Automatic Control AC.L16

Lag network design examples

1) $T(s) = \dfrac{L(s)}{1+L(s)} = \dfrac{N_L(s)}{D_L(s)+N_L(s)}$

$L(s) = \dfrac{N_L(s)}{D_L(s)}$

if $L(s)$ contains a lag network

→ N_L has a zero at low frequency

⇒ $T(s)$ has a zero at low fr.

2) at low frequency

$|T(j\omega)| \simeq 1$

LF zero in L & T

LF pole in T induced by the presence of the LF zero in L (lag fcn)

⇓

T contains a pole with a bigger time constant w.r.t. the dominating one. ⇒ tail effect

Ex. Lag 1

After steady state design

$$L'(s) = C_{ss}(s) \cdot G(s)$$

$$|L'(j\omega_{c,des})| = 19{,}7\,dB \qquad \omega_{c,des} = 1{,}9\ rad/s$$

$$\measuredangle L'(j\omega_{c,des}) = -122°$$

phase is OK $\rightarrow$ magnitude attenuation at $\omega_{c,des}$ is required

$$\left[\ |y_{d_a}^{\infty}| \le 0{,}1 \qquad d_a(t) = \delta_a t\,\varepsilon(t) \qquad |\delta_a| \le 1\right.$$

$$\longrightarrow C_{ss}(s) = \frac{K_c}{s}$$

$$\left.\left|\frac{\delta_a}{K_c}\right| \le 0{,}1 \Rightarrow |K_c| \ge 10 \ \ldots\ldots\ K_c = 10\ \right]$$

Lag network is needed

Lag ntwk design:

$$m_I = |L'(j\omega_{c,des})| = 10^{\frac{19{,}7}{20}} = \ldots \quad \sim 10$$

$$\omega_I = \frac{\omega_{c,des}}{\alpha\, m_I} \stackrel{\uparrow}{=} \frac{1{,}9}{10 \cdot 10} = 0{,}019\ rad/s$$

$\alpha = 10$

After Lag network

$$L''(s) = C_I(s)\, L'(s)$$

$$C_I = \frac{1 + \frac{s}{m_I \omega_I}}{1 + \frac{s}{\omega_I}}$$

$$\left.\begin{array}{l} |L''(j\omega_{c,des})| \cong 0\,dB \\ \angle L''(j\omega_{c,des}) = -127° \end{array}\right\} \rightarrow \text{simulation}$$

$\hat{s} = 18{,}17\,\%$ ok

$t_r = 0{,}93\,s$

$|y_{d_a}^{\infty}| = 0{,}1$ ok

$$C(s) = \frac{K_c}{s} \cdot \frac{1 + \frac{s}{m_I \omega_I}}{1 + \frac{s}{\omega_I}}$$

dc-gain form

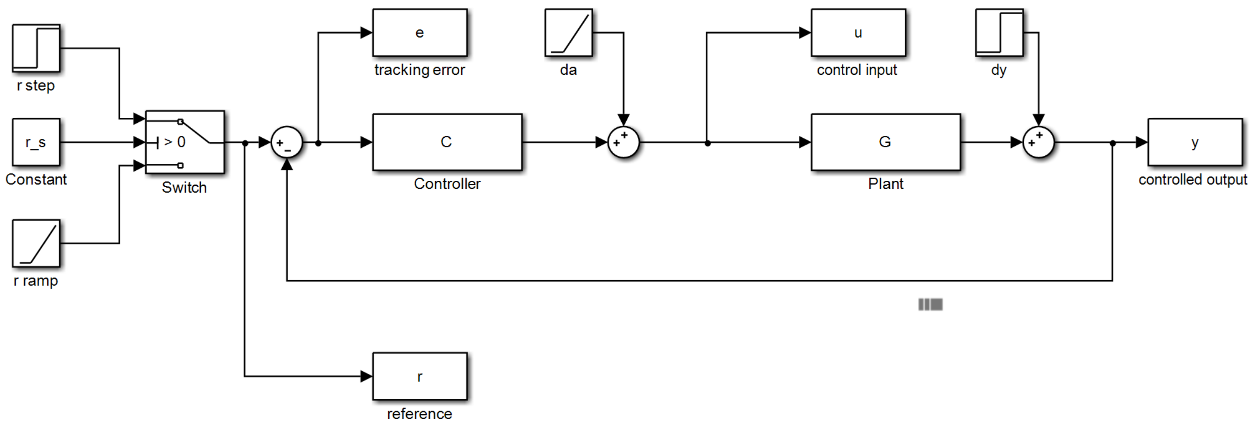
$K_c = 10$

$m_I = 9{,}6605$

$\omega_I = 0{,}0197$

```
clear all
close all
clc

s=tf('s');

% Plant tf
G=2/((0.1*s+1)*(0.2*s+1));

% steady state controller
Kc=10
C_SS=Kc/s;
L1=G*C_SS;% loop function update

% transient requirements
T_p=1.72;
S_p=3.63;
wc_des=1.9;

% nichols diagram for L1
figure(1)
nichols(L1,'b'), hold on
T_grid(T_p)
S_grid(S_p)
%return

% lag network design
mI=10^(19.7/20)
alpha=10;
wI=wc_des/(alpha*mI)
C_I=(1+s/(mI*wI))/(1+s/(wI));
L2=L1*C_I; % loop function update
C=C_SS*C_I % controller tf update
figure(1)
nichols(L2,'r')
%return

% simulation with step reference signal
r_s = 1; % switch impose step reference
rho = 1;
delta_a = 0;
delta_y = 0;
t_stop = 20;
sim('control_structure_sim_1')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
```

```
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')

%return
% simulation with disturbance da
r_s = 1; % switch impose step reference
rho = 0;
delta_a = 1;
delta_y = 0;
t_stop = 100;
sim('control_structure_sim_1') % modify control_structure_sim.slx
                               % since disturbance d_a is a ramp
figure
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
```



