

Differentiate a highly nonlinear term

$$\alpha(u) = ||\nabla u||^q \quad (1)$$

$$u = \sum c_k \psi_k \quad (2)$$

We want the derivative of α with respect to c_j :

$$\begin{aligned} \frac{\partial}{\partial c_j} ||\nabla u||^q &= \frac{\partial}{\partial c_j} (\nabla u \cdot \nabla u)^{\frac{q}{2}} = \frac{q}{2} (\nabla u \cdot \nabla u)^{\frac{q}{2}-1} \frac{\partial}{\partial c_j} (\nabla u \cdot \nabla u) \\ &= \frac{q}{2} ||\nabla u||^{q-2} \left(\frac{\partial}{\partial c_j} (\nabla u) \cdot \nabla u + \nabla u \cdot \frac{\partial}{\partial c_j} (\nabla u) \right) \\ &= q ||\nabla u||^{q-2} \left(\nabla u \cdot \nabla \frac{\partial u}{\partial c_j} \right) = q ||\nabla u||^{q-2} (\nabla u \cdot \nabla \psi_j) \end{aligned} \quad (3)$$