Detection and Mitigation of Corrupted Information in Distributed Model Predictive Control Based on Resource Allocation

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https://git.io/JEFGW

Context

- Energy Distribution System
- Traffic management
- Heat distribution
- Water distribution
- ...

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Context

- Geographically distributed
- Coupled by constraints (energy)
- Optimization objectives
  - Energy
  - User satisfaction
  - ...
- Solution $\rightarrow$ Model Predictive Control

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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$
\begin{aligned}
\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{optimize}} \quad & J(\boldsymbol{x}(k), \boldsymbol{u}(k)) \\
\text{subject to} \quad & \left.\begin{array}{c}
\boldsymbol{x}(\xi+1) = f(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \\
g_i(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \leq 0 \\
h_j(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) = 0
\end{array}\right\} \begin{array}{l}
\forall \xi \in \{1, \ldots, N_p\} \\
\forall i \in \{1, \ldots, m\} \\
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\end{array}
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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$\begin{aligned}
\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{minimize}} \quad & \textstyle\sum_{j=1}^{N_p} \|\boldsymbol{v}(k+j|k)\|_Q^2 + \|\boldsymbol{u}(k+j-1|k)\|_R^2 \\
\text{subject to} \quad & \left.\begin{array}{c} \boldsymbol{x}(\xi+1) = f(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \\ g_i(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \leq 0 \\ h_j(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) = 0 \end{array}\right\} \begin{array}{l} \forall \xi \in \{1, \dots, N_p\} \\ \forall i \in \{1, \dots, m\} \\ \forall j \in \{1, \dots, p\} \end{array}
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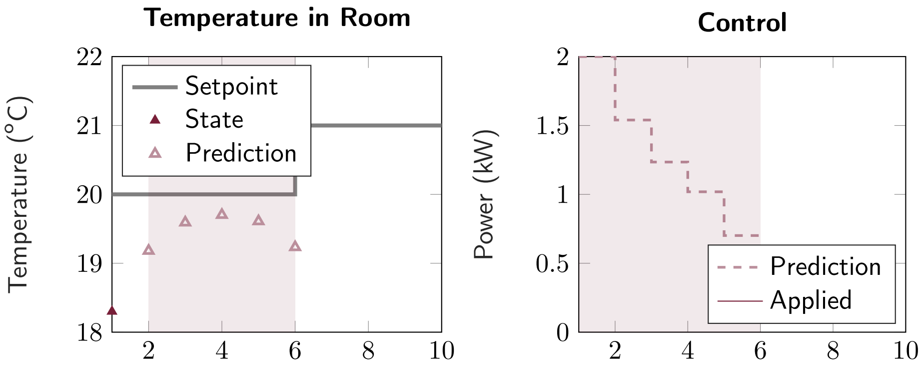
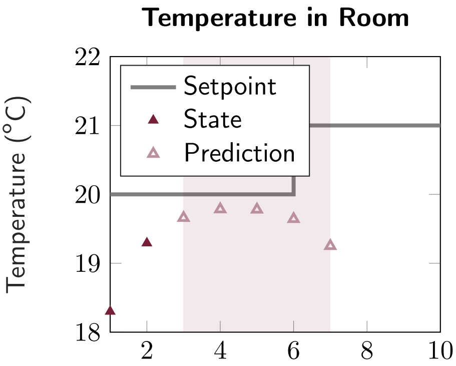
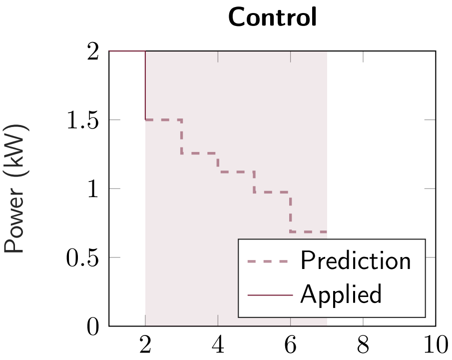
Find optimal control sequence



