Detection and Mitigation of Corrupted Information in Distributed Model Predictive Control Based on Resource Allocation

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https://git.io/JEFGW

Context

Smart City

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- Energy Distribution System
- Traffic management
- Heat distribution
- Water distribution

...

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- Geographically distributed
- Coupled by constraints (energy)
- Optimization objectives
  - Energy
  - User satisfaction
  - ...
- Solution → Model Predictive Control

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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$
\begin{aligned}
\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{optimize}} \quad & J(\boldsymbol{x}(k), \boldsymbol{u}(k)) \\
\text{subject to} \quad & \left.\begin{array}{c} \boldsymbol{x}(\xi+1) = f(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \\ g_i(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) \leq 0 \\ h_j(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi)) = 0 \end{array}\right\} \begin{array}{l} \forall \xi \in \{1, \dots, N_p\} \\ \forall i \in \{1, \dots, m\} \\ \forall j \in \{1, \dots, p\} \end{array}
\end{aligned}
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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{optimize}} \quad J(\boldsymbol{x}(k), \boldsymbol{u}(k))$$

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Model Predictive Control

Objective: Find control input sequence that optimizes an objective function

$$
\begin{array}{ll}
\underset{\boldsymbol{u}(k:k+N_p-1|k)}{\text{minimize}} & \sum_{j=1}^{N_p} \|\boldsymbol{v}(k+j|k)\|_Q^2 + \|\boldsymbol{u}(k+j-1|k)\|_R^2 \\
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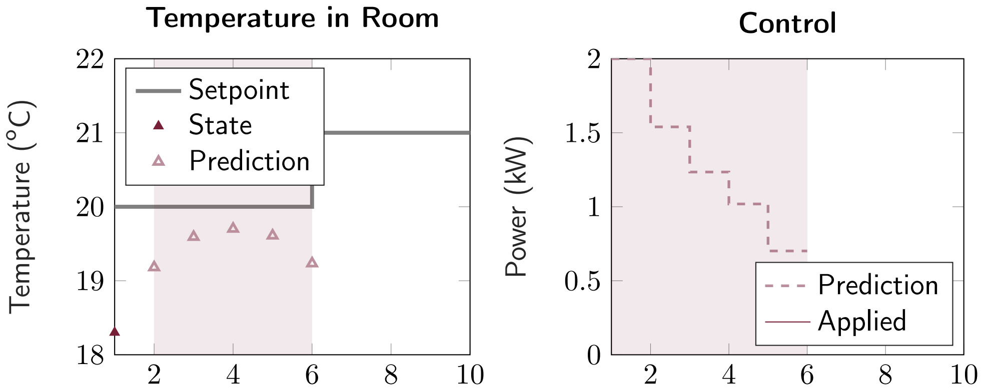
\end{array}
$$

