INF5620 EXAM

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1 PROBLEM 2: 1D finite element for approximation

1.1 a

We want to approximate $f(x) = x(1-x)$ on the domain $\Omega = [0,1]$ In general we have:

$$\tilde{A}^{(e)}_{r,s} = \int_{-1}^{1} \varphi_r(X)\varphi_s(X)\frac{h}{2}\mathrm{d}X$$

$$\tilde{b}^{(e)}_{r} = \int_{-1}^{1} f(x(X))\varphi_r(X)\frac{h}{2}\mathrm{d}X$$

These are general, and for P1 elements we have:

$$\tilde{\varphi}_0(X) = \frac{1}{2}(1-X)$$

$$\tilde{\varphi}_1(X) = \frac{1}{2}(1+X)$$

Inserting above in the integrals and solving for our function gives us:

$$\tilde{A}^{(e)}_{0,0} = \frac{h}{8}\int_{-1}^{1}(1-X)^2\mathrm{d}X = \frac{h}{3}$$

$$\tilde{A}^{(e)}_{1,0} = \frac{h}{8}\int_{-1}^{1}(1-X^2)\mathrm{d}X = \frac{h}{6}$$

$$\tilde{A}^{(e)}_{0,1} = \tilde{A}^{(e)}_{1,0}$$

$$\tilde{A}^{(e)}_{1,1} = \frac{h}{8}\int_{-1}^{1}(1+X)^2\mathrm{d}X = \frac{h}{3}$$

And the $\tilde{b}_r^{(e)}$ gives us:

$$\begin{aligned}
\tilde{b}_0^{(e)} &= \int_{-1}^{1} (x_m + \frac{1}{2}hX)(1 - (x_m + \frac{1}{2}hX))\frac{1}{2}(1-X)\frac{h}{2}\mathrm{d}X \\
&= -\frac{h^3}{24} + \frac{h^2 x_m}{6} - \frac{h^2}{12} - \frac{hx_m^2}{2} + \frac{hx_m}{2} \\
\tilde{b}_1^{(e)} &= \int_{-1}^{1} (x_m + \frac{1}{2}hX)(1 - (x_m + \frac{1}{2}hX))\frac{1}{2}(1+X)\frac{h}{2}\mathrm{d}X \\
&= -\frac{h^3}{24} - \frac{h^2 x_m}{6} + \frac{h^2}{12} - \frac{hx_m^2}{2} + \frac{hx_m}{2}
\end{aligned}$$

Using `sympy` we get the following:

$$A = \begin{bmatrix} \frac{h}{3} & \frac{h}{6} \\ \frac{h}{6} & \frac{h}{3} \end{bmatrix}$$

$$b = \begin{bmatrix} -\frac{h^3}{12} + \frac{h^2}{6} \\ -\frac{h^3}{4} + \frac{h^2}{3} \end{bmatrix}$$

$$c = \begin{bmatrix} \frac{h^2}{6} \\ \frac{1}{h}\left(-\frac{5h^3}{6} + h^2\right) \end{bmatrix}$$