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Format for Assignment Submission

For Term End Exam June/December- JUNE **(Year)** 2025

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1. **Name of the Student** : Ashu Chaurasiya
2. **Enrollment Number** : 2501321326
3. **Programme Code** : MCA_NEW
4. **Course Code** : MCS-212
   (Use this format course-wise separately)
5. **Study Centre Code** : LSC-38046
6. **Name of the Study Centre With complete address** : RAJDHANI COLLEGE RAJA GARDEN NEW DELHI
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9. **Details if this same assignment has been submitted anywhere else also** : No
10. **Above information is cross checked and it is correct: Yes/No** ✓

Date of Submission: 01-06-25

Ashu
(Signature of the student)

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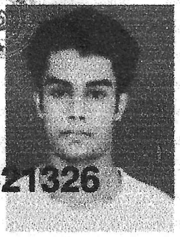

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Course Code	:	**MCS-212**
Course Title	:	**Discrete Mathematics**
Assignment Number	:	**MCA_NEW(I)/212/Assign/2025**
Maximum Marks	:	**100**
Weightage	:	**30%**
Last Dates for Submission	:	**30^{th} April 2025 (for January Session)**
		31^{st} October 2025 (for July Session)

This assignment has 20 questions of 4 Marks each, amounting to 80 marks. Answer all questions. Rest 20 marks are for viva voce. You may use illustrations and diagrams to enhance the explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.

Q1: Prove by mathematical induction that $\sum_{i=1 \, to \, n} \frac{1}{i(i+1)} = n/(n+1)$

Q2: Verify whether $\sqrt{11}$ is rational or irrational.

Q3: Write the following statements in the symbolic form.

i) Some students can not appear in exam.
ii) Everyone can not sing.

Q4: Draw logic circuit for the following Boolean Expression:
(x y z) + (x+y+z)'+(x'zy')

Q5: Explain whether function: f(x) = x^2 posses an inverse function or not.

Q6: Write the finite automata corresponding to the regular expression (a + b)*ab

Q7: If L1 and L2 are context free languages then, prove that L1 U L2 is a context free language.

Q8: Explain Decidable and Undecidable Problems. Give example for each.

Q9: What is equivalence relation? Explain use of equivalence relation with the help of an example.

Q10: There are three Companies, C1, C2 and C3. The party C1 has 4 members, C2 has 5 members and C3 has 6 members in an assembly. Suppose we want to select two persons, both from the same Company, to become president and vice president. In how many ways can this be done?

Q11: How many words can be formed using letter of DEPARTMENT using each letter at most once?

i) If each letter must be used,
ii) If some or all the letters may be omitted.

Q12: What is the probability that a number between 1 and 10,000 is divisible by neither 2, 3, 5 nor 7?

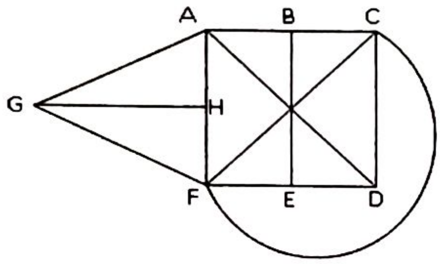
Q13: Explain inclusion-exclusion principle and Pigeon Hole Principle with example.

Q14: Find an explicit recurrence relation for minimum number of moves in which the n-disks in tower of Hanoi puzzle can be solved! Also solve the obtained recurrence relation through an iterative method.

Q15: Find the solution of the recurrentces relation $a_n = a_{n-1} + 2a_{n-1}$, $n > 2$ with $a_0 = 0$, $a_1 = 1$

Q16: Prove that the complement of $\overline{G}$ is G

Q17: What is a chromatic number of a graph? What is a chromatic number of the following graph?



Q18: Determine whether the above graph has a Hamiltonian circuit. If it has, find such a circuit. If it does not have, justify it.

Q19: Explain and prove the Handshaking Theorem, with suitable example

Q20: Explain the terms PATH, CIRCUIT and CYCLES in context of Graphs.

Enrollment No. 2501321326

Question no. 1: Prove by mathematical induction that Σ i = 1 to n 1/(i(i+1)) = n/(n+1)

Answer: To prove this statement by mathematical induction, we follow three steps.

First, the base case: For n=1, the left side of the equation is 1/(1(1+1))=1/2, and the right side is 1/(1+1)=1/2. Since both sides are equal, the base case holds.

Second, the inductive hypothesis: Assume the statement is true for some positive integer k, meaning the sum from i=1 to k of 1/(i(i+1)) equals k/(k+1).