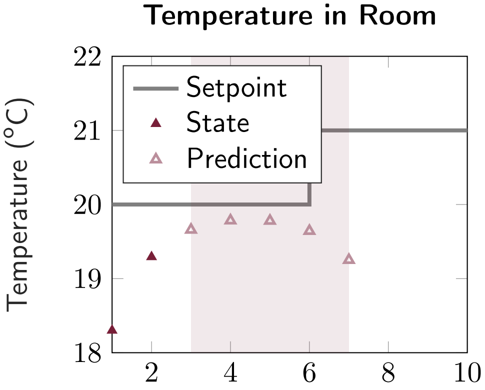
Model Predictive Control

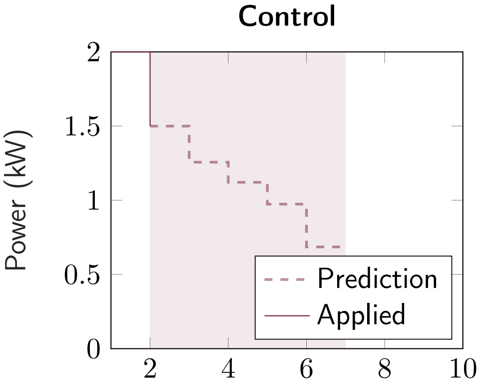
Find optimal control sequence



Model Predictive Control

Find optimal control sequence, apply first element





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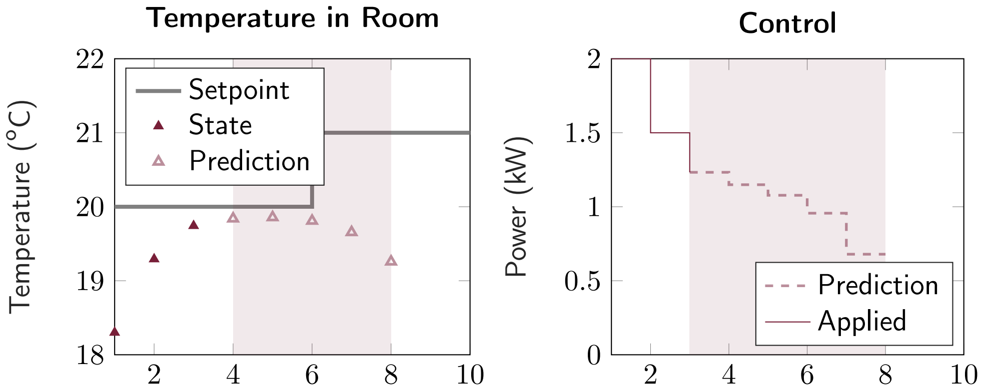
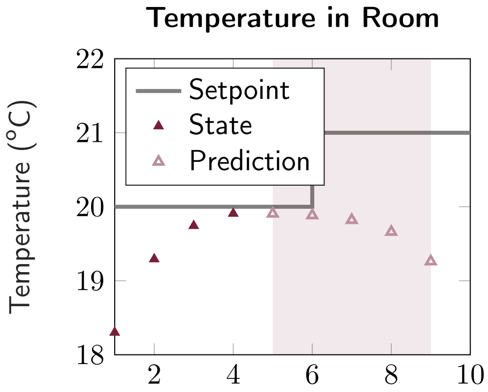
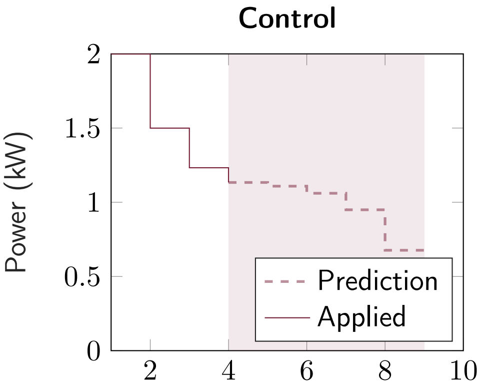
Find optimal control sequence, apply first element, rinse repeat



Model Predictive Control

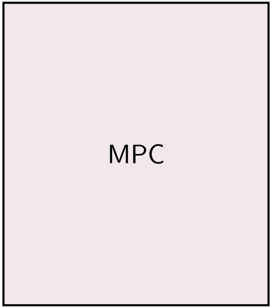
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





Distributed Model Predictive Control

- Problem: Complexity depends on N_p, m, p and sizes of $\boldsymbol{x}$ and $\boldsymbol{u}$
- Solution: Divide and Conquer



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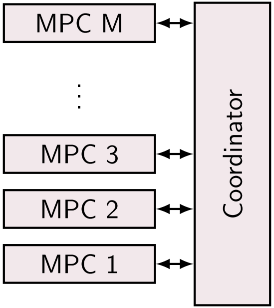

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$$\begin{array}{ll}\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} & \overbrace{\sum_{i=1}^{M} \underbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|^2_{Q_i} + \|\boldsymbol{u}_i(k+j-1|k)\|^2_{R_i}}_{J_i(k)}}^{J_G(k)} \\ \text{subject to} & \left.\begin{array}{c}\boldsymbol{x}_i(k+1) = A_i\boldsymbol{x}_i(k) + B_i\boldsymbol{u}_i(k) \\ \sum_{i=1}^{M} \Gamma_i \boldsymbol{u}_i(k) = \boldsymbol{u}_{\max}\end{array}\right\} \begin{array}{l}\forall i \in \{1,\ldots,M\} \\ \forall j \in \{1,\ldots,N_p\}\end{array}\end{array}$$

$$
\left.\begin{aligned}
J_i^\star(\boldsymbol{\theta}_i(k)) = \underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \quad & J_i(k) \\
\text{s.t.} \quad & \boldsymbol{x}_i(k+1) = A_i \boldsymbol{x}_i(k) + B_i \boldsymbol{u}_i(k) \\
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$$

$$
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J^\star = \underset{\boldsymbol{\theta}(k:k+N_p-1|k)}{\text{minimize}} \quad & \sum_{i=1}^{M} J_i^\star(\boldsymbol{\theta}_i(k)) \\
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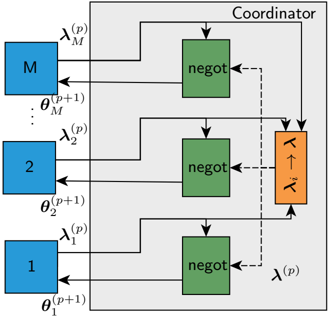
$$\boldsymbol{\theta}_i^{(p+1)} = \boldsymbol{\theta}_i^{(p)} + \rho\left(\boldsymbol{\lambda}_i^\star(\boldsymbol{\theta}_i^{(p)}) - \frac{1}{M}\sum_{j=1}^{M}\boldsymbol{\lambda}_j^\star(\boldsymbol{\theta}_j^{(p)})\right)$$



