Security of distributed Model Predictive Control under False Data Injection

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Seminar
École Centrale de Lyon / Laboratoire Ampère
26/05/2023 @ Écully

https://bit.ly/3g3S6X4

About me

Rafael Accácio Nogueira

Postdoctoral researcher at LAAS/CNRS

Garanteed relative localisation and anticollision scenario for autonomous vehicles

Project AutOCampus (GIS neOCampus)

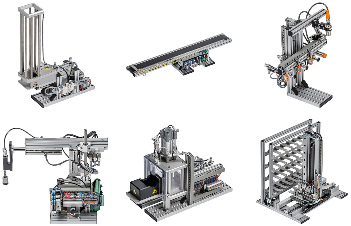
Advised by Soheib Fergani

About me

Bachelor Thesis at Escola Politécnica/UFRJ

Identification of DES for fault-diagnosis

Advised by Marcos Vicente de Brito Moreira



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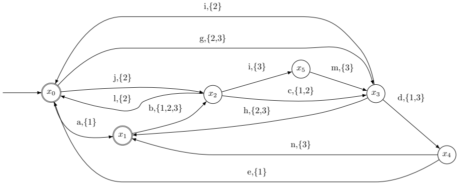
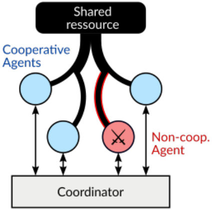


About me

Doctoral Thesis at CentraleSupélec/IETR

Security of dMPC under False Data Injection

Advised by Hervé Guéguen and Romain Bourdais



Context

Smart(er) Cities

Multiple systems interacting

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Multiple systems interacting

Context

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- Distribution:

Smart(er) Cities

Multiple systems interacting

- Distribution:
  - Electricity

Context

Smart(er) Cities

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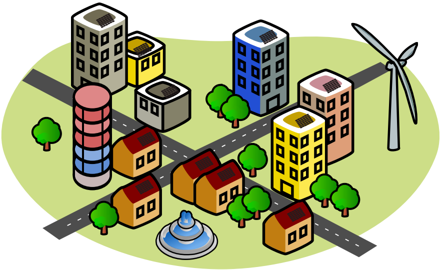

- Distribution:
  - Electricity
  - Heat
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Context

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- Distribution:
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Multiple systems interacting under

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Context

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- We also want
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Context

Smart(er) Cities

Multiple systems interacting under

- Technical/Comfort Constraints
- We also want
  - Minimize consumption
  - Maximizer satisfaction
  - Follow a trajectory
- Solution → MPC

Model-based Predictive Control

Brief recap

Model-based Predictive Control

Brief recap

Find best control sequence using predictions based on a model.

Model-based Predictive Control

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Model-based Predictive Control

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Find optimal control sequence using predictions based on a model.

Brief recap

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- We need an optimization problem

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N

$$\underset{\boldsymbol{u}[0:{\color{red}N}-1|k]}{\text{minimize}} \qquad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:{\color{red}N}-1|k])$$

Brief recap

Find optimal control sequence using predictions based on a model.

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  - Decision variable is the control sequence calculated over horizon N
  - Objective function to optimize

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N
  - Objective function to optimize
  - System's Model

$$\begin{aligned}
\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad & J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k]) \\
\text{subject to} \quad & \left.\begin{aligned} \boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ \\ \end{aligned}\right\} \forall \xi \in \{1, \dots, N\}
\end{aligned}$$

Find optimal control sequence using predictions based on a model.

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  - System's Model
  - Other constraints to respect

$$\begin{array}{ll} \underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} & J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k]) \\ \text{subject to} & \left.\begin{array}{c} \boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ g_i(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \leqslant 0 \\ h_j(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) = 0 \end{array}\right\} \begin{array}{l} \forall \xi \in \{1, \dots, N\} \\ \forall i \in \{1, \dots, m\} \\ \forall j \in \{1, \dots, p\} \end{array} \end{array}$$

Model-based Predictive Control

Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon N
  - Objective function to optimize
  - System's Model
  - Other constraints to respect (QoS, technical restrictions, ...)

$$
\begin{array}{ll}
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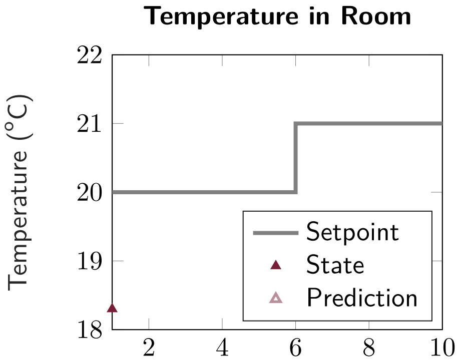
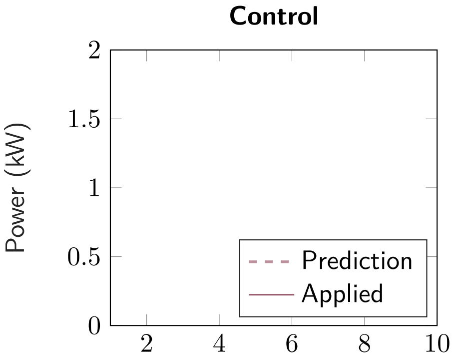
Model Predictive Control

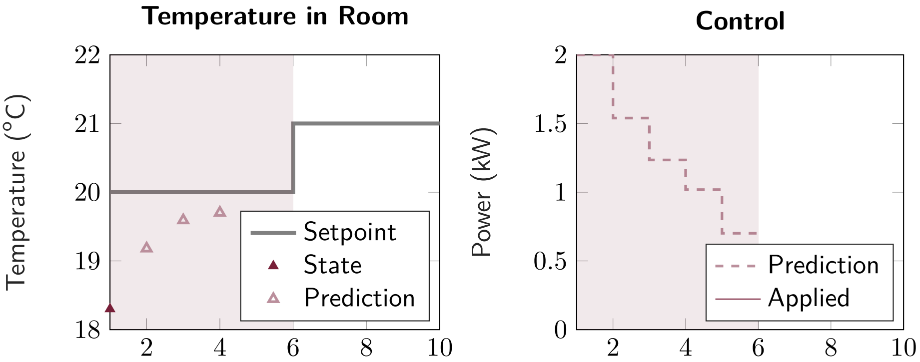
In a nutshell





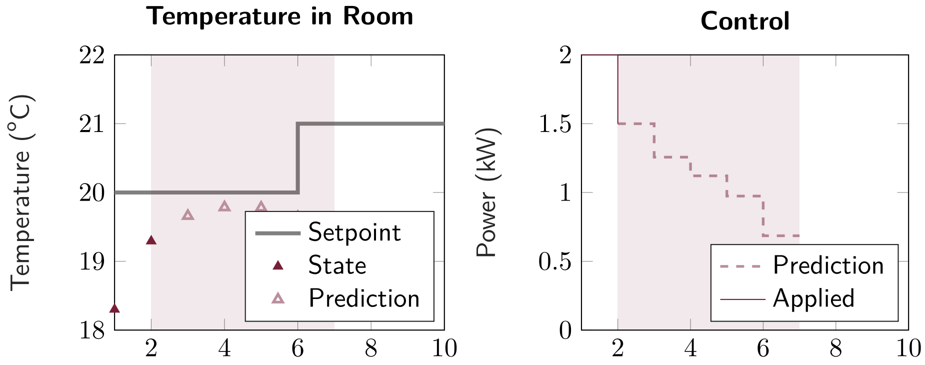
In a nutshell

Find optimal control sequence



In a nutshell

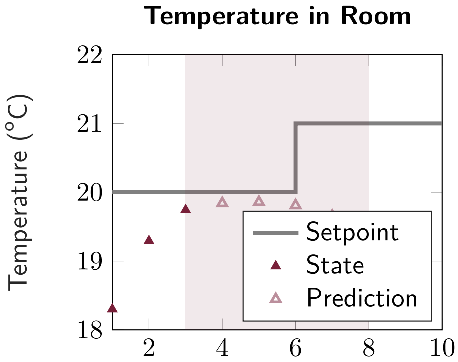
Find optimal control sequence, apply first element

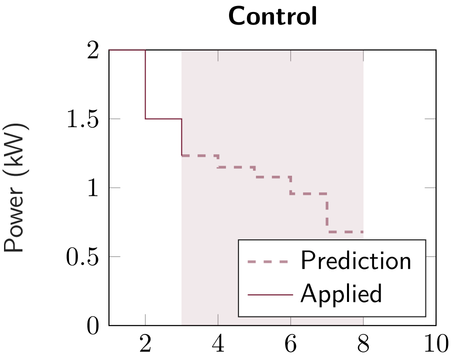


Model Predictive Control

In a nutshell

Find optimal control sequence, apply first element, rinse repeat

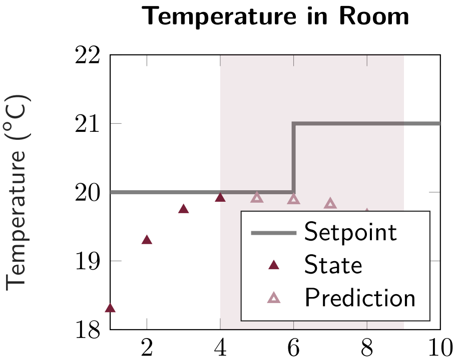


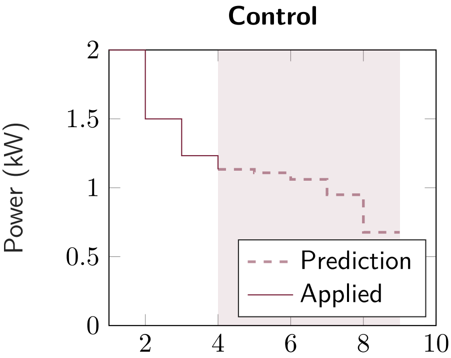


Model Predictive Control

In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





Model Predictive Control

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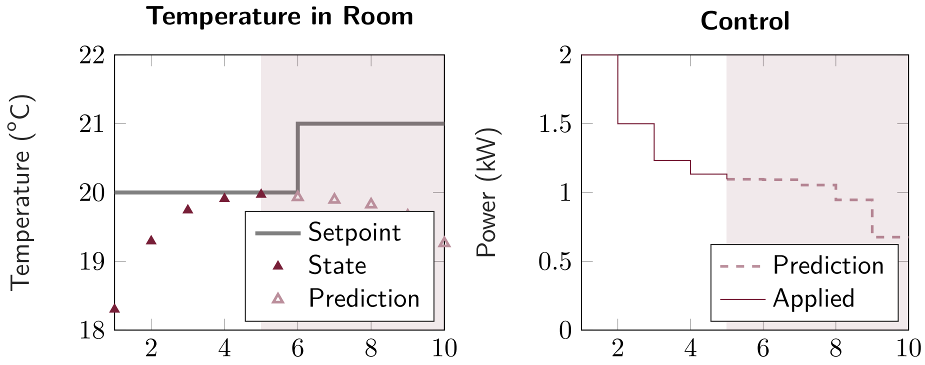
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



