The Basic Laws of Arithmetic by Gottleb Frege; Montgomery Furth; A Study of Frege by Jeremy D. B. Walker
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existence of the various elements in the substantive model in terms of which the [observation language] is formulated' (117, applied to theories). Not even the existence of observable events is therefore assured and the 'problem of the cognitive status of theories' expands into the problem of the cognitive status of *all* notions of our language. Nagel's book contains hardly a hint concerning this more general problem and this despite the fact that his own analysis of theories and his rejection of sense data leads straight to it. This is true of other accounts also: the ontological status of observation terms is a matter that is hardly examined today.[1] This certainly is a step back from the sophistication that had already been reached by Kant and an indication that the belief in sense data, though denied in *words* still seems to have a strong influence upon the *deeds* of almost all contemporary philosophers of science.

(10) The topics dealt with so far fill hardly one-third of Nagel's book. There are chapters on the quantum theory, on the science of mechanics, and there is a large amount of argument dealing with the social sciences. These arguments are very instructive and I am sure that everyone reading the book will greatly profit from examining them. The reason why I have concentrated on the chapters of the book dealing with more general matters is that the opinions expressed there are widespread, that they are presented in great detail, and supported with better arguments than I have found elsewhere.

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The Basic Laws of Arithmetic. By Gottlob Frege. Translated and edited, with an introduction, by Montgomery Furth.
University of California Press, 1964. Pp. lxiii+119. $5.00.
A Study of Frege. By Jeremy D. B. Walker.
Basil Blackwell Oxford, 1965. Pp. xiv+201. 30*s*.

THE profound influence of Frege's work on the development of formal logic and philosophy of mathematics is now well known. However, his philosophical writings, which, unlike his technical achievements, are still of more than historical interest, appear to be comparatively little known or ill-understood, at least by philosophers who are not primarily interested in the foundations of mathematics. This is a great pity, for Frege's thought is precise, penetrating, subtle and often relevant to philosophical problems which are still under discussion. Perhaps a wider recognition of his importance will be hastened by the translation of more of his works into English, and, it is to be hoped, by the publication of papers reported to be still in the possession of his literary executors.

[1] An exception is Carnap's article 'Empiricism, Semantics, and Ontology'.

Mr Furth's translation of parts of Vol. I. of *Grundgesetze der Arithmetic* is a valuable supplement to what is already available in English, though some parts overlap with papers in the collection edited by Peter Geach and Max Black. Mr Furth offers us the Preface (for some reason he translates 'Vorwort' as 'Introduction'), the Introduction ('Einleitung'), and the first 52 sections of vol. i. Two appendices are added, the first including parts of §§ 54, 55 and 91 of vol. i. These are required for a full understanding of the second appendix which is a complete translation of Frege's Appendix to vol. ii, in which he discussed Russell's paradox. In all this Frege's own symbols are reproduced, except that script letters have been used instead of Gothic. A large portion of the Preface, the first seven sections of vol. i, and a slightly 'doctored' version of the appendix on Russell's paradox (without Frege's symbols) are already available in the Geach and Black collection. The newly translated passages include a criticism of attempts to bring psychological concepts to bear on logical problems, and an attack on Idealism (Preface, pp. xv-xxiv). In addition there are Frege's detailed explanations of several of his symbols (e.g. the *wertverlauf* and definite description operators), his typographical distinctions between different sorts of variables, general remarks on definitions and *bedeutung* (translated as 'denotation'), informal discussion of his axioms and principles of inference and finally some examples of derivations within the system.

