Detection and Mitigation of Corrupted Information in Distributed Model Predictive Control Based on Resource Allocation

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https://git.io/JEFGW

Context

- Energy Distribution System
- Traffic management
- Heat distribution
- Water distribution
- . . .

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- Geographically distributed
- Coupled by constraints (energy)
- Optimization objectives
  - Energy
  - User satisfaction
  - ...
- Solution → Model Predictive Control

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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$\begin{aligned} &\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{optimize}} && J(\boldsymbol{x}(k),\boldsymbol{u}(k)) \\ &\text{subject to} && \left.\begin{aligned} \boldsymbol{x}(\xi+1) = f(\boldsymbol{x}(\xi),\boldsymbol{u}(\xi)) \\ g_i(\boldsymbol{x}(\xi),\boldsymbol{u}(\xi)) \leq 0 \\ h_j(\boldsymbol{x}(\xi),\boldsymbol{u}(\xi)) = 0 \end{aligned}\right\} \begin{aligned} &\forall \xi \in \{1,\ldots,N_p\} \\ &\forall i \in \{1,\ldots,m\} \\ &\forall j \in \{1,\ldots,p\} \end{aligned} \end{aligned}$$

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Model Predictive Control

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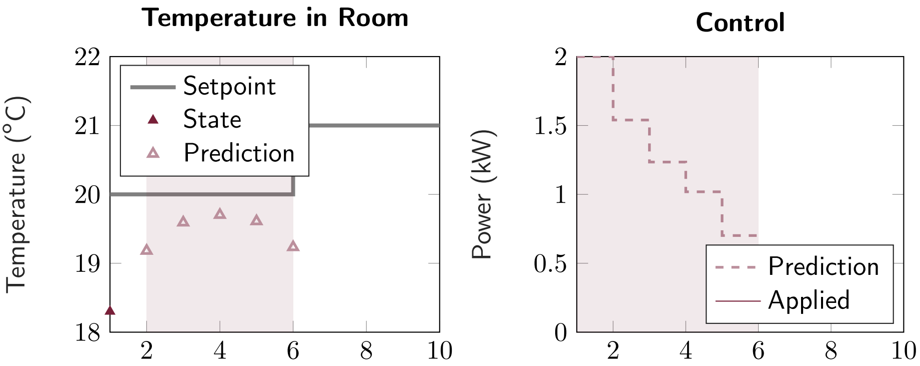
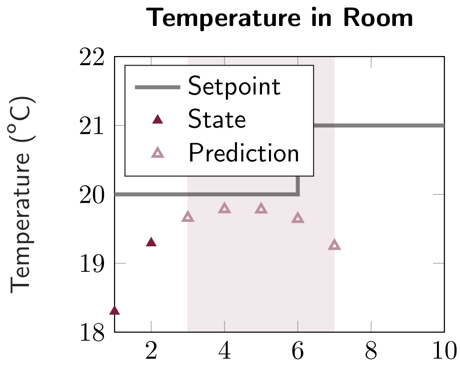
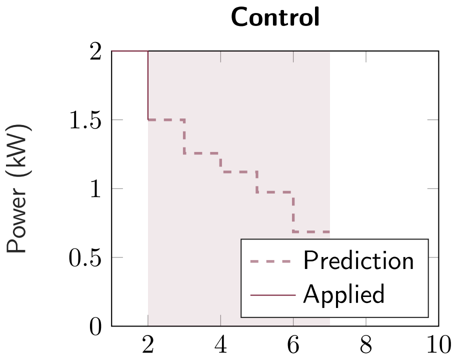
Find optimal control sequence



Model Predictive Control

Find optimal control sequence, apply first element





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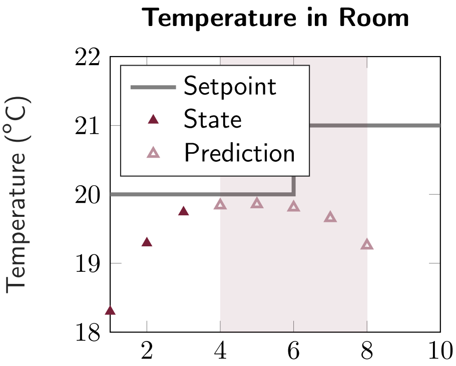
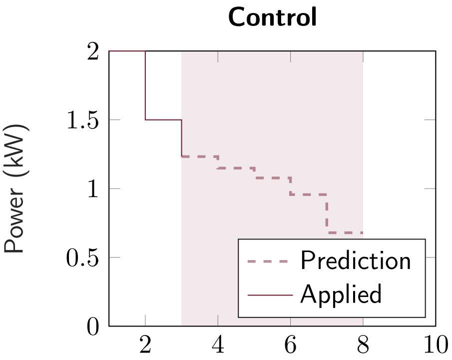
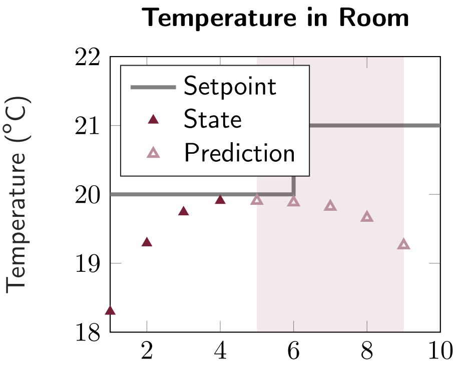
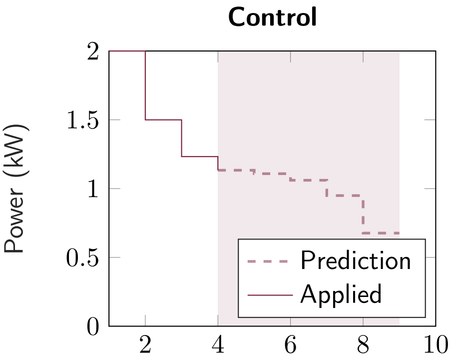
Find optimal control sequence, apply first element, rinse repeat





