GRAPHULO

Server-side Sparse Matrix Multiply in the Accumulo Database

Dylan Hutchison[1,2]*   Vijay Gadepally[1]*   Jeremy Kepner[1]*   Adam Fuchs[3]

[1]MIT Lincoln Laboratory   [2]University of Washington   [3]Sqrrl Inc.

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Massachusetts Institute of Technology

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This work is NOT

Creating the best system

for a particular task (matrix multiply)

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This work IS

Adding graph analytic capabilities

(matrix multiply) to an all-around good

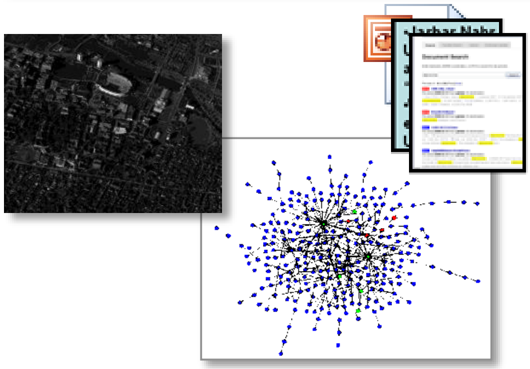
system used in practice today (Accumulo)

Outline

- Intro to Graphulo
- Intro to Matrix Multiply
- Intro to Accumulo
- Matrix Multiply pre-Graphulo
- Inner Product
- Outer Product
- Accumulo Implementation
- Performance
- Conclusions

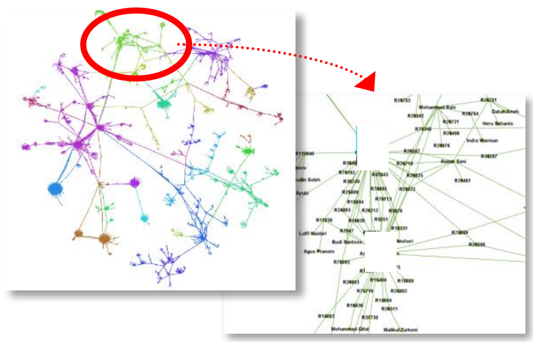
Real Graph Analytics used in Accumulo

ISR



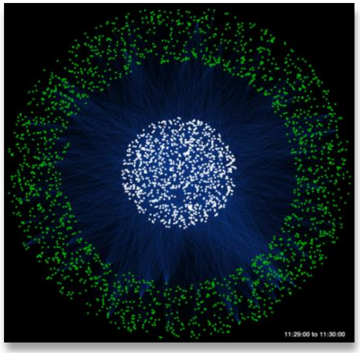
- Graphs represent entities and relationships detected through multi-INT sources
- 1,000s – 1,000,000s tracks and locations
- GOAL: Identify anomalous patterns of life

Social



- Graphs represent relationships between individuals or documents
- 10,000s – 10,000,000s individual and interactions
- GOAL: Identify hidden social networks

Cyber

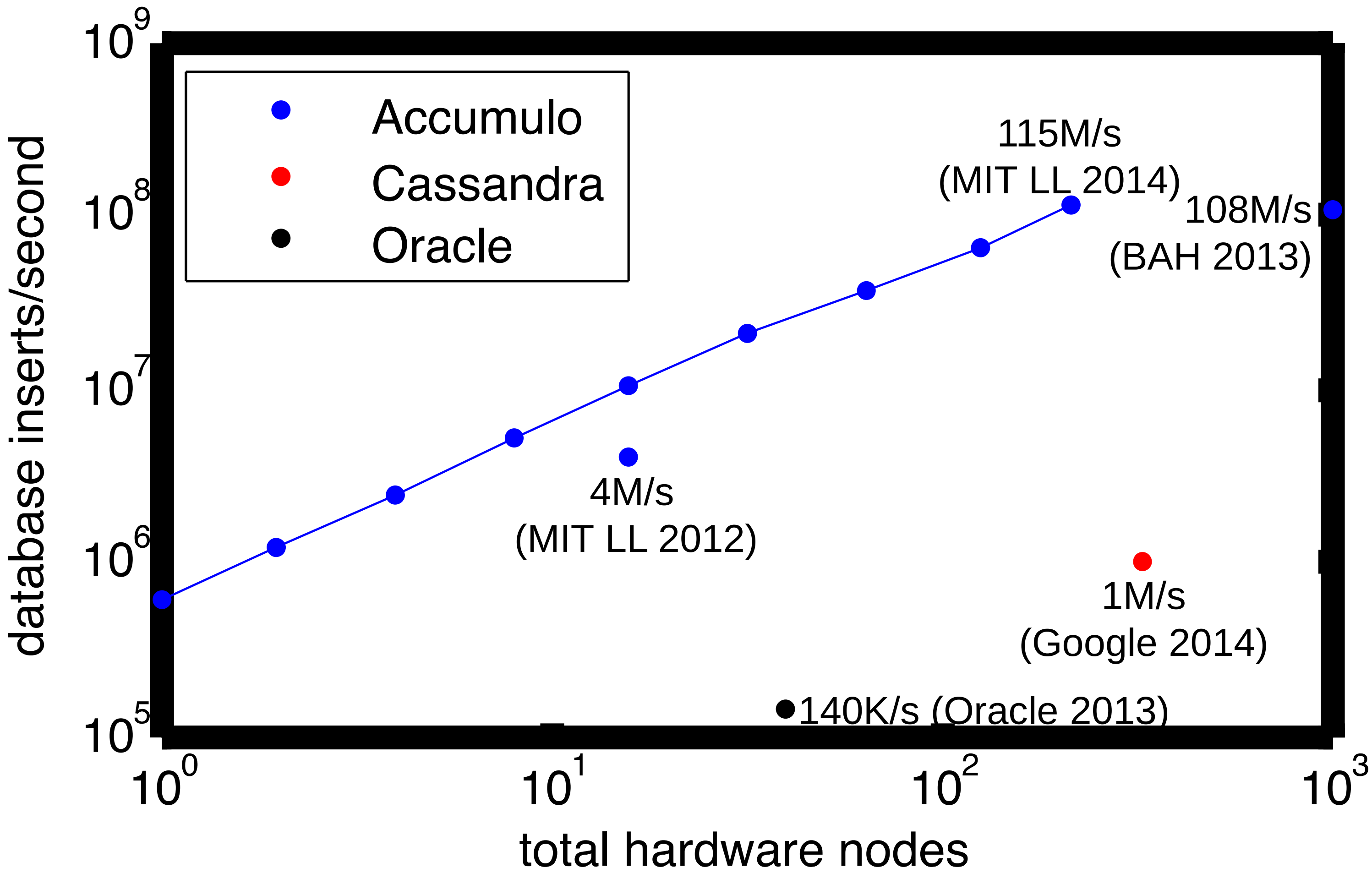


- Graphs represent communication patterns of computers on a network
- 1,000,000s – 1,000,000,000s network events
- GOAL: Detect cyber attacks or malicious software

Many groups store graph data in Accumulo

➔ Need tools for graph analysis in Accumulo

Why Accumulo?



