Lecture 7

LAST Time

$E[X+Y] = E[X] + E[Y]$

game exp. value of the game $E[X] = 4$

play it 10 times. Expected win at then end ($)?

$E[X_1 + X_2 + X_3 + \cdots + X_{10}] = \$40 = 10(4)$

$E[10X] = 10E[X] = 10 \cdot 4$

parametric families AND PMF

PMF $\underline{P_X(k)}$ defined as $P(X=k)$

$P_X(4) = \frac{1}{10}$ Dice $\overset{10 \text{ icon}}{\text{game}}$ $x = \{1, \ldots, 10\}$

<u>gemetric</u>: keep trying until you win => then stop game.

PMF $P_X(k) = \underline{P(X=k)}$ each try is independent

prob that the <u>first time</u> I win is on the kth try.

parameter:

win probability:

$\underline{P} = .25$ $\quad P_X(\underline{5}) = \underbrace{(1-P)(1-P)(1-P)(1-P)}_{\text{Loss . 4 time}} \underbrace{P}_{\text{5 time win.}}$

$= (1-P)^4 P$

$$\boxed{P_X(k) = (1-p)^{k-1} p}$$

$E[X] = \frac{1}{p}$

$var[X] = \frac{1-p}{p^2}$

Binomial: PLAY only n games k is the # of times you win out of tries. p prob of winning.

EACH game is independent.

$$P_{\underline{X}}(\underline{k}) = \binom{n}{k}(p)^k(1-p)^{n-k} \qquad \binom{n}{k} = {}_nC_k$$

$E[X] = np$

$\rightarrow \underbrace{P(X=k)}$

$Var[X] = np(1-p)$

expected X is # wins our of n games

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parametric Family: indicated / bernoulli time

A single time / single game where you win or loose parameter is p prob. of win

indicated X is 1 if you win. else 0 if you loose

$$PMF: P_X(k) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \\ 0 & \text{else} \end{cases} \qquad \underline{P(X=1)} =$$

$$E[X] = 0\cancel{P(X=0)} + 1 \cdot P(X=1) = \boxed{P} \qquad \boxed{E[X]=P}$$

~~++++~~ +

0 P

.25

Simulating $P(X=1)$ vs $\underline{E[X]}$

```
count = 0;
for (n reps) {
    if (runif <= p)
        count++;
}
print P(X=1) is count/n.reps
```

```
for (n reps) {
    if (Runif <= p) win?
        sum ++ {
}
print
```

$$Var[X] = E[X^2] - E[X]^2$$

$$E[X^2] = 0^2 P(X=0) + 1^2 \cdot P(X=1) = P$$

$$var(X) = p - p^2 = p(1-p)$$

INidcated : $P_X(k) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \end{cases}$

$E[X] = p \qquad var[X] = p(1-p)$

Binomial I play n games where I win or loose

$$\underbrace{X}_{\#\text{ of wins}} = \underbrace{X_1}_{\text{win}=1} + X_2 + X_3 + \dots X_7 \dots + X_n$$

$$E[X] = E[X_1 + X_2 + \dots + X_n]$$

$$= \underbrace{E[X_1]}_{p} + \underbrace{E[X_2]}_{p} + \dots \underbrace{E[X_n]}_{p}$$

$$= np$$

$$Var[X] = Var[X_1 + X_2 + \dots X_n]$$

FAMilies	game	Ex	Var[X]	X supp
indicated	single win or loose X=1 is win	$E[X]=p$	$var[X] = p(1-p)$	$X=1$ $X=0$
binomial	PLAY N game sum # of wins	$E[X]=np$	$var[X] = np(1-p)$	$X=\{0,1,n\}$

PMF: $P_X(k)$ $n-k = \binom{n}{k}(1-p)^{n-k}p^k$	$X = \#$ of wins			
gametric $P_X(k) = (1-p)^{k-1}p$ $k = \{1 \ldots \infty\}$	PLAY until First win X = Tky you win on	$E[X] = \frac{1}{p}$	$var[X] = \frac{1-p}{p^2}$	$X = \{1 \ldots \infty\}$

Ex1 prob of class cancelled is 15%.