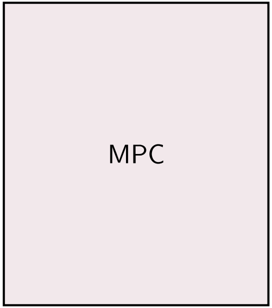
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





Distributed Model Predictive Control

- Problem: Complexity depends on N_p, m, p and sizes of $\boldsymbol{x}$ and $\boldsymbol{u}$
- Solution: Divide and Conquer



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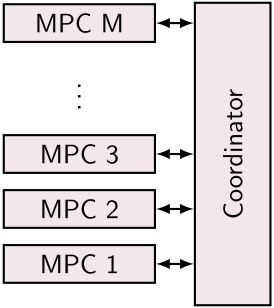

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$$
\begin{array}{ll}
\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} & \overbrace{\sum_{i=1}^{M} \underbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|^2_{Q_i} + \|\boldsymbol{u}_i(k+j-1|k)\|^2_{R_i}}_{J_i(k)}}^{J_G(k)} \\
\text{subject to} & \left.\begin{array}{c} \boldsymbol{x}_i(k+1) = A_i \boldsymbol{x}_i(k) + B_i \boldsymbol{u}_i(k) \\ \sum_{i=1}^{M} \Gamma_i \boldsymbol{u}_i(k) = \boldsymbol{u}_{\max} \end{array}\right\} \begin{array}{l} \forall i \in \{1,\dots,M\} \\ \forall j \in \{1,\dots,N_p\} \end{array}
\end{array}
$$

Distributed Model Predictive Control

$$\left.\begin{aligned} J_i^\star(\boldsymbol{\theta}_i(k)) = \underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \; & J_i(k) \\ \text{s.t.} \quad & \boldsymbol{x}_i(k+1) = A_i \boldsymbol{x}_i(k) + B_i \boldsymbol{u}_i(k) \\ & \Gamma_i \boldsymbol{u}_i(k) = \boldsymbol{\theta}_i(k) : \boldsymbol{\lambda}_i(k) \end{aligned}\right\} \begin{array}{l} \forall i \in \{1, \ldots, M\} \\ \forall j \in \{1, \ldots, N_p\} \end{array}$$

$$\begin{aligned} J^\star = \underset{\boldsymbol{\theta}(k:k+N_p-1|k)}{\text{minimize}} \; & \sum_{i=1}^{M} J_i^\star(\boldsymbol{\theta}_i(k)) \\ \text{s.t.} \quad & \sum_{i=1}^{M} \boldsymbol{\theta}_i(k) = \boldsymbol{u}_{\max} \end{aligned}$$

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$$

$$
\boldsymbol{\theta}_i^{(p+1)} = \boldsymbol{\theta}_i^{(p)} + \rho \left(\boldsymbol{\lambda}_i^\star(\boldsymbol{\theta}_i^{(p)}) - \frac{1}{M} \sum_{j=1}^{M} \boldsymbol{\lambda}_j^\star(\boldsymbol{\theta}_j^{(p)}) \right)
$$

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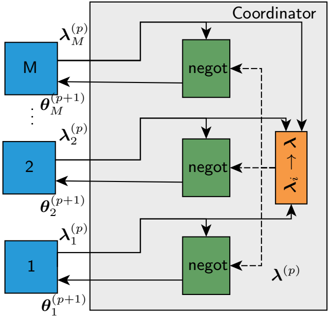

$$
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& \Gamma_i\boldsymbol{u}_i(k) = \boldsymbol{\theta}_i(k) : \boldsymbol{\lambda}_i(k)
\end{aligned}
\right\}
\begin{aligned}
&\forall i \in \{1,\dots,M\} \\
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$$

$$
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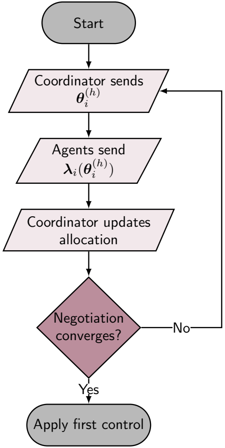
$$



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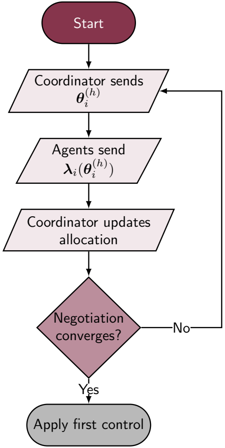
Quantity Decomposition | Resource Allocation



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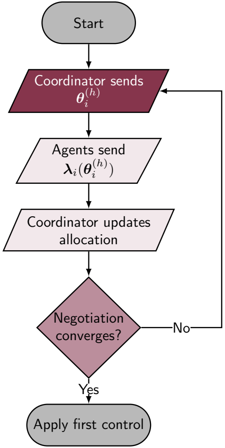

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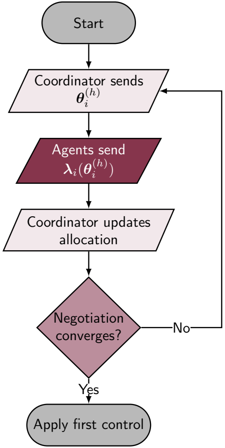
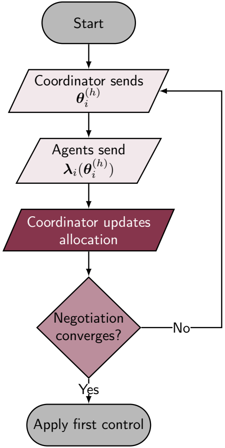