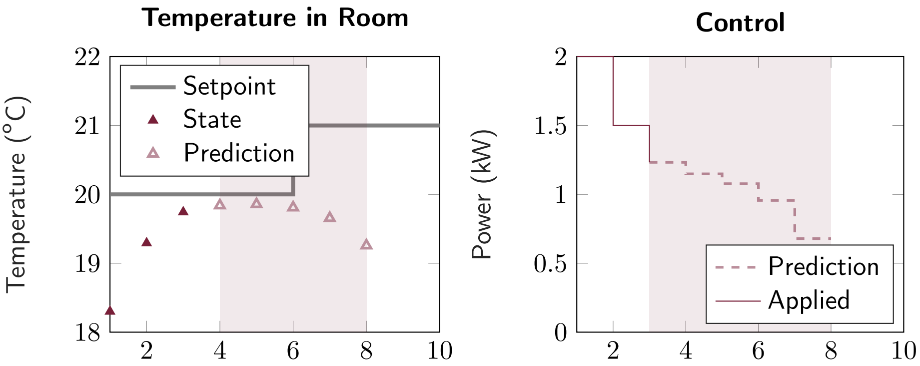
Model Predictive Control

Find optimal control sequence, apply first element



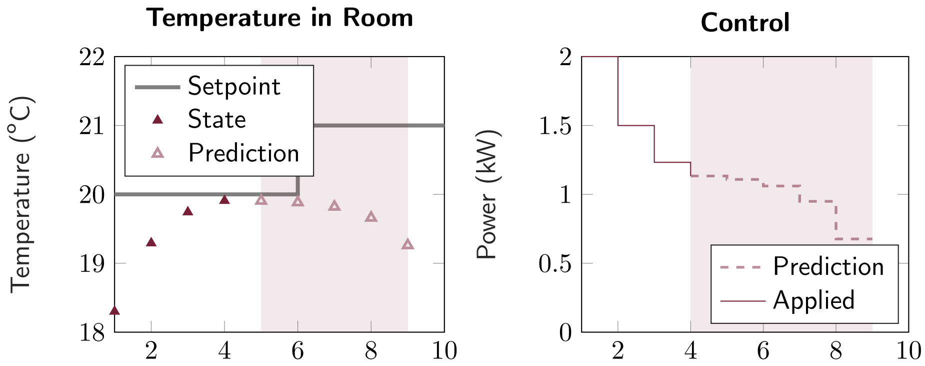


Find optimal control sequence, apply first element, rinse repeat



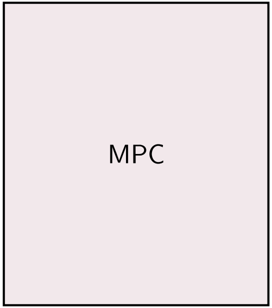
Model Predictive Control

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



Distributed Model Predictive Control

- Problem: Complexity depends on N_p, m, p and sizes of $\boldsymbol{x}$ and $\boldsymbol{u}$
- Solution: Divide and Conquer



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- Problem: Complexity depends on N_p, m, p and sizes of x and u
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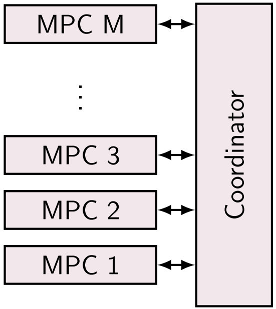

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$$
\begin{aligned}
\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \quad & \overbrace{\sum_{i=1}^{M} \underbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|^2_{Q_i} + \|\boldsymbol{u}_i(k+j-1|k)\|^2_{R_i}}^{J_i(k)}}^{J_G(k)} \\
\text{subject to} \quad & \left.\begin{array}{c} \boldsymbol{x}_i(k+1) = A_i\boldsymbol{x}_i(k) + B_i\boldsymbol{u}_i(k) \\ \sum_{i=1}^{M} \Gamma_i \boldsymbol{u}_i(k) = \boldsymbol{u}_{\max} \end{array}\right\} \begin{array}{l} \forall i \in \{1,\ldots,M\} \\ \forall j \in \{1,\ldots,N_p\} \end{array}
\end{aligned}
$$

Distributed Model Predictive Control

$$\left.\begin{aligned} J_i^\star(\boldsymbol{\theta}_i(k)) = \underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \quad & J_i(k) \\ \text{s.t.} \quad & \boldsymbol{x}_i(k+1) = A_i\boldsymbol{x}_i(k) + B_i\boldsymbol{u}_i(k) \\ & \Gamma_i\boldsymbol{u}_i(k) = \boldsymbol{\theta}_i(k) : \boldsymbol{\lambda}_i(k) \end{aligned}\right\} \begin{aligned} &\forall i \in \{1,\dots,M\} \\ &\forall j \in \{1,\dots,N_p\} \end{aligned}$$

$$\begin{aligned} J^\star = \underset{\boldsymbol{\theta}(k:k+N_p-1|k)}{\text{minimize}} \quad & \sum_{i=1}^{M} J_i^\star(\boldsymbol{\theta}_i(k)) \\ \text{s.t.} \quad & \sum_{i=1}^{M} \boldsymbol{\theta}_i(k) = \boldsymbol{u}_{\max} \end{aligned}$$

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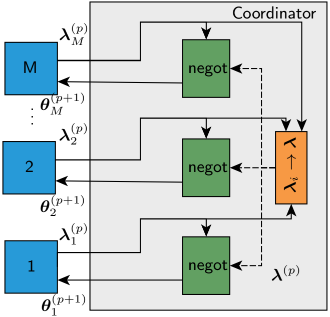
$$

$$
\boldsymbol{\theta}_i^{(p+1)} = \boldsymbol{\theta}_i^{(p)} + \rho\left(\boldsymbol{\lambda}_i^\star(\boldsymbol{\theta}_i^{(p)}) - \frac{1}{M}\sum_{j=1}^{M}\boldsymbol{\lambda}_j^\star(\boldsymbol{\theta}_j^{(p)})\right)
$$



$$
\left.
\begin{aligned}
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\end{aligned}
\right\}
\begin{aligned}
&\forall i \in \{1, \dots, M\} \\
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\end{aligned}
$$

