1、无耗传输线的特性阻抗$Z_0 = 100\ (\Omega)$。

根据给出的已知数据，分别写出传输线上电压、电流的复数和瞬时形式的表达式：

(1) $R_L = 100\ (\Omega)$，$I_L = e^{j0°}\ (mA)$；

(2) $R_L = 50\ (\Omega)$，$V_L = 100e^{j0°}\ (mV)$；

(3) $V_L = 200e^{j0°}\ (mV)$，$I_L = 0\ (mA)$。

$$\begin{bmatrix} U(z) \\ I(z) \end{bmatrix} = \begin{bmatrix} \cos\beta z & jZ_0\sin\beta z \\ j\frac{1}{Z_0}\sin\beta z & \cos\beta z \end{bmatrix} \begin{bmatrix} U_L \\ I_L \end{bmatrix}$$

解：（1）根据已知条件，可得：$V_L = I_L R_L = 100\ (mV)$，

$$U(z) = 100\cos(\beta z) + j100\sin(\beta z) = 100e^{j\beta z}\ (mV)$$
$$I(z) = \cos(\beta z) + j\sin(\beta z) = e^{j\beta z}\ (mA)$$
$$U(z,t) = \mathrm{Re}\left[U(z)e^{j\omega t}\right] = 100\cos(\omega t + \beta z)\ (mV)$$
$$I(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right] = \cos(\omega t + \beta z)\ (mA)$$

（2）根据已知条件，可得：$I_L = \frac{V_L}{R_L} = \frac{100e^{j0°}}{50} = 2\ (mA)$

复数表达式为：$U(z) = 100\cos(\beta z) + j200\sin(\beta z)\ (mV)$

$I(z) = 2\cos(\beta z) + j\sin(\beta z)\ (mA)$

瞬时表达式为：$U(z,t) = \mathrm{Re}\left[U(z)e^{j\omega t}\right] = 150\cos(\omega t + \beta z) - 50\cos(\omega t - \beta z)\ (mV)$

$I(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right] = 1.5\cos(\omega t + \beta z) + 0.5\cos(\omega t - \beta z)\ (mA)$

(3) 复数表达式为：$U(z) = 200\cos(\beta z)\ (mV)$

$I(z) = j2\sin(\beta z)\ (mA)$

瞬时表达式为：$U(z,t) = 100\cos(\omega t + \beta z) + 100\cos(\omega t - \beta z)\ (mV)$

$I(z,t) = \cos(\omega t + \beta z) - \cos(\omega t - \beta z)\ (mA)$

证明题：$Z_{in}(z) = Z_{in}(z + \frac{\lambda}{2}),\ Z_{in}(z)\cdot Z_{in}(z + \frac{\lambda}{4}) = Z_0^2$

$$Z_{in}(z + \frac{\lambda}{2}) = Z_0 \frac{Z_L + jZ_0 tg\beta(z + \frac{\lambda}{2})}{Z_0 + jZ_L tg\beta(z + \frac{\lambda}{2})}$$
$$= Z_0 \frac{Z_L + jZ_0 tg(\beta z + \frac{\lambda\beta}{2})}{Z_0 + jZ_L tg(\beta z + \frac{\lambda\beta}{2})} \xlongequal{\beta = \frac{2\pi}{\lambda}} Z_0 \frac{Z_L + jZ_0 tg(\beta z + \pi)}{Z_0 + jZ_L tg(\beta z + \pi)}$$
$$= Z_0 \frac{Z_L + jZ_0 tg\beta z}{Z_0 + jZ_L tg\beta z} = Z_{in}(z)$$

