Lecture 9 prob. dist. function

$\underline{F_X(t)} = P(X \le t)$

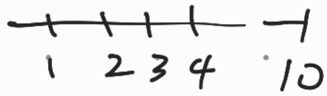
(continuous or descrete)

continuous rand var (None countable) $\equiv R$

Let X be cont rand var $\quad \underline{f_X(t)} = \frac{dF_X(t)}{dt}$

where it exist

$P(a < X \le b) = P(X \le b) - P(X \le a)$



$P(X \le b) - P(X < a)$

$P(3.2 < X \le 4.5)$

$\frac{2}{10}$

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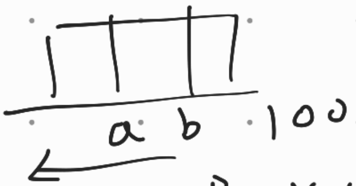


$1 - P(X < a \text{ or } X > b)$

$= F_X(b) - F_X(a)$

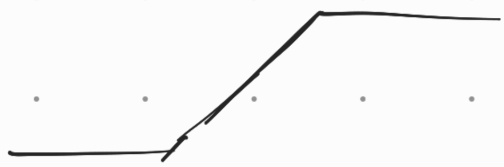
$= \int_a^b f_X(t)\,dt$

$P(a \le X \le b) = P(a < X \le b) = P(a \le X < b)$

$= P(a < X < b)$

$P(a < X \le b) = \int_a^b f_X(t)\,dt$

property c:



cont $\int_{-\infty}^{\infty} f_X(t)\,dt = 1 = F_X(+\infty) - F_X(-\infty)$

$1 - 0 = 1$

Desc $\sum_{x \in A} P(x) = 1$

Cont. what is prob $x \in [a,b]$? $\int_a^b f_X(t)\,dt$

Descrete $x \in [a,b] = P(a) + P(a+1) \dots P(b-1) + P(b)$

Decrete $E[X]: \sum_{c \in A} c P(c)$

Rand

$Var[X] = E[(X-EX)^2]$

$Var[X] = E[X^2] - (EX)^2$

cont.

prob of an.

$E[X] = \int_A t \, \overbrace{f_X(t)}^{\text{prob of an.}} dt$

$Var(X) = \int_A (X - EX^2) f_X(t) dt$

$E[X^2] = \int_A t^2 f_X(t) dt$

$f_X(t) = \frac{1}{10}$ $[10, 20]$

$= 0$ otherwise

$\int_{10}^{20} \boxed{\frac{1}{10}} dt$

$\sum_A c P(c) \to E(X) = \int_A t f_X(t) dt$ $\quad \frac{1}{10} t \Big|_{10}^{20} = \frac{(20-10)}{10}$

(9) $E[X] = \int_{10}^{20} t \cdot \frac{1}{10} dt = \frac{1}{10} \frac{t^2}{2} \Big|_{10}^{20} = \left(\frac{20^2 - 10^2}{20} \right) = \frac{300}{20} = 15$

(10) $Var[X] = E[X^2] - (EX)^2$

$(15)^2$

$Var[X] = 9 - (15)^2$

$\int_{10}^{20} t^2 \frac{1}{10} dt = \frac{1}{10} \frac{t^3}{3} \Big|_{10}^{20} = \frac{(20)^3 - 10^3}{30}$

