3.7. $X(k) = DFT[x(n)]_N$

$$\because \begin{cases} \frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot X^*(k). \\ \sum_{k=0}^{N-1}|x(n)|^2 = \sum_{k=0}^{N-1} x(n)\cdot x^*(n). \\ x(n) = IDFT[X(k)]_N = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-kn}. \end{cases}$$

$$\therefore \sum_{k=0}^{N-1}|x(n)|^2 = \sum_{k=0}^{N-1} x^*(n)\left(\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-kn}\right) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\sum_{k=0}^{N-1} x^*(n)\, W_N^{-kn}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\sum_{k=0}^{N-1}\left(x(n)\, W_N^{kn}\right)^* = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot X^*(k) = \frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$$

补充题:

长度为N的序列x[n].

$$X(z) = ZT[x(n)] = \sum_{n=0}^{N-1} x(n) z^{-1}$$

$$X(e^{j\omega}) = FT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}.$$

$$X(k) = DFT[x(n)] = \sum_{n=0}^{N-1} x(n)\, W^{kn}$$

$\because X(k) = X(e^{j\omega})|_{\omega=\frac{2\pi}{N}k}$. $k=0, 1, 2 \cdots N-1$

∴ 序列的DFT是FT在频域上区间[0. 2π]内的N点等间隔采样.

$\because X(k) = X(z)|_{z=e^{j\frac{2\pi}{N}k}}$, $k=0.1.2\cdots N-1$

∴ 序列的DFT是ZT单位圆上N点等间隔采样.

$\because FT = X(z)|_{z=e^{j\omega}}$

∴ 序列的FT是ZT单位圆上的取值.