Lecture 8

Capacitors:

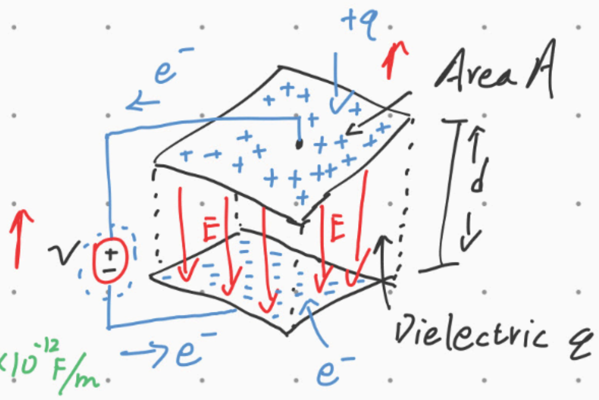
$q \propto v$

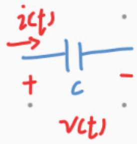
$\uparrow \uparrow q = \overset{\uparrow}{C} v$

$\hookrightarrow$ capacitance (F)

$\uparrow \uparrow C = \epsilon \dfrac{A \uparrow}{d \downarrow}$

$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$





$v(t) \overset{?}{=} c\, i(t)$

$v(t) = R i(t)$

$$\left.\begin{aligned} i(t) &= \frac{dq(t)}{dt} \\ q(t) &= c\, v(t) \end{aligned}\right\} \Rightarrow i(t) = \frac{d}{dt}[c\, \overbrace{v(t)}^{q(t)}]$$

$$\int i(t) = c \int \frac{dv(t)}{dt}$$

$$v(t) = f(i(t))$$

$$F(x) - F(x_0) = \int_{x_0}^{x} t(x')\,dx'$$

$$\int_{t_0}^{t} i(t)\,dt = c \int_{t_0}^{t} dv(t')$$

$$\int_{t_0}^{t} i(t)\,dt' = c[v(t) - v(t_0)]$$

$$\frac{1}{c}\int_{t_0}^{t} i(t')\,dt' = v(t) - v(t_0)$$

$v(t) = R i(t)$

$$\boxed{v(t) = \frac{1}{c}\int_{t_0}^{t} i(t')\,dt' + v(t_0)}$$

$$t_0 = 0: \boxed{v(t) = \frac{1}{c}\int_{0}^{t} i(t')\,dt'} + \cancel{v(0)}$$

$v(0) = 0$

No initial charge

$$p(t) = v(t) \cdot i(t) = v(t) \cdot c\frac{dv(t)}{dt}$$

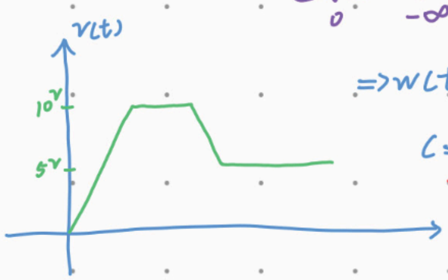
$$= \boxed{c\, v(t) \frac{dv(t)}{dt}}$$

$$p(t) = \frac{dw(t)}{dt} \Rightarrow dw(t) = p(t)\,dt = \overbrace{c\, v(t) \frac{dv(t)}{\cancel{dt}}}^{p(t)} \cdot \cancel{dt}$$

$$w(t) - \underbrace{w(-\infty)}_{0} = \int_{-\infty}^{t} dw(t') = \int_{-\infty}^{t'} c\, v(t')\,dv(t') = c \int_{-\infty}^{t} v(t')\,dv(t')$$

$$= \frac{1}{2}c[v^2(t) - \underbrace{v^2(-\infty)}_{0}]$$

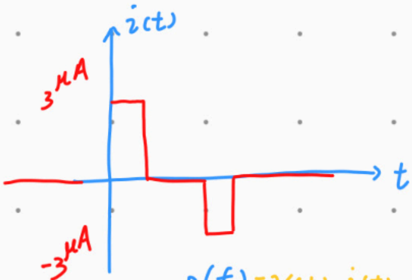
$$\Rightarrow w(t) = \frac{1}{2}c\, v^2(t)$$