Third, the inductive step: We need to prove the statement is true for k+1. This means showing that the sum from i=1 to k+1 of 1/(i(i+1)) equals (k+1)/((k+1)+1), which simplifies to (k+1)/(k+2). We can rewrite the sum for k+1 as the sum for k plus the (k+1)-th term.

By the inductive hypothesis, the sum for k is k/(k+1). The (k+1)-th term is 1/((k+1)(k+2)). Adding these, we get k/(k+1)+1/((k+1)(k+2)).

To combine these fractions, we find a common denominator, which is (k+1)(k+2). This gives us (k(k+2)+1)/((k+1)(k+2)), which simplifies to (k2+2k+1)/((k+1)(k+2)).

The numerator is a perfect square, (k+1)2. So, we have (k+1)2/((k+1)(k+2)). We can cancel one (k+1) term from the numerator and denominator, leaving us with (k+1)/(k+2). This matches the right side of the equation for $n=k+1$.

Therefore, by the principle of mathematical induction, the statement is true for all positive integers n.

Question no. 2: Verify whether 11 is rational or irrational.

Answer: To verify whether the square root of 11 is rational or irrational, we can use a proof by contradiction. A rational number is any number that can be expressed as a fraction p/q, where p and q are integers and q is not zero. An irrational number cannot be expressed in this form.

Let's assume, for the sake of contradiction, that the square root of 11 is rational. This means we can write $11=p/q$, where p and q are integers, $q=0$, and p and q have no common factors (meaning the fraction is in its simplest form).

Squaring both sides gives $11=p2/q2$, which implies $p2=11q2$.

This equation tells us that p2 is a multiple of 11. If p2 is a multiple of 11, then p itself must also be a multiple of 11. So, we can write $p=11k$ for some integer k. Substituting this back into the equation $p2=11q2$, we get $(11k)2=11q2$, which simplifies to $121k2=11q2$. Dividing both sides by 11, we get $11k2=q2$.

This means q2 is a multiple of 11, and consequently, q must also be a multiple of 11. So, both p and q are multiples of 11. However, this contradicts our initial assumption that p and q have no common factors.

Since our assumption leads to a contradiction, the initial assumption must be false. Therefore, the square root of 11 is an irrational number.

Question no. 3: Write the following statements in the symbolic form.

i) Some students can not appear in exam.

ii) Everyone can not sing.

Answer: i) Let $S(x)$ be the predicate "x is a student" and $E(x)$ be the predicate "x can appear in exam".

The statement "Some students can not appear in exam" can be written in symbolic form as: There exists an x such that $S(x)$ and not $E(x)$.

ii) Let $P(x)$ be the predicate "x is a person" and $S(x)$ be the predicate "x can sing". The statement "Everyone can not sing" means that it is not true that everyone can sing.

This can be written in symbolic form as: It is not true that for all x, $P(x)$ implies $S(x)$. Alternatively, it can be interpreted as "There exists someone who cannot sing," which is: There exists an x such that $P(x)$ and not $S(x)$.

Question no. Q4: Draw logic circuit for the following Boolean Expression:

$(x\ y\ z) + (x+y+z)' + (x'zy')$

Answer: To describe the logic circuit for the Boolean expression (x AND y AND z) OR (NOT (x OR y OR z)) OR (NOT x AND z AND NOT y), we would use a combination of AND, OR, and NOT gates.

First, for the term (x AND y AND z), you would have a three-input AND gate with inputs x, y, and z.

Second, for the term (x OR y OR z) prime, you would have a three-input OR gate with inputs x, y, and z, followed by a NOT gate.

Third, for the term (NOT x AND z AND NOT y), you would have a NOT gate for x, a NOT gate for y, and then a three-input AND gate with inputs from the NOT x output, z, and the NOT y output.

Finally, the outputs of these three intermediate gates (the AND gate for xyz, the NOT gate for $(x+y+z)$ prime, and the AND gate for $x'zy'$) would feed into a three-input OR gate. The output of this final OR gate represents the entire Boolean expression.

Question no. 5: Explain whether function: $f(x)=x2$ posses an inverse function or not.

Answer: The function $f(x)=x2$ does not possess an inverse function over its entire domain (all real numbers) because it is not a one-to-one function. A function is one-to-one if each output value corresponds to exactly one input value.

For $f(x)=x2$, different input values can produce the same output value; for example, $f(2)=4$ and $f(-2)=4$. If we try to find an inverse, for an output of 4, we wouldn't know whether the original input was 2 or -2.

For an inverse function to exist, the original function must pass the horizontal line test, meaning no horizontal line intersects the graph of the function more than once.