Start
Coordinator sends
$\boldsymbol{\theta}_i^{(h)}$
Agents send
$\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i^{(h)})$
Coordinator updates
allocation
Negotiation
converges?
No
Yes
Apply first control

Quantity Decomposition | Resource Allocation



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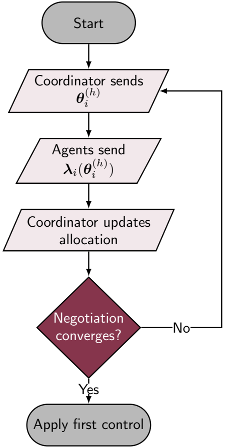

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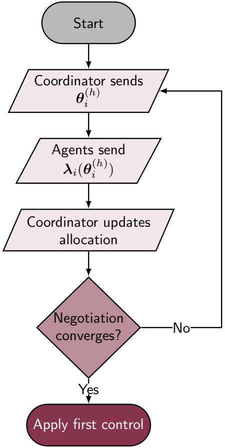

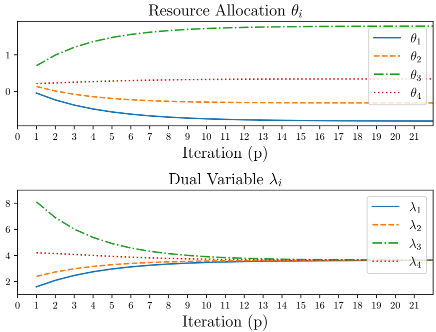

What if agents send a non-agreed $\boldsymbol{\lambda}_i$?

Outline

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Outline

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How can a non-cooperative agent attack?

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
- Non-cooperative agent sends $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$

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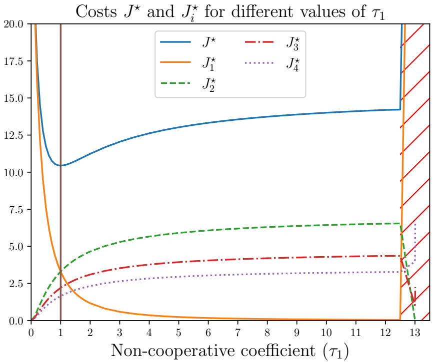
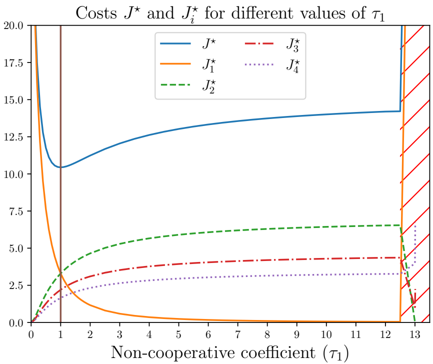
Example



4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\boldsymbol{\lambda}}_1 = \gamma_1(\boldsymbol{\lambda}_1) = \tau_1 I \boldsymbol{\lambda}_1$

Example



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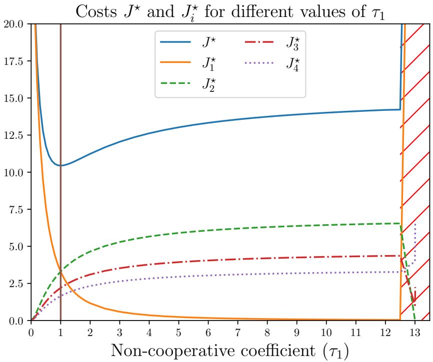

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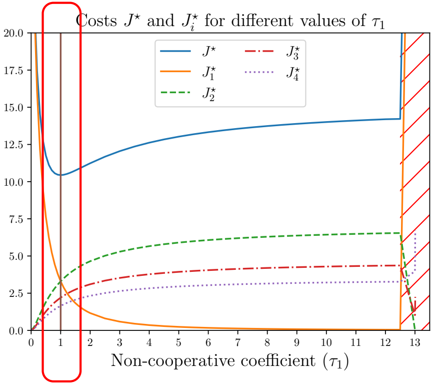

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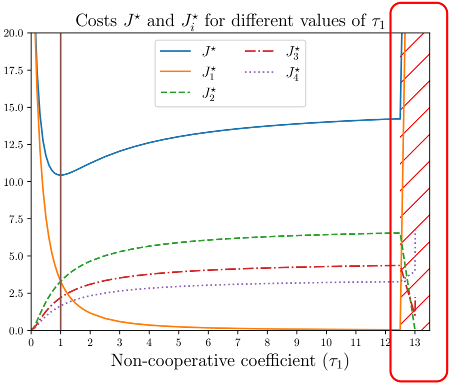

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Outline

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Quadratic Case

$$\begin{aligned}\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}}\ & \overbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|_{Q_i}^2 + \|\boldsymbol{u}_i(k+j-1|k)\|_{R_i}^2}^{J_i(k)} \\ \text{s.t.}\ & \left.\begin{aligned} \boldsymbol{x}_i(\xi+1) &= A_i\boldsymbol{x}_i(\xi) + B_i\boldsymbol{u}_i(\xi) \\ \Gamma_i\boldsymbol{u}_i(\xi) &= \boldsymbol{\theta}_i(\xi) : \boldsymbol{\lambda}_i(\xi)\end{aligned}\right\} \forall \xi \in \{1,\ldots,N_p\}\end{aligned}$$

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Quadratic Case

$$\begin{array}{cc} \underset{\boldsymbol{U}_i(k)}{\text{minimize}} & \overbrace{\frac{1}{2}\boldsymbol{U}_i(k)^T H_i \boldsymbol{U}_i(k) + \boldsymbol{f}_i(k)^T \boldsymbol{U}_i(k)}^{J_i(\boldsymbol{\theta}_i)} \\ \text{s.t.} & \Theta_i \boldsymbol{U}_i(k) = \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{array}$$

