Lecture 7

ECS 122A

$$n[a \longrightarrow a_n] \begin{cases} n_1 \\ n_2 \\ n_3 \end{cases} \Big] \neq \Big([\quad] \Big) \quad n^2 \cdot n = O(n^3)$$

$$A[n,n] \times B[n,n] = C[n,n] \quad n^{\log_2 7}$$

$\log_2 8$

MAx-Subarray prumem

input: $A[1 \ldots n]$ pos/negative numbers

output: index i, j. sum x such that

$$\boxed{x = \sum_{k=i}^{j} A[k]} \quad x \text{ is max.}$$

DAY 0, 1, 2, 3, 4, 5, 6, 7

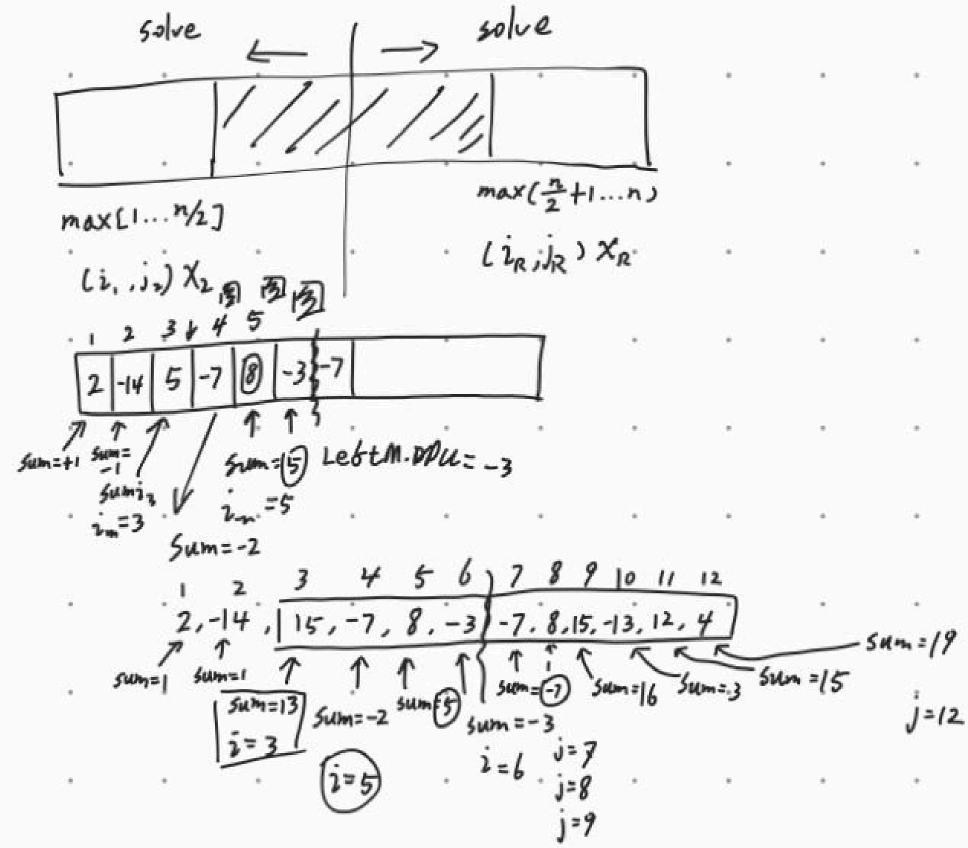
cost 10, 11, 7, 15, 11, 19, 5, +10

Array | 1, -4 | 8, -4, 8 | -14, 5 | max subarray

$(i, j) x$ [3.5] 12

$(i,j) \neq 0$ cur-max $= -\infty$

```
for(i=1 to n)
    for(j=i to n)
        x = sum(A[i...j])
        if(x > cur-max)
            cur-max = x
            (i,j) = i,j
```



$$\begin{array}{r} 19 \\ +13 \\ \hline 2 \end{array}$$

(3,12) = X middle(A[1...n])

(3,5),16 = maxsubarray(A[1...6])

(8,9),23 = max subarray(A[7...12])

<o, j, x> maxsubarry(A[1...n], i=1, j=n):

if(i=j)

Return(i, j) A[i]

else

m = i+j/2

T(n/2) => $\langle i_L, j_L, x_L \rangle$ = maxsubarray(A, i, m)

