2. 用列主元消去法解方程组

$$\begin{pmatrix} -3 & 2 & 6 \\ 10 & -7 & 0 \\ 5 & -1 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 & 6 & 4 \\ 10 & -7 & 0 & 7 \\ 5 & -1 & 5 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 & -7 & 0 & 7 \\ 5 & -1 & 5 & 6 \\ -3 & 2 & 6 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 & -7 & 0 & 7 \\ 0 & \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & -\frac{1}{10} & 6 & \frac{61}{10} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 10 & -7 & 0 & 7 \\ 0 & \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & 0 & \frac{31}{5} & \frac{31}{5} \end{pmatrix} \quad x_3 = 1 \quad x_2 = -1 \quad x_1 = 0 \quad \therefore x = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

6. 用追赶法解下列方程组：

(1) $\begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 4 \end{pmatrix}$； (2) $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \\ 0 \end{pmatrix}$.

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & \frac{5}{2} & 0 & 0 \\ 0 & 1 & \frac{3}{5} & 0 \\ 0 & 0 & 2 & -\frac{7}{3} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{2}{5} & 0 \\ 0 & 0 & 1 & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$Ly = b$

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 1 \\ 1 & \frac{5}{2} & 0 & 0 & 2 \\ 0 & 1 & \frac{3}{5} & 0 & -2 \\ 0 & 0 & 2 & -\frac{7}{3} & 0 \end{pmatrix} \qquad y = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{5} \\ -5 \\ -\frac{30}{7} \end{pmatrix}$$

$Ux = y$

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{2}{5} & 0 & \frac{3}{5} \\ 0 & 0 & 1 & \frac{5}{3} & -5 \\ 0 & 0 & 0 & 1 & -\frac{30}{7} \end{pmatrix} \qquad x = \begin{pmatrix} \frac{22}{35} \\ -\frac{9}{35} \\ \frac{65}{7} \\ -\frac{30}{7} \end{pmatrix}$$

9. 设线性方程组为

$$\begin{cases} 7x_1 + 10x_2 = 1 \\ 5x_1 + 7x_2 = 0.7 \end{cases}$$

(1) 试求系数矩阵 **A** 的条件数 $\mathrm{Cond}_\infty(A)$；

(2) 若右端向量有扰动 $\delta b = (0.01, -0.01)^T$，试估计解的相对误差

(1) $A = \begin{pmatrix} 7 & 10 \\ 5 & 7 \end{pmatrix}$ $\quad A^{-1} = \begin{pmatrix} -7 & 10 \\ 5 & -7 \end{pmatrix}$

$\mathrm{Cond}_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = 17 \times 17 = 289$

(2) $\begin{cases} 7x_1 + 10x_2 = 1 \\ 5x_1 + 7x_2 = 0.7 \end{cases}$ $\quad x = \begin{pmatrix} 0 \\ 0.1 \end{pmatrix}$

$\begin{cases} 7x_1 + 10x_2 = 1.01 \\ 5x_1 + 7x_2 = 0.69 \end{cases}$ $\quad x^* = \begin{pmatrix} -0.17 \\ 0.22 \end{pmatrix}$

$\dfrac{\|\delta x\|_\infty}{\|x\|_\infty} = \dfrac{0.17}{0.1} = 170\%$

10. 用雅可比方法和高斯-塞德尔方法解方程组

$$\begin{cases} 8x_1 - 3x_2 + 2x_3 = 20 \\ 4x_1 + 11x_2 - x_3 = 33 \\ 6x_1 + 3x_2 + 12x_3 = 36 \end{cases}$$

要求取 $\boldsymbol{x}^{(0)} = (0,0,0)^T$，计算 $\boldsymbol{x}^{(5)}$，并分别与精确分解 $\boldsymbol{x} = (3,2,1)^T$ 比较.

雅可比：$\begin{cases} x_1 = \frac{3}{8}x_2 - \frac{1}{4}x_3 + \frac{5}{2} \\ x_2 = -\frac{4}{11}x_1 + \quad + \frac{3}{11}x_3 + 3 \\ x_3 = -\frac{1}{2}x_1 - \frac{1}{4}x_2 + 3 \end{cases}$

$\begin{cases} x_1^{(k+1)} = \frac{3}{8}x_2^{(k)} - \frac{1}{4}x_3^{(k)} + \frac{5}{2} \\ x_2^{(k+1)} = -\frac{4}{11}x_1^{(k)} + \quad + \frac{3}{11}x_3^{(k)} + 3 \\ x_3^{(k+1)} = -\frac{1}{2}x_1^{(k)} - \frac{1}{4}x_2^{(k)} + 3 \end{cases}$

$x^{(5)} = \begin{pmatrix} 3.000323 \\ 1.983988 \\ 1.000968 \end{pmatrix}$ $\quad \dfrac{\|\delta x\|_\infty}{\|x\|_\infty} = \dfrac{0.016012}{3} = 0.53\%$

高斯塞德尔：$\begin{cases} x_1 = \frac{3}{8}x_2 - \frac{1}{4}x_3 + \frac{5}{2} \\ x_2 = -\frac{4}{11}x_1 + \quad + \frac{3}{11}x_3 + 3 \\ x_3 = -\frac{1}{2}x_1 - \frac{1}{4}x_2 + 3 \end{cases}$

$$\begin{cases} x_1^{(k+1)} = \frac{3}{8}x_2^{(k)} - \frac{1}{4}x_3^{(k)} + \frac{5}{2} \\ x_2^{(k+1)} = -\frac{4}{11}x_1^{(k+1)} + \frac{3}{11}x_3^{(k)} + 3 \\ x_3^{(k+1)} = -\frac{1}{2}x_1^{(k+1)} - \frac{1}{4}x_2^{(k+1)} + 3 \end{cases}$$

$$x^{(5)} = \begin{pmatrix} 2.999842 \\ 2.000072 \\ 1.000061 \end{pmatrix} \qquad \frac{\|\delta x\|_\infty}{\|x\|_\infty} = 0.0053\%$$