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\end{array}
$$

$$\boldsymbol{\lambda}_i = -P_i \boldsymbol{\theta}_i - \boldsymbol{s}_i(k)$$

where $P_i = (\Theta_i H_i^{-1} \Theta_i^{\mathrm{T}})^{-1}$ and $\boldsymbol{s}_i(k) = P_i \Theta_i H_i^{-1} \boldsymbol{f}_i(k)$

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Quadratic Case

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where $P_i = \left(\Theta_i H_i^{-1} \Theta_i^{\mathrm{T}}\right)^{-1}$ and $\boldsymbol{s}_i(k) = P_i \Theta_i H_i^{-1} \boldsymbol{f}_i(k)$

Quadratic Case

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Detection

Assumption

We know nominal $\bar{P}_i$

Assumption

Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i$
$-T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k) \rightarrow -\tilde{P}_i\boldsymbol{\theta}_i - \tilde{\boldsymbol{s}}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}}_i(k)\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

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Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i$
$-T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k) \rightarrow -\tilde{P}_i\boldsymbol{\theta}_i - \tilde{\boldsymbol{s}}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

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- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

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Detection

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About Estimation

- We estimate $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ simultaneously using Recursive Least Squares
- Problem: Estimation during negotiation fails
  - Consecutive $\boldsymbol{\lambda}_i^p$ and $\boldsymbol{\theta}_i^p$ are linearly dependent $\rightarrow$ low input excitation
- Solution: Send sequence of random values of $\boldsymbol{\theta}_i$ until estimates converge

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Detection
In detail

- Error $E_i(k) = \|\widehat{\tilde{P}}_i(k) - \bar{P}_i\|_F$
- Create threshold ϵ_P
- Indicator $d_i \in \{0, 1\}$ detects the attack in agent i.
- $d_i = 1$ if $E_i(k) > \epsilon_P$, 0 otherwise

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Mitigation

- Main idea: Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Assumption

We suppose $\tilde{\boldsymbol{\lambda}}_i = \mathbf{0}$ only if $\boldsymbol{\lambda}_i = \mathbf{0}$, which implies $T_i(k)$ invertible.

- Estimate the inverse of $T_i(k)$

$$\widehat{T_i(k)^{-1}} = \bar{P}_i \widehat{\tilde{P}}_i(k)^{-1}$$

- Reconstruct $\boldsymbol{\lambda}_i$

$$\boldsymbol{\lambda}_{i\mathrm{rec}} = \widehat{T_i(k)^{-1}} \tilde{\boldsymbol{\lambda}}_i = -\bar{P}_i \boldsymbol{\theta}_i - \widehat{T_i(k)^{-1}} \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

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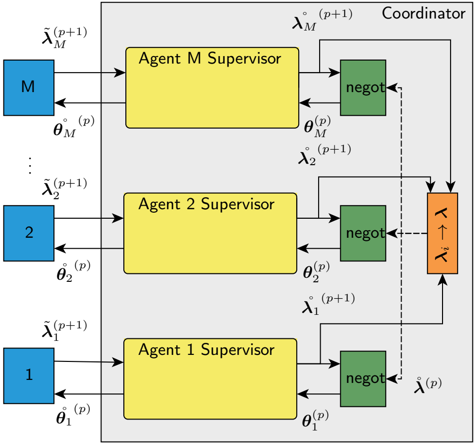
Complete Mechanism



Two phases:

1. Detect which agents are non-cooperative
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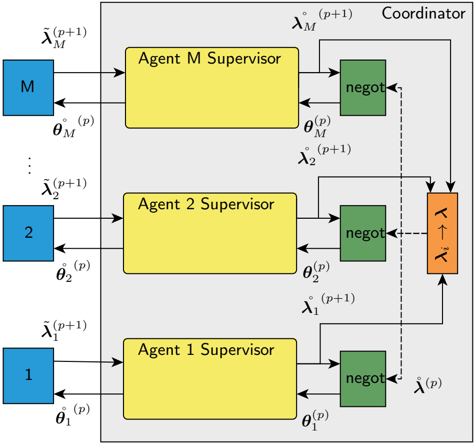

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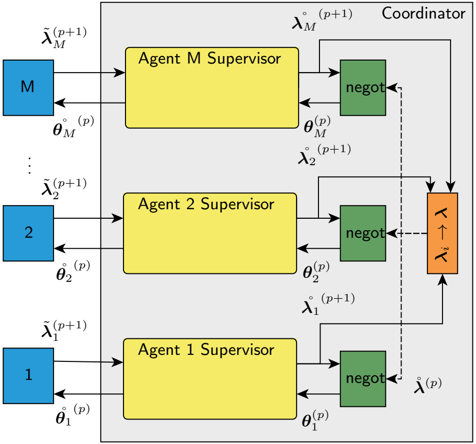

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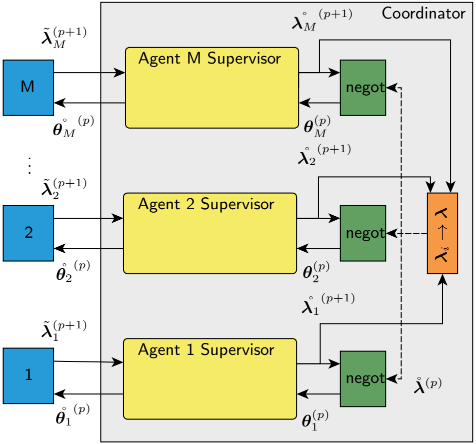

