Lecture - 3

Random sample

IDO. each person Ans is independent. every equally like to be choosen And can choosen twice /3

independent sample distribution

$\bar{X}$ $\frac{X_1+X_2+X_3+X_4}{4}$

Def: sample mean

$$\bar{X} = \frac{X_1+X_2+X_3+X_4+\dots+X_n}{n}$$

$\boxed{E[\bar{X}]}$

X: result of a 6 sided dice

$E[\bar{X}] = 3.5$

sum=0
for (sample size n=20)

X = sample (1:6)

Sum = X + sum

}

ave = sum /20

}

sum arg = Avg + SumAvg

print SumAvg / n.reps

30	$\bar{X}$
line1	5
line2	4
line3	6
line	1

$$E[\bar{X}] = E\left[\frac{X_1+X_2+X_3+\cdots X_n}{n}\right] = \frac{1}{n}E[X_1+X_2+X_3+\ldots X_n]$$

$$= \frac{1}{n}E[X_1]+E[X_2]+\cdots E[X_n]$$

$$= \frac{1}{n}E[X] = E[X]$$

$$E[\bar{X}] = E[X]$$

line1 3

line2 2

line 3 5

4

$$E[X] = \frac{\text{sum of lines of notebook}}{n \cdot \text{reps}}$$

$$\bar{X} = \frac{X_1+X_2+X_3+X_4+X_5+X_6}{6}$$

$$E[\bar{X}] = \frac{\text{sum of lines in } \bar{X}}{n \cdot \text{reps}}$$

$$E[\bar{X}] = E[X]$$

$$Var[\bar{X}] = \frac{1}{n} Var[X]$$

sample mean

$$\bar{X} = \frac{X_1+X_2+\cdots X_n}{n}$$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

sample variance $Var[X] = E[(X-E[X])^2]$

$\Downarrow$ —

$$S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

$$S^2 = \frac{\sum_{i=1}^{n} x^2}{n} - (\bar{x})^2$$

$Var(X)$

$E(X^2) - (E[X])^2$

Sample test score

70, 89, 92

Find $\bar{x}$, s^2

$$\bar{x} = \frac{70+89+92}{3} = \frac{251}{3} = B$$

$$s^2 = \frac{70^2+89^2+92^2}{3} - (B)^2$$

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

sample average vs popular

$E[X^2] = E[X]$

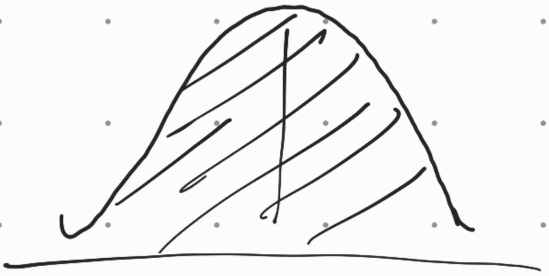
$E[s^2] = \frac{n-1}{n} Var[X]$

sample variance

$s^2 \approx Var[X]$

Chp13 you run a study sampling $n=3$ people get their test result 70. 89, 92 $\bar{x} = 83.6$

[ ] confidence where % of prob that the $E[X]$ lives in $E[X]$ [80, 85] with confidence of 95%



$\bar{X}$

$E[\bar{X}] = E[X]$

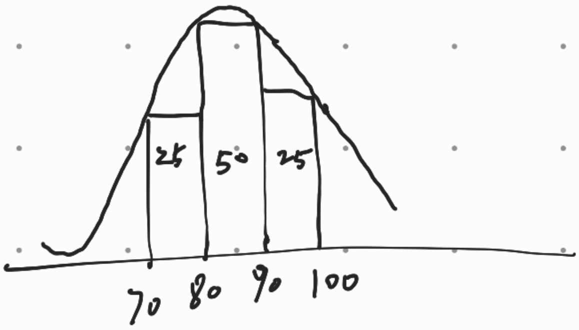
sample pop

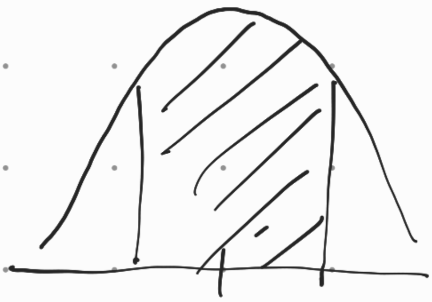
$\bar{X} \Rightarrow E[\bar{X}] = E[X]$

$var[\bar{X}] = \frac{1}{n} var(X)$

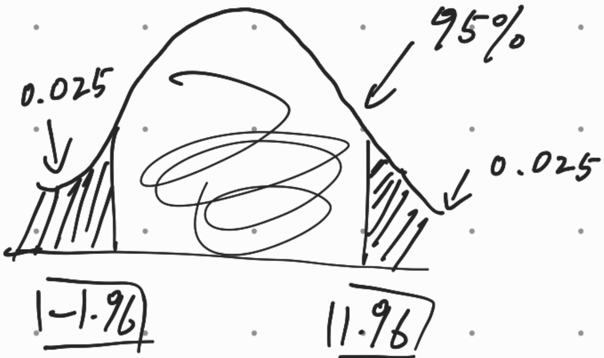
$\bar{X} \sim N(E[X], \frac{1}{n} var(X))$

E[X]





E[X]



$$P\left(-1.96 \le \frac{\bar{X} - E[X]}{\sqrt{\frac{1}{n} var(X)}} \le 1.96\right) = 95\%$$

population

$$P\left(-1.96 \le \frac{83.6 - \mu}{\sqrt{\frac{1}{n} S^2}} \le 1.96\right) = 95\%$$

$$-1.96\sqrt{\frac{1}{n}S^2} \le 83.6 - \mu \le 1.96\sqrt{\frac{1}{n}S^2}$$

$$-83.6 - 1.96\sqrt{\frac{1}{n}S^2} \le -\mu \le 1.96\sqrt{\frac{1}{n}S^2} - 83.6$$

$\mu \in [83.6 \pm 1.96\sqrt{\tfrac{1}{n}}\, S^2]$

$\mu \in [\bar{X} \pm 1.96\sqrt{\tfrac{1}{n}}\, S^2]$ with 95% confid.

population average

E(X)

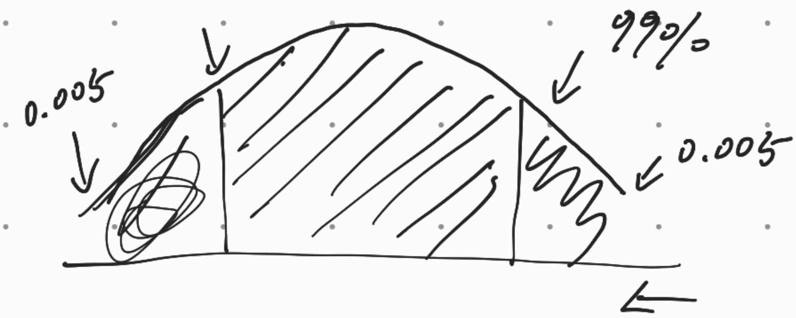
sample average

$\mu \in [83.6 \pm 1.96\sqrt{\tfrac{1}{3}\, 94.8}$

$\mu \in [83.6 \pm 11]$

$\mu \in [94.6, 72.4]$

Find 99% confidence interval for the test average.



$P(-0.005 \le \bar{X} - E[X] \le 0.005) = 99\%$