

# Security of distributed Model Predictive Control under False Data Injection

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**Seminar**

**École Centrale de Lyon / Laboratoire Ampère**

26/05/2023 @ Écully



<https://bit.ly/3g3S6X4>

Rafael Accácio Nogueira

Postdoctoral researcher at LAAS/CNRS

*Garanteed relative localisation and anticollision  
scenario for autonomous vehicles*

Project AutOCampus (GIS neOCampus)

Advised by Soheib Fergani



Bachelor Thesis at Escola Politécnica/UFRJ

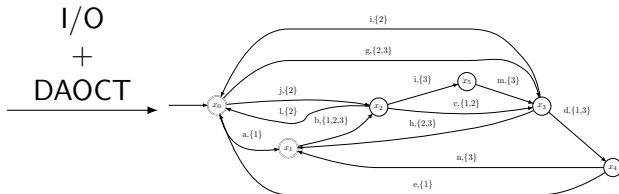
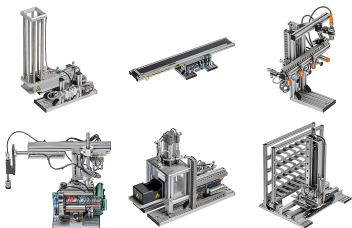
*Identification of DES for fault-diagnosis*

Advised by Marcos Vicente de Brito Moreira

Politécnica  
UFRJ



UFRJ  
UNIVERSIDADE FEDERAL  
DO RIO DE JANEIRO



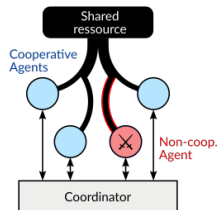
Doctoral Thesis at CentraleSupélec/IETR

*Security of dMPC under False Data Injection*

Advised by Hervé Guéguen and Romain Bourdais



CentraleSupélec



Multiple systems interacting



# Context

## Smart(er) Cities

Multiple systems interacting



Multiple systems interacting



- Distribution:

Multiple systems interacting



- Distribution:
  - Electricity



Multiple systems interacting



- Distribution:
  - Electricity
  - Heat
  - Water

Multiple systems interacting



- Distribution:
  - Electricity
  - Heat
  - Water
- Traffic

### Multiple systems interacting



- Distribution:
  - Electricity
  - Heat
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- Traffic
- ...

Multiple systems interacting under



- Technical/Comfort Constraints

Multiple systems interacting under



- Technical/Comfort Constraints
- We also want

Multiple systems interacting under



- Technical/Comfort Constraints
- We also want
  - Minimize consumption

Multiple systems interacting under



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Multiple systems interacting under



- Technical/Comfort Constraints
- We also want
  - Minimize consumption
  - Maximize satisfaction
  - Follow a trajectory
- Solution  $\rightarrow$  MPC

# Model-based Predictive Control

## Brief recap

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Find best control sequence using predictions based on a model.

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Find **best** control sequence using predictions based on a model.

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Find optimal control sequence using predictions based on a model.

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Find optimal control sequence using predictions based on a model.

- We need an optimization problem

minimize  
 $\mathbf{u}[0:N-1|k]$

$$J(\mathbf{x}[0|k], \mathbf{u}[0 : N - 1|k])$$

# Model-based Predictive Control

## Brief recap

Find optimal control sequence using predictions based on a model.

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  - Decision variable is the control sequence

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# Model-based Predictive Control

## Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon  $N$

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# Model-based Predictive Control

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Find optimal control sequence using predictions based on a model.

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# Model-based Predictive Control

## Brief recap

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence calculated over horizon  $N$
  - Objective function to optimize
  - System's Model

$$\begin{array}{ll} \underset{\mathbf{u}[0:N-1|k]}{\text{minimize}} & J(\mathbf{x}[0|k], \mathbf{u}[0 : N - 1|k]) \\ \text{subject to} & \left. \begin{array}{l} \mathbf{x}[\xi|k] = f(\mathbf{x}[\xi - 1|k], \mathbf{u}[\xi - 1|k]) \end{array} \right\} \forall \xi \in \{1, \dots, N\} \end{array}$$

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# Model-based Predictive Control

## Brief recap

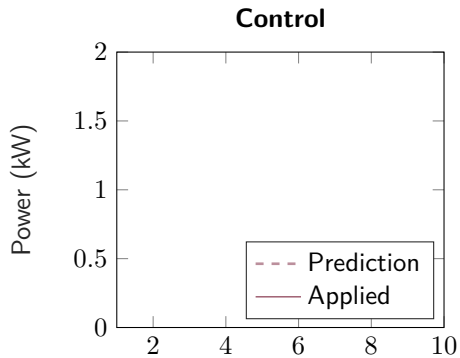
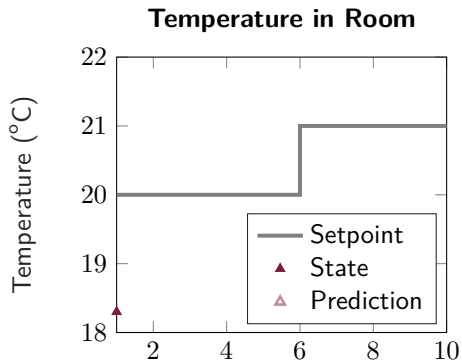
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- We need an optimization problem
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  - Other constraints to respect (QoS, technical restrictions, ...)

