Performance evaluation of intensity modulated optical OFDM system with digital baseband distortion

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Abstract: Bit-Error-Ratio (BER) of intensity modulated optical orthogonal frequency division multiplexing (OFDM) system is analytically evaluated accounting for nonlinear digital baseband distortion in the transmitter and additive noise in the photo receiver. The nonlinear distortion that is caused by signal clipping and quantization is taken into consideration. The signal clipping helps to overcome the system performance limitation related to high peak-to-average power ratio (PAPR) of the OFDM signal and to minimize the value of optical power that is required for achieving specified BER. The signal quantization due to a limited bit resolution of the digital to analog converter (DAC) causes an optical power penalty in the case when the bit resolution is too low. By introducing an effective signal to noise ratio (SNR) the optimum signal clipping ratio, system BER and required optical power at the input to the receiver is evaluated for the OFDM system with multi-level quadrature amplitude modulation (QAM) applied to the optical signal subcarriers. Minimum required DAC bit resolution versus the size of QAM constellation is identified. It is demonstrated that the bit resolution of 7 and higher causes negligibly small optical power penalty at the system BER = 10^{-3} when 256-QAM and a constellation of lower size is applied. The performance of the optical OFDM system is compared to the performance of the multi-level amplitude-shift keying (M-ASK) system for the same number of information bits transmitted per signal sample. It is demonstrated that in the case of the matched receiver the M-ASK system outperforms OFDM and requires $3 - 3.5$ dB less of optical power at BER = 10^{-3} when $1 - 4$ data bits are transmitted per signal sample.

OCIS codes: (060.0060) Fiber optics and optical communications; (060.4080) Modulation; (060.4230) Multiplexing.

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1. Introduction

Orthogonal frequency division multiplexing (OFDM), invented in 1966 [1] and patented in 1970 [2], nowadays is widely used in broadband wired and wireless radio communication systems [3], [4]. The OFDM system is characterized by superior flexibility due to dynamic bandwidth allocation and adaptive bit rate functionalities, high spectral efficiency and high tolerance to multi-path interference, channel dispersion and frequency-selective fading. The modern OFDM technique is based on high-speed digital signal processing (DSP) and utilizes discrete fast Fourier transform (FFT) to shape the signal in the transmitter as well as to recover the data from the signal at receiver site. Recent advances in the development of high-speed electronics, enabling real-time DSP at several GS/s and higher, have stimulated strong interest in applying the OFDM technique in the next generation optical fiber communication systems. Implementations of OFDM in various ways - trading off the system complexity and cost versus signal transmission performance - for applications in different segments of optical networks, including single-mode fiber (SMF), multimode fiber (MMF) and plastic optical fiber (POF) access, as well as metropolitan and core network fiber communication systems, has been recently considered [5–11]. Last year the first book on OFDM for optical communications summarizing the status of this hot research topic has been published [12].

In this paper the OFDM system for low-cost applications, mainly in the access segment of optical fiber network, employing intensity modulated (IM) laser transmitter and optical power receiver is considered. Such a type of OFDM system implementation is also often called discrete multitone modulation (DMT), for instance in [9].

To a great extend, the performance of an OFDM system is studied by applying the method of brute-force numerical simulations with direct error counting and employing a random number generator to simulate noise distortions of the signal. Usually, such a study requires large computing resources in terms of CPU time and power, due to complexity of the system and due to slow convergence of simulation results when evaluating the system bit-error-ratio (BER).

The complexity of the system modeling is related to the principle of OFDM modulation. In OFDM the data information is carried over many low-rate subcarriers by using quadrature amplitude modulation (QAM) at each subcarrier. The size of QAM constellation (how many bits are mapped into one QAM symbol) and the number of subcarriers are system design parameters. Even when the size of QAM constellation and the number of subcarriers is moderate, the system modeling requires long bit sequences in order to generate all possible signal realizations. When modeling the most practically interesting cases, i.e. when 16 to 1024 subcarriers and the modulation up to 256-QAM is used, generating such long sequences is not feasible due to limited computing resources available in modern computers. This specific

feature of the OFDM substantially complicates the system performance evaluation when using brute-force numerical simulations as well as experimental tests, because such tests are often based on off-line computer processing of measured data. In this view, achieving analytical results on the system performance evaluation such as reported e.g. in [13] and [14] has manifold value. Analytical results provide a quick answer in a wide range of system parameters, lead to deeper understanding of the system behavior and can be used as a reference for numerical simulations and experimental tests. The disadvantage of the analytical approach is identified to the complexity of analysis when considering complicated system models.

In this paper a simple and illustrative analytical model for the performance evaluation of the IM optical OFDM system with digital baseband distortion is derived. The model takes account of the digital signal clipping and quantization in the OFDM transmitter as well as optical power receiver noise. The distortion due to signal clipping is deliberate and, as we will see in the following, helps to optimize the power efficiency of the system. The distortion in the form of signal quantization is caused by limited bit resolution of digital to analog converter (DAC) and represents an important characteristic for the cost-performance optimization of the system. The optimum signal clipping ratio, required optical power and minimum required bit resolution of DAC is evaluated for any specified level of the system BER and size of the QAM constellation. The results of the analytical model are verified with brute-force (Monte Carlo) simulations.