Model Predictive Control

Nothing is perfect

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- Problems

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  - Topology (Geographical distribution)

Model Predictive Control

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  - Privacy (RGPD)

Model Predictive Control

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  - Topology (Geographical distribution)
  - Complexity of calculation
  - Flexibility (Add/remove parts)
  - Privacy (RGPD)
- Solution: Divide and Conquer (distributed MPC)

Outline

1. Decomposing the MPC

Outline

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1. Decomposing the MPC
2. Attacks on the dMPC
3. Securing the dMPC

Outline

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Distributed Model Predictive Control

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In other words

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- Agents solve local problems

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- Variables are updated

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Until Convergence

Distributed Model Predictive Control

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Until Convergence

Remark

If agents exchange same variable → consensus problem

Distributed Model Predictive Control

Optimization Frameworks

Usually based on optimization decomposition methods[1]:

[1] Boyd et al., "Notes on Decomposition Methods"

Distributed Model Predictive Control

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Distributed Model Predictive Control

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Basically 2 choices[2]:

- Modify based on dual problem[3] (Solve with dual and send primal)

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Distributed Model Predictive Control

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Security/privacy properties

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Distributed Model Predictive Control

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Distributed Model Predictive Control

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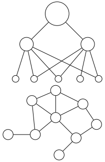
  - Many flavors to choose from
    - Hierarchical/Anarchical



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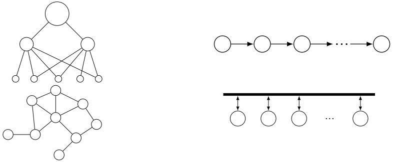

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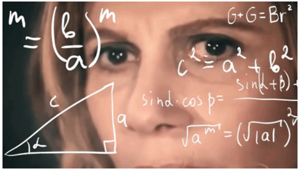

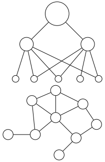

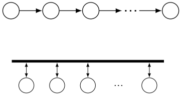

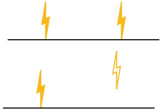

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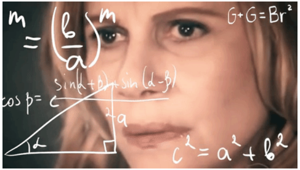

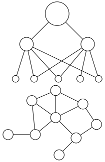

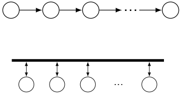

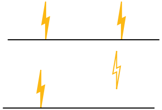

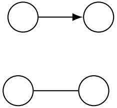

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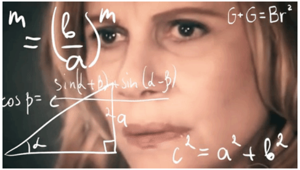

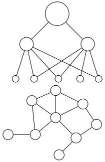

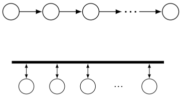

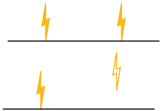

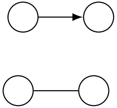

[4] José M Maestre, Negenborn, et al., Distributed Model Predictive Control made easy

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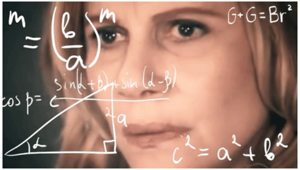

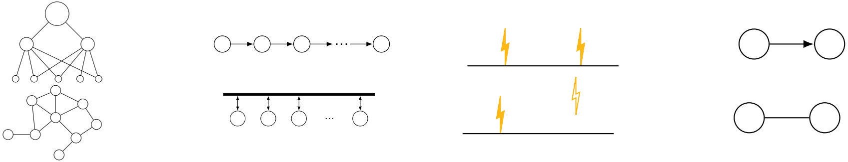

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Distributed Model Predictive Control

Optimization Decomposition

MPC

Distributed Model Predictive Control

Optimization Decomposition

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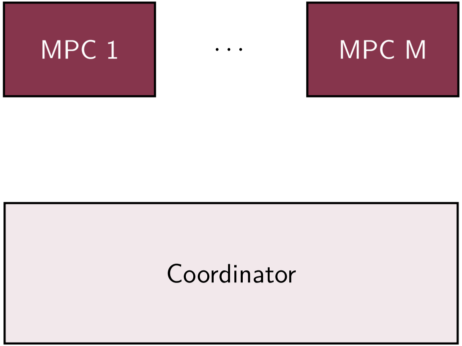
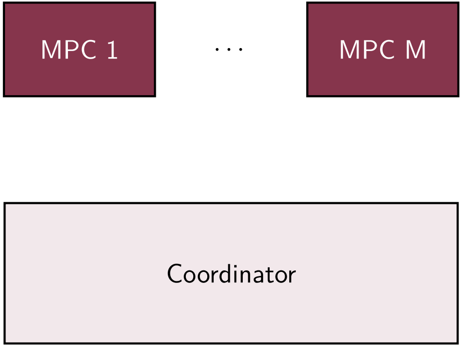
Optimization Decomposition



- Coordinator

Distributed Model Predictive Control

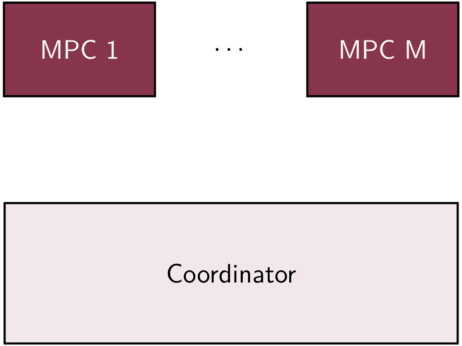
Optimization Decomposition



- Coordinator
  - Enforce global constraints

Distributed Model Predictive Control

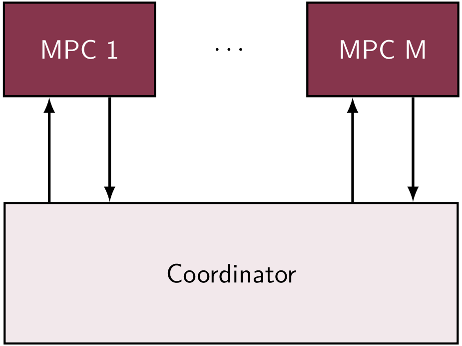
Optimization Decomposition



- Coordinator → Hierarchical
  - Enforce global constraints

Distributed Model Predictive Control

Optimization Decomposition



- Coordinator → Hierarchical
    - Enforce global constraints
- Bidirectional

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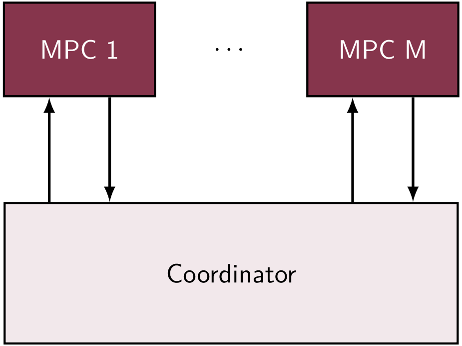
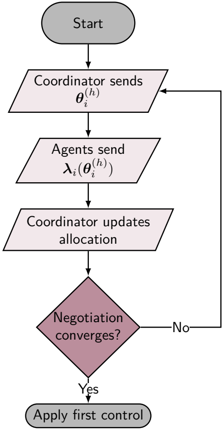


- Coordinator → Hierarchical
  - Enforce global constraints
- Bidirectional
- No delay → Synchronous

Primal Decomposition

or Quantity Decomposition | or Resource Allocation



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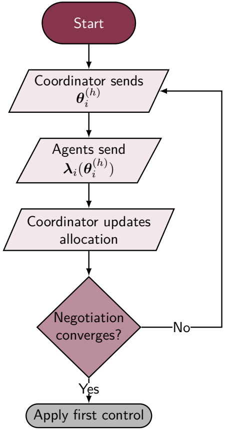

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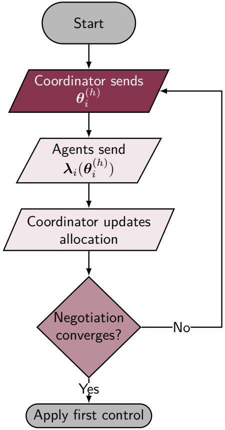

Allocation $\boldsymbol{\theta}_i$

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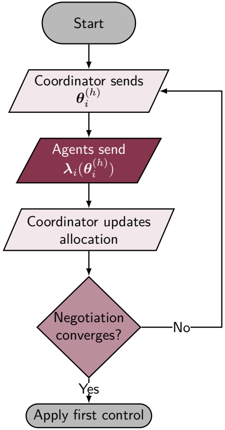