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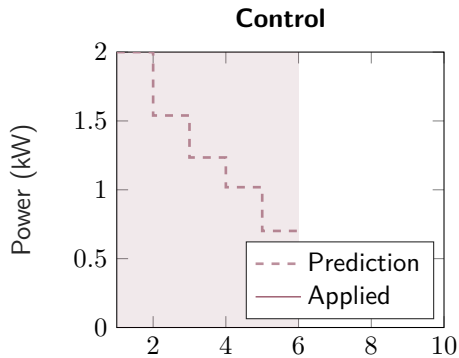
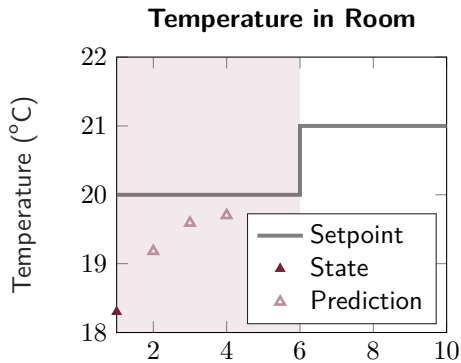
In a nutshell



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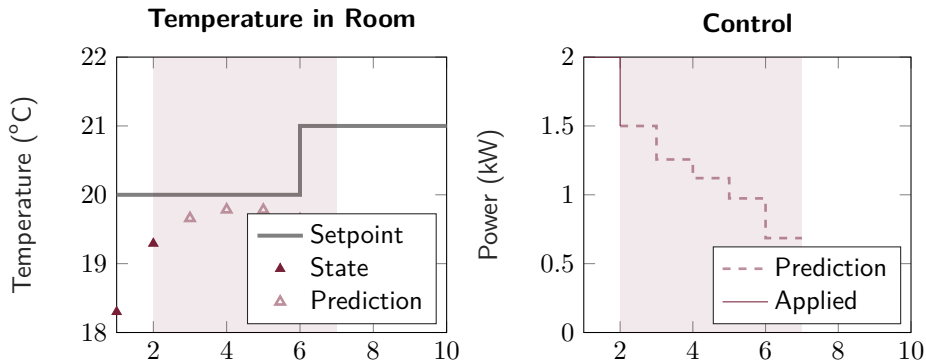
Find optimal control sequence



# Model Predictive Control

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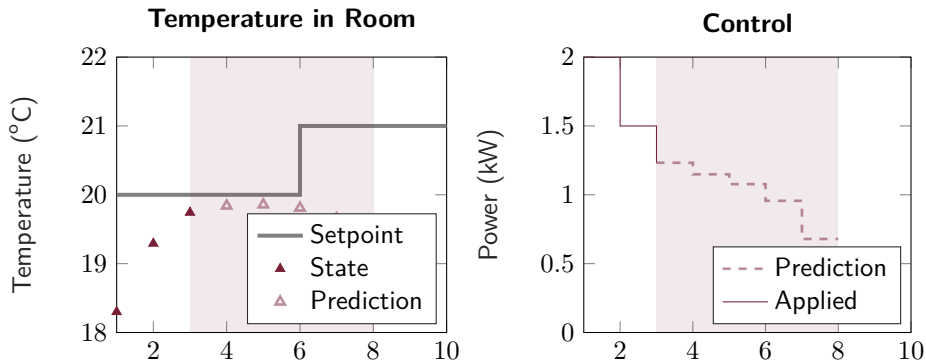
Find optimal control sequence, apply first element