Accumulo ingest performance is 100x greater than competing technologies

Graphulo Overview

- **Primary Goal**
  - **Open source Apache Accumulo Java library that enables many graph algorithms in Accumulo**
- **Core primitives: GraphBLAS**
- **3 Graph Schemas**
  - **Adjacency, Incidence, Single-Table**
- **4 Demonstration Graph Algorithms**
  - **Degree-filtered Breadth First Search, Jaccard coefficients, k-Truss subgraph, Non-negative Matrix Factorization**
- **Focus on Interactive Computing**
  - **"Queued" / Localized analytics within a neighborhood, as opposed to whole table analytics**
  - **Low latency more important than high throughput**
  - **Progress monitoring for user sanity**
    - ***Is the library working or stuck?***

GraphBLAS initial function list

Function	Parameters	Returns	Math Notation
SpGEMM	- sparse matrices **A** and **B** - unary functors (op)	sparse matrix	**C** = op(**A**) * op(**B**)
SpM{Sp}V **(Sp: sparse)**	- sparse matrix **A** - sparse/dense vector **x**	sparse/dense vector	**y** = **A** * **x**
SpEWiseX	- sparse matrices or vectors - binary functor and predicate	in place or sparse matrix/vector	**C** = **A** .* **B**
Reduce	- sparse matrix **A** and functors	dense vector	**y** = sum(**A**, op)
SpRef	- sparse matrix **A** - index vectors **p** and **q**	sparse matrix	**B** = **A**(**p**,**q**)
SpAsgn	- sparse matrices **A** and **B** - index vectors **p** and **q**	none	**A**(**p**,**q**) = **B**
Scale	- sparse matrix **A** - dense matrix or vector **X**	none	check manual
Apply	- any matrix or vector **X** - unary functor (op)	none	op(**X**)

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Matrix Multiply on Big Data

Traditional Matrix Multiply: $AB = C$

$$\begin{bmatrix} 6 & 5 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 5 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 23 \\ 0 & 12 \end{bmatrix}$$

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- **Row & Column Labels**

Database Table Multiply

$$\begin{array}{l} \\ \text{word|coffee} \\ \text{word|desert} \end{array} \begin{array}{c} \begin{array}{cccc} \text{tod|0500} & \text{tod|0800} & \text{tod|0900} & \text{tod|1400} \end{array} \\ \begin{bmatrix} 6 & 5 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{tod|0500} \\ \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400} \end{array} \begin{array}{c} \begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 5 & 0 \\ 3 & 4 \end{bmatrix} \end{array} = \begin{array}{l} \\ \text{word|coffee} \\ \text{word|desert} \end{array} \begin{array}{c} \begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\ \begin{bmatrix} 6 & 23 \\ 0 & 12 \end{bmatrix} \end{array}$$

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- **Row & Column Labels**
- **Sparse**

Database Table Multiply

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- ➢ **Row & Column Labels**
- ➢ **Sparse**

➔ **Associative Array Mathematics**[1]

Database Table Multiply

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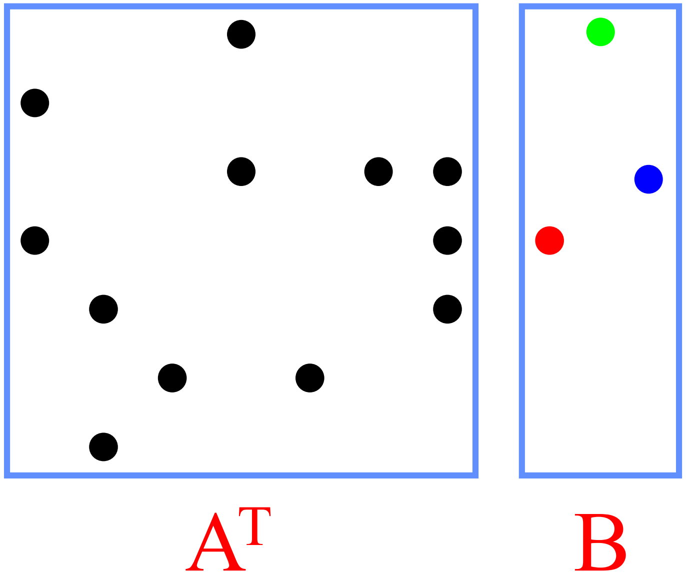
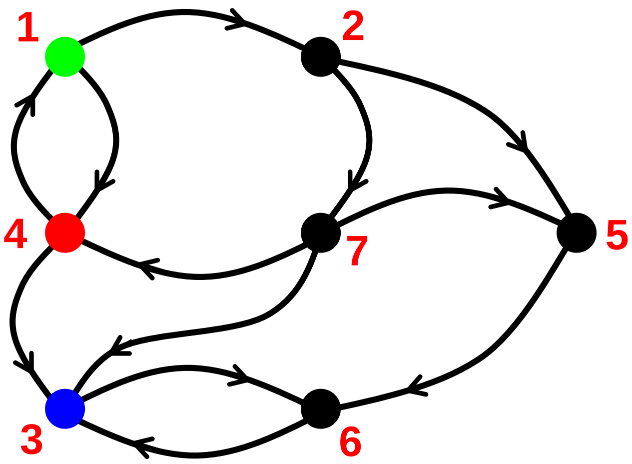
[1] J. Kepner and V. Gadepally. "*Adjacency matrices, incidence matrices, database schemas, and associative arrays*" in International Parallel & Distributed Processing Symposium Workshops (IPDPSW). IEEE, 2014

Application: Multi-Source Breadth-First Search





- Sparse array representation => space efficient
- Sparse matrix-matrix multiplication => work efficient
- Three possible levels of parallelism: searches, vertices, edges
- Basis for a wide range of graph algorithms

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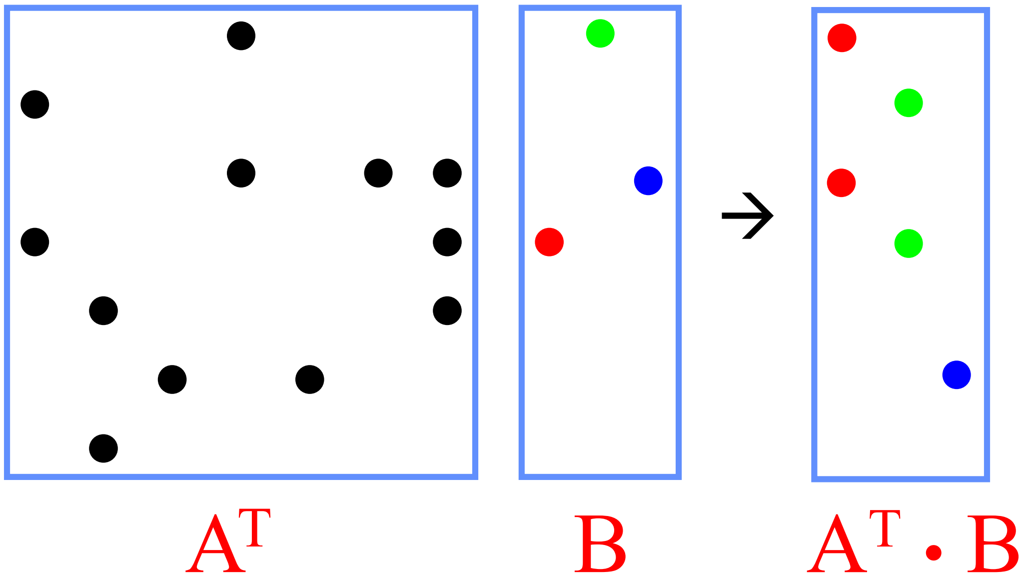

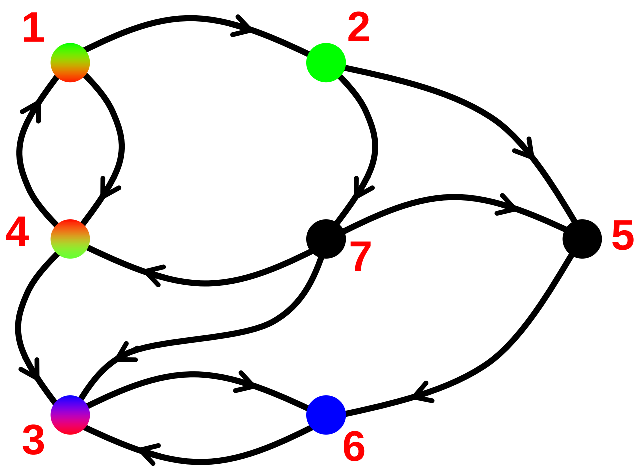

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Background on Accumulo

Key					Value
Row ID	Column			Timestamp	
	Family	Qualifier	Visibility		

Best for:

- Large, de-normalized tables (NoSQL)
- Hadoop HDFS / Java ecosystem
- Huge data volume – TBs to PBs
- Cell-level visibility
- Robust horizontal scaling