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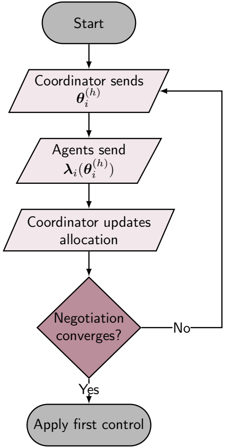
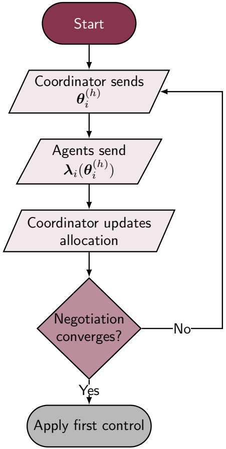


Figure 1: Quantity decomposition based DMPC



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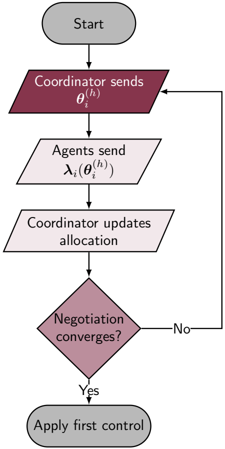
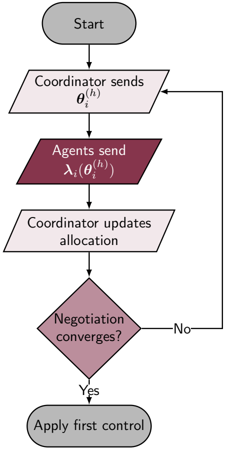


Figure 1: Quantity decomposition based DMPC

Quantity Decomposition | Resource Allocation



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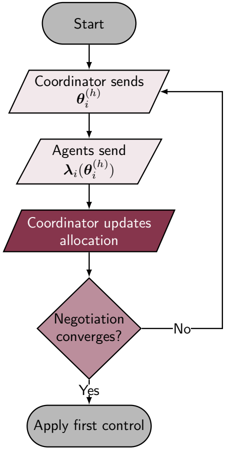

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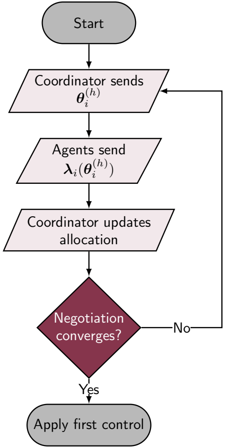

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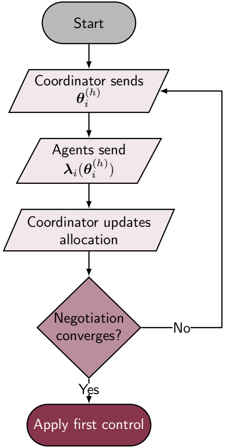

Figure 1: Quantity decomposition based DMPC

What if agents send a non-agreed $\boldsymbol{\lambda}_i$?

Outline

1. Vulnerabilities in distributed MPC based on Resource Allocation
    - Attacks
    - Consequences
2. Securing the DMPC
    - Analysis of Subproblems
    - Detection Mechanism
    - Mitigation Mechanism
    - Complete Mechanism
3. Results

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Outline

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How can a non-cooperative agent attack?

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
- Non-cooperative agent sends $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$

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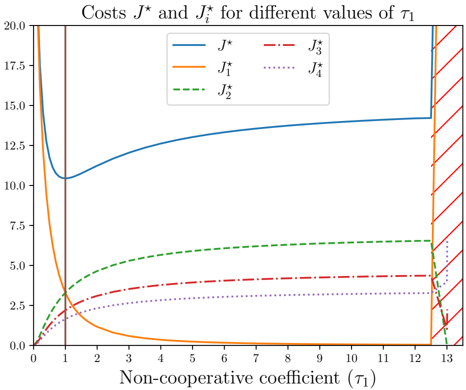
Example



4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\boldsymbol{\lambda}}_1 = \gamma_1(\boldsymbol{\lambda}_1) = \tau_1 I \boldsymbol{\lambda}_1$

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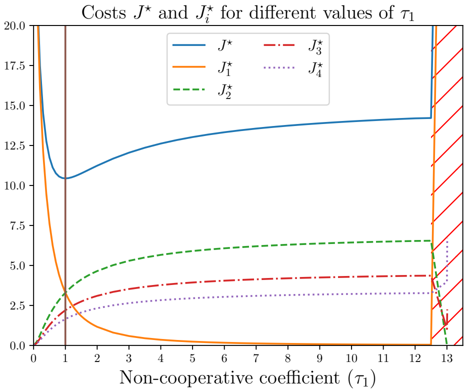
Example



