Security of distributed Model Predictive Control under False Data Injection

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Seminar
École Centrale de Lyon / Laboratoire Ampère
26/05/2023 @ Écully

https://bit.ly/43h2jms

About me

Rafael Accácio Nogueira

Postdoctoral researcher at LAAS/CNRS

Garanteed relative localisation and anticollision scenario for autonomous vehicles

Project AutOCampus (GIS neOCampus)

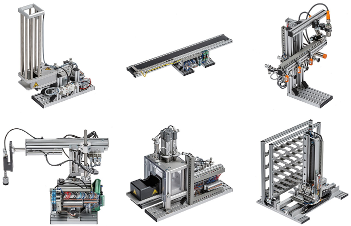
Advised by Soheib Fergani

About me

Bachelor Thesis at Escola Politécnica/UFRJ

Identification of DES for fault-diagnosis

Advised by Marcos Vicente de Brito Moreira



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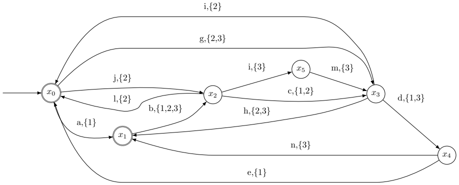
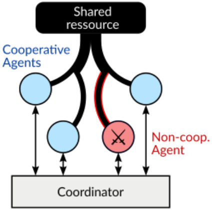


About me

Doctoral Thesis at CentraleSupélec/IETR

Security of dMPC under False Data Injection

Advised by Hervé Guéguen and Romain Bourdais



Context

Smart(er) Cities

Multiple systems interacting

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- Distribution:

Context

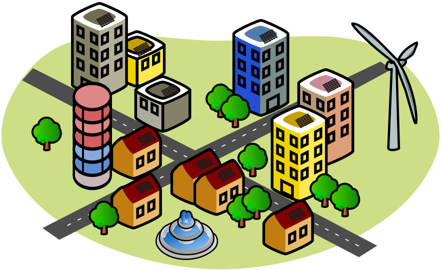
Smart(er) Cities

Multiple systems interacting

- Distribution:
  - Electricity

Smart(er) Cities

Multiple systems interacting



- Distribution:
  - Electricity
  - Heat
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Context

Smart(er) Cities

Multiple systems interacting

- Distribution:
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Smart(er) Cities

Multiple systems interacting under

- Technical/Comfort Constraints

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- We also want

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- We also want
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Context

Smart(er) Cities

Multiple systems interacting under

- Technical/Comfort Constraints
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  - Follow a trajectory
- Solution → MPC

Model-based Predictive Control

Brief recap

Model-based Predictive Control

Brief recap

Find best control sequence using predictions based on a model.

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Model-based Predictive Control

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Find optimal control sequence using predictions based on a model.

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N

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Find optimal control sequence using predictions based on a model.

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  - Decision variable is the control sequence calculated over horizon N
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Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N
  - Objective function to optimize
  - System's Model

$$\begin{aligned}
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\text{subject to} \quad & \left.\begin{aligned} \boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ \\ \end{aligned}\right\} \forall \xi \in \{1, \dots, N\}
\end{aligned}$$

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Find optimal control sequence using predictions based on a model.

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\end{aligned}
$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N
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  - System's Model
  - Other constraints to respect (QoS, technical restrictions, ...)

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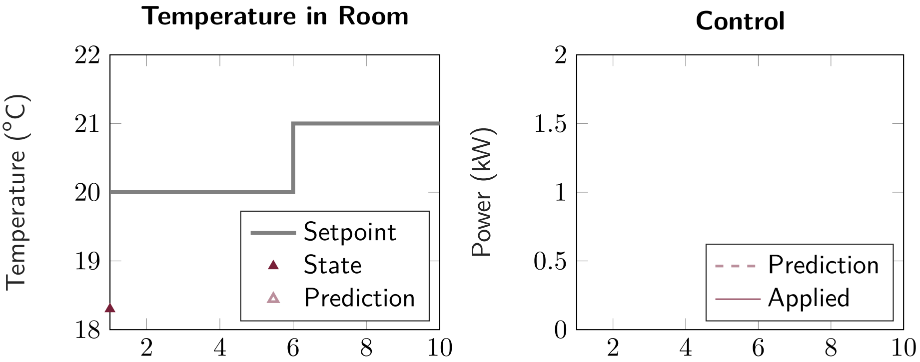
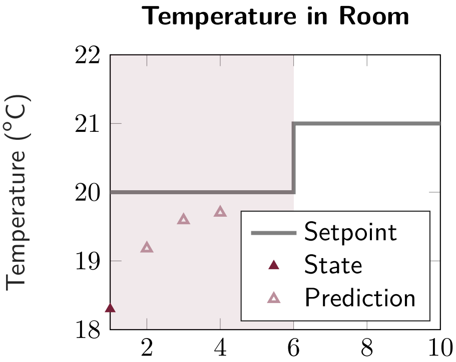
Model Predictive Control

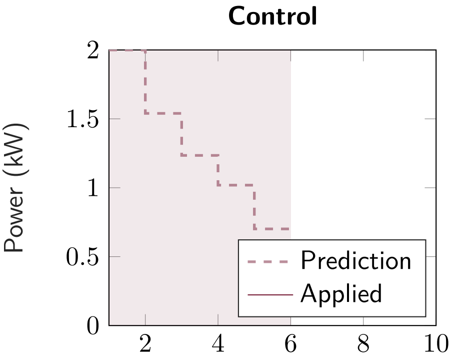
In a nutshell



In a nutshell

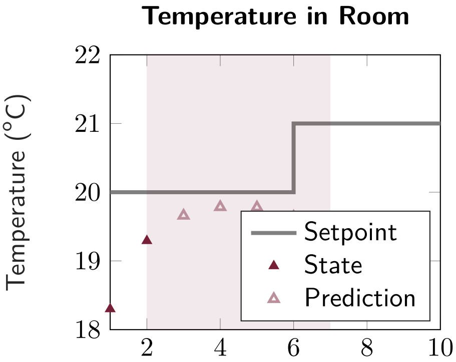
Find optimal control sequence

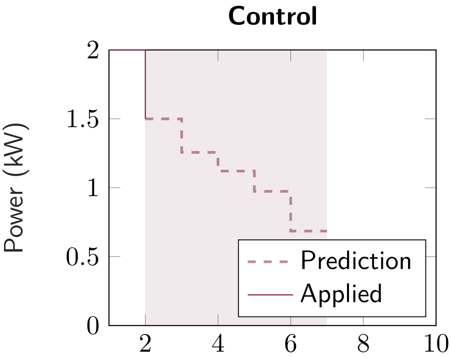




In a nutshell

Find optimal control sequence, apply first element

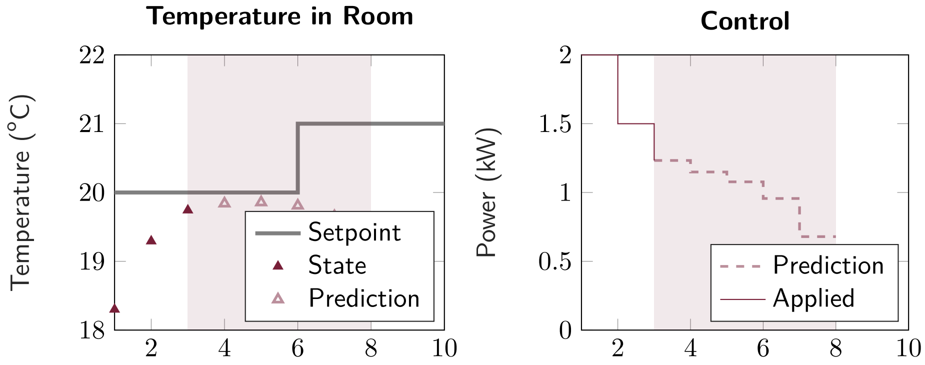




Model Predictive Control

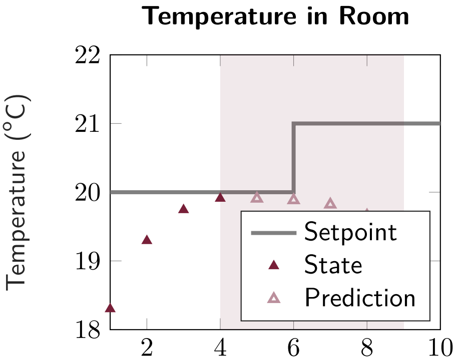
In a nutshell

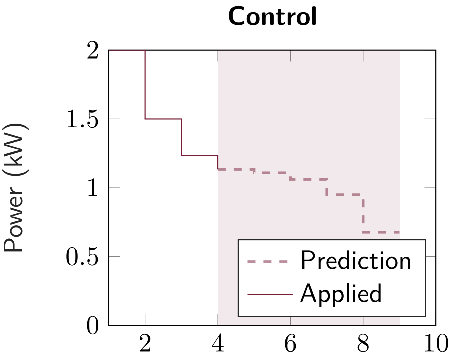
Find optimal control sequence, apply first element, rinse repeat