$$
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$$

Quantity Decomposition | Resource Allocation

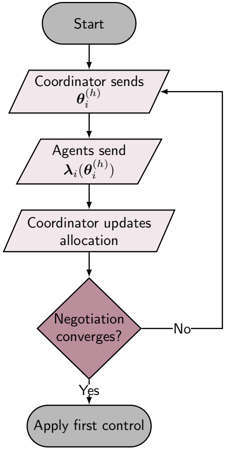

Figure 1: Quantity decomposition based DMPC

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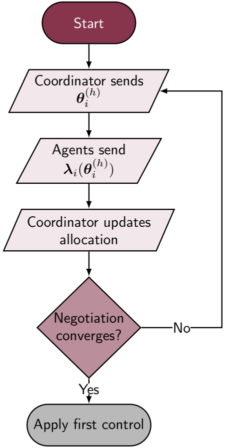


Figure 1: Quantity decomposition based DMPC

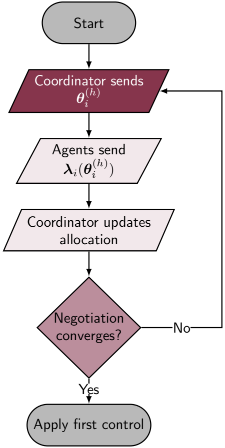
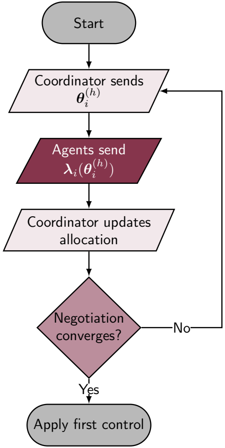


Figure 1: Quantity decomposition based DMPC



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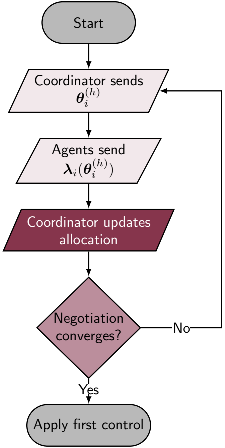

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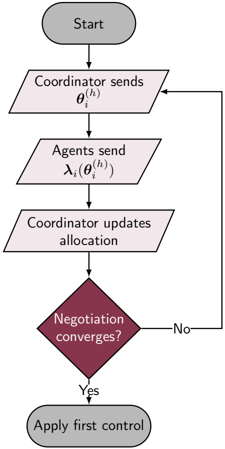

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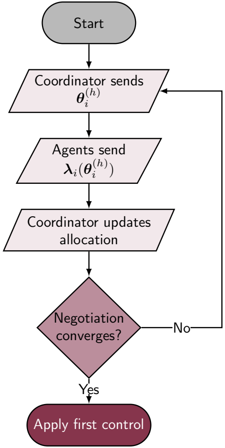

Figure 1: Quantity decomposition based DMPC

What if agents send a non-agreed $\boldsymbol{\lambda}_i$?

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Outline

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Outline

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How can a non-cooperative agent attack?

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
- Non-cooperative agent sends $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$

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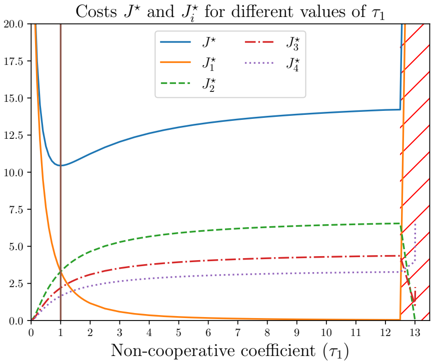
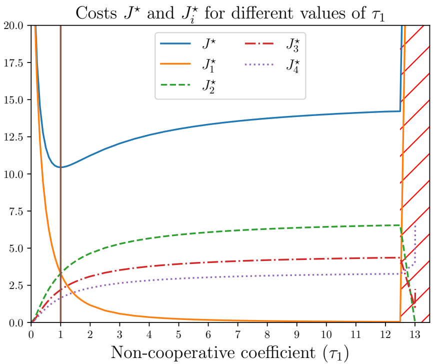
Example



4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\boldsymbol{\lambda}}_1 = \gamma_1(\boldsymbol{\lambda}_1) = \tau_1 I \boldsymbol{\lambda}_1$

Example



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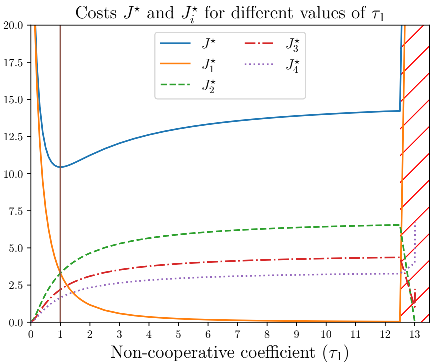

