

Team notebook

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1 DP

1.1 1D-1D optimization

```

//Batch Scheduling
#include <bits/stdc++.h>
using namespace std;
#define maxn 1000005
int dp[maxn];
int t[maxn], f[maxn];
int n, s;
int w(int i, int x){
    return ( t[x]-t[i]+s )*( f[n]-f[i] );
}
int main(){
    scanf("%d %d",&n,&s);
    for(int i=1; i<=n; i++){
        scanf("%d %d",&t[i],&f[i]);
        t[i] += t[i-1]; f[i] += f[i-1];
    }
    vector < pair<int,int> > vp; // pos , best-k
    vp.pb( mp( 0 , 0 ) );
    for(int x=1; x<=n; x++){
        int idx=upper_bound(vp.begin(), vp.end(), mp(x,n+1)) - vp.begin();
        idx--;
        dp[x]=dp[vp[idx].yy]+w(vp[idx].yy,x);
        while( (int)vp.size() > 0 ){
            if(vp.back().xx>x&&dp[x]+w(x,vp.back().xx)<=
                dp[vp.back().yy]+w(vp.back().yy,
                    vp.back().xx))vp.pop_back();
            else break;
        }
        if(vp.size()==0) vp.push_back(mp(0,x));
        else{
            int lo = max(vp.back().xx,x+1),hi=n;
            if(lo>hi||dp[vp.back().yy]+
                w(vp.back().yy,hi)<=dp[x]+w(x,hi))continue;
            while( lo < hi ){
                int mid = (lo+hi)/2;
                if( dp[vp.back().yy]+w(vp.back().yy,mid)<=
                    dp[x]+w(x,mid))lo=mid+1;
                else hi=mid;
            }
            vp.pb( mp( lo , x ) );
        }
    }

```

```

}
printf("%d\n",dp[n]) ;
return 0 ;
}

```

1.2 ConnctedComponentDP

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pi;
i64 dp[102][32][12][12][2][2] , M;
int a[102];
i64 f(int n,int r,int k,int c,int st,int en)
{
    if(n==0||c<0||st<0||en<0) return 0;
    r = (r+a[n]*(2*c+st+en))%M;
    if( dp[n][r][k][c][st][en]!=-1)
        return dp[n][r][k][c][st][en];
    i64 ans = f(n-1,r,k,c,st,en);/*itisnotused*/
    if( k==1 ){
        if( c==0 && (st||en) && r==0 ) ans++;/*if
        this is the last element to take
        then is should either connect st and en ,
        or be the first element or last */
    }
    else{
        if(st==0) ans+=(f(n-1,r,k-1,c,1,en)+f(n-1,r,
        k-1,c-1,1,en)*c); // this is starting element
        if(en==0) ans += ( f(n-1,r,k-1,c,st,1) +
        f(n-1,r,k-1,c-1,st,1)*c);// this is ending element

        ans += f( n-1 , r , k-1 , c+1 , st , en );
        // created & independent
        ans += f(n-1,r,k-1,c,st,en)*2*c+
        f(n-1,r,k-1,c,st,en)*(st+en);
        /*created and connected with some other
        component possibly start or end component */
        ans += ( f(n-1,r,k-1,c-1,st,en)*c*(c-1)+
        f(n-1,r,k-1,c-1,st,en)*c*(st+en));
        /*created and connected between two component */
    }
    return dp[n][r][k][c][st][en] = ans%mod;
}

```

1.3 Convex Hull Trick Linear

```

//Min:M inc, x dec, useless(s-1, s-2, s-3)
// M dec, x inc, useless(s-3, s-2, s-1)
//Max:M inc, x inc, useless(s-3, s-2, s-1)
// M dec, x dec, useless(s-1, s-2, s-3)
struct CHT {
    vector<LL> M; vector<LL> C; int ptr = 0;
    bool useless(int l1, int l2, int l3) {
        return (C[l3]-C[l1])*(M[l1]-M[l2])
            <= (C[l2]-C[l1])*(M[l1]-M[l3]);
    } //Use double comp if M,C is LL range
    LL f(int id, LL x) {return M[id]*x+C[id];}
    void add(LL m, LL c) {
        M.push_back(m); C.push_back(c);
        int s = M.size();
        while (s >= 3 && useless(s-3, s-2, s-1)) {
            M.erase(M.end()-2); C.erase(C.end()-2); s--;
        }
    }
    LL query(LL x) {

```

```

        if (ptr >= M.size()) ptr = M.size()-1;
        while (ptr<M.size()-1 && f(ptr,x)>f(ptr+1,x))
            ptr++;
        return f(ptr, x);
    }
};

```

1.4 Convex Hull Trick Online

```

//cht for max, for min, insert(-m,-c) and negate
bool Q;
struct Line {
    mutable ll m, c, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : m < o.m;
    }
};
struct LineContainer : multiset<Line> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->m==y->m) x->p=x->c > y->c ? -inf : -inf;
        else x->p = div(y->c - x->c, x->m - y->m);
        return x->p >= y->p;
    }
    void addLine(ll m, ll c) {
        auto z = insert({m, c, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x!=begin() && isect(--x, y))
            isect(x,y=erase(y));
        while ((y = x) != begin() && (--x->p >= y->p)
            isect(x, erase(y)));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.m * x + l.c;
    }
    bool isEmpty(){ return (empty()) ; }
    void Clear() { clear() ; }
};ch;

```

1.5 Digit DP Template

```

int call(int pos, int st, int sm,
        bool strt, bool allLarge, bool alSmall) {
    if(sm > limit) return 0;
    if(pos == 200) return 1;
    if(dp[pos][st][sm][strt][allLarge][alSmall]!=-1)
        return dp[pos][st][sm][strt][allLarge][alSmall];
    int ret = 0;
    for(int i = 0; i < base; i++) {
        if(allLarge == false && i<L[pos]-'a') continue;
        if(alSmall == false && i>R[pos]-'a') continue;
        int npos = pos + 1;
        bool nstrt = strt||i>0;
        bool nallLarge = allLarge||i>L[pos]-'a';
        bool nalSmall = alSmall||i<R[pos]-'a';
        int nst = st;
        if(nstrt) nst = Next(nst, 'a' + i);
        int nsm = sm + sts[nst].cnt;

```

```

        add(ret,
            call(npos,nst,nsm,nstrt,nalLarge,nalSmall));
    }
    dp[pos][st][sm][strt][allLarge][alSmall] = ret;
    return ret;
}

```

1.6 Knuth Optimisation

```

//There are n points on the segment (0, 1). You
//have to mark the points in some order. Cost of
//picking a point. Cost of marking is the distance
//between closest marked points to the left and
//to the right. Minimise cost.
LL a[N], dp[N][N], opt[N][N];
LL Knuth(int l, int n) {
    a[0] = 0; a[++n] = 1;
    for (int i=1;i<=n;i++) opt[i-1][i]=i-1;
    for (int len=2; len<=n; len++)
        for (int l=0; l+len<=n; l++) {
            int r = l+len, optl = opt[l][r-1];
            int optr = opt[l+1][r]; dp[l][r] = INF;
            for (int i=optl; i<=optr; i++) {
                LL c = dp[l][i] + dp[i][r] + a[r] - a[l];
                if(c<dp[l][r]) dp[l][r] = c, opt[l][r]=i;
            }
        }
    return dp[0][n];
}

```

1.7 SOS(on the fly)

```

public class TestProctoring {
    public double expectedTime(int[] p, int[] q) {
        int n = p.length;
        double[] prob = new double[n];
        for (int i = 0; i < n; i++) {
            prob[i] = p[i] * 1.0 / q[i];
        }
        double[][] t = new double[n+1][1<<n];
        double[] dp = new double[1<<n];

        /* t[i][mask] is sum of all submask of mask where
        difference of mask and submask is before i'th
        bit( 0 based ) , that means difference can be in 0
        to i-1 th bit t[0][mask] contains nothing other
        than just value of this mask t[n][mask] contains
        result of all submask of this mask */
        for (int mask = 1; mask < 1 << n; mask++){
            double fail = 1; double mult = 1;
            double am = 1;
            for (int j = 0; j < n; j++) {
                t[j+1][mask] = t[j][mask];
                if (((mask>>j)&1) == 1) {
                    t[j+1][mask] += t[j][mask^(1<<j)];
                    fail *= (1 - prob[j]);
                    mult *= prob[j];
                    am *= (1 - prob[j]) / prob[j];
                }
            }
            dp[mask] = (1+mult*t[n][mask]) / (1 - fail);
            for (int j = 0; j <= n; j++) {
                t[j][mask] += dp[mask] * am;
            }
        }
    }
}

```

```

    }
}
return dp[(1<<n)-1];
}
}

```

2 Data Structures

2.1 2D BIT Range update Range query

```

const int mx = 1002, my = 1002;
long long bit[4][mx][my];
void update( int x, int y, int val, int i ) {
    int y1;
    while( x<=mx ) {
        y1=y;
        while( y1<=my )
            bit[i][x][y1] += val, y1 += (y1&-y1);
        x += (x&-x);
    }
}
long long query( int x, int y, int i ) {
    long long ans=0; int y1;
    while( x>0 ) {
        y1 = y;
        while( y1>0 )
            ans += bit[i][x][y1], y1 -= (y1&-y1);
        x -= (x&-x);
    }
    return ans;
}
// add value k from (x1,y1) to (x2,y2) inclusive
void add( int x1, int y1, int x2, int y2, int k ) {
    update(x1,y1,k,0);
    update(x1,y2+1,-k,0);
    update(x2+1,y1,-k,0);
    update(x2+1,y2+1,k,0);
    update(x1,y1,k*(1-y1),1);
    update(x1,y2+1,k*y2,1);
    update(x2+1,y1,k*(y1-1),1);
    update(x2+1,y2+1,-y2*k,1);
    update(x1,y1,k*(1-x1),2);
    update(x1,y2+1,k*(x1-1),2);
    update(x2+1,y1,k*x2,2);
    update(x2+1,y2+1,-x2*k,2);
    update(x1,y1,(x1-1)*(y1-1)*k,3);
    update(x1,y2+1,-y2*(x1-1)*k,3);
    update(x2+1,y1,-x2*(y1-1)*k,3);
    update(x2+1,y2+1,x2*y2*k,3);
}
// get value from (x1,y1) to (x2,y2) inclusive
long long get( int x1, int y1, int x2, int y2 ) {
    LL v1=query(x2,y2,0)*x2*y2 +
        query(x2,y2,1)*x2 +
        query(x2,y2,2)*y2 +
        query(x2,y2,3);
    LL v2=query(x2,y1-1,0)*x2*(y1-1) +
        query(x2,y1-1,1)*x2 +
        query(x2,y1-1,3) +
        query(x2,y1-1,2)*(y1-1);
    LL v3=query(x1-1,y2,0)*(x1-1)*y2 +
        query(x1-1,y2,2)*y2 +
        query(x1-1,y2,1)*(x1-1) +

```

```

        query(x1-1,y2,3);
    LL v4=query(x1-1,y1-1,0)*(x1-1)*(y1-1) +
        query(x1-1,y1-1,1)*(x1-1) +
        query(x1-1,y1-1,2)*(y1-1) +
        query(x1-1,y1-1,3);
    LL ans=v1-v2-v3+v4;
    return ans;
}

```

2.2 Centroid Decomposition

```

vector<int> g[N]; int n, child[N], done[N];
void dfs_size(int u, int par) {
    child[u] = 1;
    for (int v: g[u]) {
        if (done[v] or v == par) continue;
        dfs_size(v, u); child[u] += child[v];
    }
}
int dfs_find_centroid(int u, int par, int sz) {
    for (int v: g[u]) {
        if (!done[v] and v != par and child[v] > sz) {
            return dfs_find_centroid(v, u, sz);
        }
    }
    return u;
}
void solve (int u) {/**problem specific things */}
void dfs_decompose(int u) {
    dfs_size(u, -1);
    int centroid=dfs_find_centroid(u, -1, child[u]/2);
    solve(centroid);
    done[centroid] = 1;
    for (int v : g[centroid]) {
        if (!done[v]) dfs_decompose(v);
    }
}

```

2.3 HLD

```

namespace hld{
    int in[maxn] , out[maxn] , sub[maxn] , t = 1,
        nxt[maxn] , depth[maxn], par[maxn] , n ;
    vector<int> g[maxn] ;
    void init(int _n){
        n = _n;
        for(int i=0 ; i<=n ; i++) g[i].clear() ;
    }
    void addEdge(int u, int v){
        g[u].pb(v) ; g[v].pb(u) ;
    }
    void dfsSZ(int u){
        sub[u] = 1 ;
        for(int i=0 ; i<g[u].size() ; i++){
            int v = g[u][i] ;
            for(int j=0 ; j<g[v].size() ; j++){
                if ( g[v][j] == u ){
                    g[v].erase(g[v].begin()+j);
                    break ;
                }
            }
            dfsSZ(v) ;
            sub[u] += sub[v] ;
            if(sub[v]>sub[g[u][0]])swap(g[u][0],g[u][i]);

```

```

}
}
void dfsHLD(int u){
    in[u] = ++t ;
    for(int i=0 ; i<g[u].size() ; i++){
        int v = g[u][i] ; par[v] = u ;
        depth[v] = depth[u] + 1 ;
        if( i==0 ) nxt[v] = nxt[u] ;
        else nxt[v] = v ;
        dfsHLD(v) ;
    }
    out[u] = t ;
}
void preprocess(int root){
    dfsSZ(root) ; t = 0 ; nxt[root] = root ;
    depth[root] = 1 ; dfsHLD(root) ;
}
int hldQuery( int u , int v ){
    int ans = -INF ;
    while( nxt[u] != nxt[v] ){
        if(depth[nxt[u]]<depth[nxt[v]]){
            ans=max(ans,query(1,1,n,in[nxt[v]],in[v]));
            // do you thing here ( from in[v] to in[ nxt[v]] )
            v = par[nxt[v]] ;
        }
        else{
            ans=max(ans,query(1,1,n,in[nxt[u]],in[u]));
            // do your thing here (from in[u] to in[nxt[u]])
            u = par[nxt[u]] ;
        }
    }
    int lc ;
    if( depth[u] > depth[v] ) swap(u,v) ;
    lc = u ;
    //here lc is the lca
    //if you are working on node ,
    //not on edge, then update/query upto u also
    //otherwise update/query from in[u]+1 to in[v]
    ans = max( ans , query(1,1,n,in[u]+1,in[v]) );
    return ans ;
}
void hldUpdate( int u , int v , int val ){
    while( nxt[u] != nxt[v] ){
        if( depth[ nxt[u] ] < depth[ nxt[v] ] ){
            update(1,1,n,in[ nxt[v] ] , in[v] , val );
            // do you thing here ( from in[v] to in[ nxt[v] ] )
            v = par[ nxt[v] ] ;
        }
        else{
            update(1,1,n,in[ nxt[u] ] , in[u] , val );
            // do your thing here (from in[u] to in[nxt[u]])
            u = par[ nxt[u] ] ;
        }
    }
    int lc ;
    if( depth[u] > depth[v] ) swap(u,v) ;
    lc = u ;
    //here lc is the lca
    //if you are working on node , not on edge,
    //then update/query upto u also

```

```
//otherwise update/query from in[u]+1 to in[v]
update(1,1,n,in[u]+1,in[v],val) ;
return ;
}
}
```

2.4 Persistent Segment Tree

```
int a[N], root[N];
struct node { int sm, l, r;} node[N*LOG];
int tot_nodes = 0;
int upd(int cn, int b, int e, int i, int val) {
    int cur = ++tot_nodes;
    if(b==e) {
        node[cur].sm=node[cn].sm + val; return cur;
    }
    int mid = (b+e)/2;
    if (i <= mid) {
        node[cur].l = upd(node[cn].l, b, mid, i, val);
        node[cur].r = node[cn].r;
    }
    else {
        node[cur].r = upd(node[cn].r, mid+1,e,i, val);
        node[cur].l = node[cn].l;
    }
    node[cur].sm = node[node[cur].l].sm
        + node[node[cur].r].sm;
    return cur;
}
int query(int cn , int b , int e , int i, int j) {
    if (b > j or e < i or !cn) return 0 ;
    if (b >= i and e <= j) return node[cn].sm;
    int mid = (b+e)/2;
    return query(node[cn].l,b,mid,i,j)
        + query(node[cn].r,mid+1,e,i,j);
}
```

2.5 RMQ(2D)

```
int st[K][K][N][N]; int lg[N];
void pre() {
    lg[1] = 0;
    for (int i=2; i<N; i++) lg[i] = lg[i/2]+1;
}
int query(int l1, int r1, int l2, int r2) {
    int xx = lg[l2-l1+1], yy = lg[r2-r1+1];
    return max(max(st[xx][yy][l1][r1],
        st[xx][yy][l2-(1<<xx)+1][r1]),
        max(st[xx][yy][l1][r2-(1<<yy)+1],
        st[xx][yy][l2-(1<<xx)+1][r2-(1<<yy)+1]));
}
void build() {
    for (int x=0; x<K; x++) {
        for (int y=0; y<K; y++) {
            for (int i=1; i<=n; i++) {
                for (int j=1; j<=m; j++) {
                    if (i+(1<<xx)-1>n || j+(1<<yy)-1>m)
                        continue;
                    if (!x&&!y) st[0][0][i][j]=flag[i][j];
                    else if (x>0) st[x][y][i][j] =
max(st[x-1][y][i][j],st[x-1][y][i+(1<<(x-1))][j]);
                    else if (y>0) st[x][y][i][j] =
max(st[x][y-1][i][j],st[x][y-1][i][j+(1<<(y-1))]);
                }
            }
        }
    }
```

```
}
}
}
}
```

2.6 SegTree Range Inc, Max Query

```
LL tr[4*N], lz[4*N];
void propagate(int u, int st, int en) {
    if (!lz[u]) return;
    tr[u] += lz[u];
    if (st!=en) {lz[2*u]+=lz[u]; lz[2*u+1]+=lz[u];}
    lz[u] = 0;
}
void update(int u,int st,int en,int l,int r,LL x){
    propagate(u, st, en);
    if (r<st || en<l) return;
    else if (l<=st && en<=r){
        lz[u]+=x; propagate(u, st, en);
    }
    else {
        int mid = (st+en)/2;
        update(2*u, st, mid, l, r, x);
        update(2*u+1, mid+1, en, l, r, x);
        tr[u] = max(tr[2*u], tr[2*u+1]);
    }
}
LL query(int u, int st, int en, int l, int r) {
    propagate(u, st, en);
    if (r<st || en<l) return -inf;
    else if (l<=st && en<=r) return tr[u];
    else {
        int mid = (st+en)/2;
        return max(query(2*u, st, mid, l, r),
            query(2*u+1, mid+1, en, l, r));
    }
}
```

3 Geometry

3.1 Circle Cover