Parametric families

Descrete

geom

bionomial

indicator $E[X] = p$

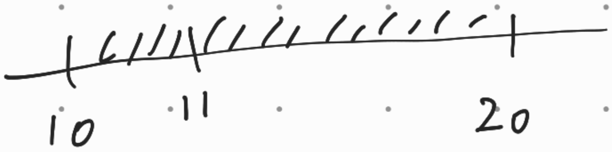
$var[X] = p(1-p)$

uniform $[a,b]$

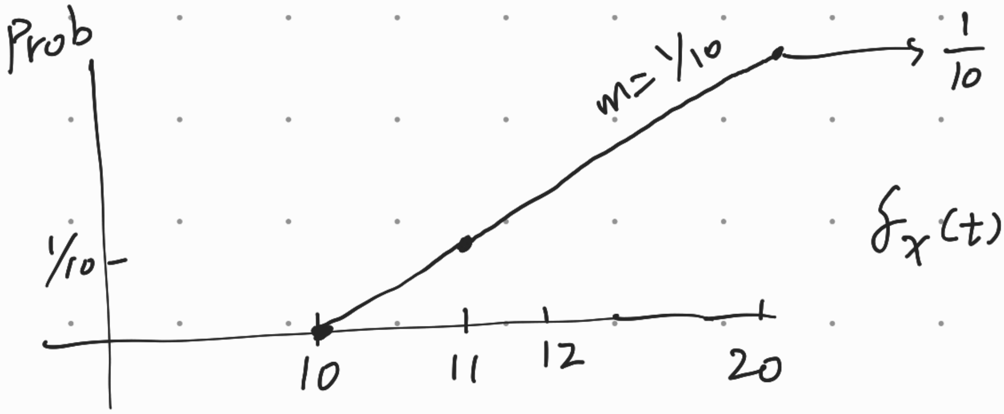
Decrete $E[X] = \frac{a+b}{2}$ $\frac{1}{12}\ \frac{2}{12}\ \frac{3}{12}\ \frac{4}{12}\ \frac{5}{12}$

PMF: $f_A \cdot P_X(t) = \frac{1}{b-a+1}$

$var[X]$



$F_X(t)$ $f_X(t)$



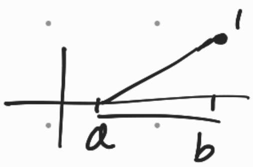
$f_X(t) = \frac{1}{10}$

$F_X(10)$ $F_X(11)$

uniform continuous parametric family

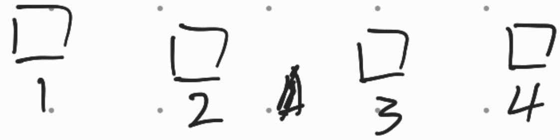
ALL outcome are equally likely in (a,b)

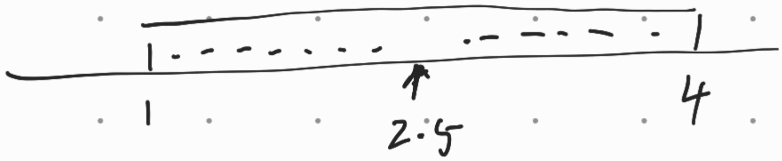
$f_X(t) = \frac{1}{b-a}$



PDF: $f_X(t) = \frac{1}{b-a}$ $E[X] = \frac{a+b}{2}$

$var[X] = \frac{1}{12}(b-a)^2$

Desc


Cont.


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uniformaly server respond speed is dist btw 1 and 7.5 milseconds

what should you expect your responce time to be to the server?

$$E[X] = \frac{7.5+1}{2} = \frac{8.5}{2} = 4.25\text{ms}.$$

② what is prob that you responce time is less than 1.5 ms or fast?

$[1, 7.5]$

PDF: $f_X \frac{1}{7.5-1} = \frac{1}{6.5}$

$$P(\underset{X}{time} \le 1.5) = \int_1^{1.5} \frac{1}{6.5}\,dt = \frac{t}{6.5}\Big|_1^{1.5} = \frac{(1.5-1)}{6.5}$$

(c) given the server responded fast.

what is the prob that it took more than 3 ms? $P(X \geq 3 | X \leq 1.5) = 0$

(d) given the server responded fast

what is the prob that it took more than 1.2 ms?

$$P(X \geq 1.2 | X \leq 1.5) = \frac{P(X \geq 1.2 \;\&\&\; X \leq 1.5)}{P(X \leq 1.5)}$$

$$F_X(1.5) - F_X(1.2) = \frac{\int_{1.2}^{1.5} \frac{1}{6.5} dt}{k}$$

The weather is uniformally dist 70F AND 100F in summer

PDF
$f(x) = \frac{1}{30}$

$E(X) = \frac{100+30}{2}$

(a) prob its more than 80 deg. today?

(b) what should I expect the temp to be to morrow? 85F

Lets say Hot is 90F or more

© what is the prob that its 81F on HOT DAY.

$P(X \leq 81 \mid X \geq 90) = 0$

(c.2) More than 95 F on HOT DAY?

$$P(X \geq 95 \mid X \geq 90) = \frac{P(X \geq 95 \,\&\&\, P \geq 90)}{P(X \geq 90)} = \frac{P(X \geq 95)}{P(X \geq 90)} = \frac{\int_{95}^{100} \frac{1}{30}\,dt}{\int_{90}^{10} \frac{1}{30}\,dt}$$

Normal / gausian

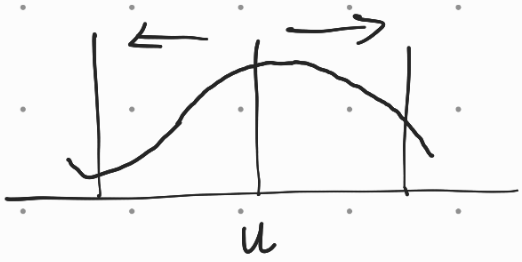
$\mu = E[X]$

$\sigma = \sqrt{var[X]}$

$$\text{PDF: } f_X(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-0.5\left(\frac{t-u}{\sigma}\right)^2}$$