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

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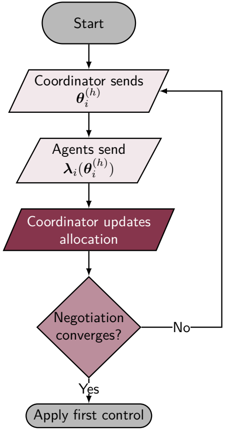

Allocation $\boldsymbol{\theta}_i$

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Update $\boldsymbol{\theta}_i^+ = f_i(\boldsymbol{\theta}_i, \boldsymbol{\lambda}_i)$

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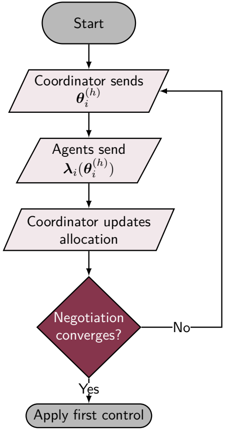

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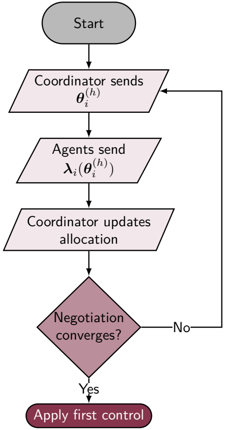

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Primal Decomposition

In detail

$$\begin{aligned} \underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\mathsf{total}} \end{aligned}$$

Primal Decomposition

In detail

- Objective is sum of local ones

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$$\Big\downarrow \ \text{For each } i \in \mathcal{M}$$

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$$\boldsymbol{\theta}[k]^{(p+1)} = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)}$$

Primal Decomposition

In detail

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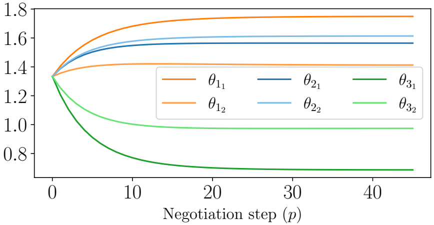
1. Allocate $\boldsymbol{\theta}_i$ for each agent
2. They solve local problems and
3. Send dual variable $\boldsymbol{\lambda}_i$
4. Allocation is updated (respect global constraint)

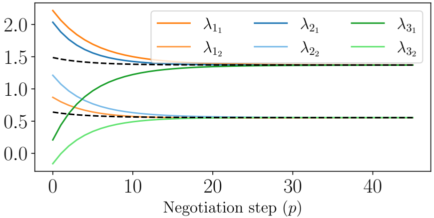
$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \text{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)})$$

Example

Until everybody is evenly[5] dissatisfied





[5] For inequality constraints dynamics are more complex

Distributed Model Predictive Control

Negotiation works if agents comply.

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But what if some agents are ill-intentioned and attack the system?

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- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects? How?

---

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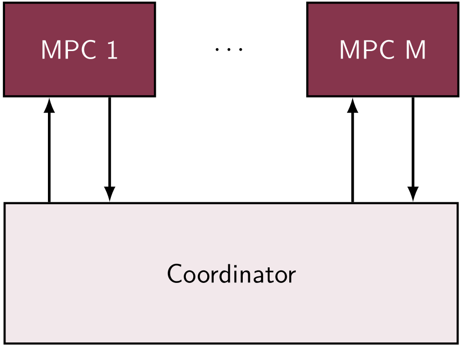
Outline

2. Attacks on the dMPC

How can a non-cooperative agent attack?

Literature



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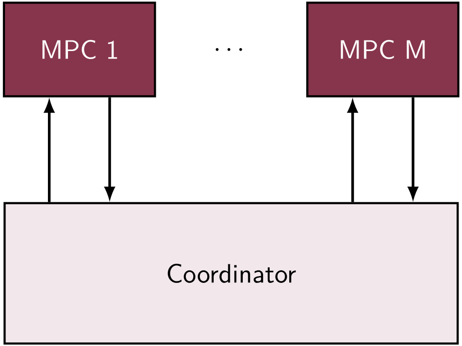

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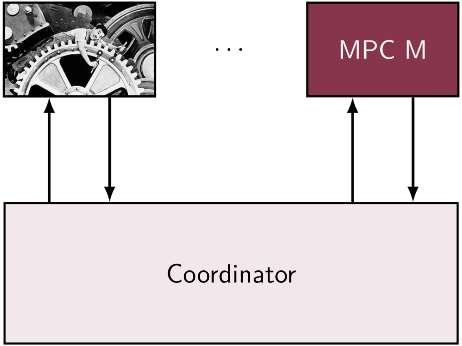

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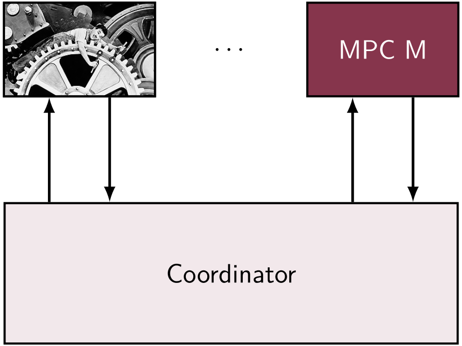

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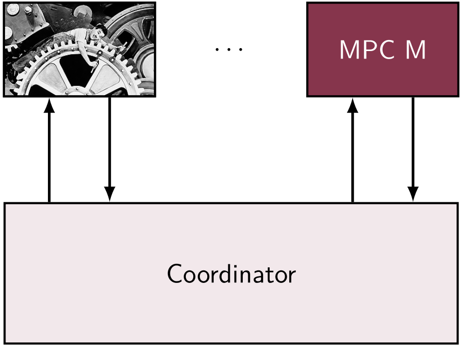

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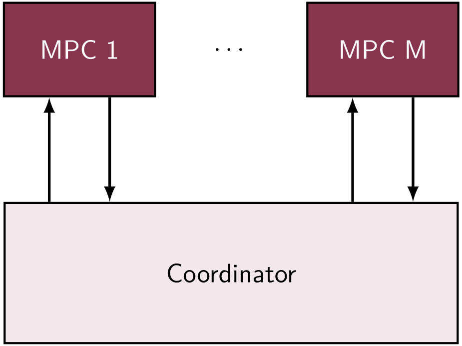

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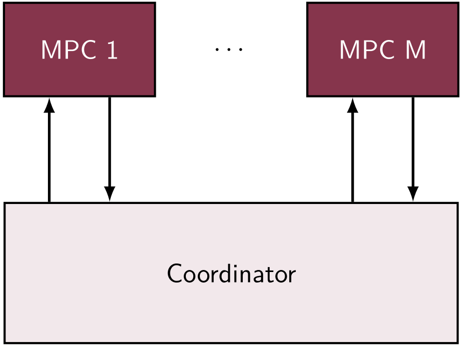

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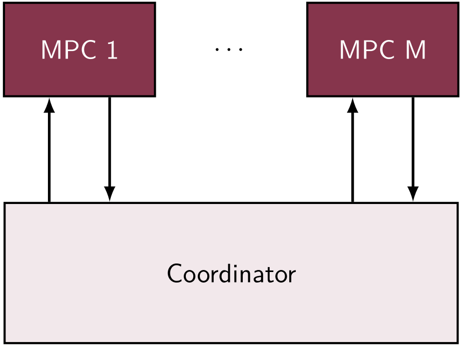

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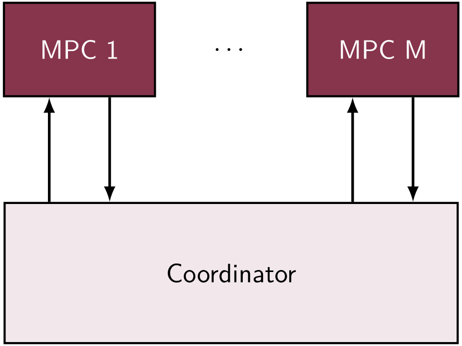

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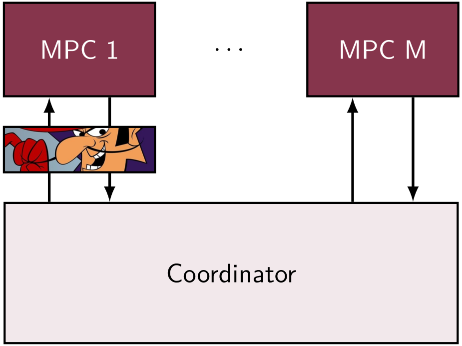

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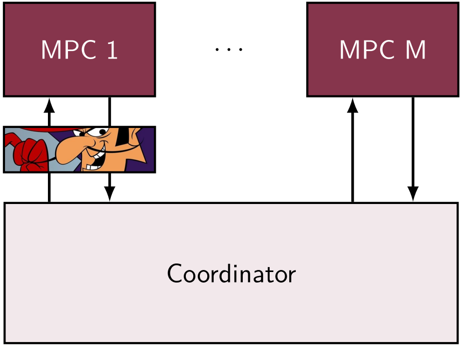

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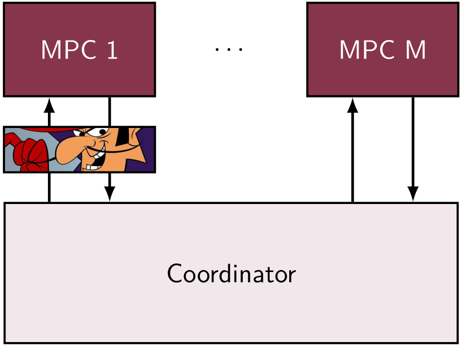