```
///Check if the all of the area of circ(0, R) in
///Circ(00, RR) is covered by some other circle
bool CoverCircle(PT O, double R, vector<PT> &cen,
    vector<double> &rad, PT OO, double RR) {
    int n = cen.size();
    vector<pair<double, double>> arcs;
    for (int i=0; i<n; i++) {
        PT P = cen[i]; double r = rad[i];
        if (i!=0 && R + sqrt(dist2(O, P))<r) return 1;
        if (i==0 && r + sqrt(dist2(O, P))<R) return 1;
        vector<PT> inter =
            CircleCircleIntersection(O, P, R, r);
        if (inter.size() <= 1) continue;
        PT X = inter[0], Y = inter[1];
        if (cross(O, X, Y) < 0) swap(X, Y);
        if (!(cross(O, X, P) >= 0 &&
            cross(O, Y, P) <= 0)) swap(X, Y);
        if (i==0) swap(X, Y);
        X = X-O; Y=Y-O;
        double l1 = atan2(X.y, X.x);
        double rr = atan2(Y.y, Y.x);
    }
```

```
if (rr < l1) rr += 2*PI;
arcs.emplace_back(l1, rr);
}
if (arcs.empty()) return false;
sort(arcs.begin(), arcs.end());
double st = arcs[0].ff, en = arcs[0].ss,ans = 0;
for (int i=1; i<arcs.size(); i++) {
    if (arcs[i].first <= en + EPS)
        en = max(en, arcs[i].second);
    else st = arcs[i].first, en = arcs[i].second;
    ans = max(ans, en-st);
}
return ans >= 2*PI;
}
```

3.2 Circle Polygon Common

```
LD areaCT(Point pa, Point pb, LD r) {
    if (pa.Norm() < pb.Norm()) swap(pa, pb);
    if (dcmp(pb.Norm()) == 0) return 0;
    LD a=pb.Norm(),b=pa.Norm(),c=(pb-pa).Norm();
    LD sinB = fabs(pb.cross(pb-pa)/a/c);
    LD cosB = pb.dot(pb-pa)/a/c;
    LD sinC = fabs(det(pa,pb)/a/b);
    LD cosC = pa.dot(pb)/a/b;
    LD B = atan2(sinB, cosB), C = atan2(sinC, cosC);
    LD S = 0.;
    if (a > r) {
        S = C / 2 * r * r;
        LD h = a * b * sinC / c;
        if (h < r && B < PI / 2) {
            S -= (Acos(h/r)*r*r - h*sqrt(r*r-h*h));
        }
    }
    else if (b > r) {
        LD theta = PI - B - Asin((sinB/r * a,-1,+1));
        S = a*r*sin(theta)/2 + (C-theta)/2*r*r;
    }
    else {
        S = sinC * a * b / 2;
    }
    return S;
}
LD poly_cross(vector<Point> P, Point cen, LD r) {
    int n = P.size(); LD ans = 0;
    for(int i = 0; i < n; i++) {
        LD cr=fabs(areaCT(P[i]-cen,P[(i+1)%n]-cen,r))*
            dcmp((P[i]-cen).cross(P[(i+1)%n]-cen));
        ans += cr;
    }
    return ans;
}
```

3.3 Circle Union Area

```
struct Point {
    LD x,y ;
    LD operator*(const Point &a)const {
        return x*a.y-y*a.x;}
    LD operator/(const Point &a)const {
        return sqrt((a.x-x)*(a.x-x)+(a.y-y)*(a.y-y));}
}po[N];
LD r[N];
int sgn(LD x) {return fabs(x)<EPS?0:(x>0.0?1:-1);}
pair<LD,bool> ARG[2*N] ;
LD cir_union(Point c[],LD r[],int n) {
```



```

LD sum = 0.0 , sum1 = 0.0 ,d,p1,p2,p3 ;
for(int i = 0 ; i < n ; i++) {
    bool f = 1 ;
    for(int j = 0 ; f&&j<n ; j++)
        if(i!=j && sgn(r[j]-r[i]-c[i]/c[j])!=-1)f=0;
    if(!f) swap(r[i],r[--n]),swap(c[i--],c[n]);
}
for(int i = 0; i < n; i++) {
    int k = 0, cnt = 0;
    for(int j = 0; j < n; j++) {
        if(i!=j&&sgn((d=c[i]/c[j])-r[i]-r[j])<=0){
            p3=acos((r[i]*r[i]+d*d-r[j]*r[j])/(
                (2.0*r[i]*d)));
            p2=atan2(c[j].y-c[i].y,c[j].x-c[i].x);
            p1 = p2-p3; p2 = p2+p3;
            if(sgn(p1+PI)==-1) p1+=2*PI,cnt++;
            if(sgn(p2-PI)==1) p2-=2*PI,cnt++;
            ARG[k++] = make_pair(p1,0);
            ARG[k++] = make_pair(p2,1);
        }
    }
    if(k) {
        sort(ARG,ARG+k) ;
        p1 = ARG[k-1].first-2*PI;
        p3 = r[i]*r[i] ;
        for(int j = 0 ; j < k ; j++) {
            p2 = ARG[j].first;
            if(cnt==0) {
                sum+=(p2-p1-sin(p2-p1))*p3 ;
                sum1+=(c[i]+Point(cos(p1),sin(p1))*
                    r[i])*(c[i]+
                    Point(cos(p2),sin(p2))*r[i]);
            }
            p1 = p2;
            ARG[j].second ? cnt--:cnt++;
        }
        else sum += 2*PI*r[i]*r[i];
    }
    return (sum+fabs(sum1))*0.5 ;
}

```

3.4 Geometry 2D Basic

```

const LD EPS = 1e-9;
const LD PI = acos(-1);
LD Sq(LD x) {return x * x;}
LD Acos(LD x){return acos(min(1.0L,max(-1.0L,x)));}
LD Asin(LD x){return asin(min(1.0L,max(-1.0L,x)));}
LD Sqrt(LD x) {return sqrt(max(0.0L, x));}
int dcmp(LD x) {
    if(fabs(x) < EPS) return 0;
    return (x > 0.0 ? +1 : -1);
}
struct Point {
    LD x, y;
    Point() {}
    Point(LD a, LD b) : x(a), y(b) {}
    Point(const Point& a) : x(a.x), y(a.y) {}
    void operator=(const Point& a) { x=a.x; y=a.y;}
    Point operator+(const Point& a) const
        { Point p(x + a.x, y + a.y); return p; }
    Point operator-(const Point& a) const

```

```

        { Point p(x - a.x, y - a.y); return p; }
    Point operator*(LD a) const
        { Point p(x*a,y*a); return p; }
    Point operator/(LD a) const
        { assert(a > EPS); Point p(x/a,y/a); return p; }
    bool IsZero() const {
        return abs(x) < EPS && abs(y) < EPS;
    }
    bool operator==(const Point& a) const {
        return (*this - a).IsZero();
    }
    LD cross(const Point& a) const {
        return x * a.y - y * a.x;
    }
    LD cross(Point a, Point b) const {
        a = a-*this; b = b-*this; return a.cross(b);
    }
    LD dot(const Point& a) const {
        return x * a.x + y * a.y;
    }
    LD Norm() const { return Sqrt(Sq(x) + Sq(y));}
    void NormalizeSelf() { *this = *this / Norm();}
    Point Normalize() {
        Point res(*this);res.NormalizeSelf();
        return res;
    }
    LD Dist(const Point& a) const
        {return (*this-a).Norm();}
    LD Angle() const { return atan2(y, x);}
    void RotateSelf(LD angle) {
        LD c = cos(angle), s = sin(angle);
        LD nx = x*c-y*s, ny = y*c+x*s; y = ny, x = nx;
    }
    Point Rotate(LD angle) const {
        Point res(*this); res.RotateSelf(angle);
        return res;
    }
    static bool LexCmp(const Point&a,const Point&b){
        if(abs(a.x - b.x) > EPS) return a.x < b.x;
        return a.y < b.y;
    }
    LD SqNorm() { return x * x + y * y;}
};
struct Circle {
    Point center; LD r;
    Circle(LD x, LD y, LD rad) {
        center = Point(x, y); r = rad;
    }
    Circle(const Point& a,LD rad):center(a),r(rad){}
    Point PointAtAngle(LD ang) const {
        return center+Point{r*cos(ang),r*sin(ang)};
    }
    bool operator==(const Circle& c) const {
        return center == c.center && abs(r-c.r) < EPS;
    }
};
struct Line {
    Point p[2]; bool is_seg;
    Line(Point a, Point b, bool is_seg_ = false) {
        p[0] = a; p[1] = b; is_seg = is_seg_;
    }
    Line() {}
    // Ax + By + C = 0, not tested

```

```

Line(LD A, LD B, LD C, bool is_seg_ = false) {
    if(fabs(A) > EPS)
        p[0]=Point(-C/A,0.),p[1]=Point(-(B+C)/A,1.);
    else
        p[0]=Point(0.,-C/B),p[1]=Point(1.,-(A+C)/B);
    is_seg = is_seg_;
}
Point& operator[](int a) { return p[a];}
Point Dir() { return p[1] - p[0];}
Point NormalVector() {
    Point perp = p[1]-p[0];perp.RotateSelf(PI/2);
    perp.NormalizeSelf(); return perp;
}
Line shift(Point q) { // not tested
    return Line(p[0] + q, p[1] + q, is_seg);
}
// (A,B,C) such that A^2+B^2=1, (A,B) > (0,0)
vector<LD> LineEqNormLD() { // seems ok
    LD A = p[1].y - p[0].y, B = p[0].x - p[1].x;
    LD C = -(A * p[0].x + B * p[0].y);
    assert(abs(A*p[1].x + B*p[1].y + C) < EPS);
    LD norm = Sqrt(Sq(A) + Sq(B));
    vector<LD> res{A, B, C};
    for (auto& x : res) x /= norm;
    if (A < -EPS || (abs(A) < EPS && B < -EPS))
        for (auto& x : res) { x *= -1; }
    return res;
}
// assumes that coordinates are integers!
vector<int> LineEqNormInt() { // seems ok
    int A = round(p[1].y - p[0].y);
    int B = round(p[0].x - p[1].x);
    int C = -(A * p[0].x + B * p[0].y);
    int gcd = abs(__gcd(A, __gcd(B, C)));
    vector<int> res{A, B, C};
    for (auto& x : res) { x /= gcd; }
    if (A < 0 || (A == 0 && B < 0))
        for (auto& x : res) { x *= -1; }
    return res;
}
};
namespace Utils {
LD Angle(Point P, Point Q, Point R) { // angle PQR
    LD ang2 = (P - Q).Angle(), ang1 = (R-Q).Angle();
    LD ans = ang1 - ang2;
    if (ans < EPS) ans += 2 * PI;
    return ans;
}
bool PtBelongToLine(Point p, Line l) {
    return abs(l[0].cross(l[1], p)) < EPS;
}
bool PtBelongToSeg(Point p, Line l) { // seems ok
    return abs(p.Dist(l[0])+p.Dist(l[1])
        -l[0].Dist(l[1])) < EPS;
}
bool AreParallel(Line l1, Line l2) { // seems ok
    LD t=l1[0].cross(l2[0],l1[1])
        -l1[0].cross(l2[1],l1[1]);
    return abs(t) < EPS;
}
bool AreCollinear(Line l1, Line l2) { // not tested

```

```

    return AreParallel(l1,l2) &&
           PtBelongToLine(l2[0],l1);
}
Point ProjPtToLine(Point p, Line l) { //Tested
    Point dir = l[1]-l[0];
    return l[0]+dir*(dir.dot(p-l[0])/dir.dot(dir));
}
Point ReflectPtWRTLine(Point p, Line l) {
    Point proj = ProjPtToLine(p, l);return proj*2-p;
}
Point ProjPtToSegment(Point p, Line l){//!tested
    LD base = (l[1]-l[0]).SqNorm();
    if (fabs(base) < EPS) return l[0];
    LD param = (p-l[0]).dot(l[1]-l[0])/base;
    if (param < 0) return l[0];
    if (param > 1) return l[1];
    return l[0] + (l[1]-l[0]) * param;
}
LD PtToLine(Point p, Line l) { // not tested
    Point v1 = l[1] - l[0], v2 = p - l[0];
    return fabs(v1.cross(v2))/v1.Norm();
}
LD PtToSegment(Point p, Line l) {
    if (l[0] == l[1]) return (p-l[0]).Norm();
    Point v1 = l[1]-l[0], v2 = p-l[0], v3 = p-l[1];
    if ((v1.dot(v2)) < -EPS) return v2.Norm();
    else if ((v1.dot(v3)) > EPS) return v3.Norm();
    else return fabs(v1.cross(v2))/v1.Norm();
}
vector<Point> InterLineLine(Line& a, Line& b){//ok
    Point vec_a = a[1] - a[0];
    Point vec_b1 = b[1] - a[0];
    Point vec_b0 = b[0] - a[0];
    LD tr_area = vec_b1.cross(vec_b0);
    LD quad_area = vec_b1.cross(vec_a)
                    + vec_a.cross(vec_b0);
    if (abs(quad_area) < EPS) { //parallel/coinciding
        if (PtBelongToLine(a[0], b)) {
            return {a[0], a[1]};
        } else {
            return {};
        }
    }
    return {a[0] + vec_a * (tr_area / quad_area)};
}
//SZ(res)==0:empty,SZ(res)=1=> intersection is
//a pt,SZ(res) == 2 => intersection is a segment
vector<Point> InterSegs(Line l1, Line l2) { // ok
    if(!Point::LexCmp(l1[0],l1[1]))
        swap(l1[0], l1[1]);
    if(!Point::LexCmp(l2[0],l2[1]))
        swap(l2[0], l2[1]);
    if(AreParallel(l1, l2)) {
        if(!PtBelongToLine(l2[0],l1))
            return vector<Point>();
        vector<Point> ends(2);
        for (int tr = 0; tr < 2; tr++)
            if (Point::LexCmp(l1[tr], l2[tr]) ^ tr)
                ends[tr] = l2[tr];
            else ends[tr] = l1[tr];
        if ((ends[1] - ends[0]).IsZero())
            ends.pop_back();
    }

```

```

    if(SZ(ends)==2&&Point::LexCmp(ends[1],ends[0]))
        return vector<Point>();
    return ends;
}
else {
    vector<Point> p = InterLineLine(l1, l2);
    if(PtBelongToSeg(p[0],l1) &&
        PtBelongToSeg(p[0],l2))
        return p;
    return vector<Point>();
}
}
LD SegmentToSegmentDistance(Line l1,Line l2){//nt
    vector<Point> inter = InterSegs(l1, l2);
    if(inter.size() > 0) return 0.0;
    LD an=min(PtToSegment(l1[0],l2),
              PtToSegment(l1[1],l2));
    an = min(an, PtToSegment(l2[0], l1));
    an = min(an, PtToSegment(l2[1], l1));
    return an;
}
//0,1,2,3 pts.If 3 pts it means they are equal
vector<Point>InterCircleCircle(Circle a,Circle b){
    if (a.r + EPS < b.r) swap(a, b);
    if (a == b) {
        return vector<Point>{a.PointAtAngle(0),
                              a.PointAtAngle(2 * PI / 3),
                              a.PointAtAngle(4 * PI / 3)};
    }
    Point diff=b.center-a.center;LD dis=diff.Norm();
    LD ang = diff.Angle();
    LD longest=max(max(a.r,b.r),dis),per=a.r+b.r+dis;
    if (2 * longest > per + EPS)
        return vector<Point>();
    if (abs(2 * longest - per) < 2 * EPS)
        return vector<Point>{a.PointAtAngle(ang)};
    LD d=Acos((Sq(a.r)+Sq(dis)-Sq(b.r))/(2*a.r*dis));
    return vector<Point>{a.PointAtAngle(ang - d),
                        a.PointAtAngle(ang+d)};
}
vector<Point>InterCircleLine(Circle c,Line l){//ok
    Point proj = ProjPtToLine(c.center, l);
    LD dis_proj = c.center.Distance(proj);
    if (dis_proj > c.r + EPS) return vector<Point>();
    LD a = Sqrt(Sq(c.r) - Sq(dis_proj));
    Point dir = l[1] - l[0];
    LD dir_norm = dir.Norm();
    vector<Point> cands{proj + dir * (a / dir_norm),
                       proj - dir * (a / dir_norm)};
    if (cands[0].Dist(cands[1])<EPS)
        return vector<Point>{proj};
    return cands;
}
vector<Point>InterCircleSeg(Circle c, Line l){//ok
    vector<Point> from_line = InterCircleLine(c, l);
    vector<Point> res;
    for(auto p:from_line)
        if(PtBelongToSeg(p,l)) res.pb(p);
    return res;
}
vector<Point>TangencyPtsToCircle(Circle c,Point p){
    LD d = c.center.Distance(p);//ok

```