In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





Model Predictive Control

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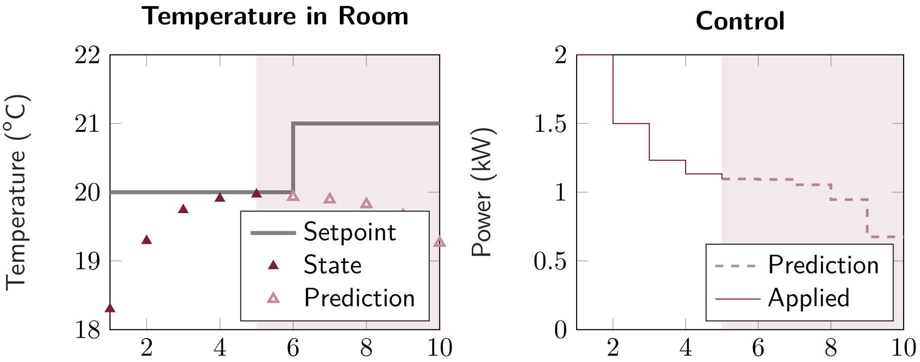
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



Model Predictive Control

Nothing is perfect

- Issues

Model Predictive Control

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- Issues
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  - Flexibility (Add/remove parts)
  - Privacy (RGPD)
- Solution: distributed MPC

Objective

Study security in dMPC context

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Security in dMPC context is relatively new[1] (First article from 2017[2])

[1] <30 documents in scopus

[2] Velarde, Jose Maria Maestre, H. Ishii, et al., "Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security"

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- CentraleSupélec Rennes - Expertise in MPC for Smart Buildings

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Security in dMPC context is relatively new[1] (First article from 2017[2])

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- How fragile are dMPC structures?
- How can agents act non-cooperatively?
- How to identify such agents and mitigate the effects?

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Outline

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1. Decomposing the MPC

Distributed Model Predictive Control

- We break the MPC optimization problem
- Make agents communicate

Distributed Model Predictive Control

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In other words

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In other words

- Agents solve local problems

Distributed Model Predictive Control

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Remark

If agents exchange same variable → consensus problem

Distributed Model Predictive Control

Optimization Frameworks

Usually based on optimization decomposition methods[3]:

[3] Boyd et al., "Notes on Decomposition Methods"

Distributed Model Predictive Control

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Distributed Model Predictive Control

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Basically 2 choices[4]:

- Modify based on dual problem[5] (Solve with dual and send primal)

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Security/privacy properties

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Distributed Model Predictive Control

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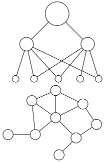

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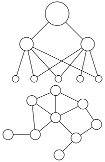

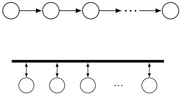

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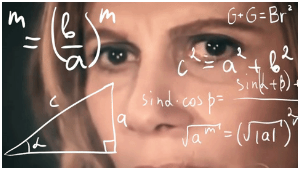

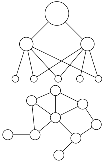

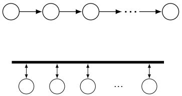

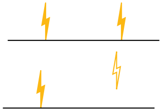

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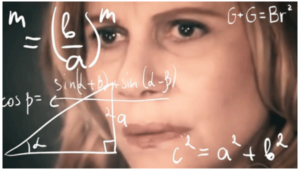

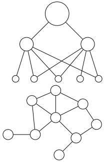

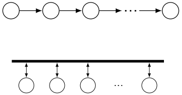

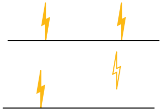

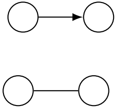

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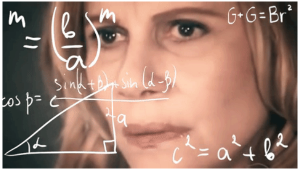

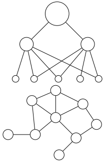

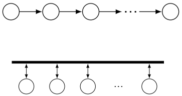

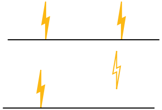

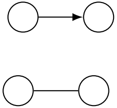

[6] José M Maestre, Negenborn, et al., Distributed Model Predictive Control made easy

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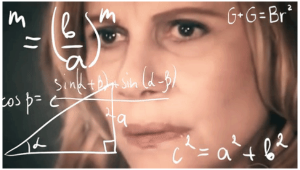
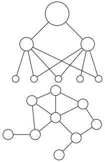
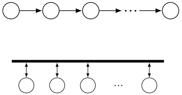
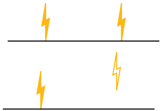
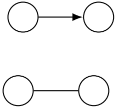










[6] José M Maestre, Negenborn, et al., Distributed Model Predictive Control made easy

Distributed Model Predictive Control

Optimization Decomposition

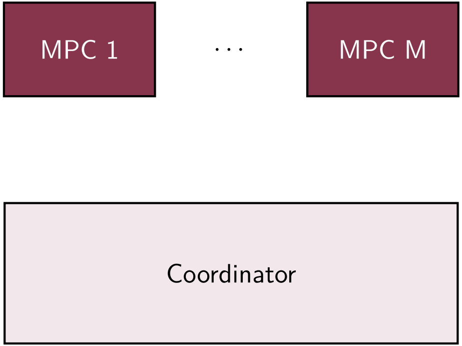
MPC

Distributed Model Predictive Control

Optimization Decomposition

Distributed Model Predictive Control

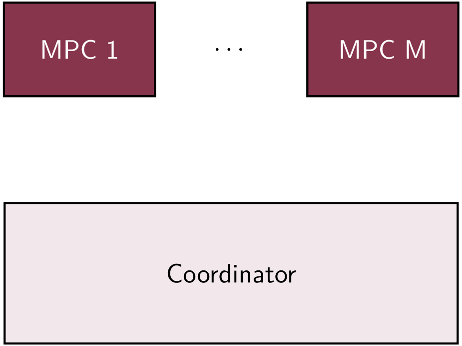
Optimization Decomposition



- Coordinator

Distributed Model Predictive Control

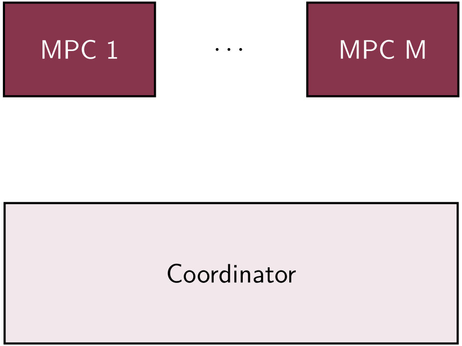
Optimization Decomposition



- Coordinator
  - Enforce global constraints

Distributed Model Predictive Control

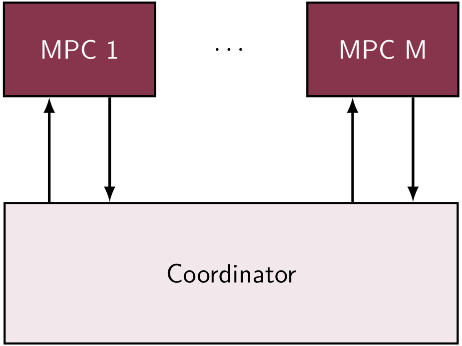
Optimization Decomposition



- Coordinator → Hierarchical
  - Enforce global constraints

Distributed Model Predictive Control

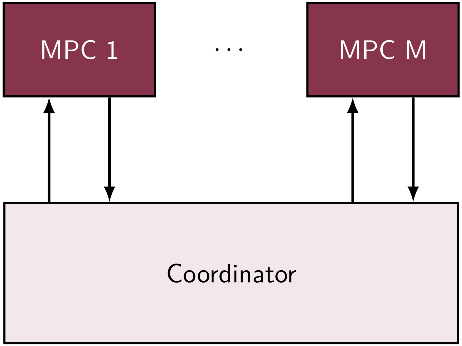
Optimization Decomposition



- Coordinator → Hierarchical
  - Enforce global constraints
- Bidirectional

Distributed Model Predictive Control

Optimization Decomposition



- Coordinator → Hierarchical
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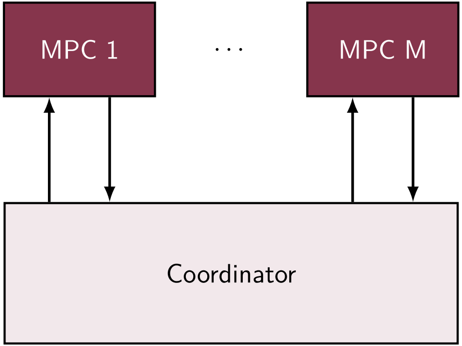
- No delay → Synchronous

Distributed Model Predictive Control

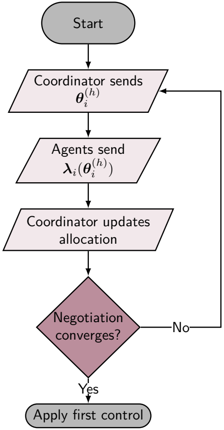
Optimization Decomposition



- Coordinator → Hierarchical
  - Enforce global constraints
- Bidirectional
- No delay → Synchronous
- But what to send?