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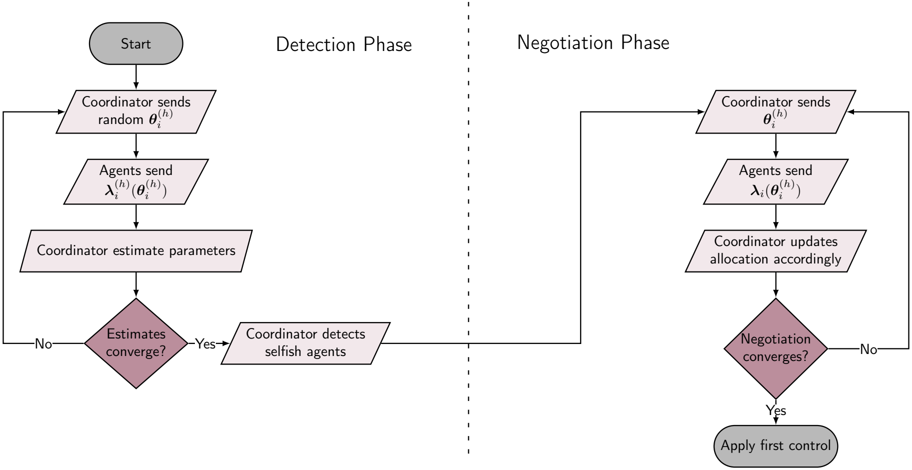
Secure DMPC



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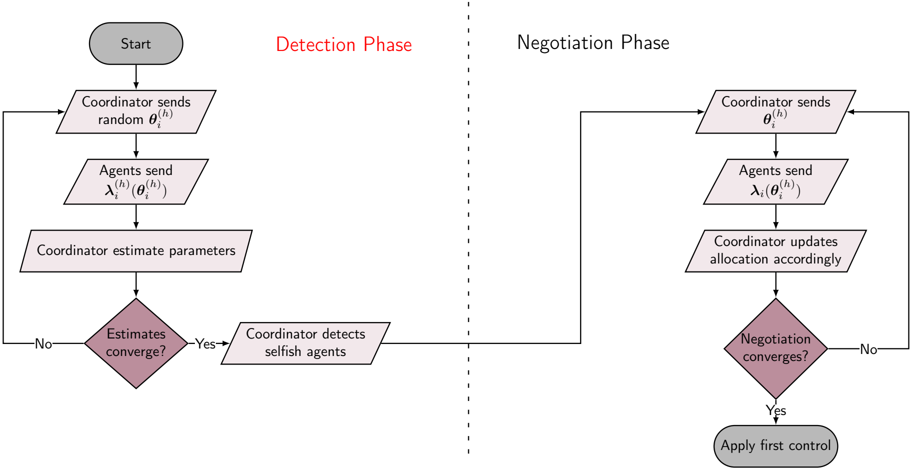

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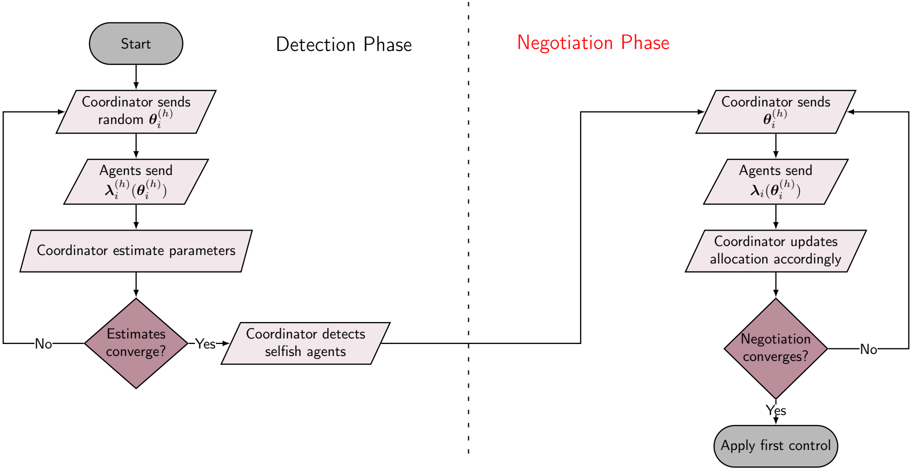

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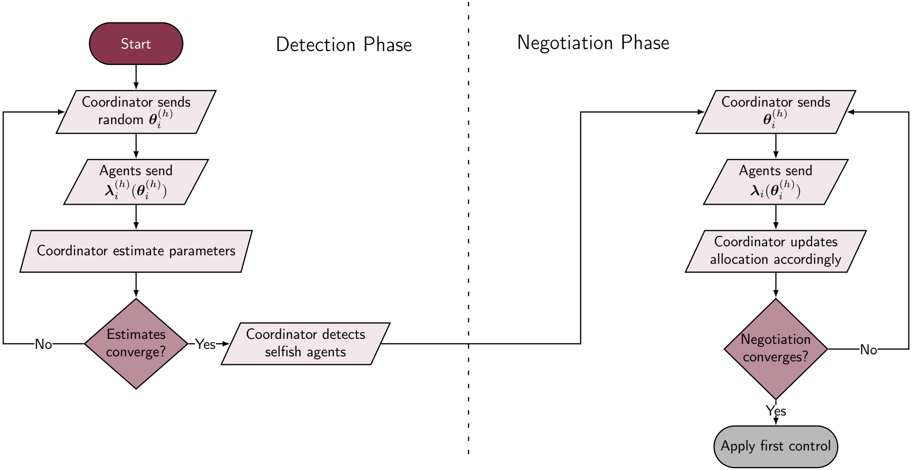

