

Matrix Project

EE17BTECH11018 & EE17BTECH11019

IIT Hyderabad

EE 1390-INTRO to AI and ML

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Geometry Question

Let P be a point on $y^2 = 8x$ which is at minimum distance from the centre C of the circle $x^2 + y^2 + 12y = 1$. Find the equation of circle passing through C and has centre P .

Matrix Equivalent of Question

Circle equation is $x^2 + y^2 + 12y = 1$ i.e $A=1$, $B=0$, $C=1$,
 $D=0$, $E=12$, $F=-1$ Comparing it with original conic equation

$$(x^T)Vx + 2(u^T)x + F = 0$$

we get $V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $u = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$. Therefore the equation of circle is as follows:

$$x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 12 \end{bmatrix} x = 1$$

Similarly parabola equation is

$$x^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} -8 & 0 \end{bmatrix}$$

where x and V are Matrices . V is 2×2 and x is 2×1 .

Solution in MATRIX form

Idea

We are given that circle's centre C is at a minimum distance from point P on the parabola i.e Circle is also at a minimum distance from the point on the parabola.

Therefore the normal drawn at the point P on the parabola should also be a normal to the circle (Since the tangents are parallel) i.e the normal at P passes through C .

We know tangent for the conic $x^T V x + 2u^T x + F = 0$ is

$$(p^T V + u^T)x + p^T u + F = 0$$

Here p is $\begin{bmatrix} 2t^2 \\ 4t \end{bmatrix}$, $V = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $u = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$

Solution in MATRIX form

Therefore equation of tangent at P is

$$\begin{bmatrix} 2t^2 & 4t \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 \end{bmatrix})x + \begin{bmatrix} 2t^2 & 4t \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 0 \end{bmatrix} = 0$$

Simplifying , we get

$$\begin{bmatrix} -4 & 4t \end{bmatrix} x = 8t^2$$

Given tangent

$$\begin{bmatrix} a & b \end{bmatrix} x = c$$

,
we can write normal equation $\begin{bmatrix} b & -a \end{bmatrix} x = c'$,Therefore the equation of normal passing through p is as follows

$$\begin{bmatrix} t & 1 \end{bmatrix} x = c'$$

Solution in MATRIX form

Now substitute $x = c$ which is centre of circle $c = \begin{bmatrix} 0 \\ -6 \end{bmatrix}$

$$\begin{bmatrix} t & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -6 \end{bmatrix} = c' \text{ that is } c' = -6.$$

Therefore the equation of normal is

$$\begin{bmatrix} t & 1 \end{bmatrix} x = -6.$$

Now substituting point P in the normal equation we get

$$\begin{bmatrix} t & 1 \end{bmatrix} \cdot \begin{bmatrix} 2t^2 \\ 4t \end{bmatrix} = -6$$

i.e $2t^2 + 4t = -6$ Therefore $t = -1$ is the only real root for the above equation Therefore the point P is

$$p = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Solution in MATRIX form

Now taking P as centre and C as point on the circle ,we calculate radius of the required circle

$$\begin{aligned}|p - c|^2 &= (2 - 0)^2 + (-4 + 6)^2 \\ r^2 &= 2^2 + 2^2 = 8\end{aligned}$$

Now, taking this radius and P as a centre, equation of the circle is,

$$\begin{aligned}|x - p|^2 &= r^2 \\ (x - p) \cdot (x - p)^T &= r^2 = 8 \\ xx^T - xp^T - px^T + pp^T &= 8\end{aligned}$$

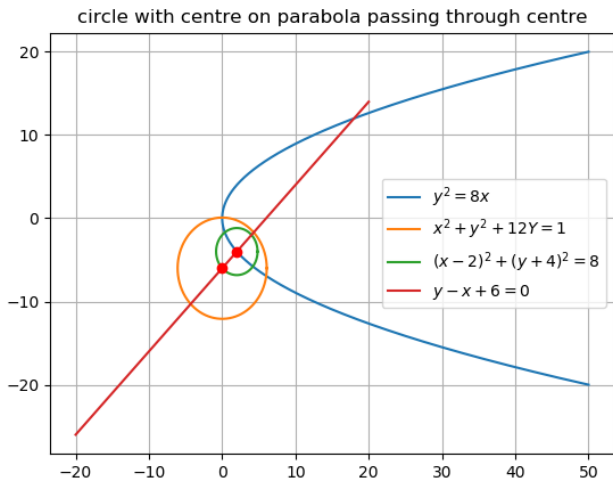
Solution in MATRIX Form

$$xx^T - x \begin{bmatrix} 2 & -4 \end{bmatrix} - \begin{bmatrix} 2 \\ -4 \end{bmatrix} x^T + 2^2 + (-4)^2 = 8$$

$$xx^T - x \begin{bmatrix} 2 & -4 \end{bmatrix} - \begin{bmatrix} 2 \\ -4 \end{bmatrix} x^T + 12 = 0$$

This is the equation of the circle required.

Figure



The End