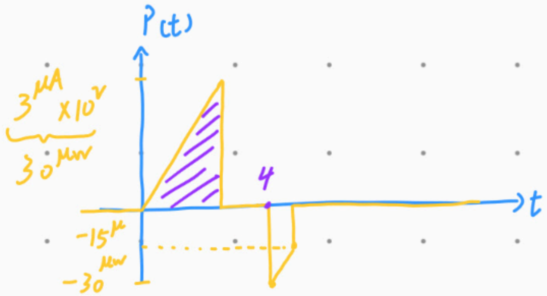
$C = 0.6\mu F$ $\quad i(t) = c\dfrac{dv(t)}{dt}$

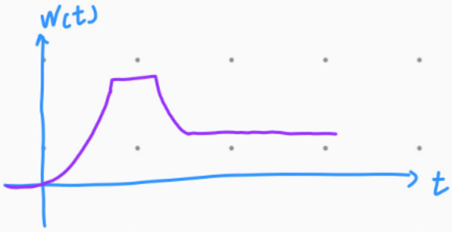
$$v(t) = \frac{1}{c}\int_{0}^{t_0} i(t')\,dt' + v(0)$$

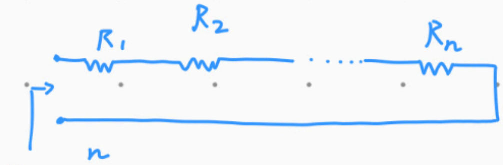
$$p(t) = c\, v(t) \frac{dv(t)}{dt} \qquad w(t) = \frac{1}{2}c\, v^2(t) = \int_{t_0}^{t} p(t')\,dt'$$



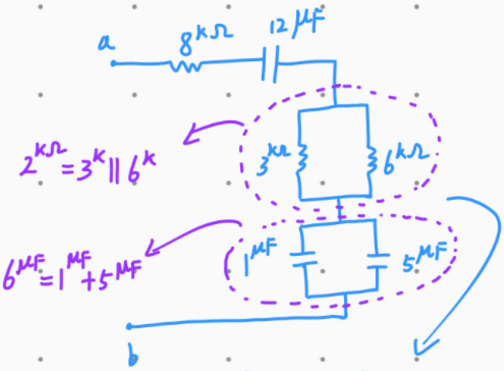
$p(t) = v(t) \cdot i(t)$

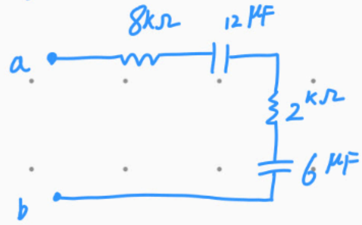


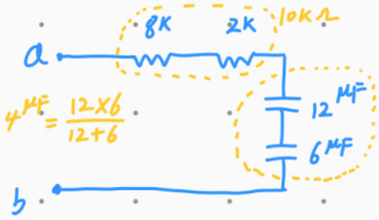




$R_{eq} = \sum_{i=1}^{n} R_i = R_1 + R_2 + \cdots + R_n$

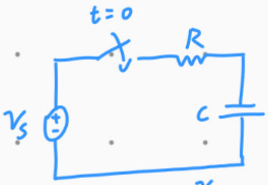






$4^{\mu F} = \frac{12 \times 6}{12+6}$

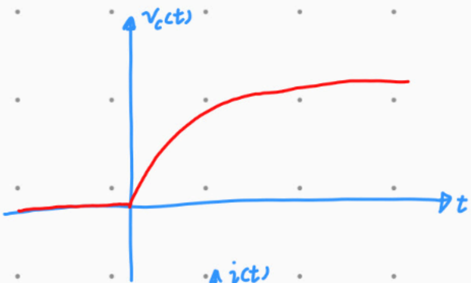
RC Circuit:

