Chapter 1

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1 Interval arithmetic

1.1 Calculate intervals

If $[x] = [-1, 3]$ and $[y] = [2, 5]$ calculate

1.1.1 $\diamond \in \{+, -, \cdot, /, \max, \min\}$

1. $+$
   $[x] + [y] = [-1, 3] + [2, 5] = [1, 8]$
2. $-$
   $[x] - [y] = [-1, 3] - [2, 5] = [-1, 3] + [-5, -2] = [-6, 1]$
3. $\cdot$
   $[x] \cdot [y] = [-1, 3] \cdot [2, 5] = [\{-2, -5, 6, 15\}] = [-5, 15]$
4. $/$
   $[x]/[y] = [-1, 3]/[2, 5] = [-\frac{1}{2}, \frac{3}{2}]$
5. max
   $\max([x], [y]) = \max([-1, 3], [2, 5]) = [\{2, 3, 5\}] = [2, 5]$
6. min
   $\min([x], [y]) = \min([-1, 3], [2, 5]) = [\{-1, 2, 3\}] = [-1, 3]$

1.2 Compute

1.2.1 $[-2, 4] \cdot [1, 3]$

$[\{-2, 4, -6, 12\}] = [-6, 12]$

1.2.2 $[-2, 4] \sqcup [6, 7]$

$[[-2, 4] \cup [6, 7]] = [-2, 7]$

1.2.3 $\max([2, 7], [1, 9])$

$\max([2, 7], [1, 9]) = [2, 9, 7] = [2, 9]$

1.2.4 $\max(\emptyset, [1, 2])$

$[1, 2]$

1.2.5 $[-1, 3]/[0, \infty]$

$[-1, 3]/[0, \infty] = [-\infty, \infty]$

1.2.6 $([1, 2] \cdot [-1, 3]) + \max([1, 3] \cap [6, 7], [1, 2])$

$[-2, 6] + \max(\emptyset, [1, 2]) = [-2, 6] + \max(\emptyset, [1, 2]) = [-2, 6] + [1, 2] = [-1, 8]$

1.3 Compute

1.3.1 **sqr**$([-1, 3])$

$\text{sqr}([-1, 3]) = [0, 9]$

1.3.2 **sqrt**$([-10, 4])$

$\text{sqrt}([-10, 4]) = [0, 2]$

1.3.3 $\log([-2, -1])$

$\log([-2, -1]) = \emptyset$

1.4 Compute

1.4.1 $([1, 2] + [-3, 4]) \cdot [-1, 5]$

$[-2, 6] \cdot [-1, 5] = [\{(2, -6, -10, 30\}] = [-10, 30]$

1.4.2 $\exp([1, 2]/[0, \infty])$

$\exp([0, \infty]) = [1, \infty]$