Primal Decomposition

or Quantity Decomposition | or Resource Allocation



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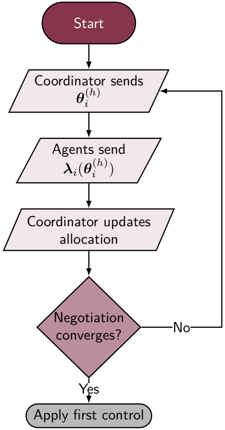

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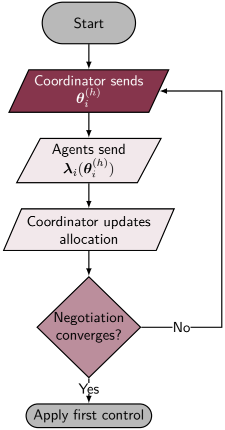

Allocation $\boldsymbol{\theta}_i$

Primal Decomposition

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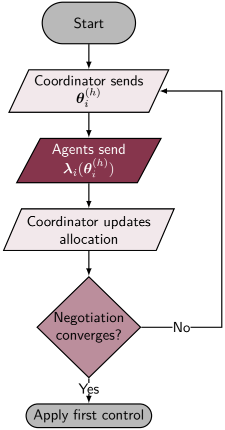

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

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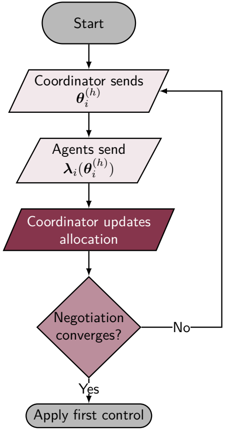

Allocation $\boldsymbol{\theta}_i$

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Update $\boldsymbol{\theta}_i^+ = f_i(\boldsymbol{\theta}_i, \boldsymbol{\lambda}_i)$

Primal Decomposition

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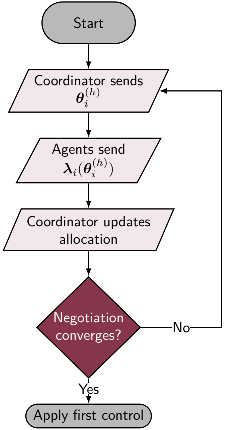

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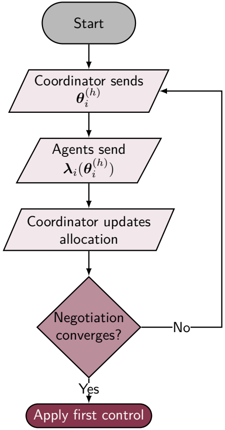

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

Update $\boldsymbol{\theta}_i^+ = f_i(\boldsymbol{\theta}_i, \boldsymbol{\lambda}_i)$

Primal Decomposition

In detail

$$\begin{array}{ll} \underset{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_M}{\text{minimize}} & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s.t.} & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{array}$$

Primal Decomposition

In detail

- Objective is sum of local ones

$$\begin{aligned} \underset{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{aligned}$$

Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

$$
\begin{aligned}
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\text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}}
\end{aligned}
$$

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$$\Big\downarrow \text{ For each } i \in \mathcal{M}$$

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i \end{aligned}$$

Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

1. Allocate $\boldsymbol{\theta}_i$ for each agent

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i \end{aligned}$$

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- Objective is sum of local ones
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2. They solve local problems and

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Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

1. Allocate $\boldsymbol{\theta}_i$ for each agent
2. They solve local problems and
3. Send dual variable $\boldsymbol{\lambda}_i$[7]

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

[8] It obfuscates system's parameters (+ Privacy)

Primal Decomposition

In detail

- Objective is sum of local ones
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1. Allocate $\boldsymbol{\theta}_i$ for each agent
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4. Allocation is updated[8]

$$
\begin{aligned}
\underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\
\text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i
\end{aligned}
$$

$$
\boldsymbol{\theta}[k]^{(p+1)} = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)}
$$

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[8] Only equation to change to add/remove agents

Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

1. Allocate $\boldsymbol{\theta}_i$ for each agent
2. They solve local problems and
3. Send dual variable $\boldsymbol{\lambda}_i$[7]
4. Allocation is updated[8] (respect global constraint)

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)})$$

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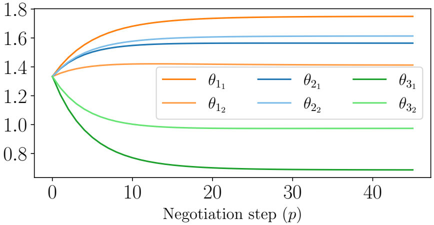
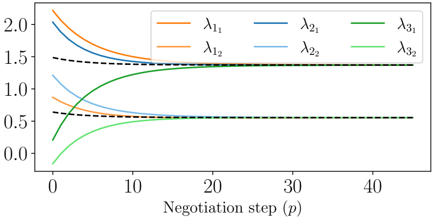
[8] Only equation to change to add/remove agents

Example

Until everybody is evenly[9] dissatisfied





[9]For inequality constraints dynamics are more complex

Distributed Model Predictive Control

Negotiation works if agents comply.

Distributed Model Predictive Control

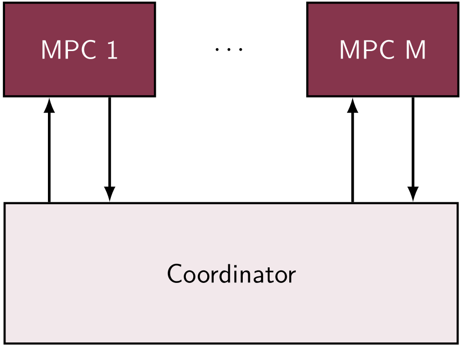
Negotiation works if agents comply.
But what if some agents are ill-intentioned and attack the system?

Outline

2 Attacks on the dMPC

How can a non-cooperative agent attack?

Literature



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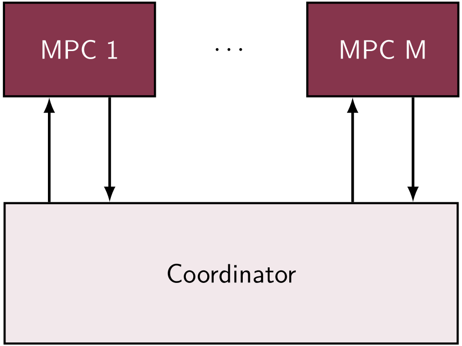

- Common attacks[10]

[10] Velarde, Jose Maria Maestre, Hideaki Ishii, et al., "Scenario-based defense mechanism for distributed model predictive control"

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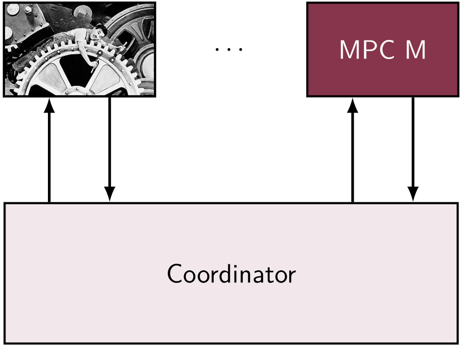

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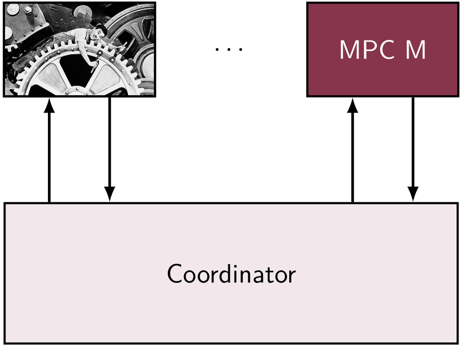

- Common attacks[10]
  - Fake objective function
  - Fake constraints
  - Use different control

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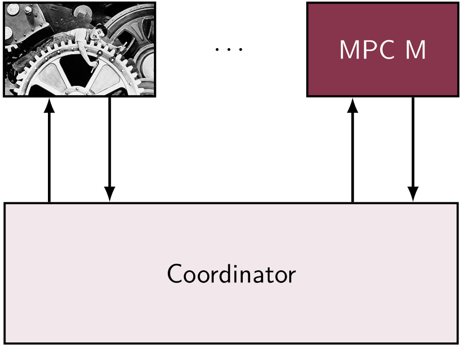

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} Deceptio

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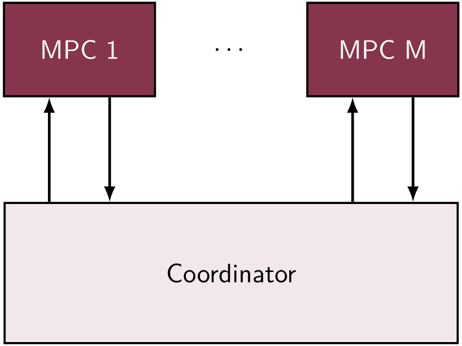
Our approach[11]



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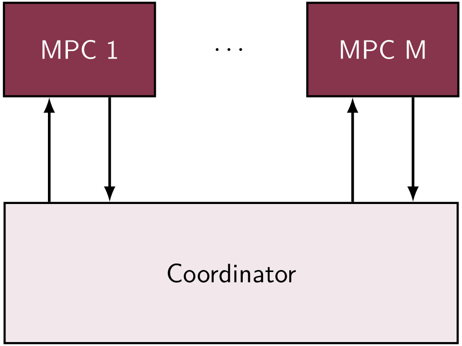