# Model Predictive Control

## In a nutshell

Find optimal control sequence, apply first element, rinse repeat

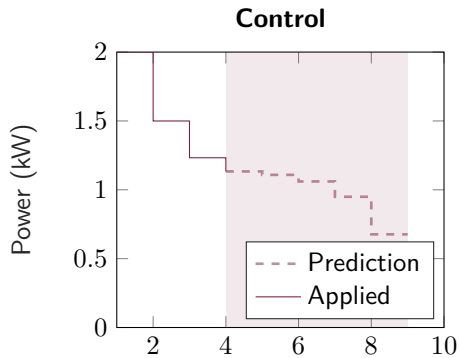
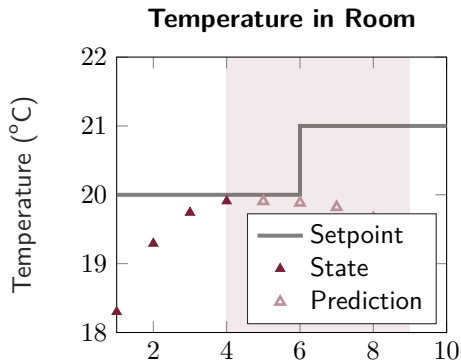




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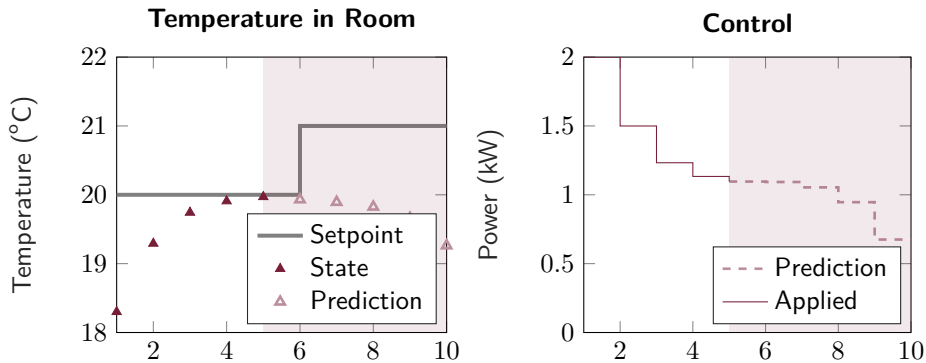
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



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In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



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# Model Predictive Control

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  - Privacy (RGPD)
- Solution: Divide and Conquer (distributed MPC)

## ① Decomposing the MPC

# Outline

- ① Decomposing the MPC
- ② Attacks on the dMPC

- ① Decomposing the MPC
- ② Attacks on the dMPC
- ③ Securing the dMPC

# Outline

## ① Decomposing the MPC

# Distributed Model Predictive Control

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## Remark


*If agents exchange same variable  $\rightarrow$  consensus problem*

# Distributed Model Predictive Control

## Optimization Frameworks

Usually based on optimization decomposition methods<sup>1</sup>:

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Security/privacy properties

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# Distributed Model Predictive Control

It is about communication

- We break the MPC optimization problem
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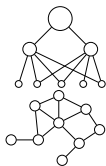
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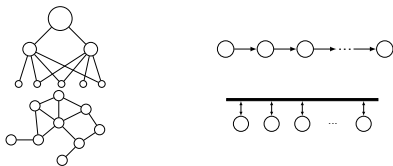
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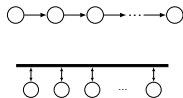
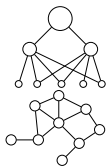
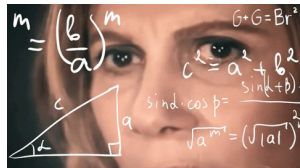
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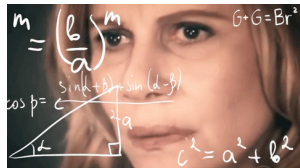
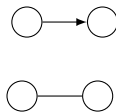
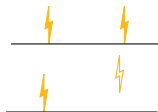
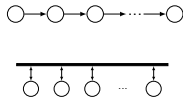
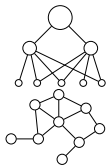
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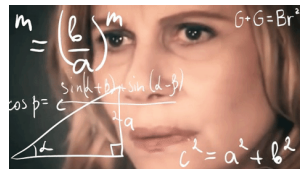
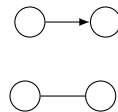
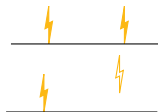
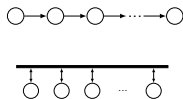
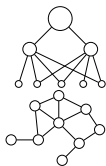
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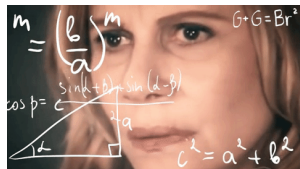
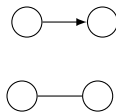
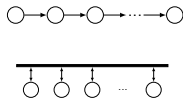
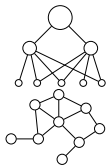
José M Maestre, Negenborn, et al., Distributed Model Predictive Control made easy



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# Distributed Model Predictive Control

