Q2. $y = 5$

5-side dice

X : the first roll

Y : the second roll

Let $X \in \{1, 2, 3, 4, 5\}$

Let $Y \in \{1, 2, 3, 4, 5\}$

X	$P(X)$
1	0.2
2	0.2
3	0.2
4	0.2
5	0.2

$4 \cdot 0.2$

Y	$P(Y \mid FAIR)$
1	0.2
2	0.2
3	0.2
4	0.2
5	0.2

Y	$P(Y \mid \text{unfair})$
1	0.125
2	0.5
3	0.125
4	0.125
5	0.125

$4 \cdot 0.125$

$$P(X+Y>2 \mid X+Y=4) = \frac{P(X+Y=4) \cdot P(X+Y>2)}{P(X+Y=4)}$$

If X and Y are fair:

$(1,3) \to X+Y=4$, $P(X+Y=4) = 0.2 \cdot 0.2 = 0.04$

$(1,2), (1,3), (1,4), (1,5) \to X+Y>2$

$P(X+Y>2) = 0.2 \cdot 0.8 = 0.16$

$$\therefore P(X+Y>2 \mid X+Y=4) = \frac{0.04 \cdot 0.16}{0.04} = 0.16$$

If X are fair and Y are unfair.

$(2,2), (3,1) \to X+Y=4$, $P(X+Y=4) = (0.2 \cdot 0.5) + (0.2 \cdot 0.125)$

$= 0.1 + 0.025 = 0.125$

$(2,1), (2,2), (2,3), (2,4), (2,5)$

$(3,1), (3,2), (3,3), (3,4), (3,5)$

$\vdots$

$X+Y>2$

$$P(X+Y>2) = (4\cdot 0.125)\cdot(4\cdot 0.2) + (4\cdot 0.2\cdot 0.5)$$
$$= 0.5\cdot 0.8 + 0.4$$
$$= 0.8$$

$$\therefore P(X+Y>2 \mid X+Y=4) = \frac{0.125\cdot 0.8}{0.125} = 0.8$$

Q 2.3

fair 5 sided dice: $S=\{1, 2, 3, 4, 5\}$

a = even number: $\{2, 4\} = \frac{2}{5}$

b = show $\{5\} = \frac{1}{5}$

c = show $\{2\} = \frac{1}{5}$

$$P(a \text{ and } b) = P(a)\cdot P(b) = \frac{2}{5}\cdot\frac{1}{5} = \frac{2}{25}$$

$$P(a \text{ or } b) = P(a) + P(b) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

$$P(a|b) = \frac{P(a)\cdot P(b)}{P(b)} = \frac{\frac{2}{25}}{\frac{1}{5}} = \frac{2}{5}$$

$$P(b \text{ and } c) = P(b)\cdot P(c) = \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25}$$

Q 3.

covid patient $y=5$

For $\underline{2000}$, $80 \cdot 5 = 400$ positive

$P(\text{covid} \mid \text{positive}) = \frac{400}{2000} = 20\%$

For $\underline{1500}$ non covid patient, $20 \cdot 5 = 100$ positive

$P(\text{non-covid} \mid \text{positive}) = \frac{100}{1500} = 6\%$

population, $(5+5)/100 = 10\%$ have covid

$$P(\text{covid}) = \frac{5\cancel{00}}{35\cancel{00}} = \frac{5}{35} = \frac{1}{7}$$

$$P(\text{covid} \mid \text{patient}) = \frac{2000}{3500} = \frac{4}{7}$$

$$P(\text{non covid}) = 1 - \frac{4}{7} = \frac{3}{7}$$

$$P(\text{positive}) = \frac{4}{7} \cdot 0.2 + \frac{3}{7} \cdot 0.06$$

$$= 0.14$$

$\therefore P(\text{have covid, tesed positive})$

$$= \frac{\frac{1}{7} \cdot 0.2}{0.14} = 0.204 = 20.4\%$$

Q6.

$\because y = 5$

$\therefore$ 5 sided dice, $E[X]$

fair dice $S = \{1, 2, 3, 4, 5\}$

$$E(X) = \sum_{i=1}^{5} i \cdot P(X = i)$$

$$= 1 \cdot \frac{1}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{1}{5}$$

$$= 3$$