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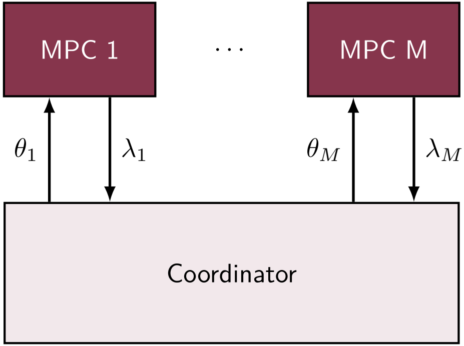

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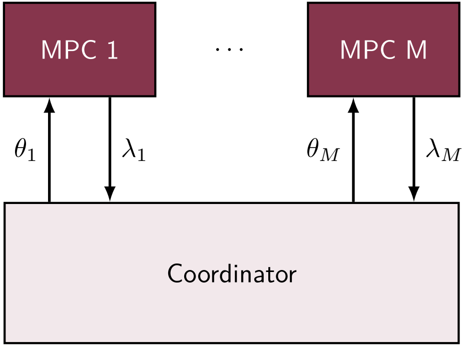

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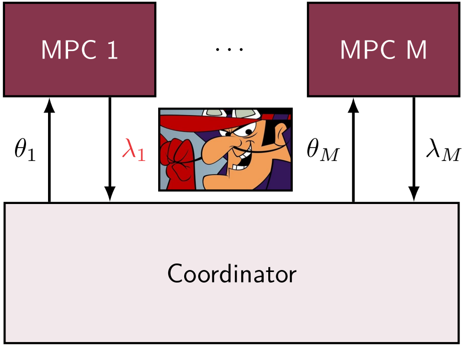

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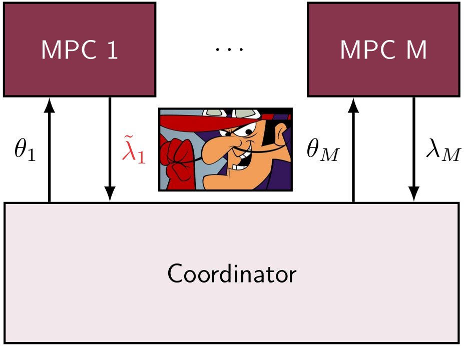

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

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Attack model

Liar, Liar, Pants of fire

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Assumptions

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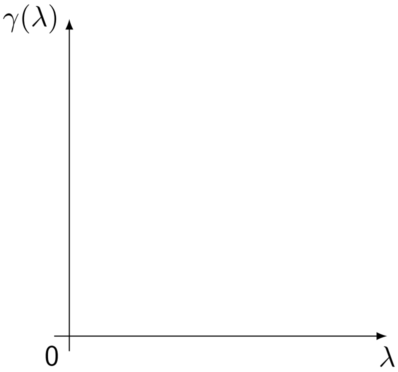

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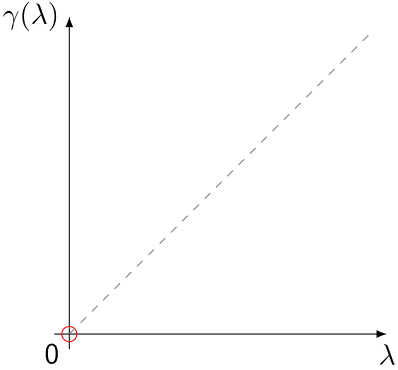

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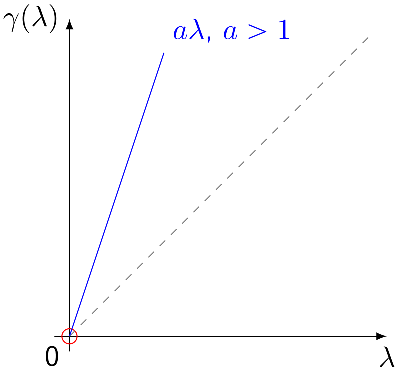


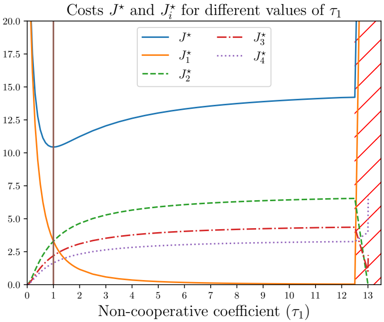
Example

Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\boldsymbol{\lambda}}_1 = \gamma_1(\boldsymbol{\lambda}_1) = \tau_1 I \boldsymbol{\lambda}_1$
- Simulate for different τ_1 get J_i

Example



4 distinct agents

- Agent 1 is non-cooperative
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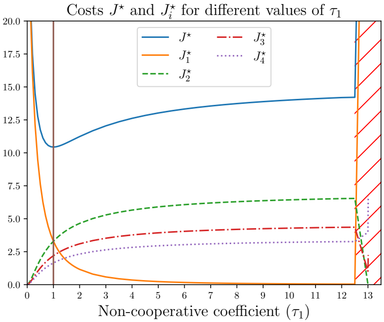

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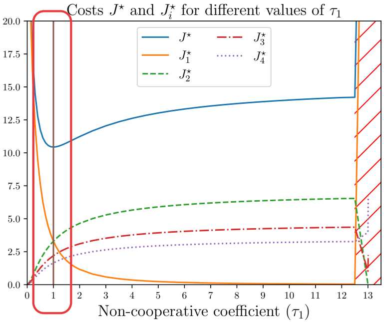

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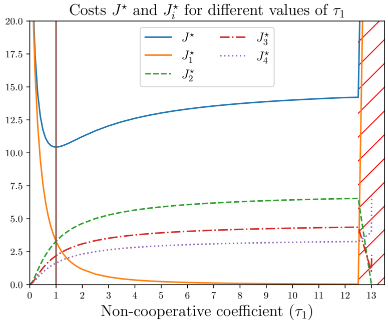

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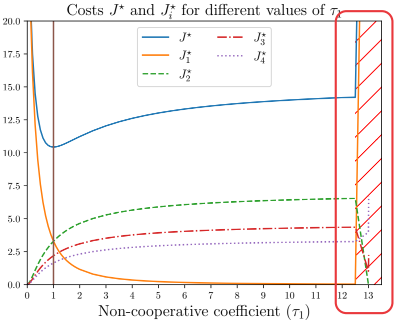

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  - Global minimum when $\tau_1 = 1$
  - Agent 1 benefits if τ_1 increases (inverse otherwise)
  - All collapses if too greedy

- But can we mitigate these effects?

- But can we mitigate these effects?
- Yes! (At least in some cases)

Outline

3 Securing the dMPC
- Classification
- State of Art
- Proposed Methods

Classification of mitigation techniques

Passive (Robust)

Active (Resilient)

Classification of mitigation techniques

Passive (Robust)

- 1 mode

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- 2 modes

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[10]	Dual	Robust (Scenario)	NA	NA
[11]	Dual	Robust (f-robust)	NA	NA
[12]	Jacobi-Gauß	–	–	–
[13]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)

[10] José M. Maestre et al., "Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc".

[11] Velarde, José M. Maestre, et al., "Vulnerabilities in Lagrange-Based Distributed Model Predictive Control".

[12] Chanfreut, J. M. Maestre, and H. Ishii, "Vulnerabilities in Distributed Model Predictive Control based on Jacobi-Gauss Decomposition".

[13] Ananduta et al., "Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids".

Our Approach

Explore Scarcity

- Resilient
- Analytical/Learning
- Data reconstruction

Our Approach

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Our Approach

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- Explore Scarcity

Outline

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Systems whose optimal solution has all constraints active

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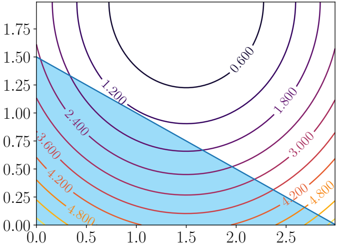

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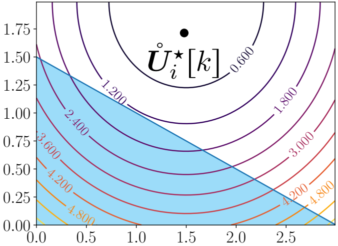

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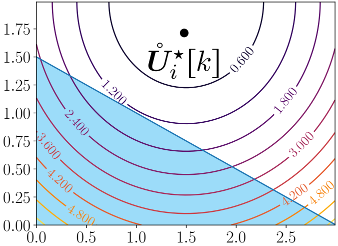

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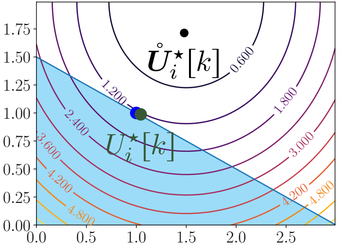

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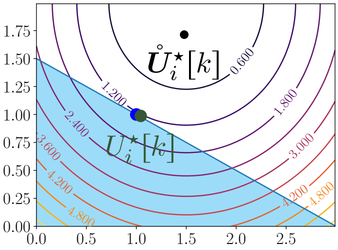
  - Same as with equality constraints[14]



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[14] If system can have all constraints active simultaneously ▸ see here

Analyzing Deprived Systems

Assumptions

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- *Quadratic local problems*

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(local parameters unknown by coordinator) $\left\{ \begin{array}{l} \bullet\ P_i \text{ is time invariant} \\ \bullet\ \boldsymbol{s}_i[k] \text{ is time variant} \end{array} \right.$

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Detection Mechanism

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- We estimate[15] $\hat{P}_i[k]$ and $\widehat{\tilde{\boldsymbol{s}}}_i[k]$ such as:

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[15] Using Recursive Least Squares for example

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- If $\left\|\widehat{\tilde{P}}_i[k] - \bar{P}_i\right\|_F > \epsilon_P \rightarrow$ Attack
- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