## Optimization Decomposition



MPC

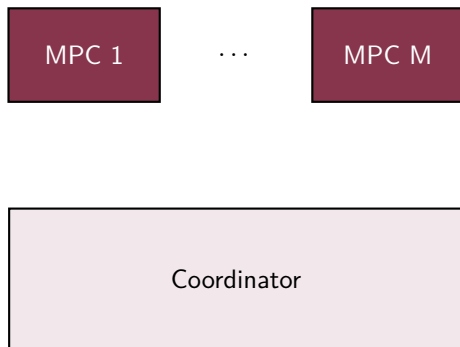
# Distributed Model Predictive Control

## Optimization Decomposition



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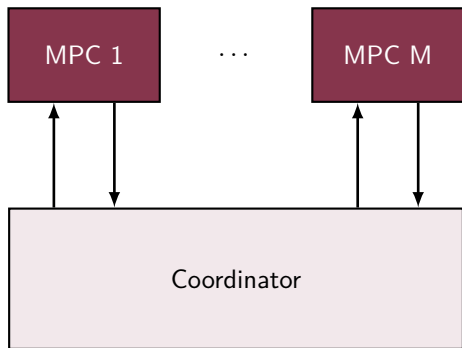
## Optimization Decomposition



- Coordinator  $\rightarrow$  Hierarchical

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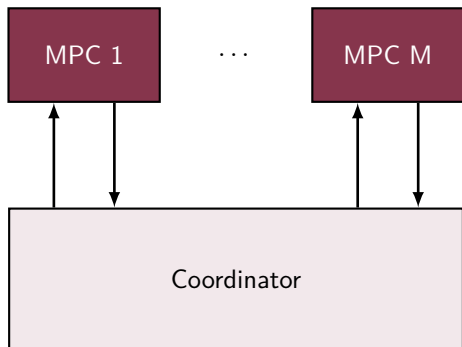
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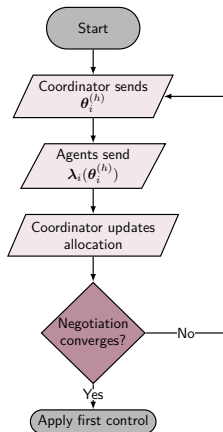
## Optimization Decomposition



- Coordinator  $\rightarrow$  Hierarchical
- Bidirectional
- No delay  $\rightarrow$  Synchronous

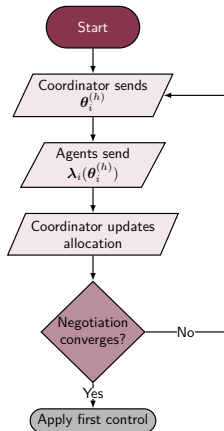
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or Quantity Decomposition | or Resource Allocation



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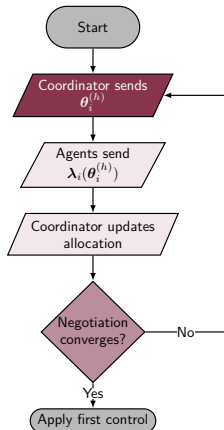
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# Primal Decomposition

or Quantity Decomposition | or Resource Allocation

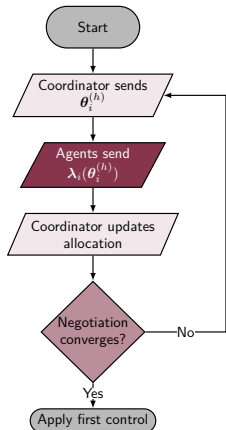


Allocation  $\theta_i$



# Primal Decomposition

or Quantity Decomposition | or Resource Allocation

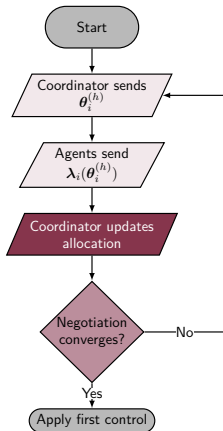


Allocation  $\theta_i$   
Dissatisfaction  $\lambda_i$



# Primal Decomposition

or Quantity Decomposition | or Resource Allocation



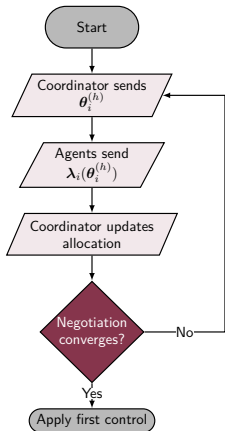
Allocation  $\theta_i$   
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Update  $\theta_i^+ = f_i(\theta_i, \lambda_i)$

# Primal Decomposition

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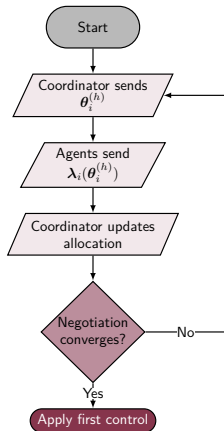
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# Primal Decomposition