This paper is organized as follows: in Section 2 the model for IM optical OFDM system with digital baseband distortion is outlined; Section 3 is devoted to the analysis of the nonlinear signal distortion due to clipping and quantization as well as the derivation of effective SNR and the system BER; in Section 4 the dependency of the system performance versus signal clipping ratio is analyzed; the system performance penalty due to signal quantization (limited bit resolution) in DAC is evaluated in Section 5; Section 6 is dedicated to the analytical model verification and presents comparison of the analytical results with the results of brute-force numerical simulations; Section 7 summarizes the analytical results on IM optical OFDM system performance evaluation as well as presents results on comparison of the OFDM system performance with multi-level amplitude shift keying (M-ASK) scheme; discussions and conclusions are presented in Section 8 and Section 9, respectively.

2. Theoretical outline

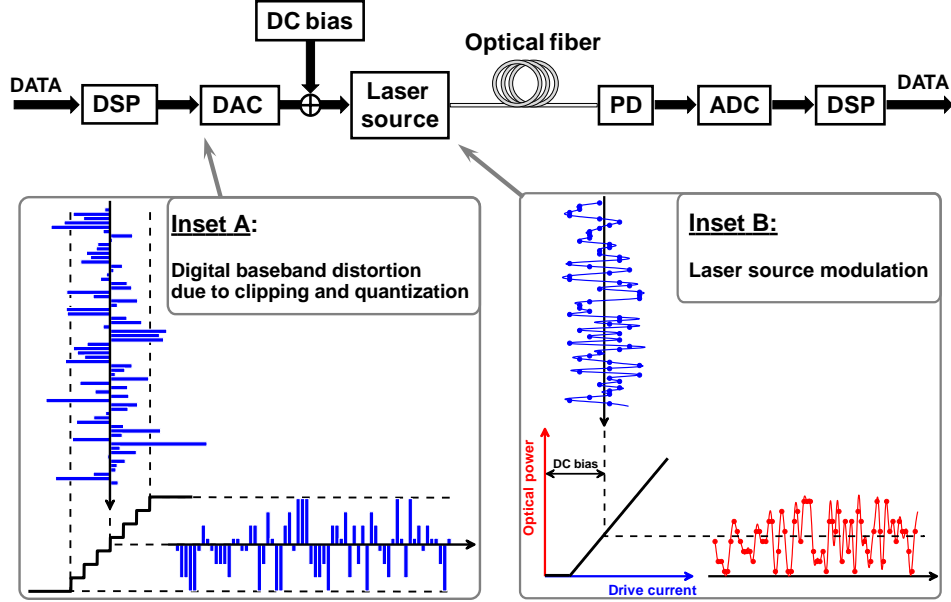The schematic of IM optical OFDM system considered in this report is illustrated in Fig. 1.

Fig. 1. (Color online) Schematic of IM optical OFDM system with digital baseband distortion. Dots on the analog waveforms shown in the inset B indicate values of digital signal samples.

In the front end DSP unit of the OFDM transmitter a sequence of data is processed in parallel by using IFFT in order to produce a real valued digital signal. This signal is distorted due to clipping and quantization (see Inset A in Fig. 1.), and then converted into analog electrical waveform in DAC. The analog waveform is biased with direct current (DC) and then applied to drive a laser source (see Inset B in Fig. 1.). The generated optical signal is transmitted over the optical fiber network and detected by the optical power receiver. The received signal is sampled in analog to digital converter (ADC) and processed in DSP unit in order to recover transmitted data. In the optical signal transmission part of the system model only the fiber loss is taken into consideration.

The digital signal at the input to the DAC is the sequence of the OFDM symbols with the time domain samples given by

$$x_k = \frac{1}{N} \sum_{n=1}^{\frac{N}{2}-1} X_n \exp\{-2\pi i k n / N\} + c.c., \ \ k = 0,1,...N-1, \tag{1}$$

where N is the size of FFT. The OFDM symbol is the superposition of Fourier harmonics (subcarriers) with complex amplitudes X_n that are constrained to have Hermitian symmetry: $X_n = X_{N-n}^*$ in order to produce real-valued signal samples x_k. Complex value X_n represents the gray coded M-ary QAM modulation. Subcarriers at $n = 0$ (average signal amplitude) and $n = N/2$ (maximum FFT harmonic) are set to zero, i.e. are not carrying any data. One OFDM symbol carries over $(N/2-1) \cdot \log_2 M$ data bits when the same QAM constellation size M is used at all non-zero subcarriers. The digital OFDM symbol given by Eq. (1) is distorted and converted into analog waveform in the DAC. The time duration of the OFDM symbol at the output of DAC is equal to N/R_{smpl}, where R_{smpl} is the sampling rate. Usually, a cyclic prefix is added in front of each OFDM symbol in order to reduce the effect of intersymbol interference due to fiber dispersion. The cyclic prefix is inserted at the input to the DAC and increases the time duration of the OFDM symbol. In this paper we assume that the cyclic prefix is negligibly small. In this case the system bit rate is estimated to

$\left(R_{smpl} \cdot \log_2 M \right) / 2$ when the number of subcarriers is large, i.e. $\left(\log_2 M \right) / 2$ data bits are transmitted with each digital signal sample.