The graph of $f(x)=x2$ (a parabola) fails this test. However, if we restrict the domain of $f(x)=x2$ to either non-negative numbers (x greater than or equal to 0) or non-positive numbers (x less than or equal to 0),

then it becomes one-to-one on that restricted domain, and an inversefunction would exist. For example, if the domain is restricted to x greater than or equal to 0, the inverse function would be the squareroot of x.

Question no. Q6: Write the finite automata corresponding to the regular expression (a+b)ab

Answer: A finite automaton for the regular expression (a OR b)ab can be described with states and transitions.

Let's define three states: Q0, Q1, and Q2. Q0 will be our initial state.

From Q0, if we read an 'a' or a 'b', we stay in Q0. This accounts for the (a OR b) part, meaning any sequence of 'a's and 'b's. If we are in Q0 and read an 'a', we can transition to Q1. This signifies that we have potentially started the 'ab' sequence. From Q1, if we read a 'b', we transition to Q2. This signifies that we have successfully completed the 'ab' sequence. Q2 will be our final or accepting state.

What happens if we read other characters? If we are in Q1 and read an 'a' (instead of a 'b'), we would go back to Q1, as this 'a' could be the start of a new 'ab' sequence. If we are in Q2 and read an 'a' or a 'b', we would transition back to Q0, as the 'ab' sequence has been completed, and any subsequent characters would start a new potential match from the beginning. So, the automaton has:

States: Q0 (start state), Q1, Q2 (accepting state).

Alphabet: {a, b}.

Transitions:

From Q0: on 'a' or 'b' go to Q0.

From Q0: on 'a' go to Q1.

From Q1: on 'b' go to Q2.

From Q1: on 'a' go to Q1.

From Q2: on 'a' or 'b' go to Q0. This setup ensures that the automaton accepts strings that end with 'ab' and have any combination of 'a's and b's before it.

Question no. 7: If L1 and L2 are context free languages then, prove that L1 U L2 is a context free language.

Answer: If you have two context-free languages (CFLs), let's call them L1 and L2, you can combine them to form a new language, L1 L2, which is also a CFL. This is because CFLs are closed under union.

Here's a simpler way to understand why:

Since L1 and L2 are CFLs, they each have their own set of grammatical rules (a context-free grammar). Let's say G1 generates L1 and G2 generates L2.

To create a grammar for L1 U L2, we simply do the following:

Create a brand new starting point: We introduce a new "start" symbol that isn't used in G1 or G2. Let's call it Snew.

Combine the rules: We take all the rules from G1 and all the rules from G2.

Add two new rules: We add two special rules involving our new start symbol:

SnewS1 (where S1 is the original start symbol for G1)

SnewS2 (where S2 is the original start symbol for G2)

Now, any word that could be formed by G1 can also be formed by our new grammar (by starting with Snew and then using the rule SnewS1). The same goes for any word from G2.

Since we've successfully created a new context-free grammar that generates L1 L2, it proves that the union of two CFLs is always a CFL.

Question no. 8: Explain Decidable and Undecidable Problems. Give example for each.

Answer: In the realm of computation theory, problems are classified based on whether an algorithm can solve them. A decidable problem is a decision problem for which an algorithm exists that can always produce a correct yes or no answer in a finite amount of time for any given input. This means there is a guaranteed procedure that will halt and provide a definitive answer. An example of a decidable problem is determining whether a given string is a valid keyword in a programming language. An algorithm can easily check the string against a predefined list of keywords and return true or false. Another example is checking if a number is prime; algorithms like trial division can determine this in a finite number of steps.

Conversely, an undecidable problem is a decision problem for which no algorithm exists that can always produce a correct yes or no answer for every possible input in a finite amount of time. It's not just that we haven't found such an algorithm yet; it has been mathematically proven that no such algorithm can ever exist. The most famous example of an undecidable problem is the Halting Problem. This problem asks whether it is possible to determine, for any arbitrary program and any arbitrary input, if that program will eventually halt (finish its execution) or run forever. Alan Turing proved that no general algorithm can solve the Halting Problem for all possible program-input pairs. Another example is determining if two context-free grammars generate the same language.

Question no. 9: What is equivalence relation? Explain use of equivalence relation with the help of an example.

Answer: An equivalence relation is a type of binary relation on a set that satisfies three specific properties: reflexivity, symmetry, and transitivity. First, reflexivity means that every element in the set is related to itself. For any element 'a' in the set, 'a' is related to 'a'. Second, symmetry means that if one element is related to another, then the second element is also related to the first. If 'a' is related to 'b', then 'b' is related to 'a'. Third, transitivity means that if a first element is related to a second, and the second element is related to a third, then the first element is also related to the third. If 'a' is related to 'b' and 'b' is related to 'c', then 'a' is related to 'c'. When a relation satisfies all three of these properties, it is an equivalencerelation. The main use of an equivalence relation is to partition a setinto disjoint subsets called equivalence classes. All elements within anequivalence class are considered "equivalent" to each other with respect tothe relation. For example, consider the set of all integers, anddefine a relation "is congruent to modulo 3". Two integers 'a' and 'b' arerelated if their difference $(a - b)$ is divisible by 3.

