Review session at 9pm - 10pm

2 at 6pm - 7pm

Lecture 6 ECS 132

→ LAST Time

① $E(X) = \sum_{c \in A} c\,P(X=c)$

② $E(X+Y) = E(X) + E(Y)$

③ $E(aX) = aE(X)$

④ $E(aX+bY) = aE(X) + bE(Y)$

⑤ $E(b) = b$

⑥ if U AND V are independent
$E(UV) = E(U) \cdot E(V)$

⑦ $E(g(X)) = \sum_{c \in A} g(c) \cdot P(X=c) \Rightarrow E(X^2) = \sum_{c \in A} c^2 P(X=c)$

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variance

⑧ $Var[X] = E((X - \underline{EX})^2)$

⑨ $Var[X] = E[X^2] - [EX]^2$

⑩ $Var[cX] = c^2\, Var[X]$ $c = 7$

⑪ $Var[X+c] = Var[X]$

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⑫ $cov(X,Y) = E((X-EX)(Y-EX)) = \underline{E[XY] - E[X]E[Y]}$

IF X, Y are independence $E[XY] = E[X] \cdot E[Y]$
$= E[X]E[Y] - E(X)E(Y) = 0$

⑬ $Var(X+Y) = Var[X] + Var[Y] + 2cov(X,Y)$

given 2 Dice X is 3 sided Y is 4 sided

Calculate $\underline{\text{var}(X+Y)}$

$$\text{var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + \cancel{\text{cov}(X,Y)}^{\,0}$$

$$\text{var}[X] = E[X^2] - (EX)^2 \qquad E(X^2) \neq (EX)^2$$

$$\text{Var}[Y] = E[Y^2] - (EY)^2 \qquad c \to \text{outcome}$$

$$\text{var}(X) + \text{var}(Y) = E[X^2] - (EX)^2 + \underline{E[Y^2] - (EY)^2}$$

$\{1, 2, 3\}$ $\{1, 2, 3, 4\}$

$$E(X^2) = \sum c^2 P(X=c) = 1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{3} + 3^2 \cdot \frac{1}{3} = \frac{1}{3} + \frac{4}{3} + \frac{9}{3} = \frac{14}{3}$$

$c \in A$

$$E(Y^2) = \sum c^2 P(Y=c) = 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4} + 4^2 \cdot \frac{1}{4}$$

$$= \frac{1}{4} + 1 + \frac{9}{4} + \frac{16}{4} = \frac{30}{4} = \frac{15}{2}$$

$$E(X) = 1\frac{1}{3} + 2\frac{1}{3} + 3\frac{1}{3} = \frac{1}{3}(1+2+3) = 2$$

$$\therefore \text{Var}(X) \Rightarrow E(X^2) - (EX)^2 = \frac{14}{3} - (2)^2$$

$$\text{var}(Y) = \frac{30}{4} - \left(\frac{10}{4}\right)^2 = 6$$

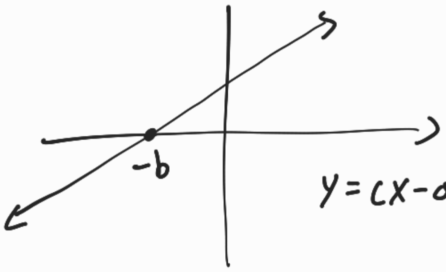
$$E[Y] = 1\frac{1}{4} + 2\frac{1}{4} + 3\frac{1}{4} + 4\frac{1}{4} = \frac{10}{4}$$

$$E[Y^2] = \frac{30}{4}$$

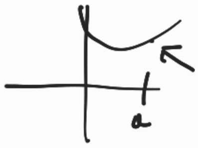
$$\text{Var}(X+Y) = \text{var}(X) + \text{var}(Y)$$

$y = \underline{m}\,(x + \underline{b})$

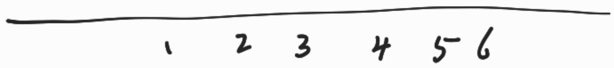
parameter



$y = (x-a)^2 + b$

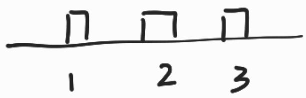


Dice 1 to 6

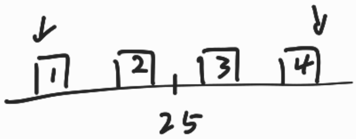


$E[X] = \frac{1+6}{2}$

1 to 3



$E[X] = \frac{1+3}{2} = 2$



$E(X) = \frac{1+4}{2} = 2.5$

$E[X] = \frac{a+b}{2}$

3.13 Distribution

Def 6: Let X be descret RAND var. Then the distribution of X is simply the support // set of all possible outcomes together with its associated prob

distribution of $X = \{(1, \frac{1}{4}), (2, \frac{1}{4}), (3, \frac{1}{4}), (4, \frac{1}{4})\}$

