

# **In-Database Analytics for NoSQL Key-Value Stores**

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Dylan Hutchison, 5 December 2016

Advisors: Bill Howe, Dan Suciu

# Tons of NoSQL Key-Value Stores!



levelDB



Microsoft Azure  
DocumentDB

ORACLE®

BERKELEY DB

APACHE  
HBASE





# Who uses a K-V DB? How?

## Basho Riak Use Cases

➔ Find a "natural key" for your application

DATA TYPE	KEY	VALUE
Session	User/Session ID	Session Data
Content	ID	Documents, Images, Posts, Videos, Texts, JSON/HTML, etc.
Advertising	Campaign ID	Ad Content
Logs	Date	Log File
Sensor	Date/Time	Sensor Updates
User Data	Login, Email, UUID	User Attributes



# Who uses a K-V DB? How?

## Timely – Time Series Event Monitoring

Row	ColumnFamily	ColumnQualifier	Value
sys.cpu.user\1447879348291	host=r001n01	instance=0,rack=r001	2.0
sys.cpu.user\1447879348291	instance=0	host=r001n01,rack=r001	2.0
sys.cpu.user\1447879348291	rack=r001	host=r001n01,instance=0	2.0

- Compound key
- Intentional order
- Multiple values per key
- Repeated values

**Many tricks to encode information into K-Vs**

# Who uses a K-V DB? How?

GeoMesa – Spatiotemporal Store

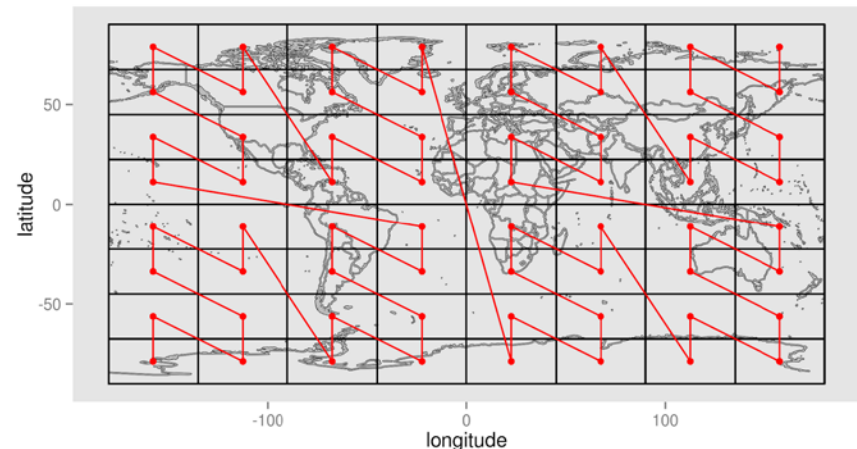
K-V encoding tricks  
can be arbitrarily complex

KEY							VALUE
ROW			COLUMN		TIMESTAMP	VIZ	Byte-encoded <b>SimpleFeature</b>
			COLUMN FAMILY	COLUMN QUALIFIER			
Epoch Week 2 bytes	<b>Z3(x,y,t)</b> 8 bytes	Unique ID (such as UUID)	"F"	-	-	Security tags	

Hierarchical  
Storage

Hash  
Functions

+ Secondary or Full Indexes on Values



# Why use K-V stores?

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- > Scale out – large commodity clusters
- > Transparent partitioning, layout, performance
  - Friendly to developers & applications that want control
- > Some have "special features"
  - entry-level access control
- > Simple solution, *if* all your application requires is to read & write entries by key



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***What if you want to do more?***

**Computing beyond read-writes**

**Computing @application client  
can have drawbacks...**



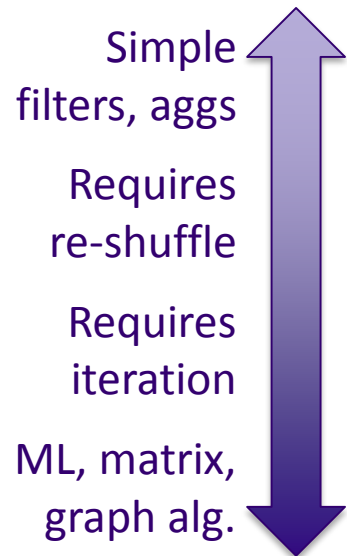


# Computing on Key-Value DB

Example from Riak Use Case: Browser Sessions (e.g. Wikipedia)

DATA TYPE	KEY	VALUE
Session	User/Session ID	Session Data (pages visited, shopping cart, ...)

- > **Fast read-writes perfect for low-latency web server**
- > **What if the website managers want to run analytics?**







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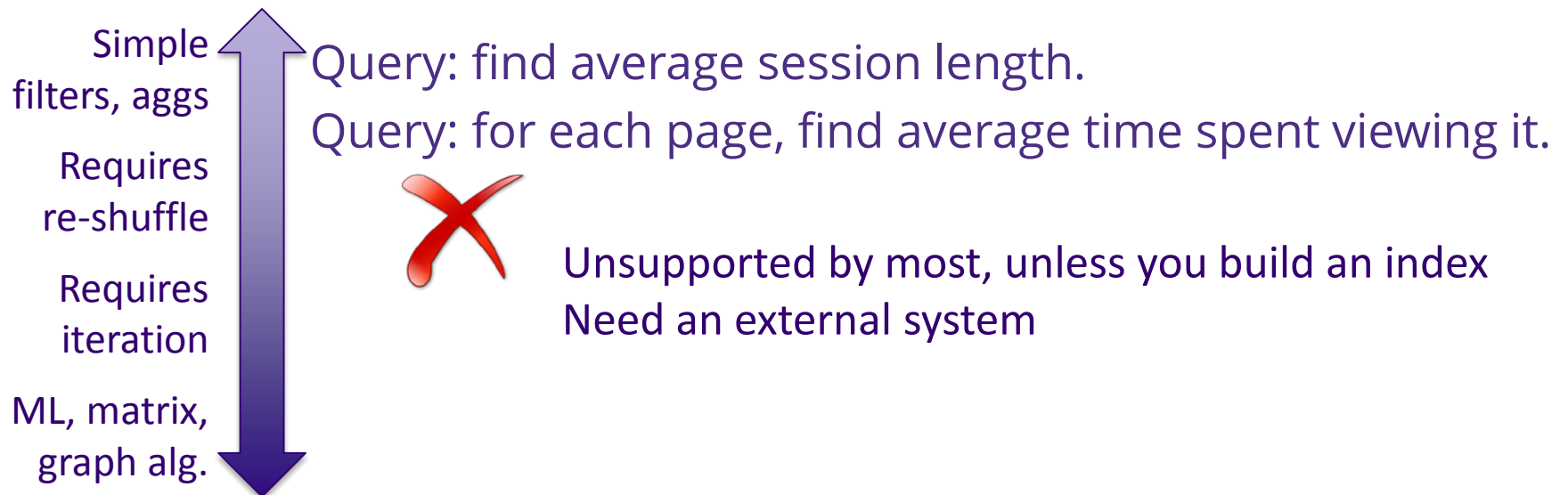


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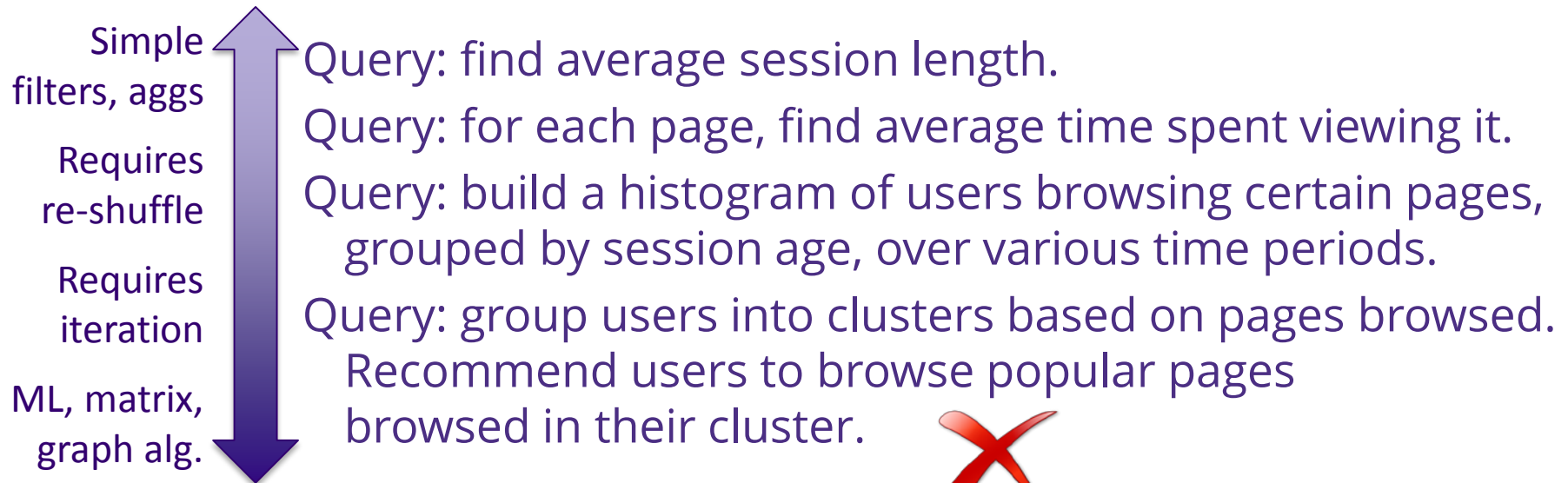


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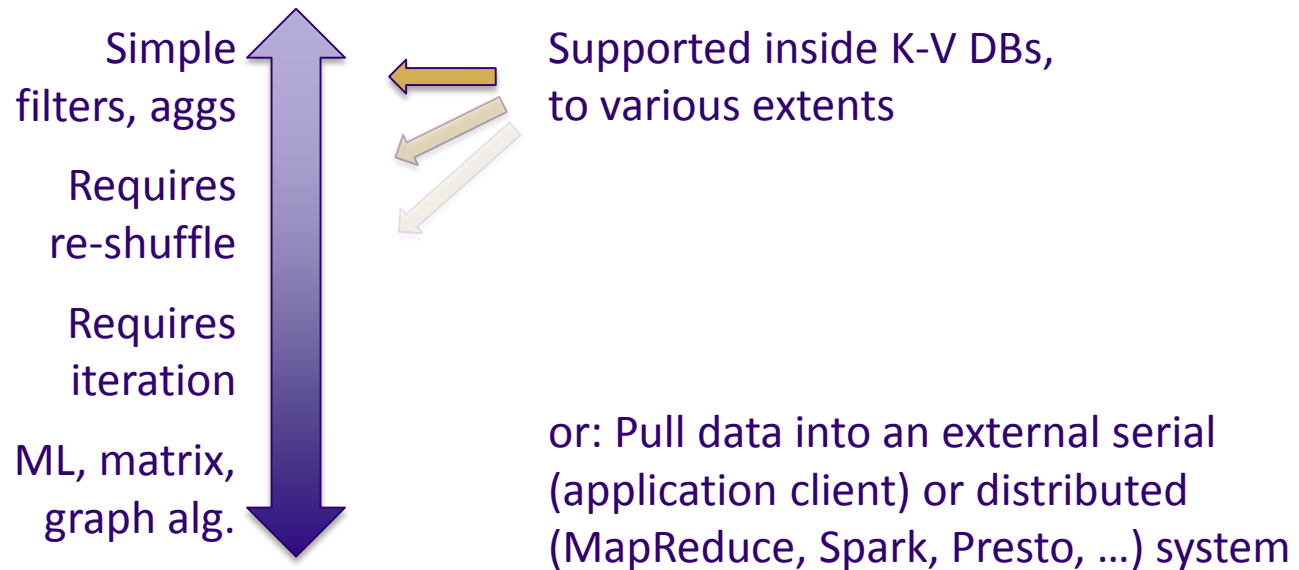
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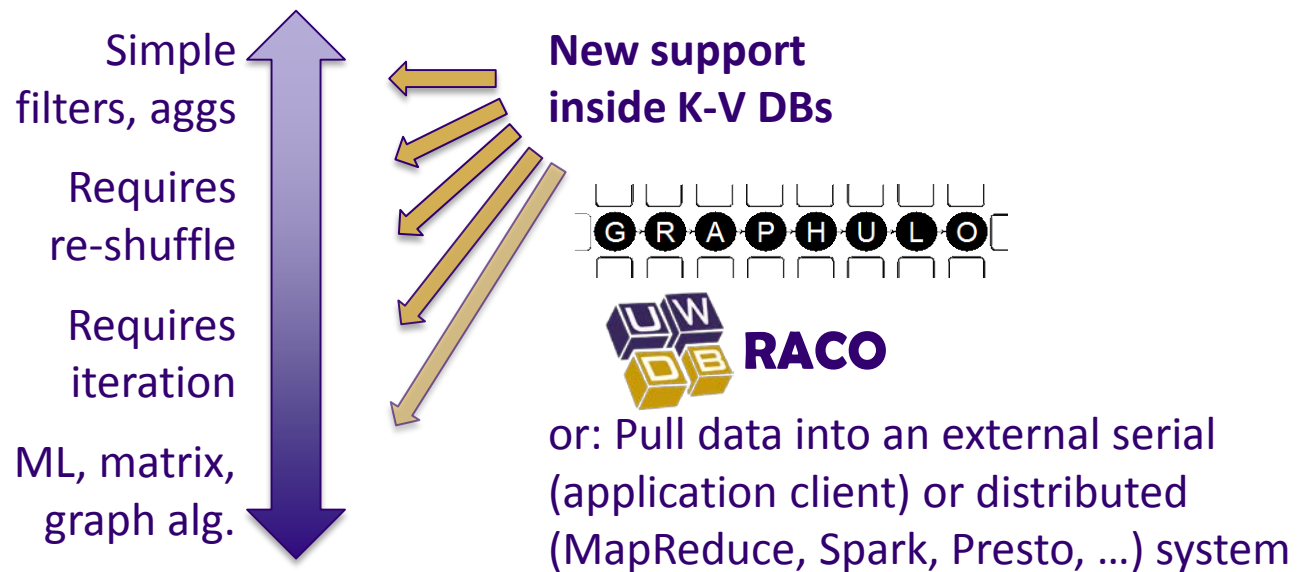
# Computing on Key-Value DB

## State of the Art



# Computing on Key-Value DB

## This Talk



- > How to implement analytics in K-V DBs?
- > When is it good to do so?
  - Consider: # of re-shuffles, access path, selectivity, ...



