MATHS340: Real and Complex Calculus

Tutorial 1

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1. Work through the MuPAD guide (instructions can be found at the end of this document).

2. Let $\mathbf{v} = [1, 2, 3, 4, 5]$ and $\mathbf{u} = [1.5, \pi, e, 0, 2]$. Calculate $\mathbf{v} \cdot \mathbf{u}$ by hand. Check your answer using MuPAD.
Solution:

$$\mathbf{v} \cdot \mathbf{u} = 1.5 + 2\pi + 3e + 10 = 25.94$$

```
v := matrix([1, 2, 3, 4, 5]):
u := matrix([1.5, PI, exp(1), 0 , 2]):
scalarProduct(v, u)
```
25.93803079

3. Let $\mathbf{a} = [1, 2, 2, 4]$. Calculate $\|\mathbf{a}\|$ by hand and then using MuPAD.
Solution:

$$\|\mathbf{a}\| = \sqrt{1^2 + 2^2 + 2^2 + 4^2} = \sqrt{25} = 5$$

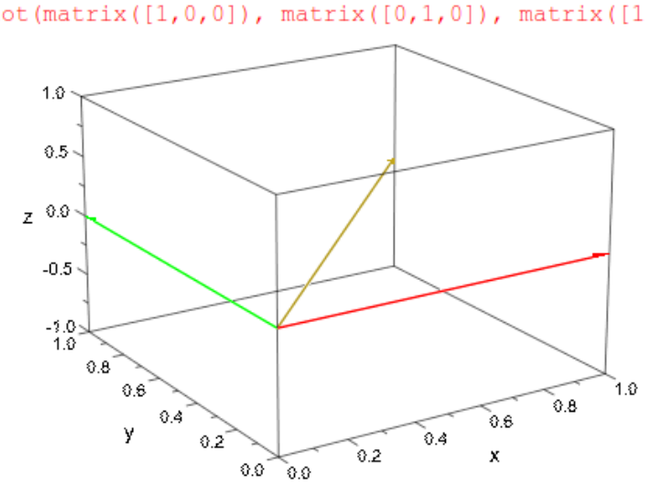
```
a := matrix([1,2,2,4]):
norm(a, 2)
```
5

4. If $\mathbf{a} = [1, 2]$ and $\mathbf{b} = [-1, -2]$, what is the angle between $\mathbf{a}$ and $\mathbf{b}$?
Solution:

```
a := matrix([1,2]):; b := matrix([-1,-2]):
angle(a,b)
```
π

5. Plot the three vectors $\mathbf{a} = [1, 0, 0]$, $\mathbf{b} = [0, 1, 0]$ and $\mathbf{c} = [1, 1, 0]$ using MuPAD. The following command can be used: plot(matrix([1,0,0]),matrix([0,1,0]),matrix([1,1,0]),3D)
Do the vectors form a basis for $\mathbb{R}^3$? Why or why not? Before you move on, explain your answer to another student or your tutor.
Solution:
Since $\mathbf{c} = \mathbf{a} + \mathbf{b}$, we only have two linearly independent vectors: $\mathbf{a}$ and $\mathbf{b}$. A basis for $\mathbb{R}^3$ would need three linearly independent vectors. If we do want a basis for $\mathbb{R}^3$, any vector with a non-zero component in the z-axis would suffice.

```
plot(matrix([1,0,0]), matrix([0,1,0]), matrix([1,1,0]), #3D)
```



6. Are $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [1, 0, 4]$ orthogonal? You can do the calculations in your head.
Solution:
If $\mathbf{a}$ and $\mathbf{b}$ were orthogonal, then $\mathbf{a} \cdot \mathbf{b} = 0$. In this case,

$$\mathbf{a} \cdot \mathbf{b} = 1 + 0 + 12 = 13 \neq 0$$

Therefore, $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal.

7. Consider the function $f(x, y) = \ln(x^2 + y^2)$.

   (a) Plot a graph of the function using MuPAD.