- Row store by default
  - Scan over rows for O(log n) lookup & sorted order
  - Log-structured Merge Tree design
- Iterator processing framework

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Use Transpose Tables
see D4M Schema[1]

- **Row store by default**
  - **Scan over rows for O(log n) lookup & sorted order**
  - **Log-structured Merge Tree design**
- **Iterator processing framework**

[1]*D4M 2.0 Schema: A General Purpose High Performance Schema for the Accumulo Database*
Kepner et al, IEEE HPEC 2013

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Table Multiply Before Graphulo

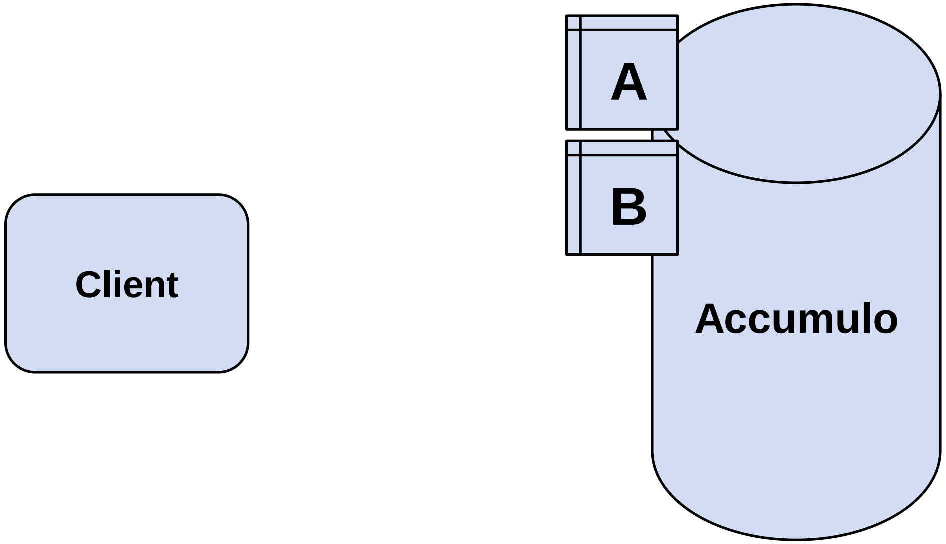


Table Multiply Before Graphulo

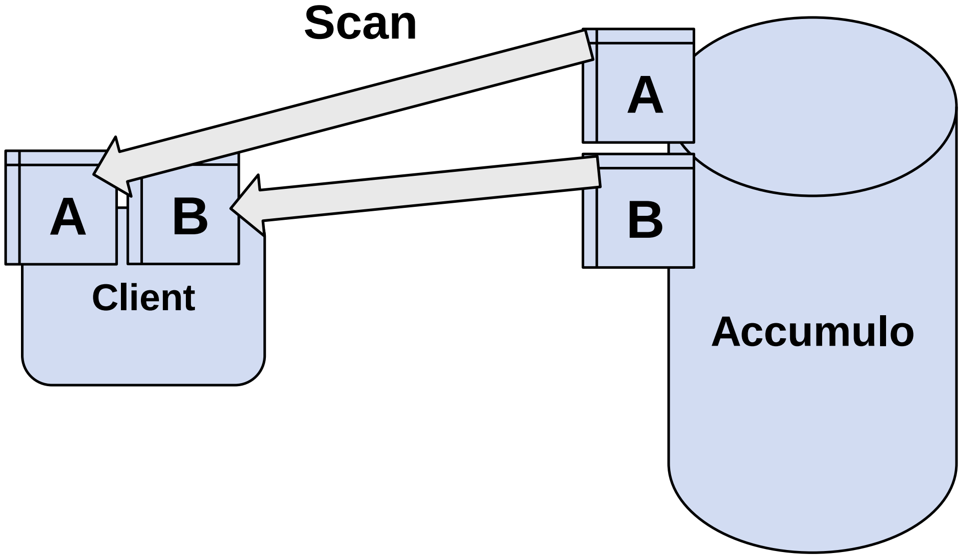
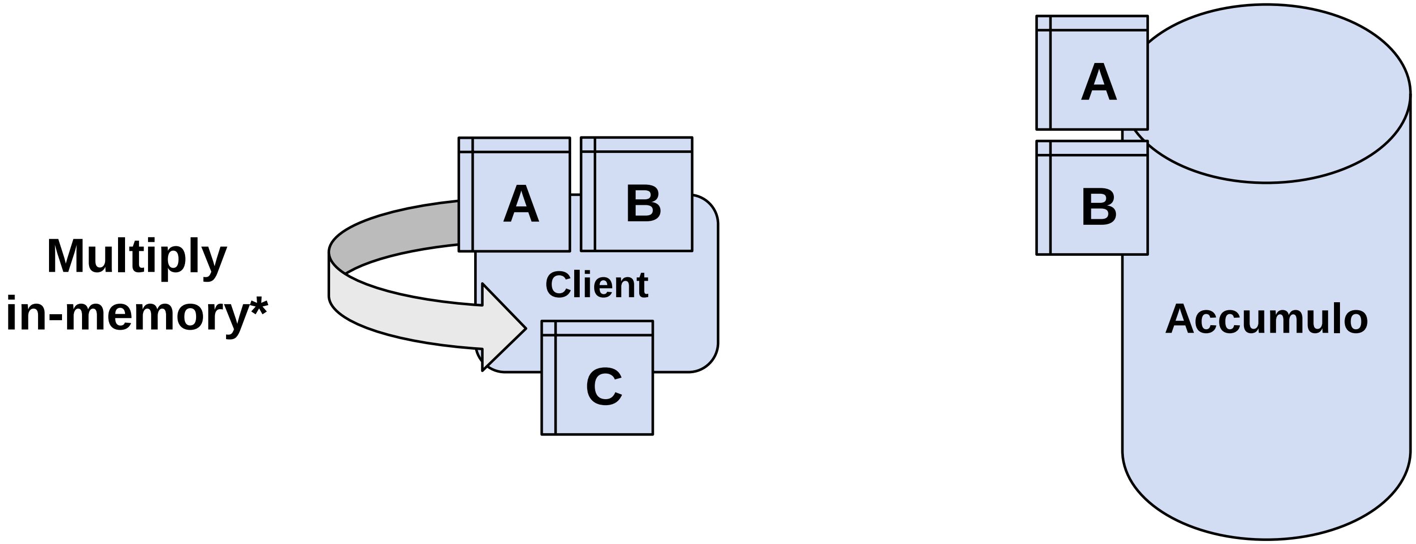


