

Homework 1

Problem 0. Warm-up

- a) How many bit strings of length seven either begin with two 0s or end with three 1s?
- b) How many bit strings of length 10 either begin with three 0s or end with two 0s?
- c.) In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
 - i.) the bride must be next to the groom?
 - ii.) the bride is not next to the groom?
- d.) A palindrome is a string whose reversal is identical to the string. How Many Bit Strings Of Length N Are Palindromes?

Problem 1.

You are playing a version of the roulette game, where the pockets are from 0 to 14 and even numbers are red and odd numbers are black (0 is green). You spin 3 times and add up the values you see. What is the probability that you get a total of 18 given on the first spin you spin a 5? What about a 8?

Solve by simulation and analytically.

Problem 2. (Matloff)

Consider the ALOHA example, but now assume that initially exactly one node is active. ($X_0=1$)

- a.) What is the probability that $X_2=2$? $P(X_2=2)$
- b.) What is the probability that $X_2=2$ given $X_1=1$. $P(X_2=2|X_1=1)$
- c.) What is the probability that $X_2=2$ given $X_1=0$. $P(X_2=2|X_1=0)$
- d.) Now assume that both nodes are active i.e. $X_0=2$.

Suppose it is known that $X_1 \neq X_2$. Find the probability that there were 0, 1 or 2 collisions during the next two epochs

Analytically and confirm via R simulation.

Problem 3.

Suppose we deal a 5-card hand from a regular 52-card deck. Which is larger, $P(\text{exactly 2 5's})$ or $P(\text{exactly 4 diamonds})$?

Before continuing, take a moment to guess which one is more likely

Please solve both analytically and use R to simulate and provide probabilities.

Problem 4.

You are running a camp of 50 students, including John and Jane.

3a.) What is the total possible ways you can arrange 2 focus groups of students one group being size 4 , and the other size 6.

3b.) What is the probability that John and Jane are not in the same group (so either not chosen or are chosen but not in the same group).

Problem 5.

You are a doctor. You have a medical test that given a person has cancer the test sees a high ($X=\text{true}$) level of protein X in 94% of patients.

Given the person is Healthy the test sees the protein X as high($X=\text{true}$) in 12% of patients.

Additionally it is known that 13% of the population is found to have cancer at the time of being screened.

A new patient with high levels of protein X wants to know how probable is it that he is cancer free.

Hint (Bayes Theorem)