# Computing on Key-Value DB

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## Benefits of Server-Side Computation

### 1. Data Locality

- Save communication

### 2. Reuse infrastructure

- One less system to adopt and maintain

### 3. Database features for free

- Indexed access
- Distributed execution



# Graphulo: Linear Algebra for Apache Accumulo

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or, conceptually, any  
**Google BigTable**-based K-V DB





# GraphBLAS: Working Spec for LA

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	GraphBLAS Kernel
	BuildMatrix ( $\oplus$ )
	ExtractTuples
	MxM ( $\oplus, \otimes$ )
	EwiseMult ( $\otimes$ )
	EwiseAdd ( $\oplus$ )
	Extract
	Apply ( $f$ )
	Assign
	Reduce ( $\oplus$ )
	Transpose

**for *Sparse* Matrices**



# Matrix as K-Vs

Key				Value	
Row	Column				Timestamp
	Family	Qualifier	Visibility		

- > Adjacency list view of Matrix
- > (Row, Column Qualifier, Value)  
= (v<sub>1</sub>, v<sub>2</sub>, weight)  
[Transpose: (v<sub>2</sub>, v<sub>1</sub>, weight)]

$$\begin{array}{c}
 1 \quad 10 \quad \dots \\
 1 \begin{bmatrix} 141 & 12 & \dots \\ 18 & \vdots & \\ \vdots & \vdots & \ddots \end{bmatrix} \\
 10 \\
 \vdots
 \end{array}$$

Example K-Vs			
row	:colq	->val	
1	:1 []	->	141
1	:10 []	->	12
1	:101 []	->	9
1	:105 []	->	3
1	:11 []	->	9
1	:110 []	->	3
10	:1 []	->	18
10	:109 []	->	2
15	:1 []	->	18
15	:109 []	->	2

Other schemas supported:

- Edge List / Incidence Matrix
- Single Table with transpose & degrees



# Matrix as K-Vs

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 \vdots \begin{bmatrix} \vdots & \dots & \ddots \end{bmatrix}
 \end{array}$$

Other schemas supported:

- Edge List / Incidence Matrix
- Single Table with transpose & degrees

		Example K-Vs		
		row :colq ->val		
Partition by Row	Tablet 1	1 :1 []	->	141
		1 :10 []	->	12
		1 :101 []	->	9
		1 :105 []	->	3
		1 :11 []	->	9
		1 :110 []	->	3
		10 :1 []	->	18
		10 :109 []	->	2
Tablet 2		15 :1 []	->	18
		15 :109 []	->	2