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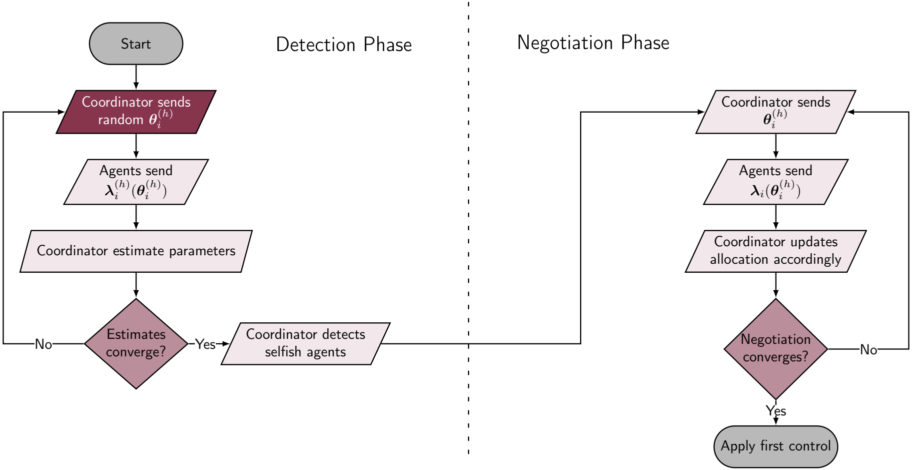

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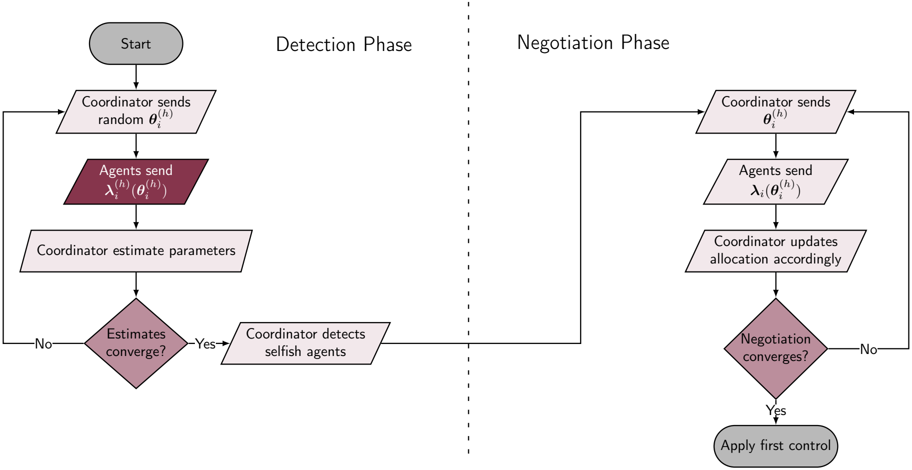

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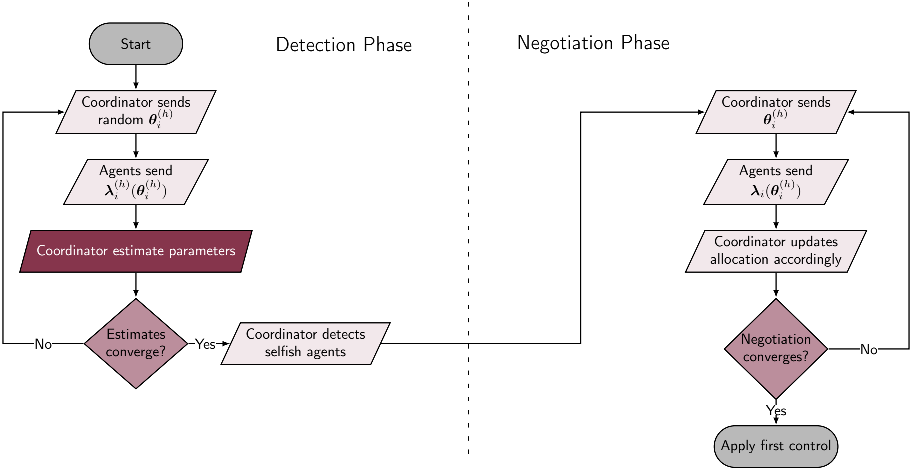

Detection Phase
Negotiation Phase
Start
Coordinator sends random $\boldsymbol{\theta}_i^{(h)}$
Agents send $\boldsymbol{\lambda}_i^{(h)}(\boldsymbol{\theta}_i^{(h)})$
Coordinator estimate parameters
Estimates converge?
No
Yes
Coordinator detects selfish agents
Coordinator sends $\boldsymbol{\theta}_i^{(h)}$
Agents send $\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i^{(h)})$
Coordinator updates allocation accordingly
Negotiation converges?
No
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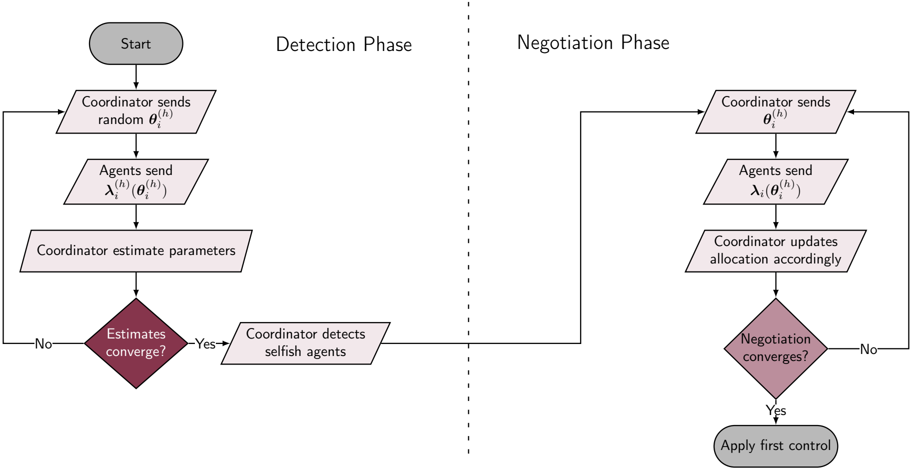
Apply first control