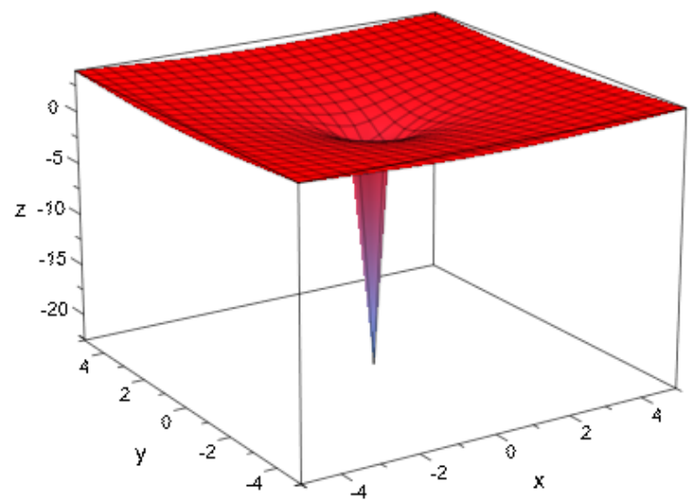
   (b) Show that $f(x, y) = \ln(x^2 + y^2)$ satisfies the partial differential equation $f_{xx} + f_{yy} = 0$ except possibly at (0, 0).

   Solution:

```
f := (x,y) -> log(x^2+y^2)
```

$$(x,\ y) \to \log\left(x^2 + y^2\right)$$

```
plot(f)
```




```
a := diff(log(x^2+y^2), x $2)
```

$$\frac{2}{x^2 + y^2} + \frac{4\,x^2}{(x^2 + y^2)^2}$$

```
b:= diff(log(x^2+y^2), y $2)
```

$$\frac{2}{x^2 + y^2} + \frac{4\,y^2}{(x^2 + y^2)^2}$$

Both are undefined at (0,0)

```
a+b
```

$$\frac{4}{x^2 + y^2} + \frac{4\,x^2}{(x^2 + y^2)^2} + \frac{4\,y^2}{(x^2 + y^2)^2}$$

```
simplify(%)
```

$$0$$

8. Find the equation of the plane through the points (1, 0, 0), (0,-2, 0) and (0, 0, 3). Plot the graph of the plane and the three points using MuPAD. The points should appear to be on the graph of the plane. Hint: To plot a point in $\mathbb{R}^3$, use the command: plot([x,y,z],3D).
Solution :
Define $\mathbf{u} = (1, 0, 0) - (0, 0, 3) = (1, 0, -3)$ and $\mathbf{v} = (0, -2, 0) - (0, 0, 3) = (0, -2, -3)$. $\mathbf{u}$ and $\mathbf{v}$ both lie on the plane and are linearly independent.

$$\mathbf{u} \times \mathbf{v} = (-6, 3, -2)$$

So the equation can be found using: $(-6, 3, -2) \cdot (x - 1, y, z) = 0$

$$-6(x - 1) + 3y - 2z = 0 \Longrightarrow -6x + 3y - 2z = -6 \Longrightarrow z = 3 - 3x + \frac{3}{2}y$$

Alternatively, we could have also used: $(-6, 3, -2) \cdot (x, y + 2, z) = 0$

$$-6x + 3(y + 2) - 2z = 0 \Longrightarrow -6x + 3y - 2z = -6 \Longrightarrow z = 3 - 3x + \frac{3}{2}y$$

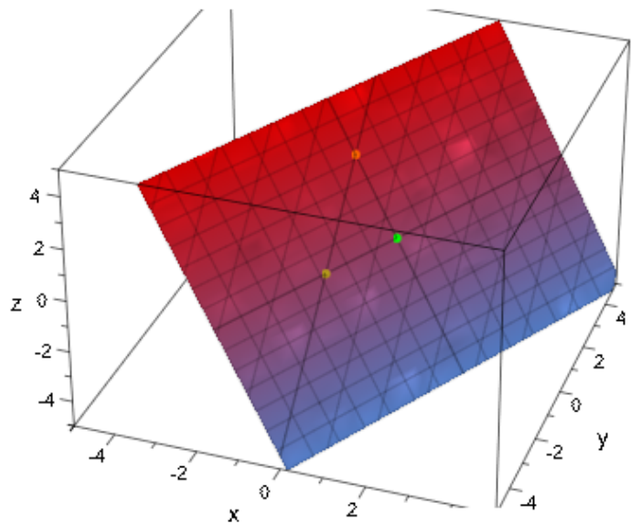
Or: $(-6, 3, -2) \cdot (x, y, z - 3) = 0$

$$-6x + 3y - 2(z - 3) = 0 \Longrightarrow -6x + 3y - 2z = -6 \Longrightarrow z = 3 - 3x + \frac{3}{2}y$$

```
u := matrix([1, 0, -3]):; v:= matrix([0, -2, -3]):
crossProduct(u,v)
```

$$\begin{pmatrix} -6 \\ 3 \\ -2 \end{pmatrix}$$

```
plot(z = 3-3*x+1.5*y, ([1,0,0]), ([0,-2,0]),([0,0,3]),#3D)
```