In detail

$$\begin{array}{ll}\text{minimize} & \sum_{i \in \mathcal{M}} J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s.t.} & \sum_{i \in \mathcal{M}} \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \mathbf{u}_{\text{total}}\end{array}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones

$$\begin{array}{ll}\text{minimize} & \sum_{i \in \mathcal{M}} J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s.t.} & \sum_{i \in \mathcal{M}} \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \mathbf{u}_{\text{total}}\end{array}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
- Constraints couple variables

$$\begin{array}{ll} \underset{\mathbf{u}_1, \dots, \mathbf{u}_M}{\text{minimize}} & \sum_{i \in \mathcal{M}} J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s.t.} & \sum_{i \in \mathcal{M}} \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \mathbf{u}_{\text{total}} \end{array}$$



# Primal Decomposition

In detail

- Objective is sum of local ones
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$$\begin{aligned}
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 & \text{s.t.} && \sum_{i \in \mathcal{M}} \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \mathbf{u}_{\text{total}}
 \end{aligned}$$

$\downarrow$  For each  $i \in \mathcal{M}$

$$\begin{aligned}
 & \underset{\mathbf{u}_i}{\text{minimize}} && J_i(\mathbf{x}_i, \mathbf{u}_i) \\
 & \text{s. t.} && \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \boldsymbol{\theta}_i
 \end{aligned}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
- Constraints couple variables

① Allocate  $\theta_i$  for each agent

$$\begin{array}{ll} \underset{\mathbf{u}_i}{\text{minimize}} & J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s. t.} & \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \boldsymbol{\theta}_i \end{array}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
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- 2 They solve local problems and

$$\begin{array}{ll} \underset{\mathbf{u}_i}{\text{minimize}} & J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s. t.} & \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \theta_i \end{array}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
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- 1 Allocate  $\theta_i$  for each agent
- 2 They solve local problems and
- 3 Send dual variable  $\lambda_i$

$$\begin{array}{ll} \underset{u_i}{\text{minimize}} & J_i(x_i, u_i) \\ \text{s. t.} & h_i(x_i, u_i) \leq \theta_i : \lambda_i \end{array}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
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$$\begin{array}{ll} \underset{\mathbf{u}_i}{\text{minimize}} & J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s. t.} & \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)}$$

# Primal Decomposition

## In detail

- Objective is sum of local ones
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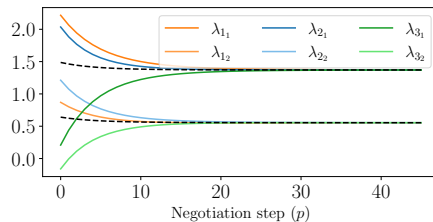
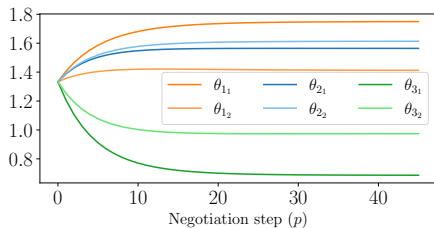
- 1 Allocate  $\theta_i$  for each agent
- 2 They solve local problems and
- 3 Send dual variable  $\lambda_i$
- 4 Allocation is updated  
(respect global constraint)

$$\begin{array}{ll} \underset{\mathbf{u}_i}{\text{minimize}} & J_i(\mathbf{x}_i, \mathbf{u}_i) \\ \text{s. t.} & \mathbf{h}_i(\mathbf{x}_i, \mathbf{u}_i) \leq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \text{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)})$$

# Example

Until everybody is evenly<sup>5</sup> dissatisfied



<sup>5</sup>For inequality constraints dynamics are more complex

# Distributed Model Predictive Control

Negotiation works if agents comply.



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But what if some agents are ill-intentioned and attack the system?

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- Incentive Brittany Region (Sustainable Energy + cybersecurity)
- CentraleSupélec Rennes - MPC for Smart Buildings
- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects? How?

---

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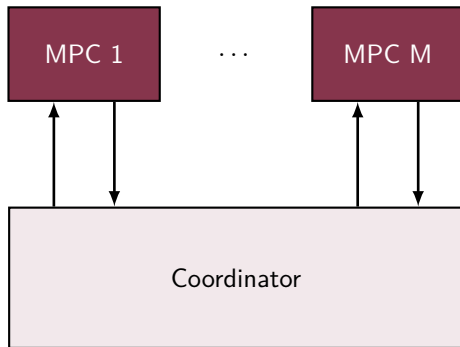
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# Outline

## ② Attacks on the dMPC

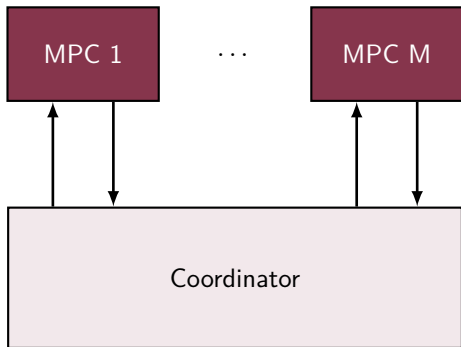
# How can a non-cooperative agent attack?