T(n/2) $\langle i_R, j_R, x_R \rangle$ = maxsubarray(A, m+1, j)

O(n) $\langle i_m, j_m, x_m \rangle$ = X middlesum(A, i, j)

Return the corresponding i, j for the max of $\underline{x_L}$, $\underline{x_R}$, $\underline{x_m}$

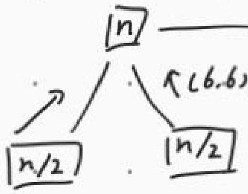
1	2	3	4	5	6	7	8
7	-8	12	-14	-12	8	-7	5

j=3 j=-1 j=9 sum=11 j=-12 -4 -11 -6

(3,3), 12 = maxsubarray([7, -8, 12, -14])

(6,6), 8 = maxsubarray([-12, 8, -7, 5])

(3,6) -6 = maxcrossing([7, -8, 12, -14, -12, 8, -7, 5])



work
→ n (3,6) -6 compared w/ 12, 8

$$\left.\begin{array}{r} 336 \\ +\ \underbrace{144}_{2n} \end{array}\right\} n^2+2n+2n$$

$|X| = n$

$|Y| = n$

$$\begin{array}{r} 9 \\ \times\ 9 \\ \hline 81 \end{array}$$

n ← 4

3

48

37

$(4\cdot 10+8)(3\cdot 10+7)$

$\boxed{4\cdot 3}\,10^2 + \boxed{4\cdot 7}\cdot 10 + \boxed{8\cdot 3}\cdot 10 + 8\cdot 7$

$12\cdot 10$

$1200+280+240+56$

$$X\cdot Y = (X\cdot 10^{n/2} + X_R)\cdot(Y_l\cdot 10^{n/2} + Y_R)$$

$$= (X_L Y_L)\cdot 10^n + (X_L Y_R)10^{n/2} + (X_R Y_L)10^{n/2} + (X_R Y_R)$$

$n=4$ $X = 58\,72$ $X_l = 58$ $X_R = 72$

$Y = 42\,85$ $Y_l = 42$ $Y_R = 85$

int multT(int X, int Y) → $|X| = |Y| = n$

n/2 In 1·1 Return y

$4\frac{n}{2} = O(n)$ ① create X_l, X_R, Y_l, Y_R.

$T(n/2)$ ② $A = \text{multT}(X_L, Y_L)$

$T(n/3)$ ③ $B = \text{MulT}(X_L, Y_R)$

$T(n/2)$ ④ $C = \text{multT}(X_R, Y_R)$ $J = \text{multT}(X_R + Y_R,\ X_L + Y_L)$

$T(n/2)$ ⑤ $D = \text{multT}(X_R, Y_R)$

$O(n)$ ⑥ $A = \text{shift}(A, n)$

O(n) ⑦ $F = B + C$

O(n) ⑧ $F = \text{shift}(F, n/2)$

O(n) ⑨ Return $\underline{A + F + D}$

}

$T(n) = 4T(n/2) + O(n)$

$A = 4 \quad B = 2 \quad f(n) = n$

$n^{\log_2 4}$

$$X \cdot Y = (X_L 10^{n/2} + X_R)(Y_L \cdot 10^{\frac{n}{2}} + Y_R)$$

$$= \underbrace{(X_L \cdot Y_L)}_{A} \cdot 10^n + \underbrace{X_L Y_R}_{B} \cdot 10^{n/2} + \underbrace{X_L \cdot Y_L}_{C} \cdot 10^{n/2} + \underbrace{X_R \cdot Y_R}_{D}$$

$$= \overbrace{\underbrace{X_L Y_L}_{A} + \underbrace{X_L Y_R}_{B} + \underbrace{X_R Y_L}_{+C} + \underbrace{X_R Y_R}_{D}}^{(X_L + X_R)\cdot(Y_L + Y_R) = J}$$

$48 \cdot 37$

$(4+8)\cdot(3+7)$

$B + C = J - A - D$

$48 \cdot 37$

A: $4 \cdot 3$ B, C: $4 \cdot 7 + 8 \cdot 3$ D: $8 \cdot 7$

10

$48 \cdot 37$

A: $4 \cdot 3$ $12 \cdot 10$ $8 \cdot 7$

$120 - 12 - 56$