Towards QC applications in weather/climate

Weather and climate predictions are obtained by integrating the governing equations forward in time numerically.

Governing equations	
Equation of motion:	$\frac{DU}{Dt} = -\frac{1}{\rho}\nabla p + g - 2\Omega \times U + F$
Continuity equation:	$\frac{D\rho}{Dt} = -\rho \nabla \cdot U$
Thermodynamic equation:	$Q = C_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$
Equation of state:	$p = \rho R T$
Water conservation:	$\frac{Dq}{Dt} = -q\nabla \cdot U$

Variables
U – wind field
p – pressure
ρ – density
T – temperature
q – water content

Wilhelm Bjerknes (1862-1951)

Problem description:

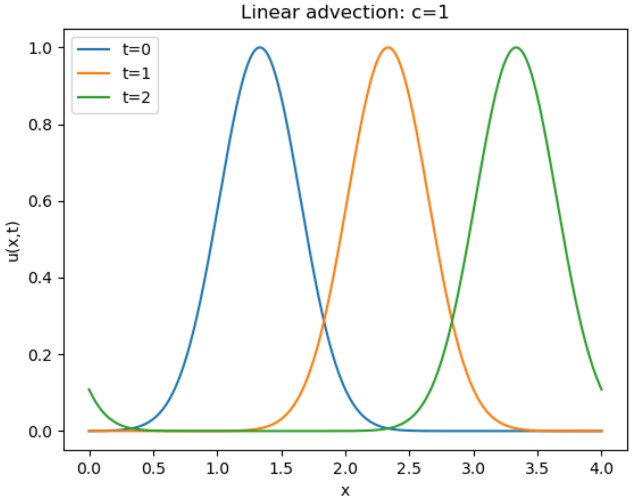
1) Create a quantum circuit that simulates (A) the 1d linear advection equation or (B) the 1d linear diffusion (or heat) equation.

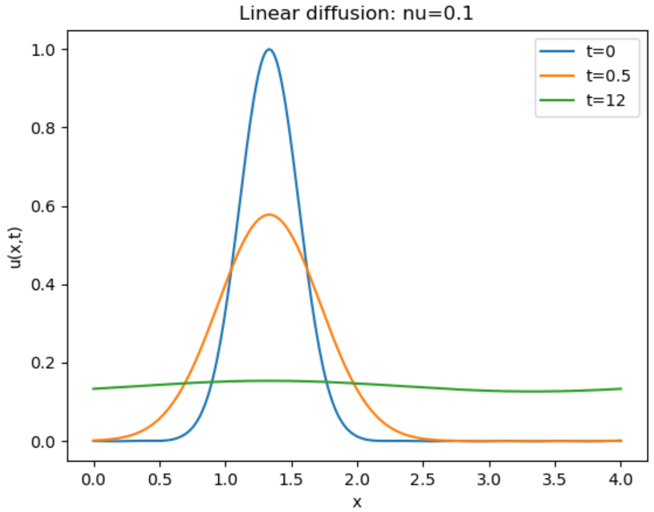
$$u\colon [0,T] \times [0,d] \to \mathbb{R}$$

$$(A) \quad \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$(B) \quad \frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

$$IC\colon u(0,x) = u_0(x)$$
$$BC\colon u(t,0) = u(t,d)$$





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The solution to 1) should be a gate efficient circuit encoding a unitary $U: \mathbb{C}^{2^n} \to \mathbb{C}^{2^n}$ such that,

$$|\psi_0\rangle \mapsto |\psi_T\rangle := U\,|\psi_0\rangle$$

where $|\psi_0\rangle$, $|\psi_T\rangle$ are discrete approximations of $u_0(x),\ u_T(x) := u(T,x)$.

$$u\colon [0,T] \times [0,d] \to \mathbb{R}$$

$$(A) \quad \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$(B) \quad \frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial x^2}$$

$$IC\colon u(0,x) = u_0(x)$$

$$BC\colon u(t,0) = u(t,d)$$

- Assume noiseless quantum computation.
- Ignore the cost of state preparation.

Problem description:

2a) Construct a quantum circuit and a measurement strategy that outputs an approximation to

$$S := \int_{d/4}^{3d/4} u(T,x)dx,$$

when the initial state is $|\psi_T\rangle$.

2b) Construct a quantum circuit and a measurement strategy that outputs 1 if

$$u(T,x) > \tau$$

for some x in the domain and some given threshold value τ , otherwise 0.

Application: Prediction of mean temperature in a specific subdomain.

Application: Prediction of extreme weather events.

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Evaluation criteria:

1) Does the program/circuit solve one or more of the problems?
2) Gate complexity: gate depth, number of 2-qubit and 1-qubit gates.
3) Creativity, efficiency and generality of solution.
4) Presentation and explanation of the program/circuit.

See the background material for additional details.

Good luck!

Extra: To increase the level of difficulty in 1), take one of the following equations instead.

E1) The 1d advection-diffusion equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}.$$

E2) The 2d advection or diffusion equations

$$\frac{\partial u}{\partial t} + C \cdot \nabla u = 0$$

$$\frac{\partial u}{\partial t} = \nu \Delta u$$