$$Z_{in}(z)\cdot Z_{in}(z + \frac{\lambda}{4}) = Z_0^2 \cdot \frac{Z_L + jZ_0 tg\beta z}{Z_0 + jZ_L tg\beta z} \cdot \frac{Z_L + jZ_0 tg\beta(z + \frac{\lambda}{4})}{Z_0 + jZ_L tg\beta(z + \frac{\lambda}{4})} = Z_0^2 \cdot \frac{Z_L + jZ_0 tg\beta z}{Z_0 + jZ_L tg\beta z} \cdot \frac{Z_L + jZ_0 tg(\beta z + \frac{\pi}{2})}{Z_0 + jZ_L tg(\beta z + \frac{\pi}{2})}$$
$$= Z_0^2 \cdot \frac{Z_L + jZ_0 tg\beta z}{Z_0 + jZ_L tg\beta z} \cdot \frac{Z_L - jZ_0 ctg\beta z}{Z_0 - jZ_L ctg\beta z}$$
$$= Z_0^2 \cdot \frac{Z_L^2 + jZ_0 Z_L tg\beta z - jZ_0 Z_L ctg\beta z + Z_0^2}{Z_0^2 + jZ_0 Z_L tg\beta z - jZ_0 Z_L ctg\beta z + Z_L^2}$$
$$= Z_0^2$$

证明：如果能测得开路和短路阻抗，则可求出 Z_0和 β 。

$$Z_0=\sqrt{Z_{in}^{sc}(z)\cdot Z_{in}^{oc}(z)}$$

$$\beta=\frac{1}{z}\arctan\sqrt{-\frac{Z_{in}^{sc}(z)}{Z_{in}^{oc}(z)}}$$

证： $Z_{\rm in}(z)=Z_0\dfrac{Z_l+{\rm j}Z_0\tan(\beta z)}{Z_0+{\rm j}Z_l\tan(\beta z)}$ ➡ $Z_{in}^{sc}(z)={\rm j}Z_0\tan(\beta z)$ (1)

$Z_{in}^{oc}(z)=-{\rm j}Z_0 c\tan(\beta z)$ (2)

$(1)*(2)$ ➡ $Z_{in}^{sc}(z)\cdot Z_{in}^{oc}(z)=Z_0^2$ ➡ $Z_0=\sqrt{Z_{in}^{sc}(z)\cdot Z_{in}^{oc}(z)}$

$(1)/(2)$ ➡ $\dfrac{Z_{in}^{sc}(z)}{Z_{in}^{oc}(z)}=-\tan^2(\beta z)$ ➡ $\beta=\dfrac{1}{z}\arctan\sqrt{-\dfrac{Z_{in}^{sc}(z)}{Z_{in}^{oc}(z)}}$

无耗传输线$Z_0=50\ (\Omega)$，已知在距负载$z_1=\lambda_p/8$*处的反射系数为* $\Gamma(z_1)={\rm j}0.5$。
*试求(1) 传输线上任意观察点z*处的反射系数 $\Gamma(z)$*和等效阻抗Z(z)；*
(2) 利用负载反射系数 Γ_L计算负载阻抗Z_L；(3) 通过等效阻抗Z(z)计算负载阻抗Z_L。

解：(1)由 $\Gamma(z)=\Gamma_{\rm L}e^{-j2\beta z}$

$$\Gamma(z_1)=\Gamma\left(\frac{\lambda_{\rm p}}{8}\right)=\Gamma_{\rm L}e^{-{\rm j}\frac{4\pi}{\lambda_{\rm p}}\cdot\frac{\lambda_{\rm p}}{8}}=\Gamma_{\rm L}e^{-{\rm j}\frac{\pi}{2}}={\rm j}0.5$$