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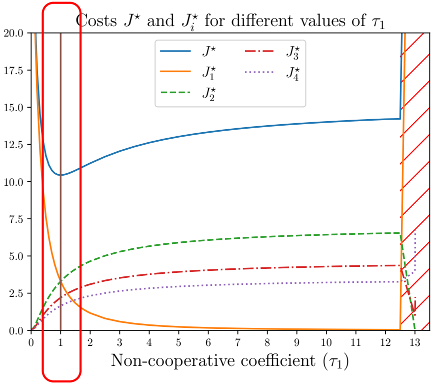

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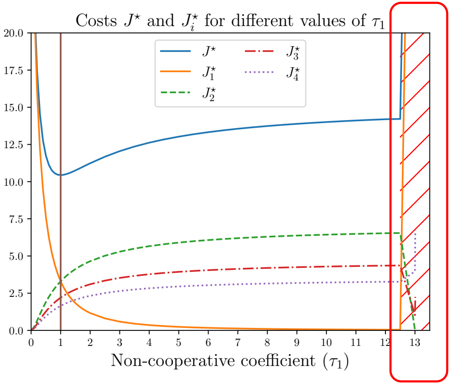

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Outline

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Quadratic Case

$$\begin{aligned}
\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \; & \overbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|^2_{Q_i} + \|\boldsymbol{u}_i(k+j-1|k)\|^2_{R_i}}^{J_i(k)} \\
\text{s.t.} \; & \left.\begin{aligned} \boldsymbol{x}_i(\xi+1) = A_i\boldsymbol{x}_i(\xi) + B_i\boldsymbol{u}_i(\xi) \\ \Gamma_i\boldsymbol{u}_i(\xi) = \boldsymbol{\theta}_i(\xi) : \boldsymbol{\lambda}_i(\xi) \end{aligned}\right\} \forall \xi \in \{1, \ldots, N_p\}
\end{aligned}$$

Quadratic Case

$$
\begin{array}{cc}
\underset{\boldsymbol{U}_i(k)}{\text{minimize}} & \overbrace{\frac{1}{2}\boldsymbol{U}_i(k)^T H_i \boldsymbol{U}_i(k) + \boldsymbol{f}_i(k)^T \boldsymbol{U}_i(k)}^{J_i(\boldsymbol{\theta}_i)} \\
\text{s.t.} & \Theta_i \boldsymbol{U}_i(k) = \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i
\end{array}
$$

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$$\boldsymbol{\lambda}_i = -P_i \boldsymbol{\theta}_i - \boldsymbol{s}_i(k)$$

where $P_i = (\Theta_i H_i^{-1} \Theta_i^{\mathrm{T}})^{-1}$ and $\boldsymbol{s}_i(k) = P_i \Theta_i H_i^{-1} \boldsymbol{f}_i(k)$

Quadratic Case

$$\begin{array}{cc} \underset{\boldsymbol{U}_i(k)}{\text{minimize}} & \overbrace{\frac{1}{2}\boldsymbol{U}_i(k)^T H_i \boldsymbol{U}_i(k) + \boldsymbol{f}_i(k)^T \boldsymbol{U}_i(k)}^{J_i(\boldsymbol{\theta}_i)} \\ \text{s.t.} & \Theta_i \boldsymbol{U}_i(k) = \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{array}$$

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where $P_i = \left(\Theta_i H_i^{-1} \Theta_i^{\mathrm{T}}\right)^{-1}$ and $\boldsymbol{s}_i(k) = P_i \Theta_i H_i^{-1} \boldsymbol{f}_i(k)$

Detection

Assumption

We know nominal $\bar{P}_i$

Assumption

Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i \rightarrow -T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}}_i(k)\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

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- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

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[1] Using Recursive Least Squares

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We know nominal $\bar{P}_i$

Assumption

Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i \to -T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}}_i(k)\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \to$ Attack

[1] Using Recursive Least Squares

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About Estimation

- We estimate $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ simultaneously using Recursive Least Squares
- Problem: Estimation during negotiation fails
  - Consecutive $\boldsymbol{\lambda}_i^p$ and $\boldsymbol{\theta}_i^p$ are linearly dependent $\rightarrow$ low input excitation
- Solution: Send sequence of random values of $\boldsymbol{\theta}_i$ until estimates converge

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Detection

In detail

- Error $E_i(k) = \|\widehat{\tilde{P}}_i(k) - \bar{P}_i\|_F$
- Create threshold ϵ_P
- Indicator $d_i \in \{0, 1\}$ detects the attack in agent i.
- $d_i = 1$ if $E_i(k) > \epsilon_P$, 0 otherwise

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Mitigation

- Main idea: Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Assumption

We suppose $\tilde{\boldsymbol{\lambda}}_i = \mathbf{0}$ only if $\boldsymbol{\lambda}_i = \mathbf{0}$, which implies $T_i(k)$ invertible.