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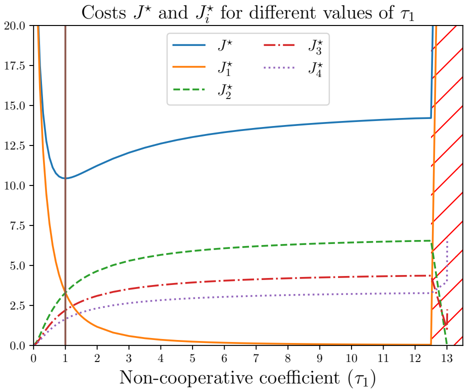

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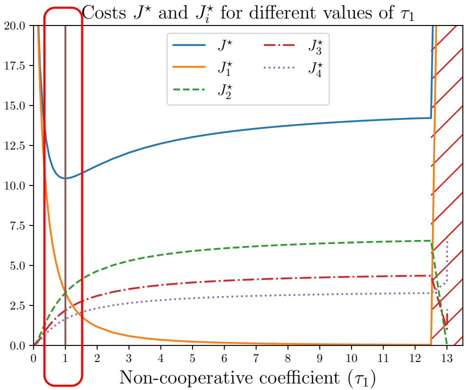

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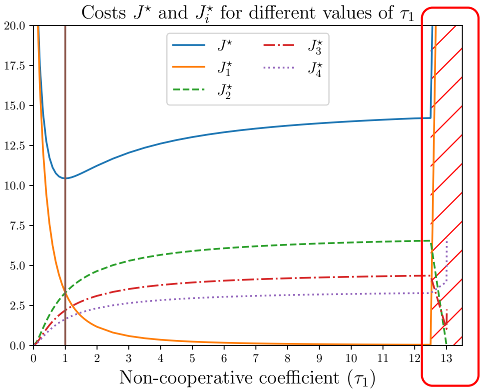

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Outline

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Quadratic Case

$$\begin{aligned}
\underset{\boldsymbol{u}_i(k:k+N_p-1|k)}{\text{minimize}} \quad & \overbrace{\sum_{j=1}^{N_p} \|\boldsymbol{v}_i(k+j|k)\|_{Q_i}^2 + \|\boldsymbol{u}_i(k+j-1|k)\|_{R_i}^2}^{J_i(k)} \\
\text{s.t.} \quad & \left.\begin{aligned} \boldsymbol{x}_i(\xi+1) = A_i\boldsymbol{x}_i(\xi) + B_i\boldsymbol{u}_i(\xi) \\ \Gamma_i\boldsymbol{u}_i(\xi) = \boldsymbol{\theta}_i(\xi) : \boldsymbol{\lambda}_i(\xi) \end{aligned}\right\} \forall \xi \in \{1, \dots, N_p\}
\end{aligned}$$

Quadratic Case

$$\begin{aligned} \underset{\boldsymbol{U}_i(k)}{\text{minimize}} \quad & \overbrace{\frac{1}{2}\boldsymbol{U}_i(k)^T H_i \boldsymbol{U}_i(k) + \boldsymbol{f}_i(k)^T \boldsymbol{U}_i(k)}^{J_i(\boldsymbol{\theta}_i)} \\ \text{s.t.} \quad & \Theta_i \boldsymbol{U}_i(k) = \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

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$$\boldsymbol{\lambda}_i = -P_i \boldsymbol{\theta}_i - \boldsymbol{s}_i(k)$$

where $P_i = \left(\Theta_i H_i^{-1} \Theta_i^{\mathrm{T}}\right)^{-1}$ and $\boldsymbol{s}_i(k) = P_i \Theta_i H_i^{-1} \boldsymbol{f}_i(k)$

Quadratic Case

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Detection

Assumption

We know nominal $\bar{P}_i$

Assumption

Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i \to -T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}}_i(k)\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \to$ Attack

[1] Using Recursive Least Squares

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[1] Using Recursive Least Squares

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About Estimation

- We estimate $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ simultaneously using Recursive Least Squares
- Problem: Estimation during negotiation fails
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Detection

In detail

- Error $E_i(k) = \|\widehat{\tilde{P}}_i(k) - \bar{P}_i\|_F$
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- Indicator $d_i \in \{0, 1\}$ detects the attack in agent i.
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Mitigation

- Main idea: Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

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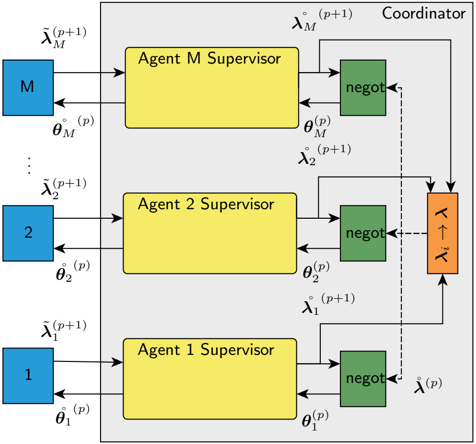
Complete Mechanism