```

    if (d < c.r - EPS) return {};
    if (d < c.r + EPS) return {p};
    LD from_cent = (p - c.center).Angle();
    LD ang_dev = Acos(c.r / d);
    return {c.PointAtAngle(from_cent - ang_dev),
           c.PointAtAngle(from_cent + ang_dev)};
}
vector<Line> OuterTangents(Circle c1, Circle c2) {
    if(c1 == c2) { return {}; }//is it best choice?
    if(c1.r < c2.r) { swap(c1, c2); }
    if(c2.r + c1.center.Distance(c2.center) < c1.r-EPS)
        return {};
    if (abs(c1.r - c2.r) < EPS) {
        Point diff = c2.center - c1.center;
        Point R = diff.Rotate(PI/2)*(c1.r/diff.Norm());
        return {{c1.center + R, c2.center + R},
               {c1.center - R, c2.center - R}};
    }
    Point I = c1.center +
              (c2.center-c1.center)*(c1.r/(c1.r-c2.r));
    if (c2.r+c1.center.Distance(c2.center)<c1.r+EPS) {
        return {{I,I+(c2.center-c1.center).Rotate(PI/2)}};
    }
    vector<Point> to1 = TangencyPtsToCircle(c1, I);
    vector<Point> to2 = TangencyPtsToCircle(c2, I);
    vector<Line>res{{to1[0],to2[0]},{to1[1],to2[1]}};
    assert(Utils::PtBelongToLine(I, res[0]));
    assert(Utils::PtBelongToLine(I, res[1]));
    return res;
}
vector<Line> InnerTangents(Circle c1, Circle c2) {
    if (c1 == c2) return {};//surely best choice
    if (c1.r < c2.r) { swap(c1, c2); }
    LD d = c1.center.Distance(c2.center);
    if (d < c1.r + c2.r - EPS) { return {}; }
    Point I = c1.center +
              (c2.center-c1.center)*(c1.r/(c1.r+c2.r));
    if(d < c1.r + c2.r + EPS) return {{I,I+
              (c2.center-c1.center).Rotate(PI/2)}};
    vector<Point> to1 = TangencyPtsToCircle(c1, I);
    vector<Point> to2 = TangencyPtsToCircle(c2, I);
    vector<Line>res{{to1[0],to2[0]},{to1[1],to2[1]}};
    assert(Utils::PtBelongToLine(I, res[0]));
    assert(Utils::PtBelongToLine(I, res[1]));
    return res;
}
LD DiskInterArea(Circle c1, Circle c2) {
    if (c1.r < c2.r) swap(c1, c2);
    LD d = c1.center.Distance(c2.center);
    if (c1.r + c2.r < d + EPS) return 0;
    if (c1.r - c2.r > d - EPS) return PI * Sq(c2.r);
    LD al=Acos((Sq(d)+Sq(c1.r)-Sq(c2.r))/(2*d*c1.r));
    LD be=Acos((Sq(d)+Sq(c2.r)-Sq(c1.r))/(2*d*c2.r));
    return al * Sq(c1.r) + be * Sq(c2.r) -
           sin(2*al)*Sq(c1.r)/2-sin(2*be)*Sq(c2.r)/2;
}
Line RadicalAxis(Circle c1, Circle c2) {
    LD d = c1.center.Distance(c2.center);
    LD a = (Sq(c1.r) - Sq(c2.r) + Sq(d)) / (2 * d);
    Point Q = c1.center+(c2.center-c1.center)*(a/d);
    Point R = Q+(c2.center-c1.center).Rotate(PI/2);
    return Line(Q, R);
}

```

```

}
vector<Point> CirThroughAPtAndTngntToALineWithRad
    (Point p, Line l, LD r) { //not tested
    vector<Point> sol;
    Point norm = l.NormalVector();
    Line l1=l.shift(norm*r);
    Line l2=l.shift(norm*(-r));
    sol = InterCircleLine(Circle(p, r), l1);
    vector<Point> t=InterCircleLine(Circle(p, r),l2);
    for(auto pp : t) sol.push_back(pp);
    return sol;
}
vector<Point> CirTngntToTwoLinesWithRad
    (Line l1,Line l2,LD r) { // not tested
    vector<Point> sol;
    Point e1 = l1.NormalVector();
    Point e2 = l2.NormalVector();
    Line L1[2]={l1.shift(e1*r),l1.shift(e1*(-r))},
    L2[2]={l2.shift(e2 * r),l2.shift(e2 * (-r))};
    for(int i = 0; i < 2; i++) {
        for(int j = 0; j < 2; j++) {
            vector<Point> t = InterLineLine(L1[i],L2[j]);
            for(auto pp : t) sol.push_back(pp);
        }
    }
    return sol;
}
vector<Point> CirTanToTwoDisjointCirclesWithRadius
    (Circle c1, Circle c2, LD r) { // not tested
    c1.r += r; c2.r += r;
    return InterCircleCircle(c1, c2);
}

// CENTERS BEGIN
Point Bary(Point A,Point B,Point C,LD a,LD b,LD c){
    return (A * a + B * b + C * c) / (a + b + c);
}
Point Centroid(Point A, Point B, Point C) {
    return Bary(A, B, C, 1, 1, 1);
}
Point Circumcenter(Point A, Point B, Point C) {
    LD a = (B - C).SqNorm(), b = (C - A).SqNorm();
    LD c = (A - B).SqNorm();
    return Bary(A,B,C,a*(b+c-a),b*(c+a-b),c*(a+b-c));
}
Point Incenter(Point A, Point B, Point C) {
    return Bary(A,B,C,(B-C).Norm(),
        (A-C).Norm(),(A-B).Norm());
}
Point Orthocenter(Point A, Point B, Point C) {
    LD a=(B-C).SqNorm(),b=(C-A).SqNorm();
    LD c=(A-B).SqNorm();
    return Bary(A,B,C,(a+b-c)*(c+a-b),
        (b+c-a)*(a+b-c),(c+a-b)*(b+c-a));
}
Point Excenter(Point A,Point B,Point C){ //opp to A
    LD a=(B-C).Norm(),b=(A-C).Norm(),c=(A-B).Norm();
    return Bary(A, B, C, -a, b, c);
}

```

3.5 Geometry 2D Polygon

```

// Cut Polygon (not tested)

```

```

void ints(vector<Point> &V,Point a,Point b,Line l){
    Point p = l[0], q = l[1];
    LD na = (a-p).cross(q-p), nb = (b-p).cross(q-p);
    if (na*nb < 0.0)
        V.push_back(a + (b-a)*(na/(na-nb)));
}
vector<Point> cut(vector<Point> polygon,Line l,
    int sign){
    vector<Point> np; int sz = polygon.size();
    for(int i = 0 ; i < sz ; i++) {
        Point p = polygon[i], q = polygon[(i+1)%sz];
        if(dcmp(l.Dir().cross(p))*sign>=0) np.pb(p);
        ints(np, p, q, l);
    }
    return np;
}
//diameter of a convex polygon p (not tested)
LD rotating_calipers(vector<Point> p) {
    int q = 1, n = p.size(); LD ans = 0;
    for( int i = 0; i < n; i++) {
        while(p[i].cross(p[(i+1)%n],p[(q+1)%n]) >
            p[i].cross(p[(i+1)%n],p[q])) q = (q+1)%n;
        LD t1=(p[i]-p[q]).Norm();
        LD t2=(p[(i+1)%n]-p[q]).Norm();
        ans = max(ans, max(t1, t2));
    }
    return ans;
}
//minimum area rect for convex polygon(!tested)
LD rec_rotating_calipers(vector<Point> p) {
    int n=p.size(),l=0,r=0,q=1;
    LD ans1=1e15, ans2=1e15;
    for( int i = 0; i < n; i++) {
        while(dcmp(p[i].cross(p[(i+1)%n],p[(q+1)%n])
            -p[i].cross(p[(i+1)%n],p[q])) > 0) q=(q+1)%n;
        while(dcmp((p[(i+1)%n]-p[i]).dot
            (p[(r+1)%n]-p[r]))>0) r=(r+1)%n;
        if (!i) l = q;
        while(dcmp((p[(i+1)%n]-p[i]).dot
            (p[(l+1)%n]-p[l]))<0) l=(l+1)%n;
        LD d = (p[(i+1)%n]-p[i]).Norm();
        LD h = p[i].cross(p[(i+1)%n],p[q])/d;
        LD w = ((p[(i+1)%n]-p[i]).dot(p[r]-p[i]))
            -((p[(i+1)%n]-p[i]).dot(p[l]-p[i])))/d;
        ans1 = min(ans1,2*(h+w)),ans2 = min(ans2,h*w);
    }
    return ans2;
}
struct Polygon {
    vector<Point> pts;
    Polygon(vector<Point> pts_) : pts(pts_) {}
    Polygon() : Polygon(vector<Point>()) {}
    void Add(Point p) { pts.push_back(p);}
    // positive for counterclockwise
    LD Area() {
        LD area = 0;
        for(int i = 0; i < SZ(pts); i++)
            area += pts[i].cross(pts[(i + 1) % SZ(pts)]);
        area /= 2; return area;
    }
}
void OrientCounterclockwise() {
    if (Area()<0) reverse(pts.begin(), pts.end());
}

```

```

}
int next(int a) {
    if (a + 1 < SZ(pts)) return a + 1;
    return 0;
}
pair<int, int> FurthestPair() {
    MakeConvexHull(); OrientCounterclockwise();
    int furth = 1;
    pair<int, int> best_pair = make_pair(0, 0);
    LD best_dis = 0;
    for (int i = 0; i < SZ(pts); i++) {
        Point side = pts[next(i)] - pts[i];
        while(side.cross(pts[furth] - pts[i])
            < side.cross(pts[next(furth)]-pts[i])){
            furth = next(furth);
        }
        vector<int> vec{i, next(i)};
        for (auto ind : vec) {
            if (pts[ind].Dist(pts[furth]) > best_dis){
                best_pair = make_pair(ind, furth);
                best_dis = pts[ind].Dist(pts[furth]);
            }
        }
    }
    return best_pair;
}
void MakeConvexHull() {
    vector<Point> one_way_hull[2];
    sort(pts.begin(), pts.end(), Point::LexCmp);
    for (int dir = -1; dir <= 1; dir += 2) {
        int hull_num = (dir + 1) / 2;
        auto& H = one_way_hull[hull_num];
        one_way_hull[hull_num].push_back(pts[0]);
        if (SZ(pts) > 1) {
            H.push_back(pts[1]);
        }
        for(int i = 2; i < SZ(pts); i++) {
            while(SZ(H)>=2&&dir*(pts[i]-H[SZ(H)-2]).
                cross(H.back()-H[SZ(H)-2]) > -EPS){
                H.pop_back();
            }
            H.push_back(pts[i]);
        }
    }
    pts.clear();
    for(auto p:one_way_hull[1])pts.push_back(p);
    for(int i = SZ(one_way_hull[0])-2; i >= 1; i--)
        pts.push_back(one_way_hull[0][i]);
}
// without sides
vector<vector<bool>> InsideDiagonalsMatrix() {
    int n = pts.size();
    vector<vector<bool>> res(n, vector<bool>(n));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            Line diag(pts[i], pts[j]);
            if(i==j || abs(i-j)==1 || abs(i-j)==n-1)
                continue;
            res[i][j] = 1;
            for (int k = 0; k < n; k++) {
                int kk = next(k);
                Line side(pts[k], pts[kk]);
            }
        }
    }
}

```



```

    if(k==i || k==j || kk==i || kk==j)
        continue;
    vector<Point>inter =
        Utils::InterSegs(diag,side);
    if (SZ(inter)) res[i][j] = 0;
}
int act = next(i), passed_j = 0;
LD areas[2] = {0, 0};
while (act != i) {
    passed_j |= (act == j);
    LD t =
        pts[i].cross(pts[act],pts[next(act)]);
    areas[passed_j] += t; act = next(act);
}
if (areas[0] * areas[1] < EPS)
    res[i][j] = 0;
}
}
return res;
};
// CLIP START
bool InUpper(Point a) {
    if (abs(a.y) > EPS) return a.y > 0;
    return a.x > 0;
}
bool angle_cmp(const Point a, const Point b) {
    bool u = InUpper(a), v = InUpper(b);
    return u!=v ? u : a.cross(b)>0;
}
LD cross(Point a, Point b, Point c, Point d) {
    return (d-c).cross(a-c) / (d - c).cross(a - b);
}
struct ClipLine { // valid side is on left
    ClipLine(Point A, Point B) {
        al = A, bl = B, a = A, b = B;
    }
    Point al,bl; // original line points
    mutable Point a,b; // actual intersection points
    Point dir() const { return bl - al; }
    bool operator<(const ClipLine& l) const {
        return angle_cmp(dir(),l.dir());
    }
    Point cross(const ClipLine& l) {
        return al+(bl-al)*::cross(al,bl,l.al,l.bl);
    }
    bool left(Point p){return(bl-al).cross(p-al)>0;}
};
struct Clip {
    Clip(LD r) : area(4*r*r) {
        Point a{-r,-r}, b{r,-r}, c{r,r}, d{-r,r};
        lines = {ClipLine(a,b), ClipLine(b,c),
            ClipLine(c,d), ClipLine(d,a)};
    }
    void insert(Line l){insert(ClipLine(l[0],l[1]));}
    void insert(ClipLine l) {
        assert(abs(l.dir().SqNorm()) > EPS); find(l);
        while(size()&&!l.left(it->a)&&!l.left(it->b))
            erase();
        if(size())
            while(prev(),size() && !l.left(it->a) &&
                !l.left(it->b)) erase();
    }
};

```

```

    if(size()&&(!l.left(it->a) || !l.left(it->b))) {
        l.a = l.cross(*it);
        area -= l.a.cross(it->b)*.5;
        it->b = l.a; next();
        l.b = l.cross(*it);
        if ((l.a-l.b).SqNorm() < EPS) l.b = l.a;
        area -= it->a.cross(l.b) * .5;
        it->a = l.b;
        if (!(l.a - l.b).IsZero()) {
            area += l.a.cross(l.b)*.5;
            lines.insert(l);
        }
    }
}
void find(const ClipLine &l) {
    it = lines.lower_bound(l);
    if(it == lines.end()) { it = lines.begin(); }
}
void recalculate() {
    area = 0;
    for(const ClipLine &l : lines)
        area+=l.a.cross(l.b);
    area *= .5;
}
int size() { return lines.size(); }
void next(){if(++it==lines.end())
    it = lines.begin();}
void prev(){
    if(it==lines.begin()) it=lines.end();--it;
}
void erase() {
    assert(it!=lines.end());
    area -= it->a.cross(it->b)*.5;
    it = lines.erase(it);
    if(it==lines.end()) it = lines.begin();
}
typename set<ClipLine>::iterator it;
set<ClipLine> lines;
LD area;
};

```

3.6 Geometry 3D Basic

```

struct Point3 {
    LD x, y, z;
    Point3() {}
    Point3(LD a, LD b, LD c) : x(a), y(b), z(c){}
    void operator=(const Point3& a) {
        x=a.x,y=a.y,z=a.z;
    }
    Point3 operator+(Point3 a) {
        Point3 p{x + a.x, y + a.y, z + a.z}; return p;
    }
    Point3 operator-(Point3 a) {
        Point3 p{x - a.x, y - a.y, z - a.z}; return p;
    }
    Point3 operator*(LD a) {
        return Point3(x*a,y*a,z*a);
    }
    Point3 operator/(LD a) {
        assert(a > EPS); Point3 p{x/a, y/a, z/a};
        return p;
    }
    LD& operator[](int a) {

```

```

        if (a == 0) return x;
        if (a == 1) return y;
        if (a == 2) return z;
        assert(false);
    }
    bool IsZero() {
        return abs(x)<EPS&& abs(y)<EPS && abs(z) < EPS;
    }
    bool operator==(Point3 a) {
        return (*this - a).IsZero();
    }
    LD dot(Point3 a) {
        return x * a.x + y * a.y + z * a.z;
    }
    LD Norm() {
        return Sqrt(x * x + y * y + z * z);
    }
    LD SqNorm() {
        return x * x + y * y + z * z;
    }
    void NormalizeSelf() {
        *this = *this/Norm();
    }
    Point3 Normalize() {
        Point3 res(*this); res.NormalizeSelf();
        return res;
    }
    LD Dis(Point3 a) {
        return (*this - a).Norm();
    }
    pair<LD, LD> SphericalAngles() {
        return {atan2(z,Sqrt(x*x+y*y)),atan2(y,x)};
    }
    LD Area(Point3 p) {
        return Norm() * p.Norm() * sin(Angle(p)) / 2;
    }
    // LD Angle(Point3 p) {
    //     LD a = Norm(), b = p.Norm(), c = Dis(p);
    //     return Acos((a*a+b*b-c*c)/(2*a*b));
    // }
    LD Angle(Point3 b) { // not tested
        Point3 a(*this);
        return Acos(abs(a.dot(b))/a.Norm()/b.Norm());
    }
    LD Angle(Point3 p, Point3 q){return p.Angle(q);}
    Point3 cross(Point3 p) {
        Point3 q(*this);
        return {q[1]*p[2] - q[2]*p[1], q[2]*p[0] -
            q[0] * p[2], q[0] * p[1] - q[1] * p[0]};
    }
    bool LexCmp(Point3& a, const Point3& b) {
        if (abs(a.x - b.x) > EPS) { return a.x < b.x;}
        if (abs(a.y - b.y) > EPS) { return a.y < b.y;}
        return a.z < b.z;
    }
};
struct Line3 {
    Point3 p[2];
    Line3() {}
    Line3(Point3 a, Point3 b) { p[0] = a, p[1] = b;}
    Point3& operator[](int a) { return p[a]; }
};

```



```

struct Plane {
    Point3 p[3];
    Point3& operator[](int a) { return p[a]; }
    Plane(Point3 p0, Point3 p1, Point3 p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
    }
    // Ax + By + Cz = D
    Plane(Point3 normal, LD D) {
        /// to do, update p[0], p[1], p[2]
    }
    Point3 GetNormal() {
        Point3 cross = (p[1]-p[0]).cross(p[2]-p[0]);
        return cross.Normalize();
    }
    void GetPlaneEq(LD& A, LD& B, LD& C, LD& D) {
        Point3 normal = GetNormal();
        A = normal[0], B = normal[1], C = normal[2];
        D = normal.dot(p[0]);
        assert(abs(D - normal.dot(p[1])) < EPS);
        assert(abs(D - normal.dot(p[2])) < EPS);
    }
    vector<Point3> GetOrtonormalBase() {
        Point3 normal = GetNormal();
        Point3 cand = {-normal.y, normal.x, 0};
        if (abs(cand.x) < EPS && abs(cand.y) < EPS)
            cand = {0, -normal.z, normal.y};
        cand.NormalizeSelf();
        Point3 third = Plane{Point3{0, 0, 0},
                               normal, cand}.GetNormal();
        return {normal, cand, third};
    }
};
struct Circle3 {
    Plane pl; Point3 cent; LD r;
};
struct Sphere {
    Point3 cent; LD r;
};
namespace Utils3 {
    //angle PQR
    LD Angle(Point3 P, Point3 Q, Point3 R) {
        return (P - Q).Angle(R - Q);
    }
    LD Area(Point3 p, Point3 q, Point3 r) { // ok
        q = q-p; r = r-p; return q.Area(r);
    }
    LD DistPtLine(Point3 p, Line3 l) { // not tested
        return ((l[1]-l[0]).cross((p-l[0])).Norm()/
                (l[1]-l[0]).Norm());
    }
    Point3 ProjPtToLine3(Point3 p, Line3 l) { // ok
        Point3 diff = l[1]-l[0]; diff.NormalizeSelf();
        return l[0] + diff * (p - l[0]).dot(diff);
    }
    Point3 ProjPtSeg3(Point3 p, Line3 l) { //!tested
        LD r = (l[1]-l[0]).dot(l[1]-l[0]);
        if(abs(r) < EPS) return l[0];
        r = (p-l[0]).dot(l[1]-l[0])/r;
        if(r < 0) return l[0];
        if(r > 1) return l[1];
        return l[0] + (l[1]-l[0]) * r;
    }
}

```