- Estimate the inverse of $T_i(k)$

$$\widehat{T_i(k)^{-1}} = \bar{P}_i \widehat{\tilde{P}}_i(k)^{-1}$$

- Reconstruct $\boldsymbol{\lambda}_i$

$$\boldsymbol{\lambda}_{i\text{rec}} = \widehat{T_i(k)^{-1}} \tilde{\boldsymbol{\lambda}}_i = -\bar{P}_i \boldsymbol{\theta}_i - \widehat{T_i(k)^{-1}} \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

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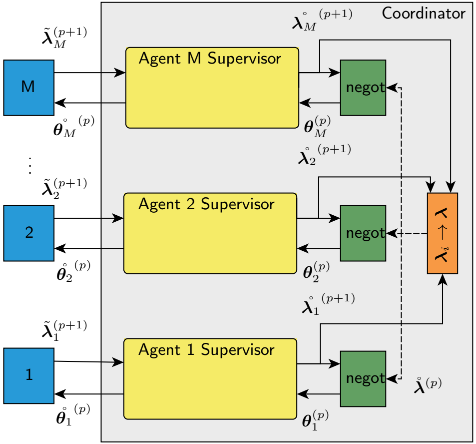
Complete Mechanism



Two phases:

1. Detect which agents are non-cooperative
2. Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

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Secure DMPC

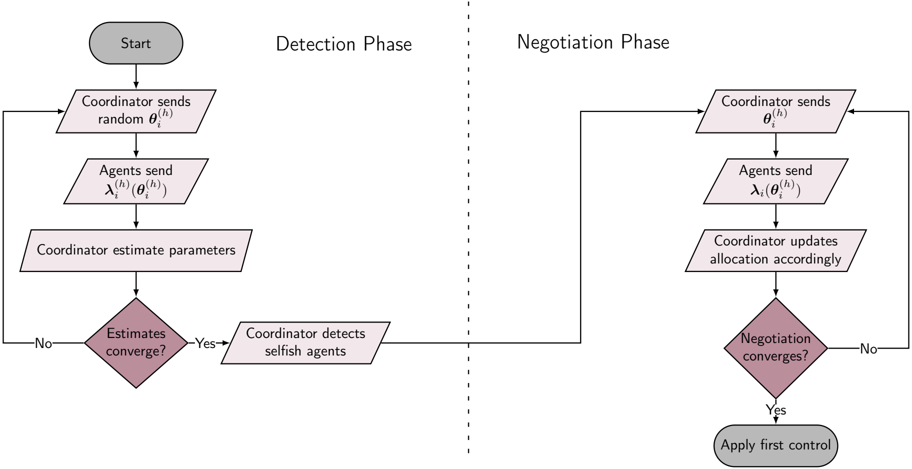
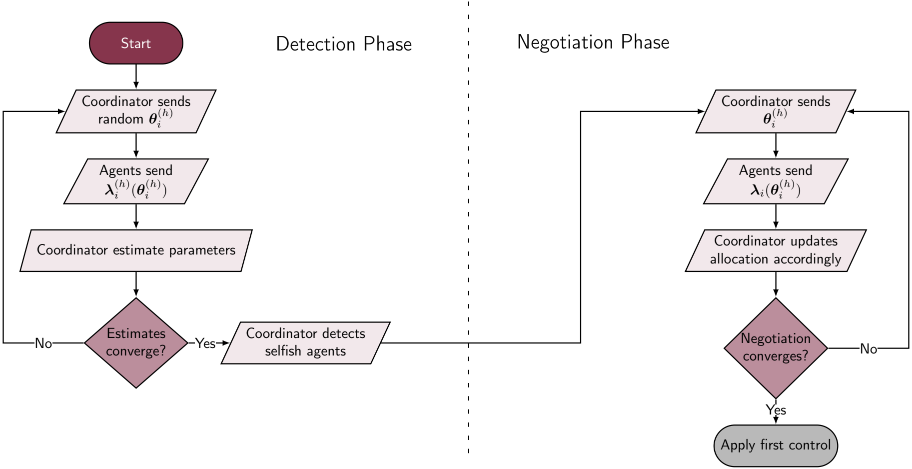


Figure 2: Secure DMPC

Secure DMPC



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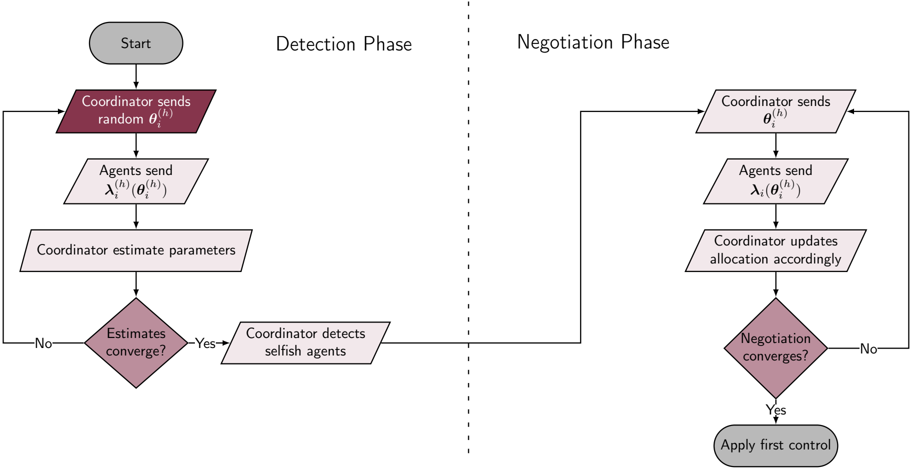