I star college what day should I expect CLAss to be concelled for the first time?

$p = .15 \quad E[x] = \frac{1}{.15} = \frac{100}{15} =$

Ex. This quater HAs 40 Lecture DAys? How many of them do I expect will be cancelled $p = .15 \quad E[X] = 40 \cdot (.15)$

What is the prob that in this quater no lecture is cancelled? $\binom{40}{0}(.85)^{40}(.15)^0 = (.85)^{40}$

what is prob that only 2 out of 40 are cancelled? $\binom{40}{2}(15)^2(.85)$

uniform descrete DET $P_X(k) = \begin{cases} \frac{1}{b-a+1} & [a, \ldots b] \end{cases}$	single but outcomes are $[a, a+1, a+2 \ldots b-1, b]$ EZ each outcome eqally likely.	$E[X] = \frac{a+b}{2}$	$var[X] = \frac{(b-a)(ba+2)}{12}$
poison	NOT NAtural Description		

$$\begin{array}{ccc} a & & b \\ 12 & 13 & \cdot\ 14 \end{array}$$

Dice$\{1 \to 6\}$ $\quad E[X] = \frac{1+6}{2} = \frac{7}{2} = 3.5$ $\quad b-a+1$

ROLL Dice 7 sided if you see a 4 you win \$10.

① what is expected value, I winec win?

IF I play this game, How much should I expect to win? 0 OR \$10

$$E[\$] = 0 \cdot p(0) + 10(\tfrac{1}{7}) = \frac{10}{7}$$

② Now you get paid for every dot you see \$1. How much Do I expect to win. (Diff from I)

$$\begin{array}{cc} a & b \\ 14 & 18 \end{array} \quad \text{ontime}: 5$$

prob $\frac{1}{6}$

$$\frac{1}{b-a+1} = \frac{1}{18+14+1} = \frac{1}{4+1} = \frac{1}{5}$$

$$p(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} \quad k = 0, 1, 2 \cdots \infty$$

$$E[X] = \lambda \qquad \mathrm{var}(X) = \lambda$$

Bug count is poison dist w/the expected value of 7. what is the prob of seeing 5 Bugs?

$$p(X=5) = \frac{e^{-7}\lambda^5}{5!} = \frac{e^{-7}(7)^5}{5!}$$

game stage1 $X \in \{4, 6\}$ $P(X=4) = 13\%$

stage 2 is $x=6$

$y \in \{1, 7\}$ $P(y=1) = 15\%$

$E[X \cdot Y]$

x	prob
4	.13
6	.87

$Y \mid X=4$

y	prob$(Y \mid X=4)$	$P(Y \mid X=6)$
0	.1	0
7	.9	.85
1	0	.15

$X \cdot Y$	X	Y	prob
28	4	7	(.13)(.9)
0	4	0	(.13)(.1)
42	6	7	(.87)(.85)
6	6	1	(.87)(.15)

$y \{7, 0, 7, 1\}$

$P(Y=7) = P(x=6 \text{ and } y=7) + P(X=4 \text{ and } y=7)$

$\underline{P(Y=7 \mid X=6)} \cdot P(X=6) + P(Y=7 \mid X=4) \cdot P(X=4)$

$= (.85)(.87) + (.13)(.9)$

$P(Y=1) = P(X=6 \text{ and } y=1)$

$(.15)(.87)$

$E[Y] = 0 \cdot P(Y=0) + 1 \cdot P(Y=1) + 7P(Y=7)$

$= (.15)(.87 + 7b)$ from part 1

Simulate to confirm $P(Y=1)$

```
// stage1
if (Runif(1) ≤ .13) X=4
else , X=6
// stage 2
  if (X==4) {

     if (Runif(Y) ≤ .1) Y=0
     else  Y=7

  }
  else
     if (Runif() ≤ .85) Y=7
     else  Y=1

   if (Y==1) count ++;

}

  print 'p (Y=1)' + count/n reps ;
```

3 nodes only 1 message can get mous

AND 2 Nodes are active $P(X_1=3)$