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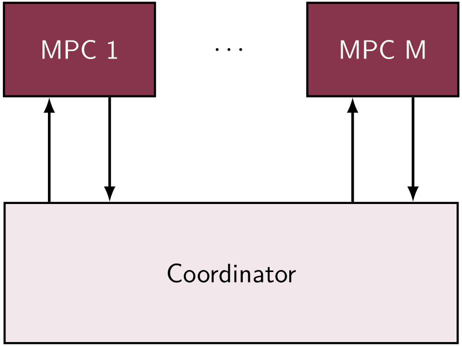

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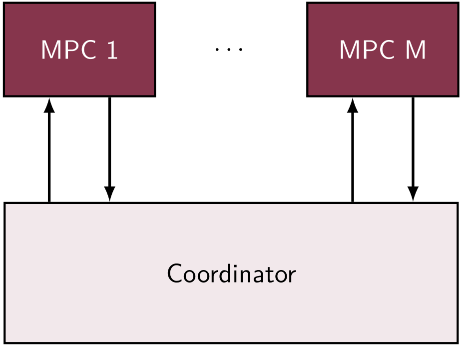

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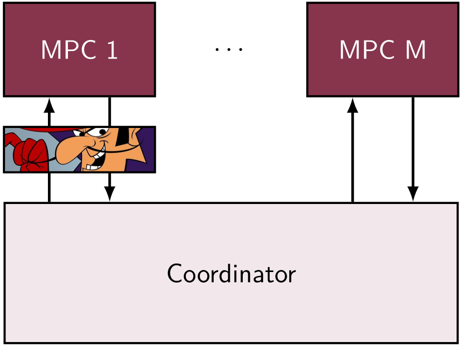

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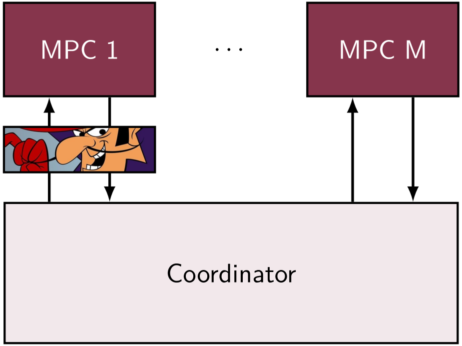

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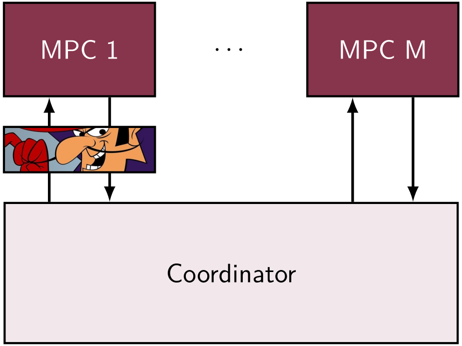

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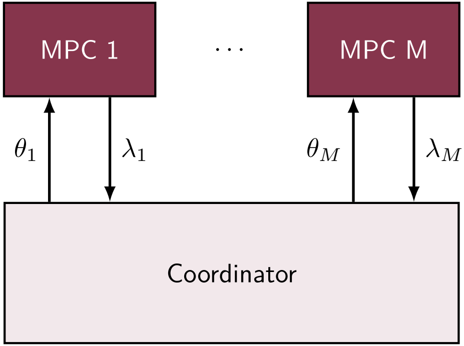

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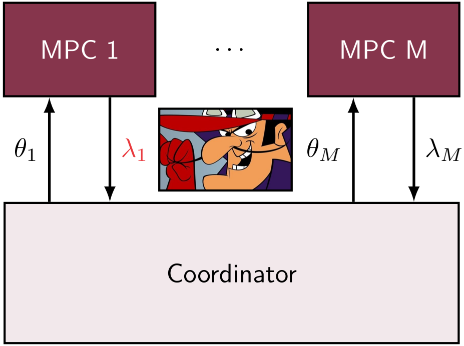

- $\boldsymbol{\lambda}_i$ is the only interface
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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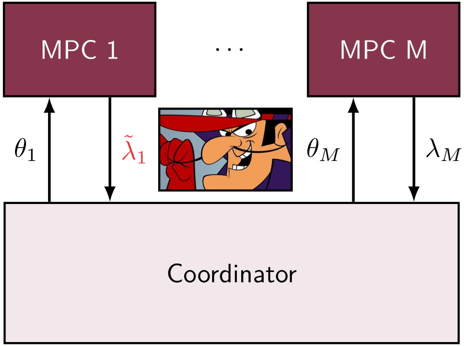

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

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Attack model

Liar, Liar, Pants of fire

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- $\lambda \geqslant 0$ means dissatisfaction

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Assumptions

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- *Same attack during negotiation*

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    - $\gamma(\lambda) = 0 \rightarrow \lambda = 0$

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- Attack is invertible $\rightarrow \exists T_i[k]^{-1}$

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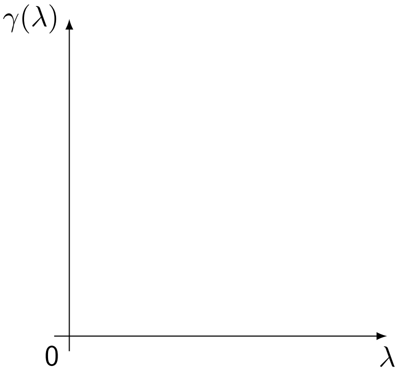

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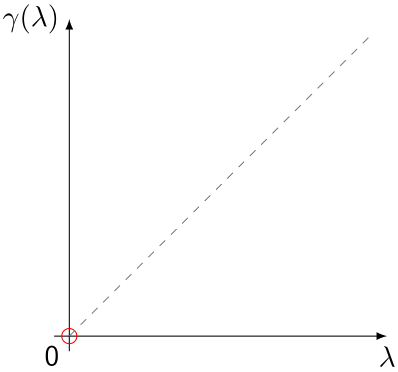

Attack model

Liar, Liar, Pants of fire

- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Same attack during negotiation*
- *Attacker satisfied only if it really is*
    - $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
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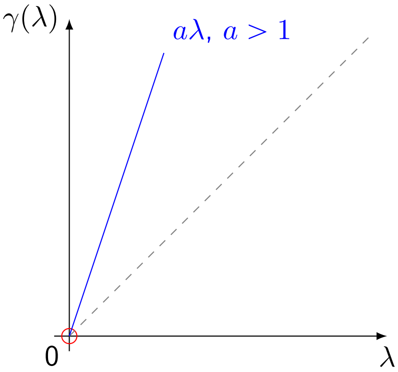

Example

Example

4 distinct agents

- Agent 1 is non-cooperative
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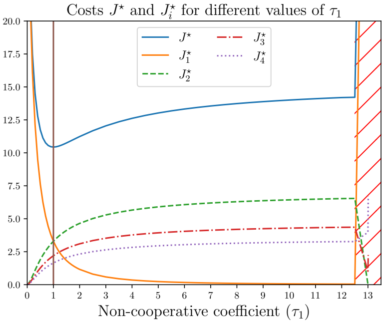

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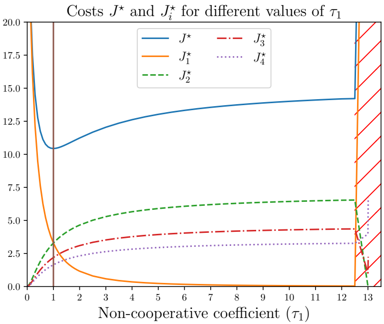

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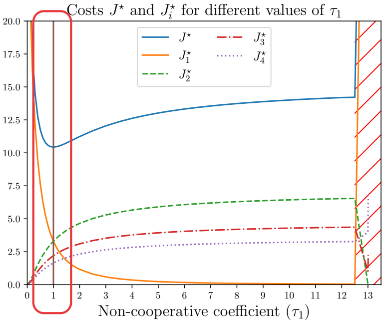

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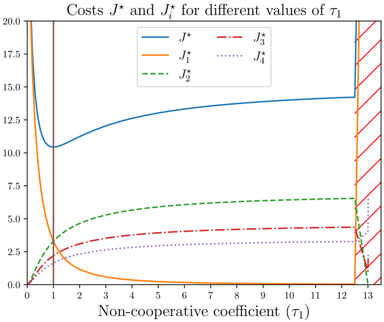

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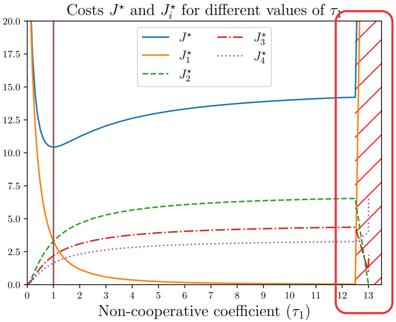

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  - Global minimum when $\tau_1 = 1$
  - Agent 1 benefits if τ_1 increases (inverse otherwise)
  - All collapses if too greedy

- But can we mitigate these effects?