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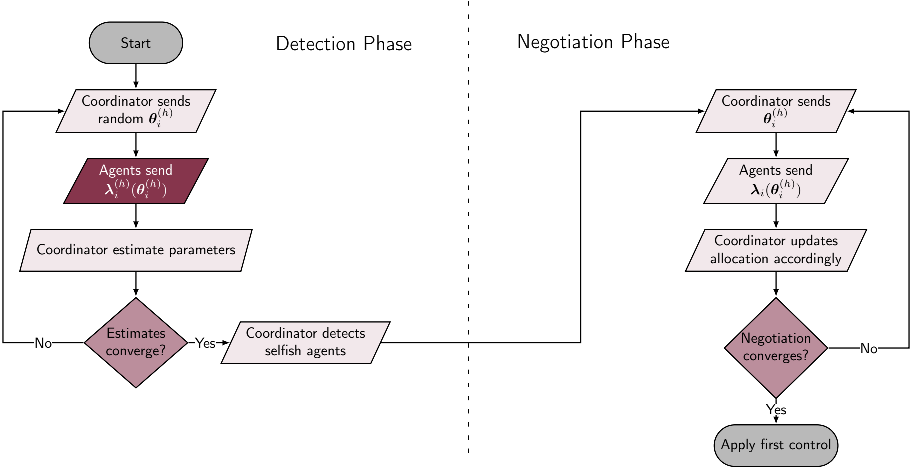

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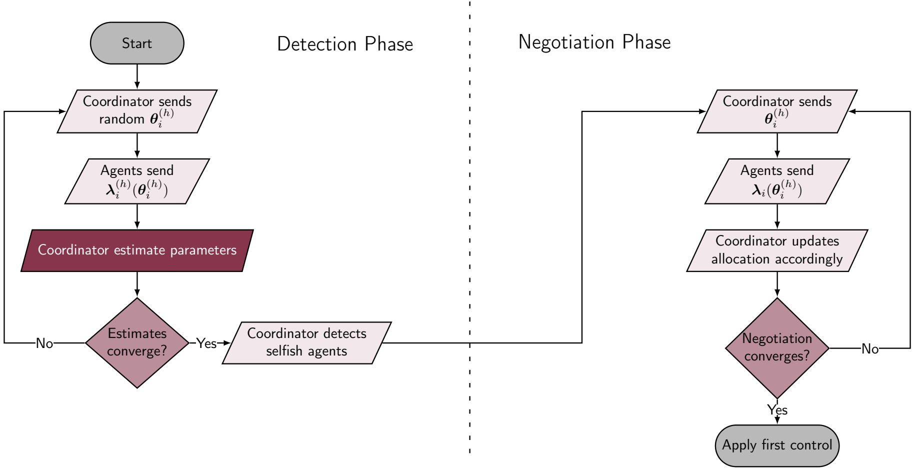

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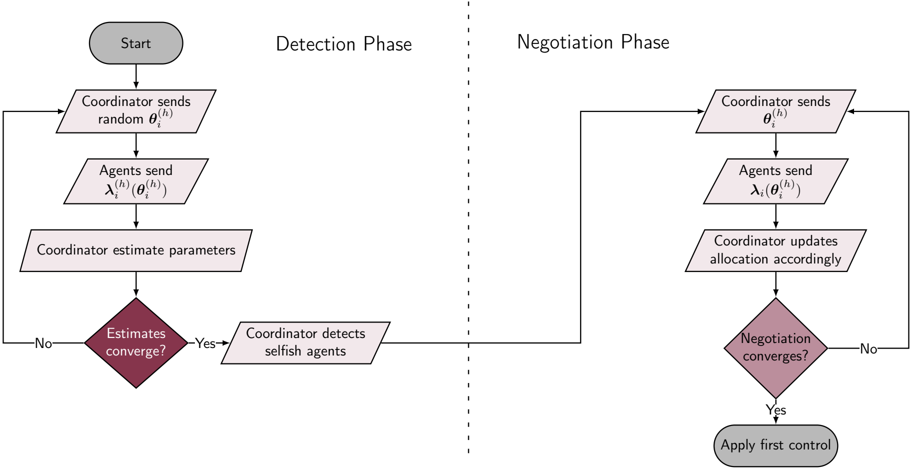