## Literature



# How can a non-cooperative agent attack?

## Literature



- Common attacks<sup>8</sup>

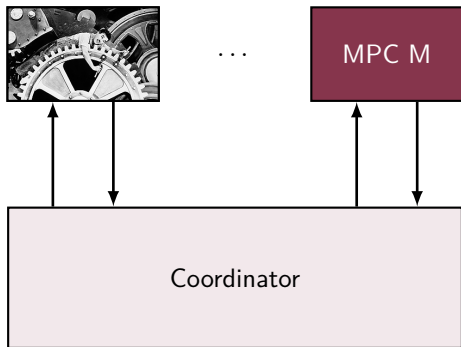
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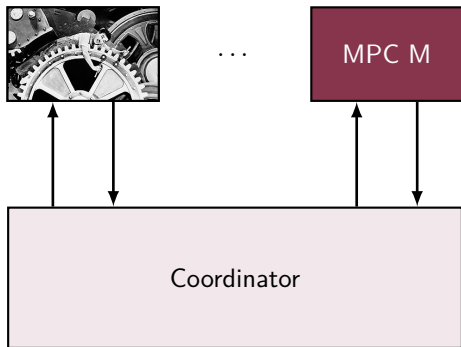


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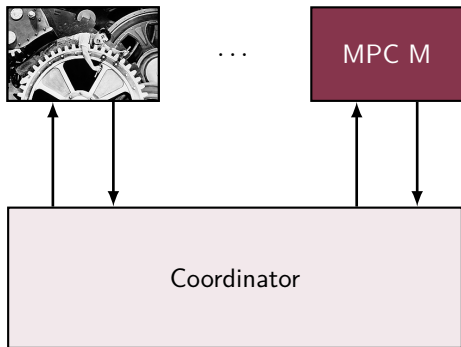


- Common attacks<sup>8</sup>
  - Fake objective function
  - Fake constraints
  - Use different control

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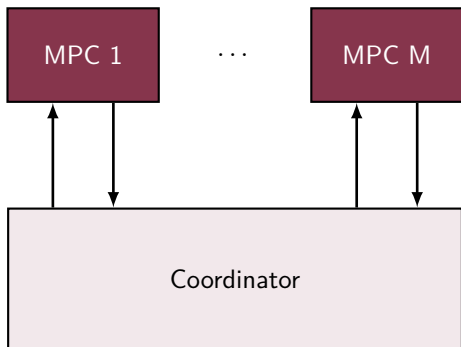


- Common attacks<sup>8</sup>
    - Fake objective function
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    - Use different control
- } Deception Attacks

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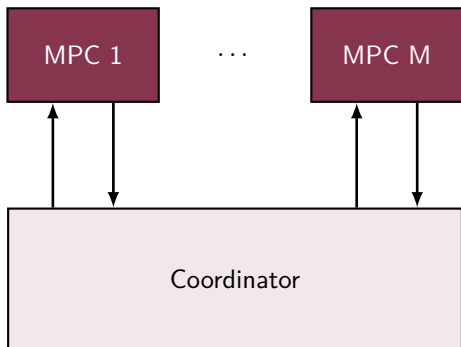
## Our approach



- Primal decomposition

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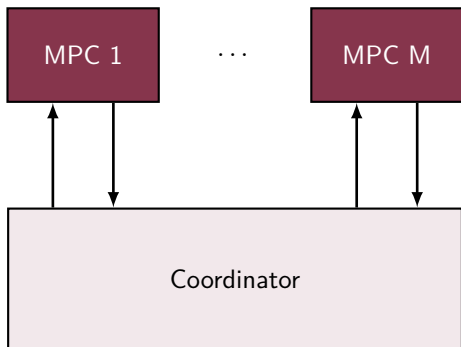
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- Primal decomposition
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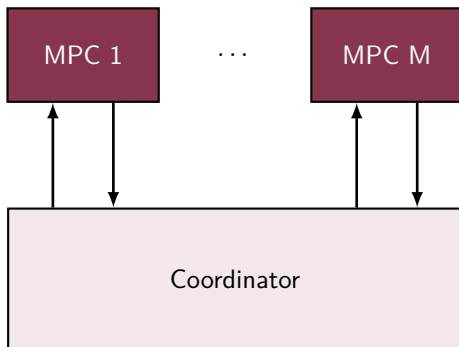
## Our approach