Table Multiply Before Graphulo



*Blocked algorithms exist for large tables at reduced efficiency

Table Multiply Before Graphulo

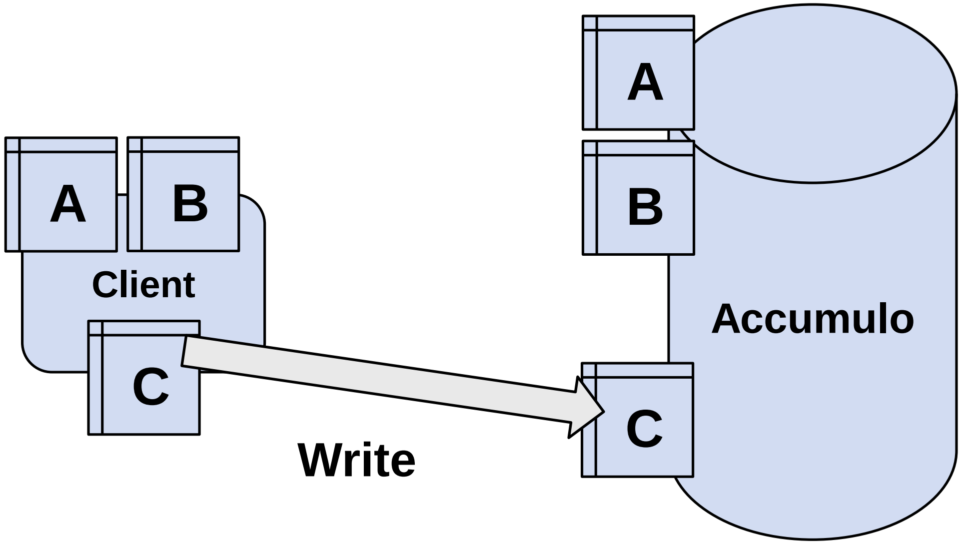
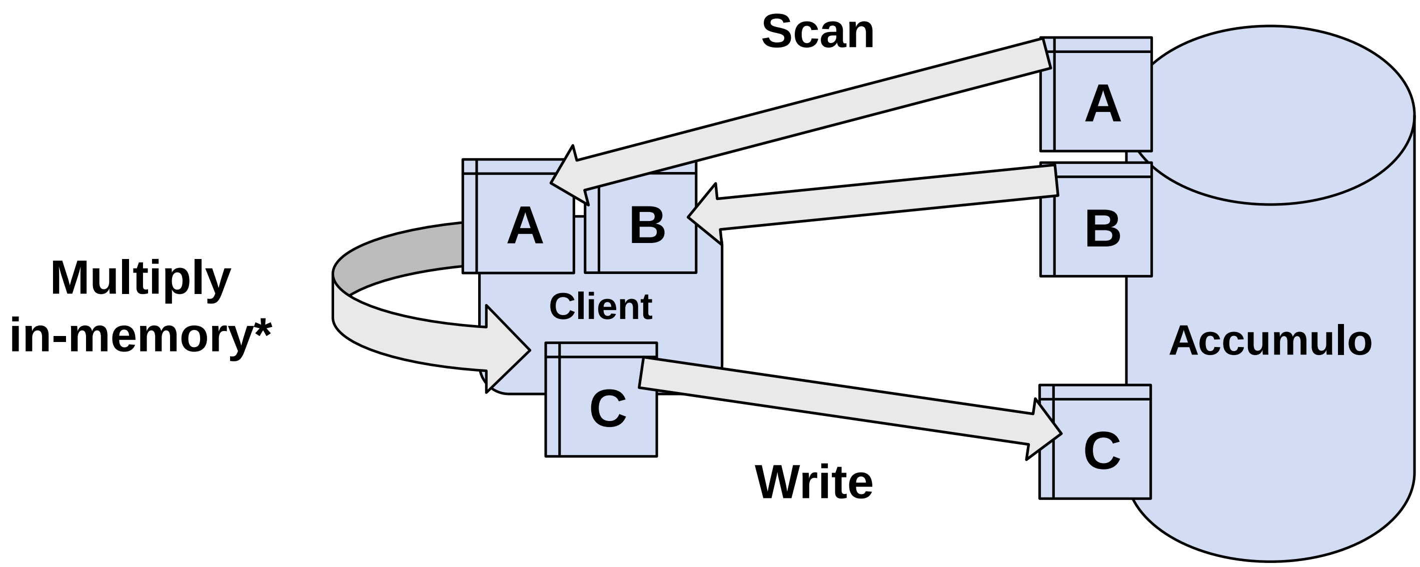


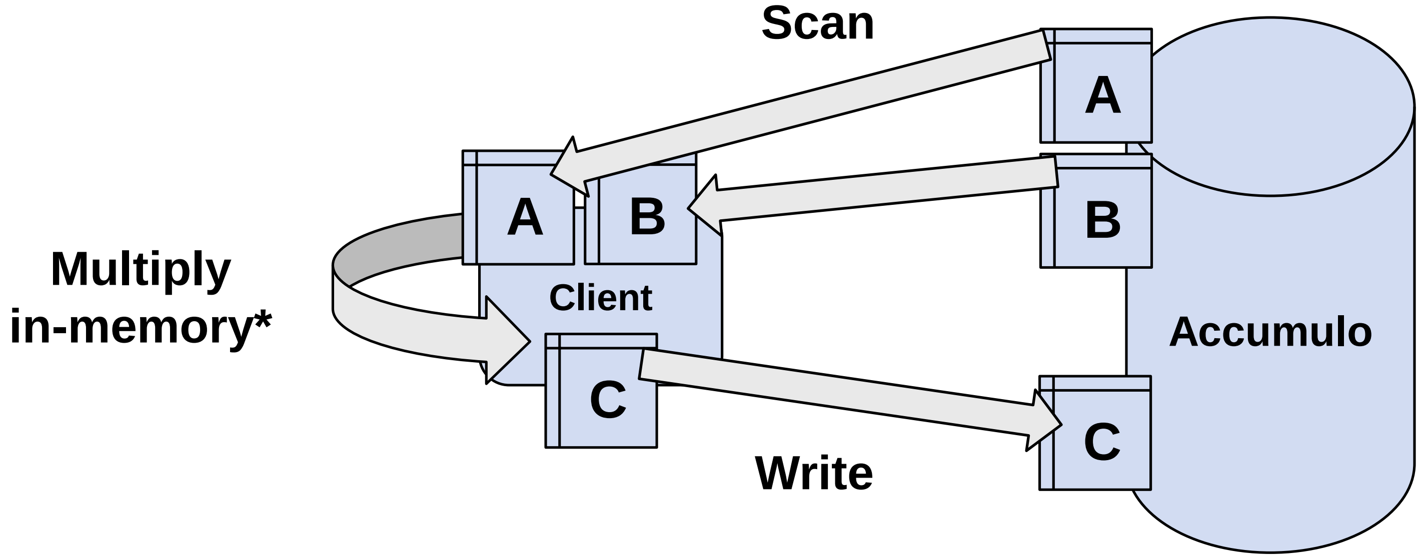
Table Multiply Before Graphulo



Old: DB = Indexed Storage

*Blocked algorithms exist for large tables at reduced efficiency

Table Multiply Before Graphulo



Old: DB = Indexed Storage

New: DB = Indexed Storage + Computation Engine

***Blocked algorithms exist for large tables at reduced efficiency**

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Inner Product

$$
\begin{array}{c|ccc}
 & \text{tod|0500} & \text{tod|0800} & \text{tod|1400} \\
\hline
\text{word|coffee} & \boxed{6} & \boxed{5} & \boxed{2} \\
\text{word|desert} & & 4 &
\end{array}
\;
\begin{array}{c|cc}
 & \text{word|dew} & \text{word|hot} \\
\hline
\text{tod|0800} & & 3 \\
\text{tod|0900} & \boxed{5} & \\
\text{tod|1400} & \boxed{3} & 4
\end{array}
=
\begin{array}{c|c}
 & \text{word|dew} \\
\hline
\text{word|coffee} & \boxed{6}
\end{array}
$$

1

for $i = 1 : N$ = 2
 for $j = 1 : L$ = 2
  for $k = 1 : M$ = 4
   $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C}(i,j) = \bigoplus_{k=1}^{M} \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$$

Inner Product

$$
\begin{array}{r}
 \\ \text{word|coffee} \\ \text{word|desert}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{tod|0500} & \text{tod|0800} & \text{tod|1400} \end{array} \\
\begin{bmatrix} \mathbf{6} & \mathbf{5} & \mathbf{2} \\ & \mathbf{4} & \end{bmatrix}
\end{array}
\begin{array}{r}
 \\ \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix}
\end{array}
=
\begin{array}{r}
 \\ \text{word|coffee}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{① word|dew} & \text{word|hot ②} \end{array} \\
\begin{bmatrix} \mathbf{6} & \boxed{\mathbf{23}} \end{bmatrix}
\end{array}
$$

1st Scan

for $i = 1 : N$ = 2

for $j = 1 : L$ = 2

for $k = 1 : M$ = 4

$\mathbf{C}(i,j) \oplus = \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C}(i,j) = \bigoplus_{k=1}^{M} \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$$