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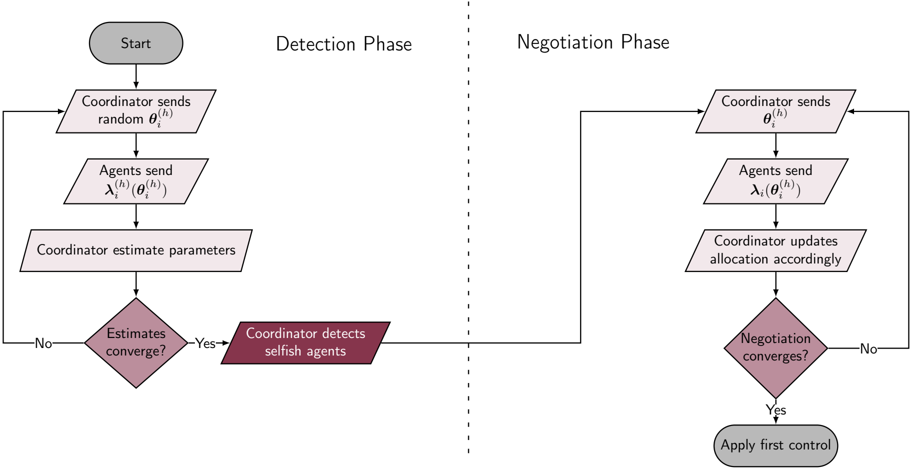

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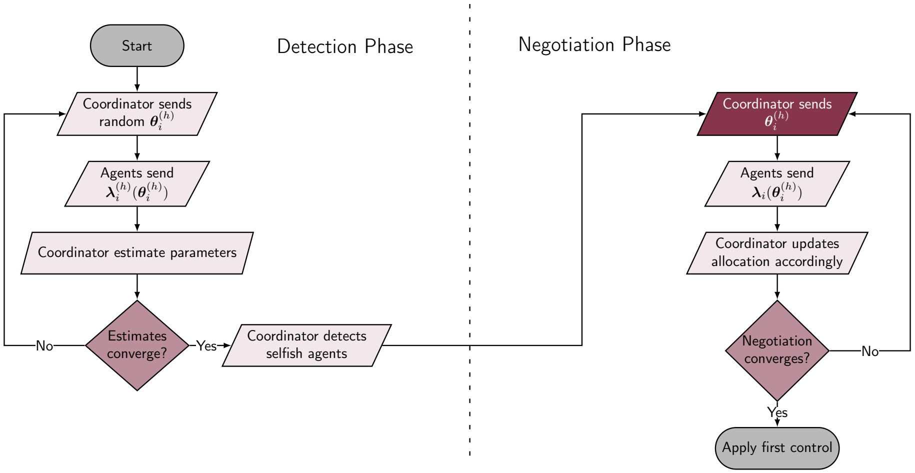

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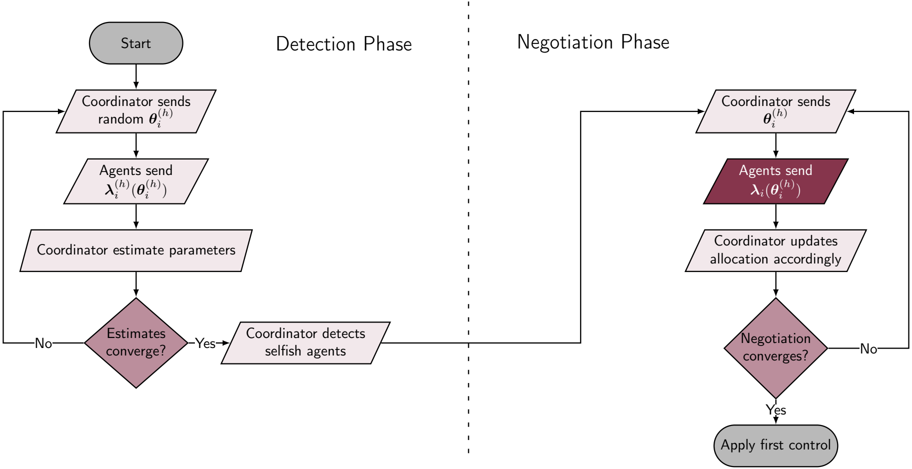

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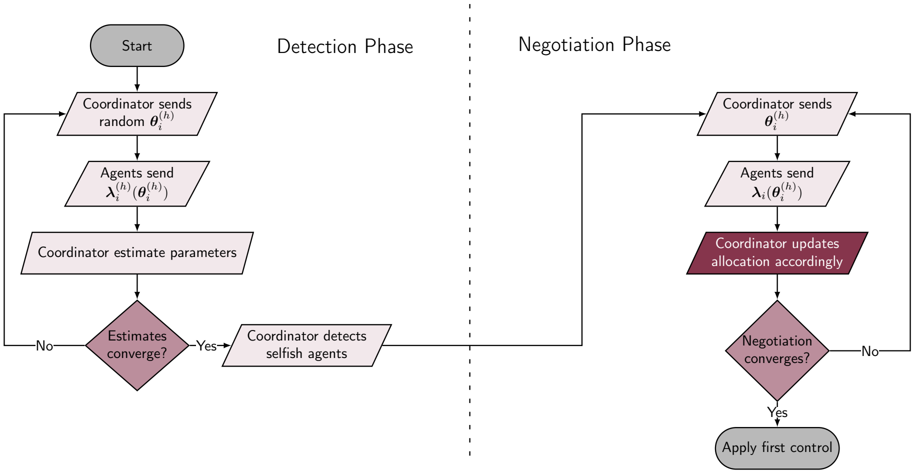