- But can we mitigate these effects?
- Yes! (At least in some cases)

Outline

Classification of mitigation techniques

Passive (Robust)

Active (Resilient)

Classification of mitigation techniques

Passive (Robust)

- 1 mode

Active (Resilient)

- 2 modes

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[12]	Dual	Robust (Scenario)	NA	NA
[13]	Dual	Robust (f-robust)	NA	NA
[14]	Jacobi-Gauß	–	–	–
[15]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)

[12] José M. Maestre et al., "Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc".

[13] Velarde, José M. Maestre, et al., "Vulnerabilities in Lagrange-Based Distributed Model Predictive Control".

[14] Chanfreut, J. M. Maestre, and H. Ishii, "Vulnerabilities in Distributed Model Predictive Control based on Jacobi-Gauss Decomposition".

[15] Ananduta et al., "Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids".

Our Approach

Explore Scarcity

- Resilient
- Analytical/Learning
- Data reconstruction

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} Parameter
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Our Approach

Explore Scarcity

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- Explore Scarcity

Outline

3 Securing the dMPC
- Proposed Methods
  - Resilient Primal Decomposition-based dMPC for deprived systems
  - Resilient Primal Decomposition-based dMPC using Artificial Scarcity

What are deprived systems?

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Systems whose optimal solution has all constraints active

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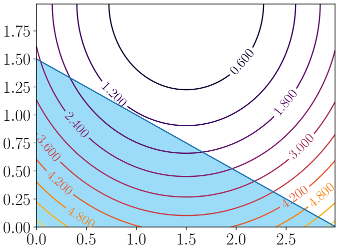

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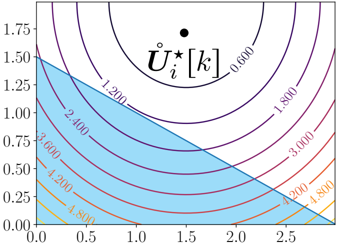
- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$



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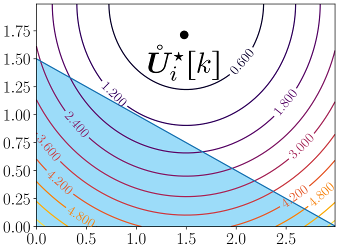

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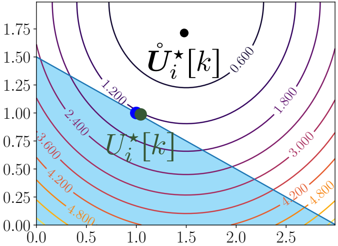

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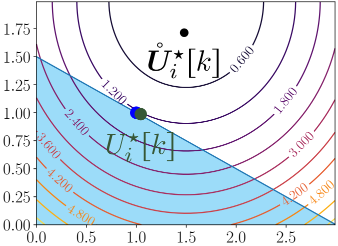
  - Same as with equality constraints[16]



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[16] If system can have all constraints active simultaneously see here

Analyzing Deprived Systems

Assumptions

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(local parameters unknown by coordinator)

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- Change $\rightarrow$ Probably an Attack! Let's take advantage of this!

Detection Mechanism

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- We estimate[17] $\hat{P}_i[k]$ and $\widehat{\tilde{\boldsymbol{s}}}_i[k]$ such as:

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[17] Using Recursive Least Squares for example

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- If $\left\| \widehat{\tilde{P}}_i[k] - \bar{P}_i \right\|_F > \epsilon_P \rightarrow$ Attack
- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

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Estimating $\widehat{\tilde{P}}_i[k]$

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[18] A random signal causes persistent excitation of any order (Adaptive Control)

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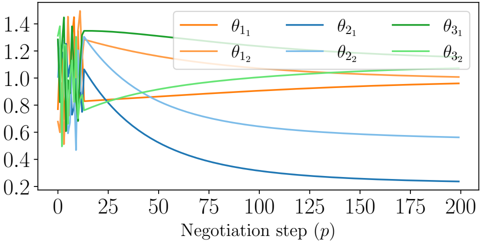

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  1. Detection/Isolation ✔
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Mitigation mechanism

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Reconstructing $\boldsymbol{\lambda}_i$

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Mitigation mechanism

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Mitigation mechanism

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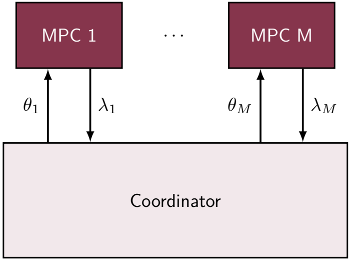
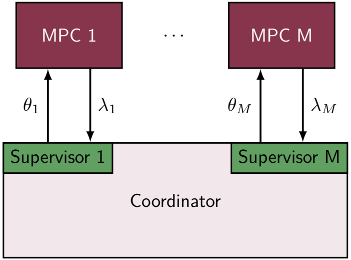
- Choose adequate version for coordination

$$\overset{\text{mod}}{\boldsymbol{\lambda}}_i = \begin{cases} \overset{\text{rec}}{\boldsymbol{\lambda}}_i, & \text{if attack detected} \\ \tilde{\boldsymbol{\lambda}}_i, & \text{otherwise} \end{cases}$$

Complete Mechanism

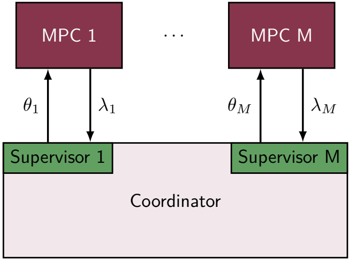


Complete Mechanism



- Supervise exchanges by inquiring the agents

Complete Mechanism



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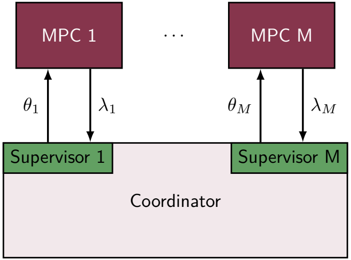

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Two Phases

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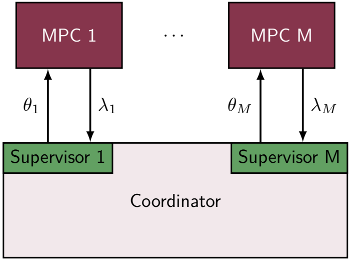

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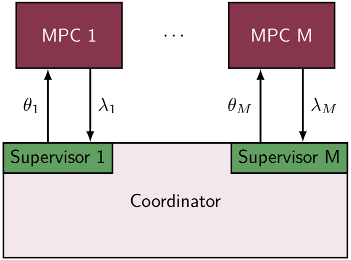
Two Phases

1. Detect which agents are non-cooperative

Complete Mechanism



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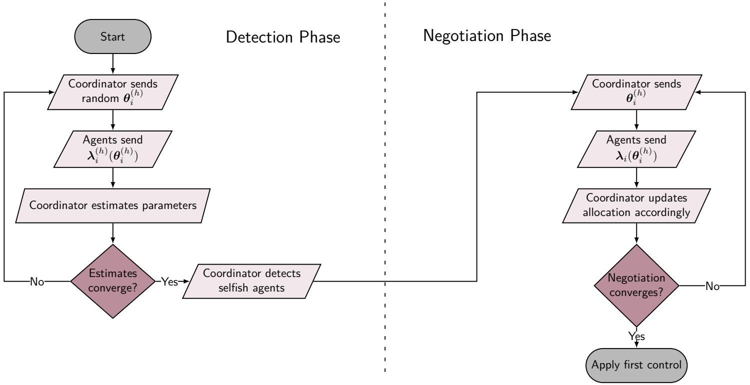
Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Complete algorithm

RPdMPC-DS[19]



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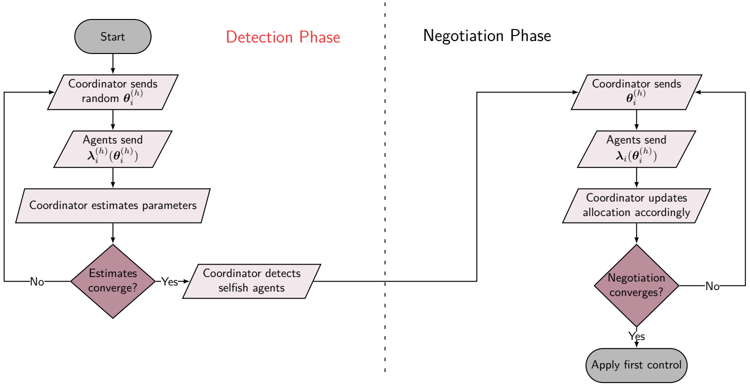

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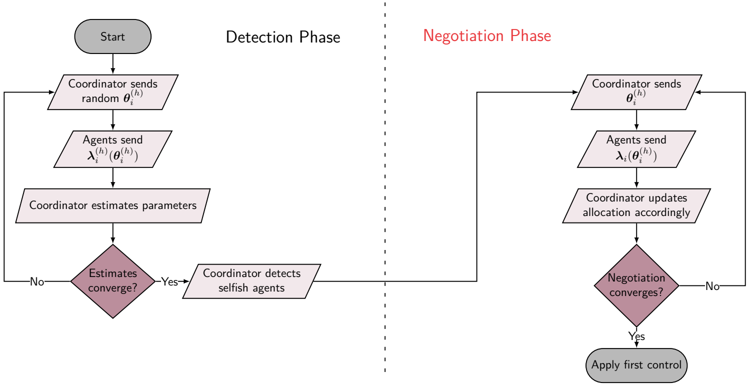