Secure DMPC



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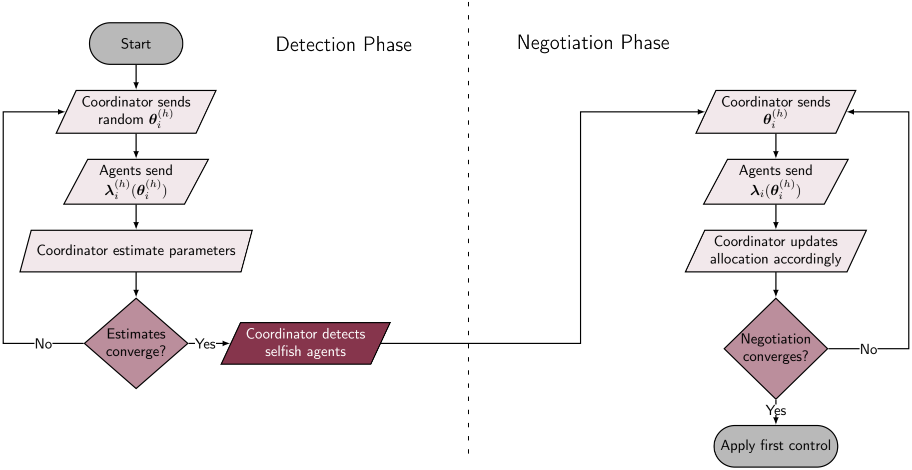
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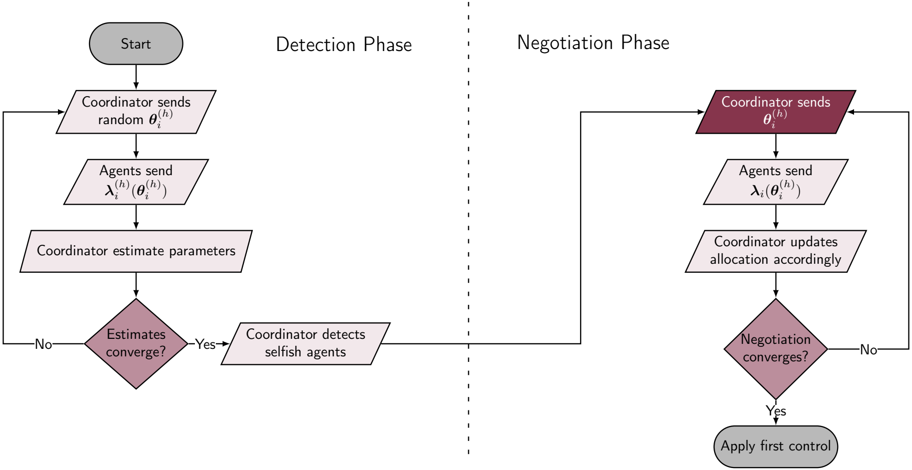
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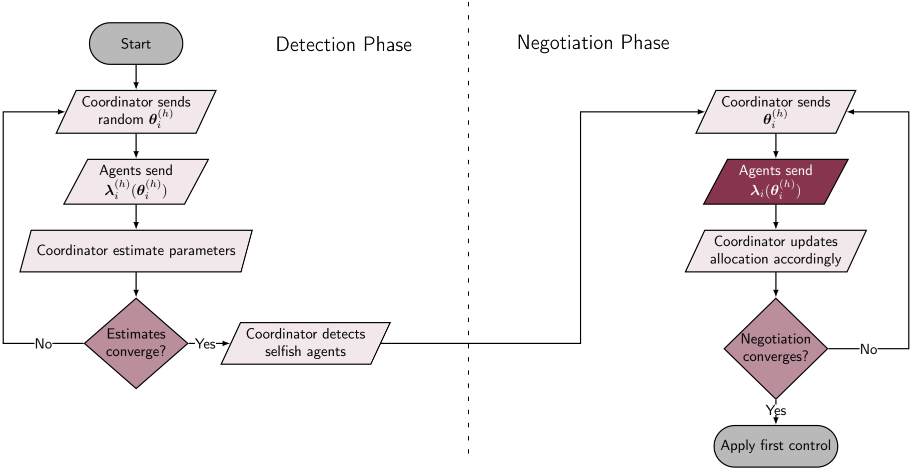
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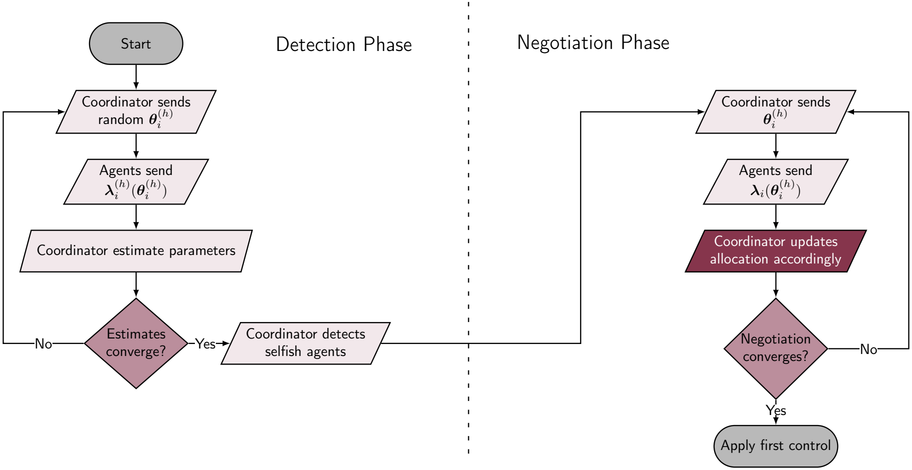