The OFDM analog signal that is produced at the output of the DAC in response to input digital samples x_k is bipolar (waveform in blue in Inset B, Fig. 1) and is characterized by high peak-to-average power ratio (PAPR) especially when the number of non-zero subcarriers is large. Applying such a signal directly to the intensity modulated laser source would lead to large distortions of the optical signal because no optical power is generated when the driving signal is below the laser threshold. In order to avoid the optical signal distortions, a positive DC component has to be added to the analog signal. Applying DC bias to the signal is the drawback of IM optical OFDM because it substantially reduces the power efficiency of the data transmission. It is easy to estimate that in the case of a large DC bias, when no signal distortion occurs in the process of laser source modulation, the power of the useful part of the signal that is carrying over the data at the output of the optical power receiver will be at least PAPR times smaller in comparison to the total signal power detected by the photo receiver. It means that in the most interesting case for practical implementations, when many subcarriers are used to transmit data, the efficiency of the IM optical OFDM system will be very low. Additionally, the response characteristics of both the transmitter and the receiver should have a linear dynamic range that is equal to or larger than PAPR. High PAPR represents serious drawback of the OFDM technique. This problem has been addressed in a number of research studies [12,15,16].

It is well know that the system performance can be substantially improved by keeping in mind that high-power spikes occur very seldom – when many subcarriers in the OFDM symbol are superimposed in phase. Indeed, limiting the bipolar signal amplitude to such a level that only rare spikes will be clipped off is not going to cause drastic signal distortions. At the same time applying signal clipping enables substantial reduction of PAPR and effective improvement of the system performance. It is intuitively clear that as long as the signal clipping distortion is smaller or comparable to the additive noise in the receiver, the PAPR can be further reduced for the sake of the system performance improvement.

Let us now consider signal distortion in the digital domain in the form of symmetric clipping, derive optimum clipping ratio of the signal and evaluate the IM optical OFDM system performance.

In the following we will also account for additional signal distortion due to a limited bit resolution of DAC. Bit resolution is an important parameter of DAC and should be correctly identified in order to optimize the cost-performance characteristic of the system.

Please observe that it is assumed in this report that clipping of analog waveform at the laser threshold does not occur at all or so small that it does not lead to any system performance penalties.

Assuming an idealized situation when the modulation characteristic of the laser transmitter is linear and the receiver is matched (does not lead to any intersymbol interference) the signal samples at the output of ADC in the OFDM receiver can be expressed in the following form

$$ z_k = \rho P_0 \left(1 + \frac{y_k}{x_{DC}} \right) + d_k \big|_{therm} + d_k \big|_{shot}, \qquad (2) $$

where ρ - optical receiver responsivity, P_0 - received average optical power, $y_k = Q(x_k)$ - distorted digital signal, and $x_{DC} = I_{DC} - I_{th}$ - difference between DC bias and laser threshold current. The receiver thermal and shot (Gaussian) noise contributions are denoted by $d_k \big|_{therm}$ and $d_k \big|_{shot}$, respectively. The nonlinear transfer function $Q(x)$ accounts for the signal distortion caused by clipping and quantization. In the case of symmetric clipping considered in this paper the nonlinear transfer function is specified in the following form

$$Q(x) \equiv Q_C(x) = \begin{cases} x, & if \ |x| < x_0 \\ x_0 \cdot \text{sign}(x), & if \ |x| \ge x_0 \end{cases} \qquad (3)$$

when the bit resolution is high and, therefore, the signal quantization is negligibly small. In the case of limited bit resolution the nonlinear transfer function given by Eq. (3) takes the following form

$$Q(x) \equiv Q_{C\&Q}(x) = \begin{cases} x_s, & if \ |x - x_s| < \Delta x / 2 \\ x_0 \cdot \text{sign}(x), & if \ |x| \ge x_0 - \Delta x / 2 \end{cases} \qquad (4)$$

where $\Delta x = 2x_0 / \left(2^{N_{bit}} - 1\right)$ - quant of signal amplitude, $x_s = -x_0 + s \cdot \Delta x$ - s-th signal level, $s = 1, 2, ..., 2^{N_{bit}} - 2$ and N_{bit} - DAC bit resolution. The amplitude of the clipped signal x_0 is assumed to be smaller or equal to x_{DC}.

3. Nonlinear distortion noise and effective signal to noise ratio

Let us now analyze the effect of nonlinear distortion specified in Eq. (3) and Eq. (4) on the OFDM system performance.

Rigorously defined, the signal x_k given by Eq. (1) takes many but finite number of discrete values. When the number of subcarriers is large these discrete values are so many that the central limit theorem applies, and the signal histogram can be accurately described by continuous Gaussian distribution: $f(x) = \exp\{-x^2 / 2\sigma_x^2\} / \sqrt{2\pi\sigma_x^2}$. The distribution in our case has zero mean, and the variance that is equal to the mean square of the OFDM signal amplitude: $\sigma_x^2 = < x^2 >$. In this case the signal distortion caused by a memory less nonlinear transfer function $Q(x)$ can be analyzed by applying the well know Bussgang theorem [17], [18]. According to Bussgang theorem the distorted signal $y = Q(x)$ can be decomposed into an undistorted component that is equal to the signal x reduced in amplitude by a factor of α, and distortion noise d:

$$y = \alpha \cdot x + d. \qquad (5)$$

The distortion noise is uncorrelated with the signal x and therefore the factor α is estimated to

$$\alpha = \frac{< x \cdot y >}{< x^2 >} = \frac{1}{\sqrt{2\pi}\sigma_x^3} \int_{-\infty}^{+\infty} x \cdot Q(x) \cdot \exp\left\{-\frac{x^2}{2\sigma_x^2}\right\} dx. \qquad (6)$$