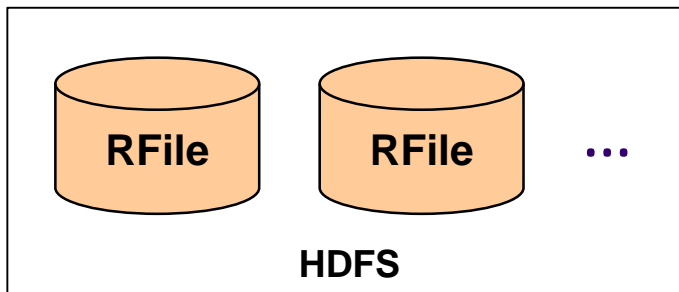


# Accumulo Scan Iterator Pipeline

**Goal: Understand Accumulo's support for in-database computation in order to re-purpose it for analytics**

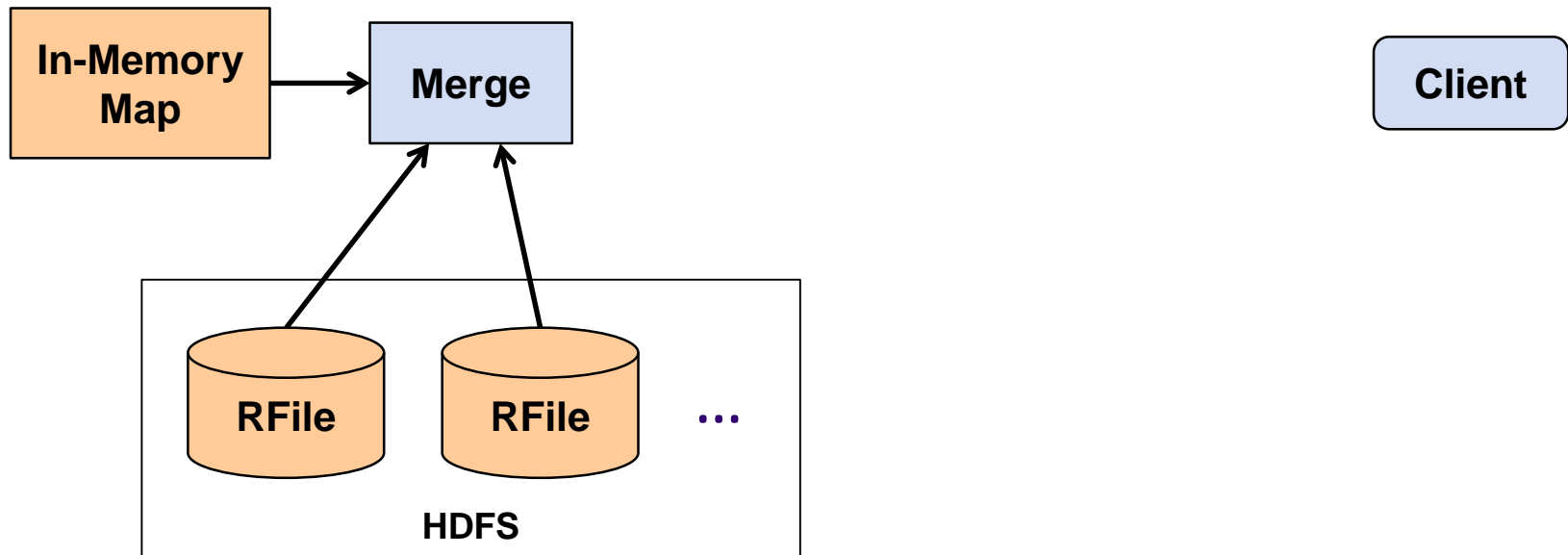
**In-Memory Map**

**Client**

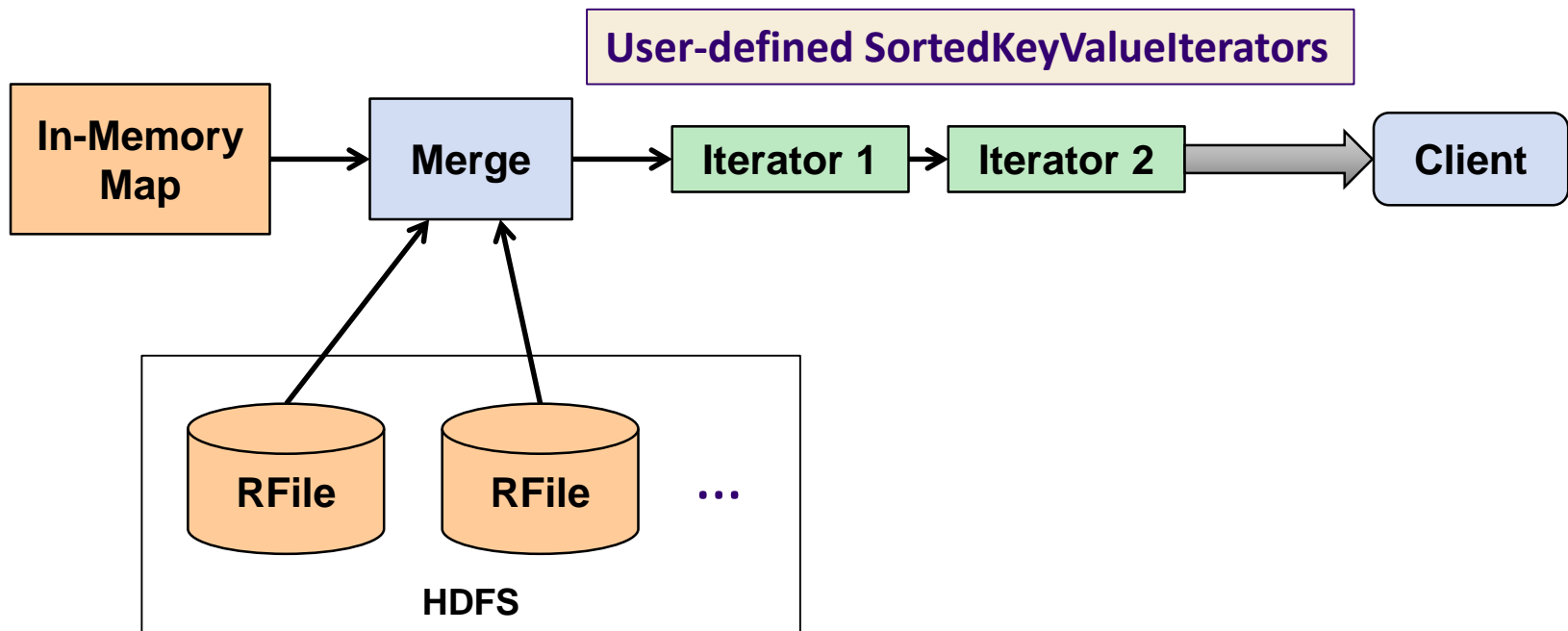




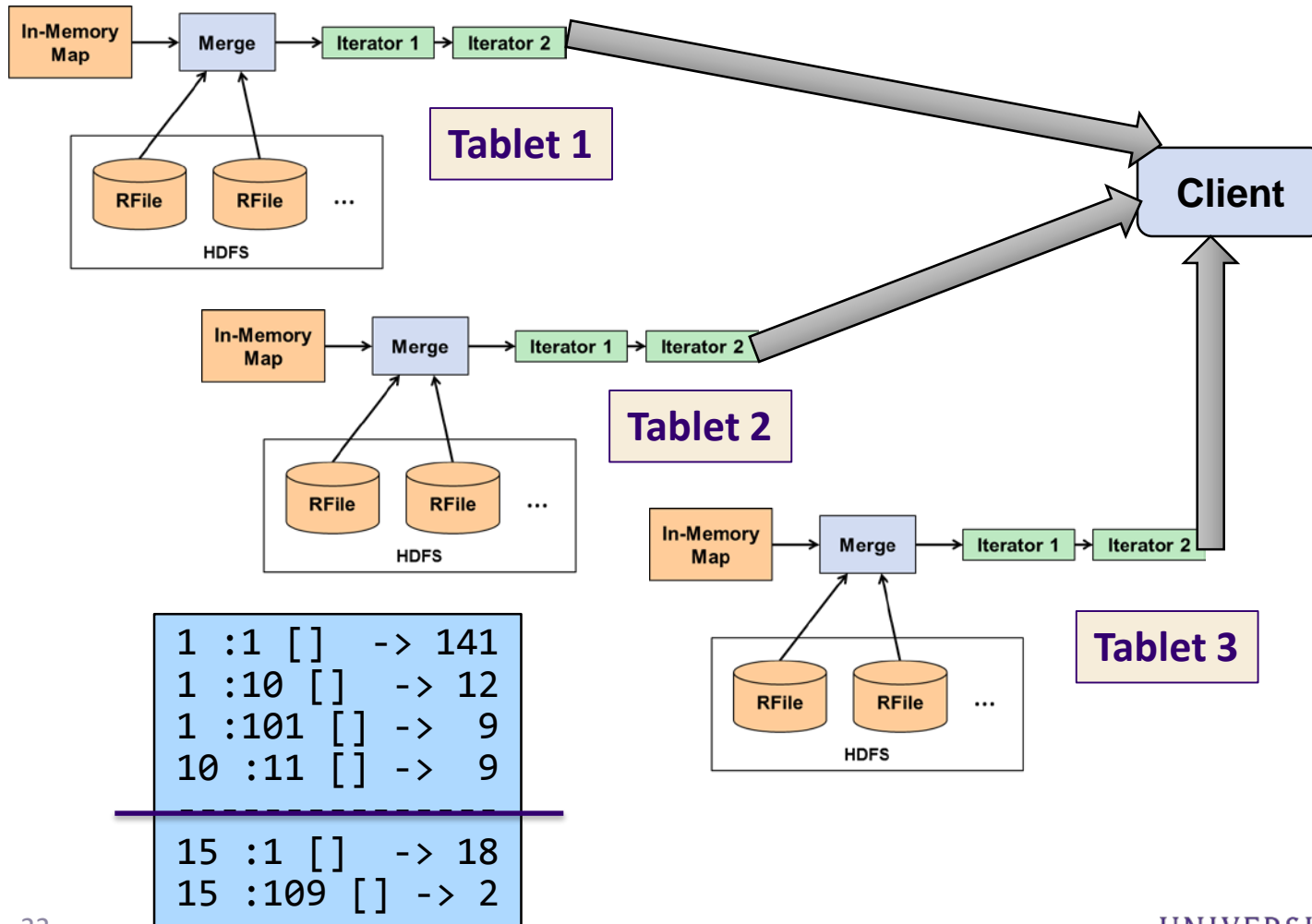
# Accumulo Scan Iterator Pipeline



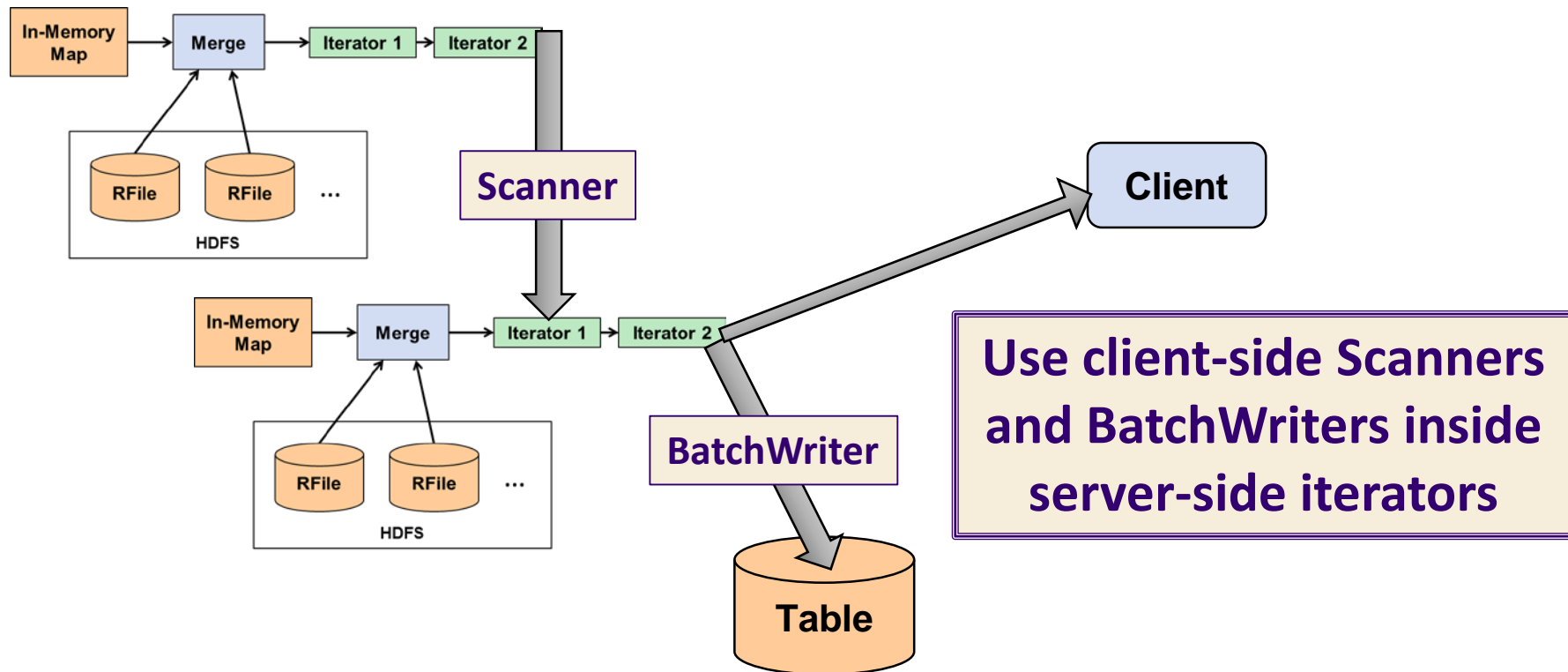
# Accumulo Scan Iterator Pipeline



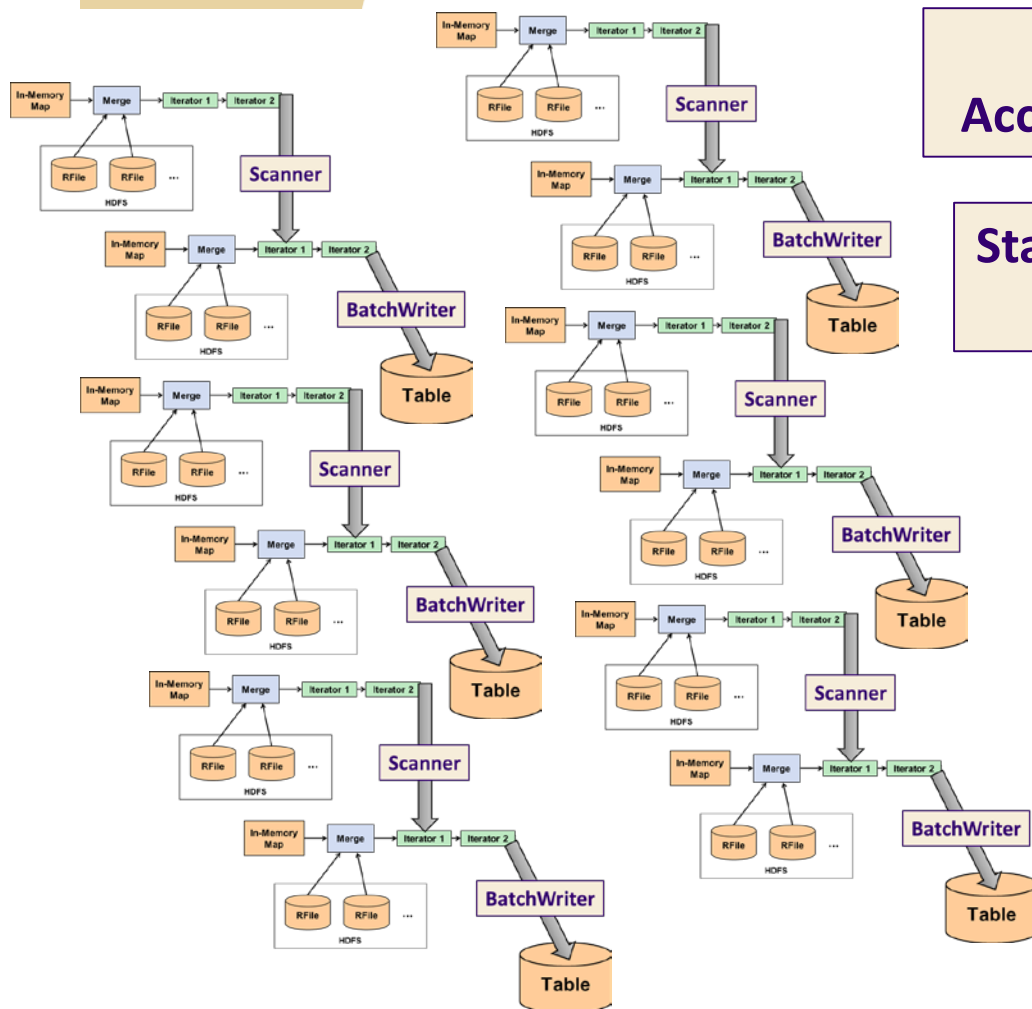
# Accumulo BatchScan Iterator Pipeline



# Graphulo addition to Iterator Pipeline



# Graphulo addition to Iterator Pipeline



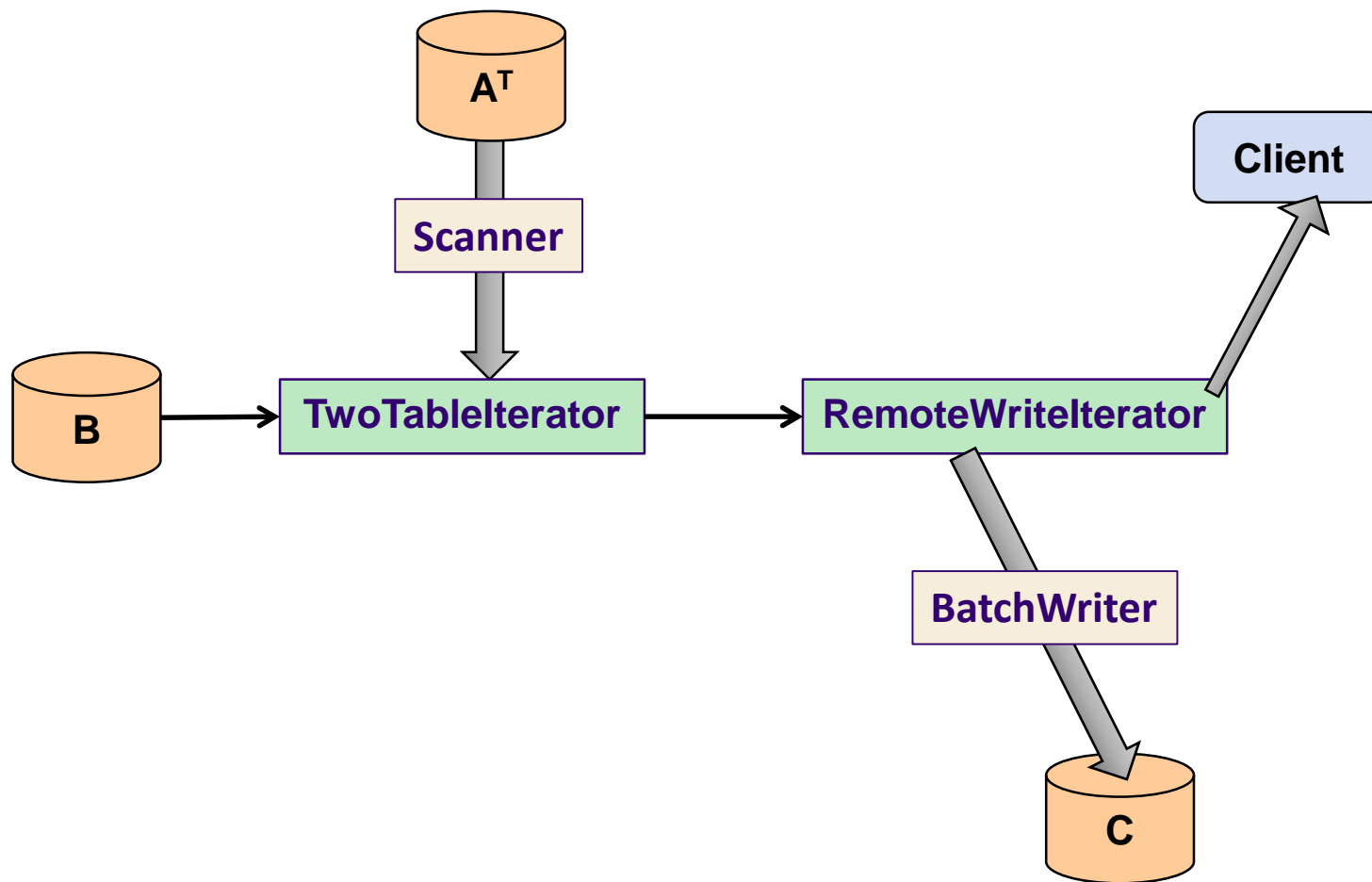
Distributes with  
Accumulo's Tablet Servers

Standing tablet server thread pools  
service queries interactively

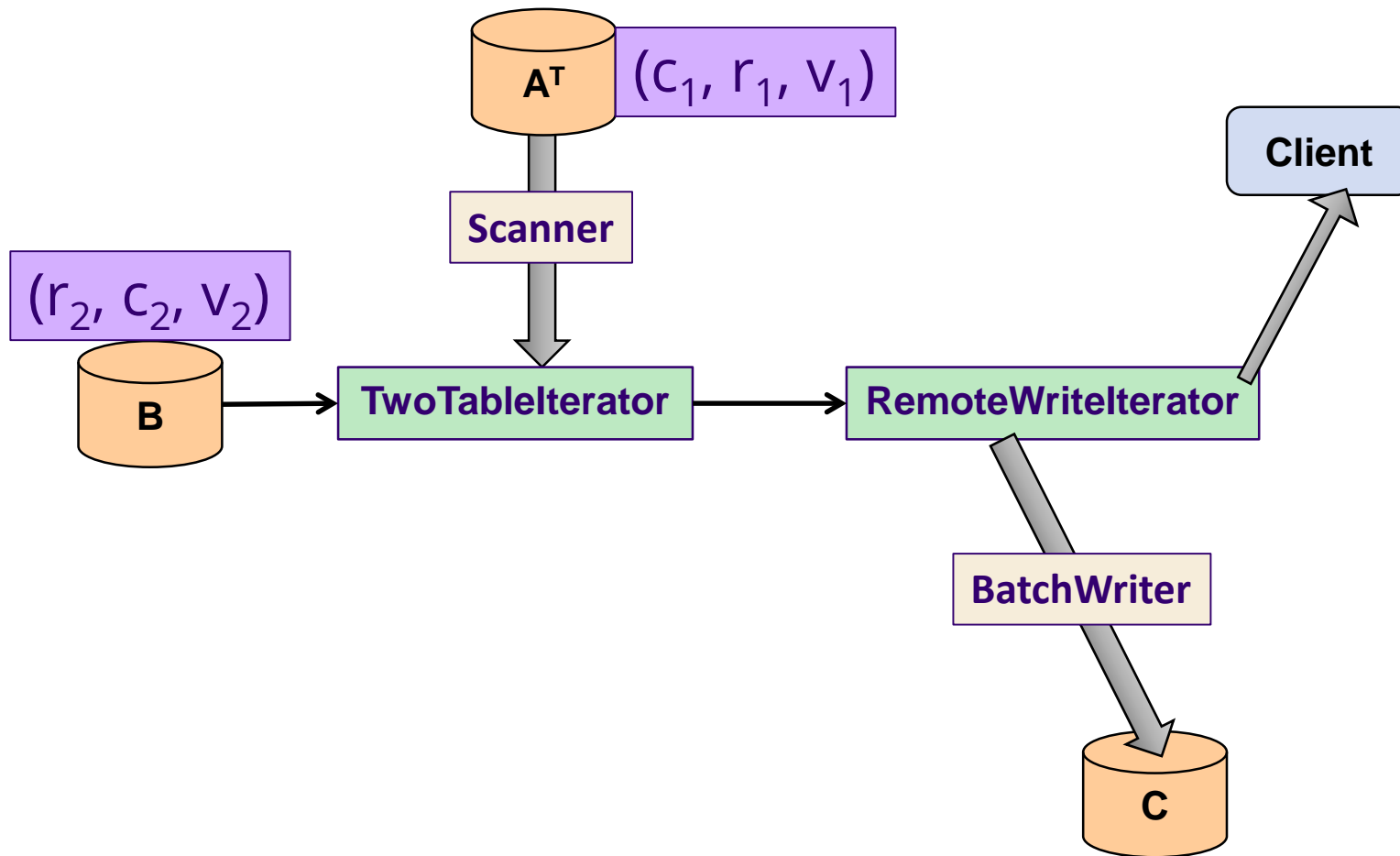
Use client-side Scanners  
and BatchWriters inside  
server-side iterators



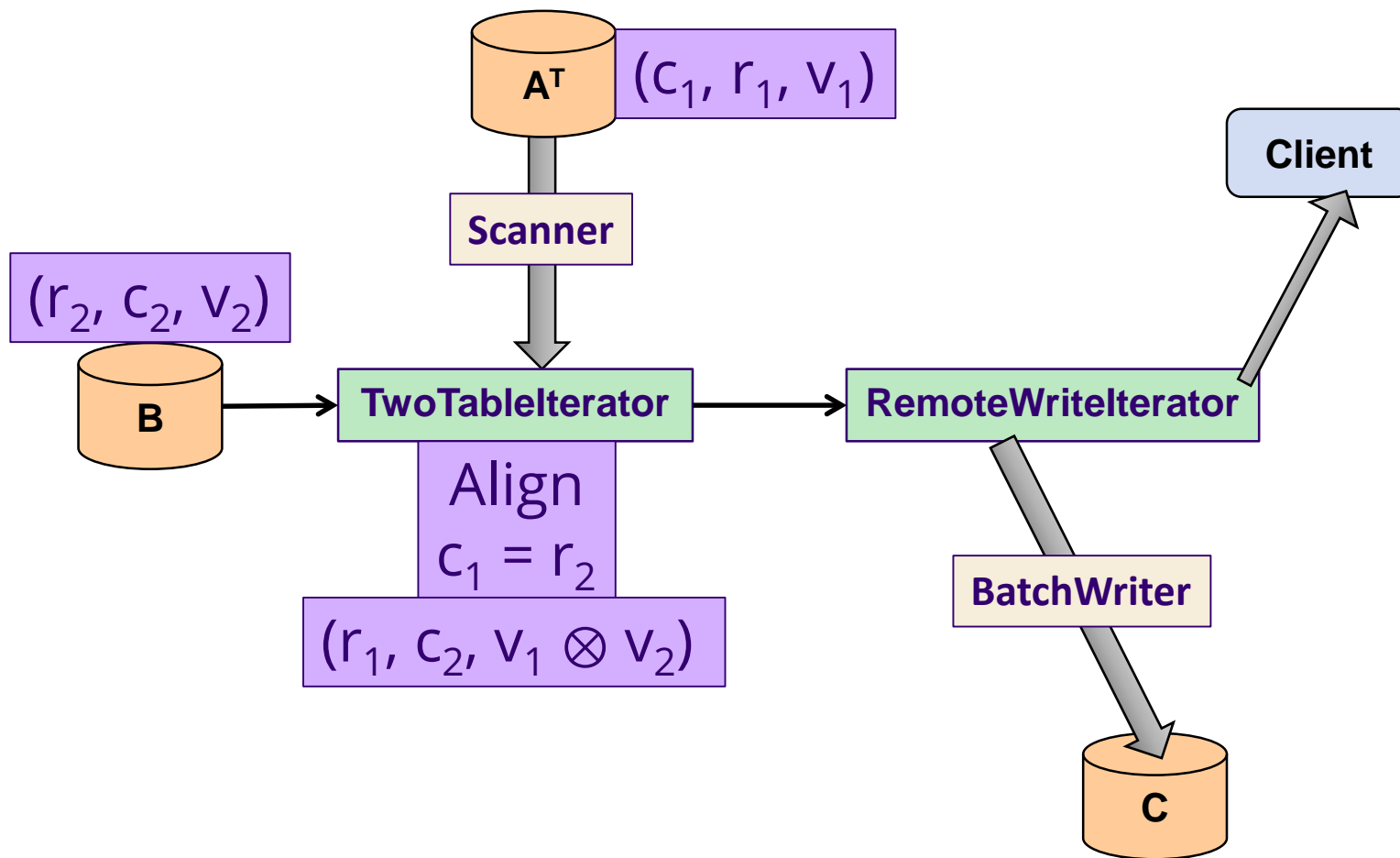
# Graphulo MxM: $A^T (\oplus.\otimes) B$