$-\infty \leq t \leq \infty$

TABle

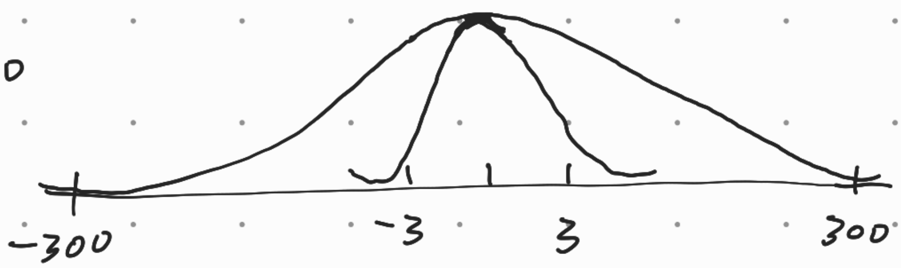


$\int_{-\infty}^{-1} f_X(t)$

$\int_{-\infty}^{\infty} f_X(t)$

d1: $\sigma = 1$ $\mu = 0$

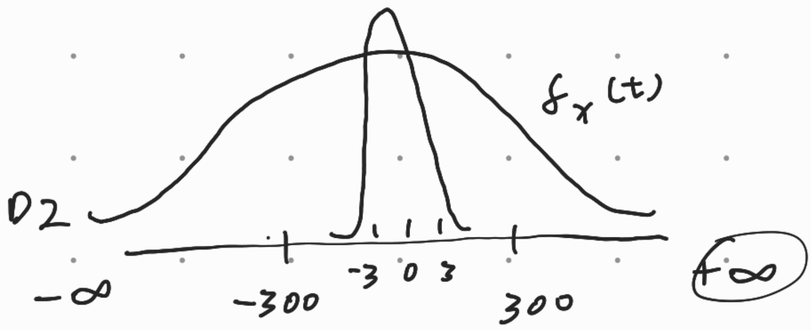
$\sigma = 100$ $\mu = 0$



$\sigma = 1$

$\mu = 0$

$\sigma = 100$



X is NORM. Dist $\mu = 3$ $\text{var}(X) = 4$

$\sigma = 2$

prob. $X \leq 2$ $\quad P(X \leq 2) = \underbrace{\int_{-\infty}^{2} \frac{1}{\sqrt{2\pi 2}} e^{-0.5\left(\frac{t-3}{2}\right)^2} dt}_{f_x(t)}$

uniform $(1, 100)$

$P(X \leq 2) = \int_1^2 f(x) dt$ $\quad \frac{1}{100}$

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rotum

$X \sim N(\underbrace{\mu, \sigma^2}_{\text{parameters}})$ $\quad$ X is normally dist w/ μ, and σ^2

$x \sim$ uniform (a, b)

STANDERED NORMAL Dist

$Z \sim N(\underset{\mu}{0}, \underset{\sigma^2}{1})$

Cont. RAND

$f_x(t) =$ prob of variable

$E[X] = \int t \underbrace{f_x(t) dt}_{\text{prob}}$

$P(a \leq X \leq b)$

① uniform cont. pramatrized fam $\int_a^b f_x dt$

② normal $\longrightarrow$ use TALE

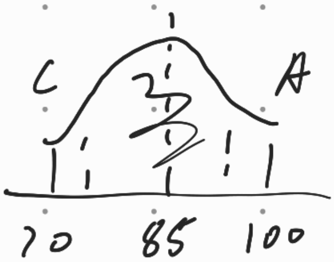
③ exponential $\rightarrow$ intergrate

general Normal $\longrightarrow$ standared normal

$X \sim N(-\mu, \sigma^2)$ $\qquad Z \sim N(0,1)$

$\mu = E[X]$

$\sigma =$ st. dev



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win 2 Heads max games 5 layed

$X = 2 \rightarrow HH \longrightarrow (1/2)^2$

$X = 3 \rightarrow THH \longrightarrow (1/2)^2$

$X = 4 \rightarrow TTHH \longrightarrow 2(\frac{1}{2})^4$

HTHH

$X = 5 \longrightarrow 1 - (\frac{1}{2})^2 - (\frac{1}{2})^3 - (\frac{1}{2})^3$