Inner Product

$$\begin{array}{r|ccc} & \text{tod|0500} & \text{tod|0800} & \text{tod|1400} \\ \hline \text{word|coffee} & \mathbf{6} & \mathbf{5} & \mathbf{2} \\ \text{word|desert} & & \mathbf{4} & \end{array} \quad \begin{array}{r|cc} & \text{word|dew} & \text{word|hot} \\ \hline \text{tod|0800} & & \mathbf{3} \\ \text{tod|0900} & \mathbf{5} & \\ \text{tod|1400} & \mathbf{3} & \mathbf{4} \end{array} = \begin{array}{r|cc} & 1 \ \text{word|dew} & \text{word|hot} \ 2 \\ \hline \text{word|coffee} & \mathbf{6} & \mathbf{23} \\ & \boxed{} & \end{array}$$

for $i = 1 : N$ = 2
  for $j = 1 : L$ = 2
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Inner Product

$$
\begin{array}{r|ccc|}
 & \text{tod|0500} & \text{tod|0800} & \text{tod|1400} \\
\text{word|coffee} & 6 & 5 & 2 \\
\text{word|desert} & & \boxed{4} &
\end{array}
\;
\begin{array}{r|cc|}
 & \text{word|dew} & \text{word|hot} \\
\text{tod|0800} & & 3 \\
\text{tod|0900} & 5 & \\
\text{tod|1400} & 3 & 4
\end{array}
=
\begin{array}{r|cc|}
 & \text{word|dew} & \text{word|hot} \\
\text{word|coffee} & 6 & 23 \\
\text{word|desert} & & \boxed{12}
\end{array}
$$

2nd Scan

for $i = 1 : N$ = 2

  for $j = 1 : L$ = 2

    for $k = 1 : M$ = 4

      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C}(i,j) = \bigoplus_{k=1}^{M} \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$$

Inner Product

+ Write locality (sorted)
+ Pre-sum partial products
  (3 entries written)

– N scans over table B

$$
\begin{array}{r}
 \\
\text{word|coffee} \\
\text{word|desert}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{tod|0500} & \text{tod|0800} & \text{tod|1400} \end{array} \\
\begin{bmatrix} 6 & 5 & 2 \\ & \boxed{4} & \end{bmatrix}
\end{array}
\begin{array}{r}
 \\
\text{tod|0800} \\
\text{tod|0900} \\
\text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} & 3 \\ 5 & \\ 3 & 4 \end{bmatrix}
\end{array}
=
\begin{array}{r}
 \\
\text{word|coffee} \\
\text{word|desert}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} 6 & 23 \\ & \boxed{12} \end{bmatrix}
\end{array}
$$

2nd Scan

$\textbf{for } i = 1 : N$ = 2
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Outer Product

Now explicitly showing A^T

$$
\begin{array}{r}
 \\
\text{tod|0500} \\
\text{tod|0800} \\
 \\
\text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\
\begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ & \\ \mathbf{2} & \end{bmatrix}
\end{array}
\quad
\begin{array}{r}
 \\
 \\
\text{tod|0800} \\
\text{tod|0900} \\
\text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} & \\ & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix}
\end{array}
=
\begin{bmatrix} \quad & \quad \end{bmatrix}
$$

$$
\begin{aligned}
&\textbf{for } k = 1:M \text{ = 4} \\
&\quad \textbf{for } i = 1:N \text{ = 2} \\
&\quad\quad \textbf{for } j = 1:L \text{ = 2} \\
&\quad\quad\quad \mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)
\end{aligned}
$$

$$
\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)
$$

Outer Product

1. Align Rows

$$\begin{array}{rc} & \begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\ \begin{array}{r} \text{tod|0500} \\ \text{tod|0800} \\ \text{tod|1400} \end{array} & \begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ \mathbf{2} & \end{bmatrix} \end{array} \begin{array}{rc} & \begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\ \begin{array}{r} \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400} \end{array} & \begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix} \end{array} = \begin{bmatrix} & \end{bmatrix}$$

for $k = 1 : M$ = 4
 for $i = 1 : N$ = 2
  for $j = 1 : L$ = 2
   $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

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$$
\begin{array}{r c}
 & \begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\
\begin{array}{r} \text{tod|0500} \\ \text{tod|0800} \\ \text{tod|1400} \end{array} &
\begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ \mathbf{2} & \end{bmatrix}
\end{array}
\begin{array}{r c}
 & \begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{array}{r} \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400} \end{array} &
\begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix}
\end{array}
= \begin{bmatrix} \quad & \quad \end{bmatrix}
$$

for $k = 1 : M$ = 4
  for $i = 1 : N$ = 2
    for $j = 1 : L$ = 2
      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

2. Cartesian Product

$$
\begin{array}{c|cc}
 & \text{word|coffee} & \text{word|desert} \\
\hline
\text{tod|0500} & \mathbf{6} & \\
\text{tod|0800} & \boxed{\mathbf{5}} & \mathbf{4} \\
\text{tod|1400} & \mathbf{2} &
\end{array}
\quad
\begin{array}{c|cc}
 & \text{word|dew} & \text{word|hot} \\
\hline
\text{tod|0800} & & \boxed{\mathbf{3}} \\
\text{tod|0900} & \mathbf{5} & \\
\text{tod|1400} & \mathbf{3} & \mathbf{4}
\end{array}
=
\begin{array}{c|c}
 & \text{word|hot} \\
\hline
\text{word|coffee} & \boxed{\mathbf{15}}
\end{array}
$$

1

for $k = 1 : M$ = 4
  for $i = 1 : N$ = 2
    for $j = 1 : L$ = 2
      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

2. Cartesian Product

$$
\begin{array}{r}
 \\ \text{tod|0500} \\ \text{tod|0800} \\ \\ \text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\
\begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \boxed{\mathbf{4}} \\ & \\ \mathbf{2} & \end{bmatrix}
\end{array}
\quad
\begin{array}{r}
 \\ \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} & \boxed{\mathbf{3}} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix}
\end{array}
=
\begin{array}{r}
 \\ \text{word|coffee} \\ \text{word|desert}
\end{array}
\begin{array}{c}
\text{word|hot} \\
\begin{bmatrix} \mathbf{15} \\ \boxed{\mathbf{12}} \end{bmatrix}
\end{array}
$$

1

2

for $k = 1 : M$ = 4
    for $i = 1 : N$ = 2
        for $j = 1 : L$ = 2
            $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

1. Align Rows

$$
\begin{array}{r}
\\ \text{tod|0500} \\ \text{tod|0800} \\ \\ \text{tod|1400}
\end{array}
\begin{array}{cc}
\text{word|coffee} & \text{word|desert} \\
\left[\begin{array}{cc} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ & \\ \mathbf{2} & \end{array}\right]
\end{array}
\quad
\begin{array}{r}
\\ \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400}
\end{array}
\begin{array}{cc}
\text{word|dew} & \text{word|hot} \\
\left[\begin{array}{cc} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{array}\right]
\end{array}
=
\begin{array}{r}
\\ \text{word|coffee} \\ \text{word|desert}
\end{array}
\begin{array}{c}
\text{word|hot} \\
\left[\begin{array}{c} \mathbf{15} \\ \mathbf{12} \end{array}\right]
\end{array}
\begin{array}{l}
\\ 1 \\ 2
\end{array}
$$

for $k = 1 : M$ = 4
  for $i = 1 : N$ = 2
    for $j = 1 : L$ = 2
      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

1. Align Rows

$$
\begin{array}{rc}
 & \begin{array}{cc} \text{word}\mid\text{coffee} & \text{word}\mid\text{desert} \end{array} \\
\begin{array}{r} \text{tod}\mid\text{0500} \\ \text{tod}\mid\text{0800} \\ \text{tod}\mid\text{1400} \end{array} &
\begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ \mathbf{2} & \end{bmatrix}
\end{array}
\quad \Rightarrow \quad
\begin{array}{rc}
 & \begin{array}{cc} \text{word}\mid\text{dew} & \text{word}\mid\text{hot} \end{array} \\
\begin{array}{r} \text{tod}\mid\text{0800} \\ \text{tod}\mid\text{0900} \\ \text{tod}\mid\text{1400} \end{array} &
\begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \mathbf{4} \end{bmatrix}
\end{array}
=
\begin{array}{rc}
 & \begin{array}{c} \text{word}\mid\text{hot} \end{array} \\
\begin{array}{r} \text{word}\mid\text{coffee} \\ \text{word}\mid\text{desert} \end{array} &
\begin{bmatrix} \mathbf{15} \\ \mathbf{12} \end{bmatrix}
\end{array}
\begin{array}{c} 1 \\ 2 \end{array}
$$

for $k = 1 : M$ = 4
for $i = 1 : N$ = 2
for $j = 1 : L$ = 2
$\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