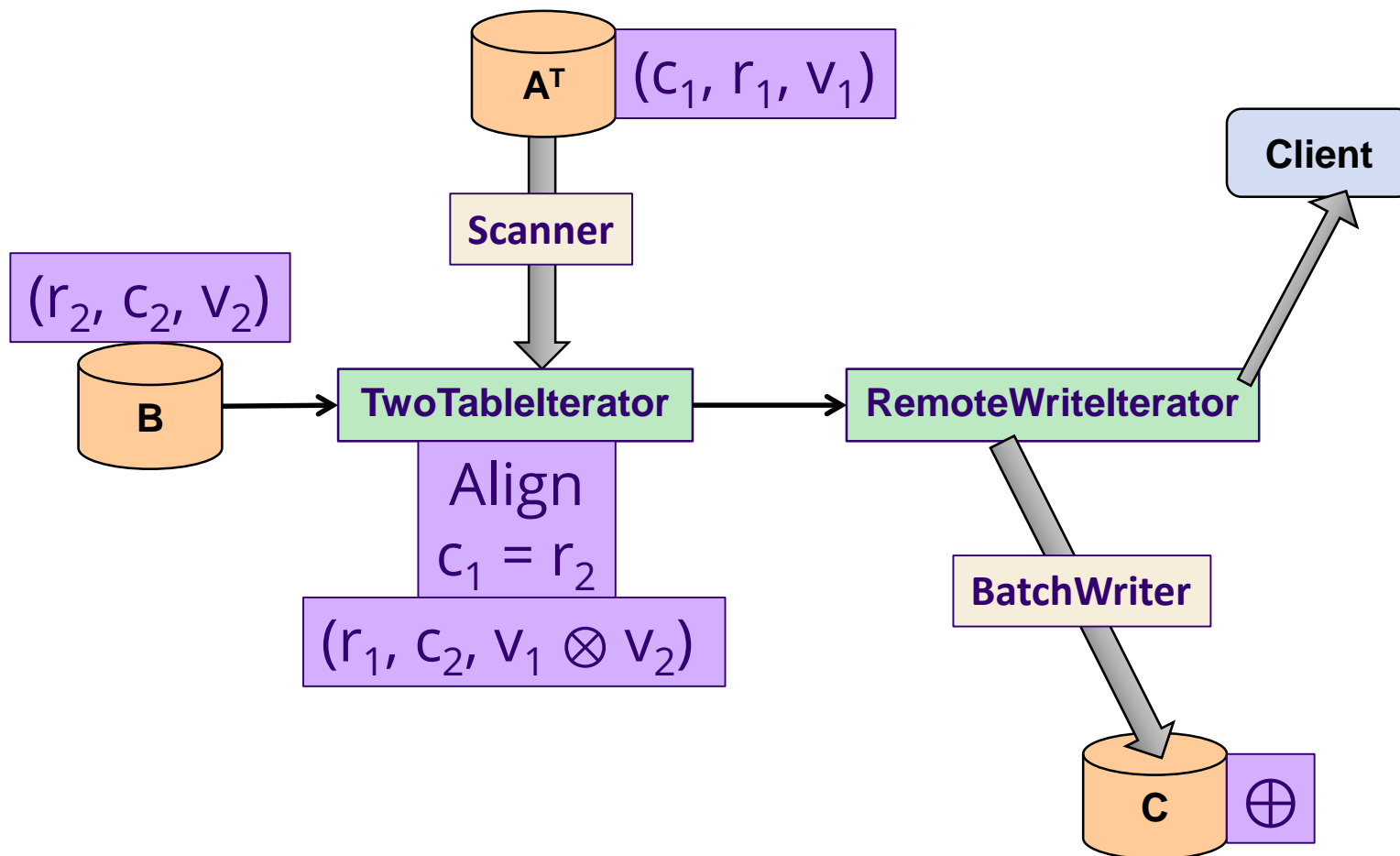
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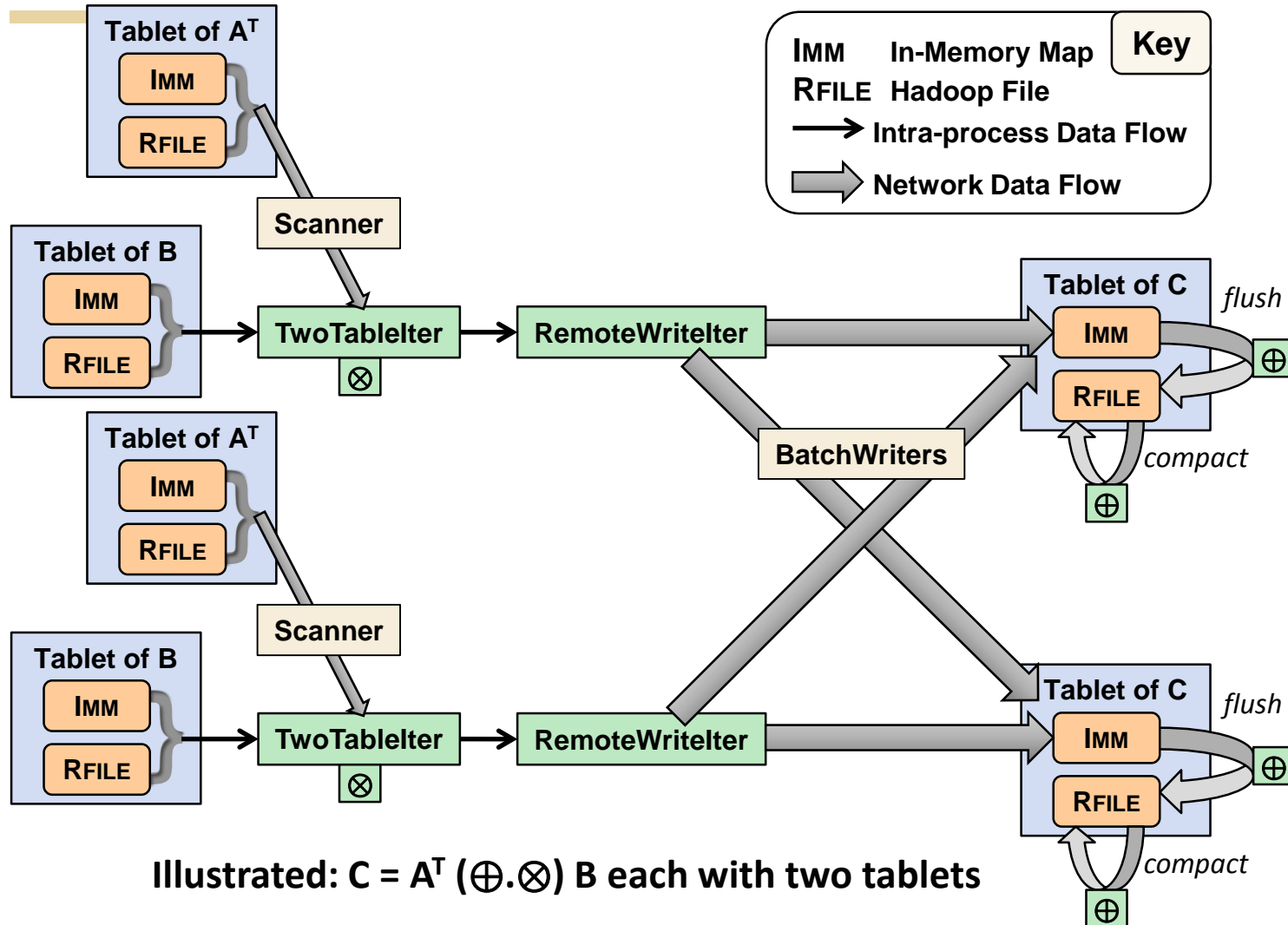
# Graphulo MxM: $A^T (\oplus.\otimes) B$



# Graphulo MxM: $A^T (\oplus.\otimes) B$



# Graphulo MxM: $A^T (\oplus.\otimes) B$ (distributed)



Illustrated:  $C = A^T (\oplus.\otimes) B$  each with two tablets



# Graphulo Client Functions

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```
long TableMult(String Atable, String Btable, String Ctable)
```

```
long SpEwiseX(String Atable, String Btable, String Ctable)
```

```
long SpEwiseSum(String Atable, String Btable, String Ctable)
```

```
...
```

**Simple API abstracts  
the iterator pipeline**

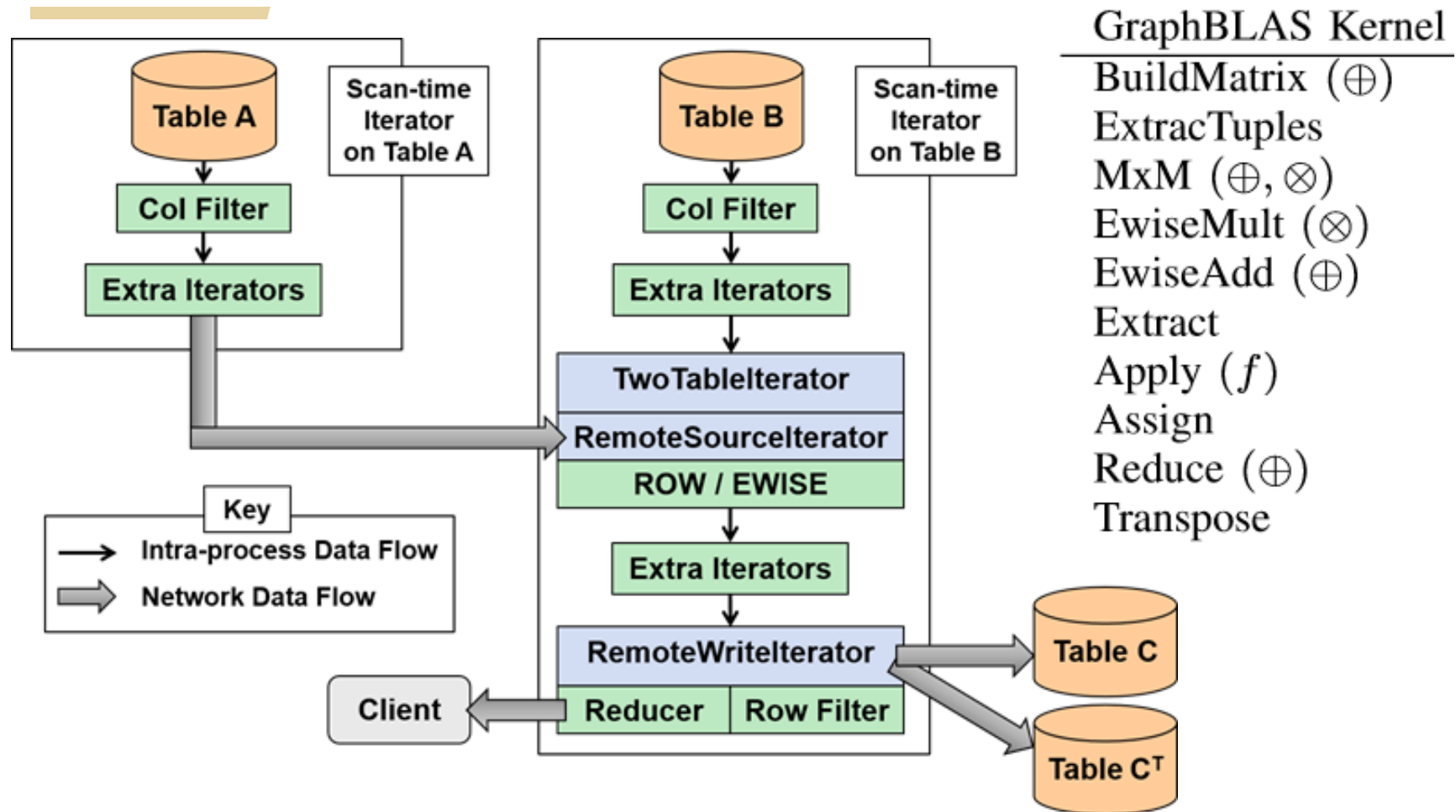


# Graphulo Client Functions

