Lecture 8

LAST Time

Descrete Pinoim variables

Parametric families

→ geometric: play until you win

→ binomial: How many times will you win out of n games

→ indicatore: $X \neq S$ An indicatore that win occure single game win/lose $X=1$ or $X=0$

→ uniform Descret: set uniform equally likely one game $X \in \{a, .a+1, .b\}$

→ poison: NULL

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property of variance

$\text{Var}[X] = E[X^2] - (EX)^2 = E((X-EX)^2)$

$\text{var}[aX] = a^2 \text{var}[X]$

$\text{var}[-1X] = (-1)^2 \text{var}[X] = \text{var}[X]$

$\text{var}[X+Y] = \text{var}[X] + \text{var}[Y] + 2\text{cov}[X,Y]$

$\text{cov}(X,Y) = E[XY] - E[X]E[Y]$ when X, Y are independent

$\text{var}(L_1 - L_2) \neq \text{var}(L_1) + \text{var}(L_2)$

$\text{var}(X + (-Y)) = \text{var}(X) + \text{var}(-Y) + 2\text{cv}(X,-Y))$

Bus Ride L_i = # of people driving on the bus when Bus Leaves stop i.

$P(L_1 = 1) = .4$

Bus stop 1

0 →

1

2

L_1	Prob
0	.5
1	.4
2	.1

$\underbrace{X = \text{sample}(0:2}_{\text{range}}, \underset{\text{samples}}{1}, \underbrace{c(.5,.4,.1)}_{\text{probility}})$

Bus stop

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if (x==1)
  count L1++;
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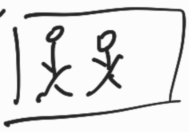
L_1	get on	L_2	$L_1=0$ and $L_2=..$
0	0	0	(.5)(.5)
0	1	1	(.4)(.5)
0	2	2	(.1)(.5)

u  get on L_2

2 SS 0 → 2, 1, 2 → 4 (.8)(.8), (.1)·(.1)

2 SF 0, 1, 2

2 FS 0, 1, 2

2 FF 0, 1, 2

$P(L_2=4 \mid L_1=2) \cdot P(L_1=2)$

Simulated ~~sample~~ data:

$E(x) = \$5$

5, 4, 7, 8

estimate variance

estimate expected value: $\frac{5+4+7+8}{4} = \frac{24}{4} = \$6$

$$\sqrt{\frac{(5-6)^2+(4-6)^2+(7-6)^2+(8-6)^2}{4}}$$

$$E((X-EX)^2) = \frac{5^2+4^2+7^2+8^2}{4} - (6)^2$$

Chapter 7.

support for x is

Descrete Random variables: any things countable

All outcome come from countable over.

N, Z, ~~[illegible]~~ # 5

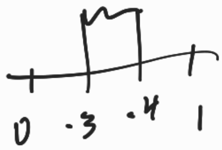
.01, .0001, .00001

	1	2	3	4
1	1	2		
2	2	1		
3				

continuous Rand variable:

exact hight, weight, speed,

time, location.



Bark thraing exact dart

.3 And .4

7.3.1 Def11

For any Rand variable. X the cummulative distribution Function (cdf) defined:

$$F_X(t) = P(X \le t) \quad -\infty \le t \le \infty$$

6-side dice

$$F_X(5) = P(X \le 5) = P(\overset{1/6}{\cancel{X=1}}) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= 5/6$$