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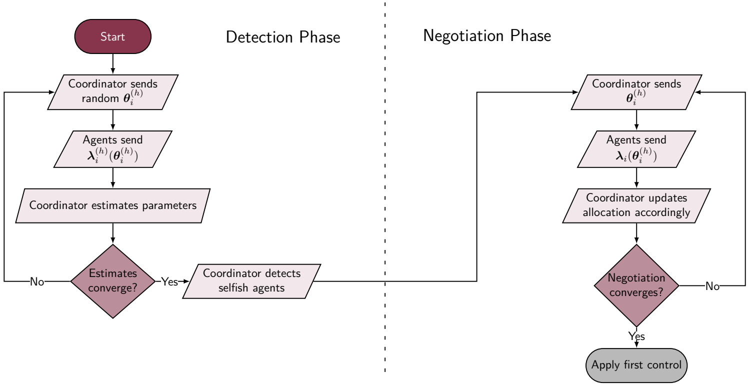

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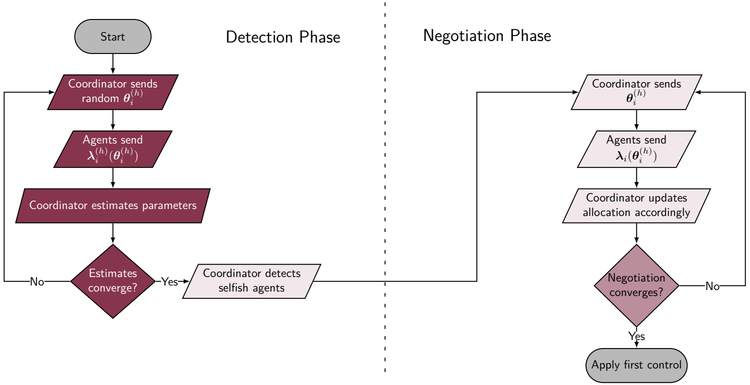

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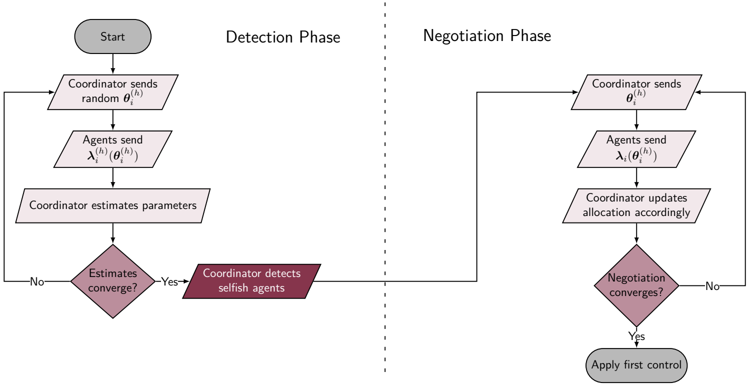

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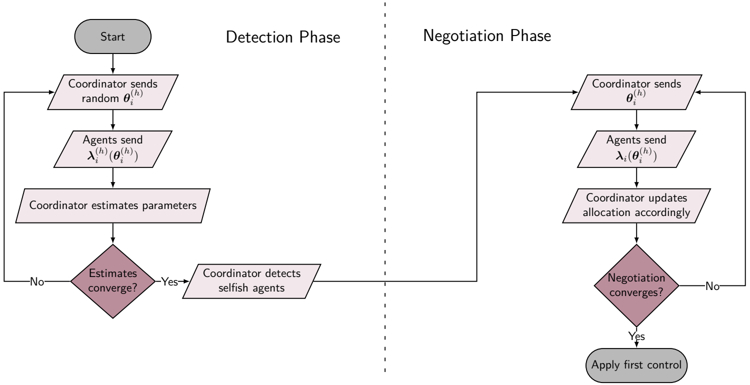

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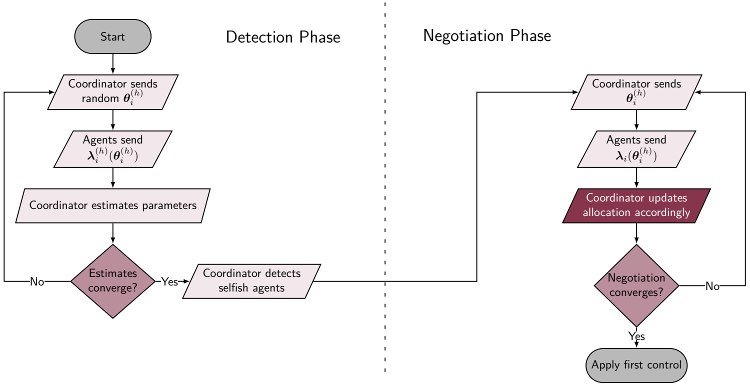

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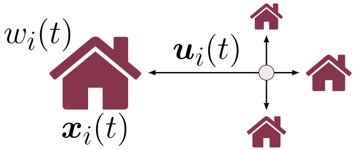
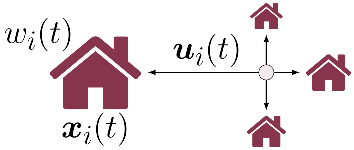
Example



District Heating Network (4 Houses)

Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)

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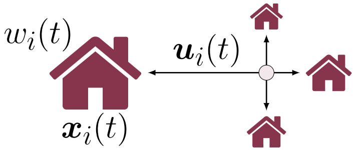

District Heating Network (4 Houses)

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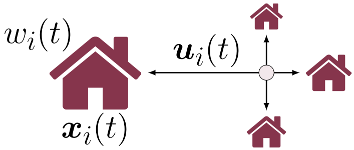
Example



District Heating Network (4 Houses)

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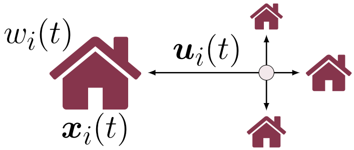

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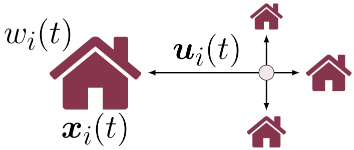

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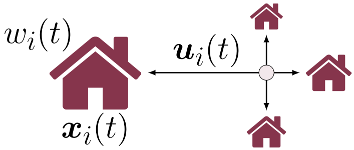

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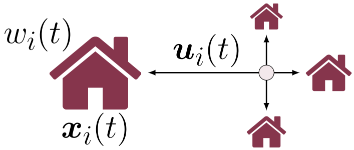

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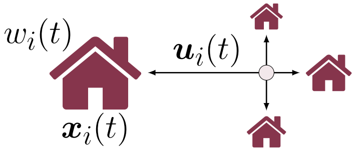

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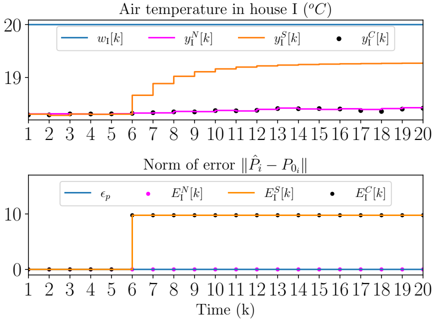
  - S Agent I cheats (RPdMPC-DS)

Results

Temporal



Temperature in house I.

Error $E_I(k)$.

N Nominal, S Selfish, C Corrected

Results

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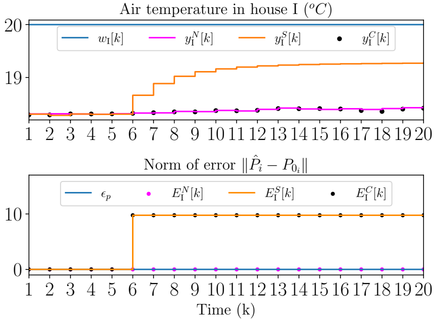

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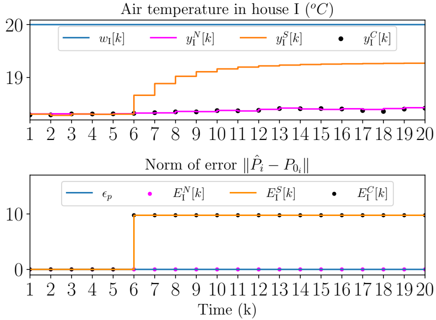

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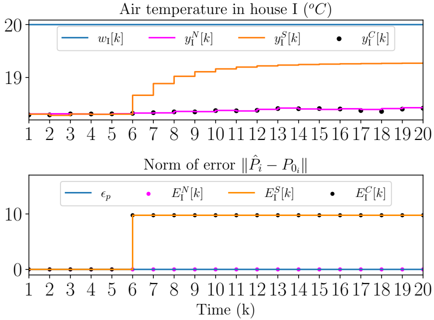
Temporal



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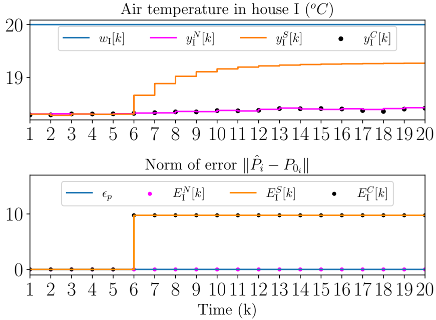

