

用 Euler 法、预报-校正法求  $\begin{cases} y' = -y \\ y(0) = 1 \end{cases}, x \in [0, 1.0]$  的数值解, 取  $h = 0.1$ , 结果保留四位小数。

1. ① Euler  $\begin{cases} y_{i+1} = y_i + h f(x_i, y_i) = y_i + 0.1(-y_i) \\ y_0 = 1 \end{cases} \quad i=0, 1, \dots, 9$

即:  $\begin{cases} y_{i+1} = 0.9y_i \\ y_0 = 1.0000 \quad y_1 = 0.9000 \quad y_2 = 0.8100 \quad y_3 = 0.7290 \\ y_4 = 0.6561 \quad y_5 = 0.5905 \quad y_6 = 0.5315 \quad y_7 = 0.4784 \\ y_8 = 0.4306 \quad y_9 = 0.3875 \quad y_{10} = 0.3488 \end{cases}$

预报校正法:  $y_{i+1} = y_i + \frac{h}{2} [k_1 + k_2] = y_i + \frac{h}{2} (h-2)y_i = 0.905y_i$   
 $k_1 = -y_i \quad k_2 = f(x_i + h, y_i + hk_1) = -y_i + h y_i$

$y_0 = 1 \quad y_1 = 0.905 \quad y_2 = 0.8190 \quad y_3 = 0.7412 \quad y_4 = 0.6708 \quad y_5 = 0.6071$   
 $y_6 = 0.5494 \quad y_7 = 0.4972 \quad y_8 = 0.4500 \quad y_9 = 0.4073 \quad y_{10} = 0.3686$

3. 用预报-校正法求  $\begin{cases} y' = x^2 \\ y(0) = 0 \end{cases}, x \in [0, 2]$  的数值解, 取  $h = 0.5$ , 结果保留四位小数; 并与精确解  $y = \frac{1}{3}x^3$  作比较。

3. 解:  $k_1 = x_i^2 \quad k_2 = (x_i + 0.5)^2 = x_i^2 + x_i + 0.25 \quad i=0, 1, 2, 3$   
 $y_{i+1} = y_i + \frac{0.5}{2} (2x_i^2 + x_i + 0.25) = y_i + 0.5x_i^2 + 0.25x_i + 0.0625$   
 $y_0 = 0$

$i$	$x_i$	预报-校正 $y_i$	精确解
0	0	0	0
1	0.5	0.0625	0.0417
2	1	0.3750	0.3333

3	1.5	1.1875	1.1250
4	2	2.7500	2.6667