geometric dist

$$F_X(2) = P(X \le 2) = P(X=1) + P(X=2)$$

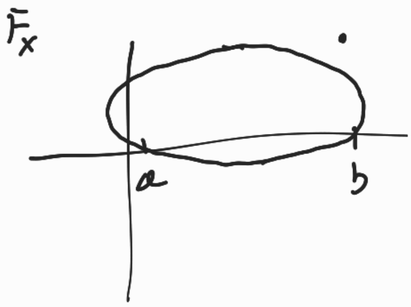
$$(.7) + (.3)(.7)$$

$\boxed{\text{Dice } 1\text{-}6}$ $F_X(8) = 1$ $F_X(-1) = 0$

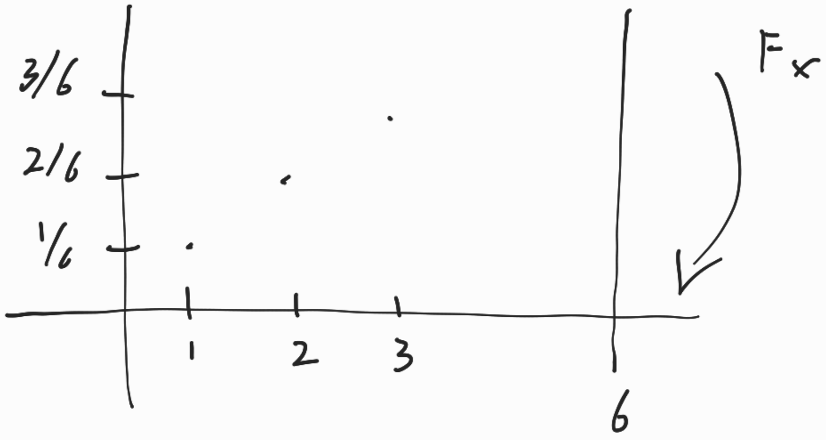
$F_X(8) =$

geometric

w/ $p = .5$

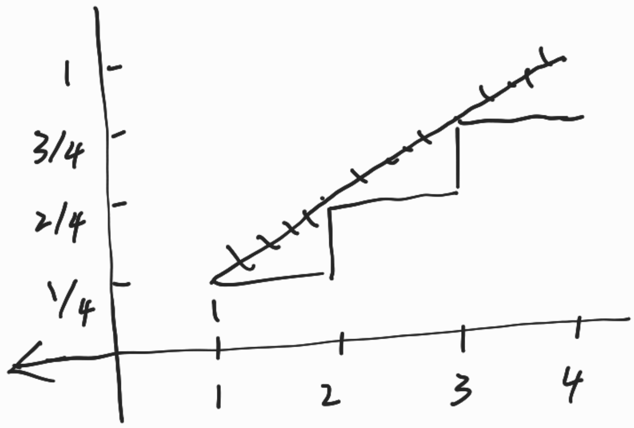


$F_X(\infty) = 1$ $\qquad$ $F_X(-\infty) = 0$



Dice

1-4



$P(X \le 1.999)$

Tossing coin Y is indicator on Heads.