- Primal decomposition
  - Maximum resources fixed
- We are in coordinator's shoes

# How can a non-cooperative agent attack?

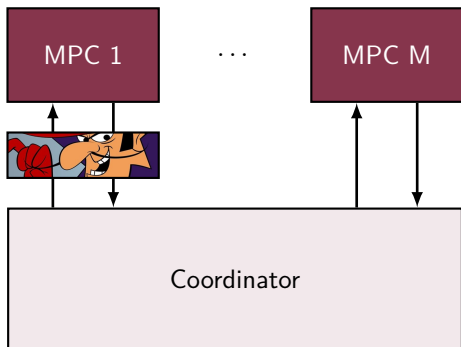
## Our approach



- Primal decomposition
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- What matters is the interface

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## Our approach

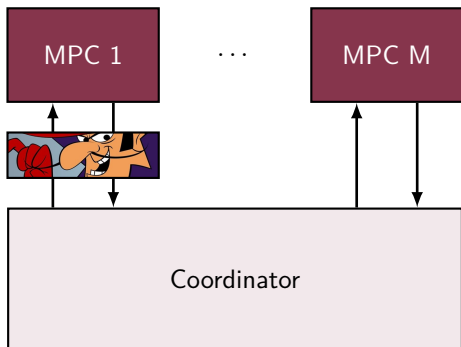


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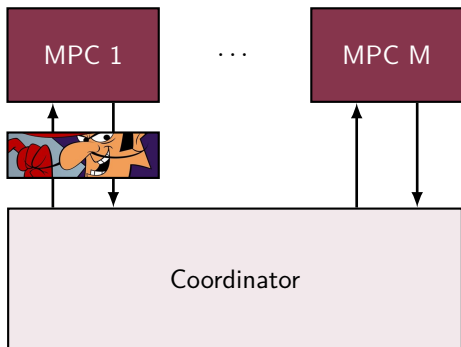
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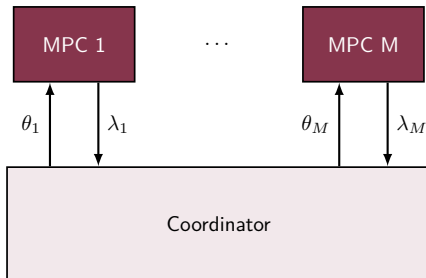
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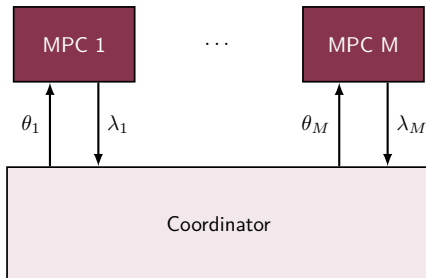
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- $\lambda_i$  is the only interface

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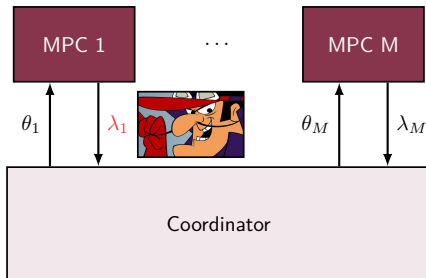
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- $\lambda_i$  is the only interface
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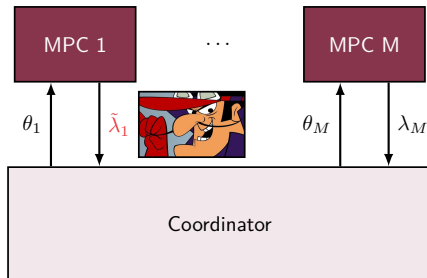
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- $\lambda_i$  is the only interface
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# How can a non-cooperative agent attack?

## Our approach



- $\lambda_i$  is the only interface
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- Malicious agent modifies  $\lambda_i$

$$\tilde{\lambda}_i = \gamma_i(\lambda_i)$$

# Example

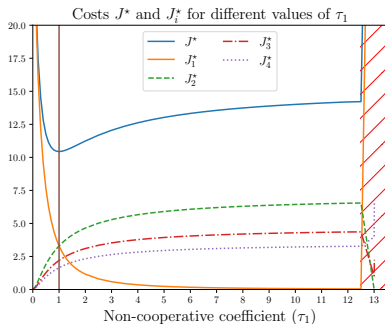
# Example

## 4 distinct agents

- Agent 1 is non-cooperative
- It uses  $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$
- Simulate for different  $\tau_1$  get  $J_i$