By applying this approach, the power of undistorted component in Eq. (5) (the power that is useful for the data recovery in the receiver) is straightforward to derive. Substituting Eq. (5) in Eq. (2) leads to the following expression for the useful signal power in terms of mean square current

$$I_{signal \ C\&Q}^2 = \left(\frac{\rho P_0}{x_{DC}} \alpha \sigma_x\right)^2. \qquad (7)$$

The distortion component d in Eq. (5) leads to uncertainty in the data recovery and is equivalent to a noise contribution with the mean square current:

$$\left\langle I_{noise \ C\&Q}^2 \right\rangle = \left(\frac{\rho P_0}{x_{DC}}\right)^2 \left(< y^2 > - \alpha^2 \sigma_x^2\right), \qquad (8)$$

where

$$< y^2 > = \frac{1}{\sqrt{2\pi}\sigma_x} \int\limits_{-\infty}^{+\infty} Q^2(x) \cdot \exp\left\{-\frac{x^2}{2\sigma_x^2}\right\} dx \ . \tag{9}$$

Finally, the system performance can be easily evaluated by introducing an effective signal to noise ratio as the ratio between the power of the undistorted part of the signal (specified by Eq. (7)) and an effective noise power, which accounts for contributions caused by clipping and quantization (specified by Eq. (8)) as well as by shot and thermal noise (last two terms in Eq. (2)):

$$SNR_{eff} = \frac{I_{signal\ C\&Q}^2}{\left\langle I_{noise\ C\&Q}^2 \right\rangle + \left\langle I_{therm}^2 \right\rangle + \left\langle I_{shot}^2 \right\rangle} = \frac{\left(\frac{\rho P_0}{x_{DC}}\alpha\sigma_x\right)^2}{\left(\frac{\rho P_0}{x_{DC}}\right)^2 \left(< y^2 > -\alpha^2\sigma_x^2\right) + S_{th}^2 \Delta f + 2q\rho P_0 \Delta f}\ , \tag{10}$$

where S_{th} - thermal noise density, Δf - receiver bandwidth, and q - electron charge. The system performance can now be easily evaluated by using well know results for the system BER evaluation. For instance, in the case of rectangular QAM the system BER is evaluated by using the following expression (see for instance [19] or [20]):

$$BER_{M-QAM} = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) \mathrm{erfc}\left(\sqrt{\frac{3\ SNR_{eff}}{2\ (M-1)}}\right). \tag{11}$$

In the case of non rectangular cross 32-QAM and 128-QAM constellations the expressions for BER can be found in [21].

4. Optimum clipping ratio

Let us now first neglect the effect of signal quantization and analyze the impact of the signal clipping on the system performance. In this case the nonlinear transfer function $Q(x)$ is specified in Eq. (3) and the integration in Eq. (6) and Eq. (9) yields the following results for factor α and mean square of clipped signal amplitude:

$$\alpha = 1 - \mathrm{erfc}\left(\sqrt{R_{cl}/2}\right), \tag{12}$$

$$< y^2 > = \sigma_x^2 \left(1 - \mathrm{erfc}\left(\sqrt{R_{cl}/2}\right) + R_{cl}\,\mathrm{erfc}\left(\sqrt{R_{cl}/2}\right) - \sqrt{2R_{cl}/\pi}\,\exp\left\{-R_{cl}/2\right\}\right), \tag{13}$$

where $R_{cl} = x_0^2/\sigma_x^2$ - clipping ratio. By substituting Eq. (12) and Eq. (13) in Eq. (10) the effective SNR can be expressed in the following form

$$SNR_{eff} = \frac{S_0\left(1 - \mathrm{erfc}\left(\sqrt{R_{cl}/2}\right)\right)^2}{R_{cl} + S_0\left((1+R_{cl})\,\mathrm{erfc}\left(\sqrt{R_{cl}/2}\right) - \mathrm{erfc}^2\left(\sqrt{R_{cl}/2}\right) - \sqrt{2R_{cl}/\pi}\,\exp\left\{-R_{cl}/2\right\}\right)}, \tag{14}$$

by introducing a dimensionless parameter S_0:

$$S_0 = \left(\frac{x_0}{x_{DC}}\right)^2 \frac{\left(\rho P_0\right)^2}{\left(S_{th}^2 + 2q\rho P_0\right)\Delta f}. \tag{15}$$

In fact the parameter S_0 is equal to a conventionally defined SNR (i.e. the ratio between average received signal power and the additive noise power at the output of photo receiver) reduced with the factor $\left(x_0/x_{DC}\right)^2$ related to the clipping amplitude and DC bias.

The effective SNR specified in Eq. (14) enables straightforward evaluation of the optimum clipping ratio that has been discussed above in Section 2. The effective SNR specified in Eq. (14) is plotted in Fig. 2 versus clipping ratio by solid curves for various
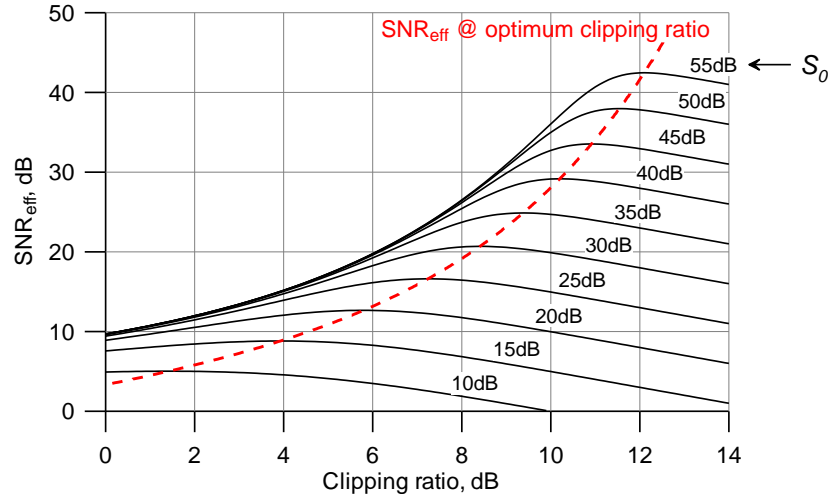