2. Cartesian Product

$$
\begin{array}{r} \\ \text{tod|0500} \\ \text{tod|0800} \\ \\ \text{tod|1400} \end{array}
\begin{array}{c} \begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\ \begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ & \\ \boxed{\mathbf{2}} & \end{bmatrix} \end{array}
\quad
\begin{array}{r} \\ \text{tod|0800} \\ \text{tod|0900} \\ \text{tod|1400} \end{array}
\begin{array}{c} \begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\ \begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \boxed{\mathbf{3}} & \mathbf{4} \end{bmatrix} \end{array}
=
\begin{array}{r} \\ \text{word|coffee} \\ \text{word|desert} \end{array}
\begin{array}{c} \begin{array}{cc} 3\ \text{word|dew} & \text{word|hot}\ 1 \end{array} \\ \begin{bmatrix} \boxed{\mathbf{6}} & \mathbf{15} \\ & \mathbf{12} \end{bmatrix} \\ 2 \end{array}
$$

$$
\begin{aligned}
&\textbf{for } k = 1:M = 4 \\
&\quad \textbf{for } i = 1:N = 2 \\
&\quad\quad \textbf{for } j = 1:L = 2 \\
&\quad\quad\quad \mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)
\end{aligned}
$$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

2. Cartesian Product

***Lazy** $\oplus$:
Accumulo stores both 15 and 8 until next scan or compaction

$$
\begin{array}{r}
 \\
\text{tod|0500} \\
\text{tod|0800} \\
\\
\text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|coffee} & \text{word|desert} \end{array} \\
\begin{bmatrix} \mathbf{6} & \\ \mathbf{5} & \mathbf{4} \\ & \\ \boxed{\mathbf{2}} & \end{bmatrix}
\end{array}
\quad
\begin{array}{r}
 \\
\text{tod|0800} \\
\text{tod|0900} \\
\text{tod|1400}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} & \mathbf{3} \\ \mathbf{5} & \\ \mathbf{3} & \boxed{\mathbf{4}} \end{bmatrix}
\end{array}
=
\begin{array}{r}
 \\
\text{word|coffee} \\
\text{word|desert}
\end{array}
\begin{array}{c}
\begin{array}{cc} \text{word|dew} & \text{word|hot} \end{array} \\
\begin{bmatrix} \mathbf{6} & \boxed{\mathbf{23}} \\ & \mathbf{12} \end{bmatrix}
\end{array}
*
$$

for $k = 1:M$ = 4
  for $i = 1:N$ = 2
    for $j = 1:L$ = 2
      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Outer Product

- **No write locality; unsorted writes**
- **Hard to pre-sum partial products (4 entries written)**

+ Single scan over table B

***Lazy ⊕:**
Accumulo stores both 15 and 8 until next scan or compaction

$$
\begin{array}{r}
\\
\text{tod|0500} \\
\text{tod|0800} \\
\\
\text{tod|1400}
\end{array}
\begin{array}{cc}
\text{word|coffee} & \text{word|desert} \\
\left[\begin{array}{c} \mathbf{6} \\ \mathbf{5} \\ \\ \boxed{\mathbf{2}} \end{array}\right.
& \left.\begin{array}{c} \\ \mathbf{4} \\ \\ \\ \end{array}\right]
\end{array}
\quad
\begin{array}{r}
\\
\text{tod|0800} \\
\text{tod|0900} \\
\text{tod|1400}
\end{array}
\begin{array}{cc}
\text{word|dew} & \text{word|hot} \\
\left[\begin{array}{c} \\ \mathbf{5} \\ \mathbf{3} \end{array}\right.
& \left.\begin{array}{c} \mathbf{3} \\ \\ \boxed{\mathbf{4}} \end{array}\right]
\end{array}
=
\begin{array}{r}
\\
\text{word|coffee} \\
\text{word|desert}
\end{array}
\begin{array}{cc}
\textcircled{3}\ \text{word|dew} & \text{word|hot}\ \textcircled{1} \\
\left[\begin{array}{c} \mathbf{6} \\ \\ \end{array}\right.
& \left.\begin{array}{c} \boxed{\mathbf{23}}\ \textcircled{4}\,* \\ \mathbf{12} \\ \textcircled{2} \end{array}\right]
\end{array}
$$

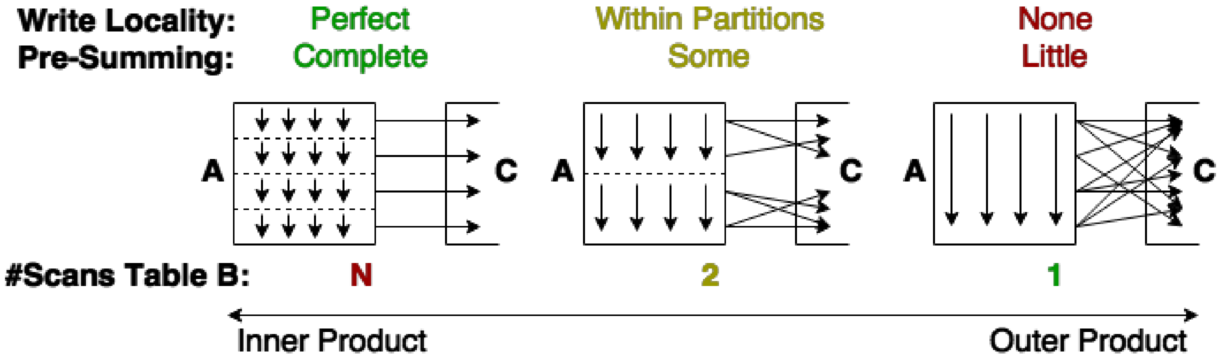
for $k = 1 : M$ = 4
  for $i = 1 : N$ = 2
    for $j = 1 : L$ = 2
      $\mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)$

$$\mathbf{C} = \bigoplus_{k=1}^{M} \mathbf{A}(:,k)\mathbf{B}(k,:)$$

Inner vs. Outer Product

- **Outer product best for Accumulo**
  - **Single pass over table B = single disk read**
  - **BatchWriter ingest handles unsorted writes**
  - **Combiners handle ⊕**
  - **Less extra partial products written for sparse data**