```

LD DistPtSeg3(Point3 p, Line3 l) {
    Point3 q = ProjPtSeg3(p, l); return p.Dis(q);
}
LD DisPtLine3(Point3 p, Line3 l) { // ok
    LD dis2 = p.Dis(ProjPtToLine3(p, l));
    return dis2;
}
bool PtBelongToLine3(Point3 p, Line3 l) {
    return DisPtLine3(p, l) < EPS;
}
bool Lines3Equal(Line3 p, Line3 l) {
    return PtBelongToLine3(p[0],l) &&
           PtBelongToLine3(p[1],l);
}
bool OrientPointPlane(Point3 t,Plane p){ //!tested
    LD dot = p.GetNormal().dot(t - p[0]);
    return dcmp(dot);
}
Point3 ProjPtToPlane(Point3 p, Plane pl) {
    Point3 normal = pl.GetNormal();
    return p - normal * normal.dot(p - pl[0]);
}
LD DisPtPlane(Point3 p, Plane pl) {
    Point3 normal = pl.GetNormal();
    return abs(normal.dot(p - pl[0]));
}
bool PtBelongToPlane(Point3 p, Plane pl) {
    return DisPtPlane(p, pl) < EPS;
}
bool Line3BelongToPlane(Line3 l, Plane pl) {
    return PtBelongToPlane(l[0], pl) &&
           PtBelongToPlane(l[1], pl);
}
Plane ShiftUpDown(Plane p, LD dist) { //!tested
    Point3 n = p.GetNormal();
    LD d = p.GetNormal().dot(p[0]);
    return Plane(n, d + dist * n.Norm());
}
Plane ParallelPlane(Plane pl, Point3 A) {
    Point3 diff = A - ProjPtToPlane(A, pl);
    return Plane{pl[0]+diff,pl[1]+diff,pl[2]+diff};
}
//undefined for parallel line and plane(!tested)
Point3 InterLinePlane(Line3 l, Plane p) {
    Point3 norm = p.GetNormal();
    LD D = norm.dot(p[0]);
    LD k =
        (D - (norm.dot(l[0])))/(norm.dot(l[1]-l[0]));
    return l[0] + (l[1]-l[0])*k;
}
// not tested, assumes planes are not parallel
Line3 InterPlanePlane(Plane p1, Plane p2) {
    Point3 n1=p1.GetNormal(),n2 = p2.GetNormal();
    LD d1 = n1.dot(p1[0]), d2 = n2.dot(p2[0]);
    Point3 dir = n1.cross(n2);
    assert(!dir.IsZero()); // parallel plane
    Point3 u =
        (n2*d1 - n1*d2).cross(dir)/dir.dot(dir);
    return Line3(u, u + dir);
}
Point PlanePtTo2D(Plane pl, Point3 p) { // ok

```

```

    assert(PtBelongToPlane(p, pl));
    vector<Point3> base = pl.GetOrtonormalBase();
    Point3 control{0, 0, 0};
    for (int tr = 0; tr < 3; tr++) {
        control=control+base[tr] * p.dot(base[tr]);
    }
    assert(PtBelongToPlane(pl[0] + base[1], pl));
    assert(PtBelongToPlane(pl[0] + base[2], pl));
    assert((p - control).IsZero());
    return {p.dot(base[1]), p.dot(base[2])};
}
Line PlaneLineTo2D(Plane pl, Line3 l) {
    return
        {PlanePtTo2D(pl,l[0]),PlanePtTo2D(pl,l[1])};
}
Point3 PlanePtTo3D(Plane pl, Point p) { // ok
    vector<Point3> base = pl.GetOrtonormalBase();
    return base[0]*base[0].dot(pl[0]) +
           base[1]*p.x+base[2]*p.y;
}
Line3 PlaneLineTo3D(Plane pl, Line l) {
    return Line3{PlanePtTo3D(pl,l[0]),
                 PlanePtTo3D(pl, l[1])};
}
Line3 ProjLineToPlane(Line3 l, Plane pl) { // ok
    return Line3{ProjPtToPlane(l[0], pl),
                 ProjPtToPlane(l[1], pl)};
}
Point3 ClosestPtOnL1FromL2(Line3 l1,Line3 l2){
    Point3 n = (l1[1]-l1[0]).cross(l2[1]-l2[0]);
    Point3 n3 = (l2[1]-l2[0]).cross(n); //!tested
    ///p is the plane including line l2 and n
    Plane p = Plane(n3, n3.dot(l2[0]));
    return InterLinePlane(l1, p);
}
vector<Point3> InterLineLine(Line3 k, Line3 l) {
    if (Lines3Equal(k, l)) { return {k[0], k[1]}; }
    if (PtBelongToLine3(l[0], k)) { return {l[0]}; }
    Plane pl{l[0], k[0], k[1]};
    if (!PtBelongToPlane(l[1], pl)) { return {}; }
    Line k2 = PlaneLineTo2D(pl, k);
    Line l2 = PlaneLineTo2D(pl, l);
    vector<Point> inter=Utils::InterLineLine(k2,l2);
    vector<Point3> res;
    for (auto P:inter) res.pb(PlanePtTo3D(pl, P));
    return res;
}
LD DisLineLine (Line3 l1, Line3 l2){ //!tested
    Point3 dir = (l1[1]-l1[0]).cross(l2[1]-l2[0]);
    if(dcmp(dir.Norm())==0)
        return DisPtLine(l2[0],l1);
    return abs((l2[0]-l1[0]).dot(dir))/dir.Norm();
}
/**
LD DisLineLine(Line3 l, Line3 k) { //ok(para fix)
    Plane together{l[0],l[1],l[0]+k[1]-k[0]};
    Line3 proj = ProjLineToPlane(k, together);
    Point3 inter =
        (Utils3::InterLineLine(l,proj))[0];
    Point3 on_k_inter = k[0] + inter - proj[0];
    return inter.Dis(on_k_inter);
}
*/

```

```

LD Det(Point3 a, Point3 b, Point3 d) { // ok
    Point3 pts[3] = {a, b, d};
    LD res = 0;
    for (int sign : {-1, 1}) {
        for (int st_col=0; st_col<3; st_col++) {
            int c = st_col;
            LD prod = 1;
            for (int r=0; r<3; r++){
                prod *= pts[r][c];
                c = (c + sign + 3) % 3;
            }
            res += sign * prod;
        }
    }
    return res;
}
Point3 PtFromSphericalAng(LD al, LD be) { // ok
    return {cos(al)*cos(be), cos(al)*sin(be), sin(al)};
}
//img of B in rot wrt line
//passing thru orig s.t. A1->A2
Point3 RotateAccordingly
    (Point3 A1, Point3 A2, Point3 B1) { // ok
    Plane pl{A1, A2, {0, 0, 0}};
    Point A12 = PlanePtTo2D(pl, A1);
    Point A22 = PlanePtTo2D(pl, A2);
    complex<LD> rat = complex<LD>(A22.x, A22.y) /
        complex<LD>(A12.x, A12.y);
    Plane plb = ParallelPlane(pl, B1);
    Point B2 = PlanePtTo2D(plb, B1);
    complex<LD> Brot = rat*complex<LD>(B2.x, B2.y);
    return
        PlanePtTo3D(plb, {Brot.real(), Brot.imag()});
}
vector<Point3> InterLineSphere(Line3 l, Sphere s){
    vector<Point3> ints; // not tested
    LD h2 = Sq(s.r) - Sq(DisPtLine3(s.cent, l));
    if(dcmp(h2) < 0) return ints;
    if(dcmp(h2) == 0){
        ints.push_back(ProjPtToLine3(s.cent, l));
        return ints;
    }
    Point3 v = ProjPtToLine3(s.cent, l);
    Point3 dir = l[1] - l[0];
    Point3 h = dir * Sqrt(h2)/dir.Norm();
    ints.push_back(v+h); ints.push_back(v-h);
    return ints;
}
vector<Circle3> InterPlaneSphere
    (Plane pl, Sphere s){//ok
    Point3 proj = ProjPtToPlane(s.cent, pl);
    LD dis = s.cent.Dis(proj);
    if (dis > s.r + EPS) {
        return {};
    }
    if (dis > s.r - EPS) {
        return {{pl, proj, 0}}; // is it best choice?
    }
    return {{pl, proj, Sqrt(s.r*s.r-dis*dis)}};
}
bool PtBelongToSphere(Sphere s, Point3 p) {
    return abs(s.r - s.cent.Dis(p)) < EPS;
}

```

```

}
LD DisOnSphere(Sphere sph, Point3 A, Point3 B) {
    assert(PtBelongToSphere(sph, A));
    assert(PtBelongToSphere(sph, B));
    LD ang = Angle(A, sph.cent, B);
    return ang * sph.r;
}
}
bool InsideATriangle
    (Point3 a, Point3 b, Point3 c, Point3 p){
    Plane abc = Plane(a, b, c);
    if(!Utils3::PtBelongToPlane(p, abc)) return 0;
    Point3 n = abc.GetNormal();
    vector<int> sign(3);
    for(int i = 0; i < 3; i++) {
        LD t = n.dot((abc[(i+1)%3]-abc[i]).
            cross(p-abc[i]));
        sign[i] = dcmp(t);
    }
    if(sign[0]>=0&&sign[1]>=0&&sign[2]>=0) return 1;
    if(sign[0]<=0&&sign[1]<=0&&sign[2]<=0) return 1;
    return 0;
}
LD PtDistOn3dTriangle
    (Point3 a, Point3 b, Point3 c, Point3 p){
    Plane abc = Plane(a, b, c);
    Point3 p_ = Utils3::ProjPtToPlane(p, abc);
    LD ret = 1e19;
    if(InsideATriangle(a, b, c, p_))
        ret = min(ret, p.Dis(p_));
    ret = min(ret, Utils3::DistPtSeg3(p, Line3(a, b)));
    ret = min(ret, Utils3::DistPtSeg3(p, Line3(b, c)));
    ret = min(ret, Utils3::DistPtSeg3(p, Line3(a, c)));
    return ret;
}
struct Face{
    Point3 a, b, c;
    Face(){
        Face(Point3 a, Point3 b, Point3 c):a(a),b(b),c(c){}
        Face(const Face &f) : a(f.a), b(f.b), c(f.c) {}
    };
}
LD polyhedronVolume(vector<Face> &vec) { //!tested
    if(vec.size() == 0) return 0;
    Point3 reff = vec[0].a; LD vol = 0;
    for(int i = 1; i < vec.size(); i++) {
        Point3 ar = (vec[i].b-vec[i].a).
            cross(vec[i].c - vec[i].a);
        vol += abs(ar.dot(reff-vec[i].a));
    }
    return vol/6.0;
}
vector<Face> Convex3dHull(vector<Point3> &V){//nt
    vector<Face> Faces;
    for(int i = 0; i < V.size(); i++) {
        for(int j = i+1; j < V.size(); j++) {
            for(int k = j+1; k < V.size(); k++) {
                if(((V[j]-V[i]).cross(V[k]-V[i])).Norm()
                    < EPS) continue;
                bool up = 0, down = 0;
                Plane P(V[i], V[j], V[k]);
                Point3 normal = P.GetNormal();
                for(int l = 0; l < V.size(); l++) {
                    if (l == i or l == j or l == k)

```

```

                        continue;
                    if(InsideATriangle(V[i],V[j],V[k],V[l])){
                        up = down = 1;
                        break;
                    }
                    else if(normal.dot(V[l]-V[i])<0) down=1;
                    else up = 1;
                }
                if(up == 0 or down == 0) {
                    Face temp;
                    temp.a=V[i],temp.b=V[j],temp.c=V[k];
                    Faces.push_back(temp);
                }
            }
        }
    }
    return Faces;
}
struct PointS {
    LD lat, lon;
    PointS(LD latt, LD lonn) {lat=latt; lon=lonn;}
    Point3 toEucl() {
        return Point3{cos(lat)*cos(lon),
            cos(lat)*sin(lon), sin(lat)};
    }
    PointS(Point3 p) {
        p.NormalizeSelf(); lat = Asin(p.z);
        lon = Acos(p.y / cos(lat));
    }
};
LD DistS(Point3 a, Point3 b) {
    return atan2l(b.cross(a).Norm(), a.dot(b));
}
struct CircleS {
    Point3 o; // center of circle on sphere
    LD r; // arc len
    LD area() const { return 2*PI*(1 - cos(r)); }
};
CircleS From3(Point3 a, Point3 b, Point3 c){
    int tmp = 1; //any 3 dif pts
    if((a-b).Norm()>(c-b).Norm()){
        swap(a, c); tmp = -tmp;
    }
    if((b-c).Norm()>(a-c).Norm()){
        swap(a, b); tmp = -tmp;
    }
    Point3 v=(c-b).cross(b-a);
    v = v * (tmp / v.Norm());
    return CircleS{v, DistS(a, v)};
}
CircleS From2(Point3 a, Point3 b){//nei same nor opp
    Point3 mid = (a + b) / 2;
    mid = mid / mid.Norm();
    return From3(a, mid, b);
}
//angle at A, no two points opposite
LD Angle(Point3 A, Point3 B, Point3 C) {
    LD a = B.dot(C), b = C.dot(A), c = A.dot(A);
    return Acos((b-a*c)/Sqrt((1-Sq(a))*(1-Sq(c))));
}
// no two points opposite
LD TriangleArea(Point3 A, Point3 B, Point3 C) {
    LD a = Angle(C, A, B), b = Angle(A, B, C);

```

```

    LD c = Angle(B,C,A);
    return a + b + c - PI;
}
// what about c1==c2 case?
vector<Point3> IntersectionS
    (CircleS c1, CircleS c2) {
    Point3 n = c2.o.cross(c1.o);
    Point3 w = c2.o * cos(c1.r) - c1.o * cos(c2.r);
    LD d = n.SqNorm();
    if (d < EPS) {
        cerr<<"parallel circles?\n";
        return {};
    }
    LD a = w.SqNorm() / d; vector<Point3> res;
    if (a >= 1 + EPS) return res;
    Point3 u = n.cross(w) / d;
    if (a > 1 - EPS) {
        res.pb(u); return res;
    }
    LD h = Sqrt((1 - a) / d);
    res.pb(u + n * h);
    res.pb(u - n * h);
    return res;
}

```

3.7 Geometry 3D Convex Hull

```

typedef vector<Point3> face;
typedef vector<Point3> edge;
typedef vector<face> hull;
#define INSIDE (-1)
#define ON (0)
#define OUTSIDE (1)
int side(Point3 a, Point3 b, Point3 c, Point3 p){
    Point3 norm = (b-a).cross(c-a);
    Point3 me = p-a;
    return dcmp(me.dot(norm));
}
hull find_hull(vector<Point3> P) {
    random_shuffle(P.begin(), P.end());
    int n = P.size();
    for(int j = 2; j < n; j++) {
        Point3 n = (P[1]-P[0]).cross(P[j]-P[0]);
        if(n.Norm() > EPS) {swap(P[j], P[2]);break;}
    }
    for(int j = 3; j < n; j++) {
        if(side(P[0],P[1],P[2],P[j])) {
            swap(P[j], P[3]); break;
        }
    }
    if(side(P[0],P[1],P[2],P[3]) == OUTSIDE)
        swap(P[0], P[1]);
    hull H{ {P[0],P[1],P[2]}, {P[0],P[3],P[1]},
        {P[0],P[2],P[3]}, {P[3],P[2],P[1]}};
    auto make_degrees = [&](const hull& H) {
        map<edge,int> ans;
        for(const auto & f : H) {
            for(int i = 0; i < 3; i++){
                Point3 a = f[i], b = f[(i+1)%3];
                ans[{a,b}]++;
            }
        }
        return ans;
    }
}

```

```

};
for(int j = 4; j < n; j++) {
    hull H2; H2.reserve((int)H.size());
    vector<face> plane;
    for(const auto & f : H) {
        int s = side(f[0],f[1],f[2],P[j]);
        if (s == INSIDE || s == ON) H2.pb(f);
    }
    //For any edge that now only has 1 incident
    //face (it's other face deleted) add a new
    //face with this vertex and that edge.
    map<edge, int> D = make_degrees(H2);
    const auto tmp = H2;
    for (const auto & f : tmp) {
        for(int i = 0; i < 3; i++) {
            Point3 a = f[i], b = f[(i+1)%3];
            int d = D[{a,b}] + D[{b,a}];
            if (d==1) H2.pb({a, P[j], b}); //a new face
        }
    }
    H = H2;
}
return H;
}

```

3.8 Geometry Hull

```

int dcmp(int x) {
    if(x < 0) return -1;
    return x > 0;
}
struct Point {
    int x, y;
    Point() {}
    Point(int a, int b) : x(a), y(b) {}
    Point(const Point& a) : x(a.x), y(a.y) {}
    void operator=(const Point& a){x=a.x;y=a.y;}
    Point operator+(const Point& a) const {
        Point p(x + a.x, y + a.y); return p;
    }
    Point operator-(const Point& a) const {
        Point p(x - a.x, y - a.y); return p;
    }
    Point operator*(int a)const {
        return Point(x*a,y*a);
    }
    Point operator/(int a)const{
        return Point(x/a, y/a);
    }
    int cross(const Point& a)const {
        return x * a.y - y * a.x;
    }
    int cross(Point a, Point b) const {
        a = a - *this; b = b - *this; return a.cross(b);
    }
    int DotProd(const Point& a) const {
        return x * a.x + y * a.y;
    }
    Point Rotate90() { return Point(-y, x); }
    bool operator < (const Point &p) const{
        return make_pair(x, y) < make_pair(p.x, p.y);
    }
    bool operator > (const Point &p) const {

```