Fig. 2. (Color online) Effective signal to noise ratio SNR_{eff} specified by Eq. (14) versus clipping ratio for various values of parameter S_0. Dashed curve in red shows SNR_{eff} at optimum clipping ratio.

value of parameter S_0. This family of curves clearly indicates the optimum clipping ratio (tradeoff between clipping distortion and useful signal power discussed above in Section 2) as well as the maximum achievable SNR_{eff} for specified value of the parameter S_0 (Fig. 2, dashed curve in red). By using the results presented in Fig. 2 the performance of the system can be evaluated in the following way. Let us assume that the OFDM subcarriers are modulated with 16-QAM and we want to estimate the optimum clipping ratio and the required optical power in order to achieve the system BER = 10^{-3}. From Eq. (11) we find that such BER is achieved when SNR_{eff} = 16.5 dB. Using this value of SNR_{eff} we find from the results shown in Fig. 2 that the optimum clipping ratio in this case is equal to 7.2 dB and $S_0 \approx 25$ dB. Using this value of parameter S_0 we can estimate, by using Eq. (15), that the required optical power is equal to -14.5 dBm, when optical receiver parameters are set to values specified in Table 1 and $x_0 = x_{DC}$.

Table 1. Optical Receiver Parameters

Responsivity	ρ	0.8 A/W
Thermal noise current	S_{th}	16 pA/$\sqrt{\text{Hz}}$
Bandwidth	Δf	10 GHz

5. Signal quantization due to finite bit resolution of DAC

Let us now take into consideration the effect of signal quantization due finite bit resolution of DAC. When the OFDM signal is distorted by both clipping and quantization the nonlinear transfer function $Q(x)$ is specified by Eq. (4). Substituting Eq. (4) in Eq. (6) and Eq. (9) and performing mathematical integration leads to the following results:

$$\alpha = \sqrt{\frac{2R_{cl}}{\pi}} \left(\exp\left\{ -\frac{R_{cl}}{2} \beta^2 (1/2) \right\} \right.$$
$$\left. + \sum_{k=1}^{2^{N_{bit}-1}-1} \beta(k) \left[\exp\left\{ -\frac{R_{cl}}{2} \beta^2 (k+1/2) \right\} - \exp\left\{ -\frac{R_{cl}}{2} \beta^2 (k-1/2) \right\} \right] \right), \qquad (16)$$

$$< y^2 > = \sigma_x^2 R_{cl} \left(\text{erfc}\left\{ \sqrt{\frac{R_{cl}}{2}} \beta(1/2) \right\} + \right.$$
$$\left. + \sum_{k=1}^{2^{N_{bit}-1}-1} \beta^2(k) \cdot \left[\text{erfc}\left\{ \sqrt{\frac{R_{cl}}{2}} \beta(k+1/2) \right\} - \text{erfc}\left\{ \sqrt{\frac{R_{cl}}{2}} \beta(k-1/2) \right\} \right] \right), \qquad (17)$$

where $\beta(s) = 1 - 2s / \left(2^{N_{bit}} - 1\right)$. The effective SNR in this case is convenient to rewrite in the following compact form

$$SNR_{eff} = \frac{S_0 \cdot \alpha^2}{R_{cl} + S_0 \cdot \left(\dfrac{< y^2 >}{\sigma_x^2} - \alpha^2 \right)} . \qquad (18)$$

The dependency of SNR_{eff} versus the clipping ratio is exemplified in Fig. 3 for the bit resolution from 2 to 8 at the value of parameter S_0 set to 45dB.
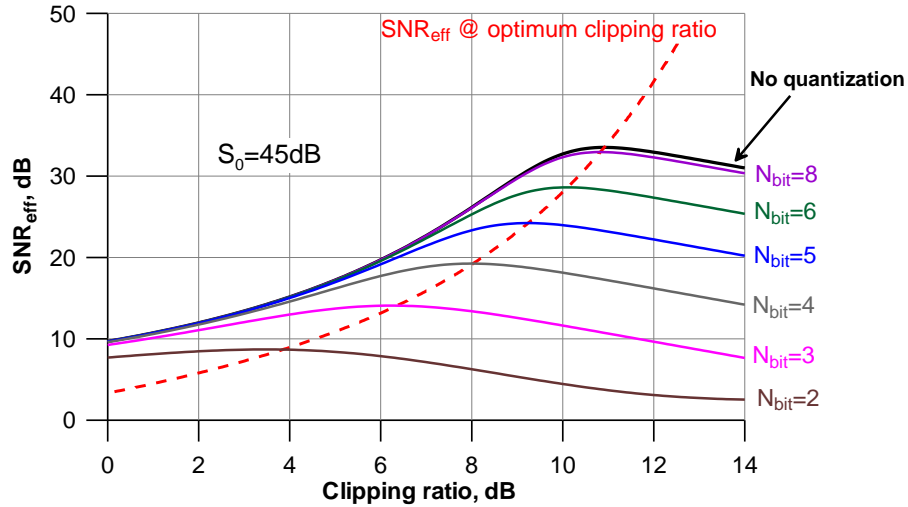

Fig. 3. (Color online) Effective signal to noise ratio SNR_{eff} specified by Eq. (18) versus clipping ratio for $S_0 = 45$dB and DAC bit resolution N_{bit} equal to 2, 3, 4, 5, 6 and 8, respectively. Dashed curve in red shows SNR_{eff} at optimum clipping ratio (the same as shown in Fig. 2). Solid curve in bold black is the same as shown in Fig. 2 for $S_0 = 45$dB .