Reflexivity: For any integer 'a', $(a - a) = 0$, which is divisible by 3. So, 'a' is congruent to 'a' modulo 3.

Symmetry: If 'a' is congruent to 'b' modulo 3, then $(a - b)$ is divisible by 3. This means $(a - b) = 3k$ for some integer k. Then $(b - a) = -3k$, which is also divisible by 3. So, 'b' is congruent to 'a' modulo 3.

Transitivity: If 'a' is congruent to 'b' modulo 3 and 'b' is congruent to 'c' modulo 3, then $(a - b)$ is divisible by 3 and $(b - c)$ is divisible by 3. This means $(a - b) = 3k1$ and $(b - c) = 3k2$ for integers $k1$, $k2$. Adding these equations, $(a - b) + (b - c) = 3k1 + 3k2$, which simplifies to $(a - c) = 3(k1 + k2)$. Since $(k1 + k2)$ is an integer, $(a - c)$ is divisible by 3. So, 'a' is congruent to 'c' modulo 3. Since all three properties hold, "is congruent to modulo 3" is an equivalence relation. This relation partitions the set of integers into three equivalence classes:

Numbers that leave a remainder of 0 when divided by 3 (e.g. -3, 0, 3, 6, ...)

Numbers that leave a remainder of 1 when divided by 3 (e.g. -2, 1, 4, 7, ...)

Numbers that leave a remainder of 2 when divided by 3 (e.g. -1, 2, 5, 8, ...) This partitioning is a fundamental use of equivalence relations in mathematics and computer science.

Question no. Q10: There are three Companies, C1, C2 and C3. The party C1 has 4 members, C2 has 5 members and C3 has 6 members in an assembly. Suppose we want to select two persons, both from the same Company, to become president and vice president. In how many ways can this be done?

Answer: To figure out the total number of ways to pick a president and vice president from the same company, we simply calculate the permutations for each company and add them up.

Company C1

With 4 members, there are $P(4,2)=12$ ways.

Company C2

With 5 members, there are $P(5,2)=20$ ways.

Company C3

With 6 members, there are $P(6,2)=30$ ways.

Total Ways

Adding these up: $12+20+30=62$ ways.

So, there are 62 ways to select a president and a vice president from the same company.

Question no. Q11: How many words can be formed using letter of DEPARTMENT using each letter at most once? i) If each letter must be used, ii) If some or all the letters may be omitted.

Answer: The word DEPARTMENT has 10 distinct letters: D, E, P, A, R, T, M, E, N, T. However, the letter 'E' appears twice and 'T' appears twice.

So, the distinct letters are D, E, P, A, R, T, M, N.

There are 8 distinct letters. The question states "using each letter at most once" and "letter of DEPARTMENT". This implies we are considering the unique letters available. Let's re-read carefully: "using letter of DEPARTMENT".

This usually means the actual letters in the word, including repetitions.

If we consider the letters available as {D, E, P, A, R, T, M, E, N, T}, then we have 10 positions.

But "using each letter at most once" implies distinct letters. Let's assume it means using the distinct letters available from the word 'DEPARTMENT'. The distinct letters are D, E, P, A, R, T, M, N. There are 8 distinct letters.

Let's clarify the interpretation of "using letter of DEPARTMENT using each letter at most once". If it means using the set of unique letters {D, E, P, A, R, T, M, N}, then there are 8 distinct letters.

i) If each letter must be used: If we are using the 8 distinct letters (D, E, P, A, R, T, M, N) and each must be used, then we are forming permutations of these 8 letters. The number of words that can be formedis 8 factorial (8!), which is 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 =40,320

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ii) If some or all the letters may be omitted: This means we can form words of length 1, length 2, .., up to length 8 using the 8 distinct letters. For length 1: 8 words (P(8,1))

For length 2: 8 * 7 = 56 words

(P(8,2)) For length 3: 8 * 7 * 6 = 336 words

(P(8,3)) For length 4: 8 * 7 * 6 * 5 = 1680 words

(P(8,4)) For length 5: 8 * 7 * 6 * 5 * 4 = 6720 words

(P(8,5)) For length 6: 8 * 7 * 6 * 5 * 4 * 3 = 20160 words

(P(8,6)) For length 7: 8 * 7 * 6 * 5 * 4 * 3 * 2 = 40320 words

(P(8,7)) For length 8: 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 40320 words

(P(8,8)) The total number of words is the sum of permutations for each length: 8 + 56 + 336 + 1680 + 6720 + 20160 + 40320 + 40320 = 109600 words.