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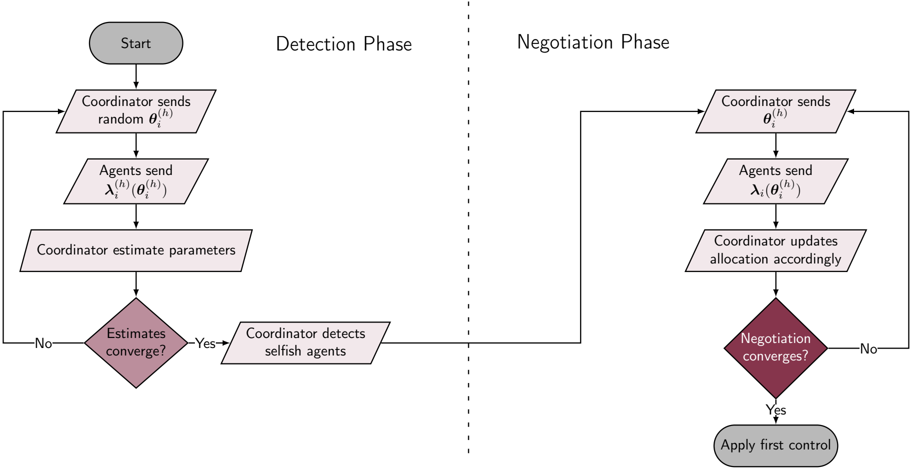

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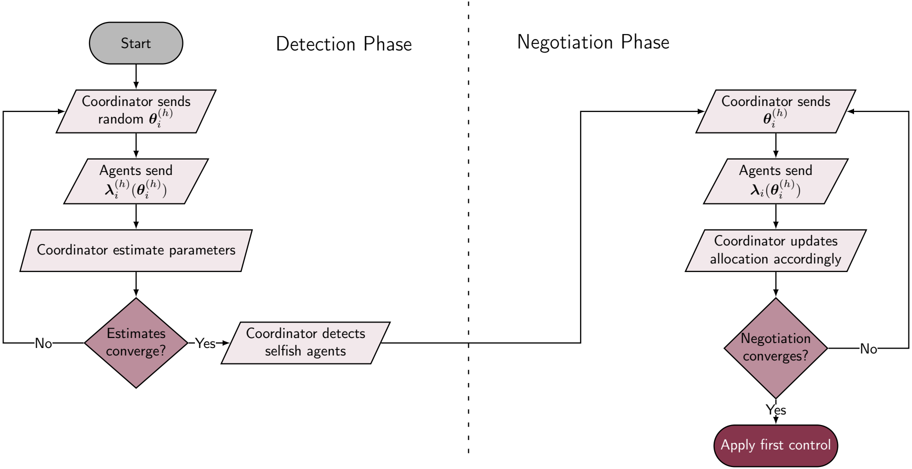

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Outline

Example

Temperature Control of 4 Distinct Rooms Under Power Scarcity

- 4 distinct rooms modeled using 3R-2C
- Initial temperature under $20^o\mathrm{C}$
- Not enough power to achieve setpoint $\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leq 4\mathrm{kW}\right)$
- Simulated for a period of $5\mathrm{h}$
- ZOH $T_s = 0.25\mathrm{h}$

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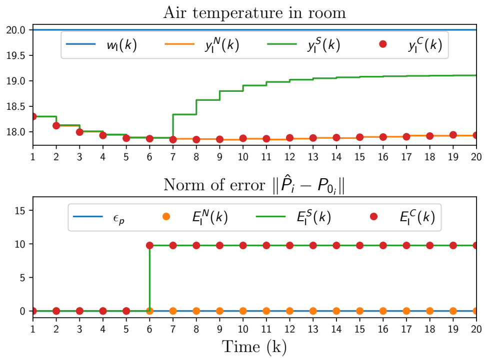
Results

Temporal



N Nominal

S Selfish behavior

C selfish behavior with Correction

Results

Table 1: Comparison of costs J_i^N and J_G^N

Agent	Nominal	Selfish	Selfish + correction
I	103	64	104
II	73	91	73
III	100	123	101
IV	132	154	131
Global	408	442	409

Summary

1. Resource allocation based DMPC is vulnerable to attacks.
2. Sub-problems' structure has time invariant parameters.
3. Attacks can be detected using these parameters.
4. Effects can be mitigated.

- Outlook
  - Inequality Constraints yield Hybrid behavior
  - Non-linear attack model

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For Further Reading I

J. M. Maestre, R. R. Negenborn *et al.*
Distributed Model Predictive Control made easy.
Springer, 2014, vol. 69.

P. Velarde, J. M. Maestre, H. Ishii, and R. R. Negenborn,
"Scenario-based defense mechanism for distributed model predictive control,"
2017 IEEE 56th Annual Conference on Decision and Control (CDC). IEEE, Dec 2017, pp. 6171–6176.

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Thank you!

Repository
https://github.com/Accacio/SysTol-21

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