9. Use substitution to evaluate the following integral:

$$\int \sin^5\theta \; d\theta$$

Solution:

$$\int \sin^5\theta \; d\theta = \int \sin^2\theta \; \sin^2\theta \; \sin\theta \; d\theta = \int (1-\cos^2\theta)^2 \; \sin\theta \; d\theta$$

Let $u = \cos\theta$, $du = -\sin\theta \; d\theta$ Therefore,

$$\begin{aligned} &\int (1-\cos^2\theta)^2 \; \sin\theta \; d\theta = -\int (1-u^2)^2 \; du = -\int (1-2u^2+u^4) \; du \\ &= -(u - \frac{2}{3}u^3 + \frac{1}{5}u^5) + c = -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + c \\ &= -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta + c \end{aligned}$$

10. Use integration by parts to evaluate the following integral:

$$\int \ln x \; dx$$

Solution:

$$\begin{aligned} \int \ln x \; dx &= x\ln x - \int x \; d(\ln x) = x\ln x - \int x\frac{1}{x} \; dx \\ &= x\ln x - \int \; dx = x\ln x - x + c \end{aligned}$$

11. Evaluate by hand and using MuPAD.

$$\int \sqrt{e^{2t}-9} \; dt$$

Solution:
Let $u = \sqrt{e^{2t}-9}$, therefore, $e^{2t} = u^2+9$,

$$\frac{du}{dt} = \frac{1}{2u}2e^{2t} = \frac{u^2+9}{u} \Longrightarrow dt = \frac{u}{u^2+9}du$$

Hence,

$$\begin{aligned} \int \sqrt{e^{2t}-9} \; dt &= \int u \times \frac{u}{u^2+9} \; du = \int \frac{u^2}{u^2+9} \; du = \int \frac{u^2+9-9}{u^2+9} \; du \\ &= \int (1 - \frac{9}{u^2+9}) \; du = u - \int (\frac{9}{u^2+9}) \; du \end{aligned}$$

Let $u = 3w$, therefore, $du = 3\ dw$,

$$u - \int (\frac{9}{u^2+9})\ du = u - \int (\frac{9}{(3w)^2+9})\ 3\ dw = u - 3\int (\frac{9}{9w^2+9})\ dw$$
$$= u - 3\int (\frac{1}{w^2+1})\ dw = u - 3\tan^{-1}(w) + c = \sqrt{e^{2t}-9} - 3\ \tan^{-1}(\frac{1}{3}\sqrt{e^{2t}-9}) + c$$

Using MuPAD, we get a mess, but the solutions are the same if you were to simplify using trigonometric identities.

```
int(sqrt(exp(2*t)-9),t)
```

$$\left(\frac{3\ e^{-t}\ \arcsin\left(3\ e^{-t}\right)}{\sqrt{1-9\ e^{-2\,t}}}+1\right)\sqrt{e^{2\,t}-9}$$

```
simplify(%)
```

$$\left(\frac{3\ e^{-t}\ \arcsin\left(3\ e^{-t}\right)}{\sqrt{1-9\ e^{-2\,t}}}+1\right)\sqrt{e^{2\,t}-9}$$

Checking the solution we obtained by hand.

```
diff(sqrt(exp(2*t)-9) - 3 * arctan((1/3) * sqrt(exp(2*t)-9)), t)
```

$$\frac{e^{2\,t}}{\sqrt{e^{2\,t}-9}}-\frac{9}{\sqrt{e^{2\,t}-9}}$$

Guide to using MuPAD

MuPAD is a computer algebra system that is very similar to Maple. Recent versions of MATLAB (2008b and later) use MuPAD to drive MATLAB's symbolic toolbox. MATLAB is available in computer labs and you may also download a free copy for your own computer by following the instructions in the file '`Downloading a free copy of MATLAB.pdf`', which can be found under the Administration Module on Canvas.

MuPAD can be initiated in by typing '`mupad`' in the MATLAB command window.

The rest of what follows in this guide are instructions walking through the various steps needed to initiate the Guide file for MuPAD.

Instructions:

1. Log on and download `guide.mn` from Canvas. It can be found under the Tutorials Module.
2. Click `guide.mn`, MATLAB will load followed by the Maths 340 MuPAD Guide.
3. Follow the instructions in the guide. Feel free to experiment with MuPAD by implementing your own commands or modifying the commands in the guide.

There are help files and online help if you get stuck or wish to try operations not in the guide.