Question no. 12: What is the probability that a number between 1 and 10,000 is divisible by neither 2, 3, 5 nor 7?

Answer: To find the probability that a number between 1 and 10,000 (inclusive) is divisible by neither 2, 3, 5 nor 7, we first need to find the count of such numbers. This is equivalent to finding the count of numbers that are coprime to the product of 2, 3, 5, and 7, which is 210

We can use the Principle of Inclusion-Exclusion, or more simply, Euler's totient function extended for a range. The numbers that are not divisible by 2, 3, 5, or 7 are those whose prime factors do not include 2, 3, 5, or 7. The total number of integers is 10,000.

We can calculate the proportion of numbers that are not divisible by these primes. The proportion of numbers not divisible by 2 is $(1 - 1/2) = 1/2$. The proportion of numbers not divisible by 3 is $(1 - 1/3) = 2/3$. The proportion of numbers not divisible by 5 is $(1 - 1/5) = 4/5$.

The proportion of numbers not divisible by 7 is $(1 - 1/7) = 6/7$. Since 2, 3, 5, and 7 are prime numbers, their divisibility properties are independent. So, the proportion of numbers not divisible by any of them is the product of these individual proportions: $(1/2) * (2/3) * (4/5) * (6/7) = 48 / 210 = 8 / 35$. Now, we multiply this proportion by the total number of integers (10,000) to find the count of such numbers:

$(8/35) * 10{,}000 = 80{,}000 / 35$

which is approximately 2285.71. Since we need an integer count, we should be careful. A more precise way is to count using inclusion-exclusion.

Let $N = 10{,}000$. Numbers divisible by 2: $N/2 = 5000$ Numbers divisible by 3: $N/3 = 3333$ (floor) Numbers divisible by 5: $N/5 = 2000$ Numbers divisible by 7: $N/7 = 1428$ (floor)

Numbers divisible by 2 and 3 (ie, by 6): $N/6 = 1666$ Numbers divisible by 2 and 5 (ie, by 10): $N/10 = 1000$ Numbers divisible by 2 and 7 (ie, by 14): $N/14 = 714$ Numbers divisible by 3 and 5 (ie, by 15): $N/15 = 666$ Numbers divisible by 3 and 7 (ie, by 21): $N/21 = 476$ Numbers divisible by 5 and 7 (ie, by 35): $N/35 = 285$

Numbers divisible by 2, 3, and 5 (ie, by 30): $N/30 = 333$ Numbers divisible by 2, 3, and 7 (ie, by 42): $N/42 = 238$ Numbers divisible by 2, 5, and 7 (ie, by 70): $N/70 = 142$ Numbers divisible by 3, 5, and 7 (ie, by 105): $N/105 = 95$

Numbers divisible by 2, 3, 5, and 7 (ie, by 210): $N/210 = 47$

Total numbers divisible by at least one of 2, 3, 5, 7: Sum of singles - Sum of pairs + Sum of triples - Sum of quadruples = $(5000+3333+2000+1428) - (1666+1000+714+666+476+285) + (333+238+142+95) - 47 = 11761 - 4807 - 47 = 7715$

Numbers not divisible by any of them = Total numbers - Numbers divisible by at least one $= 10000 - 7715 = 2285$.

The probability is the count of favorable outcomes divided by the total number of outcomes: $2285/10000 = 0.2285$.

Question no. Q13: Explain inclusion-exclusion principle and Pigeon Hole Principle with example.

Answer: The Principle of Inclusion-Exclusion is a counting technique used to determine the size of the union of multiple finite sets. It states to find the number of elements in the union of two or more sets, you sum the sizes of the individual sets, then subtract the sizes of all pairwise intersections, then add back the sizes of all three-way intersections, and so on, alternating between adding and subtracting.

This principle ensures that elements belonging to multiple sets are counted exactly once. For example, suppose we want to find the number of students in a class who like either Math or Science. Let M be the set of students who like Math, and S be the set of students who like Science. If we simply add the number of students in M and the number of students in S, we would double-count the students who like both Math and Science (those in the intersection of M and S). So, the principle of inclusion-exclusion tells us that the number of students who like Math or Science is the number of students who like Math, plus the number of students who like Science, minus the number of students who like both Math and Science. If 20 students like Math, 15 like Science, and 5 like both, then the number of students who like Math or Science is:

$20 + 15 - 5 = 30$.

The Pigeonhole Principle is a simple yet powerful concept in combinatorics. It states that if you have more pigeons than pigeonholes, and you try to put every pigeon into a pigeonhole, then at least one pigeonhole must contain more than one pigeon.