[15] Using Recursive Least Squares for example

Estimating $\widehat{\tilde{P}}_i[k]$

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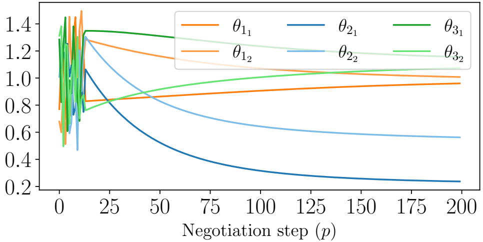

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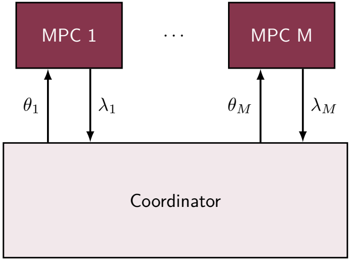
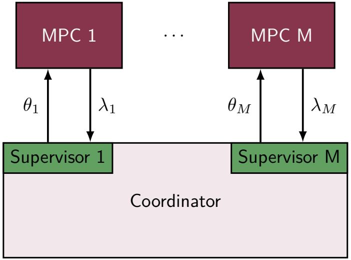
- Choose adequate version for coordination

$$\overset{\text{mod}}{\boldsymbol{\lambda}}_i = \begin{cases} \overset{\text{rec}}{\boldsymbol{\lambda}}_i, & \text{if attack detected} \\ \tilde{\boldsymbol{\lambda}}_i, & \text{otherwise} \end{cases}$$

Complete Mechanism

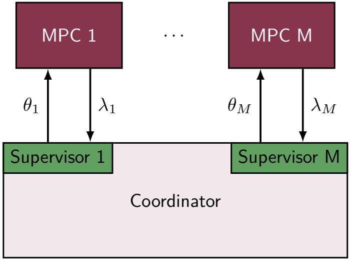


Complete Mechanism



- Supervise exchanges by inquiring the agents

Complete Mechanism



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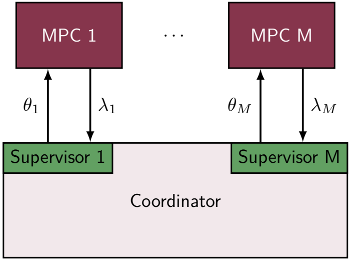

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Two Phases

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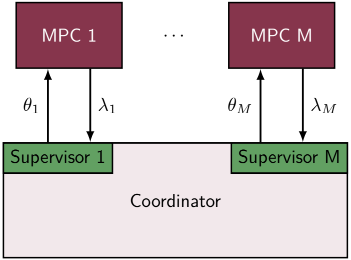

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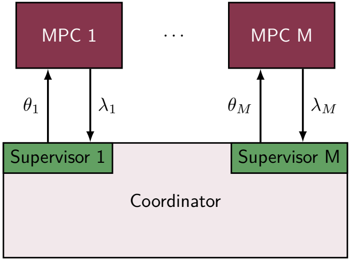
Two Phases

1. Detect which agents are non-cooperative

Complete Mechanism



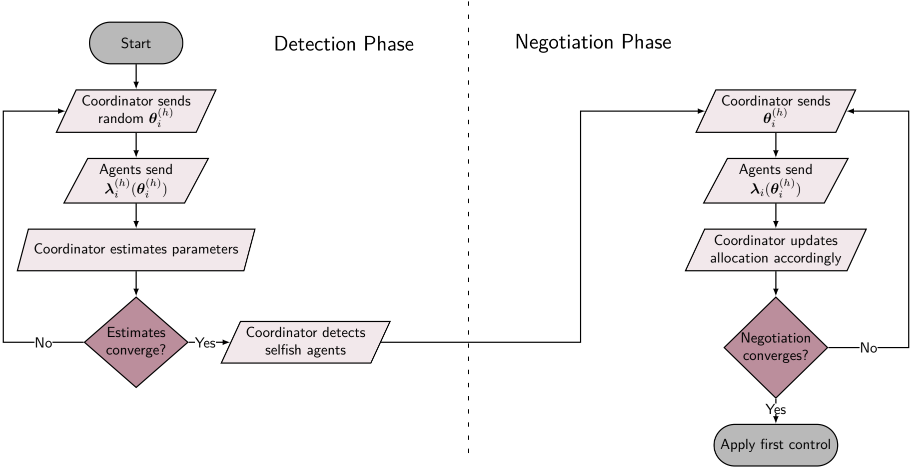
- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Complete algorithm

RPdMPC-DS



Complete algorithm

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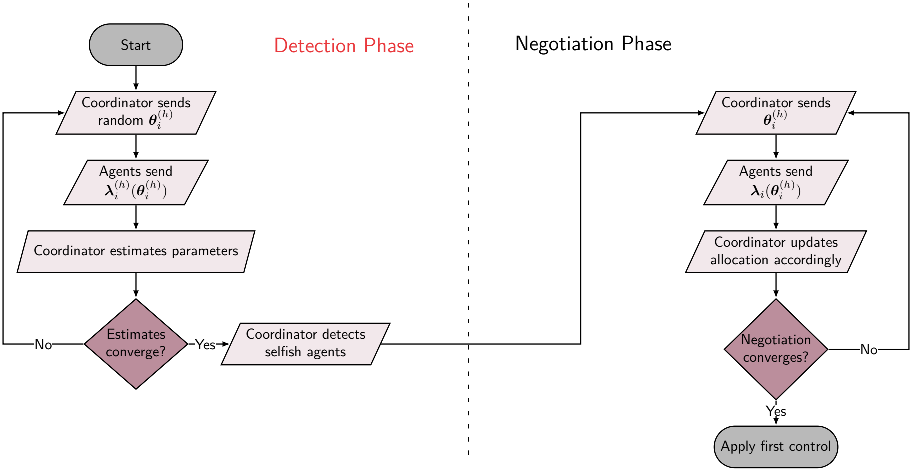

Complete algorithm

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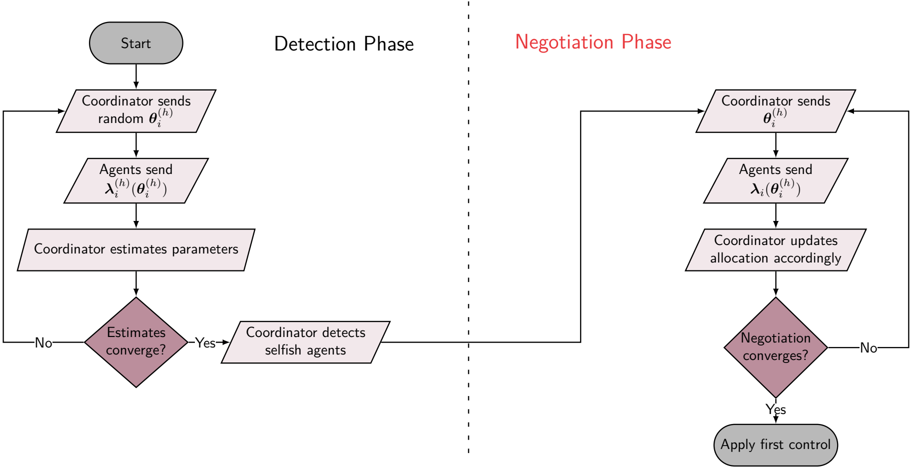

Complete algorithm

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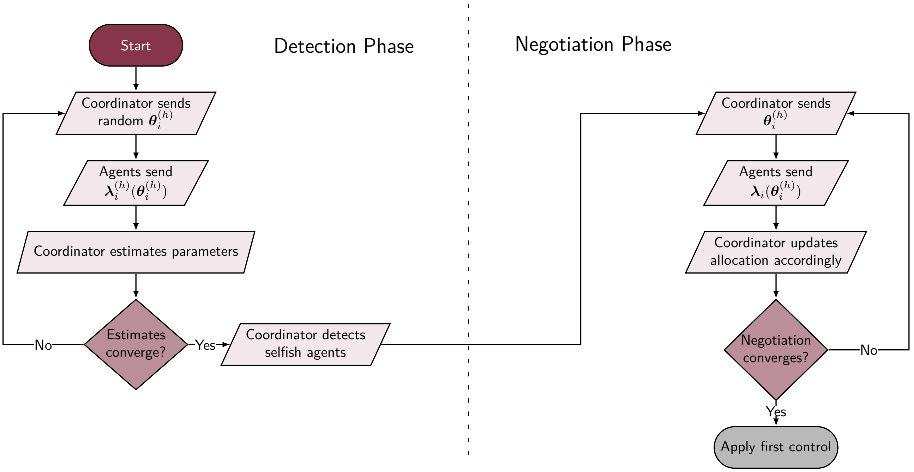

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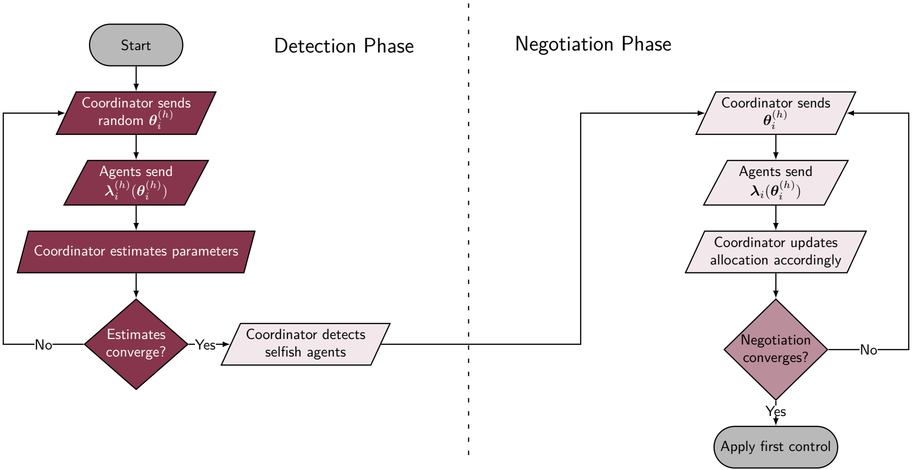