The book will not be easy for non-mathematical philosophers to read, but there are passages translated here for the first time which should help to clarify some of Frege's philosophical ideas, such as the notion of 'bedeutung' especially as applied to function-signs and concept-signs. Thus, a concept-sign (i.e. a sentence from which a name has been removed) has a *bedeutung*, which Frege calls a 'concept', if and only if every insertion of a successfully referring name into the gap yields a sentence corresponding to a definite truth-value (§ 29). This suggests, as pointed out in the translator's introduction, that if we treat 'has a bedeutung' as an indissoluble metalinguistic unit, and then interpret all Frege's talk about concepts as being a shorthand for talk about concept-expressions which 'have a bedeutung', then there is no need to get bogged down in obscure discussions of what kind of 'entity' a concept is supposed to be. Perhaps Frege's frequent reminders that he is speaking metaphorically in much of what he says about functions and concepts support this suggestion. Thus, without resorting to any obscure metaphysics we can preserve the elegance and comprehensiveness of Frege's theory of sense and bedeutung, according to which the logical features of such varied signs as names, definite descriptions, concept-words, mathematical and non-mathematical function-signs (e.g. 'the father of . . .'), sentences, sentential connectives, and second-level function-signs such as quantifiers, can all be summed up under the principle that if two or more of these signs are combined correctly then the resulting sign (i) will have a sense if and only if the components do, (ii) will have a bedeutung if and only

if the components do, (iii) the sense of the whole being fully determined by the senses of the components, and (iv) the bedeutung of the whole being fully determined by those of the components. (Perhaps it would be well to remind reader that this applies to sentences only in so far as they are stripped of their *asserting* role, which Frege assigns to a separate symbol.) The beauty and power of this theory are clearly brought out in Mr Furth's introduction and appear not yet to have been fully appreciated. They justify Frege's decision to construe sentences as naming, or denoting, truth-values. As it stands, the theory will not work in connection with a language which, unlike Frege's symbolic script, includes some concepts which are only partly determinate. This sort of limitation, arising from his pre-occupation with the foundations of mathematics, may partly explain why the theory is not better known and understood. I think such limitations can be removed, but this is not the place to show how.

Mr Furth's fifty-two page introduction is sure to be useful to those who have difficulty in understanding some of Frege's more involved and obscure discussions, both within *Grundgesetze* and elsewhere. There are eight sections, of which the following are most important: § 3, a discussion of the distinction between *sinn* and *bedeutung*, and the application of the distinction to names (complete expressions) and function-signs (incomplete expressions). There is a lucid exposition of Frege's reasons for treating complete sentences as names. Section 4 contains an explanation of Frege's distinction between function and object, including further remarks on the sense in which incomplete signs *denote* functions, and a description of the role of quantification in Frege's system. Section 5 explains the use of Roman letters as free variables. Section 6 gives a very precise account of Frege's 'wertverlauf' operator, stressing the need to distinguish his notion of the course-of-values of a function from the modern notion of a function-in-extension, or class of ordered pairs. The difference between Frege's notion of the extension of a concept and the ordinary notion of an extension is rightly stressed. A detailed analysis of some of Frege's remarks then explodes the myth that he thought of courses-of-values as *denotations* and functions as *senses* of function-signs. This section also comments on Frege's attempt to get round Russell's paradox. Section 7 explains Frege's assertion sign and raises some objections to his view of sentences as denoting truth-values. The rest of the Introduction consists of general and historical remarks, together with the usual translator's notes. All of it is clear, concise, and, in my view, mostly correct about Frege's intentions.

Mr Walker's book on Frege covers a wider range of topics and goes into them in more detail. Further, it has the advantage of an index. However, on all other points it is much inferior to Mr Furth's introduction, for it is often too vague and does not bring out the central aspects of Frege's theory clearly enough. One feels the author struggling to reach a clear understanding of Frege's work, but not quite getting there. This might

