Q2.

$Boo(n) = O(n^y) = O(n^{17})$

$y = 17$

```
1. void Foo(int A[]) {
2.     let n = A.size();          -> O(1)
3.     for (i = 1 to n) {         } O(n^18)
5.             Boo(i);
6.     } //end for
7.     j = 1;
8.     while (j < n) {
9.         for(k=0; k<n; k=k+4)   n/4 · log_17(n)
           { print "hello";}
10.         j=j*y;                -> j = j x 17   every iteration
11.     } //end while
12. }
```

Line 2 : $O(1)$, It is a constant time operation.

Line 3 : $O(n)$, Looping from 1 to n . Increase iteration

Line 5 : $O(n^{17})$

Line 7 : $O(1)$, It is a const time operation

Line 8 → Line 11 : $O(n \log n)$

Line 3 → Line 5 : $O(n^{18})$, total runtime $O(n) \cdot O(n^{17}) = O(n^{18})$

Q3.

$y = 17$, base of log

$f(n) = 5n^{17} + n^2 + 5$ is $O(n^{17}/\log n)$

So, $f(n) = O(g(n))$

$\therefore$ $g(n) = n^{17}/\log n$

$\therefore$ $n_0 = 17$ because $\log_{17}(17) = 1$

for all $n \geq 17$, $\log_{17} n \geq 1$

$\therefore$ $5n^{17} + n^2 + 5 \leq c \cdot (n^{17}/\log_{17} n)$

$\therefore$ we can find $c = 6$

$\therefore$ for $c = 6$, $n_0 = 17$ we know that $5n^{17} + n^2 + 5 \leq c \cdot (n^{17}/\log_{17} n)$

and therefore $f(n) = O(g(n))$

$\therefore$ $f(n)$ is $O(n^{17}/\log n)$

Q4.

$y = 17$

prove $f(n) = \log(n)$ is $O(n^{17/1000})$ via the limit lemma

$\because \lim_{n \to \infty} \frac{f(n)}{g(n)} \leq c$

$\therefore g(n) = n^{17/1000}$

$\therefore \lim_{n \to \infty} \frac{\log n}{n^{17/1000}} = \lim_{n \to \infty} \frac{\frac{d}{dn}(\log n)}{\frac{d}{dn}(n^{17/1000})} = \lim_{n \to \infty} \frac{1000}{17} \cdot \frac{1}{n^{17/1000}}$

$= 0$

$\therefore$ It is equal to constant

$\therefore f(n) = O(g(n))$

Q5.

```
foo(int A[]){
  n=A.size;   -> O(1)
  while (n > 1) {
    n = n-1;
    count++;      } O(n)
  }
  n=A.size;   -> O(1)
  mergeSort(A); -> O(nlogn)  sorting algorithms linearithmic
  foo(A[1..n/y]);        foo(A[1..n/17]);  -> O(n/17)
  foo(A[n/y+1...2n/y]);  foo(A[n/17+1...2n/17]);  -> O(n/17)
}
```

$$T(n) = \begin{cases} O(1), & \text{if } n \le 1 \\ 2T(n/17) + O(n\log n) + O(n), & \text{if } n > 1 \end{cases}$$