Ex. Lag 2

after steady state design $\quad L'(s) = C_{ss}(s)\,G(s)$

$$\omega_{c,des} = 4 \text{ rad/s}$$

$$|L'(j\omega_{c,des})| \cong 1,69 \text{ dB}$$

$$\angle L'(j\omega_{c,des}) = -163^\circ$$

$$\downarrow$$

$$-123^\circ \div -108^\circ$$

$$\Delta\varphi \cong 40^\circ \div 55^\circ$$

due to the need of a lag network in the next design step

→ phase lead action is needed $\quad C_D(s) = \dfrac{1 + \frac{s}{\omega_D}}{1 + \frac{s}{m_D \omega_D}}$

$m_D = 12$

$\omega_{norm} = 3$ $\quad \omega_D = \dfrac{\omega_{c,des}}{\omega_{norm}} = \dots$

After lead network $\quad L''(s) = C_D(s)\,L'(s)$

$$|L''(j\omega_{c,des})| \cong 11,4 \text{ dB}$$

$$\angle L''(j\omega_{c,des}) = -106^\circ \rightarrow \text{ok}$$

→ magnitude attenuation is needed at $\omega_{c,des}$

$$\Downarrow$$

lag network

$$C_I(s) = \frac{1+\frac{s}{m_I \omega_I}}{1+\frac{s}{\omega_I}}$$

$$m_I = 10^{\frac{11,4}{20}} = \dots$$

$$\omega_I = \frac{\omega_{c,des}}{\underset{\substack{\downarrow \\ 10}}{\alpha}\, m_I} = \dots$$

After lag network

$$L'''(s) = C_I(s)\, L''(s)$$

$$|L'''(j\omega_{c,des})| \cong 0 \text{ dB}$$

$$\angle L'''(j\omega_{c,des}) \cong -110^\circ$$

OK on Nichols plane

Simulation

$\hat{s} = 5{,}8\ \%$ OK

$t_r = 0{,}55\ s$ No

$\Downarrow$

modify lag network design to obtain a greater value of ω_c

$\Downarrow$

$$m_I = 10^{\frac{10}{20}} =$$

$\Downarrow$

$\hat{s} \cong 5{,}5\ \%$ ok

$t_r = 0{,}467\ s$ ok

$|e_r^{\infty}| = 0{,}2$ $\quad (r(t) = 2t\,\varepsilon(t))$

$$C(s) = K_c \frac{1+\frac{s}{\omega_D}}{1+\frac{s}{m_D\omega_D}} \cdot \frac{1+\frac{s}{m_I\omega_I}}{1+\frac{s}{\omega_I}}$$

$K_c = 20$ $\quad m_D = 12$ $\quad m_I = 3{,}1623$

$\omega_D = 1{,}\bar{3}$ $\quad \omega_I = 0{,}1265$

```matlab
clear all
close all
clc

s=tf('s');

% Plant tf
G=(s+0.2)/(s*(s+0.4)*(s+1));

% steady state controller
Kc=20
C_SS=Kc;
L1=G*C_SS;% loop function update

% transient requirements
T_p=0.42;
S_p=2.68;
wc_des=4;

% nichols diagram for L1
figure(1)
nichols(L1,'b'), hold on
T_grid(T_p)
S_grid(S_p)
%return

% lead network design
mD=12
wnorm=3;
wD=wc_des/wnorm
C_D=(1+s/wD)/((1+s/(mD*wD)));
L2=C_D*L1; % loop function update
C=C_D*C_SS; % controller tf update
figure(1), hold on
nichols(L2,'r')
%return

% lag network design
mI=10^(10/20)
alpha=10;
wI=wc_des/(alpha*mI)
C_I=(1+s/(mI*wI))/(1+s/(wI));
L3=L2*C_I; % loop function update
C=C*C_I % controller tf update
figure(1)
nichols(L3,'k')
%return
```

```
% simulation with step reference signal
r_s = 1; % switch impose step reference
rho = 1;
delta_a = 0;
delta_y = 0;
t_stop = 10;
sim('control_structure_sim')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')
%return
% simulation with ramp reference signal
r_s = -1; % switch imposes ramp reference
rho = 2;
delta_a = 0;
delta_y = 0;
t_stop = 100;
sim('control_structure_sim')
figure
plot(r.time,r.data,'r','linewidth',1.5)
grid on
hold on
plot(y.time,y.data,'b','linewidth',1.5)
xlabel('t (s)')
ylabel('y(t)')
legend('r(t)','y(t)')

figure
plot(e.time,e.data,'b','linewidth',1.5)
grid on
xlabel('t (s)')
ylabel('e(t)')
```