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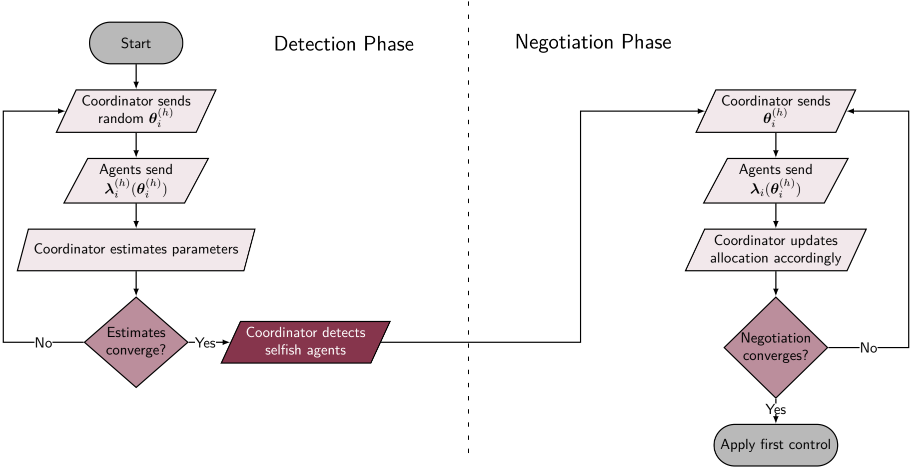

Complete algorithm

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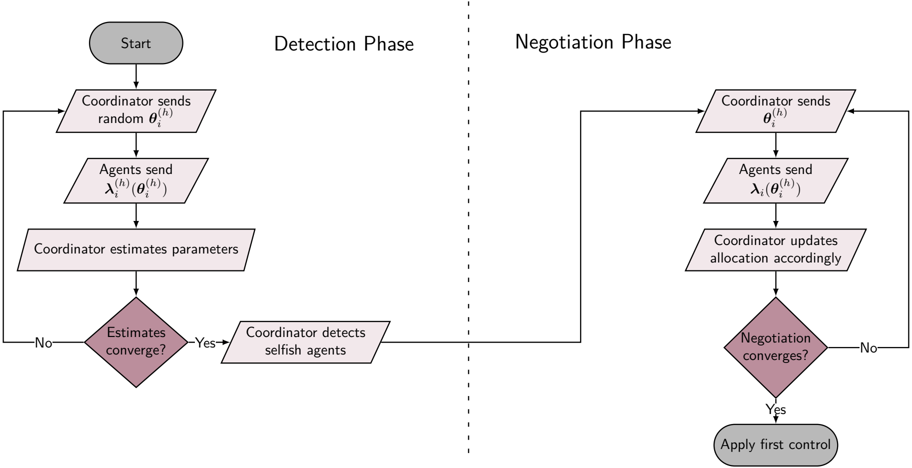

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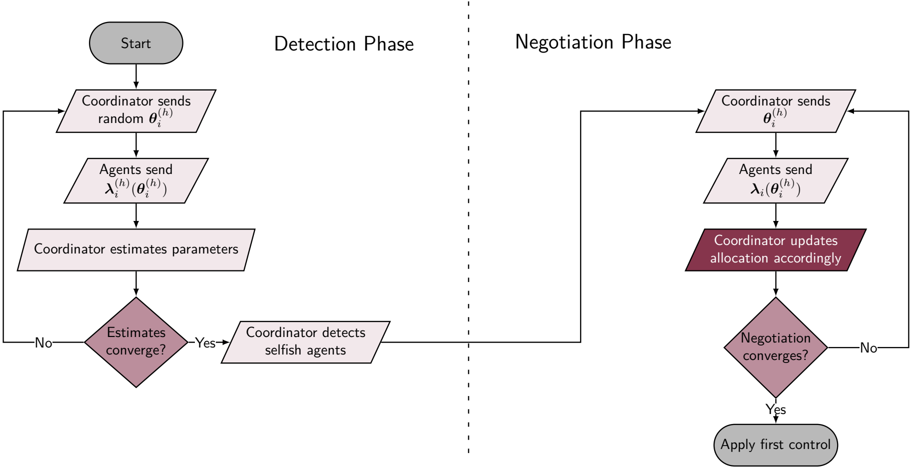
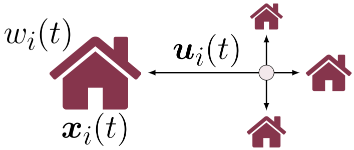
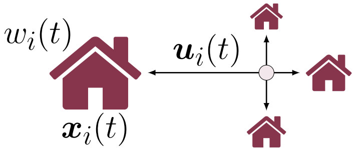


Example



District Heating Network (4 Houses)

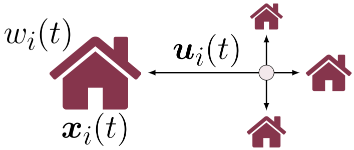
Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)

Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
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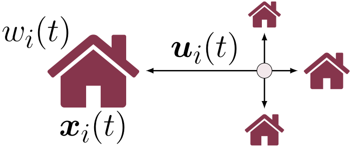
Example



District Heating Network (4 Houses)

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- Period of 5h ($T_s = 0.25h \rightarrow k = \{1 : 20\}$)

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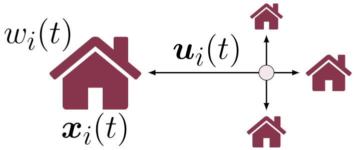

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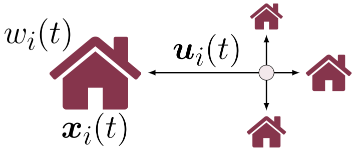
Example



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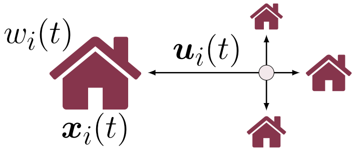
Example



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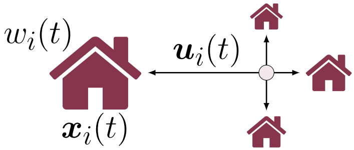
Example



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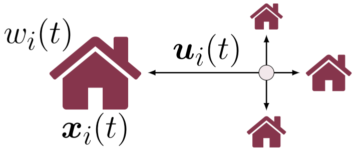

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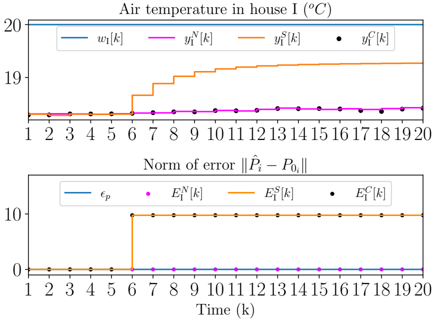
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-DS)

Results

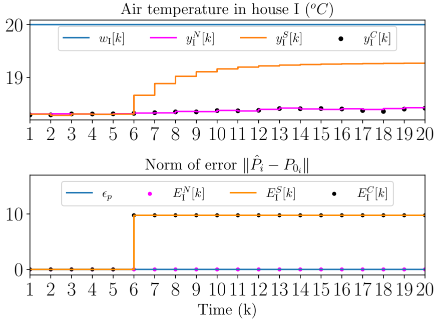
Temporal



Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selflish, C Corrected

Results

Temporal



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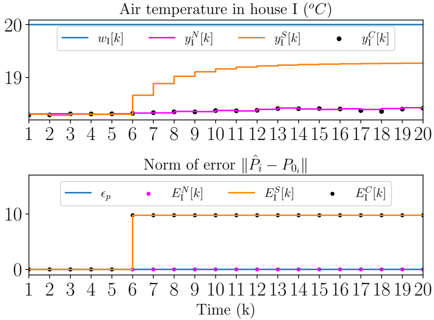

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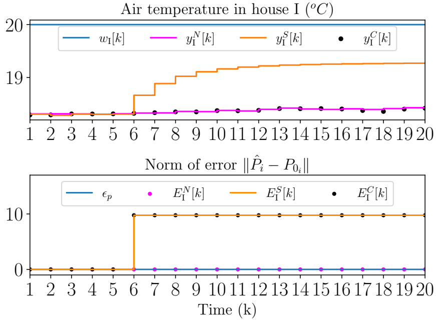
Temporal



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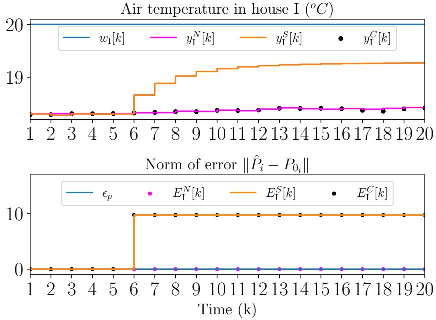
- Agent starts cheating in $k = 6$
- S Agent increases its comfort

Results

Temporal



Temperature in house I.
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S Agent increases its comfort

C Restablish behavior close to N

Results

Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.5
II	21.67	-0.55
III	17.39	-0.0
IV	17.63	-0.09
Global	3.53	0.02

Results

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Outline

Relaxing scarcity assumption

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  - We can't change our constraints to equality ones anymore

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i \boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

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Now, we may have multiple (Piecewise affine function)

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(Z)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(Z)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^Z_{\boldsymbol{\lambda}_i} \end{cases}$$

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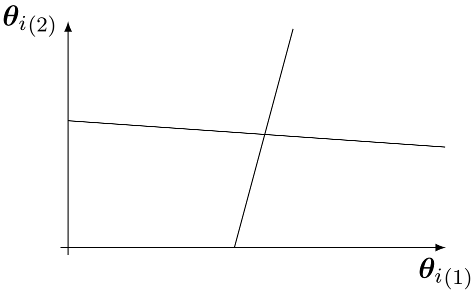
Still the $P_i^{(z)}$ are time independent

Analyzing System

Solution for $\lambda_i[k]$ (Continued)

Analyzing System

Solution for $\lambda_i[k]$ (Continued)



Separation surfaces depend on state and local parameters.

