Uniform continuous Parametric family

PDF $f_X(t)\,dt$ $\qquad P(a \le x \le b) = \int_a^b f_X(t)\,dt$

prob of cont var

parameters: $(a,b)$ $\qquad f_X(t) = \frac{1}{b-a}$

① what is the prob of $P(X \le k)$ ?

$\uparrow$

$a \le k \le b$

$P(X \le k) = F_X(k) = \int_a^k \frac{1}{b-a}$

② $E[X] = \frac{a+b}{2}$ $\qquad Var[X] = \frac{(b-a)^2}{12}$

③ what $k$ is in the top 15% or more or values ?

$\boxed{P(X \ge k) = 15\%} =$

$P(\overset{A}{X \ge k}) = \boxed{15\%}$ $\qquad 15\% = 1 - P(\underset{\bar{A}}{X \le k})$

$P(A) = 1 - P(\bar{A})$

$P(X \le k) = .85$ $\qquad A: X > k$

$\int_a^k \frac{1}{b-a} = .85$ $\qquad \bar{A}: X \le k$

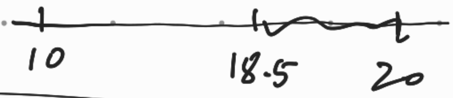
$a=10$ $b=20$

$\int_{10}^{k} \frac{1}{10} = .85$

$\frac{1}{10}\Big|_{10}^{k} = .85$ $\qquad \frac{k-10}{10} = \frac{8.5}{100}$

$\boxed{k=18.5}$

$P(X \geq 18.5) = 15\%$

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what is the bottom 10% of volus in uniform dist (18, 35)?

$\int_{a}^{k}$


$P(X \leq k) = .10$

$\int_{18}^{k} \frac{1}{35-18} = .10$

$\frac{(k-18)}{35-18} = .10$

$k = (.10)(35-18)+18$

MIDT. uniform$(.7, .85)$

① What is min score need to be in top 18% of class?

$P(X \le k) = .9$

$Z = \frac{\quad}{10}$

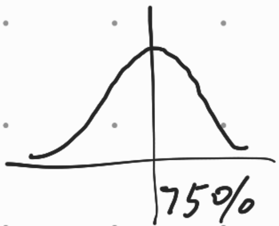
$\int_{.7}^{k} \frac{1}{.15}\, dt = .9$ $\qquad k - .7 = (.9)(.15)$

$\left.\frac{t}{.15}\right|_{.7}^{k} = .9$ $\qquad k - .7 = .135$

$k = \boxed{.835}$

$\frac{(k-.7)}{.15} = .9$

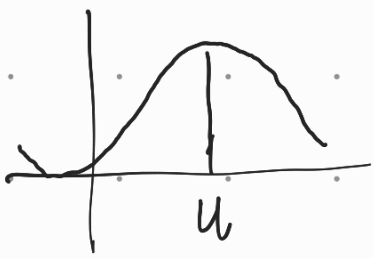
83.56 uniform test get me into top 10% of class.




$\mu = .75 \quad \sigma = 10$

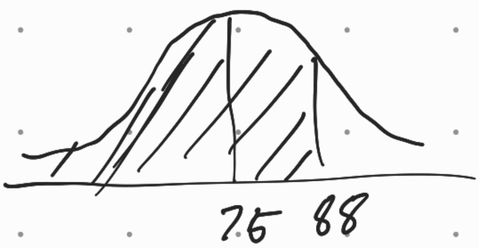
parametric family normal gausian

$[\mu, \sigma^2]$ $\qquad \mu = E[X]$

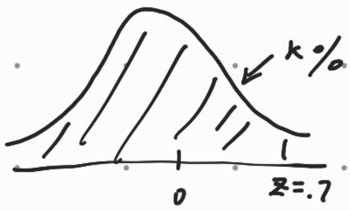



$\sigma = \sqrt{Var[X]}$ $\qquad \sigma^2 = Var[X]$

$X = 82$ ←



$\int_{\infty}^{.82}$ $\boxed{1.7}$




$\int_{-\infty}^{z}$

7.84

variable

$y = cx + d$ $\quad y \sim$

$\boxed{y \sim N(c\mu + d, c^2\sigma^2)}$

$E[X] = \mu$ $\quad E[y] =$

$var[x] = \sigma^2$ $\quad E[cx+d] = \boxed{c\mu + d}$

$cE[x] + d$

$\boxed{X \sim N(\mu, \sigma^2)}$ $\quad var[y] =$

$var[cx+d] = c^2 var[x]$

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$y = c^2x + d^2$ $\qquad x \sim N(a, b)$

$E[x]$ $\quad var(x)$

$E[y] = E[c^2x + d^2] = c^2E[x] + d^2 = c^2a + d^2$

$var[y] = var[c^2x + d^2]$

$(c^2)^2 var[x] = \boxed{c^4 b}$

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New Question

$F_y(t) = P(y \le t)$

$= P(cx + d \le t) = P(cx \le t - d)$

$= P(x \le \frac{t-d}{c})$

$\boxed{y = cx + d}$

prob. $P(y < t) = P(z < \frac{t-75}{10})$

$z \sim N(0,1)$

$x \sim N(\mu, \sigma^2)$

Derivation Es comp.