X is 4 sided dice

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{if } k \in \{0, 2\} \\ \frac{1}{2} & \text{if } k = 1 \\ 0 & \text{else} \end{cases}$$

Poss 2 coins Let X = # of Heads seen

distribution of $X = \{(0, \frac{1}{4}), (1, \frac{1}{2}), (2, \frac{1}{4})\}$

$\underline{g(x)} = \underline{x^2}$

0	1	2
↓	↓	↓
TT	TH HT	HH

Def 7 The probability mass function (pmf) of descrete RAND var X is denoted

$P_X(k) = P(X = k)$

X is 4 sided dice

$\boxed{P_X(k) = P(X=k)}$

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{if } k \in \{1, 2, 3, 4\} \\ 0 & \text{else} \end{cases}$$

$P_X(3) =$

$P_X(10) =$

X is 7 sided Dice

$$P_X(k) = \begin{cases} \frac{1}{7} & \text{if } k \in \{1, 2, 3, 4, 5, 6, 7\} \\ 0 & \text{else} \end{cases}$$

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given you are playing a game where winning 4, 5, 6, 7, 8 is equally likely. X = win amount what is the pmf for X?

$$P_X(k) = \begin{cases} \frac{1}{5} & \text{if } k \in \{4, 5, 6, 7, 8\} \\ 0 & \text{else} \end{cases}$$

4.2 game where you play until you win

prob of winning .1 (each day ind. of one another)

X = DAY you win For the first time

$X \in \{1, 2, 3, 4, 5, 6 \cdots, \infty\}$

$P(X = \overset{k}{1}) = .1$ $\qquad P(A \cdot B) = P(A)P(B)$

$P(X=2) = (.9)(.1)$

$P(X=3) = P(\text{Lossing first 2})\, P(\text{3rd win})$

$(.9)(.9)(.1) = (.9)^2(.1)$

$P(X=4) = (.9)(.9)(.9)(.1) = (.9)^3(.1)$

PMF X:

$$P_X(k) = \begin{cases} (.9)^{k-1} \cdot (.1) & k = \{1, 2, 3, \ldots \infty\} \\ 0 & \text{else} \end{cases}$$

$$P(X = 1000) = (.9)^{999}(.1)$$

One simple trial/exp. prob of winning is p

$$P(X=3) = (1-p)^2(p)$$

$$P_X(k) = (1-p)^{k-1} \cdot p$$

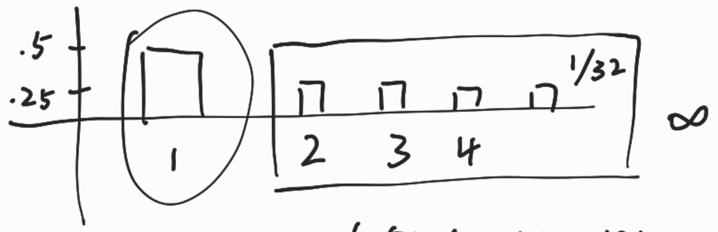
gemoetric parametric family

parameter is $\boxed{p}$ prob of winning on single trial

PMF : $P_X(k) = (1-p)^{k-1} \cdot p \quad k \in \{1, 2, 3, 4 \ldots\ldots \infty\}$

else 0

prob of win $.5$



$$E(X) = \frac{1}{p}$$

$$E(X) = \frac{1}{1/2} = 2$$

$$(.5)(.5)(.5) = .125$$

$$\boxed{E[X] = \frac{1}{p}}$$

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win $\boxed{p=1}$

$$E[X] = 1(.1) + 2(.9)(.1) + 3(.9)^2(.1) + 4(.9)^3(.1)$$

$$E[X] = \frac{1}{.1} = \frac{1}{1/10} = 10$$

$$\text{var}[X] = \frac{1-p}{p^2}$$

given prob of class being online is .15

what is the expected day I will