```
long TwoTable(  
    String ATtable, String Btable, String Ctable, String CTtable,  
    int BScanIteratorPriority, TwoTableIterator.DOTMODE dotmode,  
    Map<String, String> optsTT, IteratorSetting plusOp,  
    String rowFilter, String colFilterAT, String colFilterB,  
    boolean emitNoMatchA, boolean emitNoMatchB,  
    List<IteratorSetting> iteratorsBeforeA,  
    List<IteratorSetting> iteratorsBeforeB,  
    List<IteratorSetting> iteratorsAfterTwoTable,  
    Reducer reducer, Map<String, String> reducerOpts,  
    int numEntriesCheckpoint, Authorizations ATauthorizations,  
    Authorizations Bauthorizations, int batchWriterThreads  
)
```

**Full control  
when you need it**

# Graphulo's TwoTable Pipeline





# Evaluating Graphulo

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When is it better to use Graphulo vs. an external system?



# Past Experiments

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## Compare Graphulo to

- > **Single-node in-memory matrix libraries**
  - D4M Sparse matrix library (MATLAB)
  - MTJ Dense matrix library (Java)
- > **Itself**
  - Benchmark of Graphulo scalability with cluster size

# Graphulo Implementation of Server-Side Sparse Matrix Multiply in the Accumulo Database

Dylan Hutchison,<sup>†§\*</sup> Jeremy Kepner,<sup>†‡◇\*</sup> Vijay Gadepally,<sup>†‡\*</sup> Adam Fuchs<sup>+</sup>

<sup>†</sup>MIT Lincoln Laboratory, <sup>§</sup>University of Washington,

<sup>‡</sup>MIT Computer Science & AI Laboratory, <sup>◇</sup>MIT Mathematics Department, <sup>+</sup>Sqrrl, Inc.

2015  
HPEC

*Abstract*  
distributed storage  
graph data  
and persistent  
calculations

The GraphBLAS standard provides a compact and efficient basis for a wide range of graph applications through a small number of sparse matrix operators. Graphulo is a GraphBLAS implementation that leverages Accumulo's distributed storage and computation. We compare the matrix-vector product implementation achieved in Graphulo to the peak write rate. We use Graphulo library that graph analytics within

**Graphulo faster than single-node in-memory external systems on I/O-bound, single-pass computation**

This  
LAS  
atics

In this paper we focus on Sparse Generalized Matrix Multiply (SpGEMM), the core kernel at the heart of GraphBLAS.

- > **Task: Matrix Multiply (MxM)**
- > **Graphulo vs. D4M, Single-node**
- > **Result: Graphulo universally faster**
- > **Why?**
  - > **I/O costs dominate (locality is good)**
  - > **MxM can be formulated as a single-pass algorithm (use outer product)**

used in terms of  
addition func-  
wide range of  
[5] and many

1 in Accumulo  
accumulo tables.  
sparse matrices.

# From NoSQL Accumulo to NewSQL Graphulo: Design and Utility of Graph Algorithms inside a BigTable Database

2016  
HPEC

Dylan Hutchison<sup>†</sup> Jeremy Kepner<sup>‡§◇</sup> Vijay Gadepally<sup>‡§</sup> Bill Howe<sup>†</sup>

<sup>†</sup>University of Washington <sup>‡</sup>MIT Lincoln Laboratory

<sup>§</sup>MIT Computer Science & AI Laboratory <sup>◇</sup>MIT Mathematics Department

Best Student  
Paper

**Results hold for more complex graph algorithms**

Abstract—  
key-value store  
Recently the  
workloads that demand distributed computation local to data  
store  
a tree  
set of  
article  
kernel  
phases  
Accumulo  
two  
performance  
results  
execution  
an  
relationship

- > **Tasks: Find Jaccard coefficients (vertex similarity),  
k-Truss subgraph (community detection)**
- > **Graphulo vs. D4M vs. MTJ**
- > **Result: Graphulo universally faster at Jaccard;  
D4M/MTJ universally faster at k-Truss**
- > **Why?**
  - > Jaccard can be expressed as a fused MxM
  - > k-Truss iterations require one pass each
  - > Accumulo writes intermediary tables to disk

store) and  
before sending  
entries to a client (or a file, in the case of a compaction)  
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# Benchmarking the Graphulo Processing Framework

Timothy Weale<sup>†</sup>, Vijay Gadepally<sup>‡§\*</sup>, Dylan Hutchison<sup>°</sup> and Jeremy Kepner<sup>‡§+</sup>  
<sup>†</sup>Department of Defense, <sup>‡</sup>MIT Lincoln Laboratory, <sup>§</sup>MIT Computer Science & AI Laboratory  
<sup>+</sup>MIT Mathematics Department, <sup>°</sup>University of Washington

2016  
HPEC

**Abstract**—Graph algorithms have wide applicability to a variety of domains and are often used on massive datasets. Recent standardization efforts such as the GraphBLAS specify a set of key computational kernels that hardware and software developers can adhere to. Graphulo is a processing framework that enables GraphBLAS kernels in the Apache Accumulo database.

Graphulo multiplies the results of scaling the Graphulo engine to larger problems and scalability when a greater number of resources is used. Specifically, we present two experiments that demonstrate Graphulo scaling performance is linear with the number of available resources.

processing results on large graphs in a single experiment on a large graph. These benchmarks show that Graphulo is a framework that anyone who wish to

is provided in [12]. It is natural that a processing framework designed for databases makes use of the GraphBLAS kernels.

Graphulo [13] is a specialized graph processing framework built to work with Apache Accumulo and to conform to the GraphBLAS standard. Apache Accumulo is a NoSQL database