因此有 $\Gamma_{\rm L}=-0.5$ $\rightarrow$ $\Gamma(z)=\Gamma_{\rm L}e^{-j2\beta z}=-0.5\,{\rm e}^{-{\rm j}2\beta z}$

$$Z(z)=Z_0\frac{1+\Gamma(z)}{1-\Gamma(z)}=50\times\frac{1-0.5{\rm e}^{-{\rm j}2\beta z}}{1+0.5{\rm e}^{-{\rm j}2\beta z}}=50\times\frac{1-0.5^2+{\rm j}2\times0.5\sin(180^\circ-2\beta z)}{1+0.5^2-2\times0.5\cos(180^\circ-2\beta z)}$$

$$=50\times\frac{3+{\rm j}4\sin(2\beta z)}{5+4\cos(2\beta z)}$$

(2) 利用负载反射系数计算负载阻抗

$$\Gamma_{\rm L}=\frac{Z_{\rm L}-Z_0}{Z_{\rm L}+Z_0}\ \rightarrow\ Z_{\rm L}=Z_0\times\frac{1+\Gamma_{\rm L}}{1-\Gamma_{\rm L}}=50\times\frac{1+(-0.5)}{1-(-0.5)}=\frac{50}{3}(\Omega)$$

(3) 通过等效阻抗计算负载阻抗

$$Z_{\rm L}=Z(0)=50\times\frac{3+{\rm j}4\sin(0)}{5+4\cos(0)}=\frac{50}{3}(\Omega)$$

特性阻抗为Z_0的无耗传输线上电压波腹点的位置是z_1'，电压波节点的位置是z_1''，试证明可用下面两个公式来计算负载阻抗Z_L：

$$Z_L = Z_0 \frac{\rho - \mathrm{j}\tan(\beta z_1')}{1-\mathrm{j}\rho\tan(\beta z')} \quad 和 \quad Z_L = Z_0 \frac{k - \mathrm{j}\tan(\beta z_1'')}{1-\mathrm{j}k\tan(\beta z'')}$$

证: $$Z(z) = Z_0 \frac{Z_L + \mathrm{j}Z_0 \tan(\beta z)}{Z_0 + \mathrm{j}Z_L \tan(\beta z)} \Rightarrow Z_L = Z_0 \frac{Z(z) - \mathrm{j}Z_0 \tan(\beta z)}{Z_0 - \mathrm{j}Z(z)\tan(\beta z)}$$

当$z = z_1'$时，$Z(z_1') = Z_0\rho$，*所以得：*

$$Z_L = Z_0 \frac{Z_0\rho - \mathrm{j}Z_0 \tan(\beta z_1')}{Z_0 - \mathrm{j}Z_0\rho\tan(\beta z_1')} = Z_0 \frac{\rho - \mathrm{j}\tan(\beta z_1')}{1-\mathrm{j}\rho\tan(\beta z_1')}$$

当$z = z_1''$时，$Z(z_1'') = Z_0 k$，所以得：

$$Z_L = Z_0 \frac{Z_0 k - \mathrm{j}Z_0 \tan(\beta z_1'')}{Z_0 - \mathrm{j}Z_0 k\tan(\beta z_1'')} = Z_0 \frac{k - \mathrm{j}\tan(\beta z_1'')}{1-\mathrm{j}k\tan(\beta z_1'')}$$

有一无耗传输线，终端接负载阻抗$Z_L = 40 + \mathrm{j}30\ (\Omega)$。
试求：(1) 要使线上的驻波比最小，传输线的特性阻抗Z_0应为多少？
(2) 该最小驻波比和相应的电压反射系数之值；
(3) 距负载最近的电压波节点位置和该处的输入阻抗（等效阻抗）。

解：(1) 传输线上的反射系数最小，其驻波比就最小。
设传输线的特性阻抗为Z_0，根据已知条件，负载反射系数为

$$|\Gamma_L| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{R_L + \mathrm{j}X_L - Z_0}{R_L + \mathrm{j}X_L + Z_0}\right| = \left|\frac{(R_L - Z_0) + \mathrm{j}X_L}{(R_L + Z_0) + \mathrm{j}X_L}\right| = \sqrt{\frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}}$$

$$|\Gamma_L|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2} = \frac{R_L^2 + Z_0^2 + X_L^2 - 2R_L Z_0}{R_L^2 + Z_0^2 + X_L^2 + 2R_L Z_0} = \frac{40^2 + 30^2 + Z_0^2 - 2\times 40 + Z_0}{40^2 + 30^2 + Z_0^2 + 2\times 40 + Z_0} = \frac{Z_0^2 - 80Z_0 + 2500}{Z_0^2 + 80Z_0 + 2500}$$