The results displayed in Fig. 3 show that the SNR_{eff} penalty caused by signal quantization is negligibly small when the bit resolution is equal to or larger than 8. It is also seen that reducing bit resolution to below 8 leads to a significant SNR_{eff} penalty and decreased value for the optimum clipping ratio. It is worth to emphasize that the family of curves shown in Fig. 2 for the case when the signal quantization is negligibly small (infinite bit resolution) and

curves shown in Fig. 3 for the case of limited bit resolution have almost the same dependency of SNR_{eff} versus optimum clipping ratio.

6. Analytical model verification

Now the system BER can be evaluated in a simple way by using Eq. (11) in combination with Eq. (14) in the case of infinite bit resolution or with Eqs. (16) – (18) when the bit resolution is limited. The system BER that is analytically evaluated for the case of 16-QAM, 64-QAM and 256-QAM constellations is plotted in Fig. 4 versus signal clipping ratio for selected set of parameters S_0 and N_{bit}. Figure 4 also shows the results of brute-force Monte Carlo simulations with direct error counting for comparison.
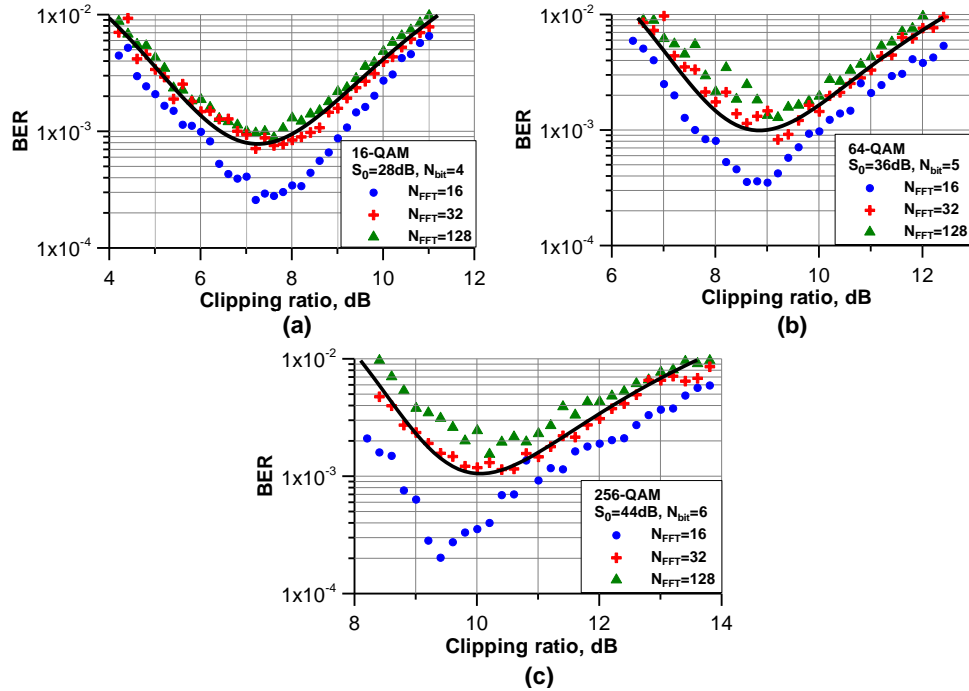

Fig. 4. (Color online) BER versus clipping ratio for (a) 16-QAM, $S_0 = 28\text{dB}$, $N_{bit} = 4$ (b) 64-QAM, $S_0 = 36\text{dB}$, $N_{bit} = 5$, and (c) 256-QAM, $S_0 = 44\text{dB}$, $N_{bit} = 6$. Solid lines show the results of analytical evaluation. Symbols display the results of brute-force numerical simulations with direct error counting for FFT size (total number of subcarriers) equal to 16 (dots in blue), 64 (crosses in red), and 128 (triangles in green).

For each QAM constellation and specified set of parameters the numerical simulations have been performed for a different number of subcarriers (FFT size of 16, 32 and 128) in order to test the validity of the central limit theorem. The random binary sequence of a few of 10^5 bits has been used in the numerical simulations in order to model the signal pattern. At the receiver site the detected signal pattern has been loaded with additive receiver noise. In each simulation run a new random binary sequence has been generated using CPU clock as the seed in order to test the BER convergence in terms of data pattern length. If simulations are performed in such a way, observing large "oscillations" of BER versus clipping ratio instead of a smooth curve would indicate that the data pattern length is too short. Small irregular oscillations in the simulation results, shown in Fig. 4 by symbols, indicate that the data pattern length of a few of 10^5 bits is long enough in order to achieve a nearly converged BER.

It is clearly seen in Fig. 4 that the results of the analytical model are in good agreement with the results of brute-force numerical simulations. A slightly increased mismatch between

analytical and numerical results is seen when the FFT size is reduced to 16. This observation indicates that applying the central limit theorem in this case leads to increased inaccuracy when describing the signal histogram with continuous Gaussian distribution.