```

        return make_pair(x, y) > make_pair(p.x, p.y);
    }
    int SqNorm() { return x * x + y * y; }
};
bool OnSegment(Point p, Point a, Point b) {
    return (a-p).cross(b-p)==0&&(a-p).DotProd(b-p)<0;
}
int isPointInPolygon(Point p,vector<Point> &poly){
    int wn = 0, n = poly.size();
    for(int i = 0; i < n; i++) {
        if(OnSegment(p,poly[i],poly[(i+1)%n]))
            return -1; //on edge
        int k=(poly[(i+1)%n]-poly[i]).cross(p-poly[i]);
        int d1 = poly[i].y-p.y;
        int d2 = poly[(i+1)%n].y-p.y;
        if (k > 0 && d1 <= 0 && d2 > 0) wn++;
        if (k < 0 && d2 <= 0 && d1 > 0) wn--;
    }
    if (wn != 0) return 1; //inside
    return 0; //outside
}
// returns 1 if p is on or inside triangle(a,b,c)
bool PointInTriangle
    (Point a,Point b,Point c,Point p) {
    int d1 = dcmp((b-a).cross(p-b));
    int d2 = dcmp((c-b).cross(p-c));
    int d3 = dcmp((a-c).cross(p-a));
    return !(((d1 < 0) || (d2 < 0) || (d3 < 0)) &&
        ((d1 > 0) || (d2 > 0) || (d3 > 0)));
}
struct ConvexHull {
    vector<Point> hull, lr, ur; int n;
    // builds convex hull of a set of points
    bool ccw(Point p, Point q, Point r) {
        return p.cross(q, r) > 0;
    }
    int cross(Point p, Point q, Point r) {
        return (q-p).cross(r-q);
    }
    Point LineLineIntersection
        (Point p1, Point p2, Point q1, Point q2) {
        int a1 = cross(q1,q2,p1), a2 = -cross(q1,q2,p2);
        return (p1 * a2 + p2 * a1) / (a1 + a2);
    }
    void init(vector<Point> &poly) {
        hull.clear(), lr.clear(), ur.clear();
        sort(poly.begin(), poly.end());
        for(int i = 0; i < poly.size(); i++) {
            while(lr.size() >= 2 &&
                !ccw(lr[lr.size()-2],lr.back(),poly[i]))
                lr.pop_back();
            lr.push_back(poly[i]);
        }
        for(int i = (int)poly.size()-1; i >= 0; i--){
            while(ur.size() >= 2 &&
                !ccw(ur[ur.size()-2],ur.back(),poly[i]))
                ur.pop_back();
            ur.push_back(poly[i]);
        }
        hull = lr;
        for(int i = 1; i+1 < ur.size(); i++)
            hull.push_back(ur[i]);
    }
}

```



```

    n = hull.size();
}
int sign(int x) {
    if (x < 0) return -1; return x > 0;
}
int crossOp(Point p, Point q, Point r) {
    int c = (q-p).cross(r-q); if (c < 0) return -1;
    return (c > 0);
}
//tests if p is inside or on the convex poly
//if Pt p is on side a,b is the idx of two ends
bool contain(Point p, int&a, int&b) {
    if (p.x < lr[0].x || p.x > lr.back().x) return 0;
    int id = lower_bound(lr.begin(),
        lr.end(), Point(p.x, -INF)) - lr.begin();
    if (lr[id].x == p.x) {
        if (lr[id].y > p.y) return 0;
    } else {
        if (crossOp(lr[id-1], lr[id], p) < 0) return 0;
        if (crossOp(lr[id-1], lr[id], p) == 0) {
            a = id - 1; b = id;
            return 1;
        }
    }
    id = lower_bound(ur.begin(), ur.end(), Point
        (p.x, INF), greater<Point>()) - ur.begin();
    if (ur[id].x == p.x) {
        if (ur[id].y < p.y) return 0;
    } else {
        if (crossOp(ur[id-1], ur[id], p) < 0) return 0;
        if (crossOp(ur[id-1], ur[id], p) == 0) {
            a = id - 1 + lr.size() - 1;
            b = id + lr.size() - 1;
            return 1;
        }
    }
    return 1;
}
int find(vector<Point> &vec, Point dir) {
    int l = 0, r = vec.size();
    while (l+5 < r) {
        int L = (l*2+r)/3, R = (l+r*2)/3;
        if (vec[L].DotProd(dir) > vec[R].DotProd(dir))
            r = R;
        else
            l = L;
    }
    int ret = l;
    for (int k = l+1; k < r; k++)
        if (vec[k].DotProd(dir) > vec[ret].DotProd(dir))
            ret = k;
    return ret;
}
//rays frm inf in dir, returns the furthest Pt
int findFarest(Point dir) {
    if (sign(dir.y) > 0 || sign(dir.y) == 0 &&
        sign(dir.x) > 0) {
        return ((int)lr.size()-1 + find(ur, dir))%n;
    } else {
        return find(lr, dir);
    }
}
Point get(int l, int r, Point p1, Point p2) {

```

```

    int s1 = crossOp(p1, p2, hull[l%n]);
    while (l+1 < r) {
        int m = (l+r)>>1;
        if (crossOp(p1, p2, hull[m%n]) == s1) l = m;
        else r = m;
    }
    return LineLineIntersection
        (p1, p2, hull[l%n], hull[(l+1)%n]);
}
//Ints between line and convex polygon. O(log(n))
//touching the hull does not count as intersection
vector<Point> Line_Hull_Intersection
    (Point p1, Point p2) {
    int X = findFarest((p2-p1).Rotate90());
    int Y = findFarest((p1-p2).Rotate90());
    if (X > Y) swap(X, Y);
    if (crossOp(p1, p2, hull[X]) *
        crossOp(p1, p2, hull[Y]) < 0) {
        return {get(X, Y, p1, p2), get(Y, X+n, p1, p2)};
    } else {
        return {};
    }
}
void update_tangent(Point p, int id, int&a, int&b) {
    if (crossOp(p, hull[a], hull[id]) > 0) a = id;
    if (crossOp(p, hull[b], hull[id]) < 0) b = id;
}
void binary_search(int l, int r, Point p,
    int&a, int&b) {
    if (l==r) return;
    update_tangent(p, l%n, a, b);
    int s1 = crossOp(p, hull[l%n], hull[(l+1)%n]);
    while (l+1 < r) {
        int m = l+r>>1;
        if (crossOp(p, hull[m%n], hull[(m+1)%n]) == s1)
            l = m;
        else r = m;
    }
    update_tangent(p, r%n, a, b);
}
void get_tangent(Point p, int&a, int&b) {
    if (contain(p, a, b)) return;
    a = b = 0;
    int id = lower_bound(lr.begin(), lr.end(), p,
        greater<Point>()) - lr.begin();
    binary_search(0, id, p, a, b);
    binary_search(id, lr.size(), p, a, b);
    id = lower_bound(ur.begin(), ur.end(), p,
        greater<Point>()) - ur.begin();
    binary_search((int)lr.size() - 1,
        (int)lr.size() - 1 + id, p, a, b);
    binary_search((int)lr.size() - 1 + id,
        (int)lr.size() - 1 + ur.size(), p, a, b);
}
};

```

3.9 Line Polygon Intersection

```

//Dist from O to OX intersect AB, div by len(OX)
double distance(PT O, PT X, PT A, PT B) {
    B = B-A; A = A-O; X=X-O;
    return 1.0*cross(A, B)/cross(X, B);
}

```

```

int sign(LL a) { return a==0 ? 0 : (a>0 ? 1:-1); }
//Special Pts are given by pair<double,int>(a,b)
//Let 0 be a special pt with OX,OY incident edges.
//Then a = AO/AB, b is an integer denoting type.
//CS: OX and OY are on the same side: ignore
//CS: OX and OY are on the different side: b=0
//Cs: OX on line, OY on side, b=sign(cross(A,B,Y))
double LinePoly(PT A, PT B, const vector<PT> &p) {
    int n = p.size();
    vector<pair<double, int>> special;
    for (int i=0; i<n; i++) {
        PT X = p[i], Y = p[(i+1)%n], W = p[(i-1+n)%n];
        LL crx = cross(A, B, X), cry = cross(A, B, Y),
            crw = cross(A, B, W);
        if (crx == 0) {
            double f;
            if (B.x != A.x) f = 1.0*(X.x-A.x)/(B.x-A.x);
            else f = 1.0*(X.y-A.y)/(B.y-A.y);
            if (sign(crw) && sign(cry))
                if (sign(crw) != sign(cry))
                    special.pb({f, 0});
            else if (sign(cry)) special.pb({f, sign(cry)});
            else if (sign(crw)) special.pb({f, sign(crw)});
        }
        else if (sign(crx) == -sign(cry)) {
            double f = distance(A, B, X, Y);
            special.push_back({f, 0});
        }
    }
    sort(special.begin(), special.end());
    bool active = false;
    int sgn = 0; //1st side sign if currtly linear,
    double prv = 0, ans = 0;
    for (auto &pr: special) {
        double d = pr.first; int tp = pr.second;
        if (sgn) {
            assert(sgn && tp);
            if (sgn != tp) active = !active;
            ans += d - prv; sgn = 0;
        }
        else {
            if (active) ans += d - prv;
            if (tp == 0) active = !active;
            else sgn = tp;
        }
        prv = d;
    }
    return ans*sqrt(dot(B-A, B-A));
}

```

3.10 Min Enclosing Circle

```

bool is_colinear(Point a, Point b, Point c) {
    return fabs((b-a).cross(c-a)) < EPS;
}
bool on(Point a, Point b, Point x) {
    LD t = (x-a).Norm()+(x-b).Norm()-(a-b).Norm();
    return fabs(t) < EPS;
}
bool in_circle(const Point& v, const Circle& C) {
    return (v - C.center).Norm() <= C.r + EPS;
}
Circle better(Circle A, Circle B) {

```



```

    if (A.r < B.r) return A;
    return B;
}
Circle find_circle(Point a) {return Circle(a,0);}
Circle find_circle(Point a, Point b) {
    return Circle((a+b)/2, (a-b).Norm()/2);
}
Circle find_circle(Point a, Point b, Point c,
                    bool force_on = false) {
    if(is_colinear(a,b,c)) {
        if(on(a,b,c))
            return Circle((a+b)/2, (a-b).Norm()/2);
        if(on(a,c,b))
            return Circle((a+c)/2, (a-c).Norm() / 2);
        if(on(c,b,a))
            return Circle((c+b)/2, (c-b).Norm() / 2 );
    }
    Point u = (b-a), v = (c-a);
    Point uperp = u.Rotate90(), vperp = v.Rotate90();
    Point ab = (a+b)/2, ac = (a+c)/2;
    Point ans=InterLineLine(ab,ab+uperp,ac,ac+vperp);
    LD rad = ((ans-a).Norm()+
              (ans-b).Norm()+ (ans-c).Norm())/3.01;
    Circle C = Circle(ans,rad);
    if (force_on) return C;
    Circle C_ab = find_circle(a,b);
    Circle C_bc = find_circle(b,c);
    Circle C_ac = find_circle(a,c);
    if(in_circle(c, C_ab)) C = better(C, C_ab);
    if(in_circle(a, C_bc)) C = better(C, C_bc);
    if(in_circle(b, C_ac)) C = better(C, C_ac);
    return C;
}
Circle find_circle(vector<Point> P, int N, int K){
    if (K >= 3)
        return find_circle(P[N-1],P[N-2],P[N-3],true);
    if (N == 1) return find_circle(P[0]);
    if (N == 2) return find_circle(P[0],P[1]);
    int i = rand()%(N-K);
    swap(P[i], P[N-1-K]); swap(P[N-1-K], P[N-1]);
    auto C = find_circle(P, N-1, K);
    swap(P[N-1-K], P[N-1]); swap(P[i], P[N-1-K]);
    if (in_circle(P[i],C)) return C;
    swap(P[i], P[N-1-K]);
    C = find_circle(P, N, K+1);
    swap(P[i], P[N-1-K]);
    return C;
}

```

3.11 Minkowski Sum

```

PT dir;
bool half(PT p){
    return cross(dir, p) < 0 ||
           (cross(dir, p) == 0 && dot(dir, p) > 0);
}
bool polarComp(PT p, PT q) {
    return make_tuple(half(p), 0)
           < make_tuple(half(q), cross(p, q));
}
void process(vector<PT> &P) {
    int mnid = 0;
    for (int i=0; i<P.size(); i++)
        if (P[i] < P[mnid])

```

```

        mnid = i;
    rotate(P.begin(), P.begin()+mnid, P.end());
}
vector<PT> MinkowskiSum(vector<PT>A, vector<PT>B){
    process(A); process(B);
    int n = A.size(), m = B.size();
    vector<PT> P(n), Q(m);
    for(int i=0; i<n; i++) P[i] = A[(i+1)%n] - A[i];
    for(int i=0; i<m; i++) Q[i] = B[(i+1)%m] - B[i];
    dir = PT(0, -1);
    vector<PT> C(n+m+1);
    merge(P.begin(), P.end(), Q.begin(), Q.end(),
          C.begin()+1, polarComp);
    C[0] = A[0] + B[0];
    for(int i=1; i<C.size(); i++) C[i]=C[i]+C[i-1];
    C.pop_back();
    return C;
}

```

3.12 Point Rotation Trick

```

struct pnt{
    int x, y, idx;
    bool operator<(const pnt &p)const{
        return pi(x, y) < pi(p.x, p.y);
    }
}a[5005];
struct line{
    int dx, dy, i1, i2;
};
vector<line> v;
int n, rev[5005];
int p, q;
LL ccw(pnt a, pnt b, pnt c){
    int dx1 = b.x - a.x;
    int dy1 = b.y - a.y;
    int dx2 = c.x - a.x;
    int dy2 = c.y - a.y;
    return abs(1ll * dx1 * dy2 - 1ll * dy1 * dx2);
}
void solve(int c1, int c2, LL l){
    ans = max(ans, ccw(a[c1], a[c2], a[0]));
    ans = max(ans, ccw(a[c1], a[c2], a[n-1]));
}
int main(){
    cin >> n;
    for(int i=0; i<n;i++) cin >> a[i].x >> a[i].y;
    sort(a, a+n);
    for(int i=0; i < n;i++) a[i].idx = rev[i] = i;
    for(int i=0; i<n; i++)
        for(int j=i+1; j<n; j++)
            v.pb({a[j].x-a[i].x,a[j].y-a[i].y,
                 a[i].idx,a[j].idx});
    sort(v.begin(), v.end(), [&]
        (const line &a, const line &b){
            LL cw = 1ll*a.dx*b.dy - 1ll * b.dx * a.dy;
            if(cw != 0) return cw > 0;
            return pi(a.i1, a.i2) < pi(b.i1, b.i2);
        });
    LL ret = 0;
    for(int i=0; i<v.size(); i++){
        int c1 = rev[v[i].i1], c2 = rev[v[i].i2];
        if(c1 > c2) swap(c1, c2);

```

```

        solve(c1, c2, p);
        swap(a[c1], a[c2]);
        swap(rev[v[i].i1], rev[v[i].i2]);
    }
}

```

3.13 Simpson

```

//We divide the integration segment[a;b] into 2n
//equal parts # of steps (already multiplied by 2)
double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) {
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}

```

3.14 Visibility Polygon

```

bool half(PT p) {
    return p.y > 0 || (p.y == 0 && p.x > 0);
}
int compare(PT a, PT b) {
    auto l = make_tuple(half(b), 0);
    auto r = make_tuple(half(a), cross(a, b));
    return l==r ? 0 : (l<r ? -1 : 1);
}
double distance(PT X, PT A, PT B) {
    B = B-A; assert(cross(X, B));
    return sqrt(dot(X, X))*cross(A, B)/cross(X, B);
}
bool compareDis(PT X, PT A, PT B, PT AA, PT BB) {
    B = B-A; BB = BB-AA;
    return 1.0*cross(A, B)/cross(X, B) <
           1.0*cross(AA, BB)/cross(X, BB);
}
pair<double, double> shoot(PT X, double len) {
    double rat = len/sqrt(dot(X, X));
    return make_pair(X.x*rat, X.y*rat);
}
vector<pair<double, double>>
getVisibilityPolygon(PT Z, vector<PT> &p) {
    for (PT &X: p) X = X-Z;
    int n = p.size(); PT O(0, 0);
    auto comp=[&](PT a,PT b){return compare(a,b)<0;};
    map<PT,vector<int>,decltype(comp)> events(comp);
    for (int i=1; i<=n; i++){
        PT X = p[i-1], Y = p[i%n];
        if (cross(O, X, Y) < 0) swap(X, Y);
        if (compare(X, Y) == 0) continue;
        events[X].push_back(i);events[Y].push_back(-i);
    }
    PT dir, last = events.rbegin() -> first;
    auto comp2 = [&dir, &p, &n](int i, int j){return
        compareDis(dir,p[i-1],p[i%n],p[j-1],p[j%n]);};
    multiset<int, decltype(comp2)> st(comp2);
    vector<bool> open(n+1);
    for (auto pr: events) {
        for (int v: pr.second) if (v>0) open[v] = 1;

```

```

    for (int v: pr.second) if (v<0) open[-v] = 0;
}
vector<int> pending;
vector<pair<double, double>> poly;
for (int i=1; i<=n; i++)
    if (open[i]) pending.push_back(i);
for (auto pr: events) {
    PT nw = pr.first;
    dir = nw+last;
    for (int i: pending) st.insert(i);
    pending.clear();
    int i = *st.begin();
    poly.push_back(shoot(last,
        distance(last,p[i-1],p[i%n])));
    poly.push_back(shoot(nw,
        distance(nw, p[i-1], p[i%n])));
    for (int i: pr.second) {
        if (i < 0) st.erase(-i);
        else pending.push_back(i);
    }
    last = nw;
}
return poly;
}

```

3.15 Voronoi