令 $$\frac{\partial}{\partial Z_0}\left(\frac{Z_0^2 - 80Z_0 + 2500}{Z_0^2 + 80Z_0 + 2500}\right) = \frac{(2Z_0 - 80)(Z_0^2 + 80Z_0 + 2500) - (2Z_0 + 80)(Z_0^2 - 80Z_0 + 2500)}{(Z_0^2 + 80Z_0 + 2500)^2} = 0$$

$$\Downarrow$$

$$Z_0 = 50\ (\Omega)$$

(2) $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 + \mathrm{j}30 - 50}{40 + \mathrm{j}30 + 50} = \mathrm{j}\frac{1}{3} = \frac{1}{3}\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}$$

$$\rho = \frac{1 + 1/3}{1 - 1/3} = 2$$

$$\Gamma(z) = |\Gamma_L|\mathrm{e}^{\mathrm{j}(\varphi_L - 2\beta z)} = \frac{1}{3}\mathrm{e}^{\mathrm{j}(\frac{\pi}{2} - 2\beta z)}$$

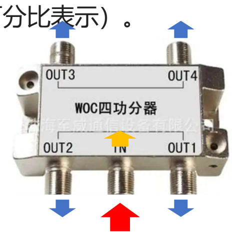
(3) $$\Gamma(z) = |\Gamma_L|\mathrm{e}^{\mathrm{j}(\varphi_L - 2\beta z_1'')} = \frac{1}{3}\mathrm{e}^{\mathrm{j}(\frac{\pi}{2} - 2\beta z_1'')} = -\frac{1}{3}$$

$$\frac{\pi}{2} - 2\beta z_1'' = -\pi \quad \rightarrow \quad z_1'' = \frac{3}{8}\lambda$$

$$Z(z_1'') = Z\left(\frac{3}{8}\lambda\right) = \frac{Z_0}{\rho} = \frac{50}{2} = 25\ (\Omega)$$

现有四路功率分配器（1入4出），设该功分器在2.5GHz-5.5GHz频率范围内其输入端的输入驻波比均小于等于1.5，插入损耗为0.7dB，设输入功率被平均地分配到各个输出端口，试计算（1）输入端的回波损耗（用分贝表示）；

（2）每个输出端口得到输出功率与输入端总输入功率的比值（用百分比表示）。



解：（1）输入端的回波损耗

$$L_r(z) = -20\lg|\Gamma_l| \quad (\mathrm{dB})$$

$$= -20\lg\frac{\rho-1}{\rho+1} \qquad = 13.98dB$$

（2）每个输出端口得到输出功率与输入端总输入功率的比值

$$L_r(z) = 10\lg\frac{P_{\mathrm{in}}}{P_r} \Rightarrow \frac{P_{\mathrm{in}}}{P_r} = 10^{L_r/10} \Rightarrow P_r = \frac{P_{\mathrm{in}}}{10^{L_r/10}} = \frac{P_{\mathrm{in}}}{10^{1.398}} \approx 0.04P_{\mathrm{in}}$$

$$\therefore P'_{\mathrm{in}} = P_{\mathrm{in}} - P_{\mathrm{r}} = 0.96P_{\mathrm{in}}$$

$$L_{\mathrm{i}} = 10\lg\frac{P_{in}}{P_t} = 10\lg\frac{P'_{\mathrm{in}}}{4P_{out}} \Rightarrow \frac{P'_{\mathrm{in}}}{4P_{out}} = 10^{L_i/10} = 10^{0.07} \approx 1.175$$

$$P_{out} = \frac{P'_{\mathrm{in}}}{4*1.175} = \frac{0.96P_{\mathrm{in}}}{4*1.175} \approx 20.4\%$$