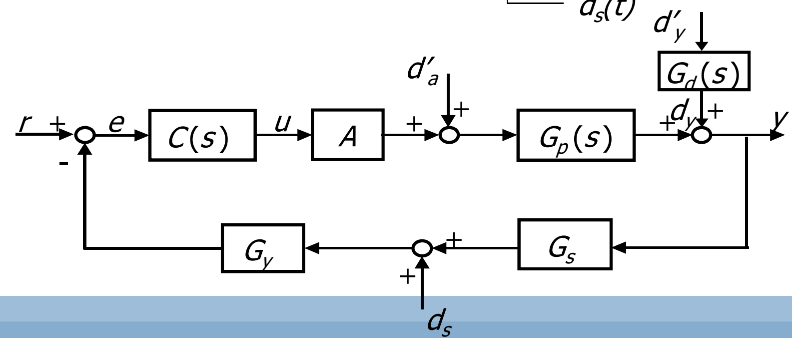
We start from the scheme of principle of a feedback control system previously introduced (see AC_L01 p. 9)



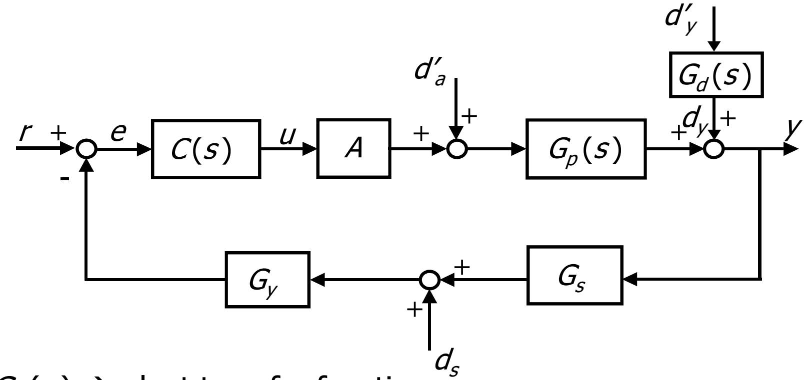
... the next step is to describe the dynamic behavior of each block

The general structure of a control system



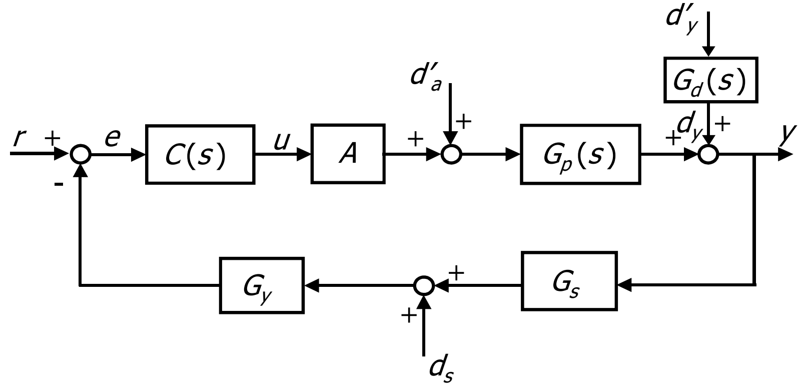


The general structure of a control system



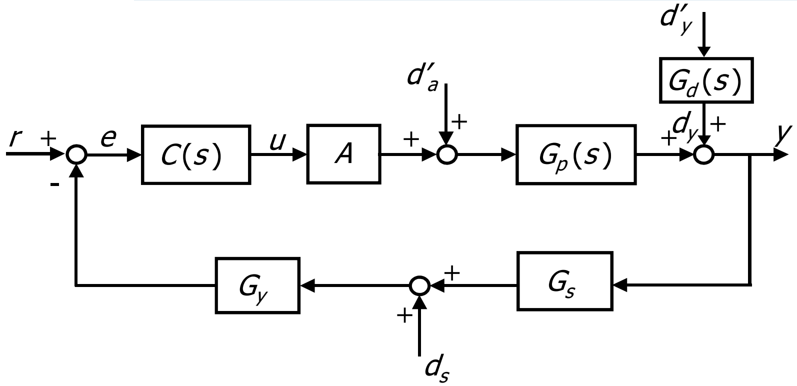
- $G_p(s)$ → plant transfer function
- $C(s)$ → controller tf
- A → actuator gain
- G_s → Sensor gain, G_y → Conditioning filter gain