More formally, if 'n' items are put into 'm' containers, with $n > m$, then at least one container must contain more than one item.

This principle is often used to prove the existence of certain conditions.

For example, consider a group of 13 people. We want to show that at least two people in this group must have been born in the same month.

Here, the "pigeons" are the 13 people, and the "pigeonholes" are the 12 months of the year. Since there are 13 people (pigeons) and only 12 possible birth months (pigeonholes), by the Pigeonhole Principle, at least two people must share the same birth month.

Question no. Q14: Find an explicit recurrence relation for minimum number of moves in which the n-disks in tower of Hanoi puzzle can be solved! Also solve the obtained recurrence relation through an iterative method.

Answer: The Tower of Hanoi is a mathematical puzzle involving three pegs and a number of disks of different sizes, which can slide onto any peg.

The puzzle starts with the disks in a neat stack in ascending order of size on one peg, the smallest at the top, thus making a conical shape. The objective of the puzzle is to move the entire stack to another peg, obeying the following rules:

Only one disk can be moved at a time.

Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty peg.

No disk may be placed on top of a smaller disk.

Let T_n be the minimum number of moves required to solve the Tower of Hanoi puzzle with 'n' disks. To move 'n' disks from a source peg to a destination peg using an auxiliary peg:

Move the top n-1 'disks from the source peg to the auxiliary peg. This requires T_{n-1} moves.

Move the largest (nth) disk from the source peg to the destination peg. This requires 1 move.

Move the n-1 'disks from the auxiliary peg to the destination peg.

This again requires T_{n-1} moves. So, the recurrence relation is

$T_n = T_{n-1} + 1 + T_{n-1}$,

It simplifies to $T_n = 2T_{n-1} + 1$. The base case is for 1 disk: $T_1 = 1$ move.

Now, let's solve this recurrence relation iteratively:

$T_1 = 1$ $T_2 = 2T_1 + 1 = 2(1) + 1 = 3$ $T_3 = 2T_2 + 1 = 2(3) + 1 = 7$ $T_4 = 2T_3 + 1 = 2(7) + 1 = 15$

We can observe a pattern here: $T_n = 2n-1$.

Let's prove this by substitution:

$T_n = 2T_{n-1} + 1$ Substitute $T_{n-1} = 2n-1-1$: $T_n = 2(2n-1-1) + 1$ $T_n = 2n-2+1$ $T_n = 2n-1$

This confirms that the explicit formula for the minimum number of moves for n'disks in the Tower of Hanoi puzzle is $2n-1$.

Question no. 15: Find the solution of the recurrences relation

$a_n = a_{n-1} + 2a_{n-1}$, $n > 2$ with $a_0 = 0$, $a_1 = 1$

Answer:

The recurrence relation is $a_n = a_{n-1} + 2a_{n-1}$ for $n > 2$, which simplifies to $a_n = 3a_{n-1}$. The initial conditions are $a_0 = 0$ and $a_1 = 1$.

Calculating the Terms

Let's find the first few terms using the simplified recurrence relation and the given initial conditions:

For $n=0$: $a_0=0$ (given)

For $n=1$: $a_1=1$ (given)

For $n=2$: $a_2=3a_1=3\times 1=3$

For $n=3$: $a_3=3a_2=3\times 3=9$

For $n=4$: $a_4=3a_3=3\times 9=27$

General Solution

We can observe a pattern for $n\geq 1$: $a_n=3^{n-1}$.

Let's verify this pattern:

For $n=1$: $a_1=3^{1-1}=3^0=1$, which matches the given a_1.

For $n=2$: $a_2=3^{2-1}=3^1=3$, which matches our calculation.

For $n=3$: $a_3=3^{3-1}=3^2=9$, which matches our calculation.

However, this formula does not hold for a_0, as $3^{0-1}=3^{-1}=1/3\neq 0$.

Conclusion

Given the strict condition $n>2$ for the recurrence relation $a_n=3a_{n-1}$, and the initial conditions $a_0=0$ and $a_1=1$, the solution can be stated as:

$a_0=0$

$a_1=1$

For $n\geq 2$, $a_n=3^{n-1}$

Question no. Q16: Prove that the complement of $\bar{G}$ is G

Answer: In graph theory, the complement of a graph G, denoted as $\bar{G}$ (read as G-bar), is a graph on the same set of vertices. The key rule for its edges is: two distinct vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

In simpler terms:

If there's an edge between two vertices in G, there's no edge between them in G^-.

If there's no edge between two vertices in G, there is an edge between them in G^-.