Two phases:

1. Detect which agents are non-cooperative
2. Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

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Secure DMPC

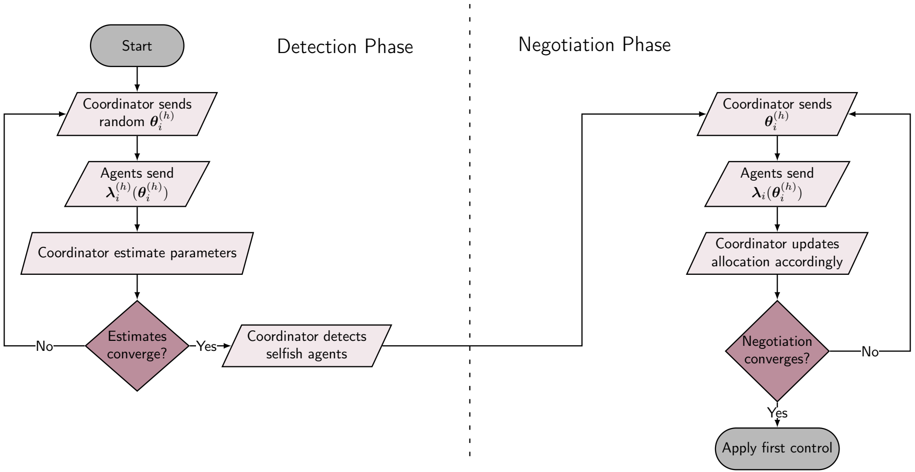
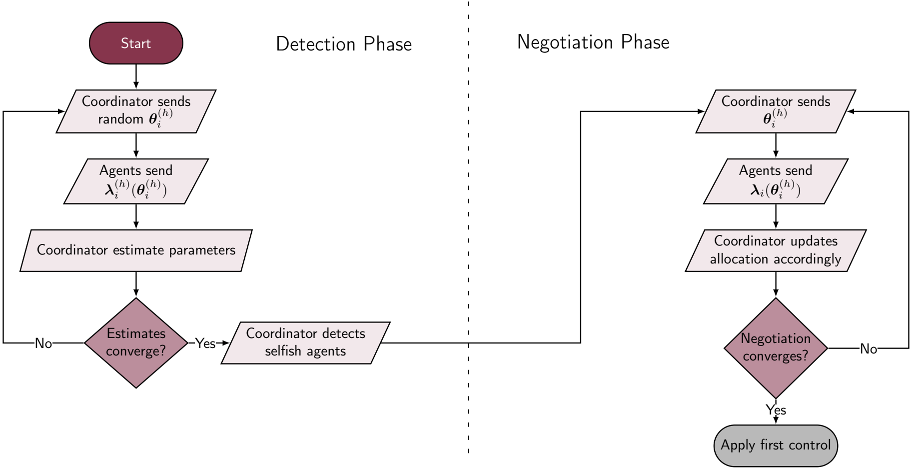


Figure 2: Secure DMPC

Secure DMPC



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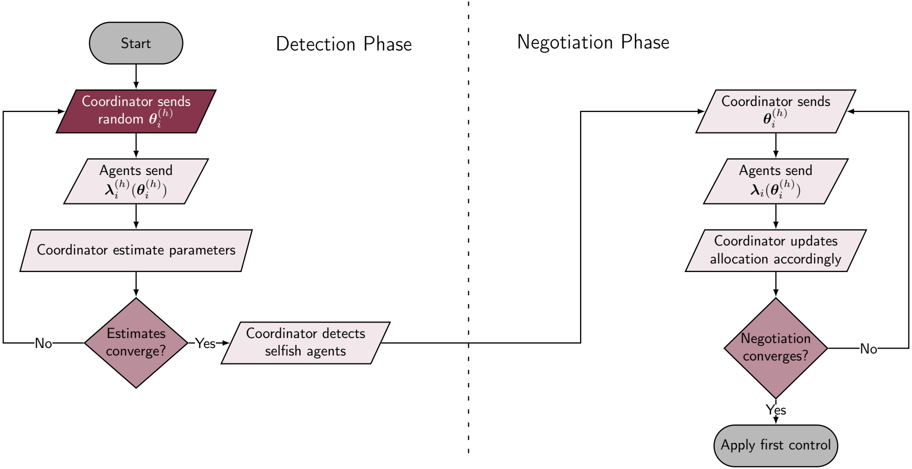