Analyzing System

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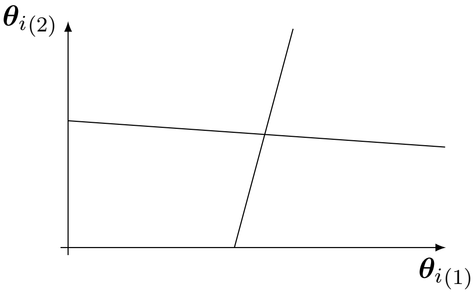

Separation surfaces depend on state and local parameters.
Unknown by the coordinator.

Analyzing System

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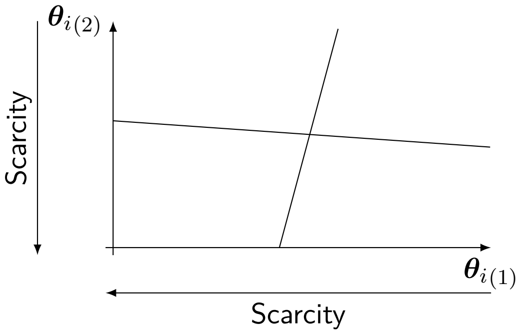

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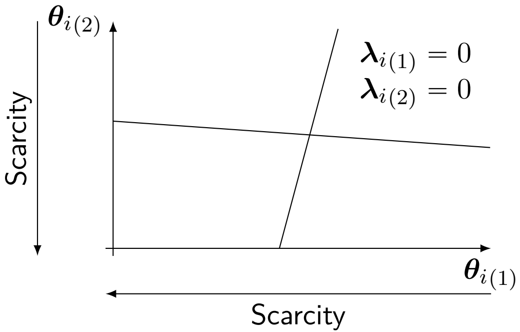

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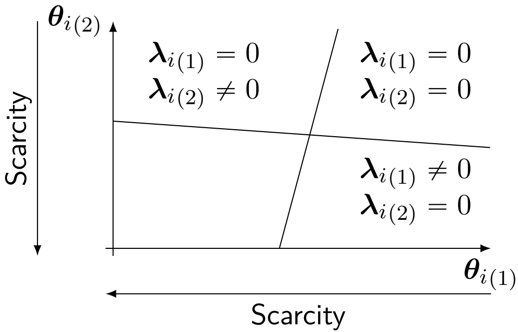

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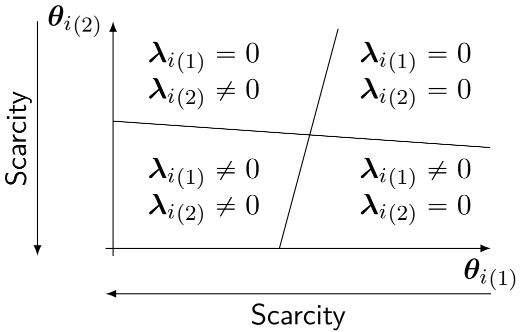

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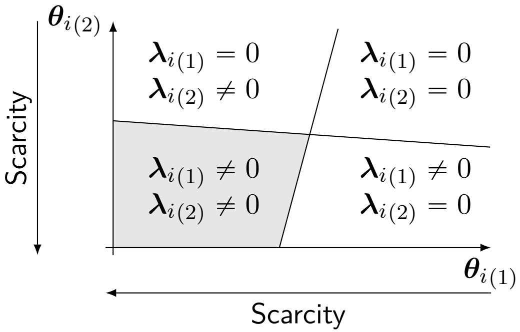

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Analyzing System

Solution for $\lambda_i[k]$ (Continued) Still?

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(Z)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(Z)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^Z_{\boldsymbol{\lambda}_i} \end{cases}$$

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$\uparrow$ Scarcity

All constraints active $\quad -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k] \quad \rightarrow \quad -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$

Analyzing System

Solution for $\lambda_i[k]$ (Continued) Still?

$$\boldsymbol{\lambda}_i[k] = \begin{cases} {\color{red}-P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k]}, & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(Z)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(Z)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^Z_{\boldsymbol{\lambda}_i} \end{cases}$$

Scarcity

All constraints active	$-P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k]$	$\rightarrow$	$-P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$
None constraints active	$-P_i^{(Z)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(Z)}[k]$	$\rightarrow$	$\mathbf{0}$

Analyzing System

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Assumptions

The region $\mathcal{R}^0_{\boldsymbol{\lambda}_i} \neq \varnothing$ *and we known a point* $\overset{\varnothing}{\boldsymbol{\theta}}_i \in \mathcal{R}^0_{\boldsymbol{\lambda}_i}$

Analyzing System

Under attack!

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$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Analyzing System

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$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Parameters are modified.

$$\tilde{\boldsymbol{\lambda}}_i[k] = \begin{cases} -\widetilde{P}_i^{(0)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0 \\ \vdots & \vdots \\ -\widetilde{P}_i^{(Z)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(Z)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^Z_{\boldsymbol{\lambda}_i} \end{cases}$$

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Parameters are modified. But not the regions' limits

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before

Analyzing System

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Parameters are modified. But not the regions' limits

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before
- Problem: We don't know in which region $\boldsymbol{\theta}_i$ is

Analyzing System

Under attack!

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- Problem: We don't know in which region $\boldsymbol{\theta}_i$ is
- Solution: Let's force it using Artificial Scarcity

Artificial Scarcity

What you thought was way too much is not enough

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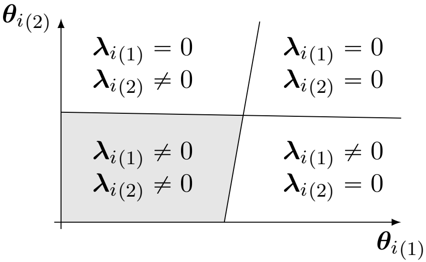

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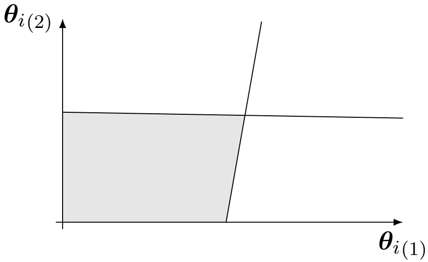

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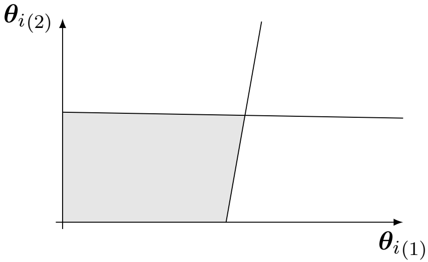

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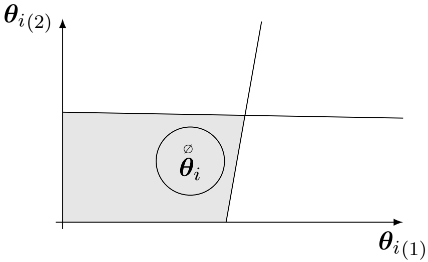

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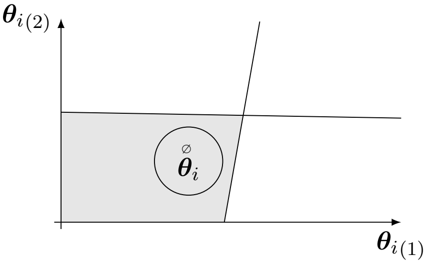

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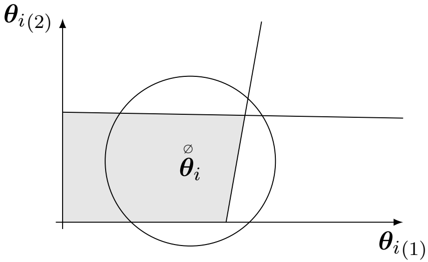

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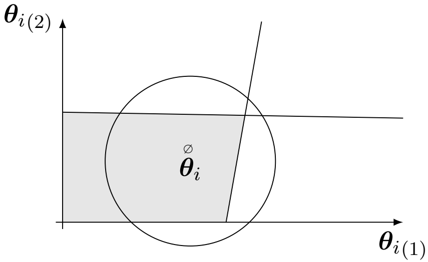

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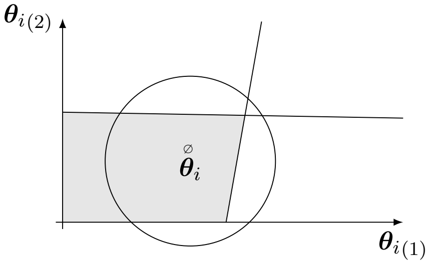

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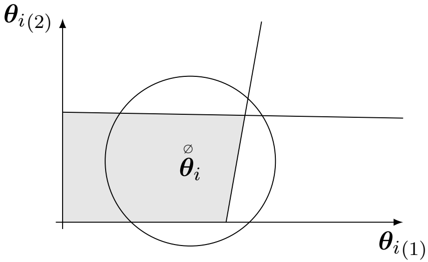

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- We recover the associated parameter, i.e., $\widehat{\widetilde{P}}_i^{(0)}[k]$

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Detection and Mitigation

Same same, but different

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Assumption

We estimate nominal $\bar{P}_i^{(0)}$ *from attack free negotiation*

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$$\left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F \geqslant \epsilon_{P_i^{(0)}}$$

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$$\widehat{T_i[k]^{-1}} = \bar{P}_i^{(0)} \widehat{\tilde{P}}_i^{(0)}[k]^{-1}.$$

