Mandatory Session exercises

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Exercise 1

1. linear
2. linear
3. nonlinear
4. linear
5. linear
6. nonlinear
7. nonlinear
8. nonlinear
9. nonlinear
10. nonlinear

Exercise 4 turn it into two first ODE:

$$v' = a$$
$$mv' + b|v|v + s(r) = F(t)$$

Where r is the position and a for acceleration. Discritized and using geometric mean gives:

$$v^{n+1} = \Delta t a + v^n$$
$$v^{n+1} = [F(t) - s(r) - b(v^n v^{n+1}]\frac{\Delta t}{m} + v^n$$

As for the Picard iteration we will assume we have an approximation v^- to the v and $v = v^{n+1}$, $v^{(1)} = v^n$. What we will end up with will be an equation on the form

$$A(u)u = b(u)$$

. And using terms introduced we will have on the form

$$A(u^-)u = b(u^-)$$

and before new iteration we will induce $u^- \leftarrow u$

For the Newton's method we can have something similar to the Picard on the form:

$$F(u) = F(u^-) + J(u^-) \cdot (u - u^-) + \mathcal{O}(||u - u^-||^2)$$

With J being the Jacobian for our F

$$J_{i,j} = \frac{\partial F_i}{\partial u_j}$$

As for the Picard we will approximate our nonlinear system of $F = 0$ by the following:

$$\hat{F}(u) = F(u^-) + J(u^-) \cdot \delta u = 0, \quad \delta u = u - u^-$$

Algorithm is straight forward where we solve the Jacobian $J(u^-)\delta u = -F(u^-)$ and then update $u = u^- + \delta u$