



Portfolio Optimisation

Problem statement for QST-Hack2025, January 28-31, 2025



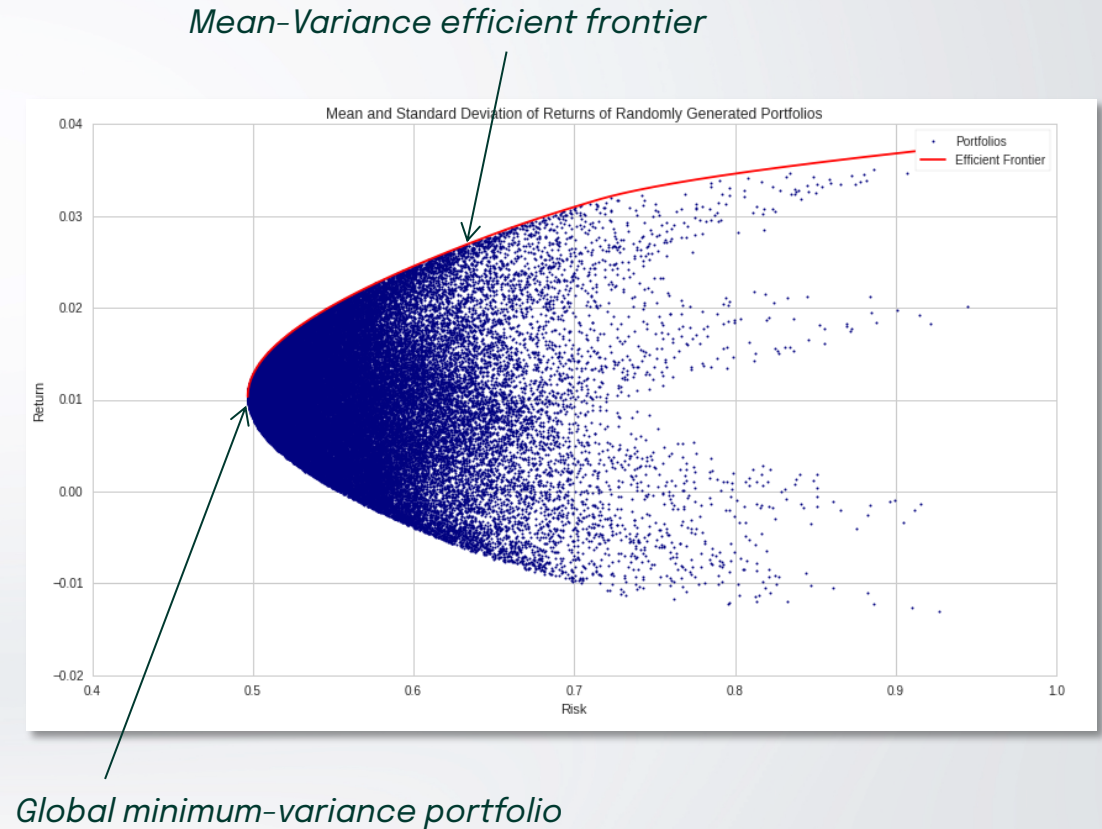
Introducing the challenge

Portfolio optimisation is the financial optimisation problem of selecting a portfolio of assets to invest in that maximizes return for a given level of risk.

In a Mean-Variance portfolio optimisation problem the selection is done based on stock market time-series data from which important statistical quantities about the asset pool is extracted:

- Expected asset returns, $\mu \in \mathbb{R}^n$
- Covariance between assets, $\Sigma \in \mathbb{R}^{n \times n}$

Correlations between assets affects the total risk of the portfolio, leading to an *efficient frontier* as illustrated in the figure.



Mean-Variance optimisation, <https://hudsonthames.org/portfolio-optimisation-with-portfolio-lab-mean-variance-optimisation/>
Sakuler et al. (2023) <https://arxiv.org/pdf/2303.12601>

Problem formulation

Consider Mean-Variance portfolio optimisation for an asset pool of n assets. A portfolio is defined as a vector of weights $\mathbf{w} \in \mathbb{R}^n$ indicating how much of the budget B is invested in each asset.

In the simplest case, \mathbf{w} is a vector of binary decision variables such that $\mathbf{w} \in \{0, 1\}^n$.

The available budget is B , which must be spent, but not exceeded. This is expressed as the budget constraint:

$$\sum_{i=1}^n w_i = B \quad \text{or in matrix form} \quad \mathbf{1}^\top \mathbf{w} = B.$$

The optimisation problem of minimizing the portfolio variance can be expressed as:

$$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w} - q \boldsymbol{\mu}^\top \mathbf{w} \quad \text{subject to} \quad \mathbf{1}^\top \mathbf{w} = B$$

where $q > 0$ is a risk tolerance factor.

Challenge

- Express the Mean-Variance portfolio optimisation problem as a QUBO problem.
- Implement a working QAOA solution to the problem and test it on data, e.g. following the procedure in this IBM Qiskit tutorial (ref. below)
- Generate new data sets with strong and weak correlations between assets and investigate the QAOA solvers performance.
- Adapt model to include constraints on e.g. budget, number of assets invested in, level of risk...

Sakuler et al. (2023) <https://arxiv.org/pdf/2303.12601>

Quadratic Unconstrained Binary Optimization (QUBO) https://pennylane.ai/qml/demos/tutorial_QUBO

IBM Qiskit tutorial https://qiskit-community.github.io/qiskit-finance/tutorials/01_portfolio_optimization.html