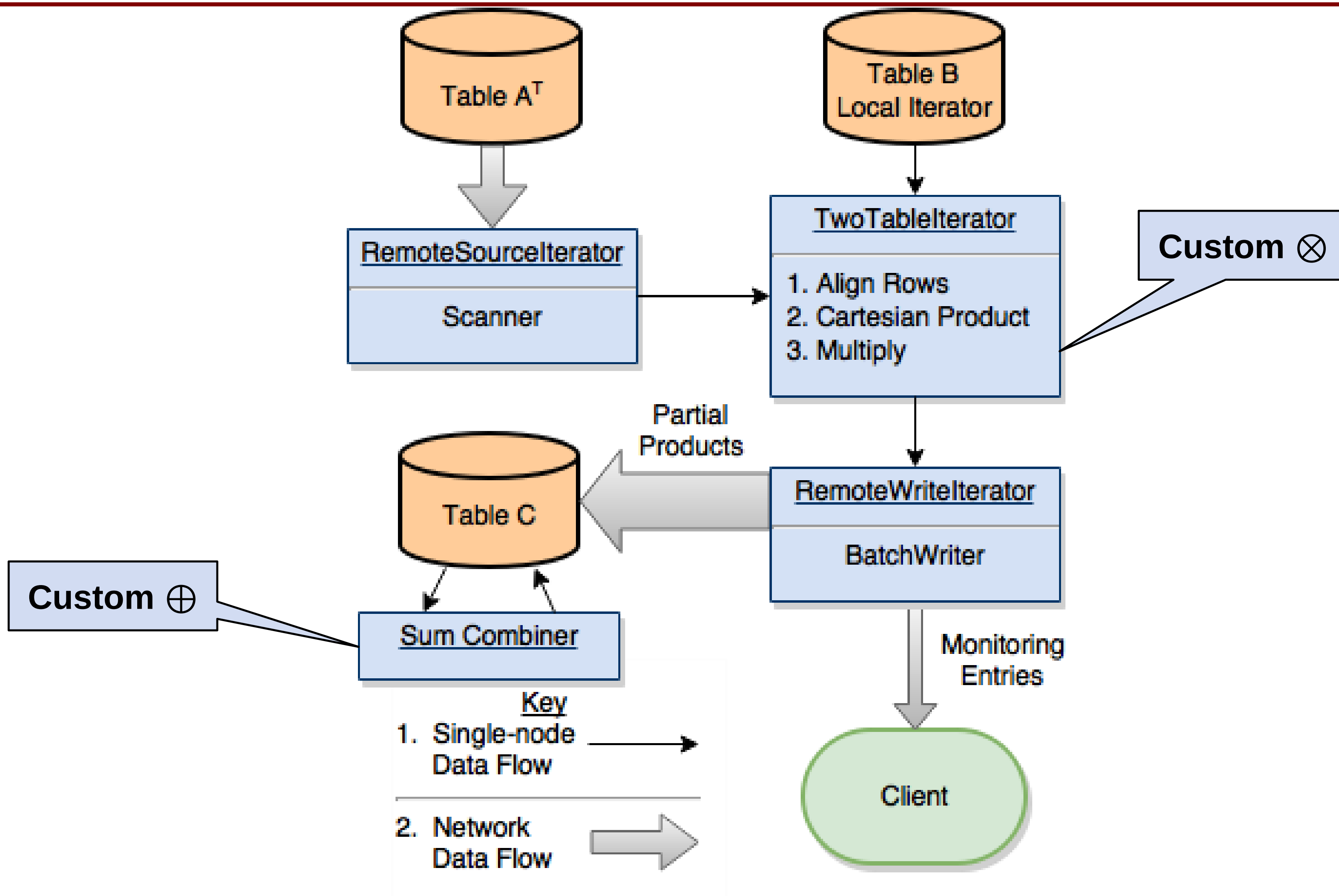
- **Inner product still has merit**
  - **Better for dense data**
  - **Hybrid 2D-like algorithm possible**



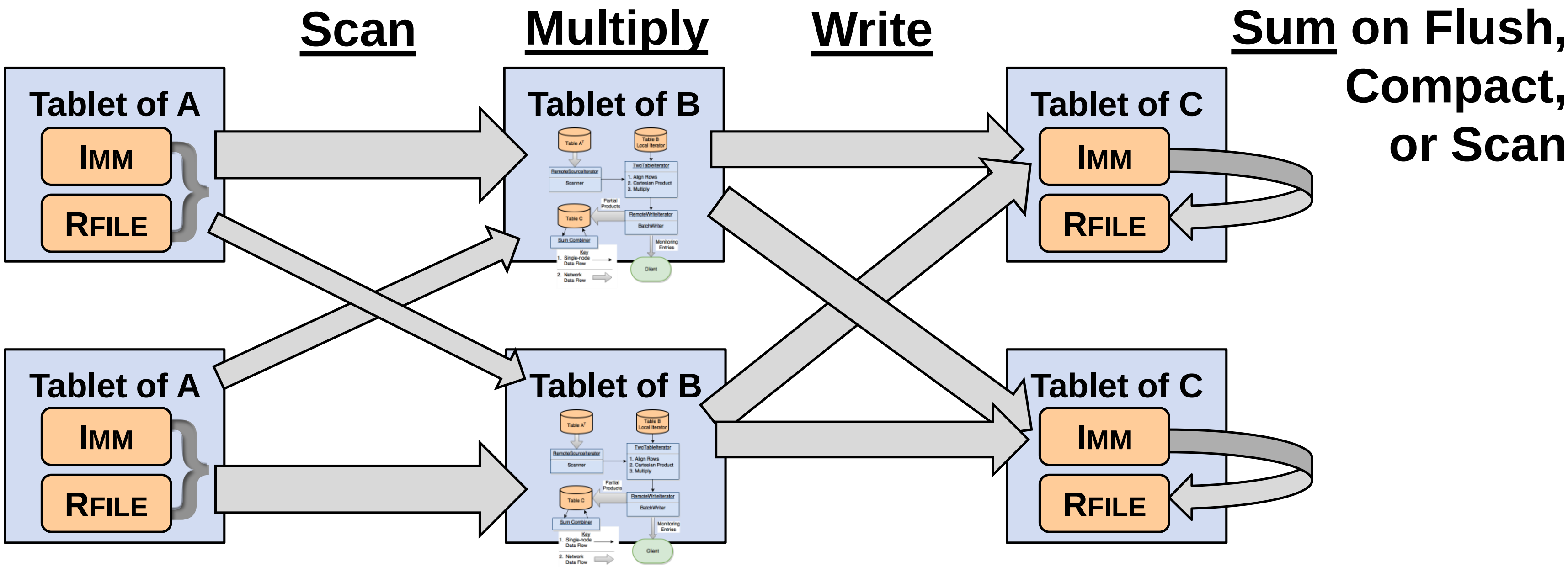
Outline

- Intro to Graphulo
- Intro to Matrix Multiply
- Intro to Accumulo
- Matrix Multiply pre-Graphulo
- Inner Product
- Outer Product
- Accumulo Implementation
- Performance
- Conclusions

Outer Product in Graphulo Iterators



Accumulo Distributes Graphulo Iterators



Key

IMM: In-Memory Map
RFILE: Hadoop File

- Tablets can be hosted on any tablet server
  - Accumulo load balances tablet allocation
- Matrix multiply iterators run on B's tablets in parallel
  - Scan from A's tablets in parallel
  - BatchWrite to C's tablets in parallel