The general structure of a control system



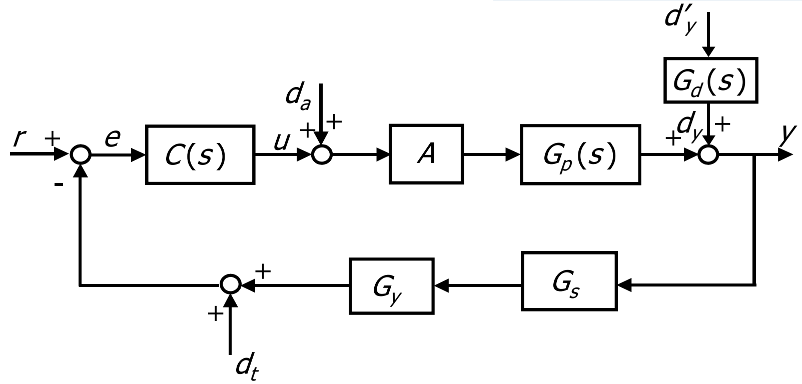
- r : reference signal → desired behavior of the controlled output
- y : (controlled) output
- $e = r - y$ → tracking error
- u : control input (command signal)

The general structure of a control system



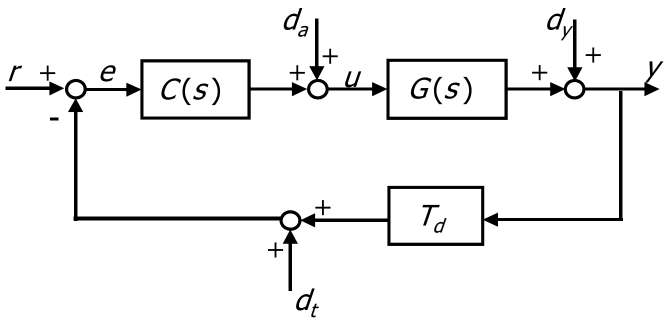
- d'_a : input disturbance
- d'_y : output disturbance, $G_d(s)$ → tf between d'_y and y
- d_s : sensor noise

A simplified structure



- $d_a = d'_a \,/\, A$ actuator disturbance
- $G_d(s) = 1$ → $d'_y = d_y$
- $d_t = d_s\, G_y$

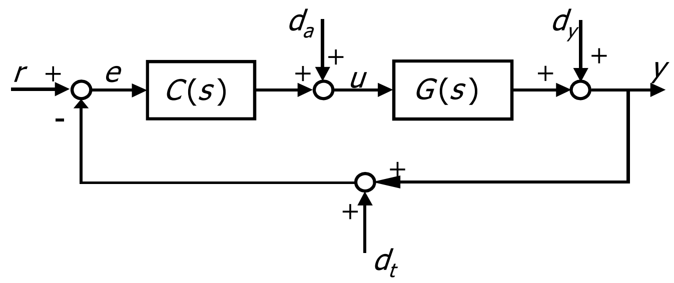
A simplified structure

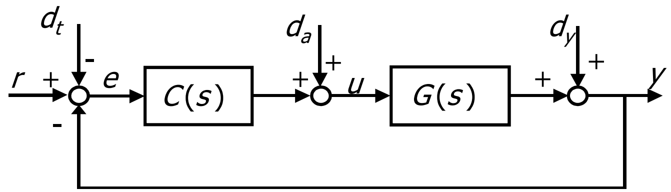


- $G(s) = G_p(s)\ A$
- $T_d = G_y G_s$

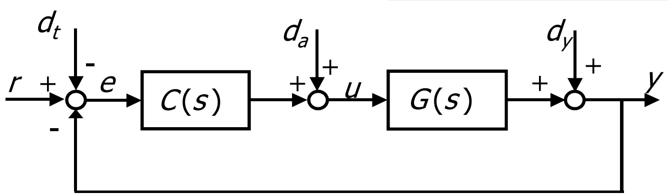
A simplified structure

- $T_d = 1 \rightarrow$





The loop function $L(s)$

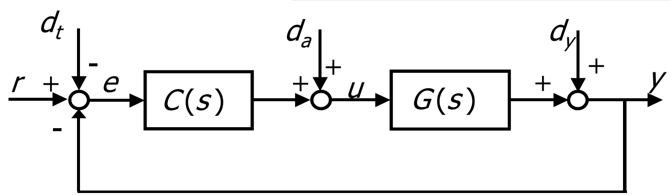


- The **loop function** is defined as

$$L(s) = G(s)\ C(s)$$

- $L(s)$ is made up by the product of all the tf in the loop

The sensitivity function $S(s)$



- The **sensitivity function** is defined as

$$S(s) = \frac{1}{1 + L(s)}$$

The sensitivity function $S(s)$

$$S(s) = \frac{1}{1+L(s)}$$

- The **sensitivity function** represents:
  - the transfer function between r and e