Proving $G^{--}=G$

To prove that the complement of the complement of G is G (i.e., $G^{--}=G$), we need to show that applying the complement operation twice brings us back to the original graph.

Let G be a graph with a set of vertices V and a set of edges E.

Consider the complement of G, which is G^-:

The vertices of G^- are the same as G (V).

The edges of G^-, let's call them E^-, are defined such that an edge (u,v) is in E^- if and only if (u,v) is not in E (assuming u and v are distinct vertices).

Now, let's take the complement of G^-, which is denoted as G^{--}:

The vertices of G^{--} are again the same set V.

The edges of G^{--}, let's call them E^{--}, are defined such that an edge (u,v) is in E^{--} if and only if (u,v) is not in E^-.

Connecting the definitions:

From our definition of E^-, we know that (u,v) is not in E^- if and only if (u,v) is in E.

Therefore, an edge (u,v) is in E^{--} if and only if (u,v) is in E.

Conclusion

This means that the set of edges E-- is exactly the same as the set of edges E. Since G-- has the same vertices as G and the same edges as G, it logically follows that G-- is equal to G. This proves that the complement of the complement of a graph is the original graph itself.

Question no. Q) 7: What is a chromatic number of a graph? What is a chromatic number of the following graph?

Answer: The chromatic number of a graph is the minimum number of colours needed to color the vertices of the graph such that no two adjacent vertices (vertices connected by an edge) share the same color. This is also known as vertex coloring. The chromatic number is usually denoted by (G), where G is the graph.

Now, let's determine the chromatic number of the graph provided in the image. The graph has vertices A, B, C, D, E, F, G, H. Let's list the vertices and their connections (adjacencies) based on the image:

A is connected to B, H, F, C.

B is connected to A, C, E.

C is connected to A, B, D.

D is connected to C, E, F.

E is connected to B, D, F.

F is connected to A, D, E, G.

G is connected to A, H, F.

H is connected to A, G.

Let's try to color the graph: We can start by assigning color 1 to vertex A.

Color A: 1 Now, its neighbors (B, H, F, C) must be different from 1.

Color B: 2 (A is 1)

Color C: 3 (A is 1, B is 2)

Color D: 1 (C is 3, E is not yet colored, F is not yet colored. D is not connected to A, so it can be 1)

Color E: 4 (B is 2, D is 1, F is not yet colored. E needs a new color)

Color F: Needs to be different from A (1), D (1), E (4), G (not yet colored). F cannot be 1. Let's try 2. (A=1, D=1, E=4, G=?) Let's restart with a systematic approach, looking for cliques (sets of vertices where every pair is connected). Consider the rectangle A-B-C-D-E-F. This is not a simple cycle. A, B, C form a triangle (A-B, B-C, A-C). This means they require at least 3 colors. Let's assign: A: Color 1 B: Color 2 C: Color 3 Now, let's color the remaining vertices:

H is connected to A and G. If G is also connected to A, then H and G cannot be color 1.

F is connected to A, D, E, G.

D is connected to C, E, F.

E is connected to B, D, F.

Let's try to use 3 colors: Color A: 1 Color B: 2 Color C: 3 Since A, B, C form a triangle, we need at least 3 colors. Now, let's try to the rest with these 3 colours if possible:

H is adjacent to A (1) and G. H cannot be 1. Let H be 2.

G is adjacent to A (1), H (2), F. G cannot be 1 or 2. So G must be 3.

F is adjacent to A (1), D, E, G (3). F cannot be 1 or 3. So F must be 2.

D is adjacent to C (3), E, F (2). D cannot be 2 or 3. So D must be 1.

E is adjacent to B (2), D (1), F (2). E cannot be 1 or 2. So E must be 3.

Let's check the coloring: A: 1 B: 2 C: 3 D: 1 (Adj to C(3), E(3), F(2) - OK) E: 3 (Adj to B(2), D(1), F(2) - OK) F: 2 (Adj to A(1), D(1), E(3), G(3) - OK) G: 3 (Adj to A(1), H(2), F(2) - OK) H: 2 (Adj to A(1), G(3) - OK)

All adjacencies are satisfied with 3 colours. Since we found a triangle (A, B, C), we know the chromatic number must be at least 3. Since we successfully colored it with 3 colours, the chromatic number is exactly 3.

The chromatic number of the given graph is 3.

Question no. 18: Determine whether the above graph has a Hamiltonian circuit. If it has, find such a circuit. If it does not have, justify it.