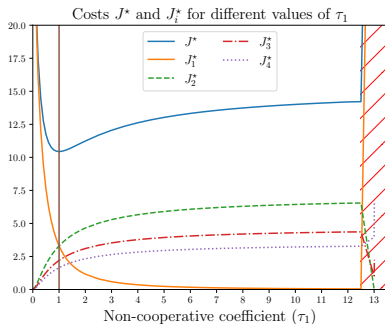
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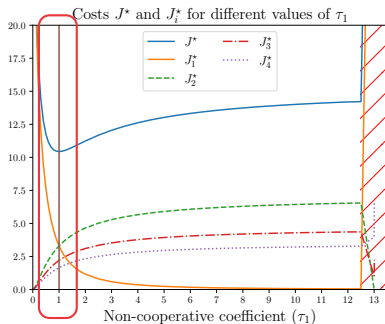
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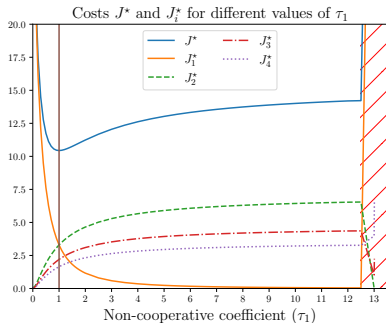
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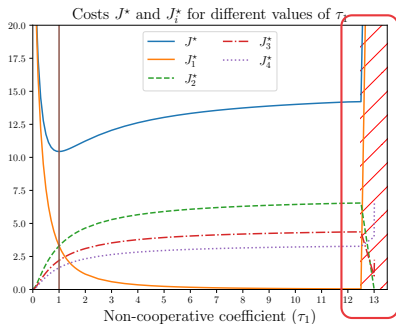
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  - Agent 1 benefits if  $\tau_1$  increases (inverse otherwise)

# Example



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- Simulate for different  $\tau_1$  get  $J_i$
- We can observe 3 things
  - Global minimum when  $\tau_1 = 1$
  - Agent 1 benefits if  $\tau_1$  increases (inverse otherwise)
  - All collapses if too greedy



- But can we mitigate these effects?

- But can we mitigate these effects?
- Yes! (At least in some cases)



# Outline

## ③ Securing the dMPC

# Classification of mitigation techniques

Passive (Robust)

Active (Resilient)

# Classification of mitigation techniques

## Passive (Robust)

- 1 mode

## Active (Resilient)

- 2 modes

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  - ① Attack free
  - ② When attack is detected

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# State of art

## Security dMPC

	Decomposition	Resilient/Robust
<sup>9</sup>	Dual	Robust (Scenario)
<sup>10</sup>	Dual	Robust (f-robust)
<sup>11</sup>	Jacobi-Gauß	–
<sup>12</sup>	Dual	Resilient

<sup>9</sup>José M. Maestre et al., “Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc”.

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# State of art

## Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
<sup>9</sup>	Dual	Robust (Scenario)	NA	NA
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# Attack model

Liar, Liar, Pants of fire

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Liar, Liar, Pants of fire

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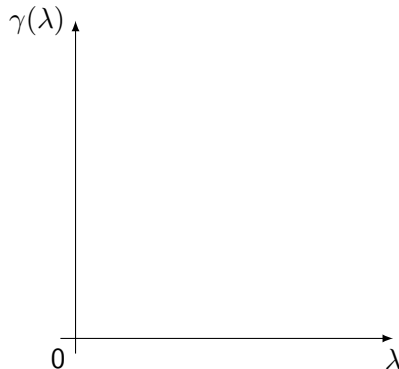
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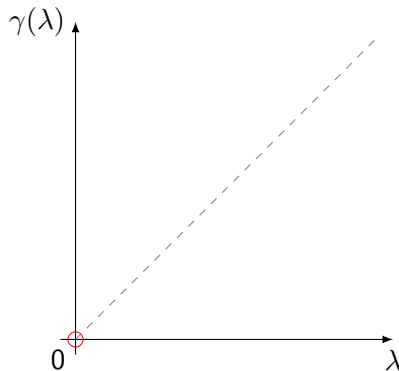
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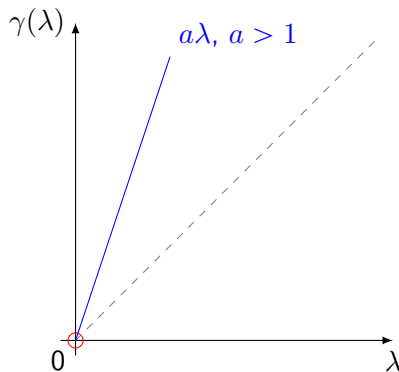
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# For Further Reading I



Maestre, José M, Rudy R Negenborn, et al.

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