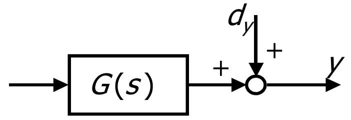
$$S(s) = \frac{e(s)}{r(s)}$$

  - the transfer function between d_y and y

$$S(s) = \frac{y(s)}{d_y(s)}$$

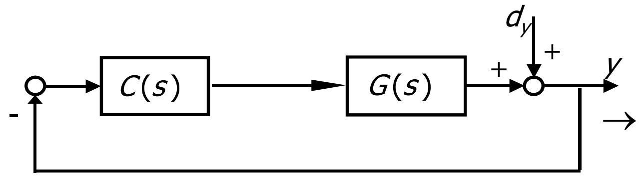
The sensitivity function $S(s)$

- The term **sensitivity function** is due to the fact that it describes the ability of the control action in reducing the "sensitivity" of the controlled system to the output disturbance w.r.t. the uncontrolled case
  - Uncontrolled system (i.e. open loop)



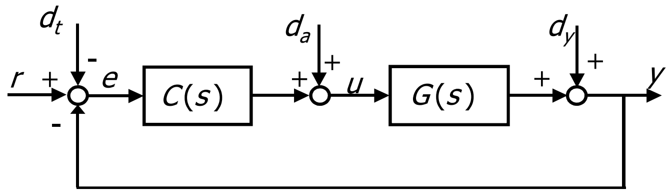
$$\rightarrow \frac{y(s)}{d_y(s)} = 1$$

  - Controlled system (i.e. closed loop)



$$\rightarrow \frac{y(s)}{d_y(s)} = \frac{1}{1+L(s)} = S(s)$$

The complementary sensitivity function $T(s)$



- The **complementary sensitivity function** is defined as

$$T(s) = \frac{L(s)}{1+L(s)}$$

- Note that:

$$S(s) + T(s) = \frac{1}{1+L(s)} + \frac{L(s)}{1+L(s)} = 1$$

The complementary sensitivity function $T(s)$

$$T(s) = \frac{L(s)}{1+L(s)}$$

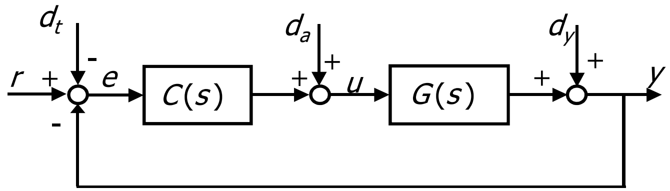
- The **complementary sensitivity function** represents:
  - the transfer function between r and y

$$T(s) = \frac{y(s)}{r(s)}$$

  - the transfer function between d_t and y (except for the sign)

$$T(s) = -\frac{y(s)}{d_t(s)}$$

The control sensitivity function $R(s)$



- The **control sensitivity function** is defined as

$$R(s) = \frac{C(s)}{1 + L(s)}$$

The control sensitivity function $R(s)$

$$R(s) = \frac{C(s)}{1 + L(s)}$$

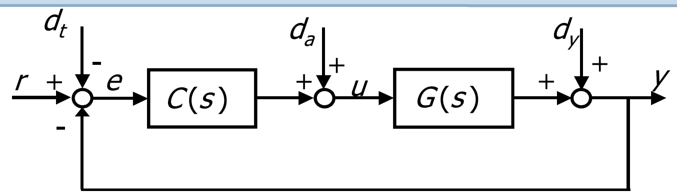
- The **control sensitivity function** represents:
  - the transfer function between r and u

$$R(s) = \frac{u(s)}{r(s)}$$

  - the transfer function between d_t and u (except for the sign)

$$R(s) = -\frac{u(s)}{d_t(s)}$$

The function $Q(s)$



- We define also the function: $Q(s) = \dfrac{G(s)}{1 + L(s)}$

which represents the transfer function between d_a and y

$$Q(s) = \frac{y(s)}{d_a(s)}$$

Function $Q(s)$ is also known as the **actuator disturbance sensitivity function**