have made it useful for students to read and criticise, if there had been more references to particular pages of Frege's writings, and if there had not been too many bad examples set in it. Here are some specimens: p. 1: 'The best way to elucidate the notion [of a function] is to make clear exactly what state of affairs is described by the equation '$y = x^2$'.' But such an equation describes *no* state of affairs, unless the variables are interpreted as bound, in which case it is unsuitable for elucidating the notion of a function. Page 17 (cf. p. 51): 'Frege insists as strongly as Russell that we are only justified in using expressions which have the form of singular definite descriptions, e.g. "The so and so", if we know that at least one thing is so and so, namely if we know the truth of some existentially quantified statement. . . .' This is a sloppy description of Russell's views, and is also misleading concerning Frege, since he said this only about 'scientific' contexts, in which speakers are concerned about the truth or falsity of what they say, and he mentions reported speech and statements about beliefs, thoughts, etc., as counter-instances. Page 30: 'For truth and falisty are notions which operate in definite ways; the ordinary truth-tables of propositional logic contain the ways in which they operate . . . when we are concerned with validity of proof and correctness of inference, we must see that both premises and conclusions conform to the laws of truth, namely the conditions implicit in the truth-tables.' No comment! Page 81: 'Replacement of a component [of an expression] by another with the same sense but a different reference would not alter the sense of the whole, but would alter its reference ("On Sense and Reference", p. 62).' (The page reference is to the Geach and Black collection.) For Frege, two expressions with the same *sinn* must have the same *bedeutung*, for the *sinn* is what ensures that the expression has this bedeutung rather than that one. It is clear from the context that I have not merely quoted a misprint, with 'sense' and 'reference' transposed. Similar remarks occur elsewhere, e.g. on p. 126. The next two quotations contradict the previous one, but are still inaccurate p. 78: 'For an object to become a sign is to acquire a sense, and to be given a reference is the same as both.' And p. 82: 'It seems then that knowing what the sense of the name 'Aristotle' means is more or less the same as knowing what its reference is.' But Frege makes it clear in 'On Sense and Reference' that one can know the sense of a name without even knowing whether there *is* a reference (bedeutung). (Further, do senses *mean* anything?) There are other topics on which Mr Walker seems to be confused. Page 107: 'Finally complete sentences are taken as functions, since like arithmetical equations they state facts.' Here we find (1) confusion between function and function-sign (2) apparent ignorance of Frege's insistence that only *incomplete* signs can be function-signs, and (3) a repetition of the confusion shown in the quotation from p. 1. On p. 118 Mr Walker claims that only in terms of a distinction between positive and negative thoughts can we understand the role of the negation sign as applied to sentences with unknown truth-value. Compare

this with what Frege says in 'Negation' (especially pp. 125 and 130 in the Geach and Black edition). Finally, on p. 180 we find Mr Walker asserting that in talking about a function's value-range (*wertverlauf*) Frege was not talking about the arguments of the function but about its values.

These specimens (and more could have been given) should make it clear to anyone familiar with Frege's work how unreliable Mr Walker is as an expositor. Perhaps he has the excuse of youth and inexperience, but some of the mistakes should never have got past a publisher's reader. There are signs, however, that given more time and some helpful criticism Mr Walker could have written a much better book, for many of his mistakes are implicitly corrected in other parts of the book, and he has usefully collected together many of Frege's scattered remarks on topics of interest (though without giving enough detailed references). The book ranges over so much of Frege's thought, and raises so many interesting questions (e.g. did his defined terms always have the same sense as well as the same reference as the definiens?) that readers already familiar with Frege's work may find reading it a useful and stimulating exercise once they have got over being infuriated.

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Philosophical Problems of Space and Time. By Adolf Grünbaum. Routledge and Kegan Paul, London, 1964. Pp. 448.

BEGINNING with the consideration of the problems that led to the theory of relativity, there has been a steadily growing realisation that our basic concepts of space and time involve a very pervasive background of tacit assumptions concerning the structure of the world. Such assumptions originate largely in that domain of thought which is generally called philosophical, rather than in the pragmatic efforts to deal with the observational facts of physics, more or less close to the level at which they are directly presented in experiment. Nevertheless, these philosophical assumptions have an enormous though almost unknown range of influence on how our more pragmatic observations are interpreted, and how the corresponding research activities are organised, directed, and shaped, in such a way that we inquire mainly into questions that are deemed to be significant because they play a part in our tacit philosophical point of view. This book, which deals with a very broad range of basic philosophical problems of space and time, is therefore very pertinent especially to the present phase of development of physics, in which our philosophical notions about space and time are being called into question in the fields of relativity, cosmology, quantum theory and theory of elementary particles.