Answer: A Hamiltonian circuit (or Hamiltonian cycle) in a graph is a cycle that visits each vertex exactly once and returns to the starting vertex. It must include every vertex in the graph. Let's examine the graph provided in the image with vertices A, B, C, D, E, F, G, H. There are 8 vertices. A Hamiltonian circuit must visit each of these 8 vertices exactly once. Let's try to find a Hamiltonian circuit by tracing paths: Start at A. A -> B -> C -> D -> E -> F -> G -> H -> A Let's check if this path visits all vertices and returns to A, and if all connections exist: A-B (exists) B-C (exists) C-D (exists) D-E (exists) E-F (exists) F-G (exists) G-H (exists) H-A (exists) This sequence (A, B, C, D, E, F, G, H, A) forms a Hamiltonian circuit. All 8 vertices are visited exactly once, and it returns to the starting vertex. Therefore, the graph does have a Hamiltonian circuit, and one such circuit is A-B-C-D-E-F-G-H-A.

Question no. Q19: Explain and prove the Handshaking Theorem, with suitable example

Answer: The Handshaking Theorem is a fundamental result in graph theory, specifically dealing with undirected graphs. It states that in any undirected graph, the sum of the degrees of all vertices is equal to twice the number of edges. In simpler terms, if you add up how many connections each point (vertex) has, that total will always be double the total number of connections (edges) in the graph. This is because each edge connects exactly two vertices, contributing 1 to the degree of each of those two vertices. Thus, each edge is counted twice in the sum of degrees. The theorem can be formally stated as: For an undirected graph G = (V, E), where V is the set of vertices and E is the set of the sum of the degrees of all vertices is equal to $2\times E$, where E is the number of edges.

Proof: Consider an undirected graph G with 'n' vertices ($v_1, v_2, \ldots, v_n$) and 'm' edges ($e_1, e_2, \ldots, e_m$). Let $deg(v_i)$ denote the degree of vertex v_i, which is the number of edges incident to v_i. We want to prove that $\sum_{i=1}^{n} deg(v_i) = 2\times m$. Let's think about how the sum of degrees is calculated. When we sum the degrees of all vertices, we are essentially counting the "ends" of all the edges. Each edge in an undirected graph connects exactly two vertices. For example, if there is an edge between vertex 'u' and vertex 'v', this edge contributes 1 to the degree of 'u' and 1 to the degree of 'v'. So, for every single edge in the graph, it contributes exactly two units to the total sum of degrees (one for each of its endpoints). Since there are 'm' edges in the graph, and each edge contributes 2 to the sum of degrees, the total sum of degrees must be $2\times m$. Therefore, $\sum_{i=1}^{n} deg(v_i) = 2\times m$.

Example: Consider a simple undirected graph with 4 vertices and 3 edges. Let the vertices be A, B, C, D. Let the edges be: (A, B), (B, C), (C, D). Now, let's find the degree of each vertex:

Degree of A ($deg(A)$) = 1 (connected to B)

Degree of B ($deg(B)$) = 2 (connected to A, C)

Degree of C ($deg(C)$) = 2 (connected to B, D)

Degree of D ($deg(D)$) = 1 (connected to C) The sum of the degrees of all vertices = $deg(A) + deg(B) + deg(C) + deg(D) = 1 + 2 + 2 +$

) = 6. The number of edges in the graph is E=3. According to theHandshaking Theorem, the sum of degrees should be 2×E. 2×3=6. Sincethe sum of degrees (6) equals twice the number of edges (6), theHandshaking Theorem holds true for this example.

Question no. 20: Explain the terms PATH, CIRCUIT and CYCLES in context of Graphs.

Answer: In the context of graphs, vertices are the points and edges are the connections between them. These terms describe different sequences of vertices and edges.

A Path in a graph is a sequence of distinct vertices such that there is an edge between consecutive vertices in the sequence. The key characteristic of a path is that no vertex is repeated. For example, in a graph with vertices A, B, C, D, if there are edges (A,B), (B,C), and (C,D), then A-B-C-D is a path. The length of a path is the number of edges it contains.

A Circuit (also sometimes called a closed walk) is a sequence of vertices that starts and ends at the same vertex, and where there is an edge between consecutive vertices. Unlike a path, a circuit can repeat vertices and edges. For example, A-B-C-B-A is a circuit if edges (A,B), (B,C), (C,B), (B,A) exist. The crucial part is that the start and end vertices are the same.

A Cycle is a special type of circuit. It is a path that starts and ends at the same vertex, with the additional condition that all vertices are distinct. In other words, a cycle is a closed path.

Thismeans that a cycle does not repeat any vertex except for the startingand ending vertex, which are the same. The minimum length of a cycle is 3(forming a triangle). For example, A-B-C-A is a cycle if edges (A,B),(B,C), and (C,A) exist, and A, B, C are distinct vertices. A cycle isa circuit, but not every circuit is a cycle (because circuits can repeatintermediate vertices).