Temperature in house I.
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- Agent starts cheating in $k = 6$
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- C Restablish behavior close to N

Results

Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.5
II	21.67	-0.55
III	17.39	-0.0
IV	17.63	-0.09
Global	3.53	0.02

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Outline

3. Securing the dMPC
   - Proposed Methods
     - Resilient Primal Decomposition-based dMPC using Artificial Scarcity

Relaxing scarcity assumption

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- Systems are not completely deprived

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  - We can't change our constraints to equality ones anymore

$$\begin{aligned} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|^2_{H_i} + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} \quad & \bar{\Gamma}_i \boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{aligned}$$

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

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Now, we may have multiple (Piecewise affine function)

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Still the $P_i^{(z)}$ are time independent

Analyzing System

Solution for $\lambda_i[k]$ (Continued)

Analyzing System

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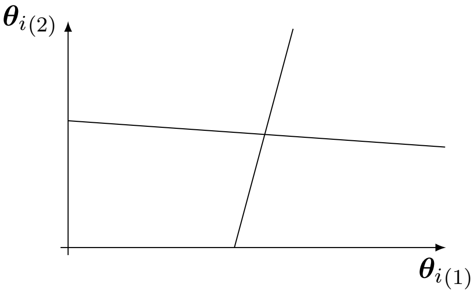

Separation surfaces depend on state and local parameters.

Analyzing System

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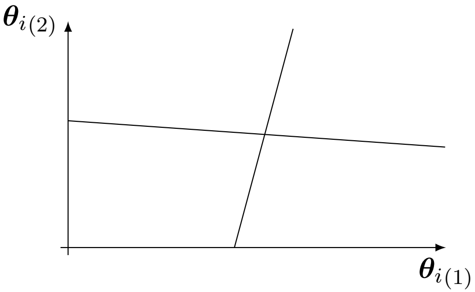

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Unknown by the coordinator.

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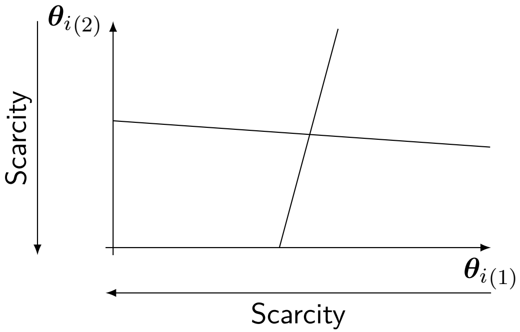

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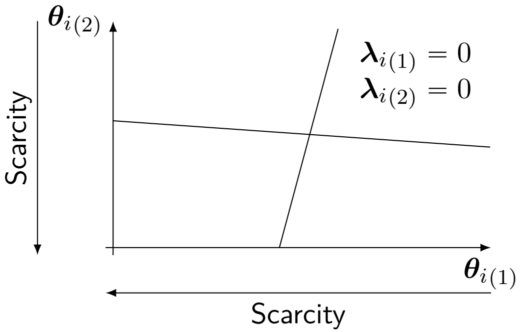

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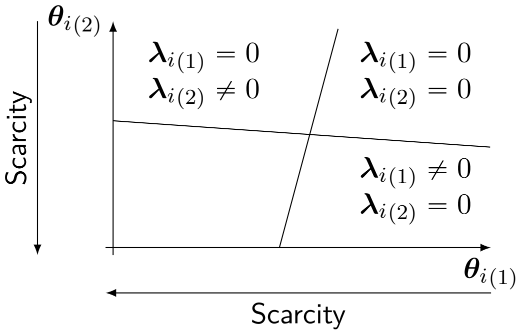

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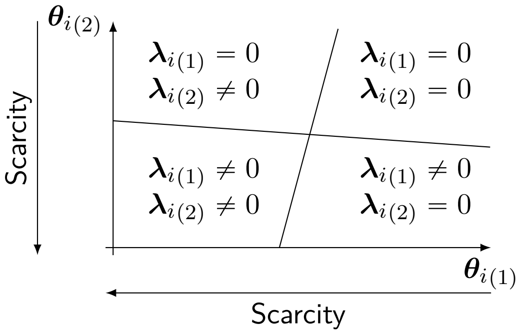

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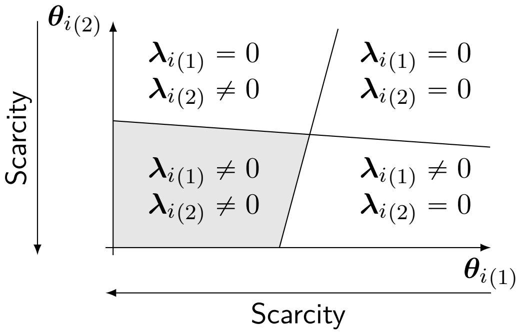

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Scarcity

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All constraints active $\quad -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k] \xrightarrow{\text{Scarcity}} -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$

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All constraints active	$-P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k]$	$\rightarrow$	$-P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$
None constraints active	$-P_i^{(Z)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(Z)}[k]$	$\rightarrow$	$\mathbf{0}$

Analyzing System

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Scarcity

Assumptions

The region $\mathcal{R}^0_{\boldsymbol{\lambda}_i} \neq \varnothing$ *and we known a point* $\overset{\varnothing}{\boldsymbol{\theta}}_i \in \mathcal{R}^0_{\boldsymbol{\lambda}_i}$

Analyzing System

Under attack!

Analyzing System

Under attack!

$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Analyzing System

Under attack!

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- Solution: Let's force it using Artificial Scarcity

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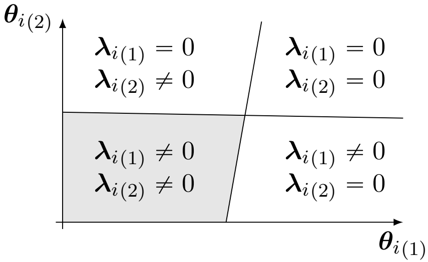

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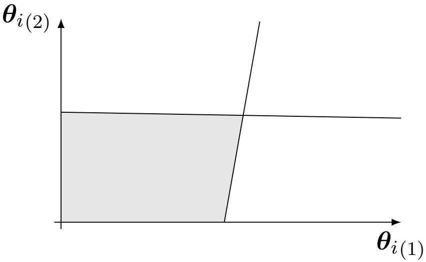

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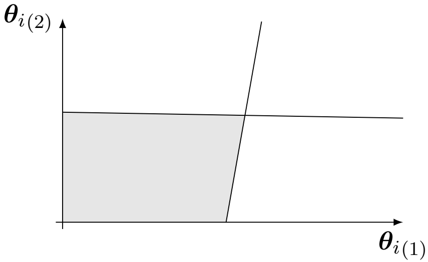

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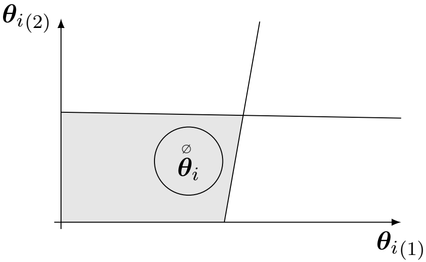

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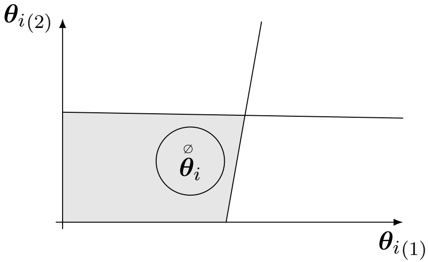

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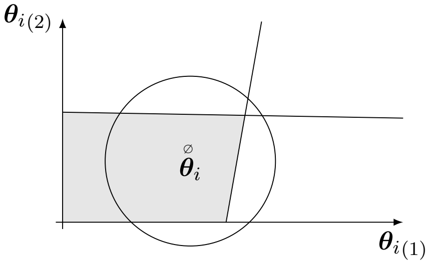

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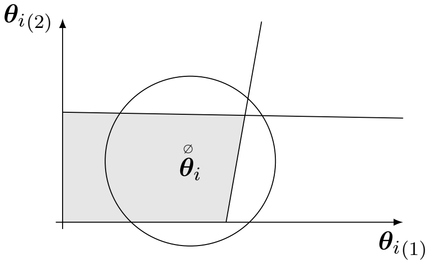

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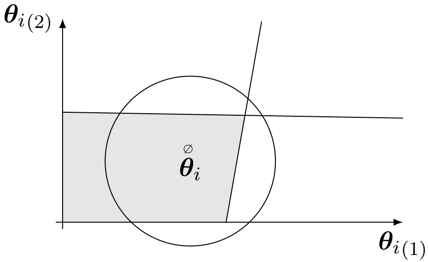

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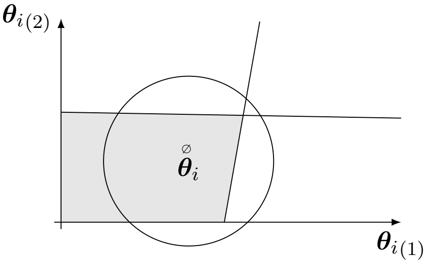