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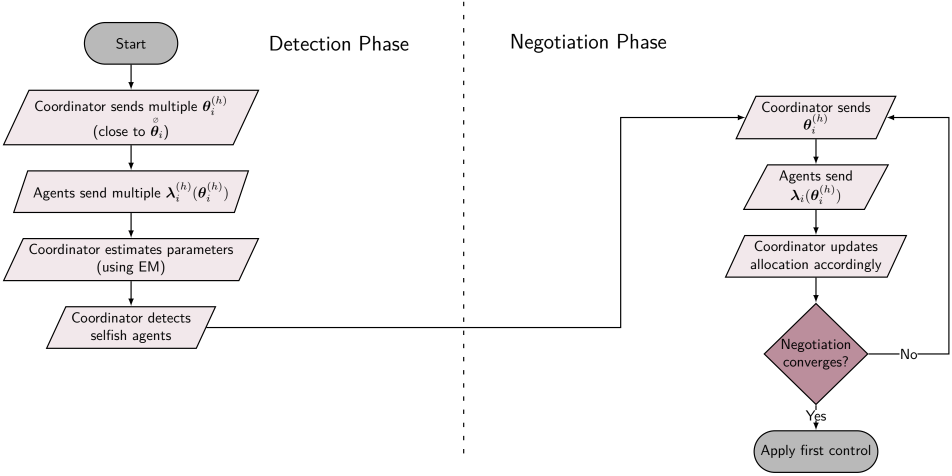
$$\overset{\text{rec}}{\boldsymbol{\lambda}_i} = \widehat{T_i[k]^{-1}} \tilde{\boldsymbol{\lambda}}_i.$$

Complete algorithm

RPdMPC-AS



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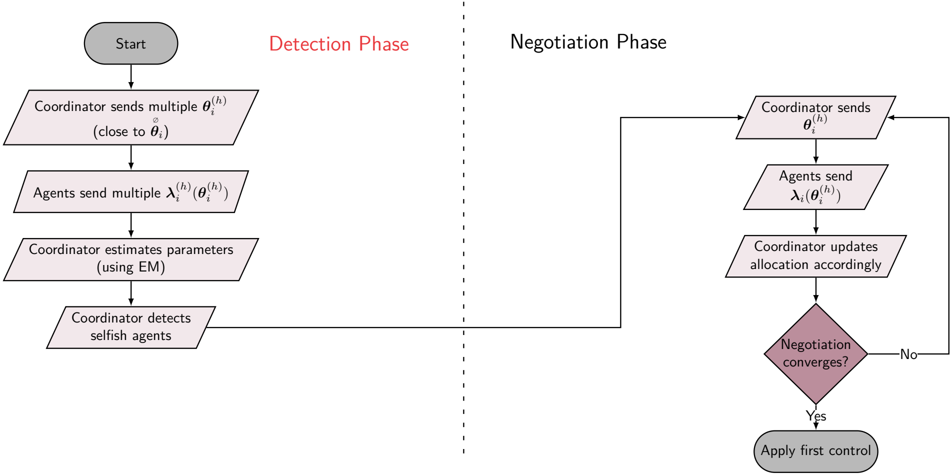

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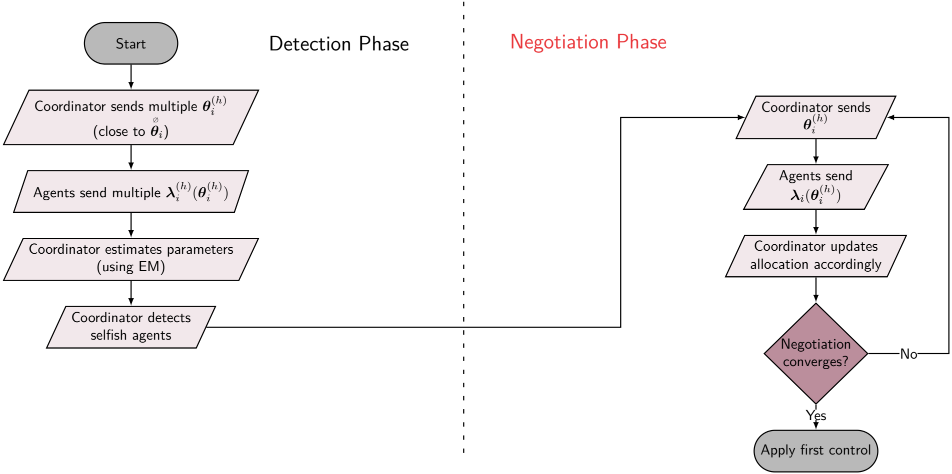

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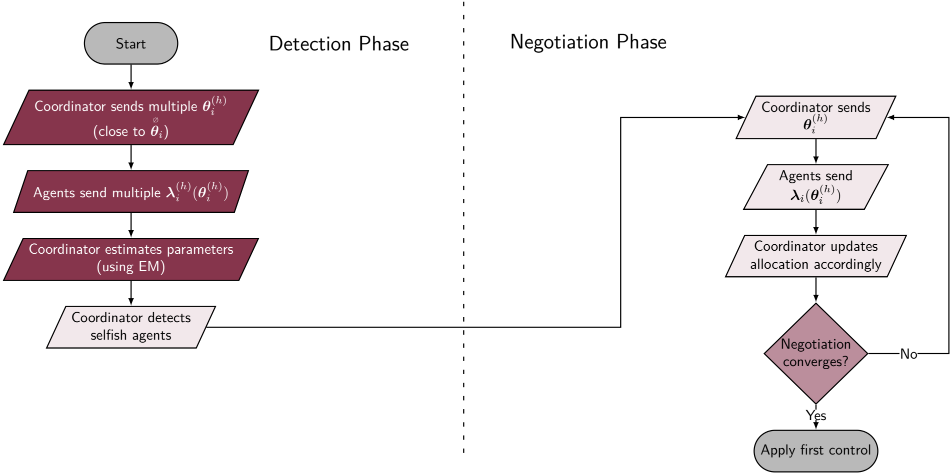

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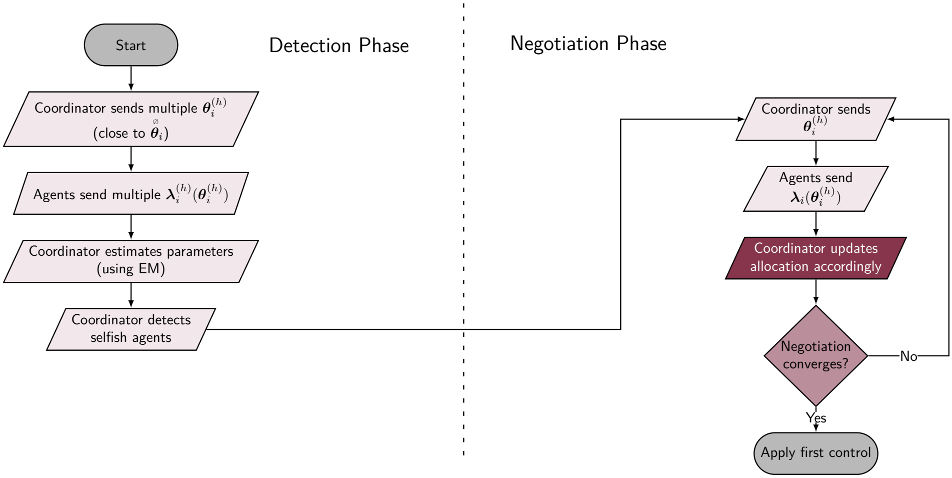
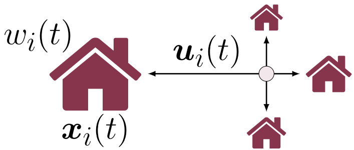


Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of 5h ($T_s = 0.25h \rightarrow k = \{1 : 20\}$)
- Prediction horizon ($N = 4$)
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-AS)

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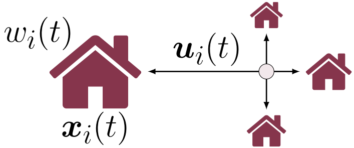

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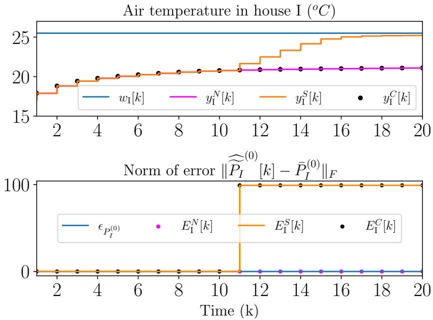
Results

Temporal



Temperature in house I.
Error $E_I(k)$.
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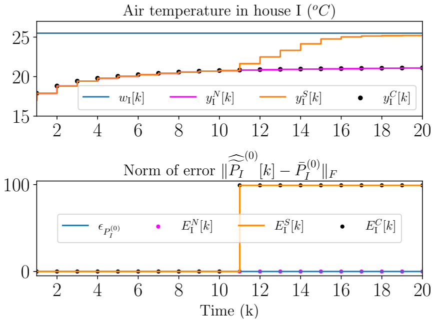

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.49	$-4.12e-05$
II	35.81	$1.74e-05$
III	29.22	$2.14e-05$
IV	37.54	$1.73e-05$
Global	10.69	$-6e-07$

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  - Associate with other methods of the same family

For Further Reading I

- Åström, K.J. and B. Wittenmark. Adaptive Control. Addison-Wesley series in electrical and computer engineering: Control engineering. Addison-Wesley, 1989. ISBN: 9780201097207. DOI: 10.1007/978-3-662-08546-2_24.
- Maestre, José M, Rudy R Negenborn, et al. Distributed Model Predictive Control made easy. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.
- Nogueira, Rafael Accácio. "Security of DMPC under False Data Injection". 2022CSUP0006. PhD thesis. CentraleSupélec, 2022. URL: http://www.theses.fr/2022CSUP0006.