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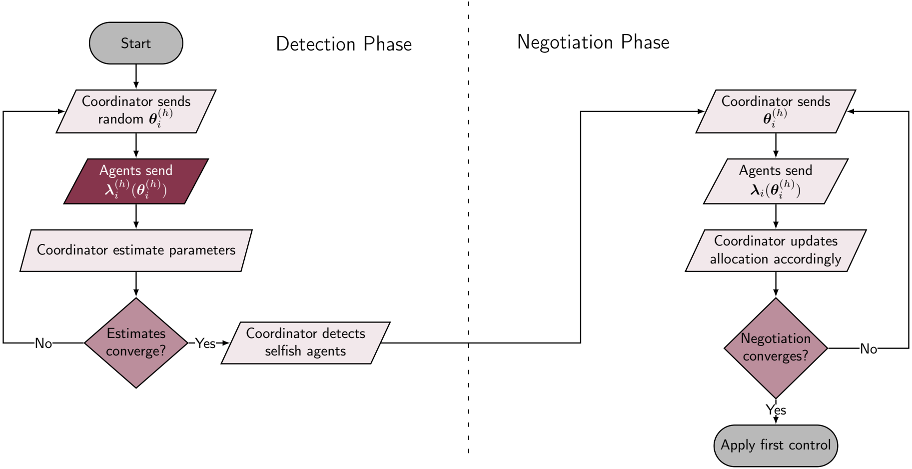

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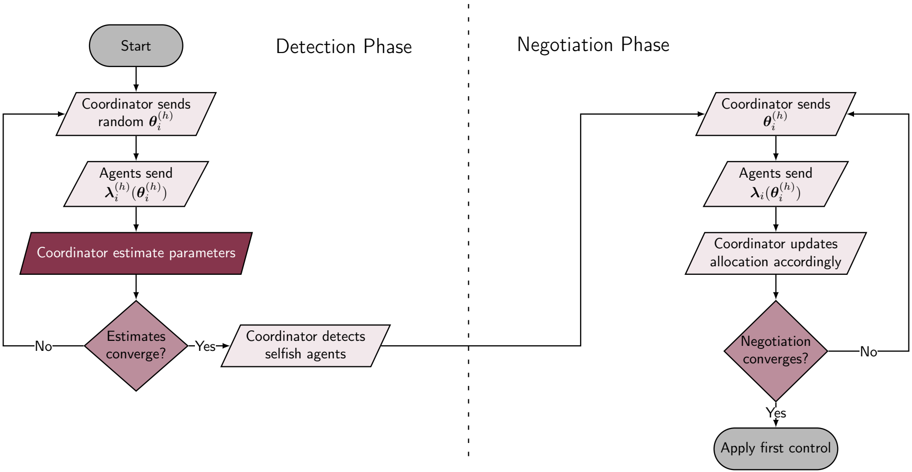

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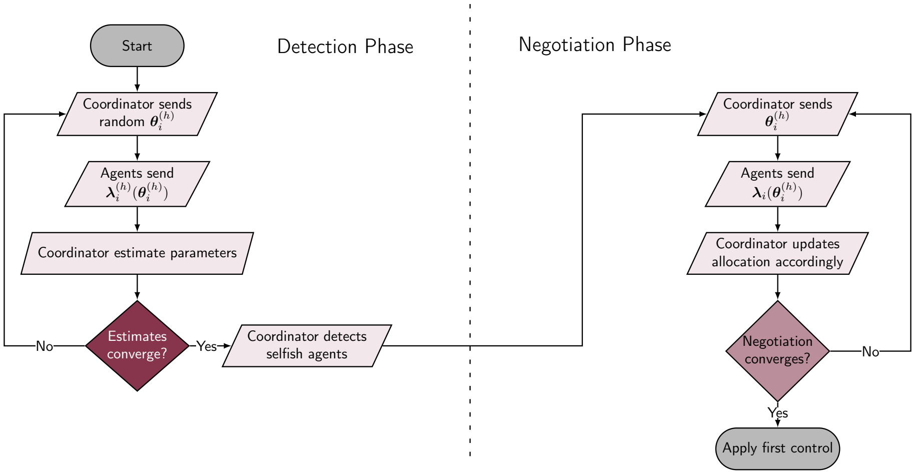

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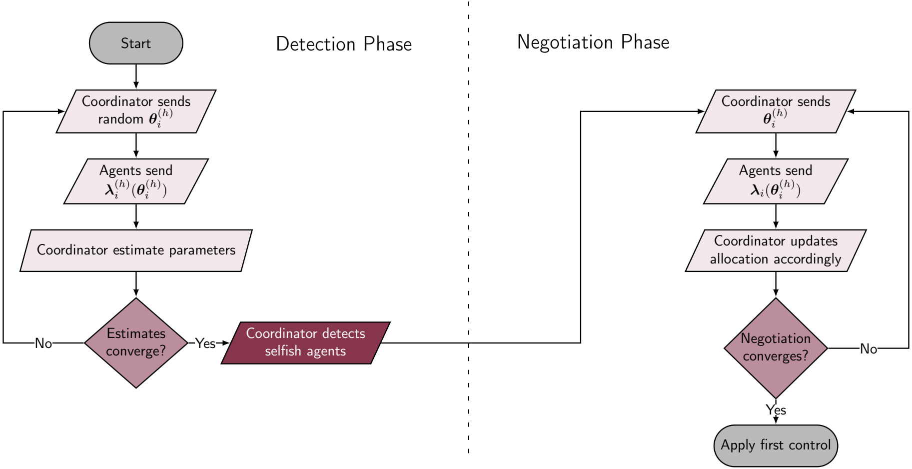