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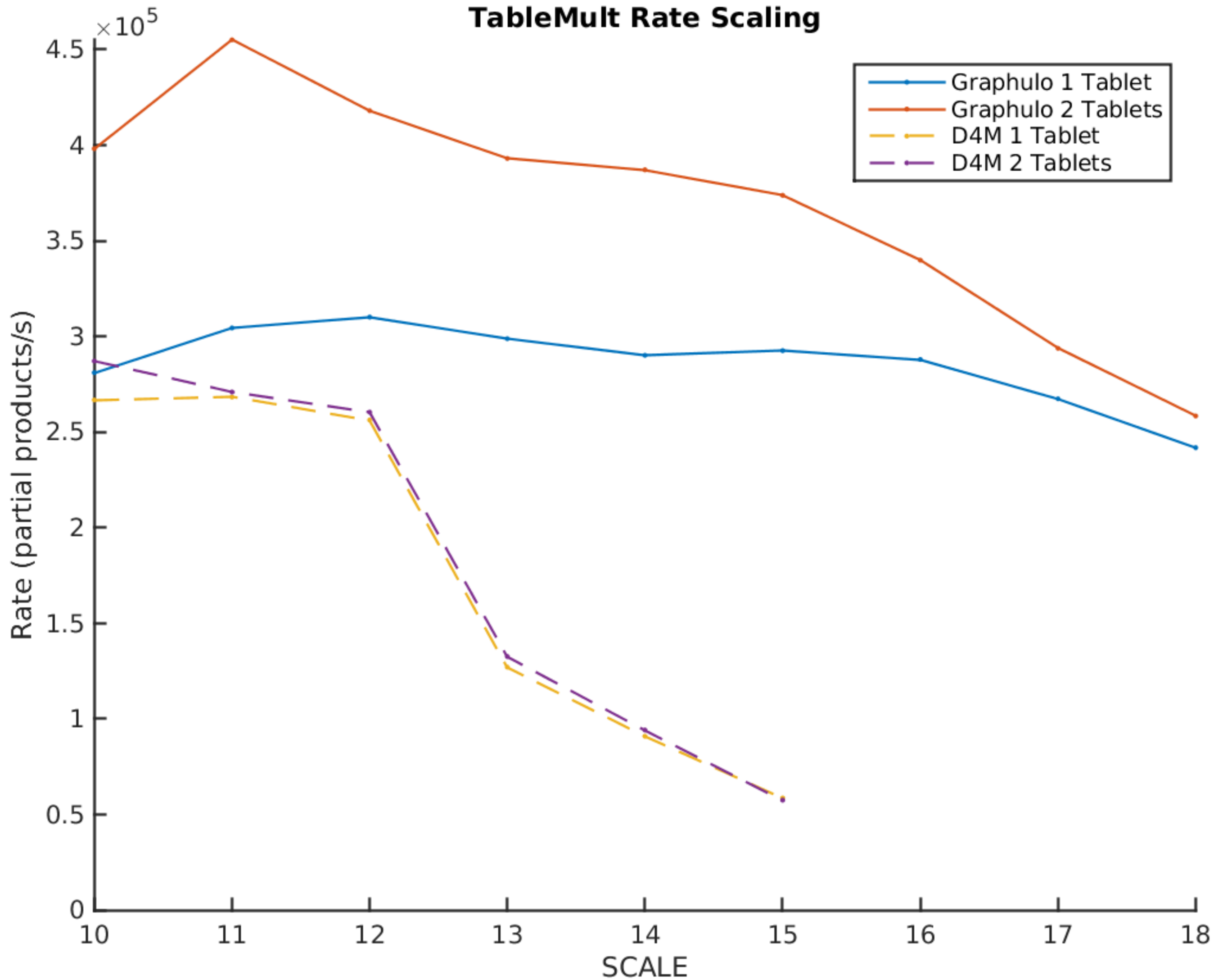
Performance Experiment

- **Compare to pre-Graphulo alternative:**
  - **D4M Matlab client as Middleman**
- **Scaled / Weak scaling study:**
  - **How multiply rate varies with increasing problem size at fixed resources**
  - **Ideal: constant multiply rate**
- **Fixed / Strong scaling study:**
  - **How multiply rate varies with increasing resources at fixed problem size**
  - **Ideal: multiply rate scales linearly with increasing resources**

- **Environment:**
  - **Laptop, 16GB RAM, 2 Dual-core i7 processors, Accumulo 1.6.1**
- **Vary problem size between SCALE 10 and 18**
  - **Unpermuted Power law graph generator**
  - **# of nodes in each input table is 2^{SCALE}. Used 16 edges/node**
- **Vary resources with # Accumulo Tablets (Varies # Threads)**

Performance Experiment



Outline

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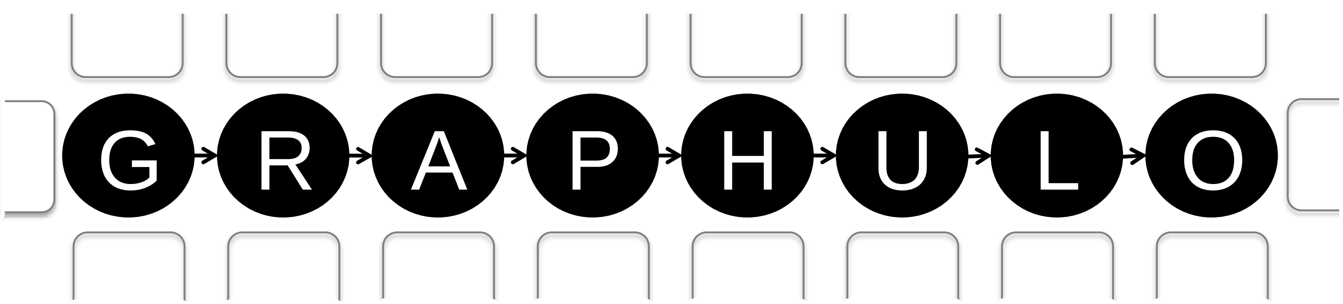
Conclusion

- **Promising performance**
  - **Write rates near 400k / sec, near highest single-node recorded rates**
  - **Experiments on a larger cluster will confirm weak & strong scaling**
- **Outer product better suited to Accumulo**
  - **Hybrid inner-outer product algorithms worth studying**
- **Current Graphulo research is**
  - **implementing remaining GraphBLAS**
  - **developing graph algorithms**



Backup

TABLE I: Output Table **C** Sizes and Experiment Timings

SCALE	Entries in Table C		Graphulo 1 Tablet		D4M 1 Tablet		Graphulo 2 Tablets		D4M 2 Tablets	
	PartialProducts	AfterSum	Time (s)	Rate (pp/s)	Time (s)	Rate (pp/s)	Time (s)	Rate (pp/s)	Time (s)	Rate (pp/s)
10	8.05×10^5	2.69×10^5	2.87	2.81×10^5	3.02	2.67×10^5	2.02	3.98×10^5	2.80	2.87×10^5
11	2.36×10^6	8.15×10^5	7.76	3.04×10^5	8.80	2.68×10^5	5.19	4.55×10^5	8.72	2.71×10^5
12	6.82×10^6	2.43×10^6	2.20×10^1	3.10×10^5	2.66×10^1	2.56×10^5	1.63×10^1	4.18×10^5	2.62×10^1	2.60×10^5
13	1.91×10^7	7.04×10^6	6.40×10^1	2.99×10^5	1.50×10^2	1.27×10^5	4.86×10^1	3.93×10^5	1.44×10^2	1.33×10^5
14	5.27×10^7	2.00×10^7	1.82×10^2	2.90×10^5	5.79×10^2	9.09×10^4	1.36×10^2	3.87×10^5	5.59×10^2	9.42×10^4
15	1.47×10^8	5.83×10^7	5.03×10^2	2.93×10^5	2.51×10^3	5.86×10^4	3.94×10^2	3.74×10^5	2.56×10^3	5.75×10^4
16	4.00×10^8	1.63×10^8	1.39×10^3	2.88×10^5			1.18×10^3	3.40×10^5		
17	1.09×10^9	4.59×10^8	4.06×10^3	2.67×10^5			3.70×10^3	2.94×10^5		
18	2.94×10^9	1.28×10^9	1.21×10^4	2.42×10^5			1.14×10^4	2.58×10^5		

Inner-Outer Hybrid Algorithm

$$
\begin{aligned}
&\textbf{for } p = 1 : P \\
&\quad \textbf{for } k = 1 : M \\
&\quad\quad \textbf{for } i = \left(\left\lfloor \frac{(p-1)N}{P} \right\rfloor + 1 \right) : \left\lfloor \frac{pN}{P} \right\rfloor \\
&\quad\quad\quad \textbf{for } j = 1 : L \\
&\quad\quad\quad\quad \mathbf{C}(i,j) \oplus= \mathbf{A}(i,k) \otimes \mathbf{B}(k,j)
\end{aligned}
$$

P = N – Inner Product

P = 1 – Outer Product

D4M Schema for Sparse Arrays in Key/Value Databases (Accumulo)

Accumulo Table: `Ttranspose`

	01-01-2001	02-01-2001	03-01-2001
Col1|a	1		
Col1|b		1	
Col2|b		1	
Col2|c			1
Col3|a	1		
Col3|c			1

Input Data

Time	Col1	Col2	Col3
2001-01-01	a		a
2001-01-02	b	b	
2001-01-03		c	c

	Col1|a	Col1|b	Col2|b	Col2|c	Col3|a	Col3|c
01-01-2001	1				1	
02-01-2001		1	1			
03-01-2001				1		1

Accumulo Table: `T`

- **Tabular data expanded to create many type/value columns**
- **Transpose pairs allows quick look up of either row or column**

[1]*D4M 2.0 Schema: A General Purpose High Performance Schema for the Accumulo Database*
Kepner et al, IEEE HPEC 2013