7. Evaluation of IM optical OFDM system performance

The results of the system performance evaluation obtained above are summarized in Fig. 5. At the top of Fig. 5 the parameter S_0 is plotted versus SNR_{eff} at the optimum clipping ratio in the case of infinite bit resolution (no quantization) as well as when the bit resolution is set to the value from 2 to 8. The optimum clipping ratio and the system BER (for various size of QAM constellation) versus SNR_{eff} is shown at the bottom of Fig. 5. Using these results the following system performance parameters are straight-forward to estimate:

- Minimum required bit resolution of DAC,
- Optimum clipping ratio,
- Required optical power at the input to the receiver,
- Power penalty due to signal quantization in DAC versus bit resolution,
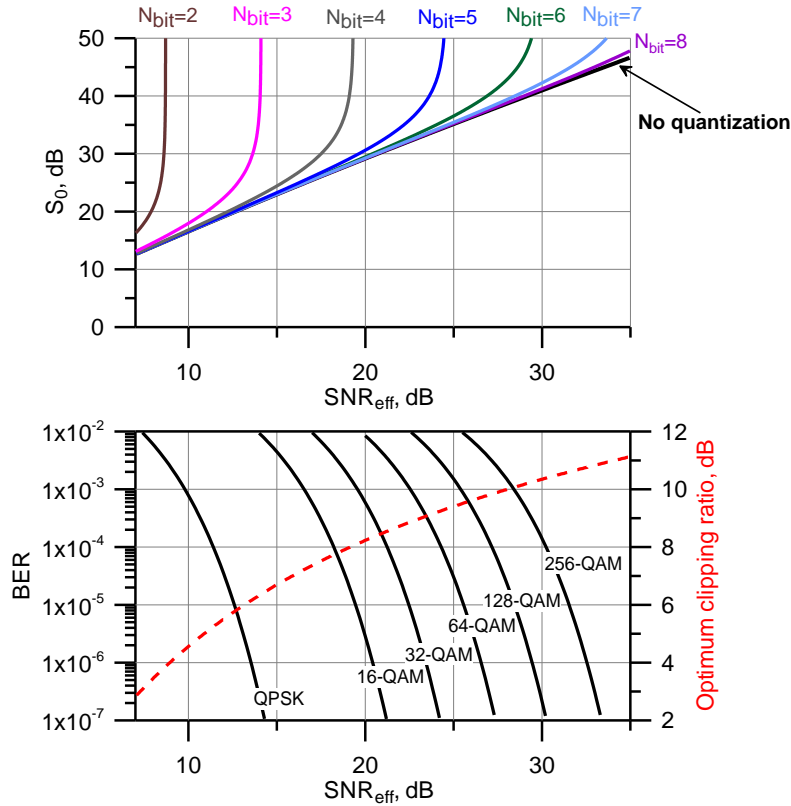- All at any specified level of BER and size of QAM constellation.

Fig. 5. (Color online) Top: Parameter S_0 defined by Eq. (15) versus effective signal to noise ratio SNR_{eff} at optimum clipping ratio and DAC bit resolution N_{bit} equal to $2 \div 8$ and infinity (no quantization). Bottom: System BER (solid curves) for various size of QAM constellation and optimum clipping ratio (dashed curve in red) versus SNR_{eff}.

Minimum required bit resolution in each specific case can easily be identified by observing that the signal quantization in DAC can lead to an unacceptable level of the system BER (BER floor) even when the received optical power (parameter S_0) is very high. For instance, in the case of 64-QAM the bit resolution of 4 is unacceptable because the BER floor in this case is above 10^{-2} and substantially exceeds the limit for the forward error correction (FEC). The minimum required bit resolution for 64-QAM is 5. At this value of the bit resolution the optimum clipping ratio is 8.8 dB at BER = 10^{-3} and the parameter S_0 is equal to 35.8 dB. The received optical power is estimated to −8.9 dBm by using the receiver parameters from Table 1 and assuming $x_0 = x_{DC}$ in Eq. (15). When the bit resolution is increased from 5 to 6 the received optical power is reduced to −10.4 dBm. Further increase of the bit resolution leads to an insignificant reduction in the required optical power.

At the level of BER = 10^{-3} the minimum required bit resolution is 3 for QPSK, 4 for 16-QAM, 5 for 32-QAM and 64-QAM, and 6 for 128-QAM and 256-QAM. When the bit resolution is equal to 7 the optical power penalty due to signal quantization is very small for all considered QAM constellations. These estimates provide a simple and practical tool for selection of DAC bit resolution when designing the IM optical OFDM system.