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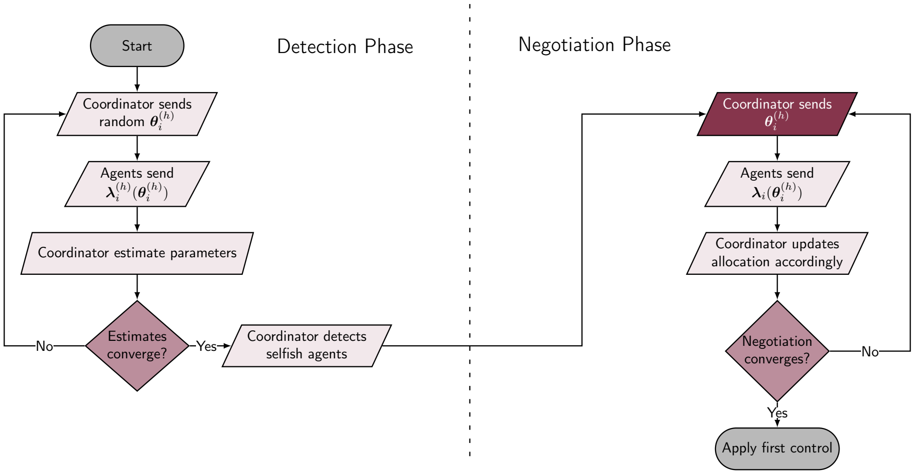

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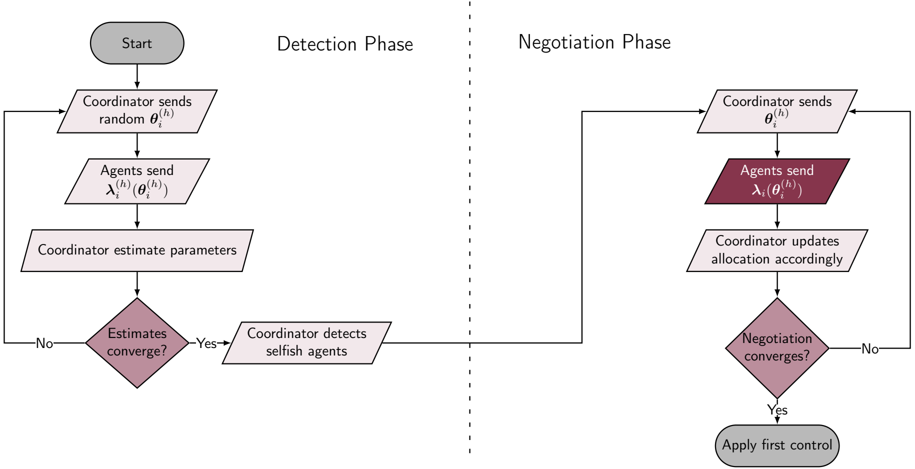

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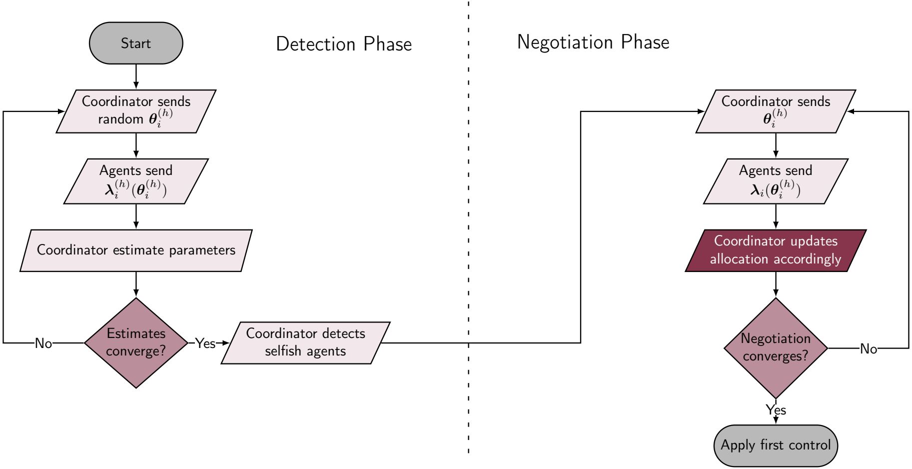