```

LD ccw(Point p, Point q, Point r) {
    return (q-p).cross(r-q);
}
// ax + by = c
struct Line{
    LD a, b, c;
    Point u, d;
    Line(LD a, LD b, LD c):a(a), b(b), c(c) {
        // careful that u, d is not updated here.
    }
    Line(Point u_, Point d_) {
        u = u_, d = d_; //anti-clock dir is the region
        a = d.y, b = -d.x, c = -u.y*d.x + u.x*d.y;
        // ax + by <= c
    }
    bool operator < (const Line &l) const {
        bool flag1 = mp(a, b) > mp(0.0L, 0.0L);
        bool flag2 = mp(l.a, l.b) > mp(0.0L, 0.0L);
        if(flag1 != flag2) return flag1 > flag2;
        LD t = ccw(Point(0.0L, 0.0L),
            Point(a, b), Point(l.a, l.b));
        return dcmp(t) == 0 ? c*hypot(l.a, l.b) <
            l.c * hypot(a, b):t>0;
    }
    Point slope() { return Point(a, b); }
};
Point cross(Line a, Line b){
    LD det = a.a * b.b - b.a * a.b;
    return Point((a.c * b.b - a.b * b.c) / det,
        (a.a * b.c - a.c * b.a) / det);
}
bool bad(Line a, Line b, Line c){
    if(ccw(Point(0, 0), a.slope(), b.slope()) <= 0)
        return false;
    Point crs = cross(a, b);
    return crs.x * c.a + crs.y * c.b >= c.c;
}

```

```

}
// ax + by <= c;
bool hpi(vector<Line> v, vector<Point> &solution){
    sort(v.begin(), v.end());
    deque<Line> dq;
    for(auto &i : v) {
        if(!dq.empty() && !dcmp(ccw(Point(0,0),
            dq.back().slope(), i.slope()))) continue;
        while(dq.size()>2 && bad(dq[dq.size()-2],
            dq.back(), i)) dq.pop_back();
        while(dq.size()>2 && bad(i, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(i);
    }
    while(dq.size()>2 && bad(dq[dq.size()-2],
        dq.back(), dq[0])) dq.pop_back();
    while(dq.size()>2 && bad(dq.back(),
        dq[0], dq[1])) dq.pop_front();
    vector<Point> tmp;
    for(int i=0; i< dq.size(); i++){
        Line cur = dq[i], nxt = dq[(i+1)%dq.size()];
        if(ccw(Point(0,0), cur.slope(), nxt.slope())
            <= EPS) return false;
        tmp.pb(cross(cur, nxt));
    }
    solution = tmp; return true;
}
int main() {
    int n; cin >> n; vector<Point> P(n);
    for(int i=0; i<n; i++) cin >> P[i].x >> P[i].y;
    LD R = 1e9;
    vector<vector<Point>> voronoi_diagram;
    for(int i = 0; i < n; i++) {
        vector<Line> lines;
        lines.pb(Line(1,0,R)); // x <= R
        lines.pb(Line(-1,0,R)); // x >= -R => -x <= R
        lines.pb(Line(0,1,R)); // y <= R
        lines.pb(Line(0,-1,R)); // y >= -R => -y <= R
        for(int j = 0; j < n; j++) {
            if(P[i] == P[j]) continue;
            Point u=(P[i]+P[j])*0.5, dir = P[j]-P[i];
            Point dir_90 = dir.Rotate90();
            Point v = u + dir_90;
            LD a = dir_90.y, b = -dir_90.x;
            LD c = -u.y*dir_90.x + u.x*dir_90.y;
            lines.pb(Line(a,b,c));
        }
        vector<Point> polygon;
        hpi(lines, polygon);
        voronoi_diagram.pb(polygon);
    }
}

```

4 Graph

4.1 BlockCutTree

```

namespace BCT{
    const int mx = 100005; //max(edge,node)
    bool isCutPoint[mx];
    int low[mx], pre[mx], cnt2vcc, used[mx], Timer = 0;
    vector<int> biComp[mx]; int n, m;
    struct Edge{ int v, id; };
}

```

```

vector<Edge> g[mx]; vector<int> bridges;
stack<int> stk;
void init(int _n, int _m){
    n = _n; m = _m;
    for(int i=1; i<=max(n,m); i++)
        g[i].clear(), biComp[i].clear();
    bridges.clear(); // for bridge
}
void addEdge(int u, int v, int id){
    g[u].pb({v,id}); g[v].pb({u,id});
}
void dfs(int u, int par){
    pre[u] = ++Timer; low[u] = pre[u]; int chCnt=0;
    for(int i=0; i<g[u].size(); i++){
        int edgeId = g[u][i].id;
        if(used[edgeId]) continue;
        used[edgeId] = true; stk.push(edgeId);
        int v = g[u][i].v;
        if(pre[v]==-1){
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if(low[v] == pre[v]) bridges.pb(edgeId);
            if(low[v] >= pre[u]){
                cnt2vcc++;
                while(stk.size()>0) /*making component*/{
                    biComp[cnt2vcc].pb(stk.top());
                    stk.pop();
                    if(biComp[cnt2vcc].back()==edgeId) break;
                }
                if(par!=0) isCutPoint[u]=true;
                //checking if non-root
            }
            chCnt++;
        }
        else low[u] = min(low[u], pre[v]);
    }
    if(chCnt > 1 && par==0) isCutPoint[u] = true;
    //checking for root
}
int find2VCC(){
    int i, j; Timer = 0;
    for(i=1; i<=n; i++) used[i] = false;
    for(i=1; i<=n; i++){
        isCutPoint[i] = false; pre[i] = -1;
    }
    cnt2vcc = 0;
    for(i=1; i<=n; i++){
        if(pre[i]==-1) dfs(i, 0);
    }
}
struct Edge{
    int u, v, id;
} edge[maxn];
int main(){
    //BCT::addEdge(u,v,i);
    BCT::find2VCC();
    int cntVcc = BCT::cnt2vcc; int ans1;
    unsigned long long int ans2;
    if(cntVcc==1){
        ans1 = 2; ans2 = (n*(n-1))/2LL;
    }
    else{
}

```

```

    ans1 = 0 , ans2=1LL ;
    for(i=1; i<=cntVcc ; i++){
        set<int> nodes ;
        for(j=0 ; j<BCT::biComp[i].size() ; j++){
            int id= BCT::biComp[i][j] ;
            nodes.insert(edge[id].u);
            nodes.insert(edge[id].v);
        }
        set<int> :: iterator it = nodes.begin() ;
        int artCnt = 0 ;
        while( it!=nodes.end() ){
            if( BCT::isCutPoint[*it] ) artCnt++ ;
            it++ ;
        }
        if( artCnt==1 ){
            ans1++ ;
            ans2 *= (1LL*(nodes.size() - artCnt )) ;
        }
    }
    return 0 ;
}

```

4.2 Blossom

```

const int N = 2020 + 1;
struct GM { /// 1-based Vertex index
    int vis[N], par[N], orig[N], match[N], aux[N], t;
    vector<int> conn[N]; queue<int> Q;
    void addEdge(int u, int v) {
        conn[u].push_back(v); conn[v].push_back(u);
    }
    void init(int n) {
        N = n; t = 0;
        for(int i=0; i<=n; ++i) {
            conn[i].clear(); match[i]=aux[i]=par[i]=0;
        }
    }
    void augment(int u, int v) {
        int pv = v, nv;
        do {
            pv=par[pv]; nv=match[pv];
            match[pv]=pv; match[pv]=v; v=nv;
        } while(u != pv);
    }
    int lca(int v, int w) {
        ++t;
        while(true) {
            if(v) {
                if(aux[v] == t) return v;
                aux[v] = t; v = orig[par[match[v]]];
            }
            swap(v, w);
        }
    }
    void blossom(int v, int w, int a) {
        while(orig[v] != a) {
            par[v] = w; w = match[v];
            if(vis[w] == 1) Q.push(w), vis[w] = 0;
            orig[v] = orig[w] = a; v = par[w];
        }
    }
    bool bfs(int u) {
        fill(vis+1, vis+1+N, -1);

```

```

        iota(orig + 1, orig + N + 1, 1);
        Q = queue<int> ();
        Q.push(u); vis[u] = 0;
        while(!Q.empty()) {
            int v = Q.front(); Q.pop();
            for(int x: conn[v]) {
                if(vis[x] == -1) {
                    par[x] = v; vis[x] = 1;
                    if(!match[x]) return augment(u, x), true;
                    Q.push(match[x]); vis[match[x]] = 0;
                }
                else if(vis[x] == 0 && orig[v] != orig[x]) {
                    int a = lca(orig[v], orig[x]);
                    blossom(x, v, a); blossom(v, x, a);
                }
            }
        }
        return false;
    }
    int Match() {
        int ans = 0;
        vector<int> V(N-1); iota(V.begin(), V.end(), 1);
        shuffle(V.begin(), V.end(), mt19937(0x94949));
        for(auto x: V) if(!match[x]){
            for(auto y: conn[x]) if(!match[y]){
                match[x] = y, match[y] = x; ++ans; break;
            }
        }
        for(int i=1; i<=N; ++i)
            if(!match[i] && bfs(i)) ++ans;
        return ans;
    }
};

```

4.3 Directed MST

```

struct Edge{
    int u, v, w; Edge(){ }
    Edge(int a, int b, int c){ u = a, v = b, w = c; }
};
//Directed minimum spanning tree in O(n * m)
//Mks a rooted tree of min weight frm da root node
//Returns -1 if no solution from root
int directed_MST(int n, vector<Edge> E, int root){
    const int INF = (1 << 30) - 30;
    int i, j, k, l, x, y, res = 0;
    vector<int> cost(n), parent(n), label(n), comp(n);
    for (; ;){
        for (i = 0; i < n; i++) cost[i] = INF;
        for (auto e: E){
            if (e.u != e.v && cost[e.v] > e.w){
                cost[e.v] = e.w; parent[e.v] = e.u;
            }
        }
        cost[root] = 0;
        for (i = 0; i < n && cost[i] != INF; i++){
            if (i != n) return -1; /// No solution
            for (i = 0, k = 0; i < n; i++) res += cost[i];
            for (i = 0; i < n; i++) label[i] = comp[i]=-1;
            for (i = 0; i < n; i++){
                for(x=i; x!=root&&comp[x]==-1; x=parent[x])
                    comp[x]=i;
                if (x != root && comp[x] == i){

```

```

                    for (k++;label[x]==-1;x=parent[x])
                        label[x]=k-1;
                }
            }
            if (k == 0) break;
            for (i = 0; i < n; i++){
                if (label[i] == -1) label[i] = k++;
            }
            for (auto &e: E){
                x = label[e.u], y = label[e.v];
                if (x != y) e.w -= cost[e.v];
                e.u = x, e.v = y;
            }
            root = label[root], n = k;
        }
        return res;
    }
}

```

4.4 Dominator Tree

```

struct ChudirBhai {
    int n, T; VVI g, tree, rg, bucket;
    VI sdom, par, dom, dsu, label, arr, rev;
    ChudirBhai(int n):n(n),g(n+1),tree(n+1),rg(n+1),
        bucket(n+1),sdom(n+1),par(n+1),dom(n+1),
        dsu(n+1),label(n+1),arr(n+1),rev(n+1),T(0) {
        for(int i=1;i<=n;i++){
            sdom[i]=dom[i]=dsu[i]=label[i]=i;
        }
        void addEdge(int u, int v) {g[u].push_back(v);}
        void dfs0(int u) {
            T++; arr[u] = T, rev[T] = u;
            label[T] = T, sdom[T] = T, dsu[T] = T;
            for(int i = 0; i < g[u].size(); i++) {
                int w = g[u][i];
                if(!arr[w]) dfs0(w), par[arr[w]] = arr[u];
                rg[arr[w]].push_back(arr[u]);
            }
        }
        int Find(int u, int x = 0) {
            if(u == dsu[u]) return x? -1: u;
            int v = Find(dsu[u], x+1);
            if(v < 0) return u;
            if(sdom[label[dsu[u]]]<sdom[label[u]])
                label[u]=label[dsu[u]];
            dsu[u] = v;
            return x? v: label[u];
        }
        void Union(int u, int v) { dsu[v] = u; }
        VVI buildAndGetTree(int s) {
            dfs0(s);
            for(int i = n; i >= 1; i--) {
                for(int j = 0; j < rg[i].size(); j++){
                    sdom[i]=min(sdom[i],sdom[Find(rg[i][j])]);
                    if(i > 1) bucket[sdom[i]].push_back(i);
                }
                for(int j = 0; j < bucket[i].size(); j++) {
                    int w = bucket[i][j], v = Find(w);
                    if(sdom[v] == sdom[w]) dom[w] = sdom[w];
                    else dom[w] = v;
                }
                if(i > 1) Union(par[i], i);
            }
            for(int i = 2; i <= n; i++) {

```

```

    if(dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    tree[rev[i]].push_back(rev[dom[i]]);
    tree[rev[dom[i]]].push_back(rev[i]);
}
return tree;
};

```

4.5 EulerPath

```

#include <bits/stdc++.h>
using namespace std;
#define maxn 500005
int c[maxn], d[maxn];
map<int, multiset<int>> g;
map<int, int> vis;
void dfs1(int u){
    vis[u] = 1;
    for( auto v : g[u] )
        if( vis.find(v) == vis.end() ) dfs1(v);
}
//-----Euler path printing-----//
//just call dfs2 with the node you want to start
//your path
//at first you need to make sure, the graph is
//connected and euler path exist
vector<int> ans;
void dfs2(int u){
    while( (int)g[u].size() != 0 ){
        int v = *g[u].begin();
        g[u].erase( g[u].find(v) );
        g[v].erase( g[v].find(u) );
        dfs2(v);
    }
    ans.pb(u);
}
//-----Euler path printing-----//
int main(){
    int n;
    scanf("%d",&n);
    for(int i=1; i<n; i++) scanf("%d",&c[i]);
    for(int i=1; i<n; i++) scanf("%d",&d[i]);
    for(int i=1; i<n; i++) {
        if( c[i] > d[i] ){
            printf("-1\n");
            return 0;
        }
        g[ c[i] ].insert( d[i] );
        g[ d[i] ].insert( c[i] );
    }
    int src = c[1], cnt = 0;
    for( auto it : g ){
        if( (int)it.second.size() & 1 ){
            cnt++;
            src = it.first;
        }
    }
    dfs1( src );
    if(vis.size()!=g.size()||(cnt!=0&&cnt!=2)){
        printf("-1\n");
        return 0;
    }
    //call for printing euler path
    dfs2(src);
}

```

```

for(int i=0; i<ans.size(); i++){
    printf("%d",ans[i]);
    if( i == (int)ans.size() - 1 ) printf("\n");
    else printf(" ");
}
return 0;
}

```

4.6 Hopcroft Karp

```

const int maxN = 50000+5, maxM = 50000+5;
struct HopcroftKarp {
    int n, vis[maxN], lev[maxN], ml[maxN], mr[maxM];
    vector<int> edge[maxN]; //edges for left art only
    HopcroftKarp(int n) : n(n) { //n=nodes in left prt
        for (int i = 1; i <= n; ++i) edge[i].clear();
    }
    void add(int u, int v) {edge[u].push_back(v);}
    bool dfs(int u) {
        vis[u] = true;
        for(auto it=edge[u].begin(); it!=edge[u].end(); ++it){
            int v = mr[*it];
            if (v==-1 || (!vis[v] && lev[u] < lev[v] &&
                dfs(v))) {
                ml[u] = *it; mr[*it] = u; return true;
            }
        }
        return false;
    }
    int matching() { // n for left
        memset(vis, 0, sizeof vis);
        memset(lev, 0, sizeof lev);
        memset(ml, -1, sizeof ml);
        memset(mr, -1, sizeof mr);
        for (int match = 0;;) {
            queue<int> que;
            for (int i = 1; i <= n; ++i) {
                if (ml[i] == -1) lev[i] = 0, que.push(i);
                else lev[i] = -1;
            }
            while (!que.empty()) {
                int u = que.front();
                que.pop();
                for (auto it = edge[u].begin();
                    it != edge[u].end(); ++it) {
                    int v=mr[*it];
                    if(v!=-1 && lev[v] < 0)
                        lev[v] = lev[u]+1, que.push(v);
                }
            }
            for (int i = 1; i <= n; ++i) vis[i] = false;
            int d = 0;
            for (int i=1;i<=n;++i)
                if(ml[i]==-1 && dfs(i)) ++d;
            if (d == 0) return match;
            match += d;
        }
    }
};

```

4.7 Hungarian Algorithm

```

namespace wm{

```

```

bool vis[N]; int U[N],V[N],P[N];
int way[N],minv[N],match[N],ar[N][N];
//n=no of row, m=no of col, 1 based,
//flag=MAXIMIZE/MINIMIZE
//match[i] = the column to which row i is matched
int hungarian(int n,int m,int mat[N][N],int flag){
    clr(U), clr(V), clr(P), clr(ar), clr(way);
    for (int i = 1; i <= n; i++){
        for (int j = 1; j <= m; j++){
            ar[i][j] = mat[i][j];
            if (flag == MAXIMIZE) ar[i][j] = -ar[i][j];
        }
    }
    if (n > m) m = n;
    int i, j, a, b, c, d, r, w;
    for (i = 1; i <= n; i++){
        P[0] = i, b = 0;
        for (j=0; j<=m; j++) minv[j]=inf, vis[j] = 0;
        do{
            vis[b] = true; a = P[b], d = 0, w = inf;
            for (j = 1; j <= m; j++){
                if (!vis[j]){
                    r = ar[a][j] - U[a] - V[j];
                    if (r < minv[j]) minv[j] = r, way[j]=b;
                    if (minv[j] < w) w = minv[j], d = j;
                }
            }
            for (j = 0; j <= m; j++){
                if (vis[j]) U[P[j]] += w, V[j] -= w;
                else minv[j] -= w;
            }
            b = d;
        } while (P[b] != 0);
        do{
            d = way[b]; P[b] = P[d], b = d;
        } while (b != 0);
    }
    for (j = 1; j <= m; j++) match[P[j]] = j;
    return (flag == MINIMIZE) ? -V[0] : V[0];
}

```

4.8 Maxflow

```

/* 0 based for directed graphs */
const LL INF = (~0ULL) >> 1, N = 30010;
namespace flow{
    struct Edge{
        int u, v; LL cap, flow;
        Edge(){
            Edge(int a, int b, LL c, LL f){
                u = a, v = b, cap = c, flow = f;
            }
        };
        vector<int> adj[N]; vector<Edge> E;
        int n, s, t, ptr[N],len[N],dis[N],Q[N];
        void init(int nodes,int src,int sink){
            clr(len); E.clear();
            n = nodes, s = src, t = sink;
            for(int i=0;i<N;i++) adj[i].clear();
        }
        void addEdge(int a, int b, LL c){
            adj[a].push_back(E.size());

```