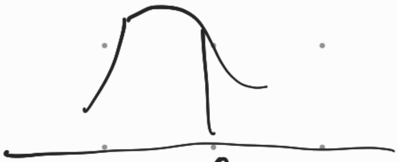
$$P(y \le t) \equiv P\left(Z \le \frac{t - E[y]}{\sqrt{var[y]}}\right)$$

④ Test $E[y] = 75$ $var[y] = 100$

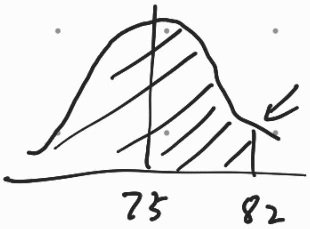
$\sigma = 10$

$$P(y \le t) = P(\hat{Z} < Z')$$

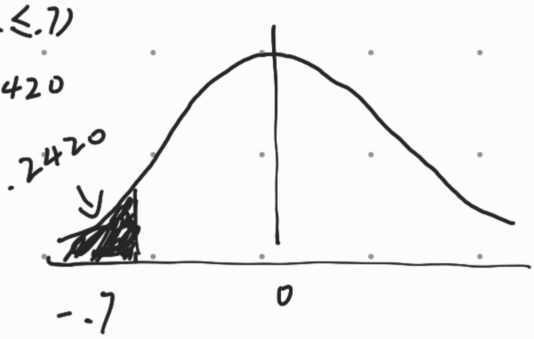
$$Z' = \frac{t - E[y]}{\sqrt{var[y]}} = \frac{t - \mu_y}{\sigma_y}$$




$$P(y \le 82) = P\left(Z \le \frac{82-75}{10}\right)$$
$$= P(Z \le .7)$$




$$P(y \le 82) = P\left(Z \le \frac{82-75}{10}\right)$$
$= P(Z \le .7)$
$= 1 - .2420$
$= \underline{.7580} =$





Read count on machine 600 files on minute.

Variance of 25 times.

intrussion detection software will flag if

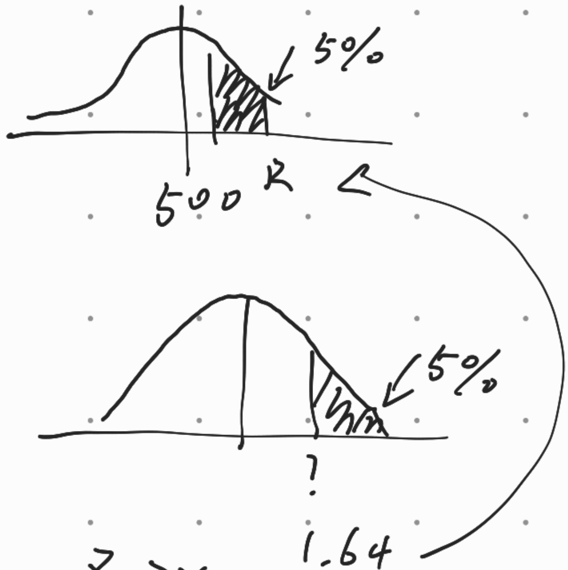
keerd to many files.

What precent of activity is reading 600 or more files.

How Likely are you to read 600 or more files

If we flag at top 5% access given $E[y] = 500$ $var[y] = 50^2$. what # access/reads or more causes a flag?

$P(y \geq R) = .05$




$$Z = 1.64 = \frac{R-500}{50}$$

$$R = (1.64)50 + 500$$

$Z \to X$.

Average Highway speed $\hat{z} = \frac{x-\mu}{\sigma}$

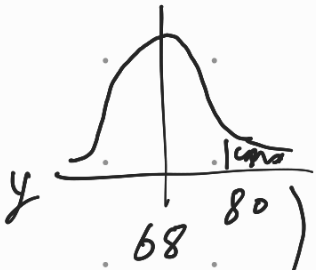
is → normally dist

68 mph var of 64 mph

$\mu$ = average

$\sigma = \sqrt{var[x]}$

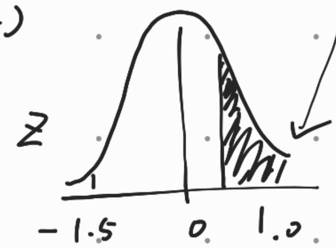
what is the prob that the car next to you is going more than 80 mph.




$P(y \geq 80) =$

$P(z \geq \frac{80-68}{\sqrt{64}}) = P(z \geq \frac{12}{8})$

$P(z \geq 1.5) = .0665$




7.25 CLT. Centrel limit theorem.

Roughly speaking sum of RAND variables of many components is approximately Normally Dist.

THM 14

suppose $X_1, X_2, X_3 \ldots X_n$ are ind rand. variable From same Distrib Distribution.

same exp

$y^n$

Let $T = X_1 + X_2 + \dots X_n \Rightarrow T \sim N(nE[X], n\,var(X))$

$E(T) = nE[X]$

$var(T) = n\,var[X]$

$var(nX_1]$

$var(X_1 + X_2 + X_3 \dots$

$= var(X_1) + var(X_2)$