Matrix Project

EE17BTECH11018 & EE17BTECH11019

IIT Hyderabad

EE 1390-INTRO to AI and ML

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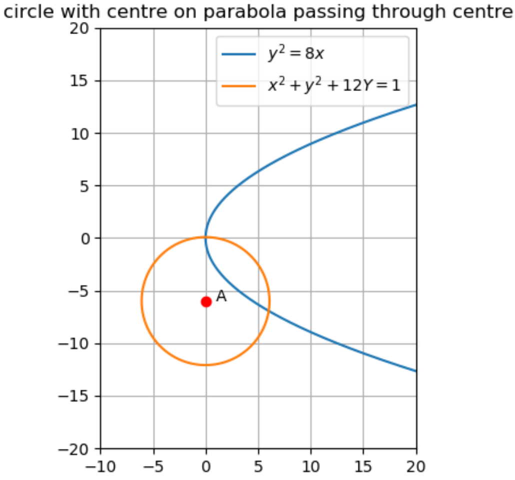
Geometry Question

Let P be a point on $y^2 = 8x$ which is at minimum distance from the centre C of the circle $x^2 + y^2 + 12y = 1$ .Find the equation of circle passing through C and has centre P .

Figure


circle with centre on parabola passing through centre
$y^2 = 8x$
$x^2 + y^2 + 12Y = 1$
A
20
15
10
5
0
−5
−10
−15
−20
−10
−5
0
5
10
15
20

Matrix Equivalent of Question

Circle equation is $x^2 + y^2 + 12y = 1$ i.e A=1 ,B=0, C=1, D=0,E=12,F=-1 Comparing it with original conic equation

$$(x^T)Vx + 2(u^T)x + F = 0$$

we get V=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and u=$\begin{bmatrix} 0 \\ 6 \end{bmatrix}$ .Therefore the equation of circle is as follows:

$$x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 12 \end{bmatrix} x = 1$$

Similarly parabola equation is

$$x^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} -8 & 0 \end{bmatrix}$$

where x and V are Matrices .V is 2*2 and x is 2*1.

Solution in MATRIX form

Idea

We are given that circle's centre C is at a minimum distance from point P on the parabola i.e Circle is also at a minimum distance from the point on the parabola.
Therefore the normal drawn at the point P on the parabola should also be a normal to the circle (Since the tangents are parallel) i.e the normal at P passes through C .

We know tangent for the conic $x^T Vx + 2u^T x + F = 0$ is

$$(p^T V + u^T)x + p^T u + F = 0$$

Here p is $\begin{bmatrix} 2t^2 \\ 4t \end{bmatrix}, V = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, u = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$

Solution in MATRIX form

Therefore equation of tangent at P is

$$\begin{bmatrix} 2t^2 & 4t \end{bmatrix} . \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 \end{bmatrix}))x + \begin{bmatrix} 2t^2 & 4t \end{bmatrix} . \begin{bmatrix} -4 \\ 0 \end{bmatrix} = 0$$

Simplifying , we get

$$\begin{bmatrix} -4 & 4t \end{bmatrix} x = 8t^2$$

Given tangent

$$\begin{bmatrix} a & b \end{bmatrix} x = c$$

,
we can write normal equation $\begin{bmatrix} b & -a \end{bmatrix} x = c'$,Therefore the equation of normal passing through p is as follows

$$\begin{bmatrix} t & 1 \end{bmatrix} x = c'$$

Solution in MATRIX form

Now substitute $x = c$ which is centre of circle $c = \begin{bmatrix} 0 \\ -6 \end{bmatrix}$

$$\begin{bmatrix} t & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -6 \end{bmatrix} = c' \text{ that is } c' = -6.$$

Therefore the equation of normal is

$$\begin{bmatrix} t & 1 \end{bmatrix} x = -6.$$

Now substituting point P in the normal equation we get

$$\begin{bmatrix} t & 1 \end{bmatrix} \cdot \begin{bmatrix} 2t^2 \\ 4t \end{bmatrix} = -6$$

i.e $2t^3 + 4t = -6$ Therefore $t = -1$ is the only real root for the above equation Therefore the point P is

$$p = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Solution in MATRIX form

Now taking P as centre and C as point on the circle ,we calculate radius of the required circle

$$|p-c|^2 = (2-0)^2 + (-4+6)^2$$
$$r^2 = 2^2 + 2^2 = 8$$

Now, taking this radius and P as a centre, equation of the circle is,

$$|x-p|^2 = r^2$$
$$(x-p)\cdot(x-p)^T = r^2 = 8$$
$$xx^T - xp^T - px^T + pp^T = 8$$

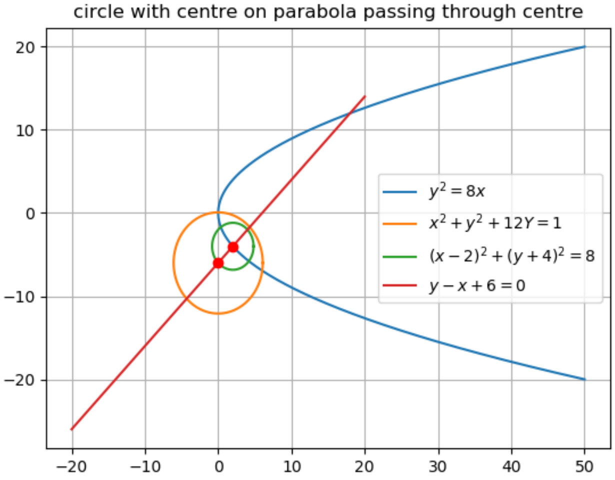
Solution in MATRIX Form

$$xx^T - x\begin{bmatrix} 2 - 4 \end{bmatrix} - \begin{bmatrix} 2 \\ -4 \end{bmatrix} x^T + 2^2 + (-4)^2 = 8$$

$$xx^T - x\begin{bmatrix} 2 - 4 \end{bmatrix} - \begin{bmatrix} 2 \\ -4 \end{bmatrix} x^T + 12 = 0$$

This is the equation of the circle required.

Figure



circle with centre on parabola passing through centre
20
10
0
−10
−20
−20
−10
0
10
20
30
40
50
$y^2 = 8x$
$x^2 + y^2 + 12Y = 1$
$(x-2)^2 + (y+4)^2 = 8$
$y - x + 6 = 0$

The End