wide adoption in a variety of government and non-government settings.

As shown in Figure 1, Accumulo's data model has a key

**Graphulo scales with Accumulo as cluster size increases**

- > **Tasks: MxM, Subgraph Extraction**
- > **Graphulo Scalability Benchmark**
- > **Results: Linear weak & strong scaling on MxM. Subgraph extraction scales in interactive range (1-2k edge extraction)**
- > **Why?**
  - > **Graphulo designed to scale with Accumulo**

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Past experiments compare Graphulo to itself & to in-memory single-node external systems.

Where's the experiment comparing Graphulo to *distributed* external systems?

(answer: this talk)



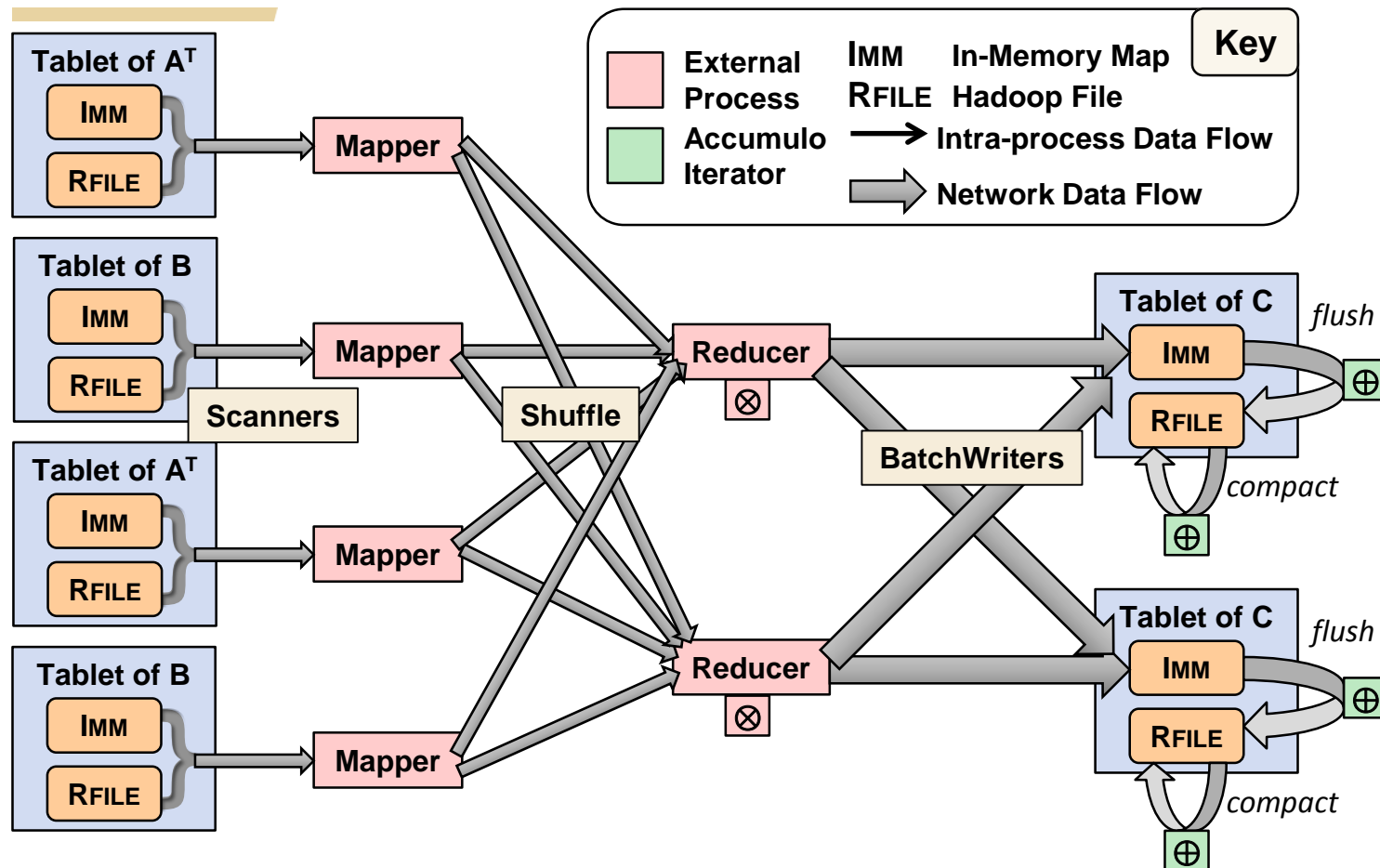


# MxM: Graphulo vs. MapReduce

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- > MapReduce well supported by Accumulo
  - "Use Accumulo for low-latency queries on subgraphs"
  - "Use MapReduce for high-throughput analytics"
  - Are these assumptions true?
- > MapReduce  $C = A^T B$  (see next slide)
  - Map phase: BatchScan & Join  $A^T$ ,  $B$  on their common key
  - Reduce phase: Multiply, BatchWrite partial products
    - > # of reducers not a major factor
    - > Disable speculative execution for correctness
  - Sum partial products via iterator on  $C$
  - Very similar to Graphulo code, except more room to optimize (earlier projection, loop fusion, fewer copies)

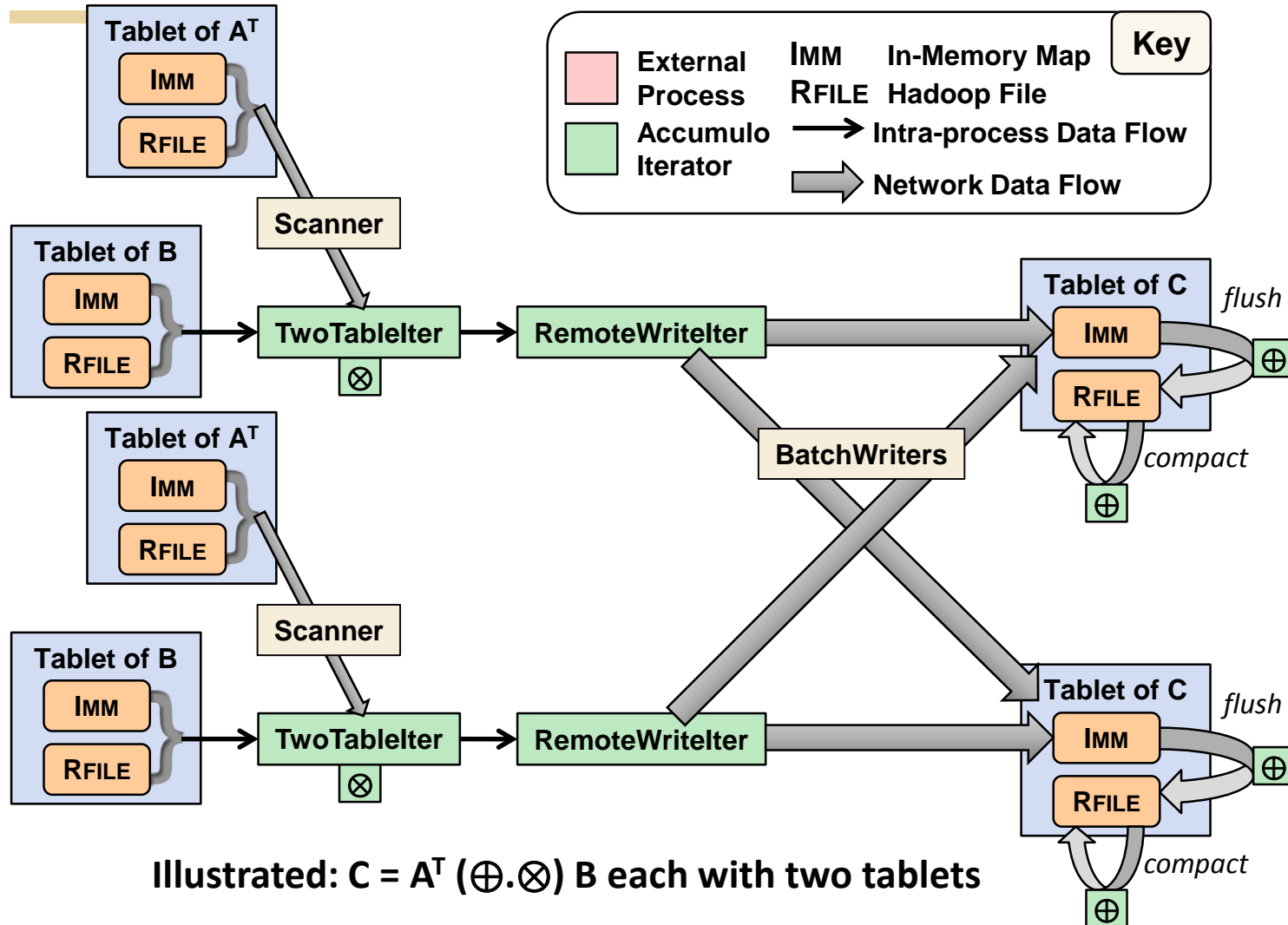
# MapReduce $C = A^T (\oplus.\otimes) B$



Illustrated:  $C = A^T (\oplus.\otimes) B$  each with two tablets, two Reducers



# Graphulo $C = A^T (\oplus.\otimes) B$ (reminder)





# MxM: Graphulo vs. MapReduce

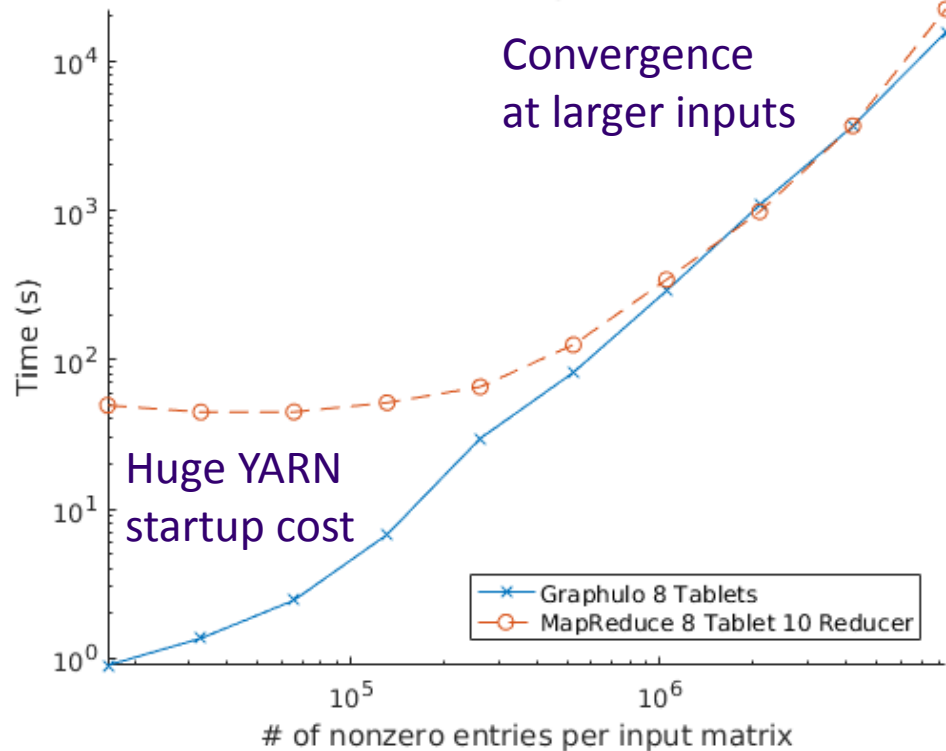
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## Hardware & Setup

- > **12 x m3.large nodes on Amazon EC2, \$38/day**
  - 7.5 GB memory, 2 vCPUs, 30 GB SSD per node
  - 8 worker tablet servers
    - > 3 GB memory for Accumulo, 3 GB for YARN
    - > One tablet per node (low intra-node parallelism)
  - 3 coordinator nodes (YARN ResourceManager, Accumulo Master, HDFS NameNode, Zookeeper)
  - 1 metric-monitoring node (Grafana with InfluxDB)
- > **Graph500 Power law matrix generator**
  - From  $2^{10}$  to  $2^{19}$  rows, 16 non-zeros/row, in each input
  - Skew!

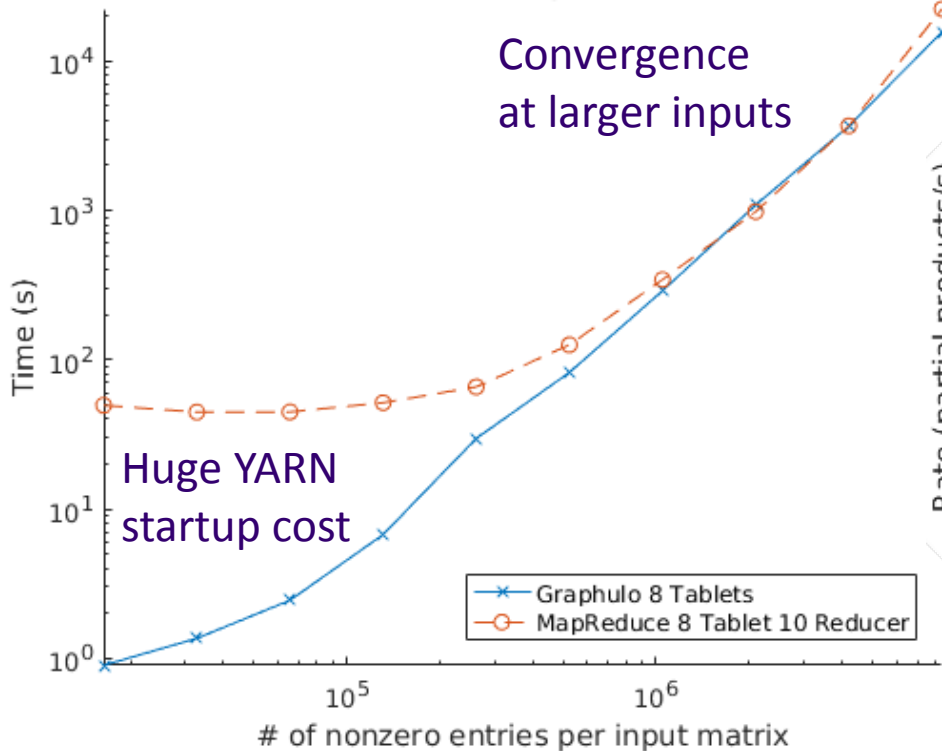
# MxM: Graphulo vs. MapReduce

TableMult Runtime, Scale 10 to 19

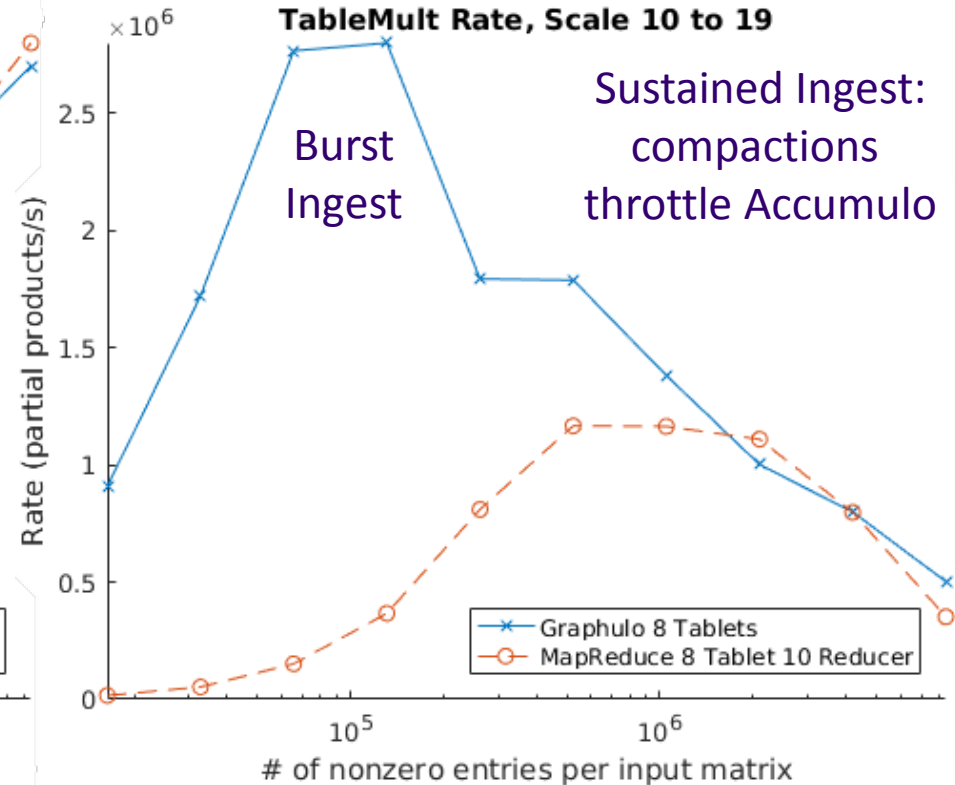


# MxM: Graphulo vs. MapReduce

TableMult Runtime, Scale 10 to 19



TableMult Rate, Scale 10 to 19





# When to use Graphulo?

---

- > Use Graphulo for I/O-bound single-pass analytics
  - Big speedup for smaller problem sizes
  - Equivalent at larger problem sizes
  - Holds against external in-memory processing (D4M, MTJ matrix libraries) & distributed (MapReduce)
- > Use an in-memory system for CPU-bound or multi-pass analytics
  - k-Truss task: expressed as a graph algorithm that loops over GraphBLAS kernels
  - Horrible to write each intermediary table to disk
    - > Yet, even HPC systems write *checkpoints*



# RACO: Relational Algebra for Apache Accumulo

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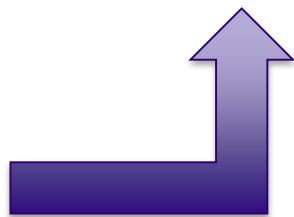
$\pi$   $\sigma$   $\bowtie$   $\gamma$   $\cup$

# How to model a relation in Accumulo?

More details: [public.adhoc.netflow](http://public.adhoc.netflow)

Name	Type
StartTime	STRING_TYPE
Dur	DOUBLE_TYPE
Proto	STRING_TYPE
SrcAddr	STRING_TYPE
Sport	STRING_TYPE
Dir	STRING_TYPE
DstAddr	STRING_TYPE
Dport	STRING_TYPE
State	STRING_TYPE
sTos	LONG_TYPE
dTos	LONG_TYPE
TotPkts	LONG_TYPE
TotBytes	DOUBLE_TYPE
SrcBytes	LONG_TYPE
Label	STRING_TYPE

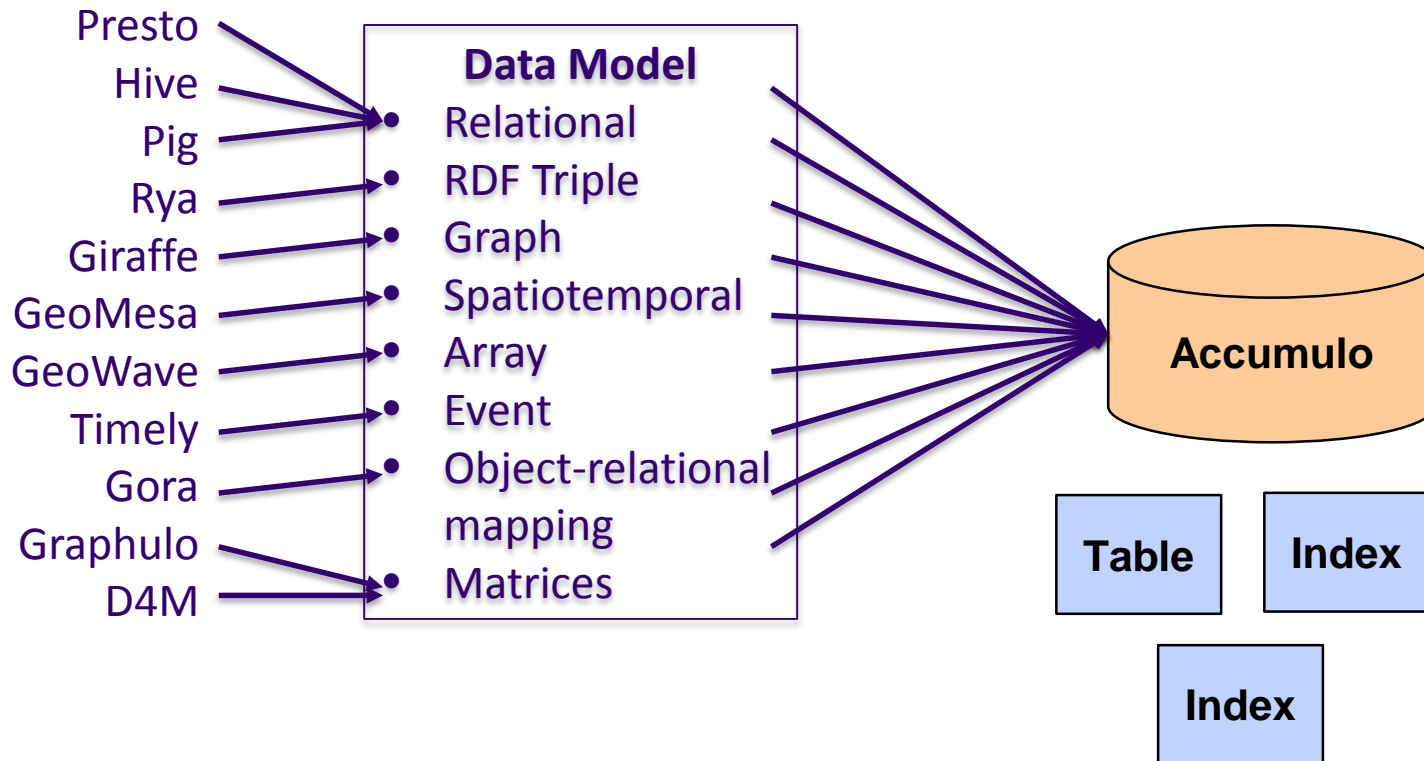
Key					Value
Row	Column			Timestamp	
	Family	Qualifier	Visibility		



- > Relations have lots of attributes
- > More choices than with matrices
  - Graphulo: adjacency, incidence, single-table schemas
- > Choice affects partitioning
- > Should work for any Accumulo data



# Accumulo Access Paths "in the wild"







# Accumulo Access Path "Physical Schema"

For each family, divide attributes into three parts:

**DAP | LAP | VAP**

**DAP: Distributed Access Path**

- How data is partitioned across servers; null disallowed

**LAP: Local Access Path**

- How data is sorted within a server; null disallowed

**VAP: Value Access Path**

- Other attributes, unsorted; null allowed

Key							Value
Row	Column				Timestamp		
	Family	Qualifier	Visibility				
DAP	FAMILY	LAP	VAP	VIS	VAP	TS	VAP





# Access Path Netflow Example

DAP | LAP | VAP

TotBytes, StartTime | | SrcAddr, DstAddr, Dur

Key					Value
Row	Column			Timestamp	
	Family	Qualifier	Visibility		
09:46:59.607825,276	""	""	public	145635001	DstAddr 94.44.127.113
09:46:59.607825,276	""	""	public	145635001	Dur 1.026539
09:46:59.607825,276	""	""	public	145635001	SrcAddr 147.32.84.59

Each tuple is a set of Key-Values

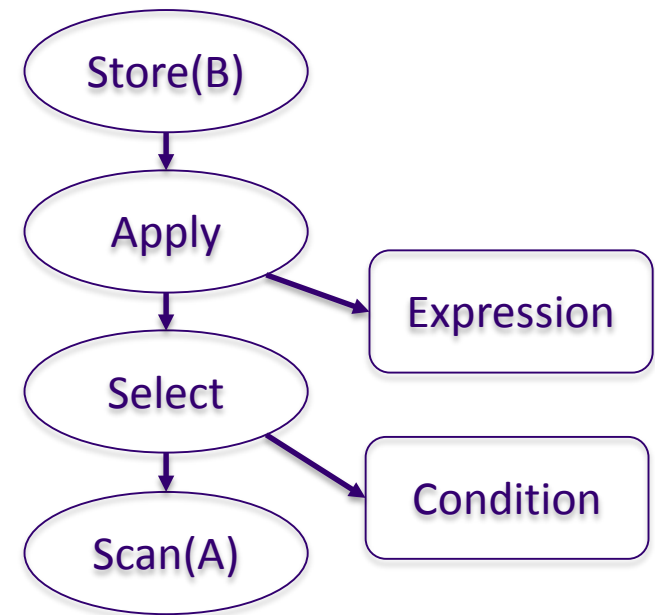
- Facilitates projection, filtering, missing values



# Compiling RA onto Accumulo

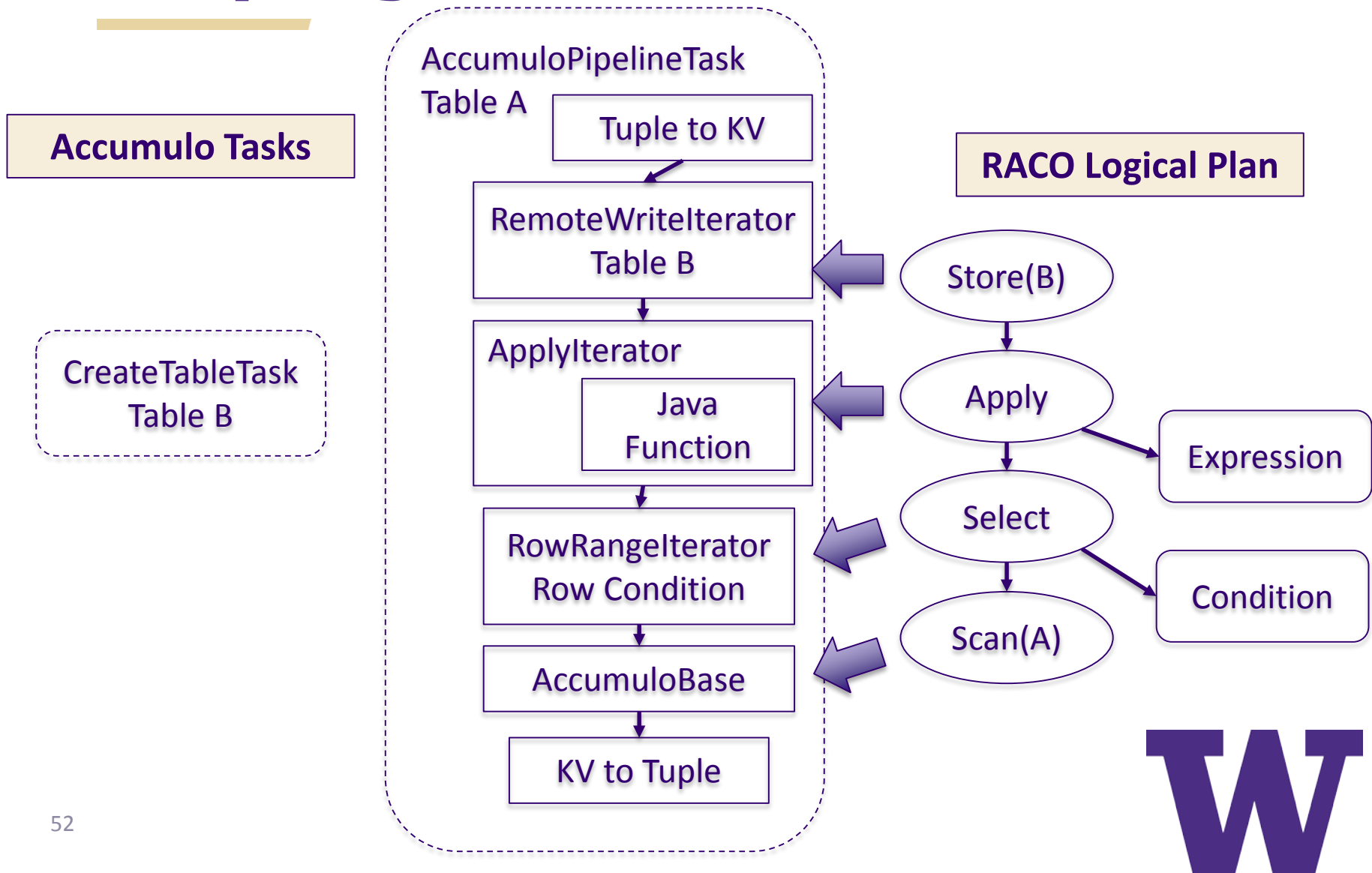
## Accumulo Tasks

## RACO Logical Plan





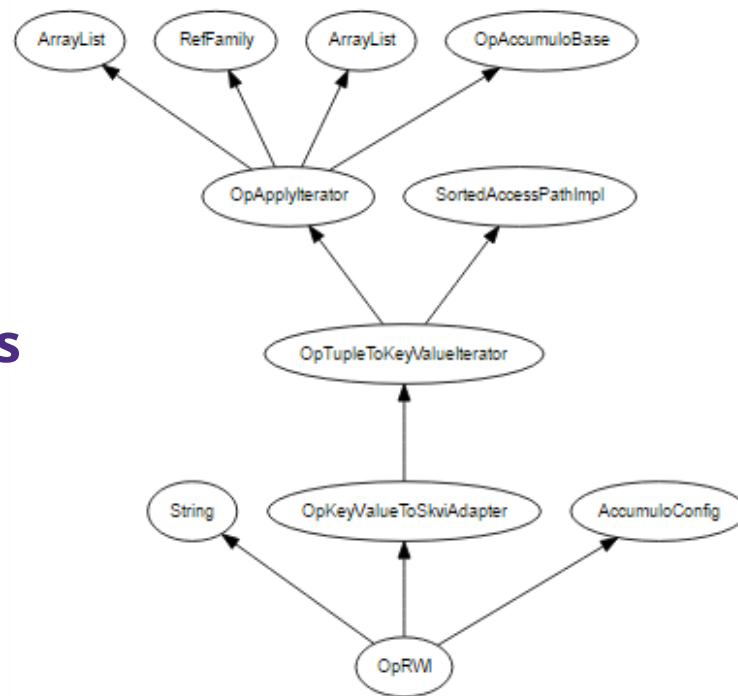
# Compiling RA onto Accumulo



# Takeaways from RA on Accumulo

## Polystore Demo

- > **RA Compilation used in an October sponsor demo**
  - Federated plan executed part Accumulo, part Spark or C