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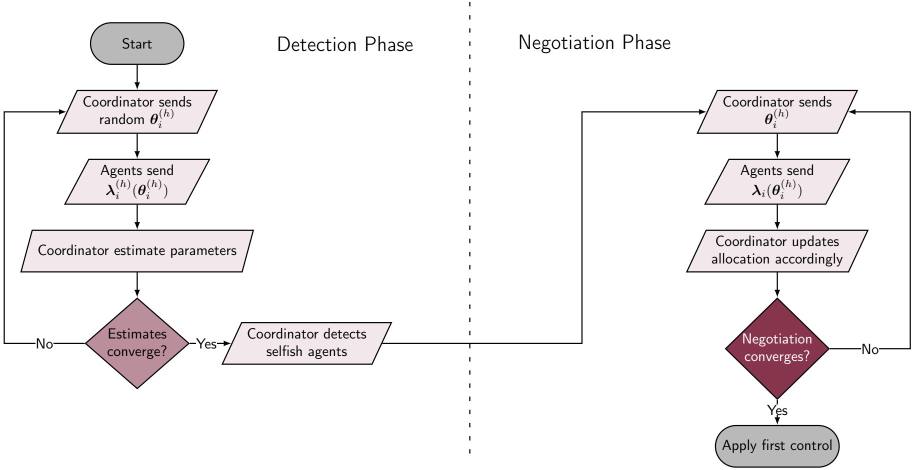
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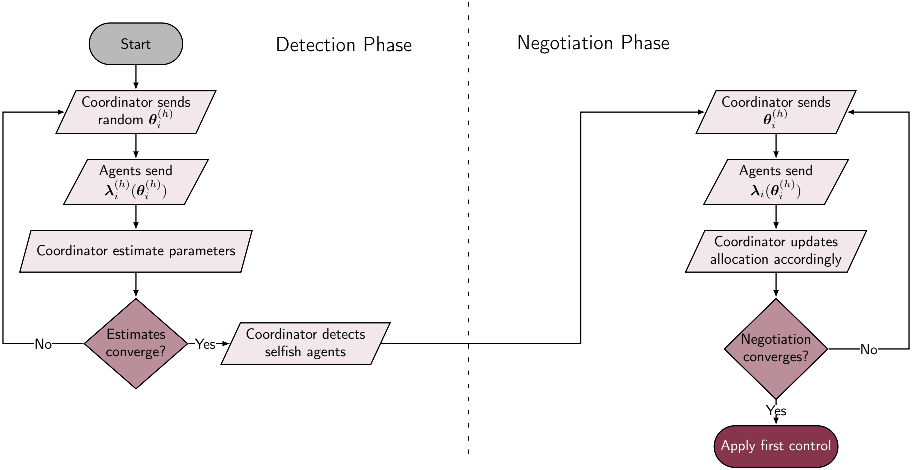
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Outline

1. Vulnerabilities in distributed MPC based on Resource Allocation
   - Attacks
   - Consequences
2. Securing the DMPC
   - Analysis of Subproblems
   - Detection Mechanism
   - Mitigation Mechanism
   - Complete Mechanism
3. Results

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Example

Temperature Control of 4 Distinct Rooms Under Power Scarcity

- 4 distinct rooms modeled using 3R-2C
- Initial temperature under $20^o\mathrm{C}$
- Not enough power to achieve setpoint $\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leq 4\mathrm{kW}\right)$
- Simulated for a period of $5\mathrm{h}$
- ZOH $T_s = 0.25\mathrm{h}$
- 3 scenarios
  1. Nominal
  2. Agent I non cooperative from k>6 with T=4*I
  3. Similar but with secure algorithm

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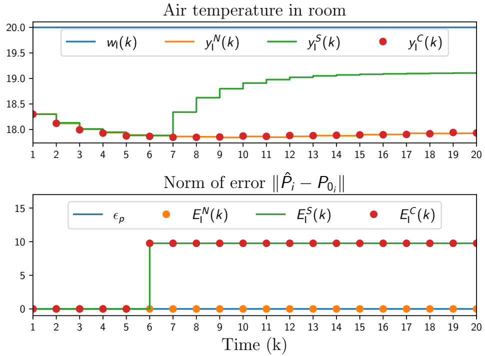
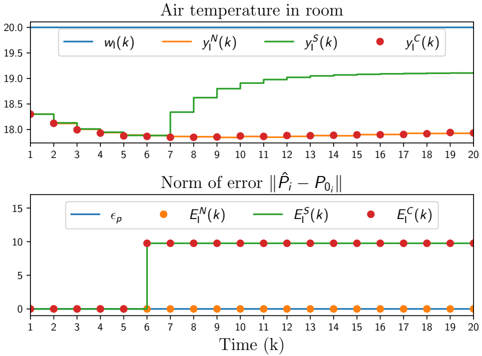
Results

Temporal



- N Nominal
- S Selfish behavior
- C selfish behavior with Correction

Results
Temporal



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Results

Table 1: Comparison of costs J_i^N and J_G^N

Agent	Nominal	Selfish	Selfish + correction
I	103	64	104
II	73	91	73
III	100	123	101
IV	132	154	131
Global	408	442	409

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1. Resource allocation based DMPC is vulnerable to attacks.
2. Sub-problems' structure has time invariant parameters.
3. Attacks can be detected using these parameters.
4. Effects can be mitigated.

- Outlook
  - Inequality Constraints yield Hybrid behavior
  - Non-linear attack model

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4. Effects can be mitigated.

- Outlook
  - Inequality Constraints yield Hybrid behavior
  - Non-linear attack model

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Summary

1. Resource allocation based DMPC is vulnerable to attacks.
2. Sub-problems' structure has time invariant parameters.
3. Attacks can be detected using these parameters.
4. Effects can be mitigated.

- Outlook
  - Inequality Constraints yield Hybrid behavior
  - Non-linear attack model

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For Further Reading I

J. M. Maestre, R. R. Negenborn *et al.*
Distributed Model Predictive Control made easy.
Springer, 2014, vol. 69.

P. Velarde, J. M. Maestre, H. Ishii, and R. R. Negenborn,
"Scenario-based defense mechanism for distributed model predictive control,"
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Thank you!

Repository

https://github.com/Accacio/SysTol-21

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