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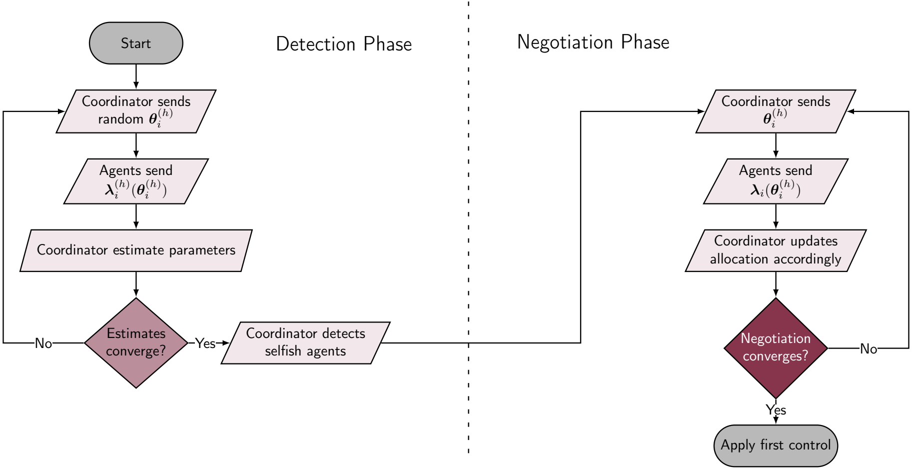

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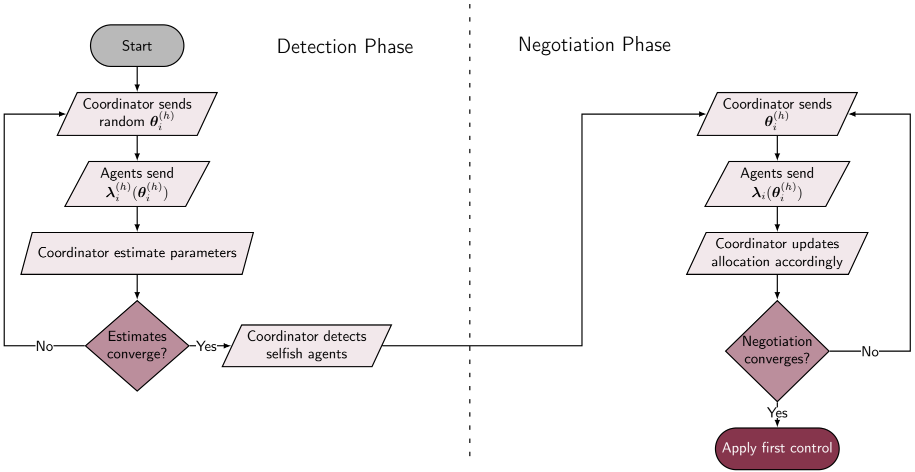

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Outline

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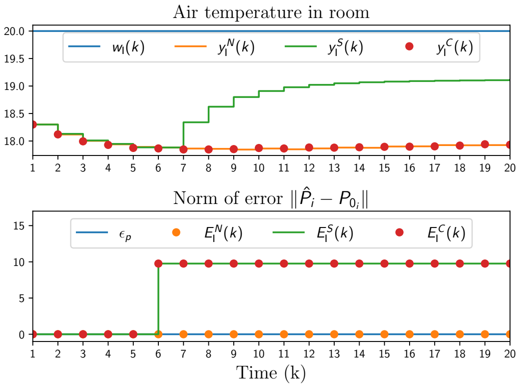
Example

Temperature Control of 4 Distinct Rooms Under Power Scarcity

- 4 distinct rooms modeled using 3R-2C
- Initial temperature under $20^o\mathrm{C}$
- Not enough power to achieve setpoint $\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leq 4\mathrm{kW}\right)$
- Simulated for a period of $5\mathrm{h}$
- ZOH $T_s = 0.25\mathrm{h}$

Results

Temporal



- N Nominal
- S Selfish behavior
- C selfish behavior with Correction

Results

Table 1: Comparison of costs J_i^N and J_G^N

Agent	Nominal	Selfish	Selfish + correction
I	103	64	104
II	73	91	73
III	100	123	101
IV	132	154	131
Global	408	442	409

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Summary

1. Resource allocation based DMPC is vulnerable to attacks.
2. Sub-problems' structure has time invariant parameters.
3. Attacks can be detected using these parameters.
4. Effects can be mitigated.

- Outlook
  - Inequality Constraints yield Hybrid behavior
  - Non-linear attack model

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For Further Reading I

J. M. Maestre, R. R. Negenborn *et al.*
Distributed Model Predictive Control made easy.
Springer, 2014, vol. 69.

P. Velarde, J. M. Maestre, H. Ishii, and R. R. Negenborn,
"Scenario-based defense mechanism for distributed model predictive control,"
2017 IEEE 56th Annual Conference on Decision and Control (CDC). IEEE, Dec 2017, pp. 6171–6176.

Questions?

Repository
https://github.com/Accacio/SysTol-21

Contact
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