  - Accumulo query stored results in a CSV file for Spark/C
  - "Prototype works!"
- Much more to do...





# Takeaways from RA on Accumulo

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Access Paths hint at a general physical schema

## > Primitives

- **Block[n](...):** Store contiguous range of n entries together
- **Group(...):** Each entry stored separately; Block[1]
- **Range(...):** All entries stored together; Block[N]

## > Examples

- **Accumulo:** Block(row) Group(family)  
Range(row, family, qualifier, visibility, timestamp) value
- **Myria hash partitioning:** Group( $a_1, \dots, a_k$ )  $a_1, \dots, a_k$
- **TileDB matrix:** Block(j, i) Range(i, j) v
- **More access paths for indices**
- **Applications pick both a logical & physical schema**

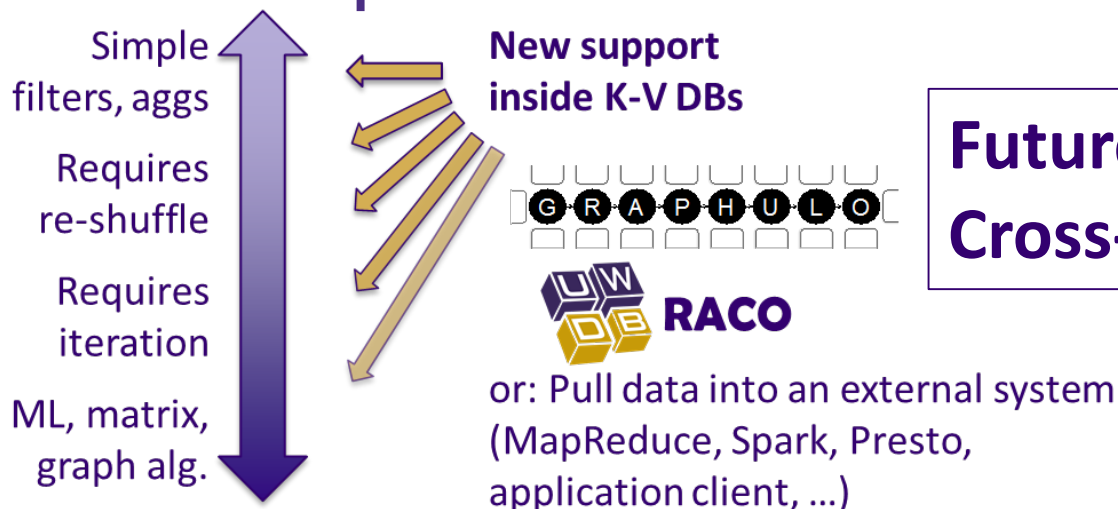
# In-Database Analytics for K-V DBs

## > Graphulo: LA on K-V DBs

- Graphulo vs. MapReduce experiment
  - > Comparable at scale; in-DB dominates "scale down"
- Use in-DB approach for I/O-bound single-pass analytics

## > RACO: RA on K-V DBs

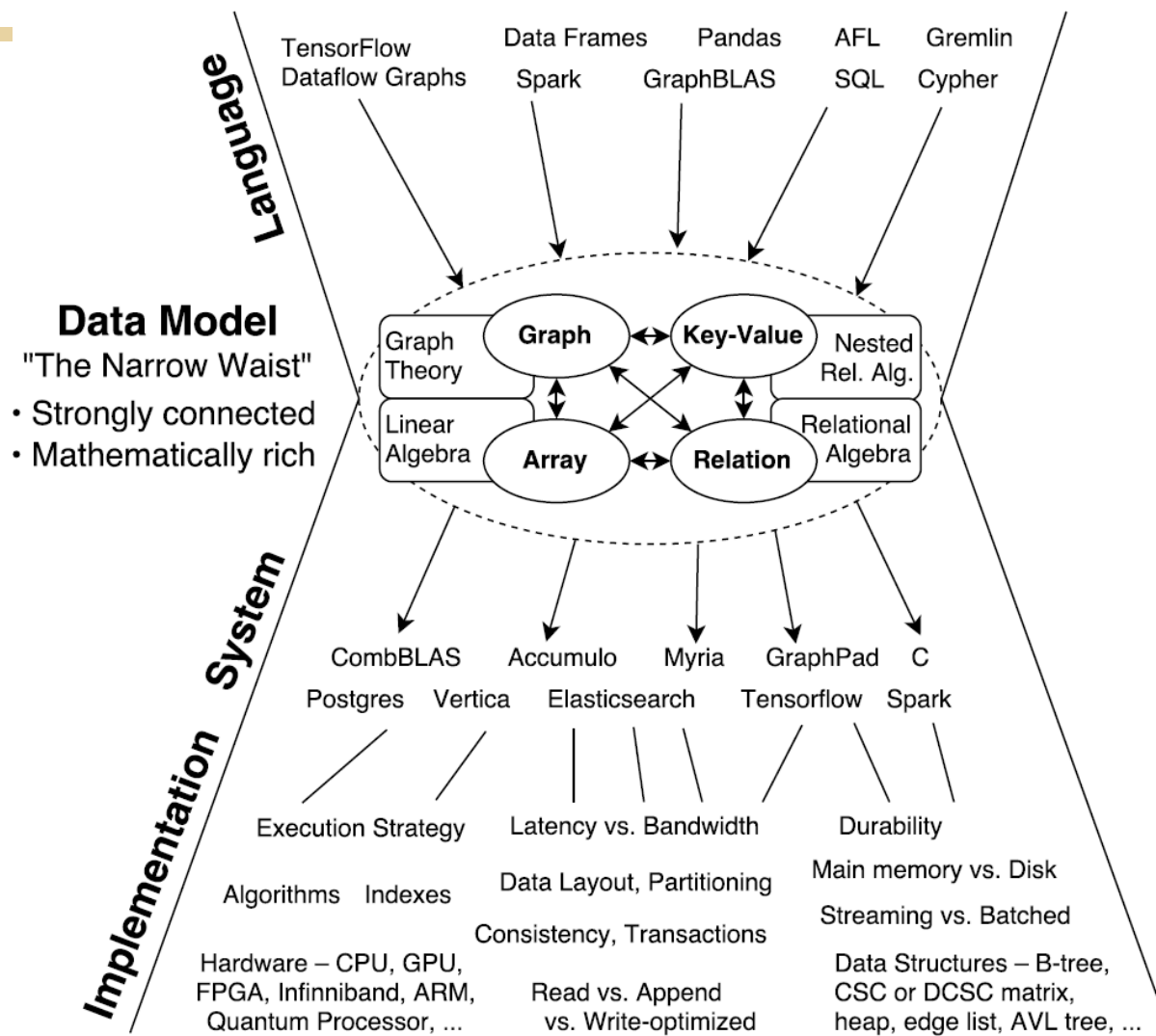
- Access Paths helpful as a "physical schema"
- Compiles into Accumulo iterators



**Future: A Polystore  
Cross-Platform Optimizer**



# Why does LA, RA work on K-V DBs?







# Backup Slides

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# Background on Accumulo

Key				Value	
Row	Column				Timestamp
	Family	Qualifier	Visibility		

Best for:

- > Large, de-normalized tables; no schema necessary
  - Unlimited columns; un-interpreted values; everything is a byte[]
- > TBs to PBs of data; robust horizontal scaling
- > Hadoop HDFS / Java ecosystem
- > Cell-level access control
- > Row store by default
  - Scan over rows for  $O(\log n)$  lookup & sorted order
  - Maintain "Transpose Table" for column-major access path
- > Iterator processing framework



# MxM: Graphulo vs. MapReduce

## Table of Results

SCALE	Entries in Table C		Graphulo		MapReduce 10 Reducers	
	PartialProducts	AfterSum	Time (s)	Rate (pp/s)	Time (s)	Rate (pp/s)
10	$8.16 \times 10^5$	$2.69 \times 10^5$	$8.98 \times 10^{-1}$	$9.09 \times 10^5$	$4.95 \times 10^1$	$1.65 \times 10^4$
11	$2.34 \times 10^6$	$7.90 \times 10^5$	1.36	$1.72 \times 10^6$	$4.45 \times 10^1$	$5.26 \times 10^4$
12	$6.77 \times 10^6$	$2.41 \times 10^6$	2.45	$2.77 \times 10^6$	$4.47 \times 10^1$	$1.52 \times 10^5$
13	$1.89 \times 10^7$	$7.04 \times 10^6$	6.74	$2.80 \times 10^6$	$5.15 \times 10^1$	$3.66 \times 10^5$
14	$5.32 \times 10^7$	$2.34 \times 10^7$	$2.97 \times 10^1$	$1.79 \times 10^6$	$6.57 \times 10^1$	$8.10 \times 10^5$
15	$1.47 \times 10^8$	$6.38 \times 10^7$	$8.21 \times 10^1$	$1.79 \times 10^6$	$1.26 \times 10^2$	$1.17 \times 10^6$
16	$4.00 \times 10^8$	$2.30 \times 10^8$	$2.89 \times 10^2$	$1.38 \times 10^6$	$3.44 \times 10^2$	$1.16 \times 10^6$
17	$1.09 \times 10^9$	$9.19 \times 10^8$	$1.09 \times 10^3$	$1.00 \times 10^6$	$9.80 \times 10^2$	$1.11 \times 10^6$
18	$2.94 \times 10^9$	$2.55 \times 10^9$	$3.67 \times 10^3$	$7.99 \times 10^5$	$3.68 \times 10^3$	$7.97 \times 10^5$
19	$7.84 \times 10^9$	$6.58 \times 10^9$	$1.56 \times 10^4$	$5.02 \times 10^5$	$2.23 \times 10^4$	$3.52 \times 10^5$



# Past Experiments

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## Compare Graphulo to

- > **Single-node in-memory matrix libraries**
  - D4M Sparse matrix library (MATLAB)
  - MTJ Dense matrix library (Java)
- > **Itself**
  - Benchmark of Graphulo scalability with cluster size

## Tasks:

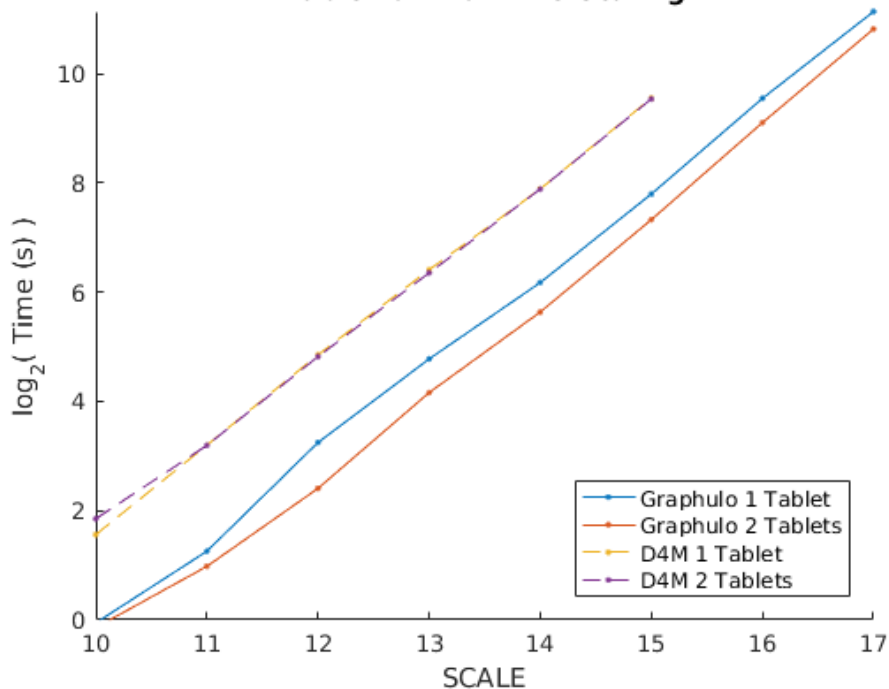
- > **Matrix Multiply (MxM), the backbone of LA**
- > **Subgraph Extraction, via MxM with a cue table**
- > **Jaccard Coefficients, a vertex similarity metric**
- > **k-Truss Subgraph, used to detect communities**

- 
- > **2015 HPEC: MxM, single-node**
    - Graphulo vs. D4M (MATLAB sparse matrix library)
    - Result: Graphulo universally better
  - > **2016 HPEC: Jaccard, k-Truss, single-node**
    - Graphulo vs. D4M vs. MTJ (Java dense matrix library)
    - Result: Graphulo universally better at Jaccard;  
D4M/MTJ universally better at k-Truss
  - > **2016 HPEC: MxM, subgraph extraction, cluster**
    - Graphulo vs. itself on increasing cluster size
    - Result: Linear weak & strong scaling on MxM  
Subgraph extraction scales, in interactive range

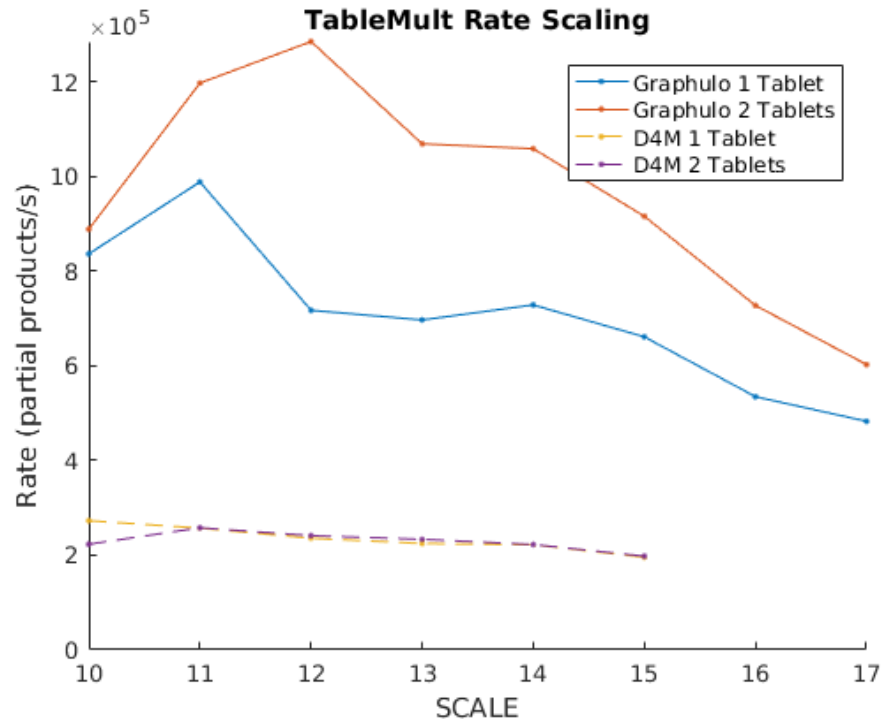


# MxM Single-Node Experiment

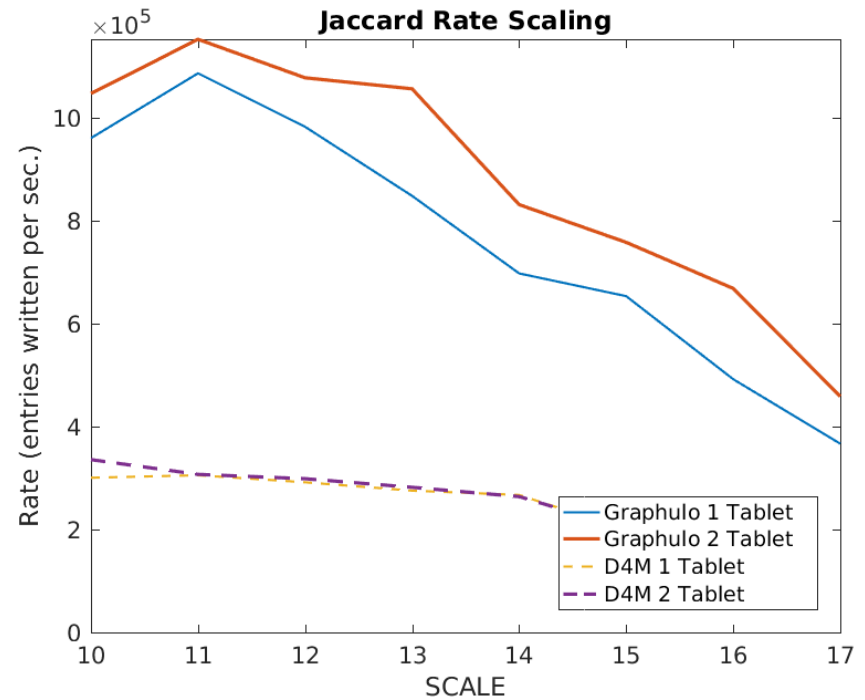
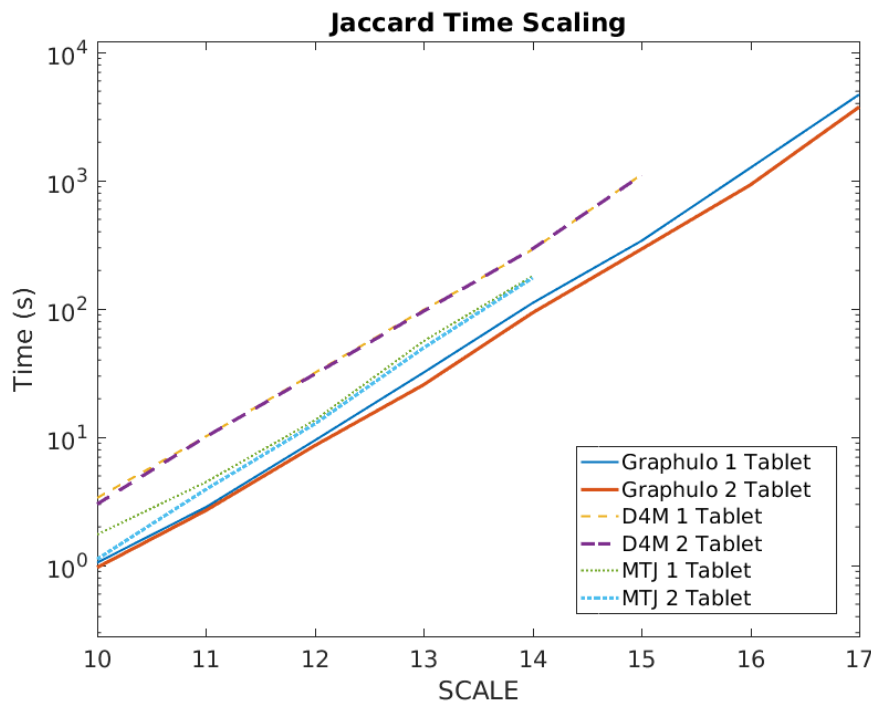
TableMult Runtime Scaling



TableMult Rate Scaling

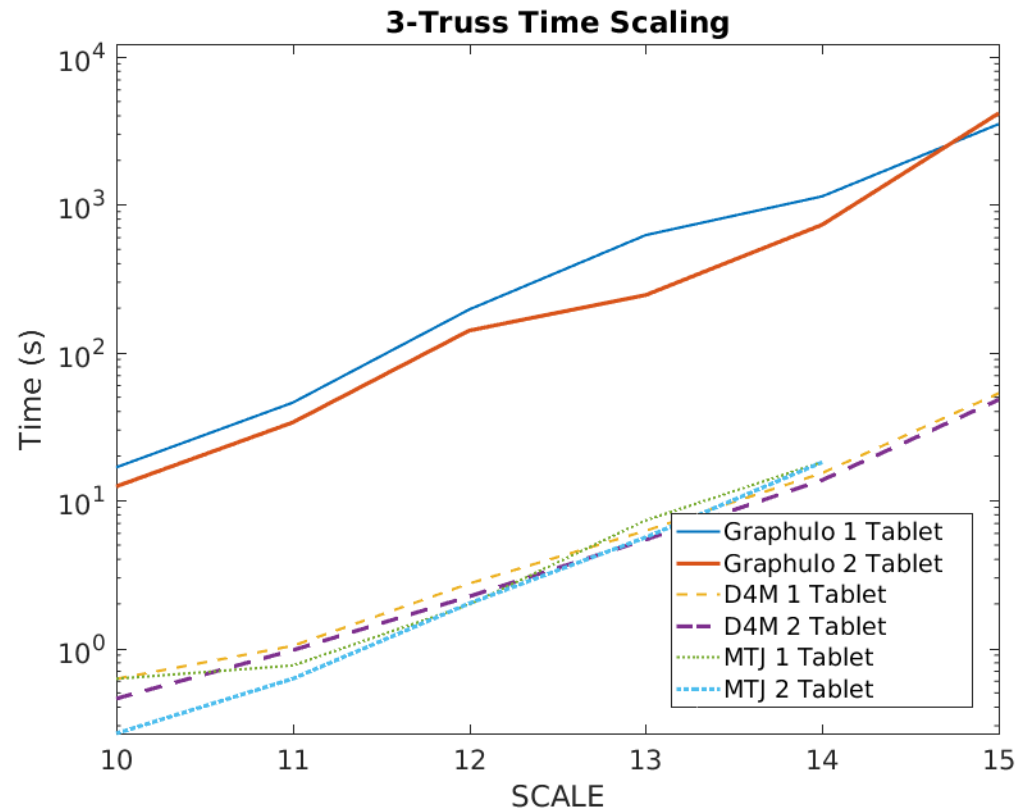


# Jaccard Experiment Graphs





# 3-Truss Experiment Graph

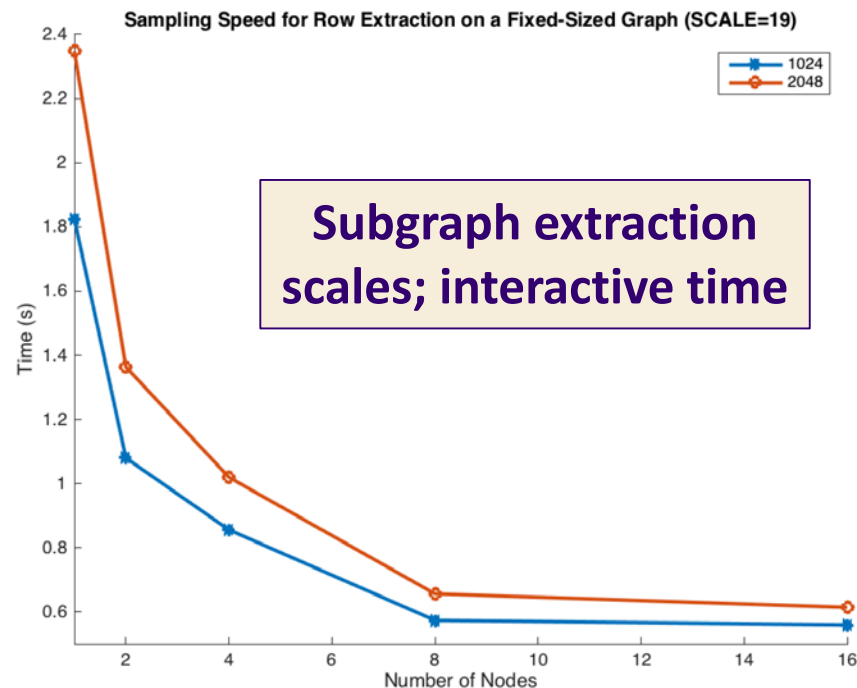
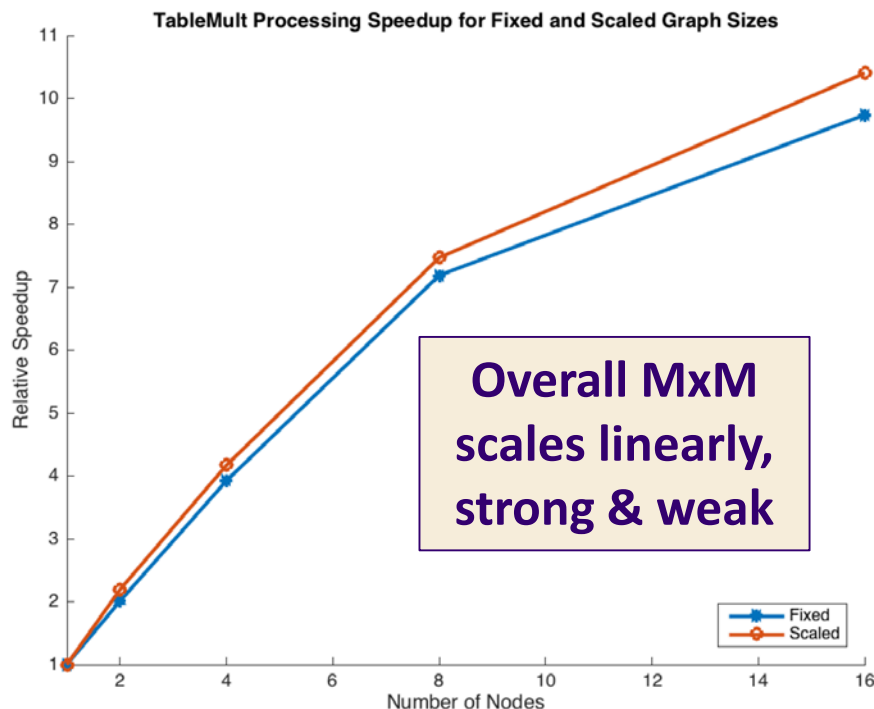


Some points may  
need re-evaluating

# Past Exp: Benchmarking Graphulo

## Benchmarking the Graphulo Processing Framework

Timothy Weale<sup>†</sup>, Vijay Gadepally<sup>‡§\*</sup>, Dylan Hutchison<sup>°</sup> and Jeremy Kepner<sup>‡§+</sup>  
<sup>†</sup>Department of Defense, <sup>‡</sup>MIT Lincoln Laboratory, <sup>§</sup>MIT Computer Science & AI Laboratory  
<sup>+</sup>MIT Mathematics Department, <sup>°</sup>University of Washington



# Alg 1: Jaccard Coefficients

**Input:** Unweighted, undirected adjacency matrix  $A$

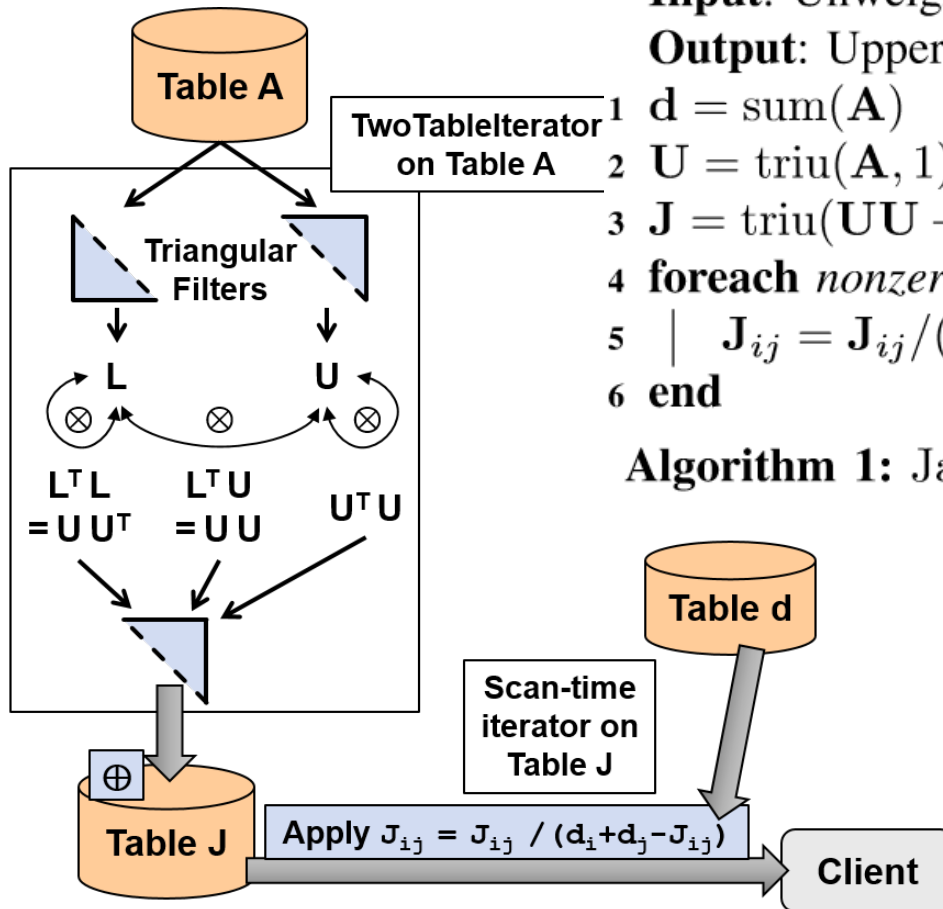
**Output:** Upper triangle of Jaccard coefficients  $J$