$F_Y(.5) = P(Y \le .5) = P(Y=0) = 1/2$

1-6 sided dice

$F_X(1.3) = P(X=1)$

$= 1/6$

PMF $\simeq$ PDF $\rightarrow$ Prob. Density Function

$\Downarrow$

Prob function descrete

$f_W(t) = \frac{d\, F_W(t)}{dt} \quad -\infty \le t \le \infty$

$f_X(i) = P(X=i) \quad i \in$ Domain

uniform Descreate

$P_X(i) = \frac{1}{b-a+1}$

geometric

$P_X(i) = (1-p)^{i-1} p$

Property A 7.33

$P(a \le X \le b)$

$= F_X(b) - F_X(a)$

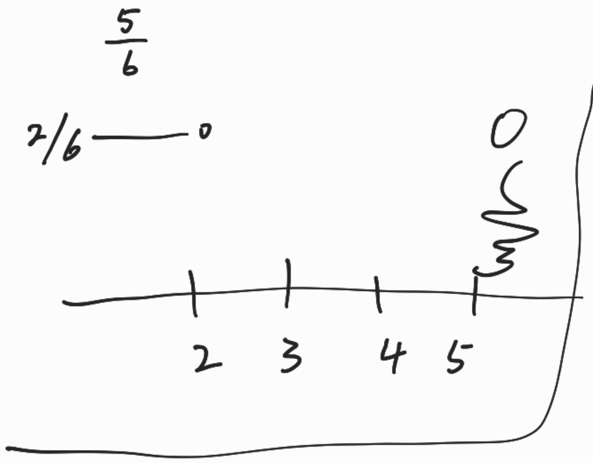
$= \int_a^b f_w(t)$

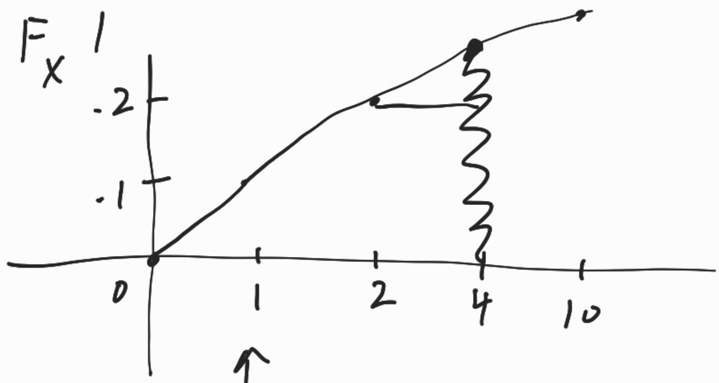
Dice game $X=2$

$P(1 < X \le 2) = \underline{F_X(2)} - \underline{F_X(1)}$

$= \frac{2}{6} - 1/6 = \frac{1}{6}$

$P(2 < X \le 5) = 5/6 - 2/6 = 3/6$



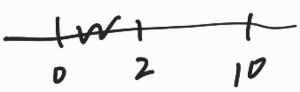


$\uparrow$

$P(X \le 1)$

$(0,0) \longrightarrow (10,1)$

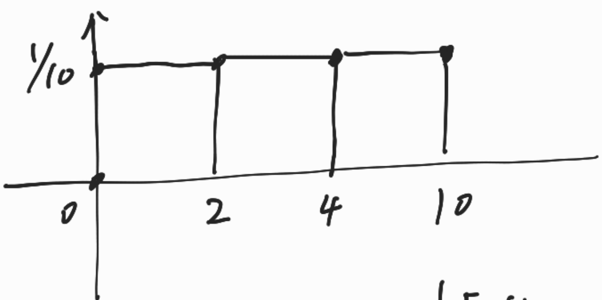
$\frac{\Delta y}{\Delta x} = \frac{1}{10}$



DART game

$P(2 < X \le 4) = F_X(4) - F_X(2)$

$\underline{f(x)}$



$\int_2^4 f_X(t) = F_X(4) - F_X(2)$

$F_X(t) = P(X \le t)$

$f_X(t) = \frac{d\, F_X(t)}{dt}$

$\underline{P(a < X \le b)} = \int_a^b f_X(t)$

in some game

$$f_X(t) = \frac{2t}{15} \text{ for } (1,4) \qquad \int at = \frac{at^2}{2}$$

$$f_X(t) = 0 \text{ other wise}$$

ⓐ $P(2 < X \leq 3) = \int_2^3 \frac{2t}{15} = \frac{2t^2}{2\cdot 15}\Big|_2^3 = \frac{9-4}{15} = \frac{5}{15}$

$3 <$

ⓑ $P(X \leq \boxed{4}) = 100\%$ $\rightarrow \int_{-\infty}^4 f_X(t) = \int_{-\infty}^1 0 + \int_1^4 \frac{2t}{15} = 1$

$-\infty \quad 1 \quad 4$

ⓒ $F_X(5) = P(X \leq 5) = P(-\infty < X \leq 5) = \int_{-\infty}^1 0 + \int_1^4 \frac{2t}{15} + \int_4^5 0$

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property A: $P(a < X \leq b) = \int_a^b f_X(t)\,dt$

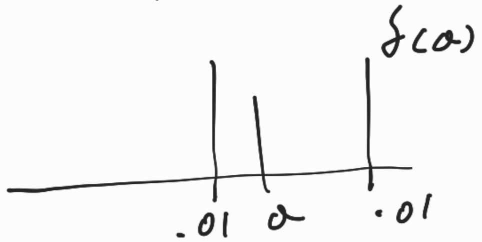
continous Rand Var: $P(a \leq X < b) = P(a < X \leq b)$

property B: $= P(a \leq X \leq b) = P(a < X < b)$

property C: $\int_{-\infty}^{\infty} f_X(t)\,dt = F_X(+\infty) - F_X(-\infty)$

$= 1$

prob $X = a$ $\delta = .01$ $P(-.01 \leq X \leq a + 0.1) \approx f(a) \cdot .02$



Descrete RAND Var continuous

$$E[X] = \sum_{c \in A} c \cdot p(X=a) \Rightarrow E(X] = \int_{t \in A} t \cdot f_X(t)$$

$$E[X^2] = \sum_{c \in A} c^2 p(X=c) \qquad E[X^2] = \int_{t \in A} t^2 f_X(t)$$

$$Var[X] = E(X^2) - E(X)^2$$
$$= E((X - (EX))^2$$

game: $f_X(t) = \frac{2t}{15}$ for $(1, 4)$ other wise 7.4 Book

calc: $E[X]: \int_1^4 t \cdot \frac{2t}{15} dt = \frac{2}{15} \cdot \frac{t^3}{3} \Big|_1^4 = \frac{2}{15} \left(\frac{4^3 \cdot 1^3}{3} \right)$

$= \int_1^4 \frac{2t^2}{15} dt \qquad = b$

calc: Var[X]

$$E[X^2] = \int_1^4 t^3 \frac{2\cancel{t}}{15} dt = \frac{2}{15} \frac{t^4}{4} \Big|_1^4 = \frac{2}{15} \frac{1}{4} (4^4 - 1^4) = a$$

$$var(X) = a - b^2$$