Infrastructure Optimization

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1 Problem Description

The problem proposed as infrastructure optimization can be modelled as a Max Coverage/Max-k-Coverage problem. The case consist in choosing the placement of the minimum number objects in some given area where at least $x\%$ of the area is covered. In our setting, there are some fixed points where the the objects can be placed, and the coverage of each single object is given as input data. Our task is to identify which objects should be cativated, and which ones should remain deactivated. The area that we would like to cover is divided into a square grid, and the unit of coverage is single pixels.
In this setting we are assuming that:

- All pixels have the same value.
- All objects have the same cost
- A pixel is considered covered if at least one object is covering it.

Such problem can be applied for EV-charger station placement, antenna placement...

1.1 Max-k-Coverage

Given n elements and m sets $S = \{S_1, S_2, ..., S_m\}$ consisting of various configurations of the n elements, we may pick at most k sets $S' \subset S$ that maximizes the number of elements covered. This translates into infrastructure optimization in the following way:

Imagine the area we wish to cover being split into n pixels. Each of m available antennas covers a set of pixels $\{S_1, S_2, ..., S_m\}$. We can then choose at most k antennas, so that the number of covered pixels is maximized. The Max-k-Coverage problem is NP-hard, has can be solved approximately with both the greedy algorithm and Integer Linear Programming.

Note that the initial setting of the problem was a Max-k-Coverage, but given the nature of the problem proposed, we will not fix k *a priori*, but rather search simultaneously for the optimal k that provides a $x\%$ coverage.

2 Create a PUBO model

The first step to apply a QIO solution, (e.g. Microsoft QIO suite, DWave quantum annealer, Toshiba SQBM+, Fujitsu Digital annealer...) is to create a QUBO/PUBO model.

QUBO stands for Quadratic Unconstrained Binary Optimization (in PUBO, P stands for Polynomial) and it is a model that can be directly fed into the solvers.

The goal of the hackaton is to create a model that tries to approach the problem described above.

Some useful references:

- A Tutorial on Formulating and Using QUBO Models, Glover et al. (arXiv:1811.11538)
- Ising formulations of many NP problems, Lucas (arXiv:1302.5843)

3 Now what...

If you have a model you are satisfied with we can provide some data to run the instances and benchmark against our current solution.

Moreover, these models can be useful for some optimization problems and fully embedded into Gate Based Quantum Computers via QAOA.