Figure 5 also illustrates the well known fact that increasing the spectral efficiency of the transmission system by using sophisticated modulation formats with many signal levels requires higher optical signal power to be detected at the receiver. Based on the results presented in Fig. 5 the IM optical OFDM system BER versus received optical power is plotted in Fig. 6 for 4-, 16-, 64- and 256-QAM (curves in red) for the case of infinite DAC bit resolution. The results show that about −7 dBm of received optical power is required for 256-QAM modulation at BER = 10^{-3}. This level of power is rather high and substantially limits the available power budget of the system.
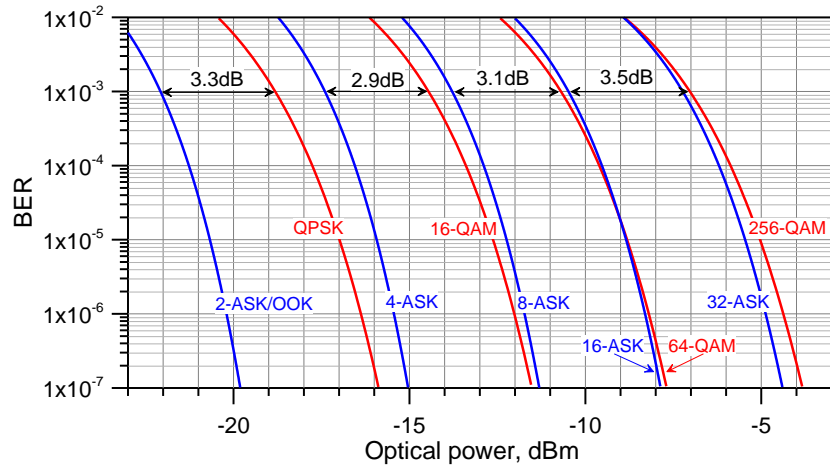

Fig. 6. (Color online) BER versus received optical power for IM optical OFDM system (infinite DAC bit resolution) with M-QAM modulated subcarriers (in red) and for the system with multi-level ASK (in blue). The performance of OFDM and ASK systems is compared in terms of required optical power at BER = 10^{-3} for the same number of data bits transmitted per signal sample.

Figure 6 also shows the results for multi-level amplitude-shift keying (M-ASK) scheme when several levels of optical power are used to transmit group of bits per each signal sample. M-ASK can be used as an alternative to IM optical OFDM for achieving increased transmission capacity and spectral efficiency of the system. In a M-ASK system the received signal samples can be expressed (by using the notations of Eq. (2)) in the form

$$z_k = 2\rho P_0 \frac{m-1}{M-1} + d_k\big|_{therm} + d_k\big|_{shot}, \quad m = 1, 2, \dots M, \quad (19)$$

where M is the number of optical power levels. The BER in this case is straightforward to derive (see for instance Section 4.2.7 in [20])

$$
\begin{aligned}
BER_{M-ASK} &\approx \frac{M-1}{M\log_2 M} \Pr\left(\left|z_k - 2\rho P_0 \frac{m-1}{M-1}\right| > \frac{\rho P_0}{M-1}\right) \approx \\
&\approx \frac{M-1}{M\log_2 M} \mathrm{erfc}\left(\frac{1}{\sqrt{2}(M-1)} \sqrt{\frac{(\rho P_0)^2}{(S_{th}^2 + 2q\rho P_0)\cdot \Delta f}}\right).
\end{aligned}
\quad (20)
$$

The BER for the M-ASK system is shown in Fig. 6 for M = 2, 4, 8, 16, 32 by the blue curves. In the case of two optical power levels this scheme is equivalent to on-off keying (OOK) when only one bit is transmitted per signal sample. It is relevant to compare the 2-ASK/OOK system with the IM optical OFDM when 4-QAM/QPSK is used in all subcarriers. This comparison is justified because $(\log_2 M)/2$ data bits are transmitted per signal sample in the OFDM system if the number of subcarriers is large and the length of cyclic prefix is negligibly small. For higher order modulations it is relevant to compare M-ASK and M-QAM OFDM when $\log_2 M_{ASK} = (\log_2 M_{QAM})/2$. Figure 6 shows that the ASK system requires about 3 dB − 3.5 dB less of received optical power than the OFDM system. It is interesting to note that the performance of 32-ASK (5 bits/sample) is very close to the performance of 256-QAM OFDM (4 bits/sample).

8. Discussions

The results of the analytical model presented in Fig. 6 quantify the drawback of the IM optical OFDM system discussed in Section 2 above: reduced power efficiency due to presence of a large optical carrier that is not used for any data transmission. This drawback should be kept in mind along with attractive features of IM optical OFDM such as dynamic bandwidth allocation, adaptive bit rate and tolerance to channel dispersion when designing the system and selecting signal modulation format.

Figure 6 represents an optimistic evaluation of the system performance because it shows the results that are obtained by assuming an idealized (matched) receiver as well as the minimum acceptable level of DC bias in the OFDM transmitter (OFDM transmitter ($x_{DC} = x_0$). Practical system design requires taking into account the system performance impairments due to limited bandwidth and nonlinear characteristics of optoelectronic components, chromatic and spatial (modal) dispersion of fiber, as well as the laser frequency chirp. The performance evaluation of both the OFDM and M-ASK systems in such a general case is the prospect for future work where the analytical model presented in this report can be employed as "idealized reference".

9. Conclusions

In conclusion, a simple and illustrative analytical model has been derived in this report in order to estimate the performance of the IM optical OFDM system with digital baseband distortion. By applying the Bussgang theorem the impact of the digital baseband distortion due to deliberate signal clipping and limited bit resolution of DAC has been analyzed. Minimum required bit resolution, optimum clipping ratio, required optical power at the input to the receiver, optical power penalty due to signal quantization in DAC versus bit resolution - all at any specified level of BER and size of QAM constellation have been estimated. The analytical results are in good agreement with brute-force Monte Carlo simulations. The analytical model points out to the drawback of the IM optical OFDM: reduced power efficiency (in comparison with M-ASK scheme) due to presence of large optical carrier that is

not used for any data transmission. This drawback should be kept in mind along with the attractive features of the IM optical OFDM when designing the system and selecting the modulation format for high-speed transmission over SMF, MMF and POF in low-cost access and home networks.

Acknowledgments