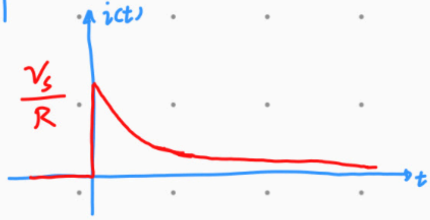


$v_c(0) = 0^V$

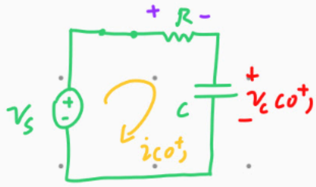
0^-: ϵ before $t=0$ $(\epsilon \to 0)$

0^+: ϵ after $t=0$ $(\epsilon \to 0)$





$t = 0^+$: $\quad v_R(0^+) = ?$



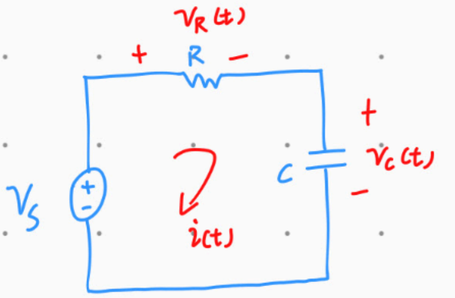
$v_c(0^+) = v_c(0^-) = 0^V$

KVL: $-v_s + v_R(0^+) + v_c(0^+) = 0$

$-v_s + v_R(0^+) + 0 = 0 \Rightarrow v_R(0^+) = v_s$

$i(0^+) = \frac{v_R(0^+)}{R} = \frac{v_s}{R} \Rightarrow q(t)\uparrow \Rightarrow \uparrow q(t) = C v_c(t) \uparrow$

$\frac{v_s - 0}{R}$



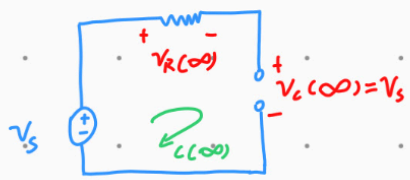
$\downarrow i_c(t) = \frac{v_R(t)}{R} = \frac{v_s - v_c(t)}{R} \uparrow$

$\downarrow i_c(t) = C\frac{dv_c(t)}{dt} \downarrow$

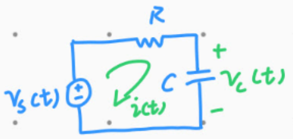
KVL: $v_s = v_R(t) + v_c(t)$

$v_R(t) = v_s - v_c(t)$

$t = \infty$:



$v_R(\infty) = 0 \Rightarrow i_R(\infty) = i_c(\infty) = 0$



KVL: $\begin{cases} R\,i(t) + v_c(t) = v_s(t) \\ i(t) = C\dfrac{dv_c(t)}{dt} \end{cases}$

$RC\frac{dv_c(t)}{dt} + v_c(t) = \overset{v_s}{\cancel{v_s(t)}}$ $\quad t>0$

$$\frac{dv_c(t)}{dt} + \overbrace{\frac{1}{RC}}^{a} v_c(t) = \overbrace{\frac{v_s}{R_c}}^{b}$$

$$\frac{dv_c(t)}{dt} + a\,v_c(t) = b$$

$$\frac{dv_c(t)}{dt} = -a(v_c(t) - b)$$

$$\int \frac{dv_c(t)}{v_c(t) - b/a} = \int -a\,dt \qquad \int \frac{dx}{x-k} = \ln(x-k)$$

$$\ln\left[v_c(t) - \frac{b}{a}\right]_{0^+}^{t} = -at\Big|_{0^+}^{t}$$

$$\ln\left[v_c(t) - \frac{b}{a}\right] - \ln\left[v_c(0^+) - \frac{b}{a}\right] = -a(t - 0^+)$$