```

E.push_back(Edge(a, b, c, 0));
len[a]++;adj[b].push_back(E.size());
E.push_back(Edge(b,a,0,0));len[b]++;
}
bool bfs(){
int i, j, k, id, f = 0, l = 0;
memset(dis, -1, sizeof(dis[0]) * n);
dis[s] = 0, Q[l++] = s;
while (f < l && dis[t] == -1){
i = Q[f++];
for (k = 0; k < len[i]; k++){
id = adj[i][k];
if (dis[E[id].v] == -1 &&
E[id].flow < E[id].cap){
Q[l++] = E[id].v;
dis[E[id].v] = dis[i] + 1;
}
}
}
return (dis[t] != -1);
}
LL dfs(int i, LL f){
if (i == t || !f) return f;
while (ptr[i] < len[i]){
int id = adj[i][ptr[i]];
if (dis[E[id].v] == dis[i] + 1){
LL ff = E[id].cap - E[id].flow;
LL x = dfs(E[id].v, min(f,ff));
if (x) {
E[id].flow+=x,E[id^1].flow-=x;
return x;
}
}
ptr[i]++;
}
return 0;
}
LL dinic(){
LL res = 0;
while (bfs()){
memset(ptr, 0, n * sizeof(ptr[0]));
while (LL f = dfs(s, INF)) {
res += f;
}
}
return res;
}
}

```

4.9 Mincost Maxflow

```

//0 Based, dir graphs (for undir add two diredge)
namespace mcmf{
const int N = 1000010; const LL INF = 1LL << 60;
LL cap[N], flow[N], cost[N], dis[N];
int n,m,s,t,Q[10000010];
int adj[N],link[N],last[N],from[N],vis[N];
void init(int nodes, int source, int sink){
m = 0, n = nodes, s = source, t = sink;
for (int i = 0; i <= n; i++) last[i] = -1;
}
void addEdge(int u,int v,LL c,LL w){
adj[m]=v, cap[m]=c, flow[m]=0, cost[m]=+w,
link[m]=last[u], last[u]=m++;
}

```

```

adj[m]=u, cap[m]=0, flow[m]=0, cost[m]=-w,
link[m]=last[v], last[v]=m++;
}
bool spfa(){
int i, j, x, f = 0, l = 0;
for (i=0; i<=n; i++) vis[i] = 0, dis[i] = INF;
dis[s] = 0, Q[l++] = s;
while (f < l){
i = Q[f++];
for (j = last[i]; j != -1; j = link[j]){
if (flow[j] < cap[j]){
x = adj[j];
if (dis[x] > dis[i] + cost[j]){
dis[x] = dis[i]+cost[j], from[x] = j;
if (!vis[x]){
vis[x] = 1;
if (f && rand() < 7) Q[--f] = x;
else Q[l++] = x;
}
}
}
}
}
vis[i] = 0;
return (dis[t] != INF);
}
pair <LL, LL> solve(){
int i, j; LL mincost = 0, maxflow = 0;
while (spfa()){
LL aug = INF;
for(i=t,j=from[i];i!=s;i=adj[j^1],j=from[i])
aug = min(aug, cap[j]-flow[j]);
for(i=t,j=from[i];i!=s;i=adj[j^1],j=from[i])
flow[j] += aug, flow[j ^ 1] -= aug;
maxflow += aug, mincost += aug * dis[t];
}
return make_pair(mincost, maxflow);
}
}

```

4.10 SCC + 2SAT

/*at first take a graph of size 2*n(for each variable two nodes). for each clause of type (a or b), add two diredge !a-->b and !b-->a. if both x_i and !x_i is in same connected component for some i, then this equations are unsatisfiable. Otherwise there is a solution. Assume, f is satisfiable. Now we want to give values to each var in order to satisfy f. It can be done with a top sort of vertices of the graph we made. If !x_i is after x_i in topological sort, x_i should be FALSE. It should be TRUE otherwise. say we have equation with three var x1,x2,x3.(x1 or !x2) and (x2 or x3) = 1. so we addx1,x2,x3 and x4(as !x1), x5(!x2) and x6(!x3). Add edge x4-->x2,x2-->x1, x5-->x3, x6-->x2.

you need to pass array to the function findSCC, in which result will be returned every node will be given a number, for nodes of a single connected component the number will be same this number representing nodes willbe topsorted*/

```

class SCC{
public:

```

```

vector<int> *g1, *g2; int maxNode, *vis1, *vis2;
stack<int> st;
SCC(int MaxNode){
maxNode = MaxNode ; vis1 = new int[maxNode+2];
vis2 = new int[maxNode+2] ;
g1 = new vector<int>[maxNode+2] ;
g2 = new vector<int>[maxNode+2] ;
}
void addEdge(int u,int v){
g1[u].push_back(v); g2[v].push_back(u);
}
void dfs1(int u){
if(vis1[u]==1) return; vis1[u]=1 ;
for(int i=0;i<g1[u].size();i++)dfs1(g1[u][i]);
st.push(u); return;
}
void dfs2(int u, int cnt , int *ans){
if(vis2[u]==1) return ; vis2[u] = 1 ;
for(int i=0;i<g2[u].size();i++)
dfs2(g2[u][i],cnt,ans);
ans[u] = cnt ;
}
int findSCC( int *ans ) {
for(int i=1 ; i<=maxNode ; i++) vis1[i] = 0 ;
for(int i=1 ; i<=maxNode ; i++)
if(vis1[i]==0) dfs1(i);
int cnt = 0 ;
for(int i=1 ; i<=maxNode ; i++) vis2[i] = 0 ;
while( !st.empty() ) {
int u = st.top() ;
if(vis2[u]==0) {++cnt ; dfs2(u, cnt, ans);}
st.pop() ;
}
for(int i=1 ; i<=maxNode ; i++) {
g1[i].clear() ; g2[i].clear() ;
}
delete vis1 ; delete vis2 ; return cnt ;
}
}

```

5 Math

5.1 FFT

```

struct FFT {
struct node {
double x,y;
node() {}
node(double a, double b): x(a), y(b) {}
node operator+(node a)const
{return node(x+a.x,y+a.y);}
node operator-(node a)const
{return node(x-a.x,y-a.y);}
node operator*(node a)const
{return node(x*a.x-y*a.y,x*a.y+a.x*y);}
};
int M; vector<node> A, B, w[2]; vector<int>rev;
const long double pi = acos(-1);
void init(int n) {
M = 1; while(M < n) M <= 1; M <= 1;
A.resize(M); B.resize(M); w[0] = w[1] = rev = B;
for (int i=0; i<M; i++) {
int j=i,y=0;

```

```

    for (int x=1; x<M; x<=1,j>=1) (y<=1)+=j&1;
    rev[i]=y;
}
for (int i=0; i<M; i++) {
    w[0][i] = node( cos(2*pi*i/M),sin(2*pi*i/M));
    w[1][i] = node( cos(2*pi*i/M),-sin(2*pi*i/M));
}
}
void ftransform( vector<node> &A, int p ) {
    for (int i=0; i<M; i++)
        if (i<rev[i]) swap(A[i],A[rev[i]]);
    for (int i=1; i<M; i<=1)
        for (int j=0,t=M/(i<<1); j<M; j+=i<<1)
            for (int k=0,l=0; k<i; k++,l+=t) {
                node x=w[p][l]*A[i+j+k], y=A[j+k];
                A[j+k]=y+x; A[j+k+i]=y-x;
            }
    if (p) for (int i=0; i<M; i++) A[i].x/=M;
}
void multiply(VI &P, VI &Q, VI &res) {
    init(max(P.size(),Q.size()));
    for(int i=0; i<M; i++)
        A[i].x=A[i].y=B[i].x=B[i].y=0;
    for(int i = 0; i < P.size(); i++) A[i].x = P[i];
    for(int i = 0; i < Q.size(); i++) B[i].x = Q[i];
    ftransform(A,0); ftransform(B,0);
    for (int k=0; k<M; k++) A[k] = A[k]*B[k];
    ftransform(A,1);
    res.resize(M);
    for( int i=0; i<M; i++) res[i] = round(A[i].x);
}
//use long double in fft if RT >= 13
const int RT = 15; //Upto M <= 4^RT
vector<LL>anymod(vector<LL>&a,vector<LL>&b,LL mod){
    init(max(a.size(),b.size()));
    vector<node> al(M), ar(M), bl(M), br(M);
    for (int i=0; i<a.size(); i++) {
        LL k = a[i]%mod; al[i] = node(k >> RT, 0);
        ar[i] = node(k & ((1<<RT)-1), 0);
    }
    for (int i=0; i<b.size(); i++) {
        LL k = b[i]%mod; bl[i] = node(k >> RT, 0);
        br[i] = node(k & ((1<<RT)-1), 0);
    }
    ftransform(al, 0); ftransform(ar, 0);
    ftransform(bl, 0); ftransform(br, 0);
    for (int i=0; i<M; i++) {
        node ll = al[i] * bl[i], lr = al[i] * br[i];
        node rl = ar[i] * bl[i], rr = ar[i] * br[i];
        al[i] = ll; ar[i] = lr;
        bl[i] = rl; br[i] = rr;
    }
    ftransform(al, true); ftransform(ar, true);
    ftransform(bl, true); ftransform(br, true);
    vector<LL> ans(M);
    for (int i=0; i<M; i++) {
        LL right = round(br[i].x);
        right %= mod;
        LL mid=round(round(bl[i].x) + round(ar[i].x));
        mid = ((mid%mod)<<RT)%mod;
        LL left=round(al[i].x);
        left = ((left%mod)<<(2*RT))%mod;
    }
}

```

```

    ans[i] = (left+mid+right)%mod;
}
return ans;
};

```

5.2 FWHT

```

void FWHT(vector<LL> &p, bool inv) {
    int n = p.size(); assert((n&(n-1))==0);
    for (int len=1; 2*len<=n; len<= 1) {
        for (int i = 0; i < n; i += len+len){
            for (int j = 0; j < len; j++) {
                LL u = p[i+j], v = p[i+len+j];
                //XOR p[i+j]=u+v; p[i+len+j]=u-v;
                //OR if(!inv) p[i+j]=v,p[i+len+j]=u+v;
                //OR else p[i+j] = -u+v,p[i+len+j]=u;
                //AND if(!inv) p[i+j]=u+v,p[i+len+j]=u;
                //AND else p[i+j]=v, p[i+len+j]=u-v;
            }
        }
    }
    //XOR if(inv)for(int i=0;i<n;i++)p[i]/=n;
}

```

5.3 Gaussian Elimination

```

//n = no of eqn, m = no of var, ar[i][m] = rhs
//returns -1 if no sol, else no of free variables
int gauss(int n,int m,double **ar,VD&res){
    res.assign(m, 0); vector<int> pos(m, -1);
    int i, j, k, l, p, free_var = 0;
    for (j = 0, i = 0; j < m && i < n; j++){
        for (k = i, p = i; k < n; k++){
            if (abs(ar[k][j]) > abs(ar[p][j])) p = k;
        }
        if (abs(ar[p][j]) > EPS){
            pos[j] = i;
            for (l = j; l <= m; l++)
                swap(ar[p][l], ar[i][l]);
            for (k = 0; k < n; k++){
                if (k != i){
                    double x = ar[k][j] / ar[i][j];
                    for (l=j; l<=m; l++)
                        ar[k][l] -= (ar[i][l] * x);
                }
            }
            i++;
        }
    }
    for (i = 0; i < m; i++){
        if (pos[i] == -1) free_var++;
        else res[i] = ar[pos[i]][m] / ar[pos[i]][i];
    }
    for (i = 0; i < n; i++) {
        double val = 0.0;
        for (j = 0; j < m; j++)
            val += (res[j] * ar[i][j]);
        if (abs(val - ar[i][m]) > EPS) return -1;
    }
    return free_var;
}
int gauss(int n,int m, bitset<MAXCOL>ar[MAXROW],
    bitset<MAXCOL>&res){
}

```

```

res.reset(); vector<int>pos(m, -1);
int i, j, k, l, v, p, free_var = 0;
for (j = 0, i = 0; j < m && i < n; j++){
    for (k = i, p = i; k < n; k++){
        if (ar[k][j]) {p = k; break;}
    }
    if (ar[p][j]){
        pos[j] = i; swap(ar[p], ar[i]);
        for (k = 0; k < n; k++){
            if (k != i && ar[k][j]) ar[k] ^= ar[i];
        }
        i++;
    }
}
for (i = 0; i < m; i++){
    if (pos[i] == -1) free_var++;
    else res[i] = ar[pos[i]][m];
}
for (i = 0; i < n; i++) {
    for (j=0, v=0; j<m; j++) v^=(res[j]&ar[i][j]);
    if (v != ar[i][m]) return -1;
}
return free_var;
}

```

5.4 NTT

```

struct NTT {
    vector<int>A, B, w[2], rev;
    int P, M, G;
    NTT(int mod, int g) {P = mod; G = g;}
    int Pow(int a, int b) {
        int res=1;
        for (;b; b>>=1,a=a*1LL*a%P)
            if (b&1) res=res*1LL*a%P;
        return res;
    }
    void init( int n ) {
        for (M=1; M<n; M<=1); M<=1;
        A.resize(M); B.resize(M); w[0]=w[1]=rev=B;
        for (int i=0; i<M; i++) {
            int x=i, &y=rev[i];
            y=0;
            for (int k=1; k<M; k<=1,x>=1)(y<=1)|=x&1;
        }
        int x=Pow(G, (P-1)/M),y=Pow(x,P-2);
        w[0][0]=w[1][0]=1;
        for (int i=1; i<M; i++) {
            w[0][i]=w[0][i-1]*1LL*x%P;
            w[1][i]=w[1][i-1]*1LL*y%P;
        }
    }
    void ntransform(vector<int> &a, int f) {
        for (int i=0; i<M; i++)
            if (i<rev[i]) swap(a[i],a[rev[i]]);
        for (int i=1; i<M; i<=1)
            for (int j=0,t=M/(i<<1); j<M; j+=i<<1)
                for (int k=0,l=0; k<i; k++,l+=t) {
                    int x=a[j+k+i]*1LL*w[f][l]%P, y=a[j+k];
                    a[j+k+i]=y-x<0?y-x+P:y-x;
                    a[j+k]=y+x>P?y+x-P:y+x;
                }
        if (f) {

```

```

    int x=Pow(M,P-2);
    for (int i=0; i<M; i++) a[i]=a[i]*111*x%P;
}
}
void multiply(VI &X, VI &Y, VI &res) {
    init(max(X.size(), Y.size()));
    for( int i = 0; i < M; i++ ) A[i]=B[i]=0;
    for( int i = 0; i < X.size(); i++) A[i]=X[i];
    for( int i = 0; i < Y.size(); i++) B[i]=Y[i];
    ntransform(A,0); ntransform(B,0);
    res.clear(); res.resize(M);
    for (int i=0;i<M;i++) res[i]=A[i]*1LL*B[i]%P;
    ntransform(res,1);
}
};

```

5.5 Number Theory

```

LL gcd(LL u, LL v) {
    if (u == 0) return v; if (v == 0) return u;
    int shift = __builtin_ctzll(u | v);
    u >>= __builtin_ctzll(u);
    do {
        v >>= __builtin_ctzll(v);
        if (u > v) swap(u, v);
        v = v - u;
    } while (v);
    return u << shift;
}
LL lcm(LL a, LL b) {return (a/gcd(a, b))*b;}
LL power(LL a, LL b, LL m) {
    a = (a%m+m)%m; LL ans = 1;
    while (b) {
        if (b & 1) ans = (ans*a)%m;
        a = (a*a)%m;
        b >>= 1;
    }
    return ans;
}
//returns g = gcd(a, b); finds x, y st d = ax+ by
LL egcd(LL a, LL b, LL &x, LL &y) {
    LL xx = y = 0; LL yy = x = 1;
    while (b) {
        LL q = a/b;
        LL t = b; b = a%b; a = t;
        t = xx; xx = x-q*xx; x = t;
        t = yy; yy = y-q*yy; y = t;
    }
    return a;
}
//Solves ax=b(mod m)
vector<LL>SolveCongruence(LL a,LL b,LL m){
    LL x, y, g = egcd(a, m, x, y); vector<LL> ans;
    if (b%g == 0) {
        x = (x*(b/g))%m; if (x<0) x+=m;
        for (LL i=0;i<g;i++) {
            ans.push_back(x); x=(x+m/g)%m;
        }
    }
    return ans;
}
}
LL inverse(LL a, LL m) {
    LL x, y, g = egcd(a, m, x, y);
    if (g > 1) return -1;
}

```

```

    return (x%m+m)%m;
}
//find z st z%m1=r1,z%m2=r2. Here, z is unique mod
//M=lcm(m1,m2), on failure, M=-1,
PLL CRT(LL m1, LL r1, LL m2, LL r2) {
    LL s, t, g = egcd(m1, m2, s, t);
    if (r1%g != r2%g) return PLL(0, -1);
    LL M = m1*m2;
    LL ss = ((s*r2)%m2)*m1, tt = ((t*r1)%m1)*m2;
    LL ans = ((ss+tt)%M+M)%M;
    return PLL(ans/g, M/g);
}
PLL CRT(const vector<LL> &m, const vector<LL> &r) {
    PLL ans = PLL(r[0], m[0]);
    for (LL i = 1; i < m.size(); i++) {
        ans = CRT(ans.second, ans.first, m[i], r[i]);
        if (ans.second == -1) break;
    }
    return ans;
}
//computes x and y such that ax + by = c
bool LinearDiophantine(LL a,LL b,LL c,LL &x,LL &y){
    if (!a && !b) {x=y=0; return !c;}
    if (!a) {x=0;y=c/b; return !(c%b);}
    if (!b) {x=c/a;y=0; return !(c%a);}
    LL g = gcd(a, b);
    x=c/g*inverse(a/g, b/g); y=(c-a*x)/b;
    return !(c%g);
}
// Min sol to a^x = b (mod M),use unmap for speed
int DiscreteLog(int a, int b, int M) {
    map<int, int> id; LL cur=1, RT=sqrt(M)+5;
    for (int i=0;i<RT;i++) id[cur]=i,cur=(cur*a)%M;
    int pp = power(cur, M-2, M);
    cur = b;
    for (int i=0; i*RT<M; i++) {
        auto it = id.find(cur);
        if (it != id.end()) return i*RT+it->second;
        cur = (cur*pp)%M;
    }
    return -1;
}
}

```

5.6 Pollard Rho