```

1  $d = \text{sum}(A)$  // pre-computed in degree table
2  $U = \text{triu}(A, 1)$  // strict upper triangle filter
3  $J = \text{triu}(UU + UU^T + U^T U, 1)$  // fused MxM
4 foreach nonzero entry  $J_{ij} \in J$  do
5 |    $J_{ij} = J_{ij} / (d_i + d_j - J_{ij})$  // stateful Apply on J
6 end
    
```

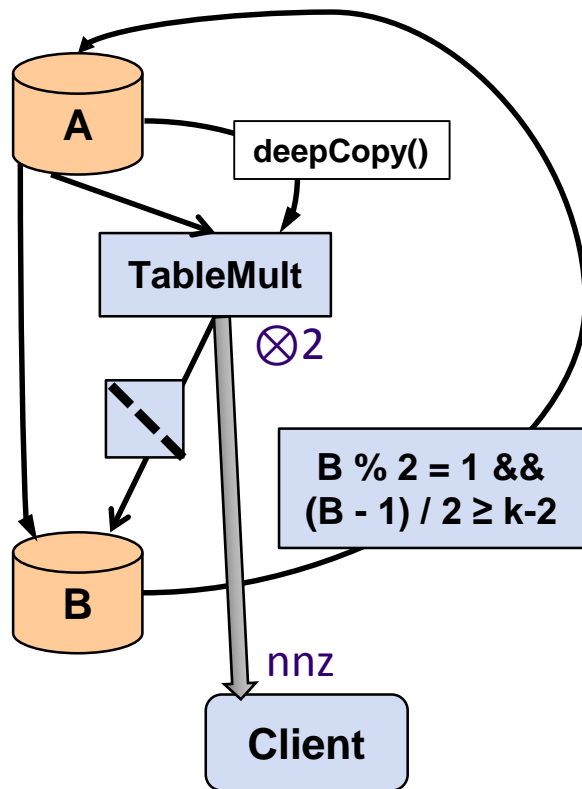
**Algorithm 1:** Jaccard

**Fusion:**  
A single Graphulo  
TwoTable pass!



Note! This has been improved since the HPEC paper. Doable as a special OneTable operation.

# Alg 2: k-Truss Subgraph



**Input:** Unweighted, undirected adjacency matrix  $A_0$ , integer  $k$

**Output:** Adjacency matrix of  $k$ -truss subgraph  $A$

```

1  $z' = \infty, A = A_0$  // table clone
2 repeat
3    $z = z'$ 
4    $B = A$  // table clone
5    $B = B + 2AA$  //  $M \times M$  with  $a \otimes b = 2$  if  $a, b \neq 0$ 
6    $B(B \% 2 == 0) = 0$  // filter on B
7    $B((B - 1)/2 < k - 2) = 0$  // filter on B
8    $A = |B|_0$  // Apply on B; switch  $A \leftrightarrow B$ 
9    $z' = \text{nnz}(A)$  // Reduce, gathering nnz at client
10 until  $z == z'$  // client controls iteration
  
```

**Algorithm 2:** kTruss

Iterations of Graphulo TwoTables