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Same same, but different

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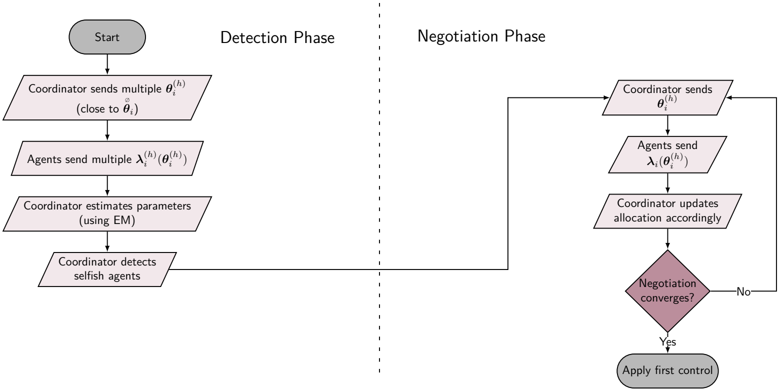
Complete algorithm

RPdMPC-AS[22]



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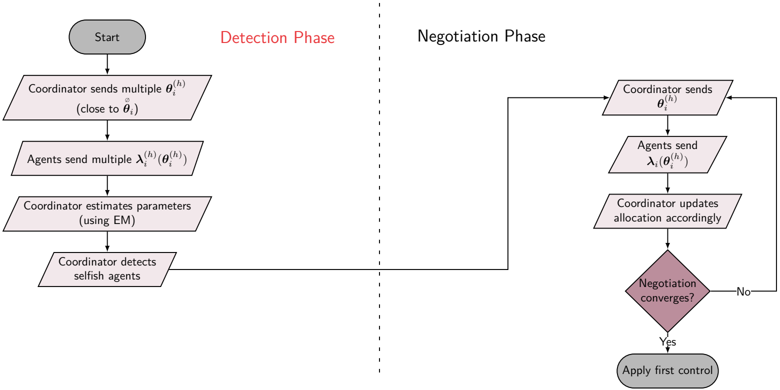

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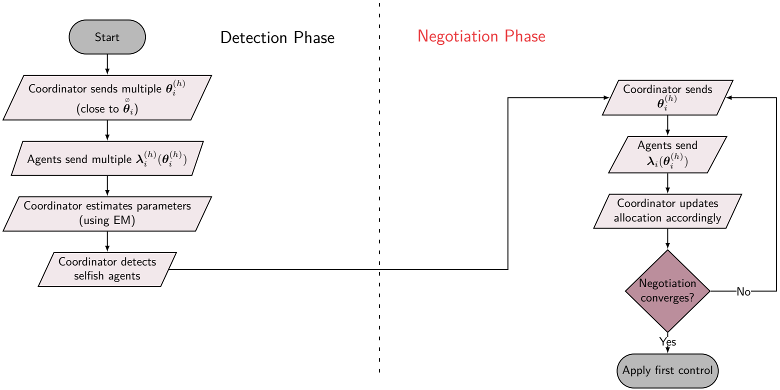

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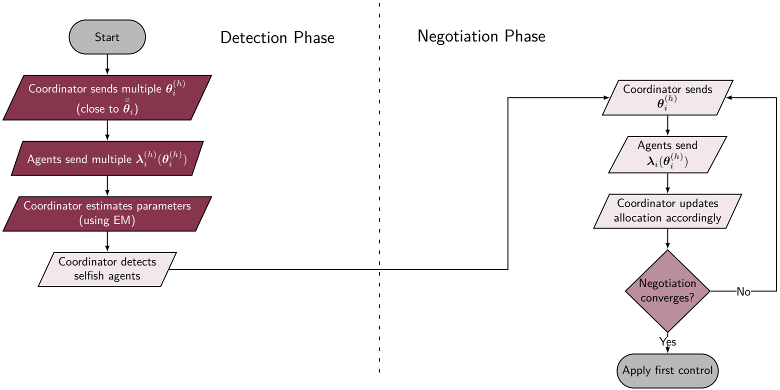

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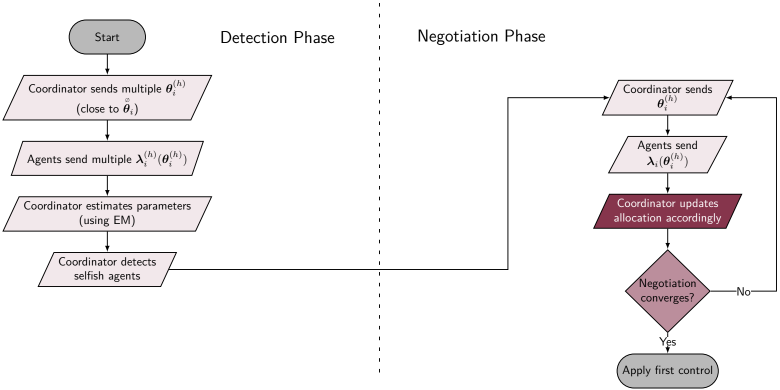

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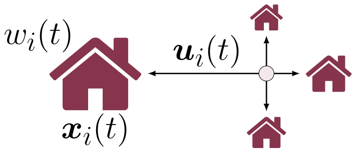

District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of $5\mathrm{h}$ $(T_s = 0.25h \rightarrow k = \{1:20\})$
- Prediction horizon $(N = 4)$
- 3 scenarios
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  - C Agent I cheats (dMPC)
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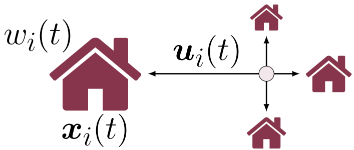

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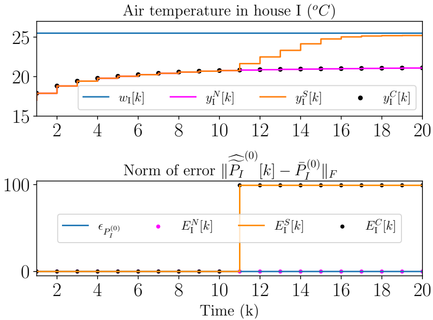

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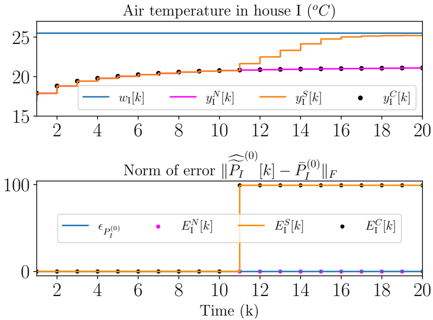

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.49	$-4.12e-05$
II	35.81	$1.74e-05$
III	29.22	$2.14e-05$
IV	37.54	$1.73e-05$
Global	10.69	$-6e-07$

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Questions? Comments?

Repository
https://github.com/Accacio/thesis

Contact
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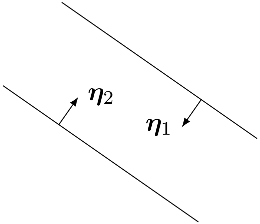
For Further Reading I

- Åström, K.J. and B. Wittenmark. Adaptive Control. Addison-Wesley series in electrical and computer engineering: Control engineering. Addison-Wesley, 1989. ISBN: 9780201097207. DOI: 10.1007/978-3-662-08546-2_24.
- Maestre, José M, Rudy R Negenborn, et al. Distributed Model Predictive Control made easy. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.
- Nogueira, Rafael Accácio. "Security of DMPC under False Data Injection". 2022CSUP0006. PhD thesis. CentraleSupélec, 2022. URL: http://www.theses.fr/2022CSUP0006.

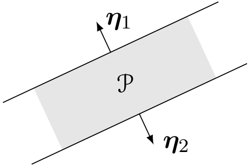
Conditions

back

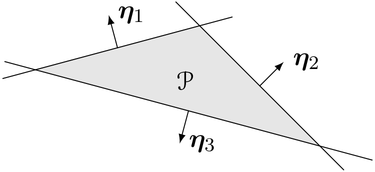
One way to ensure this, is to make the original constraints to form a cone.



No intersection



$\langle_{\boldsymbol{\eta}_1}^{\boldsymbol{\eta}_2} = 180^o$



A 3-sided polyhedron.

θ dynamics

◂ back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \frac{1}{M}\rho^{(p)}P_1 & I - \frac{M-1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)}P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \end{bmatrix}$$

Parameters estimated depending on Prediction Horizon N

$\#$ constraints depend on $\#$ global constraints c and prediction horizon N

- Number of Regions $= 2^{Nc}$
- Parameters in each region $=$ Matrix $P_i^{(z)} = (Nc)^2$ $+$ vector $\boldsymbol{s}_i^{(z)}[k] = Nc$
  - Total $((Nc)^2 + Nc)2^{Nc}$

Some examples

- 1 constraint
  - $N = 3 \rightarrow$ 96 elements
  - $N = 4 \rightarrow$ 320 elements

Remark

We can reduce number of elements estimated from $P_i^{(z)}$ if we assume $P_i^{(z)} \in \mathbb{S}$
New total $\rightarrow$ $((Nc)^2 + 3Nc)2^{Nc-1}$