```

LL mult(LL a, LL b, LL mod) {
    assert(b < mod && a < mod);
    long double x = a;
    uint64_t c = x * b / mod;
    int64_t r = (int64_t)(a*b-c*mod) % (int64_t)mod;
    return r < 0 ? r + mod : r;
}
LL power(LL x, LL p, LL mod){
    LL s=1, m=x;
    while(p) {
        if(p&1) s = mult(s, m, mod);
        p>>=1;
        m = mult(m, m, mod);
    }
    return s;
}
bool witness(LL a, LL n, LL u, int t){
    LL x = power(a,u,n);
    for(int i=0; i<t; i++) {

```

```

        LL nx = mult(x, x, n);
        if (nx==1 && x!=1 && x!=n-1) return 1;
        x = nx;
    }
    return x!=1;
}
vector<LL>bases = {2,325,9375,28178,450775,9780504,
1795265022}; //2, 13, 23, 1662803 for 10^12
bool miller_rabin(LL n) {
    if (n<2) return 0; if (n%2==0) return n==2;
    LL u = n-1; int t = 0;
    while(u%2==0) u/=2, t++; // n-1 = u*2^t
    for (LL v: bases) {
        LL a = v%(n-1) + 1;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}
mt19937_64 rng(7852365);
//returns n if prime or 1, or proper divisor of n
LL pollard_rho(LL n) {
    if (n==1) return 1; if (n%2==0) return 2;
    if (miller_rabin(n)) return n;
    while (true) {
        LL x=uniform_int_distribution<LL>(1,n-1)(rng);
        LL y = 2, res = 1;
        for (int sz=2; res==1; sz*=2) {
            for (int i=0; i<sz && res<=1; i++) {
                x = mult(x, x, n) + 1;
                res = gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

5.7 Prime Counting Function

```

#define MAXN 500
#define MAXM 100010
#define MAXP 666666
#define MAX 10000010
#define chkbit(ar, i) \
    (((ar[(i) >> 6]) & (1 << (((i) >> 1) & 31))))
#define setbit(ar, i) \
    (((ar[(i) >> 6]) |= (1 << (((i) >> 1) & 31))))
#define isprime(x)\
    (((x)&&((x)&1)&&(!chkbit(ar,(x))))||((x)==2))
namespace pcf{
    long long dp[MAXN][MAXM];
    unsigned int ar[(MAX>>6)+5] = {0};
    int len=0, primes[MAXP], counter[MAX];
    void Sieve(){
        setbit(ar,0), setbit(ar,1);
        for (int i=3;(i*i)<MAX;i++){
            if(!chkbit(ar, i)){
                int k=i<<1;
                for(int j=(i*i);j<MAX;j+=k) setbit(ar,j);
            }
        }
        for(int i=1;i<MAX;i++){
            counter[i]=counter[i-1];
            if(isprime(i)) primes[len++]=i,counter[i]++;
        }
    }
}

```

```

    }
}
void init(){
    Sieve();
    for(int n=0;n<MAXN;n++){
        for(int m=0;m<MAXM;m++){
            if(!n) dp[n][m]=m;
            else dp[n][m] =
                dp[n-1][m]-dp[n-1][m/primes[n-1]];
        }
    }
}
LL phi(LL m,int n){
    if(n==0) return m;
    if(primes[n-1]>=m) return 1;
    if(m<MAXM && n<MAXN) return dp[n][m];
    return phi(m,n-1) - phi(m/primes[n-1],n-1);
}
LL Lehmer(long long m){
    if(m<MAX) return counter[m];
    LL w,res=0;
    int i,a,s,c,x,y;
    s=sqrt(0.9+m), y=cbrt(0.9+m);
    a=counter[y], res=phi(m,a)+a-1;
    for(i=a;primes[i]<=s;i++) res =
        res-Lehmer(m/primes[i])+Lehmer(primes[i])-1;
    return res;
}
}

```

5.8 Primitive Root

/** Find primitive root of p assuming p is prime.
if not, we must add calculation of phi(p)
Complexity : $O(\text{Ans} * \log(\phi(n)) * \log n + \sqrt{p})$
Returns -1 if not found
*/

```

int primitive_root(int p) {
    vector<int> factor; int phi = p-1, n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n%i == 0) {
            factor.push_back(i);
            while (n%i==0) n/=i;
        }
    if (n>1) factor.push_back(n);
    for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (int i=0; i<factor.size() && ok; ++i)
            ok &= power(res, phi/factor[i], p) != 1;
        if (ok) return res;
    }
    return -1;
}
int nttdata(int mod,int &root,int &inv, int &pw) {
    int c = 0, n = mod-1; while (n%2==0) c++, n/=2;
    pw = (mod-1)/n; int g = primitive_root(mod);
    root = power(g, n, mod);
    inv = power(root, mod-2, mod);
    return c;
}

```

5.9 Stern Brocot Tree

//finds x/y with min y st: $L \leq (x/y) < R$

```

pair<LL,LL>solve(LL L, LL R){
    pair<LL, LL> l(0, 1), r(1, 1);
    if(L==0.0) return l; // corner case
    while(true) {
        pair<int, int> m(l.x+r.x, l.y+r.y);
        if(m.x<L*m.y){ // move to the right
            LL kl=1, kr=1;
            while(l.x+kr*r.x <= L*(l.y+kr*r.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(l.x+km*r.x < L*(l.y+km*r.y)) kl=km+1;
                else kr=km;
            }
            l={l.x+(kl-1)*r.x,l.y+(kl-1)*r.y};
        }
        else if(m.x>=R*m.y){ //move to left
            LL kl=1, kr=1;
            while(r.x+kr*l.x>=R*(r.y+kr*l.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(r.x+km*l.x>=R*(r.y+km*l.y)) kl = km+1;
                else kr = km;
            }
            r={r.x+(kl-1)*l.x,r.y+(kl-1)*l.y};
        }
        else return m;
    }
}

```

5.10 Sum of Floors

///Finds sum of floor((p*i+r)/q) from i = 0 to n-1

```

LL findSum(LL n, LL p, LL r, LL q) {
    if (p == 0) return (r / q) * n;
    if (p>=q||r>=q) return findSum(n,p%q,r%q,q) +
        ((p/q)*(n-1)+2*(r/q))*n/2;
    return findSum((p*n+r)/q, q, (p*n+r)%q,p);
}

```

6 Miscellaneous

6.1 Fast IO C++

```

#include<bits/stdc++.h>
using namespace std;
static const int buf_size = 4096;
inline int getChar() {
    static char buf[buf_size];
    static int len = 0, pos = 0;
    if(pos==len)
        pos=0, len=fread(buf, 1, buf_size, stdin);
    if (pos == len) return -1;
    return buf[pos++];
}
inline int readChar() {
    int c = getChar();
    while (c <= 32) c = getChar();
    return c;
}
template <class T>
inline T readInt() {
    int s = 1, c = readChar(); T x = 0;
    if (c == '-') s = -1, c = getChar();

```

```

    while ('0'<=c&&c<='9')
        x=x*10+c-'0',c=getChar();
    return s == 1 ? x : -x;
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar( int x ) {
    if (write_pos == buf_size)
        fwrite(write_buf, 1, buf_size,
            stdout),
        write_pos = 0;
    write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt( T x, char end ) {
    if (x < 0) writeChar('-'), x = -x;
    char s[24]; int n = 0;
    while (x || !n) s[n++] = '0' + x % 10,
        x/=10;
    while (n--) writeChar(s[n]);
    if (end) writeChar(end);
}
inline void writeWord( const char *s ) {
    while (*s) writeChar(*s++);
}

```

6.2 Fast IO JAVA

```

public class Main {
    public static void main(String[] args) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        int n = in.nextInt(); long l = in.nextLong();
        out.println(n); out.println(l);
        out.println("done"); out.close();
    }
    static class InputReader {
        public BufferedReader reader;
        public StringTokenizer tokenizer;
        public InputReader(InputStream stream) {
            reader = new BufferedReader(
                new InputStreamReader(stream),32768);
            tokenizer = null;
        }
        public String next() {
            while(tokenizer==null ||
                !tokenizer.hasMoreTokens()){
                try {
                    tokenizer =
                        new StringTokenizer(reader.readLine());
                } catch (IOException e) {
                    throw new RuntimeException(e);
                }
            }
            return tokenizer.nextToken();
        }
        public int nextInt() {
            return Integer.parseInt(next());
        }
        public long nextLong(){
            return Long.parseLong(next());
        }
    }
}

```



```

}

7 String
7.1 Aho Corasick

struct AC {
struct state {
    int to[ALPHA], depth, sLink,
    int par, parLet, cnt, nxt[ALPHA];
} states[N];
vector<int> suff_tree[N]; int tot_nodes;
void init() {
    for(int i = 0; i < N; i++) suff_tree[i].clear();
    tot_nodes = 1; clr(states); //careful, memset TLE
}
int add_string(string &str) {
    int cur = 1;
    for(int i = 0; i < str.size(); i++) {
        int c = str[i] - 'a';
        if(!states[cur].to[c]) {
            states[cur].to[c] = ++tot_nodes;
            states[tot_nodes].par = cur;
            states[tot_nodes].depth = states[cur].depth + 1;
            states[tot_nodes].parLet = c;
        }
        cur = states[cur].to[c];
    }
    return cur;
}
void push_links() {
    queue<int> qq;
    qq.push(1);
    while (!qq.empty()) {
        int node = qq.front();
        qq.pop();
        if (states[node].depth <= 1)
            states[node].sLink = 1;
        else {
            int cur = states[states[node].par].sLink;
            int parLet = states[node].parLet;
            while (cur > 1 and !states[cur].to[parLet]) {
                cur = states[cur].sLink;
            }
            if (states[cur].to[parLet]) {
                cur = states[cur].to[parLet];
            }
            states[node].sLink = cur;
        }
        if (node != 1)
            suff_tree[states[node].sLink].pb(node);
        for (int i = 0; i < ALPHA; i++) {
            if (states[node].to[i])
                qq.push(states[node].to[i]);
        }
    }
}
int next_state(int from, int c) {
    if (states[from].nxt[c])
        return states[from].nxt[c];
    int cur = from;
    while (cur > 1 && !states[cur].to[c])
        cur = states[cur].sLink;
    if (states[cur].to[c]) cur = states[cur].to[c];
}

```

```

    return states[from].nxt[c] = cur;
}
void dfs(int u) {
    for(int v : suff_tree[u]) {
        dfs(v); states[u].cnt += states[v].cnt;
    }
}
} aho;

7.2 KMP, Manacher, Z

vector<int> prefix_function (string s) {
    int n = (int) s.length(); vector<int> pi (n);
    for (int i=1; i<n; ++i) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j]) j = pi[j-1];
        if (s[i] == s[j]) ++j;
        pi[i] = j;
    }
    return pi;
}
//p[0][i] = maxlen of hlf palin arnd half idx i
//p[1][i] = maxlen of hlf palin arnd idx i, 0 based
VI p[2];
void manacher(const string s) {
    int n = s.size(); p[0] = VI(n+1); p[1] = VI(n);
    for (int z=0; z<2; z++)
        for (int i=0, l=0, r=0; i<n; i++) {
            int t = r - i + !z;
            if (i<r) p[z][i] = min(t, p[z][l+t]);
            int L = i-p[z][i], R = i+p[z][i] - !z;
            while (L>=1 && R+1<n && s[L-1] == s[R+1])
                p[z][i]++, L--, R++;
            if (R>r) l=L, r=R;
        }
}
bool ispalin(int l, int r) {
    int mid = (l+r+1)/2, sz = r-l+1;
    return 2*p[sz/2][mid] + b>=sz;
}
vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i=1; i<n; i++) {
        if (i<=r) z[i] = min(r-i+1, z[i-l]);
        while (i+z[i]<n && s[i+z[i]]==s[z[i]]) z[i]++;
        if (i+z[i]-1>r) l = i, r = i+z[i]-1;
    }
    return z;
}

7.3 Palindromic Tree

struct node { int next[26], len, sufflink; };
int len, sz, suff; char str[N]; node tree[N];
bool addLetter(int pos) {
    int cur = suff, curlen = 0, let = str[pos] - 'a';
    while (true) {
        curlen = tree[cur].len;
        if (pos-curlen>=1 && str[pos-1-curlen]==str[pos])
            break;
        cur = tree[cur].sufflink;
    }
}

```

```

    if (tree[cur].next[let]) {
        suff = tree[cur].next[let]; return false;
    }
    suff = ++sz; tree[sz].len = tree[cur].len + 2;
    tree[cur].next[let] = sz;
    if (tree[sz].len==1){
        tree[sz].sufflink = 2; return 1;
    }
    while (true) {
        cur=tree[cur].sufflink; curlen=tree[cur].len;
        if (pos-curlen>=1 && str[pos-1-curlen]==str[pos]){
            tree[sz].sufflink=tree[cur].next[let]; break;
        }
    }
    return true;
}
void initTree() {
    memset (tree, 0, sizeof tree); sz = 2; suff = 2;
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}

7.4 Suffix Array

const int N = 1e6+7, LOG = 20, ALPHA = 128;
struct SuffixArray {
    int sa[N], data[N], rnk[N], hgt[N], n;
    int wa[N], wb[N], wws[N], wv[N];
    int lg[N], rmq[N][LOG], rev_sa[N];
    int cmp(int *r, int a, int b, int l) {
        return (r[a]==r[b]) && (r[a+l]==r[b+l]);
    }
    void DA(int *r, int *sa, int n, int m) {
        int i, j, p, *x=wa, *y=wb, *t;
        for(i=0; i<m; i++) wws[i]=0;
        for(i=0; i<n; i++) wws[x[i]=r[i]]++;
        for(i=1; i<m; i++) wws[i]+=wws[i-1];
        for(i=n-1; i>=0; i--) sa[--wws[x[i]]]=i;
        for(j=1, p=1; p<n; j*=2, m=p) {
            for(p=0, i=n-j; i<n; i++) y[p++] = i;
            for(i=0; i<n; i++) if (sa[i]>=j) y[p++] = sa[i]-j;
            for(i=0; i<n; i++) wv[i] = x[y[i]];
            for(i=0; i<m; i++) wws[i]=0;
            for(i=0; i<n; i++) wws[wv[i]]++;
            for(i=1; i<m; i++) wws[i]+=wws[i-1];
            for(i=n-1; i>=0; i--) sa[--wws[wv[i]]]=y[i];
            for(t=x, x=y, y=t, p=1, i=0; i<n; i++)
                x[sa[i]] = cmp(y, sa[i-1], sa[i], j)? p-1: p++;
        }
    }
    void calhgt(int *r, int *sa, int n) {
        int i, j, k=0;
        for(i=1; i<n; i++) rnk[sa[i]]=i;
        for(i=0; i<n; hgt[rnk[i++]] = k)
            for(k? k--: 0, j=sa[rnk[i]-1]; r[i+k]==r[j+k]; k++);
    }
}
void suffix_array (string &A) {
    n = A.size();
    for(int i=0; i<max(n+5, ALPHA); i++) sa[i]=data[i]=
        rnk[i]=hgt[i]=wa[i]=wb[i]=wws[i]=wv[i]=0;
    for (int i = 0; i < n; i++) data[i] = A[i];
    DA(data, sa, n+1, ALPHA);
    calhgt(data, sa, n);
}

```

```

for(int i = 0; i < n; i++)
    sa[i]=sa[i+1], hgt[i]=hgt[i+1], rev_sa[sa[i]]=i;
range_lcp_init();
}
void range_lcp_init() {
    for(int i = 0; i < n; i++) rmq[i][0] = hgt[i];
    for(int j = 1; j < LOG; j++) {
        for(int i = 0; i < n; i++) {
            if (i+(1<<j)-1 < n) rmq[i][j] =
                min(rmq[i][j-1], rmq[i+(1<<(j-1))][j-1]);
            else break;
        }
    }
    lg[0] = lg[1] = 0;
    for(int i = 2; i <= n; i++) lg[i] = lg[i/2] + 1;
}
int query_lcp(int l, int r) {
    assert(l <= r); assert(l>=0&&l<n&&r>=0&&r<n);
    if(l == r) return n-sa[l];
    l++; int k = lg[r-l+1];
    return min(rmq[l][k], rmq[r-(1<<k)+1][k]);
}

```

```

}
}SA;

```

7.5 Suffix Automata

```

// # No of Occ of each state: init each state (except
// the clones) with cnt[state]=1, loop dec order of
// len[state], and do: cnt[link[state]]+=cnt[state]
// # First Occ of each state:
// for new state: firstpos(cur) = len(cur)-1
// for cloned state: firstpos(clone) = firstpos(q)
const int ALPHA = 26;
namespace SuffixAutomata {
    vector<vector<int>> to, nstate;
    vector<int> link, len;
    int n, sz, cur;
    void add(int c) {
        int p = cur;
        cur = ++sz; len[cur] = len[p] + 1;
        while (to[p][c]==0) {to[p][c]=cur; p = link[p];}
        if (to[p][c] == cur) {link[cur] = 0; return;}
        int q = to[p][c];
        if (len[q] == len[p] + 1) link[cur] = q; return;
    }
}

```

```

int cl = ++sz;
to[cl] = to[q]; link[cl] = link[q];
len[cl] = len[p] + 1; link[cur] = link[q] = cl;
while (to[p][c] == q) {to[p][c]=cl; p=link[p];}
}
int advance(int state, int c) {
    if(nstate[state][c]!=-1) return nstate[state][c];
    int nstate;
    if(to[state][c]) nstate = to[state][c];
    else if(state) nstate = advance(link[state], c);
    else nstate = state;
    return nstate[state][c] = nstate;
}
void build(string &s) {
    cur = sz = 0; n = s.size();
    to.assign(2*n+1, vector<int> (ALPHA, 0));
    nstate.assign(2*n+1, vector<int> (ALPHA, -1));
    link.assign(2*n+1, 0); len.assign(2*n+1, 0);
    for(int i = 0; i < n; i++) add(s[i]-'a');
}
}

```