Matrix Project

EE17BTECH11018 & EE17BTECH11019

IIT Hyderabad

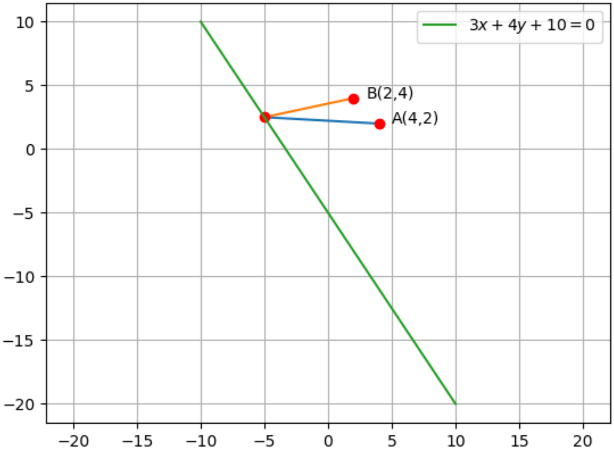
EE 1390-INTRO to AI and ML

February 18, 2019

Geometry Question

Find a point P on the line $3x+2y+10=0$ such that PA+PB is minimum when A is (4,2) and B is (2,4).

Figure



Matrix Equivalent of Question

The equation of given line is $\begin{bmatrix} 3 & 2 \end{bmatrix}$ x = -10
If we compare this with standard line equation $n^T x = c$,we can know that the direction vector of any line normal to this line is n=$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and c=-10

Solution in MATRIX form

Idea

We know A and B are on the same side of the given line. The idea is to take a point A or B and find its image in the given line A'
We know PA=PA'.Therefore PA+PB is minimum is same as PA'+PB is minimum.
A' and B are on different sides of the given line and they can form a triangle.Using triangle inequality ,we can say that PA'+PB is minimum when A',P,B are collinear.

The equation of given line is $\begin{bmatrix} 3 & 2 \end{bmatrix}$ x = -10.

Now we find a normal to this line and passing through A=$\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and let the intersection point be X. The equation of the normal is as following

$$x = A + \lambda(X - A)$$

Solution in MATRIX form

But we know the direction vector X-A$=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$.Therefore the equation

$$x = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 4+3\lambda \\ 2+2\lambda \end{bmatrix}$$

Since X lies on normal, Let

$$X = \begin{bmatrix} 4+3\lambda' \\ 2+2\lambda' \end{bmatrix}$$

Now we substitute this point X in the given line ,we get as follows

$$\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 4+3\lambda' \\ 2+2\lambda' \end{bmatrix} = -10$$

Solution in MATRIX form

$$3(4+3\lambda'+2(2+2\lambda')=-10$$
$$12+9\lambda'+4+4\lambda'=-10$$
$$13\lambda'=-26$$
$$\lambda'=-2$$

We know X is midpoint of A and A' and $X=A+n\lambda'$.Therefore,
$A'=A+2n\lambda'$
$i.e\ A'=\begin{bmatrix} 4+3(-2)(2) \\ 2+2(-2)(2) \end{bmatrix}$

Solution in MATRIX form

$\text{A}' = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$.

Now we have to find the line A′B which cuts the given line at P (We know that A',P,B are on the same line).

The line equation of $A'B$ is as follows

$$x = B + \alpha(A' - B)$$

$$x = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \alpha(\begin{bmatrix} -8 \\ -6 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \end{bmatrix})$$

$$x = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \alpha(\begin{bmatrix} -10 \\ -10 \end{bmatrix})$$

Solution in MATRIX Form

$$x = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \alpha(\begin{bmatrix} 1 \\ 1 \end{bmatrix})$$

Since P lies on the line $A'B$ say ,

$$P = \begin{bmatrix} 2+\alpha' \\ 4+\alpha' \end{bmatrix}$$

Substituting P in the given line we get,

$$\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2+\alpha' \\ 4+\alpha' \end{bmatrix} = -10$$

$$6 + 3\alpha' + 8 + 2\alpha' = -10$$

Solution in MATRIX Form

$$14 + 5\alpha' = -10$$

$$\alpha' = -24/5$$

$$P = \begin{bmatrix} 2 - (24/5) \\ 4 - (24/5) \end{bmatrix}$$

Therefore required point P is

$$\begin{bmatrix} -14/5 \\ -4/5 \end{bmatrix}$$

and the minimum distance PA+PB is equal to $A'B = ||B - A'||=$ $|| \begin{bmatrix} 2 \\ 4 \end{bmatrix} - \begin{bmatrix} -8 \\ -6 \end{bmatrix} || = || \begin{bmatrix} 10 \\ 10 \end{bmatrix} || = 10\sqrt{2}$

Figure

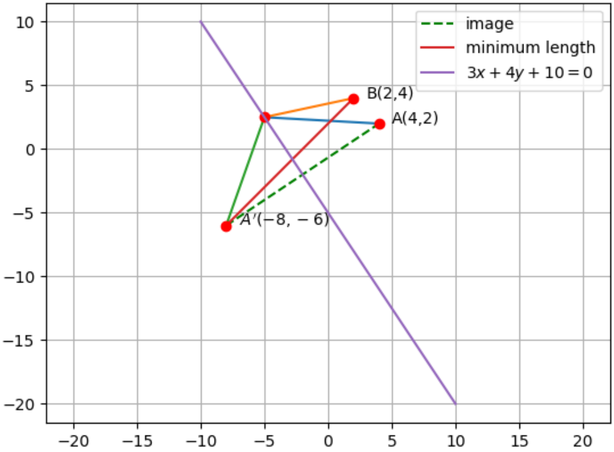


image
minimum length
$3x + 4y + 10 = 0$
B(2,4)
A(4,2)
$A'(-8,-6)$
10
5
0
−5
−10
−15
−20
−20
−15
−10
−5
0
5
10
15
20

The End